Essays on Public Capital and Economic Growth

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Abstract

In recent years, there has been an increasing interest in the fact that public capital, or public infrastructure, plays a crucial role in determining economic growth in low-income countries. This thesis studies the links among public capital, economic development, and growth, using overlapping generations (OLG) models.

Chapter 1 develops a three-period, gender-based overlapping generations model of endogenous growth with endogenous intra-household bargaining and child labour in home production by girls. Improved access to infrastructure reduces the amount of time parents find optimal for their daughters to spend on household chores, thereby allowing them to allocate more time to studying at home. The model is calibrated for a low-income country and various quantitative experiments are conducted. This includes an increase in the share of public spending on infrastructure, an increase in time allocated by mothers to their daughters, and a decrease in fathers’ preference for their daughters’ education. Our analysis shows that poor access by families to infrastructure may provide an endogenous explanation, beyond social norms and cultural values, for the persistence of child labour at home and gender inequality in low-income countries.

In Chapter 2, the link between infrastructure and industrial development is studied in an OLG model with endogenous skill acquisition. Industrial development is defined as a shift from an imitation-based, low-skill economy to an innovation-based, high-skill economy where ideas are produced domestically. Imitation generates knowledge spillovers, which enhance productivity in innovation. Changes in industrial structure are measured by the ratio of the variety of imitation-based to innovation-based intermediate goods. The model also distinguishes between basic infrastructure, which helps to promote learning-by-doing and productivity in imitation activities, and advanced infrastructure, which promotes knowledge networks and innovation. Numerical experiments, based on a calibrated version for a low-income country, show that changes in the level and composition of public investment in infrastructure may have significant effects on the structure of the labour force and the speed of industrial development.
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I declare that no portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.
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Dedication

To my beloved parents for their love and support
First and foremost, I would like to express my sincere gratitude to Professor Pierre-Richard Agénor, my research supervisor, for his invaluable guidance, patience, and advice. Without his supervision this thesis would not have been possible. I would also like to thank Professor Keith Blackburn for his helpful comments on a previous version of Chapter 1 of this thesis.

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Introduction

Human development (both economic and social) has continued to improve in recent times but at different rates in different regions, due to a variety of factors. Statistics show disparity, such as when proportional and nominal changes are compared. Although governing bodies such as the United Nations recognise basic needs, for instance, access to energy, transportation, and water and sanitation (which should be provided for everyone) there are still large numbers of people without these necessities.

The overall problem which has yet to be resolved has led to policymakers considering more holistic approaches that help to resolve the cause rather than just treat the symptoms, whilst also providing basic services which are needed to maintain basic levels of health and welfare. One possible solution which has been proposed is for an increase in investment in public capital or public infrastructure. Not only will it provide necessary services, it will also boost economic growth.

This thesis consists of two substantive chapters which seek to address the links among public capital, economic development, and growth in low-income countries. The content of each chapter has been summarised below.

The purpose of Chapter 1 is to study intra-household bargaining that matters for gender equality and economic growth: the impact of women’s bargaining power on girls’ time allocation to home schooling, that is, girls’ time allocated to school-related activities at home and therefore on their human capital accumulation by emphasising the role of access to infrastructure. To conduct our analysis, we have developed a three-period (childhood, adulthood, and retirement), gender-based overlapping generations (OLG) model of endogenous growth in which only girls’ time allocated to

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1The terms “public capital” and “public infrastructure” are often used interchangeably in the field, although the term public capital in its broadest sense refers to school buildings, hospitals, public libraries, and all other physical assets. However, we mainly focus on “core” public capital, including energy, water and sanitation, transportation, and information and communications technology (see Agénor (2012b) for a more detailed discussion).
household chores (which is a form of child labour, an expression often used to refer to work outside the home) is endogenously related to access to infrastructure, whereas the amount of time that boys spend on home schooling is exogenous. We also assume intergenerational altruism, which operates from mothers to daughters (mothers care more than fathers about the human capital of their daughters), while men care more about current consumption than women. In this chapter, gender inequality is an equilibrium outcome connected to the way household members endowed with individual preferences interact with each other and make decisions about girls’ time allocation, as well as a number of noneconomic factors such as social norms, religious beliefs, and cultural values.

We characterise the long-run properties of the model and consider the impact of an increase in spending on infrastructure. The model is calibrated for a low-income country. To illustrate the role of public policy in the model, we consider several experiments: an increase in investment in infrastructure, financed by either a cut in unproductive spending (which helps to highlight changes in girls’ time allocation) or a cut in another productive share of spending, namely, education (to emphasise the trade-offs that policymakers may face in allocating their resources), a reallocation of mothers’ time toward girls, a reduction in the sensitivity of the endogenous component of bargaining power to relative stocks of human capital, and a reduction in fathers’ preference for their daughters’ education. A sensitivity analysis for alternative values of some parameters that are critical to illustrate the effects of experiments is also reported.

The results show that policies aimed at promoting an increase in family access to infrastructure will not only improve education outcomes for girls but also boost economic growth. Improved access to infrastructure may help girls to allocate less time to household chores and therefore allow them to devote more time to school-related activities at home and invest more in human capital. It also promotes productivity and wages in their adult life, as well as improved bargaining power in
terms of resource allocation within the family. This increase in women’s bargaining power reduces the amount of time parents find optimal for their daughters to spend on household chores if mothers relatively more value their daughters’ education. The results also show these positive effects in the case where trade-offs arise when an increase in spending on infrastructure is financed by a cut in spending on education, especially if spending more on infrastructure is offset by spending reductions on an inefficient or corrupt bureaucracy, which is the case in education systems in low-income countries (see for instance UNESCO (2009, Chapter 3)).

In Chapter 2, we have developed a two-period (adulthood and old age) overlapping generations (OLG) model of endogenous growth and horizontal innovation à la Romer (1990) to study the role of public capital in industrial development. We first account for two types of design activities: imitation that involves essentially copying foreign ideas and adapting foreign technology, and innovation that involves inventing new ideas. Changes in industrial structure are measured by changes over time in the ratio of imitation-based and innovation-based production inputs, rather than changes in the relative share of manufacturing production in total final output. Industrial development is thus defined as a shift from an imitation-based, low-skill economy to an innovation-based, high-skill economy where ideas are produced domestically. Our analysis also accounts for the existence of a learning externality, associated with imitation activities, which enhances productivity in the innovation sector. If learning externalities are sufficiently strong, imitation may be the main source of productivity growth (at least in an initial phase) and investment in human capital is not a prerequisite for promoting growth in the early stages of development. Second, the process of skill acquisition is endogenised; individuals choose to acquire skills depending on the relative returns to schooling. While imitation requires only unskilled labour, innovation requires high-skilled labour. Third, the model distinguishes between basic infrastructure (roads, energy, and basic telecommunications), which helps to promote learning-by-doing and productivity in imitation activities,
and advanced infrastructure (advanced information and communication technologies (ICTs) in general, and high-speed communication networks in particular), which promotes knowledge networks and innovation.

The model is calibrated for a low-income country. Numerical experiments show that the structure of the labour force and the speed of industrial development may be significantly affected by changes in the level and composition of public infrastructure. Our experiments suggest that in the early stages of development, investing in basic infrastructure may initiate a process of industrial development and promote growth in the case where unskilled labour is abundant and skilled labour in short supply. The process of industrialisation through the learning externality associated with imitation activities increases the demand for high-skilled labour (which puts upward pressure on skilled wages) and incentives for individuals to invest in higher education, especially in the early stages of development.
Chapter 1

Child Labour, Intra-Household Bargaining and Economic Growth

1.1 Introduction

The role of women in promoting growth and development continues to occupy centre stage in policy debates and academic circles alike. Much evidence suggests that gender inequality in terms of access to education, health, formal sector employment, and income continues to be a significant constraint on human development and growth in many developing countries.\(^1\) For instance, although in many countries gender parity has been achieved in primary and secondary school enrolment, in many others—especially in Sub-Saharan Africa—girls go to school much less frequently than boys. In developing countries, nearly 1 of every 5 girls who enrol in primary school do not complete their primary education and only 43 percent of girls attend secondary school (UNICEF (2007, 2012a)). In low-income countries, only 5 to 10 percent of students are female. According to the International Labour Office (2012, p. 16), the gender gap in the labour force participation rate decreased globally in the 1990s from 27.9 to 26.1 percentage points. However, between 2002 and 2012, it remained largely constant. In 2012, the labour force participation rate for women was only 31.8 percent in South Asia, compared with 81.3 percent for men; for Latin

\(^1\)See Blackden and Bham (1999), Blackden et al. (2006), Herz and Sperling (2004), Morrison et al. (2007), Momsen (2009), Jütting et al. (2010), and World Bank (2011).
America and the Caribbean, these rates were 49.6 and 79.5, and for the Middle East, 18.7 and 74.3, respectively. When they do work, women often face less favourable employment opportunities and often end up in “bad jobs,” with poor prospects of escaping precarity and vulnerability.

The causes of gender inequality (both at home and in the workplace) are complex and include a wide range of economic and noneconomic factors, such as social norms, cultural values, religious beliefs, and inadequate social institutions. A few contributions on this issue have focused on women’s bargaining power—or lack thereof—in the family as a possible structural cause of inequality between husbands and wives. Basu (2006), for instance, developed a collective household model in which spouses have different utility functions and the power balance in the family is endogenously related to decisions made by the family itself via consumption and labour supply. His analysis shows that in some cases the equilibrium outcome may be characterised by persistence in gender inequality.

This chapter follows the same perspective, but focuses on an alternative mechanism through which intra-household (cooperative) bargaining may matter for gender equality and economic growth: the impact of women’s bargaining power on girls’ human capital accumulation. The premise of our analysis is that bargaining between spouses may bias the allocation of family resources toward girls and may have major effects on their ability to accumulate human capital when young. This is likely to affect their productivity and capacity to generate income in adulthood. In addition, we also take a macro perspective on women’s bargaining power, by emphasising the role of access to infrastructure. If such access is poor, girls may be forced by their parents to engage in household chores—an important form of child labour, as discussed by Fors (2012)—thereby limiting their ability to generate human capital in

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2 Another strand of the literature has focused on endogenising social institutions themselves, at the local or national levels. Strulik (2011) for instance studied how community attitudes affect school attendance and child labour, and how aggregate behaviour of the community feeds back onto the formation of schooling attitudes. His analysis has obvious implications for gender inequality as well.
childhood and restraining their bargaining power in adulthood. The weaker women’s intra-household bargaining is to begin with, the greater the adverse effect on girls’ time allocated to home schooling and the weaker their human capital later in life. Thus, poor access to infrastructure may explain persistence in gender inequality.

To conduct our analysis we have developed a three-period, gender-based overlapping generations (OLG) model of endogenous growth in which only girls are involved in child labour (in the form of work at home, that is, time allocated to household chores, rather than work outside the home). This is consistent with the evidence from a wide range of developing countries. Webbink et al. (2012), for instance, in an extensive study of 16 African and Asian countries, found that about 30 percent of African children and 11 percent of Asian children work over 15 hours a week in what they call hidden child labour—family and business work. Girls are more involved in housework whereas boys tend to work in the family business. In the same vein, in Bolivia, Zapata et al. (2011) found that girls are 51 percent more likely than boys to be out of school and working, mostly in domestic activities. In Guatemala, more than 90 percent of child domestic workers are girls (UNICEF (2007, p. 48)). Reggio (2011) found that in Mexico an increase in a mother’s bargaining power—measured in terms of ownership of family assets and the decision-making process related to those assets—is associated with fewer hours of work, including housework, for her daughters but not for her sons.

In the model, intra-household bargaining is endogenous and depends on the relative level of human capital of men and women. Girls’ time is combined with access to infrastructure to produce home goods and parents choose how much time their daughters must allocate to home production. The key mechanism that we highlight is that improved access to infrastructure reduces the amount of time that girls spend on child labour.

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3 Gender-based OLG models include a seminal paper by Galor and Weil (1996), and subsequent contributions by Greenwood et al. (2005), de la Croix and Vander Donckt (2010), and Agénor (2012a). Other important recent contributions on the economics of gender include Doepke and Tertilt (2011), which is further discussed later, and Fernández (2013). None of these papers, however, considers jointly the issues of child labour and gender inequality, as we do here.

4 See also the references in Edmonds (2008) and Webbink et al. (2012).
parents find optimal for their daughters to spend on household chores, which allows them therefore to allocate more time to studying at home—thereby enhancing the human capital that they build in childhood and use in adulthood. In turn, this increase in human capital, to the extent that it occurs at a relatively faster rate than boys’ human capital, may improve women’s bargaining power. If mothers relatively more value the education of their daughters, this shift in bargaining power may further reduce the amount of time that the family finds optimal for girls to spend in home production. The benefits of improved access to infrastructure over time and across generations are therefore magnified.\textsuperscript{5} Thus, our analysis shows that poor access by families to infrastructure may provide an endogenous, “macro” explanation—in addition to studies emphasising solely social norms and attitudes, and religious or cultural factors—for the persistence of child labour at home and gender inequality in low-income countries.

The remainder of the chapter is organised as follows. Section 2 describes the model, while Section 3 characterises the balanced growth equilibrium and illustrates analytically the transitional and steady-state effects of an increase in public investment in infrastructure, taking into account endogenous intra-household bargaining. Section 4 presents a benchmark calibration for low-income countries. The approach that we propose here is to calibrate the steady-state solution of the model and focus therefore on the long-run effects of public policy, because of the fact that many of these policies are structural in nature and unlikely to produce tangible economic results in the short-run. In Section 5, several experiments designed to illustrate the properties of the model are discussed, including (again) an increase in investment in infrastructure, a reallocation of mothers’ time toward girls, a reduction in the sensitivity of women’s bargaining power, and a reduction in fathers’ preference for their daughters’ education. Section 6 offers some concluding remarks and discusses

\textsuperscript{5}The shift in bargaining power may also tilt the allocation of the family’s resources toward children in general and girls in particular—a mechanism for which there is much empirical evidence, even though we do not dwell much on it in the present chapter.
possible extensions of the analysis.

1.2 The Model

We now present a three-period, gender-based overlapping generations (OLG) model of economic growth with public capital that incorporates intra-household bargaining.

Formally, we consider an OLG economy where two goods are produced, a marketed commodity and a home good. Individuals live for (at most) three periods. This represents childhood, adulthood (or middle age), and retirement (denoted \( t - 1, t, t + 1 \), respectively). The marketed commodity can be either consumed in the period it is produced or stored to yield capital at the beginning of the following period. Each individual is either male or female, and is endowed with one unit of time in childhood and adulthood, and zero units of time in old age. Children are born with the same innate abilities and depend on their parents for consumption and any spending associated with schooling. Girls and boys are endowed with one unit of time. Boys allocate their time between school and homework only, whereas girls allocate their time between school, homework, and household chores.\(^6\) The latter activities are viewed here as a form of child labour, an expression often used to refer to work outside the home. Mothers’ time allocated to child rearing and market work is considered exogenous.\(^7\)

All individuals, both males and females, work in middle age. The only source of income is therefore wages in the second period of life, which serve to finance family consumption in adulthood and old age. In adulthood, individuals also match randomly into couples with someone of the opposite sex to form a family. All income is pooled, and couples therefore become joint decision makers. For simplicity, once

\(^6\)Note that we do not consider child “domestic workers,” that is, children (girls, for the most part) who work in other people’s households, doing domestic chores, caring for children, etc.; see UNICEF (1999, 2012b) and International Labour Office (2013).

\(^7\)Endogenising women’s time allocation could of course be pursued, as for instance in Agénor (2012a) and Agénor et al. (2012). However, our focus here is solely on girls’ child labour, and we therefore abstract from that issue. We return to it in our concluding remarks.
married, individuals do not divorce; couples retire together and die together. Each couple produces a constant number \( n \geq 2 \) of children. It is also assumed that parents’ preferences over boys and girls are the same, and that they have control over the gender composition of their family, so that half of their children are daughters and half of them sons. Rearing children involves both parental time and spending on marketed commodities to feed them and send them to school. Male spouses allocate inelastically all their time to market work. Due to exogenous factors (such as social or cultural norms), mothers incur the whole time cost involved in rearing children.

In addition to individuals, the economy is populated by firms and an infinitely-lived government. Firms produce marketed commodities using public capital in infrastructure as an input, in addition to male and female labour and private capital. Home production (which positively affects the family’s utility) combines girls’ time and infrastructure services. Only girls are engaged in home production. The government invests in infrastructure and spends on education, as well as some unproductive items. It taxes the wage income of adults (males and females), but not the interest income of retirees. It cannot borrow and therefore must run a balanced budget in each period. Finally, all markets clear.

### 1.2.1 Home Production

Home production (which includes cooking dinner, doing laundry, cleaning the house, etc.) involves combining girls’ time, in proportion \( c_i^{g,P} \), and infrastructure services. For tractability, we assume that these factors are perfect substitutes and that pro-

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8 The assumption that spouses die together, or soon after the passing of the other, is consistent with the evidence on the so-called broken heart syndrome—clinically known as stress cardiomyopathy—according to which sudden emotional stress related to the loss of a close family member can trigger acute heart failure.

9 Thus, our analysis does not address that source of gender bias; see Zhang et al. (1999) for instance.

10 The model could be extended to account for the use of marketed goods as inputs in the production of home good, as for instance in Siegel (2011). However, this would significantly complicate the analysis without adding much insight, given the issue at stake.
duction, $Q_t$, takes place under decreasing returns to scale:

$$Q_t = [0.5n\varepsilon_{t}^{g,P} + \zeta^{P}(\frac{K_{t}^{I}}{K_{t}^{P}})]^{\pi^{Q}}, \quad (1.1)$$

where the superscript $g$ is used to identify girls, $0.5n$ the number of daughters in the family, $K_{t}^{I}$ is the stock of public capital in infrastructure, $K_{t}^{P}$ the aggregate stock of private capital, $\pi^{Q} \in (0, 1)$, and $\zeta^{P} \in (0, 1)$ a coefficient that parameterises the degree of substitutability between girls’ time and infrastructure services. Thus, greater access to roads or electricity allows girls to devote less time to home production. With better access to roads, for instance, girls do not need to walk long hours to fetch water and collect wood, especially in rural areas (see Food and Agriculture Organisation (2010)). Access to infrastructure is not excludable but subject to congestion (and thus partially rival), discussed in the next section.\textsuperscript{11}

\subsection*{1.2.2 Market Activity}

Firms are identical and their number is normalised to unity. They produce a single nonstorable commodity, using male effective labour, $E_{t}^{m}N_{t}^{m,i}$, where $E_{t}^{m}$ is average male human capital and female effective labour, defined as $\varepsilon_{t}^{f,w}E_{t}^{f}N_{t}^{f,i}$, where $E_{t}^{f}$ is average female human capital and $\varepsilon_{t}^{f,w}$ time allocated by mothers to market work, private capital, $K_{t}^{P,i}$ and public infrastructure. Although public capital is nonexcludable, it is partially rival because of congestion effects; for simplicity, congestion is taken to be proportional to the aggregate private capital stock, $K_{t}^{P} = \int_{0}^{1} K_{t}^{P,i}di$. Thus, the more firms use public infrastructure services in the production process (as measured by their private capital stock), the smaller the stock of those assets available for use by firms and households (see equation (1.1)).

\textsuperscript{11}Note also that the assumption of nonexcludability (no agent, individual or firm, can prevent other agents from using it concomitantly) is important here to justify the introduction of the aggregate stock of public capital in the production functions for the home and market goods.
The production function of individual firm $i$ takes the form

$$Y_i^t = \left( \frac{K_i^t}{K^F} \right)^{\alpha} (E^m_i N^m,i)^{\beta} (E^{f,W} N^{f,i})^{\beta} (K^{P,i})^{1-2\beta}, \quad (1.2)$$

where $\beta \in (0, 1)$. The elasticity of output with respect to male and female labour is assumed to be the same.\(^{12}\)

With the price of the marketed good normalised to unity, profits of firm $i$ in the final sector, $\Pi_{i,t}$, are given by

$$\Pi_{i,t} = Y_i^t - (w^m_i E^m_i N^m,i + w^f_i E^{f,W} N^{f,i}) - r_t K^{P,i},$$

where $r_t$ is the rental rate of private capital (which is also the rate of return on savings), $w^m_i$ the effective male wage, and $w^f_i$ the effective female wage.

Given our emphasis on intra-household bargaining (or gender inequality in the family), we abstract from gender discrimination in the workplace.\(^{13}\) Thus, profit maximisation with respect to private inputs, taking factor prices as given, yields

$$w^m_i = \frac{\beta Y_i^t}{E^m_i N^m,i}, \quad w^f_i = \frac{\beta Y_i^t}{E^{f,W} N^{f,i}}, \quad r_t = \frac{(1-2\beta) Y_i^t}{K^{P,i}}. \quad (1.3)$$

In equilibrium, the superscript $i$ can be dropped. And given that men and women are in equal numbers in the adult population ($N^m_i = N^f_i$),

$$w^m_i = \frac{E^{f,W} E^{f}_i}{E^m_i} w^f_i. \quad (1.4)$$

Given that all firms are identical, and that their number is normalised to 1,

\(^{12}\)In practice, $2\beta$ is in the range 0.6-0.7, consistent with the observed share of labour income in output.

\(^{13}\)The two issues may not be unrelated. Chichilnisky (2008), in particular, studies a game with incomplete information about women’s work at home and in the marketplace. Expectations about women’s lower wages lead to women bearing the brunt of household chores, and this, in turn, hampers their productivity and lowers their wages in the marketplace. Inequality at home fosters inequality in the marketplace and vice versa, and both combine to generate persistence in the gender gap.
\(K_t^P = K_t^{P,i} \forall i\), and aggregate output \(Y_t\) is, from (1.2),

\[
Y_t = \int_0^1 Y_t^i \, di = (k_t^I)\alpha (E_t^m N_t^m)^\beta (E_t^f \varepsilon_{t}^{f,W} N_t^f)^\beta (K_t^P)^{1-2\beta},
\]

where \(k_t^I = K_t^I / K_t^P\) is the public-private capital ratio. Equivalently, this expression can be rewritten as

\[
Y_t = (k_t^I)\alpha (E_t^m N_t^m / K_t^P)^\beta (E_t^f N_t^f / K_t^P)^\beta (\varepsilon_{t}^{f,W})^\beta K_t^P.
\] (1.5)

### 1.2.3 Time Allocation and Utility

To raise their children mothers must spend \(\varepsilon_{t}^{f,R} \in (0, 1)\) units of time on each of them. For simplicity, child rearing involves no direct cost in terms of marketed commodities.

In addition to raising children, mothers allocate time to market activity (in proportion \(\varepsilon_{t}^{f,W}\)). The time that females can devote to market activity is thus\(^{14}\)

\[
\varepsilon_{t}^{f,W} = 1 - n\varepsilon_{t}^{f,R}.
\] (1.6)

Let \(\varepsilon^{S}\) denote the indivisible amount of time that boys and girls must both allocate to formal (out of home) schooling. The time allocated by boys (identified with the superscript \(b\)) and girls to home schooling is thus

\[
\varepsilon^{b,L}_{t} = 1 - \varepsilon^{S},
\] (1.7)

\[
\varepsilon^{g,L}_{t} = 1 - \varepsilon^{S} - \varepsilon^{g,P}_{t},
\] (1.8)

where, as noted earlier, \(\varepsilon^{g,P}_{t}\) denotes the amount of time that girls allocate to home

\(^{14}\text{As noted earlier, because we assume that the fertility rate is exogenous and constant at } n, \text{ and } \varepsilon^{f,R} \text{ is exogenous, } \varepsilon^{f,W} \text{ is exogenous as well. We introduce them explicitly, however, because these are important to provide a realistic numerical calibration of the model, as discussed later on.}\)
production. For simplicity, we do not explicitly account for the fact that (older) girls may also allocate time to rearing their (younger) siblings, given that we consider only one period in childhood.

The family’s (collective) utility takes the composite form

\[ U_t = \xi_t U^f_t + (1 - \xi_t) U^m_t, \]

where \( U^j \) is partner \( j \)’s utility function and \( \xi_t \in (0, 1) \) is a weight that measures the wife’s bargaining power in the household decision process. Perfect equality corresponds therefore to \( \xi_t = 0.5 \). As shown by Doepke and Tertilt (2011), maximising (1.9) subject to appropriate constraints and for \( \xi_t \) given yields an outcome that is similar to the solution of a Nash bargaining problem in which the couple maximises the weighted product of the two partners’ marital surpluses and the outside option is given by the utility achieved upon divorce.16

Families consume both the marketed commodity and the good produced at home. Assuming that the consumption of children is subsumed in the family’s consumption, the sub-utility functions are given by, with \( j = m, f, \)

\[ U^j_t = \eta^j_C \ln c^j_{t-1} + \eta_Q \ln Q_t + \eta^j_E \ln e^f_{t+1} + \frac{1}{1 + \rho} \ln e^f_{t+1}, \]

where \( c^j_{t-1} \) and \( c^j_{t+1} \) are the family’s total consumption in adulthood and old age, respectively, \( e^f_{t+1} \) a unit of human capital of a female, and \( \rho > 0 \) a common discount rate. Coefficients \( \eta^j_C \) measure the relative preference for today’s consumption, \( \eta_Q \) the family’s common relative preference for the home produced good, and \( \eta^j_E \) the relative preference for girls’ education. The restrictions \( \eta^f_C < \eta^m_C \) and \( \eta^f_E > \eta^m_E \) are also imposed. Thus parents benefit equally from consumption of the home good;

---

15 Note that only girls’ time is endogenously related to public infrastructure, whereas the amount of time \( \varepsilon_{b,L} \) that boys spend on home schooling is exogenous. In addition, we abstract from leisure time for either type; this does not alter the analysis as long as such time is exogenous.

16 Doepke and Tertilt also develop a noncooperative model of household bargaining that has similar implications to the type of cooperative bargaining framework used here.
\( \eta_Q \) does not depend on \( j \). But women are less concerned than men about current consumption (\( \eta^f_E < \eta^m_E \)) and care more about the human capital that their daughters have accumulated by the time they become adults (\( \eta^f_E > \eta^m_E \)).\(^{17}\) Therefore there is intergenerational altruism, but it matters more for mothers. Note that only the marketed commodity is consumed in old age.

A male (female) adult in period \( t \) is endowed with \( e_t^m \) (\( e_t^f \)) units of human capital. Each unit of human capital earns an effective market wage, \( w_t^m \) for men and \( w_t^f \) for women, per unit of time worked.

The family’s budget constraints for periods \( t \) and \( t+1 \) are given by

\[
e_t^{t-1} + s_t = (1 - \tau)w_t^T, \tag{1.11}
\]

\[
e_t^{t-1} = (1 + r_{t+1})s_t, \tag{1.12}
\]

where \( \tau \in (0, 1) \) a constant tax rate, \( s_t \) savings, and \( w_t^T \) gross wage income of the family, defined as

\[
w_t^T = e_t^m w_t^m + \varepsilon_t^{W} e_t^f w_t^f. \tag{1.13}
\]

From equations (1.11) and (1.12), the family’s consolidated budget constraint is

\[
e_t^{t-1} + \frac{e_t^{t-1}}{1 + r_{t+1}} = (1 - \tau)w_t^T. \tag{1.14}
\]

Families maximise (1.9), taking \( z_0 \) as given, subject to (1.10) and (1.14), with respect to \( e_t^{t-1}, c_t^{t-1}, \) and \( \varepsilon_t^{g,p} \), which as shown below affects girls’ human capital in adulthood.

\(^{17}\)For some evidence on these facts, see UNICEF (2007), Doepke and Tertilt (2011), and World Bank (2011).
1.2.4 Human Capital Accumulation

Boys and girls have access to the same “out of home” learning technology. However, each group’s education outcomes depend also on the amount of time that parents devote to tutoring them at home.

Let \( e_{t+1}^j, j = m, f \) be the human capital of males and females born in period \( t \) and used in period \( t + 1 \). The production of either type of human capital requires several inputs. First, it depends on the time mothers allocate to tutoring their children. A sequential process is considered whereby mothers determine first the total amount of time allocated to child rearing, \( \varepsilon^{f,R} \), and then subdivide that time into a fraction \( \chi^R \in (0, 1) \) allocated to sons and \( 1 - \chi^R \) allocated to daughters.\(^{18}\) A bias in parental preferences toward boys can therefore be captured by assuming that \( \chi^R > 0.5 \).

Second, knowledge accumulation depends on average government spending on education per child, \( G_t^E/n0.5N_t \), where \( N_t \) is the number of adults alive in period \( t \), itself given by

\[
N_t = n0.5N_{t-1},
\]

that is, the number of children born in period \( t - 1 \), \( n \), times the number of families formed in \( t - 1, 0.5N_{t-1}.\(^{19}\)

Third, human capital accumulation depends on a mother’s human capital. Because individuals are identical within a generation, a mother’s human capital at \( t \) is equal to the average human capital of the previous generation, \( E_t^f \). Finally, although time spent in school affects equally the human capital of boys and girls at \( t + 1 \), girls’ human capital depends also on the amount of time that they allocate to school-related activities at home.\(^{20}\)

\(^{18}\)The analysis could be extended to account for boys’ education in the family’s utility function and solve optimally for the time that mothers allocate to them, \( \chi^R \). In the present setting, we take it as being determined by social norms.

\(^{19}\)We therefore abstract from the possibility that government spending on education may itself be subject to gender bias; see, for instance, Masterson (2012).

\(^{20}\)Our analysis would remain conceptually the same as long as time allocated to home schooling affects boys and girls differently, with a higher marginal effect for girls. The key point is that the
Thus, abstracting from gender-based discrimination in the public education system itself, and assuming no depreciation for simplicity, the human capital that men and women have in the second period of life is $^{21,22}$

\[
e_{t+1}^m = \left( \frac{G^E_t}{n0.5N_t} \right)^{v_1} (E^f_t)^{1-\nu_1} \left( \chi^{R,j} e^{f,R} \right)^{\nu_2} (\varepsilon^S)^{\nu_3}, \tag{1.16}
\]

\[
e_{t+1}^f = \left( \frac{G^E_t}{n0.5N_t} \right)^{v_1} (E^f_t)^{1-\nu_1} \left( \chi^{R,j} e^{f,R} \right)^{\nu_2} (\varepsilon^S + \varepsilon^g)^{\nu_3}, \tag{1.17}
\]

where $\nu_1 \in (0,1)$, $\nu_2$, $\nu_3 > 0$, and

\[
\chi^{R,j} = \begin{cases} 
\chi^R & \text{for } j = m \\
1 - \chi^R & \text{for } j = f 
\end{cases}, \tag{1.18}
\]

Combining equations (1.16) and (1.17) yields

\[
\frac{e_{t+1}^m}{e_{t+1}^f} = \left( \frac{\chi^R}{1 - \chi^R} \right)^{\nu_2} \left( \frac{\varepsilon^S}{\varepsilon^S + \varepsilon^g} \right)^{\nu_3}, \tag{1.19}
\]

which implies that if girls are involved in household chores (assuming that parents choose their daughters’ time allocated to household chores), home schooling could be positively related with the family’s access to infrastructure. Improved access to infrastructure reduces the amount of time parents find optimal for their daughters to spend on household chores, and therefore allows daughters to allocate more time to studying at home. Note also that a reduction in $\chi^R$ (that is, an increase in rearing time that girls can allocate to school-related activities at home (if any) is determined residually, given the time constraint, time spent in school, and time spent on household chores, which is determined by the family’s utility maximisation problem.

$^{21}$For tractability, the human capital technology is taken to exhibit constant returns to scale in government spending and the average human capital of mothers. Note also that we abstract from the impact of infrastructure on human capital; see for instance Yamauchi et al. (2011) for some country evidence and Agénor (2012b) for an overview. Accounting for this externality would strengthen the main policy conclusion of this chapter, if it is stronger for girls; otherwise it would not affect the relative human capital ratio and therefore would not affect women’s bargaining power.

$^{22}$In principle the term $\varepsilon^S + \varepsilon^{h,L}$, rather than $\varepsilon^S$ alone, should enter in equation (1.16), to ensure symmetry with (1.17). However, $\varepsilon^{h,L}$ is constant in the analysis and this would not have any qualitative effect on the behaviour of bargaining power and human capital accumulation.
time allocated to daughters) raises the relative level of girls’ human capital.

1.2.5 Government

As noted earlier, the government taxes only the wage income of adults. It spends $G^I_t$ on infrastructure investment, $G^E_t$ on education, and $G^U_t$ on other (not directly productive) items. All its services are provided free of charge. It cannot issue bonds and must therefore run a balanced budget:

$$G_t = \Sigma G^j_t = \tau(w^m_t E^m_t N^m_t + w^f_t \varepsilon^{J,W} E^f_t N^f_t). \quad (1.20)$$

Shares of spending are all assumed to be constant fractions of government revenues:

$$G^j_t = v_j \tau(w^m_t E^m_t N^m_t + w^f_t \varepsilon^{J,W} E^f_t N^f_t), \quad j = E, I, U \quad (1.21)$$

where $v_j \in (0, 1)$ for all $j$. Combining these equations therefore yields

$$v_E + v_I + v_U = 1,$$

or equivalently, this equation can be rewritten as

$$\sum v_j = 1. \quad (1.22)$$

Assuming full depreciation for simplicity, public capital in infrastructure evolves according to\footnote{Although here we focus on the case where only the flow of public investment determines the accumulation of public capital in infrastructure, in Appendix 1.7 we consider the more general case where existing public capital is an essential input in the production of public capital in infrastructure.}

$$K^I_{t+1} = G^I_t. \quad (1.23)$$
1.2.6 Bargaining Power

We now examine what determines women’s bargaining power, $z_t$. In the literature, women’s bargaining power has been related to, or measured by, a variety of measures: the male-female ratio of earned incomes, the share of assets that they hold within the household or patterns of decision-making within the household (as revealed by surveys), and women’s access to financial services.\textsuperscript{24} However, it has been found that several of these measures are highly correlated with relative educational outcomes (see, for instance, Frankenberg and Thomas (2003)). Accordingly, here the relative bargaining power of women is assumed to evolve as a function of an autonomous component $\tilde{z} \in (0, 1)$ and of the relative levels of human capital of husband and wife:

$$
    z_t = \tilde{z}^{1-\gamma_B} \left[ \frac{e_t^f}{e_t^m} \right]^{\mu_B} \gamma_B,
$$

where $\gamma_B \in (0, 1)$ measures the relative importance of the endogenous component of bargaining power; $1 - \gamma_B$ measures the importance of extraneous factors, such as social norms and cultural values. The parameter $\mu_B \geq 0$ measures the sensitivity of the endogenous component of bargaining power to relative stocks of human capital.

1.2.7 Market-Clearing Condition

The asset-market clearing condition requires equality between savings and investment, or equivalently, that tomorrow’s private capital stock be equal to today’s savings by adult workers. Given that $s_t$ is savings per family, that the number of families is $(N_t^m + N_t^f)/2$, and that $N_t^m = N_t^f$,

$$
    K_{t+1}^P = 0.5(N_t^m + N_t^f) s_t = N_t^f s_t,
$$

where again for simplicity full depreciation is assumed.

\textsuperscript{24}See for instance Doss (1996, 2013), Frankenberg and Thomas (2003), Anderson and Eswaran (2009), Angel-Urdinola and Wodon (2010), and Quisumbing (2010).
1.3 Balanced Growth Path

A competitive equilibrium in this model is a sequence of prices \( \{w^m_t, w^f_t, r_{t+1}\}_{t=0}^{\infty} \), allocations \( \{c^m_{t-1}, c^f_{t-1}, s_t, \epsilon_t^{gL}\}_{t=0}^{\infty} \), physical capital stocks \( \{K^p_{t+1}, K^f_{t+1}\}_{t=0}^{\infty} \), human capital stocks \( \{E^m_{t+1}, E^f_{t+1}\}_{t=0}^{\infty} \), a constant tax rate, and constant spending shares such that, given initial stocks \( K^p_0, K^f_0 > 0 \) and \( E^m_0, E^f_0 > 0 \), individuals maximise utility, firms maximise profits, markets clear, and the government budget is balanced.

In equilibrium, it must also be that \( \epsilon_t^j = E_t^j \) for \( j = m, f \). A balanced growth equilibrium is a competitive equilibrium in which \( c^m_t, c^f_t, K^p_{t+1}, K^f_{t+1}, E^m_{t+1}, E^f_{t+1} \) grow at the constant, endogenous rate \( 1 + \gamma \), the rate of return on private capital \( r_{t+1} \) is constant, and girls’ time allocation is constant.

As shown in Appendix 1.7, the solution of the model yields

\[
k^j_t = J = \frac{\nu_{jt}}{\sigma(1 - \tau)}, \quad \forall t
\]

(1.26)

where \( \sigma \) is the family’s propensity to save, defined as

\[
\sigma = \frac{1}{1 + (1 + \rho) \eta_C} < 1.
\]

(1.27)

Equation (1.26) implies that the public-private capital ratio is constant over time.

As also shown in Appendix 1.7, solving the family’s optimisation problem leads to the following solution for girls’ time allocated to home production, allowing for the fact that they may not go to school at all—in which case all their time is devoted to household chores:

\[
\epsilon^{g,P} = \min\left[\frac{\Lambda_1 - \zeta^P J}{\Lambda_2}, 1\right],
\]

(1.28)

where

\[
\Lambda_1 = 0.5 n \eta_Q \pi^Q \eta_E^{-1} \nu_3^{-1},
\]
\[ \Lambda_2 = 0.5n + \Lambda_1, \]

and, for \( h = C, E, \)

\[ \eta_h = \kappa \eta_h^f + (1 - \kappa) \eta_h^m = \eta_h^m + (\eta_h^f - \eta_h^m) \kappa. \quad (1.29) \]

Given the restrictions imposed earlier, \( \eta_C^f < \eta_C^m \) and \( \eta_E^f > \eta_E^m \), equation (1.29) implies that

\[
\frac{d\eta_C}{d\kappa} < 0, \quad \frac{d\eta_E}{d\kappa} > 0. 
\]

Combining (1.8) and (1.28) yields girls’ time allocated to school-related activities at home, assuming that they do attend school:

\[
\varepsilon^{g,L} = \max \left\{ 1 - \varepsilon^S - \frac{\Lambda_1 - \zeta^P J}{\Lambda_2}, 0 \right\}. \quad (1.30)
\]

This result shows that improved family access to infrastructure reduces the amount of time parents find optimal for their daughters to spend on household chores and therefore allows daughters to allocate more time to studying at home. Moreover, equations (1.28)-(1.30) show that a higher family preference for girls’ education (a higher \( \eta_E \)) reduces the optimal amount of time that girls must allocate to household chores. Because \( \eta_E \) depends positively on women’s bargaining power, \( \kappa, \) it follows that an increase in \( \kappa \) contributes to improving women’s human capital, independently of any other effect.

Equations (1.28) and (1.30) also show the possibility of a stagnating equilibrium, as in Bell and Gersbach (2009) for instance: indeed, if access to public capital is too low, it is possible for \( (\Lambda_1 - \zeta^P J)/\Lambda_2 > 1 - \varepsilon^S, \) even while \( (\Lambda_1 - \zeta^P J)/\Lambda_2 \leq 1, \) in which case \( \varepsilon^{g,L} = 0. \) In those conditions, parents will choose not to send their daughters to school, implying therefore no school-related activities at home.\textsuperscript{25} The

\textsuperscript{25}In that case, of course, the model generates a corner solution in which no capital is accumulated
critical value of the public-private capital ratio above which schooling takes place for girls is thus \((\Lambda_1 - \zeta^P J_C)/\Lambda_2 = 1 - \varepsilon^S\), so that

\[ J_C = \frac{\Lambda_1 - \Lambda_2 (1 - \varepsilon^S)}{\zeta^P}. \]  

(1.31)

Figure 1.1 illustrates the behaviour of \(\varepsilon^{g,P}\) and \(\varepsilon^{g,L}\) as a function of \(J\). For \(J = 0\), equation (1.28) implies that \(\varepsilon^{g,P} = \Lambda_1 / \Lambda_2 < 1\). However, for parents to actually send their daughters to school, \(\varepsilon^{g,P}\) must actually be less than \(1 - \varepsilon^S\), given that the amount of time that they must allocate to that activity is indivisible. For \(0 < J < J_C\), \(\varepsilon^{g,P}\) remains above \(1 - \varepsilon^S\), so girls do not attend school at all. As a result, \(\varepsilon^{g,P} = 1\) and \(\varepsilon^{g,L} = 0\). As \(J\) increases above \(J_C\), \(\varepsilon^{g,P}\) jumps down from 1 to either \(1 - \varepsilon^S\) or some value below that (from point A to point B, for instance) and continues to fall afterward. At the same time, \(\varepsilon^{g,L}\) starts increasing from its initial value of 0, reaching a maximum at \(1 - \varepsilon^S\), which is obtained when \(\varepsilon^{g,P} = 0\), that is, from (1.28), when \(J \geq J_H = \Lambda_1/\zeta^P\).^26

As shown in Appendix 1.7, the model can be condensed into a single first-order difference equation in \(x^f_{t+1} = K^P_t / \epsilon^f_t N^f_t\), the private capital-effective female labour ratio:

\[ x^f_{t+1} = \Gamma_5 \left( x^f_t \right)^{(1-2\beta)(1-\nu_1)}, \]  

(1.32)

where

\[ \Gamma_5 = \Gamma_4 J^\nu_1 \left( \varepsilon^S + \varepsilon^{g,L} - \nu_3[1+\beta(1-\nu_1)] \right), \]

\[ \Gamma_4 = \Gamma_3 \Gamma_1^{\nu_1}, \]

\[ \Gamma_1 = \left\{ \left( \frac{\chi^R}{1 - \chi^R} \right)^{\nu_2} \varepsilon^{f,W} (\varepsilon^S)^{\nu_3} \right\}^\beta, \]

\[ \Gamma_3 = \left\{ \frac{2\beta(1 - \tau)\sigma}{[(1 - \chi^R)\varepsilon^{f,R}]^{\nu_2} \chi^R^{1-\nu_1} 0.5} \right\} (v^E)^{2\beta} - \nu_1, \]

and output is zero.

^26For simplicity, we assume that there is no minimum amount of time that girls must allocate to home production.
with the growth rate of output given by

\[ 1 + \gamma_{t+1} = \frac{Y_{t+1}}{Y_t} = \Gamma_1(\frac{1}{\varepsilon + \varepsilon^g,L})^{\beta \nu_2} J^\alpha \left( \frac{1}{x_{t+1}^f} \right)^{2\beta} (1 - \tau) \sigma. \]

(1.33)

Stability of the adjustment process described by (1.32) requires \(|(1 - 2\beta)(1 - \nu_1)| < 1\), which always holds. The steady-state solution of (1.32) is

\[ \tilde{x}^f = \Gamma_{1/\Pi_1}, \]

(1.34)

where \(\Pi_1 = 1 - (1 - 2\beta)(1 - \nu_1) > 0\). Substituting this solution in (1.33) gives the steady-state growth rate of output:

\[ 1 + \gamma = \Gamma_1(\varepsilon^S + \varepsilon^g,L)^{-\beta \nu_2} J^\alpha (\tilde{x}^f)^{2\beta} (1 - \tau) \sigma. \]

(1.35)

The adjustment process corresponding to (1.32) is illustrated by the concave curve XX in the right-hand side panel of Figure 1.2. The left-hand side panel in the figure displays the convex curve GG, which corresponds to (1.33) and shows the relationship between the growth rate of output \(1 + \gamma_{t+1}\) and the private capital-effective female labour ratio \(x_{t+1}^f\). The initial equilibrium obtains at points \(A\) and \(B\).

Note also that using (1.19), (1.24), and (1.30), with \(J \geq J_C\),

\[ \kappa = \tilde{\kappa}^{1-\gamma_B} \left[ \frac{\chi^R}{1 - \chi^R} \right]^{\nu_2} \left( \frac{\varepsilon^S}{\varepsilon^S + \varepsilon^g,L} \right)^{\nu_3} = \kappa(J), \]

(1.36)

with \(\kappa > 0\). Thus, women’s bargaining power is also positively related to access to infrastructure. Note also that in the particular case where \(J = J_C\), with \(J_C\) defined in (1.31), \(\varepsilon^g,L = 0\) and women’s bargaining power is independent of the public-private capital ratio—even though girls are actually allowed to attend school.

To illustrate analytically the long-run effects of public capital, consider the im-
impact of a budget-neutral increase in the share of government spending on infra-
structure, financed by a cut in unproductive spending, that is, $dv_I + dv_U = 0$.\textsuperscript{27} As
shown in Appendix 1.7, an increase in $v_I$ raises the public-private capital ratio,
time allocated by girls to home schooling, and women’s bargaining power, but it
has an ambiguous effect on the private capital-effective female labour ratio and the
steady-state growth rate. The reason for the latter is as follows. The increase in
the public-private capital ratio has a direct, positive effect on growth, which reflects
its impact on overall productivity of private inputs. In turn, the increase in pro-
ductivity tends to increase the demand for (male and female) labour. At the same
time, the increase in girls’ time allocated to home schooling raises directly their
human capital and the effective supply of female labour. There is also an indirect
effect on that variable, because the initial relative increase in women’s human capi-
tal raises their bargaining power (to an extent that depends on the parameter $\nu_3$),
which increases the family’s preference for girls’ education, $\eta_E$, and induces parents
(as discussed earlier) to further reduce their daughters’ time allocated to household
chores.\textsuperscript{28} However, because both the private capital stock and the effective supply
of female labour increase, the change in the ratio of these variables is ambiguous
and so is its impact on growth.

This ambiguity is illustrated in Figure 1.2 as well. As can be inferred from
(1.30), (1.34), and (1.35), and given that both $J$ and $\varepsilon^{g,L}$ increase, curves $XX$ and
$GG$ may shift either upward or downward following an increase in $v_I$. The figure
illustrates the case where $GG$ shifts upward (which implies, for a given value of
the private capital-effective female labour ratio, that the direct effect of the public-
private capital ratio dominates its indirect effect of girls’ time allocated to home
schooling, $\varepsilon^{g,L}$), whereas $XX$ shifts either up or down. In the first case, the new

\textsuperscript{27}Assuming instead that the increase in infrastructure investment is financed by a cut in edu-
cation spending (as discussed later in the numerical experiments) would not alter the fundamental
ambiguities discussed here.

\textsuperscript{28}An increase in $\varepsilon$, as noted earlier, also tends to reduce $\eta_C$, which, from (1.27), tends to increase
the savings rate, and thus the private capital stock. This tends to mitigate the increase in the
public-private capital ratio, but not to reverse it.
equilibrium is at $A'$ and $B'$, characterised by a higher private capital-effective female labour ratio and a lower growth rate. In the second case, the new equilibrium is at $A''$ and $B''$, characterised now by both a lower private capital-effective female labour ratio and a higher growth rate. However, if curve $GG$ shifts downward, the equilibrium outcome could be a higher private capital-effective female labour ratio and a lower steady-state growth rate.

The foregoing discussion suggests therefore that, even accounting for a positive effect of improved access to infrastructure on women’s bargaining power (and thus girls’ time allocated to human capital accumulation), the net effect on growth may not be positive. To explore this issue further, we now turn to a numerical analysis.

1.4 Calibration

To further examine the conditions under which improved access to infrastructure may have an adverse effect on growth, the model is calibrated using average data for low-income countries for the period 2000-09 (unless otherwise indicated) and simulated under different parameter configurations. We use data provided by the World Development Indicators (WDI) database of the World Bank, UNESCO and UNICEF surveys, supplemented as needed with information from specific papers.

For households, the annual discount rate is set at $0.04$, which is standard in the literature. This implies that the discount factor is equal to 0.96 on a yearly basis. Interpreting a period as 20 years in this OLG framework yields the intergenerational discount factor $[1/(1 + 0.04)]^{20} = 0.456$.

To calibrate $\varpi$, as defined in (1.36), requires setting eight parameters: $\gamma_B$, $\varpi$, $\chi^R$, $\nu_2$, $\nu_3$, $\mu_B$, $\varepsilon^S$, and $\varepsilon^{g;L}$. The coefficients $\nu_2$ and $\nu_3$ are equal to 0.3 and 0.4, respectively, as discussed below. In the absence of survey-based data, the parameters $\mu_B$ and $\gamma_B$ are set at “neutral” values of 1 and 0.5, respectively. Thus, in the initial equilibrium, women’s bargaining power depends equally on factors (social norms
and values) that are outside the scope of the model and on relative human capital stocks. A sensitivity analysis with respect to $\mu_B$ is reported later on.

Even though there is much informal evidence in favour of bias in mothers’ rearing time allocation toward boys, survey data provide little information on its magnitude. We therefore assume that such bias exists in the initial equilibrium but is quite moderate; we therefore set $\chi^R = 0.6$ and treat it as a shift parameter later on.

To calibrate girls’ schooling time, we use a combination of data from UNESCO surveys and UNICEF’s Multiple Indicator Cluster Surveys (MICS), round 4, for low-income Sub-Saharan African countries.\(^{29}\) According to UNESCO data, entrance age in primary school is on average 6, and exit age from secondary school is 18. A period is 20 years, so schooling in childhood is 12 years.\(^{30}\) UNESCO also cites that the number of school days per year in developing countries typically varies between 180 and 209 days in secondary schools, with the number of teaching hours varying between 30 and 34 hours per week. Using the lower estimate, 49.3 percent of each year is spent in school; multiplied by 12, this means that the effective number of years in mandatory schooling is 5.9 years. Again, with a period representing 20 years in the model, the proportion of time spent in school could thus be measured as 0.29. However, this number may still be too high for lower-income developing countries. This data suggest also a greater number of school days lost due to illness and other factors. Accordingly, we choose a slightly lower value, $S = 0.2$.

The initial bargaining power of women is set at $\kappa = 0.3$. This ratio measures women’s relative human capital stock, and that this corresponds to the main determinant of bargaining power, as hypothesised in the analytical model. In one of the few empirical studies available on the topic, Reggio (2011) found an average estimate of women’s bargaining power in the family of the order of 0.46, with a standard deviation of 13 percent. Our benchmark value is thus well within a two-standard


error deviation confidence interval. Expression (1.36) can therefore be solved for the parameter \( \tilde{\alpha} \), so that

\[
\tilde{\alpha} = [0.3^2 \cdot (\frac{0.6}{1 - 0.6})^{0.3} \cdot (\frac{0.2}{0.2 + 0.3})^{0.4}] = 0.07.
\]

The private savings rate, \( \sigma \), is set at 12 percent, which corresponds to the average value for low-income countries reported in Agénor (2012a). Using the definition of \( \sigma \) given in (1.27) implies \( 1/\left[1 + (1 + \rho)\eta_C\right] = 0.12 \), an expression that can be solved for \( \eta_C \):

\[
\eta_C = \left(\frac{1}{1 + \rho}\right)\left[\left(\frac{1}{0.12}\right) - 1\right].
\]

With the intergenerational discount factor equal to 0.456, this expression yields \( \eta_C = 3.34 \). We assume that families value consumption of the marketed good and the home good equally, so that the parameter \( \eta_Q \) is set at the same value as \( \eta_C \).

In the home production sector, the parameter \( \zeta^P \) is set to unity to capture a high degree of substitution between girls’ time and infrastructure services, and the curvature of nonmarket production function is set initially at \( \pi^Q = 0.8 \), to capture rapidly decreasing marginal returns in terms of girls’ time and infrastructure services. This seems to be a more reasonable assumption than the relatively low values used in the literature (see for instance Kimura and Yasui (2010)). In any case, for sensitivity analysis, in the experiments reported next, a lower value of \( \pi^Q = 0.5 \) (which implies weaker marginal returns initially) will also be used.

Time allocated by girls to home production can also be estimated from a sample of recent MICS results for low-income countries. These surveys provide information on children’s time allocated to child labour, both in the home and outside the home (including as domestic workers). In general, they indicate that the majority of children aged 5-14 years who are attending school are also involved in child labour activities. Results for Ghana for instance (based on a 2011 survey) indicate that

\[^{31}\text{Using an alternative benchmark value of 0.4 makes little difference to the results.}\]
60.2 percent of girls aged 5-11, and 85.3 percent of girls aged 12-14, are engaged in household chores for less than 28 hours per week (MICS 2011). Of the 97 percent of the children aged 5-14 years attending school, 35 percent are also involved in child labour activities. In Sierra Leone (MICS 2010), 62.7 percent of girls aged 5-11, and 87.1 percent of girls aged 12-14, are engaged in household chores for less than 28 hours per week. Time allocated solely to household chores varies between 4 and 6 hours a day. Similar results are obtained for Nigeria (MICS 2011) and Gambia (MICS 2012).³² Accordingly, we set the time that girls allocate to home production, \( g^P \), to an average of 0.5. This implies therefore that \( g^L \), which is determined residually from (1.8), is equal to 0.3. Thus, initially, girls allocate 30 percent of their time to school-related activities.³³

Using the estimate of \( g^P \), \( \eta_E \) is calibrated as follows. Given the value of the fertility rate, \( n = 4.7 \), considered by Baldacci et al. (2004a, Table 1), and from the values given above and the definitions of \( \Lambda_1 \) and \( \Lambda_2 \), \( \Lambda_1 = \eta_E^{-1}15.7 \) and \( \Lambda_2 = \eta_E^{-1}15.7 + 2.35 \). Substituting these results, with \( J = 0.148 \) as shown below, in (1.28) yields

\[
g^P = 0.5 = \frac{\eta_E^{-1}15.7 - 1 \cdot 0.148}{\eta_E^{-1}15.7 + 2.35},
\]

which can be solved for the relative preference for education:

\[
\eta_E = 5.93. \quad (1.38)
\]

Having determined \( \eta_C \) and \( \eta_E \), the values \( \eta_C^n, \eta_C^f \) and \( \eta_E^n, \eta_E^f \) must be determined. Given that \( \alpha = 0.3 \) and setting \( \eta_C^n = 4.5 \) and \( \eta_E^n = 5 \), the last two values can be

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³²In the same vein, Togunde and Carter (2006) found that in Nigeria children spend on average 4 hours a day of work (some of it outside the home), while 20 percent work 5 to 6 hours a day.

³³This may be an overestimate because the model does not account explicitly for leisure. However, as long as leisure is a fixed fraction of \( g^L \) our results would be qualitatively the same.
determined residually using (1.29), (1.37), and (1.38):

\[ \eta_C^f = \frac{\eta_C - \eta_C^m(1 - \varpi)}{\varpi} = \frac{3.34 - 4.5(1 - 0.3)}{0.3} = 0.65, \]

\[ \eta_E^f = \frac{\eta_E - \eta_E^m(1 - \varpi)}{\varpi} = \frac{5.93 - 5(1 - 0.3)}{0.3} = 8.1, \]

so that by construction \( \eta_C^f < \eta_C^m \) and \( \eta_E^f > \eta_E^m \).\textsuperscript{34}

Given the value of time allocated by mothers to child rearing, \( \varepsilon^{f,R} = 0.053 \) which is taken from Agénor et al. (2012) and again the value of the fertility rate, \( n = 4.7 \), the time constraint (equation (1.6)) is solved residually, \( \varepsilon^{f,W} = 0.75 \).

In the marketed good production sector, the elasticities of production of final goods with respect to public capital and each type of labour, \( \alpha \) and \( \beta \), are set equal to 0.15 and 0.35, respectively. Both values are taken from Agénor (2011) and are consistent with the empirical evidence. The first parameter, for instance, is close to the average estimated by Bom and Ligthart (2011) from a large number of studies. This yields a value of the elasticity of output with respect to private capital equal to \( 1 - 2\beta = 0.3 \), again in line with the empirical evidence.

In the human capital sector, the elasticity with respect to government spending on education, \( \nu_1 \), is set equal to 0.4. The elasticity with respect to time allocated by mothers to child rearing, \( \nu_2 \), is set equal to a relatively low value, 0.3. Both values are consistent with those reported in Agénor (2012a). The elasticity with respect to time allocated by girls to home schooling, \( \nu_3 \), is set equal to 0.4. A sensitivity analysis with respect to \( \nu_1 \) and \( \nu_3 \) is also reported later on.\textsuperscript{35}

The effective tax rate on wages, \( \tau \), is calculated by multiplying the average ratio of tax revenues to GDP for low-income countries, equal to 15.05 percent for...
the period 2001-08, estimated by Baldacci et al. (2004b, Table 1), divided (to match the model’s definition) by the average share of labour income for developing countries estimated by Guerriero (2012), 0.701. Thus, $\tau = 21.5\%$. To estimate the initial share of government investment on infrastructure, $v_I$, we use as a starting point the ratio of total public investment to GDP in low-income countries calculated by Gupta et al. (2011, Table 1) for the period 2000-09. Because public investment includes non-infrastructure related outlays, we assume, based on the evidence reported in Foster and Briceño-Garmendia (2010), that about 40 percent of that amount (or 1.4 percent) really consists of infrastructure investment. The share $v_I$ can therefore be estimated by $0.014/0.215$, that is, $v_I = 6.5\%$. The initial share of government spending on education, $v_E$, is based on the average estimated from WDI for the years 2004, 2006, and 2007 and is set at 0.171. These numbers imply from the budget constraint that the share of spending on other items is $v_U = 0.764$.

From the model’s solution (1.26), and the above values for $\sigma$, $v_I$ and $\tau$, the equilibrium value of the public-private capital ratio is

$$J = \frac{0.065 \cdot 0.215}{0.12(1 - 0.215)} = 0.148,$$

which implies therefore that public capital is a relatively scarce factor in the economy, consistent with the evidence for low-income countries (see for instance Foster and Briceño-Garmendia (2010)).

The benchmark parameter values are summarised in Table 1.1. Based on these values, the model is solved for the steady-state value of private capital-effective labour ratio, $\tilde{x}^f$, using (1.32) and $1 + \gamma$, together with the solutions for $J$, $\varepsilon^{g,L}$, and $\tilde{x}^f$, to determine the growth rate of output. A multiplicative constant is also

$\text{36The estimate used is the corrected measure LS5 proposed by Guerriero, which (importantly for developing countries) accounts for self-employed workers, while considering the possibility for them to generate some capital income. Note also that estimating the effective tax rate by dividing the ratio of tax revenues to GDP by the elasticity of output with respect to labour, }2\beta, \text{ would give exactly the same result.}$

40
introduced, in order to yield an annual growth rate of marketed output per worker equal to 3.3 percent, the average growth rate of low-income countries during the period 1975-2000, considered by Baldacci et al. (2004b).

1.5 Quantitative Experiments

To illustrate the role of public policy in the model, we consider several experiments: an increase in investment in infrastructure (aimed at promoting access to rural roads, power grids, and so on), a reallocation of mothers’ time toward girls (which eventually improves their bargaining power in adulthood), a reduction in the sensitivity of the endogenous component of bargaining power to relative stocks of human capital, and a reduction in fathers’ preference parameter for their daughters’ education.\footnote{\(A \) number of other experiments could be conducted with the model. However, those that have been selected illustrate well a broad range of gender-based policies.}

In all cases we focus on steady-state effects and assume that the initial public-private capital ratio is sufficiently high to ensure that it remains above the critical value \(J_C\) defined in (1.31) yet below the upper value \(J_H\), above which \(e^g_P = 0\) and \(e^g_L = 1 - e^S\). Thus, we consider an initial equilibrium in which the economy experiences positive, albeit low, economic growth.

To summarise the simulation results, we focus on the following variables: girls’ time allocation, women’s bargaining power, the public-private capital ratio, and the growth rate of marketed output.

1.5.1 Investment in Infrastructure

We consider first the effects of a budget-neutral increase in the share of public expenditure on infrastructure investment, \(v_I\), from an initial value of 0.065 to 0.105, under two alternative financing assumptions: first, financing by a cut in unproductive spending, as in the analytical experiment reported earlier \((dv_I + dv_U = 0)\) and...
second, financing by a cut in spending on education \((d v_I + d v_E = 0)\). The first experiment helps to highlight changes in girls’ time allocation, whereas the second helps to emphasise the policy trade-offs that policymakers may face in allocating their resources.

**Cut in Unproductive Spending**

The results of an increase in infrastructure investment financed by a cut in unproductive spending are displayed in Table 1.2, for different values of some key structural parameters.

Consider first the impact under the benchmark case. The direct effect of the shock is of course an increase in the public-private capital ratio \(J\) (which rises overall from an initial value of 0.148 to 0.239, or 0.091 percentage points) thereby promoting growth. In addition, an increase in the share of government spending on infrastructure lowers girls’ time allocated to home production. This, in turn, raises time allocated to home schooling and girls’ human capital accumulation, and thus eventually women’s bargaining power in the family.

With the benchmark parameter values, the results (shown in bold in Table 1.2) indicate that the net effect of the increase in the share of investment spending has a net positive effect on growth, of the order of 0.15 percentage points. At the same time, time allocated by girls to home production falls (by about 1.8 percentage points), whereas both time allocated by girls to home schooling and the relative bargaining power of women in family increase (by about 1.8 percentage points and 0.2 percentage points, respectively). Table 1.2 reports results for a lower \(\pi_Q = 0.5\) as well; in that case, the policy strengthens the reduction in girls’ time allocated to home production and the positive benefit of time allocated to home schooling, the relative bargaining power of women, and output growth.

\(^{38}\)The type of offsetting cuts in education spending that we have in mind here do not involve cuts in pay or outlays on school supplies, which could affect the productivity of teachers and children—thereby mitigating the benefits of spending reallocation emphasised here. Rather, one can think of these cuts as involving reductions in spending on a bloated and possibly corrupt bureaucracy.
Table 1.2 also indicates results for two alternative values of $\nu_3$, the elasticity of human capital with respect to girls’ time allocated to home schooling, equal to 0.2 and 0.6, for comparison with the benchmark case of 0.4. A decrease, say, in $\nu_3$ has both direct and indirect effects on girls’ time allocation. On the one hand, for a given ratio of human capital, it weakens the effect of a reduction in time allocated by girls to home production, $\varepsilon^{g,P}$, on girls’ human capital and women’s relative bargaining power. On the other, the sensitivity of $\varepsilon^{g,P}$ to the public-private capital stock ratio, $J$, gets stronger. Consequently, an increase in $J$ triggered by a rise in the share of public spending on infrastructure has a larger impact on $\varepsilon^{g,P}$, due to the higher marginal benefit of additional schooling. Opposite effects hold for a higher value of $\nu_3$, although in either case there are no discernible effects on economic growth.

**Cut in Education Spending**

The results of an increase in infrastructure investment financed by a cut in education spending are displayed in Table 1.3, again for a range of values of some key structural parameters. To illustrate potential trade-offs, we focus on two key parameters: $\nu_1$ (the elasticity of human capital with respect to government spending on education) and $\nu_3$ (the elasticity of human capital with respect to girls’ time allocated to home schooling).\(^{39}\)

The intuition about the role of $\nu_1$ and $\nu_3$ is clear; the lower the elasticity of human capital with respect to government spending on education, or the higher the elasticity of human capital with respect to girls’ time allocated to home schooling, the more productive investment in infrastructure is compared to spending on education, and the more likely it is that the net impact on the growth rate is positive. The channels through which these effects operate, however, are different. This is captured by (1.36), where a change in $\nu_1$ has no effect on the ratio of human capital stocks

\(^{39}\)Values of the remaining parameters are the same as those used in the benchmark case described in Table 1.1.
(in contrast to \(\nu_3\)), so the only channel through which \(\nu_1\) can potentially affect the relative bargaining power of women is an indirect one, operating through a change in time allocated by girls to home production.

Table 1.3 illustrates two sets of outcomes: \(\nu_1\) varying between 0.1 and 0.6 for \(\nu_3\) fixed at its benchmark value of 0.4, and vice versa. The benchmark results are shown in bold in Table 1.3. When \(\nu_3\) is fixed, the effect of a change in \(\nu_1\) on girls’ time allocation and bargaining power is much the same, because \(\nu_1\) has no quantitatively significant effect on these variables. However, the important point here is that in the case where \(\nu_1 = 0.4\), as in the benchmark case, a comparison of the results in Tables 1.2 and 1.3 shows that when the increase in spending on infrastructure is financed by a cut in spending on education (which adversely affects the rate of human capital economy, for boys and girls alike), the net effect on growth is negative—despite the fact that girls are able to reallocate their time from household chores to studying. This result illustrates well the trade-offs that arise when budget-neutral changes in government expenditure involve a reallocation across productive outlays.

For a given value of \(\nu_1\), increases in \(\nu_3\) have an indirect impact on time allocation but also (as discussed earlier) a direct positive effect which gets stronger on women’s relative bargaining power; thus, an increase in \(\nu_3\) magnifies the effect of a change in \(\varepsilon^{g,P}\) on women’s bargaining power. By implication, if \(\nu_3\) is larger, then the ratio of human capital stocks has a larger, direct effect on bargaining power. There is also an indirect effect, related to the fact that when \(\nu_3\) goes up, the sensitivity of \(\varepsilon^{g,P}\) to \(J\) (the public-private capital ratio) becomes stronger. As a result, an increase in \(J\) triggered by a rise in spending on infrastructure has a larger impact on \(\varepsilon^{g,L}\), because the marginal benefit of additional schooling is higher.\(^{40}\) Indeed, when \(\nu_1\) is fixed, increases in \(\nu_3\) magnify the positive effect of improved access to infrastructure on girls’ time allocation to home schooling, and this translates into a

\(^{40}\text{As indicated in (1.28), } \Lambda_1, \text{ and thus } \Lambda_2, \text{ are negatively related to } \nu_3. \text{ When } \nu_3 \text{ rises, the marginal effect of an increase in } J \text{ on } \varepsilon^{g,P}, \text{ as measured by } \frac{\zeta^P}{\Lambda_2}, \text{ becomes therefore stronger and so is the effect on } \varepsilon^{g,L}. \text{ As a result, the relative human capital of girls goes up by more, and so does the increase in women’s bargaining power.}\)
stronger effect on women’s bargaining power. However, a result similar to the one established before obtains: the financing of higher spending on infrastructure by a concomitant reduction in spending on education translates into a negative effect on growth, despite the benefit associated with women’s higher human capital stock. As $\nu_3$ increases (falls) this adverse effect is mitigated (magnified), but the trade-off persists.

Figure 1.3 illustrates the impact of changes in $\nu_1$ and $\nu_3$, both individually and in combination, on the steady-state growth rate of output. Consistent with the results reported in Table 1.3, the negative effect on growth weakens (except for the initial increase in $\nu_1$) when either one of these parameters increases. Moreover, the figure shows that for higher values of $\nu_3$ and a relatively low value of $\nu_1$ the growth rate may actually turn out to be positive.\textsuperscript{41} Put differently, because girls are able to reallocate a larger fraction of their time toward human capital accumulation and to improve in so doing their bargaining power later in life, a policy that entails higher spending on infrastructure may still promote growth—even if it involves an offsetting cut in spending on education.

The foregoing discussion has focused on the case where the degree of substitutability between girls’ time allocation and infrastructure services is perfect, that is, $\zeta^P = 1$. As a result, the benefits of an increase in infrastructure investment on girls’ time allocation and human capital accumulation, and therefore on economic growth, are maximised. By implication, imperfect substitutability ($\zeta^P < 1$) would mitigate substantially these benefits, implying that (in contrast to the case illustrated in Figure 1.3) increases in public investment that are fully offset by cuts in education spending may not, even with high values of $\nu_3$, generate a positive effect on growth. However, a larger increase in $\nu_I$, or a smaller share of financing of a higher $\nu_I$ by a cut in education spending, would restore this result. For instance, with $\zeta^P = 0.5$, an increase in $\nu_I$ of the same magnitude as before, but combined

\textsuperscript{41}A low value of $\nu_1$ is quite often used in simulation studies focusing on developing countries; see for instance Agénor (2011) and the references therein.
now with only a 20 percent financing by a cut in education spending, would generate long-run growth of the order of 0.1 percentage points.

1.5.2 Allocation of Mothers’ Time toward Girls

Consider a reduction in time allocated to sons $\chi^R$, and thus a concomitant increase in time allocated to daughters, from an initial value of 0.6 to 0.5 (see Table 1.2). This may capture changes in social norms and attitudes toward women, unrelated to direct policy changes. By definition, this policy has no impact on mothers’ total time allocated to child rearing, which remains at $n_{e^{f,R}} = 0.25$. In the present setting (where rearing time affects schooling outcomes in childhood), if mothers allocate relatively less time to their sons, their human capital and productivity later in life will also be relatively lower when compared to their daughters. By implication, effective male labour supply will tend to fall relative to women’s effective labour supply. In turn, the relative increase in women’s human capital stock promotes growth and raises their bargaining power, which translates into a reduction in the family’s preference parameter for current consumption. The family’s propensity to save and the level of savings therefore increase, and so does the stock of private capital. This positive effect on growth is mitigated by the congestion effect associated with the higher propensity to save (which entails a fall in the public-private capital ratio), but overall the net impact on the growth rate remains positive.

1.5.3 Reduction in Sensitivity of Women’s Bargaining Power

Consider a reduction in $\mu_B$, which measures the sensitivity of women’s bargaining power to changes in their relative stock of human capital, from an initial value of unity to 0.2 (see Table 1.2). At the initial levels of human capital, the fall in $\mu_B$ reduces women’s bargaining power; in turn, this lowers the family’s preference for girls’ education and raises its preference for current consumption. The first effect translates into more time in household chores for girls, which eventually weakens their
bargaining position in adulthood—thereby magnifying the initial change in time allocation. The lower rate of human capital accumulation by girls is also detrimental to growth. The second effect translates into a lower family savings rate and a lower stock of private capital, which has an adverse effect on growth. However, this effect is mitigated by the fact that a lower private capital stock weakens the magnitude of congestion effects. Overall, the decrease in $\mu_B$ reduces the relative bargaining power of women by about 2.8 percentage points and exerts a slightly negative effect on steady-state growth. The key point is that the endogenous mechanism that relates women’s bargaining power, girls’ time allocation, and human capital accumulation tends to magnify the initial shift in the bargaining function.

1.5.4 Decrease in Fathers’ Preference for Daughters’ Education

Consider a decrease in fathers’ preference for girls’ education, $\eta^m_E$, from an initial value of 5 to a value of 3, with $\eta^f_E$ remaining constant at 8.1. As shown in Table 1.2, this shift (which leads to an immediate reduction in the family-wide preference for girls’ education, $\eta_E$) translates into an increase in the optimal amount of time that girls must allocate to household chores, or equivalently (assuming that the new optimal value for $\varepsilon^{g,P}$ remains less than $1 - \varepsilon^S$, to avoid a corner solution), a decrease in time that they allocate to school-related activities at home. In turn, this translates into a lower relative capital stock for females, and therefore a weakening in their bargaining power later in life. The initial reduction in the family-wide preference parameter $\eta_E$ is thus magnified (see (1.29)). Overall, the parameter $\eta_E$ drops from an initial value of 5.93, as given in (1.38), to 4.48.

The impact of this experiment on steady-state growth is also illustrated in Table 1.2. The fact that women accumulate less human capital is, by itself, detrimental to growth. In addition, because women’s preference for current consumption is lower than that of men ($\eta^f_C < \eta^m_C$), the reduction in their bargaining power increases the
average family preference parameter for today’s consumption, $\eta_C$ (see again (1.29)), from the initial value of 3.34 to 3.38. Thus, the family’s savings rate, defined in (1.27), decreases from an initial value of 0.12 to 0.11. At the aggregate level, the decrease in savings translates into a lower private capital stock in the steady-state, which adversely affects growth; at the same time, however, a lower private capital stock weakens the magnitude of congestion effects, which enhances the impact of public capital on growth. The net effect on the growth rate is, nevertheless, slightly negative.42

Finally, note that, given the parsimonious nature of the model, it is likely that the growth effects of changes in women’s bargaining power are underestimated. Indeed, in the foregoing analysis we abstracted from the fact that families spend a fraction of their resources on children, and that such spending may improve the quality of their education or their health (through improved nutrition and cognitive skills). Suppose that the share of family spending on each child, $\theta_R^c$, is a weighted average of the preferred shares of spending by fathers and mothers, $\eta_R^f$ and $\eta_R^m$, and that mothers have a higher preference for spending on children ($\eta_R^f > \eta_R^m$), out of concern for their well-being. This is well documented in the literature (see Schultz (2002), Smith et al. (2003), Roushyd (2004), Ahmed (2006), UNICEF (2007), Doepke and Tertilt (2011), and World Bank (2011)). Suppose also that not only education, as is the case here, but also health (as for instance in Agénor et al. (2012)), display persistence over time. In such conditions an increase in women’s bargaining power may raise children’s chances of survival through infancy, their performance in school, and their productivity in adulthood—thereby promoting growth. If these effects are strong enough to compensate for the impact of lower family savings on physical capital accumulation—a likely outcome if initial levels of health and human capital are relatively low—the growth effect of policies that are conducive to women exerting greater control over family resources would be magnified.

42As also shown in Table 1.2, symmetric results are obtained for an increase in $\eta_E^f$, from 8.1 to 15.
1.6 Concluding Remarks

The purpose of this chapter was to study the growth effects of externalities associated with intra-household bargaining and the role of access to infrastructure (or lack thereof) on girls’ time allocation. To that end we presented a three-period, gender-based overlapping generations (OLG) model that accounts for human capital accumulation, infrastructure, and growth. In contrast to boys, only girls’ time allocated to household chores was assumed to be endogenously related to access to infrastructure. Mothers care more than fathers about the human capital of their daughters (they are more intergenerationally altruistic towards girls) and men care more about current consumption than women. Fundamentally, in this chapter gender inequality is an equilibrium outcome that is linked not only to social norms and cultural values but also to the way household members endowed with individual preferences interact with each other and make decisions about girls’ time allocation.

The long-run properties of the model were characterised and its properties were illustrated by considering the impact of an increase in spending on infrastructure. The model was then calibrated using data for low-income countries and then used to analyse numerically the effects of not only an increase in spending on infrastructure, but also a reduction in fathers’ preference for their daughters’ education and a reallocation of mothers’ time toward girls. These experiments were conducted by considering alternative values of the parameters that were deemed essential to understanding their effects. The results show that policies aimed at promoting an increase in family access to infrastructure may have significant benefits for girls (in terms of education outcomes), as well as in terms of economic growth. This policy may lead to a reduction in girls’ time devoted to household chores, which may in turn allow them to build more human capital—with persistent effects on productivity and wages in their adult life, as well as improved bargaining power in terms of resource allocation within the family. If mothers have a relatively higher preference than fathers for their daughters’ education, this increase in women’s bargaining power may
further reduce the amount of time that the family finds optimal for girls to spend on household chores. The benefits of improved access to infrastructure are therefore magnified. Importantly, the analysis shows that these effects may occur even when an increase in government expenditure on infrastructure is financed by a reduction in spending on education. The practical policy implications of these results cannot be overemphasised: to promote girls’ education and reduce gender inequality, the best policy may not be to allocate more public resources to education (as advocated by Schultz (2002) for instance), but instead to invest in infrastructure. This is especially important if offsetting changes in education expenditure come from spending reductions on an inefficient or corrupt bureaucracy—a common feature of education systems in low-income countries (see for instance UNESCO (2009, Chapter 3)).

Our analysis could be extended in several directions. A first and relatively straightforward extension would be to endogenise fertility, account for family spending on children, and to relate it to parental preferences. As noted earlier, if mothers have a relatively higher preference for children’s education and health (a well-documented fact), the growth effects of policies that contribute to increasing women’s bargaining power would probably be magnified. By how much growth increases is an empirical matter that would be worth exploring quantitatively. In addition, with endogenous fertility, accounting for the fact that family resources are partly allocated to children would also help to examine how changes in intra-household bargaining affect the demographic transition, through the well-known trade-off between the quantity and the quality of offspring.

A second extension would be to consider alternative intra-household bargaining schemes. As noted in the text, several of the alternative measures of women’s bargaining power used in practice (such as relative wages or the relative share of assets that women hold within the household) are likely to be highly correlated with relative educational outcomes—the measure used in this chapter. However, one possibly important measure that we do not capture is greater access by women to financial
services. Although the addition of a financial sector would add some significant
degree of complexity to the model, it would be a fruitful way to examine the im-
pact of access to microfinance, for instance, on women’s control of family resources
and their implications for children’s health, girls’ education, gender equality, and
economic growth.

A third extension would be to endogenise mothers’ time allocation as well, along
the lines of Agénor (2012a) and Agénor et al. (2012) for instance, and assume
that home production requires mothers’ and daughters’ time—both of which are
determined optimally to maximise the family’s utility. If endogenous, mother’s time
allocation would be another margin through which the household can respond to
changes in the environment that induce the household to increase/decrease daugh-
ters’ hours of housework. Mothers’ ability to alter their housework hours could also
prevent daughters’ educational attainment to be negatively influenced by bargaining
power differences between their parents as well as changes in the environment that
increase the opportunity cost of their time. A key issue then would be how much
the family values mothers’ time (given its higher opportunity cost, in terms of the
market wage) relative to daughters’ time. In addition, the degree of intergenera-
tional altruism, which in this chapter operates from mothers to daughters, could
operate in the opposite direction, with important consequences on mothers’ time
allocation today. Indeed, if mothers expect their daughters to provide substantial
support to their parents in their old age, they may be more willing to engage in
home production today and “liberate” their daughters’ time, thereby allowing them
to engage more in human capital accumulation.43 However, in practice, it is often
boys who are groomed to provide old age support, so it is not clear that this “reverse
altruism effect” would prove to be particularly strong.

A fourth extension would be to introduce child labour for both gender types,
with parents using girls to perform household chores (as in the present setting) and boys to smooth family income by engaging in market-related activities outside the home, such as farming or a family business. This would be consistent with the evidence on the division of labour often imposed on children, as discussed earlier. And because education outcomes and access to infrastructure would probably affect (directly or indirectly) the market wage that boys earn, this would allow a richer analysis of wage gaps and gender inequality in poor countries, as well as the type of public policies that may affect their evolution. However, as long as improved access to infrastructure has a sizable effect on girls’ time allocated to education, their ultimate effect on labour market returns for women may continue to dominate the effect for men; as a result, women’s bargaining power may again improve relatively more and the main conclusions of the present chapter would not be qualitatively altered.

Finally, the model provides a number of general, qualitative implications that can be assessed with formal econometric techniques. First, persistence in gender inequality should be lower in countries where households have higher access to public infrastructure. Second, the intergenerational correlation between the educational attainment of mothers and daughters should be lower for countries where families have greater access to public infrastructure. Third, if the mechanism that relates child labour and education in adulthood applies only to girls (as hypothesised in the model), these relationships should only be significant for mothers and daughters but not for fathers and sons. Finally, time allocated to housework by girls should be negatively correlated with the education of women in future generations. In the introduction, a number of studies that have looked at some of these patterns (in Bolivia, Mexico, and Sub-Saharan Africa) were identified and used as motivation for focusing our analysis on girls’ time allocation. Other studies focusing on the relationship between child labour and educational attainment are also consistent with the predictions of the model; as documented by UNICEF (2007, p. 27) for
instance, in developing countries children with uneducated mothers are on average at least twice as likely to be out of school than children whose mothers attended primary school. Another study of children aged 7 to 14 years in Sub-Saharan Africa found that 73 percent of children with educated mothers were in school, compared with only 51 percent of children whose mothers lacked schooling. However, as far as we know there are no formal, quantitative studies focusing squarely on the relationship between public infrastructure, gender inequality, and child labour. To conduct such analysis a possible avenue would be to start with the country data from UNICEF’s Multiple Indicator Cluster Surveys, mentioned earlier. However, there are two potentially difficult issues to address in this context. First, differences across countries in social norms, religious beliefs, and cultural values with respect to the role of women need to be controlled for. Such variables may be difficult to measure and to standardise across countries. Second, the UNICEF surveys would need to be matched with comparable surveys that provide information on access to infrastructure at the household level; to our knowledge, such information is fairly limited at the moment.

\footnote{In a study of Brazil, Emerson and Souza (2007) document the fact that a mother’s education has a greater positive impact than a father’s education on daughters’ school attendance.}
1.7 Appendix

Consider first the family’s optimisation problem. Substituting (1.1) in (1.10), and the result in (1.9) yields

\[ U_t = \left[ \chi \eta_C^f + (1 - \chi) \eta_C^m \right] \ln c_t^{l-1} + \eta_Q \pi^Q \ln (0.5 n \varepsilon_i^{gP} + \zeta^P k_t^f) \]

\[ + \left[ \chi \eta_E^f + (1 - \chi) \eta_E^m \right] \ln e_{t+1}^f + \frac{1}{1 + \rho} \ln c_{t+1}^{l-1}, \] (1.39)

Define

\[ \eta_h = \chi \eta_h^f + (1 - \chi) \eta_h^m = \eta_h^m + (\eta_h^f - \eta_h^m) \chi. \quad h = C, E \]

Given the restrictions discussed in the text, \( \eta_C^f < \eta_C^m \), and \( \eta_E^f > \eta_E^m \). Thus,

\[ \frac{d\eta_C}{d\chi} < 0, \quad \frac{d\eta_E}{d\chi} > 0. \]

If women’s bargaining power increases, the family will value consumption today less and therefore spend less today (saving more in the process), and it will value the education of children more.

Using the above definitions, the collective utility function (1.39) takes the form

\[ U_t = \eta_C \ln c_t^{l-1} + \eta_Q \pi^Q \ln (0.5 n \varepsilon_i^{gP} + \zeta^P k_t^f) + \eta_E \ln e_{t+1}^f + \frac{1}{1 + \rho} \ln c_{t+1}^{l-1}, \] (1.40)

where \( k_t^f = K_t^f / K_t^P \). From equations (1.3), dropping the index \( i \) and given that \( N_t^m = N_t^f \),

\[ w_t^m = \left( \frac{\varepsilon_j W}{E_t^m} \right) w_t^f, \]

which can be substituted in (1.13) to give, with \( E_t^j = e_t^j \), for \( j = m, f \),

\[ w_t^T = e_t^m w_t^m + e_t^f \varepsilon_j W w_t^f = 2 e_t^f \varepsilon_j W w_t^f. \] (1.41)
In turn, this expression can be substituted in the budget constraint (1.14) to give
\[
2(1 - \tau)e_f^t \varepsilon^{f,w} w_t^f - c_t^{-1} - \frac{c_{t+1}^{-1}}{1 + r_{t+1}} = 0. \tag{1.42}
\]

From (1.17), together with (1.18), and noting from (1.8) that \( \varepsilon^S + \varepsilon^g_L = 1 - \varepsilon^g_P \), the human capital of females in \( t + 1 \) is,
\[
e_t^f = (\frac{G^E_t}{n0.5N_t})^{\nu_1}(E^f_t)^{1-\nu_1}[(1 - \chi^R)\varepsilon^{f,R}]^{\nu_2}(1 - \varepsilon^g_P)^{\nu_3}. \tag{1.43}
\]

Families maximise (1.40) subject to (1.42) and (1.43), with respect to \( c_t^{-1}, c_{t+1}^{-1}, \varepsilon^g_P \) and with \( \varepsilon^g_L \) solved residually from (1.8). First-order conditions yield the familiar Euler equation
\[
\eta_C \frac{c_{t+1}^{-1}}{c_t^{-1}} = \frac{1 + r_{t+1}}{1 + \rho}, \tag{1.44}
\]

\[
\frac{\eta_Q \pi Q0.5n}{0.5n\varepsilon^g_P + \zeta^P k_t^l} = \frac{\eta_E \nu_3}{1 - \varepsilon^g_P},
\]
or equivalently
\[
0.5n\varepsilon^g_P + \zeta^P k_t^l = \Lambda_1(1 - \varepsilon^g_P), \tag{1.45}
\]
where
\[
\Lambda_1 = 0.5n\eta_Q \pi Q\eta_E^{-1} \nu_3^{-1}.
\]
Substituting (1.44) in the intertemporal budget constraint (1.42) yields
\[
c_t^{-1} = \frac{(1 + \rho)\eta_C}{1 + (1 + \rho)\eta_C}2(1 - \tau)e_f^t \varepsilon^{f,w} w_t^f. \tag{1.46}
\]
Thus, from (1.11), (1.41), and (1.46), family savings, \( s_t \), is equal to
\[
s_t = 2(1 - \tau)\sigma e_f^t \varepsilon^{f,w} w_t^f. \tag{1.47}
\]
where $\sigma$ is the marginal propensity to save, defined as

$$
\sigma = \frac{1}{1 + (1 + \rho)\eta_C} < 1.
$$

(1.48)

From equation (1.45), we have

$$
\min \varepsilon_t^{g.P} = \left[ \frac{\Lambda_1 - \zeta^P_k l_t^f}{\Lambda_2}, 1 \right],
$$

(1.49)

where

$$
\Lambda_2 = 0.5n + \Lambda_1.
$$

(1.50)

This equation can be substituted in (1.8), together with (1.50), to give

$$
\varepsilon_t^{g.L} = \max \left\{ 1 - \varepsilon^S - \frac{\Lambda_1 - \zeta^P_k l_t^f}{\Lambda_2}, 0 \right\}.
$$

(1.51)

To study the dynamics in the economy, substitute (1.47) in (1.25) to give

$$
K_{t+1}^P = N_t^f s_t = N_t^f 2(1 - \tau)\sigma e_t^f \varepsilon^{f.W} w_t^f,
$$

(1.52)

that is, substituting for $w_t^f$ from (1.3) and dividing by $K_t^P$,

$$
\frac{K_{t+1}^P}{K_t^P} = 2\beta(1 - \tau)\sigma \frac{Y_t}{K_t^P}.
$$

(1.53)

Equations (1.21) can be rewritten as given that $N_t^m = N_t^f$,

$$
G_j^t = v_j \tau (w_t^m E_t^m + w_t^f \varepsilon^{f.W} E_t^f) N_t^f, \quad j = I, E, U
$$

that is, using (1.41),

$$
G_j^t = v_j \tau 2e_t^f \varepsilon^{f.W} w_t^f N_t^f.
$$
Substituting for \(w^f_t\) from (1.3) gives

\[ G^j_t = v_j \tau 2 \beta Y_t. \tag{1.54} \]

To study the dynamics, in this Appendix 1.7, we start from a more general formulation of (1.23), that is,

\[ K^I_{t+1} = (G^I_t)^{\mu_I} (K^I_t)^{1-\mu_I}, \tag{1.55} \]

where \(\mu_I \in (0, 1)\). As in Agénor (2012b, Chapter 1), we assume that the production of new public capital requires combining the flow of investment and the existing capital stock.

Substituting (1.54) for \(j = I\) in (1.55) gives

\[ K^I_{t+1} = \left( \frac{G^I_t}{K^I_t} \right)^{\mu_I} K^I_t = \left( \frac{v_I \tau 2 \beta Y_t}{K^I_t} \right)^{\mu_I} K^I_t = (v_I \tau 2 \beta)^{\mu_I} \left( \frac{Y_t}{K^I_t} \right)^{\mu_I} K^I_t, \]

or equivalently

\[ \frac{K^I_{t+1}}{K^I_t} = (v_I \tau 2 \beta)^{\mu_I} \left( \frac{k^I_t}{Y_t} \right)^{1-\mu_I} \left( \frac{Y_t}{K^P_t} \right)^{\mu_I}, \tag{1.56} \]

where \(k^I_t = K^I_t / K^P_t\) is the public-private capital ratio.

Combining (1.53) and (1.56) yields

\[ k^I_{t+1} = \frac{(v_I \tau 2 \beta)^{\mu_I}}{2 \beta (1 - \tau) \sigma} \left( k^I_t \right)^{1-\mu_I} \left( \frac{Y_t}{K^P_t} \right)^{(1-\mu_I)}. \tag{1.57} \]

To fully specify the dynamics of \(k^I_{t+1}\), the expression \(Y_t/K^P_t\) must therefore be solved. First, rewrite equation (1.5) here for convenience:

\[ \frac{Y_t}{K^P_t} = (k^I_t)^\alpha \left( \frac{E^m_t N^m_t}{K^P_t} \right)^\beta \left( \frac{E^f_t N^f_t}{K^P_t} \right)^\beta \left( \psi^f, W \right)^\beta. \]
This equation can be rewritten as

$$\frac{Y_t}{K_t^P} = (k_t^I)^\alpha (\varepsilon_{f,W})^\beta \left( \frac{1}{x_t^m} \right)^\beta \left( \frac{1}{x_t^m} \right),$$

(1.58)

where $x_t^I = K_t^P / e_t^m N_t^m$ is the private capital-effective labour $j$ ratio.

Because $N_t^m = N_t^f$, and given that from (1.19) to eliminate $e_t^m$,

$$x_t^m = \frac{K_t^P}{e_t^m N_t^m} = \frac{K_t^P}{e_t^m N_t^f} \left( \frac{e_t^f}{e_t^m} \right) = x_t^f \left( \frac{1 - \chi_R}{\chi_R} \right)^{\nu_2} \left( \frac{\varepsilon_S + \varepsilon_{g,L}}{\varepsilon_S} \right)^{\nu_3}.$$

Substituting this result in (1.58), together with (1.8), yields

$$\frac{Y_t}{K_t^P} = \Gamma_1 \left( \frac{1}{\varepsilon_S + \varepsilon_{g,L}} \right)^{\beta \nu_3} (k_t^I)^{\alpha} \left( \frac{1}{x_t^I} \right)^{2\beta},$$

(1.59)

where

$$\Gamma_1 = \left( \frac{\chi_R}{1 - \chi_R} \right)^{\nu_2} (\varepsilon_{f,W})^{\nu_3} \left( \varepsilon_S \right)^{\nu_3}.$$

Substituting (1.59) into (1.57) yields

$$k_{t+1}^I = \Gamma_2 \left( \frac{1}{\varepsilon_S + \varepsilon_{g,L}} \right)^{\beta \nu_3 (1-\mu_I)} (k_t^I)^{(1-\mu_I)(1-\alpha)} \left( \frac{1}{x_t^I} \right)^{2\beta (1-\mu_I)},$$

(1.60)

where

$$\Gamma_2 = \left( \frac{v_I \tau 2 \beta}{2 \beta (1 - \tau) \sigma} \right)^{\mu_I} \Gamma_1^{1-\mu_I}.$$

From (1.60), it is clear that as long as $\mu_I = 1$, $k_t^I$ is constant $\forall t$ at

$$J = \frac{v_I \tau 2 \beta}{2 \beta (1 - \tau) \sigma} = \frac{v_I \tau}{(1 - \tau) \sigma},$$

given the definition of $\Gamma_2$.

The dynamic equation for $x_{t+1}^f$ is now derived. From (1.54), with $j = E$,

$$\frac{G_t^E}{N_t} = v_E \tau 2 \beta \left( \frac{Y_t}{N_t} \right).$$
Substituting this result into (1.17), together with (1.18), yields
\[ e_{t+1}^f = \left( \frac{\nu + \beta}{n} \right)^{\nu_1} \left( \frac{Y_t}{0.5N_t} \right)^{\nu_1} (E_t^f)^{1-\nu_1} [(1 - \chi^R)\varepsilon^f R_t^1 \varepsilon^R + \varepsilon^g L_t^1]^{\nu_3}. \] (1.61)

From (1.15) for \( t + 1 \), (1.53), (1.61) and given that \( N_{t+1}^f = 0.5N_{t+1} \),
\[ x_{t+1}^f = \frac{K_{t+1}^P}{e_{t+1}^f N_{t+1}^f} = \Gamma_3 \left( \frac{Y_t}{0.5e_t^f N_t} \right)^{1-\nu_1} (\varepsilon^S + \varepsilon^g L_t^1)^{-\nu_3}, \] (1.62)
where
\[ \Gamma_3 = \left\{ \frac{2\beta(1-\tau)\sigma}{[(1-\chi^R)\varepsilon^f R_t^1 \varepsilon^R + 0.5]^{\nu_3}} \right\} (\nu^E \beta^2)^{-\nu_3}. \]

By definition \( Y_t/0.5e_t^f N_t = (Y_t/K_t^P)x_t^f \). Using (1.59) to substitute for \( Y_t/K_t^P \) yields therefore
\[ \frac{Y_t}{0.5e_t^f N_t} = \Gamma_4 \left( \frac{1}{\varepsilon^S + \varepsilon_t^g L} \right)^{\beta_3} (k_{t+1}^f)^{\alpha} (x_{t+1}^f)^{1-2\beta}. \]

Substituting this result in (1.62), together with (1.8), yields
\[ x_{t+1}^f = \Gamma_4 (k_{t+1}^f)^{\alpha(1-\nu_1)} (x_{t+1}^f)^{(1-2\beta)(1-\nu_1)} (\varepsilon^S + \varepsilon_t^g L)^{-\nu_3[1+\beta(1-\nu_1)]}, \] (1.63)
where
\[ \Gamma_4 = \Gamma_3 \Gamma_1^{1-\nu_1}. \]

To determine the growth rate of output per worker, it is convenient to note first that \( Y_{t+1}/N_{t+1} = (Y_{t+1}/K_{t+1}^P)(K_{t+1}^P/N_{t+1}) \). Now, using (1.15), (1.53), and (1.59) for \( t + 1 \) yields
\[ \frac{Y_{t+1}}{N_{t+1}} = \Gamma_1 \left( \frac{1}{\varepsilon^S + \varepsilon_t^g L} \right)^{\beta_3} (k_{t+1}^f)^{\alpha} (x_{t+1}^f)^{-2\beta(1-\tau)\sigma(\frac{Y_t}{n0.5N_t})}. \] (1.64)

The balanced growth rate of output per worker is thus
\[ 1 + \gamma_{Y/N} = \Gamma_1 \left( \frac{1}{\varepsilon^S + \varepsilon_t^g L} \right)^{\beta_3} (\tilde{k}_t^f)^{\alpha} \frac{1}{0.5n} (\tilde{x}_t^f)^{-2\beta(1-\tau)\sigma}, \]
where, from the equation (1.51),

\[ \bar{g}^{g.L} = \max \left\{ 1 - \varepsilon^S - \frac{\Lambda_1 - \zeta^P \tilde{k}^I}{\Lambda_2}, 0 \right\}, \]  

(1.65)

and \( \tilde{k}^I \) and \( \tilde{x}^I \) are the steady-state solutions obtained by setting \( \Delta k_{t+1}^I = \Delta x_{t+1}^I = 0 \) in (1.60) and (1.63):

\[ \tilde{k}^I = \left\{ \frac{\Gamma_2}{\varepsilon^S + \bar{g}^{g.L}} \right\}^{1/\Pi_1}, \]

(1.66)

\[ \tilde{x}^I = \left\{ \frac{\Gamma_4 (\tilde{k}^I)^{\alpha(1-\nu_1)} (\varepsilon^S + \bar{g}^{g.L})^{-\nu_3[1+\beta(1-\nu_1)]}}{1 + \gamma} \right\}^{1/\Pi_2}, \]

(1.67)

where

\[ \Pi_1 = 1 - (1 - \mu_I)(1 - \alpha), \]

\[ \Pi_2 = 1 - (1 - 2\beta)(1 - \nu_1) > 0. \]

To determine the growth rate of output proceeds in the same way. From (1.59) for \( t + 1 \),

\[ Y_{t+1} = \Gamma_1 \left( \frac{1}{\varepsilon^S + \bar{g}^{g.L}} \right)^{\beta\nu_3 (k_{t+1}^I)^{\alpha}} \left( \frac{1}{x_{t+1}^I} \right)^{2\beta} K_{t+1}^P, \]

that is, using (1.53),

\[ 1 + g_{t+1} = \frac{Y_{t+1}}{Y_t} = \Gamma_1 \left( \frac{1}{\varepsilon^S + \bar{g}^{g.L}} \right)^{\beta\nu_3 (k_{t+1}^I)^{\alpha}} \left( \frac{1}{x_{t+1}^I} \right)^{2\beta} 2\beta(1 - \tau) \sigma, \]

which yields the steady-state growth rate:

\[ 1 + \gamma = \Gamma_1 \left( \frac{1}{\varepsilon^S + \bar{g}^{g.L}} \right)^{\beta\nu_3 (\tilde{k}^I)^{\alpha}} \left( \tilde{x}^I \right)^{-2\beta} 2\beta(1 - \tau) \sigma. \]

(1.68)

Let \( \mu_I = 1 \), as in the text. This implies, as can be inferred from (1.66), that \( \tilde{k}^I \) is constant at \( J \) (as shown in (1.26)) and that from (1.67) \( \tilde{x}^I \) is equal to (1.34). Using then (1.26) and (1.34), as well as (1.65) and (1.68), it can be verified that the
log derivatives of $J$, $\tilde{x}$, $\varepsilon^{g.L}$, and $1 + \gamma$ with respect to $v_I$ are, with $\kappa$ given,

\[
\frac{d\ln J}{dv_I}
\bigg|_{dv_I + dv_U = 0} = \frac{1}{v_I} > 0,
\]

(1.69)

\[
\frac{d\ln \tilde{x}}{dv_I}
\bigg|_{dv_I + dv_U = 0} = \left\{ \alpha(1 - \nu_1) - \frac{\nu_3[1 + \beta(1 - \nu_1)]\zeta^P J}{\Lambda_2(\varepsilon^S + \varepsilon^g.L)} \right\} \frac{1}{v_I [1 - (1 - 2\beta)(1 - \nu_1)]} \leq 0,
\]

(1.70)

\[
\frac{d\ln(\varepsilon^S + \varepsilon^{g.L})}{dv_I}
\bigg|_{dv_I + dv_U = 0} = \frac{\zeta^P J}{v_I \Lambda_2(\varepsilon^S + \varepsilon^{g.L})} > 0,
\]

(1.71)

\[
\frac{d\ln(1 + \gamma)}{dv_I}
\bigg|_{dv_I + dv_U = 0} = \alpha \frac{d\ln J}{dv_I}
\bigg|_{dv_I + dv_U = 0}
\]

(1.72)

\[
-\beta\nu_3 \frac{d\ln(\varepsilon^S + \varepsilon^{g.L})}{dv_I}
\bigg|_{dv_I + dv_U = 0} - 2\beta \frac{d\ln \tilde{x}}{dv_I}
\bigg|_{dv_I + dv_U = 0} \leq 0.
\]

Substituting (1.69)-(1.71) in (1.72) gives

\[
\frac{d\ln(1 + \gamma)}{dv_I}
\bigg|_{dv_I + dv_U = 0} = \frac{\alpha}{v_I} - \frac{\beta\nu_3\zeta^P J}{v_I \Lambda_2(\varepsilon^S + \varepsilon^{g.L})}
\]

\[
-2\beta \left\{ \alpha(1 - \nu_1) - \frac{\nu_3[1 + \beta(1 - \nu_1)]\zeta^P J}{\Lambda_2(\varepsilon^S + \varepsilon^{g.L})} \right\} \frac{1}{v_I [1 - (1 - 2\beta)(1 - \nu_1)]} \leq 0.
\]

This result is discussed in the text. With $\kappa$ endogenously related to $J$, as implied by (1.36), both $\eta_C$ and $\eta_E$ become also endogenous, and the above expressions become even more complex and ambiguous.

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## 1.8 Tables and Figures

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Table 1.2: Quantitative Experiments

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<tr>
<td>Home schooling</td>
<td>0.3</td>
</tr>
<tr>
<td>Relative bargaining power of women</td>
<td>0.3</td>
</tr>
<tr>
<td>Public-private capital stock ratio</td>
<td>0.148</td>
</tr>
<tr>
<td>Output growth rate</td>
<td>0.033</td>
</tr>
</tbody>
</table>

| Other shocks 2/                        | Absolute deviations from baseline |
|                                        | Baseline | $\mu_b = 0.2$ | $\chi^b = 0.5$ | $\eta_{le}^m = 3$ | $\eta_{le}^f = 15$ |
| Time allocated by girls to             |          |                |                |                |
| Household chores                       | 0.5      | 0.0030         | -0.0020        | 0.0727         | -0.0820        |
| Home schooling                         | 0.3      | -0.0030        | 0.0020         | -0.0727        | 0.0820         |
| Relative bargaining power of women     | 0.3      | -0.0280        | 0.0190         | -0.0092        | 0.0092         |
| Public-private capital stock ratio     | 0.148    | 0.0042         | -0.0028        | 0.0013         | -0.0013        |
| Output growth rate                     | 0.033    | -0.0001        | 0.0013         | -0.0014        | 0.0014         |

Notes: $\eta^O$ is the curvature of home production function and set equal to 0.8; $\psi_3$ is the elasticity of human capital with respect to girls’ time allocated to home schooling and set equal to 0.4; $\mu_b$ is the sensitivity of bargaining power to human capital stocks and set equal to 1.0; $\chi^b$ is the proportion of mothers’ rearing time allocated to boys and set equal to 0.6; $\eta_{le}^m$ and $\eta_{le}^f$ are the preference parameters of males and females for children’s education, respectively. They are equal to 5.0 and 8.1, respectively in the benchmark case.

1/ Increase in $\psi_3$ from 0.065 to 0.105, financed by a cut in $\psi_3$.

2/ Decrease in $\mu_b$ from 1 to 0.2, decrease in $\chi^b$ from 0.6 to 0.5, decrease in $\eta_{le}^m$ from 5 to 3, and increase in $\eta_{le}^f$ from 8.1 to 15.
Table 1.3: Increase in Infrastructure Investment, Financed by a Cut in Education Spending

<table>
<thead>
<tr>
<th></th>
<th>$v_3$ fixed at 0.4</th>
<th>Baseline</th>
<th>$v_1 = 0.1$</th>
<th>$v_1 = 0.2$</th>
<th>$v_1 = 0.3$</th>
<th>$v_1 = 0.4$</th>
<th>$v_1 = 0.5$</th>
<th>$v_1 = 0.6$</th>
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<tbody>
<tr>
<td>Time allocated by girls to household chores</td>
<td>0.5</td>
<td>-0.0184</td>
<td>-0.0184</td>
<td>-0.0184</td>
<td>-0.0184</td>
<td>-0.0184</td>
<td>-0.0184</td>
<td>-0.0184</td>
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<tr>
<td>Time allocated by girls to home schooling</td>
<td>0.3</td>
<td>0.0184</td>
<td>0.0184</td>
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<td>0.0184</td>
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<tr>
<td>Relative bargaining power of women</td>
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<td>0.0021</td>
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<td>0.0021</td>
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<tr>
<td>Public-private capital stock ratio</td>
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<td>0.0907</td>
<td>0.0907</td>
<td>0.0907</td>
<td>0.0907</td>
<td>0.0907</td>
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<tr>
<td>Output growth rate</td>
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<td>-0.0004</td>
<td>-0.0017</td>
<td>-0.0017</td>
<td>-0.0014</td>
<td>-0.0011</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>$v_3$ fixed at 0.4</th>
<th>Baseline</th>
<th>$v_3 = 0.1$</th>
<th>$v_3 = 0.2$</th>
<th>$v_3 = 0.3$</th>
<th>$v_3 = 0.4$</th>
<th>$v_3 = 0.5$</th>
<th>$v_3 = 0.6$</th>
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<tbody>
<tr>
<td>Time allocated by girls to household chores</td>
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<td>-0.0186</td>
<td>-0.0188</td>
<td>-0.0187</td>
<td>-0.0184</td>
<td>-0.0180</td>
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<tr>
<td>Time allocated by girls to home schooling</td>
<td>0.3</td>
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<td>0.0184</td>
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<tr>
<td>Relative bargaining power of women</td>
<td>0.3</td>
<td>0.0005</td>
<td>0.0011</td>
<td>0.0016</td>
<td>0.0021</td>
<td>0.0026</td>
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<tr>
<td>Public-private capital stock ratio</td>
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<td>0.0907</td>
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<tr>
<td>Output growth rate</td>
<td>0.033</td>
<td>-0.0021</td>
<td>-0.0018</td>
<td>-0.0016</td>
<td>-0.0014</td>
<td>-0.0013</td>
<td>-0.0011</td>
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</table>

Notes: Increase in $v_3$ from 0.065 to 0.105, financed by a cut in $v_1$, $v_1$ is the elasticity of human capital with respect to government spending on education and $v_3$ is the elasticity of human capital with respect to girls’ time allocated to home schooling. Both are set equal to 0.4 in the benchmark case.
Figure 1.1: Access to Infrastructure and Girls’ Time Allocation
Figure 1.2: Equilibrium and Increase in Spending on Infrastructure
Figure 1.3: Increase in Infrastructure Investment, Financed by a Cut in Spending on Education (Absolute deviations from baseline)

Notes: Increase in \( \nu_3 \) from 0.065 to 0.105, financed by a cut in \( \nu_2 \). \( \nu_3 \) is the elasticity of human capital with respect to government spending on education and \( \nu_2 \) is the elasticity of human capital with respect to girls’ time allocated to home schooling. Both are set equal to 0.4 in the benchmark case.
Chapter 2

Infrastructure and Industrial Development with Endogenous Skill Acquisition

2.1 Introduction

Much research on growth and development during the past decades has focused on structural transformation. This process has been defined by the reallocation of economic activity and employment across three broadly-defined production sectors (agriculture, industry, and services) and the movement of population from rural to urban areas.\footnote{Kuznets (1959) is an early reference on structural transformation. Another notable contribution is Syrquin and Chenery (1989), who provided a comprehensive review of post-war industrialisation patterns. Rogerson et al. (2013) offer a recent review of the literature, whereas Buera and Kaboski (2012a, 2012b) and Uy et al. (2013) provide recent analytical contributions.} A key insight of the literature is that accounting for the sectoral composition of output is crucial for understanding a variety of outcomes associated with the process of development. This includes changes in productivity, the composition of the labour force, and wage inequality.\footnote{The literature on economic transformation (reviewed by Greenwood and Seshadri (2005) for instance) refers to a broader set of issues, including changes in fertility patterns and women’s labour allocation between home and market production.} In particular, it has been shown that structural transformation involves not only a decline in the share of agriculture and extractive mining, mirrored by an increase in the share of industry and modern services in output, but also a shift between and within sectors from lower- to higher-productivity activities.
Some contributions to the literature on structural transformation have focused on the process of industrialisation, and more specifically the role of manufacturing industries (as opposed to mining or construction) in promoting rapid and sustained growth.\(^3\) The evidence collected by Szirmai (2012) and others suggests that—with the exception of a few small countries that benefited from successfully managing natural resource windfalls—virtually all countries that have sustained high growth rates for decades since the 1950s did so by building highly competitive manufacturing industries and by experiencing very rapid penetration of export markets in manufacturing. In fact, economic historians, most prominently Maddison (1991), have long argued that in the history of the world economy rarely has a country been able to develop without engaging in manufacturing: transforming first from a primarily agrarian society to one that is more industry driven, before relying on services as an engine for growth and employment. Moreover, as documented by Rodrik (2013), manufacturing industries tend to exhibit unconditional convergence, in the sense that industries that start further away from the world productivity frontier tend to experience significantly faster productivity growth. This occurs even without conditioning on the usual variables, such as initial income per capita, human capital, and institutional quality. Put differently, manufacturing has often proved to be a key engine for growth, even in the presence of poor institutions, weak governance, and bad policies.\(^4\)

There are several reasons, closely related to each other, which may help to explain why manufacturing industries play such a critical role in the process of industrial change and economic development. First, manufactured goods are highly trad-

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\(^3\)The United Nations’ Department of Economic and Social Affairs (DESA) defines manufacturing as “...the physical or chemical transformation of materials, substances or components into new products.” The materials, substances or components transformed are raw materials that are products of agriculture, forestry, fishing, mining or quarrying, or products of other manufacturing activities. Substantial alteration, renovation or reconstruction of goods is also generally considered to be manufacturing.

\(^4\)Rodrik’s results also suggest that countries with better institutions and policies experience faster rates of productivity growth in manufacturing and therefore benefit from a more rapid rate of conditional convergence.
able and the technology that enables their production—at least in the case of light manufacturing—often requires abundant raw labour but relatively few specialised skills, which can be transferred across borders if they are not available locally. Tradability and limited skill requirements have indeed allowed a number of countries to initiate a rapid transition to manufacturing by engaging in labour-intensive imitation activities, based on imported technology from advanced countries. Through learning-by-doing effects, manufacturing has often proved to be a major conduit for the diffusion of technologies, not only across countries but also between sectors of the economy. More generally, by providing an ever greater variety of inputs (some of which are in the form of new capital goods), with an ever greater degree of technological sophistication, the manufacturing sector helps to build knowledge that may fuel the expansion of a variety of production sectors.

Second, large productivity differences between firms within manufacturing—often related to firm size—enhance the scope for labour reallocation, from less efficient to more efficient firms. This process helps to foster innovation and to improve competitiveness in world markets, thereby creating new opportunities to respond to foreign demand for highly differentiated manufactured products. Export performance and export-led growth have therefore often been the result of sustained shifts in productivity patterns across domestic manufacturing firms.

Third, the diversity and sophistication of the types of products produced by the manufacturing sector also promote productivity change and economic growth. This effect is consistent with the evidence suggesting that industrial diversification and export sophistication are highly correlated (UNIDO (2009)). Furthermore, countries with more diversified production and export structures have higher incomes per capita (Imbs and Wacziarg (2003)). And finally, countries that produce and export more sophisticated products—those that are primarily manufactured at higher income levels—tend to grow faster. UNIDO (2009), for instance, found that between 1975 and 2005 fast growing low- and middle-income countries diversified and
increased significantly the sophistication of their production and export structures. By contrast, slowly growing low- and middle-income countries were less successful at diversifying, and performed poorly in terms of increasing product and export sophistication.

Fourth, relative to other sectors, there are often strong forward and backward linkages, as well as spillover effects, associated with manufacturing activities (see UNIDO (2011)). In particular, machinery and fertilisers may serve as inputs in agricultural production, and manufacturing firms may be an important source of demand for other sectors, such as agriculture (as a source of inputs), banking, transport, insurance, and communication services. Consequently, at the aggregate level, manufacturing may provide significant indirect contributions to investment and production. Furthermore, it may also provide employment opportunities for workers with a variety of skills—including those with little education, at least in an initial phase where imitation activities dominate. Successful industrial transformation may therefore lead to the modernisation of other sectors in the economy, including agriculture.

This chapter contributes to the ongoing debate on industrialisation and growth by developing a two-period overlapping generations (OLG) model of endogenous growth and horizontal innovation à la Romer (1990), with three main characteristics that are broadly consistent with some of the observations we have discussed. First, in line with Currie et al. (1999), Acemoglu et al. (2006), Vandenbussche et al. (2006), and Perez-Sebastian (2007), we account for two types of design activities: those based on imitation and those based on homegrown innovation. Both of these activities produce blueprints for intermediate-good producers. Imitation involves essentially copying foreign ideas and adapting foreign technology, whereas innovation involves inventing new ideas. Thus, unlike studies focusing on structural transformation, our analysis of the process of industrial development does not focus

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5 Within manufacturing, linkages are typically stronger than in other sectors, as a large share of industrial output is in fact used as inputs for other industries.
on the relative contribution of manufacturing to the whole economy. Indeed, rather than focusing on the composition of final output, we focus on the composition of the range of intermediate inputs, created through imitation and innovation activities, that firms use in producing final goods. From that perspective, industrial development takes the form of an increase over time in the relative contribution of more technology-intensive intermediate goods in the production process of manufactured goods. Industrial transformation is thus fundamentally the result of the transition from imitation to innovation, or equivalently of a reallocation over time of resources to more skill- and technology-intensive production of designs and development activities.

Our analysis also accounts for the existence of a learning externality associated with imitation activities, which tends to stimulate (at least in an initial phase) productivity in the innovation sector. If this effect is sufficiently strong, imitation may be the main source of productivity growth in the early stages of development, with innovation taking over later on in time. This feature of our analysis highlights the fact that the international diffusion of technology, and the promotion of imitation activities based on the nonrivalrous use of ideas, may help a poor country to absorb knowledge embedded in products invented in the rich world. Technological imitation by low-income developing countries with a limited pool of skilled workers may thus account for their success at an early stage of industrialisation, as noted earlier. In that sense, imitation activities can serve as a “stepping stone” for innovation, as emphasised in some other contributions (see for instance Glass (2010) and Agénor and Dinh (2013)). Beyond a certain point, however, these benefits tend to fade, thereby reducing over time the marginal gain associated with present imitation effort. It therefore becomes crucial to find new ways to increase productivity in the

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6 Glass (2010) builds a product-cycle model in which an exogenous fraction of industries has to engage in imitation before being able to target the market for innovations. By contrast, Agénor and Dinh (2013) consider learning spillovers between separate imitation and innovation sectors, as we do here. Lorenczik and Newiak (2012) discuss how policies targeted at intellectual property rights affect the process through which imitation can help to promote innovation.
innovation sector and avoid falling into a middle-income growth trap (see Agénor and Canuto (2012)).

Second, the process of skill acquisition is endogenised. In standard fashion, individuals choose to invest in (advanced) education depending on the relative returns to schooling. While imitation requires only unskilled labour, innovation requires high-skilled labour. At the same time, the process of industrialisation itself—especially in the early stages, through the learning externality associated with imitation activities, as noted earlier—increases the demand for high-skilled labour, which puts upward pressure on skilled wages and induces more individuals to invest in education. Thus, there is a two-way interaction between industrialisation and the quality of the labour force. At the same time, however, if learning externalities are sufficiently strong, investment in human capital is not a prerequisite for promoting growth in its initial stages. Once the process of imitation takes off, it contributes to the accumulation of knowledge available to all workers in the economy, raising productivity and wages in the innovation sector, thereby promoting investment in skills and the expansion of the innovation sector. Thus, as in Vandenbussche et al. (2006), our analysis highlights the importance of distinguishing between different types of human capital and their allocation across sectors at different stages of economic development. However, we also go beyond their analysis by accounting explicitly for the process through which skill acquisition occurs, instead of assuming (as they do) that the distribution of the labour force is given exogenously and constant over time. In doing so we bring to the fore the two-way interaction between

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7 Cross-country evidence suggests the existence of a close correlation between export sophistication and the percentage of the labour force that has completed tertiary education (see World Bank (2009)). This is consistent with the specification of an innovation sector that uses skilled labour only.

8 This feature of the model is consistent with the evidence in Madsen et al. (2010). Using data for 55 developing and developed countries, they found that innovation is an important factor for growth in OECD countries, whereas growth in developing countries is driven by imitation. Furthermore, the interaction between educational attainment and the distance to the world technological frontier is a significant determinant of growth in the overall sample. A similar result is obtained by Madsen (2013) for productivity growth in industrial countries.

9 This is consistent for instance with Gómez (2011), who unlike Funke and Strulik (2000) found that the sequencing between human capital formation and innovation can be reversed.
a country’s own development process—which depends on learning spillovers across sectors, rather than distance to the technological frontier per se—and human capital formation.

Third, the model accounts for the composition of infrastructure, or public capital, and its impact on production and design activities. Specifically, we distinguish between basic infrastructure (which consists of roads, energy, and basic telecommunications) and advanced infrastructure, which consists of advanced information and communication technologies (ICTs) in general, and high-speed communication networks in particular. Basic infrastructure helps to promote productivity in the final good and imitation sectors, whereas advanced infrastructure benefits mainly the innovation sector. One reason for the latter effect is that access to high-speed broadband, in particular, facilitates the buildup of knowledge networks, thereby promoting the dissemination of ideas within and across borders (see Romer (2010) and Agénor and Canuto (2012)). This is consistent with recent research, which shows that ICTs and innovation are closely interlinked (see Cardona et al. (2013) and Baquero Forero (2013)) and that ICTs have a strong, albeit possibly nonlinear, impact on growth (Czernich et al. (2011) and Oulton (2012)). Baquero Forero (2013), in particular, found that countries with more investment in two specific technologies, mobile telephony and internet infrastructure, are closer to the global technological frontier.\footnote{The evidence also suggests that broadband internet is a necessary, but not sufficient, condition for the positive impact of ICTs on innovation and growth. As noted earlier, the model highlights the availability of high-skilled labour as an important determinant of innovation. Yet, access to high levels of human capital is neither necessary nor sufficient to promote innovation and growth; a low level of productivity in the innovation sector may be due to a lack of access to (advanced) infrastructure.}

These features of the model, and the analysis that they lead to, shed useful light on the nature of the ongoing debate on industrialisation and the role of manufacturing in low-income countries. This is especially relevant to countries in Sub-Saharan Africa, where the relative importance of the manufacturing sector has declined almost continuously in recent decades (see Page (2012)). As documented by UNIDO
(2011, Table 1), in Africa the share of manufacturing in GDP fell from 15.3 percent in 1990 to 12.8 percent in 2000 and 10.5 percent in 2008. The diversity and sophistication of the region’s manufacturing sectors have also declined, and it continues to be marginalised in global manufacturing trade: its share in global manufacturing value added fell from 1.2 percent in 2000 to 1.1 percent in 2008. The distinction between imitation and innovation, and the associated implication regarding the diversification of inputs that we emphasise, is important to highlight the phases of development that countries in the region may go through.

Accounting for endogenous skill acquisition allows us to also account for the fact that Africa lags substantially behind other developing country regions in skills and vocational training. The gross enrolment ratio at the tertiary level is only 6 percent in the region and recent increases in the number of higher education graduates have often been at the expense of quality (African Development Bank (2012, Chapter 6), UNECA (2013, Chapter 2)). Where they exist, technical and vocational skills development systems suffer from a shortage of qualified staff, obsolete equipment, poorly-designed programmes, and weak links with the labour market. According to some recent estimates (see Messinis and Ahmed (2013)), and despite substantial spending on education in several countries, synthetic measures of human capital and cognitive skills in the region have deteriorated almost continuously since the early 1980s—a potential hindrance to cross-border technology diffusion and assimilation.

Finally, introducing public capital allows us to account for the fact that African countries have very poor transport, communication and energy infrastructure. As recognised by many observers, poor access to basic infrastructure remains a major obstacle to the development of competitive manufacturing industries in the region and acts as a significant drag on economic growth. Indeed, it is estimated that Africa loses about 1 percentage point per year in per capita income growth as a result of

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11By contrast, in developing Asia, that share rose from 13 percent to 25 percent during the same period. See Memedovic and Lapadre (2009) for a structural decomposition analysis of these changes.
its infrastructure deficit (UNIDO (2012)). And according to Foster and Briceño-Garmendia (2010), the high cost of infrastructure in Africa significantly increases trade costs and reduces the productivity of African firms. At the same time, our analysis helps to emphasise that the type of infrastructure that is needed to promote growth does change in between the initial and later phases of development.

The remainder of the chapter is organised as follows. Section 2 presents the model, which is a two-period overlapping generations (OLG) model of endogenous growth and horizontal innovation (as in Romer (1990)). A key feature of the model, as noted earlier, is the learning externality associated with imitation, which tends to promote innovation activities and growth. The strength of this externality determines the speed at which industrial transformation occurs. The balanced growth equilibrium is defined in Section 3 and the steady-state solution is presented in Section 4. Section 5 calibrates the model for a low-income country, characterised by an initial situation where the proportion of skilled workers is small, the innovation sector is embryonic (so that most of the skilled workers are engaged in the production of final goods), access to basic infrastructure is limited, access to advanced infrastructure is almost non-existent, and the cost of acquiring skills is high. Section 6 focuses on experiments involving changes in the level and composition of public investment in infrastructure and studies the transitional dynamics associated with these changes. The final section draws together the policy implications of the analysis and offers some concluding remarks.

2.2 The Model

The economy that we consider is populated by individuals with different innate abilities, who live for two periods: adulthood and old age. Population is constant at \( \tilde{N} \). Each individual is endowed with one unit of time in the first period of life, and zero units in old age.
There are five production sectors in the economy: one producing a homogeneous final good, two producing intermediate goods (core and enhanced inputs), and two creating designs (imitation and innovation sectors) used for the production of each of the two categories of intermediate inputs. The final good is produced by combining both private and public inputs, and is used for consumption, private and public investment, and the production of intermediate goods. Public inputs consist of basic infrastructure (roads, electricity, water and sanitation, and basic telecommunications) and advanced infrastructure (essentially of high-speed telecommunications). Both types of services are provided free of charge but are subject to congestion. Production in the design sectors combines public and private (labour) inputs as well, but in different ways.

Firms in the final good and design sectors are perfectly competitive, whereas those in the intermediate good sectors are monopolistically competitive, each producing (as in Romer (1990)) a differentiated variety of good. The total number of blueprints existing at a certain point in time coincides with the number of intermediate input varieties available in the economy, and represents the stock of (nonrival) knowledge. Knowledge is non-appropriable, so designers have access to that information for free. However, they cannot sell the knowledge gathered from a design to future potential designers either. As in Agénor and Dinh (2013), knowledge accumulated in the imitation sector creates an externality that promotes productivity in both design sectors, but this benefit is subject to diminishing marginal returns.

There are two categories of labour in the economy, skilled and unskilled. Individuals are born unskilled and must decide at the beginning of adulthood whether to become skilled or remain unskilled for the rest of their adult life. Becoming skilled involves both time and pecuniary costs. Labour (either skilled or unskilled) is perfectly mobile between the final good and design sectors, and wages adjust to clear both segments of the labour market.
2.2.1 Individuals

Individuals have identical preferences but are born with different abilities, indexed by \(a\). Ability (or talent) is observable at birth by all and follows a continuous, time-independent distribution. For tractability, \(a\) is assumed to follow a Pareto (Type I) distribution, defined over \(a \in [a_m, \infty)\), with a density function \(f(a) = \theta a_1^\theta / a^{1+\theta}\) and a cumulative distribution function \(F(a) = 1 - (a_m/a)^\theta\), where \(a_m > 1\) denotes the lowest ability and \(\theta > 2\) is the tail index.\(^{12}\) Average ability is thus \(a_m^\theta/(\theta - 1)\). Each individual maximises utility and decides whether to enter the labour force as an unskilled worker or (after undergoing training) a skilled worker. The individual’s ability determines his or her relative cost of acquiring skills.

Specifically, an adult with ability \(a\) can enter the labour force at the beginning of period \(t\) as an unskilled worker and earn the wage \(w_t^U\), which is independent of the worker’s ability. Alternatively, the individual may choose to spend first a fraction of time \(\varepsilon \in (0, 1)\) of his or her time endowment at the beginning of adulthood in training (or higher education), incur a cost \(tc_t > 0\), and enter the labour force for the remainder of the period as a skilled worker, earning the wage \(w_t^S\). During training, workers earn no income.\(^{13}\) There are no barriers to entry in the design sectors, so any individual can work there if he or she is willing to do so.\(^{14}\)

Let \(c_{t+j}\) denote consumption at period \(t+j\) of an individual working at wage \(w_t^h\), where \(h = U, S\), born at the beginning of period \(t\), with \(j = 0, 1\). The individual’s

\(^{12}\)The Pareto distribution (which has a fat tail, that is, it is skewed to the right, like the log-normal distribution) implies that the proportion of low-ability individuals is high—a good representation of the empirical evidence. It also implies that the proportion of individuals with ability above the threshold level \(a_m\) decreases with the threshold at a constant rate. The assumption \(\theta > 2\) ensures that the ability distribution has a finite variance.

\(^{13}\)To avoid a corner solution in which all individuals become skilled, it could be assumed that individuals with ability \(a \in (0, a_L]\) never choose to undergo training. We do not impose this restriction explicitly, because in the numerical experiments that we perform later the unskilled population never shrinks to zero.

\(^{14}\)Perfect labour mobility between the final good sector and the design sectors implies that there is a single, economy-wide wage for each category of labour.
discounted utility function is given by

\[ U_t^h = \eta C \ln c_{t}^{h,t} + \ln \frac{c_{t+1}^{h,t}}{1 + \rho}, \quad h = U, S \]  

(2.1)

where \( \rho > 0 \) is the discount rate and \( \eta C > 0 \) a preference parameter.

The period-specific budget constraints, which depend on the sector of employment in adulthood, are given by

\[ c_t^{U,t} + s_t^{U} = (1 - \tau)w_t^{U}, \]  

(2.2)

\[ c_t^{S,t} + s_t^{S} = (1 - \varepsilon)(1 - \tau)(w_t^{S} - tc_t), \]  

(2.3)

\[ c_{t+1}^{h,t} = (1 + r_{t+1})s_t^{h}, \quad h = U, S \]  

(2.4)

where \( s_t^{h} \) is the savings rate of type-\( h \) worker, \( 1 + r_{t+1} \) the rate of return on holding assets between periods \( t \) and \( t+1 \), \( tc_t \) the training cost, and \( \tau \in (0, 1) \) the tax rate.

It is optimal for an individual with ability \( a \) to train and become skilled if and only if

\[ (1 - \varepsilon)w_t^{S} \geq w_t^{U} + tc_t. \]  

(2.5)

The training cost is proportional, at rate \( \mu \in (0, 1) \), to the wage that skilled workers earn once training is completed and they become employed, so that

\[ tc_t = \mu w_t^{S}/a^\chi. \]  

(2.6)

Thus, the cost of training is independent of the length of training. In addition, it is also assumed that it is easier for able individuals to learn. As a result of this, they incur a lower training cost. The productivity parameter \( \chi \in (0, 1) \) measures the strength of that effect, which is subject to diminishing returns.

Equation (2.5), holding as an equality, together with (2.6), give the threshold level of ability \( a_t^C \) such that all individuals with ability \( a < a_t^C \) choose to remain
unskilled:

\[ a_t^U = \mu^{1/\gamma}(1 - \varepsilon - \frac{w_t^U}{w_t^S})^{-1/\gamma}. \]  

(2.7)

This equation describes an increasing and convex relationship between the wage ratio \( w_t^U / w_t^S \) and \( a_t^U \).

The productivity of unskilled workers is equal to unity, independently of their abilities. The (effective) supply of unskilled labour, \( N_t^U \), is thus equal to

\[ N_t^U = \int_{a_m}^{a_C} f(a) da N = a_m^\theta \left[-a^{-\theta}\right]_{a_m}^{a_C} \tilde{N} = \left[1 - \left(\frac{a_m}{a_C}\right)^\theta\right] \tilde{N}. \]  

(2.8)

The raw supply of skilled labour is \( \int_{a_C}^{a_C} f(a) da N = (a_m/a_C)^\theta \tilde{N} \). However, we must account for the average productivity of workers with ability \( a \in [a_C, \infty) \) who have undergone training, which is equal to \( \theta a_C^\theta / (\theta - 1) \). Thus, the effective supply of skilled labour at time \( t \), \( N_t^S \), can be defined as

\[ N_t^S = \int_{a_C}^{\infty} a f(a) da \tilde{N} = \theta a_m^\theta \left[\frac{a^{1-\theta}}{1-\theta}\right]_{a_C}^{\infty} \tilde{N} = \frac{\theta a_m^\theta}{\theta - 1} (a_C^\theta)^{1-\theta} \tilde{N}. \]  

(2.9)

### 2.2.2 Final Good

Production of the final good (manufacturing), \( Y_t \), requires the use of skilled labour, \( N_t^{S,Y} \), unskilled labour, \( N_t^{U,Y} \), private capital, \( K_t^P \), basic public infrastructure, \( K_t^B \), and a combination of core intermediate inputs, \( x_t^{I,s,t} \), with \( s \in (0, M_t^I) \), and enhanced intermediate inputs, \( x_t^{R,s,t} \), with \( s \in (0, M_t^R) \). Within each category, inputs are substitutes to one another.

Let \( X_t \) be a composite intermediate input defined as

\[ X_t = \left[\int_0^{M_t^I} (x_t^{I,s,t})^\eta ds\right]^{\nu/\eta} \cdot \left[\int_0^{M_t^R} (x_t^{R,s,t})^\eta ds\right]^{(1-\nu)/\eta}, \]

(2.10)

where \( \eta \in (0, 1) \) and \( 1/(1 - \eta) > 1 \) is (the absolute value of) the price elasticity of demand for each intermediate good, and \( \nu \in (0, 1) \). Thus, the composite inter-
mediate input exhibits constant returns to scale with respect to core and enhanced inputs.

The production function is specified as

\[ Y_t = \left[ \frac{K_t^B}{(K_t^P)^{\zeta_K} N_t^{\xi_N}} \right]^\omega \left[ (1 - \varepsilon) N_t^{S,Y} \right]^{\beta_S} \left( N_t^{U,Y} \right)^{\beta_U} X_t^U (K_t^P)^\alpha, \]  \tag{2.11}

where \( \beta_S, \beta_U, \alpha, \gamma \in (0, 1), \ \omega > 0, \ \zeta_K, \zeta_N > 0, \ \alpha = 1 - (\beta_S + \beta_U) - \gamma, \) and \( K_t^P \) is the aggregate private capital stock. This specification implies that there are constant returns to scale in private inputs and that basic public capital is partially rival and subject to congestion, measured by the aggregate private capital stock and population size. The strength of congestion effects is measured by the parameters \( \zeta_K \) and \( \zeta_N \).

Assuming that private capital depreciates fully during each period, profits of the representative firm are given by

\[ \Pi_t^Y = Y_t - \int_0^{M_I} P_{t}^{I_s} x_{s,t}^I \, ds - \int_0^{M_R} P_{t}^{R_s} x_{s,t}^R \, ds - w_t^S (1 - \varepsilon) N_t^{S,Y} - w_t^U N_t^{U,Y} - r_t K_t^P, \]

where \( P_{t}^{I_s} (P_{t}^{R_s}) \) is the price of core (enhanced) intermediate good \( s \) and \( r_t \) the (net) rental rate of private capital. The final good is used as the numéraire and its price is normalised to unity.

Subject to (2.10) and (2.11), profit maximisation with respect to labour, private capital, and quantities of all intermediate goods \( x_{s,t}^j, \forall s, \) taking as given factor prices and \( M_t^j, j = I, R, \) yields

\[ w_t^S = \beta_S \frac{Y_t}{(1 - \varepsilon) N_t^{S,Y}}, \quad w_t^U = \beta_U \frac{Y_t}{N_t^{U,Y}}, \]  \tag{2.12}

\[ r_t = \alpha \left( \frac{Y_t}{K_t^P} \right), \]  \tag{2.13}

\[ x_{s,t}^j = \left( \frac{\gamma \nu^j Z_t^j}{P_{t}^{I_s}} \right)^{1/(1 - \eta)}, \quad s = 1, ... M_t^j, \]  \tag{2.14}
\[ Z^j_t = Y_t / \int_0^{M^j_t} (x^j_{s,t})^n ds, \quad (2.15) \]

where \( j = I, R, \nu^j = \nu, \) and \( \nu^R = 1 - \nu. \)

### 2.2.3 Intermediate Goods

As noted earlier, there are two sets of intermediate goods producers: those producing core inputs (index \( I \)), based on blueprints produced by the imitation sector, and those producing enhanced inputs (index \( R \)), based on designs produced by the innovation sector. Each firm produces one, and only one, horizontally-differentiated intermediate good. In both cases, production of each unit of intermediate good requires one unit of the final good.\(^{15}\)

The two sectors are treated symmetrically. Each producer in sector \( j = I, R \) must purchase a patented design from the respective design sector (imitation or innovation). Once the patent is bought, each producer sets its price to maximise profits, given the perceived demand function for its good (2.14), which determines marginal revenue. Under a symmetric equilibrium, profits are given by \( \Pi^j_t = (P^j_t - 1)x^j_t \) or using (2.14) and (2.15), \( \Pi^j_t = (P^j_t - 1)[\gamma \nu^j Y_t / P^j_t M^j_t(x^j_t)^n]^{1/(1-\eta)}, j = I, R. \)

In standard fashion, the solution yields the optimal price as

\[ P^j_t = \frac{1}{\eta}. \quad \forall s = 1, \ldots, M^j_t, \quad j = I, R \quad (2.16) \]

Using (2.14), the quantity demanded at this price is \( x^j_{s,t} = (\gamma \eta \nu^j Z^j_t)^{1/(1-\eta)}, \forall s, \) that is, noting that under symmetry \( \int_0^{M^j_t} (x^j_{s,t})^n ds = M^j_t (x^j_t)^n, \)

\[ x^j_t = \gamma \eta \nu^j \left( \frac{Y_t}{M^j_t} \right), \quad j = I, R \quad (2.17) \]

\(^{15}\)It could be assumed, as in Daido and Tabata (2013) for instance, that access to basic infrastructure helps to promote production of intermediate goods. However, as long as the production process for each type of inputs remains the same, this would not make qualitative differences to our analysis, given the specification of the production function of final goods.
with maximum profit given by

\[ \Pi_j^t = (1 - \eta) \gamma \nu^j \left( \frac{Y_t}{M_t^j} \right). \quad j = I, R \]  

(2.18)

For simplicity, it is assumed that intermediate-input producing firms last for only one period, and that patents are auctioned off randomly to a new group of firms in each period. Thus, each producer of a new intermediate good holds a patent only for the period during which it is bought, implying monopoly profits during that period only; yet patents last forever.\textsuperscript{16} By arbitrage, therefore, the patent price \( Q_j^t \) is

\[ Q_j^t = \Pi_j^t. \quad j = I, R \]  

(2.19)

### 2.2.4 Design Sectors

Designs are produced in two sectors: an imitation sector, which employs only unskilled labour, in quantity \( N_t^{U,I} \), and an innovation sector, which employs only skilled labour, in quantity \( N_t^{S,R} \). There is no aggregate uncertainty in either sector. In the imitation sector, local firms invest resources in order to absorb and adapt the information needed to replicate new products invented abroad. Thus, imitation differs from innovation in that the number of goods that can be copied at any point in time is limited in part to the rate at which imitable goods are being discovered elsewhere.

Both imitation and innovation create two kinds of knowledge. First, \( \textit{private} \) knowledge, which is acquired (for a price) by intermediate goods firms to produce a new production input. Second, \( \textit{public} \) knowledge, which spills over to other firms in the imitation and innovation sectors—in ways specified later—and increases productivity there. In addition, there is an externality from imitation for innovation: as agents learn to imitate, they also develop cognitive skills that help them to innovate later on. This is consistent with the idea, alluded to earlier, that imitation can be a

\textsuperscript{16}See Agénor and Canuto (2012) for a more detailed discussion of this assumption.
stepping stone for true innovation.

Consider the imitation sector first. The aggregate technology is defined as

\[ M_{t+1}^I - M_t^I = A_t^I \left( \frac{N_t^U I_t}{N_t} \right) (1 + g^W)^{\kappa^I}, \tag{2.20} \]

where \( A_t^I \) is a productivity factor, and \( g^W > 0 \) the growth rate of the stock of designs available internationally that can be effectively imitated in the country under consideration. The technology parameter \( \kappa^I \in (0, 1) \) is assumed to be less than unity, to capture the fact that the growth in imitable goods worldwide entails diminishing marginal benefits for domestic imitation. In addition, as in Chen and Funke (2013) for instance, the international knowledge pool available for copying is assumed to grow at an exogenous rate.\(^\text{17}\) Finally, to eliminate scale effects, it is the ratio of unskilled workers to total population that is taken to affect activity in that sector.\(^\text{18}\)

Productivity in imitation activities depends on the economy’s stock of imitated designs and access to basic infrastructure:

\[ A_t^I = (k_t^B)^{\phi_t^I} M_t^I, \tag{2.21} \]

where \( k_t^B = K_t^B / K_t^P \), and \( \phi_t^I \in (0, 1) \). Thus, as in Romer (1990), each design creates a positive externality for future imitation activities.\(^\text{19}\) Access to basic public capital is subject to (proportional) congestion, measured by the private capital stock.

Firms in the imitation sector choose labour so as to maximise profits, \( \Pi_t^I = Q_t^I (M_{t+1}^I - M_t^I) - w_t^I N_t^U I_t \), subject to (2.20), and taking the wage rate, the patent

\(^\text{17}\)A more general specification would be to assume, as in Perez-Sebastian (2007) for instance, that as distance from the world technological frontier of imitated goods decreases, imitation becomes more expensive. This would mitigate the benefits of imitation over time but would not eliminate those that occur in the early stages, as we discuss later.

\(^\text{18}\)See Dinopoulos and Segerstrom (1999). As in Di Maria and Stryszowski (2009), it could also be assumed that a fraction of the skilled labour force is also involved in adapting existing (imported) technologies. However, this would significantly complicate the analysis without adding much insight.

\(^\text{19}\)We therefore abstract from duplication externalities, as discussed in some of the literature (see for instance Gancia and Zilibotti (2005) and Gómez and Sequeira (2013)). However, much of the evidence on these externalities relates to industrial countries.
price, $Q^I_t$, and productivity $A^I_t$, as given. The first-order condition with strictly positive employment is given by

$$w^U_t = \left(\frac{Q^I_t A^I_t}{N}\right)(1 + g^W)^{\nu^I}. \quad (2.22)$$

Consider now the innovation sector. The aggregate technology is defined as

$$M^R_{t+1} - M^R_t = A^R_t \left(\frac{(1 - \varepsilon)N^{S,R}_t}{N}\right), \quad (2.23)$$

where $A^R_t$ is productivity, which depends on access to advanced infrastructure and both stocks of technological knowledge—with innovation creating a stronger spillover effect than imitation:

$$A^R_t = (k^A_t)^{\phi^R_t} (M^R_t + \phi^R_{2} M^I_t), \quad (2.24)$$

where $k^A_t = K^A_t / K^P_t$, $\phi^R_1 \in (0, 1)$, and $\phi^R_2 > 0$.\(^{20}\)

This specification accounts for an efficiency gain associated with imitation: the more a country engages initially in copying foreign technology, the more its workers become familiar with existing innovations made abroad, and the easier it is to engage in original innovation. These trade-related, knowledge spillovers and their cross-sectoral impact on the capacity to innovate have been well documented in the literature.\(^{21}\) However, this spillover may weaken over time if the ratio $M^I_t / M^R_t$ itself decreases over time, which occurs if the innovation sector—precisely as a result of knowledge spillovers—expands at a relatively faster rate.

Firms in the innovation sector also choose labour so as to maximise profits,

$$\Pi^R_t = Q^R_t (M^R_{t+1} - M^R_t) - w^S_t (1 - \varepsilon)N^{S,R}_t, \quad \text{subject to } (2.23),$$

and taking the wage rate, the patent price, $Q^R_t$, and productivity as given.

\(^{20}\)For tractability, access to advanced infrastructure is again taken to be congested by the private capital stock.

\(^{21}\)See Keller (2004) and World Trade Organisation (2008, Chapter 2). The case considered here corresponds to what is usually referred to in the trade literature as “active” spillovers, when the importing country acquires the knowledge embodied in the imported good (whether intermediate or final), often through a process of reverse engineering.
The first-order condition with positive employment is given by

\[ w_i^S = \frac{Q_i^R A_i^R}{N}. \]  \hfill (2.25)

### 2.2.5 Government

The government levies a tax on wages at the rate \( \tau \), invests a total of \( G_i^B \) and \( G_i^A \) on basic and advanced infrastructure, and spends \( G_i^U \) on other items. Its services are provided free of charge. It cannot issue debt claims and must therefore run a balanced budget:

\[ G_t = \sum_i G_i^i = \tau \left[ w_i^U N_i^U + (w_i^S - tc_i)(1 - \varepsilon)N_i^S \right]. \]  \hfill (2.26)

Shares of public spending are all assumed to be constant fractions of government revenues:

\[ G_i^i = v_i \tau \left[ w_i^U N_i^U + (w_i^S - tc_i)(1 - \varepsilon)N_i^S \right], \quad i = A, B, U \]  \hfill (2.27)

where \( v_i \in (0, 1), \forall i \).

Combining (2.26) and (2.27) therefore yields

\[ \sum_i v_i = 1. \]  \hfill (2.28)

Assuming that public capital depreciates fully during each period, both stocks evolve according to

\[ K_{i+1}^i = \varphi G_i^i, \quad i = A, B \]  \hfill (2.29)

where \( \varphi \in (0, 1) \) is an efficiency parameter, which measures the extent to which investment flows translate into actual accumulation of public capital (see Agénor (2010, 2012b)).
2.2.6 Market-Clearing Conditions

Finally, the model requires specifying the equilibrium conditions between aggregate savings and investment, and between supply and demand in the markets for skilled and unskilled labour.

The savings-investment balance requires the capital stock in $t + 1$ to be equal to savings in period $t$ by individuals born in $t - 1$:

$$K_{t+1}^P = s_t^U N_t^U + s_t^S N_t^S.$$ (2.30)

Equilibrium of the market for unskilled labour implies that workers are employed either in the production of the final good or in the imitation sector, that is, $N_t^{U,Y} + N_t^{U,I} = N_t^U$, or equivalently, in terms of ratios,

$$\theta_t^{U,Y} + \theta_t^{U,I} = \theta_t^U,$$ (2.31)

where $\theta_t^U = N_t^U / \bar{N}$ is the total supply of unskilled labour in proportion to total population. Subsequently, from (2.8) and normalising $a_m$ to unity for simplicity, we obtain

$$\theta_t^U = 1 - (a_t^C)^{-\theta}.$$ (2.32)

This equation implies a positive relationship between $a_t^C$ and $\theta_t^U$.

Similarly, equilibrium of the market for skilled labour implies that workers are employed either in the production of the final good or in the innovation sector, that is, $N_t^{S,Y} + N_t^{S,R} = N_t^S$, or equivalently, in relative terms,

$$\theta_t^{S,Y} + \theta_t^{S,R} = \theta_t^S,$$ (2.33)

where $\theta_t^S = N_t^S / \bar{N}$ is the total supply of skilled labour, measured in efficiency units,
in proportion to total population. From (2.9), this is equal to

\[ \theta^S_t = \frac{\theta}{\theta - 1} (a^C_t)^{1-\theta}. \]  

(2.34)

Because \( \theta > 1 \), this equation implies a negative relationship between \( a^C_t \) and \( \theta^S_t \).

## 2.3 Balanced Growth Equilibrium

In this economy an *equilibrium with imperfect competition* is a sequence of consumption and saving allocations \( \{c^h_t, s^h_t \}_{t=0}^\infty \) for \( h = U, S \), private capital stock \( \{K^P_t \}_{t=0}^\infty \), public capital stocks \( \{K^A_t, K^B_t \}_{t=0}^\infty \), prices of production inputs \( \{w^U_t, w^S_t, r_{t+1} \}_{t=0}^\infty \), prices and quantities of intermediate inputs \( \{P^s,j_t, x^s,j_t \}_{t=0}^\infty \), \( \forall s \in (0, M^I_t) \) and \( j = I, R \), existing varieties, \( \{M^I_t, M^R_t \}_{t=0}^\infty \), such that, given initial stocks \( K_0 > 0, K^A_0, K^B_0 > 0 \), and \( M^I_0, M^R_0 > 0 \).

a) all individuals, skilled or unskilled, maximise utility by choosing consumption subject to their intertemporal budget constraint, taking factor prices and the tax rate as given;

b) firms in the final good sector maximise profits by choosing labour, capital, and intermediate inputs, taking input prices as given;

c) intermediate input producers set prices so as to maximise profits, while internalising the effect of their decisions on the perceived aggregate demand curve for their product;

d) producers in the design sectors maximise profits by choosing how much labour to hire, taking wages, patent prices, productivity, and population, as given;

e) the equilibrium (patent) price of each blueprint extracts all profits made by the corresponding intermediate input producer; and

f) all markets clear.

A *balanced growth equilibrium* is an equilibrium with imperfect competition in which
2.4 Dynamics and Steady-State Growth

As shown earlier, the threshold level of ability above which individuals choose to acquire skills depends on the wage ratio (see equation (2.7)). In Appendix 2.8, the wage ratio is shown to be given by

\[
\frac{w_{t}^{U}}{w_{t}^{S}} = \beta \left( \frac{\theta_{t}^{S,Y}}{\theta_{t}^{U,Y}} \right),
\]

where \( \beta = (1 - \varepsilon)\beta^{U} / \beta^{S} \). Thus, the unskilled-skilled wage ratio varies inversely with the relative supplies of skilled and unskilled labour in the final good sector.

Substituting this result in (2.7) implies that in equilibrium the threshold level of ability, \( a_{t}^{C} \), is given by

\[
a_{t}^{C} = \mu^{1/\gamma} [1 - \varepsilon - \beta(\theta_{t}^{S,Y} / \theta_{t}^{U,Y})]^{-1/\gamma}.
\]

Appendix 2.8 also shows that the public-private capital ratios are constant over time and given by

\[
k_{t}^{i} = J^{i} = \frac{\varphi^{U_{i}^{T}}}{\sigma(1 - \tau)}, \quad i = A, B
\]
where \( \sigma \in (0, 1) \) is the family’s propensity to save, defined as

\[
\sigma = \frac{1}{1 + \eta (1 + \rho)} < 1.
\]  

(2.38)

To determine the growth rate, the first step is to derive the restrictions on the congestion parameters in (2.11). In a symmetric equilibrium,

\[
X_t = \left( (M^I_t)^{1/\eta} x^I_t \right)^\nu \left( (M^R_t)^{1/\eta} x^R_t \right)^{1-\nu}.
\]  

(2.39)

From (2.17), \( x^j_t = \gamma \eta \nu^j (Y_t/M^j_t) \), for \( j = I, R \). Substituting these results in (2.39) yields

\[
X_t = \gamma \eta \nu^j (1 - \nu)^{1-\nu} \left[ (M^I_t)^{\nu(1-\eta)/\eta} (M^R_t)^{(1-\nu)(1-\eta)/\eta} \right] Y_t,
\]

or equivalently, noting that \( (K^P_t)^{(1-\eta)/\eta} = (K^P_t)^{1/\eta} / K^P_t \), we obtain

\[
X_t = \Lambda_1 (m^I_t)^{\nu(1-\eta)/\eta} (m^R_t)^{(1-\nu)(1-\eta)/\eta} \left( \frac{Y_t}{K^P_t} \right) (K^P_t)^{1/\eta},
\]

where \( m^j_t = M^j_t / K^P_t \), \( j = I, R \) and \( \Lambda_1 = \gamma \eta \nu^j (1 - \nu)^{1-\nu} \). Substituting this expression in (2.11) yields

\[
Y_t = (1 - \varepsilon)^{\beta^S} (\delta^S Y^s) (\delta^U Y^s) \beta^S N_t^{\beta^S + \beta^U - \omega \zeta_N}
\]

\[
\times (K^P_t)^{\omega} \left\{ \Lambda_1 (m^I_t)^{\nu(1-\eta)/\eta} (m^R_t)^{(1-\nu)(1-\eta)/\eta} \left( \frac{Y_t}{K^P_t} \right) \right\}^\gamma (K^P_t)^{\alpha + \gamma / \eta + \omega (1 - \zeta_K)}.
\]

(2.40)

The following restrictions on the congestion parameters \( \zeta_K \) and \( \zeta_N \) are imposed: \( \beta^S + \beta^U - \omega \zeta_N = 0 \) and \( \alpha + \gamma / \eta + \omega (1 - \zeta_K) = 1 \). Thus, the level of output becomes:

\[
Y_t = \frac{(J^B)^{\omega(1-\gamma)}}{[(\delta^S)^{\beta^S} (\delta^U)^{-\gamma}]^{-1/\gamma} \Lambda_2} \left\{ (m^I_t)^{\nu(1-\eta)/\eta} (m^R_t)^{(1-\nu)(1-\eta)/\eta} \right\}^{1/(1-\gamma)} \left( K^P_t \right) \gamma^{(1-\gamma)},
\]

(2.41)

where \( \Lambda_2 = [(1 - \varepsilon)^{\beta^S} \Lambda_1^{1/\gamma}]^{1/(1-\gamma)} \). Equation (2.41) is thus linear in the private capital stock.

Appendix 2.8 shows that the dynamic system that drives the economy consists
of two first-order difference equations in $m^I_t$ and $m^R_t$ and nine static equations:

\[ m^I_{t+1} = \left[ 1 + (J^B)^{\phi^I}(1 + g^W)^{\kappa^I} \right] m^I_t \]  
\[ \times \left\{ \sigma(1 - \tau)Q^I_t \theta^U_t (J^B)^{\phi^I} m^I_t (1 + g^W)^{\kappa^I} + \sigma(1 - \tau)(1 - \varepsilon)[1 - \frac{\mu}{[\theta a^C_t / (\theta - 1)]^x}] Q^R_t \theta^S_t (J^A)^{\phi^R} m^R_t [1 + \phi^R_2 \left( \frac{m^I_t}{m^R_t} \right)] \right\}^{-1}, \]

\[ m^R_{t+1} = \left[ 1 + (J^A)^{\phi^R}[1 + \phi^R_2 \left( \frac{m^I_t}{m^R_t} \right)](1 - \varepsilon)\theta^S_t \right] m^R_t \]  
\[ \times \left\{ \sigma(1 - \tau)Q^I_t \theta^U_t (J^B)^{\phi^I} m^I_t (1 + g^W)^{\kappa^I} + \sigma(1 - \tau)(1 - \varepsilon)[1 - \frac{\mu}{[\theta a^C_t / (\theta - 1)]^x}] Q^R_t \theta^S_t (J^A)^{\phi^R} m^R_t [1 + \phi^R_2 \left( \frac{m^I_t}{m^R_t} \right)] \right\}^{-1}, \]

\[ \frac{Y_t}{K^P_t} = \frac{(J^B)^{\omega/(1-\gamma)} \Lambda_2}{\left[ (\theta^S_t)^{\beta^S} (\theta^U_t)^{\beta^U} \right]^{1/(1-\gamma)}} \left\{ (m^I_t)^{\nu(1-\eta)/\eta} (m^R_t)^{1-\nu(1-\eta)/\eta} \right\}^{\gamma/(1-\gamma)}, \]  

\[ Q^I_t = (1 - \eta)\gamma \nu^j (\frac{Y_t}{K^P_t}) (m^I_t)^{-1}, \quad j = I, R \]

\[ a^C_t = \mu^{1/\chi}[1 - \varepsilon - \beta(\frac{\theta^S_t}{\theta^U_t})]^{-1/\chi}, \]  

\[ \theta^U_t = 1 - (a^C_t)^{-\theta}, \]  

\[ \theta^S_t = \frac{\theta}{\theta - 1} (a^C_t)^{1-\theta}, \]  

\[ \theta^S_{t,Y} = \frac{\beta^S (J^A)^{-\phi^R}}{(1 - \varepsilon)(1 - \eta)(1 - \nu)^{\gamma}} [1 + \phi^R_2 \left( \frac{m^I_t}{m^R_t} \right)]^{-1}, \]  

\[ \theta^S_{t,R} = \theta^S_t - \theta^S_{t,Y}, \]  

\[ \theta^U_{t,Y} = \frac{\beta^U (J^B)^{-\phi^I}}{(1 - \eta)\nu^j} (1 + g^W)^{-\kappa^I}, \]  

\[ \theta^U_{t,I} = \theta^U_t - \theta^U_{t,Y} . \]
Given its complexity, the stability of this system cannot be studied analytically. Nevertheless, its stability can be verified numerically, once the model is calibrated.

In the steady state, the growth rates of imitation- and innovation-based knowledge are equal. From the static conditions (2.44)-(2.52), $Y_t = K^P_t$, $Q^I_t$, $Q^R_t$, $a^C_t$, $\theta^U_t$, $\theta^S_t$, $\theta^{S,Y}_t$, $\theta^{S,R}_t$, $\theta^{U,Y}_t$ and $\theta^{U,I}_t$ are also constant. Thus, the steady-state growth rate of output is the same as the growth rate of knowledge and the growth rate of the private capital stock.

The long-run growth rate, $1 + \gamma$ can be written in several equivalent ways. In particular, as shown in Appendix 2.8,

$$1 + \gamma = (J^B)^{\phi^I_t}(1 + g^W)^{\kappa^I_t}\theta^{U,I}, \tag{2.53}$$

$$1 + \gamma = (J^A)^{\phi^I_t}[1 + \phi^R_2(\frac{\tilde{m}^I_t}{m^R})(1 - \varepsilon)]\theta^{S,R} \tag{2.54}$$

From the solutions (2.42)-(2.52), we can define an index of industrial transformation as $m_t = m^I_t/(m^I_t + m^R_t)$, an index of public capital allocation, $k_t = k^B_t/(k^A_t + k^B_t)$, an index of the distribution of the unskilled labour force as $\theta^{U,I}_t/\theta^{U,Y}_t$, and an index of the distribution of the skilled labour force as $\theta^{S,R}_t/\theta^{S,Y}_t$, all of which help to characterise the process of development.

### 2.5 Calibration

To study the transitional dynamics of the model and the steady-state effects of public policy, we calibrate it as follows. For households, the annual discount rate is set at 0.04, as is standard in the literature. Interpreting a period as 25 years in this framework yields an intergenerational discount factor of $[1/(1 + 0.04)]^{25} = 0.375$. In line with the evidence on private savings for low-income countries, the family’s propensity to save, $\sigma$, is set at 0.06. Consequently, we consider a low-income country where households are close to the subsistence level of consumption.
Solving (2.38) backward for the preference parameter $\eta_C$ and given the value of the intergenerational discount factor, the calibrated value of $\eta_C$ is 5.87.

Time allocated to advanced schooling, $\varepsilon$, is set at 0.15. This corresponds to an average number of years in higher education of 3.8. The parameter that measures the efficiency of training, $\chi$, is set initially at 0.5; experiments with slightly higher or lower values of $\chi$ did not alter the results significantly. Furthermore, we assume that the cost of acquiring an education, $\mu$, is initially set at 0.08 of the skilled wage and therefore acquiring skills represents a substantial cost. The tail index parameter, $\theta$, is set at 2.1.

In the final good sector, the elasticity of production with respect to basic public capital, $\omega$, is set at 0.14, the long-run value estimated through meta-regression analysis by Bom and Ligthart (2011, Table 4) for core public capital at the national level. The elasticity of production with respect to unskilled labour, $\beta^U$, is set at 0.2, the elasticity with respect to skilled labour, $\beta^S$, at 0.35, and the elasticity of production with respect to private capital, $\alpha$, at 0.3, which is standard in the literature (see Agénor (2011)). By implication, the elasticity of output with respect to the composite intermediate good, $\gamma$, is equal to 0.15. This is substantially lower than the value of 0.36 used by Funke and Strulik (2000) and Sequeira (2011) for instance, but it is more appropriate for a low-income country where, to begin with, the share of intermediate goods is relatively small, compared to capital and, especially, labour. We also assume that the relative share of imitated goods in the composite intermediate good $X_t$, as measured by $\nu$ (which, when multiplied by $\gamma$, measures the relative share of that input in final production), is set at 0.9.

In the intermediate good sectors, the parameter $\eta$ (which determines the price elasticity of the demand for intermediate goods) is set to 0.61, similar to the value used by Iacopetta (2011) and Chen and Funke (2013). This implies an elasticity of substitution between intermediate inputs of about 2.6, which corresponds also to...
the value found by Acemoglu and Ventura (2002).

In the imitation sector, the growth rate of the international pool of blueprints available for imitation, $g^W$, is set at 0.02, as in Chen and Funke (2013). The elasticity with respect to the growth rate of imitable goods worldwide, $\kappa^I$, is set initially at 0.35, in line again with Chen and Funke (2013). The elasticity with respect to basic infrastructure, $\phi^I_1$, is set at 0.1, which is slightly lower than the Bom-Ligtart estimate referred to above for the production of final goods.

In the innovation sector, the parameter $\phi^R_1$, which measures the response to advanced infrastructure, is set initially at 0.1. The parameter measuring the externality associated with the stock of imitative knowledge, $\phi^R_2$, is set initially equal to 0.1; a sensitivity analysis is also reported later on.

Regarding the government, the tax rate on final output, $\tau$, is set equal to 0.151, which corresponds to the average ratio of tax revenues to GDP for low-income countries calculated by Baldacci et al. (2004b, p. 530). This value is divided (to match the model’s definition) by $\beta^U + \beta^S = 0.55$, given earlier, which corresponds to the share of labour income in final output. Thus, the effective domestic tax rate is $\tau = 0.151/0.55 = 27.4$ percent. By definition, because the model does not consider deficit financing, this is also the share of government spending in output. The share of government investment in basic infrastructure, $\nu_B$, is set equal initially to 6.5 percent, and the share of investment in advanced infrastructure to 0.5 percent. Consequently, we consider the case of a country where initially much of public investment in infrastructure is devoted to “core” infrastructure, which includes electricity, roads, basic telecommunications, water and sanitation, and so on. This is a natural assumption for a low-income country.

To estimate the efficiency parameter of public spending, $\varphi$, we use the median value estimated by Dabla-Norris et al. (2012) for a sample of 71 developing countries, that is, 0.4. An alternative approach is to use the governance index defined in Baldacci et al. (2008, Table 1) which, once normalised to be between 0 and 1, gives a value of 0.5. However, the results are
is “wasted”, in the sense that it does not transform into public capital.

Parameter values are summarised in Table 2.1. Given these values, the model is solved iteratively (in its nonlinear form) to generate a steady-state equilibrium with $\Delta m_{t+1}^j = 0$, $j = I, R$. The steady-state values of some of the main variables are shown in Table 2.2. The proportion of unskilled workers in the population, $\theta^U$, is equal to 0.95 whereas the (effective) proportion of skilled workers, $\theta^S$, is equal to 0.049. The absolute share of the unskilled labour force in final good production, $\theta^U;Y$, is equal to 0.7, which implies that the share of that type of labour in the imitation sector, $\theta^U;I$, is 0.25. Similarly, the share of the (effective) skilled labour force in the final good sector, $\theta^S;Y$, is equal to 0.04, which implies that the share of that type of labour in the innovation sector, $\theta^S;R$, is 0.009. The ratio of imitation-based goods to private capital is equal to $m^I = 0.05$, whereas the ratio of innovation-based goods to private capital is set equal to $m^R = 0.01$. By implication, our index of industrial structure, $m = m^I/(m^I + m^R)$, is initially equal to 0.83. The patent price for both imitation and innovation sector, $Q^j$ where $j = I, R$, is set equal to 0.1. Using formula (2.47), the threshold level of ability, $a^C$, is 4.16. The unskilled-skilled wage ratio is normalised to 0.8, which implies an education premium equal to 25 percent, in line with the evidence for developing countries (see Agénor (2006)). The solution also yields initial steady-state values of the public-private capital ratios in basic and advanced infrastructure equal to $k^B = 0.16$ and $k^A = 0.01$, respectively, implying that the index of the composition of public capital is $k = 0.928$; access to advanced infrastructure is relatively scarce to begin with. Based on these values, the steady-state growth rate of final output (on an annual basis) is calibrated at 2.4 percent. That value corresponds to the average growth rate in Sub-Saharan Africa over the period 1990-2010 (see Agénor and Dinh (2013)) and is obtained by adding a multiplicative constant in the relevant equation.

Thus, as in Agénor and Dinh (2013), the low-income economy that we calibrate

not highly sensitive to that change.
is characterised initially by a) a positive but low growth rate in income per capita; b) an embryonic innovation sector and a relatively more developed imitation sector; c) a high cost of acquiring skills; d) a labour force consisting mostly of unskilled workers, employed in both the imitation sector and final good production (and more so in the latter); e) a small fraction of skilled workers in the labour force, employed almost entirely in final good production (in line with the assumption that the innovation sector is negligible in size); f) limited availability of basic infrastructure and almost non-existent advanced infrastructure; and g) correspondingly a relatively low share of public investment in basic infrastructure and a much lower one on advanced infrastructure. At the same time, both stocks of public capital are relatively small in proportion to the private capital stock.

2.6 Policy Experiments

To illustrate the role of public policy in promoting growth and industrial transformation, we focus on public investment in basic and advanced infrastructure. As discussed earlier, a key aspect of the model is the differentiated impact of public capital: while basic infrastructure (roads, energy, and basic telecommunications) has a positive impact on the production of final goods and imitation activities, advanced infrastructure (advanced information and communication technologies, ICTs) helps to promote innovation activities only. The channels through which public investment operates depend therefore on the type of capital that is being built. In addition, there are possible trade-offs between the two types of investment. We consider both of these issues in turn.

To characterise the results of our experiments, we focus on the following variables: the share of unskilled labour, $\theta^U$, the share of unskilled labour in the imitation sector, $\theta^{U,I}$, the share of the (effective) skilled labour force, $\theta^S$, the share of skilled labour in the innovation sector, $\theta^{S,R}$, the growth rate of the stock of designs (or imitated
intermediate goods) in the imitation sector, the growth rate of the stock of designs (or innovation-based intermediate goods) in the innovation sector, the industrial structure index defined earlier, \( m = m^I/(m^I + m^R) \), and the growth rate of final output.\(^{24}\)

### 2.6.1 Changes in the Level of Public Investment

Consider first a permanent, budget-neutral increase in the share of spending on basic infrastructure, \( v_B \), from an initial value of 0.065 to 0.075, financed by a cut in unproductive spending \((dv_B + dv_U = 0)\). Figure 2.1 reports the results of this experiment for two different values of the parameter \( \phi_2^R \), which measures the strength of the externality associated with imitation activities for the innovation sector: the benchmark value of 0.1 and a higher value of 0.4. The first impact of this policy is to increase the marginal product of unskilled labour, and therefore labour demand, and the economy-wide wage for that category of workers. At the initial level of skilled wages, this tends to reduce incentives for workers to acquire skills, and therefore to reduce on impact the (effective) supply of skilled labour. This, in turn, tends to mitigate activity in both the innovation and the final good sectors. At the same time, the demand for labour rises more significantly in the imitation sector. The share of unskilled labour employed in that sector therefore rises initially—matching, to a large extent, the increase in the wage ratio, as implied by (2.35)—by a full percentage point.

However, during the transition, these initial effects are reversed. By raising productivity in the innovation sector, the learning effect contributes to raising wages for skilled labour. The unskilled-skilled wage ratio starts falling, thereby inducing more individuals to invest in higher education. The adjustment process involves therefore a falling (increasing) share of unskilled (skilled) labour in the population,\(^{24}\)\n
\(^{24}\)In performing our simulations, we imposed a partial adjustment on \( Q^I \) to its equilibrium value in order to smooth out the transitional dynamics. However, this had no effect on the steady-state results.
as illustrated in Figure 2.1. Thus, while the imitation growth rate falls during the transition, the innovation growth rate increases. As shown in the figure, the stronger the learning effect, the larger is the initial increase in the innovation growth rate. Nevertheless, and regardless of the magnitude of the learning effect, it is not strong enough to prevent the industrial structure shifting toward imitation over time. On impact, the growth rate of final output goes up by 1.2-1.3 percentage points relative to baseline (depending on the value of $\phi_R^2$), mainly as a result of the productivity effect associated with a higher stock of basic infrastructure. Over time, this effect on the marginal product of labour fades out, but the increase in the composite intermediate input helps to sustain activity. In the long run, the growth rate of output converges to a value of about 1 percentage point.

The results therefore illustrate the fact that imitation (promoted by higher public investment in basic infrastructure) may indeed be the main source of productivity growth in the early stages of development—in both the imitation and innovation sectors. In the model, the learning effect associated with imitation is magnified by the endogeneity of labour supply—as productivity increases in the innovation sector through the learning externality, wages there tend to increase; in our calibration, this effect is strong enough to reverse the initial drop in the skilled-unskilled wage ratio, and as a result the transitional dynamics are characterised by an increase in the supply of skilled labour—which further stimulates activity in the innovation sector. However, the learning spillover associated with imitation and the shift in the structure of the labour force are not strong enough (at least for the range of parameters considered here) to generate growth in the innovation sector that is fast enough to have a substantial impact on the industrial structure and induce eventual convergence toward a mature, innovation-based economy.

Consider now a permanent, budget neutral increase in the share of spending on advanced infrastructure, $u_A$, from an initial value of 0.005 to 0.02, financed again by a cut in unproductive spending ($du_A + du_U = 0$). Figure 2.2 shows the
results of this policy for the benchmark value of the parameter $\phi_1^R = 0.1$, which measures the response of innovation activity to advanced infrastructure, as well as a higher value of $\phi_1^R, 0.15$. As mentioned earlier, advanced infrastructure benefits only innovation activities; it has no direct effect on production elsewhere in the economy. Thus, on impact the higher stock of advanced infrastructure raises productivity and wages in the innovation sector. The fall in the unskilled-skilled wage ratio promotes investment in skills. On impact, the imitation growth rate falls whereas the innovation growth rate increases—the latter by about 2 percentage points with $\phi_1^R = 0.1$ and almost 3 percentage points with $\phi_1^R = 0.15$. The increase in labour demand in the innovation sector is satisfied not only by an increase in the overall supply of skilled labour but also by a reallocation of skilled workers away from the final good sector. This tends to mitigate activity in that sector, with the growth rate of output falling on impact by about 1 percentage point with $\phi_1^R = 0.1$ and almost 1.3 percentage points with $\phi_1^R = 0.15$.

Over time, once again, these initial effects are reversed. The fall in the supply of unskilled labour tends to mitigate the initial drop in the unskilled-skilled wage ratio. As a result, the proportion of individuals seeking higher education falls over time. This dampens the initial increase in the innovation growth rate and raises the imitation growth rate, as more unskilled workers shift toward imitation activities. During the transition the former rate is high enough to ensure that the industrial structure shifts toward an innovation-based economy. This is partly the consequence of the learning effect: as activity in the imitation sector expands, it also helps to sustain activity in the innovation sector. While the growth rate of final output falls initially (as noted earlier), it recovers gradually during the transition and turns positive in the long run. While a higher value of $\phi_1^R$ does not appear to make much difference in terms of the long-run effects (except for the industrial structure index) it does affect the initial phase of the transitional dynamics. In particular, the impact on the composition of the labour force and activity in the design sectors are
magnified.

To summarise, increased investment in advanced infrastructure does generate a more rapid process of industrial transformation and accelerated growth, consistent with the evidence discussed earlier. In the model, the benefits of a higher stock of public capital are magnified by a shift in the composition of the labour force toward higher skill levels. However, this process occurs endogenously through changes in productivity and higher wages—rather than, say, through education subsidies—which act as signals to individuals about the returns to education.

It is important to note also that these results do not depend on the assumption that advanced infrastructure has no effect on activity in the final good sector. In practice, of course, this type of infrastructure is likely to benefit all sectors of activity. However, as long as the effect on innovation (as measured by $\phi_1^R$) is relatively stronger, the qualitative nature of the results illustrated in Figure 2.2 would remain unchanged. Conversely, if it had been assumed that the pecuniary or time cost of education is decreasing through access to advanced infrastructure—because access to the internet facilitates learning for instance—the benefits of higher investment in that category of infrastructure in terms of growth and the share of skilled labour in the population would be magnified.

### 2.6.2 Changes in the Composition of Public Investment

To illustrate potential trade-offs between different types of public investment, we now consider the case, once again, of a permanent increase in the share of spending on advanced infrastructure, $v_A$, from an initial value of 0.005 to 0.02, but this time 50 percent of the increase in $v_A$ is financed by a cut in investment in basic infrastructure, $v_B$. The results of this experiment are displayed in Figure 2.3. Because qualitatively the effects of this policy are similar to those shown in Figure 2.2, we focus on only two variables, the industrial transformation index and the growth rate of final output. In addition, we do so for two different values of $\phi_1^R$ (the benchmark value of 0.1
and a higher value of 0.15), the parameter that measures the response of innovation activity for advanced infrastructure, and two different values of $\phi^R_2$ (the benchmark value of 0.1 and a higher value of 0.4) the parameter that measures the strength of the learning externality associated with imitation in the innovation sector. These values correspond to those used for sensitivity analysis in Figures 2.1 and 2.2. The benchmark experiment, in which the increase in the share of spending on advanced infrastructure is financed by a cut in unproductive spending, is also shown in the figure (continuous red line) for comparative purposes.

These experiments illustrate two main results. First, financing higher investment in advanced infrastructure by a cut in investment in basic infrastructure is always detrimental to growth. The reason is largely because basic infrastructure directly benefits the production of final goods; a cut in that category of investment has therefore an adverse effect on production. While this effect is highly significant on impact—the growth rate of output drops by about 2 percentage points in the case where $\phi^R_1 = \phi^R_2 = 0.1$ for instance—it is mitigated by the increase in the supply of intermediate goods over time. Second, as shown in Figure 2.3, the combination of lower spending on basic infrastructure (which has an adverse effect on activity in the imitation sector) and higher spending on advanced infrastructure (which directly promotes innovation) is quite effective in terms of inducing a shift toward an innovation-based economy—despite the fact that lower imitation activity mitigates learning spillovers. Third, higher values of $\phi^R_1 = 0.15$ and $\phi^R_2 = 0.4$ are associated with faster rates of industrial transformation, as measured by the ratio of the variety of imitation- to innovation-based intermediate goods, but this comes with little additional benefits in terms of growth in the long run.

The drop in steady-state growth displayed in Figure 2.3 could of course be mitigated, or even reversed, if the fraction of $\nu_A$ financed by a cut in $\nu_B$ is smaller, if $\phi^R_1$ is higher, or if advanced infrastructure were to affect final output. Indeed, the assumption in equation (2.11) is that only basic infrastructure directly affects the
production of final goods. This is a reasonable assumption for a low-income country where the stock of advanced public capital is scarce to begin with. But even in such conditions the marginal impact of improved access to advanced infrastructure on the manufacturing sector could be sizable. For instance, better access to high-speed telecommunication networks could help to improve significantly the management of supply chains in real time, thereby boosting efficiency in the production of manufactured goods (see Park et al. (2013)). This could be captured by replacing $K_t^B$ in (2.11) by the weighted average $(K_t^B)_{G_t^c}(K_t^A)_{1-G_t^c}$, where $\alpha_G \in (0, 1)$. The case considered so far is thus $\alpha_G = 1$. For instance, a numerical experiment involving the same increase in $u_A$ as in Figure 2.3, combined with a 20 percent financing by a reduction in $u_B$, an elasticity $\phi_1^R = 0.95$, and a coefficient $\alpha_G = 0.5$ would generate higher long-run growth of the order of 0.1 percentage points. With a lower $\alpha_G$, the impact on steady-state growth would be even stronger.

The broader lesson from these experiments is that, during an initial stage, when the supply of skilled labour is relatively scarce, investing in basic infrastructure may be a critical step to promote growth and initiate a process of industrial development. This is based on replicating foreign technologies and adapting them to local markets. As this imitation process gathers pace, and learning spillovers become significant, wage signals become strong enough to induce individuals to invest in higher education. These results are consistent with the evidence which suggests that investments in education, training and new technologies are closely related, and are associated with higher productivity (Mattalia (2012)). However, this may not be enough to promote further industrial diversification, in the form of a sustained reduction in the ratio of the variety of imitation- to innovation-based intermediate goods. To achieve this goal, a shift in the composition of public investment toward more advanced infrastructure, which is critical to foster further increases in productivity in the innovation sector, becomes essential. At the same time, the productivity-enhancing effect of advanced infrastructure raises the demand for skilled labour, which further
promotes innovation. It is possible that the resulting shift in the industrial structure may not be sufficient to promote growth—especially if the increase in spending on advanced infrastructure is financed by a cut in investment in basic infrastructure. To mitigate adverse effects on growth, governance reform aimed at improving investment spending efficiency and at reallocating spending away from non-productive uses may also be required.

Although our analysis focuses on a set of initial conditions that characterises today’s low-income countries, it also has some broader historical relevance. Some recent studies—such as Iacopetta (2010) and Gómez (2011) for instance—have argued that a transition path involving a phase of innovation first, and human capital accumulation second, is more consistent with the historical evidence for today’s developed countries, at least during the first phase of the Industrial Revolution (1760-1830). Indeed, as documented by Galor (2005), literacy rates did not increase much during that time. However, in the second phase of the Industrial Revolution, the demand for skilled labour in the growing industrial sector rose markedly. To satisfy the increasing skill requirements in the process of industrialisation, human capital formation improved substantially.25 This two-way interaction is consistent with our analysis of the interplay between imitation, innovation, and skill acquisition in promoting sustained growth. At the same time, however, there are important differences in our view between the experience of developed countries during the past two centuries, the growth process of today’s middle-income countries since the end of World War II, and the type of policies that today’s low-income countries should implement to catch up with richer economies. Our analysis has drawn attention to public infrastructure and its composition.

25Galor (2005, p. 274) also notes the possibility of a positive feedback between the rate of technological progress and the level of education, when ability is endogenous.
2.7 Concluding Remarks

In this chapter the link between public capital and industrial development was studied in an OLG model with endogenous skill acquisition. Industrial development is defined as a shift from an imitation-based, low-skill economy to an innovation-based, high-skill economy, where technological progress occurs through the domestic invention of ideas. In addition, productivity in innovation is enhanced through a knowledge spillover from imitation. Changes in industrial structure are measured by changes over time in the ratio of imitation-based and innovation-based production inputs, rather than changes in the relative share of manufacturing production in total final output. The model also distinguishes between basic infrastructure, which helps to promote learning-by-doing and productivity in imitation activities, and advanced infrastructure, which promotes knowledge networks and innovation.

Numerical experiments, based on a calibrated version for a low-income country, showed that changes in the level and composition of public investment may have significant effects on the structure of the labour force and the speed of industrial development. Our experiments suggest that the scope for imitation-based learning has significant implications for the pace of industrial change. In the early stages of development, when skilled labour is in short supply and unskilled labour is abundant, investing in core infrastructure may be a critical step to initiate a process of industrial development and growth based on imitating imported technologies. As this imitation process unfolds, learning externalities gather pace and begin to benefit skilled labour productivity and spur innovation. In doing so, wage signals are strengthened and induce more individuals to invest in higher education. In that sense, large investments in human capital are not critical to promote growth during the early stages of development; human capital accumulation becomes instead a by-product of industrial development.

A broader implication of our analysis is that during the initial phases of industrial change, whom a country trades with matters, as the potential and scope for
learning is higher with more technologically advanced trading partners. In addition, what products a country trades matters as well, as more research-intensive imports of intermediate and capital goods could be linked to more intensive learning and facilitate greater economy-wide spillovers from the discovery of more new activities (see Mendoza (2010)). At the same time, however, beyond an initial stage learning spillovers may not be enough to promote industrial diversification away from imitation-based to innovation-based activities. To that end, a shift in the composition of public investment toward more advanced information and communication technologies, which is critical to promote knowledge networks and ensure sustained increases in labour productivity in the innovation sector, becomes essential. To the extent that a lack of resources may force a reduction in the basic infrastructure investment needed to bring about this shift, governance reform (aimed at improving efficiency of investment spending and at reallocating public expenditure away from non-productive uses) may also be needed to avoid adverse effects on growth. Alternatively, public-private partnerships in infrastructure provision may be pursued, although the performance of these arrangements has been mixed in developing countries.

These results provide important lessons for policymakers in low-income countries who are now considering ways to speed up the process of industrial development. At the same time, our analysis does not imply that industrial development should be achieved at the expense of the agricultural sector. As noted in the introduction, the latter can contribute to industrial development through, for instance, the supply of production inputs. Rattsø and Torvik (2003) for instance show that discrimination against the agriculture sector could lead to the contraction of industry, through trade linkages. In addition, a rush to promote sophisticated industries without the requisite accumulation of skills is unlikely to succeed. But our emphasis on the fact that industrial development is essential in the longer run to sustain economic growth is consistent with the evidence.
2.8 Appendix

From (2.2) and (2.4), the household’s consolidated budget constraint is, for unskilled individuals who can work either in the final good sector or the imitation sector,

\[ c_t^U + \frac{c_{t+1}^U}{1 + r_{t+1}} = (1 - \tau)w_t^U, \tag{2.55} \]

whereas from (2.3) and (2.4), for those skilled individuals who can work either in the final good sector or in the innovation sector,

\[ c_t^S + \frac{c_{t+1}^S}{1 + r_{t+1}} = (1 - \tau)(1 - \varepsilon)(w_t^S - tc_t), \]

or equivalently, using (2.6),

\[ c_t^S + \frac{c_{t+1}^S}{1 + r_{t+1}} = (1 - \tau)(1 - \varepsilon) \left\{ 1 - \frac{\mu}{\theta a_t^C / (\theta - 1)} \right\} w_t^S. \tag{2.56} \]

In this expression, average ability of skilled workers, \( \theta a_t^C / (\theta - 1) \), is used.

Each individual maximises (2.1) subject to intertemporal budget constraint either (2.55) or (2.56). The first-order conditions give the standard Euler equation

\[ \frac{c_{t+1}^h}{c_t^h} = \frac{1 + r_{t+1}}{\eta_C(1 + \rho)}, \quad h = U, S \tag{2.57} \]

Substituting this result in (2.55) and (2.56), together with (2.6), yields

\[ c_t^U = \left[ \frac{\eta_C(1 + \rho)}{1 + \eta_C(1 + \rho)} \right] (1 - \tau)w_t^U, \]

\[ c_t^S = \left[ \frac{\eta_C(1 + \rho)}{1 + \eta_C(1 + \rho)} \right] (1 - \tau)(1 - \varepsilon) \left\{ 1 - \frac{\mu}{\theta a_t^C / (\theta - 1)} \right\} w_t^S, \]

or equivalently

\[ c_t^U = (1 - \sigma)(1 - \tau)w_t^U, \]
\[ c_t^{S,t} = (1 - \sigma)(1 - \tau)(1 - \varepsilon) \left( 1 - \frac{\mu}{[\theta a_t^C/(\theta - 1)]^x} \right) w_t^S, \]

so that, from (2.2) and (2.3),

\[ s_t^U = \sigma(1 - \tau)w_t^U, \quad (2.58) \]

\[ s_t^S = \sigma(1 - \tau)(1 - \varepsilon) \left( 1 - \frac{\mu}{[\theta a_t^C/(\theta - 1)]^x} \right) w_t^S, \quad (2.59) \]

where \( \sigma = 1/[1 + \eta_C(1 + \rho)] < 1. \)

Consider first the imitation sector. Productivity is given by (2.21), which is repeated here for convenience:

\[ A_t^I = (k_t^B)^{\phi_t} M_t^I. \quad (2.60) \]

Substituting this result in equation (2.20) and dividing by \( M_t^I \) yields

\[ \frac{M_{t+1}^I}{M_t^I} = 1 + (k_t^B)^{\phi_t} (1 + g^W)^{n^I} \theta_t^U. \quad (2.61) \]

From the first-order condition (2.22), using (2.60),

\[ w_t^U = \left( \frac{Q_t^I}{N^I} \right) (k_t^B)^{\phi_t} M_t^I (1 + g^W)^{n^I}. \quad (2.62) \]

From the first-order condition (2.12), \( w_t^U = \beta^U Y_t / N_t^U Y_t \). Equating this expression with (2.62) yields

\[ Y_t / M_t^I = (\beta^U)^{-1} Q_t^I \theta_t^{U,Y} (k_t^B)^{\phi_t} (1 + g^W)^{n^I}. \quad (2.63) \]

From (2.17)-(2.19), together with \( \nu^I = \nu, Q_t^I = (1 - \eta) \nu \gamma (Y_t / M_t^I) \). Substituting this result in the above expression and simplifying by \( Y_t / M_t^I \) yields a solution for \( \theta_t^{U,Y} \), the share of unskilled labour in the production of the final good:

\[ \theta_t^{U,Y} = \frac{\beta^U (k_t^B)^{-\phi_t}}{(1 - \eta) \nu \gamma (1 + g^W)^{-n^I}. \quad (2.64) \]
Using (2.31), the share of unskilled labour in the imitation sector is thus

\[ \theta_t^{Ul} = \theta_t^U - \theta_t^{Ul,Y}. \] (2.65)

Consider next the innovation sector. Productivity in equation (2.24) can be rewritten as

\[ A_t^R = (k_t^A)^{\phi_t^R} (M_t^R + \phi_2^R M_t^I). \] (2.66)

Substituting this result in equation (2.23) and dividing by \( M_t^R \) yields

\[ \frac{M_{t+1}^R}{M_t^R} = 1 + (k_t^A)^{\phi_t^R} [1 + \phi_2^R (\frac{m_l^I}{m_t^R})](1 - \varepsilon)\theta_t^{SR}, \] (2.67)

where \( m_l^I = M_t^I / K_t^P \) and \( m_t^R = M_t^R / K_t^P \).

From (2.25), and using (2.66),

\[ w_t^S = (\frac{Q_t^R}{N}) (k_t^A)^{\phi_t^R} [1 + \phi_2^R (\frac{m_l^I}{m_t^R})] M_t^R. \] (2.68)

From (2.12), \( w_t^S = \beta^S Y_t / (1 - \varepsilon) N_t^{SY} \). Equating this expression with (2.68) yields

\[ \frac{Y_t}{M_t^R} = (1 - \varepsilon) Q_t^R \theta_t^{SY} (k_t^A)^{\phi_t^R} [1 + \phi_2^R (\frac{m_l^I}{m_t^R})]. \]

From (2.17)-(2.19), together with \( \nu^R = 1 - \nu \), \( Q_t^R = (1 - \eta)(1 - \nu)\gamma(Y_t/M_t^R) \).

Substituting this result in the above expression and simplifying by \( Y_t/M_t^R \) yields a solution for \( \theta_t^{SY} \), the share of skilled labour in the production of the final good:

\[ \theta_t^{SY} = \frac{\beta^S (k_t^A)^{\phi_t^R}}{(1 - \varepsilon)(1 - \eta)(1 - \nu) \gamma [1 + \phi_2^R (\frac{m_l^I}{m_t^R})]^{-1}.} \] (2.69)

Using (2.33), the share of skilled labour in the innovation sector is thus

\[ \theta_t^{SR} = \theta_t^S - \theta_t^{SY}. \] (2.70)
From (2.6), (2.27), and (2.29), we have
\[ K_{t+1}^i = \varphi v_t \left\{ w_t^U N_t^U + (1 - \varepsilon) \left[ 1 - \frac{\mu}{[\theta a_t^U/(\theta - 1)]^x} \right] w_t^S N_t^S \right\}, \quad i = A, B \] (2.71)
where again average ability of skilled workers is used.

Now, consider the dynamics of the private capital stock. From (2.30), (2.58), and (2.59),
\[ K_{t+1}^P = \sigma (1 - \tau) \left\{ w_t^U N_t^U + (1 - \varepsilon) \left[ 1 - \frac{\mu}{[\theta a_t^U/(\theta - 1)]^x} \right] w_t^S N_t^S \right\}. \] (2.72)

Combining (2.71) and (2.72) yields the public-private capital ratio:
\[ k_{t+1}^i = \frac{\varphi v_t}{\sigma (1 - \tau)}, \quad i = A, B \] (2.73)
which is constant over time.

Substituting (2.61) and (2.68) into (2.72) and dividing by $K_t^P$ yields
\[ \frac{K_{t+1}^P}{K_t^P} = \sigma (1 - \tau) Q_t^U \theta_t^U (J_B)^{\phi_t^U} m_t^I (1 + g^W)^{\kappa^I} \] (2.74)
\[ + \sigma (1 - \tau) (1 - \varepsilon) \left[ 1 - \frac{\mu}{[\theta a_t^U/(\theta - 1)]^x} \right] Q_t^R \theta_t^S (J_A)^{\phi_t^R} m_t^R [1 + \phi_2^R \left( \frac{m_t^I}{m_t^R} \right)]. \]

Combining (2.61) and (2.74), and using (2.73), yields
\[ m_{t+1}^I = \left[ 1 + (J_B)^{\phi_t^I} (1 + g^W)^{\kappa^I} \theta_t^{U,I} \right] m_t^I \] (2.75)
\[ \times \left\{ \sigma (1 - \tau) Q_t^U \theta_t^U (J_B)^{\phi_t^U} m_t^I (1 + g^W)^{\kappa^I} \right\}^{-1}. \]
Dividing equation (2.67) by (2.74), and using (2.73), yields

\[ m_{t+1}^R = \left[ 1 + (J^A)^{\phi^R} \left[ 1 + \phi^R_2 \left( \frac{m^I_t}{m^R_t} \right) \right] (1 - \epsilon) \theta_{t,S,R} \right] m_t^R \]  

\[ \times \left\{ (1 - \tau) Q_t^U \theta_t^U (J^B)^{\phi^U} m_t^I (1 + g^W) \right\} \]

\[ + \sigma(1 - \tau)(1 - \epsilon) [1 - \frac{\mu}{\theta_a^{C_t}/(\theta - 1)}] Q_t^R \theta_t^S (J^A)^{\phi^R} m_t^R [1 + \phi^R_2 \left( \frac{m^I_t}{m^R_t} \right)]^{-1} \]  

From the first-order conditions (2.12),

\[ \frac{w_t^U}{w_t^S} = \left[ \frac{\beta^U (1 - \epsilon)}{\beta^S} \right] \left( \frac{N_t^{S,Y}}{N_t^{U,Y}} \right) = \beta \left( \frac{\theta_t^{S,Y}}{\theta_t^{U,Y}} \right), \]

where \( \beta = (1 - \epsilon) \beta^U / \beta^S \). Substituting this result in equation (2.7) yields

\[ a_t^C = \mu^{1/\chi} [1 - \epsilon - \beta \left( \frac{\theta_t^{S,Y}}{\theta_t^{U,Y}} \right)]^{-1/\chi}, \]  

which can be substituted in (2.32) and (2.34) to determine \( \theta_t^U \) and \( \theta_t^S \):

\[ \theta_t^U = 1 - (a_t^C)^{-\theta}, \quad \theta_t^S = \frac{\theta}{\theta - 1} (a_t^C)^{1 - \theta}. \]  

From (2.18) and (2.19),

\[ Q_t^j = (1 - \eta) \gamma \nu^j \left( \frac{Y_t}{M_t^j} \right), \quad j = I, R \]

where \( \nu^I = \nu \) and \( \nu^R = 1 - \nu \).

Equivalently, noting that \( Y_t/M_t^j = (Y_t/K_t^P)(m_t^j)^{-1} \),

\[ Q_t^j = (1 - \eta) \gamma \nu^j \left( \frac{Y_t}{K_t^P} \right) (m_t^j)^{-1}, \quad j = I, R \]  

(2.79)
where, from (2.41),

$$\frac{Y_t}{K^F_t} = \frac{(J^B)^{(1-\gamma)/(1-\gamma)}}{[\theta^{S,Y}_t \theta^S (\theta^{U,Y}_t)^{\rho U} - 1]/(1-\gamma)} \left\{ (m^I_t)^{\rho (1-\eta)/\eta} (m^R_t)^{1-\eta}(1-\eta) \right\}^{\gamma/(1-\gamma)}.$$

(2.80)

The steady-state growth rate can be rewritten in several equivalent ways. From (2.61), together with (2.73), for instance,

$$1 + \gamma = (J^B)^{\phi^I_t} (1 + g^W)^{\kappa} \theta^U^I.$$

Equivalently, from (2.67), together with (2.73),

$$1 + \gamma = (J^A)^{\phi^R_t} [1 + \phi^R_2 \left( \frac{\bar{m}^I_t}{\bar{m}^R_t} \right)] (1 - \varepsilon) \theta^{S,R}.$$

Equations (2.61) and (2.67), given (2.73), yield the steady-state solutions (2.53) and (2.54) in the text.

To determine the level of final output and its growth rate during the transition, note that equation (2.80) relates the path of $Y_t$ to that of $K^F_t$. In turn, to derive the path of $K^P_t$, equation (2.74) can be written for period $t$ as

$$\frac{K^P_t}{K^P_{t-1}} = \sigma(1 - \tau) Q^I_{t-1} \theta^U_{t-1} (k^B_{t-1})^{\phi^I_{t-1}} m^I_{t-1} [1 + g^W]^{\kappa}$$

(2.81)

$$+ \sigma(1 - \tau)(1 - \varepsilon) [1 - \mu \left( \frac{\theta^C - \mu}{\sigma \theta_{t-1}} \right)] Q^R_{t-1} \theta^S_{t-1} (k^A_{t-1})^{\phi^R_{t-1}} m^R_{t-1} [1 + \phi^R_2 \left( \frac{m^I_{t-1}}{m^R_{t-1}} \right)],$$

which gives the growth rate of $K^P_t$. For any given starting value $K^P_0$, the path of $K^P_t$ can be derived from (2.81). Substituting this result, together with the solution paths from (2.73), (2.75), (2.76), (2.77), (2.78), and (2.79), for $k^i_t$, where $i = A, B, m^I_t, m^R_t, \theta^S_t, \text{and } Q^I_t$, where $j = I, R$, in (2.80) gives the solution for $Y_t$, from which its growth rate can be derived.
## 2.9 Tables and Figures

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<th>Parameter</th>
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<td>Annual discount rate</td>
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### Table 2.2: Initial Steady-State Values of Key Variables

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<tr>
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<td>Share of unskilled labour force in population</td>
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<td>$\theta^{U,Y}$</td>
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<td>Share of unskilled labour force in final good production</td>
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<td>$\theta^{U,I}$</td>
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<td>Share of unskilled labour force in imitation sector</td>
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<tr>
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<td>Share of skilled labour force in population</td>
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<tr>
<td>$\theta^{S,Y}$</td>
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<td>Share of skilled labour force in final good production</td>
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<td>Patent price, imitation and innovation sectors</td>
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Figure 2.1: Permanent Increase in Share of Spending on Basic Infrastructure (Absolute deviations from baseline)
Figure 2.2: Permanent Increase in Share of Spending on Advanced Infrastructure
(Absolute deviations from baseline)
Figure 2.3: Permanent Increase in Investment in Advanced Infrastructure Financed in half by a Reduction in Investment in Basic Infrastructure (Absolute deviations from baseline)
Summary and Conclusion

Recently, researchers have shown an increased interest in the role of public capital, or public infrastructure, in economic development and growth in low-income countries. This thesis studied the links among public capital, economic development, and growth, using overlapping generations (OLG) models. The content of each chapter has been summarised below.

The purpose of Chapter 1 was to study the growth effects of externalities associated with intra-household bargaining and the role of access to infrastructure (or lack thereof) on girls’ time allocation within the context of a three-period (childhood, adulthood, and retirement), gender-based overlapping generations (OLG) model. Only girls’ time allocated to household chores (an important form of child labour) was endogenously related to access to infrastructure, whereas the amount of time that boys spend on home schooling was exogenous. Mothers are more intergenerationally altruistic towards girls, more engaged than fathers with the human capital of their daughters, while men care more about current consumption than women. The chapter treated gender inequality as an equilibrium outcome connected not only to social norms and cultural values but also to the way household members endowed with individual preferences interact with each other and make decisions about girls’ time allocation.

In this chapter, we studied the long-run properties of the model and considered the impact of an increase in spending on infrastructure. The model was then calibrated using data for a low-income country and various quantitative experiments with a consideration of alternative values of the parameters were conducted, including an increase in the share of public spending on infrastructure, an increase in time allocated by mothers to their daughters, and a decrease in fathers’ preference for their daughters’ education. The study revealed significant benefits for girls in terms of both education outcomes and economic growth as a result of policies aimed at
promoting better access to infrastructure for families. Such policies may lead to less girls’ time dedicated to household chores, which may in turn allow them to develop greater human capital—thereby affecting their productivity and wages in their adult life, as well as improved bargaining power in terms of resource allocation within the family. The aforementioned increase in women’s bargaining power may further reduce the amount of time that the family finds optimal for girls to spend on household chores, especially considering that mothers have a relatively higher preference than fathers for their daughters’ education. This consolidates the benefits made possible through improved access to infrastructure. Importantly, the analysis shows that benefits still exist even when an increase in government expenditure on infrastructure is financed by a reduction in spending on education. Consequently, the promotion of girls’ education and the elimination of gender inequality may be better served through investment in infrastructure, rather than the allocation of more public resources to education (as advocated by Schultz (2002) for instance), especially considering that offsetting changes in education expenditure come from spending reductions on an inefficient or corrupt bureaucracy—a common feature of education systems in low-income countries (see for instance UNESCO (2009, Chapter 3)).

Chapter 2 examined the connection between public capital and industrial development in a two-period (adulthood and old age) OLG model with endogenous skill acquisition. We accounted for two types of design activities: imitation (copying foreign ideas and adapting foreign technology) that requires only unskilled labour and innovation (inventing new ideas) that requires skilled labour. We focused on the ratio of the variety of imitation-based to innovation-based intermediate goods for a change in industrial structure and defined industrial development as a shift from an imitation-based, low-skill economy to an innovation-based, high-skill economy, where technological advance comes about through the domestic invention of ideas. In addition, imitation generates knowledge spillovers, which enhance productivity
in innovation. The model was calibrated for a low-income country and numerical experiments were conducted. The results showed that significant effects on the structure of the labour force and the speed of the industrial development may be induced by changes in the level and composition of public investment. Our experiments also showed that the scope for imitation-based learning plays a crucial role in the pace of industrial change. In the case where skilled labour is in short supply and unskilled labour is abundant, investing in basic infrastructure (which consists of roads, energy, and basic telecommunications) may be of importance in initiating a process of industrial development, by imitating imported technologies, and growth in the early stages of development. Once the process of imitation unfolds, it contributes to the accumulation of knowledge available to all workers in the economy, raising skilled labour productivity and wages in the innovation sector, thereby inducing more individuals to invest in skills or higher education and stimulating the innovation sector. However, investment in human capital is not critical for promoting growth during the early stages of development if learning externalities are sufficiently strong.
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