Essays on non-expected utility theory and individual decision making under risk

A thesis submitted to The University of Manchester for the degree of Doctor of Philosophy in the Faculty of Humanities

2015

Katarzyna Werner

School of Social Sciences

Economics

Contents

Final Word Count: 18,000

List of Figures

Abstract

This thesis investigates the choices under risk in the framework of non-expected utility theories. One of the key contributions of this thesis is providing an approach that allows for a complete characterisation of Cumulative Prospect Theory (CPT) preferences without prior knowledge of the reference point. The location of the reference point that separates gains from losses is derived endogenously, thus, without any additional assumptions on the decision maker's risk behaviour. This is different to the convention used in the literature, according to which, the reference point is preselected. The problem arising from imposing the location of the reference point is that the underlying preference conditions might not be alligned with the predictions made by the model. Consequently, it is difficult to verify such a model or to test it empirically. The present contribution offers a set of normatively and descriptively appealing preference conditions, which enable the elicitation of the reference point from the decision maker's behaviour. Since these conditions are derived using objective probabilities, they can also be applied to settings such as health or insurance, where the continuity of the utility function is not required. As a result, the obtained representation theorem is not only the most general foundation for CPT currently available, but it also provides further support for the use of CPT as a modelling tool in decision theory and finance.

Another contribution that this thesis can be credited with is an application of rank-dependent utility theory (RDU) to the problem of insurance demand in the monopoly market affected by adverse selection. The present approach extends the classical model of Stiglitz (1977) by accounting for an additional component of heterogeneity among consumers, the heterogeneity in risk perception. Specifically, consumers employ distinctive probability weighting functions to assess the likelihood of risky events. This aspect of consumers' behaviour highlights the importance that the probabilistic risk attitudes within the RDU framework, such as optimism and pessimism, have for the choice of insurance contract. The analysis yields a separating equilibrium, with full insurance for a sufficiently pessimistic decision maker. An important implication of this result is that any low-risk individual who sufficiently overestimates his probability of loss will induce the uninformed insurer to offer him full coverage, thereby, affecting the high-risk type adversely. This outcome is consistent with the recent empirical puzzle regarding the correlation between ex-post risk and insurance coverage, according to which, agents with low exposure to risk receive a larger amount of compensation. By providing an explanation of this pattern of individual behaviour, the current work demonstrates that theory and practice of insurance demand can be reconciled to a greater extent. The paper also provides a behavioural rationale for policy intervention in the market with RDU agents, where the initial distortions in contracts due to unobservable risks are aggravated by the non-linear weighting of probability of a risky event.

Key words: asymmetric information, expected utility, probabilistic risk attitudes, probability weighting function, prospect theory, rank-dependent utility, reference dependence.

JEL classification: D81, D82, G22

Declaration

I declare that no portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

Copyright Statement

Acknowledgement

I would like to thank my advisor, Professor Horst Zank, for his invaluable guidance throughout my graduate studies. His mentoring and advice are one of the main pillars in obtaining my degree. I would also like to thank Dr Craig Webb, who on numerous occassions provided interesting insights with regards to the improvement and the extension of my work. Finally, I thank my family, especially my husband, Mark, and my friends, for their essential support and help; the School of Social Sciences at the University of Manchester and the Hausdorff Research Institute for Mathematics at the University of Bonn for making my graduate studies possible.

Part I
Thesis Introduction

The research on non-expected utility theory models and their applications in the context of risk has been provoked by Allais (1953), who by means of a cleverly designed experiment demonstrated deficiencies of the classical theory, the expected utility theory (EUT) model of von Neumann and Morgenstern (1944, 1947, 1953). The experiment revealed that the overwhelming majority of the decision makers systematically violate the axiom of independence underlying EUT. This discovery has initiated a long string of research proposing different variants of relaxing the criticised independence condition. The purpose of these studies has been to gain an improvement in the descriptive ability of the standard model. The family of non-expected utility models emerged as one of the most prominent alternatives to, or, as many argue, generalisations of EUT. Indeed, the models such as cumulative prospect theory (CPT) by Tversky and Kahneman (1992) and its special case, the rank-dependent utility model (RDU) by Quiggin (1982, 1993),[1] share several important properties with EUT.[2] Yet, there are numerous aspects in which the CPT model departures from the expected utility hypothesis. One of such departures is the presence of the non-linear probability weighting function, which transforms cumulative probabilities of outcomes. As claimed by Fehr-Duda and Epper (2012), this feature of CPT model provides a unifying framework for explaining a large number of world phenomena and its neglect may prevent researchers from understanding and predicting further phenomena. This and another feature of CPT, the reference dependence, which implies that the utility (value) function depends on gains and losses, have proven to successfully accommodate several patterns of the behaviour unexplained within the EUT framework, including simultaneous gambling and insuring (Friedman and Savage, 1948), the discrepancy between willingness to pay and willingness to accept (Kahneman et al., 1990; Bateman et al., 1997; Viscusi et al., 1987; Viscusi and Huber, 2012), equity and overtime premium puzzles (Mehra and Prescott, 1985, and Dunn, 1996, respectively), status quo bias (Samuelson and Zeckhauser, 1988), the endowment effect (Thaler, 1980; Loewenstein and Adler, 1995) and the famous Allais paradox, itself. The aim of this thesis is to investigate both features in more depth. First, the present study addresses the question regarding the conditions on the decision maker's preferences that allow the derivation of the reference dependence from behaviour. Second, this work examines the impact that the non-linear weighting of probabilities has on the choice of insurance contracts in the market affected by asymmetric information. Providing the answers to these relevant questions enables to emphasise the contribution of CPT to decision making under risk, in both, theoretical and applied contexts.

[1] It will be clear later that RDU is a special case of CPT, which does not account for reference dependence. Hence, RDU will be labelled CPT when the aspect of reference dependence is not important.

[2] For the discussion of similarities and differences between the models see Machina (1995).

The property of reference dependence has been considered as a "cornerstone" of prospect theory (Kahneman and Tversky, 1979, p. 273) due to its importance for the derivation of further defining features of this theory. In particular, loss aversion (a stronger sensitivity towards losses as compared to equally sized gains) and sign-dependence (different treatment of probabilities associated with gain and loss outcomes) require reference dependence. Nevertheless, the popularity of reference dependence has not sheltered this feature from criticism. What has not been clarified is why this central feature of cumulative prospect theory has not yet been supported by the appropriate preference conditions. Despite a high volume of existing literature on CPT-foundations (Tversky and Kahneman, 1992; Wakker and Tversky, 1993; Chateauneuf and Wakker, 1999; Zank, 2001; Wakker and Zank, 2002; Köbberling and Wakker, 2003, 2004; Schmidt and Zank, 2009; Wakker, 2010; and Kothiyal, Spinu and Wakker, 2011), not one of these approaches has analysed how to derive the reference point - the key ingredient of and the basis for the reference dependence - endogenously. Instead, the location of the reference point has been set a priori, providing inadequate information on the decision maker's behaviour. This point has been recently challenged by Shalev (2000, 2002), Kőszegi and Rabin (2006) and Kőszegi (2010), who developed a theory about the formation of reference point in a dynamic setting. Their approach is different to the approach adopted in this thesis as it appeals to the equilibrium notions used in the classical game theory and can lead to multiple reference points. The purpose of this work is to derive CPT with a unique reference point revealed through decision maker's choice behaviour but without appealing to strategic consideration. This idea is developed by Schmidt and Zank (2012), who obtained a representation theorem for CPT without prior knowledge of the reference point. Nonetheless, like the previous CPT foundations, the setting in which they derived the reference dependence required continuity of the utility function. This assumption is sensible for monetary outcomes. Yet, when outcomes are discrete, e.g., as in health or insurance, this requirement cannot be satisfied. Consequently, it is difficult to say whether the conditions obtained based on the assumption of the continuity of the utility function are necessary and sufficient for the CPT model to hold.

The representation theorem that this study offers, avoids this problem. The present approach imposes no assumptions on the set of outcomes, but instead it employs the objective probabilities of outcomes to define the behavioural conditions underlying CPT. Specifically, the main preference condition, probability midpoint consistency, is invoked to test for sign-dependence. If the consistency test fails, the sign-dependence will hold, revealing the presence of the probability weighting functions for gains and for losses. Thus, there must be an outcome, the reference point, which separates gains from losses associated with those functions. As a result, the reference point is derived from the probabilistic risk behaviour of the decision maker implying that the obtained representation theorem for CPT delivers in one stroke conditions characterising CPT preferences and a method to elicit the reference point from behaviour.

The probabilistic risk component of the individual behaviour captured by the probability weighting function plays a crucial role in determining the location of the reference point endogenously. This feature of the CPT model is further applied to examine the robustness of one of the most fundamental results in the classical insurance theory. This result is offered by Stiglitz (1977), who investigates the demand for insurance among heterogeneous agents in the monopoly market affected by asymmetry of information. Stiglitz shows that in a market with two risk classes of consumers, an equilibrium is always reached, but at the cost of the low-risk individual being adversely affected by the presence of the high-risk type. Specifically, the equilibrium entails full insurance for the latter agent and only partial coverage for the low-risk consumer. As compared with the symmetric information benchmark, the low-risk type suffers welfare loss, an outcome which provides a justified rationale for policy intervention.

Like the vast majority of the literature on insurance economics, the work by Stiglitz is based within the EUT framework, where probabilities of risky events are evaluated in a linear fashion. Yet, there is a large number of empirical and experimental data on insurance demand that questions the assumption of linearity in probabilities. One of the recent developments demonstrates that agents with lower exposure to risk receive more insurance (e.g., Finkelstein and Poterba, 2004; Finkelstein and McGarry, 2006). This observation is at odds with insurance theory, which has promoted the positive link between the ex-post risk and the amount of insurance coverage as one of the most robust outcomes of insurance analysis. In fact, posing high-risk types to have the incentive to purchase a larger quantity of insurance served for years as an indicator or even a prerequisite of the presence of adverse selection in the insurance market (Chiappori, et al., 2000). Nevertheless, an empirical examination delivered by Finkelstein and Poterba (2004) has shown that heterogeneity in risk might instead lead to the so called advantageous selection - the case when the individuals with low exposure to risk acquire a larger quantity of an insurance asset.

In the following, a theoretical model of insurance demand is developed, which provides an account of the aforementioned behaviour. In particular, the present analysis verifies what non-expected utility theory, the RDU model in particular, can contribute to insurance economics by examining the impact that non-linear probability treatment has on the insurance choices of agents in the monopoly market. Explicitly, the agents in the current framework perceive their loss probabilities differently. While for some agents the treatment of probabilities is consistent with the linearity imposed by the EUT preferences, for others distorting probabilities is the way to rationalise the disparity between their beliefs and objective likelihood. As a result, distinctive probabilistic risk attitudes drive the difference in consumers' choices of insurance contract.

In this study of insurance demand, reference dependence is omitted. One reason for omitting this feature has been advocated by Barberis (2012), who finds that in the studies of risk involving insurance, probability weighting has proven to be the central property underlying decision maker's choice behaviour. Indeed, introducing reference dependence to the model of insurance demand adds to the complexity of the analysis without, necessarily, providing more substantial insights into the implications of the insurance demand problem. The reason behind this claim is the choice of the reference point for the analysis of insurance demand, which is not obvious and difficult to justify, but can considerably affect the resulting predictions. Another reason that prevents the use of the reference point in the present model of insurance demand is the increased mathematical complexity of the underlying analysis. The analysis with reference dependence entails a lower level of tractability by enforcing the separation of gains from losses. Such an analysis would be desirable if one wanted to determine the impact of altering the location of the reference point on the implications of the model of insurance demand. Since the current analysis focuses exclusively on the effects of heterogeneity in agents' risk perception on the market equilibrium, adding reference dependence is undesirable. Thus, to provide a model of insurance demand that is mathematically tractable, the following analysis employs RDU preferences, which are also easier to work with empirically (Machina, 1994). In that way, the present study allows to examine how agents' probabilistic risk attitudes captured by the curvature of the probability weighting function, and not the choice of the reference point, affect the equilibrium contracts received by both types of consumers.

This study of insurance demand can be credited with at least three contributions. First, it extends the analysis of Stiglitz (1977) by incorporating the heterogeneous risk perceptions among consumers. The obtained equilibrium is separating with full insurance for the agents who sufficiently overestimate their odds of loss, and partial coverage for the EU consumers. This result provides further support for the adverse effect of asymmetric information in the insurance market. Second, this study reconciles the theory and practice of insurance by providing an explanation of the negative risk-coverage relationship. It shows that the behaviour of the low-risk agent who estimates his risk beyond that of the high-risk type and who, accordingly, receives full insurance, complies with the RDU preferences. This finding is important as it implies that heterogeneity in the way agents view their risks is one of the crucial determinants of consumers' choices. Thus, not only the underlying risk, but also the weighting of this risk should be accounted for when investigating insurance decisions. Finally, this study shows how to model the heterogeneity in risk preferences by means of different probability weighting functions. Despite strong empirical and experimental support for this modelling method (Bruhin, et al., 2010; Conte, Hey and Moffat, 2011), the paper does not claim that employing different probability weighting functions is the only manner to account for subjectivity in perception of risk. However, this work hopes to convince the reader that employing the non-EUT preferences in the assessment of risky choices sheds a light on the previously ignored aspects of the individual behaviour, thus, providing an improvement in the predictive power of economic theory.

This thesis is composed of two parts. Part II is concerned with the CPT axiomatisation that enables the derivation of the property of reference dependence from behaviour. Part III provides an application of RDU, a special case of GPT, to the problem of asymmetric information in the monopoly insurance market. Part II is organised as follows. It begins by laying out the motivation for a new representation theorem for CPT and it highlights the key differences between the new approach and the existing CPT axiomatisations. Section 2 introduces the notation used in this part. This section is followed by the presentation of the set of preference conditions that are necessary for CPT. The key preference condition is introduced in Section 3, where it is shown that the combination of the offered conditions is not only necessary but also sufficient for CPT and it allows to pin down the reference point endogenously. The remaining part of the first part discusses the relevant extensions and summarises the key results. In addition, Section 5 is introduced to critically review the obtained representation theorem in the context of loss aversion. The aim of this section is to expose the difficulties arising from deriving the preference conditions for CPT based on the property of loss aversion. Section 6 concludes.

Part III analyses the impact of the non-expected utility preferences on the insurance choices in the market with heteogeneous consumers. The first section of this part reviews the theoretical literature on asymmetric information in the monopoly insurance market and examines its relation to the practice observed in that market. Particularly, this section aims to motivate the application of the RDU model to the analysis of insurance demand. Section 3 presents the key features of the RDU model focusing on the inverse-S probability weighting function. The next three sections analyse insurance demand in different contexts, including symmetric information case, asymmetric information case with heterogeneity in risk perception only, and the asymmetric information case with heterogeneity in both, risk and risk perception. In addition, Section 6 provides the implications of the two-dimensional heterogeneity for the existence of pooling equilibria, the need for policy intervention and the risk-coverage relationship. The final section of Part III gives a brief summary of the main findings.

Finally, a general conclusion following both parts is drawn upon the entire thesis, tying up the key findings in order to emphasise the contribution that the non-expected utility theory models have to offer to the study of individual decision making under risk. All the proofs can be found in the Appendices, where Appendix 1 and Appendix 2 provide the proofs of the results obtained in Part II and Part III, respectively.

Part II

Foundations for Prospect Theory through Probability Midpoint Consistency

1 Introduction

Kahneman and Tversky (1979) provided us with a powerful descriptive theory for decision under risk that integrates behavioural findings from psychology into economics. The three major advancements of original prospect theory concern reference dependence (outcomes are gains or losses relative to a reference point), loss aversion (a loss leads to greater disutility than the utility of a comparative gain) and sign dependence (decision weights for gains differ from those for losses). Later, in Tversky and Kahneman (1992), prospect theory[3] (PT) was extended to uncertainty and ambiguity by incorporating the requirements of rank-dependence introduced by Quiggin (1981, 1982) for risk and by Schmeidler (1989) for ambiguity, and it received a sound preference foundation by using the tools underlying continuous utility measurement developed in Wakker (1989); see also Wakker and Tversky (1993). Due to its ability to incorporate and account for sign-dependent probabilistic risk attitudes, ambiguity attitudes, reference dependence, loss aversion and diminishing sensitivity in outcomes and probabilities, PT has become one of the most well-known descriptive theories for risk and uncertainty (Starmer, 2000; Kahneman and Tversky, 2000; Wakker, 2010).

The aim of this work is to provide a behavioural preference foundation for PT for decision under risk without assuming prior knowledge of a reference point. To position the contribution of this work it is important to briefly recall the existing PT-foundations for risk. Remarkably, it took many years since the 1979' model to develop the first foundations of PT for decision under risk; this was done by Chateauneuf and Wakker (1999). Later, Kothiyal, Spinu and Wakker (2011) provided foundations of PT for continuous probability distributions. More recently, Schmidt and Zank (2012) derived PT with endogenous reference points by exploiting sign dependence and diminishing sensitivity of the utility. All these theoretical developments

[3]Some authors prefer to distinguish the original prospect theory of Kahneman and Tversky (1979) from the modern version, cumulative prospect theory, of Tversky and Kahneman (1992). Indeed, as Wakker (2010, Apendix 9.8) clarifies, in general these models make different predictions. Here the attention is restricted to the modern version, and hence, the shorter name *prospect theory* is used.

assumed that the set of outcomes is endowed with a sufficiently rich structure that allows for the derivation of continuous cardinal utility.

This study takes a different approach to obtain foundations for PT. It does not assume richness of the set of outcomes but, instead, it follows the traditional approach pioneered by von Neumann and Morgenstern (1944) of using the natural structure given by the probability interval. This approach has been used to derive preference foundations for rank-dependent utility (RDU) by Chateauneuf (1999), Abdellaoui (2002) and Zank (2010); specific parametric probability weighting functions were provided by Diecidue, Schmidt and Zank (2009), Abdellaoui, l'Haridon and Zank (2010) and Webb and Zank (2011). Neither of these results have looked at PT-preferences,[4] although, intuitively, most of those preference foundations for RDU can be extended to PT if the reference point is given. In the absence of this information such extensions become a challenge. This may explain why until now PT has not been derived using the "probabilistic approach". This study fills this gap and shows that PT can be obtained from preference conditions where objective probabilities are given and the set of outcomes can be very general. The reference point in this approach is revealed through probabilistic risk behavior and its existence is not assumed a priori. This shows that, also in this respect, the present model extends all existing PT-foundations for risk. There is a related literature discussing reference point formation in dynamic settings (Shalev, 2000; 2002; Kőszegi and Rabin, 2006; Kőszegi 2010). This work is complementary as it provides existence results for reference points in the traditional static framework.

The importance of having sound preference foundations for decision models, in particular for PT, has recently been reiterated by Kothiyal, Spinu and Wakker (2011, pp. 196–197). If a continuous utility is not available, as a result of outcomes being discrete (e.g., as in health or insurance), the relationship between the empirical primitive (i.e., the preference relation) and the assumption of PT becomes unclear, which is undesirable. In that case one can no longer be sure that the predictions and estimates are in line with the behaviour underlying the preferences. The conditions presented here are necessary and sufficient for PT and, therefore, they help to clarify which assumptions one makes by invoking the model. In particular, the new foundations highlight the difference between expected utility, RDU and PT in a transparent way.

The key preference condition is based on the idea of probability midpoint elicitations. If the reference point is known, this condition simply requires that elicited probability midpoints are independent of the outcomes (i.e., the stimuli) used to derive those midpoints, whenever all outcomes are of the same sign (i.e., either all outcomes are gains or all are losses). Indeed, under PT the probability weighting function for probabilities of gains may be different to the probability weighting function for probabilities of losses. This feature, called *sign-dependence*, has widely been documented (Edwards, 1953, 1954; Hogarth and Einhorn, 1990; Tversky and

[4] An exception is Prelec (1998), however, the key preference condition is his work requires a continuous utility.

Kahneman, 1992; Abdellaoui, 2000; Bleichrodt, Pinto, and Wakker, 2001; Etchard-Vincent, 2004; Payne, 2005; Abdellaoui, Vossmann and Weber, 2005; Abdellaoui, l'Haridon and Zank, 2010).[5] The original elicitation technique for nonparametric probability weighting functions was presented by Abdellaoui (2000) and Bleichrodt and Pinto (2000). They invoke utility measurements prior to the elicitation of probability weighting functions. A simplified version of this method appeared recently in van de Kuilen and Wakker (2011) and requires a single utility midpoint elicitation. In contrast, the method of Wu, Zhang and Abdellaoui (2005) can be applied in the probability triangle and does not necessitate utility midpoint elicitation. This study assumes that probabilities are given and extends these methods to derive PT axiomatically. In this way preference conditions are obtained that are empirically meaningful and provide behavioural foundations of PT for decision under risk.

The elicitation tool for probability midpoints is based on joint shifts in probabilities away from intermediate outcomes. Additionally, and central to the present preference foundation for PT, is the incorporation of a behavioural test for sign-dependence. This study invokes consistency of probability shifts to worse outcomes and consistency of probability shifts to better outcomes for given pairs of prospects. If no sign-dependence is revealed, the two midpoint consistency requirements become compatible and, thus, they imply RDU. In that case it is difficult to obtain a distinction of outcomes into gains and losses by looking at probabilistic risk attitudes. However, if sign-dependence is present, an incompatibility of the two consistency properties is revealed. As a result outcomes can be divided into two disjoint sets with consistency of probability midpoints holding on each set. That is, there must be a special outcome, i.e., the reference point, that demarcates the set of gains from the set of losses. In the presence of standard preference conditions, the two principles of consistency are sufficient to obtain either RDU (i.e., absence of reference points), a special case of PT, or genuine PT with sign-dependent probability weighting. In addition, some extreme forms of optimistic and pessimistic behaviour are permitted. Although these are compatible with PT, the corresponding preference functionals are more general. Specifically, if consistency reveals that there is only one gain, it may not be possible to separate the weighting function for gain probabilities from the utility of that gain due to asymptotic behaviour for probabilities close to one. Similarly, consistency may reveal that there is only one loss and the weighting function for loss probabilities is unbounded at 1. As the objective of the current study is to avoid any structural assumptions on the set of outcomes these extreme cases cannot be excluded. The derived class of preference functionals can be combined as *general prospect theory.*

Next we present our notation and recall the standard preference condition with implications thereof. In Section 3 we present the main preference condition and the theorem. Extensions are discussed in Section 4. Conclusion in Section 6 is followed by Appendix 1 with proofs.

[5] Sign-dependence is one of the consequences of reference dependence.

2 Preliminaries

This section recalls the standard ingredients for decision under risk and the traditional preference conditions that are shared by expected utility and prospect theory.

2.1 Notation

Let X denote the set of *outcomes*. Initially, several simplifying assumptions are made. In Section 4, these are relaxed to demonstrate the full generality of our approach. First, assume a finite set of outcomes, such that $X = \{x_1, \ldots, x_n\}$, with $n \geq 4$. A *prospect* is a finite probability distribution over X. Prospects can be represented by $P = (p_1, x_1; \ldots; p_n, x_n)$ meaning that outcome $x_j \in X$ is obtained with probability p_j, for $j = 1, \ldots, n$. Naturally, $p_j \geq 0$ for each $j = 1, \ldots, n$ and $\sum_{i=1}^{n} p_i = 1$. Let $\mathcal{L}$ denote the set of all prospects.

A *preference relation* $\succcurlyeq$ is assumed over $\mathcal{L}$, and its restriction to subsets of $\mathcal{L}$ (e.g., all degenerate prospects where one of the outcomes is received for sure) is also denoted by $\succcurlyeq$. The symbols $\succ$ (strict preference) and $\sim$ (indifference) are defined as usual. No two outcomes in X are indifferent; they are ordered from best to worst, i.e., $x_1 \succ \cdots \succ x_n$. As in this section the outcomes are fixed they are dropped from the notation without loss of generality.

Recall, that under *expected utility* (EU) prospects are evaluated by

$$EU(p_1, \ldots, p_n) = \sum_{j=1}^{n} p_j u(x_j), \tag{1}$$

with a *utility* function, u, which assigns to each outcome a real number and is strictly monotone (that is, u agrees with the preference ordering over outcomes: $u(x_i) \geq u(x_j) \Leftrightarrow x_i \succcurlyeq x_j$, $i, j \in \{1, \ldots, n\}$). Under EU the utility is *cardinal*, i.e., it is unique up to multiplication by a positive constant and translation by a location parameter.

A more general model is *rank-dependent utility* (RDU) where prospect $P = (p_1, \ldots, p_n)$ is evaluated by[6]

$$RDU(p_1, \ldots, p_n) = \sum_{j=1}^{n} [w(p_1 + \cdots + p_j) - w(p_1 + \cdots + p_{j-1})]u(x_j). \tag{2}$$

Utility is similar to EU, however, RDU involves a weighting function for probabilities, w, that is uniquely determined. Formally, the *weighting function*, w, is a mapping from the probability interval $[0, 1]$ into $[0, 1]$ that is strictly increasing with $w(0) = 0$ and $w(1) = 1$. In this study the axiomatically derived weighting functions are continuous on $[0, 1]$. There is, however, empirical and theoretical interest in discontinuous weighting functions at 0 and at 1 (Kahneman and Tversky, 1979; Birnbaum and Stegner, 1981; Bell, 1985; Cohen, 1992; Wakker, 1994, 2001;

[6] As usual, the convention used is that the sum $\sum_{j=i}^{m} f_j = 0$ when $m < i$.

Chateauneuf, Eichberger and Grant, 2007; Webb and Zank, 2011; Andreoni and Sprenger, 2009, 2012). Relaxing the continuity assumption at the extreme probabilities is discussed in Section 4. It is well known that RDU reduces to EU if $w(p) = p$.

The main model of interest in this work extends RDU by incorporating reference dependence: the model assumes an outcome $x_k \in X$, $1 \le k \le n$, exists, such that outcomes preferred to it are *gains* and outcomes worse than it are *losses*. This may have the implication that, in contrast to RDU, the weighting function will depend on whether the weighted (decumulative) probabilities are those of gains or of losses. For this reason the term *sign-dependence* is used to highlight that the nonlinear treatment of decumulative probabilities depends on the sign of the outcome attached to each probability. Under *prospect theory* (PT) prospect $P = (p_1, \ldots, p_n)$ is evaluated by

$$\begin{aligned} PT(p_1,\ldots,p_n) = & \textstyle\sum_{j=1}^{k-1}[w^+(p_1+\cdots+p_j) - w^+(p_1+\cdots+p_{j-1})]u(x_j) \\ & + \textstyle\sum_{j=k}^{n}[w^-(p_j+\cdots+p_n) - w^-(p_{j+1}+\cdots+p_n)]u(x_j), \end{aligned} \tag{3}$$

where $u(x_k) = 0$; w^+ and w^- are continuous and strictly increasing probability weighting functions for decumulative probabilities of gains and losses, respectively. Under PT the utility is a *ratio scale* (i.e., it is unique up to multiplication by a positive constant) and the weighting functions are uniquely determined. If the dual probability weighting function for losses, $\hat{w}^-(p) := 1 - w^-(1-p)$, for all $p \in [0,1]$, is identical to w^+, then PT reduces to RDU. In that case sign-dependence does not hold.

As mentioned in the introduction, several preference foundations for PT have been proposed using the approach based on continuous utility. Foundations with general continuous utility include Tversky and Kahneman (1992), Wakker and Tversky (1993), Chateauneuf and Wakker (1999), Köbberling and Wakker (2003, 2004), Wakker (2010), Kothiyal, Spinu and Wakker (2011), and Schmidt and Zank (2012). Derivations of CPT with specific forms of the utility function (linear/exponential, power, and variants of multiattribute utility) have been provided in Zank (2001), Wakker and Zank (2002), Schmidt and Zank (2009). Bleichrodt, Schmidt and Zank (2009) assume attribute specific reference points for derivations of functionals that combine PT and multiattribute utility. The next subsection presents the standard preference conditions that all functionals presented in this section have to satisfy.

2.2 Traditional Preference Conditions

This subsection presents the classical preference conditions that are necessary for EU, RDU and PT. The interest is in conditions for a preference relation, $\succcurlyeq$, on the set of prospects $\mathcal{L}$ that *represent* $\succcurlyeq$ by a function, V, that assigns a real value to each prospect, such that for all $P, Q \in \mathcal{L}$,

$$P \succcurlyeq Q \Leftrightarrow V(P) \ge V(Q).$$

A requirement for the representation is that $\succcurlyeq$ is a *weak order*, i.e., the following axiom holds:

WEAK ORDER: The preference relation $\succcurlyeq$ is *complete* ($P \succcurlyeq Q$ or $P \preccurlyeq Q$ for all $P, Q \in \mathcal{L}$) and transitive.

Further requirements are those of first order stochastic dominance and of continuity in probabilities.

DOMINANCE: The preference relation satisfies *first order stochastic dominance* (or *monotonicity* in decumulative probabilities) if $P \succ Q$ whenever $\sum_{j=1}^{i} p_j \geq \sum_{j=1}^{i} q_j$ for all $i = 1, \ldots, n$ and $P \neq Q$.

CONTINUITY: The preference relation $\succcurlyeq$ satisfies *Jensen-continuity* on the set of prospects $\mathcal{L}$ if for all prospects $P \succ Q$ and R there exist $\rho, \mu \in (0,1)$ such that $\rho P + (1-\rho)R \succ Q$ and $P \succ \mu R + (1-\mu)Q$.[7]

A monotonic weak order that satisfies Jensen-continuity on $\mathcal{L}$ also satisfies the stronger Euclidean-continuity on $\mathcal{L}$ (see, e.g., Abdellaoui 2002, Lemma 18). Further, the three conditions taken together imply the existence of a continuous function $V : \mathcal{L} \to \mathbb{R}$, strictly increasing in each decumulative probability, that represents $\succcurlyeq$.[8] The latter follows from results of Debreu (1954).

2.3 Additive Separability Over Decumulative Probabilities

This subsection presents a separability or independence property that is shared by EU, RDU and PT. It is formulated as a preference condition involving common elementary shifts in the probabilities of outcomes. Given the prospect $P \in \mathcal{L}$, the prospect resulting from an elementary shift of probability ε from outcome x_i to the adjacent outcome x_{i+1} in P is denoted by

$$\varepsilon_{i,i+1}P := (p_1, \ldots, p_i - \varepsilon, p_{i+1} + \varepsilon, p_{i+2}, \ldots, p_n).$$

Whenever this notation is used, it is implicitly assumed that $p_i \geq \varepsilon > 0$ and $i \in \{1, \ldots, n-1\}$. Similarly, write $\varepsilon_{i+1,i}P$ for the prospect that results from an elementary probability shift of ε from outcome x_{i+1} to outcome x_i in P (whereby $p_{i+1} \geq \varepsilon > 0$, $i \in \{1, \ldots, n-1\}$ is implicit in this notation). In general, one writes $\varepsilon_{i,j}P$ for a (not necessarily elementary) shift of probability from outcome x_i to x_j of prospect P.

Expected utility satisfies the following property of invariance of the preferences under common elementary probability shifts.

[7] The ρ-probability mixture of P with R is the prospect $\rho P + (1-\rho)R = (\rho p_1 + (1-\rho)r_1, \ldots, \rho p_n + (1-\rho)r_n)$.

[8] This function may be unbounded at x_n or x_1.

Independence: The preference relation $\succcurlyeq$ satisfies *independence of (common elementary) probability shifts* (IPS)

$$P \succcurlyeq Q \Leftrightarrow \varepsilon_{i,i+1}P \succcurlyeq \varepsilon_{i,i+1}Q,$$

whenever $P, Q, \varepsilon_{i,i+1}P, \varepsilon_{i,i+1}Q \in \mathcal{L}$.

One can demonstrate that IPS is necessary for EU. Substitution of Eq. (1) in the preceding equivalence gives

$$P \succcurlyeq Q \Leftrightarrow \sum_{j=1}^{n} p_j u(x_j) \geq \sum_{j=1}^{n} q_j u(x_j).$$

Adding $\varepsilon[u(x_{i+1}) - u(x_i)]$ to both sides of the latter inequality, one obtains the equivalence $\varepsilon_{i,i+1}P \succcurlyeq \varepsilon_{i,i+1}Q$, whenever $P, Q, \varepsilon_{i,i+1}P, \varepsilon_{i,i+1}Q \in \mathcal{L}$. Sufficiency of IPS, in the presence of weak order, first order stochastic dominance and J-continuity, has been shown in Webb and Zank (2011, Theorem 5).

RDU and PT generally violate IPS. However, they satisfy a restricted version of the principle:

Comonotonic Independence: The preference relation $\succcurlyeq$ satisfies *comonotonic independence of (common elementary) probability shifts* (CIS) if

$$P \succcurlyeq Q \Leftrightarrow \varepsilon_{i,i+1}P \succcurlyeq \varepsilon_{i,i+1}Q,$$

whenever $P, Q, \varepsilon_{i,i+1}P, \varepsilon_{i,i+1}Q \in \mathcal{L}$ such that $\sum_{j=1}^{i} p_j = \sum_{j=1}^{i} q_j$.

CIS says that common elementary probability shifts maintain the preference between two prospects if the two prospects offer identical "good news" probabilities of obtaining outcome x_i or better. That is, the decumulative probabilities of obtaining x_i or a better outcome is the same in both prospects. Obviously, this is equivalent to saying that the cumulative probability of obtaining x_{i+1} or a worse outcome is the same in both prospects, so they have identical "bad news" probabilities. Therefore, CIS requires that elementary shifts in probabilities between common decumulative probabilities of outcomes are permitted. If one writes prospects as (de)cumulative distributions over X, it can be immediately observed that CIS translates into an independence requirement on a rank-ordered or comonotonic set of probability distributions, hence, the name for CIS. Substitution of RDU from Eq. (2) into the preceding equivalence gives

$$P \succeq Q$$
$$\Leftrightarrow$$
$$\sum_{j=1}^{n}[w(p_1 + \cdots + p_j) - w(p_1 + \cdots + p_{j-1})]u(x_j)$$
$$\geq \sum_{j=1}^{n}[w(q_1 + \cdots + q_j) - w(q_1 + \cdots + q_{j-1})]u(x_j)$$

This inequality remains unaffected if to both sides $[w(\alpha + p_{i+1} + \varepsilon)) - w(\alpha + p_{i+1})]u(x_{i+1})$ is added and $[w(\alpha) - w(\alpha - \varepsilon)]u(x_i)$ is subtracted, where $\alpha := \sum_{j=1}^{i} p_j = \sum_{j=1}^{i} q_j$ is set. Thus, the equivalence to $\varepsilon_{i,i+1}P \succcurlyeq \varepsilon_{i,i+1}Q$ is obtained, whenever $P, Q, \varepsilon_{i,i+1}P, \varepsilon_{i,i+1}Q \in \mathcal{L}$ such that $\sum_{j=1}^{i} p_j = \sum_{j=1}^{i} q_j$.

The preceding calculations show that CIS is necessary for RDU. Similarly, it can be shown that CIS is necessary for PT. Both models require additional properties in order to distinguish them. However, CIS and the preference conditions in the previous subsection, imply that an additive separability property across outcomes holds for the representing function V. The result is formulated next and its proof follows from results of Wakker (1993) for additive representations on comonotonic sets.

LEMMA 1 *The following two statements are equivalent for a preference relation $\succcurlyeq$ on $\mathcal{L}$:*

(i) The preference relation $\succcurlyeq$ on $\mathcal{L}$ is represented by an additive function

$$V(P) = \sum_{j=1}^{n-1} V_j(\sum_{i=1}^{j} p_i), \tag{4}$$

with continuous strictly increasing functions $V_1, \ldots, V_{n-1} : [0,1] \to \mathbb{R}$ which are bounded with the exception of V_1 and V_{n-1} which could be unbounded at extreme decumulative probabilities (i.e., V_1 may be unbounded at 1 and V_{n-1} may be unbounded at 0).

(ii) The preference relation $\succcurlyeq$ is a Jensen-continuous weak order that satisfies first order stochastic dominance and comonotonic independence of common elementary probability shifts.

The functions $V_1, \ldots, V_{n-1}$ are jointly cardinal, that is, they are unique up to multiplication by a common positive constant and addition of a real number. □

Next section presents the condition that, if added to Lemma 1, delivers general PT.

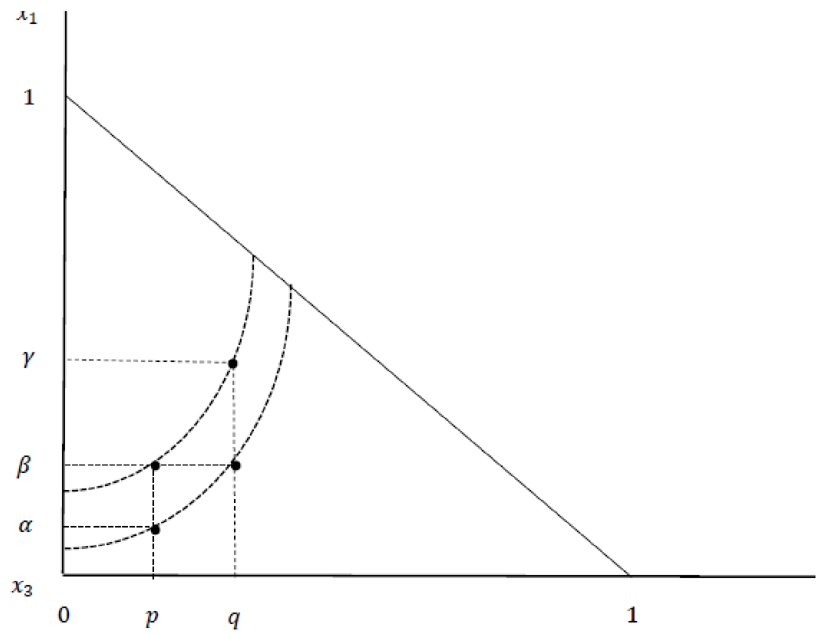

Figure 1: Elicited probability midpoint β.

3 Consistent Probability Midpoints

This section presents consistency requirements for elicited probability midpoints. To motivate the term "probability midpoint", suppose we have two prospects P, Q over outcomes $\{x_1, x_2, x_3, x_4\}$. Let $P = (\alpha, 0, 1-p-\alpha, p)$ and $Q = (\beta, 0, 1-q-\beta, q)$ with $\alpha < \beta$ such that $P \sim Q$. A probability shift of $\beta - \alpha$ from x_3 to x_1 in prospect P requires a shift of probability $\gamma - \beta$ from x_3 to x_1 in prospect Q in order to obtain indifference between the resulting prospects. Thus, we obtain $(\beta-\alpha)_{3,1}P \sim (\gamma-\beta)_{3,1}Q$. The conditions presented in the previous section ensure that such prospects P, Q and probability γ exist if α and β (and, therefore, p and q) are sufficiently close. Figure 1 above illustrates these indifferences in the probability triangle with outcomes x_1, x_3 and x_4.

Substituting the additive representation in Eq. (4) into the preceding two indifferences yields

$$P \sim Q \Leftrightarrow V_1(\alpha) + V_2(\alpha) + V_3(1-p) = V_1(\beta) + V_2(\beta) + V_3(1-q)$$

and

$$(\beta-\alpha)_{3,1}P \sim (\gamma-\beta)_{3,1}Q \Leftrightarrow V_1(\beta) + V_2(\beta) + V_3(1-p) = V_1(\gamma) + V_2(\gamma) + V_3(1-q).$$

Taking the difference between the resulting equations implies

$$[V_1(\gamma) + V_2(\gamma)] - [V_1(\beta) + V_2(\beta)] = [V_1(\beta) + V_2(\beta)] - [V_1(\alpha) + V_2(\alpha)].$$

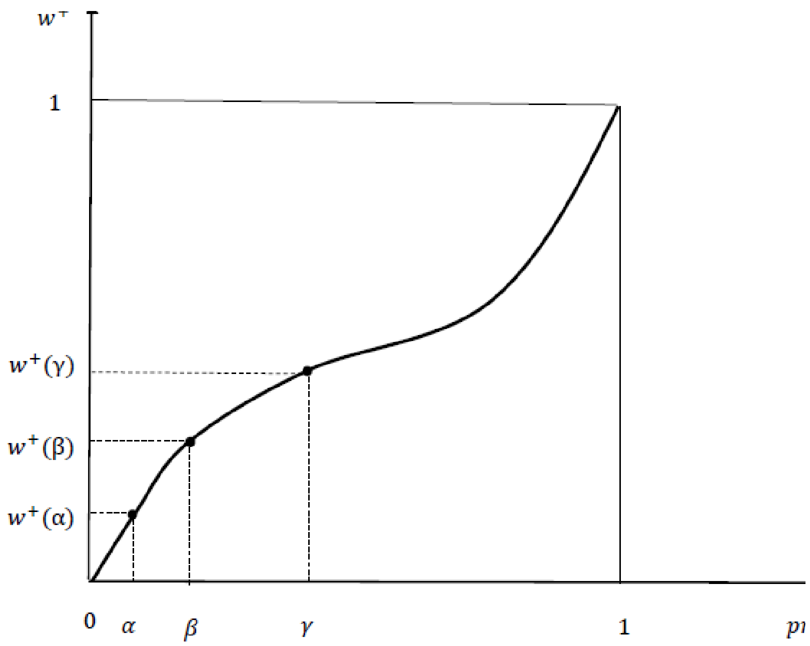

Figure 2: Equally spaced good news probabilities.

Thus, β is a probability midpoint between α and γ for the sum of functions $V_1 + V_2$ of Eq. (4).

Suppose that we know more about preferences, specifically, assume that the preference is a PT-preference and that x_3 is a gain (Case 1). Then substitution of Eq. (2) into the indifferences $P \sim Q$ and $(\beta - \alpha)_{3,1}P \sim (\gamma - \beta)_{3,1}Q$, subtraction of the second equation from the first, and cancellation of common terms give

$$w^+(\beta) - w^+(\alpha) = w^+(\gamma) - w^+(\beta). \tag{5}$$

That is, β is a probability midpoint between α and γ for the probability weighting function for gains, w^+ (see Figure 2).

In practice one elicits midpoints by fixing the probabilities α, p, q and asking for the probability β that makes a person indifferent between prospects P and Q. Figure 3 presents such an elicitation question.

Figure 3 indicates that replacing p in the left prospect with q (thus, shifting probability $q - p$ from x_3 to x_4) requires some appropriate probability being shifted from x_3 to x_1 in the prospect on the right in order to obtain indifference. The required probability shift from x_3 to x_1 is then found to be $\beta - \alpha$. Subsequently, α is replaced by β in the left prospect and one asks for the probability mass that needs to be shifted from x_3 to x_1 in order to maintain the indifference. This way one obtains $\gamma - \beta$. Continuing with this elicitation process, behaviour reveals a standard sequence of equally spaced probabilities based on the the initial probability shift $q - p$ from x_3 to x_4 as unit of measurement.

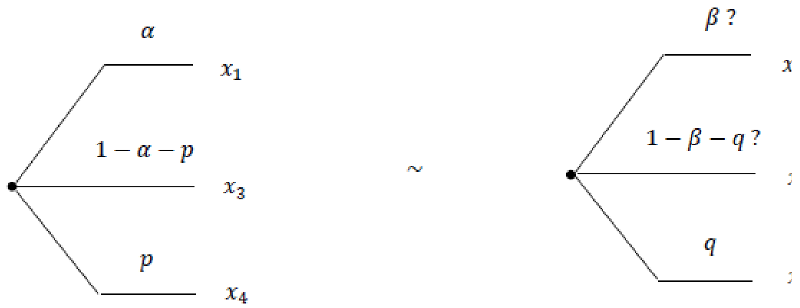

Figure 3: Eliciting standard sequences of probabilities.

Sequences of elicited probability midpoints are not meaningful unless they are independent of the outcomes used to elicit the sequence and the measurement unit $q-p$. Therefore, consistency in measuring such standard sequences is required. For example, instead of shifting $\beta-\alpha$ and $\gamma-\beta$ from x_3 to x_1, shifting the same probabilities from x_3 to x_2 should also leave the indifference unaffected. That is, $(\beta-\alpha)_{3,2}P \sim (\gamma-\beta)_{3,2}Q$ should be obtained. Indeed, Eq. (5) follows from substitution of PT in $P \sim Q$ and in $(\beta-\alpha)_{3,2}P \sim (\gamma-\beta)_{3,2}Q$, subtraction of the second equation from the first, and cancellation of common terms.

Next we continue our analysis but assume that x_2 is a gain while x_3 is the reference point (Case 2) or a loss (Case 3). In Case 2 with the indifferences $P \sim Q$ and $(\beta-\alpha)_{3,1}P \sim (\gamma-\beta)_{3,1}Q$, substitution of PT into these indifferences, subtraction of the second equation from the first and cancellation of common terms, give

$$[w^+(\alpha) - 2w^+(\beta) + w^+(\gamma)]u(x_1) = 0.$$

The latter holds only if β is a probability midpoint between α and γ for w^+. The same conclusion is obtained if PT is substituted into $P \sim Q$ and $(\beta-\alpha)_{3,2}P \sim (\gamma-\beta)_{3,2}Q$. Thus, consistency in probability shifts is obtained.

Let us now turn to Case 3 (x_2 is a gain and x_3 is a loss). Then, substituting PT into the indifferences $P \sim Q$ and $(\beta-\alpha)_{3,1}P \sim (\gamma-\beta)_{3,1}Q$ implies

$$[w^+(\alpha) - 2w^+(\beta) + w^+(\gamma)]u(x_1) = [w^-(1-\alpha) - 2w^-(1-\beta) + w^-(1-\gamma)]u(x_3)$$

and substituting PT in the second pair of indifferences $P \sim Q$ and $(\beta-\alpha)_{3,2}P \sim (\gamma-\beta)_{3,2}Q$ implies

$$[w^+(\alpha) - 2w^+(\beta) + w^+(\gamma)]u(x_2) = [w^-(1-\alpha) - 2w^-(1-\beta) + w^-(1-\gamma)]u(x_3). \qquad (6)$$

Combining the two equations yields

$$[w^+(\alpha) - 2w^+(\beta) + w^+(\gamma)]u(x_1) = [w^+(\alpha) - 2w^+(\beta) + w^+(\gamma)]u(x_2),$$

which holds only if $[w^+(\alpha) - 2w^+(\beta) + w^+(\gamma)] = 0$ (as $u(x_1) > u(x_2)$ is assumed). Equivalently, this means that β is a probability midpoint between α and γ for w^+. But then, substitution into Eq. (6) says that $1-\beta$ is a probability midpoint between $1-\gamma$ and $1-\alpha$ for the probability weighting function w^-. Reformulated in terms of the dual of w^- it means that β is a probability midpoint between α and γ for $\hat{w}^-$. If this holds for all elicited midpoints, then preferences are not sign-dependent and are represented by RDU.

For Cases 1 and 2 it can be concluded that genuine PT (that is, PT with sign-dependence) and the requirement of consistency in probability shifts leads to β being a probability midpoint between α and γ for w^+ independent of outcomes, as long as x_3 is a gain or the reference point. When x_3 is a loss, sign-dependence and consistency cannot hold jointly. The first property states the consistency requirement for general prospects but without a priori knowledge of whether sign-dependence holds.

Good News Midpoint Consistency: The preference relation $\succcurlyeq$ satisfies *consistency in probability midpoints* above x_i or *good news midpoint consistency* (GMC) at $x_i, i \in \{2, \ldots, n\}$ if

$$\begin{aligned} P = (\alpha, 0, \ldots, 0, p_i, \ldots, p_n) &\sim Q = (\beta, 0, \ldots, 0, q_i, \ldots, q_n) \\ \text{and } (\beta-\alpha)_{i,1}P &\sim (\gamma-\beta)_{i,1}Q \\ \text{imply } (\beta-\alpha)_{i,j}P &\sim (\gamma-\beta)_{i,j}Q, \end{aligned}$$

for all $j \in \{1, \ldots, i-1\}$ whenever $\alpha < \beta < \gamma$ are probabilities such that $P, Q, (\beta-\alpha)_{i,1}P$, and $(\gamma-\beta)_{i,1}Q$ are well-defined.[9]

It can be verified that RDU satisfies GMC at x_i for all $i = 2, \ldots, n$. This has been shown in Zank (2010). Further, RDU also satisfies a property, dual to GMC, defined next.

Bad News Midpoint Consistency: The preference relation $\succcurlyeq$ satisfies *consistency in probability midpoints* at x_i or *bad news midpoint consistency* (BMC) at $x_i, i \in \{1, \ldots, n-1\}$ if

$$\begin{aligned} P = (p_1, \ldots, p_i, 0, \ldots, 0, \alpha) &\sim Q = (q_1, \ldots, q_i, 0, \ldots, 0, \beta) \\ \text{and } (\beta-\alpha)_{i,n}P &\sim (\gamma-\beta)_{i,n}Q \\ \text{imply } (\beta-\alpha)_{i,j}P &\sim (\gamma-\beta)_{i,j}Q, \end{aligned}$$

[9] In this definition the case $i = 2$ is included, which holds trivially, for completeness.

for all $j \in \{i+1, \ldots, n\}$ whenever $\alpha < \beta < \gamma$ are probabilities such that $P, Q, (\beta - \alpha)_{i,n}P$, and $(\gamma - \beta)_{i,n}Q$ are well-defined.[10]

In contrast to RDU, genuine PT does satisfy GMC at x_i only for $i \in \{2, \ldots, k\}$ and it satisfies BMC at x_i only for $i \in \{k, \ldots, n-1\}$. Unless PT-preferences agree with RDU, there are no further outcomes, except the reference point x_k, where both GMC and BMC hold. Usually, in applications of PT the reference point is unknown. However, the preceding consistency properties for probability midpoints can serve as a test for detecting at which outcome one of the properties fails. Thus, GMC and BMC provide a critical test for PT-preferences through sign-dependence for elicited probability midpoints. This test is built into the next preference condition.

SIGN-DEPENDENT MIDPOINT CONSISTENCY: The preference relation $\succcurlyeq$ satisfies *sign - dependent probability midpoint consistency* (SMC) if for each outcome x_i, $i \in \{2, \ldots, n-1\}$ the preference satisfies good news midpoint consistency at x_i or bad news midpoint consistency at x_i (or both).

Let us look at the implications of SMC. First, consider that $M \in \{3, \ldots, n-1\}$ is such that GMC holds at x_M. Let M be maximal with this property. That is, there is no $j > M$ such that GMC holds at x_j. If $M \leq n-1$ then BMC holds at x_i for all $i = M, \ldots, n-1$. Let m be minimal with the property that BMC holds at x_m. Two cases can occur:

(i) $M = m$, in which case sign-dependence holds. Then $k := M$ can be set, and x_k is a (unique) reference point;

(ii) $m < M$, in which case sign-dependence does not hold, thus, there is no reference point.

Suppose now that $M = 2$ and there is no $j > M$ such that GMC holds at x_M. Then BMC holds at x_i for all $i = M, \ldots, n-1$. Then Case (i) above can be obtained with $M = m = 2$ and sign-dependence. Therefore, either sign-dependence holds and there is a unique reference point, or sign-dependence does not hold, hence, there is no reference point. As the main result below shows, Case (ii) gives RDU, the special case of PT without reference dependence, and if $3 \leq M = m \leq n-2$ then Case (i) gives genuine PT.

The cases $M = m = 2$ and $M = m = n-1$, however, warrant special attention. The reason for this is the possible unboundedness of the functions V_1 at 1 and of V_{n-1} at 0 as stated in Lemma 1. For example, representing functionals of the following form are compatible with all preference conditions presented above:

$$W(P) = V_1(p_1) + \sum_{j=2}^{n} [w^-(p_j + \cdots + p_n) - w^-(p_{j+1} + \cdots + p_n)]u(x_j), \tag{7}$$

[10] Similar to GMC, in this definition we have included the case $i = n-1$, which holds trivially, for completeness.

where u and w^- are as in PT and V_1 converging to ∞ at 1 is as in Lemma 1 above, or

$$W(P) = \sum_{j=1}^{n-2}[w^+(p_1 + \cdots + p_j) - w^+(p_1 + \cdots + p_{j-1})]u(x_j) + V_{n-1}(1 - p_{n-1}), \qquad (8)$$

where u and w^+ are as in PT and V_{n-1} converging to $-\infty$ at 0 (i.e., when p_{n-1} approaches 1) is as in Lemma 1 above.

Specifically one can choose $V_1(p_1) = [\frac{p_1}{1-p_1}]u(x_1)$ and $V_{n-1}(1 - p_n) = [\frac{1-p_n}{p_n}]u(x_n)$ in Eqs. (7) and (8) above. In the first case (Eq. (7)) one can think of a patient who has been diagnosed with a severe disease, such as cancer. The various available treatments can lead to outcomes of which the best one is $x_1 =$ "healed". It should be obvious that this outcome is so attractive that any treatment with even the smallest positive probability for x_1 will be superior to any other treatment that has zero probability for x_1. Another related example is documented in Thaler and Johnson (1990) and analysed in Barberis, Huang and Santos (2001). After having faced a series of losses, many investors attempt to break even by taking additional risks despite the chances to break even being small. By contrast, the second representation above (Eq. (8)), can be thought of an extreme form of pessimism, where the possible loss x_n is extremely unattractive and any prospect with positive likelihood for x_n will be regarded as inferior to a prospect with zero probability for x_n. Individuals exhibiting this form of pessimism are willing to buy insurance at prices far above the actuarially fair value to avoid the loss x_n.

Behaviour described above with extreme optimism for a good outcome or extreme pessimism for a bad outcome is not excluded here. Instead, the current study allows for such preferences and refers to the resulting representations as "generalised" prospect theory, that is, PT including the cases of $k = 2$ and V_1 in Lemma 1 unbounded, or $k = n-1$ and V_{n-1} in Lemma 1 unbounded. Now the main result can be presented.

Theorem 1 *The following two statements are equivalent for a preference relation $\succcurlyeq$ on $\mathcal{L}$:*

(i) The preference relation $\succcurlyeq$ on $\mathcal{L}$ is represented by generalised prospect theory, with the functions V_1 or V_{n-1} in Lemma 1 possibly unbounded.

(ii) The preference relation $\succcurlyeq$ is a Jensen-continuous weak order that satisfies first order stochastic dominance, comonotonic independence of common elementary probability shifts, and sign-dependent probability midpoint consistency.

Whenever V_1 and V_{n-1} are bounded, the probability weighting functions are uniquely determined. If further $w^+ \neq \hat{w}^-$, the reference point is unique and the utility function is a ratio scale; otherwise, if $w^+ = \hat{w}^-$, utility is cardinal. If V_1 (or V_{n-1}) is unbounded then w^- (w^+) is uniquely determined and V_1 (or V_{n-1}) and u are jointly cardinal with $u(x_k) = 0$ restricting the location parameter of u to 0. □

4 Extensions

In the previous sections it has been assumed that outcomes are strictly ordered. The strict ordering can be relaxed if there are at least four strictly ordered outcomes. If X is finite all results remain valid if one takes representatives for each set of indifferent outcomes. These outcomes will then be given the same utility value. If, however, X is infinite, then results remain valid for each finite subset of outcomes Y that contains at least four outcomes that are strictly ordered. Then the PT-representations on the sets of prospects over the different finite subsets, Y and Y', can be extended to a general PT-representation by using the fact that the representations on any such sets of prospects over Y and of prospects over Y' must agree with the representation on the set of prospects over $Y \cup Y'$. Hence, a common PT-representation must exist over prospects with finite support in the possibly infinite X.

If there are only three strictly ordered outcomes, the sign-dependent probability midpoint consistency principle is trivially satisfied. In that case, stronger tools to obtain additive separability (Lemma 1) are required. An additive representation can still be derived by using stronger conditions like the Thomsen condition or triple cancellation as in Wakker (1993, Theorem 3.2). Those additive functions can be seen as the product of utility times the corresponding weighting function and one immediately obtains generalised PT. To obtain the special case of RDU, one has to additionally invoke the probability tradeoff consistency principle of Abdellaoui (2002) or a refinement of that principle as proposed in Köbberling and Wakker (2003). For fewer than three strictly ordered outcomes first order stochastic dominance and weak order are sufficient for an ordinal representation of preferences.

In the present derivation of PT it has been essential that the weighting functions are continuous at 0 and at 1. Discontinuities at these extreme probabilities are, however, empirically meaningful. One could adopt a weaker version of Jensen-continuity that is restricted to prospects that have common best and worst outcomes with positive objective probability. Such conditions have been used in Cohen (1992) and more recently in Webb and Zank (2011) where probability weighting functions are derived that are linear and discontinuous at extreme probabilities. These weighting functions can then be described by two parameters one for optimism and one for pessimism. As Webb and Zank show, this relaxation of continuity in probabilities comes at a price. They require additional structural assumptions for the preference in order to obtain consistency of the parameters across sets of prospects with different minimal and maximal outcomes. Also, specific consistency principles that imply the uniqueness of these parameters are required. The conjecture derived from the current study is that in this framework such consistency principles can be formulated for nonlinear weighting functions that are discontinuous at 0 and at 1. A formal derivation of PT with such weighting functions is, however, beyond the scope of this work.

5 Loss aversion in Prospect Theory

The preference representation for Prospect Theory derived in this paper has been founded on the property of reference dependence. As a central feature of the PT model, the reference dependence can be exposed in various ways, and the way chosen in this study had been through sign-dependence, the principle that enables the weighting functions to depend on the sign of outcomes. Other means by which the reference dependence manifests itself in the decision maker's behaviour, such as diminishing sensitivity for outcomes (a greater sensitivity to outcome changes near the reference point than to changes remote from it) and loss aversion, have not been considered in the present analysis. Yet, one would expect that it is also possible to elicit the reference point from choice behaviour using these properties.

Indeed, Schmidt and Zank (2012) demonstrate how to identify the reference point endogenously by employing the aspect of diminishing sensitivity for outcomes. In PT with diminishing sensitivity, the utility function is concave in the domain of gains and convex in the domain of losses, with the reference point marking the change in the domains. Thus, in order to account for the preferences consistent with diminishing sensitivity under PT, the underlying axioms must incorporate the change in the shape of the utility function. The representation theorem offered by Schmidt and Zank (2012) accredits this change by implementing the so called "rich outcome set" approach, which assumes the continuum of outcomes. This assumption enables the elicitation of a continuous utility function, on which the axioms of choice are focused. Nevertheless, there are various reasons for which this approach to derive the reference point has not been extended to account for loss aversion.

Loss aversion is said to reflect a greater sensitivity to loss outcomes than to gain outcomes of equal magnitude (Tversky and Kahneman, 1992). This notion is usually interpreted by the means of the utility function for losses being steeper than the corresponding utility function for gains, and have received considerable empirical support (e.g., Odean, 1998; Rabin, 2000; Rabin and Thaler, 2001; Genesove and Mayer, 2001). Yet, despite laying down the foundations for the modern study of behavioural economics, this intrinsic property of PT has not been assigned a unique quantification method. Instead, several definitions of loss aversion have emerged in the economic literature (e.g., Wakker and Tversky, 1993; Benartzi and Thaler, 1995; Neilson, 2002; Sugden, 2003). For instance, Benartzi and Thaler (1995, p.74) suggest to measure loss aversion as a ratio of utility derivatives, which implies a kink at the reference point. Köbberling and Wakker (2005) take a similar approach and improve the earlier measure by making it independent of the units of payments. In particular, they propose an index, which separates loss aversion from other components of risk attitudes in PT, such as basic utility and probability weighting. In contrast, Schmidt and Zank (2008) do not disentangle the individual components of risk attitudes in PT, but rather they derive jointly the utility curvature and the shape of the probability weighting functions from additional preference conditions regarding individual risk behaviour. By employing the ratio between gain and loss decision weights, their proposed definition of loss aversion entails an additional element, the index of probabilistic loss aversion. This index captures the distinction between the probability weighting function for gains and the probability weighting function for losses induced by sign-dependence. In that way, the index relates the notion of probabilistic loss aversion to probabilistic risk aversion, as defined by Wakker (1994). Similarly to Köbberling and Wakker (2005), the index of loss aversion proposed by Schmidt and Zank (2008) is constructed within the PT framework, hence, it is specific to that model. Though, unlike Köbberling and Wakker (2005), this index does not require the continuity of the weighting functions. The latter implication is particularly important as it allows to uncover the relationship between the continuity of the weighting function and the curvature of the utility function. In particular, it reveals that the utility function must be concave in order to ensure the continuity in probabilities.

A more general approach to explore the notion of loss aversion requires its model-free definition. Such an approach is adopted by Brooks and Zank (2005) and Zank (2010), in the context of discrete probability distributions (lotteries), and by Ghossoub (2013), in the context of continuous distributions (objects of choice being more general than lotteries). In his purely preference-based examination of loss aversion, Ghossoub emphasises the importance of invoking the probability weighting in the definition of the phenomenon. Specifically, he claims that as a property of choice behaviour, loss aversion must surely account for the probabilistic risk attitudes of decision makers, whose characterisation is embedded in the shape of the weighting functions and not the utility function. The idea is supported by Zank (2010), who considers the sign-dependence as a "byproduct" of loss aversion. Further, Ghossoub shows that his definition of loss aversion derived in the model-free environment coincides with that of Köbbeling and Wakker (2005), for the case of sign-dependence being absent. In the framework of PT, his definition of loss aversion accounts for both, individual's tastes (as captured by the utility function) and beliefs (as evaluated by the probability weighting functions).

Among the handful of studies exploring the notion of loss aversion outside of PT, there is also the work of Blavatskyy (2011), who not only considers the model-free notion of (comparative) loss aversion, but he does it for outcomes which are not necessarily monetary. Unlike Ghossoub, this author claims that the phenomenon of loss aversion is not related to the shape of the probability weighting function. Blavatskyy, however, does not provide a clear definition of the absolute notion of loss aversion,[11] neither does he investigate this phenomenon in the PT model. Moreover, like the other studies of aversion to loss, his analysis is based on the assumption of continuous utility function.

In this environment, where the notion of loss aversion has not been uniquely agreed-upon, employing this behavioural phenomenon to test for reference dependence is challenging. In

[11] Blavatskyy's notion of absolute loss aversion is defined in terms of "more loss averse" preferences than the "loss neutral" ones. However, he does not provide a definition of loss neutrality in that context.

order to engage the utility-based definitions of loss aversion into the comparison of the marginal changes in utility for gains and losses of similar size, one needs to identify and separate these utilities first. An additional problem arises due to the fact that it is plausible that the kink of the utility function at the reference point, which shall reflect the property of loss aversion, might actually be an intrinsic kink of the utility function. As Köbberling and Wakker (2005) suggest, in such case one can no longer distinguish between the source of the kink without further information about the curvature of the utility function. Therefore, implementing loss aversion to the derivation of the reference point from behaviour, in a setup in which loss aversion itself is governed by the location of such a reference point, remains a topic for future research. In addition, the present preference foundation for PT disposes of the assumption of continuity of the utility function, on which the principle of loss aversion is built. Therefore, it is challenging to extend the "probability weighting"-based approach presented in this work to incorporate the principle of loss aversion in the characterisation of PT preferences.

6 Conclusion

The focus of this work has been on sign-dependence, the different treatment of probabilities depending on whether the latter are attached to gains or to losses. The present study has complemented existing foundations for PT in the "continuous utility approach" with preference foundations based on the "continuous weighting function approach" by adopting and extending a familiar tool from empirical measurement of probability weighting functions, the midpoint consistency principle. Preference midpoints for outcomes are a useful tool for the analysis of risk attitudes captured by utility. It was recently shown by Baillon, Driessen and Wakker (2012), how these midpoint based tools facilitate the analysis of ambiguity preferences and time preferences. This work demonstrates how similar midpoint tools can be adapted for the analysis of PT-preferences. The present method facilitates the analysis of probabilistic risk attitudes and, therefore, complements the utility-based approach. Further, it shows how the probability midpoint principle can be employed to identify reference points in an efficient and tractable manner.

Part III

Insurance Demand and Heterogeneity in Risk Perception

1 Introduction

Asymmetric information has been a core problem of economic analysis since the groundbreaking work of Akerlof (1970), who highlighted the impact of information imperfections on the existence of insurance markets. Following Akerlof, Rothschild and Stiglitz (1976) and Stiglitz (1977) provided a benchmark models for analysing the potential of asymmetric information in creating market failure in the competitive and monopoly insurance markets with heterogeneous risks, respectively. Their models employed the expected utility (EU) framework, in which several common practices of the insurance market, such as the provision of deductibles, nonlinear coinsurance and maximum limits (Young and Browne, 1997; Fluet and Pannequin, 1997), have already been explained. Yet, there is still a large number of behavioural patterns, which are inconsistent with EU. One of them concerns the risk-coverage relationship, which, contrary to the theoretical predictions, has been found to be advantageous (Hurd and McGarry, 1997; Cawley and Philipson, 1999; McCarthy and Mitchell, 2003; Fang et al., 2008; Finkelstein and McGarry, 2006; Davidoff and Welke, 2007; Wang, Huang and Tzeng, 2009). The advantageous relationship between the risk one faces and the compensation that this person receives implies that a less risky individual is offered a larger amount of insurance coverage. The present study aims to explain this puzzle in the market for low probability - large loss events, e.g., natural disasters, by incorporating the non-linear treatment of probabilities consistent with rank-dependent utility (RDU) by Quiggin (1982, 1993), which conforms to a wider class of binary separable preferences.[12]

The consumers in the model are characterised by multi-dimensional heterogeneity. They are distinguishable not only with respect to their risk exposure, which can be either low or high, but also with respect to their perception of risk. Indeed, Huck and Müller (2012) and List (2004)

[12] RDU alike, CPT of Tversky and Kahneman (1992) also incorporates the non-linear treatment of probabilities. Nevertheless, as explained in the introduction, the use of RDU preferences in the insurance demand model is more advantageous. In fact, Starmer (2000) states that RDU has emerged as the most widely adopted model in both theoretical and applied context.

show that perception of risk varies among the individuals and is driven by various factors.[13] Moreover, Bruhin et al. (2010) investigates the risk preferences of agents and find that the majority of subjects distort probabilities of risky events using a transformation rule resembling the inverse-S function found in Prospect Theory (Tversky and Kahneman, 1992; Prelec, 1998). These results suggest that there is no unique preference theory that would accurately describe the choice behaviour of all agents and a mix of such theories is more adequate for modelling purposes (Wilcox, 2006; Conte, Hey and Moffatt, 2011). Therefore, in this study, a fraction of the consumers in the population is assumed to overweigh the probability of bad events according to inverse-S probability weighting, which is consistent with the empirical evidence by Kunreuther and Hogarth (1989), who find that the propensity to pay above the actuarially fair premium for full insurance is especially marked for consumers exposed to low levels of risk. The remaining part of consumers processes the loss probabilities in a linear fashion present under EU. As a result, the two fractions of agents in the market necessarily differ with respect to their probability-based risk attitudes, driving the difference in those agents' willingness to hedge against an equal risk.

This work demonstrates that the non-expected utility maximiser is exploited by the monopolistic insurer, supporting the claim by Segal and Spivak (1990). This result follows on from observing that in the market with heterogeneous risk perceptions, the pessimistic type is always willing to pay a higher premium price than any equally exposed to risk standard utility maximiser. This implication can be further extended to the case, in which agents' exposures to risk vary. Consequently, the equilibrium in the market with multi-dimensional heterogeneity of consumers is separating, with full insurance for the pessimistic consumer and a partial coverage for the EU type. Unlike the work of Stiglitz (1977), where asymmetric information always leads to a welfare loss of the low-risk individual, this study suggests that the presence of multiple dimensions of heterogeneity does not necessarily harm the low-risk individual. In fact, asymmetric information affects negatively the low-risk individual only if this individual is characterised by EU preferences. Hence, whenever the agents' exposures to risk are sufficiently close, the expected utility maximiser will be adversely affected by the monopolist's inability to differentiate between the agents, irrespective of this agent's risk type. As such, the equilibrium contract is not necessarily tailored to match the risk that the consumers are exposed to.

To see it, consider a low-risk agent who perceives his probability of loss as being high. In such case, screening of this agent induces the monopolist to offer full insurance at a high price. As a result, the true high-risk expected utility maximiser who remains in the market is forced to underinsure. This example shows that incorporating heterogeneity in risk perception into the analysis of insurance demand allows to establish a theoretical explanation of negative risk-coverage relationship in the monopoly market offering insurance against rare hazards.

[13] Slovik (2000) provides a comprehensive survey reviewing the literature on heterogeneity in risk perception and its determinants.

Particularly, it demonstrates that irrespective of the true risk type, the agent who perceives his chance of suffering a loss as being higher than the loss probability of the other type will receive larger compensation. Henceforth, in this theoretical framework advantageous selection is attributed to the heterogeneity in the treatment of probability.[14]

In the light of the obtained equilibrium characterisation, further implications of this study are derived for the existence of pooling equilibrium and welfare. In particular, it is shown that pooling is never optimal. This finding is in stark contrast with a couple of papers that, like this one, investigate the equilibrium contracts of agents with non-expected utility preferences in the presence of adverse selection (see Ryan and Vaithianathan, 2003; and Jeleva and Villeneuve, 2004).[15] These studies show that pooling equilibrium arises whenever the fraction of low-risk pessimists sufficiently outnumbers the fraction of high-risk optimists or expected utility maximisers. Yet, both studies constrain the screening menu of contracts for the pessimistic agents and offer policies that do not account for these consumers' risk perception. In addition, these authors make no assumption on the size of the difference in risk that the two types of consumers are exposed to. As a result, individuals within the same market might be facing very different levels of risk, a scenario which is not realistic in the context of natural disasters, such as an earthquake, a flood or a hurricane, which are a focus of the present analysis. In contrast to those studies, this work conforms to the observations from real-life by emphasising the influence that probabilistic risk attitudes exert on the individuals' choices of insurance against low probability extreme outcomes (see Peacock et al., 2005, who find that people in Florida perceive the risk of a hurricane damaging their property very differently despite similar risk exposures). Therefore, the results of this paper are original in that they provide the assessment of the impact that asymmetric information has on the demand for insurance in the under-researched market for unlikely hazards.

This work also sheds a light on the welfare in the monopoly market in the context of multi-dimensional heterogeneity of consumers. Specifically, the result of welfare analysis exposes the need for an implementation of some welfare-improving measures. This necessity arises due to the fact that with heterogeneity in both risk exposure and risk perception the dispersion in the agents' willingness to pay for insurance against rare catastrophes increases, forcing the monopolist to distort to a greater extent the contracts offered to the agents. Hence, the welfare gains from intervening are larger than those derived in the case of homogeneous risk perceptions, thereby, providing another important implication of multi-dimensionality of heterogeneity in

[14] Advantageous selection has also been explained by means of the multi-dimensional heterogeneity of customers in the work of Liu and Browne (2007) and Netzer and Scheuer (2010). Both of these papers, however, employ exclusively EU preferences.

[15] Al-Nowaihi and Dhami (2010) also analyse the problem of asymmetric information in the RDU and PT frameworks, but they employ Prelec probability weighting function, which in addition to the standard (inverse-S) probability weighting function allows for underweighting of very small probabilities close to 0, while still overweighting low probabilities. This function captures empirically observed underinsurance for low probability - risky events (Kunreuther et al., 1978). They do not not discuss pooling.

consumers' characteristics.

The rest of this paper is organised as follows. The next section outlines the key assumptions of the model. This section is followed by the formal introduction of rank-dependent utility theory and its characteristic features in Section 3. Section 4 analyses the insurance demand problem with the symmetry of information in the monopoly market. The analysis of the asymmetric case with heterogeneity solely in risk perception is presented in Section 5. The robustness of these results in the presence of both dimensions of heterogeneity is investigated in Section 6. This section also provides an implication of the current analysis for the existence of pooling equilibrium, the need for policy intervention and the risk-coverage relationship. Finally, Section 7 concludes. All the proofs can be found in Appendix 2.

2 Basic framework

Consider a simple model of an insurance policy to cover a potential loss. Let there be two states of the world, good state s_1 and bad state s_2. The states occur with fixed probabilities, such that p is the probability of s_1 and $1-p$ is the probability of s_2 for $p \in (0,1)$. In this risky world, the consumers decide whether to hedge against the risk of loss by purchasing insurance. These consumers are characterised next.

2.1 Consumers

The consumers' population is non-uniform - the individuals are characterised by either heterogeneous risk perception (see Sections 4 and 5) or both: heterogeneous risk perception and heterogeneous risk exposure (see Section 6). The former category entails expected utility (EU) maximisers and consumers with rank-dependent utility (RDU) preferences who transform the objective loss probability using a non-linear function. For simplicity, these individuals are labelled EU and RDU, respectively. The heterogeneity in risk is captured by the presence of the low-risk and high-risk agents, who suffer an accident with probabilities $1-p_l$ and $1-p_h$, respectively, where $1-p_l < 1-p_h$.[16] To distinguish the low/high-risk agents from the EU/RDU consumers, they are referred to as risk exposure and risk preference types, respectively.

The initial wealth of every consumer is given by $\bar{x}_1 = W$ in the good state and by $\bar{x}_2 = W - L > 0$ in the bad state, where L is an exogenous parameter denoting monetary loss. A utility maximising agent can insure against the loss by purchasing an insurance policy. Once insured, an agent pays the price per unit of insurance, $1-r$, independently of which state occurs. The total premium paid by the agent amounts to $R = (1-r)C$, where C is the amount of compensation, such that $0 \leq C \leq L$. The compensation is paid out only if a loss occurs.

[16] All loss probabilities are exogenous, hence, the problem of moral hazard does not arise.

Thus, the final wealth of an insuree is given by:

$$\begin{array}{ccc} \text{Without insurance} & \text{With insurance} & \\ \bar{x}_1 = W & x_1 = W - R & \text{in } s_1, \\ \bar{x}_2 = W - L & x_2 = W - R - L + C & \text{in } s_2, \end{array}$$

where $x_1,\ x_2 \in \mathbb{R}_+$ and $x_1 \geq x_2$, so that overinsuring is excluded.

2.2 Insurer

A monopolistic risk neutral firm supplies insurance to the market. It sells a contract $\psi = (R, C)$, which specifies the amount of premium paid and the compensation received by an agent. The profit the firm earns from this contract is denoted by Π and is defined as follows:

$$\Pi = (p - r)C.$$

The insurer requires $p \geq r$ in order to avoid negative profit. No additional cost is incurred in writing insurance policies. The likelihood of selling a contract to an RDU individual equals θ, while α denotes the probability of selling a contract to a high-risk agent. The residual probabilities $1 - \theta$ and $1 - \alpha$ denote the likelihoods of acquiring the contract by an EU agent and by a low-risk type, respectively. Probabilities θ and α are not correlated, and it holds that $\alpha, \theta \in (0, 1)$. The distribution of consumers' types in the population is known to all decision makers.

2.3 Equilibrium

In the monopoly market with asymmetric information the insurer offers a pair of policies, $(\psi_i^m,\ \psi_j^k)$, to a random agent, where i (j) and m (k) denote the risk- and preference-types, respectively,[17] and $i, j \in \{l, h\}$, $i \neq j$; $m, k \in \{EU, RDU\}$, $m \neq k$. These policies act as a screening device, so that each consumer selects a single policy that best corresponds to his type.[18] Contract $\Psi^* = (\psi_i^{*m}, \psi_j^{*k})$ entailing a utility maximising policy for each agent is an equilibrium contract if it satisfies the following properties: (i) the expected profit of the insurer from selling policy Ψ^* is non-negative, such that $\Pi(\Psi^*) \geq 0$, and (ii) there is no other Ψ contract, such that the monopolist's profit from selling Ψ is at least as large as the profit from contract Ψ^*.

[17] Notice that no insurance is a special case of $\psi_i^m = 0$.

[18] See also "self-selection" mechanism described by Salop and Salop (1976).

3 Rank-dependent Utility preferences

3.1 RDU and two-outcome lotteries

Each insured consumer faces a binary lottery:

$$P = (x_1, p; x_2, 1 - p), \tag{9}$$

where the outcomes are ranked from best to worst, such that $x_1 \geq x_2$.

Lottery P is evaluated differently by agents with heterogeneous risk perceptions. The evaluations of the EU and RDU consumers, respectively, are given by:

$$EU(P) = u(x_1)p + u(x_2)(1 - p),$$

$$RDU(P) = u(x_1)w(p) + u(x_2)\left[1 - w(p)\right], \tag{10}$$

where u is the von Neumann-Morgenstern utility function mapping from $\mathbb{R}_+$ to $\mathbb{R}$, while w is a probability weighting function which enables the non-linear processing of probabilities. This function is strictly increasing and continuous, mapping from $[0, 1]$ to $[0, 1]$, and it satisfies $w(0) = 0$ and $w(1) = 1$. Additionally, $w(p)$ can be interpreted as a *subjective* probability of not having an accident. For linear w, the EU and RDU evaluations of lottery P coincide.

3.2 Risk attitude

To describe the consumers' attitude to risk a notion of *second-order stochastic dominance* (S-SD) is introduced.

Consider two lotteries with distribution functions A and B and an equal mean.

Definition 1 *For any lotteries A and B, A second-order stochastically dominates B if and only if the individual weakly prefers A to B under every weakly increasing concave utility function u.*

S-SD implies that the individual prefers A to B as long as he is strongly risk averse.[19]

Assume the following:

ASSUMPTION 1: For all $p \in (\delta, 1)$, where $\delta \in (0, 1)$, the preferences of all individuals in the market are consistent with second-order stochastic dominance.

[19] This implication is well-known in the literature and has been shown in the EU framework by Rothschild and Stiglitz (1970), and in the PT framework by Baucells and Heukamp (2006) and Brooks, Peters and Zank (2014).

In order to explore the implications of the S-SD in the RDU framework a notion of mean-preserving spread (MPS) is defined for lottery P in (9) in the following manner:

$$MPS(P) = (x_1 + \frac{\gamma}{p}, p; x_2 - \frac{\gamma}{1-p}, 1-p),$$

where $\gamma > 0$, is sufficiently small to preserve the original ordering of outcomes.

Lemma 1 unravels the relationship between S-SD and MPS and provides a key implication for the shape of the weighting function w in the RDU framework.

Lemma 1 *Assume that Assumption 1 holds and A and B have an equal mean. Then, for any lotteries A and B the following statements are equivalent:*

(i) A second-order stochastically dominates B

(ii) B is a mean-preserving spread of A

(iii) u is concave for all p and w is convex for $p \in (\delta, 1)$, where $\delta \in (0, 1)$.

Lemma 1 shows that holding preferences consistent with S-SD has distinctive implications within the EU and RDU frameworks: whereas under EU it implies the concavity of the utility function, in the RDU framework preferences satisfying S-SD additionally require the convexity of the probability weighting function. Thus, for any strongly risk averse EU and RDU agents who hold an identical utility function, the RDU agent is more risk averse than the EU consumer. This is due to the presence of the probabilistic risk component captured by the convexity of w in the probability interval $(\delta, 1)$. This conjecture plays an important role in determining the agents' willingness to pay.

3.3 Probability weighting function

Empirical evidence suggests that the probability weighting function is inversely S-shaped (Tversky and Kahneman, 1992; Tversky and Wakker, 1995; Gonzales and Wu, 1999; Abdellaoui, 2000).[20] This shape entails concavity for a range of small probabilities and convexity for intermediate and large probabilities. In this way, inverse-S accounts for probabilistic risk attitudes, such as optimism and pessimism. In particular, the overweighting of a small probability of the best (worst) event reflects optimism (pessimism). The simultaneous presence of optimism and pessimism enables the inverse-S to capture the coexistence of contradicting attitudes to risk.[21] In fact, an individual can simultaneously exhibit probabilistic risk

[20]For a review of extensive literature providing the evidence for the inverse-S shape probability weighting function see Wakker (2001).

[21]The simultaneous presence of concavity and convexity of the probability weighting function does not allow for global risk aversion under RDU.

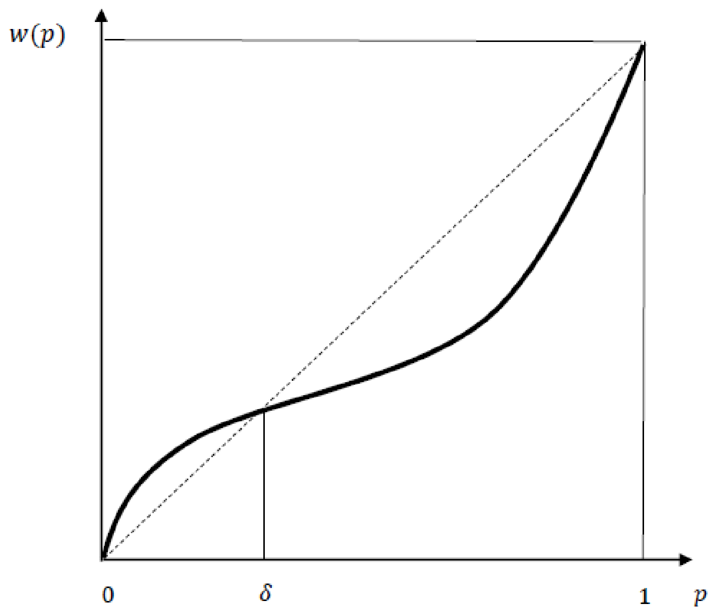

Figure 4: Inverse-S probability weighting function.

aversion and probabilistic risk loving. This situation is shown in Figure 4, where δ denotes the point of intersection of the probability weighting function and the 45 degree (certainty) line. For all probabilities smaller than δ, the agent overestimates probability p of event s_1, implying optimistic behaviour. For all probabilities larger than δ, the agent underestimates probability p of the same event, exposing pessimism.[22] Consequently, the underweighting of a large probability of a good event (equivalently, the overweighting of a small likelihood of a bad event) consititutes pessimism. This type of behaviour is equivalent to being strongly averse to risk in the RDU framework (see Lemma 1). Hence, pessimism drives the difference between the EU and RDU agents' willingness to pay for insurance against the small probability large consequence events, such as an earthquake, a flood or a hurricane.

3.4 Indifference curves

To see how pessimism affects the choice of insurance by an RDU type, as compared with that of an EU agent, consider the indifference curves in Figure 5 (Hirschleifer-Yaari diagram). The two states with probabilities p and $1-p$, respectively, have been fixed. Consistently with the definition of strong risk aversion under RDU, p lies in the interval $(\delta, 1)$. The horizontal axis in the diagram measures wealth level x_1, while the vertical axis shows wealth level x_2. The

[22]In the direct preference-based tests of the shape of w, Camerer and Ho (1994), Wu and Gonzales (1996), Gonzales and Wu (1999) and Abdellaoui (2000) estimate that this point is situated approximately in the $[0.3, 0.4]$ probability interval.

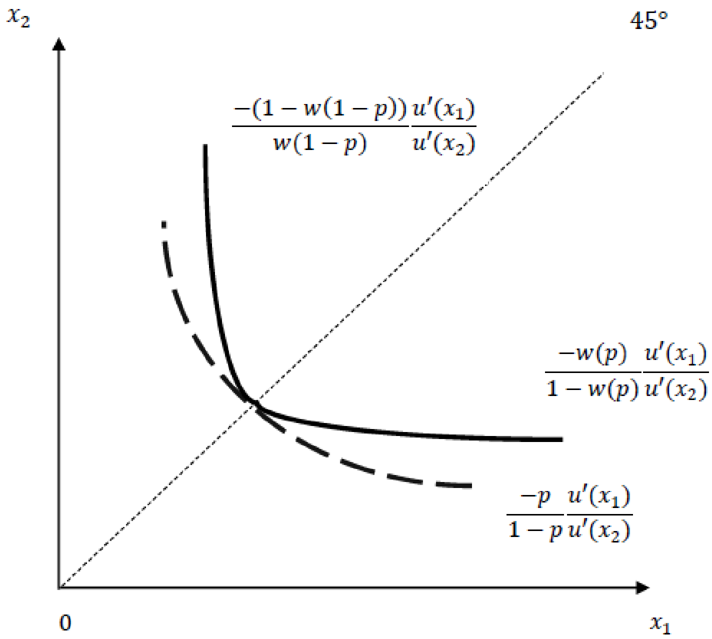

Figure 5: Indifference curves of strongly risk averse agents.

slope of the RDU indifference curve below the certainty line equals $-\frac{w(p)}{1-w(p)}\frac{u'(x_1)}{u'(x_2)}$, while above that line equals $-\frac{1-w(1-p)}{w(1-p)}\frac{u'(x_1)}{u'(x_2)}$. The slope of the EU indifference curve is everywhere equal to $-\frac{p}{1-p}\frac{u'(x_1)}{u'(x_2)}$. Utility function u is strictly increasing and twice continuously differentiable. The kink of the RDU indifference curve on the dotted certainty line depicts the non-smootheness property of RDU indifference curves induced by the change of the rank of outcome (from x_1 to x_2 or vice versa). Figure 5 shows that due to pessimism the indifference curve of the RDU agent (the solid curve) is flatter than the indifference curve of the EU individual (the dashed line) for $x_1 > x_2$, such that $-\frac{w(p)}{1-w(p)}\frac{u'(x_1)}{u'(x_2)} > -\frac{p}{1-p}\frac{u'(x_1)}{u'(x_2)}$. This property implies that an RDU agent is willing to pay a higher premium than an EU type for the equivalent level of compensation.

4 Symmetry of information

This section briefly considers the case of the informed insurer who observes heterogeneous risk perceptions of consumers.

The profit maximisation from selling a policy to agent m requires solving the following problem by the insurer:

$$\max_{(R^m,C^m)\in\Psi} R^m - (1-p^m)C^m, \tag{11}$$

where $m \in \{EU, RDU\}$, $p^{RDU} = w(p)$ and $p^{EU} = p$.[23] It holds that $p \in (\delta, 1)$. The profit is maximised subject to individual rationality constraint (IR) of the agent. This constraint

[23] The assumptions of $p^{RDU} = w(p)$ and $p^{EU} = p$ will hold throughout the paper.

ensures that having insurance yields at least as high utility as the utility from being uninsured:

$$u(x_1)p^m + u(x_2)(1-p^m) \geq u(\bar{x}_1)p^m + u(\bar{x}_2)(1-p^m) \quad \text{(IR)}$$

With the individual rationality constraint binding, *full insurance* is the optimal solution to the problem of the informed monopolist.

Proposition 1 *Under symmetry of information, the monopoly equilibrium contract* $\Psi^* = (\psi^{*EU}, \psi^{*RDU})$ *entails complete coverage.*

Recall that analogous result is obtained in the work by Stiglitz (1977), where instead of heterogeneity in risk perception, the heterogeneity is present in risk exposure. The similarity of the results might suggest that the type of heterogeneity does not affect the qualitative predictions regarding the optimal coverage level. The next section shows that this is the case only if the RDU consumer is sufficiently pessimistic.

5 Asymmetry of information

Asymmetry of information refers to the situation, in which either the supply side of the market (insurer) is less informed than the demand side (insurees), or vice versa is true. In this section the former case is considered, where consumers are screened based on their perceptions of risk.

Recall the results of the benchmark model of asymmetric information by Stiglitz (1977). Assuming heterogenity in risk exposure, Stiglitz establishes that the high-risk type is served full insurance (*Property 1*), and the low-risk type receives only partial coverage (*Property 2*) or no insurance coverage at all (*Property 4*). The latter solution is obtained if the fraction of the high-risk agents in the population is sufficiently large. In addition, heterogenous risk types never pool (*Property 3*). This section shows that these results remain robust even if one replaces the heterogeneity in risk exposure with the heterogeneity in risk perception. In particular, $p \in (\delta, 1)$ is assumed to account for strong aversion to risk.

The profit maximisation problem of the insurer who no longer observes the consumer's perception of risk takes the following form

$$\max_{(R^{RDU},C^{RDU}),\,(R^{EU},C^{EU})\in\Psi} \theta\left[R^{RDU} - (1-p^{RDU})C^{RDU}\right] + (1-\theta)\left[R^{EU} - (1-p^{EU})C^{EU}\right], \quad (12)$$

where the unknown agent is subjected to his individual's rationality constraint (IR) and the incentive compatibility constraint (IC). The latter one is given by:

$$u(x_1^m)p^m + u(x_2^m)(1-p^m) \geq u(x_1^k)p^m + u(x_2^k)(1-p^m), \quad \text{(IC)}$$

where $x_1^m = W - R^m$, $x_2^m = W - R^m - L + C^m$; $k, m \in \{EU, RDU\}$, $k \neq m$.

The incentive compatibility constraint (IC) enables the uninformed insurer to screen between RDU and EU types. In particular, this constraint is binding for the pessimistic RDU type. In contrast, the individual rationality constraint is binding for the EU type. As such, one finds an immediate correspondence between the RDU and EU types in this framework and the high-risk and low-risk agents in the model by Stiglitz (1977). This correspondence confirms that the qualitative predictions regarding the equilibrium contracts of EU and pessimistic RDU consumers do not differ from those obtained for low- and high-risk agents, respectively. Proposition 2 highlights this result:

PROPOSITION 2 *Under Assumption 1, the equilibrium contract in the monopoly market is separating: the RDU type receives full insurance, whereas the EU type purchases partial or no coverage.*

By analogy with *Property 4* (Stiglitz), the solution with no insurance for the EU type, $\psi^{EU*} = (0, 0)$, applies whenever the fraction of pessimistic agents in the population, θ, becomes sufficiently large.

Proposition 1 and Proposition 2 provide similar insights into the role of heterogeneity in determining the equilibrium contracts - they show that the type of consumers' heterogeneity does not affect the qualitative implications derived from the analysis of insurance demand as long as the RDU agent is a pessimist. As a result, it is immaterial whether heterogeneity is hidden in risk exposure or in risk perception, because the individual with a higher evaluation of the likelihood of loss will always acquire a larger quantity of insurance. Next section shows that in the presence of multi-dimensional heterogeneity in agents' characteristics this conclusion may no longer be true.

6 Multiple asymmetry of information

In this section distinctive risk exposures of the consumers are incorporated into the analysis of insurance demand. In that way two types of heterogeneity are relevant for the uninformed monopolist: the heterogeneity in loss probabilities (risk exposure) and the heterogeneity in perception of these probabilities (risk perception).

The main result of this section is governed by Assumption 2, which provides the comparison of the two types of heterogeneity.

ASSUMPTION 2: For all $p \in (\delta, 1)$ and $\delta \in (0, 1)$, the loss probabilities of the low-risk and high-risk types are sufficiently close, so that $w(1 - p_l) > 1 - p_h$.

Whenever the actual loss probabilities of heterogenous agents are sufficiently close, pessimism of the RDU type results in overestimating his chance of loss beyond the actual loss probability of the other type. Hence, Assumption 2 implies that even when exposed to a low probability of a bad event a pessimistic RDU agent will be willing to pay more for an insurance asset than a high-risk expected utility maximiser. The implications of Assumption 2 for the equilibrium allocation of contracts are established next.

6.1 Analysis

Assuming the presence of both types of heterogeneity, the uninformed insurer solves the following optimisation problem:

$$\begin{aligned}
&\max_{\substack{\{(R_l^m, C_l^m),(R_h^k, C_h^k)\}\in\Psi \\ \{(R_l^m, C_l^m),(R_h^m, C_h^m)\}\in\Psi}} \Pi(C_l^{RDU}, R_l^{RDU}, C_h^{EU}, R_h^{EU}, C_l^{EU}, R_l^{EU}, C_h^{RDU}, R_h^{RDU}) \\
= \; & \theta\left[(1-\alpha)\left[R_l^{RDU} - (1-p_l^{RDU})C_l^{RDU}\right] + \alpha\left[R_h^{RDU} - (1-p_h^{RDU})C_h^{RDU}\right]\right] \\
& +(1-\theta)\left[(1-\alpha)\left[R_l^{EU} - (1-p_l^{EU})C_l^{EU}\right] + \alpha\left[R_h^{EU} - (1-p_h^{EU})C_h^{EU}\right]\right],
\end{aligned}$$

where the usual notation applies with the superscript denoting the preference type (EU, RDU) and the subscript denoting the risk exposure (low, high), so that $k, m \in \{EU, RDU\}$, $k \neq m$. The two cases of profit maximisation which entail homogenous risk perception (either both agents are RDU or both agents are EU types) are trivial to solve and their solutions coincide qualitatively with the predictions by Stiglitz (1977). For this reason, the current analysis focuses solely on the situations in which agents differ with respect to both, their risk exposure as well as their risk perception. Hence, the profit maximisation problem of the monopolist is reduced to one of the two optimisation problems:

$$\begin{aligned}
&\max_{\{(R_h^{RDU}, C_h^{RDU}),\,(R_l^{EU}, C_l^{EU})\}\in\Psi} \Pi(R_h^{RDU}, C_h^{RDU}, R_l^{EU}, C_l^{EU}) \\
= \; & \theta\alpha\left[R_h^{RDU} - (1-p_h^{RDU})C_h^{RDU}\right] + (1-\theta)(1-\alpha)\left[R_l^{EU} - (1-p_l^{EU})C_l^{EU}\right],
\end{aligned} \tag{5a}$$

$$\begin{aligned}
&\max_{\{(R_l^{RDU}, C_l^{RDU}),\,(R_h^{EU}, C_h^{EU})\}\in\Psi} \Pi(R_l^{RDU}, C_l^{RDU}, R_h^{EU}, C_h^{EU}) \\
= \; & \theta(1-\alpha)\left[R_l^{RDU} - (1-p_l^{RDU})C_l^{RDU}\right] + (1-\theta)\alpha\left[R_h^{EU} - (1-p_h^{EU})C_h^{EU}\right].
\end{aligned} \tag{5b}$$

The individual rationality constraint (IR_2) and the incentive compatibility constraint (IC_2) apply to both problems, and are given by:

$$u(x_{1i}^m)p_i^m + u(x_{2i}^m)(1-p_i^m) \geq u(\bar{x}_1)p_i^m + u(\bar{x}_2)(1-p_i^m), \tag{IR$_2$}$$

$$u(x_{1i}^m)p_i^m + u(x_{2i}^m)(1-p_i^m) \geq u(x_{1j}^k)p_i^m + u(x_{2j}^k)(1-p_i^m), \tag{IC$_2$}$$

where x_{1i}^m denotes the final wealth of the individual with risk type i and preference type m in the good state, s_1, and $m, k \in \{EU, RDU\}$, $m \neq k$; $i, j \in \{l, h\}$, $j \neq i$.

Under Assumption 2, the following equilibrium contracts follow.

- *The case of a low-risk EU type and a pessimistic high-risk RDU type* (5a)

There is no difference in the qualitative implications obtained for this case and the model by Stiglitz (1977). The pessimistic high-risk individual overweights his likelihood of an accident. Since agents are not allowed to overinsure, this type is sold full insurance at a higher than the actuarially fair price, being exploited by the monopolist. In order to separate both types, the insurer offers partial or no insurance contract to the low-risk expected utility maximiser. In that way, Stiglitz' results alike, the low-risk type is adversely affected by the presence of a high-risk individual. In addition, the two types never purchase the same contract.

- *The case of a pessimistic low-risk RDU type and a high-risk EU type* (5b)

Here Assumption 2 is at work: since the pessimistic agent with low exposure to risk evaluates his chance of loss as being higher than the loss likelihood of the high-risk EU agent, he is the type who receives full insurance. To separate different types the insurer offers the EU agent only partial coverage. Thus, for the case of consumers' loss probabilities being sufficiently close, such that $w(1-p_l) > 1-p_h$, the risk types of those consumers play marginal role in determining the equilibrium contract.

6.2 Results

The analysis leads to the following characterisation of the equilibrium contract.

PROPOSITION 3 *Under Assumptions 1 and 2, and irrespective of the consumers' risk types, the monopoly equilibrium is separating with partial or no insurance for the EU agent and full insurance for the RDU consumer.*

Proposition 3 not only confirms the findings summarised in Proposition 2, but it extends them to the case with two sources of heterogeneity in consumers' population. It shows that sufficient convexity of the probability weighting function in the region $(\delta, 1)$ eliminates the influence that agents' risk exposure has on the characterisation of the equilibrium contracts in the model by Stiglitz (1977). This implies that the current findings do not comply with the intuition behind Stiglitz' properties, where it is solely the risk class of an agent that governs what insurance contract this agent receives.

The next section gives reasons for the absence of pooling equilibrium in the present framework.

6.3 Pooling equilibrium

The equilibrium results obtained in the present analysis suggest that separation is preferred to pooling. This outcome is induced by the ability of the monopolistic insurer to exploit the pessimistic low-risk agent, who overweighs his probability of loss beyond that of the high-risk expected utility maximiser. Yet, two studies that also focus on insurance demand in the framework of non-expected utility theory find that pooling might be efficient if the number of high-risk consumers is sufficiently low. The authors of these studies, Ryan and Vaithianathan (2003) and Jeleva and Villeneuve (2004), investigate how different configurations of the consumers' risk exposures and risk perceptions affect the equilibrium in the monopoly insurance market. For the configuration involving pessimistic low-risk types, they constrain the screening menu of contracts available to this group by not allowing for overinsurance. Nevertheless, their notion of overinsurance differs from the commonly adopted definition, which refers to the situation, in which the amount of the compensation surpasses the value of the risk's damage. Using the current terminology, the standard definition of overinsurance can be conceptualised as $C > L$, with C denoting compensation and L, loss. This interpretation is adopted in the present study. The notion of overinsurance employed by Ryan and Vaithianathan (2003) and Jeleva and Villeneuve (2004) is different and corresponds to overweighting of the true probability of loss, hence, $w(1 - p_l) > 1 - p_l$. This interpretation of overinsurance neither involves the level of compensation, nor loss. Consequently, the equilibrium established in the present framework, hence, the separating contract promising complete coverage for the pessimistic type, is considered as overinsurance in the nomenclature applied by Ryan and Vaithianathan (2003) and Jeleva and Villeneuve (2004).

Whether it is righteous to say that this separating contract indeed entails overinsurance for the pessimistic type is disputable in the market with non-expected utility agents. On one hand, the full-coverage solution provided by Ryan and Vaithianathan (2003) and Jeleva and Villeneuve (2004), in which the pessimistic type receives the amount of insurance as if he were an expected utility maximiser, does not allow for maximisation of his rank-dependent utility, leading to an insufficient amount of insurance (as measured based on this agent's perception of risk). Moreover, in equilibrium obtained by these authors, the monopoly insurer is unable to exploit the excessive willingness of the pessimistic agent to pay, contrary to the proposition by Segal and Spivak (1990). On the other hand, their notion of equilibrium prevents the agents from paying a premium price that exceeds the value of expected loss. However, since the level of premium is proportional to the level of compensation, the agent in the present study can count with the anticipated (perceived) level of compensation, accordingly, avoiding the violation of the standard definition of full insurance, $C = L$.

Thus, the point that this study makes is that there is no commonly agreed upon notion of overinsurance in the non-expected utility framework. According to the definition adopted in this paper, the separating contract of the pessimistic agent entails no more than full coverage. In this way, the present analysis suggests that the consumers' loss perception should not be disregarded by the monopolist when tailoring the contracts to these consumers' needs. Providing that the profitability of most insurance companies is built around the availability of information regarding agents' attitudes to risk, the probabilistic risk attitude such, as pessimism, should be incorporated in an equal manner as the utility-based attitude.

The next section provides an analysis of consumers' welfare following on from the separating equilibrium characterisation in Proposition 3.

6.4 Welfare analysis

The presence of agents distorting their probabilities imposes a negative externality on the types with expected utility preferences, as shown in Sections 5 and 6.2. This externality implies that the contract offered to the EU individual is inefficient, in that it does not provide sufficient amount of insurance. Consequently, the market equilibrium results in the reduction of welfare as compared with the full-information benchmark analysed in Section 4. This outcome of asymmetric information provides a justified rationale for government intervention. Therefore, the aim of this section is to evaluate how far a policy intervention can help improve the welfare in the monopoly market exposed to the asymmetry of information.

An important consequence of the presence of two dimensions of heterogeneity in consumers' characteristics is the increased need for screening by the profit maximising monopolist. Since the screening process is costly, it reduces the welfare in the market. Thus, the increased discrepancy between the perceived and actual risk provides a justified rationale for government intervention. Notice that the presence of a rising dispertion between the contracts of the low-risk and high-risk agents is supported through cases (5a) and (5b), where the initial distortions due to heterogeneity in consumers' risk exposure are deepened by distinctive risk perceptions of the consumers. In case (5a), the pessimistic high-risk agent is exploited by the monopolist, while in case (5b), the high-risk expected utility maximiser receives insufficient amount of compensation. In addition, the pessimistic low-risk individual in scenario (5b) pays a premium price for a unit of insurance coverage that greatly exceeds the actuarially fair premium. In both cases the consumers would have been better off had they shared homogeneous risk perceptions. Consequently, welfare gains from policy intervention are larger when one accounts for heterogeneity in risk perception. This welfare improving result is summarised in Corollary 1.

Corollary 1 *Under Assumption 2, the gains from policy intervention are larger when the consumers in the monopoly market for insurance perceive their risk differently.*

The next section demonstrates further implications of equilibrium allocation for the correlation between risk and insurance coverage.

6.5 Ex-post risk and insurance coverage

A long string of empirical literature suggests that a presence of various biases[24] in agents' assessment of risk implies that the correlation between risk and coverage is either negative or not statistically significant.[25] Using the presence of heterogeneity in both risk and risk perception, this study offers an explanation of this phenomenon.

Consider the cases, which have been neglected in the analysis of equilibrium contracts due to the presence of homogeneity in risk perception (recall Section 6.1). For these cases, irrespective of whether both consumers are characterised by the EU or RDU preferences, the correlation between risk and insurance coverage is positive. The reason is that an agent with higher exposure to risk always receives a larger compensation ex-post. Similar result is obtained for the case (5a), where the high-risk type, whose likelihood of loss initially exceeded the loss probability of the low-risk type, overweighs it even further.

A different result is derived in scenario (5b), in which the population consists of low-risk RDU and high-risk EU consumers. Recall that despite being exposed to an unlikely hazard, a pessimistic low-risk agent is willing to pay more for a unit of insurance than the high-risk expected utility maximiser (Assumption 2). As a consequence, the high-risk agent strictly prefers his contract to the contract of a pessimistic low-risk type; the incentive compatibility constraint of the high-risk EU agent is not binding. Unlike the pessimistic low-risk type, the high-risk individual receives only partial coverage, inducing the negative relationship between risk faced and coverage received. Thus, the following implication proceeds from the analysis of the two cases.

Corollary 2 *The correlation between ex-post risk and insurance coverage is positive (negative) if an agent characterised by a higher risk exposure is willing to pay more (less) for his insurance premium.*

Corollary 2 shows that the amount of compensation one receives is positively correlated with this person's willingness to pay. Since the willingness to pay is determined through agents' perception of risk (Assumption 2), the agent who believes to be more exposed to risk will be offered more coverage. Therefore, positive correlation between risk and insurance arises only if the agent who is willing to pay more for insurance is in fact more exposed to risk. If, however, an individual demands more coverage but his exposure to risk is low, the sign of the correlation will be negative. This result supports a large amount of data on risk-coverage relationship,

[24] These biases include cognitive ability (Fang, Keane and Silverman, 2008), biased risk perceptions (Abaluck and Gruber, 2011 and Barseghyan et al. 2012), information frictions (Handel and Kolstad, 2013), and inertia (Handel, 2010).

[25] The relationship between the riskiness and coverage in various insurance markets has been shown to be negative, e.g., by Finkelstein and Poterba (2004), Finkelstein and McGarry (2006). Additionally, Cawley and Phillipson (1999), Cardon and Hendel (2001) and Chiappori and Salanié (2012) provided evidence for the lack of the statistical significance between the risk and the insurance coverage.

which suggests a negative link between risk exposure and coverage received. Thus, it is not only the utility-based risk attitude but also the probabilistic risk attitude that drives the sign of the relationship between the risk and the amount of insurance one receives. As such, the probability weighting function is the key to understand insurance choices.

7 Conclusion

This paper was concerned with the problem of adverse selection in the market with heterogeneous agents. All agents were strongly risk averse, but they differed in a way they perceive the risk, hence, they had different risk preferences. Consistently with the recent evidence showing that modern insurance companies try to manipulate consumers' perception of risk by making an event look riskier than it is in reality, e.g., by showing the advertisements focusing exclusively on the accident part of a risky event, this work has suggested to incorporate the perception of risk into the analysis of insurance demand as an important factor influencing consumers' choices of insurance policy.

The present model has also reconciled the theoretical predictions and the empirical evidence behind one of the most puzzling patterns of individual behaviour in the insurance market, the negative correlation between ex-post risk and the amount of coverage. The puzzle has been explained by means of probabilistic risk attitudes, which allow for different treatment of probability of a risky event than within the EUT framework. This result calls for more attention being given to the probabilistic risk attitudes of individuals in the insurance markets. Specifically, one shall recognise the contribution that perception of probability has to offer in explaining consumers' choices in risky situations.

Part IV

Future Research Agenda

Undoubtedly, insurance markets have served, for several decades, as a fertile ground for the study of applications of various theoretical models of decision making under risk. These markets turned out to be at least equally useful in analysing decisions made in the situation of subjective uncertainty, that is, when the probabilities of events are unknown. The great majority of models adopted to test predictions of insurance economics under uncertainty have been nested within the framework of subjective expected utility (SEU) theory. Under SEU, an agent is said to have a single subjective prior belief about the fundamentals concerning the insurance contract. Consequently, the agent cannot express any concern about his subjective prior being incorrect. This assumption, however, does not comply well with real insuring situations, in which often the decision maker has a vague idea about the underlying probability distribution of the event of interest (see also Camerer, 1995, for experimental results inconsistent with the assumption of a single prior). The uncertainty regarding the choice of the correct probability distribution calls for incorporating ambiguity into the analysis of insurance demand.

The behavioural implications of ambiguity have been first demonstrated by Ellsberg (1961),[26] who through a series of hypothetical choices showed that it is not always possible to infer subjective probabilities, as defined by Savage (1954), from the decision maker's choices among gambles. In fact, the experiment revealed that individuals prefer to bet on gambles with known odds, hence, they exhibit aversion to ambiguity. Based on this observation a large number of decision models incorporating various forms of ambiguity aversion have been developed (for the review of these models see Etner et al., 2012). One of the most prominent among those models is the smooth ambiguity (SA) model by Klibanoff et al., (2005). The SA model captures the idea that without the knowledge of the underlying distribution function the utility corresponds to the expected ambiguity function over the ambiguous beliefs. This-two stage decomposition of the decision process allows for a clear separation between ambiguity and ambiguity aversion. In addition, the smooth ambiguity model incorporates the maxmin expected utility model of Gilboa and Schmeidler (1989) as a limiting case, in which ambiguity aversion is infinite. A natural avenue for further research is, thus, to analyse the implications of the insurer's ambiguity on the equilibrium contracts of non-expected utility agents in the market characterised by asymmetry of information.

In order to model the effects of ambiguity aversion on the optimal design of contracts in the insurance market, on one hand, the monopolistic supplier of insurance is assumed to have no information about the distribution of low- and high-risk types in the population. On the

[26] Becker and Brownson (1964) demonstrated behavioural implications of ambiguity shortly after Ellsberg (1961).

other hand, different types of agents know their proportions in the market. These agents are assumed to have preferences consistent with the theory dual to expected utility developed by Yaari (1987). In contrast to EU, where attitudes to risk are captured by the shape of the utility function for wealth, in dual theory (DT) of risk, attitudes towards risks are characterised solely by the distorted probability weighting function. Consequently, by applying DT, one can immediately verify the robustness of the predictions derived under expected utility. This has already been done in competitive insurance markets by Young and Browne (2000), who showed that optimal insurance provides an indemnity benefit that is piecewise linear function of the loss. Yet, these authors have not invoked ambiguity aversion into their analysis of insurance demand. Thus, consistently with the experimental results of Hogarth and Kunreuther (1985, 1989), which demonstrate that insurers are characterised by a large degree of ambiguity aversion, the paper aims to verify whether embedding ambiguity into the model of asymmetric information with heterogeneous agents alters the predictions of the traditional economic analysis of the monopoly insurance market. It is anticipated that ambiguity aversion on the side of the monopolist will lead to further distortions in the equilibrium contracts of heterogeneous agents, with the low-risk type being more negatively affected than in the case without ambiguity aversion. Therefore, the contribution of the paper is to unravel the impact of ambiguity averse attitudes on the monopoly insurance market where the individuals have private information.

Part V

Thesis Conclusion

This thesis has analysed how two central features of the non-expected utility theory models, the reference dependence and the non-linear probability weighting, affect individual choices under risk.

Part II provided an axiomatisation of Prospect Theory. Other representations for the PT model have been offered in the literature, yet the key contribution of the present study was that the preference conditions have been derived without prior knowledge of the reference point that separates gains from losses. The central axiom which has enabled the elicitation of the reference point from the decision maker's behaviour employed so-called consistent probability midpoints, a tool from the empirical measurement of probability weighting functions. This study has shown that in combination with other preference conditions, including weak order, first-order stochastic dominance, Jensen continuity and comonotonic independence of common elementary probability shifts, the consistency requirement for probability midpoints not only enables the elicitation of the reference point from the decision maker's behaviour, but it also implies that a binary preference relation is representable by the PT functional. Deriving behavioural conditions in the current framework has had another advantage. By using a naturally given structure of the probability interval, this axiomatisation has disposed of the richness assumption for the set of outcomes. Consequently, the preference conditions have been provided for the arbitrary set of outcomes, including degenerate sets. Providing the preference conditions underlying PT is important as it enables the identification of the empirical plausibility of this model. Additionally, obtaining the representation theorem for PT while disposing of prior knowledge of the reference point provides further support for the use of the PT model as a modelling tool in decision theory and finance.

Part III has analysed the application of rank-dependent utility theory to the problem of insurance demand in the monopoly market with asymmetric information. Unlike other studies, this work accounted not only for the utility-driven risk attitudes, but also for their probabilistic counterpart. Consistently with empirical evidence on risk taking behaviour, probabilistic attitudes to risk have been modeled by the inverse-S probability weighting function, allowing to expose risk attitudes, such as optimism and pessimism. By accommodating the probabilistic risk attitudes of the individuals in the insurance market, this study succeeded in capturing the importance of risk perception for the choice of insurance contracts. In particular, this examination has identified and characterised the equilibrium in the market, in which agents differ not only with respect to the underlying risk but also their risk perception. Since heterogeneity in risk perception reflects distinctive treatment of probabilities, this work exposed how the fact that some agents employ the inverse-S probability weighting function in the evaluation of risk affects the equilibrium in the market with asymmetric information. The results of the present analysis suggest that equilibrium might have the opposite properties to the equilibrium obtained in the standard expected utility model, where probability of risk is assessed linearly. Particularly, this model supports the idea of the negative correlation between ex-post risk and insurance coverage found in the empirical studies on insurance demand. Moreover, this paper emphasises how integrating the probability weighting into the analysis on insurance demand can reconcile the theory and practice of insurance economics, providing important implications for policy makers.

Both contributions have shown how the principles of reference dependence and probability weighting embedded in the framework of non-expected utility theories can help explain individual decision making in the situation of risk. It has been clarified that the property of reference dependence can be identified from the probabilistic risk behaviour of agents. Making this property testable allows to identify the empirical plausibility of the PT model and enables a further application of this decision-making theory to the analysis of individual behaviour under risk. This, in turn, allows to verify how well the theory explains the behaviour observed in the markets. With this respect, it is shown that incorporating a probability weighting function into the analysis of the behaviour of agents in the monopoly insurance market helps reconcile the theory and practice of insurance demand. In particular, the patterns of behaviour which EUT cannot explain but which are frequently observed in the market have received a theoretical account within the RDU framework. In that way, the non-expected utility theory models, in particular PT and RDU, improve the understanding of risky choices made by the decision makers.

Appendices

Appendix 1

As can be observed from the equations for RDU, PT and Eqs. (7) and (8), and also from the additively separable preference representation in Lemma 1, cumulated probabilities are frequently used as variables. For the proofs it will be convenient to use an alternative notation for prospects, following Abdellaoui (2002) and Zank (2010). In the *decumulative probabilities* notation $P = (\tilde{p}_1, \ldots, \tilde{p}_n)$, where $\tilde{p}_j = \sum_{i=1}^{j} p_i$ denotes the probability of obtaining outcome x_j or better, $j = 1, \ldots, n$.[27] Obviously, $\tilde{p}_n = 1$. Naturally, all preference conditions can be re-written in terms of decumulative probabilities.

Proof of Lemma 1: The proof of the lemma follows from results for additive representations on rank-ordered sets in Wakker (1993, Theorem 3.2 and Corollary 3.6). That statement (i) implies statement (ii) is immediate from the properties of the functions $V_j, j = \{1, \ldots, n-1\}$. As the preference relation $\succcurlyeq$ is defined on a rank-ordered set of decumulative probabilities (i.e., a rank-ordered subset of $[0,1]^{n-1}$) and $\succcurlyeq$ satisfies weak order, Jensen-continuity and first order stochastic dominance, it follows that $\succcurlyeq$ also satisfied Euclidean continuity (by Lemma 18 in Abdellaoui 2002). First order stochastic dominance comes down to strong monotonicity in decumulative probabilities. Further, as $n \geq 4$, and independence of common elementary probability shifts comes down to coordinate independence of Wakker (1993), statement (ii) of Theorem 3.2 of Wakker is satisfied. Then statement (i) of the lemma follows from statement (i) of Theorem 3.2 of Wakker, the only difference being that strong monotonicity in this framework implies that the functions $V_j, j = \{1, \ldots, n-1\}$ are strictly increasing. Uniqueness results are as in Wakker's Theorem 3.2. This concludes the proof of Lemma 1. □

Proof of Theorem 1: The derivation of statement (ii) from statement (i) follows from Lemma 1 and the analysis preceding the theorem in the main text on the consistency of elicited probability midpoints under PT.

In the following it is proved that statement (ii) implies statement (i) of the theorem. Assume that $\succcurlyeq$ on $\mathcal{L}$ is a weak order that satisfies first order stochastic dominance, independence of common elementary probability shifts and sign-dependent probability midpoint consistency. Then, by statement (i) of Lemma 1 the preference $\succcurlyeq$ on $\mathcal{L}$ is represented by an additive function

$$V(P) = \sum_{j=1}^{n-1} V_j(\tilde{p}_j), \tag{13}$$

with continuous strictly increasing functions $V_1, \ldots, V_{n-1} : [0,1] \to \mathbb{R}$ which are bounded except V_1 and V_{n-1} which could be unbounded at extreme probabilities.

[27] Similarly, in the *cumulative probabilities* notation $P = (1, 1 - \tilde{p}_1, \ldots, 1 - \tilde{p}_{n-1})$ where entries denote the probability of obtaining outcome x_j or less, $j = 1, \ldots, n$.

Next the analysis is restricted to decumulative probabilities different from 0 or 1 to, for now, avoid the problems with the unboundedness of V_1 and V_{n-1}. Following the analysis in the main text preceding Theorem 1, SMC implies that either there is no sign-dependence or there is a unique reference point $x_k, k \in \{2, \ldots, n-1\}$. If sign-dependence does not hold, SMC comes down to the consistency in probability attitudes of Zank (2010), which implies that RDU holds. Therefore, consider the case when sign-dependence holds.

Assume first that $2 < k \leq n-1$. For any $\delta \in (0,1)$ and $\varepsilon > 0$ let $B_\varepsilon(\delta)$ be the open neighborhood around δ with Euclidean distance ε. Take any $\alpha, \beta, \gamma \in B_\varepsilon(\delta)$ such that

$$\sum_{i=1}^{k-1}[V_i(\beta) - V_i(\alpha)] = \sum_{i=1}^{k-1}[V_i(\gamma) - V_i(\beta)]. \tag{14}$$

For sufficiently small $\varepsilon > 0$, by continuity of the functions $V_i, i = k, \ldots, n-1$, there exists lotteries $P, Q \in \mathcal{L}$ with

$$\sum_{i=1}^{k-1} V_i(\alpha) + \sum_{i=k}^{n-1} V_i(\tilde{p}_i) = \sum_{i=1}^{k-1} V_i(\beta) + \sum_{i=k}^{n-1} V_i(\tilde{q}_i)$$

and

$$\sum_{i=1}^{k-1} V_i(\beta) + \sum_{i=k}^{n-1} V_i(\tilde{p}_i) = \sum_{i=1}^{k-1} V_i(\gamma) + \sum_{i=k}^{n-1} V_i(\tilde{q}_i).$$

Before proceeding with the proof, some simplifying notation is introduced. For any nonempty subset $I \subset \{1, \ldots, n-1\}$ we write $\sigma_I P$ for prospect P with $\tilde{p}_i$ replaced by $\sigma \in [0,1]$ for all $i \in I$. Clearly, for $\sigma_I P$ to be a well-defined prospect, I must include all indices between and including the smallest $(\min\{i : i \in I\})$ and the largest $(\max\{i : i \in I\})$ in I. With this notation, the latter two equations are equivalent to the respective indifferences

$$\alpha_I P \sim \beta_I Q \text{ and } \beta_I P \sim \gamma_I Q,$$

where $I = \{1, \ldots, k-1\}$, meaning that the decumulative probabilities α, β, γ are attached to gains. Consider the case $\alpha < \beta$ (and note that the case $\alpha > \beta$ is completely analogous). By first order stochastic dominance it follows that $\gamma > \beta$. Further, sign-dependent probability midpoint consistency requires that

$$\alpha_J \beta_{I \setminus J} P \sim \beta_J \gamma_{I \setminus J} Q$$

for all $J = \{1, \ldots, j\}, j \in I \setminus \{k-1\}$. First take $j = 1$. Then, substitution of Equation (13) into $\alpha_I P \sim \beta_I Q$ implies

$$\sum_{i=1}^{k-1} V_i(\alpha) + \sum_{i=k}^{n-1} V_i(\tilde{p}_i) = \sum_{i=1}^{k-1} V_i(\beta) + \sum_{i=k}^{n-1} V_i(\tilde{q}_i),$$

and substitution of Equation (13) into $\alpha_1\beta_{I\setminus\{1\}}P \sim \beta_1\gamma_{I\setminus\{1\}}Q$ gives

$$V_1(\alpha) + \sum_{i=2}^{k-1} V_i(\beta) + \sum_{i=k}^{n-1} V_i(\tilde{p}_i) = V_1(\beta) + \sum_{i=2}^{k-1} V_i(\gamma) + \sum_{i=k}^{n-1} V_i(\tilde{q}_i).$$

Taking the difference of the two latter equations and cancelling common terms implies

$$\sum_{i=2}^{k-1}[V_i(\beta) - V_i(\alpha)] = \sum_{i=2}^{k-1}[V_i(\gamma) - V_i(\beta)].$$

Similarly, joint substitution of Equation (13) into $\beta_I P \sim \gamma_I Q$ and $\alpha_1\beta_{I\setminus\{1\}}P \sim \beta_1\gamma_{I\setminus\{1\}}Q$, taking differences and cancelling common terms, imply

$$V_1(\beta) - V_1(\alpha) = V_1(\gamma) - V_1(\beta). \tag{15}$$

Similarly, if $j = 2$, one obtains

$$\sum_{i=3}^{k-1}[V_i(\beta) - V_i(\alpha)] = \sum_{i=3}^{k-1}[V_i(\gamma) - V_i(\beta)]$$

and

$$\sum_{i=1}^{2}[V_i(\beta) - V_i(\alpha)] = \sum_{i=1}^{2}[V_i(\gamma) - V_i(\beta)],$$

and using Equation (15) one obtains

$$V_2(\beta) - V_2(\alpha) = V_2(\gamma) - V_2(\beta).$$

By induction on j it can be concluded that if Equation (14) holds then for all $j = 1, \ldots, k-1$ it holds that

$$V_j(\beta) - V_j(\alpha) = V_j(\gamma) - V_j(\beta).$$

That the converse holds is immediate. One concludes that for any $\delta \in (0, 1)$ and sufficiently small $\varepsilon > 0$ for $\alpha, \beta, \gamma \in B_\varepsilon(\delta)$ it holds that

$$\begin{aligned}\sum_{i=1}^{k-1}[V_i(\beta) - V_i(\alpha)] &= \sum_{i=1}^{k-1}[V_i(\gamma) - V_i(\beta)] \\ &\Leftrightarrow \\ V_j(\beta) - V_j(\alpha) &= V_j(\gamma) - V_j(\beta) \text{ for all } j = 1, \ldots, k-1.\end{aligned}$$

This means that locally the functions V_j, $j = 1, \ldots, k-1$, are proportional and also proportional to their sum, which we denote V^+. Global proportionality follows from local proportionality and continuity. It follows that positive constants $s_1, \ldots, s_{k-1}$ and real numbers $t_1, \ldots, t_{k-1}$ exist such that

$$V_j(\cdot) = s_j V^+(\cdot) + t_j, j = 1, \ldots, k-1.$$

Following Proposition 3.5 of Wakker (1993) the functions V_j can be taken finite at 0 and 1, and can continuously be extended to all of $[0, 1]$.

Similar arguments, now applying consistency for probability midpoints of losses, can be used to derive proportionality of the functions $V^- := \sum_{j=k}^{n-1} V_j$ and V_j, $j = k, \ldots, n-1$ whenever $2 \leq k < n-1$. Proposition 3.5 of Wakker (1993) applies again saying that the functions V_j can be taken finite at 0 and 1, and can continuously be extended to all of $[0, 1]$. Thus, it can be concluded that positive constants $s_k, \ldots, s_{n-1}$ and real numbers $t_k, \ldots, t_{n-1}$ exist such that

$$V_j(\cdot) = s_j V^-(\cdot) + t_j, j = k, \ldots, n-1.$$

Next, for the case that $2 < k < n-1$, derive the weighting functions for probabilities of gains and losses and the utility for outcomes. Fix $V^+(1) + V^-(1) = 1$ and $V_j(0) = 0$ for $j = 1, \ldots, k-1$ and $V_j(1) = 0$ for $j = k, \ldots, n-1$, thereby fixing the scale and location of the otherwise jointly cardinal functions V_j. Then, $t_1 = \cdots = t_{n-1} = 0$ must hold and it follows that $V^+(1) = 1$. Define

$$w^+(\tilde{p}) := V^+(\tilde{p}) = \sum_{j=1}^{k-1} V_j(\tilde{p}) + \sum_{j=k}^{n-1} V_j(1).$$

Therefore, $w^+(0) = 0, w^+(1) = 1$ and w^+ is strictly increasing and continuous on $[0, 1]$, and is, indeed, a well-defined probability weighting function. It is the probability weighting function for probabilities of gains.

Next derive w^- by defining

$$\hat{w}(\tilde{p}) := \frac{V^-(\tilde{p})}{V^-(0)} = \frac{\sum_{j=1}^{k-1} V_j(0) + \sum_{j=k}^{n-1} V_j(\tilde{p})}{\sum_{j=1}^{k-1} V_j(0) + \sum_{j=k}^{n-1} V_j(0)}.$$

This is a well-defined function given that the V_j's, $j = k, \ldots, n-1$, are strictly increasing and bounded, and thus, $V_j(\tilde{p}) < 0$ for all $j = k, \ldots, n-1$, whenever $\tilde{p} < 1$, such that the denominator $\sum_{j=1}^{k-1} V_j(0) + \sum_{j=k}^{n-1} V_j(0) \neq 0$ and finite. It follows that the function $\hat{w}$ has the following properties: $\hat{w}(1) = 0$ and $\hat{w}(0) = 1$ and $\hat{w}$ is strictly decreasing and continuous on $[0, 1]$. Set

$$\tilde{w}^-(\tilde{p}) := 1 - \hat{w}(\tilde{p}) = \frac{V^-(0) - V^-(\tilde{p})}{V^-(0)},$$

for each $\tilde{p} \in [0, 1]$, which gives the dual weighting function for probabilities of losses. A useful rearrangement of this equation gives

$$V^-(\tilde{p}) = V^-(0)[1 - \tilde{w}^-(\tilde{p})].$$

From $\tilde{w}^-$ one obtains w^- through $w^-(\tilde{p}) = 1 - \tilde{w}^-(1 - \tilde{p})$ for all $\tilde{p} \in [0, 1]$.

Next the utility function for outcomes is derived. From the derivation of w^+ and $V_j(\cdot) = s_j V^+(\cdot), j = 1, \ldots, k-1$, one obtains

$$V_j(\cdot) = s_j w^+(\cdot), j = 1, \ldots, k-1,$$

and from the derivation of $\tilde{w}^-$ and $V_j(\cdot) = s_j V^-(\cdot), j = k, \ldots, n-1$, one obtains

$$V_j(\cdot) = s_j V^-(0)[1 - \tilde{w}^-(\cdot)], j = k, \ldots, n-1.$$

Noting that the degenerate prospect that gives x_i for sure is expressed as the prospect $0_{\{1,\ldots,i-1\}}(1, \ldots, 1)$, for each $i = 1, \ldots, n$, define utility as follows:

$$\begin{aligned} u(x_k) :=& \quad V(0_{\{1,\ldots,k-1\}}(1, \ldots, 1)) \\ =& \quad V^-(1) \\ =& \quad 0. \end{aligned}$$

Moving backwards, for $i = k-1, \ldots, 1$ one can iteratively define

$$u(x_i) := u(x_{i+1}) + s_i.$$

And for $i = k+1, \ldots, n$ one can iteratively define

$$u(x_i) := u(x_{i-1}) + s_{i-1} V^-(0).$$

These definitions of utility for gains and losses imply that the ordering of the utility for outcomes is $u(x_1) > \cdots > u(x_n)$, thus in agreement with the ordering according to the preference $\succcurlyeq$.

Next, substitution into $V(P)$, gives

$$\begin{aligned} V(P) &= \sum_{j=1}^{k-1} V_j(\tilde{p}_j) + \sum_{j=k}^{n-1} V_j(\tilde{p}_j) \\ &= \sum_{j=1}^{k-1} s_j w^+(\tilde{p}_j) + \sum_{j=k}^{n-1} s_j V^-(0)[1 - \tilde{w}^-(\tilde{p}_j)]. \end{aligned}$$

Note that for $i = 1, \ldots, k-1$ one has $s_i = u(x_i) - u(x_{i+1})$ and for $i = k, \ldots, n-1$ one has $s_i V^-(0) = u(x_{i+1}) - u(x_i)$. Substitution into the preceding equation gives

$$\begin{aligned} V(P) &= \sum_{j=1}^{k-1} w^+(\tilde{p}_j)[u(x_j) - u(x_{j+1})] + \sum_{j=k}^{n-1}[u(x_{j+1}) - u(x_j)][1 - \tilde{w}^-(\tilde{p}_j)] \\ &= \sum_{j=1}^{k-1}[w^+(\tilde{p}_j) - w^+(\tilde{p}_{j-1})]u(x_j) + \sum_{j=k}^{n-1}[u(x_{j+1}) - u(x_j)][1 - \tilde{w}^-(\tilde{p}_j)], \end{aligned}$$

where, in the latter equation, the term relating to probabilities of gains has been rearranged using the properties that $w^+(\tilde{p}_{j-1}) = 0$ for $j = 1$ and $u(x_{j+1}) = 0$ for $j = k-1$. Next the term relating to probabilities of losses is rearranged. After substitution of $\tilde{w}^-(p) = 1 - w^-(1-p)$, one obtains

$$V(P) = \sum_{j=1}^{k-1}[w^+(\tilde{p}_j) - w^+(\tilde{p}_{j-1})]u(x_j) + \sum_{j=k+1}^{n} [u(x_j) - u(x_{j-1})]w^-(1 - \tilde{p}_{j-1}).$$

Rearranging yields

$$V(P) = \sum_{j=1}^{k-1}[w^+(\tilde{p}_j) - w^+(\tilde{p}_{j-1})]u(x_j) + \sum_{j=k+1}^{n} [w^-(1-\tilde{p}_{j-1}) - w^-(1-\tilde{p}_j)]u(x_j) = PT(P), \quad (16)$$

where in the derivation of the latter expression for loss probabilities one has used that $u(x_{j-1}) = 0$ for $j = k+1$ and $w^-(1-\tilde{p}_j) = 0$ for $j = n$ (recall that $\tilde{p}_n = 1$). The conclusion is that the representation V of $\succcurlyeq$ on $\mathcal{L}$ is, in fact, a genuine PT-functional.

Cases $k = 2$ and $k = n-1$ are problematic as unboundedness is possible at 0 or 1. If $k = 2$ and V_1 is bounded at 1, or if $k = n-1$ and V_{n-1} is bounded at 0, one can simply repeat the preceding analysis and derive genuine PT. If, however, $k = 2$ and V_1 is unbounded at 1, one can derive w^- and u for losses (i.e., for $x_i, i = 3, \ldots, n$) by using similar arguments as in the preceding analysis (i.e., following the case $2 \leq k < n-1$); nothing more can be said about V_1, thus, generalised PT as in Eq. (8) is obtained. Similarly, if $k = n-1$ and V_{n-1} is unbounded at 0, one can derive w^+ and u for gains (i.e., for $x_i, i = 1, \ldots, n-1$) by using similar arguments as in the preceding analysis (i.e., following the case $2 < k \leq n-1$), thus, generalised PT as in

Eq. (7) is obtained. Hence, generalised PT represents $\succcurlyeq$ on $\mathcal{L}$. This concludes the derivation of statement (i) from statement (ii) in Theorem 1.

To complete the proof of the theorem the uniqueness results for the weighting functions and utility must be derived. For bounded V_1 and V_{n-1} one has fixed the scale and location of the otherwise jointly cardinal functions $V_j, j = 1, \ldots, n-1$ in order to derive w^+ and w^-. That is, given any other representation of preferences that is additively separable as in Lemma 1, fixing scale and location as required in the proof above will lead to the same probability weighting functions. This shows that the weighting functions w^+ and w^- are uniquely determined. From the definition of the utility function u it is clear that the only freedom one has in defining utility is the starting value at the reference point x_k (i.e., the location parameter) and a scaling parameter due to the jointly cardinal functions $V_j, j = 1, \ldots, n-1$. So, utility can, at most, be cardinal. However, in order to rewrite V in the form of the PT-functional it is critical that $u(x_k) = 0$. Otherwise, if $u(x_k) \neq 0$, the terms $w^+(p_k)u(x_k)$ and $u(x_k)w^-(1-p_k)$ will appear in Equation (16). With these terms added in Equation (16) a functional is obtained that violates first order stochastic dominance and continuity, hence, it cannot be a representation of $\succcurlyeq$ on $\mathcal{L}$. This means that u must be a ratio scale. This is somewhat different if one of V_1 or V_{n-1} are unbounded. In the first case w^- is uniquely determined but u, which must satisfy $u(x_2) = 0$, and V_1 are jointly cardinal. In the second case w^+ is uniquely determined and u, which must satisfy $u(x_{n-1}) = 0$, and V_{n-1} are jointly cardinal.

This concludes the proof of Theorem 1. □

Appendix 2

Proof of Proposition 1: Consider the Langrange function $\mathcal{L}$, which has been set up to solve the profit maximisation problem of the monopolist in 11 subject to constraint IR:

$$\mathcal{L} = R^m - (1-p^m)C^m + \lambda \left[\begin{array}{c} u(W-R^m)p^m + u(W-R^m-L+C^m)(1-p^m) \\ -u(W)p^m + u(W-L)(1-p^m) \end{array} \right]$$

First-order conditions (FOC) are given by:

$$\frac{\partial \mathcal{L}}{\partial R^m} = 1 - \lambda \left[u'(W-R^m)p^m + u'(W-R^m-L+C^m)(1-p^m) \right] = 0$$

$$\frac{\partial \mathcal{L}}{\partial C^m} = -(1-p^m) + \lambda u'(W-R^m-L+C^m)(1-p^m) = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \left[u(W-R^m) - u(W) \right] p^m + \left[u(W-R^m-L+C^m) - u(W-L) \right] (1-p^m) = 0$$

Rearranging yields:

$$u'(W-R^m) = u'(W-R^m-L+C^m),$$

which implies that $L = C^m$. Hence, an agent receives full insurance irrespective of whether he is an EU type, so that $p^m = p$, or an RDU type, $p^m = w(p)$. □

Proof of Proposition 2: Proposition 2 claims that the equilibrium in the monopoly market entails separation of EU and RDU agents. The proof derives the separation of equilibrium first, and later it shows that pooling is not optimal.

The proof of the separating case extends the proof of Stiglitz (1977). First, it shows that under Assumption 1, the contract in the monopoly market, $\Psi^* = (\psi^{EU*}, \psi^{RDU*})$ is a separating equilibrium if it holds that $C^{RDU*} = L$,

$$u(x_1^{RDU})p^{RDU} + u(x_2^{RDU})(1-p^{RDU}) = u(x_1^{EU})p^{RDU} + u(x_2^{EU})(1-p^{RDU}), \qquad (\text{IC}^{RDU})$$

and that

$$u(x_1^{EU})p^{EU} + u(x_2^{EU})(1-p^{EU}) = u(\bar{x}_1)p^{EU} + u(\bar{x}_2)(1-p^{EU}). \qquad (\text{IR}^{EU})$$

By contradiction, assume that IC^{RDU} is not binding and the RDU type strictly prefers his contract to the contract of the EU agent. In such a case the insurer can increase his profit by offering a different contract, $\psi' = (R', C')$, such that

$$\begin{aligned} u(x_1^{RDU})p^{RDU} + u(x_2^{RDU})(1-p^{RDU}) &> u(x_1^{'})p^{RDU} + u(x_2^{'})(1-p^{RDU}) \qquad (\text{A1}) \\ &> u(x_1^{EU})p^{RDU} + u(x_2^{EU})(1-p^{RDU}). \end{aligned}$$

The insurer keeps introducing different contracts satisfying condition A1 to increase his profit further. Profit-maximization requires that IC^{RDU} binds, hence, eventually A1 must be relaxed. Again, by contradiction, assume that

$$u(x_1^{EU})p^{EU} + u(x_2^{EU})(1 - p^{EU}) \geq u(x_1^{RDU})p^{EU} + u(x_2^{RDU})(1 - p^{EU}), \qquad \text{(IC}^{EU}\text{)}$$

holds. Hence, the IC^{EU} binds for $\psi^{RDU*} = (R^{RDU*}, C^{RDU*})$, which maximises profit of the insurer earned on the RDU type. In addition, binding IC^{RDU} implies that $\psi^{RDU*} > \psi^{EU*}$, so that the RDU type receives more compensation in equilibrium. This, however, and the IC^{EU} holding with equality does not imply that the EU type prefers the contract with a higher compensation (as assumed by IC^{EU}). Hence, the EU type always (strictly) prefers his contract to the contract of the RDU agent. It follows that $C^{RDU*} = L$, since changes of ψ^{RDU*} in the presence of (not binding) incentive compatibility constraint of the EU type are not profitable.

Finally, it is shown that IR^{EU} holds. In particular, since the increase in the amount of coverage for the EU type decreases the monopolist profit, the insurer optimises when IR^{EU} holds.

In the second part of the proof, I demonstrate that pooling equilibrium is not optimal. Hence, assume that there is a pooling contract, $\Psi^P = (\psi^P, \psi^P)$, which offers to both agents the same amount of coverage at an equal price. If the agents are offered a partial insurance, such that $C^P < L$, the insurer can earn more by offering a contract, Ψ^d, entaling a higher coverage, $C^d > C^P$ (without changing the price, hence, implying that $R^d > R^P$). Irrespective of the EU type's choice of contract, the RDU agent accepts contract $\Psi^d = (R^d, C^d)$ because of the higher coverage. Hence, this type does not prefer the pooling contract.

A pooling contract might also entail full insurance, such that $C^P = L$. In such a case, the monopolist can still increase his profit by offering a contract, Ψ^e, which satisfies the following condition:

$$\begin{aligned} u(x_1^P)p^P + u(x_2^P)(1 - p^P) &= u(x_1^P)p^P + u(x_2^e)(1 - p^P), \\ u(x_1^P) &= u(x_1^e)p^P + u(x_2^e)(1 - p^P), \end{aligned}$$

where $L - C^e < \eta$, for a small enough η. Additionally, it holds that $x_2^e = W - R^P - L + C^e$. Since contract Ψ^e offers less insurance, it will be preferred by the EU type. This, however, implies that the per unit price of contract Ψ^e is higher than the price of contract Ψ^P, since for the former contract offers less insurance at the same price. Consequently, the profit on the EU type increases, while the profit on the RDU type (who purchases the contract entailing more compensation, Ψ^P) remains unchanged. This completes the proof of Proposition 2. □

Proof of Proposition 3: The proof of Proposition 3 is analogous to the proof of Proposition 2. This is due to Assumption 2, which states that the willigness to pay for insurance is determined by agent's risk perception and not by their risk types. Hence, the proof immediately reduces to the case of the separating equilibrium in the presence of heterogeneity in risk perception only. This problem has been solved in Section 5 and the corresponding proof is given in the proof of Proposition 2. □

Proof of Lemma 1: Lemma 1 is not original to this work and a similar result has been derived previously (e.g., Cohen, 1995). Nevertheless, as this result plays a key role in the analysis, the sketch of its proof is provided.

First, suppose that lottery z is distributed with B. Then, for every realisation z one can write

$$y(z) = A^{-1}(B(z)).$$

Hence, statement (i) of Lemma 1 can be rewritten to:

$$\int u(z)\ dA(z) \geq \int u(z)\ dB(z)$$

for every weakly increasing concave utility function u. This notation will be useful for the remaining part of the proof.

First, it is proved that statement (ii) implies (i). The proof is undertaken solely under the assumption of EU as an equivalent proof under RDU is trivial. Notice that (ii) is given by

$$\int u(y)\ dB(y),$$

which is equivalent to

$$\int E\ \left[u(z+|\gamma|)|z\right]\ dA(z)$$

since $y = z + |\gamma|$ with $|\gamma|$ being the noise with zero expectation, $E\ [\gamma|z] = 0$, introduced in the definition of MPS. Notice that

$$\begin{aligned} \int E\ \left[u(z+|\gamma|)|z\right]\ dA(z) \ &\leq \ \int u\ \left(E\left[z+|\gamma||z\right]\right) dA(z) \\ &= \ \int u(z)\ dA(z), \end{aligned}$$

which is equivalent to statement (i). This completes the proof of equivalence between (i) and (ii). Next the proof of the equivalence between (i) and (iii) is briefly sketched.

Recall that the utility function, u, is strictly increasing and differentiable, and the probability weighting function, w, is strictly increasing and continuous. Chew, Karni and Safra (1987) show that an RDU decision maker with utility function u and probability weighting function w exhibits strong aversion to risk (hence, S-SD), if and only if, his utility function, u, is concave and his probability weighting function, w, is convex. Under Assumption 1, convexity of w is required only for all $p \in (\delta, 1)$, where $\delta \in (0, 1)$. Notice that under Assumption 1, in the RDU framework with the inverse-S probability weighting function preferences consistent with S-SD imply that $w(p) < p$ for $p \in (\delta, 1)$, hence, convexity of w in that range of probabilities. Thus, the equivalence of part (i) and (iii) of Lemma 1 follows. Since statement (ii) implies (i) and (i) implies (iii), statement (ii) must also imply (iii). This completes the proof of Lemma 1. $\square$

Bibliography

Abaluck, J., and Gruber, J. (2011). Heterogeneity in Choice Inconsistencies Among the Elderly: Evidence from Prescription Drug Plan Choice. *American Economic Review.* 101, 377-385.

Abdellaoui, M. (2000). Parameter-Free Elicitation of Utility and Probability Weighting Functions. *Management Science.* 46, 1497–1512.

Abdellaoui, M. (2002). A Genuine Rank-Dependent Generalization of the von Neumann-Morgenstern Expected Utility Theorem. *Econometrica.* 70, 717–736.

Abdellaoui, M., l'Haridon, O., and Zank, H. (2010). Separating Curvature and Elevation: A Parametric Probability Weighting Function. *Journal of Risk and Uncertainty.* 41, 39–65.

Abdellaoui, M., Vossmann, F., and Weber, M. (2005). Choice-Based Elicitation and Decomposition of Decision Weights for Gains and Losses Under Uncertainty. *Management Science.* 51, 1384-1399.

Akerlof, G. (1970). The Market for 'Lemons': Quality Uncertainty and the Market Mechanism. *Quarterly Journal of Economics.* 84, 488-500.

al-Nowaihi, A., and Dhami, S. (2010). The Behavioral Economics of Insurance. University of Leicester, Department of Economics. *Working Paper No. 10/12.*

Allais, M. (1953). Le Comportement de l'Homme Rationnel devant le Risque: Critique des Postulats et Axiomes de l'Ecole Americaine. *Econometrica.* 21, 503-546.

Andreoni, J., and Sprenger, C. (2009). Certain and Uncertain Utility: The Allais Paradox and Five Decision Theory Phenomena. *Working Paper UCSD.*

Andreoni, J., and Sprenger, C. (2012). Risk Preferences Are Not Time Preferences. *American Economic Review.* 102, 3357–3376.

Baillon, A., Driesen, B., and Wakker, P. (2012). Relative Concave Utility for Risk and Ambiguity. *Games and Economic Behavior.* 75, 481–489.

Barberis, N. (2012). Thirty Years of Prospect Theory in Economics: A Review and Assessment. *NBER Working Paper No.* 18621.

Barberis, N., Huang, M., and Santos, T. (2001). Prospect Theory and Asset Prices. *Quarterly Journal of Economics.* 116, 1–53.

Bateman, I., Munro, A., Rhodes, B., Starmer, C., and Sugden, R. (1997). A Test of the Theory of Reference-Dependent Preferences. *Quarterly Journal of Economics.* 112, 647–661.

Baucells, M., and Heukamp, F. (2006). Stochastic Dominance and Cumulative Prospect Theory. *Management Science.* 52, 1409-1423.

Becker, S., and Brownson, F. (1964). What Price Ambiguity? Or the Role of Ambiguity in Decision-Making? *Journal of Political Economy.* 72, 62-73.

Bell, D., (1985). Disappointment in Decision Making under Uncertainty. *Operations Research.* 33, 1–27.

Benartzi, S., and Thaler, R. (1995). Myopic Loss Aversion and the Equity Premium Puzzle. *Quarterly Journal of Economics.* 110, 73–92.

Birnbaum, M., and Stegner, S. (1981). Measuring the Importance of Cues in Judgment for Individuals: Subjective Theories of IQ as a Function of Heredity and Environment. *Journal of Experimental Social Psychology.* 17, 159–182.

Blavatskyy, P. (2011). Loss Aversion. *Economic Theory.* 46, 127–148.

Bleichrodt, H., and Pinto, J. (2000). A Parameter-Free Elicitation of the Probability Weighting Function in Medical Decision Analysis. *Management Science.* 46, 1485–1496.

Bleichrodt, H., Pinto, J., and Wakker, P. (2001). Making Descriptive Use of Prospect Theory to Improve the Prescriptive Use of Expected Utility. *Management Science.* 47, 1498–1514.

Bleichrodt, H., Schmidt, U., and Zank, H. (2009). Additive Utility in Prospect Theory. *Management Science.* 55, 863–873.

Brooks, P., Peters, S., and Zank, H. (2014). Risk Behavior for Gain, Loss and Mixed Prospects. *Theory and Decision.* 77, 153-182.

Brooks, P., and Zank, H. (2005). Loss Averse Behavior. *Journal of Risk and Uncertainty.* 31, 301–325.

Bruhin, A., Fehr-Duda, H., and Epper, T. (2010). Risk and Rationality: Uncovering Heterogeneity in Probability Distortion. *Econometrica.* 78, 1375-1412.

Camerer, C. (1995). Individual Decision Making. In: Handbook of Experimental Economics. Princeton University Press.

Camerer, C., and Ho, T. (1994). Violations of the Betweenness Axiom and Nonlinearity in Probability. *Journal of Risk and Uncertainty.* 8, 167-196.

Cawley, J., and Philipson, T. (1999). An Empirical Examination of Information Barriers to Trade in Insurance. *American Economic Review.* 89, 827-846.

Chateauneuf, A., (1999). Comonotonicity Axioms and Rank-Dependent Expected Utility for Arbitrary Consequences. *Journal of Mathematical Economics.* 32, 21–45.

Chateauneuf, A., Eichberger, J., and Grant, S. (2007). Choice under Uncertainty with the Best and Worst in Mind: NEO-Additive Capacities. *Journal of Economic Theory.* 137, 538–567.

Chateauneuf, A., and Wakker, P. (1999). An Axiomatization of Cumulative Prospect Theory for Decision under Risk. *Journal of Risk Uncertainty.* 18, 137–145.

Chiappori, P., and Salanié, B. (2000). Testing for Asymmetric Information in the Insurance Markets, *Journal of Political Economy.* 108, 55-78.

Chiappori, P., and Salanié, B. (2012). Asymmetric Information in Insurance: Empirical Assessments. Handbook of Insurance, 2nd edition (Dionee, ed.).

Cohen, M. (1992). Security Level, Potential Level, Expected Utility: A Three-Criteria Decision Model under Risk. *Theory and Decision.* 33, 101–134.

Cohen, M. (1995). Risk-aversion Concepts in Expected- and Non-Expected-Utility Models. *Geneva Papers on Risk and Insurance Theory.* 20, 73-91.

Conte, A., Hey, J., and Moffatt, P. (2011). Mixture Models of Choice under Risk. *Journal of Econometrics.* 162, 79-88.

Debreu, G. (1954). Representation of a Preference Ordering by a Numerical Function. In Thrall, R., Coombs, C., and Davis, R. (Eds.) *Decision Processes*, 159–165, Wiley, New York.

Davidoff, T., and Welke, G. (2007). Selection and Moral Hazard in the Reverse Mortgage Market. *Working Paper.*

Diecidue, E., Schmidt, U., and Zank, H. (2009). Parametric Weighting Functions. *Journal of Economic Theory.* 144, 1102–1118.

Dunn, L. (1996). Loss Aversion and Adaptation in the Labor Market: Empirical Indifference Functions and Labor Supply. *Review of Economics and Statistics.* 78, 441-450.

Edwards, W. (1953). Probability-Preferences in Gambling. *American Journal of Psychology.* 66, 349–364.

Edwards, W. (1954). Probability-Preferences Among Bets With Differing Expected Values. *American Journal of Psychology.* 67, 56–67.

Ellsberg, D. (1961). Risk, Ambiguity and the Savage Axioms. *Quarterly Journal of Economics.* 75, 643-669.

Etchart-Vincent, N. (2004). Is Probability Weighting Sensitive to the Magnitude of Consequences? An Experimental Investigation on Losses. *Journal of Risk Uncertainty.* 28, 217-235.

Etner, J., Jeleva, M., and Tallon, J. (2012). Decision Theory under Ambiguity. *Journal of Economic Surveys.* 26, 234-270.

Fang, H., Keane, M., and Silverman, D. (2008). Sources of Advantageous Selection: Evidence from the Medigap Insurance Market. *Journal of Political Economy.* 116, 303-350.

Fehr-Duda, H., and Epper, T. (2012). Probability and Risk: Foundations and Economic Implications of Probability-Dependent Risk Preferences. *Annual Review of Economics.* 4, 567-593.

Finkelstein, A., and McGarry, K. (2006). Multiple Dimensions of Private Information: Evidence from the Long-term Care Insurance Market. *Americal Economic Review.* 96, 938-995.

Finkelstein, A., and Poterba, J. (2004). Adverse Selection in Insurance Markets: Policyholder Evidence from the U.K. Annuity Market. *Journal of Political Economy.* 112, 183-208.

Fluet, C., and Pannequin, F. (1997). Complete Versus Incomplete Insurance Contracts under Adverse Selection with Multiple Risks. *Geneva Papers on Risk and Insurance Theory.* 22, 81-101.

Friedman, M., and Savage, L. (1948). The Utility Analysis of Choices Involving Risk. *Journal of Political Economy.* 56, 279-304.

Genesove, D., and Mayer, C. (2001). Loss Aversion and Seller Behavior: Evidence from the Housing Market. *NBER Working Paper.*

Ghossoub, M. (2013). Loss Aversion for Decision under Risk. *SSRN Working Paper.*

Gilboa, I., and Schmeidler, D. (1989). Maxmin Expected Utility with a Non-Unique Prior. *Journal of Mathematical Economics.* 18, 141-153.

Gonzales, R., and Wu, G. (1999). On the Shape of the Probability Weighting Function. *Cognitive Psychology.* 38, 129-166.

Handel, B. (2010). Adverse Selection and Inertia in Health Insurance Markets: When Nudging Hurts. Forthcoming in *American Economic Review.*

Handel, B., and Kolstad, J. (2013). Health Insurance for Humans: Information Frictions, Plan Choice, and Consumer Welfare. mimeo.

Hogarth, R., and Einhorn, H. (1990). Venture Theory: A Model of Decision Weights. *Management Science.* 36, 780–803.

Hogarth, R., and Kunreuther, H. (1985). Ambiguity and Insurance Decisions. *American Economic Review.* 75, 386-390.

Hogarth, R., and Kunreuther, H. (1989). Risk, Ambiguity, and Insurance. *Journal of Risk and Uncertainty.* 2, 5-35.

Huck, S., and Müller, W. (2012). Allais for all: Revisiting the Paradox in a Large Representative Sample. *Journal of Risk and Uncertainty.* 44, 261-293.

Hurd, M., and McGarry, K. (1997). Medical Insurance in the Use of Health Care Services by the Elderly. *Journal of Health Economics.* 16, 129-154.

Jeleva, M., and Villeneuve, B. (2004). Insurance Contracts with Imprecise Probabilities and Adverse Selection. *Economic Theory.* 23, 777-794.

Kahneman, D., Knetsch, J., and Thaler, R. (1990). Experimental Tests of the Endowment Effect and the Coase Theorem. *Journal of Political Economy.* 98, 1325–1348.

Kahneman, D., and Tversky, A. (1979). Prospect Theory: An Analysis of Decision under Risk. *Econometrica.* 47, 263–291.

Kahneman, D., and Tversky, A. (2000). Choices, Values, and Frames. Cambridge University Press, New York.

Klibanoff, P., Marinacci, M., and Mukerji, S. (2005). A Smooth Model of Decision Making under Ambiguity. *Econometrica.* 73, 1849-1892.

Köbberling, V., and Wakker, P. (2003). Preference Foundations for Nonexpected Utility: A Generalized and Simplified Technique. *Mathematical Operations Research.* 28, 395–423.

Köbberling, V., and Wakker, P. (2004). A Simple Tool for Qualitatively Testing, Quantitatively Measuring, and Normatively Justifying Savage's Subjective Expected Utility. *Journal of Risk and Uncertainty.* 28, 135–145.

Köbberling, V., and Wakker, P. (2005). An Index of Loss Aversion. *Journal of Economic Theory.* 122, 119–131.

Kőszegi, B., and Rabin, M. (2006). A Model of Reference-dependent Preferences. *Quarterly Journal of Economics.* 121, 1133–1165.

Kőszegi, B. (2010). Utility from Anticipation and Personal Equilibrium. *Economic Theory.* 44, 415–444.

Kothiyal, A., Spinu, V., and Wakker, P. (2011). Prospect Theory for Continuous Distributions: A Preference Foundation. *Journal of Risk Uncertainty.* 42, 195–210.

Kunreuther, H., Ginsberg, R., Miller, L., Sagi, P., Slovic, P., Borkan, B., and Katz, N. (1978). Disaster Insurance Protection: Public Policy Lessons. *Wiley* Interscience, New York, NY.

Kunreuther, H., and Hogarth, R. (1989). Risk, Ambiguity and Insurance. *Journal of Risk and Uncertainty.* 2, 5-35.

List, J. (2004). Neoclassical Theory Versus Prospect Theory: Evidence from the Marketplace. *Econometrica.* 72, 615-625.

Liu, J., and Browne, M. (2007). First-best Equilibrium In Insurance Markets with Transaction Costs and Heterogeneity. *Journal of Risk and Insurance.* 74, 739-760.

Loewenstein, G., and Adler, D. (1995). A Bias in the Prediction of Tastes. *Economic Journal.* 105, 929–937.

Machina, M. (1994).

McCarthy, D., and Mitchell, O. (2003). International Adverse Selection in Life Insurance and Annuities, *Working Paper.*

Meier, V. (1999). Why The Young Do Not Buy Long-Term Care Insurance. *Journal of Risk and Uncertainty.* 8, 83-98.

Neilson, W. (2002). Comparative Risk Sensitivity with Reference-Dependent Preferences. *Journal of Risk and Uncertainty.* 24, 131–142.

Netzer, N., and Scheuer, F. (2010). Competitive Screening in Insurance Markets with Endogenous Wealth Heterogeneity. *Economic Theory.* 44, 187-211.

Odean, T. (1998). Are Investors Reluctant to Realize Their Losses? *Journal of Finance.* LIII, 1775-1798.

Payne, J. (2005). It is Whether You Win or Lose: The Importance of the Overall Probabilities of Winning or Losing in Risky Choice. *Journal of Risk Uncertainty.* 30, 5–19.

Peacock, W., Brody, S., and Highfield, W. (2005). Hurricane Risk Perceptions among Florida's Single Family Homeowners. *Landscape and Urban Planning.* 73(2-3), 120-135.

Prelec, D. (1998). The Probability Weighting Function. *Econometrica.* 66, 497-527.

Quiggin, J. (1981). Risk Perception and Risk Aversion among Australian Farmers. *Australian Journal of Agricultural Economics.* 25, 160-169.

Quiggin, J. (1982). A Theory of Anticipated Utility. *Journal of Economic Behavior and Organization.* 3, 323–343.

Quiggin, J. (1993). Generalized Expected Utility Theory: The Rank Dependent Model. Kluwer Academic Publishers.

Rabin, M. (2000). Risk Aversion and Expected-Utility Theory: A Calibration Theorem. *Econometrica.* 68, 1281-1292.

Rabin, M., and Thaler, R. (2001). Anomalies: Risk Aversion. *Journal of Economic Perspectives.* 15, 219-232.

Ryan, M., and Vaithianathan, R. (2003). Adverse Selection and Insurance Contracting: A Rank-Dependent Utility Analysis. *Contributions in Theoretical Economics.* 3, 1534-5971.

Rothschild, M., and Stiglitz, J. (1970). Increasing Risk: A Definition. *Journal of Economic Theory.* 2, 225-243.

Rothschild, M., and Stiglitz, J. (1976). Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information. *Quarterly Journal of Economics.* 90, 629-649.

Samuelson, W., and Zeckhauser, R. (1988). Status Quo Bias in Decision Making. *Journal of Risk and Uncertainty.* 1, 7–59.

Savage, L. (1954). The Foundations of Statistics. *Wiley.*

Segal, U., and Spivak, A. (1990). First-order Versus Second-order Risk Aversion. *Journal of Economic Theory.* 51, 111–125.

Schmeidler, D. (1989). Subjective Probability and Expected Utility without Additivity. *Econometrica.* 57, 571–587.

Schmidt, U., and Zank, H. (2005). What is Loss Aversion? *Journal of Risk and Uncertainty.* 30, 157–167.

Schmidt, U., and Zank, H. (2008). Risk Aversion in Cumulative Prospect Theory. *Management Science.* 54, 208-216.

Schmidt, U., and Zank, H. (2009). A Simple Model of Cumulative Prospect Theory. *Journal of Mathematical Economics.* 45, 308–319.

Schmidt, U., and Zank, H. (2012). A Genuine Foundation for Prospect Theory. *Journal of Risk and Uncertainty.* 45, 97–113.

Shalev, J. (2000). Loss Aversion Equilibrium. *International Journal of Game Theory.* 29, 269–287

Shalev, J. (2002). Loss Aversion and Bargaining. *Theory and Decision.* 52, 201–232.

Slovik, P. (2000). The Perception of Risk. London: Earthscan.

Starmer, C. (2000). Developments in Non-Expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk. *Journal of Economic Literature.* 38, 332–382.

Stiglitz, J. (1977). Monopoly, Non-linear Pricing and Imperfect Information: The insurance market. *Review of Economic Studies.* 44, 407-430.

Sugden, R. (2003). Reference-dependent Subjective Expected Utility. *Journal of Economic Theory.* 111, 172-191.

Tversky, A., and Kahneman, D. (1992). Advances in Prospect Theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty.* 5, 297-323.

Thaler, R., (1980). Toward a Positive Theory of Consumer Choice. *Journal of Economic Behavior and Organization.* 1, 39–60.

Thaler, R., and Johnson, E. (1990). Gambling with the House Money and Trying to Break Even: The Effects of Prior Outcomes on Risky Choice. *Management Science.* 36, 643–660.

van de Kuilen, G., and Wakker, P. (2011). The Midweight Method to Measure Attitudes toward Risk and Ambiguity. *Management Science.* 57, 582–598.

Viscusi, K., and Huber, J. (2012). Reference-Dependent Valuations of Risk: Why Willingness-to-Accept Exceeds Willingness-to-Pay. J. Risk Uncertainty 44, 19–44.

Viscusi, K., Magat, W., and Huber, J. (1987). An Investigation of the Rationality of Consumer Valuations of Multiple Health Risks. *Rand Journal of Economics.* 18, 465–479.

von Neumann, J., and Morgenstern, O. (1944, 1947, 1953). Theory of Games and Economic Behavior. *Princeton University Press*, Princeton NJ.

Wakker, P. (1989). Additive Representations of Preferences, A New Foundation of Decision Analysis. Kluwer Academic Publishers, Dordrecht.

Wakker, P. (1993). Additive Representations on Rank-Ordered Sets II. The Topological Approach. *Journal of Mathematical Economics.* 22, 1–26.

Wakker, P. (1994). Separating Marginal Utility and Probabilistic Risk Aversion. *Theory and Decision.* 36, 1–44.

Wakker, P. (2001). Testing and Characterizing Properties of Nonadditive Measures through Violations of the Sure-thing Principle. *Econometrica.* 69, 1039–1059.

Wakker, P. (2010). Prospect Theory for Risk and Ambiguity. Cambridge University Press, Cambridge, UK.

Wakker, P., and Tversky, A. (1993). An Axiomatization of Cumulative Prospect Theory. *Journal of Risk and Uncertainty.* 7, 147–176.

Wakker, P., and Zank, H. (2002). A Simple Preference-foundation of Cumulative Prospect Theory with Power Utility. *European Economic Review.* 46, 1253–1271.

Wang, K., Huang, R., and Tzeng, L. (2009). Empirical Evidence for Advantageous Selection in the Commercial Fire Insurance Market. *Geneva Risk and Insurance Review.* 34, 1-19.

Webb, C., and Zank, H. (2011). Accounting for Optimism and Pessimism in Expected Utility. *Journal of Mathematical Economics.* 47, 706–717.

Wilcox, N. (2006). Theories of Learning in Games and Heterogeneity Bias. *Econometrica.* 74, 1271-1292.

Wu, G., and Gonzales, R. (1996). Curvature of the Probability Weighting Function. *Management Science.* 42, 1676-1690.

Wu, G., Zhang, J., and Abdellaoui, M. (2005). Testing Prospect Theories using Probability Tradeoff Consistency. *Journal of Risk and Uncertainty.* 30, 107–131.

Yaari, M. (1987). The Dual Theory of Choice under Risk. *Econometrica.* 55, 95-115.

Young, V., and Browne, M. (1997). Explaining Insurance Policy Provisions via Adverse Selection, *Geneva Papers on Risk and Insurance Theory.* 22, 121–134.

Young, V., and Browne, M. (2000). Equilibrium in Competitive Insurance Markets under Adverse Selection and Yaari's Dual Theory of Choice. *Geneva Papers on Risk and Insurance Theory.* 25, 141–157.

Zank, H. (2001). Cumulative Prospect Theory for Parametric and Multiattribute Utilities. *Mathematical Operations Research.* 26, 67–81.

Zank, H. (2010). Consistent Probability Attitudes. *Economic Theory.* 44, 167–185.

Zank, H. (2010). On Probabilities and Loss Aversion. *Theory and Decision.* 68, 243–261.