A COMPARISON BETWEEN
OBSERVED AND SIMULATED X-RAY
PROFILES IN CLUSTERS OF
GALAXIES

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By
Julien Dassa-Pizzinat
School of Physics and Astronomy
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Abstract

Clusters of galaxies provide us with crucial information for the understanding of the cosmology of our Universe. Observations are becoming increasingly precise and simulations improving their accuracy, challenging theories. We have analysed how the new Millennium Gas Simulation concurs with theory and observation. We first introduce the basic principles of the ΛCDM cosmological model, including the FRW metric. We then explain how structure formation leads to galaxy and cluster formation. Clusters are the most massive gravitationally stable objects in the Universe; we describe their typical properties and key observables and how the Intracluster medium (a very hot, low density, optically thin, plasma) X-ray emission informs us about the characteristics of these objects. The use and necessity of cosmological simulations is also discussed. We give an overview of the dark matter Millennium Simulation and the Millennium Gas Simulations which also include baryonic matter to the equation. We briefly go through every step necessary to produce these simulations and show simulated X-ray surface brightness maps eventually produced. We study the catalogues associated to each cluster, then we test the ability of the simulation to reproduce the expected mass function and scaling relations of clusters of galaxies. Our main results focus on the surface brightness profiles of clusters; we define this property as well as the fitting model known as β model. A series of case studies leads us to discuss whether the key parameters are correlated with one another and what they can tell us about the clusters. We establish that working with a large number of clusters will soften the influence of the radial range we are using to fit the β model. Finally we compare the simulated sample to three recent observational data sets, and show that they are in reasonable agreement.
Declaration

No portion of the work referred to in this thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institution of learning.

Julien Dassa Pizzinat
University of Manchester
Jodrell Bank Observatory
Macclesfield
Cheshire
SK11 9DL
U.K.
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The Author

The author was born in Pithiviers and grew up near Paris. He passed the scientific baccalaureate and pursued his studies in the field of physics at the Universite Pierre et Marie Curie (Paris 6) where he passes his Licence de Physique Fondamentale. He then got involved in the ERASMUS exchange program and move to Manchester where he stays and graduate in June 2011 with a MPhys, Physics with Theoretical Physics. He began his MSc in September 2011 in the Astrophysics Department of the University of Manchester with Dr Scott Kay for supervisor. This thesis is the result of his research work during that year.
The most exciting phrase to hear in science, the one that heralds the most discoveries, is not "Eureka!" but "That’s funny..."

-Isaac Asimov
Chapter 1

Introduction

1.1 The Λ cold dark matter paradigm

To date, no paradigm could bring a more satisfying description of our Universe than the Cold Dark Matter (CDM; Blumenthal et al., 1984) model with a cosmological constant. This very recent model eventually prevailed among the community mostly because of three aspects (Baugh, 2006): the cold dark matter particle has many potential candidates; its predictive power is huge and last, but not least, its predictions happen to be strikingly successful on large scales.

This model relies on the cosmological principle: our Universe is homogeneous, isotropic and we do not hold any special position in it. Those are the three most fundamental assumptions we make.

The addition of a cosmological constant to the CDM model, even though it was originally postulated and then withdrawn by Albert Einstein for totally different reasons, is fairly recent. Arguments in favour of dark energy were already stated in the late 80s (Yoshii & Takahara, 1988) but the strongest was the Hubble diagram of distant type-Ia supernovae which showed that the expansion of the Universe is accelerating (Riess et al., 1998; Perlmutter et al., 1999).

The subsequent ΛCDM model, where Λ is the cosmological constant, the dark energy component, is now considered as standard. This model could still change but, so far, this description of our Universe have explained with great success the
cosmic microwave background (CMB) and the large scale structures (Benson, 2010).

![Pie chart showing the distribution of components of the Universe today and 13.7 billion years ago. Source: NASA.]

Figure 1.1: The distribution between the components of the Universe today and 13.7 billions years ago. Source: NASA.

In ΛCDM, we assume a Friedmann-Robertson-Walker (FRW) metric where space expands, a behaviour which is observed. This expansion is characterized by the Hubble parameter which is a function of the scale factor $a(z) = 1/(1 + z)$ and its time derivative:

$$H = \frac{\dot{a}}{a}$$  \hspace{1cm} (1.1)

The Hubble parameter is formalised by the Friedmann equation:

$$H^2 = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3} - \frac{k c^2}{a^2}$$  \hspace{1cm} (1.2)

where $G$ is the universal gravitational constant, $\rho$ the density of matter, $\Lambda$ the cosmological constant, $k$ the curvature and $c$ the speed of light. Among other things, the Friedmann equation allows us to define a few most useful parameters, the cosmological parameters, which describe the components of the Universe and their density.
First, we need to consider the Friedmann equation for $\Lambda = k = 0$, a Universe without cosmological constant and with Euclidean geometry:

$$H^2 = \frac{8\pi G}{3}\rho_{cr}$$ \hspace{1cm} (1.3)

This leads us to the definition of the critical density:

$$\rho_{cr} = \frac{3H^2}{8\pi G}$$ \hspace{1cm} (1.4)

In the case where $z = 0$, we have $\rho_0$ the density now and the equation (1.2) becomes:

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1$$ \hspace{1cm} (1.5)

where $\Omega_m = \rho_0/\rho_{cr,0}$, $\Omega_\Lambda = \Lambda/3H_0^2$ and $\Omega_k = -kc^2/H_0^2$ represent the matter, dark energy and curvature respectively. Note that $\Omega_k = 0$ in our case and that:

$$\Omega_m = \Omega_b + \Omega_{dm}$$ \hspace{1cm} (1.6)

where $\Omega_b$ and $\Omega_{dm}$ are the baryonic and dark matter component respectively.

This summarizes the composition of the Universe: dark energy, dark matter and baryonic matter (see figure 1.1).

These cosmological parameters (see table 1.1) can be constrained using multiple approaches (Benson, 2010) such as studies of the CMB (Dunkley et al., 2009), large scale structures (Tegmark et al., 2004; Cole et al., 2005; Tegmark et al., 2006; Percival et al., 2007a,b), the type Ia supernovae magnitude-redshift relation (Kowalski et al.,

<table>
<thead>
<tr>
<th>Parameter</th>
<th>WMAP Seven-year ML</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_b$</td>
<td>0.0451</td>
</tr>
<tr>
<td>$\Omega_{dm}$</td>
<td>0.226</td>
</tr>
<tr>
<td>$\Omega_\Lambda$</td>
<td>0.729</td>
</tr>
<tr>
<td>$\sigma_8$</td>
<td>0.809</td>
</tr>
<tr>
<td>$H_0$</td>
<td>70.3 $kms^{-1}Mpc^{-1}$</td>
</tr>
<tr>
<td>$t_0$</td>
<td>13.79 Gyr</td>
</tr>
</tbody>
</table>

Table 1.1: The most recent data for the cosmological parameters from WMAP7 (Komatsu et al., 2011).
Constraining properly the cosmological parameters is crucial for cosmological simulations as we will see later in this thesis.

1.2 Structure formation

Even if the observational data concurs with the cosmological principle, we know of the existence of inhomogeneities even on large scales, referred to as large-scale structures. These structures are vital to our understanding of galaxies and clusters of galaxies and find their origins in the early Universe.

After recombination, when \( z < 1000 \), the roughly homogeneous Universe contains overdense regions. From then and until galaxy formation occurs (at \( z \approx 10 \)), the matter is only subject to gravitational collapse since the dark matter is, by definition, pressureless and the baryonic matter consists of gas without much pressure at that point.

These overdensities, also called perturbations, grow by this process. When the density of one of these regions becomes higher than the critical density of the Universe, \( \rho_{cr} \) defined in equation (1.4), it decouples from the expansion of the Universe to form a halo, an area in a quasi-stable state where gravity is balanced by the random motion of its particles (Benson, 2010).

This phenomenon had been theorized in the Spherical Top-Hat Collapse Model (STHCM; Press & Schechter, 1974). In this model we consider an overdense sphere (i.e. where the density, \( \rho \), of the sphere is higher than the critical density, \( \rho_{cr} \) ) in space, the curvature in this sphere will be positive, leading to a collapse. The sphere will not collapse to a point but will virialize, with random motions stopping the process. The virial theorem can be used to characterise a halo or predict when it should collapse:

\[
2K + V = 0
\]  

(1.7)

where \( K \) is the kinetic energy and \( V \) the potential energy of the halo. Then the
virial radius, the radius of the virialized region, can be defined as:

\[ r_{\text{vir}} = \frac{r_{\text{ta}}}{2} \]  

(1.8)

where \( r_{\text{ta}} \) is the turnaround radius, the radius at which the overdensity region decouples from the expansion. The mean density of the typical virialized halo is approximately of \( 200 \rho_{\text{cr}} \). Now, we can define the mass of a cluster within the STHCM as:

\[ M = \frac{4}{3} \pi r_{\text{vir}}^3 \Delta_c \rho_{\text{cr}} \]  

(1.9)

where \( \Delta_c \rho_{\text{cr}} \) is the mean density of a spherical volume of radius \( r_{\text{vir}} \) (Bryan & Norman, 1998). \( \Delta_c \) depends on cosmology and can be obtained by solving the collapse of a spherical top-hat perturbation under the assumption that the cluster has just virialized (Peebles, 1980) and is \( \Delta_c = 18 \pi^2 \) for \( \Omega_m = 0 \).

At this point, we expect the baryonic matter to accrete within the deep potential well of the dark matter halo, leading to an accretion shock (Binney, 1977). The subsequent cooling allows galaxy formation to occur. In haloes where \( M < 10^{12} M_\odot \), cooling is very efficient but in cluster haloes, where \( M > 10^{14} M_\odot \), most baryons are in shocked phase. In the latter case, the stellar mass represents only \( 12 \pm 2\% \) of the baryonic mass because radiative cooling is overcome by feedback (Giodini et al., 2009; Allen et al., 2011).

### 1.3 Clusters of galaxies

Clusters of galaxies are the largest stable structures in the observable Universe. They are formed through the subsequent mergers of the haloes and are the most massive. Their radius is about one megaparsec, their mass equals \( 10^{13} \) solar mass or more, and they contain between 100 and 1000 galaxies. The extremely deep potential well makes them quasi-stable closed systems. They happen to be very recent objects with a formation redshift rarely reaching \( z = 2 \). This statement was predictable within the \( \Lambda \)CDM model, as it is believed that structures grow even more massive.
The baryons which did not cool down to form galaxies makes up the so-called IntraCluster Medium (ICM; see figure 1.2), a very hot gas \((10^7 - 10^8 \text{ K})\) that we cannot see in the optical spectrum. Two tools are available for ICM observation: the X-ray emission and the Sunyaev-Zel’dovich effect. The latter relies on the detectable distortions in the CMB spectrum, the inverse compton scattering of CMB photons off ICM free electrons, and will not be developed in this document (e.g. Sunyaev & Zeldovich, 1972; Kay et al., 2012).

Instead, we decide to focus on the X-ray surface brightness. Since clusters of galaxies are the largest collections of diffuse, highly ionized baryons directly observable in the X-ray spectrum (Tozzi & Norman, 2001), this is a particularly sensible choice.

The ICM is a hot, optically thin plasma and it reaches the X-ray emission temperature by undergoing gravitational heating. The radiation mechanism involved is mostly thermal bremsstrahlung (see figure 1.3 for a brief definition). This is a convenient tool to detect a cluster and its structure. We can formalize the X-ray
surface brightness:

\[ \Sigma_X(\theta, \phi) = \frac{1}{4\pi(1+z)^4} \int n^2 \Lambda(T, Z)dl \]  

(1.10)

where \( n \) is the number density of the ICM, \( \Lambda \) a cooling function, \( T \) the temperature and \( Z \) the metallicity. The surface brightness highly correlates to the density, which makes it a useful tool to map the mass distribution of both visible and invisible matter in a cluster. It also points out at two principal qualities: the surface brightness is dominated by the cluster (the background vanishes quickly as \( \Sigma_X \) decreases to the \(-4\) inverse power of the redshift, an effect of the expansion of the Universe) and regions with high density stand out because the emissivity correlates to the density squared (Sanderson & Ponman, 2010).

1.4 Aims of the Thesis

In the second section, we shall introduce our main tool: the Millennium Gas Simulation (MGS). We will give a short overview of its basic architecture, how it was conceived and designed. The numerous data generated by this simulation will be summarized, and the most relevant to our work will be explained and analysed thoroughly. We will defend the Millennium Gas Project as a powerful tool for cluster related research. In that spirit, we will study the scaling relations in clusters of
galaxies in both the simulation and the literature and show that they are in good agreement within the theoretical assumptions.

The third section is the bulk of our work and focus on the surface brightness profile and the beta model within the Millennium Gas Simulation. We shall explain these in more detail and present our approach on the topic. We will start from specific case studies and then draw general conclusions to try establishing that statistical behaviour and correlations can be found.

Eventually, the fourth section will challenge the Millennium Gas Simulation by comparing it to several sample of observation drawn from the literature and recent survey. We hope to show that significant properties can be seen in all cases and that the Millennium Gas Simulation is successful.

To perform our research, we want to find the most relevant parameters and cuts on the MGS database and understand how they behave within the simulation, how the models fit these. If the models are up to our expectations, it may help us to extrapolate on the properties of clusters.
Chapter 2

The Millennium Gas Project

Earlier, we explained that galaxies and clusters of galaxies are a powerful tool to study the $\Lambda$CDM model and dark energy. To achieve this, observational data needs to be compared to reliable theoretical results. While it is possible to describe the initial linear growth of density perturbation, the structure formation process is highly non-linear. Hence the need for elaborate cosmological simulations.

2.1 Overview

The Millennium Simulation (MS) was designed in that perspective (Springel et al., 2005) and made possible by the fast evolution of computing science. Since we want the simulation to generate both a large enough number of clusters to get the rare objects such as quasars and enough resolution to resolve low-luminosity galaxies, the dynamic range is huge. Even though very elaborate and powerful the Millennium Simulation is based on a dark matter gravitation-only situation.

The hydrodynamics involved in the galaxy formation process still have to be modeled. Recently, Hartley et al. (2008) presented the Millennium Gas Simulations (MGS), a successful attempt at including gas physics within the MS (Stanek et al., 2009; Short et al., 2010; Stanek et al., 2010; Young et al., 2011; Kay et al., 2012). Even more recently, a new generation of MGS was introduced, all our results will be drawn from this last simulation.
2.2 Methodology

2.2.1 Initial conditions

The initial conditions are identical in both MS and MGS first generation, except the latter has fewer particles and contains gas. The cosmological parameters, based on the \( \Lambda \)CDM cosmology, are consistent with combined analyses from WMAP early data (Spergel et al., 2003) and the 2dF Galaxy Redshift Survey (Colless et al., 2001): \( \Omega_m = \Omega_{dm} + \Omega_b = 0.25, \Omega_b = 0.045, \Omega_\Lambda = 0.75, h = 0.73, \sigma_8 = 0.9 \) and \( H_0 = 100h{\text{km}}{\text{s}}^{-1}{\text{Mpc}}^{-1} \), where \( \sigma_8 \) is the root-mean-square (r.m.s.) linear mass fluctuation within a sphere of radius \( 8h^{-1}\text{Mpc} \) extrapolated to \( z = 0 \) (Springel et al., 2005; Hartley et al., 2008).

An unperturbed distribution of \( N = 2160^3 \approx 10^{10} \) particles with collisionless dynamics in MS, \( 500 \times 10^6 \) dark matter particles and the same number of gas particles in MGS, is assumed within a comoving cube of linear size \( 500h^{-1}\text{Mpc} \).

The new MGS is using a slightly different cosmology. The greatest difference is the \( \sigma_8 \approx 0.8 \) (instead of 0.9) in accordance with WMAP7 (Komatsu et al., 2011; summarized in table 1.1). We expect this modification to reduce the number of clusters produced by the simulation. Also, the simulation runs a first time with \( 1080^3 \) dark matter particles within a \( 250h^{-1}\text{Mpc} \) box and a second time including baryonic matter (in the same proportion as the first generation MGS).

This distribution is perturbed by a Gaussian random field with the \( \Lambda \)CDM linear power spectrum as given by the Boltzmann code, CMBFast, proposed in Seljak & Zaldarriaga (1996). The initial redshift is \( z = 127 \) and the simulations run to \( z = 0 \).

2.2.2 Gravity

The action of gravitation on each particle can be computed by three methods (Dolag et al., 2008):

(i) The direct summation is the most straight forward. It consists of summing the contribution of all other particle on each particles for a very accurate result but very demanding computational power since it scales as \( N^2 \).
(ii) Particle-Mesh method relies on solving the Poisson equations on a cubic mesh. Much faster than the direct summation, it is, though, memory intensive and not adapted to system involving a very heterogeneous density distribution (Hockney & Eastwood, 1981).

(iii) The oct-tree proceeds through a tree structure that is illustrated in figure 2.1. It also presents the advantage of being a fast calculation (it scales to $N \log(N)$) but tends not to be as accurate as PM for uniform particle distributions (Barnes & Hut, 1986).

Figure 2.1: A 2D illustration of the oct-tree. The root node is divided in four squares. At each step, each square is divided in four more squares (forming branches) until there is only one particle left (leaves). Then we will calculate the forces by performing a tree walk. First, the algorithm compute the distance from a particle to the center of mass d of the branch. Then is calculate the opening angle $\theta = \frac{x}{d}$ ($x$ is the length of the branch) and compare it to a $\theta_{\text{crit}} \approx 0.5$ typically. If $\theta < \theta_{\text{crit}}$, there is no need for further iterations in the branch and its whole mass is used to calculate the acceleration no matter how many particles it contains. If $\theta > \theta_{\text{crit}}$, the process must continue. This same routine is performed on each particle. Source: Springel et al. (2001)

MS and MGS rely on the contribution of both PM and oct-tree methods, the hybrid TreePM (Xu, 1995). In this method, the long-range potential is calculated with the PM approach while the short-range potential relies on the oct-tree approach, providing the final result with the best of both worlds. Not only does the hybrid method have an improved performance over the oct-tree, it also accurately works for ranges at which the density tends to be more uniform and still have a wide dynamic range. MS and MGS use a customized version of the GADGET2 code which uses the TreePM method (Springel et al., 2005).
Then we can integrate the acceleration to find out how the velocity and position of each particle is evolving. For this the choice of the right time-step is critical. Typically, we choose:

\[ \Delta t = \alpha \sqrt{\frac{\epsilon}{|a|}} \quad (2.1) \]

where \(|a|\) is the acceleration obtained at the previous time step, \(\epsilon\) the scattering length scale and \(\alpha\) the tolerance parameter. For MS, the scattering length is set to \(\epsilon = 5h^{-1}\text{kpc}\).

### 2.2.3 Hydrodynamics

Baryons do not respond only to gravitation, so it is necessary to add hydrodynamics in order to simulate their behaviour properly. They can be described as an ideal fluid which allows to use the following set of hydrodynamic equations:

\[
\frac{dv}{dt} = -\nabla P + \Phi \quad (2.2)
\]

\[
\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{v} = 0 \quad (2.3)
\]

\[
\frac{du}{dt} = -\frac{P}{\rho} \nabla \cdot \mathbf{v} - \frac{\Lambda(u, \rho)}{\rho} \quad (2.4)
\]

which are the Euler equation, continuity equation and the first law of thermodynamics and where \(P\) is the pressure, \(\Phi\) the total flux of mass flowing through surface and \(\mathbf{v}\) the velocity. Solving this set of equation result in a highly non-linear problem. Two categories of numerical schemes were developed (Dolag et al., 2008). The first one is the mesh approach and the second one is the smoothed particle hydrodynamics (SPH; Gingold & Monaghan, 1977). MS and MGS use the latter whose basic principle is to discretise the fluid by mass elements (while the mesh approach does it by volume elements).

Three different models were proposed in the first generation of MGS: the Gravitational Heating Only (GO), the Preheating and radiative Cooling (PC) and the Feedback Only (FO) models. In GO, the only source of gas entropy is gravity. This
is a useful run to generate self-similar clusters. In PC, an excess of entropy is generated by preheating the gas at high redshift. The entropy of every particle is raised to $200\text{keVcm}^2$ at $z = 4$. Ideally, this will break the self-similarity in accordance with the observation.

In FO, the approach is different. It needs two runs to be performed. The first uses a dark matter only simulation based on MS to generate a catalogue of galaxies through halo identification. This catalogue serves as an input in the run of the MGS, that will generate feedback on the corresponding regions. This method provides a reasonably good match to observations but cannot resolve cool core clusters. All these methods are explained in more detail in Short et al. (2010).

### 2.2.4 Halo identification

With velocities and positions, we can identify bound structures, in others terms : dark matter haloes. This is a three stages process. First, friends-of-friends (FOF) program identifies bound structures (by grouping particles within a fixed comoving separation; Davis et al., 1985), then SUBFIND split the FOF group in the main halo, bound sub-haloes and unbound particle (Springel et al., 2001), finally a spherical overdensity algorithm grow a sphere around local density maxima until the mean density reach a critical value. This is done using equation 1.9 which becomes here:

$$M_{\Delta} = \frac{4}{3} \pi r_{\Delta}^3 \Delta \rho_{cr}$$

(2.5)

where $r_{\Delta}$ is the proper radius of the sphere and $\Delta$ a specified density contrast. We choose $\Delta = 500$ since it is a value typically adopted in observations (Kay et al., 2012).

To know more about cosmological simulations and their algorithm in general, Benson (2010) presents a wide overview of techniques and a vast literature review on this topic.
2.2.5 New generation of MGS

The simulation runs a first time for dark matter only, with $1080^3$ particles within a $250h^{-1}\text{Mpc}$ box. At each step, FOF and SUBFIND find the haloes and subhaloes as it does in the MS but then the semi-analytic galaxy code run on the merger tree to generate galaxy catalogues (one per snapshot).

This first stage only aimed to generate these catalogues. Now the simulation runs again with both dark matter and baryons and it uses the galaxy catalogues to heat the gas at the centre of haloes (position of galaxies).

We expect these modifications to make the simulation even more precise.

2.3 Basic results

The data from MGS can be accessed through Hierarchical Data Format 5 (hdf5) files, a format developed to manage extremely large and complex data sets.

2.3.1 The sample and cuts

We have more than five thousand clusters at $z = 0$ but after applying the relevant cuts to our work, only 144 remain. Though the available redshifts range from 0 to more than 2, the highest redshift sample we will use is 0.9 because there is not enough clusters left for higher redshift to provide statistically meaningful conclusions.

The cuts were focused on two aspects: the rank of the subgroup and the spectroscopic-like temperature of the hot gas ($T_{\text{sl}}$; Mazzotta et al., 2004; Kay et al., 2012). The rank of the subgroup allows us to keep only the most massive member of an FOF group (the mass being determined by SUBFIND) to avoid having overlapping maps of the same clusters. We keep only clusters with $T_{\text{sl}} > 2\text{keV}$ which is the case where we get accurate results for bremsstrahlung radiation. If we had clusters with temperature below $T_{\text{sl}} > 2\text{keV}$, we could get bound-bound or bound-free reactions.
Figure 2.2: The mass function of our sample for 5 different redshifts, black being redshift $z = 0$ and magenta redshift $z = 0.9$. We can see that the general shape of the curve stays similar but there is an offset in the highest values for the mass. This agrees with our expectations, there are fewer massive clusters at high redshift than at low redshift.
2.3.2 Mass Function

The mass function can be defined as the number density clusters with mass greater than M in a comoving volume element. This tool provides a way to describe the evolution of a large population of clusters and it is strongly correlated to the cosmology chosen (Voit, 2005).

In figure 2.2, we show the mass function of the sample we extracted from the MGS. This result is in good agreement with our expectations, the most massive clusters mass depends on the redshift but the general shape of the mass function stays similar. Note that the cosmology is still the one from the results of Komatsu et al. (2011) and does not change.

The estimation of the mass from observational data is a challenge. It can be done from the X-ray measurement of the ICM provided that that clusters are bound, hydrostatic, self-gravitating systems (Sarazin, 1988). To achieve this, it is necessary to measure high quality gas density and temperature profiles, which is still hard to do as it is observation intensive.

2.3.3 Scaling relations

Relations between the X-ray luminosity $L_x$, the temperature $T$ and the mass $M$ were predicted by the self-similar model of cluster formation (Kaiser, 1986). Hence, we are interested in the three scaling relations $L_x - T$, $L_x - M$, $T - M$ which can be described by these approximation of self-similar population:

\begin{align}
E(z)^{-1}L_x & \propto T^2 \quad (2.6) \\
E(z)^{-7/3}L_x & \propto M^{4/3} \quad (2.7) \\
T & \propto M^{2/3}E(z)^{2/3} \quad (2.8)
\end{align}

where $E(z) = \Omega_m(1 + z)^3 + \Omega_\Lambda$.

Unfortunately, clusters are not self-similar. Observations give steeper scaling
relations for both $L_x - T$ and $L_x - M$. We expect this behaviour to be reproduced by the MGS since the feedback tends to reduce the luminosity and boost the temperature (see values, measured by linear regression in log space, for the normalizations and slopes of the scaling relations in table 2.1 and 2.2 respectively). In figure 2.3, we confirm this assumption. We fixed the slope on the value for $z = 0$, which is a relevant choice since it stays very similar. Hence, we focus our analysis of the evolution of the scaling relations with redshift on the change in the normalisation as shown in figure 2.4 for the case of the $L_x - T$ relation. We find $A_{LT}(z) = -0.153 \pm 0.002 + 1.94 \pm 0.03 \log_{10}(1 + z)$. In the same figure, we also show the scattering.

The results we show in table 2.1 and 2.2 are in reasonable agreement with Hilton et al. (2012) who uses the sample XCS-DR1 from the XMM Cluster Survey (see 3.5 for more information about the survey). The slope found for the $L_x - T$ relation ranges from 2.17 to 3.18 depending on the redshift and method of fitting.

### 2.3.4 Maps

Each redshift has its own catalogue which gives data for the whole cluster (total mass, average temperature, etc...) and a map of $200 \times 200$ pixels and is scaled to have a full width of $4R_{500}$. Several maps, for several variables (bolometric luminosity, temperature, etc...) are created for each cluster and are centred on the most bound particle. It allows us to make images, as shown in figure 2.5, to have a visual check on the target. This will prove to be most useful in the case study in section 3.2.

The bulk of our work will rely on these maps. We expect that maps where clusters have a well defined centre and a unique peak of brightness may be easier to describe accurately with a simple model while maps with an extended centre or multiple clusters will see more complicated behaviour (see figure 2.6).

A convenient way to know if the map contains several clusters, a cluster with extended core or a merger is to calculate the so-called centroid shift. To do so, one will find out the position of the pixel with maximum surface brightness ($X_{\text{max}}, Y_{\text{max}}$)
Figure 2.3: The three scaling relations on a log scale. The color scheme correspond to different redshift, black is $z = 0$, blue is $z \approx 0.5$ and magenta is $z \approx 0.9$. The linear fit have a fixed slope based on the slope at $z = 0$. We also choose $T_{\text{piv}} = 3\text{KeV}$ in order to limit the impact of extreme values.
Figure 2.4: The evolution of the normalization of the $L_x - T$ scaling relation with redshift (left) can be fitted with a power law. It has a slope of $1.94 \pm 0.03$ and a normalization of $-0.153 \pm 0.002$. On the right, we display the associated scattering.

<table>
<thead>
<tr>
<th>Redshift</th>
<th>$A_{LT}$ in $10^{44} h^{-2} \text{erg/s}$</th>
<th>$A_{LM}$ in $10^{44} h^{-2} \text{erg/s}$</th>
<th>$A_{TM}$ in $10^{44} h^{-2} \text{erg/s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$-0.15$</td>
<td>$-22.87$</td>
<td>$-7.642$</td>
</tr>
<tr>
<td>0.02</td>
<td>$-0.13$</td>
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<td>$-7.644$</td>
</tr>
<tr>
<td>0.04</td>
<td>$-0.12$</td>
<td>$-22.85$</td>
<td>$-7.643$</td>
</tr>
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<td>$-0.11$</td>
<td>$-22.83$</td>
<td>$-7.640$</td>
</tr>
<tr>
<td>0.09</td>
<td>$-0.08$</td>
<td>$-22.82$</td>
<td>$-7.641$</td>
</tr>
<tr>
<td>0.12</td>
<td>$-0.06$</td>
<td>$-22.82$</td>
<td>$-7.647$</td>
</tr>
<tr>
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<td>$-22.79$</td>
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</tr>
<tr>
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</tr>
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<td>$-7.636$</td>
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<td>$-7.652$</td>
</tr>
<tr>
<td>0.76</td>
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</tr>
<tr>
<td>0.83</td>
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<td>$-7.619$</td>
</tr>
<tr>
<td>0.91</td>
<td>$0.40$</td>
<td>$-22.51$</td>
<td>$-7.635$</td>
</tr>
</tbody>
</table>

Table 2.1: The values of the normalizations of the fitting function on the three scaling relations.

<table>
<thead>
<tr>
<th>$B_{LT}$</th>
<th>$B_{LM}$</th>
<th>$B_{TM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.92</td>
<td>1.6</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Table 2.2: The values of the slope of the fitting function on the three scaling relations.
CHAPTER 2. THE MILLENNIUM GAS PROJECT

Figure 2.5: Three maps displaying different properties of the most massive cluster at $z = 0$. Its mass is $M = 1.53 \times 10^{15} h^{-1} M_\odot$ and its spectroscopic-like temperature is $T_{sl} = 12.7 \text{keV}$.
and then calculate the centroid:

\[
\begin{align*}
X_{\text{cen}} &= \frac{\sum_{i=1}^{N_{\text{pix}}} S_{x,i} x_i}{\sum_{i=1}^{N_{\text{pix}}} S_{x,i}}, \\
Y_{\text{cen}} &= \frac{\sum_{i=1}^{N_{\text{pix}}} S_{x,i} y_i}{\sum_{i=1}^{N_{\text{pix}}} S_{x,i}}
\end{align*}
\]  

(2.9)

where \(S_{x,i}\) is the surface brightness of pixel \(i\) and \((x_i, y_i)\) the pixel coordinates. Then it is easy to calculate the centroid shift:

\[
C_{\text{shift}} = \frac{|R_{\text{cen}} - R_{\text{max}}|}{R_{500}}
\]  

(2.10)

where \(R_{\text{cen}} = (X_{\text{cen}}, Y_{\text{cen}})\), \(R_{\text{max}} = (X_{\text{max}}, Y_{\text{max}})\).

Our values for the centroid shift at redshift 0 range from 0.005 to 0.34 and the median is 0.055 (see figure 2.7). Note that the evolution of the \(C_{\text{shift}}\) with redshift (also displayed in figure 2.7) can be explained by the fact that the mergers are, on average, less advanced at high redshift.
Figure 2.7: On the left: the centroid shifts distribution at $z = 0$ for a $C_{\text{shift}}$ calculated over $R_{500}$ (in black) and a over $2R_{500}$ (in blue). On the right: The evolution of the $C_{\text{shift}}$ with redshift. The points are the median values, the straight line is the associated linear fit, the solid curves are the 16th and 84th percentiles and the dashed curve is the 5th and 95th percentiles.
Chapter 3

Surface brightness profiles

3.1 Definition and model

Our work will focus on the so-called surface brightness profile, very often used by observers to characterize clusters. Its high correlation to density and strong dependence on redshift makes it very interesting but the need for a high resolution and a clear definition of the borders of the clusters makes it challenging to obtain at high redshift. If the MGS is able to produce large samples of clusters with known surface brightness profiles, it could help us to draw a general picture of cluster behaviour.

To study a surface brightness profile, one needs a map with a sensible resolution, as in figure 3.1. We explained earlier that we are working with $200 \times 200$ pixel maps. The pixel area in kpc varies to make sure that each map is defined between 0 and $2R_{500}$ and we choose 22 bins. Each bin is an annulus of the approximate same logarithmic size ($\Delta \log R/R_{500} \approx 0.09$) centered on the peak of surface brightness. Note that near the center of the cluster, we are dealing with very small radius and the need to round from fraction to integer can break this fixed size. Figure 3.2 shows how this translates into surface brightness profile.

In observations, we rarely achieve such a high resolution, particularly at high redshift. It is then interesting to find if this profile can be fitted by a relevant model that would allow us to extrapolate. In Cavaliere & Fusco-Femiano (1976) such a model was proposed: the so-called $\beta$ model, which can be used to fit the density
Figure 3.1: On this map of a typical cluster generated by the MGS, we show how we define the annuli and calculate the average surface brightness in each of them. This logarithmic binning is the one we are using on all clusters in our work. Solid lines are the annuli within $R_{500}$ while dashed lines are annuli between $R_{500}$ and $2R_{500}$. The cluster have the following properties at $z = 0$: $M = 2.45 \times 10^{14}M_\odot, T_{sl} = 4.35\text{keV}$.
Figure 3.2: A surface brightness profile for the cluster in figure 3.1. The dashed vertical lines are the inner and outer radius of each annulus and the cross are the average surface brightness in the region. At small radius, the bins are not of identical size, because we had to convert the logarithmic coordinates of the annuli in pixels coordinates, leading to a loss of accuracy. Fortunately, the precision is sufficient for our work even with this inconvenience.
profile and the surface brightness profile:

\[
\frac{\rho_{\text{gas}}(r)}{\rho_0} = \frac{1}{[1 + (\frac{r}{r_c})^2]^{3\beta}}
\]  \hspace{1cm} (3.1)

where \(\rho_0\) is the normalization constant, \(r_c\) is the core radius, and \(\beta\) is the ratio of the energy per unit mass in galaxies to the energy per unit mass in gas:

\[
\beta = \frac{\mu m_H \sigma^2}{3kT_{\text{gas}}}
\]  \hspace{1cm} (3.2)

where \(\mu\) is the mean molecular weight, \(m_H\) is the mass of the hydrogen atom, \(\sigma\) the cluster velocity dispersion and \(T_{\text{gas}}\) the X-ray gas temperature (Jones & Forman, 1984).

From equation 3.1, we can derive the surface brightness for an isothermal gas (Sarazin, 1986):

\[
\Sigma_X \propto \Sigma_{X,0} \frac{1}{[1 + (\frac{r}{r_c})^2]^{3\beta + \frac{1}{2}}}
\]  \hspace{1cm} (3.3)

This model has the advantage of being straightforward and does not contain too many variables, avoiding excessive degeneracies. Unfortunately, we know the beta model fails in the inner and outer regions of the clusters. Cool cores are responsible for a rise in the surface brightness at the centre of the cluster and the decrease is steeper outside of \(R_{500}\), two phenomena that the \(\beta\) model cannot take into account.

There are investigations to develop alternative models: Vikhlinin et al. (2006, V06) uses a model with 17 independent free parameters and describes very accurately the observational data; Ascasibar & Diego (2008) proposes a simpler model with only five, physically motivated, free parameters with less accuracy but also less degeneracies than V06. Nevertheless, we shall focus on the historical \(\beta\) model which is simple and readily comparable to observational data.

We will also need the local surface brightness slope at \(R_{500}\), \(\beta_{500}\), which is used to analyse chandra observations by Maughan et al. (2008). It is found by fitting a straight line in log space to the data in the radial range \([0.7, 1.3]R_{500}\) and the
following equation:

\[
\beta_{500} = \frac{1}{6} \left( 1 - \frac{d \log(\Sigma_X)}{d \log(r)} \bigg|_{r=R_{500}} \right).
\]  \quad (3.4)

One particularly interesting property of \( \beta_{500} \) is its independence from the core properties which means that the fact that MGS does not simulate cool cores should not influence it, improving our ability to compare samples.

The last property we are particularly interested in is the goodness of fit that we define as follow:

\[
Err = \sqrt{\frac{\sum_{i=1}^{N_{bins}} (\log(\Sigma_i) - \log(\Sigma_{\beta,i}))^2}{N_{bins}}}
\]  \quad (3.5)

where \( N_{bins} \) is the number of bins, \( \Sigma_i \) the average surface brightness in the bin \( i \), \( \Sigma_{\beta,i} \) the surface brightness value given by the \( \beta \) model in the bin \( i \). Minimizing \( Err \) permits to fit \( \beta \).

### 3.2 Case studies

In the first instance, we focus on single specific clusters that are extreme cases. We have investigated three observables (mass, temperature and luminosity) and five variables that could be derived from our maps (\( \beta \), \( \beta_{500} \), \( r_c \), \( C_{shift} \) and \( Err \)). We will focus on extreme cases of the latter.

The extreme cases displayed in figure 3.3 are for the best and worst fits within the radial range \([0, 1]R_{500}\) and figure 3.4 for fits within the radial range \([0, 2]R_{500}\). In figure 3.3, we can see that a good fit does not necessarily mean that the map contains a single, well defined, cluster. We also see that the choice of the radial range is crucial as \( Err_{[0, 1]} = 0.0059 \) is the best value for the goodness of fit on our whole sample while \( Err_{[0, 2]} = 0.126 \) for the exact same cluster, a value even higher than the worst goodness of fit on the \([0, 1]\) radial range. This situation is also present in figure 3.4, where the worst fit case (\( Err_{[0, 2]} = 0.234 \)) gives \( Err_{[0, 1]} = 0.0464 \) which is an average to good fit.

It does not come as a big surprise since we have known for some time that the \( \beta \) model poorly fits the surface brightness profile at large radii. Also, if you go
CHAPTER 3. SURFACE BRIGHTNESS PROFILES

Figure 3.3: Maps and surface brightness profiles for both best (top) and worst (bottom) fits over the [0, 1]\(R_{500}\) radial range. The black curve is the \(\beta\) model fit and the red straight line is the linear fit within the [0.7, 1.3]\(R_{500}\) used to find \(\beta_{500}\). We observe a merger in both maps and that in the case of the best fit, we have a good fit on [0, 1] (\(Err_{0,1} = 0.0059\)) and a bad fit on [0, 2] (\(Err_{0,2} = 0.126\)).
Figure 3.4: Maps and surface brightness profiles for both best (top) and worst (bottom) fits over the [0, 2]R_{500} radial range. The blue curve is the β model fit within [0, 2]R_{500} while the black dashed curve is the fit within [0, 1]R_{500}. We note that the radial range where we fit the β model can strongly change the goodness of fit as also shown in figure 3.3 (for the bottom cluster, Err[0,1] = 0.0117 and Err[0,2] = 0.134). It can be explained in this case by the presence of an object lying just beyond R_{500}. 
to $2R_{500}$, it is more probable that you will run into another cluster, hence perturb the profile. Another value that we know to be sensitive to the radial range is the centroid shift and we would like to see if we can correlate it to the goodness of fit.

Figure 3.5: Maps and surface brightness profiles for both the highest (top) and lowest (bottom) centroid shift, $C_{\text{shift}}$, calculated within $R_{500}$. Note that the highest $C_{\text{shift}}$ is a very extended source and is likely to be an ongoing merger. An early merger can be seen in the case of the lowest $C_{\text{shift}}$ but the second cluster is not within the $R_{500}$ of the target, which is why it does not influence the centroid. Hence, within $2R_{500}$, $C_{\text{shift}}$ becomes high.

In figure 3.5, we can see that, similarly to the goodness of fit, the centroid shift can vary strongly according to the area within which you calculate the centroid. For example, in the lowest centroid shift case (see table 3.1) we have $C_{\text{shift}[0,1]} = 0.001$ and $C_{\text{shift}[0,2]} = 0.11$.

We are also interested in the extreme cases for our $\beta$ model parameter, especially situations where the model fails. Figure 3.6 shows the two clusters with the highest
and lowest $\beta$. While in the first case the model provides a decent fit to the profile, the latter is a straight line, a very non-standard profile. A quick look at the map allows us to understand that the image is not clear enough to authorize a proper analysis. Not only is this the lowest value for $\beta$ in our sample, this case also reaches the threshold value $r_c = 0.01 \text{kpc}$ that we fixed. If we try fitting to $2R_{500}$, (table 3.1), we get $\beta = 1.2$ and $r_c = 1.11 \text{kpc}$, confirming that the extended core is the problem. The model tries to approximate a power law while, in this case, the surface brightness profile behave as a linear function, hence it wants $r_c$ to be small.

Figure 3.6: Maps and surface brightness profiles for the clusters with the highest (top) and lowest (bottom) values of $\beta$, when fit within $R_{500}$.

A similar situation can be observed in figure 3.7, where we plot the extreme cases for $r_c$. In both cases, the lack of steepening of the profile makes the model fail. In the largest $r_c$ case, extending the fit to the radial range $[0, 2]R_{500}$ gives more
Figure 3.7: Maps and surface brightness profiles for the clusters with the highest (top) and lowest (bottom) values of $r_c$, when fit within $R_{500}$. Similarly to the case in figure 3.6, a merger can create a flat, power-law, profile.
Figure 3.8: Maps and surface brightness profiles for the clusters with the highest (top) and lowest (bottom) values of $\beta_{500}$. We see that in a merger case, the value can become negative.
### CHAPTER 3. SURFACE BRIGHTNESS PROFILES

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Table 3.1: All major properties of the case studies are summarized. $LC_{shift[0,1]} \& HErr_{[0,1]}$ stands for low $C_{shift}$ and high $Err$ over $[0,1]R_{500}$
Radial ranges | Normalization | Slope
--- | --- | ---
[0, 0.75] | 0.68 | 0.09
[0, 1] | 0.68 | 0.06
[0, 2] | 0.73 | -0.01
[0.15, 0.75] | 0.7 | 0.07
[0.15, 1] | 0.7 | 0.09
[0.15, 2] | 0.75 | -0.03

Table 3.2: The values for the slope and the normalization for the fits of the $\beta$ medians against redshift (see figure 3.10).

sensible values of both variables. Finally, in figure 3.8, our attention is drawn to the possibility of a negative $\beta_{500}$ if another cluster is located exactly within the radial range [0.7, 1, 3].

### 3.3 Distribution of the variables and correlation

In the case studies, the variables derived from the $\beta$ model often depends on the radial range over which the fit is performed. In figure 3.9, we explore the distribution of $\beta$, $r_c$ and the goodness of fit for various radial ranges. The distributions of $\beta$ and $r_c$ are very similar for all radial ranges; the only significant difference is for the goodness of fit which is considerably worse when the outer radius is large ($2R_{500}$). In figure 3.10, we check that the evolution of $\beta$ with redshift is not sensitive to the radial range of the fit and also note that the slope for $\beta$ evolution is close to 0 (see table 3.2).

We also investigate whether there are correlations between the different variables, shown in figure 3.11 and figure 3.12. While only $\beta$ against $r_c$ stands out and have a spearman coefficient above 0.5, other relations (such $\beta_{[0,1]}$ vs $\beta_{500}$ and $T_{sl}$ vs $\beta_{500}$) have a noticeable coefficient with strong significance. More generally, numerous variable correlations, even with small coefficient, are significant (see table 3.3). It is important to note that a high significance coupled with a low spearman coefficient just tells us that we are reasonably convinced that the correlation is low.

A surprising case, shown in figure 3.13, is the lack of correlation between the goodness of fit and the centroid shift. This has a low Spearman coefficient of $0.2 - 0.3$.
Figure 3.9: The normalized distributions of several variables. The histograms on the left and the right show the values for a fit where 0 and 0.15 are respectively the inner radius. The colour scheme distinguish the outer radius and the dashed vertical lines are the median values.
Figure 3.10: The evolution of $\beta$ with redshift for fits over several radial ranges. The points are the median values at each redshift, the straight line is the associated linear fit, the solid curves are the 16th and 84th percentiles and the colour scheme differentiate the radial ranges. We see a slight growth of $\beta$ with redshift but no striking divergence between radial ranges.

(assuming radial range) with low significance as shown in table 3.3. The analysis of figure 3.14 might help us to understand why there is no correlation to be found. In this plot of a cluster where we have a bad fit with low centroid shift, we see that the cluster is a merger which explains the bad fit. Still, the merger is advanced so it influences the centroid shift only slightly. Going back to figure 3.5 where the highest $C_{\text{shift}}$ also incidentally happens to be a good fit ($Err_{[0.1]} = 0.018$), we see how this very extended source leads to a high centroid shift and how the fit with $R_{500}$ is good. Furthermore, if we were going to $2R_{500}$ the fit gets worse while the centroid shift gets better. We conclude that the centroid shift and the goodness of fit give us little information about each other because the type and stage of merger does not influence these two variable in the same way.
Figure 3.11: Scatter plots of the $M$, $T_{sl}$, $r_c$, $\beta_{500}$ and $\beta$ against $Err$ and $C_{shift}$. The Spearman coefficients of these relations can be found in table 3.3.
Figure 3.12: Scatter plots of $\beta$ and $\beta_{500}$ against $r_c$ and $\beta$ respectively.

Figure 3.13: A scatter plot of the goodness of fit against centroid shift. The black points are the values of $Err_{[0,1]}$ and $C_{\text{shift}[0,1]}$ while the green ones are for $Err_{[0,2]}$ and $C_{\text{shift}[0,2]}$. 
Table 3.3: The spearman coefficients and the significance of its deviation from zero for several parameters. The two-sided significance level of its deviation from zero is a value in the interval $[0.0,1.0]$ where small values indicate a significant correlation.

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Figure 3.14: Map and surface brightness profile for a cluster with low $C_{\text{shift}}$ and a bad fit.
3.4 Soft band versus bolometric luminosity

Before comparing the results from the simulation to observations, we want to make sure that we are working with the same kind of variables. We have always used the X-ray bolometric surface brightness so far while observations often relies on the soft [0.5 – 2keV] band.

We expect the surface brightness profile to be only slightly changed. This is explained by the fact that when you switch from the soft band to the bolometric luminosity, it only changes the cooling function, while the surface brightness is more sensitive to the density. We check the validity of that expectation on the values of $\beta$ in figure 3.15 and see that we were correct. Whether we fit our results on the radial range $[0, 1]$ or the $[0, 2]R_{500}$, the distribution of $\beta$ is very similar in both bolometric and soft-band cases. We can compare our results to observations without switching to using the soft-band for all clusters.

![Figure 3.15: The distribution of the $\beta$ values in our sample, at $z = 0$ calculated with the bolometric (black) and soft-band (blue) X-ray luminosity for $[0, 1]$ (left) and $[0, 2]$ (right) $R_{500}$. The dashed line is the median value of $\beta$. Both distributions stay very similar.](image)

3.5 Comparison to observation

We have investigated the properties of our simulated cluster profiles and tested the $\beta$ model on this sample. The next step is to compare with observations, analyse
the similarities and explain the differences. We chose three samples to compare our results to, from Maughan et al. (2008), Alshino et al. (2010) and unpublished preliminary results from the XMM Cluster Survey (XCS).

### 3.5.1 The observational samples

The sample presented in Maughan et al. (2008) consists of a sample of 115 clusters of galaxies at $0.1 < z < 1.3$ and is based on Chandra data from November 2006. It uses $\beta_{500}$ from equation 3.4. Alshino et al. (2010) selected 27 clusters from the X-ray Multi-Mirror Large-Scale Structure (XMM-LSS) survey. The clusters have redshift $0.05 < z < 1.05$ and their temperature range from 0.6 to 4.8 keV. After cuts on the minimum temperature ($T_{\text{sl}} > 2\text{keV}$) and the maximum redshift ($z < 0.9$), we are left with 9 clusters. The XMM Cluster Survey (XCS) is one of the largest X-ray cluster surveys and its methodology is explained in Lloyd-Davies et al. (2011). In its first data release, presented by Mehrtens et al. (2012), we are introduced to the objectives of the XCS: to offer a way to measure cosmological parameters, to measure the evolution of the X-ray luminosity temperature scaling relation, to study the galaxy properties in high redshift clusters ($z > 1$) and eventually to propose a high quality and vast sample of X-ray clusters, homogeneously selected. Of the 502 clusters currently contained in the data base, we choose to keep only 14 clusters with values for $\beta$, $r_c$ and spectroscopic redshift.

### 3.5.2 Results

Since Maughan et al. (2008) uses $\beta_{500}$ instead of $\beta$, we will work with its data separately from Alshino et al. (2010) and XCS.

In figure 3.16, we compare results for $\beta$, $r_c$ and $T_{\text{sl}}$. Our simulated clusters show some similarities to the observations within the uncertainties. Unfortunately, we also see that the XCS sample tends to avoid the region where the MGS lies. In figure 3.17, we display the evolution of $\beta$ and $r_c$ with redshift. While the values of $\beta$ are in reasonable agreement with the observations, the values for the core radius
Figure 3.16: $\beta$ against core radius and temperature. The black squares are the MGS data, the blue are from Alshino et al. (2010) and the magenta from XCS.
are not as similar. It should be pointed out that the data from MGS are median values and that the observations are singular clusters. The scatter of MGS values is included to take this into account.

In figure 3.18, we are interested in the evolution of $\beta_{500}$ with respect to redshift and temperature. This time, both sets show a strong agreement with each other. The difference in temperature range can be explained by the fact that Maughan et al. (2008) uses clusters which stand out from the noise (i.e. clusters with high luminosity and high temperature) while we include all clusters with $T > 2\text{keV}$ in our MGS sample.

### 3.6 Discussion

The $\beta$ model appears convincing on average. Even though it it not always very accurate, it only fails in cases of extremely extended sources, with a large core or abundant noise. The distribution of the goodness of fit supports this but we need to be careful on individual clusters as we saw that a good fit did not necessarily mean a clean map. The centroid shift alone is not enough to solve this problem since we could not find any correlation. Still, with the knowledge of both goodness of fit and centroid shift of a cluster, we can draw deductions about the situation. The use of $\beta_{500}$ is promising but we need to be careful as a merger located around $R_{500}$ will have a very strong impact on its value. This fact, associated with the other variables, could also be a tool to establish the presence of a merger.

While the $\beta$ model seems very sensitive to the radial range over which we choose to fit our data on individual clusters, working with large samples averages out this property. The median values and distributions of $\beta$ and $r_c$ does not show any strong dependence on the radial range. It is still interesting to note that the value of beta grows slightly with redshift. It has been observed that clusters grow more luminous and their surface brightness profile gets steeper Hilton et al. (2012), an information that match our study. The feedback might be an explanation for this phenomenon.
Figure 3.17: The $\beta$ and $r_c$ values with respect to redshift $z$. The data in black is the MGS data, the crosses are the median values at each redshift, the straight line is the associated linear fit, the solid curves are the 16th and 84th percentiles and the dashed curve is the 5th and 95th percentiles. The blue stars are the values from Alshino et al. (2010) and the magenta from XCS.
Figure 3.18: $\beta_{500}$ against redshift and temperature. The data in black is the MGS data, the crosses are the median values at each redshift, the straight line is the associated linear fit, the solid curves are the 16th and 84th percentiles and the dashed curve is the 5th and 95th percentiles. The purple from Maughan et al. (2008).
Chapter 4

Conclusion

4.1 Summary

We began by giving an overview of the ΛCDM model, why it is so strongly supported and its mains properties and features. We introduced the concept of overdensity, structure growth and the dark matter halo and how this leads to galaxy and cluster formation. We explained why the study of clusters of galaxies is vital in cosmology and what we aim to achieve with this thesis.

We then introduced the Millenium Gas Project, its history, the steps from MS to MGS and the difference between the first generation of MGS and the second one. We gave a brief summary of the algorithms used in MGS, covering initial conditions, gravity, hydrodynamics and halo targeting. We introduced our tools: the catalogues and maps produced by MGS and studied a few properties of clusters: the mass function and cluster scaling relations. This work produced the expected results. The mass function changes with redshift and the \( L_x - T \) and \( L_x - M \) relations are steeper than the self-similar model predicts while the \( T - M \) relation is similar. The concept of centroid shift was also introduced.

In the bulk of the thesis, we presented the results of our research, after a short introduction about the surface brightness profiles and the \( \beta \) model. We explained how this model was designed, its flaws and qualities. In the case study, we focused on extreme situations, associated with a profile and a map for each of these. All
cases properties are summarized in table 3.1 which allows us to have a full description of the cluster and draw conclusions. The case study helped us to understand unexpected results such as the case with a high centroid shift but a good fit. The analysis of the distribution of the $\beta$ model parameters for the whole sample let us distinguish conclusions which make sense in general cases and the one that only applies if we study an individual cluster. We also tried to find correlations between variables but can only isolate a strong one between $\beta$ and $r_c$ and a few others with smaller spearman coefficients but strong significances.

Eventually, we compared the results from our analysis on the MGS with observations found in the literature and show that they concur with the uncertainty expected from the assumptions made in the simulation and the observations.

4.2 Future research

The investigation of alternatives to the $\beta$ model is a popular topic and several promising attempts do exist. A better model would improve the extrapolation power and description of the surface brightness profiles in both simulation and observation. The use of $\beta$ with the MGS is interesting but have to be done with caution. Some observational samples show better agreement than others, it is important to establish why and what criteria are critical if we want to extrapolate from MGS.

Many developments are possible from this research. The second generation MGS we used was released very recently and another sample with a $500h^{-1}$Mpc box is even more recent. Even though the results should be similar as the one we found with the $250h^{-1}$Mpc box, this larger sample will improve the statistics.

The results we used from XCS are very recent also and have yet to be published. We expect the sample to get larger and more accurate. Our simulated sample and analysis will be made available to the XCS collaboration.
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