

# Essays on the Measurement of Poverty

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# Abstract

This thesis is comprised of three distinct chapters, each of which is concerned in some way with the measurement of poverty.

The first chapter provides social preference conditions which are both necessary and sufficient for a poverty line to arise endogenously. In so doing, it turns out that the apparently independent ‘identification’ and ‘aggregation’ problems in poverty measurement are subtly intertwined. Necessary and sufficient conditions are provided for the existence of both relative and absolute poverty lines. In each case, one of the conditions is a familiar weak monotonicity property. The other conditions are simple consistency requirements.

In the second chapter, we propose classes of intertemporal poverty measures which take into account both the debilitating impact of prolonged spells in poverty and the mitigating effect of periods of affluence on subsequent poverty. The weight assigned to the level of poverty in each time period depends on the length of the preceding spell of poverty or of non-poverty. The proposed classes of intertemporal poverty measures are quite general and allow for a range of possible judgements as to the overall impact on a poor period of preceding spells of poverty or affluence. We discuss the properties of the proposed classes of measures and axiomatically characterize them.

The third chapter is an empirical application of the intertemporal poverty measures proposed in the second chapter. The new measures, together with an existing intertemporal poverty measure from the literature, are used to analyse intertemporal poverty in Great Britain during the period 1991-2005, using data from the British Household Panel Survey. Previous studies on poverty using this data-set have employed static measures of poverty. We illustrate how the use of intertemporal poverty measures makes it possible to analyse aspects of poverty which cannot be captured by static, annual, measures of poverty. We then model the determinants of intertemporal poverty, conditional upon being poor, using a Heckman two-step selection model.

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I declare that no portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.



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*For Leo Stanley Roope*

# Introduction

The important question of how poverty should best be measured has been discussed widely in economics and the topic has generated a large literature. Interest in the subject extends far beyond academia. Better measurement of poverty can provide better guidance to policy makers as to where resources are most in need. It can also lead to a better understanding of the underlying causes of poverty and what can be done to reduce it, the importance of which is perhaps best spelt out by the grim statistics. According to World Bank (2008), 1.4 billion people, 25% of the global population, live in extreme poverty, getting by on less than \$1.25 a day.<sup>1</sup>

In a seminal contribution, Sen (1976) provided the first axiomatic characterization of a poverty measure. The approach introduced by Sen (1976), of proving mathematically that his metric satisfied various normatively desirable properties, paved the way for a greater understanding of all prior and subsequent poverty metrics.

Since then, the adoption of similar approaches in the literature has led to results that have greatly improved our ability to measure poverty. The majority of contributions have focused on measuring income (or consumption) poverty at a particular point in time, taking a ‘snapshot’ of the current situation. Notable examples include the measures of Watts (1968), Clark, Hemming and Ulph (1981), Chakravarty (1983) and Foster, Greer and Thorbecke (1984). A great deal has been achieved in this way because it has been possible to evaluate the respective merits of different poverty measures by illuminating the properties and principles that underlie them.

This theoretical research has had a large influence on empirical work and on policy evaluations. For example, the class of measures developed by Foster et al. (1984) is routinely used by international organisations such as the World Bank and the United Nations Development Programme (UNDP) to assess the level of poverty in a wide range of countries. By employing these indices as standards for measuring poverty at different points in time, they also provide a useful starting point for evaluating the effectiveness of policy interventions, with respect to poverty reduction.

The popularity of the measures of Foster et al. (1984) stems largely from the fact that they have been shown to satisfy a wide range of normatively attractive

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<sup>1</sup>In US Dollars at 2005 Purchasing Power Parity.

properties. Consider, for example, an eminent member of this class of measures known as the ‘mean of squared deprivation gaps,’ or simply as the ‘FGT2’ index. It satisfies ‘income monotonicity,’ meaning that when a poor person becomes poorer, poverty is deemed to increase. It satisfies Sen (1976)’s ‘transfer principle,’ meaning that a transfer of income to a poor individual from a better-off individual is deemed to reduce poverty. It is also ‘subgroup consistent,’ which ensures that when poverty increases in some particular subgroup of a population, for example in a particular city or among a particular demographic, *ceteris paribus*, poverty is deemed to increase in the population as a whole.

These properties may appear both uncontroversial and undemanding yet, strikingly, the simple but still popular headcount measure, defined as the proportion of the population which is below the poverty line, does not satisfy either ‘income monotonicity’ or the ‘transfer principle.’ With this in mind, it is not difficult to see how over-reliance on a tool with dubious axiomatic foundations might potentially result in perverse policy prescriptions. For example, taking income away from the poorest individual and redistributing it to someone only slightly below the poverty line might, according to the headcount measure, result in a reduction of poverty, by reducing the number of people with incomes below the poverty line. By using a tool with firmer theoretical foundations, a policy maker can have some confidence that his indicator of poverty will respond to changes in income in a more reasonable fashion.

While much has been achieved in the theoretical literature on poverty measurement, a number of important conceptual, philosophical and methodological aspects have yet to be addressed. Nothing can be more fundamental to the measurement of poverty than the identification of those who are poor. The vast majority of empirical studies have focused on income (or consumption) poverty and the identification process has typically been performed through the imposition of a poverty line. Individuals are regarded as poor if their income is below the poverty line and non-poor otherwise. Naturally, any measure of poverty is typically highly sensitive to the choice of poverty line and so it is not surprising that a large literature has emerged on what methodologies should be used to set poverty lines.<sup>2</sup> What is perhaps surprising is that relatively little attention has been given to this topic in the theoretical literature on poverty measurement.

Since Sen (1976)’s seminal paper, the focus has been on deriving aggregate measures of poverty, under the assumption that the poor have already been identified via the imposition of an exogenously determined poverty line. In Chapter 1, we propose normatively appealing social preference axioms and show that they can be used to identify which individuals are poor. To put it another way, we endogenize the poverty line within a social preference framework, proving its existence from

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<sup>2</sup>Callan and Nolan (1991) provide a good introduction to this literature.

normative values. The axioms we propose are consistent with those that have been used to characterize the well-known poverty measures from the literature. We are, therefore, able to demonstrate that a common axiomatic framework can be used both to identify the poor and to characterize the existing measures of aggregate poverty in the literature.

There is a broad consensus that many of the most important issues in poverty analysis that have still to be resolved are related in some way to either the dynamics of poverty or to its multi-dimensional nature.<sup>3</sup> In Chapter 2, we focus on the measurement of poverty as a dynamic phenomenon. As such, Chapter 2 adds to a recent literature with notable contributions by Foster (2009), Hoy and Zheng (2011), Bossert, Chakravarty and D'Ambrosio (2012), Mendola, Busetta and Milito (2011) and Zheng (2011), among others. Each of these papers essentially attempts to address the following question. Suppose that two individuals are poor during some, but not all, of a given number of time periods. Suppose, furthermore, that they are poor in different periods from one another. From a policy perspective, it is important to be able to distinguish which person is poorer and, therefore, in need of extra attention. The traditional literature on static poverty analysis is of limited assistance here. In some periods of time the first individual would be regarded as being better off than the second person while in other periods of time he would be considered poorer. However, no guidance is provided as to which individual is poorer overall, when poverty is aggregated over time. Clearly, there is a need to supplement the axioms of the literature on static poverty measurement with new principles addressing the dynamic aspects of poverty.

In Chapter 2, we introduce a new class of intertemporal poverty measures. The key benefit of these new measures, relative to existing measures in the literature, is that they have the property that periods of relative affluence can have a mitigating impact on subsequent spells of poverty. Implicit in our approach is the notion that the mitigating effect of affluent periods can be transmitted through non-income dimensions such as assets, health, housing, social networks, human capital and so on. In this sense, it can be inferred that there is a link between the measurement of uni-dimensional income (or consumption) poverty, in an intertemporal context, and the measurement of poverty as a multi-dimensional phenomenon.

In our third and final chapter, the intertemporal poverty measures proposed in Chapter 2 are applied to evaluate poverty levels in Great Britain, using data from the British Household Panel Survey (BHPS). Regional patterns of poverty are analysed during each of three separate eras - 1991 to 1995, 1996 to 2000 and 2001 to 2005. After estimating the overall levels of intertemporal poverty during each of these eras, we analyse the determinants of intertemporal poverty. Our approach allows for the possibility that the determinants of being intertemporally poor or non-poor

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<sup>3</sup>See, for example, Thorbecke (2005).

might differ from the determinants of the overall severity of intertemporal poverty, conditional upon being poor. This is achieved by employing the Heckman two-step selection model. This well-known technique, and related approaches, have been used in a number of studies on the determinants of poverty and living standards. However, to the best of our knowledge, this is the first study which uses it to model the determinants of the severity of poverty, conditional upon being poor.

# Chapter 1

## The Endogenous Poverty Line: Existence and Implications

### 1.1 Introduction

Following a seminal paper by Sen (1976), the literature has traditionally considered the measurement of poverty to consist of

“...two distinct problems...viz., (i) identifying the poor among the total population, and (ii) constructing an index of poverty using the available information on the poor.” (p. 219)<sup>1</sup>

The former problem involves choosing a criterion of poverty and then assigning individuals as poor or non-poor accordingly. Typically, some minimum level of income, or ‘poverty line’ is chosen. An individual is poor if his income lies below the poverty line and non-poor otherwise. Since Sen (1976), the main emphasis in the theoretical literature has been on the latter problem. The standard approach has been to assume the poverty line to be exogenously determined and then require a poverty index to satisfy a range of normatively appealing axioms. This has led to the development of poverty indices with broad ranges of properties that are widely considered attractive on ethical grounds.<sup>2</sup> Little, if any, attempt has been made to develop ethical principles that would endogenize the poverty line and prove its existence from normative values.

This chapter uses normatively appealing social preference axioms to identify which individuals are poor. The axioms provided are consistent with those that have been used to characterize popular indices from the literature on aggregation. Our approach thus demonstrates that the apparently independent identification and

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<sup>1</sup>These have come to be known, respectively, as the ‘identification’ and ‘aggregation’ problems.

<sup>2</sup>Examples include the indices of Watts (1968), Clark, Hemming and Ulph (1981), Chakravarty (1983) and Foster, Greer and Thorbecke (1984). The approach has also recently been extended to multi-dimensional and intertemporal frameworks. For examples of the former, see Tsui (2002), Bourguignon and Chakravarty (2003) and Alkire and Foster (2011). Intertemporal poverty measurement is discussed at length in Chapter 2.



aggregation problems are, in fact, subtly intertwined as a common set of axioms can be employed to address both problems.

As Foster, Greer and Thorbecke (2010) have recently noted, the identification method has, in contrast to the aggregation method, changed remarkably little in over a century. They suggest that with so much having now been accomplished in aggregation, now may be a good time to re-evaluate the concept of a poverty line and its role in identifying and targeting the poor. They go on to pose the central question of endogenizing the poverty line, which this chapter seeks to address:

“Can axioms be reformulated to apply to the overall methodology of poverty measurement, including the identification and aggregation step? This is a subtle problem, but one that deserves additional thought.” (p. 518)

The identification method has a long history, going back at least as far as 1899, when Seebohm Rowntree defined those families whose “total earnings are insufficient to obtain the minimum necessities for the maintenance of merely physical efficiency as being in primary poverty” (Rowntree (1901)). To this day, the identification of poverty in developing countries is typically performed in a conceptually similar manner, employing the notion of an ‘absolute’ poverty line. Individuals are regarded as poor if their incomes lie below this line.<sup>3</sup>

In developed countries, where the standard of living is generally much higher, ‘relative’ poverty lines are more commonly used. This is not necessarily to focus attention on inequality. It can also reflect the idea that there is a certain minimum standard of living that is particularly desirable in a given society in order to avoid shame, or social exclusion, and to be able to contribute to the life of the society in a meaningful way. The popularity of this approach owes much to the influential work of Townsend (1979) who found evidence from the UK that below a certain income level there was a disproportionate increase in a variety of indicators of deprivation.<sup>4</sup> With such an interpretation, relative, as well as absolute, poverty lines can be viewed as being objectively important thresholds.

A number of attempts have been made to embed the concept of a poverty line, or more generally a reference income, within a broader social welfare framework.<sup>5</sup> However, these works too have assumed the poverty line / reference income to be

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<sup>3</sup>In practice, a number of different poverty lines are sometimes used. Sensitivity to the choice of poverty line can then be analysed using stochastic dominance results by Foster and Shorrocks (1988a, 1988b) and Atkinson (1987). However, this approach still uses absolute poverty lines as a starting point.

<sup>4</sup>As Townsend (1979) noted, a similar line of thought had been put forward by many social scientists in the past, including Adam Smith who regarded as “necessaries”, “...not only the commodities which are indispensably necessary for the support of life, but whatever the custom of the country renders it indecent for creditable people, even of the lowest order, to be without.” (See Smith (1776).)

<sup>5</sup>Blackorby and Donaldson (1982), for example, provided results on interpersonal comparisons of utilities where zero plays an important role as a natural reference point. See also Blackorby and Donaldson (1980), who studied the relationship between reference-level free and translatable social welfare functions. More recently, Zank (2007) provided reference income dependent extensions of traditional classes of social welfare functions.

exogenously determined. Given the degree of importance that has been attached in the literature to ensuring that aggregate measures of poverty display socially and ethically desirable properties, the question as to what social preferences might give rise to the existence of a poverty line almost seems conspicuous by its absence.

This chapter provides social preference conditions under which a poverty line arises endogenously. Results are provided for both relative and absolute poverty lines. In each case, one of the conditions is a familiar weak monotonicity property. The other conditions are simple consistency requirements with regard to the impact of incremental increases in income. These follow the basic principle that if an increase in a given income leads to some social improvement, so too an increase in any lower income should result in some improvement. Conversely, if an increase in a given income leads to no social improvement, then an increase in any higher income should also bring no improvement. Existence of an absolute poverty line requires a stronger consistency condition than is needed for a relative poverty line. The stronger condition ensures that equal levels of income are treated in a similar way, regardless of the particular distribution of income. Some implications of our findings for the derivation of poverty measures are discussed.

The rest of the chapter is organised as follows. Section 1.2 introduces our framework for studying social preferences with regard to poverty and shows how certain preferences can imply the existence of a poverty line. Implications of an endogenous poverty line for the measurement of poverty are discussed. Concluding remarks are offered in Section 1.3. All proofs are deferred to the Appendix, 1.A.

## 1.2 Deriving Poverty Lines from Social Preferences

### 1.2.1 Notation and Basic Framework

We consider a society of  $n \geq 2$  individuals.<sup>6</sup> A profile  $\mathbf{x} = (x_1, \dots, x_n) \in \mathbb{R}^n$  represents the distribution of incomes within the society. A social preference relation,  $\succsim$ , on  $\mathbb{R}^n$  is assumed.<sup>7</sup> The symbol  $\succ$  denotes strict preference while  $\sim$  denotes indifference. The symbols  $\preceq$  and  $\prec$  are defined, respectively, so that  $\mathbf{x} \preceq \mathbf{y}$  means  $\mathbf{y} \succsim \mathbf{x}$ , and  $\mathbf{x} \prec \mathbf{y}$  means  $\mathbf{y} \succ \mathbf{x}$ .

The relation  $\succsim$  is said to be *complete* if for every  $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ ,  $\mathbf{x} \succsim \mathbf{y}$  or  $\mathbf{y} \succsim \mathbf{x}$ .

The social preference relation  $\succsim$  is said to be *symmetric* if, for any permutation  $\rho$  and any income profile  $\mathbf{x} \in \mathbb{R}^n$ , it holds that  $\mathbf{x} \sim \rho(\mathbf{x})$ . From now on we assume

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<sup>6</sup>The case  $n = 1$  is trivial.

<sup>7</sup>It should be emphasised that we are considering social preferences with a particular focus on poverty. Just as a social planner may employ different indices for different purposes, so too the ranking of income distributions may depend on the purpose of the ranking. For example, distributions might instead be ranked according to inequality, polarisation, or mean income.

symmetry. Without loss of generality, we can then assume that incomes are always ordered from lowest to highest. This is indicated using the notation  $\mathbf{x} \in \mathbb{R}_\uparrow^n$ , where the subscript ‘ $\uparrow$ ’ signifies the direction of the ordering.

We use the notation  $\alpha_j \mathbf{x}$  to represent the profile obtained by replacing the  $j$ ’th element of  $\mathbf{x}$  with  $\alpha$ . When we use this notation, it is implicitly assumed that this replacement leaves the rank order unchanged.<sup>8</sup> We will often consider profiles of the form  $(x_i + \epsilon)_i \mathbf{x}$  defined for some *permissible*  $\epsilon > 0$  or any *permissible*  $\epsilon \geq 0$ . Here the term ‘permissible’ is used merely to make explicit our assumption that any such  $\epsilon$  must be small enough to ensure that the profile  $(x_i + \epsilon)_i \mathbf{x}$  is in  $\mathbb{R}_\uparrow^n$ .

## 1.2.2 Derivation of Poverty Lines

The following axioms impose constraints on the social preference relation  $\succsim$ . Our first axiom imposes the mild requirement that, *ceteris paribus*, an increase in an individual’s income cannot lead to a new distribution of income which is strictly less socially desirable than the original one. It would be rather counter-intuitive for a poverty measure not to be consistent with this property, and all well-known measures are.

**Axiom 1.1** ‘*Weak Monotonicity.*’ For every  $\mathbf{x} \in \mathbb{R}_\uparrow^n$  and every  $i \in \{1, \dots, n\}$ , for any permissible  $\delta \geq 0$ ,  $(x_i + \delta)_i \mathbf{x} \succsim \mathbf{x}$ .

Our next axiom is a simple consistency requirement. Consider a situation where some individual’s income increases by a certain amount. Suppose that this increase leads to no improvement in the social ranking of the resulting income profile compared to the original one. Our axiom says that if this is the case, then no social progress would have been made by increasing this individual’s income by some other amount. Moreover, increasing the income of a more affluent individual must also bring no social benefit.

Conversely, suppose that some increase in an individual’s income does lead to an improvement in the social ranking of the resulting income profile compared to the original one. Then increasing that individual’s income by some different amount would also have led to some improvement. Moreover, increasing the income of a less well-off individual must also be socially beneficial.

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<sup>8</sup>So if  $1 < j < n$ , we have  $x_{j-1} \leq \alpha \leq x_{j+1}$ . If  $j = 1$ , we have  $\alpha \leq x_{j+1}$ . If  $j = n$ , it must be that  $x_{j-1} \leq \alpha$ .

**Axiom 1.2** ‘Consistency.’ The preference relation  $\succsim$  on  $\mathbb{R}_+^n$  satisfies consistency (in income increments) if for any profile  $\mathbf{x} \in \mathbb{R}_+^n$ , any individual  $i \in \{1, \dots, n\}$  and any permissible  $\epsilon > 0$ :

- (i)  $(x_i + \epsilon)_i \mathbf{x} \sim \mathbf{x} \implies (x_j + \delta)_j \mathbf{x} \sim \mathbf{x}$  for all  $j \geq i$  and any permissible  $\delta > 0$ .
- (ii)  $(x_i + \epsilon)_i \mathbf{x} \succ \mathbf{x} \implies (x_j + \delta)_j \mathbf{x} \succ \mathbf{x}$  for all  $j \leq i$  and any permissible  $\delta > 0$ .

With the notable exception of the headcount ratio, all the well-known poverty measures from the literature are consistent with this property.<sup>9</sup>

We can now state our first result.

**Proposition 1.1** Suppose that  $\succsim$  is a complete preference relation on  $\mathbb{R}_+^n$ . Then the following two statements are equivalent:

- (a) The social preference relation  $\succsim$  satisfies weak monotonicity and consistency.
- (b) For any profile  $\mathbf{x} \in \mathbb{R}_+^n$  there exists  $z_{\mathbf{x}} \in \mathbb{R}$  such that we have (i)  $(x_i + \delta)_i \mathbf{x} \succ \mathbf{x}$  for all  $x_i < z_{\mathbf{x}}$ ,  $i \in \{1, \dots, n\}$  and permissible  $\delta > 0$  and (ii)  $(x_i + \delta)_i \mathbf{x} \sim \mathbf{x}$  for all  $x_i \geq z_{\mathbf{x}}$ ,  $i \in \{1, \dots, n\}$  and permissible  $\delta \geq 0$ .

**Proof.** See Appendix. ■

**Remark** It can be inferred from the proof of Proposition 1.1 that, given  $\mathbf{x} \in \mathbb{R}_+^n$ ,  $z_{\mathbf{x}}$  is not unique and need not lie within the range of incomes  $[x_1, x_n]$ . However, given  $\mathbf{x} \in \mathbb{R}_+^n$ , any  $z_{\mathbf{x}}, \hat{z}_{\mathbf{x}} \in \mathbb{R}$  which satisfy (i) and (ii) are such that for all  $i \in \{1, \dots, n\}$ ,  $x_i < \hat{z}_{\mathbf{x}}$  whenever  $x_i < z_{\mathbf{x}}$  and  $x_i \geq \hat{z}_{\mathbf{x}}$  whenever  $x_i \geq z_{\mathbf{x}}$ .

There is a strong consensus in the literature that poverty measures should satisfy a ‘focus’ axiom, which states that the level of poverty in a society is independent of the incomes of those above the poverty line.<sup>10</sup> There is also some consensus that poverty measures should be sensitive to the size of the income shortfall of those below the poverty line.<sup>11</sup> With these observations in mind, it seems reasonable that an income such as  $z_{\mathbf{x}}$  in Proposition 1.1 might be interpreted as being a poverty line. Given the dependence on the distribution  $\mathbf{x} \in \mathbb{R}_+^n$ , it would be natural to regard such a poverty line as a relative one. We can now provide a formal definition for a relative poverty line, based on this intuition.

<sup>9</sup>The headcount measure is not sensitive to the size of the income shortfall of those below the poverty line - only the fact that an income is below the poverty line. It is therefore quite common for the headcount measure to indicate that a small increase in an income would not lead to any improvement, while a larger increase would do. Similarly, increasing a poor individual’s income by some amount  $\epsilon$  may lead to no improvement, while adding the same amount to a less poor individual’s income would. This lack of sensitivity is widely regarded as a serious drawback of the headcount measure.

<sup>10</sup>All the well-known measures from the literature satisfy this property.

<sup>11</sup>As highlighted above, the headcount ratio is the only well-known measure which is insensitive to how far an income is below the poverty line. All the other popular measures satisfy Sen (1976)’s ‘monotonicity’ axiom which implies that any increase in a poor income reduces poverty.

**Definition 1.1** Given any profile  $\mathbf{x} \in \mathbb{R}_+^n$ , income  $z_{\mathbf{x}} \in \mathbb{R}$  is a relative poverty line if the following holds. For all  $x_i < z_{\mathbf{x}}$ ,  $i \in \{1, \dots, n\}$  and permissible  $\delta > 0$ ,  $(x_i + \delta)_i \mathbf{x} \succ \mathbf{x}$  and for all  $x_i \geq z_{\mathbf{x}}$ ,  $i \in \{1, \dots, n\}$  and permissible  $\delta \geq 0$ ,  $(x_i + \delta)_i \mathbf{x} \sim \mathbf{x}$ .

We now propose a stronger version of our Consistency axiom. The stronger version captures the intuition that the consistency of social preferences, with respect to income increments, is not based exclusively on relative considerations. Some importance is attached to the absolute level of income. The axiom says the following. Suppose that in some distribution, increasing some individual  $i$ 's income from  $x_i$  to  $x_i + \delta$  leads to no improvement in the social ranking of the income profile compared to the original one. Then in any income distribution in which an individual has an income at least as high as  $x_i$ , it must be that an increase in that individual's income also brings no improvement to that distribution.

Conversely, suppose that in some income distribution, increasing an individual  $i$ 's income from  $x_i$  to  $x_i + \delta$  results in the new income profile being socially preferable to the original one. Then in any income distribution in which an individual has an income equal to or less than  $x_i$ , it must be that an increase in that individual's income must also be socially beneficial.

**Axiom 1.3** ‘Global Consistency.’ The preference relation  $\succsim$  on  $\mathbb{R}_+^n$  satisfies global consistency (in income increments) if for any profiles  $\mathbf{x}, \mathbf{y} \in \mathbb{R}_+^n$ , any individuals  $i, j \in \{1, \dots, n\}$  and any permissible  $\epsilon > 0$ :

- (i)  $(x_i + \epsilon)_i \mathbf{x} \sim \mathbf{x} \Rightarrow (y_j + \delta)_j \mathbf{y} \sim \mathbf{y}$  whenever  $y_j \geq x_i$  and  $\delta > 0$  is such that  $(y_j + \delta)_j \mathbf{y} \in \mathbb{R}_+^n$ ;
- (ii)  $(x_i + \epsilon)_i \mathbf{x} \succ \mathbf{x} \Rightarrow (y_j + \delta)_j \mathbf{y} \succ \mathbf{y}$  whenever  $y_j \leq x_i$  and  $\delta > 0$  is such that  $(y_j + \delta)_j \mathbf{y} \in \mathbb{R}_+^n$ .

Apart from the headcount ratio, all the well-known measures from the literature, when applied using an absolute poverty line which remains fixed across different societies, are consistent with this property.<sup>12</sup>

We can now state the following result.

**Theorem 1.2** Suppose that  $\succsim$  is a complete preference relation on  $\mathbb{R}_+^n$ . Then the following two statements are equivalent:

- (a) The social preference relation  $\succsim$  satisfies weak monotonicity and global consistency.
- (b) There exists some  $z \in \mathbb{R}$  such that the following holds for every  $\mathbf{x} \in \mathbb{R}_+^n$  and for every  $i \in \{1, \dots, n\}$ .

- (i)  $(x_i + \delta)_i \mathbf{x} \succ \mathbf{x}$  for all  $x_i < z$  and permissible  $\delta > 0$  and
- (ii)  $(x_i + \delta)_i \mathbf{x} \sim \mathbf{x}$  for all  $x_i > z$  and permissible  $\delta \geq 0$ .

If  $z$  is finite it is unique.

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<sup>12</sup>An example of such an approach can be seen in World Bank (2008), which evaluates poverty using an international poverty line of \$1.25 per day in 2005 Purchasing Power Parity.

**Proof.** See Appendix. ■

It is clear that the  $z$  in Theorem 1.2 has some properties in common with the  $z_{\mathbf{x}}$  in Proposition 1.1, which we have defined as being a relative poverty line. The key difference is that whereas the  $z_{\mathbf{x}}$  in Proposition 1.1 was dependent on  $\mathbf{x} \in \mathbb{R}_{\uparrow}^n$ , and is in this sense relative, the  $z$  in Theorem 1.2 is independent of the distribution.<sup>13</sup> It seems natural that such an income  $z$  might be interpreted as being an absolute poverty line. As long as the social preference relation  $\succsim$  itself remains unchanged, so too  $z$  remains fixed. We can now provide a formal definition for an absolute poverty line, based on this intuition.

**Definition 1.2** *Income  $z \in \mathbb{R}$  is an absolute poverty line if the following holds for every  $\mathbf{x} \in \mathbb{R}_{\uparrow}^n$ . For all  $x_i < z$ ,  $i \in \{1, \dots, n\}$  and permissible  $\delta > 0$ ,  $(x_i + \delta)_i \mathbf{x} \succ \mathbf{x}$  and for all  $x_i > z$ ,  $i \in \{1, \dots, n\}$  and permissible  $\delta \geq 0$ ,  $(x_i + \delta)_i \mathbf{x} \sim \mathbf{x}$ .*

### 1.2.3 Implications of an endogenous poverty line

Thus far we have introduced a social preference framework and provided conditions under which both relative and absolute poverty lines arise endogenously. A natural question to ask at this stage is what the implications of an endogenous poverty line might be.

Note that the existence of a poverty line does not necessarily imply that the poverty line lies within the range of actual incomes in society; it doesn't rule out the possibility that all individuals are poor or that all individuals are non-poor. Furthermore, in the case of relative poverty lines, they are not unique. However, it can be inferred from the proof of Proposition 1.1 that every relative poverty line for a given income distribution will identify the same individuals as poor or non-poor.

In the case of absolute poverty lines, the poverty line may not be finite. The existence of poverty lines at  $+\infty$  and  $-\infty$  requires more than merely the income profile being such that 'all individuals are poor' and 'no individuals are poor,' respectively. It requires that in all possible income profiles, 'all individuals are poor' and 'no individuals are poor,' respectively. When there is a poverty line at  $+\infty$  or  $-\infty$ , the social preference relation  $\succsim$  itself is such that no finite poverty line exists, though for different reasons. A poverty line at  $+\infty$  implies that society always attaches some importance to an increase in any individual's income, regardless of the income profile. This is a strong monotonicity requirement. Such preferences do not necessarily embody a concern for poverty.<sup>14</sup> When there is a poverty line at  $-\infty$ ,

<sup>13</sup>In addition, a small technical difference between the  $z$  of Theorem 1.2 and the  $z_{\mathbf{x}}$  of Proposition 1.1, is that Theorem 1.2 doesn't stipulate the social impact of incremental increases to incomes exactly equal to  $z$ . Incomes exactly equal to the poverty line are sometimes regarded in the literature as poor and sometimes as non-poor. The headcount ratio is the only well-known measure of poverty for which this decision may make any practical difference. The other popular measures, which take into account the depth of poverty, are unaffected by this choice.

<sup>14</sup>They may, however, embody some concern for inequality. For example, strong monotonicity

society is always indifferent to any increase in any individual's income, regardless of the income profile. This seems a most unrealistic scenario; such a society does not care at all about either the level or the distribution of income. For a finite absolute poverty line to exist, the preference relation  $\succsim$  must lie somewhere between these two extremes.

Suppose that a finite poverty line does arise endogenously. One might ask whether its existence is consistent with the various axioms which underpin popular poverty measures from the literature, or whether it serves to illuminate some previously hidden weaknesses. It should be clear from the discussion of our axioms, which characterize the poverty line's existence, that apart from the headcount measure, all the popular measures in the literature, and therefore the axioms which characterize them, are consistent with our approach. Furthermore, by removing the arbitrary nature of the poverty line, and showing that it arises naturally through observed social preferences, our framework serves to enhance the attractiveness of the popular 'focus' axiom.

### 1.3 Concluding remarks

Despite its ubiquity, there has until now been little attempt to endogenize the concept of the poverty line within a broader social welfare framework. This is surprising, especially given the volume of literature on the social preference axioms which characterize poverty indices. This chapter is, to the best of our knowledge, the first study to provide necessary and sufficient social preference conditions for the existence of a poverty line.

Our approach raises some interesting points for discussion. The apparently distinct identification and aggregation problems are shown to be subtly intertwined, co-existing in a common axiomatic framework. It turns out that both relative and absolute poverty lines can be characterized by a familiar weak monotonicity property, together with some consistency properties regarding the impact of incremental increases in income.

By demonstrating that poverty lines can arise naturally, and need not be chosen arbitrarily, our results add to the appeal of the popular 'focus' axiom.

We envisage at least two possible avenues for future research.

Firstly, it may be that social preferences can be shown to give rise to a number of different reference levels, reflecting different levels of income at which new opportunities and capabilities become attainable, for example, affording secondary education or private health insurance.

Secondly, it might be possible to use a similar method to ours to endogenize 'reference ranges.' This might reflect, for example, the notion that at certain ranges 

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 is not inconsistent with even a strict version of the well-known Pigou-Dalton transfer principle.

of income, the welfare returns to income are greater than at incomes below and above these ranges. It could, alternatively, reflect the possibility that there are ranges of incomes in which individuals are close to crossing important meaningful thresholds, such as poverty lines. A social planner might welcome an increase in the income of an individual who is close to such a threshold, while being indifferent to comparable increases in the incomes of other individuals.

We hope that this chapter will encourage future research in this largely neglected, but conceptually important, area.



# 1.A Appendix

## 1.A.1 Proof of Proposition 1.1

Clearly if statement (b) is satisfied, then weak monotonicity holds. Also, for any profile  $\mathbf{x} \in \mathbb{R}_\uparrow^n$ , if  $i$  is such that  $(x_i + \epsilon)_i \mathbf{x} \sim \mathbf{x}$  for some  $\epsilon > 0$ , then  $x_i \geq z_{\mathbf{x}}$ . Therefore, it follows that  $(x_j + \delta)_j \mathbf{x} \sim \mathbf{x}$  for all  $j \geq i$  and any permissible  $\delta > 0$  as  $j \geq i \iff x_j \geq x_i$ . Similarly, for any profile  $\mathbf{x} \in \mathbb{R}_\uparrow^n$ , if  $i$  is such that  $(x_i + \epsilon)_i \mathbf{x} \succ \mathbf{x}$  for some  $\epsilon > 0$ , then  $x_i < z_{\mathbf{x}}$ . Therefore, it follows that  $(x_j + \delta)_j \mathbf{x} \succ \mathbf{x}$  for all  $j \leq i$  and any permissible  $\delta > 0$  as  $j \leq i \iff x_j \leq x_i$ . Hence, statement (a) holds.

Next we prove that statement (a) implies statement (b). Take any profile  $\mathbf{x} \in \mathbb{R}_\uparrow^n$ . By completeness and weak monotonicity it follows that for any  $\epsilon > 0$  either  $(x_n + \epsilon)_n \mathbf{x} \succ \mathbf{x}$  or  $(x_n + \epsilon)_n \mathbf{x} \sim \mathbf{x}$ .

If  $x_n$  is such that  $(x_n + \epsilon)_n \mathbf{x} \succ \mathbf{x}$  then consistency implies that  $(x_j + \delta)_j \mathbf{x} \succ \mathbf{x}$  for all  $j \leq n$  and any permissible  $\delta > 0$ . Set  $z_{\mathbf{x}} := x_n + \epsilon$  and statement (b) follows.

If, however,  $x_n$  is such that  $(x_n + \epsilon)_n \mathbf{x} \sim \mathbf{x}$  then consistency implies that  $(x_n + \delta)_n \mathbf{x} \sim \mathbf{x}$  for all  $\delta > 0$ . Then, if all  $x_i = x_n$  set  $z_{\mathbf{x}} = x_n$ . In this case there exist no  $\delta > 0$  such that  $(x_i + \delta)_i \mathbf{x} \in \mathbb{R}_\uparrow^n$  if  $i < n$ . Then statement (b) follows.

Else, if  $x_i \neq x_n$  for some  $i < n$ , then take the largest index  $k \in \{1, \dots, n-1\}$  such that  $x_n > x_k$ . We next consider all  $x_j$  with  $k < j \leq n$ . For any  $j$  such that  $k < j < n$  we have  $(x_j + \delta)_j \mathbf{x} \in \mathbb{R}_\uparrow^n$  only for  $\delta = 0$  such that  $(x_j + \delta)_j \mathbf{x} \sim \mathbf{x}$  follows by the reflexivity of  $\succsim$ . When  $j = n$ , if  $\delta = 0$ , it again follows trivially that  $(x_j + \delta)_j \mathbf{x} \sim \mathbf{x}$ . Further, we have already established that  $(x_j + \delta)_j \mathbf{x} \sim \mathbf{x}$  for all  $\delta > 0$  when  $j = n$ . Moreover, as  $x_n = x_{k+1} > x_k$  it follows that there exists  $\epsilon' > 0$  such that  $(x_k + \epsilon')_k \mathbf{x} \in \mathbb{R}_\uparrow^n$ . By completeness and weak monotonicity it follows that either  $(x_k + \epsilon')_k \mathbf{x} \succ \mathbf{x}$  or  $(x_k + \epsilon')_k \mathbf{x} \sim \mathbf{x}$ .

If  $x_k$  is such that  $(x_k + \epsilon')_k \mathbf{x} \succ \mathbf{x}$  then consistency implies that  $(x_j + \delta)_j \mathbf{x} \succ \mathbf{x}$  for all  $j \leq k$  and any permissible  $\delta > 0$ . Set  $z_{\mathbf{x}} := (x_k + x_n)/2$  and statement (b) follows.

If  $x_k$  is such that  $(x_k + \epsilon')_k \mathbf{x} \sim \mathbf{x}$  then consistency implies that  $(x_j + \delta)_j \mathbf{x} \sim \mathbf{x}$  for all  $j \geq k$  and any permissible  $\delta \geq 0$ . As shown before, for all  $i < k$  such that  $x_i = x_k$  it follows trivially by reflexivity that  $(x_i + \delta)_i \mathbf{x} \sim \mathbf{x}$  for all permissible  $\delta \geq 0$  as only  $\delta = 0$  is allowed. The proof then proceeds as above. If there is no  $x_i < x_k$  then set  $z_{\mathbf{x}} = x_k$  and statement (b) holds. Else, take the largest  $k'$  such that  $x_{k'} < x_k$ . Then  $(x_{k'} + \epsilon'')_{k'} \mathbf{x}$  is a profile in  $\mathbb{R}_\uparrow^n$  for some  $\epsilon'' > 0$  and we proceed with a further iteration on the basis of whether  $(x_{k'} + \epsilon'')_{k'} \mathbf{x} \succ \mathbf{x}$  or  $(x_{k'} + \epsilon'')_{k'} \mathbf{x} \sim \mathbf{x}$ . This process stops after finitely many iterations as  $n$  was a positive integer.

Therefore, and because  $\mathbf{x}$  was arbitrary, we can conclude that statement (b) holds. This completes the proof of Proposition 1.1.

□

## 1.A.2 Proof of Theorem 1.2

Clearly if statement (b) is satisfied, then weak monotonicity holds. Also, for any profile  $\mathbf{x} \in \mathbb{R}_+^n$ , if  $i$  is such that  $(x_i + \epsilon)_i \mathbf{x} \sim \mathbf{x}$  for some  $\epsilon > 0$ , then  $x_i \geq z$ . Therefore, it follows that  $(y_j + \delta)_j \mathbf{y} \sim \mathbf{y}$  for any permissible  $\delta > 0$  whenever  $y_j \geq x_i$ . Similarly, for any profile  $\mathbf{x} \in \mathbb{R}_+^n$ , if  $i$  is such that  $(x_i + \epsilon)_i \mathbf{x} \succ \mathbf{x}$  for some  $\epsilon > 0$ , then  $x_i < z$ . Therefore, it follows that  $(y_j + \delta)_j \mathbf{y} \succ \mathbf{y}$  for any permissible  $\delta > 0$  whenever  $y_j \leq x_i$ . Hence, statement (a) holds.

Next we prove that statement (a) implies statement (b).

By the completeness of the preference relation  $\succsim$  together with weak monotonicity, precisely one of the following three statements must be true:

(A) There exists some  $\mathbf{x} \in \mathbb{R}_+^n$ , some  $i \in \{1, \dots, n\}$  and some permissible  $\delta > 0$ , such that  $(x_i + \delta)_i \mathbf{x} \succ \mathbf{x}$ . There also exists some  $\mathbf{y} \in \mathbb{R}_+^n$ , some  $j \in \{1, \dots, n\}$  and some permissible  $\epsilon > 0$ , such that  $(y_j + \epsilon)_j \mathbf{y} \sim \mathbf{y}$ .

(B) For every  $\mathbf{x} \in \mathbb{R}_+^n$  and every  $i \in \{1, \dots, n\}$ , for any permissible  $\delta > 0$ ,  $(x_i + \delta)_i \mathbf{x} \succ \mathbf{x}$ .

(C) For every  $\mathbf{x} \in \mathbb{R}_+^n$  and every  $i \in \{1, \dots, n\}$ , for any permissible  $\delta > 0$ ,  $(x_i + \delta)_i \mathbf{x} \sim \mathbf{x}$ .

If statement (B) is true, then (b) holds for  $z = +\infty$ .

If statement (C) is true, then (b) holds for  $z = -\infty$ .

Suppose then that statement (A) holds. So for some  $\mathbf{x} \in \mathbb{R}_+^n$ , some  $i \in \{1, \dots, n\}$  and some permissible  $\delta > 0$ ,  $(x_i + \delta)_i \mathbf{x} \succ \mathbf{x}$ .

Let us define  $z_0 := x_i$ . Then by global consistency it follows that for any distribution  $\widehat{\mathbf{v}} \in \mathbb{R}_+^n$  and any individual  $m \in \{1, \dots, n\}$  such that  $\widehat{v}_m \leq z_0$ , for any permissible  $\widehat{\epsilon} > 0$ ,

$$(\widehat{v}_m + \widehat{\epsilon})_m \widehat{\mathbf{v}} \succ \widehat{\mathbf{v}}. \quad (1.1)$$

We also know from (A) that for some  $\mathbf{y} \in \mathbb{R}_+^n$ , some  $j \in \{1, \dots, n\}$ , and some permissible  $\epsilon > 0$ ,  $(y_j + \epsilon)_j \mathbf{y} \sim \mathbf{y}$ .

Let us define  $z_0 := y_j$ . Then by global consistency it follows that for any distribution  $\mathbf{v} \in \mathbb{R}_+^n$  and any individual  $l \in \{1, \dots, n\}$  such that  $v_l \geq z_0$ , for any permissible  $\widehat{\epsilon} > 0$ ,

$$(v_l + \widehat{\epsilon})_l \mathbf{v} \sim \mathbf{v}. \quad (1.2)$$

Thus far, from (1.1) and (1.2), we have shown that, in any distribution where an individual has an income less than or equal to  $z_0$ , an incremental increase in that individual's income is beneficial, while in any distribution where an individual has an income greater than or equal to  $z_0$ , an incremental increase in that individual's income brings no benefit.

The following lemma will now be useful. It simply states the following. Suppose that in all possible income distributions, an increase in the income of an individual

with an income at least as high as some amount  $z_0$  is not deemed socially beneficial. Suppose also, that in all possible income distributions, an increase in the income of an individual with an income less than some amount  $\underline{z}_0$  is deemed socially beneficial. Then it must be that  $z_0$  is higher than  $\underline{z}_0$ .

**Lemma 1.3** *Suppose that  $\succsim$  is a preference relation on  $\mathbb{R}_+^n$  and that conditions (i) and (ii) hold.*

(i) *For any distribution  $\mathbf{v} \in \mathbb{R}_+^n$  and any individual  $l \in \{1, \dots, n\}$  such that  $v_l \geq z_0$ , for any permissible  $\delta > 0$ ,  $(v_l + \delta)_l \mathbf{v} \sim \mathbf{v}$ .*

(ii) *For any distribution  $\hat{\mathbf{v}} \in \mathbb{R}_+^n$  and any individual  $m \in \{1, \dots, n\}$  such that  $\hat{v}_m \leq \underline{z}_0$ , for any permissible  $\epsilon > 0$ ,  $(\hat{v}_m + \epsilon)_m \hat{\mathbf{v}} \succ \hat{\mathbf{v}}$ .*

*Then  $z_0 > \underline{z}_0$ .*

*Proof of Lemma 1.3:* We prove by contradiction. Suppose that  $z_0 \leq \underline{z}_0$ . According to (ii), for any distribution  $\hat{\mathbf{v}} \in \mathbb{R}_+^n$  and any individual  $m \in \{1, \dots, n\}$  such that  $\hat{v}_m \leq \underline{z}_0$ , for any permissible  $\epsilon > 0$ ,  $(\hat{v}_m + \epsilon)_m \hat{\mathbf{v}} \succ \hat{\mathbf{v}}$ . Now since, by assumption,  $z_0 \leq \underline{z}_0$  it must be that for any distribution  $\mathbf{v}' \in \mathbb{R}_+^n$  and any individual  $m \in \{1, \dots, n\}$  such that  $v'_m = z_0$ , for any permissible  $\epsilon > 0$ ,  $(v'_m + \epsilon)_m \mathbf{v}' \succ \mathbf{v}'$ . But (i) implies that if  $v'_m = z_0$  then for any permissible  $\epsilon > 0$ ,  $(v'_m + \epsilon)_m \mathbf{v}' \sim \mathbf{v}'$  so we have a contradiction.

□

We now return to the main part of the proof of Theorem 1.2. Applying Lemma 1.3 to (1.1) and (1.2) note that it follows immediately that  $\underline{z}_0 < z_0$ . We now need to consider what happens in those income distributions where we increase the incomes of individuals who have incomes in the interval  $(\underline{z}_0, z_0)$ .

It is clear from the nature of  $\mathbb{R}_+^n$  that there must exist some income distribution  $\mathbf{t}_{(1)} \in \mathbb{R}_+^n$  such that for some  $i \in \{1, \dots, n-1\}$ ,  $t_{(1)i} = \frac{\underline{z}_0 + z_0}{2} < t_{(1)i+1}$ . (In fact there must be infinitely many such vectors). By the completeness of the preference relation  $\succsim$  together with weak monotonicity and global consistency, precisely one of the following two statements must be true:

For all permissible  $\delta > 0$ ,

$$(t_{(1)i} + \delta)_i \mathbf{t}_{(1)} \succ \mathbf{t}_{(1)}. \quad (1.3)$$

For all permissible  $\delta > 0$ ,

$$(t_{(1)i} + \delta)_i \mathbf{t}_{(1)} \sim \mathbf{t}_{(1)}. \quad (1.4)$$

Suppose firstly that (1.3) holds. Then by global consistency, the following statement, analogous to (1.1), must be true.

For any distribution  $\hat{\mathbf{q}} \in \mathbb{R}_+^n$  and any individual  $m \in \{1, \dots, n\}$  such that  $\hat{q}_m \leq$

$t_{(1)i} = \frac{z_0 + z_0}{2}$ , for any permissible  $\epsilon > 0$ ,

$$(\widehat{q}_m + \epsilon)_m \widehat{\mathbf{q}} \succ \widehat{\mathbf{q}}. \quad (1.5)$$

Suppose instead that (1.4) holds. Then by global consistency, the following statement, analogous to (1.2), must be true.

For any distribution  $\mathbf{q} \in \mathbb{R}_+^n$  and any individual  $l \in \{1, \dots, n\}$  such that  $q_l \geq t_{(1)i} = \frac{z_0 + z_0}{2}$ , for any permissible  $\epsilon > 0$ ,

$$(q_l + \epsilon)_l \mathbf{q} \sim \mathbf{q}. \quad (1.6)$$

If (1.3) is true then combining (1.2) and (1.5) we know that in any distribution where an individual has an income less than  $t_{(1)i} = \frac{z_0 + z_0}{2}$  that an incremental increase in that individual's income is beneficial, while in any distribution where an individual has an income greater than or equal to  $z_0$ , an incremental increase in that individual's income brings no benefit. If (1.4) is true then combining (1.1) and (1.6) we know that in any distribution where an individual has an income less than  $z_0$  that an incremental increase in that individual's income is beneficial, while in any distribution where an individual has an income greater than or equal to  $t_{(1)i} = \frac{z_0 + z_0}{2}$ , an incremental increase in that individual's income brings no benefit. Either way, it is clear that we have halved the range of incomes between which we still need to consider the impact of incremental increases in an individual's income.

Let us define

$$z_1 := \begin{cases} t_{(1)i}, & \text{if (1.3) holds;} \\ z_0, & \text{if (1.4) holds.} \end{cases}$$

Let us define

$$z_1 := \begin{cases} z_0, & \text{if (1.3) holds;} \\ t_{(1)i}, & \text{if (1.4) holds.} \end{cases}$$

We can then write the following. There exist some  $z_1, z_1 \in \mathbb{R}$  such that  $z_1 - z_1 = \frac{z_0 - z_0}{2}$  and that for every  $\mathbf{w} \in \mathbb{R}_+^n$ , the following holds. For every  $i \in \{1, \dots, n\}$ , if  $w_i \leq z_1$  then for any permissible  $\delta > 0$ ,  $(w_i + \delta)_i \mathbf{w} \succ \mathbf{w}$  and if  $w_i \geq z_1$  then for any permissible  $\delta \geq 0$ ,  $(w_i + \delta)_i \mathbf{w} \sim \mathbf{w}$ .

Now by following  $N$  times an analogous approach to that used above to yield  $z_1$  and  $z_1$ , we can continue  $N$  times to halve the range of incomes between which we still need to consider the impact of incremental increases in an individual's income. For example, starting with  $N = 2$ , we note that there must exist some income distribution  $\mathbf{t}_{(2)} \in \mathbb{R}_+^n$  such that for some  $j \in \{1, \dots, n - 1\}$ ,  $t_{(2)j} = \frac{z_1 + z_1}{2} < t_{(2)j+1}$ .

We can then follow analogous steps to those which were applied above to  $t_{(1)i}$  in order to yield incomes  $z_2, z_2 \in \mathbb{R}$  such that  $z_2 - z_2 = \frac{z_0 - z_0}{2^2}$  and that for every

$\mathbf{w} \in \mathbb{R}_\uparrow^n$ , the following holds. For every  $i \in \{1, \dots, n\}$ , if  $w_i \leq \underline{z}_2$  then for any permissible  $\delta > 0$ ,  $(w_i + \delta)_i \mathbf{w} \succ \mathbf{w}$  and if  $w_i \geq z_2$  then for any permissible  $\delta \geq 0$ ,  $(w_i + \delta)_i \mathbf{w} \sim \mathbf{w}$ .

Following this approach  $N$  times yields incomes  $\underline{z}_N, z_N \in \mathbb{R}$  such that  $z_N - \underline{z}_N = \frac{z_0 - \underline{z}_0}{2^N}$  and that for every  $\mathbf{w} \in \mathbb{R}_\uparrow^n$ , the following holds. For every  $i \in \{1, \dots, n\}$ , if  $w_i \leq \underline{z}_N$  then for any permissible  $\delta > 0$ ,  $(w_i + \delta)_i \mathbf{w} \succ \mathbf{w}$  and if  $w_i \geq z_N$  then for any permissible  $\delta \geq 0$ ,  $(w_i + \delta)_i \mathbf{w} \sim \mathbf{w}$ .

We then define a sequence  $(s_N)$  such that  $s_N = z_N - \underline{z}_N$  for  $N \in \mathbb{Z}_+$ . It is a standard result that the sequence  $(s_N)$  converges to zero as  $N \rightarrow \infty$ . That is, as  $N \rightarrow \infty$ ,  $z_N \rightarrow \underline{z}_N$ . Define  $z := \lim_{N \rightarrow \infty} z_N$ .

Suppose that for some  $\mathbf{w}' \in \mathbb{R}_\uparrow^n$  and some  $i \in \{1, \dots, n\}$ ,  $w'_i < z$ . Since  $\underline{z}_N \rightarrow z$  as  $N \rightarrow \infty$ , there must exist some  $N'$  such that  $w'_i < \underline{z}_{N'} < z$ .

Suppose that for some  $\mathbf{w}'' \in \mathbb{R}_\uparrow^n$  and some  $i \in \{1, \dots, n\}$ ,  $w''_i > z$ . Since  $z_N \rightarrow z$  as  $N \rightarrow \infty$ , there must exist some  $N''$  such that  $z < z_{N''} < w''_i$ .

We can therefore write that the following holds for every  $\mathbf{w} \in \mathbb{R}_\uparrow^n$ . For every  $i \in \{1, \dots, n\}$ , if  $w_i < z$  then for any permissible  $\delta > 0$ ,

$$(w_i + \delta)_i \mathbf{w} \succ \mathbf{w} \tag{1.7}$$

and if  $w_i > z$  then for any permissible  $\delta \geq 0$ ,

$$(w_i + \delta)_i \mathbf{w} \sim \mathbf{w}. \tag{1.8}$$

All that remains is to show that  $z$  is unique if it is finite, which we prove by contradiction. Suppose that analogous statements to (1.7) and (1.8) hold for some  $\bar{z} \in \mathbb{R}$  such that  $z \neq \bar{z}$ . That is, the following holds for  $\bar{z}$ . For every  $\mathbf{w} \in \mathbb{R}_\uparrow^n$  and every  $i \in \{1, \dots, n\}$ , if  $w_i < \bar{z}$  then for any permissible  $\delta > 0$ ,

$$(w_i + \delta)_i \mathbf{w} \succ \mathbf{w} \tag{1.9}$$

and if  $w_i > \bar{z}$  then for any permissible  $\delta \geq 0$ ,

$$(w_i + \delta)_i \mathbf{w} \sim \mathbf{w}. \tag{1.10}$$

Without loss of generality we can assume that  $\bar{z} > z$  and, furthermore, that  $\bar{z} = z + \gamma$  for some  $\gamma > 0$ . It is clear from the structure of  $\mathbb{R}_\uparrow^n$  that there must exist a distribution  $\widehat{\mathbf{w}} \in \mathbb{R}_\uparrow^n$  such that, for some  $i \in \{1, \dots, n-1\}$ ,  $\widehat{w}_i = z + \frac{\gamma}{2} < \widehat{w}_{i+1}$ . Since  $\widehat{w}_i > z$ , it follows immediately from (1.8) that for any permissible  $\delta > 0$ ,

$$(\widehat{w}_i + \delta)_i \widehat{\mathbf{w}} \sim \widehat{\mathbf{w}}. \tag{1.11}$$

But since  $\bar{z}$  satisfies (1.9) it must be that for any  $\mathbf{w} \in \mathbb{R}_+^n$ , any  $i \in \{1, \dots, n\}$  and any permissible  $\delta > 0$ ,  $(w_i + \delta)_i \mathbf{w} \succ \mathbf{w}$  holds if  $w_i < \bar{z}$ . But this is contradicted by (1.11) since  $\hat{w}_i < \bar{z}$ .

Statement (b) therefore holds and the theorem is proved.

□

# Chapter 2

## On Intertemporal Poverty Measures: The Role of Affluence and Want

### 2.1 Introduction

The important question of how poverty should best be measured has generated wide interest both in policy circles and in academia, leading to a large discourse on the level of poverty and the different poverty measures. Most of these poverty indices, however, capture poverty only at a given point in time (Watts (1968), Sen (1976), Clark, Hemming and Ulph (1981), Chakravarty (1983), Foster, Greer and Thorbecke (1984); see also Zheng (1997)). An increasing number of studies, however, indicate that measuring poverty at any single point in time is inadequate for capturing the true level of poverty, since a far greater proportion of people may experience poverty when observed over a longer term (Baulch and Hoddinott (2000)).

In this chapter, we develop classes of intertemporal poverty measures which take into account previous poverty experiences of individuals. Our focus is exclusively with the measurement of intertemporal poverty at the individual level, rather than at the societal level. Thus, we attempt to address the question of distinguishing between two individuals who may have the same level of poverty in the current period, but had different levels of poverty, or were poor at different times from one another, in the past.

Given this objective, the chapter adds to a recent literature with notable contributions by Foster (2009), Hoy and Zheng (2011), Bossert, Chakravarty and D'Ambrósio (2012), Mendola, Busetta and Milito (2011) and Zheng (2011), among others.<sup>1</sup> In Foster (2009), the spread of poor episodes over time is of no importance; only the proportion of spells in which the individual is in poverty is taken into account.

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<sup>1</sup>See also Cruces (2005), Calvo and Dercon (2009), Grab and Grimm (2007), Carter and Ikegami (2007), Porter and Quinn (2008), Foster and Santos (2012) and Gradín, del Río and Cantó (2012).

Overall poverty for each individual is a simple average of the generalized poverty gaps in each period. Bossert et al. (2012), on the other hand, assign particular importance to the extent to which individuals are in a state of poverty for consecutive periods. They argue for a principle in which people facing periods of poverty which are interrupted by relatively affluent periods are deemed as being able to manage more easily than those who are exposed to longer consecutive periods of poverty (even if the total number of periods of poverty and non-poverty are the same in each case). In this context, one can argue that while the bunching of poor episodes is important in its own right, there is also a case for explicitly taking into account the role of affluent periods in mitigating subsequent poverty.

More recently, Mendola et al. (2011) and Zheng (2011) have proposed measures, which by considering each pair of poor episodes, take into account the damaging impact of consecutive poor episodes as well as the mitigating effect of affluent spells. In both these papers, proximity of one poor episode to another serves to intensify the overall experience of poverty. While Mendola et al. (2011) consider the number of non-poor periods between two periods of poverty as an indicator of the mitigating effect, Zheng (2011), in a general framework, allows for non-poor periods to directly interact with the poor periods. In both papers, the mitigation depends on the distance between two poor episodes. Thus, it is difficult to disentangle whether affluent periods have an independent mitigating impact, separate from simply indicating that the poor episodes are not close together. A similar issue arises in Hoy and Zheng (2011), where a person who is poor in periods which are either consecutive, or separated by only short spells of relative wellbeing, is worse off than a person with similar incomes but more widely dispersed poor episodes.

The classes of measures proposed in this chapter are motivated by the relevance of consecutive poor periods, but afford a richer interpretation of the dynamics which cause closely bunched poor spells to be debilitating. The method used differs from that of the existing literature in how both the mitigating effect of affluent periods and the debilitating impact of consecutive episodes of poverty are accounted for. It is motivated by the observation that the longer the spell of relative affluence experienced prior to becoming poor, the better equipped an individual is to deal with that period of poverty. In our measures, the impact of a poor period is discounted according to the number of affluent periods directly preceding it.

This approach allows us to account for the mitigating impact of affluence independently from the intensification of poverty arising from the bunching of poor episodes. To illustrate the advantage of our method, consider the following stylised example. Suppose that there are two individuals who both live over three time periods. Each is poor, and to a similar extent, in only one of the three periods. The first person is poor in the first period, while the second person is poor in the last period. Our measures would indicate that the second person is better off since he had



an opportunity to accumulate resources before facing the poor episode. However, most of the existing measures, such as those by Foster (2009), Bossert et al. (2012), Mendola et al. (2011) and the Newtonian measure by Zheng (2011), are unable to distinguish between such cases.<sup>2</sup>

We capture the intensification of poverty due to bunching by weighting each poor period according to the number of directly preceding poor periods. Consider another example where two individuals live for two time periods and are poor in each period. Suppose that the first person is poor in the first period by a certain amount but only half as poor in the second period. Suppose that it is the other way around for the second person. Intuitively, it seems reasonable to expect the two profiles to be ranked differently, since in one case poverty is decreasing, while in the other case it is increasing. In contrast to our measures, most existing measures, including those of Foster (2009), Bossert et al. (2012), Mendola et al. (2011) and Zheng (2011) are unable to distinguish between these two profiles.

Like Foster (2009), Bossert et al. (2012), Mendola et al. (2011) and others, we do not allow the level of income in affluent periods to mitigate poverty in other periods. Yet one can argue that affluent periods should have some mitigating effect on poverty. Our measures are general and do not specify the mitigating attributes explicitly (e.g., income as in Hoy and Zheng (2011) and Zheng (2011)). Implicit in our approach is that, in the absence of income smoothing opportunities, the mitigating effect of affluent periods is transmitted through non-income dimensions such as assets, health, social networks, human capital and so on.<sup>3</sup> For instance, in a rich and detailed study, Narayan, Pritchett and Kapoor (2009) have found that, in India, those who move out of poverty have “almost always made investment in land.” Typically these investments would also include acquiring education and building social networks, the latter of which is regarded by Woolcock and Narayan (2000) as being one of the primary resources the poor have for managing risk and vulnerability. There is also strong evidence that in rural areas of developing countries body weight varies significantly between peak and off-peak seasons (Behrman and Deolalikar (1989); Dercon and Krishnan (2000)). People who have a possibility of falling into hardship often use their “body as a store of energy” during affluent times by employing a “feast now fast later” strategy (see Dercon and Hoddinott (2003), pp. 7-8). There is significant evidence that these factors do indeed, in general, have some impact on mitigating poverty (Narayan, Chambers, Shah and Petesch (2000), Sen and Hulme (2004)). Thus, from a social planner’s perspective, when evaluating

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<sup>2</sup>Hoy and Zheng (2011) would rank these cases differently. However, the motivation for doing so is very different from that here. They explicitly consider poverty early in life to be more damaging than poverty later on. We have no such assumption here. Zheng (2011) proposes several classes of measures. Here, and for the rest of the paper, we are concerned mainly with his Newtonian poverty measure (p. 10).

<sup>3</sup>Our notion of income smoothing is based on Morduch (1995, p. 104) where households can smooth income by “making conservative production and employment choices and diversifying economic activities.”

intertemporal poverty, there might be sufficient reason to take into account the mitigating impact of affluent periods on subsequent poverty even in the absence of income smoothing.

In our measures, the mitigating impact of a spell of affluence does not last long. There is strong evidence that, in the face of poverty, households draw down their existing resources quite significantly (Hulme (2003), Moser (1996)). Davis (2006) and Davis and Baulch (2009), for example, provide empirical evidence that in Bangladesh, while improvements in life conditions of individuals typically occur slowly, over long periods of time, they decline suddenly following shocks. With regard to social networks, Beall (2001) points out that such social resources can be quickly eroded by poverty. In contrast, both Mendola et al. (2011) and Zheng (2011) allow a single episode of affluence to have a mitigating impact on all subsequent episodes of poverty.

Our approach retains much of the appealing intuition of Bossert et al. (2012), with respect to the exacerbating impact of bunching of poor episodes, but adds to this the characteristics of mitigation of poverty by preceding non-poor periods and an increasing intensification of the impact of consecutive episodes of poverty. The proposed intertemporal poverty measures are quite general and allow for a range of possible judgements as to the overall impact of a poor period that is preceded by a non-poor spell and also to the overall effect of consecutive periods of poverty. When no significance is attached to either the relatively affluent periods or to the exacerbating impact of consecutive periods of poverty, our class of relative intertemporal poverty measures reduces to the simple average of per period (static) poverty measures advocated by Foster (2009). In this sense we provide axiomatic foundations for a class of measures that encompasses the latter.<sup>4</sup>

The remainder of the chapter is organised as follows. Section 2.2 introduces our notation and the basic framework. Section 2.3 formally introduces our classes of individual intertemporal poverty measures and provides axiomatic characterizations. The following section discusses two important extensions of the measures presented in Section 2.3. Section 2.5 concludes the chapter. All proofs are deferred to the Appendix, 2.A.

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<sup>4</sup>Note that here, and throughout the chapter, we are referring to Foster (2009)'s total intertemporal poverty measure, not his chronic poverty measure. Foster (2009) defines a poverty duration cut-off line as the minimum proportion of time periods a person must be poor in order to be deemed chronically poor; individuals who are in poverty for a proportion of periods less than this threshold are considered transiently poor. Foster (2009)'s total intertemporal poverty measure is obtained by choosing a poverty duration cut-off line of zero.

## 2.2 Notation and Basic Framework

We focus on the measurement of an individual's aggregate poverty over finitely many time periods. This requires the determination of a static poverty index for each time period and an aggregation of the latter across time. Subsequently, one can construct measures of poverty for an entire society by aggregating across individuals. We focus on the former two steps.

For  $T \in \mathbb{N}$  let  $t \in \{1, \dots, T\}$  denote a particular time period. An individual has income  $x_t \geq 0$  in each period  $t = 1, \dots, T$ . The income is net of any taxes and transfers.<sup>5</sup> It is important to note that since this is an ex-post measure the income in each period is the realised income for that period.

There is a poverty line  $z_t$  for each time period  $t$ , where  $0 < z_t < \infty$ , and  $\mathbf{z} = (z_1, \dots, z_T) \in \mathbb{R}_{++}^T$  denotes the profile of poverty lines. If  $x_t < z_t$  in period  $t \in \{1, \dots, T\}$ , the individual is *poor* and has an income shortfall  $p_t \in [0, 1]$ .<sup>6</sup> If  $x_t \geq z_t$ , the individual is non-poor and  $p_t = 0$ . We do not specify how the poverty line  $z_t$  is obtained. It could be exogenously determined, as is commonly assumed in the theoretical literature on poverty measurement, or it could arise endogenously within a social preference framework, in a manner such as that described in Chapter 1. Our approach is consistent with the use of either relative or absolute poverty lines.<sup>7</sup>

The individual's poverty profile is  $\mathbf{p} = (p_1, \dots, p_T)$ , representing the income shortfalls that the individual faces in each of the  $T$  time periods. Thus, a poverty profile is a  $T$ -vector where  $\mathbf{p} \in [0, 1]^T$ . We use  $\mathbf{0}^T$  to represent the poverty profile in which there is no poor period, i.e.  $p_t = 0$ , for all  $t \in \{1, \dots, T\}$ . Further, a  $T$ -period poverty profile with only one poor period such as  $\mathbf{p} = (0, \dots, 0, p_s, 0, \dots, 0)$ ,  $1 \leq s \leq T$ , is represented as  $\mathbf{p} = p_s \cdot \mathbf{e}_s^T$ , where  $\mathbf{e}_s^T$  is the profile with  $e_t = 0$  for all  $t \in \{1, \dots, T\} \setminus \{s\}$  and  $e_s = 1$ .

For a profile  $\mathbf{p}$  we define  $n_t$  to be the number of consecutive non-poor periods immediately prior to a poor period  $t$ , and we let  $k_t$  be the number of preceding periods of uninterrupted poverty, up to and including the poor episode in period  $t$ . Formally,

$$n_t := \begin{cases} 0, & \text{if } t = 1 \text{ or } p_{t-1} > 0 \\ t - \min\{s : s < t \text{ and } p_s = \dots = p_{t-1} = 0\}, & \text{otherwise,} \end{cases}$$

<sup>5</sup>Net income can be thought of as consumption, but then our interpretation of the measure has to change in line with this. If we assume consumption as our primitive, then the mitigating impact of affluent periods reflects non-consumption smoothing mechanisms since presumably any consumption smoothing is already reflected in the consumption vector.

<sup>6</sup>For example,  $p_t$  could be any static poverty measure from the literature, such as a normalized poverty gap. In fact, with some minor amendments, our results will go through for a more general definition, where  $p_t \in \mathbb{R}_+$ .

<sup>7</sup>See Chapter 3 for an empirical application with a relative poverty line  $z_t$ .

and

$$k_t := \begin{cases} 1, & \text{if } t = 1 \text{ or } p_{t-1} = 0, \\ t - \min\{s - 1 : s < t \text{ and } p_{t'} > 0, \forall t' = s, \dots, t\}, & \text{otherwise.} \end{cases}$$

For example, for  $T = 5$ , the poverty profile  $\mathbf{p} = (p_1, 0, p_3, p_4, 0)$  has  $n_1 = 0$ ,  $k_1 = 1$ ,  $n_3 = 1$ ,  $k_3 = 1$ , and  $n_4 = 0$ ,  $k_4 = 2$ . It will later become clear that there is no need to define  $n_t$  and  $k_t$  for non-poor periods.

## 2.3 Individual Intertemporal Poverty Indices

An intertemporal poverty measure for an individual is a function that assigns to each poverty profile a non-negative number. Thus,  $P : \cup_{T \in \mathbb{N}} [0, 1]^T \rightarrow \mathbb{R}_+$ . The class of individual intertemporal poverty measures that we consider are close in structure to the measures of Foster (2009) and Bossert et al. (2012).

We propose a class of measures that takes into account both the poverty mitigation arising from the presence of affluent periods ( $n_t$ ) and the intensification of poverty due to consecutive poor periods ( $k_t$ ). The *constant-relative affluence-dependent intertemporal poverty measure*  $P_R$  is defined as

$$P_R(\mathbf{p}) = \frac{1}{T} \sum_{t=1}^T \frac{k_t^\alpha}{(1 + n_t)^\beta} p_t^\theta, \text{ where } \alpha, \beta, \theta \geq 0. \quad (2.1)$$

The parameter  $\theta$  captures the sensitivity of the poverty experienced in each time period to the income shortfall. The damaging impact of consecutive periods of poverty, which serve to intensify the overall impact of poverty, is captured by  $k_t$ . The parameter  $\alpha$  determines the extent of this intensification of poverty. If  $\alpha = 0$ , there is no intensification. Similarly,  $\beta$  can be interpreted as an index representing how much one chooses to discount the impact of an individual's poor episodes according to preceding uninterrupted spells of non-poverty. When  $\beta = 0$ , there is no mitigation.<sup>8</sup>

The mitigating effect of an affluent spell on subsequent poverty is determined primarily by its duration. In the example below, we evaluate an individual's intertemporal poverty level for a four period poverty profile  $\mathbf{p} = (p_1, p_2, p_3, p_4)$ , where  $\alpha = 1$ ,  $\beta = 1$  and  $\theta = 1$ .

**Example 2.1** For  $\mathbf{p}_1 = (1/2, 0, 1/4, 0)$  we have

$$P_R(\mathbf{p}_1) = \frac{1}{4} \left( 1 \cdot \frac{1}{2} + 0 + \frac{1}{2} \cdot \frac{1}{4} + 0 \right) = \frac{5}{32} (\approx 0.156)$$

<sup>8</sup>If both  $\alpha = 0$  and  $\beta = 0$ , provided  $p_t$  is a normalized poverty gap, the measure reduces to the simple average of static poverty measures advocated by Foster (2009).

and for  $\mathbf{p}_2 = (1/2, 0, 0, 1/4)$  we have

$$P_R(\mathbf{p}_2) = \frac{1}{4}(1 \cdot \frac{1}{2} + 0 + 0 + \frac{1}{3} \cdot \frac{1}{4}) = \frac{7}{48} (\approx 0.146).$$

The measure  $P_R$  differentiates between these two profiles by attaching lower weight to the snapshot poverty level  $1/4$  in the second profile, thereby indicating that the second profile represents less intertemporal poverty. While Mendola et al. (2011) and Zheng (2011) would rank the two profiles in a similar manner to  $P_R$ , the measures of Foster (2009) and Bossert et al. (2012) rank the two profiles as being equally poor.

Implicit in the importance of the duration of affluent spells is the ‘focus axiom,’ which states that if income in a non-poor period is increased, this has no effect on overall poverty (see Foster 2009).<sup>9</sup> Although this may, at first glance, seem limiting in an intertemporal context, opportunities for income (or consumption) smoothing between periods is limited, particularly in a developing country context (Hulme and McKay (2005)). Furthermore, since this is an ex-post measure, in each time period we observe the realised income, which incorporates any income smoothing that has taken place. Thus, if an individual had a high income (well above the poverty line) in one period followed by a low income (just below the poverty line) in the next, this reflects the constraints that the individual faced in those periods. Allowing further income smoothing between these two periods to take place, which would be the result if the focus axiom were discarded, would be quite difficult to justify, especially when we know that in reality such smoothing has not occurred. As in our approach, Mendola et al. (2011) also adopt the focus axiom and resort to the duration of affluent spells as a mitigating factor. On the other hand, there are also some compelling arguments for why income levels during affluent periods may matter, in particular through investment in assets (McKay (2009)). Hence, we will later relax this assumption.

The mitigating impact of a spell of affluence dissipates after the initial period of poverty. Individuals may have non-income resources left after the first period, but these may be so limited that their effectiveness for mitigating poverty further is considered to be negligible. This follows evidence gathered by Hulme (2003) and Moser (1996), among others, where households draw down their existing resources quite significantly when faced with poverty.

The mitigation due to an affluent spell is a *proportion* of the poverty level, irrespective of the depth of poverty. In other words, if we consider two different poverty levels, immediately following the same number of affluent periods, then they should have the same constant proportion of their poverty levels mitigated. In

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<sup>9</sup>The approach of effectively censoring the income in each time period at the poverty line is common in the literature on intertemporal poverty measurement and is also adopted by Bossert et al. (2012) and Mendola et al. (2011), among others. However, this leads to a discontinuity in the measure at the poverty line. For a continuous measure see Hoy and Zheng (2011).

this sense, our measures belong to a class based on relative mitigation. Note that the mitigation of poverty is not occurring through changing the person's income in the poor period - if such a change had occurred, this would already be reflected in the person's income for that period.

Next we illustrate some additional properties of  $P_R$  through an example. Again let  $\alpha = \beta = \theta = 1$ .

**Example 2.2** For  $\mathbf{p}_3 = (0, 3/4, 1/2, 1/4)$  we have

$$P_R(\mathbf{p}_3) = \frac{1}{4}(0 + \frac{1}{2} \cdot \frac{3}{4} + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{4}) = \frac{17}{32},$$

for  $\mathbf{p}_4 = (0, 1/4, 1/2, 3/4)$  we have

$$P_R(\mathbf{p}_4) = \frac{1}{4}(0 + \frac{1}{2} \cdot \frac{1}{4} + 2 \cdot \frac{1}{2} + 3 \cdot \frac{3}{4}) = \frac{27}{32},$$

and for  $\mathbf{p}_5 = (3/4, 1/2, 1/4, 0)$  we have

$$P_R(\mathbf{p}_5) = \frac{1}{4}(1 \cdot \frac{3}{4} + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{4} + 0) = \frac{20}{32}.$$

One might reasonably expect the three profiles considered here to be ranked differently, yet the measures of Foster (2009), Bossert et al. (2012) and Mendola et al. (2011), and the Newtonian measure of Zheng (2011), all rank them as equal. Our measures distinguish between the poverty profiles  $\mathbf{p}_3$  and  $\mathbf{p}_5$ . Thus, in our framework, it matters whether the mitigating affluence occurs early, or later on. This, however, does not imply that it is better for less severe poverty to occur early on, and more severe poverty to occur later, as is the case in Hoy and Zheng (2011). We do not make any such assumption and this is evident from a comparison of  $\mathbf{p}_3$  and  $\mathbf{p}_4$ . Here, although the latter profile has less severe poverty in early periods, it is deemed by our measures to have greater intertemporal poverty because the episode with the highest severity of poverty in  $\mathbf{p}_4$  occurs after two periods of poverty while in  $\mathbf{p}_3$  it occurs after a (mitigating) period of affluence. What is important in our measures is the interplay between the mitigating effect and the intensification effect. The ranking of poverty profiles depends on the overall result of this interaction. The proposed measures can also distinguish between  $\mathbf{p}_4$  and  $\mathbf{p}_5$ , where in one case poverty has steadily increased and in the other case it has steadily declined.

We now turn to the axiomatic foundations for the proposed chronic affluence-dependent intertemporal poverty measures.

### 2.3.1 A Foundation for $P_R$

Our first requirement for an individual intertemporal poverty measure is that in trivial cases, where there is only one time period, the individual intertemporal poverty

measure is a reflection of the income shortfall in that period. This axiom is similar to the single period equivalence axiom proposed by Bossert et al. (2012).

**Axiom 2.1** *Single period equivalence holds if  $\forall p \in (0, 1]$ ,  $P(p) = p^\theta$ , where  $\theta \geq 0$ .*

Even in the context of a single period, there is scope for a range of judgements as to the relationship between an income shortfall and poverty. By allowing for a broad range of possible values of  $\theta$ , we allow for a broad range of judgements as to the precise nature of this relationship.

The second axiom considers the possibility of partitioning a longer poverty profile into shorter ones and the relation of the sub-profile measures with the overall measure. This is an additive separability condition. It is clear from our objective that different periods of poverty will be given different weights when poverty is aggregated over time. Hence, only a restricted version of separability into specific sub-profiles is permitted and the timing of the periods which have a non-zero income shortfall is critical. Similar restrictions on separability were used by Bossert et al. (2012).

**Axiom 2.2** *Time decomposability holds if for all periods of length  $T \in \mathbb{N} \setminus \{1\}$ , all poverty profiles  $\mathbf{p} \in [0, 1]^T$  and all  $t \in \{1, \dots, T - 1\}$  such that  $p_t > 0$  and  $p_{t+1} = 0$  then*

$$P(\mathbf{p}) = \frac{t}{T}P(p_1, \dots, p_t) + \frac{T-t}{T}P(p_{t+1}, \dots, p_T).$$

The axiom means that intertemporal poverty must be equal to a weighted average of two sub-profiles, where the weights are proportional to the lengths of the two sub-profiles. However, it can be applied only to situations in which the first sub-profile ends with a poor period and the second sub-profile starts with a non-poor period.

The next requirement fixes the location of our measurement scale. It concerns only the desirable case of non-poor profiles.

**Axiom 2.3** *Normalization holds if for all  $T \in \mathbb{N}$  we have  $P(\mathbf{0}^T) = 0$ .*

Next we focus on axioms which reflect our motivation that poverty in some periods can be mitigated by preceding periods of affluence. Recall Examples 2.1 and 2.2, which demonstrated how an additional preceding period of non-poverty leads to smaller weights being given to an immediately subsequent period of poverty. Our next axiom formalises the intuition that the discounting of a poor period's poverty is proportional to the length of the immediately preceding non-poor spell.

**Axiom 2.4** *Constant-relative poverty mitigation holds if for all  $T \in \mathbb{N} \setminus \{1\}$  and all  $p \in (0, 1]$  we have  $P(p \cdot \mathbf{e}_t^T) = P(p \cdot \mathbf{e}_1^T)/t^\beta$  for some  $\beta \geq 0$  and any  $t \in \{1, \dots, T\}$ .*

When  $\beta = 0$ , there is no mitigating effect of periods of affluence. On the other hand,  $\beta > 0$  ensures that non-poor episodes will have an impact on immediately

subsequent poor episodes. The axiom also implies that the greater the length of non-poor spells, the larger will be the discount. However, the incremental benefit arising from each additional period of affluence (non-poverty) diminishes.

To help clarify these concepts, consider three profiles  $\mathbf{p} = (0, 0, 2/3)$ ,  $\bar{\mathbf{p}} = (0, 2/3, 0)$ , and  $\tilde{\mathbf{p}} = (2/3, 0, 0)$ . For  $\mathbf{p}$  there are two affluent periods ( $n_3 = 2$ ) before the poor episode, for  $\bar{\mathbf{p}}$  there is only one affluent period ( $\bar{n}_2 = 1$ ) and for  $\tilde{\mathbf{p}}$  there are none. Thus, unlike the profiles  $\mathbf{p}$  or  $\bar{\mathbf{p}}$ , the poor episode in  $\tilde{\mathbf{p}}$  has no possibility of being mitigated by previous affluent periods. Note that for any profile  $\mathbf{p} = p \cdot \mathbf{e}_t^T$  and  $p > 0$ , it will always be the case that  $t = n_t + 1$ . The axiom of constant-relative poverty mitigation would then say that the poverty of  $\mathbf{p}$  and  $\bar{\mathbf{p}}$  should be less than that of  $\tilde{\mathbf{p}}$ , and given by the following rule (when  $\beta = 1$ )

$$\begin{aligned} P(\mathbf{p}) &= \frac{1}{(1 + n_3)} P(\tilde{\mathbf{p}}) = \frac{1}{3} P(\tilde{\mathbf{p}}), \\ P(\bar{\mathbf{p}}) &= \frac{1}{(1 + \bar{n}_2)} P(\tilde{\mathbf{p}}) = \frac{1}{2} P(\tilde{\mathbf{p}}). \end{aligned}$$

Although the effective rate of discount will depend on the value of  $\beta$ , what this axiom essentially proposes is that the level of discount should depend on the number of immediately preceding affluent periods. Importantly, it does not take into consideration the amount of income during the non-poor episodes. As discussed above, there may be legitimate reasons for ignoring the amount of income in non-poor periods, based on lack of income-smoothing opportunities across periods, particularly in a developing country context, but this is quite a restrictive assumption and we shall later relax this condition.

It should be noted that there are a number of other possible ways of capturing the mitigating effect of affluent periods on subsequent poor episodes. A more general axiom could easily be provided that would accommodate a broader range of methodologies.<sup>10</sup> Our goal here, however, is to provide an axiomatic basis for a specific functional form. Thus we concentrate on a particular form of discounting, as stipulated in the above axiom.

If non-poor episodes have a mitigating effect on poverty, then by a similar intuition, poor episodes should serve to intensify the experience of subsequent poverty. Our next axiom captures the intuition that not only do spells of poverty have an exacerbating impact on subsequent poverty, but that the detrimental impact increases as the length of the poor spell increases. Recall that we use  $k_t$  to denote the number of consecutive poor periods, up to and including period  $t$ .

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<sup>10</sup>See Dutta, Roope and Zank (2011) for a more general structure for poverty mitigation.



**Axiom 2.5** *Relative poverty intensification holds if for all  $T \in \mathbb{N} \setminus \{1\}$ , all  $\mathbf{p} \in [0, 1]^T$  such that  $p_{T-1} > 0$  and  $p_T > 0$ , and  $\alpha \geq 0$ ,*

$$P(\mathbf{p}) = P(p_1, \dots, p_{T-1}, 0) + k_T^\alpha P(p_T \cdot \mathbf{e}_1^T).$$

Consider a poverty profile where the penultimate period is a poor one. This axiom ensures that the difference in impact between having a poor and having a non-poor episode in the last period is not determined solely by the size of the income shortfall in the last period - it is also weighted by a factor depending on the number of poor periods preceding it. Thus the impact of being poor in the last period, when there was also poverty previously, is greater than the poverty corresponding to the income shortfall that is being added. The parameter  $\alpha$  allows for a range of judgements as to the precise impact of the preceding poor periods. For  $\alpha = 0$ , previous poor episodes should carry no additional weight. The larger the value of  $\alpha$ , the greater the exacerbating impact of the previous episodes of poverty.

Combining the above axioms we obtain the following result.

**Proposition 2.1** *An intertemporal poverty measure satisfies single period equivalence (Axiom 2.1), time decomposability (Axiom 2.2), normalization (Axiom 2.3), constant-relative poverty mitigation (Axiom 2.4) and relative poverty intensification (Axiom 2.5) if and only if it is the constant-relative affluence-dependent intertemporal poverty measure  $P_R$ .*

The following proposition demonstrates that the axioms in Proposition 2.1 are independent.

**Proposition 2.2** *The axioms single period equivalence (Axiom 2.1), time decomposability (Axiom 2.2), normalization (Axiom 2.3), constant-relative poverty mitigation (Axiom 2.4) and relative poverty intensification (Axiom 2.5) are independent.*

Before moving to the next section, we briefly draw attention to the important issue of truncation. In general, an individual will have lived prior to the first period for which we have data, and he or she could have been either affluent or poor in those periods. In our chapter, however, we do not take into consideration any such information. Consider any poverty profile  $\mathbf{p} = (p_1, \dots, p_T)$ . If the poverty in the first time period  $p_1 > 0$ , then by definition  $n_1 = 0$ , and  $k_1 = 1$ . Together they imply that poverty in the first time period will not be mitigated, and neither will it be intensified.

## 2.4 Discussion

In this section we discuss two broad extensions of the proposed measures  $P_R$ . Firstly, we relax the focus axiom and allow for income levels in affluent periods to have some

mitigating effect on subsequent poverty. Secondly, we present a class of measures where the mitigating effect of affluent periods is an absolute amount, rather than a constant proportion of subsequent poverty levels.

### 2.4.1 Focus Axiom

Previously we have highlighted that the  $P_R$  measures implicitly adopt the focus axiom and have provided arguments as to why it may be reasonable to do so. Nevertheless, it is also possible to give a justification for the amount of income in affluent periods to have a role in mitigating subsequent poverty. As we have discussed earlier, the mitigating effect in our chapter is transmitted through non-income resources, the quantity of which may clearly depend on the levels of income received. The amount of assets one can purchase, the level of skills one can acquire or the kind of location that one may be able to move to clearly may depend on the level of income one has during affluent times. Thus individuals with higher levels of income may be ‘better prepared’ for future hardships. It is this aspect that we now try to capture by relaxing the focus axiom.<sup>11</sup>

Although the amount of income above the poverty line no doubt helps, it does not necessarily have a strong correlation with achievements along non-income dimensions (see Sen (1985, 1987), UNDP (1990)). Thus while we want to capture some impact of income levels in non-poor periods, we wish to ensure that it does not have a one-to-one effect on mitigating subsequent poverty. We would also like to relax the focus axiom in such a way that we do not lose the duration aspect inherent in our  $P_R$  measure.

With this in mind, for any profile  $\mathbf{p}$ , let us define,

$$\tilde{n}_t = \begin{cases} \sum_{t=s}^{t'} \lambda_t, & \text{if } p_t = 0 \text{ for all } t = s, \dots, t' \\ 0 & \text{otherwise} \end{cases}$$

where

$$\lambda_t = \begin{cases} \gamma, & \text{if } x_t > \delta z \\ 1 & \text{otherwise} \end{cases}$$

and  $\gamma \geq 1$  and  $\delta > 1$ . If  $\gamma = 1$ , then  $\tilde{n}_t = n_t$ , i.e. it is the sum of all the non-poor periods directly preceding period  $t$ . On the other hand, if  $\gamma > 1$ , then the mitigating factor is the weighted sum of all those periods, with the weights being higher in periods where income was  $\delta$  times the poverty line. The parameter  $\delta$  captures how far above the poverty line income must be for it to carry a higher weight in the mitigating factor. An alternative axiom, in the spirit of our constant-relative poverty mitigation axiom (Axiom 2.4) is the following:

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<sup>11</sup>Note that explicitly incorporating the non-income dimensions is not feasible in this context, since all we observe is the ex-post income distribution for the individual across time.

**Axiom 2.6** *Relative poverty mitigation holds if for all  $T \in \mathbb{N} \setminus \{1\}$  and all  $p \in (0, 1]$  we have  $P(\mathbf{pe}_1^T) = P(\mathbf{pe}_1^T)/(1 + \tilde{n}_t)^\beta$  for  $\beta \geq 0$  and any  $t \in \{1, \dots, T\}$ .*

Note that, in contrast to Axiom 2.4, if income in non-poor periods is higher than a certain amount, the discount will be greater.

By replacing the constant-relative poverty mitigation axiom (Axiom 2.4) in Proposition 2.1 by Axiom 2.6, we can derive the *relative affluence-dependent intertemporal poverty measure*  $\tilde{P}_R$  which is defined as follows.

**Proposition 2.3** *An intertemporal poverty measure satisfies single period equivalence (Axiom 2.1), time decomposability (Axiom 2.2), normalization (Axiom 2.3), relative poverty intensification (Axiom 2.5) and relative poverty mitigation (Axiom 2.6), if and only if it is*

$$\tilde{P}_R(\mathbf{p}) = \frac{1}{T} \sum_{t=1}^T \frac{k_t^\alpha}{(1 + \tilde{n}_t)^\beta} p_t^\theta, \text{ where } \alpha, \beta, \theta \geq 0.$$

## 2.4.2 Absolute Mitigation

One may raise two other valid objections with the measure  $P_R$  that we have proposed in this chapter. The first issue is that the extent of the mitigation, in absolute terms, depends on the level of poverty. Consider the profile  $\mathbf{p} = (0, p)$ . When  $p = 1$ , the absolute size of the mitigation under  $P_R$  (given  $\alpha = \beta = \theta = 1$ ) is 0.5. On the other hand when  $p = 0.5$ , under the same parameters,  $P_R$  would determine the mitigation to be only 0.25. The greater the size of  $p$  is, the greater is the size of the mitigation.<sup>12</sup>

A second possible criticism is that there is no possibility of the mitigating effect of affluence being able to wipe out subsequent poverty completely, no matter how small the income shortfall is and no matter how many non-poor episodes precede it. There is room for differing judgements on this point too. It might instead be argued that the fact that there is still poverty in period  $t$  means that, notwithstanding the mitigating effects of the preceding affluence, there remains a definite problem in period  $t$  that cannot be ameliorated completely. These issues are not unique to  $P_R$  and any measure belonging to the relative mitigation class of measures would be prone to the same criticisms.

To obtain the desired relaxation for the manner in which a poor period can be discounted if preceded by periods of affluence, we need to replace constant-relative poverty mitigation (Axiom 2.4) with an axiom in which there is an absolute mitigating effect.

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<sup>12</sup>A natural counter-argument of course is that the proportion of poverty which is mitigated remains the same, regardless of the poverty level. This possible criticism is somewhat reminiscent of the charge often made against relative inequality measures, which register no change in inequality when all incomes are increased by the same proportion. Those who regard absolute differences in income to be important with respect to inequality would reject such measures.

Before stating the axiom, let us define a function  $h : \mathbb{Z}_+ \rightarrow \mathbb{R}_+$ , such that  $h(1) = 0$ . The ‘absolute’ poverty mitigation axiom can be formally stated as follows.

**Axiom 2.7** *Absolute poverty mitigation holds if for all  $T \in \mathbb{N} \setminus \{1\}$  and all  $p \in (0, 1]$  we have, for any  $t \in \{1, \dots, T\}$ ,  $P(p\mathbf{e}_t^T) = \max(P(p\mathbf{e}_1^T) - h(t), 0)$ .*

Thus we allow for the possibility that the mitigation from affluent periods may be so high that poverty arising from a subsequent income shortfall can be completely mitigated. The condition  $h(1) = 0$  ensures that in the absence of any prior periods, there is no discounting.

Our final axiom is a monotonicity condition. It considers two poor profiles of length  $T$ , each with only one episode of poverty, of level  $p$ . In one profile the poor episode occurs in period  $t \leq T$  and in the other profile the poor episode takes place in period  $(t - 1) \geq 1$ . Note that the static level of poverty in the poor period is the same in each profile. The axiom stipulates that  $t - 1$  directly preceding periods of affluence have a greater mitigating impact than  $t - 2$  periods.

**Axiom 2.8** *Monotonic poverty mitigation holds if for all  $T \in \mathbb{N} \setminus \{1\}$  and all  $p \in (0, 1]$  we have  $P(p \cdot \mathbf{e}_{t-1}^T) \geq P(p \cdot \mathbf{e}_t^T)$  for any  $t \in \{2, \dots, T\}$ .*

The above two axioms along with single period equivalence (Axiom 2.1), time decomposability (Axiom 2.2), normalization (Axiom 2.3) and relative poverty intensification (Axiom 2.5), characterize a class of *absolute affluence-dependent intertemporal poverty* measures.

**Proposition 2.4** *An intertemporal poverty measure satisfies single period equivalence (Axiom 2.1), time decomposability (Axiom 2.2), normalization (Axiom 2.3), relative poverty intensification (Axiom 2.5), absolute poverty mitigation (Axiom 2.7), and monotonic poverty mitigation (Axiom 2.8) if and only if it is*

$$P_A(\mathbf{p}) = \frac{1}{T} \sum_{\substack{t=1 \\ p_t \neq 0}}^T \max(k_t^\alpha p_t^\theta - f(n_t), 0), \text{ where } \alpha, \theta \geq 0$$

and  $f : \mathbb{Z}_+ \rightarrow \mathbb{R}_+$  such that  $f(0) = 0$  and  $f(n_t + 1) \geq f(n_t)$ .<sup>13</sup>

## 2.5 Conclusions

In this chapter we have proposed and characterized new classes of individual-level intertemporal poverty measures. Our main objective was to account for the mitigating effect of affluent periods, as well as the debilitating impact of prolonged spells

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<sup>13</sup>Note that the summation in  $P_A$  is over only those periods where  $p_t \neq 0$ . This is a technical requirement since in our paper  $n_t$  (the number of immediately preceding affluent periods) is not defined when  $p_t = 0$ .

of poverty. We have proposed three broad classes of measures which, collectively, incorporate a broad range of views as to the impact of affluent periods on subsequent poor episodes. The first proposed class of measures,  $P_R$ , incorporates an intertemporal version of the focus axiom, as used by Foster (2009) and Bossert et al. (2012), where the mitigating impact of non-poor episodes is the same, irrespective of the level of affluence in those periods. We relax this stringent restriction in another class of measures,  $\tilde{P}_R$ , and allow for the level of income in non-poor periods to have a role in determining the amount of mitigating impact such periods can bring. Within the classes of measures proposed, we also allow for two broad views as to the way in which affluent periods mitigate the level of poverty. In  $P_R$ , the mitigating impact is ‘proportional’ to the level of poverty and, as such, can never ameliorate poverty fully. We also characterize a class of measures,  $P_A$ , where the mitigating impact is ‘absolute’ and thus has the potential to completely eradicate subsequent poverty.

Our measures build on the individual-level measures of Bossert et al. (2012), which also evaluate individual intertemporal poverty as a weighted average across time of snapshot poverty. The central innovation in this chapter lies in using both the number of preceding affluent periods and the number of consecutive poor periods to determine the weights. We allow for this to be conducted in a quite general way, so that different judgements regarding the precise composition of the weights can be accommodated. Our approach provides added sensitivity, allowing one to distinguish between poverty profiles which other measures in the literature are unable to. This enhanced sensitivity can be useful for policy purposes as it allows for a more diversified approach when it comes to allocation of resources. How precisely a redistribution of wealth or consumption should be implemented in an intertemporal framework, among the poor and also between the time periods, is an important issue to which a solution is not immediately apparent. This issue is left for future research.

## 2.A Appendix

### 2.A.1 Proof of Proposition 2.1

We concentrate on the “only if” part of the proof, as it is immediate to verify that  $P_R$  satisfies the axioms stated in Proposition 2.1.

Suppose that  $P$  satisfies single period equivalence (Axiom 2.1), time decomposability (Axiom 2.2), normalization (Axiom 2.3) constant-relative poverty mitigation (Axiom 2.4) and relative poverty intensification (Axiom 2.5). We need to show that for any time period  $T \in \mathbb{N}$  and any poverty profile  $\mathbf{p} \in [0, 1]^T$  we have  $P(\mathbf{p}) = P_R(\mathbf{p})$  for an exogenously determined  $\alpha, \beta, \theta \geq 0$ . So, take any  $T \in \mathbb{N}$  and any poverty profile  $\mathbf{p} \in [0, 1]^T$ .

Suppose  $T = 1$ . In this case single period equivalence holds and  $P(\mathbf{p}) = P(p) = P_R(p)$  for some  $p \in (0, 1]$ . Hence,  $P(\mathbf{p}) = P_R(\mathbf{p})$  follows.

Assume now that  $T > 1$ . If  $\mathbf{p} = \mathbf{0}^T$ , then by normalization we obtain  $P(\mathbf{p}) = P(\mathbf{0}^T) = 0 = P_R(\mathbf{0})$ . Thus,  $P(\mathbf{p}) = P_R(\mathbf{p})$  follows.

Next we proceed by induction on the number of poor periods, when  $\mathbf{p} \neq \mathbf{0}^T$ . Consider first the case where there is exactly one poor period. Recall that we represent a  $T$ -period poverty profile  $\mathbf{p} = (p_1, \dots, p_s, \dots, p_T)$ , where  $\forall t \neq s, p_t = 0$  and  $p_s = 1$  as  $\mathbf{e}_s^T$ . Thus  $\mathbf{p} = p \cdot \mathbf{e}_s^T$  would stand for a  $T$ -period poverty profile  $\mathbf{p} = (p_1, \dots, p_s, \dots, p_T)$ , where  $\forall t \neq s, p_t = 0$  and  $p_s = p$ .

Without loss of generality, we can write  $\mathbf{p} = p \cdot \mathbf{e}_t^T$  where  $p > 0$  and  $t \in \{1, \dots, T\}$ . Thus  $P(\mathbf{p}) = P(p \cdot \mathbf{e}_t^T)$ . Then

$$\begin{aligned}
 P(p \cdot \mathbf{e}_t^T) &= \frac{t}{T}P(p \cdot \mathbf{e}_t^t) + \frac{T-t}{T}P(\mathbf{0}^{T-t}), \text{ by time decomposability,} \\
 &= \frac{t}{T}P(p \cdot \mathbf{e}_t^t), \text{ by normalization,} \\
 &= \frac{t}{T} \frac{P(p \cdot \mathbf{e}_1^t)}{t^\beta}, \text{ by constant-relative poverty mitigation,} \\
 &= \frac{1}{Tt^{\beta-1}} \cdot \left[ \frac{1}{t}P(p) + \frac{t-1}{t}P(\mathbf{0}^{t-1}) \right], \text{ by time decomposability,} \\
 &= \frac{P(p)}{Tt^\beta}, \text{ by normalization,} \\
 &= \frac{p^\theta}{Tt^\beta}, \text{ by single period equivalence.} \tag{2.2}
 \end{aligned}$$

Since  $k_t = 1$ ,  $t = (1 + n_t)$  and  $\forall i \neq t, p_i = 0$  we can write (2.2) as

$$P(\mathbf{p}) = P(p \cdot \mathbf{e}_t^T) = \frac{1}{T} \sum_{i=1}^T \frac{k_t^\alpha p_i^\theta}{(1 + n_t)^\beta} = P_R(\mathbf{p}).$$

Thus we obtain  $P(\mathbf{p}) = P_R(\mathbf{p})$ .

Suppose now that  $P(\hat{\mathbf{p}}) = P_R(\hat{\mathbf{p}})$  whenever  $\hat{\mathbf{p}}$  contains  $m$  poor periods, for some  $m \in \{1, \dots, T-1\}$ . Let  $\mathbf{p} \in [0, 1]^T$  be any poverty profile, such that the number

of poor periods is  $m + 1$ . Let  $t \in \{2, \dots, T\}$  be such that the final poor period is period  $t$ . Thus  $t = \max\{s : 2 \leq s \leq T, p_s > 0\}$ . From time decomposability we derive

$$\begin{aligned} P(\mathbf{p}) &= P(p_1, \dots, p_t, \dots, p_T) \\ &= \frac{t}{T}P(p_1, \dots, p_t) + \frac{T-t}{T}P(p_{t+1}, \dots, p_T). \end{aligned}$$

Now  $t$  being the final poor period means that by normalization we get

$$P(\mathbf{p}) = \frac{t}{T}P(p_1, \dots, p_t). \quad (2.3)$$

Let  $s \neq t$  be maximal with  $p_s > 0$ . So  $s$  is the last poor period prior to  $t$ . Suppose  $s \neq t - 1$ . Then by time decomposability we obtain

$$P(p_1, \dots, p_s, \dots, p_t) = \frac{s}{t}P(p_1, \dots, p_s) + \frac{t-s}{t}P(p_{s+1}, \dots, p_t). \quad (2.4)$$

Further,  $P(p_{s+1}, \dots, p_t) = P(p_t \cdot \mathbf{e}_{t-s}^{t-s})$  since  $p_i = 0$  for all  $i \in \{s+1, \dots, t-1\}$ . Applying single period equivalence, time decomposability, normalization and constant-relative poverty mitigation and noting that  $k_t = 1$  and  $n_t = t - s - 1$ , we obtain

$$P(p_{s+1}, \dots, p_t) = \frac{k_t^\alpha}{(1+n_t)^{1+\beta}} p_t^\theta. \quad (2.5)$$

Now consider the case when  $s = t - 1$ . Then using relative poverty intensification we get

$$P(p_1, \dots, p_s, \dots, p_t) = P(p_1, \dots, p_s, 0) + k_t^\alpha P(p_t \mathbf{e}_1^t). \quad (2.6)$$

Using time decomposability, single period equivalence and noting that  $n_t = 0$ , (2.6) can be written as

$$P(p_1, \dots, p_s, \dots, p_t) = \frac{s}{t}P(p_1, \dots, p_s) + \frac{k_t^\alpha}{(1+n_t)^\beta} \frac{p_t^\theta}{t} \quad (2.7)$$

Now  $(p_1, \dots, p_s)$  contains  $m$  poor periods. Thus, by the induction hypothesis, we have

$$P(p_1, \dots, p_s) = P_R(p_1, \dots, p_s) = \frac{1}{s} \sum_{i=1}^s w_i p_i^\theta \text{ where } w_i = \frac{k_i^\alpha}{(1+n_i)^\beta}. \quad (2.8)$$

Substituting (2.8) and (2.5) into (2.4) (for the  $s \neq t - 1$  case) or substituting (2.8) in (2.7) (for the  $s = t - 1$  case) yields

$$P(p_1, \dots, p_s, \dots, p_t) = \left[ \frac{1}{t} \sum_{i=1}^s \frac{k_i^\alpha p_i^\theta}{(1+n_i)^\beta} \right] + \frac{k_t^\alpha}{(1+n_t)^\beta} \frac{p_t^\theta}{t}. \quad (2.9)$$

Further, substituting (2.9) into (2.3) we obtain

$$P(\mathbf{p}) = \frac{1}{T} \left[ \left( \sum_{i=1}^s \frac{k_i^\alpha p_i^\theta}{(1+n_i)^\beta} \right) + \frac{k_t^\alpha p_t^\theta}{(1+n_t)^\beta} \right].$$

Finally, since  $p_i = 0$  for all  $i \in \{s+1, \dots, t-1\}$  and all  $i \in \{t+1, \dots, T\}$ , we have

$$P(\mathbf{p}) = \frac{1}{T} \sum_{i=1}^T \frac{k_i^\alpha p_i^\theta}{(1+n_i)^\beta} = P_R(\mathbf{p}).$$

This concludes the proof for the case of  $m+1$  poor periods, and by induction it follows that  $P(\mathbf{p}) = P_R(\mathbf{p})$  for any poverty profile  $\mathbf{p}$ . This completes the proof of Proposition 2.1.  $\square$

## 2.A.2 Proof of Proposition 2.2

We demonstrate that axioms single period equivalence (Axiom 2.1), time decomposability (Axiom 2.2), normalization (Axiom 2.3), constant-relative poverty mitigation (Axiom 2.4) and relative poverty intensification (Axiom 2.5) are independent by presenting a separate poverty measure that satisfies all the axioms except one. We do this one at a time for each of the five axioms.

Consider a poverty profile  $\mathbf{p} \in [0, 1]^T$ . Then the following measure violates single period equivalence (Axiom 2.1) but satisfies the other axioms,

$$P_1(\mathbf{p}) = \frac{1}{T} \sum_{i=1}^T \frac{k_i^\alpha 2p_i}{(1+n_i)^\beta}.$$

The next measure violates time decomposability (Axiom 2.2) but satisfies the other axioms.

$$P_2(\mathbf{p}) = \sum_{i=1}^T \frac{k_i^\alpha p_i^\theta}{(1+n_i)^\beta}.$$

A measure which violates normalization (Axiom 2.3) but satisfies the other axioms is given by

$$P_3(\mathbf{p}) = \begin{cases} 10 & \text{if } \mathbf{p} \in \mathbf{0}^T \\ \frac{1}{T} \sum_{i=1}^T \frac{k_i^\alpha p_i^\theta}{(1+n_i)^\beta} & \text{otherwise} \end{cases}.$$

A measure which violates constant-relative poverty mitigation (Axiom 2.4) but satisfies the others is

$$P_4(\mathbf{p}) = \frac{1}{T} \sum_{i=1}^T \frac{k_i^\alpha p_i^\theta}{\ln(1+n_i)}.$$

A measure which violates relative poverty intensification (Axiom 2.5) and satisfies



the rest is as follows

$$P_5(\mathbf{p}) = \frac{1}{T} \sum_{i=1}^T \frac{p_i^\theta}{(1+n_i)^\beta}.$$

□

### 2.A.3 Proof of Proposition 2.3

The proof is similar to that of Proposition 2.1 and is omitted. □

### 2.A.4 Proof of Proposition 2.4

We concentrate on the “only if” part of the proof, as it is immediate to verify that  $P_A$  satisfies the axioms stated in Proposition 2.4.

Suppose that  $P$  satisfies single period equivalence (Axiom 2.1), time decomposability (Axiom 2.2), normalization (Axiom 2.3), relative poverty intensification (Axiom 2.5), absolute poverty mitigation (Axiom 2.7), and monotonic poverty mitigation (Axiom 2.8). We need to show that for any time period  $T \in \mathbb{N}$  and any poverty profile  $\mathbf{p} \in [0, 1]^T$  we have  $P(\mathbf{p}) = P_A(\mathbf{p})$  for a monotonically increasing function  $f : \mathbb{Z}_+ \rightarrow \mathbb{R}_+$  such that  $f(0) = 0$  and  $f(n_t + 1) \geq f(n_t)$ .

Take any  $T \in \mathbb{N}$  and any poverty profile  $\mathbf{p} \in [0, 1]^T$ . Suppose  $T = 1$ . Note that  $k_1 = 1$  and  $n_1 = 0$ . By absolute poverty mitigation, we have

$$P(p\mathbf{e}_1^1) = \max(P(p\mathbf{e}_1^1) - h(1), 0). \quad (2.10)$$

By construction let  $f(n_t) = th(t)$ ,  $t = n_t + 1$ . Given  $h(t) \in \mathbb{R}_+$  and  $t \in \{1, \dots, T\}$ ,  $f : \mathbb{Z}_+ \rightarrow \mathbb{R}_+$ . When  $t = 1$ , it implies  $f(0) = h(1) = 0$ . Due to single period equivalence and  $k_1 = 1$  we can write equation (2.10) as

$$P(\mathbf{p}) = \max(k_1^\alpha p^\theta, 0) = P_A(\mathbf{p}).$$

Assume now that  $T > 1$ . If  $\mathbf{p} = \mathbf{0}^T$ , then by normalization we obtain  $P(\mathbf{p}) = P(\mathbf{0}^T) = 0 = P_A(\mathbf{p})$ . Thus,  $P(\mathbf{p}) = P_A(\mathbf{p})$ .

Next, we proceed by induction on the number of poor periods when  $\mathbf{p} \neq \mathbf{0}^T$ . Consider the case where there is exactly one poor period. Without loss of generality, we can write  $\mathbf{p} = p \cdot \mathbf{e}_t^T$  where  $p \in (0, 1]$  and  $t \in \{1, \dots, T\}$ . Thus  $P(\mathbf{p}) = P(p \cdot \mathbf{e}_t^T)$ . If  $t = 1$  then, applying single period equivalence, time decomposability and normalization, and noting that  $k_1 = 1$  and  $f(0) = 0$ , yields

$$\begin{aligned} P(p \cdot \mathbf{e}_1^T) &= \frac{1}{T} p^\theta, \\ &= \frac{1}{T} \max(k_1^\alpha p^\theta - f(0), 0). \end{aligned}$$

For  $t > 1$  applying time decomposability and normalization, we obtain

$$\begin{aligned}
P(p \cdot \mathbf{e}_t^T) &= \frac{t}{T} P(p \cdot \mathbf{e}_t^t). \\
&= \frac{t}{T} \max(P(p \cdot \mathbf{e}_1^t) - h(t), 0), \text{ by absolute poverty mitigation,} \\
&= \frac{1}{T} \max(t(P(p \cdot \mathbf{e}_1^t) - h(t)), 0) \\
&= \frac{1}{T} \max\left(t\left(\frac{1}{t}P(p) - h(t)\right), 0\right), \text{ by time decomposability,} \\
&= \frac{1}{T} \max((P(p) - f(n_t)), 0), \tag{2.11}
\end{aligned}$$

where  $f(n_t) = th(t)$ , by construction.

To show that  $f(n_t)$  is monotonic, consider another profile  $P(p \cdot \mathbf{e}_{t-1}^T)$ . Applying (2.11) we get

$$P(p \cdot \mathbf{e}_{t-1}^T) = \frac{1}{T} \max(P(p) - f(n_{t-1}), 0). \tag{2.12}$$

By monotonic poverty mitigation it must be the case that  $P(p \cdot \mathbf{e}_{t-1}^T) \geq P(p \cdot \mathbf{e}_t^T)$ . Comparing (2.11) and (2.12) and noting that  $n_t = n_{t-1} + 1$ , we can show  $f(n_t) \geq f(n_{t-1})$ . Applying single period equivalence in (2.11) and noting that  $k_t = 1$ , we can show

$$P(p \cdot \mathbf{e}_t^T) = \frac{1}{T} \max(k_t^\alpha p_t^\theta - f(n_t), 0). \tag{2.13}$$

Since  $p_i = 0$  for all  $i \neq t$ , we can write (2.13) as

$$P(\mathbf{p}) = P(p \cdot \mathbf{e}_t^T) = \frac{1}{T} \sum_{\substack{i=1 \\ p_i \neq 0}}^T \max(k_i^\alpha p_i^\theta - f(n_i), 0) = P_A(\mathbf{p}). \tag{2.14}$$

Suppose now that  $P(\hat{\mathbf{p}}) = P_A(\hat{\mathbf{p}})$  whenever  $\hat{\mathbf{p}}$  contains  $m$  poor periods, for some  $m \in \{1, \dots, T-1\}$ . Let  $\mathbf{p} \in [0, 1]^T$  be any poverty profile, such that the number of poor periods is  $m+1$ . Let  $t \in \{2, \dots, T\}$  be such that the final poor period is period  $t$ . Thus  $t = \max\{s : 2 \leq s \leq T, p_s > 0\}$ . From time decomposability we derive

$$\begin{aligned}
P(\mathbf{p}) &= P(p_1, \dots, p_t, \dots, p_T), \\
&= \frac{t}{T} P(p_1, \dots, p_t) + \frac{T-t}{T} P(p_{t+1}, \dots, p_T).
\end{aligned}$$

Now  $t$  being the final poor period means that by normalization we get

$$P(\mathbf{p}) = \frac{t}{T} P(p_1, \dots, p_t). \tag{2.15}$$

Let  $s \neq t$  be maximal with  $p_s > 0$ . So  $s$  is the last poor period prior to  $t$ . Suppose

$s \neq t - 1$ . Then by time decomposability we obtain

$$P(p_1, \dots, p_s, \dots, p_t) = \frac{s}{t} P(p_1, \dots, p_s) + \frac{t-s}{t} P(p_{s+1}, \dots, p_t). \quad (2.16)$$

Further,  $P(p_{s+1}, \dots, p_t) = P(p_t \cdot \mathbf{e}_{t-s}^{t-s})$  since  $p_i = 0$  for all  $i \in \{s+1, \dots, t-1\}$ . Using (2.14) and noting that  $k_t = 1$  and  $n_t = t - s - 1$ , we obtain

$$P(p_{s+1}, \dots, p_t) = \frac{1}{t-s} \sum_{\substack{i=s+1 \\ p_i \neq 0}}^t \max(k_i^\alpha p_i^\theta - f(n_i), 0). \quad (2.17)$$

Substituting (2.17) into (2.16) we get

$$P(p_1, \dots, p_s, \dots, p_t) = \frac{s}{t} P(p_1, \dots, p_s) + \frac{1}{t} \sum_{\substack{i=s+1 \\ p_i \neq 0}}^t \max(k_i^\alpha p_i^\theta - f(n_i), 0). \quad (2.18)$$

Now consider the case when  $s = t - 1$ . Using relative poverty intensification we get

$$P(p_1, \dots, p_s, p_t) = P(p_1, \dots, p_s, 0) + k_t^\alpha P(p_t \cdot \mathbf{e}_1^t). \quad (2.19)$$

Applying time decomposability, single period equivalence and using (2.11) we can write (2.19) as

$$\begin{aligned} P(p_1, \dots, p_s, p_t) &= \frac{s}{t} P(p_1, \dots, p_s) + \frac{k_t^\alpha}{t} \max(p_t^\theta - f(n_1), 0), \\ &= \frac{s}{t} P(p_1, \dots, p_s) + \frac{1}{t} \max(k_t^\alpha p_t^\theta - f(n_1), 0). \end{aligned} \quad (2.20)$$

Note that in (2.20)  $f(n_1) = 0$ , since  $n_1 = 0$  by definition. Now  $(p_1, \dots, p_s)$  contains  $m$  poor periods. By the induction hypothesis, we have

$$P(p_1, \dots, p_s) = P_A(p_1, \dots, p_s) = \frac{1}{s} \sum_{\substack{i=1 \\ p_i \neq 0}}^s \max(k_i^\alpha p_i^\theta - f(n_i), 0). \quad (2.21)$$

Substituting (2.21) into (2.18) (for the  $s \neq t - 1$  case) or substituting (2.21) into (2.20) (for the  $s = t - 1$  case) yields

$$P(p_1, \dots, p_s, \dots, p_t) = \frac{1}{t} \sum_{\substack{i=1 \\ p_i \neq 0}}^s \max(k_i^\alpha p_i^\theta - f(n_i), 0) + \frac{1}{t} \sum_{\substack{i=s+1 \\ p_i \neq 0}}^t \max(k_i^\alpha p_i^\theta - f(n_i), 0). \quad (2.22)$$

Further, substituting (2.22) into (2.15) we obtain

$$P(\mathbf{p}) = \frac{1}{T} \left[ \sum_{\substack{i=1 \\ p_i \neq 0}}^s \max(k_i^\alpha p_i^\theta - f(n_i), 0) + \sum_{\substack{i=s+1 \\ p_i \neq 0}}^t \max(k_i^\alpha p_i^\theta - f(n_i), 0) \right].$$

Finally, since  $p_i = 0$  for all  $i \in \{t+1, \dots, T\}$ , we have

$$P(\mathbf{p}) = \frac{1}{T} \sum_{\substack{t=1 \\ p_t \neq 0}}^T \max(k_t^\alpha p_t^\theta - f(n_t), 0) = P_A(\mathbf{p}).$$

This concludes the proof for the case of  $m+1$  poor periods, and by induction it follows that  $P(\mathbf{p}) = P_A(\mathbf{p})$  for any poverty profile  $\mathbf{p}$ . It therefore follows that  $P = P_A$ , which completes the proof of Proposition 2.4.  $\square$

# Chapter 3

## Intertemporal Poverty in Great Britain

### 3.1 Introduction

As discussed in Chapter 2, in the recent literature on poverty measurement, there has been a significant emphasis on developing indices designed to capture dynamic aspects of poverty, where income data is available for a number of time periods. This new approach has enabled the construction of poverty indices which are sensitive to a number of important aspects of poverty that cannot be captured by static measures. These aspects include, for example, the particularly damaging impact of poverty early in life (Hoy and Zheng (2011), Zheng (2011)), the detrimental impact of spending a high proportion of one's time in poverty (Foster (2009)), the debilitating impact of prolonged periods spent in poverty (Bossert et al. (2012)) and the mitigating impact that affluent spells might have on subsequent periods of poverty (Chapter 2, Zheng (2011)). The various measures that have been proposed differ in the underlying assumptions that are made regarding how the time dimension should be dealt with and, in some cases, these assumptions have been made explicit through the provision of axiomatic characterizations.<sup>1</sup>

In this chapter, the measures introduced in Chapter 2 and, as a special case of these, those of Foster (2009), are applied to analyse intertemporal poverty and its determinants in Great Britain during the period 1991-2005, using data from the British Household Panel Survey (BHPS).<sup>2</sup> A number of empirical studies on poverty dynamics have been undertaken using this data-set and are discussed below. However, most of these studies pre-date the recent advances that have been made

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<sup>1</sup>Axiomatic characterizations have been provided for the measures proposed in Chapter 2 and for those of Bossert et al. (2012), Hoy and Zheng (2011) and Zheng (2011). Measures have also been proposed by Jalan and Ravallion (2000), Cruces (2005), Calvo and Dercon (2007), Grab and Grimm (2007), Carter and Ikegami (2007), Foster (2009), Porter and Quinn (2008), Foster and Santos (2009), Gradín, del Río and Cantó (2011) and Duclos, Araar and Giles (2010).

<sup>2</sup>Throughout this chapter, as in Chapter 2, we are referring to Foster (2009)'s total intertemporal poverty measure, not his chronic poverty measure.

in measuring intertemporal poverty and have typically been conducted using static poverty measures for each time period. As such, they are able to consider movements into and out of poverty and the determinants of such movements, where poverty is measured over a relatively short term. Evaluating overall levels of poverty, and its determinants, across a longer time-frame requires a new set of tools.

As discussed by Jenkins and Rigg (2001), previous research on poverty dynamics using the BHPS has to date focused broadly on three main aspects. Firstly, a number of papers have studied the extent of movement into and out of poverty from one year to the next. These papers have generally found a significant amount of such movement. For example, Jarvis and Jenkins (1995, 1997) found that roughly half of those who are poor in any given year are non-poor in the following year. Nevertheless, significant numbers of individuals have been found to be stuck in poverty for a number of consecutive years - see Department of Social Security (2000) and Jarvis and Jenkins (1997). A second strand of literature studies the determinants of movements into and out of poverty, from one year to the next. Jenkins (2000), for example, found that while changes in income were the primary route to escaping poverty, falling into poverty was often a result of changes in household demographics. Jenkins and Rigg (2001) also emphasised the importance of the labour market for providing a route to escape poverty for individuals of working age.<sup>3</sup> A third area of research has been on attempting to model the lengths of poor spells and non-poor spells. Antolín et al (1999), Devicienti (2001, 2002) and Jenkins and Rigg (2001) have found that after controlling for differences in personal characteristics, the longer an individual has been poor for, the less likely is a subsequent escape from poverty. Antolín et al (1999) and Devicienti (2001, 2002) found that certain demographics, notably individuals who live in households with single parents or pensioners, are particularly likely to spend long periods in poverty. Jenkins and Rigg (2001)'s analysis found that shorter poverty spells were associated with having more working individuals in the household, while longer spells (and shorter recovery times) were associated with the presence of children in the household. More recently, and using a longer panel of the BHPS (Waves 1-16, which corresponds to 1991-2006), Devicienti (2011) has found a number of correlates of long spells in poverty. Living in a household with relatively many children and few adults, living in a household headed by a female with a low level of education and travelling to work in areas with high local unemployment rates are all associated with prolonged spells in poverty. Devicienti (2011) also found that young and elderly individuals face a relatively high risk of remaining poor for long periods, as do those from an ethnic minority group.

All of the literature above is concerned with dynamic aspects of poverty and has provided valuable insights into the patterns and determinants of movements into

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<sup>3</sup>See also Antonín et al. (1999).

and out of poverty in Great Britain. However, to the best of our knowledge, none of the literature to date has explicitly accounted for the dynamic nature of poverty in its actual measurement using BHPS data. This chapter seeks to begin to fill this gap. Using the measures proposed in Chapter 2, we aim to provide a richer ordering of poverty profiles in Great Britain, explicitly taking account of the time dimension.

Having estimated the level of intertemporal poverty in Great Britain, the focus of the chapter then shifts to attempting to model the determinants of intertemporal poverty. Various econometric approaches have been used in the literature to model determinants of poverty. Most research has been in a static framework, where the dependent variable has typically been a standard ‘snapshot’ poverty measure defined on the  $[0,1]$  interval, such as a member of the popular ‘FGT’ class of measures introduced by Foster et al. (1984). A common approach has been to perform a Tobit regression, where the poverty measure is treated as a variable which is left-censored at zero, observed only if an individual’s income is below the poverty line. This method has been adopted, for example, by Bhaumik, Gang and Yun (2006) and Walker et al. (2006). Appleton (2001), in a study using household survey data from Uganda, also performed Tobit regressions to model the determinants of poverty but took a rather different approach. In that paper, the dependent variable in the poverty regressions was the logarithm of real consumption, the variable being right censored at the median income.<sup>4</sup>

As Jalan and Ravallion (2000) have discussed, in a study on chronic and transient poverty in rural China, applying Tobit models in the context of poverty regressions can be problematic. Citing Arabmazar and Schmidt (1982), they pointed out that Tobit estimates are not robust to misspecifications of the error distribution. In particular, if there is heteroskedasticity or non-normality in the errors, the estimates will be both inconsistent and inefficient. Jalan and Ravallion (2000) preferred instead to use a semi-parametric method, employing the Censored Quantile Regression model (Powell (1984, 1986)). In a study on chronic and transient seasonal poverty in Rwanda, Muller (2003) considered a Tobit specification but found the error terms to be both heteroskedastic and non-normal. As a consequence, Muller (2003) also rejected the Tobit model in favour of Censored Quantile Regressions and noted that non-normality and heteroskedasticity in the errors is usually to be expected in the context of poverty regressions.

Both Jalan and Ravallion (2000) and Muller (2003) are concerned with whether chronic poverty and transient poverty have fundamentally different determinants. Rather than focusing on decomposing intertemporal poverty into chronic and transient components, most of the recent contributions to the theoretical literature on

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<sup>4</sup>This approach is possible in a static poverty setting, where poverty can be defined as a simple function of income. With intertemporal measures, this is usually not the case. As will be clear from Chapter 2, such measures typically depend on incomes in a number of periods and on the sequencing of those incomes.

poverty measurement have concentrated on capturing the overall severity of intertemporal poverty. This was the focus in Chapter 2 and also in Bossert et al. (2012), Hoy and Zheng (2011) and Zheng (2011), among others.<sup>5</sup> Consistent with this recent approach to measurement, we attempt to analyse the determinants of the overall severity of an individual's intertemporal poverty. Nevertheless, we take seriously the possibility that the phenomenon of having a non-zero level of intertemporal poverty might be determined by factors which differ somewhat from those which determine the overall degree of intertemporal poverty. This suggests that the determinants of intertemporal poverty should be obtained by modelling the level of intertemporal poverty conditional on being intertemporally poor.<sup>6</sup> Ignoring this possibility would run the risk that there might be a type of 'selection bias.' Our approach is to use a Heckman two-step selection model (Heckman (1979)). Heckman selection models have been used in a number of studies on the determinants of poverty but, to the best of our knowledge, not in quite the manner employed here, where the severity of individuals' poverty is modelled conditional on their being poor.

Our approach bears some resemblance to that used by Coulombe and McKay (1996) in a study on the determinants of poverty in Mauritania. They modelled the socioeconomic group to which individuals belonged to and, conditional on that choice, the determinants of living standards (and hence poverty) within that group. A multinomial logit selection model was used in the first stage to capture the determinants of choosing a particular socioeconomic group. The determinants of living standards were then modelled, conditional on this choice, by including a Heckman-like selection term in an OLS regression of living standards for that group. Related approaches to that employed in this chapter are also sometimes used to evaluate the impact of various programmes on poverty, if it is suspected that there may be a sample selection bias associated with access to the programme. For example, in a recent paper, Imai, Arun and Annim (2010) studied the impact of microfinance in India and used a Heckman sample selection model to account for possible sample selection bias or endogeneity associated with household access to microfinance institutions.

The rest of the chapter is organised as follows. In Section 3.2, we describe the data and apply a class of measures introduced in Chapter 2 and, as a special case of these, those of Foster (2009), to evaluate the overall levels of intertemporal poverty

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<sup>5</sup>As indicated previously, Foster (2009) is a notable exception.

<sup>6</sup>Note that throughout this chapter, an individual is regarded as being 'intertemporally poor' if they have a non-zero level of intertemporal poverty. Moreover, being 'poor' during some given time spell, and being 'intertemporally poor' during that same time spell are taken to mean the same thing. This reflects the fact that the intertemporal poverty measures used are weighted averages of the per-period static poverty measures. Therefore, an individual has a non-zero level of intertemporal poverty in a given spell if and only if he has a non-zero level of static poverty in at least one of the time periods of which the spell is composed. To put it another way, an individual is poor, or intertemporally poor, if he is poor during any time period.



in Great Britain, using data from the BHPS. In Section 3.3, econometric techniques are used to model the determinants of intertemporal poverty. Concluding remarks are offered in Section 3.4.

## 3.2 Intertemporal Poverty in Great Britain during 1991-2005

### 3.2.1 Intertemporal Poverty Measures

We begin this section by providing a brief recap of one of the classes of measures discussed in Chapter 2 and specifying the particular parameters which are used in this empirical application. In Chapter 2, we proposed a class of measures which take into account both the poverty mitigation arising from the presence of affluent periods and the intensification of poverty due to consecutive poor periods. Recall that our *constant-relative affluence-dependent intertemporal poverty measure*  $P_R$  is defined as

$$P_R(\mathbf{p}) = \frac{1}{T} \sum_{t=1}^T \frac{k_t^\alpha}{(1+n_t)^\beta} p_t^\theta, \text{ where } \alpha, \beta, \theta \geq 0. \quad (3.1)$$

In general,  $p_t$  can be any static poverty measure from the literature, but for the purposes of this chapter we will use the popular normalized poverty gap. If  $x_t \geq z_t$ , the individual is non-poor and  $p_t = 0$ . This static measure of an individual's poverty has some appealing properties. It is decreasing in  $x_t$  and is scale invariant since for any  $\lambda \neq 0$ ,  $(\lambda z_t - \lambda x_t)/\lambda z_t = (z_t - x_t)/z_t$ . It also has a money-metric interpretation. When denormalized it can be interpreted as the minimum cost to society of removing the individual from poverty.

Recall that the parameter  $\theta$  captures the degree of sensitivity of the poverty experienced in each time period to the income shortfall. The detrimental impact of consecutive periods of poverty, which serve to intensify the overall impact of poverty, is captured by  $k_t$ . The parameter  $\alpha$  determines the extent of this intensification of poverty. If  $\alpha = 0$ , there is no intensification. Similarly,  $\beta$  can be interpreted as an index representing how much one chooses to discount the impact of an individual's poor episodes according to preceding uninterrupted spells of non-poverty. When  $\beta = 0$ , there is no mitigation. As noted in Chapter 2, if both  $\alpha = 0$  and  $\beta = 0$ , the measure reduces to the simple average of static poverty measures advocated by Foster (2009) and defined as

$$P_F(\mathbf{p}) = \frac{1}{T} \sum_{t=1}^T p_t^\theta, \text{ where } \theta \geq 0. \quad (3.2)$$

For the remainder of this study, we will set the parameters for  $P_R$  to be  $\alpha = \beta =$

$\theta = 1$ . From here on, we will refer to  $P_R$ , with these particular parameters, as being  $P_{DRZ}$ . By way of comparison, we will also provide results using the  $P_R$  measures with  $\alpha = \beta = 0$  and  $\theta = 1$ , or, equivalently, the measures of Foster (2009) with  $\theta = 1$ . From here on, we will refer to the latter measures as  $P_{FOS}$ . We might very readily have also used a number of other measures from the literature to provide further comparisons with  $P_{DRZ}$ , such as, for example, those of Bossert et al. (2012) or Hoy and Zheng (2011). However, an exhaustive study, using all the attractive measures from the literature, is beyond the scope of this chapter. Rather, our intention is to demonstrate the kind of analysis that can be performed using intertemporal poverty measures generally. We illustrate this using a class of measures proposed in Chapter 2 which, as we have noted, have some attractive properties. The  $P_{FOS}$  measure represents an interesting special case of these measures, and is useful for comparative purposes with  $P_{DRZ}$ , since it embodies very different normative judgements. In contrast to  $P_{DRZ}$ , no specific importance is attached to the precise ordering of poor periods or non-poor periods; all that matters is the level of static poverty in each time period and the proportion of periods which are poor.

The main focus in Chapter 2 was on evaluating intertemporal poverty at an individual level. As noted in Chapter 2, subsequently measures of societal intertemporal poverty can be constructed by aggregating across individuals. In this chapter, the focus is also mainly on intertemporal poverty at an individual-level. We do, however, make reference to aggregate intertemporal poverty at a regional level. In this case, the aggregate level of intertemporal poverty is evaluated as a simple average of individual-level intertemporal poverty.

### 3.2.2 Data and Measures

The data are derived from the BHPS, Waves 1 to 15, which cover the period 1991 to 2005.<sup>7</sup> The BHPS was designed as an annual survey of each adult member (aged 16 years and over) of a nationally representative sample of over 5,000 households. The Wave 1 panel consists of 5,500 households and 10,300 individuals drawn from throughout Great Britain. The same individuals were re-interviewed in successive waves. If and when an individual left their original household, all adult members of their new households were also interviewed. Individuals were re-interviewed at approximately annual intervals. In 1999, for Wave 9 onwards, the main sample was supplemented with additional samples of 1,500 households in each of Scotland and Wales. Few panels have individual-level income data at such regular time intervals and over such a long time-frame. This makes the BHPS a particularly suitable data-set for an intertemporal poverty study, the data requirements of which are

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<sup>7</sup>The period studied in each wave started on 1st September and finished on 31st August. So, for example, Wave 1 covers the period from 1st September 1990 to 31st August 1991.

quite demanding.<sup>8</sup>

An individual's income is taken to be their equivalised household net income. The household net income variable used was provided by Bardasi et al. (2008) as an unofficial supplement to the set of derived income variables in the official BHPS release (which provide gross income rather than net income). The variable (referred to in the data-set as 'whhnetde2') was obtained by summing across all household members, cash income from all sources (except for any earnings from a second job) and deducting direct taxes (except for local taxes such as the community charge and council tax) and occupational pension contributions. The variable uses the Modified OECD equivalence scale to adjust for differences in household size and composition, and a monthly 'before housing costs' price index to express incomes in January 2008 prices.<sup>910</sup>

As highlighted in the introduction, the purpose of this study is two-fold. Firstly, by using measures designed to evaluate poverty over a longer time-frame than static measures, we aim to provide a more nuanced view of poverty in Great Britain than previous studies have been able to do. Secondly, having evaluated intertemporal poverty at an individual-level, we seek to analyse its determinants. This two-fold task presents considerable challenges from a practical point of view. For example, suppose we begin with the premise that the starting point must be to obtain a definitive measure of intertemporal poverty, across the full fifteen years of analysis. An immediate drawback is that this necessarily dramatically reduces the size of the sample, since, as discussed above, income observations in every time period are required for computation of the intertemporal poverty measures employed. Another serious difficulty is the following. Many likely determinants of poverty, such as an individual's age, employment status, number of children in household and in many cases even their level of education, can change dramatically over such long periods of time. Moreover, while it may be appropriate to treat such possible determinants of poverty over a relatively short time-frame as being exogenous factors, it may be increasingly difficult to maintain this assumption as the time-frame increases.<sup>11</sup> The approach taken in this chapter is something of a compromise between our desire to

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<sup>8</sup>It is clear from the definitions of the intertemporal poverty measures referred to in the previous subsection that it is highly desirable to have income observations in each time period. If an individual's income data is missing for some time periods,  $P_{FOS}$  can still be estimated because each time period receives an equal weight. However, missing values for most of the other measures in the literature, including  $P_{DRZ}$ , are more problematic because the weights assigned to certain time periods will be dependent on the missing information.

<sup>9</sup>The variable was constructed using the same definition of net income as that used in Britain's official income distribution statistics, as published annually in the Households Below Average Income from the Department for Work and Pensions (formerly the Department of Social Security). See for example Department for Work and Pensions (2008).

<sup>10</sup>For further information on the data-set and the construction of the 'whhnetde2' variable, see Bardasi, Jenkins and Rigg (1999) and Levy and Jenkins (2008). For more detailed information on the main BHPS data-set, see Taylor et al. (2010). Details on how to order the data, and an order form, can be obtained at <http://www.data-archive.ac.uk>.

<sup>11</sup>See Rodgers (1989) for a discussion of such issues.

measure poverty over a long time-frame and being able to successfully evaluate its possible determinants. We use the measures  $P_{DRZ}$  and  $P_{FOS}$  to provide an indication of intertemporal poverty over 5-year stretches, which we loosely refer to as ‘eras.’

Our study is therefore divided into three sections or eras, corresponding to Waves 1-5, Waves 6-10 and Waves 11-15 and we evaluate both intertemporal poverty and its determinants separately for these three eras.

After losing individuals due to attrition and non-response, we are left with panels of the following sizes in the three eras. In Waves 1-5, there are 5,968 individuals of which 2,904 are male and 3,064 are female. In Waves 6-10, there are 6,386 individuals of which 3,134 are male and 3,252 are female. In Waves 11-15, there are 8,341 individuals of which 4,075 are male and 4,266 are female.<sup>12</sup>

We recognize that the loss of individuals due to attrition and other types of non-response are likely to bias our results in ways that are difficult to predict. This is a common problem in studies using panel data and there is a large literature on the subject but not, unfortunately, a comprehensive solution. In a study on the nature and causes of attrition in the BHPS, Uhrig (2008) found that there was no impact of being at the low end of the income distribution on non-response generally, but a slightly increased chance of being unable to contact individuals and a slightly decreased chance of refusal. Uhrig (2008, p. 39) concluded from this that “...low income respondents in Britain are happy to participate in an ongoing survey in which income and financial well-being are central themes but can be somewhat difficult to maintain in the sample.” If this is the case, the poverty estimates in this study might be expected to have a slight downward bias.

The measures  $P_{DRZ}$  and  $P_{FOS}$  allow for the poverty line to change in each time period. The measures are computed by estimating the official poverty line, of 60% of the median equivalised household net income (defined as above), in each time period. The poverty lines used in each year, expressed as annual equivalised household net incomes, are displayed in Table 3.1. As this table indicates, apart from a slight dip in the late 1990s, the official poverty line has increased steadily over time.

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<sup>12</sup>The large increase in the sample size in the third era is due to the additional samples introduced in Scotland and Wales in Wave 9, as mentioned above. These individuals are not represented in the second era, since we only consider those individuals for whom we have income data for all five of an era’s constituent years. This requirement has some further impact on the variation in sample sizes between eras. For example, despite losing individuals due to attrition and non-response, the total sample in the second era is 418 higher than in the first era. This is because there are some individuals for whom there is no information on income during at least one of the first five waves but no missing data during Waves 6-10. In fact, there are only 2,734 individuals for whom we have income data during all 15 waves and who are, therefore, represented in the analysis for all three eras. The numbers of individuals who are represented in both the first and second eras and in the second and third eras are 4,256 and 4,067, respectively.

Table 3.1: Poverty Lines in Each Year

Year	Poverty Line (£)
1991	7,002
1992	7,217
1993	7,409
1994	7,435
1995	7,575
1996	7,676
1997	7,437
1998	7,480
1999	7,401
2000	7,796
2001	7,890
2002	8,551
2003	8,605
2004	8,787
2005	8,908

Note: Poverty lines are in terms of the equivalised household net income variable ‘whh-netde2,’ as described in the text.

Estimates of intertemporal poverty in Great Britain, using both  $P_{DRZ}$  and  $P_{FOS}$ , are displayed in Tables 3.2-3.4. Estimates are provided both on a regional basis and for Great Britain as a whole. The three tables correspond to the three eras.

Table 3.2: Regional Sample Sizes and Poverty Estimates During 1991-1995

Region	Sample	% poor	$P_{DRZ}$	$P_{FOS}$
Inner London	144	46.5	0.118	0.072
Outer London	313	31.3	0.086	0.046
Rest of South-East	1,074	30.0	0.074	0.039
South-West	506	32.6	0.102	0.047
East Anglia	272	38.2	0.161	0.070
East Midlands	527	42.9	0.132	0.060
West Midlands Conurb	225	42.7	0.251	0.096
Rest of West Midlands	376	39.6	0.118	0.057
Greater Manchester	206	25.7	0.069	0.035
Merseyside	120	34.2	0.122	0.052
Rest of North-West	243	27.2	0.086	0.036
South Yorkshire	171	38.0	0.213	0.087
West Yorkshire	201	40.3	0.166	0.069
Rest of Yorks & Hum	180	32.8	0.140	0.061
Tyne & Wear	134	46.3	0.134	0.064
Rest of North	240	26.7	0.092	0.040
Wales	312	44.6	0.109	0.056
Scotland	475	34.9	0.129	0.058
Total Sample	5,968	35.2	0.117	0.054
Males	2,904	34.0	0.109	0.051
Females	3,064	36.4	0.125	0.058

Note: Individuals were defined as living in a given region if they lived in that region during both Wave 1 and Wave 5. The sum of the regional sample sizes add up to 5,719, which is 249 less than the total sample of 5,968. The remaining 249 individuals lived in different regions during Waves 1 and 5 and so were excluded from the regional analysis.

Table 3.3: Regional Sample Sizes and Poverty Estimates During 1996-2000

Region	Sample	% poor	$P_{DRZ}$	$P_{FOS}$
Inner London	121	29.8	0.069	0.042
Outer London	338	25.1	0.083	0.037
Rest of South-East	1,151	23.0	0.042	0.024
South-West	577	25.8	0.062	0.031
East Anglia	263	35.4	0.149	0.067
East Midlands	553	36.9	0.096	0.050
West Midlands Conurb	221	35.3	0.181	0.077
Rest of West Midlands	374	31.6	0.070	0.035
Greater Manchester	202	20.8	0.064	0.036
Merseyside	146	34.9	0.098	0.045
Rest of North-West	283	29.7	0.076	0.038
South Yorkshire	182	37.9	0.113	0.056
West Yorkshire	211	35.1	0.102	0.054
Rest of Yorks & Hum	216	31.0	0.114	0.048
Tyne & Wear	138	41.3	0.155	0.064
Rest of North	253	25.3	0.056	0.030
Wales	336	34.2	0.095	0.047
Scotland	502	35.3	0.089	0.048
Total Sample	6,386	30.2	0.083	0.041
Males	3,134	28.8	0.075	0.038
Females	3,252	31.4	0.090	0.045

Note: Individuals were defined as living in a given region if they lived in that region during both Wave 6 and Wave 10. The sum of the regional sample sizes add up to 6067, which is 319 less than the total sample of 6,386. The remaining 319 individuals lived in different regions during Waves 6 and 10 and so were excluded from the regional analysis.

Table 3.4: Regional Sample Sizes and Poverty Estimates During 2001-2005

Region	Sample	% poor	$P_{DRZ}$	$P_{FOS}$
Inner London	85	18.8	0.050	0.038
Outer London	265	26.0	0.073	0.038
Rest of South-East	1,063	18.2	0.015	0.013
South-West	557	28.5	0.042	0.027
East Anglia	255	27.8	0.102	0.046
East Midlands	523	33.5	0.056	0.033
West Midlands Conurb	190	32.6	0.074	0.037
Rest of West Midlands	298	20.8	0.051	0.025
Greater Manchester	214	31.8	0.056	0.034
Merseyside	137	39.4	0.050	0.037
Rest of North-West	286	26.9	0.042	0.026
South Yorkshire	160	23.8	0.084	0.036
West Yorkshire	144	36.1	0.104	0.050
Rest of Yorks & Hum	215	26.5	0.036	0.027
Tyne & Wear	107	35.5	0.064	0.036
Rest of North	213	30.0	0.056	0.034
Wales	1,608	35.5	0.084	0.045
Scotland	1,692	33.9	0.066	0.038
Total Sample	8,341	29.7	0.060	0.034
Males	4,075	28.1	0.052	0.030
Females	4,266	31.2	0.068	0.038

Note: Individuals were defined as living in a given region if they lived in that region during both Wave 11 and Wave 15. The sum of the regional sample sizes add up to 8,012, which is 329 less than the total sample of 8,341. The remaining 329 individuals lived in different regions during Waves 11 and 15 and so were excluded from the regional analysis.

Tables 3.2-3.4 indicate that the percentage of poor individuals declined from each era to the next, for both males and females. There is a substantial decline in the percentage of poor from the first era to the second and a relatively modest decline from the second to the third. Only in Scotland, Merseyside and the Rest of the North-West was there an increase in the percentage of individuals who were poor from the first era to the second. From the second era to the third era, the percentage of poor individuals increased in a number of regions, namely Wales, Outer London, the South-West, West Yorkshire, Merseyside, Greater Manchester and the Rest of the North.

There is a striking change in the regional ranking of Inner London over the three eras, with respect to its percentage of poor individuals. However, this result should be treated with a good deal of caution as the sample sizes for Inner London are very small.

Some regions, such as Tyne & Wear, remained consistently among the poorest regions throughout the three eras. Merseyside and West Yorkshire became relatively poorer regions over time. The Rest of the West Midlands saw a steady improvement



in its regional ranking over the three eras. The Rest of the South-East is ranked consistently over the three eras as an area with a relatively low proportion of poor individuals. The ranking of South Yorkshire fluctuated somewhat, worsening from the first to the second era but later improving. The rankings of Greater Manchester, the Rest of the North, the Rest of the North-West and Scotland all deteriorated somewhat over the three eras.

Intertemporal poverty levels also declined from each era to the next, for both males and females and according to both measures. Focusing firstly on the  $P_{DRZ}$  measure, intertemporal poverty was found to decrease between the first and second era in all regions apart from Tyne and Wear. The simple percentages of poor individuals portrayed a very mixed picture with regard to the changes in poverty from the second era to the third. The percentage of poor decreased only very slightly overall and with significant numbers of regions in which poverty increased. In marked contrast, the  $P_{DRZ}$  measures indicate that poverty fell in all regions apart from the Rest of the North, where it remained unchanged and in West Yorkshire, where it rose very slightly. This indicates that although the overall percentages of poor individuals changed little from the second to the third era, when both the extent of individual-level per-period poverty and the sequencing of those poor episodes is accounted for in the manner advocated in Chapter 2, the overall level of intertemporal poverty decreased nearly everywhere. The relative ranking of the different regions according to  $P_{DRZ}$  also displays some notable differences to those indicated simply by the percentages of poor individuals. West Midlands Conurbations, which had the fifth highest percentage of poor individuals in each of the first two eras, is ranked as the intertemporally poorest region by  $P_{DRZ}$  in these eras. Whilst the percentage of poor in Merseyside increased by 4.5 percentage points from the second era to the third to become the region with the highest proportion of poor people, both the extent of intertemporal poverty and the regional ranking substantially improved over this time-frame according to  $P_{DRZ}$ . The  $P_{DRZ}$  measures paint a relatively bleaker picture of poverty in East Anglia than the simple percentages of poor people indicate. Although the level of intertemporal poverty decreased in successive waves, the regional ranking according to  $P_{DRZ}$  is worse than the percentages suggest and, moreover, deteriorated from each era to the next. South Yorkshire's ranking improved dramatically between the second and third era in terms of the percentage of poor individuals but, according to  $P_{DRZ}$ , its ranking actually deteriorated. Tyne and Wear's relative ranking fares better with  $P_{DRZ}$  than is indicated by the simple percentages of poor individuals. Inner London's ranking fluctuated less under  $P_{DRZ}$  than indicated by the simple percentages of poor, moving from the tenth ranked region in the first era to the fourteenth in the second and third eras.

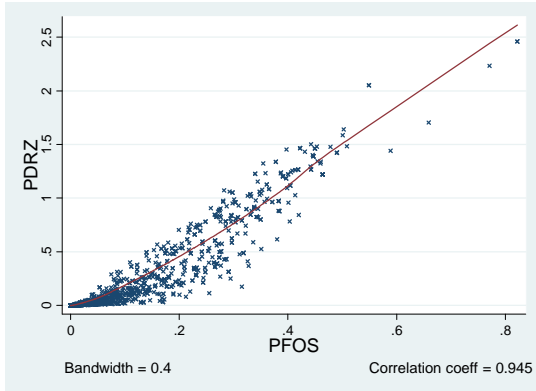
The measures of  $P_{FOS}$  paint a broadly similar picture to those of  $P_{DRZ}$  but with a few notable differences. There is a greater fluctuation in Inner London's

regional ranking between eras than indicated by  $P_{DRZ}$  (though less fluctuation than is indicated simply by focusing on the percentage of poor people). Conversely, there is less fluctuation in Merseyside's regional ranking between eras, it ranging from twelfth in the first era to eighth in the third era. In the third era, South Yorkshire, the East Midlands and the Rest of the West Midlands are all ranked more favourably by  $P_{FOS}$  than by  $P_{DRZ}$ . Since the extent of each individual's static poverty in each wave is the same for  $P_{FOS}$  and  $P_{DRZ}$ , the fact that  $P_{DRZ}$  ranks these regions as relatively poor ones compared to  $P_{FOS}$  suggests that when individuals are poor in these regions, their poor spells tend to be relatively more bunched together and with relatively fewer preceding periods of non-poverty than is the case in some of the other regions. Such differences in the  $P_{FOS}$  and  $P_{DRZ}$  rankings reflect the different normative judgements embodied by the respective measures; in particular, whether or not the precise ordering of poor episodes is important.

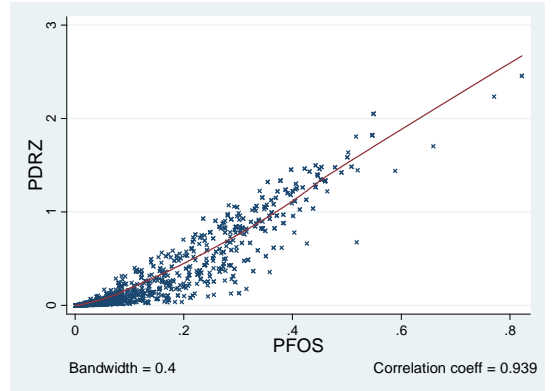
As an overall comparison between the estimates of poverty of  $P_{FOS}$  and of  $P_{DRZ}$  at an individual level, the plots in Figure 3.1 chart the relationship between the two variables over the three eras. Separate plots are displayed for Males and Females. It is clear from these plots that the two measures are very highly correlated and this is confirmed by the correlation coefficients. Simple OLS regressions of  $P_{DRZ}$  on  $P_{FOS}$  were also performed and are displayed in Tables 3.5 and 3.6. The high  $R^2$  values serve as further confirmation of the high degree of correlation between the two measures. In each of these regressions, the constant has a negative and highly significant sign. This indicates that the  $P_{DRZ}$  measures are displaced downwards from the  $P_{FOS}$  measures. However, as can be seen from the plots in Figure 3.1, except at very low levels of poverty, the  $P_{DRZ}$  measures tend to have a higher value than the  $P_{FOS}$  measures. This is consistent with the relatively high coefficients of  $P_{FOS}$  in the regression results. Except at very low values of  $P_{FOS}$ , the effect of the comparatively high  $P_{FOS}$  coefficient dominates that of the negative regression constant. These results are not surprising and reflect the functional form of the respective measures. For example, at very low levels of poverty, where an individual is poor in perhaps just one of five time periods, the  $P_{DRZ}$  measures will typically have a lower value than  $P_{FOS}$ , since the poverty in the poor period is discounted by the number of preceding periods of relative affluence. At higher levels of poverty, where an individual is poor in most time periods, the  $P_{DRZ}$  measures are typically higher than  $P_{FOS}$  due to the effect of the parameter  $k_t$ , which is intended to explicitly account for the exacerbating impact of consecutive periods of poverty.  $P_{DRZ}$  Against  $P_{FOS}$

Figure 3.1: Plots Of PDRZ Against PFOS

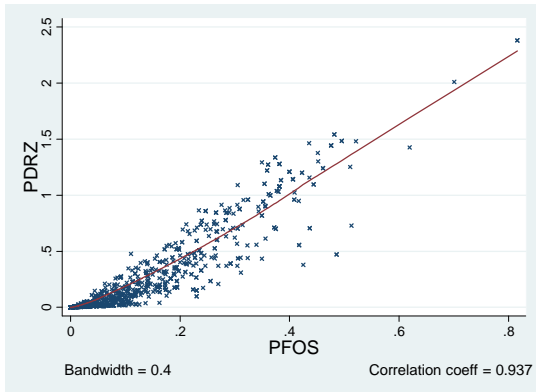
(a)  $P_{DRZ} \text{ v } P_{FOS}$  For Males In 1991-1995



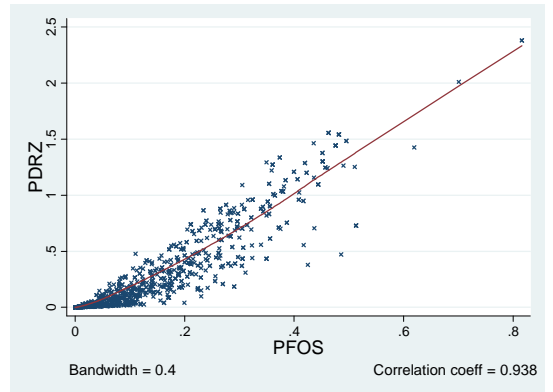
(b)  $P_{DRZ} \text{ v } P_{FOS}$  For Females In 1991-1995



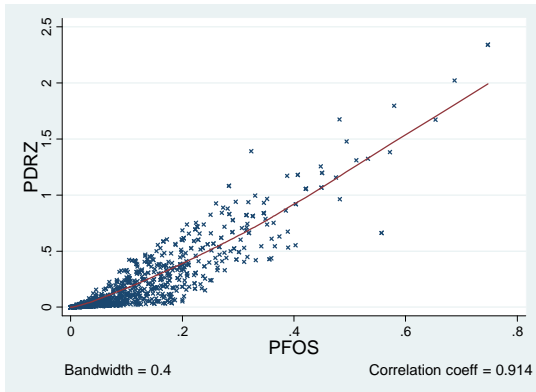
(c)  $P_{DRZ} \text{ v } P_{FOS}$  For Males In 1996-2000



(d)  $P_{DRZ} \text{ v } P_{FOS}$  For Females In 1996-2000



(e)  $P_{DRZ} \text{ v } P_{FOS}$  For Males In 2001-2005



(f)  $P_{DRZ} \text{ v } P_{FOS}$  For Females In 2001-2005

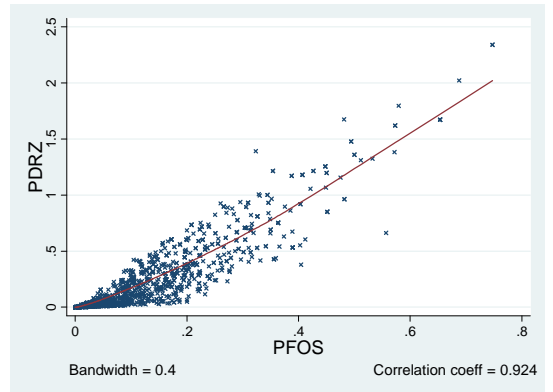


Table 3.5: OLS Regressions of PDRZ on PFOS for Males During Each Era

	$P_{DRZ}$ (1991-1995)		$P_{DRZ}$ (1996-2000)		$P_{DRZ}$ (2001-2005)	
	Coeff.	t-statistic	Coeff.	t-stat	Coeff.	t-statistic
$P_{FOS}$	2.567	64.65	2.348	45.85	2.095	34.68
Constant	-0.021	-18.33	-0.014	-12.93	-0.011	-9.59
Observations	2,904		3,134		4,075	
$R^2$	0.8933		0.8777		0.8345	

Note: The standard errors used to compute the t-statistics are White-corrected for heteroskedasticity.

Table 3.6: OLS Regressions of PDRZ on PFOS for Females During Each Era

	$P_{DRZ}$ (1991-1995)		$P_{DRZ}$ (1996-2000)		$P_{DRZ}$ (2001-2005)	
	Coeff.	t-statistic	Coeff.	t-statistic	Coeff.	t-statistic
$P_{FOS}$	2.593	65.72	2.356	52.95	2.144	45.03
Constant	-0.025	-19.00	-0.016	-14.14	-0.013	-11.62
Observations	3,064		3,252		4,266	
$R^2$	0.8825		0.8802		0.8533	

Note: The standard errors used to compute the t-statistics are White-corrected for heteroskedasticity.

### 3.3 Determinants of Intertemporal Poverty in Great Britain

#### 3.3.1 Econometric Models

Thus far we have presented a brief descriptive summary of the patterns of intertemporal poverty in Great Britain. We now turn our attention to the determinants of intertemporal poverty. As discussed in the introduction, our interest is in understanding which factors determine the degree of severity of an individual's intertemporal poverty. We wish to allow for the possibility that the determinants of having a non-zero level of intertemporal poverty may differ somewhat from the factors which shape the overall extent of intertemporal poverty.<sup>13</sup> The approach taken is therefore to model the degree of intertemporal poverty conditional on being intertemporally poor. Ignoring this possibility would run the risk that there might be a type of 'selection bias.' The method adopted is the Heckman two-step procedure. In the first

<sup>13</sup>Note that both the  $P_{DRZ}$  and the  $P_{FOS}$  measures regard an individual to have a non-zero level of intertemporal poverty if he is poor in at least one time period; otherwise he is intertemporally non-poor.

stage, we perform a Probit regression, for the probability of being intertemporally poor. This regression is of the form

$$Pr(I = 1|\mathbf{W}) = \Phi(\mathbf{W}\boldsymbol{\gamma}). \quad (3.3)$$

In this specification,  $I$  indicates whether or not an individual is intertemporally poor. If an individual has a non-zero level of intertemporal poverty,  $I = 1$ ; otherwise  $I = 0$ .  $\mathbf{W}$  is a vector of explanatory variables,  $\boldsymbol{\gamma}$  is a vector of unknown parameters, and  $\Phi$  is the cumulative distribution function of the standard normal distribution. Estimation of (3.3) yields results which can be used to predict the probability that any given individual is intertemporally poor. In the second stage, we correct for possible selection bias by including a transformation of the predicted individual probabilities as an extra explanatory variable. The intertemporal poverty equation can be specified as

$$P^* = \mathbf{X}\boldsymbol{\beta} + u \quad (3.4)$$

where  $P^*$  denotes the individual's level of intertemporal poverty, which may or may not take a non-zero value. However, it is only included in the second stage of the regression if it is non-zero, that is if  $I = 1$ . The conditional expectation of the level of intertemporal poverty given that the person is intertemporally poor is then

$$E(P|\mathbf{X}, I = 1) = \mathbf{X}\boldsymbol{\beta} + E(u|\mathbf{X}, I = 1). \quad (3.5)$$

Assuming that the error terms are jointly normal, we then have that

$$E(P|\mathbf{X}, I = 1) = \mathbf{X}\boldsymbol{\beta} + \rho\sigma_u\lambda(\mathbf{W}\boldsymbol{\gamma}) \quad (3.6)$$

where  $\rho$  is the correlation between unobserved determinants of an individual having a non-zero level of intertemporal poverty, and unobserved determinants of the overall level of intertemporal poverty  $P^*$  (i.e.  $u$ ),  $\sigma_u$  is the standard deviation of  $u$ , and  $\lambda$  is the inverse Mills ratio evaluated at  $\mathbf{W}\boldsymbol{\gamma}$ .<sup>14</sup>

Here the interpretation of 'selection' is rather different from more common usages of this method, such as propensity to work in the labour market context. Individuals do not, of course, self-select whether or not to be poor. Nevertheless, sample 'selection' of including only non-zero intertemporally poor observations in (3.6) can, as in the more familiar labour market context, be viewed as a form of omitted-variables bias.

Unbiased estimates of the determinants of the severity of intertemporal poverty, conditional on being intertemporally poor, can then be obtained simply by including the inverse Mills ratio  $\lambda$  as an additional explanatory variable in OLS estimation of

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<sup>14</sup>This follows in a similar way to that described in Greene (2000), pp. 928-929.

(3.4). It is clear that the coefficient of  $\lambda$  can only be zero if the correlation  $\rho = 0$ . We can therefore test the null hypothesis that there is no selection bias by the equivalent null hypothesis that the coefficient of  $\lambda$  equals zero.

The method described in this subsection was applied to all three eras, running separate regressions for males and females and using both poverty measures  $P_{DRZ}$  and  $P_{FOS}$ .

### 3.3.2 Misspecification tests

Two related types of misspecification tests were performed on the output of all the Heckman regressions described in the previous subsection. Each test was a type of reset test. The first one was a link test, of the form suggested by Pregibon (1979), which in turn was based on an earlier idea by Tukey (1949). A link error is a common form of specification error which occurs when the dependent variable requires a transformation in order to appropriately relate or ‘link’ to the independent variables. Consider again equation (3.4). Let  $\hat{\beta}$  be the parameter estimates. A standard link test is performed by regressing  $P^*$  on  $\mathbf{X}\hat{\beta}$  and  $(\mathbf{X}\hat{\beta})^2$ . The idea behind the test is that the  $(\mathbf{X}\hat{\beta})^2$  term is likely to be significant if there is a link error, whereas under the null hypothesis of no misspecification it should not be.

To apply this test in the context of the Heckman regressions above we regressed  $P^*$  on  $(\mathbf{X}\hat{\beta} + \lambda\hat{\alpha})$  and  $(\mathbf{X}\hat{\beta} + \lambda\hat{\alpha})^2$ , where  $\alpha = \rho\sigma_u$ , and tested whether the second term was significant. However, Pregibon (1979)’s link test was designed to apply to single equation systems and incorporating the inverse Mills ratio  $\lambda$  to the test in this manner may not be valid. Because of these doubts, we also performed a modified version of the test, regressing  $P^*$  on  $(\mathbf{X}\hat{\beta} + \lambda\hat{\alpha})$  and  $(\mathbf{X}\hat{\beta})^2$  and testing whether the latter term was significantly different from zero.

### 3.3.3 Variables used in study

The possible explanatory variables considered in the study are displayed in Table 3.7.

Table 3.7: Variables Considered For Inclusion In Probit/Selection and Heckman/OLS Models

Variable	Description	Probit	Heckman
age	Individual's age	Y	Y
agesq	Square of individual's age	Y	Y
retire	Retired	Y	Y
mortgage	Have a mortgage	Y	Y
degree	Highest educ qual: Degree	Y	Y
hndcteach	Highest educ qual: HNC, HND or teaching	Y	Y
alevel	Highest educ qual: A-Levels	Y	Y
nkids	Number of children in household	Y	Y
inactive	Economically inactive	Y	Y
evermarryliv	Have ever married or lived with partner	Y	Y
hidegree	Have a higher degree	Y	Y
unemploy	Unemployed	Y	Y
laha	Live in a local authority/housing authority	Y	Y
student	Student	Y	Y
ownedout	Home owned outright	Y	Y
immigrant	Immigrant	N	Y
r1	Live in Inner London	Y	Y
r2	Live in Outer London	Y	Y
r3	Live in rest of South-East	Y	Y
r4	Live in South-West	Y	Y
r5	Live in East Anglia	Y	Y
r6	Live in East Midlands	N	Y
r7	Live in West Midland Cities	N	Y
r9	Live in Greater Manchester	Y	N
r10	Live in Merseyside	N	Y
r11	Live in rest of North-West	Y	Y
r12	Live in South Yorkshire	Y	Y
r13	Live in West Yorkshire	Y	N
r14	Live in rest of Yorkshire and Humberside	N	Y
r15	Live in Tyne & Wear	Y	Y
r16	Live in rest of North	Y	Y

In our choice of variables, we closely followed the approach of Clark and Peters (2005). The variables employed are also broadly in line with those used in other studies on the determinants of poverty using BHPS data. In fact the variables in Table 3.7 were chosen from a slightly larger set of variables by stepwise regressing, at the 10% level of significance, (3.3) and (3.4) using Probit and OLS regressions respectively. The stepwise regressions were performed for each of the three eras. Variables were omitted from the subsequent analysis only if they were dropped in the stepwise regressions for all three eras. It follows that the variables listed in Table 3.7 in the Probit/Selection column were all found to be significant at the 10% level or lower in the Probit regression of at least one of the three eras. Similarly, the variables listed in the Heckman/OLS column of Table 3.7 were all found to be

significant at the 10% level or lower in the OLS regression of at least one of the three eras. One additional variable, a dummy for Wales, survived the stepwise regression analysis but was subsequently dropped as it was found to be insignificant in all of the subsequent analysis. The dummy variable for being an immigrant was dropped in the second era as the sample size for the number of immigrants who were also poor was extremely small during this period.

Descriptions of the variables are contained in Table 3.7. Apart from ‘age,’ ‘agesq’ and ‘nkids,’ which refer, respectively, to the age of the individual, the square of the age of the individual and the number of children in the individual’s household, all the remaining variables are dummies.

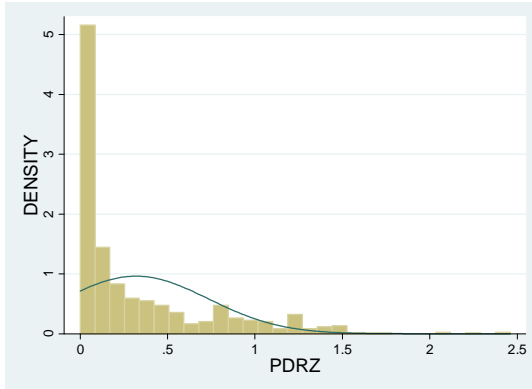
It is clear that most of the variables in Table 3.7 can change over time. The approach taken in our analysis was to use each variable’s value in the first wave of each of the three eras. For example, in the first era studied, the variable ‘degree’ has a value of 1 if the individual held a degree in Wave 1; in the second era, the variable has a value of 1 if the individual held a degree in Wave 6.

We now turn our attention to the dependent variable. An important assumption in Heckman regressions is that the dependent variable is normally distributed. Histograms were plotted for both poverty measures in each of the three eras, for males and females separately, and the charts overlaid with appropriately scaled normal density functions. These results are displayed in Figure 3.2 and Figure 3.3 for the  $P_{DRZ}$  and  $P_{FOS}$  measures respectively. It is clear from these histograms that the densities are heavily skewed towards lower levels of poverty and that a normality assumption cannot be maintained. Natural logarithms of each of the poverty measures were then taken and the corresponding histograms plotted for the logged poverty measures. These are displayed, for  $P_{DRZ}$  and  $P_{FOS}$ , in Figure 3.4 and Figure 3.5 respectively. Compared to the overlaid normal density functions, these histograms are slightly skewed to the right and there is some evidence of a possible bi-modality. Nevertheless, a normal approximation does not appear to be a bad one. Overall, the results in Figure 3.2, Figure 3.3, Figure 3.4 and Figure 3.5 strongly suggest that the ensuing analysis be conducted with the dependent variables in their logarithmic form and this is the approach taken.

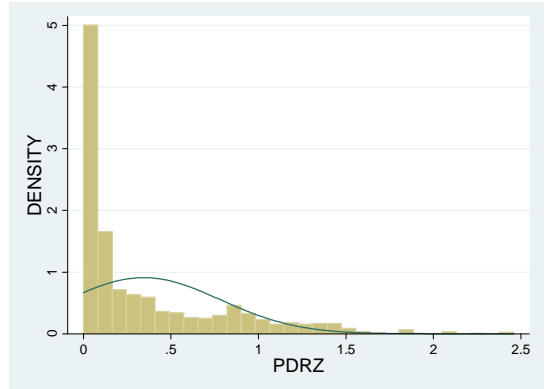


Figure 3.2: Histograms For PDRZ Measures

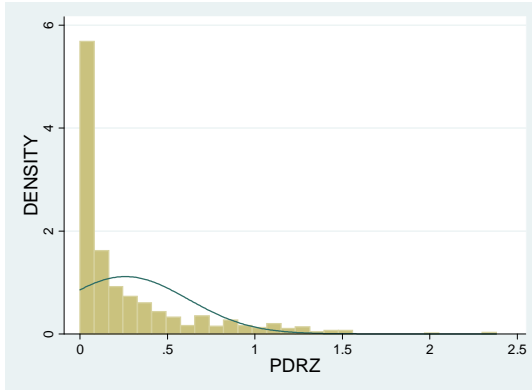
(a)  $P_{DRZ}$  Histogram For Males During 1991-1995



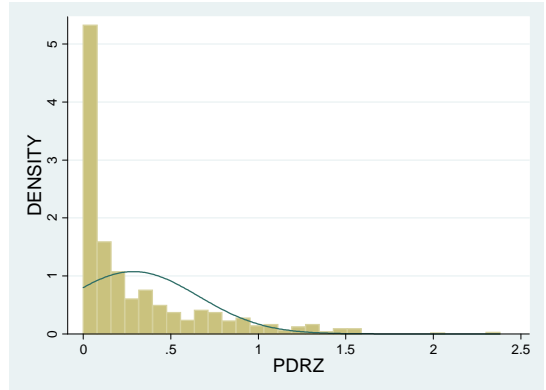
(b)  $P_{DRZ}$  Histogram For Females During 1991-1995



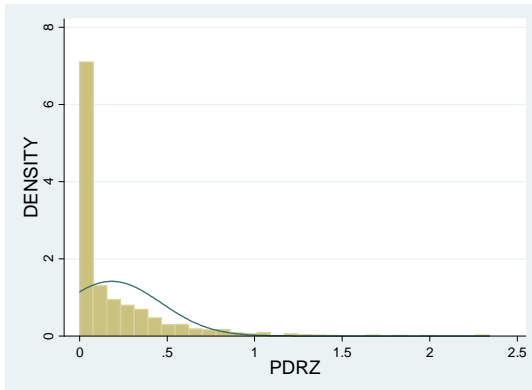
(c)  $P_{DRZ}$  Histogram For Males During 1996-2000



(d)  $P_{DRZ}$  Histogram For Females During 1996-2000



(e)  $P_{DRZ}$  Histogram For Males During 2001-2005



(f)  $P_{DRZ}$  Histogram For Females During 2001-2005

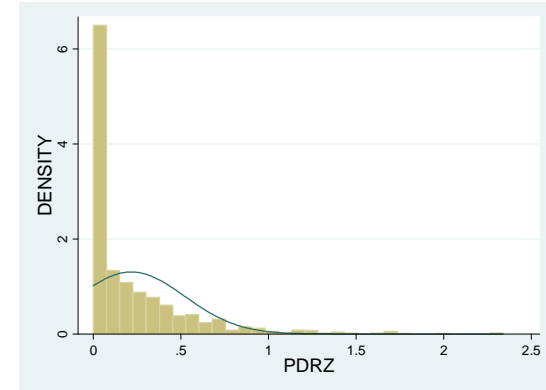
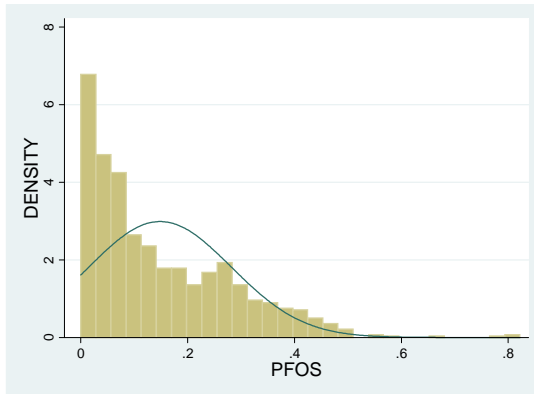
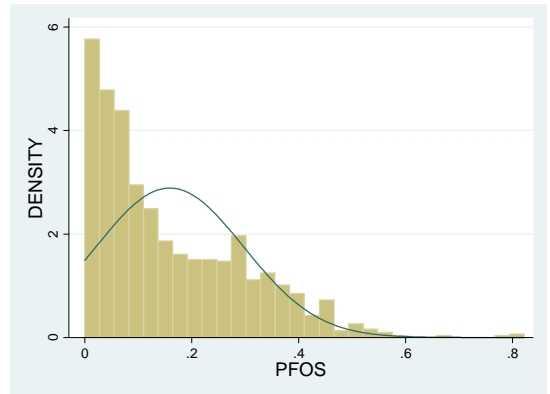


Figure 3.3: Histograms For PFOS Measures

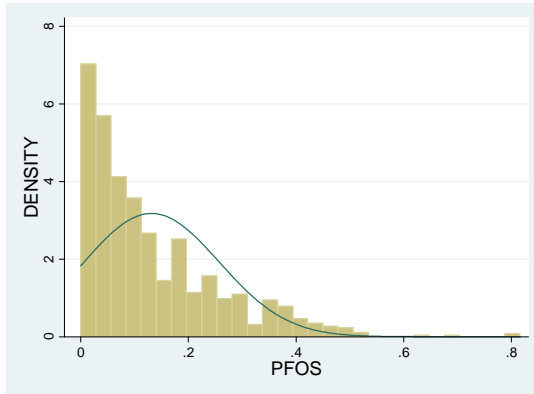
(a)  $P_{FOS}$  Histogram For Males During 1991-1995



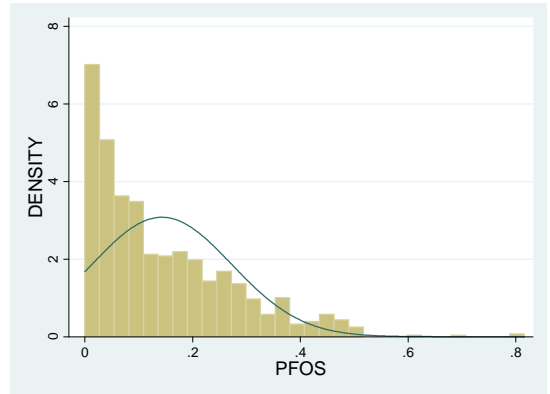
(b)  $P_{FOS}$  Histogram For Females During 1991-1995



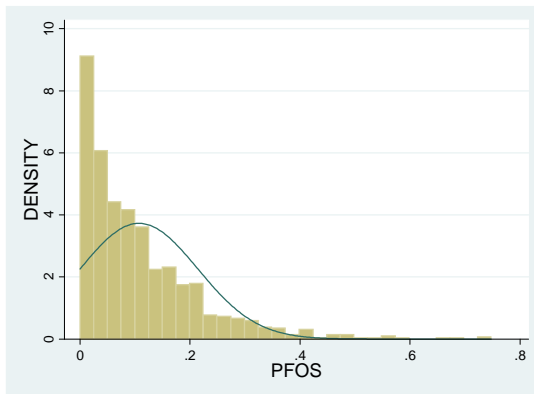
(c)  $P_{FOS}$  Histogram For Males During 1996-2000



(d)  $P_{FOS}$  Histogram For Females During 1996-2000



(e)  $P_{FOS}$  Histogram For Males During 2001-2005



(f)  $P_{FOS}$  Histogram For Females During 2001-2005

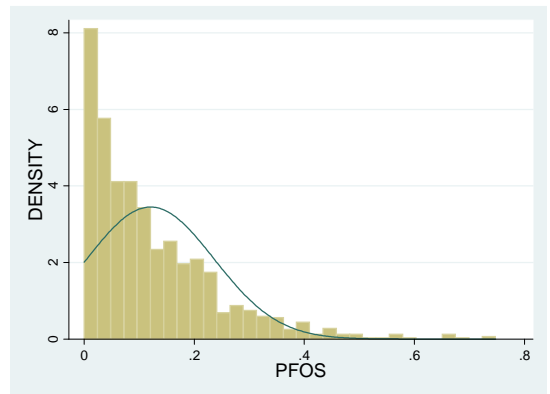
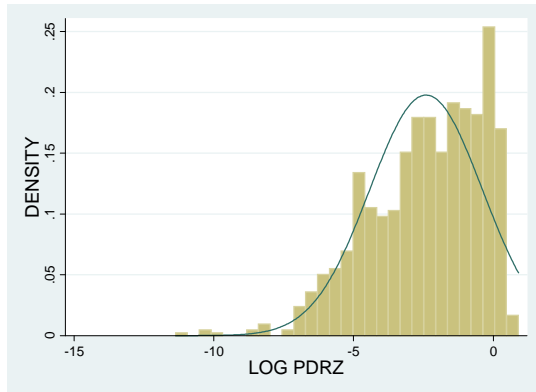
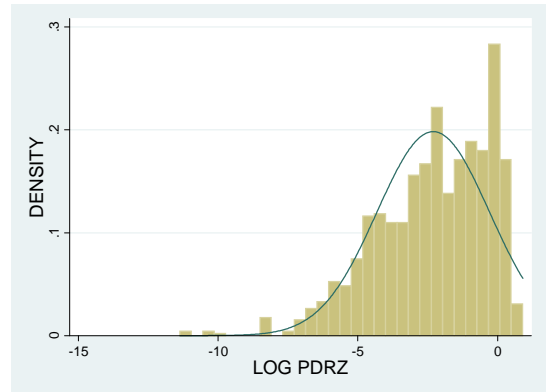


Figure 3.4: Histograms For Logged PDRZ Measures

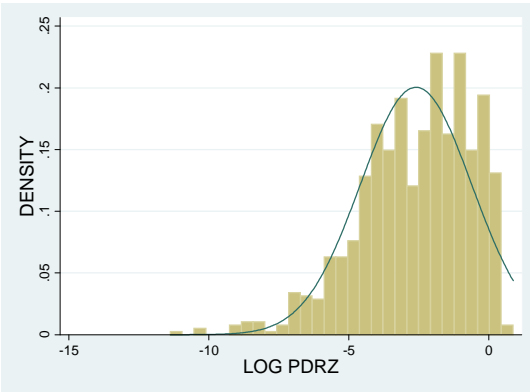
(a)  $\text{Log } P_{DRZ}$  Histogram For Males During 1991-1995



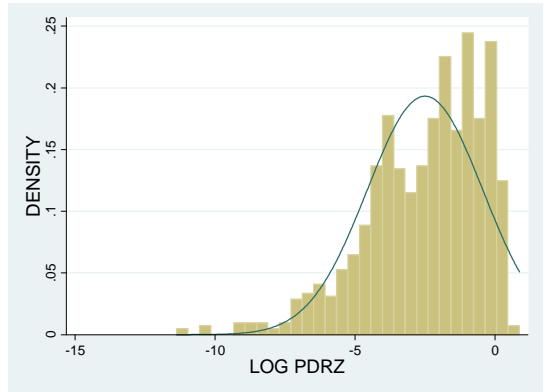
(b)  $\text{Log } P_{DRZ}$  Histogram For Females During 1991-1995



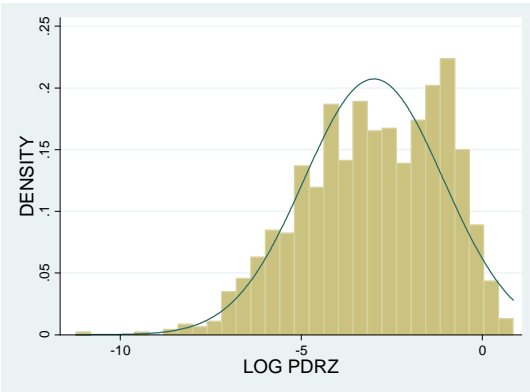
(c)  $\text{Log } P_{DRZ}$  Histogram For Males During 1996-2000



(d)  $\text{Log } P_{DRZ}$  Histogram For Females During 1996-2000



(e)  $\text{Log } P_{DRZ}$  Histogram For Males During 2001-2005



(f)  $\text{Log } P_{DRZ}$  Histogram For Females During 2001-2005

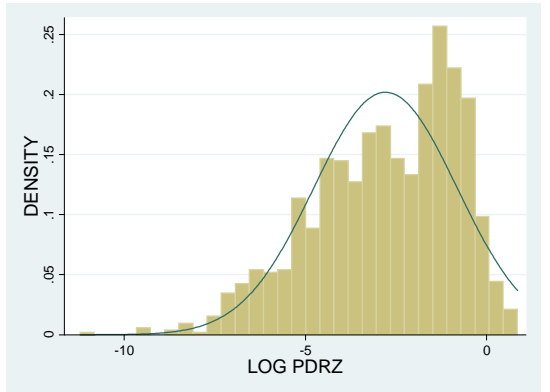
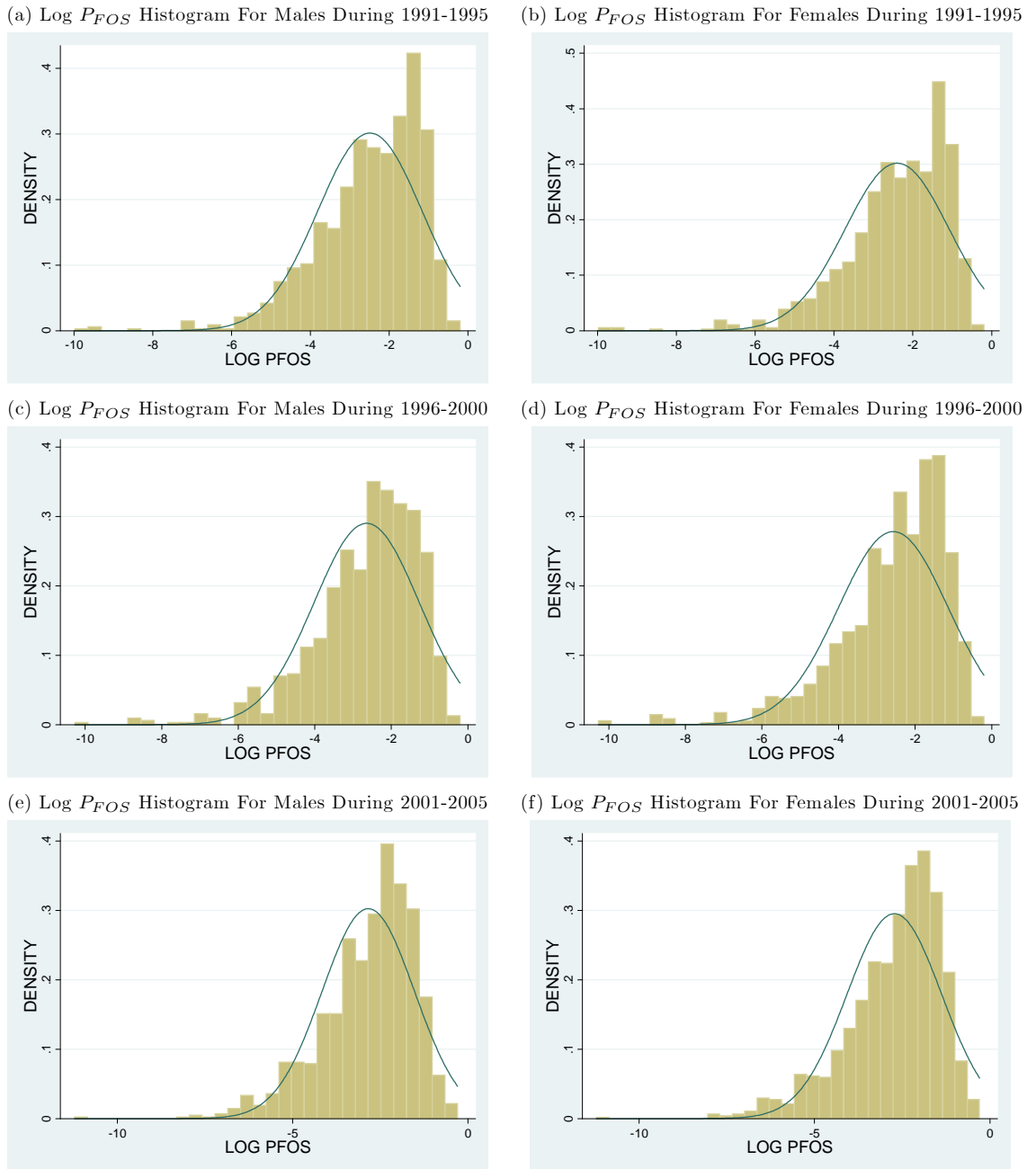


Figure 3.5: Histograms For Logged PFOS Measures



The results of the regressions for each of the three eras are presented in the next subsection. By way of comparison, results are also presented in Appendix 3.A.2 for simple OLS estimation of (3.4).

### 3.3.4 Empirical Results

The results for all the regressions are displayed in Tables 3.8 through to 3.13.<sup>15</sup> Scotland was the omitted region in all the regressions. The omitted category with respect

<sup>15</sup>In these tables, and throughout the rest of the paper, statistical significance at the 5% and 1% levels are denoted by \* and \*\* respectively.

to housing tenure was ‘any other tenure,’ such as, for example, living in privately rented accommodation. The omitted category with respect to highest educational qualification was having O-Levels, CSEs or below as the highest qualification attained.

Both the standard link test and the amended version described in Section 3.3.2 were performed on all the Heckman regressions displayed in Tables 3.8 through to 3.13. In all cases, the tests failed to reject the null hypothesis of no misspecification.<sup>16</sup>

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<sup>16</sup>The p-values for these misspecification tests are displayed in Appendix 3.A.3. The lowest p-value was 0.175.

Table 3.8: Heckman Regressions For Logged PDRZ Measures By Gender For 1991-1995

Variables	Probit Male	Heckman Male	Probit Female	Heckman Female
age	-0.026	-0.098*	-0.031*	-0.138**
agesq	0.0003*	0.001	0.0003*	0.001**
retire	0.312*	1.367**	0.625**	1.590*
mortgage	-0.574**	-1.312*	-0.648**	-1.706**
degree	-0.659**	-1.279	-0.802**	-2.447**
hndcteach	-0.496**	-0.305	-0.952**	-1.727
alevel	-0.300**	-0.902**	-0.265**	-0.800*
nkids	0.360**	0.582*	0.280**	0.699**
inactive	0.594**	1.465**	0.647**	2.129**
evermarryliv	0.123	1.535**	0.278*	0.574
hidegree	-1.156**	-0.694	-0.670	-2.053
unemploy	1.054**	2.339**	0.816**	2.742**
laha	0.192	0.582	0.281*	0.784
student	0.545**	1.906**	0.537**	1.377
ownedout	-0.183	-0.349	-0.335*	-0.992
immigrant		0.154		0.004
r1	0.029	0.257	-0.089	-0.405
r2	0.078	0.250	-0.102	-0.756
r3	-0.072	-0.135	-0.087	-0.414
r4	-0.037	0.150	-0.142	-0.539
r5	0.093	0.328	-0.029	-0.202
r6		-0.033		-0.375
r7		0.103		-0.336
r9	-0.180		-0.090	
r10		-1.022		-0.033
r11	-0.182	-0.759	-0.219	-1.037
r12	-0.047	1.164*	-0.022	0.883
r13	0.196		0.220	
r14		0.451		0.260
r15	0.226	1.091*	0.414*	1.009
r16	-0.294	-1.010	-0.188	-0.386
constant	-0.272	-4.384**	-0.032	-3.827**
No. individuals	2,047	577	2,228	723
Mills Ratio $\lambda$		2.000		3.158*
R-squared		0.2178		0.1855

Table 3.9: Heckman Regressions For Logged PDRZ Measures By Gender For 1996-2000

Variables	Probit Male	Heckman Male	Probit Female	Heckman Female
age	-0.049**	-0.065	-0.030*	-0.032
agesq	0.0006**	0.001	0.0003*	0.000
retire	0.214	0.583	0.413**	1.140**
mortgage	-0.754**	-0.701	-0.922**	-1.238*
degree	-0.566**	0.240	-0.680**	-0.604
hndcteach	-0.746**	-0.714	-0.447**	-0.149
alevel	-0.151	0.104	-0.316**	-0.062
nkids	0.378**	0.429	0.376**	0.513**
inactive	0.915**	0.724	0.642**	1.357**
evermarryliv	0.450**	0.703	-0.226	-0.317
hidegree	-0.497*	0.074	-0.298	-1.149
unemploy	1.202**	1.493*	1.053**	1.848**
laha	0.051	-0.115	0.092	0.136
student	0.757**	0.802	0.198	-0.214
ownedout	-0.174	0.095	-0.346*	-0.354
r1	-0.037	-0.557	-0.383	-0.448
r2	-0.253	-0.524	-0.336*	-0.615
r3	-0.157	-0.143	-0.300**	-0.296
r4	-0.077	-0.319	-0.232*	-0.519
r5	0.271	0.534	-0.039	0.397
r6		0.143		-0.077
r7		0.142		0.077
r9	-0.023		-0.307	
r10		-0.151		-0.360
r11	-0.121	-0.479	-0.270	-0.400
r12	0.040	0.871	0.152	0.559
r13	-0.036		-0.102	
r14		0.975*		0.540
r15	0.044	0.644	0.015	-0.011
r16	-0.016	-0.715	0.051	-0.309
constant	-0.122	-3.296**	0.428	-3.305**
No. individuals	2,238	495	2,403	660
Mills Ratio $\lambda$		0.301		0.892
R-squared		0.1548		0.1814

Table 3.10: Heckman Regressions For Logged PDRZ Measures By Gender For 2001-2005

Variables	Probit Male	Heckman Male	Probit Female	Heckman Female
age	-0.027*	0.008	-0.002	0.037
agesq	0.0004**	0.000	0.000	-0.000
retire	0.172	0.686*	0.429**	1.490**
mortgage	-0.670**	-1.009	-0.680**	-1.546**
degree	-0.570**	-0.908	-0.725**	-1.652**
hndcteach	-0.514**	-0.340	-0.354**	-0.489
alevel	-0.188*	-0.063	-0.219**	-0.432
nkids	0.317**	0.417	0.297**	0.606**
inactive	0.817**	1.282	0.711**	1.583**
evermarryliv	0.124	0.291	-0.360**	-0.754*
hidegree	-0.676**	-1.268	-0.757**	0.582
unemploy	0.891**	1.466	0.803**	2.108**
laha	0.233	-0.075	0.359**	-0.024
student	0.477*	1.640*	0.737**	1.861**
ownedout	-0.290*	-0.208	-0.334**	-0.649
immigrant		1.574		0.265
r1	-0.394	0.284	-0.522	-1.140
r2	-0.002	0.781	-0.126	0.500
r3	-0.323**	-0.772	-0.336**	-1.353**
r4	-0.086	-0.177	-0.091	-0.249
r5	-0.095	0.371	-0.119	0.018
r6		-0.314		-0.295
r7		-0.811		-0.475
r9	0.127		0.260	
r10		-0.174		-0.451
r11	-0.153	-0.573	-0.102	-0.484
r12	-0.584*	0.906	-0.373	-0.152
r13	-0.031		-0.021	
r14		-0.045		0.047
r15	-0.217	-0.104	-0.225	-1.085
r16	-0.114	-0.018	0.096	0.168
constant	-0.268	-5.357**	-0.344	-5.854**
No. individuals	2,903	682	3,135	882
Mills Ratio $\lambda$		1.215		2.029*
R-squared		0.0812		0.1081



Table 3.11: Heckman Regressions For Logged PFOS Measures By Gender For 1991-1995

Variables	Probit Male	Heckman Male	Probit Female	Heckman Female
age	-0.026	-0.062*	-0.031*	-0.094*
agesq	0.0003*	0.001	0.0003*	0.001*
retire	0.312*	0.670*	0.625**	1.272*
mortgage	-0.574**	-0.817*	-0.648**	-1.259*
degree	-0.659**	-1.024*	-0.802**	-1.881**
hndcteach	-0.496**	-0.197	-0.952**	-1.388
alevel	-0.300**	-0.576*	-0.265**	-0.563
nkids	0.360**	0.367*	0.280**	0.473*
inactive	0.594**	0.970*	0.647**	1.599**
evermarryliv	0.123	1.050**	0.278*	0.483
hidegree	-1.156**	-0.809	-0.670	-1.594
unemploy	1.054**	1.607**	0.816**	2.034**
laha	0.192	0.429	0.281*	0.542
student	0.545**	1.384**	0.537**	1.197
ownedout	-0.183	-0.222	-0.335*	-0.784
immigrant		0.161		0.185
r1	0.029	0.266	-0.089	-0.128
r2	0.078	0.325	-0.102	-0.447
r3	-0.072	0.054	-0.087	-0.218
r4	-0.037	0.213	-0.142	-0.373
r5	0.093	0.230	-0.029	-0.110
r6		-0.031		-0.278
r7		0.038		-0.412
r9	-0.180		-0.090	
r10		-0.606		0.014
r11	-0.182	-0.407	-0.219	-0.740
r12	-0.047	0.762*	-0.022	0.560
r13	0.196		0.220	
r14		0.304		0.210
r15	0.226	0.741*	0.414*	0.810
r16	-0.294	-0.555	-0.188	-0.276
constant	-0.272	-4.138**	-0.032	-3.827**
No. individuals	2,047	577	2,228	723
Mills Ratio $\lambda$		1.482*		2.572*
R-squared		0.1838		0.1678

Table 3.12: Heckman Regressions For Logged PFOS Measures By Gender For 1996-2000

Variables	Probit Male	Heckman Male	Probit Female	Heckman Female
age	-0.049**	-0.035	-0.030*	-0.025
agesq	0.001**	0.000	0.0003*	0.000
retire	0.214	0.210	0.413**	0.738**
mortgage	-0.754**	-0.730	-0.922**	-0.988**
degree	-0.566**	0.414	-0.680**	-0.344
hndcteach	-0.746**	-0.657	-0.447**	0.013
alevel	-0.151	0.040	-0.316**	-0.139
nkids	0.378**	0.250	0.376**	0.342**
inactive	0.915**	0.572	0.642**	0.953**
evermarryliv	0.450**	0.394	-0.226	-0.316
hidegree	-0.497*	0.078	-0.298	-0.302
unemploy	1.202**	1.148*	1.053**	1.299**
laha	0.051	-0.292	0.092	-0.028
student	0.757**	0.408	0.198	-0.125
ownedout	-0.174	-0.080	-0.346*	-0.354
r1	-0.037	-0.338	-0.383	-0.234
r2	-0.253	-0.328	-0.336*	-0.357
r3	-0.157	-0.063	-0.300**	-0.200
r4	-0.077	-0.316	-0.232*	-0.438*
r5	0.271	0.525	-0.039	0.305
r6		0.140		-0.023
r7		0.218		0.135
r9	-0.023		-0.307	
r10		-0.018		-0.131
r11	-0.121	-0.395	-0.270	-0.288
r12	0.040	0.727*	0.152	0.390
r13	-0.036		-0.102	
r14		0.807*		0.496
r15	0.044	0.473	0.015	-0.045
r16	-0.016	-0.424	0.051	-0.103
constant	-0.122	-3.155**	0.428	-2.981**
No. individuals	2,238	495	2,403	660
Mills Ratio $\lambda$		0.343		0.831
R-squared		0.1398		0.1446

Table 3.13: Heckman Regressions For Logged PFOS Measures By Gender For 2001-2005

Variables	Probit Male	Heckman Male	Probit Female	Heckman Female
age	-0.027*	0.012	-0.002	0.020
agesq	0.0004**	-0.000	0.000	-0.000
retire	0.172	0.271	0.429**	0.742*
mortgage	-0.670**	-0.910	-0.680**	-1.176**
degree	-0.570**	-0.440	-0.725**	-0.909*
hndcteach	-0.514**	-0.443	-0.354**	-0.291
alevel	-0.188*	0.035	-0.219**	-0.277
nkids	0.317**	0.229	0.297**	0.360**
inactive	0.817**	0.658	0.711**	0.927**
evermarryliv	0.124	0.312	-0.360**	-0.291
hidegree	-0.676**	-1.021	-0.757**	0.573
unemploy	0.891**	1.188*	0.803**	1.346**
laha	0.233	-0.263	0.359**	-0.194
student	0.477*	1.328**	0.737**	1.143**
ownedout	-0.290*	-0.310	-0.334**	-0.519
immigrant		1.132		0.263
r1	-0.394	-0.083	-0.522	-0.859
r2	-0.002	0.578*	-0.126	0.389
r3	-0.323**	-0.550	-0.336**	-0.840**
r4	-0.086	-0.137	-0.091	-0.183
r5	-0.095	0.146	-0.119	-0.069
r6		-0.232		-0.199
r7		-0.488		-0.232
r9	0.127		0.260	
r10		-0.059		-0.275
r11	-0.153	-0.279	-0.102	-0.175
r12	-0.584*	0.419	-0.373	-0.176
r13	-0.031		-0.021	
r14		0.126		0.173
r15	-0.217	-0.322	-0.225	-0.790
r16	-0.114	0.002	0.096	0.102
constant	-0.268	-4.489**	-0.344	-4.544**
No. individuals	2,903	682	3,135	882
Mills Ratio $\lambda$		0.951		1.391*
R-squared		0.0653		0.0748

Before analysing the results in detail, it is perhaps worth drawing attention to the fact that the respective Probit/Selection regressions for both males and females in each of the three eras are exactly the same for both  $P_{DRZ}$  and  $P_{FOS}$ . This is necessarily the case since both measures are equal to zero for a given individual if and only if they have incomes above the poverty line during each of the five years.

There are a number of notable trends in the results. Firstly, it is interesting to note that the R-squared values of the Heckman regressions display a marked and steady decline from each era to the next. This is true for both males and females and

using both the  $P_{DRZ}$  and the  $P_{FOS}$  measures. It is not clear why this should be so but there are a number of possibilities. It might simply be that some of the missing variables which impact upon poverty became more important predictors of poverty in the later eras. Another possibility, as alluded to in Section 3.2.2, is that in later waves the poverty estimates may be biased downwards due to a higher probability of non-response from less well-off individuals. This could affect the predicted values and so too the R-squared values. In any case, there appears to be some evidence that the model specification is not exactly the same during each of the three different eras. This is corroborated by the fact there is also a fair degree of variation between eras in both the magnitudes of coefficients and of their statistical significance.

The R-squared values are higher in all the regressions for  $P_{DRZ}$  than for the corresponding regressions for  $P_{FOS}$ . This suggests that  $P_{DRZ}$  is a more precise estimator of intertemporal poverty than  $P_{FOS}$  - at least in so far as intertemporal poverty is satisfactorily explained by the variables in our models. In Chapter 2, the justification for the manner in which measures from the  $P_R$  class, such as  $P_{DRZ}$ , account for the sequencing of poor and non-poor periods was on purely axiomatic grounds. The results in this chapter lend some empirical support to the approach. Relative to taking a neutral stance on the impact of the sequencing of poor spells on overall intertemporal poverty, as in Foster (2009), penalising consecutive periods of poverty and allowing affluent spells to have a mitigating impact on subsequent poverty results in a measure more closely correlated with a number of plausible correlates of poverty.

The coefficient of the inverse Mills ratio is statistically significant at the 5% level in five of the twelve Heckman regressions. It can be inferred from this that, at least in these five regressions, performing a simple OLS regression on the dependent variable would have resulted in sample selection bias. As noted above, by way of comparison, results for OLS regressions corresponding to all twelve specifications are displayed in Appendix 3.A.2.

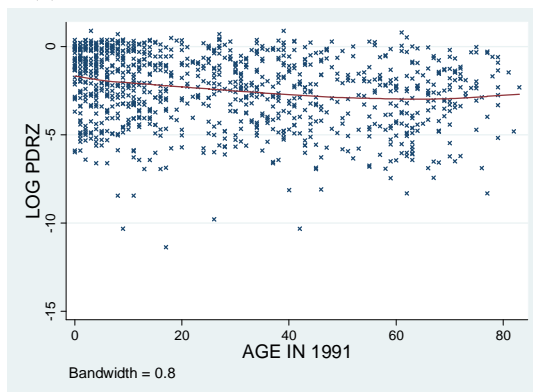
Leaving aside the regional dummy variables, with a few notable exceptions, the other explanatory variables have the expected signs in most specifications. The coefficient of ‘age’ is negative in all of the Probit/Selection regressions, suggesting that being older decreases the probability of being poor. Moreover, this variable is found to be significant at the 5% level or lower in eight of the twelve selection regressions. However, the coefficient of ‘agesq’ is positive in all the selection regressions, indicating that the impact of age is non-linear. This coefficient is significant at the 5% level or lower in ten of the twelve selection regressions. Our results suggest that even after age has been accounted for in the ‘selection’ of poverty, there is still some effect of age on the extent of intertemporal poverty. However, these results are less emphatic. The coefficient of ‘age’ is negative in eight of the twelve Heckman regressions and significant at the 5% level or lower in just four of these. There is

again some evidence of a non-linear impact of age but the evidence is rather weak. The coefficient of 'agesq' is positive in nine of the twelve Heckman regressions but significant at the 5% level or lower in just two of these.

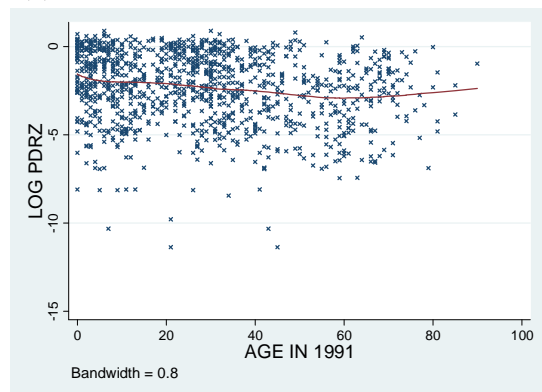
To further study the relationship between the level of intertemporal poverty and age, the logs of  $P_{DRZ}$  and  $P_{FOS}$  were plotted against 'age' and Lowess smoothers fitted. This was done for each of the three eras, separately for males and females. The plots for  $P_{DRZ}$  and  $P_{FOS}$  respectively against age are displayed in Figure 3.6 and Figure 3.7.

Figure 3.6: Plots Of Log PDRZ Against Age

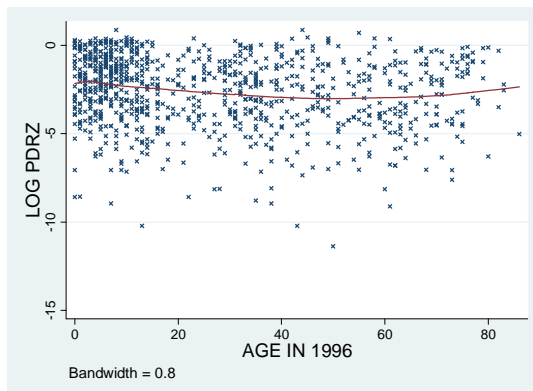
(a) Log  $P_{DRZ}$  v Age for Males During 1991-1995



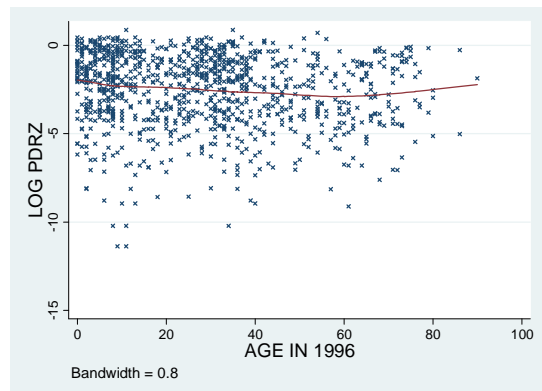
(b) Log  $P_{DRZ}$  v Age for Females During 1991-1995



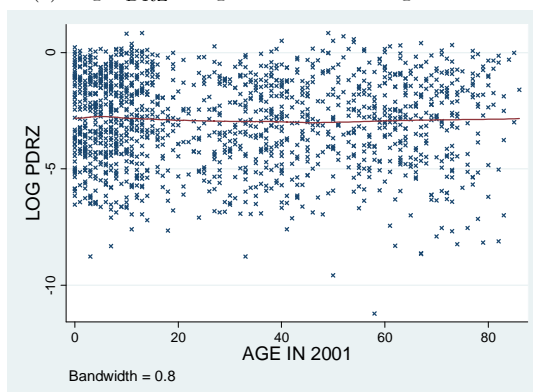
(c) Log  $P_{DRZ}$  v Age for Males During 1996-2000



(d) Log  $P_{DRZ}$  v Age for Females During 1996-2000



(e) Log  $P_{DRZ}$  v Age for Males During 2001-2005



(f) Log  $P_{DRZ}$  v Age for Females During 2001-2005

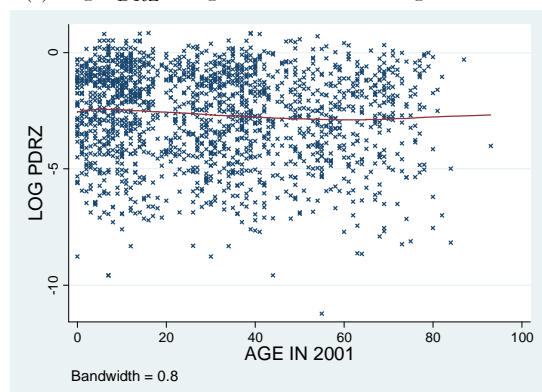
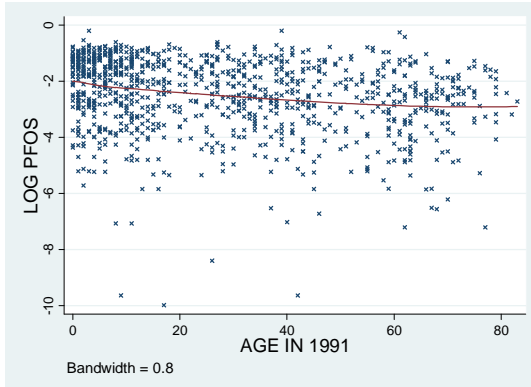
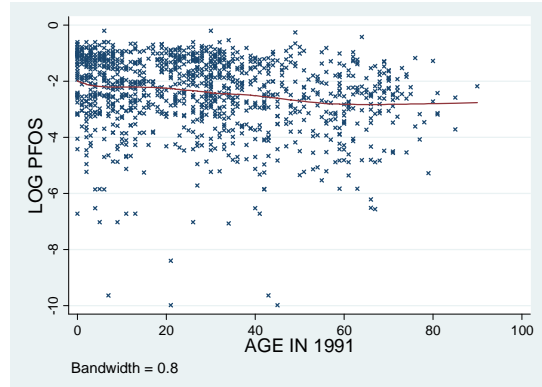


Figure 3.7: Plots Of Log PFOS Against Age

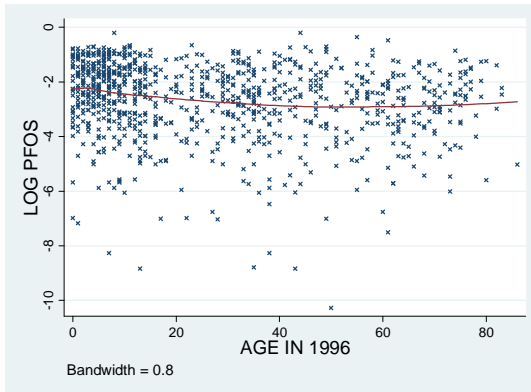
(a)  $\text{Log } P_{FOS}$  v Age for Males During 1991-1995



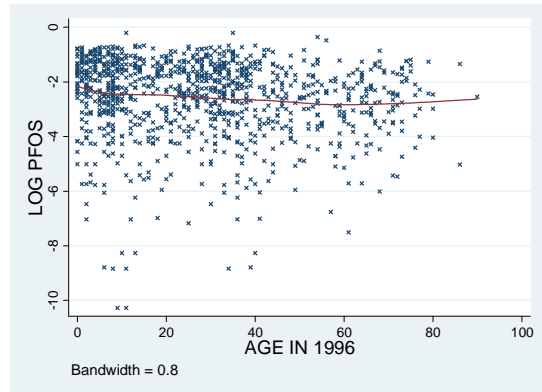
(b)  $\text{Log } P_{FOS}$  v Age for Females During 1991-1995



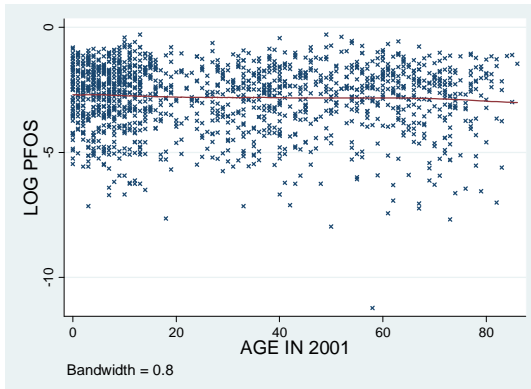
(c)  $\text{Log } P_{FOS}$  v Age for Males During 1996-2000



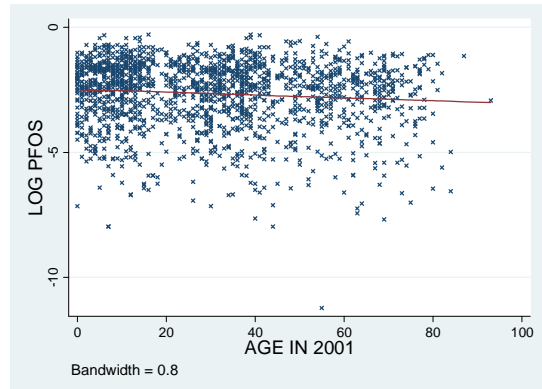
(d)  $\text{Log } P_{FOS}$  v Age for Females During 1996-2000



(e)  $\text{Log } P_{FOS}$  v Age for Males During 2001-2005



(f)  $\text{Log } P_{FOS}$  v Age for Females During 2001-2005



Even in these simple plots, there appears to be some evidence of a possible non-linear relationship between poverty and age, which is decreasing at lower levels of age and increasing at higher levels. Interestingly, in a number of the plots, notably 3.6(a), 3.6(b), 3.6(d), 3.6(f), 3.7(b), 3.7(c) and 3.7(d), there appear to be turning points (minima) that might correspond roughly to ages close to the official retirement age in Great Britain, which was 65 for men and 60 for women throughout the period of analysis. To investigate this possibility further, the dependent variable in each of the twelve Heckman regression models displayed in Tables 3.8 through to 3.13 was differentiated with respect to the age variable and set equal to zero, in order to

determine whether, *ceteris paribus*, there are ‘optimal’ ages for minimising poverty. In other words, after controlling for a number of other possible causal factors of poverty, is there a particular age at which poverty is likely to be lowest. The results are displayed in Table 3.14.

Table 3.14: Estimates Of Optimal Ages For Minimising Poverty

Era	Sex	‘Optimal’ Age for $P_{DRZ}$ Measure	‘Optimal’ Age for $P_{FOS}$ Measure
1	Male	60	58
1	Female	53	56
2	Male	46	46
2	Female	54	63
3	Male	No optimum	95
3	Female	64	57

Although the numbers in Table 3.14 do not correspond exactly to the apparent turning points in the respective plots in Figure 3.6 and Figure 3.7, it is interesting to note that in two thirds of the twelve regressions, an apparent turning point emerges at ages between 53 and 63, in and around the ages at which many people retire. It would be naive to read too much into these results, especially given the relatively large confidence intervals of some of the parameter estimates of both the ‘age’ and (especially) the ‘agesq’ variables. Nevertheless, comparing the results for each of the three eras as a whole in Table 3.14, it is interesting to note that the most plausible sounding results emerge for Era 1, and it is in Era 1 that almost all the coefficients for ‘age’ and ‘agesq’ are statistically significant in the Heckman regressions.

The coefficients of ‘retire’ are positive in all twelve of the Probit/Selection regressions, suggesting that being retired increases the probability of being poor. This variable is significant at the 5% level or lower in eight of the twelve selection regressions. The coefficients of ‘retire’ are also positive in all twelve of the Heckman regressions, and are significant at the 5% level or lower in all but three. This suggests that being retired tends to increase the extent of intertemporal poverty, conditional on being intertemporally poor. It is interesting to note that in all cases, the magnitude of the coefficient is larger in the Heckman regressions for females than in the corresponding regressions for males. This is likely to be largely due to differences between the sexes in entitlement to a full basic state pension. The state pension in Great Britain is a contributory system. In order to be entitled to a full basic state pension, individuals need to have made contributions for at least 90% of their working lives. Credits are given not only for participation in the labour market, but also for being registered as unemployed or as long-term ill or disabled. Prior to 1978, no credits were given to individuals who were out of the labour market in order to look after children.<sup>17</sup> This disproportionately affected women and resulted in women having

<sup>17</sup>In 1978, this changed through the introduction of the Home Responsibilities Protection (HRP) system.

lower state pensions than men. Given the long-term nature of contributory pension schemes, inequality between male and female state pension eligibility persisted for many years after the introduction of the HRP system. According to Blundell and Johnson (1998), at the time of their writing, whilst almost all men aged 65 and over received a full basic pension, there were "...low rates of entitlement among married women [reflecting] long periods spent out of the labor market by older cohorts." (p. 169). Blundell and Johnson (1998) expected this inequality to disappear in the early years of the twenty-first century.

Unsurprisingly, being unemployed is found to have a highly significant impact both on the probability of being intertemporally poor, and conditional on being poor, on the extent of intertemporal poverty. In fact our results suggest that this is the single most important explanatory variable for intertemporal poverty. The variable 'unemploy' is significant at the 1% level in all of the Probit/Selection regressions and in all cases has a bigger coefficient than any of the other variables which tend to increase the probability of being poor. The variable also has a significantly positive coefficient in all but one of the Heckman regressions and in eight of the twelve regressions it is significant at the 1% level. With just one exception, the magnitudes of the coefficients of 'unemploy' in the Heckman regressions are higher than for any of the other variables which tend to increase poverty. It is also interesting to note that the magnitudes of the coefficients are somewhat higher in the Heckman regressions for females than in the respective regressions for males.

Another unsurprising implication of our results is that being economically inactive increases the probability of being poor. According to our results, this variable is second only to being unemployed in its importance for increasing the probability of being intertemporally poor. The variable 'inactive' is significantly positive at the 1% level in all of the Probit/Selection regressions. It is also significantly positive, at the 5% level or lower, in two thirds of the Heckman regressions, suggesting that, conditional on being poor, being economically inactive also tends to increase the extent of intertemporal poverty. As was the case with both retirement and unemployment, the coefficient of the dummy variable for being economically inactive has a greater magnitude in the Heckman regressions for females than in the corresponding regressions for males.

Being a student also significantly increases the probability of being poor according to our results. The variable student is significantly positive at the 5% level in ten of our twelve Probit/Selection regressions, and significant at the 1% level in eight of these. Being a student also tends to increase the extent of intertemporal poverty, conditional upon being poor, according to our results; the variable 'student' has a positive coefficient in all but two of the Heckman regressions and is significant at the 5% level or lower in six of them.

We now turn attention to housing tenure. The coefficients of 'mortgage' are



negative, and significant at the 1% level, in all twelve of the Probit/Selection regressions. This suggests that having a mortgage is an important predictor of being intertemporally non-poor. The coefficients of ‘mortgage’ are also negative in each of the Heckman regressions, and significantly so, at the 5% level or lower, in two thirds of them. This suggests that having a mortgage tends to reduce the extent of intertemporal poverty, conditional on being intertemporally poor. It is also worth noting that the magnitudes of the coefficients of this variable are relatively large, and larger in the regressions for females than in those for males.

Owning a home outright is also found to significantly decrease the probability of being poor. The coefficient of ‘ownedout’ is negative in all of the Probit/Selection regressions. It is significant at the 5% level or lower in two thirds of these regressions. The coefficient of ‘ownedout’ is also negative in all but one of the Heckman regressions, but it is not significantly so in any, so providing only tentative evidence to suggest that owning one’s own home decreases the extent of intertemporal poverty, conditional on being poor.

Our results also provide some evidence to suggest that living in a local authority/housing authority dwelling increases the probability of being poor. The coefficient of the variable ‘laha’ is positive in each of the Probit/Selection regressions. It is significantly positive at the 5% level in one third of these regressions. However the results provide no evidence to suggest that living in a local authority/housing authority dwelling increases the extent of intertemporal poverty, conditional on being poor.

Overall, the results with respect to housing tenure seem broadly in line with what one might expect.

We now consider the impact of educational qualifications. Note that all of the educational dummy variables are for the highest level of qualification attained. These results are also generally in keeping with what one might expect. Having a degree is found to be an important determinant of being intertemporally non-poor. The coefficient of ‘degree’ has a negative coefficient, significant at the 1% level, in all twelve of the Probit/Selection regressions. The coefficients of ‘degree’ are also negative in all but two of the Heckman regressions, and significantly so, at the 5% level or lower, in nearly half of them. This suggests that having a degree also tends to reduce the extent of intertemporal poverty, conditional on being intertemporally poor. It is interesting to note that the magnitudes of the coefficients of this variable too are higher in the Heckman regressions for females than in the corresponding regressions for males.

Having a higher degree also appears to reduce the probability of being intertemporally poor. The coefficient of ‘hidegree’ is negative in all twelve of the Probit/Selection regressions and is significant at the 5% level or lower in two thirds of them. Conditional on being intertemporally poor, our results do not provide any

firm evidence that having a higher degree tends to reduce the level of intertemporal poverty. The variable is not significant in any of the twelve Heckman regressions and has a positive coefficient in one third of them. However, this result should not be considered robust as there are very small numbers of individuals in the sample who have a higher degree and are intertemporally poor. For example, in Waves 1-5 only 4 of the 55 individuals with a higher degree are poor.

Our results suggest that having an HNC, HND or teaching qualification significantly reduces the probability of being poor. The coefficient of the variable ‘hndcteach’ is significantly negative at the 1% level in all the Probit/Selection regressions. The variable also has a negative coefficient in all but one of the Heckman regressions, suggesting that conditional upon being poor, it tends to reduce the extent of intertemporal poverty. However this evidence is extremely weak as the variable is not significant in any of these regressions.

The coefficients of ‘alevel’ are negative in all the Probit/Selection regressions. The coefficients are significant at the 5% level or lower in all but two regressions, and at the 1% level in two thirds of them. This suggests that having A-Levels tends to reduce the probability of being intertemporally poor. In three quarters of the Heckman regressions, ‘alevel’ has a negative coefficient, suggesting that conditional upon being poor, having A-Levels also tends to reduce the extent of intertemporal poverty. However, the coefficient is only statistically significant in three of these regressions so the evidence is relatively weak.

Our results imply that the number of children in the household has a significant impact both on the probability of being intertemporally poor and, conditional on being poor, on the extent of intertemporal poverty. The coefficient of the variable ‘nkids’ is significantly positive, at the 1% level, in all twelve of the Probit/Selection regressions. The coefficients are also positive in all the Heckman regressions, and are significant at the 5% level or lower in two thirds of them. Also of interest is that the magnitudes of the coefficients are higher in all the Heckman regressions for females than in the corresponding regressions for males.

The results on the impact on poverty of ever having been married or having lived with a partner are rather mixed and it is difficult to draw firm conclusions here. This may well be due to the nature of the variable, which arguably covers too wide a range of possible domestic situations. Nevertheless, it is interesting to note that the results do provide tentative evidence for a differing impact among the sexes. In all six of the Probit regressions for males, the coefficient of the variable ‘evermarryliv’ is positive, indicating that ever having married or lived together increases the probability of being intertemporally poor. However, the coefficient is only statistically significant in two of the six regressions, so the evidence is not overwhelming. Conversely, in four of the six Probit regressions for females, the coefficient is negative, suggesting that ever having married or lived together decreases the probability

of being intertemporally poor. Again, the evidence is not overwhelming as the coefficient is only statistically significant in one of these regressions. A similar story emerges with regard to the impact of ever having been married or having lived with a partner on the extent of intertemporal poverty, conditional on being intertemporally poor. The coefficient of ‘evermarryliv’ is positive for all six of the Heckman regressions for males and statistically significant at the 5% level or lower in two of them. This provides some evidence that ever having been married or having lived with a partner also tends to increase the extent of intertemporal poverty for males, conditional on being intertemporally poor. The coefficient of ‘evermarryliv’ is negative in four of the six Heckman regressions for females but is statistically significant in just one of these. This provides some very tentative evidence to suggest that that ever having been married or having lived with a partner tends to reduce the extent of intertemporal poverty for females, conditional on being intertemporally poor.

As can be inferred from the absence of the variable ‘immigrant’ in the Probit/Selection column of Table 3.7, we found no evidence to suggest that being an immigrant either increases or decreases the probability of being poor. There is very tentative evidence to suggest that conditional upon being poor, being an immigrant tends to increase the extent of intertemporal poverty. In the two eras for which the variable was included, it was found to have a positive coefficient in all eight of the regressions. However, none of these coefficients were statistically significant.

Finally, we turn our attention to the regional dummy variables. In most cases, our results provide only tentative evidence at most to suggest that living in a given region has a significant impact either on the probability of an individual being poor or, conditional on being poor, on the extent of poverty. Living in the rest of the South-East (i.e. the South-East excluding London) is a notable exception. The variable ‘r3’ has a negative coefficient in all of the Probit/Selection regressions and is significant (at the 1% level) in half of them. This suggests that living in this part of England tends to reduce the probability of being poor. The variable ‘r3’ also has a negative coefficient in eleven of the twelve Heckman regressions but is statistically significant (at the 1% level) in just two of them. This provides some evidence to suggest that conditional on being poor, living in this region also tends to reduce the extent of intertemporal poverty. Much of this region lies within London’s commuter belt and is home to a relatively high proportion of professional people so these results are perhaps not surprising.

Our results also provide some evidence that living in the South-West of the country reduces the probability of being poor, however these results are less strong. Although the coefficient for the variable ‘r4’ is negative in all the Probit/Selection regressions, it is only statistically significant in two of them. The variable is also negative in ten of the twelve Heckman regressions, but is only statistically significant at the 5% level in one of them. This provides tentative evidence to suggest that

conditional upon being poor, living in the South-West may also tend to reduce the extent of intertemporal poverty.

In two thirds of the Probit/Selection regressions, the dummy variable for living in South Yorkshire ('r12') is found to have a negative coefficient. The negative coefficient is statistically significant negative at the 5% level in two of these regressions. This provides tentative evidence to suggest that living in this region reduces the probability of being intertemporally poor. Interestingly, the same variable has a positive coefficient in all but two of the Heckman regressions. The positive coefficient is statistically significant at the 5% level in three of these regressions. This suggests that whilst living in South Yorkshire tends to reduce the probability of being intertemporally poor, conditional on being poor, it tends to increase the extent of intertemporal poverty.

In ten out of the twelve Probit/Selection regressions, the dummy variable for living in Outer London has a negative coefficient, indicating that living there tends to reduce the probability of being intertemporally poor. However, the coefficient is only significant at the 5% level for the two female regressions in the second era. There is no clear picture with regard to the impact of living in this region on intertemporal poverty, conditional on being poor. The coefficient is negative in half of the Heckman regressions and positive in the other half.

There is very tentative evidence to suggest that living in the rest of the North-West reduces both the probability of being poor and also, conditional on being poor, the extent of intertemporal poverty. The coefficients of the variable 'r11' are negative in all the Probit/Selection regressions and in all the Heckman regressions. However, none of these coefficients are statistically significant at conventional levels.

The dummy variable for living in the rest of Yorkshire and Humberside ('r14') was omitted from our Probit/Selection regressions but included in our Heckman models. There is some evidence to suggest that, conditional upon being poor, living in this region tends to increase the extent of poverty. The variable has a positive coefficient in all but one of the Heckman regressions. However, it is only statistically significant at the 5% level in the two regressions for males in the second era.

Our results provide tentative evidence to suggest that living in Tyne & Wear increased the probability of being poor during the first two eras - but not during the third. The coefficients of the variable 'r15' are positive in all the Probit/Selection regressions for the first two eras and significantly so at the 5% level in a quarter of these. In the third era the coefficients are negative (but not significantly so) for all the Probit/Selection regressions. Overall, there is no compelling evidence regarding the impact of living in Tyne & Wear on the extent of intertemporal poverty, conditional upon being poor. However, the variable's coefficients are positive and statistically significant for the two male Heckman regressions for the first era, indicating a possible detrimental impact of living in this region for males during that

period of time.

There is no real evidence to suggest that living in East Anglia has any significant impact on either the probability of being intertemporally poor or, conditional on being poor, on its extent. The coefficient of ‘r5’ is negative in two thirds of the Probit/Selection regressions but is not statistically significant in any of them. It is positive in three quarters of the Heckman regressions but again, not statistically significant in any.

The dummy variable for living in conurbations in the West Midlands only came into our Heckman regressions. However, there is no real evidence to suggest that, conditional upon being poor, living in this region tends to either increase or decrease the extent of intertemporal poverty.

No compelling results emerge either regarding the impact of living in the Rest of the North on poverty. The ‘r16’ variable coefficients are negative for all the male Probit/Selection regressions but positive for all the female Probit regressions in the second and third eras. This might be interpreted as suggesting that the impact on the probability of being poor of living in this region is different for the two sexes, but none of the results are statistically significant so such an inference would be rather tenuous. The variable has a negative coefficient in three quarters of the corresponding Heckman regressions, but none of the coefficients are significant so there is no clear impact of living in this region on the extent of intertemporal poverty, conditional upon being poor.

### 3.4 Conclusions

It has long been recognised that poverty is a dynamic phenomenon. In the last few years there have been significant developments in the theoretical literature on attempting to measure it as such. The poverty analyst now has at his disposal a number of tools specifically designed to evaluate poverty over a relatively long time-frame, in a more nuanced way than is possible with conventional static indicators of poverty.

In this chapter, two of the indices proposed in this recent literature were applied to measure intertemporal poverty in Great Britain, using data from the BHPS. As far as we are aware, this is the first study to apply any of the new intertemporal poverty measures to this data-set. Using measures introduced in Chapter 2 and, as a special case of these, those of Foster (2009), we analysed regional patterns of poverty in Great Britain during three separate eras - 1991 to 1995, 1996 to 2000 and 2001 to 2005. The new measures provide a richer picture of poverty than can be captured simply by using static, annual, measures of poverty.

Having estimated the overall levels of intertemporal poverty in Great Britain, we then analysed the determinants of an individual’s level of intertemporal poverty. In

order to account for possible sample selection bias, where the determinants of being intertemporally poor or non-poor might differ from the determinants of the overall severity of intertemporal poverty, we adopted the Heckman two-step selection model. This well-known technique, and related approaches, have been used in a number of studies on the determinants of poverty and living standards. However, to the best of our knowledge, this is the first study which uses it to model the determinants of the severity of poverty, conditional upon being poor.

There is some evidence to support the suitability of our method. Firstly, in almost half of our Heckman regressions, the inverse Mills ratio was found to be statistically significant, which suggests that a simple OLS approach would have suffered from sample selection bias and yielded biased and inconsistent estimates. Secondly, we subjected all our regressions to two different kinds of link tests to test for misspecification. In all cases, the null hypothesis of no misspecification was strongly rejected. Thirdly, in most cases, the signs of the explanatory variables in our regressions were broadly in line with what one might expect. All this gives us a degree of confidence in our method and in our results.

We found that being unemployed, retired, economically inactive or a student all increase one's probability of being intertemporally poor and further, conditional on being intertemporally poor, tend to increase its severity. The type of housing tenure (especially home ownership) and the level of educational attainment were also found to be broadly important determinants of the probability of being intertemporally poor and, conditional on this, of the extent of intertemporal poverty.

It can also be observed from our results that, again broadly speaking, the magnitudes of the coefficients of the explanatory variables tend to be larger for females than for males. This is true both for variables which tend to increase poverty, such as unemployment and being retired, and for those variables which tend to reduce poverty, such as home ownership and educational attainment. In the case of retirement, we have discussed a plausible explanation for why the magnitude of the coefficient might be expected to be higher for females. It is not, however, immediately apparent why this feature should hold across such a range of variables. Further analysis of this phenomenon could form the basis for future research.

Our results and analysis provide some evidence to suggest that age has a non-linear impact on the extent of intertemporal poverty. Intertemporal poverty tends to decrease with age initially but later increase, with possible turning points in and around the age at which people typically retire.

Another interesting observation emerged from comparing the fit of each model for  $P_{DRZ}$  with the corresponding one for  $P_{FOS}$ . Since the explanatory variables and the data are exactly the same for both measures, the fact that the R-squared values were higher in every regression for  $P_{DRZ}$  suggests that it is a more precise estimator of intertemporal poverty than  $P_{FOS}$  - at least in so far as intertemporal poverty is

satisfactorily explained by the variables in our models. It would be interesting to see whether a similar result emerges when these measures are applied to other data-sets. It would also be interesting to explore whether, in general, measures which penalise the chronicity, or ‘bunching,’ of poor spells, typically provide more precise (in the same sense as above) estimates of intertemporal poverty than those, such as  $P_{FOS}$ , which do not.

This study differs from earlier works that have used the BHPS data-set by measuring poverty over a longer term, in a more nuanced way. As such, the results in this chapter are not directly comparable with those of other studies. Nevertheless, we conclude by noting that a number of our findings do seem broadly consistent with those of earlier papers. For example, our results on the detrimental impact of retirement echo those of Antolín et al. (1999) and Devicienti (2001, 2002), who found that individuals who live in households with pensioners are particularly likely to spend long periods in poverty. Unemployment was found to be the single most important explanatory variable for intertemporal poverty in this study. This is at least broadly consistent with Jenkins and Rigg (2001)’s findings on shorter poverty spells being associated with having more working individuals in the household. Finally, our results on an apparent non-linear impact of age on intertemporal poverty is consistent with the results of Devicienti (2011), who found that young and elderly individuals face a relatively high risk of remaining poor for long periods.

## 3.A Appendix

### 3.A.1 Dummy Variable Sample Sizes

Table 3.15: Dummy Variable Sample Sizes For Poor And Non-Poor Males

Variable	Era 1		Era 2		Era 3	
	Poor	Non-Poor	Poor	Non-Poor	Poor	Non-Poor
retire	111	164	100	229	218	393
mortgage	401	1,343	321	1,547	490	2,268
degree	19	151	17	185	36	336
hndcteach	18	116	11	159	34	246
alevel	72	302	82	379	117	552
inactive	42	41	61	53	89	79
evermarryliv	500	1,273	437	1,510	738	2,250
hidegree	2	34	6	52	9	107
unemploy	91	47	70	41	49	35
laha	337	158	336	170	496	232
student	26	38	28	56	29	70
ownedout	169	323	163	404	302	711
immigrant	29	61	0	1	5	16
r1	40	52	22	54	9	34
r2	52	117	44	142	40	103
r3	156	365	133	441	95	446
r4	78	176	75	222	79	201
r5	48	84	49	88	37	94
r6	122	158	101	180	82	173
r7	45	66	32	75	28	67
r9	25	88	20	85	34	79
r10	17	40	23	53	31	45
r11	33	86	42	103	29	110
r12	31	56	32	65	16	71
r13	38	61	41	71	25	52
r14	31	66	38	85	26	86
r15	33	40	32	46	22	37
r16	33	92	33	108	28	80



Table 3.16: Dummy Variable Sample Sizes For Poor And Non-Poor Females

Variable	Era 1		Era 2		Era 3	
	Poor	Non-Poor	Poor	Non-Poor	Poor	Non-Poor
retire	99	133	90	206	209	374
mortgage	426	1,349	368	1,556	568	2,264
degree	15	132	15	162	36	350
hndcteach	9	106	19	126	49	202
alevel	64	201	72	285	141	476
inactive	308	264	270	263	434	360
evermarryliv	643	1,344	557	1,568	927	2,369
hidegree	2	17	5	31	5	84
unemploy	32	28	26	23	46	24
laha	422	169	398	199	659	250
student	24	37	37	72	61	75
ownedout	170	338	159	382	305	708
immigrant	40	73	1	4	16	19
r1	35	57	17	51	7	38
r2	49	117	50	135	42	122
r3	184	410	154	489	110	459
r4	91	181	82	218	80	208
r5	57	92	57	95	39	91
r6	115	156	110	179	95	180
r7	53	67	48	87	34	69
r9	31	75	23	78	37	70
r10	27	41	28	45	23	43
r11	40	93	53	108	50	109
r12	34	54	39	54	22	64
r13	49	62	40	70	31	48
r14	29	62	34	75	32	75
r15	29	34	25	45	16	40
r16	34	88	38	85	39	72

### 3.A.2 Regression Results

Table 3.17: Regressions For Logged PDRZ Measures For Males For 1991-1995

Variables	Probit	Heckman	OLS
age	-0.026	-0.098*	-0.063
agesq	0.0003*	0.001	0.000
retire	0.312*	1.367**	0.956**
mortgage	-0.574**	-1.312*	-0.538
degree	-0.659**	-1.279	-0.356
hndcteach	-0.496**	-0.305	0.500
alevel	-0.300**	-0.902**	-0.473*
nkids	0.360**	0.582*	0.149
inactive	0.594**	1.465**	0.688*
evermarryliv	0.123	1.535**	1.361**
hidegree	-1.156**	-0.694	1.058**
unemploy	1.054**	2.339**	1.138**
laha	0.192	0.582	0.405
student	0.545**	1.906**	1.177**
ownedout	-0.183	-0.349	-0.106
immigrant		0.154	0.197
r1	0.029	0.257	0.284
r2	0.078	0.250	0.166
r3	-0.072	-0.135	-0.012
r4	-0.037	0.150	0.196
r5	0.093	0.328	0.233
r6		-0.033	-0.003
r7		0.103	0.193
r9	-0.180		
r10		-1.022	-1.033
r11	-0.182	-0.759	-0.452
r12	-0.047	1.164*	1.245**
r13	0.196		
r14		0.451	0.482
r15	0.226	1.091*	0.836*
r16	-0.294	-1.010	-0.610
constant	-0.272	-4.384**	-2.466**
No. individuals	2,047	577	577
Mills Ratio $\lambda$		2.000	
R-squared		0.2178	0.2114

Table 3.18: Regressions For Logged PDRZ Measures For Females For 1991-1995

Variables	Probit	Heckman	OLS
age	-0.031*	-0.138**	-0.079**
agesq	0.0003*	0.001**	0.001*
retire	0.625**	1.590*	0.214
mortgage	-0.648**	-1.706**	-0.385
degree	-0.802**	-2.447**	-0.724
hndcteach	-0.952**	-1.727	0.380
alevel	-0.265**	-0.800*	-0.283
nkids	0.280**	0.699**	0.211**
inactive	0.647**	2.129**	0.811**
evermarryliv	0.278*	0.574	0.087
hidegree	-0.670	-2.053	-0.962
unemploy	0.816**	2.742**	1.121**
laha	0.281*	0.784	0.360
student	0.537**	1.377	0.307
ownedout	-0.335*	-0.992	-0.302
immigrant		0.004	0.066
r1	-0.089	-0.405	-0.231
r2	-0.102	-0.756	-0.508
r3	-0.087	-0.414	-0.204
r4	-0.142	-0.539	-0.218
r5	-0.029	-0.202	-0.092
r6		-0.375	-0.335
r7		-0.336	-0.205
r9	-0.090		
r10		-0.033	-0.017
r11	-0.219	-1.037	-0.525
r12	-0.022	0.883	0.951**
r13	0.220		
r14		0.260	0.374
r15	0.414*	1.009	0.232
r16	-0.188	-0.386	0.047
constant	-0.032	-3.827**	-1.237*
No. individuals	2,228	723	723
Mills Ratio $\lambda$		3.158*	
R-squared		0.1855	0.1734

Table 3.19: Regressions For Logged PDRZ Measures For Males For 1996-2000

Variables	Probit	Heckman	OLS
age	-0.049**	-0.065	-0.055
agesq	0.0006**	0.001	0.001
retire	0.214	0.583	0.539
mortgage	-0.754**	-0.701	-0.548
degree	-0.566**	0.240	0.368
hndcteach	-0.746**	-0.714	-0.537
alevel	-0.151	0.104	0.134
nkids	0.378**	0.429	0.362**
inactive	0.915**	0.724	0.553*
evermarryliv	0.450**	0.703	0.608
hidegree	-0.497*	0.074	0.170
unemploy	1.202**	1.493*	1.276**
laha	0.051	-0.115	-0.118
student	0.757**	0.802	0.646
ownedout	-0.174	0.095	0.130
r1	-0.037	-0.557	-0.534
r2	-0.253	-0.524	-0.486
r3	-0.157	-0.143	-0.120
r4	-0.077	-0.319	-0.304
r5	0.271	0.534	0.486
r6		0.143	0.139
r7		0.142	0.134
r9	-0.023		
r10		-0.151	-0.128
r11	-0.121	-0.479	-0.461
r12	0.040	0.871	0.870*
r13	-0.036		
r14		0.975*	0.968*
r15	0.044	0.644	0.626
r16	-0.016	-0.715	-0.715
constant	-0.122	-3.296**	-3.029**
No. individuals	2,238	495	495
Mills Ratio $\lambda$		0.301	
R-squared		0.1548	0.1546

Table 3.20: Regressions For Logged PDRZ Measures For Females For 1996-2000

Variables	Probit	Heckman	OLS
age	-0.030*	-0.032	-0.014
agesq	0.0003*	0.000	0.000
retire	0.413**	1.140**	0.913*
mortgage	-0.922**	-1.238*	-0.703**
degree	-0.680**	-0.604	-0.186
hndcteach	-0.447**	-0.149	0.117
alevel	-0.316**	-0.062	0.103
nkids	0.376**	0.513**	0.325**
inactive	0.642**	1.357**	0.997**
evermarryliv	-0.226	-0.317	-0.201
hidegree	-0.298	-1.149	-0.964
unemploy	1.053**	1.848**	1.265**
laha	0.092	0.136	0.114
student	0.198	-0.214	-0.339
ownedout	-0.346*	-0.354	-0.158
r1	-0.383	-0.448	-0.251
r2	-0.336*	-0.615	-0.475
r3	-0.300**	-0.296	-0.160
r4	-0.232*	-0.519	-0.409
r5	-0.039	0.397	0.403
r6		-0.077	-0.091
r7		0.077	0.073
r9	-0.307		
r10		-0.360	-0.362
r11	-0.270	-0.400	-0.286
r12	0.152	0.559	0.458
r13	-0.102		
r14		0.540	0.552
r15	0.015	-0.011	-0.065
r16	0.051	-0.309	-0.374
constant	0.428	-3.305**	-2.809**
No. individuals	2,403	660	660
Mills Ratio $\lambda$		0.892	
R-squared		0.1814	0.1796

Table 3.21: Regressions For Logged PDRZ Measures For Males For 2001-2005

Variables	Probit Male	Heckman Male	OLS
age	-0.027*	0.008	0.032
agesq	0.0004**	0.000	-0.000
retire	0.172	0.686*	0.534
mortgage	-0.670**	-1.009	-0.437
degree	-0.570**	-0.908	-0.357
hndcteach	-0.514**	-0.340	0.116
alevel	-0.188*	-0.063	0.107
nkids	0.317**	0.417	0.172*
inactive	0.817**	1.282	0.646**
evermarryliv	0.124	0.291	0.170
hidegree	-0.676**	-1.268	-0.629
unemploy	0.891**	1.466	0.833**
laha	0.233	-0.075	-0.226
student	0.477*	1.640*	1.263*
ownedout	-0.290*	-0.208	0.029
immigrant		1.574	1.505**
r1	-0.394	0.284	0.633
r2	-0.002	0.781	0.804*
r3	-0.323**	-0.772	-0.480
r4	-0.086	-0.177	-0.107
r5	-0.095	0.371	0.468
r6		-0.314	-0.284
r7		-0.811	-0.805
r9	0.127		
r10		-0.174	-0.142
r11	-0.153	-0.573	-0.443
r12	-0.584*	0.906	1.405**
r13	-0.031		
r14		-0.045	-0.056
r15	-0.217	-0.104	0.033
r16	-0.114	-0.018	0.069
constant	-0.268	-5.357**	-4.226**
No. individuals	2,903	682	682
Mills Ratio $\lambda$		1.215	
R-squared		0.0812	0.0798

Table 3.22: Regressions For Logged PDRZ Measures For Females For 2001-2005

Variables	Probit Female	Heckman Female	OLS
age	-0.002	0.037	0.045
agesq	0.000	-0.000	-0.000
retire	0.429**	1.490**	0.892**
mortgage	-0.680**	-1.546**	-0.604*
degree	-0.725**	-1.652**	-0.516
hndcteach	-0.354**	-0.489	-0.009
alevel	-0.219**	-0.432	-0.160
nkids	0.297**	0.606**	0.262**
inactive	0.711**	1.583**	0.648**
evermarryliv	-0.360**	-0.754*	-0.386
hidegree	-0.757**	0.582	1.777**
unemploy	0.803**	2.108**	1.160**
laha	0.359**	-0.024	-0.379
student	0.737**	1.861**	0.862*
ownedout	-0.334**	-0.649	-0.188
immigrant		0.265	0.220
r1	-0.522	-1.140	-0.514
r2	-0.126	0.500	0.726*
r3	-0.336**	-1.353**	-0.861**
r4	-0.091	-0.249	-0.107
r5	-0.119	0.018	0.215
r6		-0.295	-0.255
r7		-0.475	-0.427
r9	0.260		
r10		-0.451	-0.412
r11	-0.102	-0.484	-0.303
r12	-0.373	-0.152	0.342
r13	-0.021		
r14		0.047	0.100
r15	-0.225	-1.085	-0.876
r16	0.096	0.168	0.041
constant	-0.344	-5.854**	-3.846**
No. individuals	3,135	882	882
Mills Ratio $\lambda$		2.029*	
R-squared		0.1081	0.1017

Table 3.23: Regressions For Logged PFOS Measures For Males For 1991-1995

Variables	Probit	Heckman	OLS
age	-0.026	-0.062*	-0.036
agesq	0.0003*	0.001	0.000
retire	0.312*	0.670*	0.366
mortgage	-0.574**	-0.817*	-0.243
degree	-0.659**	-1.024*	-0.340
hndcteach	-0.496**	-0.197	0.399
alevel	-0.300**	-0.576*	-0.259
nkids	0.360**	0.367*	0.046
inactive	0.594**	0.970*	0.395*
evermarryliv	0.123	1.050**	0.921**
hidegree	-1.156**	-0.809	0.489*
unemploy	1.054**	1.607**	0.717**
laha	0.192	0.429	0.298
student	0.545**	1.384**	0.844**
ownedout	-0.183	-0.222	-0.042
immigrant		0.161	0.193
r1	0.029	0.266	0.287
r2	0.078	0.325	0.263
r3	-0.072	0.054	0.145
r4	-0.037	0.213	0.247
r5	0.093	0.230	0.159
r6		-0.031	-0.010
r7		0.038	0.104
r9	-0.180		
r10		-0.606	-0.614
r11	-0.182	-0.407	-0.180
r12	-0.047	0.762*	0.822**
r13	0.196		
r14		0.304	0.327
r15	0.226	0.741*	0.552**
r16	-0.294	-0.555	-0.259
constant	-0.272	-4.138**	-2.717**
No. individuals	2,047	577	577
Mills Ratio $\lambda$		1.482*	
R-squared		0.1838	0.1758



Table 3.24: Regressions For Logged PFOS Measures For Females For 1991-1995

Variables	Probit	Heckman	OLS
age	-0.031*	-0.094*	-0.047*
agesq	0.0003*	0.001*	0.0004*
retire	0.625**	1.272*	0.152
mortgage	-0.648**	-1.259*	-0.183
degree	-0.802**	-1.881**	-0.478
hndcteach	-0.952**	-1.388	0.327
alevel	-0.265**	-0.563	-0.142
nkids	0.280**	0.473*	0.076
inactive	0.647**	1.599**	0.527**
evermarryliv	0.278*	0.483	0.086
hidegree	-0.670	-1.594	-0.706
unemploy	0.816**	2.034**	0.715*
laha	0.281*	0.542	0.197
student	0.537**	1.197	0.326
ownedout	-0.335*	-0.784	-0.222
immigrant		0.185	0.236
r1	-0.089	-0.128	0.013
r2	-0.102	-0.447	-0.246
r3	-0.087	-0.218	-0.047
r4	-0.142	-0.373	-0.112
r5	-0.029	-0.110	-0.020
r6		-0.278	-0.245
r7		-0.412	-0.305
r9	-0.090		
r10		0.014	0.027
r11	-0.219	-0.740	-0.323
r12	-0.022	0.560	0.616**
r13	0.220		
r14		0.210	0.303
r15	0.414*	0.810	0.177
r16	-0.188	-0.276	0.076
constant	-0.032	-3.827**	-1.718**
No. individuals	2,228	723	723
Mills Ratio $\lambda$		2.572*	
R-squared		0.1678	0.1494

Table 3.25: Regressions For Logged PFOS Measures For Males For 1996-2000

Variables	Probit	Heckman	OLS
age	-0.049**	-0.035	-0.024
agesq	0.001**	0.000	0.000
retire	0.214	0.210	0.160
mortgage	-0.754**	-0.730	-0.556*
degree	-0.566**	0.414	0.561*
hndcteach	-0.746**	-0.657	-0.455
alevel	-0.151	0.040	0.075
nkids	0.378**	0.250	0.173**
inactive	0.915**	0.572	0.378
evermarryliv	0.450**	0.394	0.285
hidegree	-0.497*	0.078	0.188
unemploy	1.202**	1.148*	0.901**
laha	0.051	-0.292	-0.294
student	0.757**	0.408	0.230
ownedout	-0.174	-0.080	-0.040
r1	-0.037	-0.338	-0.312
r2	-0.253	-0.328	-0.284
r3	-0.157	-0.063	-0.036
r4	-0.077	-0.316	-0.299
r5	0.271	0.525	0.471
r6		0.140	0.136
r7		0.218	0.210
r9	-0.023		
r10		-0.018	0.010
r11	-0.121	-0.395	-0.375
r12	0.040	0.727*	0.726**
r13	-0.036		
r14		0.807*	0.799**
r15	0.044	0.473	0.453
r16	-0.016	-0.424	-0.423
constant	-0.122	-3.155**	-2.851**
No. individuals	2,238	495	495
Mills Ratio $\lambda$		0.343	
R-squared		0.1398	0.1393

Table 3.26: Regressions For Logged PFOS Measures For Females For 1996-2000

Variables	Probit	Heckman	OLS
age	-0.030*	-0.025	-0.008
agesq	0.0003*	0.000	0.000
retire	0.413**	0.738**	0.527*
mortgage	-0.922**	-0.988**	-0.491**
degree	-0.680**	-0.344	0.045
hndcteach	-0.447**	0.013	0.260
alevel	-0.316**	-0.139	0.014
nkids	0.376**	0.342**	0.166**
inactive	0.642**	0.953**	0.618**
evermarryliv	-0.226	-0.316	-0.209
hidegree	-0.298	-0.302	-0.129
unemploy	1.053**	1.299**	0.758**
laha	0.092	-0.028	-0.048
student	0.198	-0.125	-0.242
ownedout	-0.346*	-0.354	-0.172
r1	-0.383	-0.234	-0.051
r2	-0.336*	-0.357	-0.227
r3	-0.300**	-0.200	-0.073
r4	-0.232*	-0.438*	-0.335
r5	-0.039	0.305	0.311
r6		-0.023	-0.036
r7		0.135	0.131
r9	-0.307		
r10		-0.131	-0.134
r11	-0.270	-0.288	-0.182
r12	0.152	0.390	0.296
r13	-0.102		
r14		0.496	0.507*
r15	0.015	-0.045	-0.096
r16	0.051	-0.103	-0.164
constant	0.428	-2.981**	-2.520**
No. individuals	2,403	660	660
Mills Ratio $\lambda$		0.831	
R-squared		0.1446	0.1413

Table 3.27: Regressions For Logged PFOS Measures For Males For 2001-2005

Variables	Probit	Heckman	OLS
age	-0.027*	0.012	0.032
agesq	0.0004**	-0.000	-0.000
retire	0.172	0.271	0.153
mortgage	-0.670**	-0.910	-0.463*
degree	-0.570**	-0.440	-0.009
hndcteach	-0.514**	-0.443	-0.086
alevel	-0.188*	0.035	0.168
nkids	0.317**	0.229	0.037
inactive	0.817**	0.658	0.160
evermarryliv	0.124	0.312	0.217
hidegree	-0.676**	-1.021	-0.521
unemploy	0.891**	1.188*	0.693**
laha	0.233	-0.263	-0.381*
student	0.477*	1.328**	1.033**
ownedout	-0.290*	-0.310	-0.124
immigrant		1.132	1.078**
r1	-0.394	-0.083	0.190
r2	-0.002	0.578*	0.596**
r3	-0.323**	-0.550	-0.321
r4	-0.086	-0.137	-0.082
r5	-0.095	0.146	0.222
r6		-0.232	-0.209
r7		-0.488	-0.483
r9	0.127		
r10		-0.059	-0.033
r11	-0.153	-0.279	-0.177
r12	-0.584*	0.419	0.809**
r13	-0.031		
r14		0.126	0.117
r15	-0.217	-0.322	-0.215
r16	-0.114	0.002	0.070
constant	-0.268	-4.489**	-3.604**
No. individuals	2,903	682	682
Mills Ratio $\lambda$		0.951	
R-squared		0.0653	0.0635

Table 3.28: Regressions For Logged PFOS Measures For Females For 2001-2005

Variables	Probit	Heckman	OLS
age	-0.002	0.020	0.026
agesq	0.000	-0.000	-0.000
retire	0.429**	0.742*	0.332
mortgage	-0.680**	-1.176**	-0.530**
degree	-0.725**	-0.909*	-0.130
hndcteach	-0.354**	-0.291	0.038
alevel	-0.219**	-0.277	-0.091
nkids	0.297**	0.360**	0.124**
inactive	0.711**	0.927**	0.286*
evermarryliv	-0.360**	-0.291	-0.039
hidegree	-0.757**	0.573	1.392**
unemploy	0.803**	1.346**	0.697**
laha	0.359**	-0.194	-0.437**
student	0.737**	1.143**	0.458
ownedout	-0.334**	-0.519	-0.204
immigrant		0.263	0.232
r1	-0.522	-0.859	-0.429
r2	-0.126	0.389	0.544**
r3	-0.336**	-0.840**	-0.503*
r4	-0.091	-0.183	-0.086
r5	-0.119	-0.069	0.065
r6		-0.199	-0.172
r7		-0.232	-0.199
r9	0.260		
r10		-0.275	-0.248
r11	-0.102	-0.175	-0.051
r12	-0.373	-0.176	0.162
r13	-0.021		
r14		0.173	0.209
r15	-0.225	-0.790	-0.646
r16	0.096	0.102	0.015
constant	-0.344	-4.544**	-3.168**
No. individuals	3,135	882	882
Mills Ratio $\lambda$		1.391*	
R-squared		0.0748	0.0685

### 3.A.3 Misspecification Test Results

Table 3.29: Results of Misspecification Tests For Heckman Regressions of Logged PDRZ Measures

	p-Values from Standard Link Test	p-Values from Amended Link Test
Males in Era 1	0.266	0.857
Females in Era 1	0.333	0.857
Males in Era 2	0.688	0.784
Females in Era 2	0.532	0.908
Males in Era 3	0.680	0.885
Females in Era 3	0.222	0.967

Table 3.30: Results of Misspecification Tests For Heckman Regressions of Logged PFOS Measures

	p-Values from Standard Link Test	p-Values from Amended Link Test
Males in Era 1	0.175	0.806
Females in Era 1	0.369	0.892
Males in Era 2	0.278	0.606
Females in Era 2	0.668	0.970
Males in Era 3	0.757	0.938
Females in Era 3	0.209	0.993

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