Vibration-based Damage Detection in Structures

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<th>Description</th>
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<tbody>
<tr>
<td>SHM</td>
<td>Structural Health Monitoring</td>
</tr>
<tr>
<td>NDT</td>
<td>Non-Destructive Testing</td>
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<tr>
<td>AE</td>
<td>Acoustic Emissions</td>
</tr>
<tr>
<td>GA</td>
<td>Genetic Algorithm</td>
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<tr>
<td>NN</td>
<td>Neural Network</td>
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<tr>
<td>PDF</td>
<td>Probability Distribution Function</td>
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<tr>
<td>NPP</td>
<td>Normal Probability Plot</td>
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<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
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<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
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<tr>
<td>FRF</td>
<td>Frequency Response Function</td>
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<tr>
<td>FEA</td>
<td>Finite Element Analysis</td>
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<tr>
<td>ODS</td>
<td>Operational Deflection shape</td>
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<tr>
<td>RODS</td>
<td>Residual Operational Deflection Shape</td>
</tr>
<tr>
<td>ODSC</td>
<td>Operational Deflection Shape Curvature</td>
</tr>
<tr>
<td>DND</td>
<td>Deviation from Normal Distribution</td>
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<tr>
<td>SLDV</td>
<td>Scanning Laser Doppler Vibrometer</td>
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## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>E</td>
<td>Young's Modulus</td>
</tr>
<tr>
<td>I</td>
<td>Second Moment of Area</td>
</tr>
<tr>
<td>$T_e$</td>
<td>Kinetic Energy of Beam Element</td>
</tr>
<tr>
<td>$\rho_e$</td>
<td>Density of Beam Element</td>
</tr>
<tr>
<td>$A_e$</td>
<td>Cross-sectional Area</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Natural Coordinate</td>
</tr>
<tr>
<td>$u_e$</td>
<td>Translational Displacement</td>
</tr>
<tr>
<td>$U_e$</td>
<td>Strain Energy of Beam Element</td>
</tr>
<tr>
<td>$M_e$</td>
<td>Mass Matrix of Beam Element</td>
</tr>
<tr>
<td>$K_e$</td>
<td>Stiffness Matrix of Beam Element</td>
</tr>
<tr>
<td>$K_c$</td>
<td>Stiffness Matrix of Cracked Element</td>
</tr>
<tr>
<td>$Z_r$</td>
<td>Dynamic Stiffness Matrix of Rotational Degrees of Freedom</td>
</tr>
<tr>
<td>$Z_t$</td>
<td>Dynamic Stiffness Matrix of Translational Degrees of Freedom</td>
</tr>
<tr>
<td>$r$</td>
<td>Vibration Responses</td>
</tr>
<tr>
<td>F</td>
<td>External Force</td>
</tr>
<tr>
<td>$F_{ct}$</td>
<td>Crack Forces</td>
</tr>
<tr>
<td>$F_{cr}$</td>
<td>Crack Moments</td>
</tr>
<tr>
<td>$Z_m$</td>
<td>Dynamic Stiffness Matrix of Measured Degrees of Freedom</td>
</tr>
<tr>
<td>$Z_i$</td>
<td>Dynamic Stiffness Matrix of Non-Measured Degrees of Freedom</td>
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List of Publications

Journal of Publications


Conference Publications


Abstract

Structural health monitoring systems have a great potential for cost saving and safety improvement in different types of structures. One of the most important tasks of these systems is to identify damage at an early stage of its development. A variety of methods may be used to identify, locate, or quantify the extent of damage or fault in a structural or mechanical component. However, the preferable method is the one which maximises the probability of detecting the flaw, while also considering feasibility of in-situ testing, ease of use and economic factors.

Cracks are one of the common defects in structural components that may ultimately lead to failure of structures if not detected. The presence of cracks in a structure brings about local variations in the stiffness of the structure. These variations cause the dynamic behaviour of the cracked structure to be different from that of a healthy one. Vibration-based damage detection methods have attracted considerable attention over the past few decades. These methods generally use changes to the physical properties of structures for the purpose of crack detection. In this thesis, two new vibration-based methods have been developed for damage detection in beam-like and rotor-type structures. The first method performs the entire signal processing required for crack detection in time domain. It is based on assessing the normality of vibration responses using the normal probability plot (NPP). The amount of deviation between the actual and normal distribution of measured vibration responses was calculated along the length of the structure to localise the crack.

The second proposed method converts the vibration responses into frequency domain for further processing. Excitation of the cracked structure at a given frequency always generates higher harmonic components of the exciting frequency due to the breathing of the crack. This method uses the operational deflection shape of the structure at the exciting frequency and its higher harmonics to identify the crack location.

Avoiding complicated signal processing in frequency domain is the main advantage of the first method. However, more precise identification of crack locations can be obtained through the second method. Generally, both methods have the advantage of being easy, reference-free and applicable to in-situ testing for any structure. The concept and computational approach of both methods along with their validations through numerical and experimental examples have been presented. Moreover, different input excitations have been used to evaluate the capability of the developed methods in detecting the crack location(s).
Declaration

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Acknowledgement

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I am also very thankful to my dear friends, Benyamin Amirrasouli, Mahdi Zolghadriha, Seyed Behnam Razavi, Amir Abbas Bahador and Mohammad Hassan Mollazadeh who made this journey worthwhile and interesting.

I would like to give special thanks to the School of Mechanical, Aerospace and Civil Engineering, University of Manchester for granting me a full scholarship to pursue my PhD studies.
Dedication

To my lovely mother, supportive father and wonderful siblings
CHAPTER 1: INTRODUCTION

1.1. Overview

Damage is generally defined as changes to the material or geometric properties of structural or mechanical systems which negatively affect current or future performance of those systems [1]. Ageing, excessive loads, severe environmental conditions, and unintentional incidents are the main causes of damage to many structures during their design life. It is an important task to identify the presence of damage at an early stage of its development in order to avoid unexpected failures in structures prior to the end of their design-life. An unexpected failure in a large structure such as a building or a bridge can lead to a disastrous loss of life and have considerable cost consequences. A Norwegian semi-submersible drilling rig (Alexander L. Kielland) capsized in March 1980 (Figure 1.1 (a)), which killed 123 people. The accident happened when one of the bracings failed due to propagation of fatigue cracks caused by severe wave action [2].

Analysis of pre-existing structures is essential for the purpose of damage or fault detection which determines their structural integrity and prevents catastrophic failures. This analysis is also necessary to decide whether a structure can perform its intended function reliably even after its expected design life. This is possible when the frequency of use or loading on the structure is less than originally estimated, or when an excessive level of safety has been built into the design. Therefore, the authorities can avoid the unnecessary process of replacing the structure which would result in high cost. Although a careful visual inspection can provide valuable information regarding the integrity of a structure, access to all critical areas of the structure is not always possible, which allows some problems to go unnoticed until they become serious. The fact that periodic visual inspections are not sufficient can be understood better by considering the tragic Mississippi River bridge collapse (Figure 1.1 (b)) in 2007, killing 13 people and injuring 145, where the bridge had passed a visual inspection not long before failure. As a result, continuous structural health monitoring (SHM) is necessary to determine the
Chapter 1: Introduction

integrity of structures. The SHM systems contribute to the continuing maintenance of structures and show a great potential for cost saving and safety improvement in civil infrastructure and machines [3]. Damage detection, which is one of the most important tasks of SHM systems, intends not only to identify the presence of damage but also to detect its severity at an early stage of its development.

![Disastrous failures](image)

**Figure 1.1** Disastrous failures happened to (a) the Alexander L. Kielland drilling rig in Norway and (b) St.Anthony bridge in Mississippi, USA

Among different methods of damage detection, vibration-based methods have attracted much attention over the past few decades because of their ability to identify and locate damage globally in large, complicated, and inaccessible structures.

1.2. **Aim and Objectives**

The main aim of this research is to identify and locate cracks in structures by developing practical vibration-based damage detection methods. The specific objectives of the thesis are as follows:

1. Development of reference-free damage detection methods.

2. Detection of small cracks which are not sensitive to the change in natural frequency using vibration responses of structures.
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3. Development of damage detection methods in the time and frequency domain.

4. Experimental verification of the methods in objective 3.

5. Application of the developed methods to rotor-type structures.

6. Robustness of the proposed methods to different input excitations.

1.3. Layout of thesis

This thesis has been accepted by the School of Mechanical, Aerospace and Civil Engineering, University of Manchester to be presented in an alternative format. It is divided into 8 chapters of which 5 chapters are presented as already published or submitted papers with an abstract, introduction, materials and methods, results, discussion and references according to the presentation of theses policy. All the references originally given at the end of each paper are collected together and grouped under “References” at the end of the thesis. The following paragraphs give a brief overview of the content of each chapter.

Chapter 1 provides an overview of the importance of damage identification within industry, the major objectives and the thesis structure.

Chapter 2 presents a literature review in which existing damage detection techniques are reviewed. The advantages and limitations of each method are discussed.

- Chapters 3 to 7 consist of five accepted or submitted papers which report the candidate’s own work.

Chapter 3 provides the details of the first damage detection method, which utilises vibration responses in the time domain to locate a single crack in structures.

Chapter 4 provides the development of the second damage detection method to identify single crack location in beams. The methodology to simulate cracked beams and the way of obtaining operational deflection shapes are discussed.
Chapter 1: Introduction

Chapter 5 presents an improvement to the method developed in chapter 4 to extend its application to beams with multiple cracks.

Chapter 6 applies the improved method to a rotor system consisting of a stepped shaft and a number of balance disks.

Chapter 7 presents a comparative study between the two damage detection methods. Beams and rotors with multiple cracks were considered and the effect of different types of excitations on the results was investigated.

Chapter 8 consists of a conclusion on the results obtained through this research and suggestions for areas of future work.
CHAPTER 2: LITERATURE REVIEW

2.1. Introduction

This chapter presents an overview of available condition monitoring techniques which can be applied to different structures and machines. A variety of methods may be used to identify, locate, or quantify the extent of damage or fault in a structural or mechanical component. However, the favourable method is the one which maximises the probability of detecting the flaw, while also considering feasibility of in-situ testing, ease of use and economic factors. In order to employ any condition monitoring technique, a basic knowledge of how the technique works along with its potential and limitations is needed. In general, different methods can be categorised into techniques based on non-destructive testing and vibratory information. The following sections explain these two categories, with more focus on vibration-based techniques.

2.2. Non-destructive testing methods

According to the American Society of Nondestructive Testing (ASNT), non-destructive evaluation is the inspection of an object to ensure its integrity without affecting the object’s future usefulness. Although some Non-destructive Testing (NDT) techniques such as acoustic emission may alter the test specimen permanently, they are still considered NDT since the specimen’s function is unaffected and it can be returned to service.

2.2.1. Eddy current

The eddy current method measures the response of a material to electromagnetic fields over a specific frequency range, normally a few KHz to several MHz [4]. It is widely used in the inspection of piping networks, vessels, and aircraft safety [5].
The method works on the principles of electromagnetics. As current passes through the wire loop of an Eddy current probe, an electro-magnetic field is generated. Then the electro-magnetic field is measured while the probe scans the surface of a specimen. The method is based on the idea that the measured field would change as the probe passes over a defect. From this change in the electromagnetic field, the presence of the defect can be identified [6].

Winchesky et al [7] applied the eddy current technique to Reinforced Carbon-Carbon (RCC) components. They found that the technique is particularly useful for identification of near surface cracking and voids in the RCC matrix.

Underhill et al [8] examined the signal response to corner and mid-bore cracks using eddy current testing. They used standard split-D differential probes and showed that they are sensitive to cracks at the corners of the Ds. The method gives rise to a double maximum of the response for small mid-bore cracks as the corners pass over the crack.

The eddy current inspection technique has many advantages, for example portability of sensor equipment, non-contact evaluation, high temperature applications, sensitivity to small defects, and low cost [4]. However, the technique can only be used to inspect conductive materials and will detect only surface and near surface cracks which are perpendicular to the interrogating surface [9].

### 2.2.2. Radiography

Radiographic techniques, especially X-rays, are one of the most common methods used for nondestructive evaluation [4]. These techniques are able to examine the interior of an object, and they are the only non-destructive evaluation methods which can be applied on all materials [4].

X-rays are electromagnetic waves with a wavelength in the range of 0.01 to 10 nanometres. Because of the inverse relationship between energy and wavelength, X-rays have a very high energy which allows them to pass through a material and see what is happening inside it. This key feature can be used to detect voids, cracks, and any other defects in the material. Parameters such as density and energy of the X-rays, as well as density and thickness of the material, determine the amount of X-rays that can pass through a material. The data given by the X-rays can be in the one dimensional (1-D)
form called a gauge measurement, two dimensional (2-D), referred to as a projection radiograph, or three dimensional (3-D) called computed tomography (CT) [4].

Fiori et al [10] conducted a phase-contrast neutron tomography experiment in order to investigate the ability of this imaging technique to identify cracks in an aluminium alloy which is a poor neutron absorber. They used 2024 Al alloy fatigued specimens and found that the experimental method can identify the presence of the crack as well as estimate some crack features such as crack length and thickness.

Xu et al [11] presented an automatic X-ray system to detect radial and circular cracks in aircraft wing fastener holes. They employed a robot guided X-ray imaging system equipped with a digital detector to capture high resolution reference images of fastener holes in different locations on the aircraft wing. Circular and radial scanning around fastener holes was then performed to find crack locations (Figure 2.1). Ultimately, three crack filters were applied to remove noise in crack images.

![Figure 2.1 2D X-ray scanning of an aircraft wing [11]](image_url)

Albuquerque et al [12] showed a successful evaluation of delamination between inner plies of laminate plates using radiographic images. They employed an artificial neural network to segment the radiographic images and analyse them in an efficient way.

The advantages of the radiographic technique include examining almost all kinds of materials, rapid area inspection, and visual internal inspection. However, being expensive, dangerous, and requiring a highly skilled operator can be considered disadvantages of this method.
2.2.3. Thermography

There are two main approaches available in thermal non-destructive evaluation. The first one, called active thermography, incorporates infrared imaging of a sample with an external heating source. The thermal response of the sample is then used to assess subsurface structure. In the second approach, used in the predictive maintenance community, images of temperature distributions of operating machinery or electrical equipment are captured (Figure 2.2). The locations of hot spots in the images indicate operating problems. Therefore, the difference between these two approaches is that the latter does not involve the use of an external heating source.

The variation in thermal properties between the flaw and the host material is utilised by the active thermography method to detect the flaw. In this technique, infrared cameras are used to take images of the surface temperature distribution of a structure at video rates. The surface temperature variation, which is a function of time, is then interpreted by developing analytical models.

Walle et al [13] investigated the application of active thermography using inductive heating on different steel components from the automotive industry. They were successful in detecting perpendicular and slanted surface cracks with a depth down to 200μm. They also set up a theoretical model for the temperature profile around a crack which resulted from a given induction field and compared this model with the experimental results. The theoretical model predicted a linear dependence of the crack signal on crack depth, which could be verified by the experimental results. However, for crack depths greater than 0.8mm, the model and the experiment showed different behaviour. According to Walle et al, this difference could be due to the discrepancy between the length of cracks in the model and experimental investigations.

Moropoulou et al [14] conducted an investigation into the determination of delaminations in asphalt pavements located at the international airport of Athens, Greece, by means of infrared thermography. They conclude that detection of delaminations is possible in airport pavements using infrared thermography. The advantages of the technique include large area scanning, risk-free devices, and its ability to be performed during both day and night-time hours depending upon the environmental conditions. The main limitation of the technique is its inability to determine the exact dimensions (depth and thickness) of the delaminations.
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Younus et al [15] applied thermography in the area of machine condition monitoring and fault diagnosis (Figure 2.2). They investigated different sorts of machine faults (misalignment, bearing fault, and mass unbalance) by taking into account the thermographic data. Experiments were carried out for the above faults as well as for a normal machine without fault. They found histogram features based on statistical images as appropriate features to be employed for thermal image data. Since the amount of available data was too large to be processed, feature extraction methods were used in order to transform the input data to a reduced representation set of features by using principal component analysis (PCA) and Independent component analysis (ICA). Different types of classifiers were also employed. In the end, they concluded that the combination of ICA and support vector machines (SVM) as the classifier is practical to diagnose the machine condition using thermal imagery.

Figure 2. 2 Temperature distribution of an operating machine [15]

Whilst active thermography provides quick identification of damage in structures, it is restricted to simple planar geometries, and there are difficulties in applying the active thermography to complex geometries [16].

2.2.4. Acoustic emissions

Acoustic Emission (AE) can be described as a kind of phenomena whereby transient elastic waves are produced by rapid release of energy from sources within a material
[17]. In the acoustic emission test, the signals are generated within the material itself. Therefore, an external excitation is needed to induce the source to send out acoustic waves. This is considered as a key feature of acoustic emission technique compared to other NDE techniques. Changes in load, pressure, temperature or strain can be considered as the external excitation. Although the induction usually has permanent and irreversible effects on the structure, the amount of damage associated with a given acoustic emission is very small compared to the final failure of the structure, which allows the method to be considered as a NDE technique. In brief, the external excitation causes a developing defect such as a crack to release a burst of energy in the form of high frequency sound waves which travel within the structure and are received by piezoelectric sensors. The received signals will then be analysed in order to identify the defect.

Holford et al [18] conducted an investigation into the application of acoustic emission for damage detection in bridges. They obtained damage assessment of a steel-concrete composite bridge using time-of-arrival location techniques in both global and local monitoring trials. The local monitoring aspect of the field investigation was simulated under laboratory conditions on a 12m I-beam to explore the use of Lamb waves as an alternative method to the time of arrival method. Also, a finite element study of a component of the bridge was presented and compared with location results from the field study confirming regions of possible crack location. They observed the existence and dispersive behaviour of the extensional and flexural Lamb modes in digitised acoustic emission signals and separated the modes via bandpass frequency filtering. The separation was used to estimate the source to sensor distance. The results showed that the acoustic emission method is able to successfully locate damage in steel bridges and that conventional time of arrival source location techniques are capable of locating damage due to both fatigue cracks in welds and shear stud movement in the steel-concrete interface locally and globally. Laboratory studies offered major insight into the nature of different source events, and finite element analysis proved to be a valuable tool for identifying areas likely to be of concern.

Tian et al [19] explored the likelihood of acoustic emission source localisation in thin plates using near-field acoustic emission beamforming. A series of tests were conducted to validate the effectiveness of this method. Acoustic beamforming is a technique that locates sound sources by analysing the measured signals coming from microphone
arrays [20]. Among all the beamforming techniques, they utilised delay-and-sum beamforming, which is a simple but strong array signal processing algorithm. This algorithm works by applying proper delay to each transducer so that the overall system has a maximum response at a certain direction of arrival [21]. The method was applied on a thin steel plate by a series of pencil lead break tests at various regions of the plate to analyse the accuracy of this method for acoustic emission source localisation. By comparing the positions of the identified sources and those of the real sources, they concluded that the proposed method is able to localise AE sources and it is an appropriate tool for damage detection. The main advantages of the AE beamforming method to conventional methods such as time difference of arrival, is reduction of cost and ease of design implementation [19]. Moreover, the AE beamforming method uses a small number of sensors placed close together in a local region of the plate, while conventional methods utilise a distributed array of sensors.

Hall and Mba [22] presented a diagnosis of continuous rotor-stator rubbing in a 500 MW turbine unit using acoustic emissions. Continuous rubbing between the shafts and surrounding seals of a turbo-generator can cause very severe vibration as well as costly rotor damage. Therefore, early diagnosis of such rotor-stator contact is required to minimise the negative financial effect of any unplanned shutdown. They used a sinusoidal modulation within the raw acoustic emission response to reveal a continuous rub contact at a seal. After that, by synchronous measurement at adjacent bearings, they calculated the estimation of the location of the rub using the phase delay between the adjacent AE modulations. It is clearly observed that the amplitude modulation highlights the fact that continuous rubbing is basically linked to rotor dynamics and the measured AE response at the bearings will consequently change.

Elforjani and Mba [23] assessed the ability of the acoustic emission method to detect and locate natural defects in rolling element bearing and demonstrated its applicability to locate crack initiation and propagation on bearing races whilst in operation. A range of data analysis such as spectrum analysis and information entropy was employed to detect the presence of a crack onset and its propagation by the AE technique. They were also successful in showing the capability of determining the size of natural defects on bearings using AE technology. Figure 2.3 shows typical acoustic emission waveforms at different hours of operations.
Apart from all these advantages, the major drawback of the acoustic emission technique is its poor performance in noisy environments where it is difficult to separate signal from noise [9]. This is due to the fact that the sensitivity of AE sensor networks is such that incidents unrelated to damage are widespread in most applications [24].

![Acoustic emission waveforms at different hours of operation](image)

**Figure 2.3** Acoustic emission waveforms at different hours of operation [23]

### 2.2.5. Ultrasonic waves

Ultrasonic waves are high frequency sound waves vibrating at a frequency above 20,000 Hz. They are inaudible to the human ear since it can only recognise frequencies between 20 Hz and 17 KHz. In NDE applications, ultrasonic frequencies usually range from 50 KHz to as high as several GHz [4]. These waves can be used to characterise a material’s geometry, composition, structure, density, and elastic properties. Moreover, they can be utilised to detect defects in a material. The method is based on the idea that the scattering of ultrasonic waves due to defects can be detected as an echo. The properties of the echo help to determine the location, size, and shape of a defect.
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During an ultrasonic inspection, a transducer, placed onto a specimen, transmits an ultrasonic wave into the test object. The wave travels through the object in order to respond to mechanical properties and geometry of the object. The signal is then transmitted to another transducer (pitch-catch method) or reflected back to the initial transducer (pulse-echo method). An oscilloscope transforms the received signal back to an electrical pulse and enables the signal to be observed. Since the impedance of the received signal is influenced by material characteristics such as voids and cracks, the presence of a defect and its size, shape, and location can be determined.

Gupta et al [9] developed online monitoring of fatigue damage in polycrystalline alloy structures based on statistical pattern analysis of ultrasonic signals. They identified small changes in the statistical pattern of ultrasonic data due to gradual growth of anomalies in material structure. Symbol sequences, generated from ultrasonic sensors on the structure under stress cycles, are used to derive the statistical patterns in terms of the escort distribution from statistical mechanics. The information obtained is able to provide early warning of incoming failure due to widespread crack propagation. They used a computer-controlled fatigue damage testing apparatus in a laboratory to validate their damage monitoring method in real time. The conclusion was that ultrasonic sensing is appropriate for real-time applications and it is easy to install sensing probes on site.

Harri et al [25] proposed an online method to identify surface cracks on a wing structure during its loading. They transmitted a multi-sine ultrasonic wave continuously and measured the variability of the received signal. The opening and the closing of the crack during loading causes the variability of the received signal to grow. The method is useful for applications where the critical length of crack has to be avoided. The sensors were placed at a certain distance in front of the critical crack length for the purpose of monitoring the health of the structure. They noticed that the applied load should not be sinusoidal but should contain both tension and compression stresses so that the crack can open and close. In order to validate their method, a propagation fatigue crack in a sinusoidally loaded wing panel was considered and it was shown that the method is very sensitive to crack propagation. However, the drawback of this method is that the crack has to pass in between the transmitting and receiving transducers.
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The significant advantage of the ultrasonic wave method over the acoustic emission method is its strength in noisy environments, since the externally excited waves are of high frequency and they are not interfered with by small disturbances. However, ultrasonic sensing is usually limited to surface scanning; and more importantly; ultrasonic waves are not able to detect planar cracks whose length lies parallel to the direction of the travelling wave. Also, it is not always suitable to identify damage for on-line SHM of in-service structures using ultrasonic waves [4].

2.3. Vibration-based methods

Vibration-based methods are classified as global damage detection methods which are able to identify and locate damage in large, complicated, and inaccessible structures. In general, different types of vibration-based methods can be categorised into methods which are based on modal parameters, mathematical or finite element models, signal processing, and pattern recognition techniques.

Methods based on modal parameters use the dynamic characteristics of structures such as natural frequencies and mode shapes. These methods utilize difference of structural dynamic characteristics between intact and damaged structures in order to determine the location and severity of damage. Difference in the strain energy of intact and damaged structures is another parameter used for the purpose of damage detection.

Some other vibration-based damage/fault detection methods benefit from using modern signal processing and pattern recognition techniques in order to analyse measured vibration responses of structures or machines. They are called modern type vibration-based methods or sometimes called intelligent damage diagnosis techniques [26]. By employing these modern methods, it is possible to get vibration responses at a few points on a structure or machine which may be sufficient for the purpose of damage or fault detection. Modern methods are less dependent on structural shape and are capable of identifying smaller defects compared to the modal-based ones. Wavelet analysis, neural networks, and genetic algorithms are the most popular tools which are applied to vibration responses.

Model-based damage/fault detection methods try to build mathematical or finite element models of structures and to compare updated models with nominal ones in order to investigate the location and the extent of damage.
2.3.1. Change of natural frequency

One of the vibration-based damage detection techniques is based on the changes in natural frequencies of a structure. The presence of damage in a structure reduces its stiffness and consequently decreases the natural frequencies of the structure. Most of the time, it is very appealing to measure the natural frequencies of a structure because they can be obtained at one point of the structure and are independent of the point selected [27]. Generally, the presence of damage can be concluded if the measured data of natural frequencies are different from those of the undamaged component [26].

Lee et al [28] presented a simple method to identify the location and size of a crack in a cantilever beam using the natural frequency data. They found the first four natural frequencies of the cracked beam and used them along with the Armon’s Rank-ordering method to approximately locate the crack. The result of the crack location range was used to create an appropriate finite element model to determine the crack size. Ultimately, Gudmundson's equation based on the determined crack size and the natural frequencies was used to find the actual crack location. Their results show that the maximum error related to the size of the crack is 25%. Also, there is a 12% error in predicting the location of the crack when it is near the clamped end.

Kim et al [29] presented a methodology to locate and estimate the extent of damage for a structure for which only two natural frequencies were available. They formulated two algorithms; one for damage localisation that locates damage from changes in natural frequencies, and one for damage sizing that estimates crack size from natural frequency perturbation. The approach was applied to 3.6 metre concrete beams and it was observed that the damage could be located with relatively small errors. Location errors for the cracks near the mid-span and the cracks near the left quarter-span fell in 1 cm and 13.7 cm respectively. The size of the cracks located near the mid-span was also predicted accurately, while the accuracy of size prediction decreased for the cracks near the left quarter-span.

Although using the change of natural frequency to detect the presence of damage is convenient, cheap and highly accurate, it cannot always provide enough information for identifying the size and location of the structural damage. This is due to the fact that the structural damage in different positions can cause the same frequency change [26].
2.3.2. Change of stiffness and flexibility

Stiffness is the ability of an elastic body to resist deflection under an applied load while flexibility is defined as the inverse of the stiffness matrix [30]. The method is based on the idea that the existence of damage in a structure alters the structural parameters such as the mass, stiffness, damping, and flexibility matrices.

Lin [31] has shown that higher modal frequencies make a great contribution to the stiffness matrix values. Therefore, in order to obtain a good estimation of the stiffness matrix, all the modes of a structure, especially the high frequency modes, need to be measured. However, difficulties in obtaining higher frequency data due to the limitations of the experimental instrumentation can be considered as the major drawback in applying the stiffness matrix technique to detect damage.

In contrast to the stiffness matrix, the flexibility matrix has an inverse relationship to the square of the modal frequencies. Therefore, the effect of high-order modes in the flexibility matrix will greatly decrease with the increase of natural frequency [26]. This feature enables the flexibility matrix to be calculated accurately enough by determining only low-order modes with low frequencies. Damage can be identified by comparing the flexibility matrices obtained using the low-frequency modes of the damaged and undamaged structures. It is also possible to use finite element analysis to obtain the flexibility matrix of the undamaged structure [32].

Pandey et al [30] presented a successful investigation into damage detection of a cantilever, simply supported, and free-free beams using changes in flexibility matrix with the assumption that damage in a structure will not affect the inertia matrix. They also illustrated how different boundary conditions can affect the flexibility. This method works more effectively when damage is located at a section where the maximum bending moments happen [30]. However if the damage is very small, the stiffness and flexibility method cannot work properly.

Yan et al [33] applied the covariance-driven subspace technique to an aircraft model made of steel beams to identify modal parameters, from which the measured flexibility matrix was constructed. The stiffness matrix was then obtained by a pseudo-inversion of the flexibility matrix. They used the difference in the diagonal entries of the stiffness matrices between the healthy and damaged states to locate damage. The disadvantages
of this approach include the necessity of having a sufficient number of well distributed sensors and its usefulness in the case of small amounts of damage in the structure.

### 2.3.3. Power flow

The power flow in a damaged structure is influenced by changes to the characteristics of the propagating wave due to the presence of damage. In other words, small local changes in the mass, stiffness and damping properties affect the energy flow pattern of the structure. Li et al. [34] investigated the use of vibrational power flow for damaged beam structures. The crack was modelled as a joint of a local spring and the damage point and beam element transfer matrices were obtained. They used the periodic structure theory to find the relationships between the power flow and the location and size of the damage. Lee et al. [35] examined the overall behaviour of power flow patterns of a plate for the purpose of crack detection. The vibrational power flow per unit area of a dynamically loaded plate which is called structural intensity (SI) was obtained. They observed that the SI vectors had been changed near the crack location as shown in Figure 2.4. The SI method was also employed to find out the position of dampers in vibrating plates to distract the vibration energy flow away from the crack tip as a provisional measure to prevent the crack from propagating.

The main limitation of the power flow method is its dependency to the orientation of damage. This means that the detection is not possible when the damage length in is parallel to the energy flow [35].

![Figure 2.4](image_url)
2.3.4. Strain energy

Another way to detect the presence of damage in a structure is to use the change in strain energy of damaged and intact structure subjected to vibration. The strain energy of an Euler-Bernoulli beam associated with a particular mode shape is given by:

\[ U_i = \frac{1}{2} \int_0^l EI \left( \frac{\partial^2 \psi_i}{\partial x^2} \right)^2 dx \]  

(2.1)

where \( EI \) is the flexural rigidity of the beam and \( \psi_i(x) \) is the mode shape. If the beam is subdivided into \( N \) divisions, then the energy associated with each sub-region \( j \) due to the \( i \)th mode is given by:

\[ U_{ij} = \frac{1}{2} \int_{a_j}^{a_{j+1}} (EI)_j \left( \frac{\partial^2 \psi_i}{\partial x^2} \right)^2 dx \]  

(2.2)

It is possible to make a comparison between sections of damaged and undamaged beam by calculating the change in strain energy in each subdivision. Any variation will be the consequence of a change in the flexural rigidity of the beam which is due to a localised reduction in its second moment of area at the crack [36].

Cornwell et al [36] investigated the suitability of using a method based on the changes in strain energy to detect and locate damage in structures. They generalised the method to plate-like structures which are characterised by two-dimensional curvature. Mode shapes of the intact and damaged structures are required for this method and they do not need to be mass normalised. The method was able to measure a reduction in structural stiffness as low as 10%. However, reduction in the sampling frequency resulted in the incorrect identification of damage at node lines. This can be considered as the major drawback of the method.

Hu et al [37] developed a damage index by using modal analysis and strain energy methods to detect a surface crack in composite laminates. First they obtained the mode shapes from both experimental and finite element analysis results. After that, the differential quadrature method (DQM) was used to calculate strain energy from the mode shapes. They used the strain energies of the healthy and damaged laminated plates to define the damage index. From the experimental results, they showed successful
identification of surface crack locations in different composite laminates by the damage indices. A reliable finite element model was also validated.

2.3.5. Curvature mode shape

As already mentioned, the existence of any crack or localized damage in a structure reduces the stiffness and the second moment of area of the structure. The consequence of this is the reduction in the natural frequencies and the flexural rigidity of the structure. The second derivative of a beam's mode shape is defined as mode shape curvature [32]. It is directly related to the flexural rigidity of the structure, which means a reduction in the flexural rigidity creates a reduction in the curvature mode shape. Therefore, the difference in the curvature mode shapes between the intact and damaged structures can be utilized for the purpose of damage detection.

The two main approaches used to compare mode shapes are the Modal Assurance Criterion (MAC) [38] and the Coordinate Modal Assurance Criterion (COMAC) [39]. MAC specifies the correlation between two sets of mode shapes to study their overall differences, while COMAC specifies the correlation between the mode shapes at selected measurement points of a structure.

Pandey et al [40] investigated the use of curvature mode shape as a possible candidate for identifying and locating damage in a structure. They analysed a cantilever and a simply supported beam and used finite element analysis to obtain the displacement mode shapes of the two models. Mode shapes were compared using MAC and COMAC methods. Their work demonstrated that changes in the curvature mode shapes were localized in the region of damage, while changes in the displacement mode shapes were not localized and hence could not give any information of the location of damage. Finally, the usefulness of the curvature mode shapes in detecting and locating the state of damage was demonstrated.

Qiao et al [41] employed two measurement systems for extraction of modal parameters of a composite laminated plate with embedded delamination. One of the measurement systems used PZT (lead-zirconate-titanate) actuators and PVDF (polyvinylidene fluoride) sensors, while the other system used PZT actuators and scanning laser vibrometer (SLV). A finite element analysis was also presented to validate the damage detection algorithms; uniform load surface (ULS), gapped smoothing method (GSM), strain
energy method (SEM), and generalised fractal dimension (GFD). Both measurement systems were shown to be successful in extracting the modal data by the experimental approach. However, the second system was found to be better in the finite element approach. In general, it was concluded that the GSM algorithm could detect and isolate the delamination of composite plate better than the other algorithms. Figure 2.4 compares longitudinal curvature mode shapes for healthy and damaged composite plates.

![Figure 2.5](image)

**Figure 2.5** Longitudinal curvature mode shapes for healthy (left) and damaged (right) plates [41]

### 2.3.6. Frequency response function

Frequency response function (FRF) is the characteristic of a system that expresses its response to excitation as a function of frequency. It is defined as the ratio of the complex spectrum of response to the complex spectrum of excitation. The FRF consists of two plots, the amplitude and the phase-lag. At resonant frequencies, the FRF indicates sharp peaks and the phase-lag of 90 degree in the amplitude and phase-lag plots respectively.

Huang et al [42] proposed new damage detection methods based on using semi-active friction dampers and frequency response function for controlled building structures. They called the new methods the full-excitation FRF based and the single-excitation FRF based methods. A five storey shear building was simulated for numerical investigation (Figure 2.5). The numerical results indicated that the maximum value of errors in the detection of the location and the extent of damage using the full-excitation FRF based and the single-excitation FRF based methods was less than 1 and 2 percent respectively. According to the paper, the FRF-based methods were more accurate in
determining the stiffness of the building than the natural frequency and mode shape-based methods. Also, the FRF-based methods were much less sensitive to measurement noise than the natural frequency and mode shape-based methods.

The fact that the methods using FRF do not require vibration mode measurement, mathematical model, and experimental knowledge can be considered as the advantage of the methods. However, it is shown that the amount and position of measurement points have an influence on the accuracy of damage detection [26].

![Figure 2.6](image.png)

**Figure 2.6** The FRF curves of the top storey of a building with and without added stiffness [42]

### 2.3.7. Operational deflection shape

According to Richardson [43], operational deflection shape (ODS) can be described as any forced motion of two or more points on a structure. As a result of this motion, a shape is created by considering the motion of one point relative to all others. ODSs are a useful means to show how much a structure is really moving at a specific time or frequency.

Waldron et al [44] presented a successful method of damage detection using a scanning laser Doppler vibrometer (SLDV) and ODS. In their paper, the ODSs are represented as summations of scaled mode shapes. They obtained the ODSs both numerically and experimentally using finite element analysis and the SLDV respectively. Using finite
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element simulations, they investigated the effects of boundary conditions, position and type of load, mixed mode, and higher levels of damage on detection of damage using ODS, and indicated that the ODS method is sensitive enough to detect at least 10% damage. Two aluminium beams were also tested in the intact and damaged cases using SLDV. They did the scanning on the front edge of the beams while the crack was on the interior side which was on the opposite side to the scanned edge. The test demonstrated differences in deflection amplitude between the healthy and damaged ODS at the natural frequencies of the beams.

Figure 2.7 ODS for healthy (left) and damaged (right) fixed-fixed beams encompassing the first mode [44]

Pai and Young [45] studied a boundary effect detection (BED) method to localise small amounts of damage in beams using ODS measured by a SLVD. The advantage of the BED method is that it does not need any historical data to locate structural damage. The method is based on using a sliding-window least-squares curve-fitting technique to decompose a measured ODS into central and boundary-layer solutions. Boundary-layer solutions are sensitive to damage and provide several indicators, which can be checked against each other to guarantee the identified damage locations. For example, at a damage location, the boundary-layer solution of slope changed and the boundary-layer solution of displacement showed either a positive or negative peak. Several experiments were carried out on various beams with different types of damage in the form of surface slots, edge slots, surface holes, internal holes, and fatigue cracks. Both numerical and experimental results show the sensitivity and effectiveness of the BED method to localise small damages. However, more studies are required to perform this method for on-site structural damage detection[45].

2.3.8. Detection method using wavelet transform

Wavelet analysis is a signal processing technique that is highly sensitive to singularities in a signal. Wavelets are actually waveforms of limited duration which have an average value of zero (Figure 2.8). The wavelet transform decomposes a signal into a scaled or shifted version of the mother wavelet (or the wavelet prototype function), and unlike other signal analysis techniques, wavelet analysis is capable of zooming-in on a signal to reveal some of its hidden features such as discontinuities [46]. The method is based on the idea that a local defect such as a crack reduces the local stiffness and causes local discontinuities in the derivatives of data. When wavelet transform is applied on such a signal, those discontinuities that correspond to the presence of damage appear in the transform coefficients in the vicinity of the defect. As a result, the wavelet transform is quite sensitive to small changes in vibration characteristics created by small defects which are impossible for many other methods to identify [47]. A typical waveform of a wavelet is demonstrated in Figure 2.7.

![Wavelet (db10)](image)

**Figure 2.8** Waveform of a wavelet [44]

Loutridis et al [48] presented a method to identify a crack in a doubled-cracked cantilever beam using wavelet analysis. They used continuous wavelet transform to analyse the vibration mode of the beam. Sudden changes in the spatial variation of the transformed response was utilized to locate the cracks while the relative depth of the cracks was estimated by establishing an intensity factor which relates the coefficients of the wavelet transform to the size of the cracks. The method was successful in locating the cracks accurately. However, the estimation of the crack depths provided difficulties when the values of the intensity factor were high.

Li et al [47] applied a continuous wavelet transform to analyse the flexural wave in a cracked cantilever beam subjected to an impact load at its free end. They acquired the
flexural waves by both finite element and experiments. The method is based on extracting the reflected and transmitted flexural waves by the continuous wavelet transform to detect damage. Both finite element and experimental results showed the sufficient accuracy of the method for moderate cracks and high sensitivity for small cracks. Moreover, they found that as the crack depth increases, the estimated error of the crack location decreases.

Gokdag [49] developed a damage detection method based on the combination of orthogonal and continuous wavelet transforms (OWT & CWT) for beam-like structures. His method proposed to consider only the damaged mode of a structure to extract the approximation function. CWT can then be computed for both the approximation function and the damaged mode. The difference between these two results is an index to detect damage. Gokdag showed the effectiveness of his method for a pinned-pinned beam, while poor results were obtained for a fixed end beam.

Zhong et al [50] successfully investigated crack detection in simply supported beams using stationary wavelet transform (SWT). The key feature of SWT is its ability to facilitate the identification of hidden aspects in a signal, especially for recognising noise or signal rupture. The method is based on the idea that the mode shape of the cracked beam demonstrates a local discontinuity in the vicinity of damage, which can be approximately considered as the mode shape of the intact beam contaminated by noise; and the response of the crack is hidden in the noise. SWT can decompose the cracked beam modal data into a smooth curve, called the approximation coefficient, and a detail coefficient. The difference of these two coefficients can be used for the purpose of damage detection. As a result, the proposed method does not need the vibration characteristics of an intact beam as a baseline. Zhong et al showed that the method is successful in identifying crack depths as small as 4% of the depth of the beam.

Al-Badour et al [51] studied the application of wavelets to fault detection of rotating machinery. They used the continuous wavelet and wavelet packet transforms and a suitable choice of a mother wavelet function for the analysis of vibration signals. The vibration signals were obtained in two cases: stator-to-blade rubbing, and fast start-up and coast-down of a rotor. A rotor kit was built and used on a laboratory scale to obtain the signals experimentally. It was concluded that in the case of stator-to-blade rubbing, where non stationary signals were generated, employing the combination of continuous
wavelet and wavelet packet transforms gave a suitable method for the fault detection purpose. Moreover, they found that windowing the signals into a number of shaft revolutions and analysing each window separately to localize the defect was completely effective in tests and could efficiently be used in the analysis of blade vibration problems or in detection of cracked gear teeth.

Although damage detection using the wavelet transform seems to be practical, it requires to be tested for further applications. Moreover, wavelet transform cannot provide accurate phase information requiring for non-stationary signal behaviour.

2.3.9. Detection method using neural networks

Neural Networks (NN) have been widely used in structural analysis due to their great ability in non-linear mapping. They usually consist of three layers, an input layer, a hidden layer, and an output layer (Figure 2.9). The steps needed to follow in order to detect damage in a structure using NN include determining the network structure, selecting the network parameters, normalising the learning samples, giving initial weight value and detecting structural damage.

The constructed NN will be trained according to its input and the corresponding output of structural damage called train samples. Train samples which represent damage information in a structure can be determined by experiments or numerical simulations. When a NN has been trained, experimental measurements of real structure can be put into it, and the NN output will be able to determine the location and severity of the damage. Artificial NNs are the modern usage of the term NN which are mainly used to match damage patterns in order to detect the location and the severity of damage [26].

Kao et al [52] presented an artificial neural network-based approach for detecting structural damage. First, they used neural system identification networks (NSINs) to identify the damaged and undamaged states of a structural system. They then utilized the abovementioned NSINs to produce free vibration responses. The method is based on the fact that changes in structural properties such as stiffness and damping cause changes in periods and amplitudes of the free vibration response of a structural system. Therefore, Kao et al compared the periods and amplitudes of the free vibration responses for the damaged and undamaged structures to detect damage. The results of
experimental and numerical investigation verified that the method could be applied successfully to detect damage in the structure [52].

Elshafey et al [53] investigated the application of combined neural network and random decrement signature to detect damage in offshore jacket platforms subjected to random loads. First of all, they used the random decrement technique to obtain the free decay of the structure from its online response while it was in service. After that, the free decay and its time derivative were used as an input for a neural network. The output of the neural network was employed as an index for damage detection. Experimental studies were conducted on a reduced model for a real jacket structure with the scale of 1:30. The results indicate that the method was more successful in predicting damage occurrence in diagonal members and was less sensitive to damage occurring in horizontal members. As the authors explained, this was due to the higher sensitivity of the natural frequency of the structure to changes in the stiffness of the diagonal members than to the changes in the stiffness of the horizontal ones.

Hoffman and Merwe [54] applied neural networks to vibrational diagnostics for multiple fault conditions. They demonstrated that it is impossible to determine the
degree of imbalance due to the presence of a bearing defect based on a single vibration feature such as the peak at rotational frequency. Therefore, it is necessary to use diagnostic techniques that are appropriate for the parallel processing of multiple vibration features. Neural networks were said to be the best known technique to approach such a problem. They evaluated three different neural classification techniques, self-organising maps (SOM), nearest neighbour rule (NNR), and radial basis function (RBF) for their performance on the identification of a multiple fault mechanism. It was shown that a neural classifier using the X and Y components of both the peak at rotational frequency and the peak at bipolar frequency as input features can identify the presence of a bearing defect and can indicate the degree of imbalance at the same time.

Neural networks are capable of identifying damage in a structural system accurately. However, their accuracy depends on how well they have been trained. In order to supply good training, a variety of detailed information of the vibration characteristics of a structure is needed, which is obviously time-consuming to obtain [26].

### 2.3.10. Detection method using genetic algorithm

In most vibration-based methods, the vibration characteristics of the damaged and intact structures are used to determine the location of damage. These methods can be reduced to the solution of constrained optimisation problems [55]. However, it is difficult to find the optimum solution using conventional optimisation algorithms for problems in which the objective function depends on many discrete and continuous variables, for example, structural damage detection problems. Genetic algorithms (GAs) are powerful searching methods that can solve complex optimisation problems and have been used in the last two decades to solve difficult optimisation problems [56].

In brief, the GA mimics the evolutionary process as a problem-solving strategy. For a particular problem, the input to the GA is a set of probable solutions to that problem. A metric called fitness function evaluates each random candidate quantitatively. Then, capable candidates are kept and allowed to regenerate and the process repeats.

Panigrahi et al. [57] developed a vibration-based damage detection method for a uniform strength beam using a genetic algorithm. They formulated an objective function for the steady-state genetic search optimisation procedure and presented the residual force
method for the identification of damage in a uniform strength beam. Two cases were investigated; in the first case the width was varied while the strength of the beam was kept uniform throughout and in the second case both width and depth were varied. The developed model needed experimentally determined data as input. The experimental data were simulated numerically by using finite element models of structures while considering random noise on the vibration characteristics. Excellent agreement of the damage factors identified for the beam problems by using GA was demonstrated with that of the chosen values of the parameters for simulated experimental data of damaged structures.

Baghmisheh et al [55] proposed a fault diagnosis method based on genetic algorithms for a cracked beam structure. They obtained the natural frequencies of the beam numerically and utilised genetic algorithms to monitor the changes in the natural frequencies of the structure. After formulating the identification of the crack location and depth as an optimisation problem, binary and continuous genetic algorithms (BGA, CGA) were employed to determine the optimal location and depth by minimising the difference of measured and calculated natural frequencies. An experimental investigation was also carried out in order to validate the proposed method. The average numerical errors for the prediction of location and depth of damage were found to be less than 2% for both BGA and CGA. However, the average experimental errors for both BGA and CGA were in the range of 10 to 11 percent.

The main disadvantage of the GA is that the response time difference between the shortest and longest optimisation is often very large. Moreover, it is not practical to use GA for online health monitoring without applying it first to a simulated model due to its random solutions and convergence.

2.3.11. **Model-based Damage Detection Method**

The term model-based identification refers to the use of mathematical or finite element models in order to identify the location or size of damage in structures and machines. Although all identification methods seem to have some models supporting them, there is a great difference between methods based on statistical techniques and those based on physics. For example, neural networks, genetic algorithms, or purely empirical techniques employ entirely statistical models for damage identification and do not make any attempt to model the underlying physics; therefore, they are practical for
interpolation but not for extrapolation [58]. In brief, model-based identification methods utilize either a full or partial mathematical model of a structure or machine along with the measured vibration responses and at the end estimate damage/fault-related parameters by optimisation.

Fritzen and Bohle [59] presented a model-based identification approach to detect cracks in a two storey building under seismic load by means of a computational model and measured vibration responses. They formulated an inverse sensitivity problem which led to a large number of damage parameters. Since very few parameters concerned the damaged areas, it was essential to select a parameter subset. The method was successful in finding the location of three real cracks. However, another crack location was identified while there was no crack visible. The authors explained that the misidentification may have been due to the noise in measurement signals, modelling errors, or the geometry changes because of plastic deformation. Simulation studies showed that using more sensor locations resulted in better identification results. The optimal number of sensor locations could be obtained by maximising the determinant of the Fisher information matrix.

Abu Husain et al [60] proposed a finite element model updating damage detection method to investigate a welded structure intended to represent a configuration used in automotive body construction. They first developed the finite element model of a benchmark structure and updated to its experimental data. The obtained parameters were then used in modelling the damaged structure. The process of updating the initial finite element model for the damaged structure was done by choosing three updating parameters of weld diameter, Young’s modulus of the weld, and Young’s modulus of the patch to avoid an ill-conditioning problem. The process was followed by solving a structural optimisation problem in NASTRAN for predicting the size of damage. First, five measured natural frequencies were used to evaluate the initial model. It was demonstrated that the initial FE model for the damaged structure showed excellent correlation with the experimental data. This was due to the fact that most of the uncertainties were successfully identified while updating the benchmark model. Moreover, the natural frequencies for the updated damaged model corresponded very well with the experimental data.
Sinha et al [61] presented a model-based identification approach to estimate both rotor unbalance and misalignment of a rotor-bearing-foundation system. They used pre-created models for rotor and bearing, as well as measured vibration data at the bearing pedestals from a single run-down. The frequency-band-dependant foundation parameters were estimated first, to account for the dynamics of the foundation. Different regularization methods were used to solve the least square problem. They applied their method to a small experimental rig and showed that the estimated results were excellent. Some errors were introduced into the parameters of the rotor and fluid bearing models as a sensitivity analysis in order to see how robust the proposed method was. They observed that the maximum errors in estimated unbalance amplitude and in the fluid bearing models were less than 45% and 10% respectively for the random error of 5% in the rotor model.

The limitation of model-based damage detection method is its accuracy which highly depends on appropriate selection of updating parameters representing the underlying physics of the structure.

2.4. Summary

In general, different damage detection techniques are based on either NDT methods or vibratory information. The NDT methods are mainly useful for detecting local flaws. Since the existence of flaws is an unknown, the decision of where to inspect in a structure or a machine is always a difficult question. In contrast, the vibration-based methods are capable of identifying and locating flaws in large and complicated structures which are the focus of this thesis.

The disadvantage of the method based on changes in natural frequency is its impassibility to small defects such as cracks. The mode shape curvature method requires very smooth mode shapes which are difficult to obtain experimentally due to the environmental noise. The accuracy of the methods based on pattern recognition such as neural networks and genetic algorithms depends on the theoretical models and how precisely these models are trained. The detection method using the wavelet transform demonstrates to be practical but needs to be tested for further applications. As a result, there is always a need for simple but robust methods for the purpose of damage detection in structural components or machines.
CHAPTER 3: DEVELOPMENT OF A TIME-DOMAIN METHOD

Reformatted version of the following paper:


Abstract

Cracks are one of the common defects in structural components that may ultimately lead to failure of structures if not detected. Generally, most of the vibration based crack detection methods transform measured vibration responses from time-domain into frequency-domain using Fourier or wavelet transform for damage detection. However, it would be more convenient if the vibration responses could be analysed in their original time-domain. Therefore, a practical method based on probability distribution function is proposed which performs all the data processing in time-domain for the purpose of crack detection in beam-like structures. The application of the proposed method to both numerical and experimental examples and their results are presented.

3.1. Introduction

Structural health monitoring focuses on developing methods to identify damage in structures such as aircraft, bridges, ships and buildings. It is an important area of research which has great potential for cost saving and safety improvement in different types of structures. Cracks are one of the common defects in structural components that may ultimately lead to failure of structures if not detected. The presence of cracks in a
structure brings about local variations in the stiffness of the structure. The extent of such variations mainly depends on the depth and location of the cracks [62] which affects the dynamic behaviour of the whole cracked structure. Vibration-based damage detection methods have attracted considerable attention over the past few decades. They generally use changes to the physical properties of structures for the purpose of crack detection. Doebling et al. [63], Yan et al. [64], Fan and Qiao [65] and Wang et al. [66] have presented comprehensive damage detection methods based on vibration features for different types of structures.

In vibration tests, external excitation and vibration responses of structures are measured in the form of time history. However, it has usually been difficult to use time domain data for identifying or locating damage in structures. That is why time domain data are generally transformed into the frequency domain for further processing. Nevertheless, there are a few studies which used time domain data for the purpose of damage or fault detection. Cattarius and Inman [67] utilised time histories of vibration response of structures to identify the presence of damage. They first obtained the healthy signal of a structure and the measured time response for an identical structure with some local damage. These two signals were then compared by subtracting them from one another. The resulting signal provided an indication of the existence and extent of damage reflected in local mass or stiffness changes. The limitation of this method is its requirement for the data of a healthy structure. Majumder and Manohar [68] developed a time domain approach to detect damage in bridge structures using vibration data generated by a moving vehicle. It was assumed that a validated finite element model for the undamaged bridge was available. Then a time domain approach was used to determine required alternations to be made to the initial finite element model to reflect the changes in bridge behaviour due to presence of damage. This approach led to a set of overdetermined linear algebraic equations for damage indicator variables which were solved using pseudo-inverse theory. This method is limited to availability of a validated finite element model of the healthy bridge which is very subjective. In addition, the approach was not verified experimentally. Chandrashekhar and Ganguli [69] developed a fuzzy logic system with a sliding window for damage detection in structures. They used the changes in the mode shape curvature vectors of a numerically simulated cantilever beam to calculate the curvature damage factor vector (CDF) as a damage indicator. Another methodology was presented by Choi and Stubbs [70] to locate
damage in structures using the time-domain response. They expanded the time-domain response over the structure and obtained the mean strain energy for each element of the structure. Then the damage index that represents the ratio of the stiffness parameter of the pre-damaged to the post-damaged was used to identify possible locations. Roy and Ganguli [71] used a finite element model of the helicopter rotor blade to analyse the effect of damage growth on the modal frequencies. They processed the signals using weighted recursive median (WRM) and radial basis function (RBF) filters and concluded that the change in the frequencies is sufficient to identify the presence of damage in the blade

In this paper, a method based on probability distribution function is presented in order to analyse vibration responses of structures in time domain for crack detection. Rzeszucinski et al. [72] suggested a new condition indicator based on the deviation in the normal probability density function (PDF) of measured vibration data for gearbox diagnosis. Umesh and Ganguli [71] have performed probabilistic analysis to study the effect of uncertainty on the structural health monitoring of a smart composite plate with matrix cracks using control gain shifts as damage indicators. They used the probability distribution function (PDF) and normal probability plot (NPP) to show the difference between the dispersions of damaged and undamaged cases. NPP is also employed by Rzeszucinski et al. [73] to develop a new technique for condition monitoring of helicopter gearboxes. The technique used the amount of deviation from normal distribution of a given signal in the NPP and therefore was called deviation from normal distribution (DND). The results of the proposed DND correctly indicated a gear fault for the duration of the deterioration process.

This paper aims to investigate the application of the DND method to beam-like structures for the purpose of damage detection. It is assumed the vibration responses are available at a number of locations along the beams. Off-line testing can also be used for inaccessible structure(s). For example, a crack in the rotor of rotating machines can be identified by off-line measurements after removing the casing. The advantage of the DND method is that it enables us to perform all the required processing in the time domain; hence complicated signal processing techniques in frequency domain can be avoided. Initially, the DND method is applied to simulated beams with breathing cracks; thus, finite element modelling and crack breathing is discussed in section 2. The
method is then validated through experimental examples. This paper presents the details of the DND method as well as numerical and experimental results.

3.2. Finite Element Modelling

The Euler-Bernoulli beam theory is used to construct the stiffness and mass matrices of healthy and cracked beams (Figure 3.1). This theory assumes that the cross-section of the beam is infinitely rigid and normal to the deformed axis of the beam. Also, the cross-section remains plane after deformation. Other beam theories such as Timoshenko beam model can also be used. In fact, the beam theory is employed here only for the finite element model development; however, real experiments without any assumptions are carried out for verification of the proposed method. Other assumptions made in the finite element models include crack modelling and crack breathing which are discussed in this section. It should be noted that this thesis focuses on using low frequency excitations for the purpose of surface crack detection in structures. Both the numerical and experimental beam examples are excited at their first or second resonant frequencies. In some cases, off-resonance excitations at frequencies between the first two natural frequencies are also investigated.

Mass- and stiffness-proportional damping is used to obtain the damping matrix of the beams. Each element has two nodes and there are two degrees of freedom (DOF) per node, the translational displacement and bending rotation (Figure 3.2). Let N be the number of nodes; thus, 2N DOFs are considered for the beam.

![Figure 3.1 A typical cracked beam](image-url)
The next step is to use translational displacement of the beam plane in order to approximate the strain and kinetic energy within the beam element. It is assumed that the cross-section of the beam remains planar and perpendicular to the centreline. Also, it is assumed that the material is linear elastic obeying the Hook’s law. The mass and stiffness matrices of each element can be obtained by calculating the kinetic and strain energy respectively. The kinetic energy of the beam element without considering the rotational effect is:

\[ T_e = \frac{1}{2} \int_0^{l_e} \rho_e A_e(\xi) \dot{\xi}^2(\xi, t) \, d\xi \quad (3.1) \]

where \( \rho_e \) is the density, \( A_e \) is the cross-sectional area of the beam and \( \xi \) is the natural coordinate. By using shape functions, Equation (3.1) can be written as:

\[ T_e = \begin{pmatrix} \dot{u}_{e1}(t) \\ \dot{u}_{e2}(t) \\ \dot{\theta}_{e1}(t) \\ \dot{\theta}_{e2}(t) \end{pmatrix}^T \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \begin{pmatrix} \dot{u}_{e1}(t) \\ \dot{u}_{e2}(t) \\ \dot{\theta}_{e1}(t) \\ \dot{\theta}_{e2}(t) \end{pmatrix} \quad (3.2) \]

where the elements of mass matrix are:

\[ M_e = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \]

\[ = \frac{\rho_e A_e l_e}{420} \begin{bmatrix} 156 & 22l_e & 54 & -13l_e \\ 22l_e & 4l_e^2 & 13l_e & -3l_e^2 \\ 54 & 13l_e & 156 & -22l_e \\ -13l_e & -3l_e^2 & -22l_e & 4l_e^2 \end{bmatrix} \quad (3.3) \]

The strain energy of the beam element is:
Chapter 3: Development of a time-domain method

\[ U_e = \frac{1}{2} E_e l_e \int_0^{l_e} \left( \frac{\partial^2 u_e(\xi, t)}{\partial \xi^2} \right)^2 d\xi \]  

(3.4)

Where \( E \) is the Young’s modulus and \( I \) is the second moment of area of the cross-section. Again by using shape functions, Equation (3.4) can be written as:

\[ U_e = \frac{1}{2} \begin{pmatrix} u_{e1}(t) \\ \theta_{e1}(t) \\ u_{e2}(t) \\ \theta_{e2}(t) \end{pmatrix}^T \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{pmatrix} u_{e1}(t) \\ \theta_{e1}(t) \\ u_{e2}(t) \\ \theta_{e2}(t) \end{pmatrix} \]  

(3.5)

where the element of stiffness matrix are:

\[ k_e = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} = \frac{E_e l_e}{I_e^3} \begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ 6l_e & 4l_e^2 & -6l_e & 2l_e^2 \\ -12 & -6l_e & 12 & -6l_e \\ 6l_e & 2l_e^2 & -6l_e & 4l_e^2 \end{bmatrix} \]  

(3.6)

Finally, the finite element model of the beam is constructed by allocating the calculated mass and stiffness matrices to each element. A mathematical model is also developed using the above concepts for the purpose of crack detection which is presented in Appendices A and B.

A number of approaches are available to model cracks in beam-like structures. For example, one approach is to reduce the stiffness of whole element where a crack has occurred [74-75]. Another approach divides a cracked beam into two parts at the crack location and connects them together using a pin joint. The crack is then simulated by adding a rotational spring [76-77]. These approaches simplify the dynamics of cracks to a great extent and therefore are not very accurate. On the other hand, there are approaches that use two or three-dimensional meshes for cracked beam-like structures [78-79]. Although these methods are very accurate, they are complicated and time-consuming for modelling simple structures like beams.

Here, the approach proposed by Sinha et al. [80] is used to model the crack. This approach modifies the local flexibility in the vicinity of a crack within Euler-Bernoulli
beam elements, and is based on the concept of Christides and Barr [81]. The approximation used in this model is the linear variation in flexibility from the uncracked to the cracked beam section (Figure 3.3) which considers triangular reduction in the stiffness of the cracked element. It is assumed that the stiffness reduction falls within a single element but if that reduction occurs over more than one element, the nodes of the model will be moved so that the crack effect remains within a single element.

### 3.2.1. Crack Breathing

A breathing crack opens and closes alternatively during every cycle of loading. Therefore, in order to have an accurate finite element model, crack breathing needs to be taken into account in the equation of motion. Here, nodal displacements are considered to model the breathing phenomenon (Figure 3.5). Let $\delta_n$ be the nodal displacement of the cracked element (either A or B in Figure 3.4) during external excitation of the beam. Then, the crack is assumed to be open when $\delta_n$ is less than zero and to be closed when $\delta_n$ is greater than or equal to zero. Figure 3.5 shows the graphical concept for modelling the crack breathing.

Based on this assumption, the following equation of motion can be used when the crack is closed:

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = F(t) \quad (3.7)$$

where $M$, $C$ and $K$ are the mass, damping and stiffness matrices for the healthy beam respectively and $F$ is the external excitation.

![Figure 3.3 Variation in the stiffness near a crack for triangular reduction [16]](image)
In the case of an open crack, the stiffness matrix of the cracked beam \((K_c)\) can be obtained as:

\[
K_c(t) = K - \Delta K
\]  

(3.8)

where \(K\) is the stiffness matrix for a healthy case or when the crack is closed and \(\Delta K\) is the reduction in the stiffness of the cracked element which depends on crack depth and location. For \(j^{th}\) crack the stiffness reduction is calculated as below:

\[
K_{cj} = \begin{bmatrix}
k_{11} & k_{12} & -k_{11} & k_{14} \\
k_{21} & k_{22} & -k_{12} & k_{24} \\
-k_{11} & -k_{12} & k_{11} & -k_{14} \\
k_{14} & k_{24} & -k_{14} & k_{44}
\end{bmatrix}
\]  

(3.9)

where

\[
k_{11} = \frac{12E}{l_e^4} \left[ \frac{2l_c^3}{l_e^2} + 3l_c \left( \frac{2\xi_j}{l_e} - 1 \right)^2 \right],
\]

\[
k_{12} = \frac{12E}{l_e^3} \left[ \frac{l_c^3}{l_e^2} + l_c \left( 2 - \frac{7\xi_j}{l_e} + \frac{6\xi_j^2}{l_e^2} \right)^2 \right],
\]

\[
k_{14} = \frac{12E}{l_e^3} \left[ \frac{l_c^3}{l_e^2} + l_c \left( 1 - \frac{5\xi_j}{l_e} + \frac{6\xi_j^2}{l_e^2} \right)^2 \right],
\]

\[
k_{22} = \frac{12E}{l_e^2} \left[ \frac{3l_c^3}{l_e} + 2l_c \left( \frac{3\xi_j}{l_e} - 2 \right)^2 \right],
\]

\[
k_{24} = \frac{12E}{l_e^2} \left[ \frac{3l_c^3}{l_e} + 2l_c \left( 2 - \frac{9\xi_j}{l_e} + \frac{9\xi_j^2}{l_e^2} \right) \right],
\]

\[
k_{44} = \frac{12E}{l_e^2} \left[ \frac{3l_c^3}{l_e} + 2l_c \left( \frac{3\xi_j}{l_e} - 1 \right)^2 \right]
\]

\(I_o\) and \(I_{cj}\) are the second moment of areas for the healthy and damaged beam respectively.
Finally, cracked stiffness matrix would be substituted in the equation of motion for the open crack:

\[
M \ddot{x}(t) + C \dot{x}(t) + K_c x(t) = F(t)
\]  
(3.10)

The Newmark-\(\beta\) method is used to solve Equations (3.7) and (3.10) for closed and open cracks respectively with the time steps of 200\(\mu\)s.

![Figure 3.4](image)

**Figure 3.4** Beam elements with (a) closed crack and (b) open crack

![Figure 3.5](image)

**Figure 3.5** Schematic representation of modelling the crack breathing in every cycle of vibration. \(\delta_n\) is the nodal displacement of the cracked element.

### 3.3. DND Concept and Computational Approach

The DND method is based on the normal probability plot (NPP) which is a graphical technique for assessing normality of a dataset. The idea here is that the deviation of vibration responses from their corresponding normal distribution is different for healthy and cracked structures. Considering this fact, the aim is to investigate whether the amount of deviation from normal distribution of vibration responses can be used for localising cracks in beam-like structures.
Chapter 3: Development of a time-domain method

NPP demonstrates the relative cumulative frequencies of the data by means of a specific plotting convention [82]. It compares quantiles of the observed population with quantiles of a theoretical population [73]. In this paper the populations under investigation are vibration responses in terms of acceleration. Assume that \( \{x_1, x_2, \ldots, x_n\} \) denotes an ordered sample of data and \( F(X) = \frac{x-\mu}{\sigma} \) corresponds to their cumulative distribution function where \( \mu \) and \( \sigma \) are the mean and standard deviation parameters respectively. The horizontal axis of the NPP is the sample order statistic \( x_i \), and the vertical axis is the inverse of normal cumulative distribution function \( Z_i = F^{-1}(p_i) \) where \( p_i \) refers to the cumulative probability associated with each rank-ordered data. According to Blom [83], the cumulative probability is:

\[
p_i = \frac{i - 0.375}{n + 0.25} \quad (3.11)
\]

where \( i \) is the rank order number and \( n \) is the number of points.

Figure 3. 6 and Figure 3. 7 show a random signal in time domain and its NPP respectively. In Figure 3. 7, the solid line is the actual distribution of the data and the dashed line is the signal’s normal distribution which is obtained by joining the first and third quartiles of the time domain data and afterwards extrapolating out to the ends of the data set.

In this paper, NPP is utilised to compare the values obtained from the distribution of vibration responses with the same number of values if the given dataset was distributed normally. If the dataset under review follows a normal distribution, the majority of data points on the NPP will fall along a straight line. However, deviations from the straight line (dashed area in Figure 3. 7) indicate non-linearity which can be obtained by calculating the area between actual signal distribution and its normal distribution as follows:

\[
DND = \sum_{i=1}^{n} \left( \frac{|Z_{a,i} - Z_{n,i}| + |Z_{a,i+1} - Z_{n,i+1}|}{2} \right) \times (\Delta X) \quad (3.12)
\]
where \( n \) is the number of data points in a signal, \( Z_{a,i} \) and \( Z_{n,i} \) are the values of actual and normal distribution at \( i \)-th data point respectively and \( \Delta X \) is the distance between adjacent data points.

The presence of cracks in a structure may cause measured vibration responses near the crack location to have less or greater value of DND compared to others. This can be quite useful for the purpose of crack localisation; hence, numerical and experimental examples are used to examine the proposed method.

![Figure 3.6 Typical time domain response](image)

### 3.4. Simulated Example 1- Cantilever beam

An Aluminium beam with cross-section of \( 25 \times 100 \text{mm}^2 \) and length of 3m has been considered. The density and Young’s modulus of the beam are 2700 Kg/m3 and 74 GPa respectively. The cracked beam is simulated as explained in section 2. Modal analysis is carried out for fixed-free boundary condition using open cracks for crack locations of 0.8m, 1.0m and 1.6m from the fixed end. Two crack depths \( \left( \frac{d_c}{D} = 0.2, 0.3 \right) \) are also considered in each analysis, where \( d_c, D \) are the depths of the crack and the beam respectively, as shown in Figure 3.1.

Table 3.1 shows the results of computed natural frequencies.
Chapter 3: Development of a time-domain method

Table 3.1 Computed natural frequency for the cantilever beam with different crack locations and depths

<table>
<thead>
<tr>
<th>Case</th>
<th>Crack Size ((\frac{d^2}{D}))</th>
<th>Distance from fixed end [m]</th>
<th>Mode 1 [Hz]</th>
<th>Mode 2 [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed-Free</td>
<td>----</td>
<td>----</td>
<td>2.335</td>
<td>14.637</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>0.8</td>
<td>2.321</td>
<td>14.629</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>0.8</td>
<td>2.314</td>
<td>14.624</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>1.0</td>
<td>2.324</td>
<td>14.598</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>1.0</td>
<td>2.319</td>
<td>14.578</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>2.0</td>
<td>2.332</td>
<td>14.521</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>2.0</td>
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<td>14.465</td>
</tr>
</tbody>
</table>

The beam is then excited by a sinusoidal input at the first and second natural frequency for each case according to Table 3.1. The location of excitation is away from the nodal point(s) of the exciting mode. Calculated acceleration responses are polluted with normalised random noise with signal to noise ratio of 20dB in order to simulate experimental conditions.

Figure 3. 8(a, c) demonstrates the acceleration response with added noise and its NPP at 0.8m away from the fixed end of the healthy cantilever beam. Figure 3. 8(b, d) shows the acceleration response and its NPP for the cracked cantilever beam at the same
location (0.8m) close to the location of the crack. As can be seen, the value of DND in Figure 3. 8(d) is greater than the DND in Figure 3. 8(c). Therefore, obtaining the values of DND over the entire length of the beam can be useful for locating the crack.

![Figure 3.8](image_url)

**Figure 3.8** Acceleration response and its NPP at 0.8m from the fixed end for healthy (a, c) and cracked (b, d) cantilever beam

### 3.4.1. Results and observations

The DND method is applied to vibration responses in terms of acceleration obtained from numerical simulation of the cantilever beam. Figures 3.9-3.10 illustrate the results for different excitation frequencies, crack locations and depths. Small discontinuities can be observed at the locations of cracks compared to the healthy mode shapes (Figures 3.9-3.10(a)). However, for the second mode of excitation (Figures 3.10 (b-d)), large discontinuities can be seen in addition to the crack effect in the DND of responses. These large peaks occur at the nodal point of the second mode shape of the beam where there is no displacement during the external excitation. As a result of these large peaks, small peaks due to the crack presence are not clearly recognisable. Therefore, the method needs to be improved so that discontinuities corresponding to either crack location or nodal points can be distinguished easily.
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3.5. Simulated example 2- Simply supported beam

A simply supported beam is selected here to investigate the effect of boundary condition on the DND results. Modal analysis is carried out using open cracks for crack locations of 0.8m, 1.2m and 2.5m (Table 3.2). Two crack depths \( \frac{d_c}{D} = 0.2, 0.3 \) and two modes of excitation are considered again.

Figures 3.11-3.12 show the obtained results for the case of a simply supported beam. Once again small discontinuities can be seen at the crack locations. The location of nodal point(s) is shown clearly in Figure 3.12 (b-d) for the second mode of vibration.

![Figure 3.9](image-url) (a) Mode shape and the DND plots for the cantilever beam with crack depths of 0.2D & 0.3D and crack locations of (b) 0.8m (c) 1.0m and (d) 2m at mode 1

3.6. Improved DND method

The results obtained from the DND method need to be improved further in order to recognise the discontinuities related to crack locations from those related to nodal points. Therefore, tangent directions (first derivative) and curvature (second derivative) of the DND data are employed here to observe their capability to magnify the crack effect in the DND signals. The derivatives can be calculated as follows:
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\[ dDND_j = \frac{DND_j - DND_{j-1}}{\Delta X}, \quad j = 2,3,4, ... \]  

\[ d^2DND_j = \frac{DND_{j+1} - 2DND_j + DND_{j-1}}{\Delta X^2}, \quad j = 2,3,4, ... \]

where \( j \) refers to each DND data point and \( \Delta X \) is the difference between adjacent data points.

![Figure 3.10 Mode shape and the DND plots for the cantilever beam with crack depths of 0.2D & 0.3D and crack locations of (b) 0.8m (c) 1.0m and (d) 2m at mode 2](image)

Table 3.2 Computed natural frequency for the simply supported beam with different crack locations and depths

<table>
<thead>
<tr>
<th>Case</th>
<th>Crack Location [m]</th>
<th>Frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Size (( \frac{d}{D} ))</td>
<td>Mode 1</td>
</tr>
<tr>
<td>Simply supported</td>
<td>No crack 0.2</td>
<td>6.558</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>6.516</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>6.521</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>6.490</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>6.520</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>6.539</td>
</tr>
</tbody>
</table>

Figures 3.13-3.14 illustrate the application of first and second derivatives to the DND data for the first three natural frequencies of the cantilever beam. The left (a, c, e) and right-hand side figures (b, d, f) correspond to the first and second derivatives respectively for different crack locations. The crack depth is 0.2D in all the figures.
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Figure 3.11 (a) Mode shape and the DND plots for the simply supported beam with crack depths of 0.2D & 0.3D and crack locations of (b) 0.8m (c) 1.0m and (d) 2m at mode 1

As can be seen, the first derivative can detect the location of cracks successfully when the beam is excited at its first resonant frequency. However, for the second mode, the first derivative does not provide a good indication of crack locations. In contrast, the second derivative presents a clear indication of both the crack location and the nodal point for both exciting modes. This is due to the ability of the second derivative to

Figure 3.12 (a) Mode shape and the DND plots for the simply supported beam with crack depths of 0.2D & 0.3D and crack locations of (b) 0.8m (c) 1.0m and (d) 2m at mode 2
remove curvature from the DND data. As a result, the second derivative is selected to apply to the DND data of the simply supported beam. Results are shown in Figures 3.15-3.16.

According to Figures 3.13-3.16, numerical simulations confirm that second derivatives of the DND data can accurately detect the crack location in beam structures for different excitation modes, boundary conditions and crack locations. Experimental tests are also carried out to investigate the reliability of the proposed method.

**Figure 3.13** dDND (a, c, e) and second d²DND (b, d, f) for the cantilever beam at mode 1 with crack locations of 0.8m (a, b), 1.0m (c, d) and 2.0m (e, f) from the fixed end

**Figure 3.14** dDND (a, c, e) and second d²DND (b, d, f) for the cantilever beam at mode 2 with crack locations of 0.8m (a, b), 1.0m (c, d) and 2.0m (e, f) from the fixed end
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Figure 3.15 $d^2$DND for the simply supported beam at mode 1 with crack locations of (a) 0.8m, (b) 1.2m and (c) 2.5m

Figure 3.16 $d^2$DND for the simply supported beam at mode 2 with crack locations of (a) 0.8m, (b) 1.2m and (c) 2.5m

3.7. Crack size effect

Further study has been carried out to understand whether the DND method is also sensitive to the crack size. Crack depths ratio ($\frac{d}{D}$) of 0.1, 0.2, 0.3 and 0.5 are simulated for both the cantilever and the simply supported beams. Figures 3.17-3.18 show the curvature of DND results for the cantilever and simply supported beams respectively.
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The results confirm that the DND method is sensitive to the size of the crack and its amplitude can be used for the size prediction.

![Graph showing the effect of different crack sizes on the d²DND results for the cantilever beam.](image)

**Figure 3.17** Effect of different crack sizes on the $d^2$DND results for the cantilever beam

![Graph showing the effect of different crack sizes on the d²DND results for the simply supported beam.](image)

**Figure 3.18** Effect of different crack sizes on the $d^2$DND results for the simply supported beam

### 3.8. Experimental validation

Experimental tests were conducted on healthy and cracked solid aluminium beams having dimensions of $25 \times 100 \times 3000 \text{mm}^3$ to show the effectiveness of the proposed DND method. Laser cutting was used to create cracks in the form of small slots at
different locations along the beams. Table 3.3 shows the crack sizes and locations for all the beams used in the experiment.

Table 3.3 Crack Sizes and locations used in the experimental beams

<table>
<thead>
<tr>
<th>Case</th>
<th>Crack Location [cm]</th>
<th>Crack Depth [mm]</th>
<th>Crack Width [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam 1 (healthy)</td>
<td>No Crack</td>
<td>No Crack</td>
<td>No Crack</td>
</tr>
<tr>
<td>Beam 2</td>
<td>150</td>
<td>6</td>
<td>0.2</td>
</tr>
<tr>
<td>Beam 3</td>
<td>100</td>
<td>6</td>
<td>0.2</td>
</tr>
</tbody>
</table>

3.8.1. Test setup

The aluminium beams were suspended from a hollow steel beam through two bicycle brake cables to provide free-free boundary conditions. The cables were connected to the beams using two clamps. Since the beams were excited at their first natural frequencies, the clamps were placed at nodal points of the first mode (free-free) where there were no displacements during external excitation. Laptop 1 (Figure 3.19) was used to send the input signals in the form of simple sinusoidal waves to the shaker. 15 accelerometers were installed at equal intervals of 21 cm along the beams to acquire vibration responses (Figure 3.20).

A force gauge was also used to measure the input force applied by the shaker to the beams. Amplifiers were employed to amplify all the output signals and send them into a 16 bit data acquisition card which was connected to laptop 2 (Figure 3.19). All the response channels of the data acquisition card were acquired simultaneously so that valid ODSs with correct magnitudes and phases could be guaranteed. Figures 3.20-3.21 show the experimental setup and the schematic view of it respectively for vibration data acquisition.

3.8.2. Experiment

Modal testing was carried out using laptop 1 in order to obtain the frequency response function (FRF) of each beam. The aluminium beams were then excited at their first natural frequencies according to the FRF results. The excitations were in the form of continuous sinusoidal waves applied to the beams through the shaker. The results obtained from the modal analysis are summarised in Table 3.4.
Table 3.4 Measured natural frequencies of the experimental beams

<table>
<thead>
<tr>
<th>Case</th>
<th>Mode 1 [Hz]</th>
<th>Mode 2 [Hz]</th>
<th>Mode 3 [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam 1 (healthy)</td>
<td>14.95</td>
<td>39.98</td>
<td>78.43</td>
</tr>
<tr>
<td>Beam 2</td>
<td>14.95</td>
<td>39.98</td>
<td>78.13</td>
</tr>
<tr>
<td>Beam 3</td>
<td>15.26</td>
<td>40.26</td>
<td>79.65</td>
</tr>
</tbody>
</table>

It should be noted that Beams 1-3 are not a single sample in which the cracks were introduced in-situ for the experiments. They are basically 3 identical samples but slightly different in the experimental set-up in terms of (a) clamped force on either side, (b) insignificant different in the beam length, and (c) mounting place of the
accelerometers might not be exactly the same in all the cases. These effects resulted in the minor inconsistency in the natural frequencies listed in Table 3.4.

Figure 3. 21 Schematic view of the experiment [21]

Figure 3. 22 Experimental NPP for (a) the healthy and (b) cracked free-free beam at the distance of 1.5m
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Figure 3.23 Experimental DND and $d^2$DND for Beam1 (a,b), Beam2 (c,d) and Beam3 (e,f)

Vibration responses from all the sensors were recorded using the data acquisition card for each beam. Sampling frequency of 5000 Hz was used to convert time domain responses into frequency domain.

3.8.3. Analysis and results

Figure 3.22 illustrates the NPP for the healthy and cracked free-free Aluminium beams. Here it can be observed that the value of DND near the crack location (Figure 3.22(b)) is different from that value at exactly the same location for the healthy beam without any crack (Figure 3.22(a)).

The experimental DNDs and their corresponding $d^2$DNDs for all three beams are plotted in Figure 3.23 for mode 1. As expected, the DND plots (Figure 3.23(a, c, e)) clearly show two sharp discontinuities at the location of the nodal points. A small discontinuity can be seen at the centre of Beam 2 in Figure 3.23(c). The reason is that in Beam 2, the crack is located at the centre of the beam where maximum crack breathing at the first mode has occurred. However, beam 3 has its crack at the location where crack breathing has not been prominent enough to affect the overall trend of the DND plot. By removing the curvature from the DND plots, the $d^2$DND plots (Figure 3.23(b, d, f)), present peaks at the nodal points and at the sensors nearest to the crack locations. The
results confirm that the second derivative of the DND data from time-domain responses can be employed as a useful tool for crack detection in beam-like structures.

### 3.9. Concluding remarks

The DND method has been proposed for processing the vibration responses of beam-like structures in time-domain for the purpose of crack detection. The method is based on the deviation of actual distribution of vibration responses from their normal distributions. The obtained results show large discontinuities at the nodal point(s) of the exciting modes where there are no vibrations and small discontinuities at the crack location. The second derivative is used to remove curvature from the DND plots so that small discontinuities due to the presence of cracks can be distinguished clearly from those of the nodal points. The application of the proposed method to both numerical and experimental examples demonstrates accurate location of cracks in beams with different crack locations and under different boundary conditions.
CHAPTER 4: DEVELOPMENT OF A NOVEL FREQUENCY-DOMAIN METHOD

Reformatted version of the following papers:


**Paper No. 7:** E. Asnaashari, J.K. Sinha, "Operational deflection shape for crack detection in structures", Presented at the 10th International Conference on Damage Assessment of Structures, Dublin, Ireland, July 2013.

**Abstract**

Excitation of a cracked structure at a given frequency always generates higher harmonic components of the exciting frequency due to the breathing of the crack. In this paper, the deflection of cracked structures at the exciting frequency and the second harmonic component is mapped by a new method based on the operational deflection shape (ODS) for the purpose of crack detection. While the ODS is helpful in understanding dynamic behaviour of structures and machines, it is not always possible to determine the location of cracks in structures or machines based on the ODS itself. Therefore, a new concept called residual ODS (RODS) has been defined for crack detection in beam-like structures. This paper presents the details of the proposed method and its results when applied to numerical and experimental examples.
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4.1. Introduction

One of the important tasks of structural health monitoring systems is the detection of cracks in structures such as buildings and bridges. A number of vibration-based methods are available in the literature [26] that can identify the presence and location of cracks globally in any structure.

The presence of crack in a structure changes flexibility in the vicinity of the crack. This change causes the dynamic behaviour for the whole cracked structure to be different from that of a healthy one [85]. Vibration-based crack detection methods generally consider either open or breathing cracks in their approaches. Gentile and Messina [86] identified open cracks in beam structures by minimising measurement data and baseline of the structures and using continuous wavelet transform (CWT). Although the ability of this method to efficiently identify locations of open cracks was shown, the method was not verified experimentally. Khaji et al [87] presented an analytical approach for identifying cracks in uniform beams with open edge cracks based on vibration measurements. The method was able to predict the crack location and depth with errors of less than 8% and 25% respectively. Xiang et al [88] proposed a model-based crack identification method to estimate the crack location and size in a static shaft with an open crack. The method investigated the effects of rotary inertia on the natural frequencies of the rotor system to construct a B-spline wavelet on the interval and was verified experimentally.

While some approaches use open cracks in their analysis, the existence of a breathing fatigue crack in a structure can be represented accurately by considering its non-linear behaviour. Yan et al [89] identified breathing-fatigue cracks in beams by employing the difference between natural frequencies of each stiffness region of a beam according to the stiffness interface. Surace et al [90] used higher order frequency response functions (FRFs), based on the Volterra series for the purpose of fatigue crack detection in a cantilever beam. Tsyfansky and Beresnevich [91] used the higher harmonic components of external harmonic excitation to identify fatigue cracks in geometrically flexible bars. The same approach was utilised by Semperlotti et al [92] to locate a breathing fatigue crack in an isotropic rod.

In this paper, vibration responses at the higher harmonics of exciting frequency for the structure, generated due to the breathing of fatigue cracks (opening and closing), have
again been considered. These responses can be analysed by considering either amplitude of deflection (AOD) or operational deflection shape (ODS). Ullah and Sinha [93] utilised the AOD method to map the deflection of E-glass fibre and epoxy resin composite plates with centre and off-centre delaminations. Based on the experimental results, it was shown that the normalised summation of higher harmonics (NSH) of the exciting frequency at each mode and its cumulative NSH (CNSH) were able to detect and locate the delamination respectively. The disadvantage of the AOD method at higher harmonics is that only absolute values of vibration amplitude at a specific mode are considered. Therefore, a true representation of the deflection shape of the structure, which requires both the amplitude and phase information, cannot be constructed. The other drawback of the AOD method at higher harmonics is that it may not be successful in detecting cracks in all types of structures [94]. Therefore, a new method for identifying and locating the crack based on the ODS at higher harmonics, which considers both amplitude and phase, is presented here.

Vibration amplitudes need to be measured at a number of locations along a structure during external excitation in order to construct the ODS for that structure. This can be done practically using a scanning laser Doppler vibrometer (SLVD). The key advantage of the ODS is that it uses both the amplitude and phase of vibration responses at a frequency to map the deflection pattern of structures. This results in realistic vibration patterns of structures and/or machines at a specific frequency. The ODS method is well-known in the vibration analysis [43, 95]. Applications of the ODS analysis to different structures e.g., machines, bridges, gearboxes, and wind turbines, etc. are presented in the literature for different purposes. Pai and Young [45] investigated the use of a boundary effect detection (BED) method for identifying small amounts of damage in beams based on ODSs. Boundary layer solutions were extracted from experimental ODSs using a sliding-window least-squares curve-fitting method. It was showed both numerically and experimentally that the BED method is reliable for locating small amounts of damage; however, some difficulties and limitations were observed during the application of this method. Sundaresan et al [96] used a scanning laser vibrometer to compute and then compare the ODSs of healthy and damaged turbine blades. Changes in curvature of the ODSs were used to locate the damage. Zhang et al [97] proposed a new damage detection algorithm called global filtering method (GFM) for beam and plate like structures based on ODS curvature (ODSC). A vehicle was used to move
along a line on the damaged structure as an exciter. The ODSCs were then constructed from dynamic responses of the vehicle and not the structure. The GFM, which is based on wavelet decomposition, was employed to make the experimental ODSC smoother so that it could be used as baseline data. The effect of the damage could then be detected through comparing the ODSC and the filtered one. The method does not require a number of preinstalled sensors on the structure, which can be considered an advantage; however, the optimum velocity of the vehicle needs to be determined, since the detection quality of the GFM depends on it. Moreover, the results from the GFM were sensitive to the chosen decomposition level of filtering and the method may not be helpful in detecting small amounts of damage.

The ODS method is generally helpful in identifying vibration related problems in structures and machines; however, it may not be straightforward to locate cracks in structures by using only the ODS. Recently, the residual ODS (RODS) has been defined for the purpose of crack detection [94]. The preliminary study on simulated numerical examples showed encouraging results for identifying cracks in beam-like structures. In this paper, details of the RODS method as well as the results of applying it to both numerical and experimental examples are presented.

4.2. Proposed Method

A breathing crack opens and closes alternatively during every cycle of loading and consequently produces nonlinear dynamics of the cracked structure (Figure 4.1). Due to this phenomenon, excitation of the cracked structure at a given frequency always generates higher harmonic components of the exciting frequency. A typical spectrum for the beam with a breathing crack is shown in Figure 4.2, where the exciting frequency (1x) as well as its higher harmonics (2x, 3x, 4x ...) can be seen.

The ODS of the cracked beam can be generated at the exciting frequency and the second harmonic components to map the deflection of the cracked structure. However, it has been observed from numerical simulations [94] that the ODSs at the higher harmonic components are greatly affected by the ODS at the frequency of excitation. As a result, the ODSs at 2x and other higher harmonics cannot always distinguish the crack location, because these data may contain the effect of the first harmonic component (1x; representing the mode shape) in addition to non-linearity due to the crack breathing.
Based on this assumption, a simple approach is proposed here to eliminate the effect of the first harmonic component from the ODSs at higher harmonics. In order to make such elimination possible, all the ODS values at 1x and higher harmonics are normalised first using Equation (4.1):

\[
(NODS)_{i,m} = \frac{(ODS_k)_{i,m}}{(MODS)_{i,m}}, \quad i = 1, 2, 3, 4, \ldots \tag{4.1}
\]

where \((NODS)_{i,m}\) is the normalised ODS at \(i\)-th harmonic for mode \(m\). The subscript \(k\) refers to the node number, and \((MODS)_{i,m}\) is the maximum value of ODS data at \(i\)-th harmonic for mode \(m\). Either displacement, velocity or acceleration can be used to construct the ODS of the structure. In this paper, measured and simulated acceleration responses are used.

![Figure 4.1](image)

**Figure 4.1** (a) A typical cracked beam (b) open crack and (c) closed crack

The residual operational deflection shape \((RODS)\) is then defined as the \(NODS\) at the \(p\)-th harmonic with respect to the first harmonic, as expressed in Equation (4.2). This removes the effect of the first harmonic from the ODS at \(p\)-th harmonic, so that only the non-linear effect remains which is useful for the purpose of crack detection.

\[
(RODS)_{p,m} = (NODS)_{p,m} - (NODS)_{1,m}, \quad p = 2, 3, 4, \ldots \tag{4.2}
\]
Therefore, when a cracked beam is excited at its first natural frequency and the RODS is calculated for 2x data, \( p \) and \( m \) are equal to 2 and 1 respectively.

**Figure 4.2** A typical acceleration spectrum for the free-free beam

### 4.3. Numerical Example

An example of a steel beam with the geometrical properties – length, 1000mm and square cross-section 15mm×15mm and material properties – Young’s modulus 210 GPa and density 7800Kg/m³ was considered. A finite element model of the beam under free-free boundary conditions was constructed using two node Euler-Bernoulli beam elements. The beam was divided into 10 elements, each having a length of 100mm. Each node had two degrees of freedom, the translational displacement and bending rotation. Therefore, the beam consisted of 11 nodes and 22 degrees of freedom in total. Figure 4.3 shows the finite element model of the free-free beam and the location of the external excitation. It is important to note that the excitation location must be away from the nodal point(s) of the exciting mode. In this model, the approach proposed by Sinha et al [80] is used to introduce a crack to the finite element model of the beam. Two crack depths \( (\frac{d_c}{D} = 0.2, 0.3) \) and locations \( (\frac{l_c}{L} = 0.25, 0.35) \) are utilised in each analysis, where \( d_c, l_c \) are the depth and location of the crack and \( D, L \) are the depth and length of the beam respectively, as shown in Figure 4.1(a). Natural frequencies of the cracked beam under free-free boundary conditions were computed by carrying out modal analysis while the crack was assumed to be open. Table 4.1 shows the results of
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the modal analysis. The modelled cracked beam is then excited by a sinusoidal input at its first natural frequency according to Table 4.1. The location of the excitation force was at node 7, as shown in Figure 4.3. Time steps of 0.0002s were used in the Newmark-beta numerical integration method to solve the equation of motion of the excited beam.

Breathing of the crack is also simulated causing the generation of higher harmonic components of the exciting frequency in the frequency spectrum. The crack was assumed to be closed when the displacement of the nodes of the cracked element in y-direction was less than zero for the applied excitation and to be open when that displacement was greater than zero. The sampling frequency of 5000Hz was used to convert the time-domain signals into frequency-domain. It is assumed here that the vibration of entire structure can be scanned through a laser vibrometer or a number of accelerometers attached along the beam. Therefore, taking vibration measurements in terms of velocity or acceleration at a large number of locations is possible. Figure 4.2 shows a typical frequency spectrum of the velocity response of the cracked beam at a specific node where the higher harmonics of the exciting frequency can be observed clearly.

<table>
<thead>
<tr>
<th>Case</th>
<th>Crack</th>
<th>Frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mode 1</td>
</tr>
<tr>
<td>Free-Free</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Size</td>
<td>Location</td>
</tr>
<tr>
<td></td>
<td>$\frac{d_1}{D}$</td>
<td>$\frac{d_2}{L}$</td>
</tr>
<tr>
<td>No crack</td>
<td>No crack</td>
<td>No crack</td>
</tr>
<tr>
<td>0.2</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>0.35</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.3 Finite element model of the beam and the location of excitation force
Simulated data were initially analysed using the ODS at higher harmonic components of the first resonant frequency for the cracked beam under free-free boundary conditions at all crack depths and locations (Figure 4.4). The obtained ODSs from measured data are generally complex quantities; therefore, the method explained in [94] was used to convert them into normal ODSs. It can be observed that the ODSs at 1x data (Figure 4.4(a), (b)) correspond to the mode shape of the beam while the results acquired from the ODS analysis at 2x data (Figure 4.4(c), (d)) present some indications regarding the location of the crack, but are not clear enough. The Proposed RODS method is now applied to the numerical examples and the results are shown in Figure 4.5. It can be observed that the first and the second highest values of $(\text{RODS})_{2,1}$ occurred at two consecutive nodes. In fact, the nodes corresponding to those two highest values determine the location of the cracks. As shown in Figure 4.5, the intersections of the dashed lines provide accurate identifications of the crack location.

The influence of noise on the numerical results was also investigated. Calculated acceleration responses were polluted with normalised random noise with signal-to-noise ratio (SNR) of 20 dB. Figure 4.6 presents the application of the RODS method to noisy simulated data. The RODS results locate the crack successfully which verifies that the detection is not affected by the noisy signal.

![Figure 4.4 ODSs at 1x (a, b) and 2x (c, d) for the free-free beam with crack depths of 0.2D and 0.3D at the location of 0.25m (a,c) and 0.35m (b,d) from one end](image-url)
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Figure 4.5 The \((\text{RODS})_{2,1}\) for calculated vibration responses without noise at mode 1 for the free-free beam with crack depths of 0.2D & 0.3D and crack locations of (a) 0.25m and (b) 0.35m from one end

Figure 4.6 The \((\text{RODS})_{2,1}\) for calculated vibration responses with noise at mode 1 for the free-free beam with crack depths of 0.2D & 0.3D and crack locations of (a) 0.25m and (b) 0.35m from one end

4.3.1. Off-resonance Excitation

In the previous section, the proposed method was tested when the beams were excited at their first natural frequency. However, depending on the situation, excitation at resonant frequency may not be possible in practice. Hence, off-resonance excitation is used here to examine the usefulness of the proposed method. Sinusoidal input with the frequency
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of 70 Hz, which is below the natural frequency (Table 4.1), was used to excite the beams. The \((\text{RODS})_{2,0}\) was then computed for 2x components. Subscript “0” indicates off-resonance excitation. Once again, acceleration responses were polluted with SNR of 20 dB. Figure 4.7 illustrates the obtained results which confirm the accuracy of the proposed method in locating cracks using off-resonance excitation.

4.4. Experimental Verification of Proposed Method

Experimental tests were conducted on healthy and cracked solid aluminium beams having dimensions of 25×100×3000mm$^3$ to show the effectiveness of the proposed RODS method. Laser cutting was used to create cracks in the form of small slots at different locations along the beams. Table 4.2 shows the crack sizes and locations for all the beams used in the experiment.

![Figure 4.7](image)

**Figure 4.7** The \((\text{RODS})_{2,0}\) for calculated vibration responses with noise from off-resonance excitation for the free-free beam with crack depths of 0.2D & 0.3D and crack locations of (a) 0.25m and (b) 0.35m from one end

**Table 4.2** Crack sizes and locations used in the experimental beams

<table>
<thead>
<tr>
<th>Case</th>
<th>Crack Location [cm]</th>
<th>Crack Depth [mm]</th>
<th>Crack Width [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam 1 (healthy)</td>
<td>No Crack</td>
<td>No Crack</td>
<td>No Crack</td>
</tr>
<tr>
<td>Beam 2</td>
<td>150</td>
<td>6</td>
<td>0.2</td>
</tr>
<tr>
<td>Beam 3</td>
<td>100</td>
<td>6</td>
<td>0.2</td>
</tr>
<tr>
<td>Beam 4</td>
<td>37.5</td>
<td>6</td>
<td>0.2</td>
</tr>
</tbody>
</table>
4.4.1. Test Setup

Figures 3.18-3.20 show the test setup and the method of acquiring the vibration data. More details can be found in [84].

4.4.2. Experiment

Modal testing was carried out in order to obtain the frequency response function (FRF) of each beam. The aluminium beams were then excited at their first natural frequencies according to the FRF results. The excitations were in the form of continuous sinusoidal waves applied to the beams through the shaker. The results obtained from the modal analysis are summarised in Table 4.3.

<table>
<thead>
<tr>
<th>Case</th>
<th>Mode 1 [Hz]</th>
<th>Mode 2 [Hz]</th>
<th>Mode 3 [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam 1 (healthy)</td>
<td>14.95</td>
<td>39.98</td>
<td>78.43</td>
</tr>
<tr>
<td>Beam 2</td>
<td>14.95</td>
<td>39.98</td>
<td>78.13</td>
</tr>
<tr>
<td>Beam 3</td>
<td>15.26</td>
<td>40.26</td>
<td>79.65</td>
</tr>
<tr>
<td>Beam 4</td>
<td>15.26</td>
<td>39.98</td>
<td>78.74</td>
</tr>
</tbody>
</table>

It should be noted that the dimensions of each experimental beam slightly differ from the others. Also, the mounting place of the accelerometers might not be exactly the same in all cases, causing slightly different mass distribution along each beam. Therefore, minor inconsistencies can be seen in the results for natural frequencies.

Vibration responses from all the sensors were recorded using the data acquisition card for each beam. A sampling frequency of 5000 Hz was used to convert time domain responses into frequency domain. Figure 4.8 demonstrates typical acceleration spectra obtained from the 4th accelerometer (distance of 63cm from one end) for Beam 1-4 when excited at their first mode. Higher harmonic components can be seen clearly for Beam 2-4. However, the spectrum of Beam 1 (healthy) does not show most of the harmonics. This confirms the breathing of the cracks during the experiments for Beam 2-4. The peak at the frequency of 40Hz in Figure 4.8 may be either a shaker harmonic or a natural frequency in a degree of freedom other than that in the direction of excitation.
Chapter 4: Development of a novel frequency-domain method

4.4.3. Analysis and Results

The experimental ODSs at 1x and 2x data for all three cracked beams are plotted in Figures 4.12-4.14. As can be seen, only Figure 4.12(b) shows a negative peak at the crack location. All the other ODSs display the first mode shape of the free-free beams. The reason is that in Beam 2, the crack is located at the centre of the beam, where maximum crack breathing at the first mode has occurred. However, Beams 3 and 4 have their cracks at locations where crack breathing has not been prominent enough to affect the overall trend of the ODS plots. As a result, it is not always possible to determine the location of cracks based on the ODS itself.

The RODS method, explained in section 4.2, is applied to all the experimental frequency-domain responses of the cracked Aluminium beams. Figure 4.15 presents these results. As can be observed, positive peaks have appeared after removing the effect of mode 1 from the ODSs at second higher harmonics (RODS). These peaks show the location of sensors which had the closest distance to the crack location. This means that the RODS method is capable of locating the cracks even in situations where
the crack breathing is not very high. Therefore, the method can be employed as a trustworthy tool for crack detection in beam-like structures.

![Figure 4.9 Experimental ODSs at (a) 1x and (b) 2x data for Beam 2](image)

**Figure 4.9** Experimental ODSs at (a) 1x and (b) 2x data for Beam 2

![Figure 4.10 Experimental ODSs at (a) 1x and (b) 2x data for Beam 3](image)

**Figure 4.10** Experimental ODSs at (a) 1x and (b) 2x data for Beam 3

![Figure 4.11 Experimental ODSs at (a) 1x and (b) 2x data for Beam 4](image)

**Figure 4.11** Experimental ODSs at (a) 1x and (b) 2x data for Beam 4
4.5. Finite Element Simulation of the Experiments

Finite element models of cracked Aluminium beams which were used in the experiments are simulated in this section. The beams are divided into 15 elements and two degrees of freedom are considered for each node. The same approaches mentioned in section 4.3 for modelling and simulating the crack and its non-linear breathing behaviour are utilised once more. The masses of all the mounted accelerometers in the experiment as well as the two clamps are considered in this simulation.

The first natural frequencies of the cracked Aluminium beams under free-free boundary conditions were found to be 14.926, 14.781, 14.837, and 14.921Hz for beams 1-4 respectively. Open crack was assumed during the modal analysis. The modelled cracked beams are then excited by sinusoidal inputs at their first natural frequencies.

Figure 4.12 Experimental (RODS)_{2,1} at mode 1 for (a) beam 2 (b) beam 3 and (c) beam 4
Figure 4.16 demonstrates the results obtained from the application of the RODS method to numerical data. As shown, the numerical $(\text{RODS})_{2,1}$ show proper correlations with the experimental ones in Figure 4.15. Good indications of the crack location were also observed when the RODS method was applied to higher modes of vibration. Figure 4.17 illustrates typical results obtained for mode 2 of the Aluminium beams. This confirms that the proposed RODS method is also applicable to higher modes of vibration for the purpose of crack detection, provided that the crack is not located at or close to the nodal point(s) of the exciting mode. Beam 4 is not included in Figure 4.17, since its crack is located near one of the nodal points at mode 2.

**Figure 4.13** Numerical $(\text{RODS})_{2,1}$ at mode 1 for (a) beam 2 (b) beam 3 and (c) beam 4

**Figure 4.14** Numerical $(\text{RODS})_{2,2}$ at mode 2 for (a) beam 2 and (b) beam 3
4.6. Concluding Remarks

It is known that the breathing of crack in a structure which is excited at a given frequency generates vibration responses at the frequency of excitation and its higher harmonics. The proposed method utilises the concept of the ODS analysis at the exciting frequency and its higher harmonics to identify the location of the crack. The external excitation can be at resonance or off-resonance frequency as per field convenience. A new term called RODS is defined which eliminates the effect of the ODS at exciting frequency from those at higher harmonic components for clear and accurate identification of crack location. The method has been successfully validated by applying it to both numerical and experimental beam-like examples. The advantage of the RODS method is its simplicity as well as its ability to identify the location of cracks accurately.
CHAPTER 5: IMPROVEMENT TO THE RODS METHOD USING CURVATURE APPROACH

Reformatted version of the following papers:


Abstract

Detection of fatigue cracks at an early stage of their development is important in structural health monitoring. The breathing of cracks in a structure generates higher harmonic components of the exciting frequency in the frequency spectrum. Previously, the residual operational deflection shape (RODS) method was successfully applied to beams with a single crack. The method is based on the ODSs at the exciting frequency and its higher harmonic components which consider both amplitude and phase information of responses to map the deflection pattern of structures. Although the RODS method shows the location of a single crack clearly, its identification for the location of multiple cracks in a structure is not always obvious. Therefore, an improvement to the RODS method is presented here to make the identification process distinct for the beams with multiple cracks. Numerical and experimental examples are utilised to investigate the effectiveness of the improved method.
5.1. Introduction

Initiation and propagation of fatigue cracks are very common in structures subjected to cyclic stresses, most of which are below the ultimate tensile stress. Therefore, it is essential to detect fatigue cracks at an early stage of their development to prevent structural failures. The presence of cracks changes the physical properties and consequently vibration behaviour of structures. This feature of cracks has encouraged many researchers to employ vibration-based methods in identifying and locating cracks in different types of structures. A comprehensive overview of various vibration based methods for damage detection in structures can be found in [63-66].

External excitation of a structure with fatigue cracks generates non-linear vibration responses along the structure. The non-linear responses are due to repetitive opening and closing (breathing) of the fatigue crack during every cycle of loading. Peng et al [98] used non-linear output frequency response functions (NOFRFs) to detect breathing cracks in beams based on frequency domain information. The method was sufficiently sensitive to identify the presence of cracks provided that the external excitation with appropriate intensity had been employed. Blunt and Keller [99] developed planet carrier and planet separation methods based on changes to the modulation of the fundamental gear mesh vibration due to the crack for the purpose of fatigue crack detection in a UH-60A planet gear carrier. They used vibration measurements of healthy and cracked transmissions in test-cell and on-aircraft conditions but only the results for test-cell condition were satisfactory. Bouboulas and Anifantis [62] constructed the finite element model of a cracked cantilever beam to investigate its vibration behaviour. The breathing of the crack was modelled by defining frictional contact between the surfaces of the crack. They applied fast Fourier transform (FFT) and continuous wavelet transform (CWT) to vibration responses obtained from the finite element model and concluded that both methods can be used for crack detection. Andreaus and Baragatti [100] investigated the free vibration of healthy and cracked cantilever beams to identify the presence of damage. They first carried out a three-point bending test on aluminium and steel beams to create fatigue cracks. The beams were then excited by an impact hammer to compare the natural frequencies of the intact and cracked beams to recognise the existence of cracks. The need to obtain the information of intact structures can be considered the major limitation of their detection method. Razi et al.[101] presented a method based on empirical mode decomposition (EMD) for fatigue crack detection in
an aluminium beam. Here again, vibration response of the healthy structure is used along with that of the cracked structure to produce damage indices. These indices have been shown to be sensitive in terms of identifying the presence of damage. Sinha [102] utilised bi- and tri-higher order coherence to detect fatigue cracks in a cantilever beam. Yan et al. [103] used the difference between natural frequencies of stiffness regions of beams to detect breathing fatigue cracks. Johnson et al. [104] detailed an approach to capture subharmonic responses generated due to non-linearity of fatigue cracks which can be used for the purpose of damage detection.

Breathing of the fatigue crack also generates higher harmonic components (superharmonics) together with the frequency of excitation in the frequency spectrum of vibration responses. These higher harmonics were used by Tsyfansky and Beresnevich [91] as well as Semperlotti et al. [92] to identify fatigue cracks in geometrically flexible bars and an isotropic rod respectively. Ullah and Sinha [105] utilised absolute amplitude of higher harmonics to locate centre and off-centre delamination in composite plates. However, their results lacked phase information of responses to construct the true deflection pattern of structures. By considering both amplitude and phase information of vibration responses, the accurate deflection patterns of structures during external excitation can be obtained, which is called operational deflection shape (ODS). This requires taking measurements at multiple locations along the structure. The use of the ODS for damage detection in different types of structures is presented in Pai and Young [45], Sundaresan et al.[106] and Zhang et al.[97]. However, Asnaashari and Sinha [94, 107] have shown the limitations of utilising the ODS alone in locating cracks, and they proposed a new term called residual operational deflection shape (RODS) for crack detection in beam-like structures. For beams with multiple cracks, the RODS method still shows discontinuities at the location of cracks, but these discontinuities are not clear enough for all conditions. In order to have distinct identification of multiple crack locations for any condition, the second derivative or the curvature of the RODS is proposed in this paper and applied to numerical and experimental examples.

5.2. RODS Method

The effect of crack breathing on vibration responses in the time domain can be noticed through generation of higher harmonic components of exciting frequency in the frequency domain. Figure 5.1(b) shows a typical frequency spectrum for a cracked
beam where the exciting frequency (1x) as well as its higher harmonics (2x, 3x, 4x ...) can be seen. The RODS method is based on constructing the ODS at the frequency of excitation and its higher harmonics to map the deflection pattern of the cracked structure. Asnaashari and Sinha [94] concluded from numerical simulations that in addition to nonlinearity due to the crack breathing, the ODSs at 2x and other higher harmonics contain the effect of the first harmonic component. The RODS method removes the effect of the exciting frequency from the ODSs at higher harmonics initially by normalising all the ODSs at 1x and higher harmonics using Equation (5.1):

$$(NODS)_{i,m} = \frac{(ODS_k)_{i,m}}{(MODS)_{i,m}} \quad i = 1,2,3,4,\ldots$$  \hspace{1cm} (5.1)$$

where $(NODS)_{i,m}$ is the normalised ODS at $i$-th harmonic for mode $m$. The subscript $k$ refers to the node number, and $(MODS)_{i,m}$ is the maximum value of ODS data at $i$-th harmonic for mode $m$.

The residual operational deflection shape (RODS) is then defined as the $NODS$ at the $p$-th harmonic with respect to the first harmonic as expressed in Equation (5.2):

$$(RODS)_{p,m} = (NODS)_{p,m} - (NODS)_{1,m} \quad p = 2,3,4,\ldots$$  \hspace{1cm} (5.2)$$

Therefore, when a cracked beam is excited at its first natural frequency and the RODS is calculated for 2x data, $p$ and $m$ are equal to 2 and 1 respectively. In this paper acceleration responses are used to construct the ODSs; however, either displacement, velocity or acceleration responses could be utilised.

![Image](image.png)

**Figure 5.1** Calculated acceleration response with noise in (a) time domain and (b) frequency domain for the Fixed-Fixed beam at mode 1.
5.3. Numerical examples

An aluminium beam with cross-section of 25×100 mm$^2$ and length of 3 m was used. The density and Young’s modulus of the beam were 2700 Kg/m$^3$ and 74 GPa respectively. Multiple cracks with depth of 5mm were introduced at different locations along both fixed-fixed and simply supported aluminium beams. Table 5.1 shows the details of all the investigated beams.

<table>
<thead>
<tr>
<th>Case</th>
<th>Boundary Condition</th>
<th>No. of Cracks</th>
<th>Crack Locations [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam 1-N</td>
<td>Fixed-Fixed</td>
<td>2</td>
<td>$X_1=0.5$, $X_2=1.6$</td>
</tr>
<tr>
<td>Beam 2-N</td>
<td>Fixed-Fixed</td>
<td>2</td>
<td>$X_1=1.0$, $X_2=2.2$</td>
</tr>
<tr>
<td>Beam 3-N</td>
<td>Fixed-Fixed</td>
<td>3</td>
<td>$X_1=0.7$, $X_2=1.2$, $X_3=2.0$</td>
</tr>
<tr>
<td>Beam 4-N</td>
<td>Simply supported</td>
<td>2</td>
<td>$X_1=0.8$, $X_2=2.0$</td>
</tr>
<tr>
<td>Beam 5-N</td>
<td>Simply supported</td>
<td>2</td>
<td>$X_1=1.2$, $X_2=2.5$</td>
</tr>
<tr>
<td>Beam 6-N</td>
<td>Simply supported</td>
<td>3</td>
<td>$X_1=0.4$, $X_2=2.0$, $X_3=2.3$</td>
</tr>
</tbody>
</table>

* N refers to numerical beam

Euler-Bernoulli beam elements are employed to model the beam. There are two nodes per element and each node has two degrees of freedom; translational displacement and bending rotation. Proportional damping is obtained using the constructed mass and stiffness matrices.

Various approaches are available to model cracks in beam-like structures [74-80]. In this paper, the method proposed in [80] is used to introduce cracks to the finite element model of the beams. This approach uses the concept of Christides and Barr [81] to modify the local flexibility in the vicinity of the crack within Euler-Bernoulli beam elements. It assumes that the flexibility from uncracked to cracked section of the beam varies linearly and considers triangular reduction in the stiffness of cracked element. This reduction may happen over more than one element depending upon the crack location. However, the length of each element is modelled to be flexible in such a way that the crack effect remains within a single element.

The breathing of the crack is modelled based on nodal displacements of cracked elements. When the displacement of nodes of the cracked element is greater than zero
then the crack is assumed to be open. Otherwise, the crack is considered to be closed. The equation of motion for the cracked beam under external excitation is:

\[ M\ddot{y}(t) + C\dot{y}(t) + K(t)y(t) = F(t) \] (5.3)

where \( M, C \) and \( K \) are the mass, damping and stiffness matrices for the beam respectively, \( y \) denotes vertical displacement and \( F \) is the vector of external force.

In Equation (5.3), stiffness matrix is time dependent, because it changes with the opening and closing of the cracks during the external excitation. When the crack is closed, no change happens in the stiffness; therefore, a healthy (global) stiffness matrix can be used in Equation (5.3). However, an open crack reduces the stiffness of the cracked element and this reduction needs to be taken into consideration. In the case where multiple cracks exist in the beam (Figure 5.2), the number and location of all open cracks at every time step should be determined to consider the stiffness reduction corresponding to each open crack in the global stiffness matrix as follows:

\[ K_c(t) = K - \sum_{p=1}^{n} \Delta K_{c,p}(t) \] (5.4)

where \( K_c \) is the stiffness matrix for the cracked beam, \( K \) is the stiffness matrix for the healthy beam (global), \( n \) is the number of open cracks and \( \Delta K_{c,p} \) is the reduction in the stiffness due to \( p \)-th open crack.

This reduction depends on the location and depth of the crack. The Newmark-\( \beta \) method is used to solve Equation (5.3) with time steps of 200\( \mu \)s assuming that the initial displacement and velocity is zero.

---

**Figure 5.2** Schematic view of a beam with multiple open and closed cracks (\( Cr_1 \) to \( Cr_n \) indicate the number of cracks)
Modal analysis is carried out to compute the natural frequencies of the beams, assuming all the cracks are open (Table 5.2). Beams are then excited by a sinusoidal input at their first natural frequency. It is quite important to avoid choosing the nodal point(s) of mode shapes as either the crack or excitation location. In order to simulate experimental conditions, a signal to noise ratio (SNR) of 20 dB is used to pollute the calculated acceleration responses with normalised random noise.

Fast Fourier transform (FFT) was used to convert the time-domain signals into frequency-domain with sampling frequency of 5 KHz. Figure 5.1 shows a typical noisy acceleration response from a specific node in time domain and its frequency spectrum for the fixed-fixed beam at mode 1 where the higher harmonics of the exciting frequency can be observed clearly.

<table>
<thead>
<tr>
<th>Case</th>
<th>Mode 1 [Hz]</th>
<th>Mode 2 [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam 1-N</td>
<td>14.754</td>
<td>40.922</td>
</tr>
<tr>
<td>Beam 2-N</td>
<td>14.823</td>
<td>40.382</td>
</tr>
<tr>
<td>Beam 3-N</td>
<td>14.757</td>
<td>40.292</td>
</tr>
<tr>
<td>Beam 4-N</td>
<td>6.475</td>
<td>25.775</td>
</tr>
<tr>
<td>Beam 5-N</td>
<td>6.485</td>
<td>25.957</td>
</tr>
<tr>
<td>Beam 6-N</td>
<td>6.482</td>
<td>25.788</td>
</tr>
</tbody>
</table>

5.4. Curvature of RODS

Figures 5.3-5.4 depict the numerical RODS obtained for the fixed-fixed and simply supported beams with external excitation at the first resonant frequency. Figures 5.3-5.4(a),(b) show the results for two cracks and Figs. 3-4(c) illustrate the RODSs when three cracks exist in the beams. It can be observed that the RODS figures show discontinuities at the location of cracks. However, these discontinuities are not sufficiently apparent for all the crack locations. For example, although Fig. 3(c) indicates sharp peaks at the location of the second and third cracks, the location of the first crack remains inconspicuous due to its lower amplitude compared to the other two cracks. In fact, the amplitude of discontinuity for most of the crack locations in Figs. 3-4 is affected by the curvature of the RODSs. Therefore, a better indication for the location of all cracks can be obtained by using the second derivative of the RODSs to
remove their curvature. The same concept has already been applied to mode shapes, and the second derivative of a beam’s mode shape is defined as mode shape curvature (Doebling et al. [108]). Pandey et al. [74] analysed a cantilever and a simply supported beam and used finite element analysis to obtain the displacement mode shapes of the two models. Their work demonstrated that changes in the curvature mode shapes were localized in the region of damage, while changes in the displacement mode shapes were not localized and hence could not give any information on the location of the damage. Finally, they confirmed the usefulness of the curvature mode shapes in detecting and locating the state of damage. Lestari et al. [109] developed a method for identifying damage in carbon/epoxy composite beams using curvature mode shapes. They also used a combined analytical and experimental approach to locate the damage through measured curvature mode shapes. The greatest disadvantage of the mode shape curvature method is the difficulty in obtaining the displacement mode shapes experimentally without noise. Because of the noise, the second derivative of the mode shapes may produce many sharp discontinuities which make the crack identification process complicated. Filtering the noise from the mode shapes is an option, but is not practical since the effects of the crack may also be removed. In contrast to mode shape, the RODS method uses the difference of the ODSs at the exciting frequency and its higher harmonic which consequently removes the effect of noise from measured responses. As a result, the curvature of the RODS is useful for amplifying the effect of cracks and determining their locations.

Figure 5. 3 The (RODS)_{2,1} results for (a) Beam 1-N, (b) Beam 2-N and (c) Beam 3-N at mode 1
Chapter 5: Improvement to the RODS method using curvature approach

5.5. Results and observations

The curvature (second derivative) of the RODS results is employed here to investigate its ability to magnify the crack effect. The second derivative can be calculated as:

\[
d^2(RODS_j) = \frac{(RODS)_{j+1} - 2(RODS)_j + (RODS)_{j-1}}{(\Delta x)^2}, \quad j = 2,3, \ldots \quad (5.5)
\]

where \( j \) refers to each RODS data point and \( \Delta X \) is the difference between adjacent data points.

Figures 5.5-5.6 present the application of the second derivative to the RODS results for the fixed-fixed and simply supported beams. As can be seen, the second derivative removes curvature from the RODSs and consequently provides a clear indication of the crack locations along the beams. Every single peak in the curvature of RODSs for mode 1 represents the location of a crack. According to the numerical simulations, curvature of RODS can be used as an accurate method to locate multiple cracks in beams irrespective of their boundary conditions.

5.6. Experimental examples

Experimental tests were carried out on clamped-clamped and free-free aluminium beams to investigate the reliability of the curvature of RODS method. Laser cutting was used to create a single crack in the free-free beam and two cracks in the clamped-
clamped beam. Details of the beams and the location of cracks are presented in Table 5.3. The depth and width of all the cracks are 6 mm and 0.2 mm respectively.

Figure 5.5 Curvature of RODS for (a) Beam 1-N, (b) Beam 2-N and (c) Beam 3-N at mode 1

Figure 5.6 Curvature of RODS for (a) Beam 4-N, (b) Beam 5-N and (c) Beam 6-N at mode 1

Figure 5.7 shows the test setup for the experiments. The complete description for setting up the free-free beam can be found in [107]. Figure 5.7(b) depicts the test setup for the clamped-clamped beam where fourteen accelerometers were installed at equal intervals of 19 cm along the beam to acquire vibration responses. The beam was placed on the supports and two U channels were used to clamp the sides and top of both ends in order to represent the fixed-fixed boundary condition to a feasible extent. The excitation force applied by the shaker to the beam was measured using a force gauge. The measured vibration signals were amplified and acquired simultaneously by a 16 bit data
acquisition card (Figure 5.8) to guarantee valid ODSs with correct magnitude and phase.

Table 5.3 Specifications of the healthy and cracked beams used in the experiment

<table>
<thead>
<tr>
<th>Case</th>
<th>Boundary condition</th>
<th>Length [mm]</th>
<th>Cross-section [mm$^2$]</th>
<th>No. of crack(s)</th>
<th>Crack location(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam 1-E (Healthy)</td>
<td>clamped-clamped</td>
<td>2550</td>
<td>25x100</td>
<td>--</td>
<td>----</td>
</tr>
<tr>
<td>Beam 2-E (Healthy)</td>
<td>Free-Free</td>
<td>3000</td>
<td>25x100</td>
<td>--</td>
<td>----</td>
</tr>
<tr>
<td>Beam 3-E</td>
<td>clamped-clamped</td>
<td>2550</td>
<td>25x100</td>
<td>2</td>
<td>X1=770, X2=1780</td>
</tr>
<tr>
<td>Beam 4-E</td>
<td>Free-Free</td>
<td>3000</td>
<td>25x100</td>
<td>1</td>
<td>X1=1500</td>
</tr>
</tbody>
</table>

* E refers to experimental beam

Figure 5.7 Test setup for (a) Beam 3-E and (b) Beam 4-E [107]

Figure 5.8 Schematic view of the experiment for the clamped-clamped beam
Modal testing was performed on both healthy and cracked beams in order to obtain the frequency response functions (FRFs). Figures 5.9-5.10 present the FRF amplitude and phase for the healthy and cracked clamped-clamped beams. The peak at the frequency of 37.23 Hz in Figure 5.9 and that at the frequency of 39.06 Hz in Figure 5.10 are related to the second natural frequency of the beams in a vertical direction (perpendicular to the direction of excitation). Table 5.4 summarises the first three natural frequencies of the beams.

<table>
<thead>
<tr>
<th>Case</th>
<th>Mode 1 [Hz]</th>
<th>Mode 2 [Hz]</th>
<th>Mode 3 [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam 1-E</td>
<td>17.70</td>
<td>54.93</td>
<td>103.10</td>
</tr>
<tr>
<td>Beam 2-E</td>
<td>14.95</td>
<td>39.98</td>
<td>78.43</td>
</tr>
<tr>
<td>Beam 3-E</td>
<td>17.09</td>
<td>63.78</td>
<td>105.00</td>
</tr>
<tr>
<td>Beam 4-E</td>
<td>14.95</td>
<td>39.98</td>
<td>78.74</td>
</tr>
</tbody>
</table>

It should be noted that the dimensions of the healthy and cracked beams differ slightly from each other. Also, the location of the clamps and the mounting places of the accelerometers might not be exactly the same in both, which might cause a slightly different mass distribution along each beam. Therefore, inconsistencies can be seen in the results for the second and third natural frequencies.

![Figure 5.9](image-url) Frequency response function at 80 cm from the right end for Beam 1-E
Chapter 5: Improvement to the RODS method using curvature approach

Based on the FRF results, the aluminium beams were excited by continuous sinusoidal wave at their first natural frequency through the shaker.

Since the RODS method is based on the ODSs at higher harmonic components, it is important to make sure that crack breathing has occurred during the experiments. Figure 5.11 demonstrates typical ordered frequency spectra for Beams 2-4-E when excited at their first mode. As can be observed, Beam 2-E (healthy) does not show most of the higher harmonic components. However, the spectra for Beam 3-E and 4-E confirm the breathing of crack(s) during external excitation by showing the higher harmonics components clearly.

Figure 5.12 illustrates the experimental RODS and its curvature at first mode for the free-free and clamped-clamped aluminium beams. The RODS plot for Beam 4-E (Figure 5.12(b)) clearly shows the location of the crack. This is due to the fact that the crack is located at the centre of the beam with maximum breathing at the first mode. However, discontinuities related to the crack locations in Beam 3-E are not noticeable in the RODS plot (Figure 5.12 (a)). The better indication of the crack locations in Beam 3-E can be seen in Figure 5.12(c) where the second derivative is used to remove the curvature and magnify the discontinuities in the RODS plots. The nearest sensors to the crack locations locate the cracks by presenting sharp peaks. The experimental results
confirm that the curvature of the RODS can be used to detect the location of multiple cracks in beam-like structures.

**Figure 5.11** Ordered frequency spectrum for (a) Beam 2-E, (b) Beam 3-E and (c) Beam 4-E

**Figure 5.12** Experimental RODS at mode 1 for Beam 3-E and 4-E (a, b) and their corresponding $d^2$RODS (c, d)

### 5.7. Concluding remarks

The application of the RODS method to beams with multiple cracks is presented. The method utilises the difference between the ODSs at the exciting frequency and its higher
Chapter 5: Improvement to the RODS method using curvature approach

harmonics to identify the location of the cracks. The numerical results indicated that in most of the cases, small discontinuities exist in the RODS plots at the location of cracks. As a result, the second derivative of the RODSs was used to eradicate the curvature and magnify those discontinuities that related to the cracks. Experimental tests were also carried out on free-free and fixed-fixed aluminium beams with single and multiple cracks respectively. It is shown that both the numerical and experimental examples confirm the appropriateness of the curvature of the RODS to detect the location of multiple cracks in beam-like structures.
CHAPTER 6: APPLICATION TO CRACKED ROTORS

Reformatted version of the following papers:


Abstract

This paper intends to extend the application of the residual operational deflection shape (RODS) to a simplified model of the LP turbine of a steam turbo generator. The numerical model consists of a stepped shaft which accommodates some balance disks to represent the blades at different stages of the LP turbine. The ability of the RODS method in detecting cracks in beam-like structures has already been presented. The method removes the effect of the first harmonic component (1x) from the ODSs at higher harmonic components to highlight the non-linear behaviour of cracks. In this paper, single and multiple cracks are introduced at different locations to the finite element model of the rotor. It is assumed the rotor can be taken out from the rig and mounted on a simply supported boundary condition. After carrying out the modal analysis, the rotor is excited at its first resonant frequency and acceleration responses along the rotor are computed to construct the ODSs. The RODS method uses the normalised ODSs for the purpose of crack detection. The curvature of RODS method is also presented for further improvement in indication of crack locations especially for multiple-crack rotors.
6.1. Introduction

Shafts play an important role in power transmission in rotating machines. Initiation and propagation of fatigue cracks are very common in rotating shafts since they are constantly subjected to fluctuating stresses and broad variations in temperature and environmental conditions. Early detection of fatigue cracks in the shaft of a rotor system is vital to prevent fatigue failure and consequent potential damage to other equipment and huge costs in down time.

Crack development changes the dynamic characteristics of the shaft. These changes are the main idea of many vibration-based methods for crack detection. A comprehensive overview of the vibration-based crack detection in shafts is presented in [110]. Breathing is the key feature of fatigue cracks which produces non-linear behaviour. Pennacchi et al [111] presented a model-based identification method for transverse breathing cracks in industrial machines. They simulated the crack effect on static and dynamic behaviour of the rotor in the frequency domain by applying equivalent forces of the cracked beam element to the rotor. The crack was then identified by an external force identification procedure. Chasalevris and Papadopoulos [112] investigated the cross-coupled bending vibrations of a rotating shaft with a breathing crack and resilient bearings. A continuous model for the cracked rotor-bearing system was offered to detect the existence of the crack by monitoring the change in the stiffness due to the change of crack local compliance. Darpe [113] proposed a detection method based on the breathing of the crack. The method uses the presence or absence of bending and torsional vibrations coupling at various orientations of the crack. Darpe applied a transient torsional excitation for a very short period of time and investigated the lateral vibrations. The transient features of the resonant bending vibrations were revealed by wavelet transforms in order to detect the crack. Sinou [114] used higher harmonic components of exciting frequency and the crack-unbalance interactions to identify the presence of a single breathing crack in a rotor system. By means of numerical simulations, it was demonstrated that for a given crack depth, the unbalance affects the amplitude of the $\frac{1}{2}$ and $\frac{2}{3}$ sub-critical resonant peaks as well as the 1x component which could be used to identify the crack. Sawicki et al [115] presented the modelling and analysis of rotors with breathing cracks. They utilised sinusoidal excitations to the shaft through active magnetic bearings (AMB). These excitations generated response
Chapter 6: Application to cracked rotors

frequencies combining the rotor spin speed and excitation frequency. The system was finally analysed using an approach based on the harmonic balance method and could be used with inverse methods for the purpose of damage detection.

Recently, the residual operational deflection shape (RODS) method has been proposed for crack detection in beam-like structures [84]. The RODS method removes the effect of first harmonic component (1x) from the ODSs at higher harmonics (2x, 3x, etc) to highlight the non-linear behaviour of the crack. This study intends to extend the application of the method by using a very simplified model of the LP turbine of a steam turbo generator. The model consists of a stepped shaft which accommodates some balance disks to represent the blades at different stages of the LP turbine. The reliability of the RODS method in detecting the location of single and multiple cracks in the shaft is investigated.

6.2. Finite element simulation

A rotor consisting of a stepped shaft with mounting balance disks is considered. The density and Young’s modulus of the rotor are 7850 Kg/m$^3$ and 210 GPa respectively. Figure 6.1 shows the dimensions of the rotor with a single crack. Here, it is assumed the rotor can be taken out from the rig and mounted on a simply supported boundary condition. Single and multiple cracks are introduced at different locations with depth ratio of 20% ($\frac{d_c}{D} = 0.2$). Table 6.1 shows the details of all the investigated cases.

A finite element model was created for the rotor using Euler-Bernoulli beam elements. There are two nodes defined per element and each node has two degrees of freedom; translational displacement and bending rotation. Stiffness proportional damping is obtained using the constructed stiffness matrix. The approach proposed in [80] is used here to introduce cracks to the finite element model of the rotor. This approach modifies the local flexibility in the vicinity of the crack within Euler-Bernoulli beam elements based on the concept explained in [81]. It assumes that the flexibility from uncracked to cracked section of the beam varies linearly and considers triangular reduction in the stiffness of cracked element. If this reduction happens over more than one element, the nodes of the model will be moved so that the crack effect remains within a single element.
Chapter 6: Application to cracked rotors

Table 6.1 Number and locations of cracks used in the rotor for numerical simulations

<table>
<thead>
<tr>
<th>Case</th>
<th>No. of Cracks</th>
<th>Crack Location [m]</th>
<th>Crack Depth [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>1</td>
<td>0.35</td>
<td>1.0</td>
</tr>
<tr>
<td>Case 2</td>
<td>1</td>
<td>0.85</td>
<td>1.2</td>
</tr>
<tr>
<td>Case 3</td>
<td>2</td>
<td>0.4, 0.9</td>
<td>1.0, 1.2</td>
</tr>
<tr>
<td>Case 4</td>
<td>2</td>
<td>0.4, 1.2</td>
<td>1.0, 1.0</td>
</tr>
</tbody>
</table>

The breathing of the crack(s) is modelled through sign recognition of nodal displacements of the cracked element(s). This means that when the displacement of nodes of the cracked element is greater than zero, the crack is open. Otherwise, the crack is assumed to be closed. The equation of motion for the cracked rotor under external excitation is:

\[ M \ddot{x}(t) + C \dot{x}(t) + Kx(t) = F(t) \]  \hspace{1cm} (6.1)

where M, C and K are the mass, damping and stiffness matrices for the rotor respectively and F is the external force.

Figure 6.1 Schematic view of the simply supported cracked rotor, (a) side view, (b) top view
Chapter 6: Application to cracked rotors

In Equation (6.1), the stiffness matrix is time dependent because it changes with opening and closing of the crack(s) during the external excitation. When the crack is closed, a healthy (global) stiffness matrix is used in Equation (6.1). However, the existence of cracks reduces the stiffness of the cracked elements and consequently the stiffness of the whole rotor. In order to consider the stiffness reduction corresponding to each open crack in the global stiffness matrix, the number and location of all open cracks are determined at every time step. The stiffness matrix in the presence of open cracks can be written as:

$$ K_c(t) = K - \sum_{p=1}^{n} \Delta K_{c,p}(t) $$  \hspace{1cm} (6.2)

where $K_c$ is the stiffness matrix for the cracked rotor, $K$ is the stiffness matrix for the healthy rotor (global), $n$ is the number of open cracks and $\Delta K_{c,p}$ is the maximum reduction in the stiffness due to $p$-th open crack.

Equation (6.1) is solved numerically using the Newmark-$\beta$ method with time steps of 200$\mu$s assuming that the initial displacement and velocity is zero. Modal analysis is carried out to compute the natural frequencies of the rotor assuming all the existing cracks are open. The first three calculated natural frequencies of the rotor in the vertical direction for all the cases are shown in Table 6.2. The rotor is then excited by a sinusoidal input at its first natural frequency. The calculated acceleration responses are polluted with normalised random noise of 20dB in order to simulate experimental condition. By using the Fast Fourier transform (FFT) the time-domain acceleration responses are converted into the frequency-domain with sampling frequency of 5 KHz. Figure 6.2 shows a typical noisy acceleration response in time domain and its frequency spectrum from a specific node at mode 1 where the higher harmonics of the exciting frequency can be observed clearly. These higher harmonics confirm the breathing of the crack during external excitation.
Chapter 6: Application to cracked rotors

Table 6.2 Computed natural frequencies

<table>
<thead>
<tr>
<th></th>
<th>Mode 1 [Hz]</th>
<th>Mode 2 [Hz]</th>
<th>Mode 3 [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>38.88</td>
<td>137.22</td>
<td>321.76</td>
</tr>
<tr>
<td>Case 2</td>
<td>38.77</td>
<td>138.81</td>
<td>315.49</td>
</tr>
<tr>
<td>Case 3</td>
<td>38.35</td>
<td>136.95</td>
<td>311.59</td>
</tr>
<tr>
<td>Case 4</td>
<td>38.50</td>
<td>134.67</td>
<td>314.66</td>
</tr>
</tbody>
</table>

Figure 6.2 Calculated acceleration response with noise in (a) time domain and (b) frequency domain for the cracked rotor at mode 1

6.3. The Curvature of RODS Method

Asnaashari and Sinha [116] proposed the curvature of RODS method in order to magnify the effect of crack in the RODS signals which is expressed in Equation (6.3):

\[
d^2(RODS)_j = \frac{(RODS)_{j+1} - 2(RODS)_j + (RODS)_{j-1}}{(\Delta x)^2}, j = 2, 3, \ldots \quad (6.3)
\]

where \( j \) refers to each RODS data point and \( \Delta X \) is the difference between adjacent data points.
Chapter 6: Application to cracked rotors

In fact Equation (6.3) is the second derivative of the RODS data which removes the effect of curvature for better indication of discontinuities related to the crack(s) in the RODS signal. The RODS data can be obtained as follows:

\[
(RODS)_{p,m} = (NODS)_{p,m} - (NODS)_{1,m}, \quad p = 2,3,4, \ldots \quad (6.4)
\]

where \((NODS)_{p,m}\) is the normalised ODS at \(p\)-th harmonic for mode \(m\).

Generally, as stated in Equation (6.4), the RODS method eliminates the effect of the exciting frequency (1x) from the normalised ODSs at the higher harmonic components. Therefore, The RODS only contains the non-linearity due to the crack(s) breathing. Indices \(p\) and \(m\) in Equation (6.4) are equal to 2 and 1 respectively in this paper because firstly the RODSs are calculated for 2x data and secondly the cracked rotors are excited at their first resonant frequency.

6.4. Results and Discussion

Figures 6.3-6.4 demonstrate the obtained RODS and their second derivatives for shafts with single and multiple cracks respectively. The method explained in [117] was used to obtain normal ODSs at the exciting frequency and its higher harmonic components. In the case of the rotor with a single crack (Figure 6.3(a), (c)) the RODS plot itself indicates the crack location clearly and the curvature magnifies the discontinuity even further. However, when there is more than one crack in the rotor the RODS plot may not provide a clear indication of all the crack locations, since the amplitude of discontinuity depends on the breathing intensity of each crack. This fact can be seen in Figure 6.4 for the rotor with multiple cracks. In such cases, the curvature of the RODS (Figure 6.4 (b), (d)) plots presents encouraging results for detecting the crack locations.
The rotor in Case 2 is used here to assess the ability of the curvature of RODS method to detect the severity of cracks. For this purpose crack sizes of 1.2cm, 2.4 cm and 3.0cm were used in the rotor. As Figure 6.5 demonstrates, increase in the crack depth reduces the magnitude of the RODS curvature.

Figure 6. 3 The $(\text{RODS})_{2,1}$ for Cases 1 and 2 (a,c) and their $d^2(\text{RODS})_{2,1}$ (b,d) respectively

Figure 6. 4 The $(\text{RODS})_{2,1}$ for Cases 3 and 4 (a,c) and their $d^2(\text{RODS})_{2,1}$ (b,d) respectively
Chapter 6: Application to cracked rotors

6.5. Conclusion

The application of the curvature of RODS method to the simplified LP turbine of a steam generator is presented in this paper. The numerical model consists of a stationary stepped shaft with multiple balance disks representing the blades of the turbine. The details of modelling the breathing fatigue crack(s) are discussed. Shafts with different numbers of cracks at various locations were excited at their resonant frequency. Afterwards, the acceleration response at different locations along the shafts was computed to construct the ODS of the cracked shafts. Normalised ODSs were then utilised to obtain the RODSs. The numerical results illustrated that the RODS method indicates the location of single-crack shaft by showing a sharp discontinuity. However, when multiple cracks exist in the shaft, the amplitudes of discontinuity relating to each crack may not have the same magnitude in the RODS plot. In other words, depending upon the intensity of breathing, the discontinuity generated by one crack might be significantly smaller than the other crack in the RODS plot. Therefore, the second derivative is used to remove the curvature of RODS plots so that all the discontinuities relating to the crack locations can be observed clearly.

Figure 6.5 Effect of crack size on the curvature of RODS
CHAPTER 7: COMPARATIVE STUDY BETWEEN THE DEVELOPED METHODS

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Abstract

Measured vibration responses can be processed either in their original time domain or after converting to the frequency domain for the purpose of damage detection. Recently, two new vibration-based methods have been presented separately which successfully identified the location of damage in beam-like structures. One of the methods utilises the vibration responses of cracked structures in time domain and is called deviation from normal distribution (DND). The DND method calculates the difference between actual and normal distribution of vibration responses in time domain to locate cracks. The other method is called residual operational deflection shape (RODS) which uses the amplitude and phase of the exciting frequency and its higher harmonics in the frequency domain as a method of crack detection. Previously, these methods were applied only to single cracked beams with sinusoidal excitation and their results were encouraging. Here, the application of the two methods has been extended to multiple cracked beams as well as rotors. Also, the effect of different types of excitations on the detection process using the two methods has been investigated through numerical and experimental examples.
Chapter 7: Comparative study between the developed methods

7.1. Introduction

Signal processing of measured vibration data plays an important role in damage or fault detection of structures/machines. Although vibration responses are recorded in the form of time history, the processing of responses can be performed either in the time or frequency domain.

Cattarius and Inman [67] presented a time domain procedure relying on measured time response of a healthy structure to examine structural damage. They compared the healthy signal to the measured time response of the identical structure with some local damage by subtracting them from one another. The resulting signal indicated the existence and extent of damage due to the change in local mass and stiffness of the structure. The requirement for data of a healthy structure can be considered the main limitation of this procedure. Nair et al [118] proposed a damage detection algorithm based on time series modelling. A new damage sensitive feature (DSF) based on the autoregressive (AR) coefficients was presented and a difference in the mean values of the DSF for the damaged and undamaged cases was observed. This difference was found to be useful for identifying the damage. For localising the damage, the authors investigated two localisation indices defined in the AR coefficient space; but only one index was found to be conclusive when there is minor damage in the structure. The validity of the proposed method requires further investigations [118]. Statistical classification algorithm for analysing a structure’s response in the time domain has also been used by Carden and Brownjohn [119]. They were able to identify the occurrence of damage by fitting the time-series responses with autoregressive moving average (ARMA) models and coefficients. Rzeszucinski et al. [120] developed a new technique for condition monitoring of helicopter gearboxes. Normal probability plot (NPP) was used to obtain the amount of deviation from normal distribution (DND) of a given signal in the time domain. The results of the proposed DND method correctly indicated a gear fault for the duration of the deterioration process. Asnaashari and Sinha [121] presented the application of the DND method to beam-like structures for crack detection. They assumed the vibration responses were available at a number of locations along the beam. The deviation of actual distribution of vibration responses from their normal distributions was then obtained. It was observed that measured vibration responses near the crack location indicated a higher or lower value of DND compared to other locations along the structure. Numerical and experimental investigations
demonstrated the ability of the DND method to identify the crack location accurately in beams with different crack locations and boundary conditions.

In many vibration-based damage detection methods, the vibration responses in the time domain are converted into the frequency domain using Fourier or wavelet transform for further processing. This is due to the fact that the frequency contents of vibration responses provide useful information regarding the existing damage in the structures or different types of fault in rotating machinery components.

Solis et al [122] used continuous wavelet transform to get time information of the frequency content of measured vibration signals. Their method requires the mode shapes of a reference undamaged structure as well as the cracked structure. Recently, Huda et al [123] proposed a vibration testing and health monitoring system to detect damage in membrane structures. The structure was excited by an impulse using a high power Nd: YAG laser. They analysed the vibration mode shapes of the structure by 2-D continuous transformation. The location of the crack was determined by applying a boundary treatment and the concept of an iso-surface to the 2-D wavelet coefficient. Hester and Gonzalez [124] investigated the possibility of using a bridge acceleration signal due to a vehicle crossing the bridge to determine if the bridge is damaged. They developed a wavelet-based approach using wavelet energy content at each bridge section, which was found to be more sensitive to damage than wavelet coefficient plots. Mark et al [125] suggested a simple frequency domain algorithm for early detection of damaged gear teeth. They utilised rotational-harmonic amplitudes of transducer responses caused by gear manufacturing errors and potential tooth damage. Absolute values of the logarithm of the ratio of after potential damage and before potential rotational-harmonic amplitudes were used to equally weight either increases or decreases in rotational-harmonic amplitudes. Finally, time-windowing tailored to the total contact ratio of the meshing gear pair was developed to allow damage detection based on threshold exceedances. Alamdari et al [126] proposed a method for localisation of structural damage in presence of heavy noise influences. The method uses the frequency response functions (FRFs) along with Gaussian kernel to suppress the noise. Shape signals were then developed using the denoised signals according to the second derivative of the operational mode shapes at frequencies in the half-power bandwidth of the centre resonant frequencies. A two-dimensional map created by normalization of the shape signals indicated the damage pattern.
Operational deflection shape (ODS) is another well-known tool [43, 95] for the purpose of damage identification and localisation in structures and machines. Applications of the ODS analysis to different structures can be found in [45, 106, 127-128]. Recently, a new method called residual operational deflection shape (RODS) has been developed [107] which removes the effect of exciting frequency from the ODSs at higher harmonic components of the exciting frequency. This paper intends to present a comparative study between the DND and the RODS methods for the purpose of crack detection in different structures. Previously, these methods were applied only to single cracked beams with sinusoidal excitation and their results were encouraging. Here, the application of the two methods has been extended to multiple cracked beams as well as rotors. In addition, the effect of different types of excitations on the detection process using the two methods has been investigated.

7.2. Concept of the two methods

Using the DND method, all the data processing for locating cracks in structures can be performed in the time domain. In fact, the DND method utilises the normal probability plot (NPP) to assess the normality of a dataset. The amount which the vibration data measured at a point on the structure deviate from normal distribution in the NPP is calculated and then compared to the amount of deviation at other measurement points along the structure (Equation 7.1). The DND plot indicated a single discontinuity showing the crack location in the structure with reasonable accuracy [121].

\[
DND = \sum_{i=1}^{n} \left( \frac{|Z_{a,i} - Z_{n,i}| + |Z_{a,i+1} - Z_{n,i+1}|}{2} \right) \times (\Delta X)
\]  

(7.1)

where \( n \) is the number of data points in a signal, \( Z_{a,i} \) and \( Z_{n,i} \) are the values of actual and normal distribution at \( i \)-th data point respectively and \( \Delta X \) is the distance between adjacent data points.

In the RODS method, measured vibration responses are converted into the frequency domain for further processing. The method is based on constructing the ODS of a cracked structure at the exciting frequency and its higher harmonic components in the frequency spectra obtained at different locations. The key feature of the RODS method is its ability in removing experimental noise as well as the effect of the ODS at exciting
frequency from those at higher harmonic components. As a result, non-linearity due to breathing of the crack remains in the resultant signal which has been found to be useful for detection (Equation 7.2).

\[(\text{RODS})_{p,m} = (\text{NODS})_{p,m} - (\text{NODS})_{1,m}, \quad p = 2, 4, 6, \ldots \quad (7.2)\]

where \((\text{NODS})_{p,m}\) is the normalised ODS at \(p\)-th harmonic for mode \(m\).

More details in computing the DND and the RODS methods can be found in [121] and [107] respectively. The intensity of discontinuities in the DND or RODS plots may vary depending upon the boundary condition and the number of cracks which exist in beam structures. In other words, as the number of cracks increases in the beam, the breathing strength for each single crack decreases during the beam vibration, which ultimately shrinks the discontinuities corresponding to the crack locations. Therefore, the second derivative of DND or RODS is used to magnify those discontinuities for better indication of crack locations.

### 7.3. Simulated examples

A comparative study between the DND and RODS methods has been carried out in this section. Initially, finite element models of an aluminium beam with rectangular cross-section and a rotor consisting of a steel stepped shaft with balancing disks were constructed using Euler-Bernoulli beam theory. The aluminium beam was fixed at both ends and it was assumed that the rotor could be tested offline as a simply-supported beam. Figure 7.1 shows schematic views of the two cases including important dimensions. Translational and rotational degrees of freedom were considered for each node of the beam elements.

The density and Young’s modulus of the beam and the rotor are 2700 Kg/m³, 74 GPa and respectively. The approach used to model the crack and also its breathing feature into the finite element simulation is detailed in [107]. For each case, different numbers of cracks at various locations along the beam/rotor were considered (Table 7.1) to present a comprehensive comparison between the two damage detection methods.
Chapter 7: Comparative study between the developed methods

Figure 7.1 Schematic view of the beam (a) and the rotor (b) for numerical simulation

Table 7.1 Details of the beams and rotor under investigation

<table>
<thead>
<tr>
<th>Case</th>
<th>No. of Crack(s)</th>
<th>Crack location[m]</th>
<th>Mode 1 [Hz]</th>
<th>Mode 2 [Hz]</th>
<th>Mode 3 [Hz]</th>
<th>Mode 4 [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam 1</td>
<td>1</td>
<td>0.8</td>
<td>14.86</td>
<td>40.74</td>
<td>79.96</td>
<td>132.79</td>
</tr>
<tr>
<td>Beam 2</td>
<td>2</td>
<td>1.0, 2.4</td>
<td>14.83</td>
<td>40.62</td>
<td>79.80</td>
<td>131.62</td>
</tr>
<tr>
<td>Rotor 1</td>
<td>1</td>
<td>1.3</td>
<td>13.37</td>
<td>48.04</td>
<td>108.47</td>
<td>197.88</td>
</tr>
<tr>
<td>Rotor 2</td>
<td>2</td>
<td>0.55, 1.7</td>
<td>13.30</td>
<td>47.48</td>
<td>108.74</td>
<td>196.49</td>
</tr>
</tbody>
</table>

Table 7.1 also illustrates the first four natural frequencies obtained from modal analysis for each scenario. It is necessary to know these frequencies for generating the excitation force. Finally, the equations of motion for the beam and rotor were solved using the Newmark-β method in order to obtain vibration responses. In this paper, acceleration responses are used for further processing.

Previously, the results for the DND and the RODS methods were investigated only for single frequency sine wave excitations at one of the modes. However, exciting a structure at its resonant frequency may not always be possible. On the other hand, most structures are subjected to different types of excitations during their operating life. For example, a bridge is continuously experiencing random excitations due to vehicles passing over it. As a result, this paper aims to assess the ability of the two methods in locating cracks when the structures are subjected to hammer and random excitations in
addition to the sine waves. In addition, the DND method is tested when multiple cracks are present in the structures.

### 7.3.1. External excitation

Single frequency sine wave, impulse hammer and random excitations were used in the numerical analyses. In the case of the sine wave, the structures were excited at their first resonant frequency to provide maximum breathing of the crack(s). However, the frequency variation for the impulse hammer and random excitations was selected to be from 0 to almost 200 Hz. This range covers the first four natural frequencies of the beam and the rotor. Figures 7.2-7.3 demonstrate the time and frequency domain for generated hammer and random excitations. Low-pass filter with the cut off frequency of 200 Hz was applied to the random signal.

---

**Figure 7.2** Impulse hammer excitation in the time domain (a) and frequency domain (b) used for numerical simulations

**Figure 7.3** Random excitation in the time domain (a) and frequency domain (b) used for numerical simulations
7.3.2. Simulated example 1: Fixed-Fixed beam

Vibration responses corresponding to each type of excitation were calculated in terms of acceleration at a number of locations along the fixed-fixed beam. The computed acceleration responses were then processed using both the DND and RODS methods for localising the crack(s). Figures 7.4-7.5 present the second derivative or curvature of DND and RODS results for single and multiple cracked beams respectively. It can be seen in Figure 7.4 that the RODS plots provide excellent identification of crack location(s) irrespective of the type of excitation. Subscript \( h \) in \((RODS)_{2,h}\) refers to the hammer excitation. Apart from some small differences in the amplitude, the curvature of RODS results for both the hammer and random excitations follow the same trend, which is considered one of the method’s advantages. Conversely, the DND curvatures show different numbers of discontinuities for the hammer and random excitations. As observed before in [121], the DND curvature illustrates nodal points of the beam’s mode shapes in addition to the crack locations. This feature of the DND method can also be seen in Figure 7.5. In the case of hammer excitation, the nodal point(s) for the second (at 1.5 m) and third (at 1.0 m and 2.0 m) modes are dominant in the DND plots in which corresponding peaks have appeared. In Figure 7.5(a) the crack was located at 0.8m, where a peak has appeared very close to the first nodal point of the third mode shape. Figure 7.5(b) presents the result for two cracks. Due to the presence of the first crack near the nodal point at 1.0m, the peak amplitude at this location became higher compared to the other nodal points. The location of the second crack has also been identified clearly. Poor indication of crack locations was observed when the DND curvature method was applied to vibration responses of the beam subjected to the random excitation (Figure 7.5(c,d)). In fact, in Figure 7.5(c, d) the nodal points of all the excited modes have affected the DND curvature results, which make the detection process almost impossible.
Chapter 7: Comparative study between the developed methods

Figure 7.4 Curvature of RODS obtained from hammer (a,b) and random (c, d) excitations for Beam 1 (a, c) and Beam 2 (b, d)

Figure 7.5 Curvature of DND obtained from hammer (a,b) and random (c, d) excitations for Beam 1 (a, c) and Beam 2 (b, d)
Chapter 7: Comparative study between the developed methods

7.3.3. Simulated example 2: Simply-supported rotor

The aim here is to present the application of the two methods to the rotor system shown in Figure 7.1 with a stepped shaft and a number of balance disks. The rotor is excited by the sine wave and hammer excitations since random excitation is very unlikely to happen in rotor systems. As in the beam case, the curvature of RODS is successful in localising the damage for both single and multiple cracked rotors. The results for the curvature of DND method are subjective. Two peaks at 0.8m and 2.0m can be seen in all cases, representing the steps between the 5cm and 6cm diameter shafts. For the single cracked rotor, a negative peak can be seen at the crack location when the sine wave was used to vibrate the beam; but no clear indication can be observed for the hammer excitation. In the case of multiple cracked rotors, the results for both of the excitations provide accurate indication for the location of the first crack. However, the second crack was dominated by the sharp peak at 2.0m.

Figure 7.6 Curvature of RODS obtained from hammer (a,b) and random (c,d) excitations for Rotor 1 (a, c) and Rotor 2 (b, d)
Chapter 7: Comparative study between the developed methods

7.4. Experiment

Experimental investigations were conducted on a clamped-clamped aluminium beam in order to compare the DND and the RODS methods. Two types of excitations; single frequency sine wave and impulse hammer, were used (Figure 7.8). The dimensions of the beam were the same as those in the numerical analysis. Two cracks with a depth and width of 6 mm and 0.2 mm respectively were created in the beam at the locations of 77 cm and 178 cm.

As shown in Figure 7.8, the beam was placed on the supports and two U channels were used to clamp the sides and top of both ends in order to represent the fixed-fixed boundary condition. A number of accelerometers were installed at equal intervals along the beam to acquire vibration responses. Correct magnitude and phase of ODSs were
guaranteed by acquiring all the responses simultaneously using a 16 bit data acquisition card. The frequency response functions (FRF) of the cracked beam are shown in Figure 5.10.

The impulse hammer has the frequency of 0 to 200 Hz and the sine wave has the single frequency of Hz which is the first resonant frequency of the cracked beam. Time synchronous averaging of responses with triggered level of applied force was used. Figures 7.9-7.10 present the results for the application of the DND and the RODS methods to the experimental data. The impulse hammer excitation was also applied to a healthy clamped-clamped beam with the same dimensions (Figure 7.9(a)). It can be observed that since the two cracks are located very close to the third mode nodal points of the clamped-clamped beam, small discontinuities at the nodal points for the healthy beam have become sharp peaks in the DND curvature plot for the cracked beam (Figure 7.9(b)). Simultaneously, Figure 7.9(b) shows that the discontinuity at the nodal point for the second mode (1.275m) has not been changed because there was no crack near to that point. In the case of sinusoidal excitation (Figure 7.9(c)), the beam was excited at its first mode, which does not have any nodal point; therefore, the curvature of DND plot only demonstrates the location of the two cracks. As expected from the numerical analyses, the results for the curvature of RODS (Figure 7.10) provide a precise indication of crack locations which are impervious to nodal points.
Chapter 7: Comparative study between the developed methods

Figure 7.9 Experimental results for curvature of DND for (a) healthy beam (b) cracked beam with hammer excitation and (c) cracked beam with sinusoidal excitation.

Figure 7.10 Experimental results for curvature of RODS for (a) cracked beam with sine excitation and (b) cracked beam with hammer excitation.
Chapter 7: Comparative study between the developed methods

7.5. Concluding remarks

This paper presents a comparative study between the DND and the RODS methods for crack detection in beams and rotors. Different types of external excitations are used in the numerical and experimental examples in order to provide a comprehensive comparison.

The obtained results for the beams and the rotors confirm that the curvature of RODS method can detect the crack location successfully irrespective of the type of excitation. Processing the data in the frequency domain enables the RODS method to focus on the ODS at the exciting frequency and its second harmonic only. The RODS method not only removes the effect of the exciting frequency but also eliminates the noise and the effect of the steps in the shaft of the rotors investigated from the ODSs at the second harmonic component. As a result, only the nonlinearity due to the crack breathing remains in the signal, which is magnified when the second derivative is applied.

Although the effectiveness of the curvature of DND method has already been shown [121], there are some limitations in using this method. The method processes the vibration data in their original time domain and works quite well when applied to vibration data of beam-like structures at the first mode of excitation. However, in addition to the crack, the second derivative of the DND plot is sensitive to the location of nodal point(s) and also changes in structural geometry like stepped shafts. Thus, if the crack is located near any nodal point or geometry change, detection of the crack becomes difficult. This fact can be observed specifically in the case of random excitation.

Both the DND and RODS methods are based on the breathing of the crack(s). This means that the external excitation should be high enough to make the breathing happen. Otherwise, the two proposed methods cannot be used for crack detection. Also, a number of sensors are required to be mounted along the structure under investigation. This can be done practically by using a laser Doppler vibrometer which is capable of measuring vibration responses at a very large number of points. The location of the crack can then be identified by the sensor which has the closest distance to the crack. Therefore, the accuracy of identifying the crack location(s) increases by utilising more number of vibration sensors.
CHAPTER 8: CONCLUSIONS AND FUTURE WORK

8.1. Summary

Detection of cracks at an early stage of their development is an important part of any structural health monitoring system to prevent failures. Over the past few decades, vibration-based methods have been widely used in identifying and locating cracks in different types of structures. These methods are mainly based on variations in local flexibility of the structure due to the presence of cracks. To determine the most suitable method to use, parameters such as robustness, feasibility of in-situ testing, simplicity and economic factors should be considered.

In this thesis, two new vibration-based methods have been developed for crack detection in beam-like structures and rotors. Initially, the finite element models of the beams and rotors were constructed using Euler-Bernoulli beam elements. Afterwards, the mass and stiffness matrices were calculated for each element in order to obtain the equations of motion of the structures. Proportional damping was used for numerical simulations and measured modal damping was utilised for experimental simulations. For the case of cracked structures, the triangular reduction in the stiffness of the cracked
element(s) was used to introduce the crack to the finite element model. Another step in modelling the cracked structures was to consider the non-linear behaviour of fatigue cracks due to the breathing phenomenon. This behaviour was expressed mathematically (Chapters 2, 3) in order to be used for numerical simulations. Modal analysis was carried out for each case study to understand the dynamics by obtaining its natural frequencies, mode shapes and damping ratios. Finally, vibration responses of the structures in terms of displacement, velocity and acceleration due to an external excitation were computed by employing a numerical integration method to solve the equations of motion.

8.2. Achieved objectives

The following paragraphs present the main objectives of this thesis which have been achieved:

- **Objective 1:** Many damage detection methods need information about the intact state of the structure to be compared to its damaged state. However, for most structures, especially older ones, the desired data for the intact state may not be available. The two developed methods in this thesis are completely reference-free, which means they do not require information on the undamaged state of structures for damage detection.

- **Objective 2:** Although the presence of a crack reduces the stiffness and consequently the natural frequencies of the damaged structure, this change is usually negligible unless the crack is of a certain size. In this research, the results confirmed the sensitivity of the proposed methods even to small cracks.

- **Objective 3:** Vibration responses are generally acquired in the form of a time history. However, the signal processing required to obtain information regarding the crack location can be performed either in the time or frequency domain. In this thesis, the two developed methods processed the vibration responses in the two aforementioned domains. The methods are also compared to each other to understand the advantages and limitations of each domain.
Chapter 8: Conclusions and Future Work

- **Objective 4**: Various laboratory experiments were conducted as part of this research on beams under different boundary conditions and different input excitations to validate the proposed methods.

- **Objective 5**: Numerical simulations were carried out in order to extend the application of the DND and RODS methods to rotor-type structures.

- **Objective 6**: The effect of the type of input excitation on the accuracy of locating the crack in beams and rotors was investigated. Sinusoidal waves, hammer impulse and random excitation were used to investigate the robustness of the proposed methods.

### 8.3. Overall conclusion

The two developed methods processed the vibration responses in two different domains. The first one, which is called the DND method, used the normal probability plot (NPP) to assess the normality of a given response. The amount of deviation from normal distribution was then used as the means of locating cracks. RODS is the second method, which performed the data processing in the frequency domain. The method mapped the deflection of cracked structures at the exciting frequency and the second harmonic component to obtain the ODSs. The effect of the ODS at the exciting frequency was then eliminated from those at the second harmonic components, to make the identification of cracks much clearer.

First of all, beams with a single crack under different boundary conditions, including cantilever, free-free and simply supported, were modelled numerically and tested experimentally to investigate the effectiveness of the developed methods. All the beams were excited at their first and/or second modes of vibration. It was shown that the DND plots indicated a small discontinuity at the crack location; however, larger discontinuities were found at the nodal point(s) of the exciting modes where there were no vibrations. After applying the first and second derivatives to the DND data, the latter was selected due to its better ability in removing the curvature from the DND plots. As a result, the second derivative of the DND data (or curvature of DND) enabled the crack related discontinuities to be clearly distinguishable from those of the nodal points. Both
the numerical and experimental examples validate the effectiveness of the RODS method to identify the location of the crack accurately. In addition to the simplicity of the method, the obtained results showed that the RODS is not sensitive to numerical and experimental noise; moreover, off-resonance excitation can be used as per field convenience.

Secondly, the RODS method was applied to beams with multiple cracks. It was observed that the indication of crack location was not sufficiently apparent for all the cracks due to different breathing intensity. Therefore, an improved method called curvature of RODS was introduced and applied to the numerical and experimental beams. The improved method removed curvature from the RODSs and consequently provided a clear indication of all the crack locations along the beams. The robustness of the curvature of RODS method was also tested on a simplified LP turbine of a steam generator modelled as a stepped shaft with multiple balance disks representing the blades of the turbine. Single and multiple cracks were introduced at various locations to the finite element model of the rotor. The results obtained from the numerical simulations confirmed the accuracy of the applied method for the purpose of crack detection in stationary (off-line) rotors.

Finally, a comparative study between the DND and RODS methods was carried out for different input excitations. The results obtained for the beams and the rotors confirmed that the curvature of RODS method can detect the crack location successfully irrespective of the type of excitation. The RODS method successfully eliminated the noise and the effect of the steps in the shaft of the rotors from the ODSs at the second harmonic component. Therefore, only the nonlinearity due to the crack breathing remains in the signal which is magnified when the second derivative is applied. On the other hand, the DND method worked quite well when applied to vibration responses at first mode of excitation. However, in addition to the crack, the second derivative of the DND plot indicated the location of nodal point(s) and also changes in structural geometry such as the steps of the shaft. Thus, if the crack was located near any nodal point or geometry change, detection of the crack would become difficult. This fact could be observed specifically in the case of random excitation where multiple modes with their corresponding nodal points were covered.
8.4. Future Areas of Research

The methods developed were shown to be robust for application to beam- and rotor-like structures. However, it is necessary to test these methods further on other types of structures. Moreover, other signal processing techniques such as wavelet transform can be used along with the two methods. Thus, the future areas of research include:

- Application of the proposed methods could be extended to 3D frame and plate-like structures with cracks. Also, these methods could be tested for detection of delamination in composite plates.

- Other types of cracks such as inclined or horizontal cracks could be modelled and examined using the developed methods.

- The reliability of the developed methods could be investigated and qualified further by applying them to real-life or industrial structures. For example, they could be employed as part of an online vibration-based condition monitoring system for key structures such as bridges and wind turbine towers.

- The indication of the crack location using the proposed methods could be improved to obtain better discrimination between the negative and positive peaks.

- Wavelet transform is another tool to compute the harmonic responses of cracked structures for which the RODS concept could be implemented.
References


[43] M. Richardson, "Is It a Mode Shape, or an Operating Deflection Shape?," *Sound & Vibration Magazine*, 1997.


APPENDIX A: MATHEMATICAL MODEL-BASED DAMAGE DETECTION

A.1. Introduction

A model-based approach is proposed here in order to identify the location of cracks in beam-like structures. A mathematical model is constructed from vibration responses of beams in frequency-domain. The presence of damage changes the dynamic behaviour of a structure. The approach is based on the idea that the damage-induced change in the vibrational behaviour can be represented by additional loads acting on the linear undamaged structure. These additional loads are in the form of forces and moments which generate the same dynamic behaviour as the actual non-linear damaged structure does. An investigation is done to understand whether it is possible to detect cracks in structures through the estimation of the mentioned forces and moments.

A.2. Finite element model

A finite element model of an Euler–Bernoulli beam with different boundary conditions is constructed. Table A.1 shows the specifications of the beam with square cross-section.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus</td>
<td>210 GPa</td>
</tr>
<tr>
<td>Density</td>
<td>7800 Kg/m³</td>
</tr>
<tr>
<td>Length</td>
<td>1 m</td>
</tr>
<tr>
<td>Width</td>
<td>0.015 m</td>
</tr>
<tr>
<td>Depth</td>
<td>0.015 m</td>
</tr>
</tbody>
</table>

The steel beam is divided into 10 elements each has a length of 0.1m. There are two nodes per element and two degrees of freedom are considered per node; the translational displacement and bending rotational.
A.3. Model theory

The equation of motion of the beam subjected to an external force can be written as:

\[
\begin{bmatrix}
Z_r & Z_{rt} \\
Z_{tr} & Z_t
\end{bmatrix}
\begin{bmatrix}
r_r \\
r_t
\end{bmatrix} = \begin{bmatrix}
F_r \\
F_t
\end{bmatrix} + \begin{bmatrix}
F_{cr} \\
F_{ct}
\end{bmatrix}
\tag{A.1}
\]

where \(Z\) are the dynamic stiffness matrices, the subscripts \(r\) and \(t\) refer to rotational and translational degrees of freedom (RDOF, TDOF) respectively, and \(r\) are the responses. \(F_r\) and \(F_t\) are external forces acting on the RDOF and TDOF respectively. Forces \(F_{ct}\) and moments \(F_{cr}\), generated due to the existence of crack, are to be estimated.

Solving Equation (A.1) to eliminate the unknown rotational responses gives:

\[
AF_{cr} - F_{ct} = AF_r + Br + F_t
\tag{A.2}
\]

where \(A = Z_{tr}Z_r^{-1}\) and \(B = Z_{tt}Z_r^{-1}Z_{rt} - Z_t\).

The external excitation is applied at a TDOF of the beam; therefore, \(F_t\) is zero. It is assumed that vibration responses at all TDOF are available. Thus, the only unknown quantities in Equation (A.2) are the crack’s forces and moments.

A.3.1. Modelling the dynamic stiffness matrices and external excitation

Dynamic stiffness matrix can be written as:

\[
Z = [-\omega^2 M + j\omega C + K]
\tag{A.3}
\]

where \(M\), \(C\), and \(K\) are the mass, damping, and stiffness matrices respectively, and \(\omega\) is the angular frequency of the external excitation.

As it can be understood from Equation (A.1), the RDOF and TDOF are separated since there is no information regarding the RDOF. As a result of this, four different dynamic stiffness matrices are created in Equation (A.1), each of which takes the related DOF from the rows and columns of the mass, damping, and stiffness matrices.

Single frequency excitation is applied at the TDOF of node 7. This means that \(F_t\) is a 10 by 1 matrix in size which all its values are zero except the 7th one.
A.3.2. Forces and moments estimation and regularization

By changing the right-hand side of Equation (A.2) into matrix format and considering \( F_r \) being zero we have:

\[
\begin{bmatrix} A & -I \end{bmatrix} \begin{bmatrix} F_{cr} \\ F_{ct} \end{bmatrix} = Br_t + F_t \quad (A.4)
\]

The crack forces \( (F_{ct}) \) and moments \( (F_{cr}) \) are placed together in Equation (A.4). However, it is possible to separate them from each other so that the number of unknowns can be reduced. This reduction in the number of unknowns results in better estimation of the unknown parameters. To do this, two transformation matrices, \( T_m \) and \( T_f \), are used to separate the crack moments and forces from Equation (A.4) respectively as expressed below:

\[
\begin{bmatrix} F_{cr} \\ F_{ct} \end{bmatrix} = [T_m]{F_{cr}} \quad (A.5)
\]

and

\[
\begin{bmatrix} F_{cr} \\ F_{ct} \end{bmatrix} = [T_f]{F_{ct}} \quad (A.6)
\]

Substituting Equations (A.5) and (A.6) into Equation (A.4) gives:

\[
\begin{bmatrix} A & -I \end{bmatrix}[T_m]{F_{cr}} = Br_t + F_t
\]

\[
\begin{bmatrix} A & -I \end{bmatrix}[T_f]{F_{ct}} = Br_t + F_t \quad (A.7)
\]

Equation (A.7) is a least-square problem and its solution is likely to be ill-conditioned. Therefore, regularization needs to be done in order to solve the problem. Clustering and truncated singular value decomposition (CSVD, TSVD) are used to solve the equations.

A.4. Final results and discussion

According to the explained theory, estimations of forces and moments for the beam with three different boundary conditions are presented in the following figures. It should be noted that all the estimations are based on 2x responses as other higher harmonics were unable to generate reliable estimations.
Figure A.1 corresponds to the cantilever beam. As it can be seen, estimated crack moments can clearly show the location of the cracks while estimated forces do not show clear indications. It is also demonstrated in Figure A.1 that the estimated moments are sensitive to the change of the location of cracks.

In contrast to the cantilever beam, estimated crack forces provide better indications for the location of cracks in the case of fixed-fixed boundary condition. They are also sensitive to the crack locations. Figure A.2(b-1) shows a negative sharp peak close to the location of crack. However, moments in Figure A.2 are not able to identify the cracks and not sensitive to different crack locations either.

As illustrated in Figure A.3, neither the estimated forces nor the moments provide clear identifications of the location of cracks for the free-free beam. Although Figure A.3(c-1) shows a sharp peak at 0.5m which is close to the location of crack (0.45), there is another peak at 0.8m where no crack exists.

In conclusion, the mathematical model seems to be robust for crack detection in cantilever beams and beams with fixed ends. However, its estimations need to be improved for free-free beams.
Figure A.15 The estimations of crack forces (a-1, b-1, c-1) and crack moments (a-2, b-2, c-2) for the cantilever beam with different crack locations, (a) 0.25m, (b) 0.35m, (c) 0.45m from the fixed end.
Figure A.16 The estimations of crack forces (a-1, b-1, c-1) and crack moments (a-2, b-2, c-2) for the fixed-fixed beam with different crack locations, (a) 0.25m, (b) 0.35m, (c) 0.45m from one end.
Figure A.17 The estimations of crack forces (a-1, b-1, c-1) and crack moments (a-2, b-2, c-2) for the free-free beam with different crack locations, (a) 0.25m, (b) 0.35m, (c) 0.45m from one end.
APPENDIX B: MODEL-BASED DAMAGE DETECTION USING A LIMITED NUMBER OF MEASUREMENT POINTS

B.1. Introduction

Appendix A presented a mathematical model for the purpose of crack detection in beams with different boundary conditions. The model estimated the crack forces and moments by assuming that the vibration data at all the translational degrees of freedom are available. This is possible by using a laser vibrometer which is able to scan all the vibration data of surfaces or plates completely. Although laser vibrometers are very useful for collecting a large amount of vibration data, there are some limitations in using them. Firstly, they cannot get the vibration data of all points on a structure which is not fully accessible. Secondly, laser vibrometers need to be placed at a specific minimal distance from the structure being scanned. However, such minimal space may not be always available in industrial sites. In these situations, a limited number of typical vibration sensors like accelerometers can be used for data collection. This appendix presents a mathematical model aiming to identify the location of crack in a cantilever beam while measurements have been taken at a limited number of points on it.

B.2. Model Theory

Considering the same beam in the Appendix A, the equations of motion can be written as:

\[
\begin{bmatrix}
Z_m & Z_{mi} \\
Z_{im} & Z_i
\end{bmatrix}
\begin{bmatrix}
\mathbf{r}_m \\
\mathbf{r}_i
\end{bmatrix}
= 
\begin{bmatrix}
F_m \\
F_i
\end{bmatrix} + 
\begin{bmatrix}
F_{cm} \\
F_{ci}
\end{bmatrix}
\]

(B.1)

where \(Z\) are the dynamic stiffness matrices, the subscripts \(m\) and \(i\) refer to measured and non-measured degrees of freedom (MDOF, NMDOF) respectively, and \(\mathbf{r}\) are the responses. \(F_m\) and \(F_i\) are external forces acting on the MDOF and NMDOF respectively.
Forces ($F_{cm}$) and moments ($F_{ci}$), generated due to the existence of crack, are to be estimated.

Solving Equation (B.1) to eliminate the unknown non-measured responses gives:

$$AF_{ci} - F_{cm} = Br_m - AF_i + F_m$$  \hspace{1cm} (B.2)

where $A = Z_m Z_i^{-1}$ and $B = Z_m Z_i^{-1} Z_{im} - Z_m$.

The external excitation is applied at one NMDOF of the beam; therefore, $F_m$ is zero. In this chapter, it is assumed that vibration responses at three MDOF of a cantilever beam are available. Thus, the only unknown quantities in Equation (B.2) are the crack’s forces and moments. Table B.1 shows the different sets of measurement nodes selected for the investigation.

**Table B.5 Different sets of measurement nodes**

<table>
<thead>
<tr>
<th>Set</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement nodes</td>
<td>3,8,10</td>
<td>2,8,10</td>
<td>2,7,10</td>
<td>2,5,9</td>
</tr>
</tbody>
</table>

Single frequency excitation is applied at node 7 again. Since measurement is not taken at the node of the excitation force, $F_i$ is a 19 by 1 matrix in size because of having 19 NMDOF.

**B.2.1. Forces and moments estimation and regularization**

By changing the right-hand side of Equation (B.2) into matrix format and considering $F_m$ being zero we have:

$$[I - A]\begin{bmatrix} F_{cm} \\ F_{ci} \end{bmatrix} = AF_i - Br_m$$  \hspace{1cm} (B.3)

$F_{cm}$ and $F_{ci}$ are separated from each other to reduce the number of unknowns. This reduction in the number of unknowns results in better estimation of the unknown parameters. For this reason, two transformation matrices, $T_m$ and $T_f$ are used as below:

$$\begin{bmatrix} F_{cm} \\ F_{ci} \end{bmatrix} = [T_m][F_{ri}]$$  \hspace{1cm} (B.4)

and
\[
\begin{bmatrix} F_{cm} \\ F_{ci} \end{bmatrix} = \begin{bmatrix} T_f \end{bmatrix} \begin{bmatrix} F_{cm} \end{bmatrix}
\] (B.5)

Substituting Equations (B.4) and (B.5) into Equation (B.4) gives:

\[
[A - I][T_m] \begin{bmatrix} F_{ci} \end{bmatrix} = AF_i - Br_m
\]

\[
[A - I][T_f] \begin{bmatrix} F_{cm} \end{bmatrix} = AF_i - Br_m
\] (B.6)

Equation (B.6) is a least-square problem and its solution is likely to be ill-conditioned. Therefore, regularization needs to be done in order to solve the problem. Clustering and truncated singular value decomposition (CSVD, TSVD) are used to solve the equations.

### B.3. Results and discussion

The following figures show the results obtained from 2x data for different sets of measurement nodes on the cantilever beam. Each figure consists of crack forces and moments estimated by the mathematical model. Crack is located at the distance of 0.35m along the beam.
Figure B.13 The estimated crack forces (a-1) and moments (a-2) for set 1.

The estimated crack forces and moments for set 1 are shown in Figure B.1. A sharp peak at 0.3m which is very close to the location of crack can be seen in the moment estimations (Figure B.1 (a-2)). However, there is another unwanted peak at 0.7m where no crack is present. Almost the same trend as in Figure B.1 can be observed in Figure B.2. Again, two peaks are appeared in Figure B.2 (a-2), one of which is near to the crack location. Neither the force estimations nor the moment estimations of Figures B.3 and B.4 can clearly indicate where the crack is located at.
**Figure B.14** The estimated crack forces (a-1) and moments (a-2) for set 2.

**Figure B.15** The estimated crack forces (a-1) and moments (a-2) for set 3.
Figure B.16 The estimated crack forces (a-1) and moments (a-2) for set 4.

Considering Figures B.1 to B.4., it can be concluded that the force and moment estimations of the proposed model are dependent on the set of nodes at which measurements have been taken. Therefore, the model is not robust yet in order to be used for different applications.