BEHAVIOUR AND DESIGN OF STEEL COLUMNS SUBJECTED TO VEHICLE IMPACT

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List of Abbreviations and Symbols

\(A_w\) The web area
\(A_f\) The flange area
\(A\) A stiffness coefficient of the vehicle
\(b_f\) The width of the box and H sections
\(B\) A stiffness coefficients of the vehicle
\(b_1\) and \(b_0\) Experimental parameters used to calculate vehicle stiffness coefficients \(A\) and \(B\)
\(|\mathbf{C}|\) The damping matrix of the structure
\(C\) The vehicle deformation
\(C_{max}\) The maximum deformation of the vehicle
\(c\) The damping constant of the system
\(c_c\) The critical damping value
\(c_d\) The material wave speed
\(D\) A material parameter used in the Cowper-Symonds equation
\(d\) The damage variable
\(E\) The modulus of elasticity
\(EI\) The flexural stiffness of the column section
\(E_{TOTAL}\) The total conserved energy of the system
\(F_{D}\) The frictional dissipation energy of the system
\(F\) The equivalent design horizontal characteristic force used in Eqs. 2.1 and 2.2
\(F_{max}\) The maximum transverse static force resistance of the steel column at the impact location
\(F_{el}\) The equivalent quasi-static transverse force for the elastic phase of column response
\(F_{pl}\) The equivalent quasi-static transverse force for the plastic phase of column response
\(F_y\) The yield strength of the steel material
\(F_{(t)}\) The elements’ externally applied forces at the start of the current time step \((t)\)
\(h_w\) The depth of the box and H sections
\(h_c\) The width of the column perpendicular to the direction of impact
\(I\) The moment of inertia about the axis under consideration.
\(I_i\) The nodal internal forces matrix of the system
\(IE\) The internal energy of the system
\(IE_{col}\) The energy absorbed by column deformations
\(IE_v\) The energy absorbed by vehicle deformations
The stiffness matrices of the structural system

\[
\begin{bmatrix}
K
\end{bmatrix}
\]

The residual kinetic energy of impact

\(KE\)

The slopes (stiffness) for the first and second stage of the proposed bilinear spring impact force - deformation relationships

\(K_1\) and \(K_2\)

The effective length factor of the column depending on its supporting condition

\(k\)

The equivalent elastic stiffness of the object for the case of hard impact and that of the impacted structure for the case of soft impact

\(k_e\)

The lateral stiffness of the steel column under axial load

\(k_{col.}\)

An interaction factor to take account of the secondary bending moment due to an axial compression force acting on the column lateral deformation (defined in section 6.3.1 of Eurocode 3)

\(k_{zz}\)

Total span of the column

\(L\)

The characteristic length of the element

\(L_c\)

The element length taken as the shortest element distance

\(L_e\)

The total mass of the impacting body

\(M\)

The elastic bending moment capacities of the column section at the plastic hinge (i).

\(M_i\)

The elastic bending moment of the column at section \((x)\) along its longitudinal axis

\(M_x\)

The design values of the maximum moment about the weak axis (z-z axis)

\(M_z\)

The nodal mass matrix of the structural system

\[
\begin{bmatrix}
M
\end{bmatrix}
\]

The total mass of the structural system

\(M_T\)

The plastic moment capacity of the column’s section

\(M_p\)

The full plastic moment capacity of the column cross-section about the weak axis (z-z axis)

\(M_{pc}\)

The reduced plastic moment capacity of the column’s section at the plastic hinge (i)

\(M_{PRi}\)

The number of plastic hinges required to develop a plastic failure mechanism

\(N\)

A material parameter used in the Cowper-Symonds equation

\(n\)

The column cross-sectional axial resistance (defined in section 6.3.1 of Eurocode 3)

\(N_{Rk}\)

The axial compressive load applied on the column.

\(P\)

The full axial yield load (squash load) of the columns section

\(P_Y\)

The full yield force of the section web

\(P_w\)

The Euler buckling load for columns

\(P_{cr}\)

The plastic strain energy of the system

\(PD\)

The design axial load capacity of the steel column

\(P_{Design}\)

The reaction of the column at end 1

\(R_i\)

The outer and inner radii of the hollow circular section respectively

\(r_2\) and \(r_1\)
\( SE \) The elastic strain energy
\( t_w \) The web thickness for H-sections
\( t_f \) The flange thickness of the box and H sections.
\( u_{pl} \) The effective plastic displacement
\( u_{f,pl} \) The total plastic displacement at the point of failure
\( v_r \) The velocity of the vehicle normal to the impacted structure used in Eqs. 2.1 and 2.2
\( V \) The standard vehicle impact velocity used in crash barrier tests
\( V_{cr} \) The critical impact velocity of the impacting body
\( VD \) The viscous dissipation energy of the system
\( \omega_n \) The frequency of the vibration mode \( m \)
\( \omega_{max} \) The maximum frequency of the dynamic system
\( w_{(x)} \) The deformation shape of the first mode of column bucking
\( w_v \) A variable over the width of the vehicle
\( W \) The amplitude of the deformation shape
\( W_p \) The total width of the vehicle
\( W_{el} \) The maximum column deflections to enable the bending moment in the column to reach the plastic bending moment capacity
\( W_{cr} \) The maximum displacement at which collapse occurs due to the combined effect of plastic mechanism and axial compressive force
\( WK \) The work done by the external forces
\( x \) The distance along the longitudinal axis of the column measured from the column base
\( x' \) The plastic hinge location measured from the column base
\( \bar{x} \) The position of the load application measured from the column base.
\( y, y' \) and \( y'' \) The nodal acceleration, velocity and deformation respectively
\( \theta_{Elastic} \) The maximum elastic rotation of the column at the point of plastic hinge formation at the plastic hinge (i)
\( \theta_{Critical} \) The maximum rotation of the plastic hinge when plastic failure mechanism occurs at the plastic hinge (i)
\( \theta_s \) The shear stress ratio
\( \delta_i \) The deformation of the vehicle used in Eq. 2.1
\( \delta_b \) The deformation of the impacted structure used in Eq. 2.1
\( \phi \) A quantity in the equation of first mode of bucking shape of the propped cantilever column defined by \( \phi = \frac{1.4318\pi}{L} \)
\( \chi_c \) The reduction factor for the compression due to flexural buckling about the weak axis (defined in section 6.3.1 of Eurocode 3)
\( \eta \)  
The stress triaxiality

\( \Delta \)  
The axial shortening of the column

\( \Delta t \)  
The duration of the equivalent rectangular impulse of vehicle impact

\( \epsilon^p \)  
The uniaxial equivalent plastic strain rate

\( \epsilon^{pl} \)  
The tensile damage initiation criterion

\( \epsilon_s^{pl} \)  
The shear damage initiation criterion

\( \epsilon_f^{pl} \)  
The equivalent plastic strain at the complete failure of the element

\( \epsilon_{\text{axial}} \)  
The axial strain of the column caused by membrane action

\( \sigma(t) \)  
The element stress at the current time step

\( \sigma_o \)  
The value of the static flow (yield) stress

\( \bar{\sigma} \)  
The value of the dynamic flow (yield) stress

\( \Phi_f \)  
The material degradation due to tensile damage

\( \Phi_s \)  
The material degradation due to shear damage

\( \sum \Delta \epsilon^{pl} \)  
The accumulative value of the equivalent plastic strain

\( \Omega \)  
A parameter defines the shape of interaction curve

\( \alpha \) \text{ and } \beta  
The mass and stiffness proportional Rayleigh damping factors respectively

\( \xi_m \)  
The damping ratio specified for the vibration mode \( m \)

\( \rho \)  
The material density
Columns are critical elements of any structure and their failure can lead to the catastrophic consequences of progressive failure. In structural design, procedures to design structures to resist conventional loads are well established. But design for accidental loading conditions is increasingly requested by clients and occupants in modern engineering designs. Among many accidental causes that induce column failure, impact (e.g. vehicular impact, ship impact, crane impact, impact by flying debris after an explosion) has rarely been considered in the modern engineering designs of civil engineering structures such as buildings and bridges. Therefore, most of the design requirements for structural members under vehicle impact as suggested by the current standards and codes such as Eurocode 1 are based on simple equations or procedures that make gross assumptions and they may be highly inaccurate. This research aims to develop more accurate methods of assessing steel column behaviour under vehicle impact.

The first main objective of this study is to numerically simulate the dynamic impact response of axially loaded steel columns under vehicle impact, including the prediction of failure modes, using the finite element method. To achieve this goal, a numerical model has been proposed and validated to simulate the behaviour and failure modes of axially loaded steel columns under rigid body impact using the commercial finite element code ABAQUS/Explicit. Afterwards, an extensive parametric study was conducted to provide a comprehensive database of results covering different impact masses, impact velocities and impact locations in addition to different column boundary conditions, axial load ratios and section sizes. The parametric study results show that global buckling is the predominant failure mode of axially unrestrained compressed steel columns under transverse impact. The parametric study results have also revealed that column failure was mainly dependent on the value of the kinetic energy of impact. The parametric study has also shown that strain rate has a minor effect on the behaviour and failure of steel columns under low to medium velocity impact. The parametric study results have been used to develop an understanding of the detailed behaviour of steel columns under transverse impact in order to inform the assumptions of the proposed analytical method.
To account for the effect of vehicle impact on the behaviour of steel columns, a simplified numerical vehicle model was developed and validated in this study using a spring mass system. In this spring mass system, the spring represents the stiffness characteristics of the vehicle. The vehicle stiffness characteristics can be assumed to be bilinear, with the first part representing the vehicle deformation behaviour up to the engine box and the second part representing the stiffness of the engine box, which is almost rigid.

The second main objective of this research is to develop a simplified analytical approach that can be used to predict the critical velocity of impact on steel columns. The proposed method utilizes the energy balance principle with a quasi-static approximation of the steel column response and assumes global plastic buckling as the main failure mode of the impacted column. The validation results show very good agreement between the analytical method results and the ABAQUS simulation results with the analytical results tending to be on the safe side.

A detailed assessment of the design requirements suggested by Eurocode 1, regarding the design of steel columns to resist vehicle impact, has shown that the equivalent static design force approach can be used in the design of moderately sized columns that are typically used in low multi-storey buildings (less than 10 storeys). For bigger columns, it is unsafe to use the Eurocode 1 equivalent static forces. It is acceptable to use a dynamic impulse in a dynamic analysis to represent the dynamic action of vehicle impact on columns, but it is important that both the column and vehicle stiffness values should be included when calculating the equivalent impulse force – time relationship. It is also necessary to consider the two stage behaviour of the impacting vehicle, before and after the column is in contact with the vehicle engine. A method has been developed to implement these changes.
Declaration

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Dedication

To the spirit of my father who has given me all the support I need to be what I wanted to be and who has been the source of inspiration to me throughout my life;

To my mother for her love and her prayers for me during my life;

To my brother and sisters for their encouragement and their support;

To my wife for her personal support, encouragement and great patience during the research period;

Finally, to my beloved children, Tiba and Ahmed, who are the glow of my life;

I dedicate this work.
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Chapter One

Introduction

1.1. Statement of the research problem

Columns are critical elements of any infrastructure and their failure can lead to the catastrophic consequences of progressive failure. In structural design, procedures to design structures to resist conventional loads (e.g. self weight, wind, normal imposed loads) are well established. But design for accidental loading conditions is increasingly requested by clients and occupants in modern engineering designs. Among many accidental causes that induce columns’ failure, impact (e.g. vehicular impact, ship impact, crane impact, impact by flying debris after an explosion) has rarely been considered in the modern engineering designs for civil engineering structures such as buildings and bridges.

Underground and multi-storey car parks’ columns, ground floor columns in buildings located along busy roads and bridge pairs are highly vulnerable to impact loads due to moving vehicles (see Fig. 1.1). The failure of these supporting structures, as a result of impact, may lead to progressive collapse. Therefore, a proper analysis technique is required. This analysis technique should be suitable for routine structural design so that this mode of failure can be dealt with by a majority of engineers. Despite extensive past research on structural behaviour under impact, the behaviour of columns under extreme loading conditions is still not well understood.
Figure 1.1: Collision of a vehicle with a reinforced concrete support (left); Impact test (right). Pictures from (Ghose, 2009)

A number of industry standards and codes currently address the effects of vehicle impacts on buildings in their specifications. However, their guidance is rather rudimentary, treating a transverse impact as an equivalent static force on the structure as in EN 1991-1-1 (Eurocode1, 2002), EN 1991-1-7 (Eurocode1, 2006), EN 1991-2 (Eurocode1, 2003) and ACI Committee 358 (ACI, 1992) or as an approximate dynamic impulse as in Annex C of Eurocode 1(Eurocode1, 2006). These rudimentary procedures are based on gross simplifications. Because of the varying nature of structures and impacting vehicles, it is highly unlikely that these assumptions and the existing design approaches are suitable.

1.2. Objectives and methodology of the research

Motivated by the above statement, this research will assess the accuracy of current design methods and will develop more accurate methods of dealing with column behaviour under vehicle impact. The first main goal of this study is to numerically simulate the dynamic impact response of axially loaded steel columns under vehicle impact, including the prediction of failure modes, using the finite element method. The effects of different geometrical and material parameters on column behaviour under such a dynamic load will be considered in an extensive parametric study. The purpose of this part of the study is to gain a thorough understanding of how steel columns respond to transverse impact loads.

Although the finite element method can provide a powerful and efficient approach for modelling column behaviour and failure modes, its employment requires significant effort and expertise. This is particularly true if nonlinear dynamic analysis, material
nonlinearity and strain rate dependence are included. Furthermore, in many situations, obtaining finite element solutions requires a considerable amount of computation time which prevents its use during routine design. Therefore, the second main goal of this research is to develop a simplified analytical or semi-analytical approach that can be used to predict the critical impact velocity of vehicle impact on axially loaded steel columns. This proposed method will utilize the energy balance approach assuming quasi-static behaviour for the impacted steel column. Based on the aforementioned two goals, the detailed objectives are as follows:

1. To suggest and validate a numerical model for simulating axially loaded steel columns subjected to transverse impact using the commercial finite element code ABAQUS/Explicit. The validation should be based on a comparison between the proposed simulation model and relevant experiments;

2. To conduct extensive parametric study to provide a comprehensive database of results covering different impact masses, impact velocities and impact locations in addition to different column boundary conditions, axial load ratios and section sizes. The numerical simulation results will also be used to develop an understanding of the detailed behaviour of steel columns under transverse impact in order to inform the assumptions of the proposed analytical method;

3. To develop a simplified numerical vehicle model which can be used to simulate the effects of vehicle impact on steel columns by using the commercial finite element code ABAQUS/Explicit without having to use a full scale numerical vehicle model;

4. To develop and validate a simplified analytical method for predicting the critical impact velocities of vehicle impact on axially compressed steel columns.

5. To use the numerical simulation results to assess the accuracy of the current Eurocode 1 design methods.

1.3. Layout of the thesis

This thesis presents the detailed results of the author’s work over a three year PhD research programme which was aimed at achieving the objectives of the research. It includes a detailed literature review of the different aspects of the research project, a
validation of the numerical simulations, an extensive parametric study and the
development of the analytical approach. The thesis is divided into the following nine
chapters.

**Chapter one:** Presents an introduction of the research problem with a statement of the
objectives and aims of the research.

**Chapter two:** Reviews the previous studies relevant to the research problem in addition
to the approaches suggested in the current design codes of practice. The main focus of
this chapter is to present and discuss the previous research studies in this field so as to
identify gaps in knowledge in order to justify the originality of this research and to
formulate an appropriate research methodology for this research.

**Chapter three:** Presents a detailed description of the techniques used in numerical
simulations to model the transverse impact problem of axially compressed columns
using the finite element code ABAQUS/Explicit. It will also describe the geometrical,
material, and loading application parameters used in the numerical model. The chapter
will present the numerical simulations of relevant tests and compare the numerical
simulation results against tests’ results.

**Chapter four:** Presents the parametric study conducted based on the numerical model
validated in chapter three to investigate the effects of several parameters on the response
of axially loaded steel columns. The chapter discusses the parametric study results and
ends up with important conclusions, based on which simplifying assumptions on
column behaviour under impact can be made to develop appropriate design calculation
methods.

**Chapter five:** Presents and validates a simplified numerical vehicle model that can be
used to simulate the effects of vehicle frontal impact on steel columns. The simplified
numerical vehicle model treats the vehicle as a spring-mass system. The chapter also
presents the derivations and validation of a simplified equation to predict the equivalent
linear stiffness of a vehicle that can be used in the simplified analytical model.
Chapter six: Presents the development of a simplified analytical method to predict the critical velocities of transverse impact on steel columns under axial load. This method is based on an energy balance assuming a quasi-static approximation of column behaviour.

Chapter seven: Validates the accuracy of the proposed simplified analytical method presented in chapter six by comparison against numerical simulation results using ABAQUS/Explicit.

Chapter eight: Evaluates the accuracy of the design requirements suggested by the current Eurocode 1, based on the numerical simulation results.

Chapter nine: Presents the main findings and conclusions of the research with suggestions for future research work.
Chapter Two: Literature Review

2.1.  The research problem

Civil engineering structures are frequently being subjected to various types of transverse dynamic impact loads. The following impact events may cause significant damage to structures and they should be included in the structural design of structural members that are prone to such events.

1. Transverse impact caused by ships or vehicles on bridge piers (El-Tawil et al., 2005, Sharma et al., 2012), see Fig. 2.1;
2. Transverse impact caused by travelling vehicles on buildings situated near busy roads (Ellis, 2003), see Fig. 2.2;
3. Transverse impact caused by travelling vehicles on traffic light or lighting poles (Elmarakbi et al., 2006, Klyavin et al., 2008), (see Fig. 2.3) and on road safety barriers (Jiang et al., 2004).
4. Transverse impact caused by moving vehicles on ground floor columns of multi-storey car parks (Ferrer et al., 2010),
5. Transverse impact caused by aeroplanes on multi-storey buildings or skyscrapers.

Figure 2.1: Collapse of a bridge after being struck by tractor-trailer (courtesy of NDOR) (El-Tawil et al., 2005)
In most cases, the effect of transverse impact caused by a vehicle is usually combined with the effects of the axial compressive load transmitted to the column/wall from the storeys above. The effect of an axial compressive load further complicates the impact problem because, on the one hand, it reduces the column stiffness and resistance and, on the other hand, it increases the geometrical and dynamic instabilities of the impacted column.

The transverse impact problem of structural members has been the subject of attention and investigation by a number of researchers during past decades. Different methods and approaches have been developed to investigate the behaviour and failure of these members under impact. In each approach, several assumptions have to be made in the analysis procedure according to the geometry of, and type of material used in, the
structure under consideration; the dynamic characteristics of the impacting body (impact velocity, impact duration, impact mass); the deformations expected to develop during the short period of the impact event; and the failure mode which either involves local failure or the geometrical instability of the whole structural member. However, as will be discerned from the following literature review, research on this specific topic is still rudimentary and lacks sufficient depth and detail to inform the design process.

It is the aim of this chapter to present and discuss previous research studies relating to the above mentioned impact problem in order to assist in gaining a clearer understanding of the behaviour and failure modes of axially loaded steel columns under vehicle impact. The discussions will focus on the main assumptions used in these studies and the main findings reached, and how they can be incorporated in the methodology used in the present study.

2.2. Current codes of practice

A number of current industry standards and codes of practice include rudimentary guidance on how to address the effects of vehicle impact on buildings (Eurocode1, 2002, Eurocode1, 2006, ACI, 1992) and bridge structures (Eurocode1, 2003, Highways Agency, 2004, AASHTO, 2007) by treating the transverse impact as an equivalent static force on the structure. For example, the informative Annex B of Eurocode1: Part1-1 (Eurocode1, 2002) suggests the following simple and general equation, to calculate the design horizontal characteristic force on barriers in car parks to withstand the impact of a vehicle:

\[
F = \frac{M \cdot v_r^2}{2(\delta_c + \delta_b)}
\]  

Where \(M\) is the total mass of the vehicle, \(v_r\) is velocity of the vehicle normal to the impacted structure, \(\delta_c\) is deformation of the vehicle, \(\delta_b\) is the deformation of the impacted structure.

Conceptually, this equation (Eq. 2.1) is easily derived based on conservation of energy in the system before and after the impact, but it assumes plastic force-displacement response for both the barrier and the impacting vehicle. The impact energy of the vehicle \((0.5Mv_r^2)\) is assumed to be entirely absorbed by the plastic deformation of the vehicle \((\delta_c)\) and the structure \((\delta_b)\) which have a force \((F)\) at the interface between the
two. According to this Annex, the maximum impact velocity at which the equation is valid is 4.5 m/s (16.2 km/h) and the maximum vehicle deformation is 100 mm (Eurocode1, 2002).

Sections 4.3 and 4.4 of Eurocode1: Part1-7 (Eurocode1, 2006) define the accidental action due to impact from vehicles and forklift trucks respectively. Section 4.3 recommends that impact may be represented by an equivalent static force giving the equivalent dynamic action effects on the structures. The design values of the equivalent static force are given in two tables as constant values corresponding to each category of traffic. Each table represents a particular kind of structure, namely supporting-structures and super-structures, see Table 2.1 and Table 2.2. In both tables, the effects of vehicle mass, vehicle velocity and vehicle stiffness characteristics, in addition to those of the impacted structure are all ignored.

Table 2.1: Indicative equivalent static design forces due to vehicular impact on members supporting structures over, or adjacent to, roadways, (Eurocode1, 2006)

<table>
<thead>
<tr>
<th>Category of traffic</th>
<th>Force $F_{dx}$ (kN)</th>
<th>Force $F_{dy}$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motorways and country national and main roads</td>
<td>1000*</td>
<td>500</td>
</tr>
<tr>
<td>Country roads in rural area</td>
<td>700</td>
<td>375</td>
</tr>
<tr>
<td>Roads in urban area</td>
<td>500</td>
<td>250</td>
</tr>
<tr>
<td>Courtyards and parking garages with access to:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Cars</td>
<td>50</td>
<td>25</td>
</tr>
<tr>
<td>- Lorries $^b$</td>
<td>150</td>
<td>75</td>
</tr>
</tbody>
</table>

$^a$ $x$ = direction of normal travel, $y$ = perpendicular to the direction of normal travel.  
$^b$ The term “lorry” refers to vehicles with a maximum gross weight greater than 3.5 tonnes.

Table 2.2: Indicative equivalent static design forces due to impact on superstructures (Eurocode1, 2006)

<table>
<thead>
<tr>
<th>Category of traffic</th>
<th>Equivalent static design force $F_{dx}^a$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motorways and country national and main roads</td>
<td>500</td>
</tr>
<tr>
<td>Country roads in rural area</td>
<td>375</td>
</tr>
<tr>
<td>Roads in urban area</td>
<td>250</td>
</tr>
<tr>
<td>Courtyards and parking garages</td>
<td>75</td>
</tr>
</tbody>
</table>

$^a$ $x$ = direction of normal travel
Alternatively, the informative Annex C of Eurocode 1: Part1-7 (Eurocode1, 2006) provides guidance on how to approximate the dynamic design action of structures subject to accidental impact by road vehicles. This Annex distinguishes between hard impact, under which the impact energy is mainly dissipated by the impacting body, and the soft impact where the structure is designed to deform in order to absorb the impact energy (Eurocode1, 2006). The maximum impact force is derived by simply equating the initial kinetic energy of the impacting vehicle to either the strain energy of the vehicle itself in the case of hard impact, or the strain energy of the impacted structure in the case of soft impact, to give the following equation.

\[
F = v_r \sqrt{k_e M}
\]  

2.2

Where: \(v_r\) is the vehicle velocity at impact; \(k_e\) is the equivalent elastic stiffness of the vehicle in the case of hard impact and that of the impacted structure in the case of soft impact; \(M\) is the mass of the impacting vehicle.

It is evident that Eq. 2.2 is based on a simple elastic single degree of freedom idealization of the problem assuming that one of the impacting bodies is rigid and immovable as shown in Fig. 2.4. Hence, it can easily be found by equating the initial kinetic energy of the impact \(0.5Mv_r^2\) and the maximum elastic strain energy of the vehicle column system under force \(F\) \(\left(\frac{F^2}{2k_e}\right)\).

\[\text{Figure 2.4: Single degree of freedom SDOF model idealization for the derivation of Eq. 2.2}\]

In addition, Annex C of Eurocode1: Part1-7 (Eurocode1, 2006) suggests that the impact force calculated from Eq. 2.2 can be treated as a rectangle dynamic impulse on the surface of the structure as shown in Fig. 2.5 with the duration calculated using Eq. 2.3.
This equation is derived by substituting Eq. 2.2 as the impact force (F) in the following momentum conservation equation:

\[ F \Delta t = Mv_r \] \hspace{1cm} \text{2.4}

It should be pointed out that Eq. 2.2 neglects the interaction between the vehicle and the structure, resulting in underestimated values of the impact force for the hard impact cases and overestimated values for the soft impact cases. In addition, the plastic behaviour of either the impacting vehicle or the structure is not considered.

Sections 4.7.2.1 and 5.6.2.1 of Eurocode1: Part 2 (Eurocode1, 2003) give nominal design values of an equivalent static force for impact on road bridges, footbridges and railway bridges due to vehicle collision. These forces are constant, being 1000 kN in the direction of vehicle travel or 500 kN perpendicular to that direction.

Similarly, the American Concrete Institute standard for the analysis and design of reinforced and pre-stressed-concrete guideway structures (ACI, 1992) suggests an equivalent horizontal static force value of 1000kN to represent vehicle impact on piers or other guideway support structures which are located less than 3m from the edge of an adjacent street or highway. The location of the applied horizontal force is suggested to be 1.2 m above ground level.
The basis of these values has been questioned by various researchers, for example Vrouwenvelder (Vrouwenvelder, 2000). As will be assessed in this thesis, it is not appropriate to use a constant force.

El-Tawil et al. (2005) presented a numerical simulation study of a vehicle colliding with a concrete bridge pier using the commercial finite element code LS-DYNA. The analysis results revealed that the dynamic impact forces are much higher than the design impact forces suggested by the AASHTO code (AASHTO, 2007). Moreover, the equivalent static design force, which is defined as the force that causes the same amount of deformation as the dynamic impact force, is also higher than that in the AASHTO code. Therefore, El-Tawil et al. suggested that the current design specifications may be un-conservative.

Ghose, (2009) presented a background document to the development of the current design and assessment requirements for bridges under vehicular impact. The document presented the findings of research and development projects undertaken by Network Rail and the Highways Agency (HA) in the UK to quantify impact forces and to estimate the extent of bridge damage based on finite element simulations using the FE code LS-DYNA (Ghose, 2009). One major conclusion from these researches is that, for vehicular impacts on bridge columns and piers, the static design force given in part 5 (BD 60/04) of the Design Manual For Roads And Bridges (Highways Agency, 2004) is adequate only for the average impact force while the peak impact force is very high in head-on collisions. Therefore, the statically applied design loads, in addition to having to consider the dynamic amplification factor, would need to be significantly increased in order to accurately predict the dynamic effect on a bridge support.

Ferrer et al (2010) have also presented a numerical study to assess the current European regulations and standards (Eurocode1, 2002, Eurocode1, 2006). This study was undertaken by comparing the static force values suggested by these codes with numerical FEM dynamic analysis. Ferrer et al. found that the equivalent static load of a vehicle impact on a structure is mainly dependent on the impact speed. Therefore, it is not realistic to give one value for this load regardless of the vehicle speed.

Sharma et al. (2012) have proposed a new requirement for the current standards’ specifications regarding the analysis and design of RC columns under vehicle impact. The suggested requirement is mainly based on the determination of the dynamic shear
resistance of the column to prevent a certain level of damage to occur in the column rather than to prevent the column collapse mechanism as currently suggested by most of the current standard and codes. It has been found that the column dynamic shear resistance varies with the mass and velocity of the vehicle in addition to the column configuration. Moreover, it has been concluded that the estimated values of dynamic shear force is greater that the static or quasi-static quantity currently suggested by most of the codes of practice. Therefore, the study suggests that the dynamic resistance must be used for the analysis and design of structures subjected to vehicle impact.

It is clear from the previous discussions that although representing the transverse impact as an equivalent static force, or using a dynamic impulse approximation as in the current standards and codes, may provide design engineers with a relatively simple empirical design or assessment procedure; it involves many uncertainties that may be grossly inaccurate. For instance, none of the above methods considers the interactions between the structure and the impacting body. In addition, none of the above methods includes work undertaken by the axial force in the structural member as it deforms. Also the existing research studies have focused on bridge support structures and there is a lack of such investigation for building structures.

The main objective of the present study is to develop a more realistic analysis procedure to predict the dynamic impact effects and to quantify the failure conditions of axially loaded steel columns under a transverse vehicle impact. Different parameters will be considered in the analysis including column boundary conditions, column axial load ratios, vehicle masses, vehicle velocities and impact locations and directions. The behaviour and failure of the impacted column under such a dynamic event will be predicted numerically and analytically in terms of the critical impact velocity at which the column would be considered to have failed. Both global and local failure modes will be included in the numerical analysis.

2.3. Previous research studies

Due to resource limitation, this study will be conducted numerically and analytically. The numerical results will be used to generate extensive data for developing the analytical methods. The numerical simulations will need to be validated by comparison against the experimental results obtained by others. Thus, it is necessary to review the
developments related to steel column behaviour under transverse impact and the analytical methods used to predict such dynamic behaviour.

In addition, this research is concerned with steel columns under vehicle impact and it is not desirable to introduce a detailed simulation of the vehicle when studying the column behaviour. Therefore, simplification of the vehicle is required and this chapter will also provide a review on vehicles characterises used in this simplifications.

2.3.1. Behaviour and failure modes of axially compressed columns under a transverse impact load

A considerable amount of literature has been published on the experimental and numerical studies of the behaviour and failure modes of structural members under transverse impact. In particular, the behaviour of axially restrained beams under transverse impact has been studied intensively (Menkes and Opat, 1973, Liu and Jones, 1987, Yu and Jones, 1989, Yu and Jones, 1991, Yu and Jones, 1997, Mannan et al., 2008, Bambach et al., 2008). This is understandable as axially restrained beams are frequently used as members to absorb impact energy in such major applications as vehicle crash barriers. On the other hand, although columns under compressive load may also be involved in an accidental vehicle crash, the emphasis has been almost wholly on vehicle crashworthiness for occupant protection. However, with structural robustness now an important topic for the structural engineering research community, the behaviour of columns under vehicle impact deserves attention. Nevertheless, although the behaviour of axially compressed columns under transverse impact will be different from that of axially restrained beams undergoing axial tension, a review of beam behaviour can help to understand some aspects of the steel column behaviour.

The experimental study of Menkes and Opat (1973) identified three modes of failure of clamped aluminium beams subjected to transverse impulsive dynamic load. These failure modes essentially depended on the impulse intensity (see Fig. 2.6)

I- large plastic deformation of the whole beam with the formation of a plastic hinge mechanism;
II- tensile tearing failure under catenary action, and
III- transverse shear failure at the supports.
The pure transverse shear failure was characterised by non-significant deformations at the central section. However, some overlapping was observed between the tensile and transverse shear failure modes.

A large number of research studies have investigated how these three failure modes may be quantified under the influence of different parameters, such as the influence of pre-tensioning (Chen and Yu, 2000), material type and impact location (Liu and Jones, 1987, Yu and Jones, 1989, Yu and Jones, 1991, Yu and Jones, 1997), the impact speed (Mannan et al., 2008), and different types of cross-section (Bambach et al., 2008). Of these three failure modes, global failure and shear failure may occur in axially compressed columns under transverse impact.

![Figure 2.6: Tensile tearing failure and transverse shear failure of clamped steel beams under the impact of drop mass (Liu and Jones, 1987) (a): Transverse shear failure at the impact point, (b)-(d): Tensile tearing failure at the impact](image)

Very few studies have been carried out on the behaviour and failure modes of axially compressed columns under transverse impact. Amongst these studies, Zeinoddini et al conducted a series of experimental and numerical investigations to study the response of axially pre-compressed steel tubes under low velocity transverse impact (Zeinoddini et al., 1999, Zeinoddini et al., 2002, Zeinoddini et al., 2008a, Zeinoddini et al., 2008b). Two failure modes were identified from the experimental results: plastic global buckling under high axial compression load, and local indentation and damage of the impact zone when the axial load is low and the tube is thin (Fig. 2.7).
Adachi et al (2004) and Sastranegara et al. (2005) investigated experimentally and numerically the buckling and post buckling behaviour of a number of small scale axially compressed aluminium columns subjected to lateral impact loads. They also identified global instability as the main failure mode of the columns, see Fig. 2.8. Due to the failure mode being global buckling, they found that the critical condition of column buckling was controlled only by the kinetic energy of the transverse impact, but was independent of the history of the transverse impact or its impulse. It has been observed that when a column is subjected to transverse impact, it buckles in the lower bucking mode under pure axial load irrespective of the location of impact.

Typically, as the above studies were among the first on this topic, they are limited in scope and their conclusions are preliminary. Consequently, there is insufficient data to
develop comprehensive understanding of the effects of various parameters on steel column behaviour for the development of practical design methods. The aim of this research is to build on these preliminary research investigations and to develop an extensive database of column behaviour under transverse impact. Some of the above mentioned experimental results also provide data for the validation of the numerical modelling presented in chapter three.

As a summary, the following failure modes are the ones most likely to occur in axially loaded steel columns under transverse impact:

a) Shear failure (either at the impact zone or at the supports)
b) Local failure and indentation at the impact zone, and
c) Global buckling of the whole impacted column.

This research will attempt to identify the conditions under which these failure modes occur and to focus attention on the most relevant failure mode, i.e. global buckling.

2.3.2. Analytical research studies

Although the finite element method can provide an accurate and versatile approach for modelling structural behaviour under impact, its employment requires significant effort and expertise. This is particularly true if nonlinear dynamic analysis, material nonlinearity and strain rate dependence are included in the numerical simulations. Moreover, in many situations, obtaining such a complex finite element solution consumes a great amount of computation time, which precludes its use in general structural design. It is, therefore, desirable to develop an approximate, but simplified analytical method.

This section will present a detailed review and description of a number of selected analytical researches that are directly relevant to the present research study, and which will later be used as background for developing the analytical or semi-analytical modelling approach, to be reported in detail in chapter six.

Developing a simplified method for analysing structural response under impact load has received significant research effort (Parkes, 1958, Symonds and Mentel, 1958, Nonaka, 1967, Symonds and Jones, 1972, Liu and Jones, 1988, Jones, 1997, Chen and Yu, 2000). However, as in the experimental studies, these investigations have all
concentrated on beam or plate structures under pure bending or axial tension (when axially restrained) under impact.

The quasi-static approach was also adopted in the development of the analytical methods. In this approach, the static equilibrium state is assumed for the structural system and the resultant static deformation shape is then used to express the energy absorbed due to the elastic and/or plastic deformations of the impacted structural member. The equation of the external work done by the axial load can also be derived based on the assumed static deformation shape. The static deformation shape could be assumed to be the deformation shape of the structural member under transverse loads (Biggs, 1964, Humar, 2002), or the buckling mode shape of the column (Adachi et al., 2004, Sastranegara et al., 2006, Shope, 2006). Afterwards, the energy quantities are used to write the energy conservation equation for the original dynamic system. Experimental validation of the quasi-static assumption was provided by Liu and Jones (1987) who conducted tests on small scale clamped metal beams impacted transversely by a rigid mass travelling with low impact velocities at different locations along the beam length. They revealed that the locations and types of failure of the impacted beams were similar to those of the beams loaded statically. This is an important conclusion which can be employed to simplify the analytical approaches to beams and columns under such transverse impacts. Further experimental and numerical confirmations of the quasi-static assumption may be traced to Zeinoddini et al. (2002, Zeinoddini et al., 2008a) who, crucially, indicated that quasi-static analysis may be used for the impact velocity used (25.9 km/h).

The analytical study undertaken by Jones (Jones, 1995) on a clamped beam has shown that the quasi-static analysis can be used to predict the inelastic behaviour of clamped beams subjected to low velocity transverse impact provided that the ratio of the striker mass to the beam mass is not less than one in order to ensure that most of the impact kinetic energy would be absorbed as plastic deformations during the global plastic mechanism phase. A comparison between the quasi-static solution results with the theoretical dynamic solution obtained by Shen and Jones (1991) has revealed that the error introduced by the quasi-static procedure was within acceptable ranges provided that the ratio of the striker mass to the beam mass is larger than about three. The error is significant when the striker masses are smaller than the total beam mass. It has also been concluded that, for many engineering applications, the rigid-plastic assumption of
the material behaviour may be used to simplify the quasi-static analysis. Even in the case of a light vehicle impacting a heavy steel column section, it is expected that the ratio of the vehicle mass to the steel column mass will not be less than one. Therefore, the quasi-static assumption can be adopted for the research problem of this study.

The quasi-static method has also been adopted by Wen et al (1995) and Bambach et al. (2008) to establish the transverse force - transverse displacement relationship. In the Wen et al. study (1995) an analytical method was presented to predict the critical transverse deformations, the critical absorbed energy and failure modes of the clamped beam made from a rigid –perfectly plastic material subjected to transverse impact by a mass travelling at a low velocity. The quasi-static approximation has been adopted in the analysis to derive the force-displacement relationship using a virtual work method. The plastic deformation shape was assumed in order to derive the transverse and axial deformations’ relationships. As the clamped beam is more vulnerable to shear and membrane action effects, both deformations were included in deriving the force-displacement relationship alongside the bending deformations. The energy conservation principle was then employed to determine the maximum plastic transverse deformation at which the steel beam failure occurs under both tensile tearing and/or shear failure modes. The comparison with the experimental results confirmed the validity of the proposed method.

Bambach et al. (2008) used the quasi-static method to establish the transverse force - transverse displacement relationship of hollow and concrete filled steel beam sections under transverse impact using a procedure similar to that adopted by Jones (1995), taking into account the effect of the local reduction in the sectional depth at the impact point on the plastic moment capacity of the section at that point by applying the interaction curve equation for this type of steel sections, (see Fig.2.9).

![Figure 2.9: Theoretical model of Bambach et al. (2008)](image-url)
The present study aims to address the problem of axially compressed steel columns under transverse impact on which there is very little research. The presence of an axial compressive load significantly changes the behaviour of the structural member and failure mode, from local or shear failure of structural beams/plates with in-plane axial restraint to global buckling failure under compression. Among a few relevant research studies, Shope (Shope, 2006) has used the energy conservation principle with quasi-static approximation to develop a simplified analytical and design model to predict the critical impulse values for a wide flanged steel column section (W8x4) under a static axial force subjected to a blast load with different boundary conditions and slenderness ratios. In that study, Shope assumed that the system behaves as a single degree of freedom in an elastic perfectly plastic manner, as shown in Fig. 2.10. The critical impulse-axial load relationship was obtained by setting the kinetic energy of the impulse equal to the total strain energy of the column at the maximum plastic displacement point. Since the numerical model utilized a beam element to model the steel column, the effect of local deformation or flange buckling was not considered in both the numerical simulations and the suggested analytical method.

![Figure 2.10: Idealized system used by Shope (2006) with elastic–perfectly plastic material behaviour](image)

Tsang and Lam (2008) have employed the quasi–static nonlinear approach combined with the energy conservation principle to investigate the impact resistance of reinforced concrete columns subjected to road vehicle impact at the column mid-height. The impact resistance of the column was estimated by determining the frontal impact velocity of the vehicle to cause global instability of the column. The energy absorbed by the column at failure was used in the energy balance equation. The transverse impact force resistance function was derived based on a quasi-static approximation of the reinforced concrete column behaviour under the dynamic impact load (Fig. 2.11).
Chapter Two: Literature Review

![Graph showing force-deformation behavior of a concrete column, with and without strain rate effects (Tsang and Lam, 2008)](image)

The effect of the axial compressive force was taken into account by including the \( P-W \) effect in the moment equation at the critical section as follows:

\[
M = \frac{FL}{8} + \frac{PW}{2} \tag{2.5}
\]

Where \( F \) is the transverse impact force at the column’s mid-span, \( L \) is the column length; \( P \) is the axial compressive force.

This gives the transverse impact force as:

\[
F = \frac{8M}{L} - \frac{2PW}{L} \tag{2.6}
\]

The energy absorbed by the vehicle at the failure point was calculated by assuming a linear relationship between the impact force and the uniform shortening of the vehicle frontal as below:

\[
U_{veh} = \frac{F_{max}^2}{2K_{veh}} \tag{2.7}
\]

Where: \( F_{max} \) is the maximum impact force generated between the impacting vehicle and the column and \( K_{veh} \) is the vehicle frontal stiffness. No information was given about how the value of \( K_{veh} \) can be obtained but its value was taken to be 1500kN/m.
A comparison with non-linear dynamic simulation results showed that the quasi-static method underestimated the column impact resistance by 40%. This underestimation of column resistance was as a result of neglecting the contribution of the inertia force on the column resistance (Tsang and Lam, 2008). However, for steel columns that are much lighter in weight than RC columns, the effects of the inertial force will be small and it may acceptable to discard them.

It is evident from the aforementioned literature review that there are gross assumptions in the design of axially loaded steel columns under transverse impact. Therefore, one of the main objectives of this research is to present the full development of an analytical method for predicting the critical velocity of vehicle impact on steel columns under axial compression that may be adopted in design. This proposed method will utilize the energy balance approach assuming quasi-static behaviour of the impacted steel column. The steel stress-strain curve will be assumed to be elastic-perfectly plastic. The energy balance approach will account for both elastic and plastic lateral deformations of the steel column, axial movement of the column and the energy absorbed by vehicle deformations.

2.3.3. Vehicle characteristics

Many research studies have used the finite element method to simulate vehicle impact on structures. In some of these studies, the vehicle impact was simulated using a time varying load with an assumed maximum amplitude (Thilakarathna et al., 2010, Varat and Husher, 2000) as shown in Fig. 2.12. The impact load-time function of the vehicle may be obtained from experimental crash tests. However, this approach involves many approximations and uncertainties which affect the level of accuracy achieved. For example, the impact force-time history of the vehicle cannot be accurately predicted because it depends on a large number of variables including the vehicle dynamic characteristics such as vehicle mass, vehicle velocity and the strain rate sensitivity of vehicle components (see Fig. 2.12(B)) in addition to the geometrical and material characteristics of the impacted column itself. Moreover, this approach cannot accurately account for the local deformations in the vehicle at the impact point because it does not simulate the contact interaction generated between the vehicle and the column. Furthermore, the effect of kinetic impact energy on column behaviour and failure cannot be considered if the vehicle impact is input as an impulsive load.
Figure 2.12: (A) A frontal collision test of a Honda Accord at a speed of 48.3 km/h; (B) Force-time histories of full scale crash tests for different types of vehicle (from Thilakarathna et al., 2010).

On the other hand, full-scale finite element models of vehicles have been frequently used to simulate vehicle impact under different impact scenarios to assess vehicle integrity. Full scale vehicle modelling is necessary because the focus is on the vehicle. However, the simulation is demanding, involving a very large number of elements, material nonlinearity, strain rate dependency and very large strains and deformations. For structural engineering applications of vehicle impact on structural members such as the columns of a building, it is doubtful whether full-scale vehicle modelling is necessary when the focus of the study is on the structural member, not the vehicle. Nevertheless, some numerical studies have been attempted using full-scale numerical vehicle models to assess the design approaches and equations used to investigate vehicle impact on columns and bridge piers under vehicle impact. These studies include a numerical study of vehicle impact on concrete bridge piers conducted by El-Tawil et al. (2005), a numerical study of vehicle impact on a traffic light steel pole by Elmarakbi et al. (2006) and the numerical study conducted by Ferrer et al. (2010) to simulate vehicle impact on underground car park steel columns. In all these studies, some limitations had to be applied in order to reduce the simulation effort and time. For example, in the El-Tawil et al. study, the effects of material nonlinearity and the failure of the structure were neglected and in the Elmarakbi et al. and Ferrer et al. studies, material failure was also neglected in the simulation and only one direction of impact was considered in the numerical simulations. These limitations severely restrict the applications of such investigations. Furthermore, in many situations, a full-scale model of a particular vehicle type is either not yet available or is not accessible to engineers.
Chapter Two: Literature Review

The above discussion leads to the conclusion that there is a need to develop a simplified numerical vehicle model which can be utilized for studying the behaviour and failure of structures under vehicle impact. This has already been recognized by others and the following sections review work already undertaken in this area to identify the gap for further development.

2.3.3.1. Simplifying approaches

Based on an experimental study of vehicle frontal collision on a rigid barrier, Emori (Emori, 1968) suggested that the undamaged or intact portion of the impacting vehicle may be considered as a rigid body while the crushed portion of the vehicle absorbs all the kinetic energy of the impacting vehicle just before the impact. Therefore, the vehicle impact processes may be modelled by a spring-mass system in which the rigid mass represents the vehicle total mass and the one-way linear spring represents the vehicle resistance, see Fig. 2.13.

![Spring-mass system used by Emori (Emori, 1968).](image)

Tani and Emori (1970) proposed a more sophisticated mathematical model having three masses and three springs simulating the three parts of a vehicle: the engine box, the structure in front of the engine and the frontal structure. The load-deflection characteristics of each spring were determined based on experimental observations.

Kamal (1970) and Lin (1973) proposed further refinements to the previous two models by using eight nonlinear springs with three masses to simulate frontal impact (Kamal, 1970) and seven nonlinear springs to simulate rear barrier impact (Lin, 1973).

In the above-mentioned studies, the vehicle impact was against a wide, flat and rigid barrier rather than a deformable structure with a limited width such as a column considered in the present study. Nevertheless, the concept of simplifying the vehicle
using a spring-mass system is still applicable as long as the spring can capture the dynamic and stiffness characteristics of the impacting vehicle.

Milner et al. (2001) presented a simplified theoretical study based on the dynamic analysis of two and three degrees of freedom system of a vehicle impacting on wooden poles. In their analytical model, the vehicle was modelled by a mass representing the vehicle total mass and a bi-linear stiffness representing the vehicle stiffness characteristics before and after the engine location, see Fig. 2.14. No information was given in the study about how the bilinear stiffness values were obtained. However, their validation results suggested validity of the bilinear stiffness assumption.

![Figure 2.14: Vehicle stiffness characteristics defined by Milner et al. (Milner et al., 2001)](image)

**2.3.3.2. Quantifying vehicle stiffness**

Campbell (1976) proposed a linear equation to estimate the peak impact force and the energy absorbed by a vehicle in terms of its frontal plastic deformation. A linear force-displacement relationship is proposed based upon full frontal impact on a crash barrier at velocities ranging from 24 km/h to 97 km/h. The maximum impact force is given by:

\[ F = A + BC \] \hspace{5cm} 2.8

Where: \( A \) and \( B \) are stiffness coefficients of the vehicle and \( C \) is vehicle deformation.

Fig. 2.15 shows an example of the impact force - vehicle deformation relationship, in which \( C_{\text{max}} \) is the maximum deformation of the vehicle. The two main assumptions are that the damage of the vehicle is uniform and the force-deflection
relationship does not significantly vary across the vehicle width. A minimum limit of 25\% of the vehicle frontal width in contact with the struck object was suggested.

Figure 2.15: An example of a vehicle impact force-crush distance relationship

Campbell also proposed equations to relate the stiffness coefficients A and B in Eq. 2.8 to the vehicle width ($W_v$), vehicle mass ($M$), using experimental parameters ($b_1$ and $b_0$) as follows:

\[
A = \frac{Mb_1}{W_v} \tag{2.9}
\]

\[
B = \frac{Mb_1^2}{W_v} \tag{2.10}
\]

Following Campbell’s study, many other research studies were conducted to suggest alternative approaches to compute the values of $A$, $B$, $b_0$, and $b_1$ without having to resort to experimental data.

In one of these attempts, Jiang et al. (2004) used Campbell’s equation to develop an analytical approach to predict the peak impact force caused by a vehicle frontal and inclined crash into concrete road safety barriers perpendicular to the barrier. They assumed that the value of $b_0$ could be taken as 2.2m/s while $b_1$ could be determined using the following equation:
\[ b_i = \frac{V - b_o}{C_{\text{max}}} \]

Where: \( V \) is the standard vehicle impact velocity used in crash barrier tests (around 56 km/h for most vehicle crash tests).

In their derivations, Jiang et al. (2004) assumed that all the impact energy was absorbed by vehicle deformation without any contribution from the impacted concrete barrier. The energy absorbed by vehicle deformation was obtained by integrating equation 2.8 over the damage profile of the vehicle after impact.

On the other hand, Siddall and Day (1996) defined five classes of passenger vehicles and two classes of pickup, vans and multi-purpose vehicles, and proposed the A and B values in Table 2.3.

Table 2.3: Generic stiffness coefficients A and B to be used in Campbell’s equation (Siddall and Day, 1996)

<table>
<thead>
<tr>
<th>Vehicle type</th>
<th>Passenger vehicles</th>
<th>Pickup Vehicles</th>
<th>Van Vehicles</th>
<th>Multi-Purpose Vehicles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class No.</td>
<td>A (N/m)</td>
<td>B (N/m²)</td>
<td>A (N/m)</td>
<td>B (N/m²)</td>
</tr>
<tr>
<td>1</td>
<td>31566</td>
<td>497180</td>
<td>46597</td>
<td>750976</td>
</tr>
<tr>
<td>2</td>
<td>32344</td>
<td>457673</td>
<td>38457</td>
<td>471601</td>
</tr>
<tr>
<td>3</td>
<td>36188</td>
<td>482426</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>4</td>
<td>37722</td>
<td>459880</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>5</td>
<td>50564</td>
<td>782210</td>
<td>n/a</td>
<td>n/a</td>
</tr>
</tbody>
</table>

Wågström et al (2004) proposed a stiffness value of 1000kN/m for light vehicles (1200kg) and medium weight vehicles (1600kg) and 2000 kN/m for heavy vehicles (2000kg). The proposed values were validated against full frontal rigid barrier crash tests conducted by the National Highway Traffic Safety Administration (NHTSA) for different vehicle models, weights and velocities.

In these previous studies, the structure was almost rigid whereas, in the present study, the structure (column) is flexible and will deform and absorb energy during the impact process. Nevertheless, this research study will investigate the applicability of the existing approach. In this study, the method of investigation will include the following steps:
a) Identifying vehicle characteristics affecting the behaviour of steel columns under impact;

b) Proposing and validating a simplified vehicle model, based on a comparison of column behaviour between using a full vehicle model and a simplified vehicle model. The numerical simulations will be carried out using the finite element code ABAQUS/Explicit;

c) Suggesting a method to estimate the simplified vehicle characteristics.

2.4. **Research methodology and originality**

Although many research studies have investigated the behaviour of structures under impact, very few studies have considered axially compressed columns without longitudinal restraint. Consequently, the practical design of this type of structure is based on very simple rudimentary rules using an equivalent static load. The aim of this research is to investigate this problem in detail so as to provide designers with a more accurate approach. This research will be conducted using numerical simulations to generate extensive data to guide the development of simplified analytical methods. Specifically, the objectives of this research are as follows:

1. To suggest and validate a numerical model for the simulation of axially loaded steel columns subjected to transverse impact using the commercial finite element code ABAQUS/Explicit. The validation should be based on a comparison between the proposed simulation model and the relevant experiments conducted by others;

2. To conduct extensive parametric studies to provide a comprehensive database of results covering different impact masses, impact velocities and impact locations in addition to different column boundary conditions, axial load ratios and section sizes;

3. To develop a simplified numerical vehicle model which can be used to simulate the effects of vehicle impact on steel columns by using the commercial finite element code ABAQUS/Explicit;

4. To develop a simplified analytical method for predicting the critical velocity of rigid body and vehicle impact on the axially compressed steel columns. This method will be based on the energy balance approach with a quasi-static
approximation of the column behaviour. The numerical simulations’ results will be used to develop an understanding of the detailed behaviour of steel columns under transverse impact in order to inform the assumptions of the analytical method. The numerical simulations’ results will also be used to ascertain the accuracy of the proposed analytical method;

5. To use the numerical simulations’ results to assess the accuracy of the various design methods.

2.5. Summary

This chapter has presented an overview of the behaviour and the failure modes of axially loaded columns under transverse impact. The main focus of this chapter has been to present and discuss the previous research studies in this field so as to identify gaps in knowledge in order to justify the originality of this research. A further detailed review of different aspects of this research project will be presented in individual chapters.

A review of different theoretical developments relating to columns under lateral impact has been also presented. Clearly, very few of the previous research studies have addressed the impact behaviour of columns under axial compression and no axial restraint. The analytical method may be developed based on the energy conservation principle with a quasi-static approximation of column behaviour. This chapter has reviewed the basis of this analytical approach and the assumptions and conclusions of previous related studies. This approach will be fully developed in chapter six.
Chapter Three

Validation of Finite Element Modelling Using ABAQUS/Explicit

3.1. Introduction

One of the most effective and accurate numerical methods to handle the problem of dynamic analysis of structures under impact is the finite element method (Zienkiewicz and Taylor, 1991, Bonet and Wood, 1997, Crisfield, 1997, Belytschko et al., 2000). For the problem under consideration in this study, dynamic simulation will be required.

The main objective of this chapter is to validate the application of the general finite element package ABAQUS/Explicit to the impact problem of this research so that it can be used to generate extensive numerical results through parametric studies for the development of a design method. For this, the simulation results will be compared with relevant experimental and/or numerical results of others available in the literature.

This chapter will present a detailed description of the procedure used in this research study for modelling axially compressed steel columns subjected to short duration impact by a rigid mass. This chapter will also describe the geometrical, material, and loading application parameters used in the numerical model. Special attention will also be paid to modelling the contact interaction between the impacting body (rigid mass or vehicle) and the impacted steel column due to its importance to impact analysis. A considerable sub-section of this chapter will be allocated to describing the material model used in this study to predict different failure modes of steel under such a transverse impact load.

3.2. Dynamic impact analysis using finite element commercial code ABAQUS/Explicit

ABAQUS/Explicit is a finite element analysis programme that can be adapted to solve special cases of transient dynamic problems such as blast and impact by utilizing an explicit dynamic finite element formulation. In addition to its availability, ABAQUS/Explicit is considered more appropriate and will be used during this research project for the following reasons.
a) The explicit dynamics method is more suitable to analyzing high-speed and low-duration dynamic events such as the traverse impact problems investigated in the present study (SIMULIA, 2010c);

b) Impact load is generated due to the contact interaction between the contacted bodies. The contact interaction can be formulated more easily in ABAQUS using an explicit, rather than an implicit dynamics method;

c) In order to predict different failure modes (global and local) that the impacted steel column may undergo, the analysis procedure along with the material model must be capable of tracing the response of the column up to failure point. While capturing local failure occurring in the material usually results in convergence problems in the implicit analysis procedure, these problems are mitigated considerably in ABAQUS/Explicit owing to the explicit procedure used in the analysis.

3.2.1. Summary of the explicit dynamics algorithm (SIMULIA, 2010c)

ABAQUS/Explicit does not require solving a system of simultaneous equations as in the standard finite element procedure. Instead, it integrates the dynamic quantities (accelerations, velocities, dynamic stresses and strains) over the time increment by employing an explicit dynamic finite element formulation in which the dynamic quantities are extracted kinematically from one current time increment to the next one, as illustrated in Fig. 3.1.
Calculate the nodal acceleration at the beginning of the time step by applying the dynamic equilibrium equation.

\[
\ddot{y}(t) = [M]^{-1} (F(t) - I(t))
\]

Integrate the current acceleration explicitly through the time using central finite difference method to obtain the nodal velocity and displacement.

\[
\begin{align*}
\dot{y}(t + \Delta t/2) &= \dot{y}(t - \Delta t/2) + \frac{(\Delta t/\Delta t) + \Delta t/\Delta t}{2} \ddot{y}(t) \\
y(t + \Delta t) &= y(t) + \Delta I_{t+\Delta t} \dot{y}(t + \Delta t/2)
\end{align*}
\]

Compute stresses, \(\sigma\), from constitutive equations based on element strain increments, \(\Delta \epsilon\), computed from the strain rate, \(\dot{\epsilon}\).

Assemble nodal internal forces, \(I_{t+\Delta t}\).

Set \(t + \Delta t\) to \(t\) and return to the first step.

Figure 3.1: The computational algorithm used in ABAQUS/Explicit. (Reproduced from (SIMULIA, 2010c ))

Where:

\[
\begin{align*}
\ddot{y}, \dot{y} and y & \text{ are the nodal acceleration, velocity and deformation respectively;} \\
\begin{bmatrix} M \end{bmatrix} & \text{represents the total nodal mass matrix of the system;} \\
I & \text{represents the nodal internal forces of the system;} \\
F(t) & \text{represents the elements’ externally applied forces at the start of the current time step \(t\);} \\
\sigma(t) & \text{represents the element stress at the current time step.}
\end{align*}
\]

It can be concluded from the above procedure that the values of nodal accelerations, nodal velocities and nodal displacements at the end of any time increment are merely based on the same quantities as at the beginning of the current time step, which explains why this method is considered explicit. Furthermore, it is evident from the previous
procedure that, in order to gain accurate results, the time increment must be small enough to assume the acceleration to be nearly constant throughout that time increment.

3.2.2. Modelling parameters for structural impact simulation using ABAQUS/Explicit

Simulating the behaviour of structural members under dynamic impact using ABAQUS/Explicit requires the careful selection of the proper geometrical and material modelling parameters so as to produce accurate results that are as close as possible to the actual behaviour of the impacted members. Due to the nature of the problem investigated in this study which involves applying a static axial force on a steel column during impact events, a suitable procedure is adopted to apply the axial force using the quasi-static procedure available in ABAQUS/Explicit. The following sections describe the modelling approaches and techniques used in the present study:

1. Geometrical modelling;
2. Material modelling;
3. Modelling of the contact between the impacting body and the steel column;
4. Stability limits and time increment control.
5. Damping effect in the impact analysis;
6. Applying the axial force using the quasi-static procedure.

3.2.2.1. Geometrical modelling

Fig. 3.2 shows the main element types used in the present study to simulate the impact problem which includes solid, shell and spring elements. The differences in these element types reflect the differences in the geometrical shapes of the structural members and bodies simulated in the study. These elements belong to the stress/displacement element library which is the most suitable to model the complex dynamic problems involving contact, plasticity and large deformations (SIMULIA, 2010b). Linear (first order) interpolation is used to calculate the internal stresses and strains at any point in the element which is the only interpolation order offered by ABAQUS/Explicit for solid and shell elements.
Solid elements are used in most of the numerical simulations. Shell elements are used in simulating hollow steel sections.

For solid and shell elements, ABAQUS/Explicit adopts a reduced integration technique to integrate various response outputs (stresses and strains) over the element. This integration technique uses fewer Gaussian integration points than the full integration scheme. However, the combination of using a reduced integration technique with first order (linear) interpolation elements leads to what is called the hourglass numerical problem. To overcome the hourglass problem, ABAQUS/Explicit introduces a small value of artificial stiffness called “hourglass stiffness” to an element. Moreover, at least four elements should be used over the thickness of any structural part modelled in order to obtain accurate results, as shown in Fig. 3.3.

Figure 3.2: Linear brick, shell and spring elements used in the present numerical simulation using ABAQUS/Explicit (SIMULIA, 2010b).

Figure 3.3: Meshing technique used over each thickness of a steel beam using linear elements with reduced integration.

Linear elements with reduced integration have also been frequently used by most of the previous numerical studies to model structural impact problems (Yu and Jones, 1997, Zeinoddini et al., 2008, Dorogoy and Rittel, 2008, Thilakarathna et al., 2010).

3.2.2.2. Material modelling

When the impacting body impacts the column, a high concentration of stress waves propagate from the impact point towards the ends of the column in a short period of
time (Johnson, 1972, Jones, 1997). This concentration of stresses causes highly nonlinear local behaviour at the impact zone (Johnson, 1985). With the continuation of the impact event, a stress wave is generated and spread along the entire column length causing global deformations and possibly global instability. To account for both local and global deformations, the adopted material behaviour model must be capable of tracing the development and propagation of the yielding and inelastic flow of the material up to the failure point. In addition, the strain rate and the strain hardening effects are other important issues which must also be simulated properly in the dynamic impact analysis of strain rate sensitive materials such as steel which is considered in this study.

For the dynamic analysis of ductile members, ABAQUS/Explicit provides two material models to account for the inelastic behaviour and the local failure of the structural members under dynamic loads. These are the classical metal plasticity model and the progressive damage and failure model. The two models work in conjunction with each other to trace the full response of the material up to failure. These models can consider the effects of strain hardening, strain rate sensitivity and material failure. The following two sections will describe in detail the important characteristics of each model.

A. Classical metal plasticity model (SIMULIA, 2010a)
This model uses von Mises or Hill yield stress with the corresponding plastic flow to simulate both isotropic and anisotropic yielding of the material. The model uses the true stress-true strain curve of the material to describe the inelastic response and defines the elastic response in terms of the material modulus of elasticity, see Fig. 3.4. The strain hardening and the strain rate effects can be accounted for in this material model with an acceptable accuracy especially for low strain rates. Hence, it can be adopted for the vehicle impact problem investigated in the present study. In addition, the classical metal model can be used in conjunction with the progressive damage and failure model available in ABAQUS/Explicit to simulate shear and tensile failures. Both failure modes can occur in transversely impacted steel members (Menkes and Opat, 1973, Yu and Jones, 1991, Yu and Jones, 1997).

B. Strain hardening
Strain hardening plays a significant role in the nonlinear response of structural metals such as steel under dynamic impact loads. It has been shown by Bai and Pedersen
(1993) that when strain hardening is included in the material behaviour of a structural member subjected to transverse impact, the structure becomes stiffer, resulting in a smaller strain energy absorption. In this study, the strain hardening behaviour of the steel is modelled using isotropic strain hardening in which the increase in yield stress is assumed to be equal in all directions with the increasing plastic strain. The strain hardening behaviour is defined in ABAQUS/Explicit as the yield stress-plastic strain relationship. Fig. 3.4 shows the effect of strain hardening on the typical stress-plastic strain behaviour of steel.

![Figure 3.4: Typical true plastic stress-true plastic strain relationship of steel, including strain hardening and strain rate effects.](image)

C. Strain rate dependence

Strain-rate dependence or strain-rate sensitivity is one of the most important material dynamic phenomena that must be included in the impact analysis of structures. For strain-rate sensitive material such as steel or aluminium, the yield stress increases considerably due to the rapid increase in strain, see Fig. 3.4. This increase in yield stress depends on the material type and the rate of strain increase (Cowper and Symonds, 1957).

Two methods are offered by ABAQUS/Explicit to introduce the effects of strain-rate dependence in the material model. These are using the Cowper-Symonds over-stress power law and using the tubular input of yield ratios. The Cowper-Symonds equation is used in this research. It has been used by many numerical studies to simulate the strain rate dependence of steel subjected to impact loads (Yu and Jones, 1997, Zeinoddini et al., 2008, Thilakarathna et al., 2010, Bambach, 2011).
Cowper-Symonds over-stress power law
Cowper and Symonds (1957) suggested the following equation as the constitutive relationship between the dynamic flow stress and the uniaxial plastic strain rate:

\[ \dot{\varepsilon}^{pl} = D (\frac{\bar{\sigma}}{\sigma_o} - 1)^n \] ................................................... ................................................... .......................

3.1

Or in another form:

\[ \bar{\sigma} = \sigma_o \left[ 1 + \left( \frac{\dot{\varepsilon}}{D} \right)^n \right] \] ................................................... ................................................... ..................

3.2

Where

\( \dot{\varepsilon}^{pl} \) is the uniaxial equivalent plastic strain rate,

\( \sigma_o \) is the value of the static flow (yield) stress, and,

\( \bar{\sigma} \) is the value of the dynamic flow (yield) stress at a non-zero strain rate.

The material parameters \( D \) and \( n \) can be obtained based on the uniaxial compression test data (Jones, 1997).

Table 3.1 shows the Cowper-Symonds equation parameters for three of the most common structural materials obtained by comparing Eq. 3.2 with relevant experimental data (Cowper and Symonds, 1957, Jones, 1997).

<table>
<thead>
<tr>
<th>Material</th>
<th>( D )</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mild-Steel</td>
<td>40.4</td>
<td>5</td>
</tr>
<tr>
<td>Aluminium</td>
<td>6500</td>
<td>4</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>100</td>
<td>10</td>
</tr>
</tbody>
</table>

It can be noticed from Eq. 3.2 and Table 3.1 that for mild steel, when the strain rate goes to \( 40s^{-1} \), the dynamic yield stress value becomes twice the static stress. It is common to reach a strain rate value of \( 40s^{-1} \) even in low velocity impacts.

D. Progressive damage and failure for ductile metal (SIMULIA, 2010a)
The progressive damage and failure model in ABAQUS/Explicit is used to trace the material behaviour from fracture initiation toward complete failure. This failure model can allow the specification of different failure initiation criteria including tensile and
shear failure. In this failure model, the material stiffness is degraded progressively after
damage initiation according to a specified damage evolution response. This progressive
damage allows for a smooth degradation of the material stiffness, making it suitable for
both quasi-static and dynamic situations. The model offers two choices after complete
failure, these being the removal of elements from the mesh and keeping it in the model
but with a zero stress value.

The progressive damage and failure model provided in ABAQUS/Explicit is employed
in the present numerical model in conjunction with the isotropic metal plasticity model.
According to this model, the typical stress–strain behaviour of steel with strain
hardening and strain rate effects undergoing tensile or shear failure is shown in Fig. 3.5
(SIMULIA, 2010a).

![Figure 3.5: Typical uniaxial stress-strain response of the steel with progressive damage
evolution up to failure.](image)

Fig. 3.5 shows a typical stress-strain curve for a progressive damage and failure model
for a ductile material. It consists of three parts: the undamaged material behaviour (a-b-
c-d′ in Fig 3.5), the damage initiation criterion (point c in Fig 3.5), and the damage
evolution law (c-d in Fig 3.5). The first part represents the material stress-strain
response in the absence of damage which can be obtained from uniaxial tensile test
while the following sub-sections describe the approach used in the present study to
define the other two parts which deal with damage.
Chapter Three: Validation of Finite Element Modelling

Part 2. Damage initiation criterion
The damage initiation criterion represents the point in the material true stress-true strain response where damage starts to develop and progress. For metals, two damage initiation criteria are offered by ABAQUS/Explicit according to the fracture mechanism. Both of these criteria are employed in the present study. They are:

1. The ductile damage criterion (for tensile fracture)
The ductile damage criterion uses the equivalent plastic strain limit at the onset of damage $\varepsilon_{D}^{pl}$ as the failure initiation criterion. The value of this strain is described as a function of plastic strain rate $\dot{\varepsilon}^{pl}$ and stress triaxiality $\eta$ as follows:

$$\varepsilon_{D}^{pl} (\eta, \dot{\varepsilon}^{pl})$$

Where the stress triaxiality $\eta$ is defined by ABAQUS as the ratio of the pressure stress to the equivalent Mises stress ($\eta = \frac{-P}{q}$).

Material degradation is assumed to start when the damage initiation criterion $\Phi_{f}$ is met based on the following condition:

$$\Phi_{f} = \frac{\sum \Delta \varepsilon^{pl}}{\varepsilon_{D}} = 1$$

Where $\sum \Delta \varepsilon^{pl}$ is the accumulative value of the equivalent plastic strain.

Implementing the above damage criterion is undertaken in two steps in this study. In the first step, a nonlinear dynamic analysis is performed using ABAQUS/Explicit under the same loading condition but using the isotropic metal plasticity constitutive model to obtain the values of the maximum stress triaxiality and strain rate corresponding to the loading condition for the entire model. Then, the fracture strain is obtained from the true stress versus the true strain curve extracted from the uniaxial tensile test of the material. In the second step, the values of fracture strain together with the values of strain rate and stress triaxiality are used as input data in the material damage model.
2. **Shear damage initiation criterion.**

Here, the damage initiation criterion is described in terms of the equivalent plastic strain at the onset of shear damage ($\varepsilon_{pl}^S$). The value of this strain is a function of the strain rate $\dot{\varepsilon}^{pl}$ and the shear stress ratio $\theta_S$ (Boh et al., 2004, SIMULIA, 2010a).

\[
\varepsilon_{pl}^S (\theta_S, \varepsilon^{pl}) \quad \text{.................................................. ................................................... .............................. 3.5}
\]

Where shear stress ratio $\theta_S$ is defined as:

\[
\theta_S = \frac{(q + k_s p)}{\tau_{max}} \quad \text{.................................................. ................................................... .............................. 3.6}
\]

Where $\tau_{max}$ is the maximum shear stress and $k_s$ is a material parameter.

Material degradation starts when the damage initiation criterion $\Phi_S$ is met based on the following condition:

\[
\Phi_S = \frac{\sum \Delta \varepsilon_{pl}^S}{\varepsilon_S} = 1 \quad \text{.................................................. ................................................... .............................. 3.7}
\]

Where $\sum \Delta \varepsilon_{pl}^S$ is the accumulative value of the equivalent plastic strain.

The same procedure that is used to implement the ductile damage criterion is used to implement the shear damage criterion, except that the maximum shear stress ratio is extracted from the first step of the nonlinear dynamic analysis.

**Part 3. Damage evolution and degradation**

Once damage initiation is detected at any element using any of the aforementioned damage initiation criteria, the damage process starts and continues to cause progressive degradation of the material stiffness of the element until failure. The rate of degradation with time is referred to as the damage evolution law which can be specified either as effective plastic displacement or fracture energy dissipation (SIMULIA, 2010a). In this study, the damage evolution law in all numerical simulations uses an effective plastic displacement assuming a linear relationship between the damage variable ($d$) and the effective plastic displacement $u^{pl}$ (Qiao et al., 2006). The damage variable represents the accumulated ratio of the effective plastic displacement $u^{pl}$ to the total plastic displacement at the point of failure (full degradation), $u_{f}^{pl}$, as shown in the following equation:
The effective plastic displacement \( u^{pl} \) is defined with the following evolution equations:

\[
\dot{d} = \frac{u^{pl}}{u_f^{pl}} \tag{3.8}
\]

\[
u^{pl} = L_c \varepsilon^{pl} \tag{3.9}
\]

\[
u_f^{pl} = L_c \varepsilon_f^{pl} \tag{3.10}
\]

Where \( L_c \) is the characteristic length of the element defined as the square root of the integration point area for shell elements and the cubic root of the integration point volume for solid elements (SIMULIA, 2010a); \( \varepsilon_f^{pl} \) is taken as the strain at the complete failure of the material taken from the uniaxial stress strain curve.

Using the effective plastic displacement approach as the damage evolution law helps to reduce the mesh dependency of the results. By using this approach, the degradation response of the material is characterized by a stress-displacement behaviour rather than a stress-strain behaviour. According to the definition of the damage variable \( d^* \), its value ranges from 0 corresponding to damage initiation to 1 representing complete failure. For solid element, failure is assumed to take place when \( (d^* = 1) \) at any one integration point of an element in the model. However, in a shell element all through the thickness section points at any one integration location of an element must fail before the element failure. In any case, when complete failure occurs, ABAQUS/Explicit offers two procedures to complete the analysis: either by removing the failed element from the model mesh or by keeping it but setting its stress to zero in the next analysis step. The first option is used in the present study.

In this research study, the material model combining classical metal plasticity with progressive damage and failure is used because it is the most general, effective and easy one to use.

### 3.2.2.3. Modelling of contact (SIMULIA, 2010d)

When two bodies impact on each other, a contact interaction develops between the contacted surfaces at the impact zone. This interaction generates a concentrated stress and/or pressure at each surface with a value depending on the geometrical, material and dynamic characteristics of each body at the time of impact in addition to the mechanical
properties of the contacted surfaces. The pressure resulting from the contact interaction is referred to as contact pressure and consequently the resulting force is referred to as the contact or impact force. The contact force takes a very short duration to develop and then vanishes after the two contacting surfaces separate from each other. However, the contact force has critical effects on the behaviour and failure of the structural members under impact. Thus, an accurate and realistic modelling of the contact force is necessary.

Using ABAQUS/Explicit two approaches can be followed to simulate contact. In the first approach, the impact force is simulated as a physical contact interaction. Alternatively, the impact force may be directly input as a time dependent impulse function using the AMPLITUDE option available in ABAQUS/Explicit (Dorogoy and Rittel, 2008) as shown in Fig. 3.6. The first approach is adopted in all the numerical simulations of this study because it gives more realistic results for column behaviour under impact (Yu and Jones, 1997, Zeinoddini et al., 2008, Thilakarathna et al., 2010).

![Figure 3.6: Simulating impact force as a time dependent function force (SIMULIA, 2010d)](image)

**Defining the contact interaction**

The contact pair algorithm available in ABAQUS/Explicit is employed in the numerical model to generate the contact. Although the contact pair algorithm is more restrictive concerning the types of surfaces involved in contact than the general contact algorithm which is also available in ABAQUS/Explicit, it is used as the default option in this study because it offers better accuracy. The general contact approach is also used in part of the numerical simulations conducted in chapter five for generating contact between the numerical vehicle model and the steel column. In such a contact problem, the contact pair algorithm cannot be utilized due to several restrictions relating to the characteristics of the surfaces involved in the contact. The following sub-sections describe the main contact characteristics adopted in the present study.
A. Defining the contact using a contact pair algorithm

The contact pair algorithm is defined by specifying the following interaction properties:

- **Selection of the surfaces used in the contact interaction**

The element based surface is used in most of the numerical simulations of this research. It can be used to define surfaces on the external facet of the body as a deformable or rigid surface. The selected contact surfaces are defined in a form of master and slave surface, see Fig. 3.7

![Figure 3.7: A contact pairs as master and slave surfaces](SIMULIA, 2010d)

Defining the contact surfaces using a pure master-slave approach requires refining the mesh of the model at the contact surfaces to prevent the master surface facet from overly penetrating the slave surface as shown in Fig. 3.8. This problem is most likely to occur when there is contact between a deformable body and a relatively rigid body which is common in the numerical simulations of this study.

![Figure 3.8: Master surface penetrating into the slave surface of a pure master-slave contact pair due to improper meshing](SIMULIA, 2010d)
Chapter Three: Validation of Finite Element Modelling

- **Selecting the contact formulation** (constraint enforcement formulation)

Two methods can be used to enforce the contact in numerical simulations depending on the nature of the contact surfaces: the kinematic contact formulation and the penalty contact formulation. The former utilizes the kinematic properties of the node (the mass associated with the node, the distance the slave node has slipped and the time increment) in the calculations. The kinematic contact formulation is able to conserve the kinetic energy of the contact. In addition, it does not allow the penetration problem to occur. Hence, it is considered more suitable for this study. The penalty contact formulation is used to model rigid surface contact that cannot be modelled using the kinematic contact method.

- **Selecting sliding formulation**

The default option of the finite sliding in ABAQUS/Explicit is used to define relative motion between the contact surfaces. This is a more general approach and assumes that the relative incremental motion between the two contact surfaces does not significantly exceed the characteristic length of the master surface faces. Moreover, this formulation allows for arbitrary separation, sliding, and the rotation of the surfaces during contact. Since the numerical model of this study aims to simulate the behaviour of steel columns under low to medium velocity impact, the finite sliding assumption is more suitable for this analysis.

- **Selecting the mechanical contact properties**

These contact properties define the mechanical surface interaction models that control the tangential and normal stress behaviour of surfaces when they are in contact. The default ABAQUS/Explicit option of hard contact is utilized to describe the pressure-overclosure relationship of the contact interaction in the normal direction. In hard contact behaviour when the distance between the two surfaces (clearance) becomes zero, impact pressure is generated and the contact constraint is applied, see Fig. 3.9. Afterwards, the contact pressure becomes zero. No penetration is allowed in the hard contact model and there is no limit for the value of the contact pressure generated from the impact.
For the tangential direction, ABAQUS/Explicit offers more than one model to describe the friction formulation. The isotropic penalty friction formulation is used throughout this study. This model uses the Coulomb friction model to relate the maximum allowable frictional stress to the contact pressure. The model allows for the specifying of the coefficient between the contact surfaces and assumes that the coefficient is the same in all directions. Frictionless and rough friction models are also available in ABAQUS/Explicit and they can be used to simulate the non-friction and nonslip behaviour of contact respectively. Nevertheless, because of the very short duration of the impact event, the effect of friction behaviour on contact interaction is very low.

Other characteristics of contact, such as the tracking approach and contact weighing algorithm, are chosen automatically by ABAQUS/Explicit.

**B. Defining the contact using the general contact approach**

ABAQUS/Explicit has several requirements on the surfaces involved in the contact problem in order to be able to use the contact pair algorithm. Some of these restrictions are not satisfied in the full scale numerical vehicle model, used in chapter five, such as the fact that the contact surfaces are not continuous and deformable and rigid bodies are combined to define a single surface at some of the contact surfaces. This is because the vehicle model comprises different components modelled using deformable bodies, rigid bodies and rigid surfaces with different element types. This variety in the element types, in addition to the nature of the geometrical shape of the vehicle, restricts the use of the contact pair algorithm. Therefore, general contact approach is used in the numerical simulations performed in chapter five to simulate the contact because it allows simple definitions of contact with very few restrictions on the types of surfaces involved.
The same modelling parameters used to define the contact pair algorithm are used to define the general contact in ABAQUS/Explicit, except that the general contact algorithm uses only the penalty enforcement method to enforce contact constraints between the contacting surfaces.

3.2.2.4. Stability limit and time increment control

The stability limit in ABAQUS/Explicit can be defined as the maximum time increment that can be used in the dynamic explicit analysis procedure (SIMULIA, 2010c). The highly geometrical and material nonlinearities of the dynamic impact problem of this research require the stability limit of the model to be changed continuously. Therefore, the full automatic time increment is used in all numerical simulations in this study to automatically adjust the time increment. In this time increment strategy, the analysis starts by calculating the stability limit based on the maximum frequency of all the individual elements in the system using the element by element search method as in the following:

\[ \Delta t_{\text{stable}} = \frac{L_e}{c_d} \] \hspace{1cm} (3.11)

Where:

- \( L_e \) is the element length taken as the shortest element distance, and
- \( c_d \) is the material wave speed calculated from the equation:

\[ c_d = \sqrt{\frac{E}{\rho}} \] \hspace{1cm} (3.12)

Where \( E \) is the modulus of elasticity of the material, and \( \rho \) is the material density.

Afterwards and according to the nature of the problem under investigation, the stability limit may be defined in terms of the maximum frequency of the dynamic system which is referred to as the global stability limit using the following equation:

\[ \Delta t_{\text{stable}} = \frac{2}{\omega_{\text{max}}} \left( \sqrt{1 + \xi_{\text{max}}^2} - \xi_{\text{max}} \right) \] \hspace{1cm} (3.13)

Where \( \omega_{\text{max}} \) is the maximum frequency of the dynamic system and \( \xi_{\text{max}} \) is the corresponding damping ratio defined by:
\[ \xi = \frac{c}{c_c} \] .......................................................... ........................................... .................................................. 3.14

Where \( c \) is the damping constant of the system and \( c_c \) is the critical damping value described by:

\[ c_c = 2M_T \omega_{\text{max}} \] .......................................................... .................................................. ........................................... ........................................... .................................................. 3.15

Where \( M_T \) is the total mass of the structural system.

For an un-damped system \( \xi = 0 \) which is often used in impact simulations. Then the stability limit according to Eq. 3.13 becomes:

\[ \Delta t_{\text{stable}} = \frac{2}{\omega_{\text{max}}} \] .......................................................... .................................................. ........................................... ........................................... ........................................... ........................................... .................................................. 3.16

The full automatic increment control is more conservative, especially during the initial part of the analysis when using the element by element basis to estimate the stability limit. It also has more control on the progress of the analysis. Therefore, it is adopted in this study.

3.2.2.5. Damping effects

Damping has no significant effect on structural behaviour for the simulated cases in this study because the duration of the impact is very short compared to the natural period of the structure system (Jones, 1997, Sastranegara et al., 2006, Zeinoddini et al., 2008, Thilakarathna et al., 2010). However, to investigate any possible effect of damping, the procedure below is used to determine the damping coefficient.

In ABAQUS/Explicit the damping effect can be accounted for in the material modelling phase by employing the Rayleigh damping procedure (SIMULIA, 2010a). Rayleigh damping assumes that the damping matrix is a linear combination of mass and stiffness matrices, as follows (Clough and Penzien, 1975, Humar, 2002):

\[ \begin{bmatrix} \bar{C} \end{bmatrix} = \alpha \begin{bmatrix} \bar{M} \end{bmatrix} + \beta \begin{bmatrix} \bar{K} \end{bmatrix} \] .......................................................... .................................................. ........................................... ........................................... ........................................... ........................................... .................................................. 3.17

Where:

\[ \begin{bmatrix} \bar{M} \end{bmatrix} \] and \[ \begin{bmatrix} \bar{K} \end{bmatrix} \] are the nodal mass and stiffness matrices of the structural system, 
\[ \begin{bmatrix} \bar{C} \end{bmatrix} \] is the damping matrix of the structure, and 
\( \alpha \) and \( \beta \) are the mass and stiffness proportional Rayleigh damping factors respectively.
For a given mode of vibration \( m \), the Rayleigh damping factors (\( \alpha \) and \( \beta \)) can be related to each other using the following expression:

\[
\xi_m = \frac{\alpha}{2\omega_m^2} + \frac{\beta \times \omega_m}{2}
\]

Where \( \omega_m \) is the frequency of the vibration mode \( m \) and \( \xi_m \) is the damping ratio specified for the vibration mode \( m \).

Eq. 3.18 indicates that the mass proportional Rayleigh damping, \( \alpha \), affects damping of the lower frequency modes while the stiffness proportional Rayleigh damping, \( \beta \), affects damping of the higher frequency mode (SIMULIA, 2010a). Moreover, Eq. 3.18 shows that, for the maximum frequency mode of the structure, \( \omega_{\text{max}} \), the value of \( \xi_{\text{max}} \) increases when the stiffness proportional Rayleigh damping, \( \beta \), increases as in the following equation.

\[
\xi_{\text{max}} = \frac{\alpha}{2\omega_{\text{max}}^2} + \frac{\beta \times \omega_{\text{max}}}{2}
\]

On the other hand, Eq. 3.13 shows that the time increment for numerical stability in ABAQUS/Explicit decreases with increasing \( \xi_{\text{max}} \). Therefore, using the stiffness proportional Rayleigh damping, \( \beta \), to damp out the lowest frequency mode in ABAQUS/Explicit significantly decreases time increment for stable dynamic analysis. Hence, using the mass proportional damping, \( \alpha \), is more suitable to damp out the low frequency response in ABAQUS/Explicit analysis (Thilakarathna et al., 2010, SIMULIA, 2010a).

For low frequency response, the mass proportional damping, \( \alpha \), can be calculated by neglecting the contribution of the stiffness proportional damping (i.e. \( \beta = 0 \)), in Eq. 3.18, to obtain the following equation:

\[
\alpha = 2\omega_m^2 \xi_m
\]

The natural frequency of the structural dynamic system, \( \omega_m \), can be calculated in the linear perturbation analysis available in ABAQUS/Standard while the damping ratio
needs to be assumed in advance according to the nature of the structural system. Once the damping ratio is assumed and the frequency is determined, the Rayleigh mass proportional damping factor can be subsequently calculated from Eq. 3.20. Section 3.3 gives an example of the effects of damping.

3.2.2.6. Sequence of axial load application

The numerical model developed in this study is mainly intended to simulate the behaviour and failure modes of an impacted steel column under a static compressive axial load. Therefore, the model must be able to maintain a static axial load on the structural member while the member is subjected to dynamic impact. This means that both the static and dynamic loads should be exerted simultaneously during the entire dynamic analysis duration. In ABAQUS/Explicit, only one dynamic analysis is allowed. Therefore, the only way to simulate the static load effect is to apply the axial load as a quasi-static load using the quasi-static analysis procedure available in ABAQUS/Explicit (SIMULIA, 2010e). To perform a quasi-static analysis, the load should be applied as a time dependant function using the SMOOTH AMPLITUDE option (SIMULIA, 2010d), see Fig. 3.10. Furthermore, the time period during which the load is applied must not be less than the natural period of the structural system to produce accurate static results by eliminating pseudo-dynamic effects (SIMULIA, 2010e, Biggs, 1964).

Figure 3.10: Smooth step amplitude curve used to define a quasi-static load (SIMULIA, 2010e)

To ensure that the load effect is static rather than dynamic, the kinetic energy of the deformable structural system should not exceed 10% of the total strain energy(SIMULIA, 2010e), see Fig. 3.11.
Figure 3.11: Energy histories for a quasi-static structural system (SIMULIA, 2010e)

Where:

- $IE$ is the internal energy;
- $VD$ is the viscous dissipation energy;
- $KE$ is the residual kinetic energy;
- $FD$ is the frictional dissipation energy;
- $WK$ is the work done by the external forces; and
- $ETOTAL$ is the total conserved energy of the system.

Hence, for the numerical simulations involving axially loaded members under impact, the sequence of load application is in two separate analysis steps as follows:

- a) Quasi-static step: the axial compressive load is applied during the natural period of the column;
- b) Impact dynamic step: after the column has achieved equilibrium, the impact load is applied by establishing a contact interaction between the impacting body and the impacted member.

### 3.3. Validation of the numerical model

Although the dynamic impact behaviour of structures has been investigated previously by many research studies, the problem of axially preloaded steel columns subjected to transverse impact loads has been very rarely considered either experimentally or numerically. Therefore, validation of the numerical simulations of such a problem is somewhat difficult. However, confidence in the correct implementation of the
simulation procedure can be developed through a simulation of the general dynamic transverse impact problem by checking the modelling approaches used in this study as described in the previous sections.

In this section, the ABAQUS/Explicit simulation results will be assessed against three series of published experimental tests. These three series of experiments are selected to ensure that different possible column failure modes are covered. These validation test series are:

- Transverse impact tests on steel tubes under an axial compressive load (Zeinoddini et al., 2002, Zeinoddini et al., 2008);
- Transverse impact tests of clamped steel beams with a rectangular hollow section (Bambach et al., 2008); and
- Transverse impact tests of rectangular clamped steel beams (Yu and Jones, 1991, Yu and Jones, 1997).

### 3.3.1. Global plastic buckling failure

Tests were conducted by Zeinoddini et al.(2002). In these tests, the steel tube was one metre long; it was pre-compressed and then impacted at the midpoint using a drop weight of 25.45 kg with a falling velocity of about 7 m/s (about 25 km/hour). The testing rig of the experiment was set up to provide fixed support at one end of each specimen and sliding support at the other. The tubes were loaded to different levels of axial compression loads. The test set up is shown in Fig. 3.12 together with the present ABAQUS model.
3.3.1.1. Model description

As shown in Fig. 3.12, the present numerical model consists of four parts: the steel tube, the impacting mass, the axial spring and the lateral spring. The steel tube was modelled using linear three dimensional four-node doubly curved general purpose shell elements with reduced integration and hourglass control that account for the finite membrane strains (S4R), (SIMULIA, 2010b). The impacting body was modelled using a three dimensional eight-node brick element with reduced integration and hourglass control (C3D8R). The isotropic classical metal plasticity model available in ABAQUS/Explicit was used for the steel tube with elastic-perfectly plastic stress-strain curve (Zeinoddini et al., 2002). The density and modulus of elasticity for the steel tube were taken as 7850 kg/m$^3$ and 200000 N/mm$^2$ respectively as specified in the experimental tests (Zeinoddini et al., 2002). The input yield stress of the steel tube was 500N/mm$^2$ with Poisson's ratio of 0.3. The density of the impacting body material (Steel EN24) was adjusted to give a total weight of 25.45 Kg according to the experiments. Because no
material failure was encountered during the experiments, no failure modelling was necessary in this example. Since the tubes were made of high tensile steel which usually shows very low sensitivity to strain rate (Jones, 1997) no strain rate effect was considered in the numerical model.

The support condition of the tube was assigned using the BOUNDARY option available in ABAQUS(SIMULIA, 2010d) by constraining and releasing the corresponding degrees of freedom at each end. A linear axial spring was used to apply the axial compressive load, as shown in Fig. 3.12. One of the linear spring’s ends was linked to a fixed reference point while the other end was attached to a reference point which was constrained with the circumference of the sliding end of the tube using the COUPLING TIE option in ABAQUS(SIMULIA, 2010d). The axial linear spring was introduced to the sliding end of the tube to account for the decrease in axial force caused by the movement of this end towards the other end (fixed end) as observed in the tests (Zeinoddini et al., 2002). A lateral stiffness with a very low stiffness value was also introduced in the model to account for the friction force of the vertical guide used to drop the weight (Zeinoddini et al., 2002). One end of the lateral spring was attached to the impacting body and the other was linked to a fixed reference point as can be seen in Fig 3.12.

The axial load was applied through the linear spring using a displacement control approach. The values of the linear spring stiffness and the initial axial displacement were specified to give the desired axial compressive load. The initial axial displacement was applied in a smooth amplitude function during the quasi-static analysis step with a time period equal 0.0025413 second representing the lower natural period of the system, see Fig. 3.13.

To ensure the quasi-static application of the axial load, the energy histories of the tube were plotted during the time step at which the axial load was applied as shown in Fig. 3.14. It is clear from this figure that the kinetic energy of the system (KE) during the quasi-static step is very small compared to the internal energy (IE) and the external work (WK).
Chapter Three: Validation of Finite Element Modelling

Figure 3.13: Smooth amplitude functions used to apply the quasi-static load

Figure 3.14: Energy histories during the quasi-static load application, (P/Py) = 0.5.

The contact pair algorithm available in ABAQUS/Explicit was used to simulate the contact interaction between the impacting mass and the steel tube. Hard contact and penalty friction formulation were used to describe normal and tangential behaviours respectively as mechanical interaction properties with the coefficient of friction was assumed to be 0.47. Kinematic contact enforcement method was adopted to detect contact between the two bodies with small sliding formulation.

### 3.3.1.2. Simulation results

The experimental results indicated global column failure in the columns with an axial load value above 65% of the tube squash load (Zeinoddini et al., 2002). In this failure mode, the column lost its stability and large lateral deformations developed causing the column to shorten and slide towards the fixed end. For lower axial load ratios, no global failure occurred but local and plastic deformations and indentations were recorded at the impact zone (Zeinoddini et al., 2002, Zeinoddini et al., 2008). In the present numerical
model, global failure was predicted at axial load ratios greater than 60% of the tube squash load as shown in Fig. 3.15. For the other axial load values, there was no column failure, which conforms to the experimental results. Fig. 3.16 compares the recorded and simulated impact force-time relationships for different axial load ratios. The recorded and simulation deformation shapes of the column at failure are also shown. The agreement is very good.

![Axial displacement – the time history of the impacted steel tube for different axial load levels.](image1)

Figure 3.15: Axial displacement – the time history of the impacted steel tube for different axial load levels.

![Comparison of the contact force; Comparison of the deformation shape for (P/Py)=0.6; for the tests of Zeinoddini et al (2002)](image2)

Figure 3.16: (A) Comparison of the contact force; (B) Comparison of the deformation shape for (P/Py)=0.6; for the tests of Zeinoddini et al (2002)

### 3.3.1.3. Damping effect

According to Zeinoddini et al. (2002), the viscous damping ratio of the steel tubes was 2.3% and 5% for the first and second natural mode respectively. To investigate the
possible effects of damping, three damping ratios (1%, 5% and 10%) were investigated to cover all possible damping ratios. The Rayleigh mass proportional damping factors $\alpha$ was determined using the procedure described in section 3.2.2.5 as shown in Table 3.2. This value was then introduced in the material behaviour model of the steel tube to determine the sensitivity of the contact force against the damping effects. Fig. 3.17 shows a comparison of the contact forces generated from the impact at each damping ratio. It can be observed from this figure that there is only a minor effect from material damping on the contact force and on the behaviour and failure of the steel tube. This figure indicates that increasing the damping ratio from 1% to 10% increased the maximum contact force by about 8%.

<table>
<thead>
<tr>
<th>Damping ratios ($\xi$)</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>46.7</td>
</tr>
<tr>
<td>0.05</td>
<td>23.35</td>
</tr>
<tr>
<td>0.01</td>
<td>4.67</td>
</tr>
</tbody>
</table>

Figure 3.17: Effects of damping on the contact force history of the steel tube with (P/Py)=0.5

### 3.3.2. Tensile tearing failure

Bambach et al. (2008) performed tests on 700mm long clamped steel hollow section size 50SHS impacted laterally by a 600 kg mass falling with a velocity of 6.2 m/s (about 22.32km/h). Fig. 3.18 gives details of the experimental specimen together with the numerical simulation model. There was no axial load in the specimens.
3.3.2.1. Model description

The material behaviour of the hollow section (C350 steel) were simulated using the isotropic classical metal plasticity model in conjunction with the progressive ductile damage and failure model described in section 3.2.2.2 to simulate tensile failure. Isotropic strain hardening and the strain rate effect were accounted for by utilizing the stress and strain values provided from the experiment (Bambach et al., 2008). No plastic behaviour was included in the material behaviour model of the impacting mass and the supporting gusset plates because they did not show any sign of deformation during the impact test. The steel density was \(7850 \text{ kg/m}^3\) and modulus of elasticity \(200000 \text{ N/mm}^2\). The engineering yield stress for the beam was \(455 \text{ N/mm}^2\) and the Poisson’s ratio was 0.3, (Bambach et al., 2008). The material strain rate sensitivity was taken into account by employing the Cowper-Symonds equation with \(D= 40.4\) and \(q=5\), (Jones, 1997). Table 3.3 gives the true stress-strain values.

Table 3.3: Material properties for C350 used in the numerical simulation (Bambach et al., 2008)

<table>
<thead>
<tr>
<th>Section dimension (mm)</th>
<th>Beam length (mm)</th>
<th>True yield stress N/mm²</th>
<th>True ultimate stress N/mm²</th>
<th>True failure strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 × 50 × 1.6</td>
<td>700</td>
<td>456</td>
<td>584.64</td>
<td>0.145</td>
</tr>
</tbody>
</table>

3.3.2.2. Modelling of tensile failure

The plastic strain at the ductile damage initiation was obtained from the uniaxial tensile test of the beam material C350 to be \(\varepsilon_{D, pl}^t =0.115\). A nonlinear FE analysis was
performed on the impacted hollow steel beam under the same loading conditions but using the isotropic classical metal plasticity constitutive model to obtain the values of maximum stress triaxiality and the strain rate in the steel beam model. The values of these quantities together with the values of other material failure quantities used in the present numerical model are shown in Table 3.4.

<table>
<thead>
<tr>
<th>Plastic strain at damage initiation</th>
<th>Maximum stress triaxiality</th>
<th>Maximum strain rate ( (\text{sce}^{-1}) )</th>
<th>( \varepsilon_{\text{pl}}^f )</th>
<th>( u_{\text{f}}^f ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.115</td>
<td>0.7</td>
<td>14.2</td>
<td>0.145</td>
<td>0.000291</td>
</tr>
</tbody>
</table>

Since the mass and stiffness of the impacting body are much higher than the mass and stiffness of the impacted beam, the mesh size was refined at the impact zone to prevent the impactor surface from penetrating the steel column surface as discussed in section 3.2.2.3 of this chapter. According to the mesh size used in the numerical model for this test, the value of the characteristic length \( L \) was assumed to be 2 mm.

**3.3.2.3. Simulation results**

Table 3.5 and Fig. 3.19 compare the experimental and the numerical simulation results in terms of the peak contact force (Table 3.5) and the failure mode (Fig. 3.19). Table 3.5 indicates a close correlation for the peak contact force and Fig. 3.19 shows an accurate simulation of the complete tensile tearing failure of the beam section at the supports.

<table>
<thead>
<tr>
<th>Source of results</th>
<th>Peak contact force</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental Result (Bambach et al., 2008)</td>
<td>40.5kN</td>
</tr>
<tr>
<td>Present numerical results</td>
<td>45kN</td>
</tr>
</tbody>
</table>

Figure 3.19: Deformation shape and tensile fracture at the supports at 20 ms after impact. Top: experimental result (Bambach et al., 2008). Bottom: numerical simulation
Fig. 3.20 plots the damage initiation criterion profiles at different times and shows that the damage initiation criterion was satisfied at the supports only by reaching the maximum value of 1. Fig. 3.21 compares the damage evolution between the supports and the impact point. While the impact region experienced higher damage initially due to direct contact, the supports experienced a drastic increase in damage due to the lateral deformation of the structural member inducing a large axial tensile stress at around 8 ms.

![Figure 3.20: Ductile damage initiation profile history along the top surface of the beam](image1)

![Figure 3.21: Ductile damage initiation at the support and at the point of impact](image2)

From the comparisons between the experimental and simulation results, it can be concluded that the damage initiation and failure criteria have been employed correctly in the present model to simulate material failure under tension.

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3.3.3. Shear failure

Experiments were conducted by Yu and Jones (1991) on two mild steel beams with a solid rectangular cross-section, clamped ends and 101.6 mm in clear span, see Table 3.6. The clamped length of each end was 50.8 mm to ensure full fixity of the supports. The two steel beams were impacted transversely at distances and velocities shown in Table 3.6 by a rigid mass of 5kg.

Table 3.6: Technical details of the impact test (Yu and Jones, 1991)

<table>
<thead>
<tr>
<th>Beam No. in the test</th>
<th>Width, B mm</th>
<th>Thickness, H mm</th>
<th>Impact location* mm</th>
<th>Impact velocity m/sec</th>
<th>Failure condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>SB07</td>
<td>6.2</td>
<td>10.17</td>
<td>25.4</td>
<td>8.8</td>
<td>Just broken</td>
</tr>
<tr>
<td>SB08</td>
<td>6.2</td>
<td>10.13</td>
<td>49.9</td>
<td>10.6</td>
<td>Crack and sever necking</td>
</tr>
</tbody>
</table>

*measured from the left support

3.3.3.1. Model description

The ABAQUS brick element C3D8R was used, as in previous models, to simulate both the solid beams and the impacting body. The element size for the beam was selected to be 2.5 mm based on the mesh sensitivity check shown in the simulation results. As shown in Fig. 3.22, the size of the elements was reduced near the impact zone and the supports to be 0.5 and 1 mm respectively to ensure an accurate simulation of nonlinear behaviour, contact interaction and shear failure path as discussed earlier.

Figure 3.22: Numerical model and mesh size of the steel beams with a close-up view of the mesh at the impact point.
Fig. 3.23 shows the true stress-strain relationship of the mild steel recorded from the experimental test (Yu and Jones, 1991). The classical metal (Mises) plasticity model available in ABAQUS/Explicit was used to simulate the material behaviour with isotropic strain hardening and strain rate effects. The strain rate effects were described by employing the Cowper-Symonds equation with material parameters $D=1.05 \times 10^7$ s$^{-1}$ and $q = 8.3$ (Yu and Jones, 1991, Yu and Jones, 1997). The values of $D$ and $q$ have been chosen by Liu and Jones to describe the strain rate-sensitive behaviour of the steel material to fit the true stress-strain curve recorded from the experimental tests for different strain rates (Yu and Jones, 1991, Yu and Jones, 1997). The same contact interaction model as in the previous simulations was used to simulate the contact between the rigid mass and the solid beam except that, in the current model, the tangential behaviour of the contact was assumed to be frictionless, based on the experimental observation that slipping occurred during the tests between the impacting mass and the steel beam at the impact zone (Yu and Jones, 1991). Other properties (steel density, modulus of elasticity and Poisson’s ratio) were the same as in previous simulations.

![Figure 3.23: True stress-true strain curve of steel plastic (Yu and Jones, 1991)](image)

### 3.3.3.2. Modelling of shear failure

For this exercise, the damage initiation criterion was described in terms of the equivalent plastic strain at the onset of shear damage ($\varepsilon_{s}^{pl}$). Table 3.7 presents the values of these quantities used in the simulation.
Table 3.7: Material failure parameters used in the present numerical model

<table>
<thead>
<tr>
<th>Plastic strain at damage initiation</th>
<th>Maximum shear stress ratio</th>
<th>Maximum strain rate (s^(-1))</th>
<th>$\varepsilon_f^{pl}$</th>
<th>$u_f^{pl}$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.172</td>
<td>1.8</td>
<td>120</td>
<td>0.83</td>
<td>0.000415</td>
</tr>
</tbody>
</table>

**3.3.3.3. Simulation results**

A mesh sensitive analysis was carried out firstly to select the optimum mesh sizes at the impact zone, the supports and the rest of the beam as shown in Fig. 3.22. Fig. 3.24 compares the numerical maximum transverse displacement history at the impact point of steel beam SB07 with the experimental test results for different mesh sizes. It can be seen from this figure that the transverse displacement history of the impacted beam is more sensitive to the mesh size at the impact zone and the support than to the rest of the beam since most stresses and deformations occurred at these locations. Differences in numerical simulation results are small for the different mesh sizes used, confirming that the numerical simulation results are not mesh sensitive. Excellent matching can be seen between the experimental results and the numerical results corresponding to the mesh sizes shown in Fig. 3.22 (0.5mm and 1mm at impact zone and the supports respectively and 2.5 at the rest of the beam). The maximum displacement in both results was 16.2mm and both histories indicate that no failure was experienced in the beam. The maximum transverse displacement was either maintained constantly after 3 msec. as in the present numerical simulation or slightly decreased as in the experimental test.

![Figure 3.24: Comparison of the displacement at the impact point of the steel beam SB07 between the experimental results (Yu and Jones, 1991) and the present numerical simulation](image-url)
Fig. 3.25 shows the deformed shapes of the steel specimen SB08 after complete shear failure from both the test and the present numerical simulation. Good agreement can be seen between the two shapes in terms of the location and angle of the shear failure surface. In fact, correlation between the results is highly satisfying. Table 3.8 compares the maximum permanent transverse deformation of the same beam between the present numerical results, the experimental results of Yu and Jones (1991) and the numerical results of Yu and Jones which did not incorporate failure simulation (Yu and Jones, 1997). Fig. 3.26 further compares the axial normal strain of the steel beam SB08 underneath the impact point obtained by the present numerical model with that recorded experimentally by Yu and Jones (1991). Excellent agreement can be noticed throughout. This exercise can be used to confirm the accuracy and efficiency of the present numerical model to simulate shear failure mode.

Figure 3.25: Comparison of the deformation shape of the steel specimen SB08 after shear failure between the experimental test (Yu and Jones, 1991) (top) and the numerical simulation (bottom).

Table 3.8: Comparison of the maximum transverse displacement of the steel specimen SB08 between the results from the present numerical simulation with the experimental results, (Yu and Jones, 1991) and the numerical simulation results, (Yu and Jones, 1997)

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Experiment, (Yu and Jones, 1991)</th>
<th>Numerical, no failure criteria,(Yu and Jones, 1997)</th>
<th>Numerical, present study</th>
</tr>
</thead>
<tbody>
<tr>
<td>maximum permanent transverse deformation (mm)</td>
<td>21.8</td>
<td>21.26</td>
<td>21.5</td>
</tr>
</tbody>
</table>
Chapter Three: Validation of Finite Element Modelling

Figure 3.26: Comparison of axial strain of steel specimen SB08 on the lower surface underneath the striker between the experimental results of Yu and Jones (1991) and the present numerical simulation results

Fig. 3.27 plots the numerical results of the shear damage initiation criterion profiles along the beam length at the top and bottom surfaces. At the mid span where the shear fracture occurred, the shear damage initiation criterion was satisfied for both surfaces; at the supports, shear failure had started but had not progressed through the entire section. This behaviour agrees with the experimental observation.

Figure 3.27: Shear damage initiation profile at the top and bottom surfaces of the beam of the steel specimen SB08 along its length
Finally, Fig. 3.28 plots the stress-strain behaviour of a failed element at the impact zone of the steel beam specimen B08. As clearly shown in the figure, the material behaviour of the failed element follows the full failure initiation and propagation mechanism defined in the material behaviour model of the steel element shown in Fig. 3.23.

![Stress-strain behaviour of a failed element at the impact zone showing damage initiation and propagation of the element.](image)

**3.4. Summary**

This chapter has presented details of modelling methodology using ABAQUS/Explicit to simulate the behaviour and failure of structural members under transverse impact. To confirm the correct implementation of the simulation methodology, the present analysis has been compared with three series of published experimental impact tests. Comparisons have been made for a variety of quantities, including contact force, axial displacement, transverse displacement, failure modes, deformation shape, axial strain and stress-strain of a damaged element.

The following conclusions can be extracted from this chapter:

a) It may be accepted that the present ABAQUS/Explicit model with the associated element and material behaviour and failure models is capable of simulating the behaviour and different failure modes of axially compressed columns under transverse impact.
b) The quasi-static analysis procedure in ABAQUS/Explicit can be used to simulate the static force effect.

c) The results presented in this chapter may be considered to have provided an extensive body of evidence that ABAQUS/Explicit is capable of modelling axially loaded columns under transverse impact and that this model has been correctly implemented in the current research.

d) It has been proven numerically that damping has only a minor effect on the response and contact force of pre-compressed columns subjected to transverse impact load (see Fig. 3.17). This conclusion has also been reached by other researchers (Zeinoddini et al., 2008, Thilakarathna et al., 2010).
Chapter Four

A Parametric Study of the Behaviour and Failure Modes of Axially Loaded Steel Columns Subjected to a Rigid Mass Impact

4.1. Introduction

An important objective of this research is to develop a thorough understanding of the effects of different parameters on the response and failure modes of axially compressed steel columns under transverse impact. This can then enable simple methods of analysis to be developed so that complicated numerical analyses such as the ones employed in this research may be dispensed with within the practical design procedure. On the other hand, simplifying assumptions will be necessary when developing any design calculation method, and it is important that such assumptions are based on a comprehensive understanding of the effects of different parameters on column behaviour and failure modes such that the limitations of these assumptions are clearly defined.

This chapter presents the results of an extensive parametric study to investigate the effects of several parameters on the response of axially loaded steel columns under transverse impact by a rigid mass. The chapter intends to provide results, based on which simplifying assumptions can be made for column behaviour under impact. Chapter six will use these assumptions to develop appropriate design calculation methods.

4.2. Parametric study

The following six important parameters have been identified for investigation in the parametric study presented in this chapter:

a) The impact velocity;

b) The impact location;

c) The impact direction;

d) The axial compressive load ratio;
e) The column boundary condition;

f) The column slenderness ratio (section size and column length).

The following will describe in detail the numerical models for the steel column and the impacting mass.

### 4.2.1. Steel columns

The parametric study used simply supported and propped cantilever H-section steel columns designed according to the British Standard BS 5950: Part 1:2000 (BS, 2001) to resist the total loads shown in Table 4.1. These loads represent the approximate total axial compressive loads exerted on interior ground floor columns of 10 and 5 storeys respectively of typical commercial steel buildings. To account for the slenderness effect, two column lengths and sections were used. Table 4.1 lists the columns’ dimensions and load carrying properties.

<table>
<thead>
<tr>
<th>Column length (m)</th>
<th>Column section</th>
<th>Slenderness ratio ( \frac{kL}{r} )</th>
<th>Slenderness ( \frac{F}{r} \sqrt{\frac{E}{kL}} )</th>
<th>Boundary condition</th>
<th>Design axial compressive load (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>UC 305\times305\times118</td>
<td>51.50</td>
<td>0.68</td>
<td>S.S</td>
<td>3800</td>
</tr>
<tr>
<td></td>
<td></td>
<td>36.05</td>
<td>0.476</td>
<td>Prop</td>
<td>4250</td>
</tr>
<tr>
<td>8</td>
<td>UC 356\times406\times340</td>
<td>76.9</td>
<td>1.02</td>
<td>S.S</td>
<td>6800</td>
</tr>
<tr>
<td></td>
<td></td>
<td>53.85</td>
<td>0.711</td>
<td>Prop</td>
<td>9400</td>
</tr>
</tbody>
</table>

S.S.: Simply supported; Prop: Propped cantilever.

The steel was assumed to be of grade S355 and the steel modulus of elasticity was assumed to be 206000 N/mm². The transverse impact load was mainly applied to cause bending about the weak (minor) direction (z-z axis) of the steel column, which is assumed to represent the most critical situation for design purposes. This assumption has been confirmed in a separate parametric study in this chapter investigating the effects of column impact from different directions.
4.2.1.1. Modelling properties

The solid element offered by ABAQUS/Explicit with reduced integration and hourglass control C3D8R was used to model the geometrical behaviour of the steel column. To achieve the required accuracy of the simulation, the thickness of both flanges and the web of the column H-section were divided into four layers as shown in Fig 4.1 to overcome the problems that could be raised from using the first order and the reduced integration formulation of this element (SIMULIA, 2010b).

![Figure 4.1: Meshing technique used for the steel column: A) Longitudinal direction; B) Cross sectional direction for two steel column sections.](image)

The material behaviour of the steel was modelled using an isotropic classical metal plasticity model taking into account strain hardening and strain rate effects. The strain rate effect was described by employing the Cowper-Symonds equation with material parameters of $D=40.4 \, s^{-1}$ and $q=5$ (Jones, 1997). The progressive damage and failure model was used to account for the possibility of developing shear failure mode in the steel column with the values of damage initiation criteria shown in Table 4.2. These values were selected according to the uniaxial tensile test of S350 material shown in Fig. 4.2. Other material failure quantities such maximum shear stress ratio, and maximum strain rate were determined from a nonlinear finite element analysis carried out before this simulation and these are presented in Table 4.2. It should be mentioned that because this parametric study considered only columns with free axial movement, tensile tearing failure is not likely to happen. Therefore, Table 4.2 only gives properties for simulating transverse shear failure.
Figure 4.2: Assumed true stress-strain curve used to simulate S355 material behaviour in the parametric study.

Table 4.2: Material shear failure parameters for S355 steel used in the parametric study

<table>
<thead>
<tr>
<th>Plastic strain at damage initiation</th>
<th>Maximum shear stress ratio</th>
<th>Maximum strain rate (sec(^{-1}))</th>
<th>(\epsilon_f^{pl})</th>
<th>(u_f^{pl}) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.295</td>
<td>1.85</td>
<td>16.5</td>
<td>0.65</td>
<td>0.0065</td>
</tr>
</tbody>
</table>

The axial compressive load was applied on the steel column as a pressure load distributed over the cross section area using the quasi-static analysis procedure described in chapter three. The quasi-static analysis step was then followed by an explicit dynamic step during which the rigid mass impacted the steel column at the specified impact velocity and location by generating the contact interaction between the mass and the steel column. The BOUNDARY option was used to specify the columns’ supporting condition by constraining and releasing the associated degree of freedom at each column’s end.

4.2.1.2. Mesh size sensitivity

The sensitivity of the numerical simulation results against the column elements’ size was firstly examined by determining the column’s minimum natural frequencies and minimum buckling loads for different element sizes using the linear perturbation analysis procedure available in ABAQUS (SIMULIA, 2010c) of UC 356×406×340 and UC 305×305×118 columns with total lengths of 8 and 4 metres respectively. The numerical values are compared with the corresponding theoretical values (Timoshenko, 1961) as listed in Table 4.3. It can be seen that the element sizes shown in Table 4.3 are all appropriate for the static and global behaviour simulation of the columns.
Table 4.3: Sensitivity of some static and dynamic results against the element size of the simply supported column model

<table>
<thead>
<tr>
<th>Element size (mm)</th>
<th>UC 356×406×340, L= 8m</th>
<th>UC 305×305×118, L= 4m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency (Numer./Theor.)</td>
<td>( P_{\text{buckling}} ) (Numer./Theor.)</td>
</tr>
<tr>
<td>250</td>
<td>1.042693</td>
<td>1.190373</td>
</tr>
<tr>
<td>100</td>
<td>1.022265</td>
<td>1.045299</td>
</tr>
<tr>
<td>50</td>
<td>1.015226</td>
<td>1.031459</td>
</tr>
<tr>
<td>40</td>
<td>1.013849</td>
<td>1.028637</td>
</tr>
</tbody>
</table>

Afterwards, the effect of mesh size along the web height and over the flange length on column behaviour was also verified by changing the number of divisions for each part and plotting the corresponding column displacement histories. Fig. 4.3 (A and B) shows the sensitivity of the results to each element size. It can be observed from this figure that the column lateral displacement is more sensitive to the number of elements used for the flange width than the web height. Dividing the flange width into 12 divisions gives reasonable results compared to 18 divisions while element numbers along the web height did not affect the results. This can be attributed to the direction of bending. Therefore 12 and 10 divisions were used as minimum values for the flange width and the web height respectively as previously shown in Fig.4.1
On the other hand, to gain an accurate simulation of the contact interaction between the column and the impacting mass and to reasonably predict the local damage at the impact zone, the mesh size must be reduced properly at the impact zone. To achieve the optimum mesh size along the impact zone in addition to the rest of the column, a sensitivity analysis was carried out and Figs. 4.4 and 4.5 plot the time history of the column’s longitudinal movement, the shear damage evolution and the lateral movement at the impact point respectively for each element size. It can be seen from these figures
that a mesh size corresponding to 25600 elements for the steel column section UC 305×305×118 and 38845 elements for the steel column UC 356×406×340 (10 mm within one meter of the impact point and 50 mm for the rest of the column) can be adopted. Fig 4.6 shows such a mesh.

Figure 4.4: Sensitivity of the column axial displacement history, V=40 km/h (A) and the shear damage history at the impact point; V=60 km/h (B) for different mesh sizes, column section UC 356×406×340, P=50%P_{\text{design}}, Impacting mass=6 tonnes.
Figure 4.5: Sensitivity of the column lateral displacement history at the impact point for different mesh sizes, column section UC $305 \times 305 \times 118$, $P=50\%P_{\text{Design}}$, Impacting mass=6 tonnes, $V=40$ km/h.

Figure 4.6: A close-up view of the element size of the steel column model adopted in the parametric study.

4.2.2. The impacting mass

Since the current study aims to investigate the behaviour and failure modes of axially loaded steel columns under transverse impact, the emphasis will be on the column side rather than the impactor side. Nevertheless, a brief description of the impactor properties is required.
The impactor was assumed to be rigid mass with a cubic section of a dimensions (0.5 × 1.5 × 0.3) m as shown in Fig. 4.7.

Solid elements C3D8R were used to model the impactor body while its modulus of elasticity was selected to make it behave as an almost rigid body so that all the kinetic energy of the impact would be absorbed by the steel column without any contribution from the impactor. On the other hand, the density of the impactor was adjusted to give the required masses. The mesh size of the impactor was selected to be 100mm which was intended to be larger than what was specified for the steel column at the impact point (10mm) to prevent penetration of the master surface into the slave surface (SIMULIA, 2010d), see Fig. 4.7. The impact velocity was assigned to the mass as an initial boundary condition using the PREDEFINED FIELD option in ABAQUS/Explicit.

4.2.3. Modelling of contact
The contact pair algorithm discussed in chapter three was used to simulate the contact interaction between the steel column and the impacting mass. The surface of the impacting mass involved in the contact was defined in the numerical model as the master surface while the steel column surface was considered the slave surface as shown in Fig. 4.8. Hard contact was used to describe normal behaviour while the penalty friction formulation was used for the tangential behaviour with a coefficient of friction of 0.6 based on the nature of the contacted surfaces. The kinematic contact enforcement method was also adopted in the contact model to detect the contact between the two bodies with a small sliding formulation.
4.2.4. Analysis of simulation results

Table 4.4 lists the values of the parameters used in the numerical simulations. The impact velocities shown in the table may represent the average velocities of vehicles passing through urban, residential and commercial areas. The different weights may represent those of a typical car, a light truck and a lorry and the associated impact locations represent the possible location at which each vehicle type struck the column.

<table>
<thead>
<tr>
<th>Design Case</th>
<th>(P/P_{Design})%</th>
<th>Impacting mass (tonnes)</th>
<th>Impact velocity (km/h)</th>
<th>Impact location from bottom of column (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L=4 m, UC 305×305×118</td>
<td>0, 0.3, 0.5, and 0.7</td>
<td>1.0, 3.0, and 6.0</td>
<td>20, 40 and 80</td>
<td>1.0 for Mass =1.0 and 3.0 tonnes</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.5 for Mass = 3.0 and 6.0 tonnes</td>
</tr>
<tr>
<td>L=8 m, UC 356×406×340</td>
<td>0, 0.3, 0.5, and 0.7</td>
<td>1.0, 3.0, and 6.0</td>
<td>20, 40 and 80</td>
<td>1.0 for Mass =1.0 and 3.0 tonnes</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2 for Mass = 3.0 and 6.0 tonnes</td>
</tr>
</tbody>
</table>

4.2.4.1. Failure modes

Tables 4.5 to 4.8 present the failure modes for the two different column sizes and boundary conditions. It can be seen from these tables that global plastic buckling was the predominant failure mode except for the one case corresponding to the most heavily loaded (0.7P_{Design}) stocky column (4m, UC 305×305×118) subjected to the heaviest mass impacting at the highest velocity of 80 km/h. This indicates that although shear
damage may occur to axially compressed steel columns under lateral impact, the simulated scenario is unlikely to occur as it represents the very rare case of high impact velocity/high impact mass/high axial load. In addition, it has been noticed that for this particular case, it is most likely that shear failure occurred simultaneously with or just before global buckling failure of the column. Therefore, this mode of failure (shear failure) may be ignored when developing an analytical approach for quantifying the critical failure conditions of steel columns in buildings located in urban areas.

Table 4.5: Failure modes for a simply supported column section UC 356×406×340

<table>
<thead>
<tr>
<th>(P/P_{Design})%</th>
<th>L=8m, Impact location =2 m</th>
<th>L=8m, Impact Location =1m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Impact velocity (km/h)</td>
<td>Impact velocity (km/h)</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>Mass (tonnes)</td>
<td>Mass (tonnes)</td>
</tr>
<tr>
<td>0.7</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>G</td>
<td>N</td>
</tr>
<tr>
<td>0.5</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>0.3</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>0</td>
<td>N</td>
<td>N</td>
</tr>
</tbody>
</table>

\(G=\) Global plastic failure, \(N=\) No failure.

Table 4.6: Failure modes for a propped cantilever column section UC 356×406×340

<table>
<thead>
<tr>
<th>(P/P_{Design})%</th>
<th>L=8m, Impact location =2 m</th>
<th>L=8m, Impact Location =1m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Impact velocity (km/h)</td>
<td>Impact velocity (km/h)</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>Mass (tonnes)</td>
<td>Mass (tonnes)</td>
</tr>
<tr>
<td>0.7</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>0.5</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>0.3</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>0</td>
<td>N</td>
<td>N</td>
</tr>
</tbody>
</table>

\(G=\) Global plastic failure, \(N=\) No failure.
Table 4.7: Failure modes for a simply supported column section UC 305×305×118

<table>
<thead>
<tr>
<th>(P/P_{Design})</th>
<th>Impact velocity (km/h)</th>
<th></th>
<th>Impact velocity (km/h)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>G+FD N</td>
<td>G+FD N</td>
<td>G+FD S</td>
<td>G+FD N</td>
</tr>
<tr>
<td>0.5</td>
<td>G+FD N</td>
<td>G+FD N</td>
<td>G+FD G+FD</td>
<td>N N G+FD N</td>
</tr>
<tr>
<td>0.3</td>
<td>N N G+FD N</td>
<td>N G+FD G+FD</td>
<td>N N G+FD G+FD</td>
<td>N N N N G</td>
</tr>
<tr>
<td>0</td>
<td>N N N G</td>
<td>N N N G</td>
<td>N N N N G</td>
<td>N N N N N N</td>
</tr>
</tbody>
</table>

G=Global plastic failure, S=Shear failure, N=No failure, G+FD=Global plastic failure + local flange distortion

Table 4.8: Failure modes for a propped cantilever column section UC 305×305×118

<table>
<thead>
<tr>
<th>(P/P_{Design})</th>
<th>Impact velocity (km/h)</th>
<th></th>
<th>Impact velocity (km/h)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>N N G+FD G</td>
<td>G G G+FD G</td>
<td>N N G+FD N</td>
<td>N N G+FD N</td>
</tr>
<tr>
<td>0.5</td>
<td>N N G+FD N</td>
<td>N G+FD G+FD</td>
<td>N N N N N</td>
<td>N N N N N</td>
</tr>
<tr>
<td>0.3</td>
<td>N N FD N</td>
<td>N G+FD G</td>
<td>N N N N G</td>
<td>N N N N G</td>
</tr>
<tr>
<td>0</td>
<td>N N N N N</td>
<td>N N N N N</td>
<td>N N N N N</td>
<td>N N N N N</td>
</tr>
</tbody>
</table>

G=Global plastic failure, S=Shear failure, N=No failure, G+FD=Global plastic failure + local flange distortion

For column section UC 305×305×118, Tables 4.7 and 4.8 indicate that global plastic buckling was the predominant failure mode for the two boundary conditions used in the simulation, but it was accompanied by local distortion in the flanges at the impact zone, see Fig. 4.9. This figure also shows that the severity of the flange distortion increases with increasing axial load. However, a detailed examination of column behaviour from the simulation models in terms of the deformed shape at different time intervals after the impact (see Fig. 4.10) confirms that flange distortion occurred after column global instability. For example, Fig. 4.11 presents the axial displacement - time history for three columns. Rapid acceleration of deformation of the columns at about t= 45msec and t= 90msec for simply supported columns and t= 45msec for the propped cantilever column indicates onset of column failure. Fig. 4.10 shows the deformed shapes of these three columns, indicating no significant local flange distortion at the corresponding
times. This suggests that local flange distortion is a result, not the cause, of column global failure. Analysing column behaviour without considering local flange distortion would considerably simplify the analytical model.

![Diagram showing local flange distortion at different load levels and impact locations](image)

Figure 4.9: Local flange distortion at the impact zone for the section UC305×305×118.
Figure 4.10: Deformed shape history of the columns. (A): a simply supported section UC 305×305×118, Impact location= 1.0 m, P/P_{Design} =0.5, Impact mass =1 tonnes, Impact velocity =80 km/h; (B): a simply supported section UC 305×305×118, Impact location= 1.5 m, P/P_{Design} =0.7, Impact mass =3 tonnes, Impact velocity =20 km/h; (C) a propped cantilever section UC 305×305×118, Impact location= 1.5 m, P/P_{Design} =0.7, Impact mass =6 tonnes, Impact velocity =40 km/h
Chapter Four: Parametric Study

Figs. 4.12 to 4.14 show the shear damage profiles along the column height for the simply supported and propped cantilever columns that are most vulnerable to shear failure due to high impacting mass and velocity (Yu and Jones, 1991, Jones, 1997). It can be seen that, apart from two cases which have shown a high tendency to local shear failure (Fig. 4.12(A) and Fig. 4.13(B)), the damage initiation criteria were lower than 1.0. For the propped cantilever column, (see Fig. 4.14), it can be observed that the shear damage criterion at the fixed support is greater than that at the impact point due to the high stresses developed but the damage criteria is still below the failure limit. It is apparent from these figures that the transverse shear failure is unlikely to occur in the transversely impacted steel column when the transverse impact speed is within the range of low to intermediate velocities.
4.2.4.2. Impact energy

The most simplistic analytical method for columns subjected to transverse impact loads would be to assume quasi-static behaviour. This approach is based on the energy balance principle and the most important parameter for the impactor is its kinetic energy. To investigate this assumption, the numerical simulations considered a constant level of impact energy but different combinations of impacting mass and velocity. Figs. 4.15 and 4.16 present the simply supported and propped cantilever column behaviour. Each figure shows the column displacement histories for two different levels of impact kinetic energy (KE).

It can be noticed from Figs. 4.15 and 4.16 that in both levels of impact kinetic energy, the deformation behaviour of the same column under the same level of external impact energy but with different combinations of impact mass and velocity are different. Both figures indicate that a smaller velocity with a higher mass tends to give more severe
column response (larger displacement). Nevertheless, whether or not the impacted column would fail (which is the most important design decision) appears not to be very sensitive to the different values of the impact mass and velocity as long as the external impact energy is the same. For example, in Fig. 4.15(A), for the simply supported column, the impact energy of 80 KJ was about 70% of the critical impact energy to cause column failure. Therefore, none of the columns experienced failure. In contrast, in Fig. 4.15(B), the impact energy of 117 KJ was at the level of the critical impact energy of the column. All cases indicate column failure even though the case with the highest velocity, 55km/hour, took a longer time for the column to reach failure. The same behaviour can be observed in Fig. 4.16 which represents the same column section but with a fixed support at the column base (propped cantilever).

Figure 4.15: Behaviour of the simply supported column (section UC 305×305×118, L=4m, impact location = 1m) under the same impact energy but with different combinations of impactor mass and velocity.
Figure 4.16: Behaviour of the propped cantilever column (section UC 305×305×118, L=4m, impact location = 1.5 m) under the same impact energy but with different combinations of impactor mass and velocity.

The effect of impact energy on column failure may be confirmed by studying the change in the kinetic energy of the column. Fig. 4.17 presents the total kinetic energy histories of the whole structural system, including both the column and the impactor, for the columns subjected to the critical impact energy (117 KJ) but with different combinations of impactor mass and velocity under an axial compressive load of 50% of the design load. From this figure it can be seen that, for all combinations, the total kinetic energy decreases after impact due to increasing strain energy in the column.
After reaching the minimum value, the total kinetic energy increases. This increase is caused by the accelerated movement of the column, indicating the column is losing its stability.

After impact, if the column is stable, the kinetic energy of the system will become zero when both the column and the impactor come to rest. In contrast, if the column fails after impact, then the column will accelerate in deformation and the kinetic energy will increase. For the column, at the critical situation, its kinetic energy will decrease to zero but will then increase. The energy histories of these three situations are exemplified in Fig. 4.18. According to the trend shown in Fig. 4.18, the results in Fig. 4.17 suggest that only one column was at the critical situation while all the other columns lost global stability without coming to rest. The difference in behaviour of these columns is mainly due to the different amounts of energy absorption of these columns as a result of their difference in deformation pattern under different combinations of impacting mass and velocity (thus different momentum when keeping the impact energy the same). Nevertheless, the minimum total kinetic energies of these columns were only a small fraction of the initial total kinetic energy. Therefore, it may be accepted that the column deformation patterns are very similar. This will lead to considerable simplification to aid the development of an analytical model for the calculation of the critical velocity of impact that will just cause the column to fail.

Figure 4.17: The kinetic energy history of the axially loaded simply supported steel column (section UC 305×305×118, L=4 m, impact location =1m, P/P_{Design}=50%, Impact energy = 117KJ).
Figure 4.18: Comparison of the total kinetic energy history of the columns without failure, at the critical condition, with clear failures

### 4.2.4.3. Critical impact velocity

From the numerical simulations results presented in previous sections, it is possible to establish the axial force - critical impact velocity relationship for a given column section and length. Here, the critical velocity is defined as the minimum impact velocity that causes the column to fail. This critical velocity is of particular interest in the design of columns to transverse impact. Fig. 4.19 shows examples of column mid-height displacement versus time relationship under different impact speeds and these were used to obtain the critical velocity.

Figure 4.19: Column mid-height deformation history under different impact speeds, steel column height L=4 m, Impact mass= 1 tonnes, P/P_{Design}=50%, impact position =1m, for a simply support column
From this figure, it can be noticed that the column remains stable for velocities up to 50 km/hour. But the column fails at a velocity of 55 km/h or above. To confirm this, Fig. 4.20 compares the corresponding energy histories for the different terms of energy between impact velocities of 55 km/h and 50 km/h. For the case of 55 km/h, the kinetic energy increases drastically from about 0.20s, indicating rapid movement (instability) of the structure (Fig.4.20-A), while at the velocity of 50 km/h, there is no column movement after about 0.06s so the kinetic energy maintains at zero (stability) (Fig.4.20-B).

![Energy histories corresponding to impact velocities of (A) 55km/h and (B) 50km/h.](image)

Figure 4.20: Energy histories corresponding to impact velocities of (A) 55km/h and (B) 50km/h.
Using the same procedure, the axial force - critical impact velocity interaction curves can be obtained for different levels of axial compressive load ratios and for the two column boundary conditions investigated in this chapter as shown in the Figure 4.21.

![Figure 4.21: Axial force - critical impact velocity interaction curves of the steel columns used in the parametric study: (A) section UC 356×406×340, L=8 m, impact mass=6tonnes, impact location =2m); (B) section UC 305×305×118, L=4 m, impact mass=3tonnes, impact location =1.5m)](image)

4.2.4.4. Plastic hinge location

When developing analytical solutions to the transverse impact problem, it is necessary to know where the plastic hinge forms so as to quantify the plastic dissipation energy of the column. Fig. 4.22 shows how the relationship between the axial compressive load as a percentage of the design load (vertical axis) and the location of the plastic hinge, measured from the column base, as a percentage of the total column length, for columns failed in global mode for the two column sections, two boundary conditions and different impact locations. Here, additional numerical simulations were carried out using an impact location at the middle of the span of each column section to cover the effect of different possible impact locations on the location of the intermediate plastic hinge. Fig. 4.22 (A and B) shows that the plastic hinge location is not significantly affected by the axial load values or the impact location because it is always close to the column mid-span (0.375-0.5)L for the simply supported columns and (0.5-0.65)L for the propped cantilever columns. This is because when the column fails due to global plastic instability, the deformation shape of the column is more likely to follow the first mode for static buckling (Adachi et al., 2004, Sastrapnagara et al., 2006, Shope, 2006),
especially for high levels of axial compressive load (> 25%P_{\text{design}}) as can be shown in Figs. 4.23 and 4.24.

Figure 4.22: Effects of axial load level on the intermediate plastic hinge location on (A): simply supported columns; (B): Propped cantilever columns. Impact mass = 3 tonnes.
Chapter Four: Parametric Study

Figure 4.23: Collapse shapes showing the intermediate plastic hinge location for different axial load ratios of simply supported columns (a) L = 8 m, impact location = 2 m, Mass = 6 tonnes; (b) L = 8 m, impact location = 1 m, Mass = 3 Ton; (c) L = 4 m, impact location = 1.5 m, Mass = 6 tonnes; (d) L = 4 m, impact location = 1 m, Mass = 3 tonnes.

Figure 4.24: Collapse shapes showing the intermediate plastic hinge location for different axial load ratios of propped cantilever columns; (a) L = 4 m, impact location = 1.5 m, Mass = 3 tonnes; (b) L = 8 m, impact location = 2 m, Mass = 6 tonnes.
4.2.4.5. Effect of impact direction

The numerical simulations conducted in this chapter have assumed the impact direction causing bending about the weak axis of the column to be the most critical situation in terms of the column vulnerability to global failure. To validate this assumption, the numerical simulations in this section considered two additional impact angles of 45 and 30 degrees as shown in Fig. 4.25. The axial load - critical impact velocity curves resulting from these two cases are compared with that obtained from the 90 degree impact as shown in Fig 4.26. This figure suggests that the critical velocity of the impact on steel column subjected to 45 and 30 degrees’ impact are higher than that of 90 degrees’ impact. Therefore, for impact design, considering this direction (90 degrees) should give more conservative results.

![Figure 4.25: 30, 45 and 90 degrees of impact](image)

![Figure 4.26: Effect of impact direction on the critical impact velocity of a simply supported column section UC 305×305×118, Impact location=1.5m, Impact mass 6 tonnes.](image)
4.2.4.6. Damping effects

To investigate any material damping effect on column behaviour, three damping ratios (2.5%, 5% and 10%) were introduced into the analysis for one case from the previous parametric study that just experienced failure. The damping ratios were defined using the Rayleigh mass proportional damping coefficient since it is the most suitable for dynamic analysis to damp out the structure response in the lower frequency modes (SIMULIA, 2010a, Thilakarathna et al., 2010). Fig. 4.27 compares the axial displacement at the column top and the kinetic energy histories for different ratios of damping.

Figure 4.27: Effects of damping on the behaviour and failure of the impacted steel column (section UC 356×406×340, L=8 m, impact location = 2 m, P/P_{Design}=70%), (A): Axial displacement – time history of the steel column; (B) Kinetic energy - time history.
It can be seen from Fig. 4.27, that the material damping effect is almost trivial. With increasing damping, column failure was slightly delayed but not prevented. Fig. 4.28 shows column kinetic energies and the damping energies of the columns. Damping energies increase only after the columns have failed, as indicated by the increase in kinetic energies. Hence, it can be concluded that the effect of damping can be neglected when determining the critical velocity of impact causing column failure.

![Kinetic energy and damping energy histories](image)

Figure 4.28: Kinetic energy and damping energy histories of the impacted steel columns with the damping effect for the impacted steel column shown in Fig. 4.27.

### 4.2.4.7. Effects of strain hardening and strain rate

The numerical simulations presented in the previous sections have included strain hardening and strain rate effects in the steel material behaviour. To investigate the effects of these material properties on column behaviour, particularly the critical impact velocity, numerical simulations were carried out in this section without considering strain hardening and strain rate effects.

Fig. 4.29 (A and B) shows the effect of strain hardening on the axial load-critical impact velocity curve of the column. The figure shows a large reduction in the critical impact velocity values if the strain hardening effect is ignored for the steel section UC305×305×118 (see Fig. 4.29(B)). For the steel section UC356×406×340, L=8m, no remarkable effect can be noticed (see Fig. 4.29(A)). Whilst ignoring this beneficial effect of strain hardening may give a conservative design, the results are inaccurate. Chapter six will suggest a possible approach to incorporate this effect in the simplified analytical method.
Figure 4.29: Effect of strain hardening on critical impact velocity of the simply supported steel column section (A) section UC 356×406×340, L=8m, impact mass=6 tonnes, impact location =2m; (B) section UC 305×305×118, L=4 m, impact mass=6 tonnes, impact location =1.5m)

One the other hand, the effects of strain rate may be ignored because of the low strain rate encountered in this type of impact. For example, Fig. 4.30 shows the maximum strain rate along the column length for column section UC 356×406×340 for a high impact velocity of 90 km/h. The maximum value is 0.1 sec\(^{-1}\). At this value, the maximum enhancement in the steel yield stress is 50% according to the Cowper-Symonds’ strain rate model. Whilst this change from the static yield stress of 355N/mm\(^2\) is considerable, the fact is that different rates occurring at different locations of the column would make it very difficult to be incorporated in the simplified model. Since ignoring this effect will result in a conservative design, the simplified model to be developed in chapter six will not include this effect.
4.3. Summary

This chapter has presented in detail the results of a numerical simulation study using ABAQUS/Explicit to investigate the effects of several parameters on the behaviour and failure modes of axially pre-loaded steel columns subjected to transverse impact. From the results of this parametric study, the following conclusions may be drawn:

1. The predominate failure mode for axially unrestrained compressed steel columns under transverse impact was global buckling of the column.

2. Some column failure involved large local flange distortion at, and around, the impact area. However, this local flange distortion is a result, not the cause, of global column failure.

3. Column failure was primarily dependent on the level of impact kinetic energy. At the same impact kinetic energy, different values of impacting mass and velocity had a minor effect on column failure.

4. Except for a very low level of axial compression (<25% design resistance), the formation of a plastic hinge was almost independent of the impact position, with the plastic hinge location being close to the centre of the column.
(5) The impact direction to cause bending of the column about the minor axis was found to be the most critical direction of impact.

(6) Damping has little effect on the failure of the column and hence can be neglected when calculating the critical impact velocity.

(7) Both strain hardening and strain rate have beneficial effects on column behaviour and critical impact velocity. The effect of strain hardening will be included in the development of a simplified method. However, the effect of strain rate will be discarded because of the relatively low influence of this parameter and the difficulty of implementing this effect in the simplified model.
Chapter Five

A simplified FE Vehicle Model for Assessing the Vulnerability of Axially Compressed Steel Columns Against Vehicle Frontal Impact

5.1. Introduction

Vehicle impact mechanics have been an active research area for many years due to a demand for vehicle crashworthiness and passenger safety. For this type of research, it is necessary to understand the detailed mechanical behaviour of vehicles. However, when studying column behaviour under a vehicle impact, as in the present research study, the emphasis is on the impacted column rather than on the impacting vehicle, so it is only necessary to develop an understanding of the global and external load resistance characteristics of the vehicle frontal structure involved in such impact event.

The main objective of this chapter is to present and validate a simplified numerical vehicle model that can be used to simulate the effects of vehicle frontal impact on steel columns by using the commercial finite element code ABAQUS/Explicit. The simplified numerical vehicle model treats the vehicle as a spring-mass system. This model has, in fact, already been exploited by other researchers in the preliminary stages of vehicle design and occupant safety assessment (Emori, 1968, Tani and Emori, 1970, Kamal, 1970).

Simulating vehicle impact on a column using the spring-mass system requires defining an accurate enough value of the vehicle stiffness against column impact. Vehicle stiffness is a common term used as an important parameter in the field of vehicle safety (Brell, 2005). Multiple definitions of vehicle stiffness have been established to describe how to define and quantify this parameter and how to relate it to other important vehicle parameters such as vehicle mass, vehicle velocity, vehicle deformation and vehicle energy. Nevertheless, these definitions are all based on the assumption that the impacted object has an infinite stiffness and width, and they cannot be directly used to express the
vehicle stiffness against a deformable body with a finite stiffness and width such as the steel columns considered in this study. The objective of this chapter is to develop a method to predict the equivalent stiffness of a vehicle that can be used in analyzing impacted column behaviour.

To achieve the objective of this chapter, the method of investigation will include the following steps:

a) To identify the vehicle characteristics affecting vehicle impact on steel columns;

b) To propose and validate a simplified numerical vehicle model to simulate the effects of vehicle impact on the behaviour and failure of steel columns under axial compressive load using the finite element code ABAQUS/explicit; and,

c) To suggest and validate a simplified analytical approach to estimate vehicle linear stiffness to be used in the vehicle model derived in b).

5.2. Vehicle characteristics

When a vehicle impacts a column, a considerable portion of the impact energy will be absorbed by the impacting vehicle, see Fig.5.1. The amount of impact energy absorbed by the vehicle depends on the stiffness characteristics of the vehicle before and after the vehicle engine (Tani and Emori, 1970), in addition to the stiffness of the struck column.

![Figure 5.1: Crumpling and deformation of a vehicle frontal structure after impact into a steel column](image)

To develop a simplified vehicle model, the following key features of the impacted vehicle must be simulated properly:

a) Kinetic energy of the impact (vehicle mass and velocity).

b) Vehicle stiffness.
c) Contact interaction between the vehicle and the impacted structure.

Among all the previous techniques used to simulate vehicle impact, the spring-mass model is deemed the most simplistic to represent the dynamic and load-deformation characteristics of the vehicle. Since most of the previous experimental and analytical studies have suggested a linear or bilinear relationship for the vehicle stiffness (Tani and Emori, 1970, Milner et al., 2001), a single degree of freedom spring will be implemented in the proposed simplified numerical vehicle model. Although numerical modelling of the spring-mass system of one degree of freedom may be regarded as simple, it is a highly challenging task when dealing with contact interaction and the nonlinear load-deformation characteristics of the vehicle. The proposed model should be able to generate the same kinetic energy that the original vehicle would generate at the time of impact and it must also be able to follow the assumed load-deformation characteristics of the vehicle under consideration up to the maximum impact force and transfer this force to the impacted column through the contact zone.

5.3. Simplified vehicle model

The proposed simplified model is shown in Fig. 5.2, comprising a rigid body, a nonlinear spring and a rigid massless surface. The mass of the rigid body represents the total vehicle mass; the nonlinear spring represents the dynamic load-deformation characteristics of the vehicle structure and the rigid surface simulates the contact between the vehicle and the structure. The nonlinear spring was introduced in the model using a NONLINEAR SPRING option in ABAQUS/Explicit with the load-deformation characteristics defined in the input file of the model (SIMULIA, 2010a). The rigid surface was defined as a DISCRETE RIGID SURFACE (SIMULIA, 2010b) which is intended to prevent any energy absorption by the contact surface of the simplified vehicle model. The curved contact surface has no sharp corners and ensures that the contact point of the vehicle with the impacted structure does not cause any local stress concentration in the structure.
5.3.1. Validation

Validation of the simplified vehicle model was carried out by the following two steps:

a) Comparison between the simulation results using the simplified model and full-scale vehicle tests for vehicle impact on a rigid barrier. This is to ensure that the proposed spring-mass model is capable of converting the impact energy of the vehicle to the internal energy of the simplified vehicle model;

b) Comparison between simulation results using the simplified model and using a full-scale numerical vehicle model impacting on the columns. This is to ensure that the proposed spring-mass model is capable of simulating the dynamic and structural effects on the impacted steel columns.

The following sub-sections describe in detail each of the above two validation steps.

5.3.1.1. Vehicle impact on a rigid barrier

The impact tests were conducted under the New Car Assessment Program (NCAP) organized by the US National High Traffic Safety Administration (NHTSA, 2011) involving vehicle full frontal impact on a flat rigid barrier. Eqs. (2.8-2.11) presented in chapter two, which are suggested by Campbell (1976) and developed by Jiang et al. (2004), were used to estimate the vehicle impact force – crush deformation relationship. An example is given below for a Toyota Echo 2001 vehicle model.
Chapter Five: Simplified FE Vehicle Model

**Determination of the stiffness coefficients and maximum impact force generated from a full frontal crash of a Toyota Echo 2001 (first test in Table 5.1)**

Total vehicle mass \( M = 1136 \text{ kg} \), impact velocity \( V = 56.3 \text{ km/h} \) (15.63 m/s), the maximum vehicle deformation \( C_{\text{max}} = 0.464 \text{ m} \) (NHTSA Test No. 3647), \( W_v = 1.572 \text{ m} \). The stiffness coefficient \( A \) and \( B \) can be determined using Eqs. 2.8 to 2.11 presented in chapter two as follows (Campbell, 1976, Jiang et al., 2004):

\[
b_o \approx 2.2 \text{ m/s} = 15.63 - 2.2 \times \frac{0.464}{29} \approx 29 \text{ sec}^{-1}
\]

\[
A = \frac{Mb_o b_1}{W_v} = \frac{1136 \times 2.2 \times 29}{1.572} = 46104.83 \text{ N/m}
\]

\[
B = \frac{Mb_1^2}{W_v} = \frac{1136 \times 29^2}{1.572} = 607745.55 \text{ N/m/m}
\]

Since the whole vehicle width is involved in the impact, the above calculated stiffness coefficients must be multiplied by the vehicle width. Therefore:

\[
A_{\text{total}} = 46104.83 \times 1.572 = 72476.8 \text{ N},
\]

\[
B_{\text{total}} = 607745.55 \times 1.572 = 955376 \text{ N/m}
\]

The maximum impact force is:

\[
F_{\text{max}} = A + B \times C = 72476.8 + 955376 \times 0.464 = 515771 \text{ N}
\]

The above calculations can be used to define the stiffness characteristics of the spring used in the simplified vehicle model. Fig. 5.3 shows the impact force – vehicle deformation relationship assigned to the nonlinear spring to simulate vehicle impact on a rigid crash barrier for the above example.
Figure 5.3: Force-deformation characteristics of the spring used to represent the frontal impact behaviour of a Toyota Echo 2001.

Similar calculations were performed for the other vehicles in Table 5.1 and Fig. 5.4 shows the force-deformation relationships for the vehicles in Table 5.1.

Figure 5.4: Force-deformation relationship used to define the stiffness characteristics of the nonlinear springs used to simulate the vehicles in Table 5.1.

Table 5.1 compares the maximum impact forces (contact force) obtained from ABAQUS/Explicit’s numerical simulations using the proposed spring-mass system with those recorded experimentally (where available) or calculated from the equations of Campbell. The agreement between the numerical results and the experimental/
calculation results shown in Table 5.1 is excellent, demonstrating the validity of the proposed simplified vehicle model provided the spring characteristics can be provided.

<table>
<thead>
<tr>
<th>No.</th>
<th>Test Properties</th>
<th>Vehicle type</th>
<th>Vehicle Mass (kg)</th>
<th>Impact Velocity (km/h)</th>
<th>Vehicle crush (m)</th>
<th>Spring displacement (m)</th>
<th>Impact Force (kN)</th>
<th>Contact Force (kN)</th>
<th>Spring displacement (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NHTSA Test No. 3647 (NHTSA, 2011)</td>
<td>Toyota Echo 2001</td>
<td>1136</td>
<td>56.3</td>
<td>0.464</td>
<td>515.771*</td>
<td>525.15</td>
<td>0.449</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>NHTSA Test No. 373 (NHTSA, 2011)</td>
<td>Ford Escort 1981</td>
<td>1100</td>
<td>48.6</td>
<td>0.460</td>
<td>364.79*</td>
<td>363.12</td>
<td>0.460</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>NHTSA, paper No. 393 (NHTSA, 2011)</td>
<td>Ford Explorer</td>
<td>2100</td>
<td>56</td>
<td>Assumed 0.5</td>
<td>980.0*</td>
<td>873.1</td>
<td>0.499</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>NHTSA Test No.: MB5208, 2011 (NHTSA, 2011)</td>
<td>2011 Nissan Murano</td>
<td>2000</td>
<td>56</td>
<td>0.322</td>
<td>1319.43*</td>
<td>1295.0</td>
<td>0.315</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>NHTSA Test No. 5820 (NHTSA, 2011)</td>
<td>Ford, F250, 2006</td>
<td>3054</td>
<td>55.7</td>
<td>0.78</td>
<td>1050.0**</td>
<td>809.03</td>
<td>0.762</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>NHTSA Test No. 1741 (NHTSA, 2011)</td>
<td>C2500, Pick-up Truck, 1994</td>
<td>2013</td>
<td>55.8</td>
<td>0.486</td>
<td>853.53**</td>
<td>857</td>
<td>0.488</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>NHTSA Test No. 5877 (NHTSA, 2011)</td>
<td>Chevy Silverado, 2007</td>
<td>2622</td>
<td>56.15</td>
<td>0.65</td>
<td>700.0**</td>
<td>846.79</td>
<td>0.654</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>NHTSA Test No. 3730 (NHTSA, 2011)</td>
<td>Ford Explorer, 2003</td>
<td>2323</td>
<td>55.3</td>
<td>0.570</td>
<td>875.0**</td>
<td>829.01</td>
<td>0.573</td>
<td></td>
</tr>
</tbody>
</table>

* Using Equations (2.8 to 2.11).

**Experimental result
5.3.1.2. Validation of the spring-mass system against the numerical simulation of a vehicle impact on a column using a full-scale numerical vehicle model

The previous section has demonstrated that it is appropriate to simplify a vehicle into a mass-spring system for the calculation of the maximum impact force under a full-width vehicle impact. This section provides a validation of the simplified vehicle model and the spring stiffness calculation method by comparing more detailed structural (column) behaviour of a vehicle impact on a column (a small contact area). The validation is carried out in two stages. Firstly, the vehicle stiffness is obtained from numerical simulations of a full-scale vehicle impact on a rigid column. Afterwards, simulation results using the simplified vehicle model and the full-scale vehicle model are compared for steel columns with different steel sections, boundary conditions and vehicle weights.

A. Full scale numerical vehicle model

The full scale numerical vehicle model used is for a Chevrolet C2500 Pick-Up made in 1994 (Fig. 5.5) that has been used by many researchers in vehicle crash simulations (El-Tawil et al., 2005, Ferrer et al., 2010, Zaouk et al., 1996). This numerical model can be downloaded from the National Crash Analysis Centre (NCAC) at George Washington University (GWU) (NCAC, 2011). The reduced numerical model of the C2500 vehicle has a total weight of 1840 kg and consists of 41,062 nodes and 10,500 elements arranged in 59 parts with different element types and material models. It should be pointed out that the origin input file of the model was written using the commercial FE code LS-Dyna. The author converted the origin file from the LS-Dyna input file format to the ABAQUS input file format using a special software developed by SIMULIA (SIMULIA, 2010c).

Figure 5.5: Full-scale numerical model of a 1994 Chevrolet Pick-up C2500, based on (NCAC, 2011)
Although the numerical model was developed and validated by NCAC, verification is necessary to ensure that the ABAQUS version of this model is working properly and that the conversion of the input file from LS-Dyna to ABAQUS/Explicit was performed correctly. For this, a simulation of one test of the frontal vehicle impact on a flat and rigid barrier conducted by NTHSA (test no. 1741) was performed. Details of the test are available on the web site of NHTSA (NHTSA, 2011, NCAC, 2011). Fig. 5.6 compares the author’s simulation result (contact force history) with the test result (NHTSA, 2011) and the numerical simulation result by the National Crash Analysis Centre (NCAC)(NCAC, 2011) which used a more detailed vehicle model (58313 element). The agreement between the author’s simulation result and the test result is very good, better than that achieved between the NCAC simulation result and the test result.

Figure 5.6: A comparison of the contact force history between the test (NHTSA, 2011), the numerical simulation using a detailed FE vehicle model (NCAC, 2011) and the numerical simulation using the reduced FE vehicle model of the present study

**B. Validation for vehicle impact on steel columns**

Three H-section steel columns were used in the numerical simulations. One column has already been used in the numerical simulations described in chapter four (UC 305×305×118) and the other two columns were also designed according to the British Standard BS 5950: Part 1:2000 in order to resist the total axial compressive loads exerted on interior ground floor columns of artificial 10 storey, and 3 storey commercial steel framed buildings. The resulting column sizes were UC 356×368×202 and UC 254×254×89. Columns with section sizes UC 305×305×118 and UC 254×254×89
had simply supported or propped cantilever boundary conditions while columns with section size UC 356×368×202 were simply supported column at both ends.

The steel was assumed to be of grade S355 and the steel modulus of elasticity was 206000 N/mm². The stress – strain curve of the steel is the same as that used in the parametric study in chapter four (see Fig. 4.2). The vehicle impact caused bending about the weak (minor) axis for the three column sizes. Table 5.2 lists the column details used in the numerical simulations.

Table 5.2: Steel column properties used in the numerical simulations

<table>
<thead>
<tr>
<th>Column length (m)</th>
<th>Column section</th>
<th>Slenderness ratio ( \left( \frac{kL}{r} \right)_{z-z} )</th>
<th>Relative slenderness ( \lambda_{z-z} = \frac{kL}{r\pi} \sqrt{\frac{F_y}{E}} )</th>
<th>Design axial compressive load (kN)</th>
<th>Boundary conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>UC 254×254×89</td>
<td>61</td>
<td>0.81</td>
<td>2580</td>
<td>S.S</td>
</tr>
<tr>
<td></td>
<td></td>
<td>42.7</td>
<td>0.564</td>
<td>3200</td>
<td>Prop.</td>
</tr>
<tr>
<td>4</td>
<td>UC 305×305×118</td>
<td>51.50</td>
<td>0.68</td>
<td>3800</td>
<td>S.S</td>
</tr>
<tr>
<td></td>
<td></td>
<td>36.05</td>
<td>0.476</td>
<td>4250</td>
<td>Prop.</td>
</tr>
<tr>
<td>4</td>
<td>UC U356×368×202</td>
<td>41.66</td>
<td>0.55</td>
<td>6780</td>
<td>S.S</td>
</tr>
</tbody>
</table>

S.S. = Simply supported; Prop. = Propped cantilever

The simulations were performed to investigate the form of the spring characteristics in the spring-mass model for the vehicle and whether these spring characteristics would change under different conditions. For the vehicle considered, the spring characteristics were obtained based on the results of vehicle impact on a rigid column. Fig. 5.7 shows the impact force - vehicle displacement relationships for different impact velocities on a rigid column of the same size as UC 305×305×118. It should be mentioned that the general contact was used here to generate the interaction between the numerical vehicle model and the steel column, as discussed in chapter three, with hard and penalty friction formulations to describe the mechanical properties for the normal and tangential directions respectively.
It can be seen from Fig. 5.7 that the results are not sensitive to the impact velocity. For vehicle deformation up to 625mm-650mm, the impact force-vehicle deformation relationships are almost linear. This deformation limit corresponds to the position of the engine. After exceeding this distance, the impact force increases sharply with the maximum amplitude depending on the impact velocity. This sudden increase in the contact force is a result of the column being in contact with the vehicle engine and other stiffer parts of the vehicle which are much more rigid than the vehicle frontal zone before the engine, causing a rapid increase in vehicle stiffness as can be seen in Figs. 5.8 and 5.9. It can be seen from Fig. 5.8 that, during the early stage of impact and before the engine contacts the column (t=1.2msec to t=3.6msec), the vehicle frontal structure crumples and cushions the impact energy. The impact force time history at this stage of impact is almost linear as shown in Fig. 5.9.(B) Thereafter, from t= 6msec to t=8.4msec, the vehicle stiffness increases rapidly with a sudden rise in the contact force due to the column being in contact with the engine and the stiffer parts, see Fig. 5.8. Afterwards, at t=12.0msec to t=14.4msec, the contact force drops abruptly (see Fig. 5.9 (B)) and rebound of the vehicle occurs due to the recovery of the elastic axial deformation of the vehicle (see Fig. 5.9(A)).
Figure 5.8: A longitudinal cross section of the C2500 vehicle at different times of the impact history showing the vehicle deformations before and after engine contact with a rigid column of size UC $305 \times 305 \times 118$ and the vehicle rebound thereafter, impact velocity = 56kN/m.
Figure 5.9: (A) Axial displacement and (B) contact force time histories of the C2500 vehicle impacting a rigid column of size UC 305×305×118 at an impact velocity equal to 56km/h

Similarly, Fig. 5.10 shows the simulation results for the other two column sizes in Table 5.3 and the proposed bilinear spring impact force - deformation relationships. Although Figs 5.7 and 5.10 indicate that the impact force - deformation relationship of the vehicle is more close to a parabolic curve, a bilinear relationship has been assumed to represent the vehicle force-displacement relationship as shown in Fig. 5.11. Assuming a bi-linear stiffness relationship results in little loss of accuracy, but makes the vehicle load-deformation relationship much easier to implement in both the numerical simulation model and the analytical model.

Fig. 5.11 shows the idealized spring force - displacement relationship and Table 5.4 lists the slopes (stiffness) for the first ($K_1$) and second ($K_2$) stages of the Chevrolet C2500 Pick-up vehicle. The changes in the stiffness values in Table 5.3 reflect the changes in the contact area between the vehicle and the column, with a larger contact area giving higher stiffness.
Figure 5.10: Impact force - displacement relationships for a Chevrolet C2500 Pick-up vehicle impacting on a rigid column of (A) section size UC $254 \times 254 \times 89$, (B) section size UC $356 \times 368 \times 202$. 
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Figure 5.11: Proposed force - displacement relationship for the simplified spring-mass model of a vehicle.

Table 5.3: $K_1$ and $K_2$ values for the Chevrolet C2500 Pick-up vehicle

<table>
<thead>
<tr>
<th>Steel column section</th>
<th>$K_1$(kN/m)</th>
<th>$K_2$(kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UC 254×254×89</td>
<td>463</td>
<td>40.3×10³</td>
</tr>
<tr>
<td>UC 305×305×118</td>
<td>510</td>
<td>46.8×10³</td>
</tr>
<tr>
<td>UC 356×368×202</td>
<td>546</td>
<td>19.42×10³</td>
</tr>
</tbody>
</table>

The vehicle spring stiffness values obtained from impacting on rigid columns are then used to simulate realistic column behaviour. To investigate whether the proposed vehicle spring force - deformation relationship is suitable for the same column size but under different loading and boundary conditions, numerical simulations using the proposed spring vehicle model were carried out for simply supported steel columns of section UC305×305×118 in the weak direction under different levels of axial compressive load and different impact velocities. Fig 5.12 compares the simulation results between using the full vehicle model and the Simplified Vehicle Model (SVM) up to the peak impact force. The agreement between using the full-scale vehicle model and the SVM is very good, especially considering the extremely high computation costs of implementing a full-scale vehicle model.
Figure 5.12: Typical comparison of simulation results between the full-scale vehicle model and the Simplified Vehicle Model (SVM) for impacts on a simply supported steel column section UC 305×305×118 at different impact velocities.

5.3.2. Sensitivity of column behaviour to stiffness parameters \( K_1 \) and \( K_2 \)

As can be seen from Figs. 5.7 and 5.10, there are some variations in the vehicle impact force - displacement relationship, particularly in the second stage after the column is in contact with the vehicle engine. To investigate the practical implications of these variations, simulations have been carried out to examine how the critical velocity (the velocity that is just sufficient to cause column failure) changes with different vehicle stiffness values. Table 5.4 compares the critical velocity results for different changes in \( K_1 \) and \( K_2 \) of the proposed vehicle spring force - displacement relationship for a simply supported steel column section UC 305×305×118 under 50% of the axial design load.
The critical velocity is practically insensitive to the stiffness value $K_2$ and it is acceptable to assume that the vehicle is rigid once its displacement has reached the engine position. Not surprisingly, the critical velocity shows some sensitivity to the stiffness value $K_1$ because this value directly determines the energy absorbed by the vehicle, but by changing this value by $\pm$ 50%, the critical velocity value did not change by more than 11%. As will be demonstrated in section 5.4, the stiffness value $K_1$ can be estimated to be within 25% of the true value based on full-scale vehicle modelling.

Table 5.4: Sensitivity of simply supported steel column behaviour (UC 305×305×118) to stiffness parameters $K_1$ and $K_2$.

<table>
<thead>
<tr>
<th>Sensitivity to $K_1$</th>
<th>Sensitivity to $K_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_1/K_{1\text{Nominal}}$</td>
<td>$V_{cr^*}$ (km/h)</td>
</tr>
<tr>
<td>1</td>
<td>50.4</td>
</tr>
<tr>
<td>0.5</td>
<td>45</td>
</tr>
<tr>
<td>0.8</td>
<td>47.4</td>
</tr>
<tr>
<td>1.2</td>
<td>52.2</td>
</tr>
<tr>
<td>1.5</td>
<td>54</td>
</tr>
</tbody>
</table>

*$V_{cr}$ = Critical impact velocity

Fig. 5.13 presents further results on the sensitivity of critical velocity to stiffness $K_1$, for a simply supported steel column section UC 305×305×118. If the value of $K_1$ changes by $\pm$50%, the maximum change in the critical velocity for all axial load ratios is 16%. At 20% change in $K_1$, the maximum difference in critical velocity is only 11% for all axial load ratios.
5.3.3. Determining an appropriate simplified vehicle model

Since the vehicle becomes very stiff when its displacement has reached the engine box, it may be considered acceptable to model the vehicle simply as a rigid impactor. To assess this claim, an extensive amount of numerical simulations have been performed to compare the critical impact velocities for a number of columns between the following four vehicle models:

a) full-scale numerical vehicle model;
b) spring mass system with bilinear vehicle force - displacement relationship and finite stiffness value $K_2$;
c) bilinear spring force - displacement curve with the same stiffness value $K_1$ as in (b) but $K_2$ being infinite (rigid);
d) whole vehicle as a rigid impactor throughout.

The simulation results are presented in Figs. 5.14 to 5.18 for the different column conditions as specified in Table 5.2. In all cases, buckling was about the weak axis of the column. The results are presented as a column axial load ratio - critical velocity relationship.
Figure 5.14: The axial load ratio - critical velocity relationships of a simply supported steel column section UC254×254×89

Figure 5.15: The axial load ratio - critical velocity relationships of a simply supported steel column section UC305×305×118
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Figure 5.16: The axial load ratio - critical velocity relationships of a simply supported steel column section UC356×368×202.

Figure 5.17: The axial load ratio - critical velocity relationships of a propped cantilever steel column section UC254×254×89
In all cases, there is considerable difference between the simulation results using the full-vehicle model and assuming the vehicle as a rigid impactor. This suggests that it is not appropriate to treat the vehicle as a rigid impactor. There is practically no difference in results between using a finite and an infinite stiffness value $K_2$, clearly indicating that it is acceptable to assume that the vehicle is rigid once the vehicle deformation has reached the engine box.

Using the simplified vehicle model produces critical velocities lower than using the full-scale vehicle model; therefore using the simplified vehicle model gives safe results. In most cases, using the simplified vehicle model gives simulation results in good agreement with those obtained from using the full-scale vehicle model. Energy partition to different parts of the system clearly supports this conclusion. For example, for the simulation results in Figure 5.15, Table 5.5 lists the total energy ($E_{TOTAL}$), the internal energy of the whole vehicle-column system ($IE$), the internal energy absorbed by the vehicle ($IE_v$) and the external work ($WK$) done on the column under an axial load ratio of $0.5P_{Design}$. Treating the vehicle as a rigid impactor (case d) means that the vehicle does not absorb any energy. In the other three cases, the ratios of the energy absorbed by the vehicle (the eighth column) from the simplified vehicle models (b and c) are very close to those from the full-scale vehicle model (a).
Table 5.5: Energy partition for different vehicle models impacting on a simply supported steel column section UC 305×305×118 under an axial load ratio of 0.5P_{Design}; (a) full-scale model; (b) spring with finite stiffness $K_1$ and $K_2$; (c) spring with finite stiffness $K_1$ and infinite stiffness $K_2$ (rigid), (d) rigid impactor.

<table>
<thead>
<tr>
<th>case</th>
<th>$V_{cr}$ (m/s)</th>
<th>ETOTAL (Joule)</th>
<th>WK (Joule)</th>
<th>IE (Joule)</th>
<th>$IE_v$ (Joule)</th>
<th>$(ETOTAL + WK)/IE$</th>
<th>$(IE_v/ETOTAL)%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>15.6</td>
<td>2.24×10^5</td>
<td>1.2E×10^5</td>
<td>3.173E×10^5</td>
<td>1.16×10^5</td>
<td>1.08</td>
<td>51.8</td>
</tr>
<tr>
<td>b</td>
<td>14.0</td>
<td>1.8×10^5</td>
<td>5.81×10^4</td>
<td>2.36×10^5</td>
<td>1.2×10^5</td>
<td>1.01</td>
<td>66.7</td>
</tr>
<tr>
<td>c</td>
<td>14.25</td>
<td>1.86×10^5</td>
<td>5.63×10^4</td>
<td>2.4×10^5</td>
<td>1.066×10^5</td>
<td>1.01</td>
<td>57.3</td>
</tr>
<tr>
<td>d</td>
<td>11.0</td>
<td>1.11×10^5</td>
<td>1.58×10^5</td>
<td>2.63×10^5</td>
<td>0</td>
<td>1.02</td>
<td>0</td>
</tr>
</tbody>
</table>

*Calculated from the force - deformation relationships of the vehicle model up to column buckling.

Fig. 5.19 shows the energy components’ histories for the simulating cases a and c. A similar behaviour and similar trends can be seen in the two cases, particularly the time of the column failure as characterized by the zero slope of the kinetic energy history. Using the simplified vehicle model with finite stiffness $K_1$ and infinite stiffness $K_2$ (case c) causes the column to fail at time $t=0.126$ sec, which is very close to the time of failure achieved using the full scale vehicle model (case a) of $t=0.122$ sec.

Figure 5.19: Energy histories corresponding to (A) case a at an impact velocity of 15.6m/s (B) case c at an impact velocity of 14.25m/s, both for the simply supported column section UC305×305×118

In cases where the column in strong (characterized by low slenderness, low load ratio, and impact point close to ground) the simulation results using the simplified vehicle model approach those assuming the vehicle as a rigid impactor and deviate from those
when using the full-scale vehicle model. This is because the failure of such columns requires a large amount of impact energy. When this happens, a large part of the impact energy is absorbed by deformations of other parts of the full-scale vehicle (such as in the rear part of the vehicle, shown in Fig. 5.22) which are not included in the simplified vehicle model which is based on deformation in front of the engine box.

5.3.4. Effect of increasing vehicle weight

The proposed simplified vehicle model was obtained for an empty vehicle. To examine whether the same spring force-displacement relationship can be used when there are goods in the vehicle, further simulations have been carried out using the same 1994 Chevrolet Pick-up vehicle, but by increasing the weight of the vehicle by 1 tonnes and 1.5 tonnes. Figs. 5.20 and 5.21 compare the simulation results. Compared with Fig. 5.15, the correlation of the simulation results using the simplified vehicle model with the full-scale vehicle model is slightly worsened. This is caused by increased energy absorption in other parts of the vehicle that are not included in the simplified vehicle model. Increasing the vehicle weight increases the tendency for this to happen.
Chapter Five: Simplified FE Vehicle Model

Figure 5.20: The axial load ratio - critical velocity relationships of a simply supported column section UC305×305×118, additional weight = 1tonnes.

Figure 5.21: The axial load ratio - critical velocity relationships of a simply supported column section UC305×305×118, additional weight = 1.5tonnes.
(a) A simply supported steel column section UC 356×368×202, P=0.3P_{Design}, V=170km/h.

(b) A propped cantilever steel column section UC 305×305×118, P=0.3P_{Design}, V=180km/h

(c) A simply supported steel column section UC 305×305×118, P=0.15P_{Design}, V=130km/h, additional weight = 1.5tonnes

Figure 5.22: Deformation shape of the C2500 vehicle after a steel column impact for low axial load ratios
5.4. Determination of the equivalent vehicle linear stiffness

Clearly, the initial stiffness value \( K_1 \) in the simplified vehicle model should be evaluated accurately. This section will present a detailed derivation for vehicle impact on columns, which has not been undertaken by others before, and will assess the accuracy of this estimate.

5.4.1. Derivation of the vehicle stiffness equation

Fig. 5.23 shows the typical damage pattern to a vehicle after impact by a column in the central portion of the vehicle. The vehicle deformation profile for this pattern may be assumed as shown in Fig. 5.24. Using Campbell’s linear equation to express the impact force exerted to the vehicle, the energy absorbed by vehicle deformations during full frontal impact can be calculated using the following double integral:

\[
IE_v = \int_0^W \int_0^C F(c) \cdot dc \cdot dw_v
\]

Where \( W_v \) is the vehicle total width perpendicular to the impact, \( C \) is the vehicle deformation which varies across the vehicle width and \( F(c) \) is the impact force as a function of the vehicle crush distance defined by Campbell (1976) as:

\[
F(c) = A + B \times C
\]

Where \( A, B \) and \( C \) were defined in chapter two.

Substituting the equation of \( F(c) \) into Eq. 5.1 results in the following equation:

\[
IE_v = \int_0^W \int_0^C (A + B \times C) \cdot dc \cdot dw_v
\]
Figure 5.23: Damage profile of the C2500 vehicle after a frontal impact in the central region on a column

![Damage profile of the C2500 vehicle after a frontal impact in the central region on a column](image)

Figure 5.24: Simplified damage profile of a vehicle after frontal impact in the central region on a column

![Simplified damage profile of a vehicle after frontal impact in the central region on a column](image)

For frontal impact with a column in the middle of the vehicle, see Fig. 5.24 above, the vehicle displacement can be expressed as follows:

\[
C_{m_{ax}} = \frac{C}{w_y} \quad \text{For} \quad 0 \leq w_y \leq \frac{(W_v - h_v)}{2}
\]

\[
\frac{C_{m_{ax}}}{(W_v - h_v)/2} = \frac{C}{w_y}
\]

150
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\[ C = \frac{2C_{\text{max}} \times w_c}{(W_v - h_c)} \] .................................................................5.5

For \( \frac{(W_v - h_c)}{2} \leq w_c \leq \frac{(W_v + h_c)}{2} \)

\[ C = C_{\text{max}} \] .................................................................5.6

Where: \( h_c \) is the width of the column perpendicular to the direction of impact.

By substituting Eqs. 5.5 and 5.6 into Eq. 5.3 to compute the energy absorbed by vehicle deformations, the following is obtained:

\[ IE_v = h_c \int_0^{C_{\text{max}}} (A + B C) \, dc + 2 \int_0^{C_{\text{max}}} (A + B C) \, dc \cdot dw_v \] ............................................5.7

\[ IE_v = [A C_{\text{max}} + \frac{B C_{\text{max}}^2}{2}] h_c + 2 \int_0^{C_{\text{max}}} [A C + \frac{B C^2}{2}] \, dw_v \] ............................................5.8

\[ IE_v = [A C_{\text{max}} + \frac{B C_{\text{max}}^2}{2}] h_c + \frac{(W_v - h_c)}{2} \int_0^{C_{\text{max}}} \left[ A \left( \frac{2C_{\text{max}}}{W_v - h_c} \right) w_v + \frac{1}{2} B \left( \frac{2C_{\text{max}}}{W_v - h_c} \right)^2 w_v^2 \right] \, dw_v \] ............................................5.9

\[ IE_v = [A C_{\text{max}} + \frac{B C_{\text{max}}^2}{2}] h_c + 2 [A \left( \frac{2C_{\text{max}}}{W_v - h_c} \right) \frac{w_v^2}{2} + \frac{1}{6} B \left( \frac{2C_{\text{max}}}{W_v - h_c} \right)^2 w_v^3 ] \bigg|_0^{\frac{(W_v - h_c)}{2}} \] ............................................5.10

\[ IE_v = [A C_{\text{max}} + \frac{B C_{\text{max}}^2}{2}] h_c + \frac{(W_v - h_c)}{2} [A C_{\text{max}} + \frac{B}{3} C_{\text{max}}^2] \] ............................................5.11

\[ IE_v = \frac{(w_v + h_c)}{2} + \frac{B C_{\text{max}}^2}{6} \left( \frac{w_v + 2h_c}{6} \right) \] ............................................5.12

The potential energy of a spring with linear spring stiffness is:
Chapter Five: Simplified FE Vehicle Model

\[ IE_v = \frac{1}{2} K_1 C_{max}^2 \] \hspace{1cm} \text{5.13}

Equating Eq.5.12 to Eq.5.13 gives:

\[ K_1 = \frac{2\left[ A C_{max} \left( \frac{W_v + h_v}{2} \right) + B C_{max}^2 \left( \frac{W_v + 2h_v}{6} \right) \right]}{C_{max}^2} \] \hspace{1cm} \text{5.14}

In Eq. 5.14, \( C_{max} \) is the maximum displacement of the vehicle in front of the engine box.

### 5.4.2. Validation of the suggested stiffness equation

There is a lack of data to enable the accuracy of Eq.5.14 to be directly and comprehensively assessed. As partial validation, the calculation results for the Chevrolet C2500 Pick-up using Eq.5.14 are compared with the stiffness values obtained from the numerical simulations reported in section 5.3.1.2 in addition to the stiffness values obtained from the numerical simulations using two larger virtual column sizes (500 mm and 750mm).

For this calculation, the values of A and B are A = 52.686kN/m and B = 509.623kN/m² according Eqs. 2.8 to 2.11 in chapter two. The value of \( C_{max} \) is 0.625m according to the full-scale numerical simulation results in section 5.3.1.2 \( W_v = 1.63m \). Table 5.6 compares the calculated and the numerically extracted stiffness results for the different column dimensions (\( h_c \)) in contact with the vehicle. The difference between the calculation results using Eq.5.14 and the numerically extracted results is within 25%. Also Campbell (Campbell, 1976) indicated that the applicability of Eq. 5.2 should be limited to \( h_c/W_v > 25\% \). The results in Table 5.6 for smaller column dimensions (\( h/W_v = 16\% \) and 19.3%) indicate that Eq. 5.2 can be used for a \( h/W_v \) ratio lower than 25%.
### Table 5.6: A comparison between calculated and numerically extracted linear stiffness for a Chevrolet 2500 Pick-up

<table>
<thead>
<tr>
<th>Column depth* (m)</th>
<th>% (h/W&lt;sub&gt;v&lt;/sub&gt;)</th>
<th>K&lt;sub&gt;1&lt;/sub&gt; (ABAQUS) kN/m</th>
<th>K&lt;sub&gt;1&lt;/sub&gt; using Eq.5.14 for C&lt;sub&gt;max&lt;/sub&gt;=0.625m</th>
<th>%Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.260</td>
<td>16</td>
<td>463</td>
<td>524.6</td>
<td>13.3</td>
</tr>
<tr>
<td>0.3145</td>
<td>19.3</td>
<td>510</td>
<td>549.5</td>
<td>7.8</td>
</tr>
<tr>
<td>0.375</td>
<td>23</td>
<td>546</td>
<td>575</td>
<td>5.33</td>
</tr>
<tr>
<td>0.50</td>
<td>30.6</td>
<td>610</td>
<td>628</td>
<td>2.9</td>
</tr>
<tr>
<td>0.75</td>
<td>46</td>
<td>983</td>
<td>733.8</td>
<td>-25.0</td>
</tr>
</tbody>
</table>

*Perpendicular to impact direction

### 5.5. Summary

This chapter has presented and validated a simplified approach to simulate the effects of vehicle frontal impact on steel columns under a compressive axial load. The impacting vehicle was simplified as a spring mass system with the linear spring representing the stiffness characteristics of the vehicle. The spring force-deformation relationship is assumed to be bilinear, with the first part representing the vehicle deformation behaviour up to the engine box and the second part representing the stiffness of the engine box, which is almost rigid. To validate the proposed numerical model, comparisons were made in terms of the load-deformation relationships of the vehicle and the axial load-critical impact velocity curves of the three steel columns, between the numerical simulations’ results using a full-scale vehicle model and the simplified vehicle model. Very good agreement was achieved. Furthermore, it has been found that the second part of the simplified vehicle model can be assumed to be rigid. However, it is not appropriate to assume vehicles as rigid impactors.

This chapter has also presented a method to obtain the stiffness value of a vehicle before reaching the engine box. This is based on using Eqs. 2.8-2.11 (Campbell’s original equations plus Jiang et al.’s proposal to obtain bo and b1) to obtain the vehicle force-deformation relationship per unit width and integrating this force-deformation relationship over the deformation profile of the vehicle after impact on a column with a finite width. Comparison between the vehicle stiffness values derived in such a way and those extracted from the numerical simulation indicates that the difference is less than...
25% for different column section sizes. Also Campbell’s original condition that the column width be at least 25% of the vehicle width may be removed.
Chapter Six

The Derivations of a Simplified Analytical Method for Predicting the Critical Velocity of Transverse Rigid Body Impact and Vehicle Impact on Steel Columns

6.1. Introduction

This chapter presents the development of a simplified analytical method to predict the critical velocity of rigid body impact and vehicle impact on steel columns under axial compressive load. This method is based on the energy balance with a quasi-static approximation of the column behaviour. This general method has been widely used for beams under lateral impact without any axial load (Jones, 1995, Wen et al., 1995, Bambach et al., 2008) but column buckling adds complexity to the problem. For simplification, the observations and conclusions drawn from the parametric study and numerical simulations conducted in chapters four and five have been used to provide guidance on establishing several assumptions.

Although assuming that the impacting mass behaves as a rigid body may give more conservative design results in terms of the critical impact velocity because all the impact energy is only absorbed by column deformations, it neglects the amount of impact energy that could be absorbed by vehicle deformations which may affect the column behaviour and failure. Moreover, the results presented in chapter five have shown that vehicle deformations absorb a considerable percentage of the impact energy which can significantly reduce the kinetic energy imparted to the column. The objective of this chapter is to present a simple but effective approach to account for vehicle deformations in the energy balance equation.
6.2. **Derivation of the simplified analytical method**

It should be pointed out that the main emphasis of this analytical method is to obtain the column axial load - critical impact velocity relationship. The critical impact velocity is defined in chapter four as the minimum velocity of the impact body that causes the column to lose its stability. From reviewing the available literature (chapter two), it can be concluded that the most simplistic analytical method for structures subjected to transverse impact load would be to assume a quasi-static behaviour (Jones, 1995, Wen et al., 1995, Bambach et al., 2008, Shope, 2006). Therefore, the combination of an energy balance approach with a quasi-static approximation of column behaviour will be used in the current study to provide a simplified method which is intended to provide an accurate enough prediction of the critical velocity of a vehicular impact on a steel column.

The energy balance approach is more appropriate for impact events with a very short duration compared with the natural period of the whole structural system. This is because, in such a dynamic event, the impact loading history is unimportant compared to the total impact energy imparted to the impacted structural member (Jones, 1995). This approach can be used by either assuming all the impact kinetic energy is absorbed only by the impacted column which means the impacting object behaves as rigid body or by assuming that both the impacted column and the impacting object absorb the kinetic energy of the impact. Both assumptions will be used in this chapter to determine the critical impact velocity.

6.2.1. **Developing the energy balance equation**

The general energy balance equation, as employed in ABAQUS/Explicit, is:

\[ IE + VD + KE + FD - WK = ETOTAL = Energy Balance = CONSTANT \] ........................................6.1

Where \( IE \) is the internal energy (consisting of both the recoverable or elastic strain energy, \( SE \), and the plastic strain energy, \( PD \)), \( VD \) the viscous dissipation energy, \( KE \) the residual kinetic energy, \( FD \) the frictional dissipation energy at the contact zone, \( WK \) the work done by the external forces, and \( ETOTAL \) the total conserved energy of the system (the energy balance of the system).
Chapter Six: The Derivations of a Simplified Analytical Method

For the critical situation, the column and the impactor are at rest, therefore $KE=0$. As shown in chapter four, due to the short duration of impact, the viscous dissipation energy at the critical condition is negligible compared to the initial impact energy even after introducing a damping effect, making $VD=0$. Assuming there is no friction under direct impact, then $FD=0$.

Hence, Eq. 6.1 becomes:

$$IE - WK = ETOTAL = \text{Total conserved energy} = \text{Total impact energy}$$ ......................................... 6.2

Or

$$IE = \text{Total impact energy} + WK$$ ........................................................................................................... 6.3

For the case of rigid impact, the $IE$ results from the column’s deformations only (i.e. $IE=IE_{\text{col}}$), whilst for the flexible vehicle impact, this term also includes the energy absorbed by vehicle deformation (i.e. $IE=IE_{\text{col}} + IE_{v}$).

The derivation of each term in the above energy balance equation for both rigid impact and vehicle impact will be presented in the following subsections.

**6.2.2. Energy absorbed by the column’s deformation ($IE_{\text{col}}$)**

In the majority of previous theoretical studies which have adopted the energy balance approach to predict structural behaviour under transverse impact, the material behaviour was assumed to be rigid-plastic, making $SE=0$. This assumption would lead to an over prediction of the structural permanent displacements (Jones, 1995, Samuelides and Frieze, 1984), particularly if the transverse displacements are small or moderate. Dorogoy (2008) confirmed this conclusion by undertaking a numerical investigation into the dynamic impact response of an aluminium beam subjected to transverse impact with no axial restraint. Moreover, the parametric study results of chapter four on steel columns under transverse impact indicate that the elastic strain energy of a column at failure is a considerable portion of the total internal energy of the column. Therefore, both the elastic and plastic strain energies will be included in the proposed simplified method.
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Figure 6.1: The column model used in the simplified analysis. A: Elastic phase, B: Plastic phase.

Figure 6.1 illustrates the elastic and plastic phases of a column under impact loading. The following main assumptions are adopted when calculating the column internal energy.

1. The moment-rotation curve of a plastic hinge formed in the column is elastic-perfectly plastic as shown in Fig. 6.2;
2. The column is in a quasi-static state of equilibrium; and
3. The column has a uniform cross section along its length with equal plastic bending moment capacity everywhere.

Figure 6.2: The assumed elastic-perfectly plastic moment-rotation \((M - \theta)\) relationship for the column.
Therefore, the internal energy absorbed by the column can be expressed as:

\[
IE_{\text{col}} = \sum_{i=1}^{n} \left( M_{PRi} \left( \theta_{i \text{Critical}} - \frac{\theta_{i \text{Elastic}}}{2} \right) \right) 
\]

Where \( n \) is the number of plastic hinges required to cause a plastic hinge mechanism in the column; \( M_{PRi} \) is the plastic moment capacity of the column section at the plastic hinge \( i \), taking into consideration the presence of an axial load; \( \theta_{\text{Elastic}} \) is the maximum elastic rotation of the column at the plastic hinge \( i \); \( \theta_{\text{Critical}} \) is the maximum rotation of the column at the plastic hinge \( i \).

The following section will derive expressions to calculate the rotations \( \theta_{\text{Elastic}} \) and \( \theta_{\text{Critical}} \).

**6.2.2.1. Derivations of the maximum elastic and critical rotations**

\( (\theta_{\text{Elastic}} \text{ and } \theta_{\text{Critical}}) \)

The maximum elastic rotation \( \theta_{\text{Elastic}} \) is calculated based on the column deformation when the maximum bending moment in the column has just reached the plastic bending moment capacity. Up to this stage, the column behaviour is assumed to be elastic and governed by:

\[
M_x = -\left( \frac{d^2 w}{dx^2} EI \right)_x 
\]

Provided that \( |M_x| \leq |M_{PRi}| \)

Where \( M_x \) is the elastic bending moment of the column at section \( (x) \) along its longitudinal axis, \( \frac{d^2 w}{dx^2} \) is the second derivative of the column deformation \( w(x) \) with respect to the distance \( x \), \( EI \) is the flexural stiffness of the column section (\( E = \) modulus of elasticity of the column material, \( I = 2^{nd} \) moment of area).

Hence, the maximum elastic rotation \( \theta_{\text{Elastic}} \) is calculated based on the deformations when the following conditions are reached:
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\[ (-\frac{d^2w}{dx^2}EI)_{x=x'} = +M_{PR} \] for the plastic hinge within the column height; ..............6.6A

\[ (-\frac{d^2w}{dx^2}EI)_{x=0,x=L} = -M_{PR} \] for the fix-ended column end ..............................................6.6 B

Where \( x' \) is the location of the plastic hinge within the column height (see Fig. 6.1).

The above two equations will give two different column deformations. It is proposed to use the larger deformation to calculate the elastic rotation to ensure that all the plastic hinges have formed in the cross-section. Because the elastic energy (although substantial) is still only a minor part of the total strain energy, the above approximation is acceptable.

Now, referring to Fig. 6.1(B) which represents the deformed shape of the column during the plastic deformation phase, the maximum or critical plastic rotation \( \theta_{Critical} \) at the three potential plastic hinge locations can be determined based on the critical or maximum transverse displacement. The critical displacement will be derived in section 6.2.2.2. It represents the displacement value at which the effect of the bending moment caused by the axial load becomes just equal to the plastic resistance of the column to cause global failure of the column.

**A. Selecting the elastic deformation shape**

To obtain the column deformation using Eq. 6.6 when the maximum bending moment in the column reaches its plastic bending moment capacity, the deformation shape of the column should be established first. Previous researchers have suggested using either the deformation shape of the column under a static load at the position of impact (Biggs, 1964, Humar, 2002) or the buckling mode shape of the column (Sastrapradja et al., 2006, Shope, 2006). For a structural member without any axial load, there is no considerable difference (Shope, 2006). The numerical simulations presented in chapter four have revealed that, for columns under moderate to high axial load (>25% column design load), the static buckling shape is more appropriate.

**B. Determination of the intermediate plastic hinge location**

The assumed plastic hinge mechanism shown in Fig. 6.1(B) requires the formation of a plastic hinge between the supports. Without the compressive load, the location of the intermediate plastic hinge will be at the location of the impacting mass. However, with
the presence of an axial compressive load, the column behaviour and deformation shape will be different due to the P-Δ effect. This was experimentally observed by Adachi et al. (2004) who conducted impact tests on propped cantilever aluminum columns. Their observations in Fig.6.3 clearly show that the intermediate plastic hinge location was not at the impact location. The intermediate plastic hinge location was at the position of the maximum column lateral deformation according to the static buckling mode.

Figure 6.3: Effect of the impact location on plastic hinge location of the transversely impacted column (Adachi et al., 2004)

The experimental observations of Adachi et al. (2004) were numerically confirmed and quantified by the parametric study results in chapter four which show that, for columns subjected to moderate and high levels of axial compressive load (≥25%P_{Design}), the column deformation under the axial load dominates. Therefore, the location of the intermediate plastic hinge was always close to the location of the maximum column traverse displacement according to the static buckling mode shape (0.5×L for a simply supported column, about 0.6×L from the column base for the propped cantilever case) regardless of the impact velocity or the impact mass. Some results from the numerical simulations are repeated in Fig. 6.4. For columns under a lower axial compressive load, the deformation shape under lateral loading is more influential; therefore, the location of the intermediate plastic hinge was near the impact location. In this analysis, for an axial compressive load not exceeding 25% of the column design load, it will be assumed that the intermediate plastic hinge will occur at the impact location.
Figure 6.4: Collapse shape of columns showing the locations of the plastic hinge: (A) L=4m, axial load ratio \( \left( \frac{P}{P_{\text{Design}}} \right) = 50\% \), (B) L=8m, axial load ratio \( \left( \frac{P}{P_{\text{Design}}} \right) = 70\% \)

6.2.2.2. Derivations of the internal energy equations for columns under moderate to high axial loads \( (P \geq 25\% P_{\text{Design}}) \)

A. Maximum elastic rotation \( \theta_{\text{Elastic}} \).

Assume the buckling mode shape of the columns is:

\[
w_{(x)} = f(W, x) \tag{6.7}
\]

Where \( W \) is the amplitude of the deformation shape, which is taken as the maximum transverse displacement of the column, and \( x \) is the distance along the longitudinal axis of the column measured from the column base.

Substituting Eq. 6.7 into Eq. 6.6 (A and B) for the three possible plastic hinge locations, the maximum column deflections to enable the bending moment in the column to reach the plastic bending moment capacity at each of the three possible plastic hinge locations \( (x=0, x=x' \text{ and } x=L) \) respectively can be calculated using the following equations:

\[
M_{(x=0)} = M_{PR1} \Rightarrow \left( -\frac{d^2w}{dx^2} EI \right)_{x=0} = M_{PR1} \Rightarrow W_{el(1)} = f \left( M_{PR1}, x \right) \tag{6.8}
\]

\[
M_{(x=x')} = M_{PR2} \Rightarrow \left( -\frac{d^2w}{dx^2} EI \right)_{x=x'} = M_{PR2} \Rightarrow W_{el(2)} = f \left( M_{PR2}, x \right) \tag{6.9}
\]

\[
M_{(x=L)} = M_{PR3} \Rightarrow \left( -\frac{d^2w}{dx^2} EI \right)_{x=L} = M_{PR3} \Rightarrow W_{el(3)} = f \left( M_{PR3}, x \right) \tag{6.10}
\]
As explained previously (section 6.2.2.1), the maximum value of the above three deflections should be used to determine the maximum elastic rotation at the end of the elastic deformation phase. However, as will be demonstrated by the example below, the three elastic deformations above converge to one single value.

Once the value of $W_{el}$ is determined, the maximum elastic rotation can be determined based on the plastic deformation shape in Fig. 6.1 (B), giving

$$\theta_{(x=0)\,elastic} = \frac{W_{el}}{x'}, \quad \theta_{(x=x')\,elastic} = \frac{L \times W_{el}}{x' (L - x')}, \quad \theta_{(x=L)\,elastic} = \frac{W_{el}}{(L - x')}$$ ........................................ 6.11

**B. Critical rotation $\theta_{\text{Critical}}$.**

The derivation that follows is adopted from Shope (Shope, 2006) who applied the procedure to columns subjected to blast loads. Replace the impact by a nominal static force $F_{pl}$. Referring to the left part of Fig. 6.1(B), the reaction of the column at end 1, $(R_1)$ can be determined by assuming a quasi-static equilibrium condition and taking moment about end 2:

$$R_1 = \frac{F_{pl} \times (L - \bar{x}) - M_{PR1} + M_{PR3}}{L}$$ .................................................. ............................................. 6.12

Where $\bar{x}$ is the position of the impact load application measured from the column base.

Now, referring to the right part of Fig. 6.1(B), the relationship between the equivalent quasi-static transverse force at the impact location ($F_{pl}$), the axial compressive load $P$ and the maximum transverse displacement $W$ can be determined by the moment equilibrium condition of that part:

When $\bar{x} \geq x'$

$$R_1 \times x' + M_{PR1} - M_{PR2} + P \times W = 0$$ ................................................................. 6.13

And when $\bar{x} < x'$

$$R_1 \times x' + M_{PR1} - M_{PR2} + P \times W - F_{pl} \times (x' - \bar{x}) = 0$$ .................................................. 6.14

Substituting the value of $R_1$ from Eq. 6.12 into Eqs. 6.13 and 6.14 and solving for $F_{pl}$ gives the following equations:
For \( \bar{x} \geq x' \)

\[
F_{pl} = \frac{(M_{PR1} \left( \frac{x'}{L} - 1 \right) + M_{PR2} - \frac{M_{PR3} x'}{L} - P \times W) L}{(L - \bar{x}) x'} \tag{6.15}
\]

For \( \bar{x} < x' \)

\[
F_{pl} = \frac{(M_{PR1} \left( \frac{x'}{L} - 1 \right) + M_{PR2} - \frac{M_{PR3} x'}{L} - P \times W) L}{(L - x') \bar{x}} \tag{6.16}
\]

Fig. 6.5 illustrates the decreasing relationship between the equivalent quasi-static transverse force \( F_{pl} \) with increasing the transverse displacement \( W \) according to Eqs. 6.15 or 6.16. This figure shows that the maximum displacement at which collapse occurs due to the combined effect of plastic mechanism and axial compressive force \( (W_{cr}) \) can be determined by equating the equivalent plastic quasi-static force in Eqs. 6.15 or 6.16 to zero and solving for \( W_{cr} \) as in Eq. 6.17:

\[
W_{cr} = \frac{M_{PR1} \left( \frac{x'}{L} - 1 \right) + M_{PR2} - \frac{M_{PR3} x'}{L}}{P} \tag{6.17}
\]

Hence, the plastic critical rotations are:

\[
\theta_{(x=0)_{Critical}} = \frac{W_{cr}}{x'}; \theta_{(x=x')_{Critical}} = \frac{L \times W_{cr}}{x' \left( L - x' \right)}; \theta_{(x=L)_{Critical}} = \frac{W_{cr}}{\left( L - x' \right)} \tag{6.18}
\]

It should be pointed out that since Eq. 6.18 does not contain any reference to the deformation capacity of the material (ultimate strain), it is only applicable to columns.
where global buckling governs. The numerical results in chapter four indicate that column global buckling was indeed the governing failure model.

Substituting the values of elastic rotations (Eq. 6.11) and of critical rotations (Eq. 6.18) into Eq. 6.4, the general expression for the internal energy of the column is obtained as:

\[
IE_{col} = \frac{1}{x'} M_{PR_1} (W_{cr} - W_{el} / 2) + \frac{L}{x'(L - x')} M_{PR_2} (W_{cr} - W_{el} / 2) \\
+ \frac{1}{(L - x')} M_{PR_3} (W_{cr} - W_{el} / 2) \quad ..6.19
\]

The following example illustrates the application of the above mentioned procedure to calculate the internal energy of a propped cantilever steel column.

**Example of how to determine the internal energy equation of a propped cantilever column**

1. **Elastic rotation** \(\theta_{\text{Elastic}}\)

The equation for the elastic buckling shape of a propped cantilever column can be expressed as (Timoshenko, 1961, Shope, 2006):

\[
w(x) = \frac{W}{6.2824} \left[ \sin(\phi x) - \phi L \cos(\phi x) + \phi L (1 - \frac{x}{L}) \right] \quad \text{6.20}
\]

Where \(\phi = \frac{1.4318\pi}{L}\) \quad \text{6.21}

From Eq. 6.6:

\[
M(x) = EI \times \frac{-W}{6.2824} \times \left[ -\phi^2 \sin(\phi x) + \phi' L \cos(\phi x) \right] = M_{PR} \quad \text{6.22}
\]

According to Eq. 6.20, the location of the maximum transverse displacement is \((x \approx 0.6L)\). Substituting this value into Eq. 6.22 gives the following equations:

\[
M_{(x=0)} = -\frac{1.4676\pi^2 EI}{L^2} \times W = M_{PR} \quad \text{Giving: } W_{el(i)} = \frac{(-M_{PR})}{-1.4676\pi^2 EI} = \frac{M_{PR}}{0.7P_{cr}}
\]
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And

\[ M_{(x=0.6L)} = \frac{1.466\pi^2 EI}{L^2} \times W = M_{PR2} \]

Giving: \( W_{el(2)} = \frac{M_{PR}}{1.466\pi^2 EI} \), or \( W_{el(2)} \approx \frac{M_{PR}}{0.7P_{cr}} \)

Where \( P_{cr} \) is the Euler buckling load of the propped cantilever column defined by:

\[ P_{cr} \approx \frac{2.046\pi^2 EI}{L^2} \]

2. Critical rotation \( \theta_{\text{Critical}} \)

Substituting \( x' = 0.6L \) and \( M_{PR1} = -M_{PR} \), \( M_{PR2} = M_{PR} \) and \( M_{PR3} = 0 \) into Eq. 6.17 for this column boundary condition (Fig. 6.1(B), assumption 3) gives:

\[ W_{cr} = \frac{1.4M_{PR}}{P} \]

3. Internal Energy

Substituting the values of \( x' \), \( W_{cr} \), \( W_{el} \) and the corresponding values of \( M_{PR1} \), \( M_{PR2} \) and \( M_{PR3} \) into Eq. 6.19 gives:

\[ IE_{col} = \left( -\frac{M_{PR}}{0.6L} \left( 1.4 \frac{M_{PR}}{P} - \frac{M_{PR}}{2 \times 0.7P_{cr}} \right) \right) + \left( \frac{M_{PR}}{0.24L} \times \left( 1.4 \frac{M_{PR}}{P} - \frac{M_{PR}}{2 \times 0.7P_{cr}} \right) \right) \]

\[ IE_{col} = \left( -\frac{2.334M_{PR}^2}{L} \left( \frac{1}{P} - \frac{1}{2P_{cr}} \right) \right) + \left( \frac{5.834M_{PR}^2}{L} \left( \frac{1}{P} - \frac{1}{2P_{cr}} \right) \right) \]

\[ IE_{col} = \frac{8.16634M_{PR}^2}{L} \left( \frac{1}{P} - \frac{1}{2P_{cr}} \right) \approx \frac{4M_{PR}^2}{(0.7)^2 L} \left( \frac{1}{P} - \frac{1}{2P_{cr}} \right) \]

Following the same procedure, the internal energy for the two other boundary conditions (simple supports at both ends, fixed supports at both ends) have been calculated. Table 6.1 summarizes the results.
Table 6.1: Internal energy equation of the steel column for three boundary conditions

<table>
<thead>
<tr>
<th>Column boundary condition</th>
<th>Mode shape equation (Timoshenko, 1961, Shope, 2006)</th>
<th>Internal energy equation (IE\textsubscript{col})</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-S</td>
<td>( w_{(x)} = W \times \sin\left(\frac{\pi x}{L}\right) )</td>
<td>( \frac{4M^2_{cr}}{L} \left( \frac{1}{P} - \frac{1}{2P_{cr}} \right) )</td>
</tr>
<tr>
<td>F-F</td>
<td>( w_{(x)} = \frac{W}{2} \times \left[ 1 - \cos\left(\frac{2\pi x}{L}\right) \right] )</td>
<td>( \frac{4M^2_{cr}}{(0.5)^2 L} \left( \frac{1}{P} - \frac{1}{2P_{cr}} \right) )</td>
</tr>
<tr>
<td>H-F</td>
<td>( w_{(x)} = \frac{W}{6.2824} \times \left[ \sin(\phi x) - \phi L \cos(\phi x) + \phi L(1 - \frac{x}{L}) \right] )</td>
<td>( \frac{4M^2_{cr}}{(0.7)^2 L} \left( \frac{1}{P} - \frac{1}{2P_{cr}} \right) )</td>
</tr>
</tbody>
</table>

S-S: Simple supports, F-F: Fixed-fixed supports, H-F: Hinged or pinned-fixed supports

By observing the above three equations in Table 6.1, a general equation can be written to express the internal energy of the steel column as follows:

\[
IE_{col} = \frac{4M^2_{cr}}{k^2 L} \left( \frac{1}{P} - \frac{1}{2P_{cr}} \right) \]

6.2.2.3. Derivations for the internal energy equation for columns under low axial load levels (\( P < 25\% P_{\text{Design}} \))

Apart from changing the plastic hinge location, the numerical simulation results of chapter four also indicate that, for low levels of axial compressive force, the elastic strain energy absorbed by the impacted steel column is very low compared to the plastic dissipation energy as exemplified by Fig. 6.6. Therefore, the elastic strain energy will be excluded from the energy balance equation (i.e. \( \theta_{\text{elastic}} = 0 \)).
Therefore:

\[ IE_{col} = \sum_{i=1}^{N} M_{PR_i} \theta_{Critical} \]

Figure 6.6: The time history of the energy quantities of impacted simply supported steel columns under a low level of axial compressive load \((P=10\%P_{Design})\), (A): \(L=8\)m, (B): \(L=4\)m

\[ IE_{col} = \sum_{i=1}^{N} M_{PR_i} \theta_{Critical} \]

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\[ IE_{col} = \frac{1}{x} M_{PR1} \times W_{cr} + \frac{L}{x'(L-x')} M_{PR2} \times W_{cr} + \frac{1}{(L-x')} M_{PR3} \times W_{cr} \]

Following the same procedure as for higher levels of axial compressive force, the following equations can be derived.

A. For simply supported and fix-ended columns:

\[ IE_{col} = \frac{L M_{PR}^2}{k x' P (L-x')} \]

B. For propped cantilever columns:

\[ IE_{col} \approx \frac{M_{PR}^2 (2L-x')^2}{P x' (L-x') L} \]

In Eqs. 6.26 and 6.27, the plastic hinge location, \( x' \), is equal to the impact location measured from the column base, \( \bar{x} \), as discussed before.

6.2.2.4. Reduced plastic moment capacity

Due to the axial compressive force, the plastic bending moment capacity of the column cross-section is reduced. Table 6.2 presents the exact formulae of the reduced plastic moment capacity \( M_{PR} \) for the most commonly used structural steel sections (H sections, rectangular hollow sections and hollow circular sections).
Table 6.2: Axial force-bending moment interaction equations for common structural steel sections

<table>
<thead>
<tr>
<th>Cross section shape</th>
<th>Limitations</th>
<th>Reduced plastic moment equation, $M_{PR}$</th>
<th>Eq. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box sections bent about any axis and H sections bent about major axis only (Bambach et al., 2008, Shope, 2006)</td>
<td>$P \leq P_w$</td>
<td>$M_p - \frac{h_w P^2}{4P_w}$</td>
<td>6.28</td>
</tr>
<tr>
<td></td>
<td>$P &gt; P_w$</td>
<td>$b_f t_f h_w F_y - \frac{h_w (P - P_w)}{2}$</td>
<td>6.29</td>
</tr>
<tr>
<td>For H sections bent about their minor axis only (Shope, 2006)</td>
<td>$P \leq P_w$</td>
<td>$\frac{b_f^2 t_f}{2} F_y$</td>
<td>6.30</td>
</tr>
<tr>
<td></td>
<td>$P &gt; P_w$</td>
<td>$M_p \cdot (1 - \left( \frac{P - P_w}{P_y - P_w}\right)^2)$</td>
<td>6.31</td>
</tr>
<tr>
<td>Circular sections, (Wong, 2009)</td>
<td>any value of $P$</td>
<td>$M_p \cdot \cos \left(\frac{P \times \pi}{P_y \times 2}\right)$</td>
<td>6.32</td>
</tr>
</tbody>
</table>

Where $b_f$ and $h_w$ are the section width and the web depth of the box and H sections respectively; $t_f$ is the flange thickness and $F_y$ is the yield strength of the steel material. $P_w$ is the full yield force of the section web defined by $P_w = h_w t_w F_y$; $P_y$ is the full yield load of the steel section defined by $P_y = \left(2 b_f t_f + h_w t_w\right) F_y$ for box and H sections and $P_y = \left(r_2^2 - r_1^2\right) \pi F_y$ for circular sections. $t_w$ is the web thickness for H-sections or the total wall thickness for box sections.

Alternatively, Duan and Chen (1990) suggested the following approximate interaction equation to calculate the reduced plastic moment capacity:

$$M_{PR} = M_p \left(1 - \left\lvert \frac{P}{P_y}\right\rvert^\Omega \right)$$ ................................................................. 6.33

Where $\Omega$ is a parameter that defines the shape of the interaction curve depending on the cross sectional shape of the column. The values of $\Omega$ are shown in Table 6.3.
Table 6.3: Values of $\Omega$ for different cross sectional shapes (Duan and Chen, 1990).

<table>
<thead>
<tr>
<th>Section shape</th>
<th>Circular</th>
<th>Wide-flange (major axis)</th>
<th>Wide-flange (minor axis)</th>
<th>Box</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega$</td>
<td>1.75</td>
<td>1.3</td>
<td>*2 +1.2 $(A_w/A_f)$</td>
<td>$2 - 0.5 \frac{\text{width}}{\text{depth}}$</td>
</tr>
</tbody>
</table>

*A_w = web area; A_f = flange area

The approximate interaction equation is sufficiently accurate and is easier to implement in an analytical analysis (Duan and Chen, 1990). It will be used in the proposed method.

**6.2.3. The energy absorbed by the vehicle ($IE_v$)**

According to the linear behaviour of the vehicle frontal structure observed from the numerical simulations in chapter five and referring to Fig. 6.7(A), the energy absorbed by the vehicle can be expressed by the following equation:

$$IE_v = \frac{1}{2} K_1 C^2$$  \hspace{1cm} \text{6.34}

Where $K_1$ is the linear stiffness of the frontal part of the vehicle and $C$ is the vehicle crush displacement at the column failure which can be determined according to Fig. 6.7(A and B) by:

$$C = \frac{F_{max}}{K_1}$$  \hspace{1cm} \text{6.35}

Where $F_{max}$ is the maximum transverse static force resistance of the steel column at the impact location.

Fig. 6.7 also shows that when the value of $C$ calculated from Eq. 6.35 exceeds the maximum crush distance of the vehicle, $C_{max}$, the energy absorbed by the vehicle should be determined by substituting for $C$ by $C_{max}$ in Eq. 6.34 as follows:

$$IE_v = \frac{1}{2} K_1 C_{max}^2$$  \hspace{1cm} \text{6.36}
Where $C_{\text{max}}$ is the maximum distance between the vehicle frontal structure and the vehicle engine. In other words, $C_{\text{max}}$ represents the maximum displacement at which a vehicle may be crushed before the engine impacts on the column. For the Chevrolet 1994 Pick-up vehicle considered in chapter five, the value of $C_{\text{max}}$ is 0.625m.

![Diagram of vehicle and column deformation](image)

Figure 6.7: Determination of the maximum vehicle deformation at column global failure: A) energy absorbed by the vehicle; B) energy absorbed by the column.

The maximum transverse static resistance of the steel column corresponding to each impact location and axial load ratio can be determined from a nonlinear finite element static analysis such as ABAQUS. However, for practical design purposes, it is desirable to adopt a simplified approach to calculate the value of this force and the following subsection will present two possible alternative approaches.

**A. EN 1993-1-1 (Eurocode 3)**

For steel columns under combined axial and lateral loads, Part 1-1 of Eurocode 3 (Eurocode3, 2005) may be used. Under uniaxial bending and in combination with axial compression, and on the assumption of no lateral torsional buckling, the following equation may be used:

\[
\frac{P}{\chi_{\text{z}} N_{Rk}} + k_{\text{z}} \frac{M_{\text{z}}}{M_{Pc}} \leq 1 \tag{6.37}
\]

Where $P$, $M_{\text{z}}$ are design values of the axial compression force and the maximum moment about the weak axis (z-z axis) of the member respectively; $\chi_{\text{z}}$ is the reduction factor for the compression due to flexural buckling about the weak axis (defined in
section 6.3.1 of Eurocode 3, \( N_{Rk} \) is the column cross-sectional axial resistance; \( M_{pc} \) is the full plastic moment capacity of the column cross-section about the weak axis; and \( k_{zz} \) is an interaction factor to take account of the secondary bending moment due to an axial compression force acting on the column lateral deformation.

The above equation can be used to determine the maximum bending moment that can be applied to the column in the presence of an axial compression and this bending moment can be then used to determine the corresponding maximum transverse static force. However, as will be shown in the next chapter, this method contains a high degree of conservatism. For an accidental design of the column under transverse vehicle impact, a less conservative, but more accurate, method is required.

**B. New proposal**

The column transverse resistance - maximum transverse deflection relationship, as shown in Fig. 6.5, represents the column resistance-deflection envelope corresponding to a particular axial load level. In order to determine the column maximum transverse resistance, the column transverse load (not resistance) - transverse deflection (not maximum deflection) should be determined. As shown in Fig. 6.8, it is assumed that the column maximum transverse resistance is the resistance value when the transverse load-transverse deflection line intersects the transverse resistance – the maximum transverse deflection envelope. It is further assumed that the column transverse load - lateral deflection curve can be determined based on elastic behaviour. To obtain the elastic transverse load - transverse deflection curve, Eqs. 6.15 and 6.16 can be used to give the following equations.

Consider a propped cantilever column. Since the elastic lateral load-displacement slope is sought, substitute \( M_{PR1} \), \( M_{PR2} \) and \( M_{PR3} \) by the elastic bending moment capacities \( M_1 \), \( M_2 \) and \( M_3 \) respectively in Eq. 6.15. This gives the following equation for \( \bar{x} \geq x' \):

\[
F_{el} = \frac{(M_1 \left( \frac{x'}{L} - 1 \right) + M_2 \left( x' - P \times W \right) L}{(L-x')x'} \tag{6.38}
\]

From section 6.2.2.2, \( x' = 0.6L \), \( M_1 = -\frac{1.4676 \pi^2 EI}{L^2} \times W \), \( M_2 = \frac{1.466 \pi^2 EI}{L^2} \times W \) and \( M_3 = 0 \). Therefore:
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\[ F_{el} = \frac{(-1.4676 \pi^2 EI \times W \left( \frac{0.6L}{L} - 1 \right) + 1.466 \pi^2 EI \times W - P \times W) L}{(L - \bar{x}) x'} \]

\[ \therefore F_{el} = \frac{\left( \frac{2.0528 \pi^2 EI}{L^2} \times W - P \times W \right) L}{(L - \bar{x}) x'} \]

Similarly, for \( \bar{x} \leq x' \):

\[ \therefore F_{pl} = \frac{\left( \frac{2.0528 \pi^2 EI}{L^2} \times W - P \times W \right) L}{(L - x') \bar{x}} \]

Going through the same procedure, equations have been derived for simply supported and fix-ended boundary conditions and the results are given in Table 6.4.

<table>
<thead>
<tr>
<th>Column boundary condition</th>
<th>Load-deflection equations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>For ( \bar{x} \geq x' )</td>
</tr>
<tr>
<td>S-S</td>
<td>[ F_{el} = \frac{\left( \frac{\pi^2 EI}{L^2} \times W - P \times W \right) L}{(L - \bar{x}) x'} ]</td>
</tr>
<tr>
<td>F-F</td>
<td>[ F_{el} = \frac{\left( \frac{4\pi^2 EI}{L^2} \times W - P \times W \right) L}{(L - \bar{x}) x'} ]</td>
</tr>
<tr>
<td>H-F</td>
<td>[ F_{el} = \frac{\left( \frac{2.0528 \pi^2 EI}{L^2} \times W - P \times W \right) L}{(L - \bar{x}) x'} ]</td>
</tr>
</tbody>
</table>

The above three equations can be represented by a general expression to relate the transverse force (F) with the transverse deflection (W), as follows:

For \( \bar{x} \geq x' \)

\[ F = \frac{\left( \frac{\pi^2 EI}{k^2 L^2} - P \right) W \times L}{(L - \bar{x}) x'} \]

................................................................................................................. 6.41

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Chapter Six: The Derivations of a Simplified Analytical Method

For \( \bar{x} \leq x' \)

\[
F = \frac{\left( \frac{\pi^2 EI}{k^2 L^2} - P \right) W \times L}{(L - x')\bar{x}}
\]

Fig. 6.8 demonstrates the behaviour of a simply supported steel column according to Eqs. 6.16 and 6.42 for the case of \( \bar{x} \leq x' \).

![Graph](image)

Figure 6.8: The transverse load - transverse deflection relationship of a simply supported steel column according to the proposed equations.

Referring to Fig. 6.8, the transverse displacement at which the column reaches its maximum static transverse resistance can be obtained by setting \( F \) equal to \( F_{pl} \) as follows:

\[
\left( \frac{\pi^2 EI}{k^2 L^2} - P \right) W \times L = \frac{\left( M_{PR1} \frac{x'}{L} - 1 \right) + \frac{M_{PR2}}{L} \left( x' - P \times W \right) L}{(L - x')\bar{x}}
\]

\[
\therefore W = \frac{\pi^2 EI}{k^2 L^2} \frac{\left( M_{PR1} \frac{x'}{L} - 1 \right) + \frac{M_{PR2}}{L} \left( x' - P \times W \right)}{\frac{P_{cr}}{L}}
\]

\[\text{..................6.43}\]
Substituting Eq. 6.43 into the general elastic transverse force equations (Eqs. 6.41 and 6.42) or the plastic resistance equations (Eq. 6.15 and Eq. 6.16), the following equations can be obtained to calculate the column maximum transverse resistance:

For $x \geq x'$

$$F_{\text{max}} = \frac{(M_{PR1} \left( \frac{x'}{L} - 1 \right) + M_{PR2} \left( \frac{x'}{L} \right)(1 - \frac{P}{P_{cr}})L}{(L - x')x'}$$

For $x \leq x'$

$$F_{\text{max}} = \frac{(M_{PR1} \left( \frac{x'}{L} - 1 \right) + M_{PR2} \left( \frac{x'}{L} \right)(1 - \frac{P}{P_{cr}})L}{(L - x')x'}$$

Eq. 6.45 is more appropriate in the present study since the impact location ($\bar{x}$) is always lower than or equal to the plastic hinge location ($x'$) as observed from numerical simulations presented in chapter four.

### 6.2.4. Derivations of the external work equation

The external work done by the axial compressive force can be calculated by multiplying the value of the axial compressive load by the axial shortening of the column as follows:

$$WK = P \times \Delta$$

Where $\Delta$ is the axial shortening of the column.

Referring to Fig. 6.1(B), the axial movement of the column can be calculated from the following equation:

$$\Delta = \int_{0}^{L} \varepsilon_{\text{axial}} \times dx$$

Where $\varepsilon_{\text{axial}}$ is the axial strain of the column caused by membrane action defined by continuum mechanics (Wen et al., 1995, Bambach et al., 2008) as:

$$\varepsilon_{\text{axial}} = \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2$$
The value of $\varepsilon_{axial}$ for the elastic range is very small and can be neglected without affecting the external work value. The derivation of $\varepsilon_{axial}$ for the plastic phase can be obtained from Fig. 6.1(B). From this figure, the equations for the plastic deformation shape for all boundary conditions can be expressed as follows:

$$w_x = \begin{cases} \frac{W_{cr}x}{x'} & \text{for } 0 \leq x \leq x' \\ \frac{W_{cr}(L-x)}{(L-x')} & \text{for } x' \leq x \leq L \end{cases}$$

$$\therefore \frac{dw_x}{dx}_{plastic} = \begin{cases} \frac{W_{cr}}{x'} & \text{for } 0 \leq x \leq x' \\ -\frac{W_{cr}}{(L-x')} & \text{for } x' \leq x \leq L \end{cases}$$

$$\therefore \Delta = \int_0^L \varepsilon_{axial} \times dx = \frac{1}{2} \int_0^{x'} \left( \frac{W_{cr}}{x'} \right)^2 dx + \frac{1}{2} \int_{x'}^L \left( -\frac{W_{cr}}{(L-x')} \right)^2 dx$$

$$\Delta = \frac{1}{2} \left( \frac{W_{cr}^2}{x'} \right) + \frac{1}{2} \left( -\frac{W_{cr}^2}{(L-x')} \right) = \frac{L \times W_{cr}^2}{2 x' (L - x')}$$

Giving the work done by the axial force as:

$$\therefore WK = \frac{P \times L \times W_{cr}^2}{2 x' (L - x')}$$

### 6.2.5. General energy balance equation

Referring to Eq. 6.3, the general energy balance equation can now be rewritten as:

$$IE_{col} + IE_{v} = \frac{1}{2} MV_{cr}^2 + WK$$

Where $M$ is the total mass of the impacting body and $V_{cr}$ is the critical velocity of the impacting vehicle.
Case a: \( P \geq 25\% P_{\text{Design}} \)

Substituting the general equation of the internal energy absorbed by the column (Eq. 6.23), the equation of the work done by the axial force (Eq. 6.53) and the energy absorbed by the vehicle (Eq. 6.34), into Eq. 6.54, the following general equation is obtained:

\[
\frac{4M^2_{PR}}{k^2L} \left( \frac{1}{P} - \frac{1}{2P_{cr}} \right) + \frac{1}{2} K_1 C^2 = \frac{1}{2} M V_{cr}^2 + \frac{P \times L \times W_{cr}^2}{2x'(L-x')} \tag{6.55}
\]

Case b: \( P < 25\% P_{\text{Design}} \)

Substituting the general equations of the internal energy absorbed by the column (Eqs. 6.26 and 6.27), the equation of the work done by the axial force (Eq. 6.53) and the energy absorbed by the vehicle (Eq. 6.34), into Eq. 6.54, the following general equations are obtained:

For simply supported columns:

\[
\frac{LM^2_{PR}}{k \cdot x'(L-x')P} + \frac{1}{2} K_1 C^2 = \frac{1}{2} M V_{cr}^2 + \frac{P \times L \times W_{cr}^2}{2x'(L-x')} \tag{6.56}
\]

For propped cantilever columns:

\[
\frac{M^2_{PR} \cdot (2L-x')^2}{P \cdot x'(L-x')L} + \frac{1}{2} K_1 C^2 = \frac{1}{2} M V_{cr}^2 + \frac{P \times L \times W_{cr}^2}{2x'(L-x')} \tag{6.57}
\]

In Eqs. 6.55, 6.56 and 6.57 \( C \) is substituted by \( C_{\text{max}} \) when column failure occurs after contact with the engine and this situation is indicated by Eq. 6.35. Hence, the above three equations can be used to determine the critical impact velocity \( V_{cr} \) of the impacted steel column under a specific value of axial compressive load.

6.2.6. Accounting for the strain hardening effects

The numerical simulations presented in chapter four have shown that strain hardening of the steel material can increase the resistance of the column under transverse impact because of the increase in column stiffness (Bai and Pedersen, 1993). However, the proposed simplified analytical method presented in this chapter assumes that the material behaviour is elastic-perfectly plastic. Nevertheless, a simple approach can be
used to include the steel hardening effect within the assumption of elastic-perfectly plastic behaviour by assuming that the yield stress is the average value of the yield and the ultimate stresses of the actual material behaviour as shown in Fig 6.9. This approach will be validated in a sub-section of the next chapter.

Figure 6.9: Idealised material behaviour of steel used to account for the strain hardening effect

### 6.3. Summary

This chapter has presented the full derivations of a simplified analytical method to calculate the critical velocity of a rigid body and a flexible vehicle impact on a steel column. This method used the energy balance principle and assumed a quasi-static response from the impacted steel column. The main failure mode of the column was global plastic buckling with the plastic hinge mechanism losing equilibrium under an axial compressive force. For moderate to high levels of axial load (not less than 0.25 of column design resistance), the location of the plastic hinge within the column height was assumed to be at the position of the maximum column lateral deformation according to the column buckling mode under axial load. For lower levels of axial load, the position of the plastic hinge within the column height was assumed to be at the position of impact. To obtain the maximum plastic hinge rotations, the method used by Shope was followed, based on the assumption that the column with plastic bending moment capacities at the plastic hinges loses equilibrium under the axial load.
A simple approach was suggested in this chapter to calculate the maximum energy absorbed by the vehicle at column failure. In this approach, the linear behaviour of the vehicle frontal structure up to the deformation to the engine box is assumed based on the numerical simulations’ results of chapter five. The maximum vehicle deformation is determined based on the maximum transverse static resistance of the column. For design purposes, two alternative approaches were presented in this chapter to calculate the maximum transverse static resistance of the column, one based on EC3 and the other proposed by the author.

Chapter seven will present a validation of the proposed method by comparing it against the numerical simulation results of chapters four and five.
Chapter Seven: Validation of the Simplified Analytical Method

7.1. Introduction

The numerical simulation results presented in chapters four and five included the effects of strain hardening and strain rate. The analytical model proposed in chapter six assumes an elastic-plastic stress-strain curve without a strain rate effect. A set of new numerical simulations were carried out using the steel material model assumed in the analytical approach. The results of these new numerical simulations will be used in this chapter to ascertain the validity and the accuracy of the proposed analytical method presented in chapter six. This will be established by comparisons for the column axial force-critical velocity relationship, the maximum column transverse and axial displacements-axial force relationship, and for the various energy quantities used in the simplified energy balance equation. Section 7.2 presents the results for a rigid body impact and section 7.3 provides additional comparisons for vehicle impact. An approximation is then proposed in section 7.4 to take into consideration the effect of strain-hardening. As has been demonstrated in section 4.2.4.7 in chapter four, the effect of strain rate may be safely ignored for vehicle impact on columns.

In the validation examples, the steel material is assumed to be S355, with a modulus of elasticity of $206 \times 10^9$ N/m$^2$ and a yield stress of $355 \times 10^6$ N/m$^2$. Table 7.1 lists the numerical simulations conducted and the column properties used in the validation study.
Table 7.1: Properties of the columns used to validate the simplified analytical method

<table>
<thead>
<tr>
<th>Impact analysis</th>
<th>Column length (m)</th>
<th>Column section</th>
<th>Slenderness ratio (\frac{KL}{r})</th>
<th>Relative slenderness (\lambda_{zz})</th>
<th>Design axial compressive load (kN)</th>
<th>Boundary conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid impact</td>
<td>4</td>
<td>UC 305×305×118</td>
<td>51.50</td>
<td>0.68</td>
<td>3800</td>
<td>S.S Prop.</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>UC 356×406×340</td>
<td>76.9</td>
<td>1.02</td>
<td>6800</td>
<td>S.S</td>
</tr>
<tr>
<td>Vehicle impact</td>
<td>4</td>
<td>UC 254×254×89</td>
<td>61</td>
<td>0.81</td>
<td>2580</td>
<td>S.S Prop.</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>UC 305×305×118</td>
<td>51.50</td>
<td>0.68</td>
<td>3800</td>
<td>S.S Prop.</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>UC 356×368×202</td>
<td>41.66</td>
<td>0.55</td>
<td>6780</td>
<td>S.S</td>
</tr>
</tbody>
</table>

### 7.2 Validation of the analytical method against rigid body impact

This section aims to validate the analytical method for a range of values for impact location, impact mass, slenderness ratio and for two boundary conditions. Comparisons are made between the analytical and the ABAQUS simulation results for the following quantities: maximum column displacement, critical impact velocity - axial force relationship, and various energy quantities involved in the analytical method. The correct calculation of the maximum column displacements is important because it directly determines the energy absorption of the column and the work undertaken by the axial force.

#### 7.2.1. Maximum column displacements

The column’s internal energy and the external work done by the column axial load, as derived in chapter six, depend critically on the value of the maximum column transverse displacement and the axial displacement (shortening). The maximum energy absorption capacity of the column corresponds to the maximum column deformation before losing stability. In the analytical approach, this displacement was derived based on a quasi-static approximation of column behaviour under a compressive axial load and in global buckling failure mode. It represents the state of column behaviour which has a zero
transverse load resistance. In the numerical model, the maximum column transverse
displacement should be taken as the column permanent deformation for the critical case
when the column changes from being stable to not being stable after impact. To validate
this assumption, Figs. 7.1 to 7.3 compare the maximum column displacements
calculated using the proposed analytical method and the numerical simulations using
ABAQUS/Explicit. An excellent agreement can be seen between the two sets of results
for all levels of axial load, for different impact locations and impact energies for both
simply supported and for the propped cantilever steel columns. The results shown in
these figures further confirm the assumption that the impact location has negligible
influence on column deformation, as indicated by the near coincidence of different
curves representing different impact locations in the same graph.

Figure 7.1: Comparison between analytical and ABAQUS predictions of column maximum
displacements at different levels of axial force for the simply supported column UC
305×305×118, L=4, Impact mass = 3tonnes: (A) transverse displacement; (B) axial
displacement
Figure 7.2: Comparison between analytical and ABAQUS predictions of column maximum displacements at different levels of axial force for the propped cantilever column UC 305×305×118, L=4, Impact mass = 3 tonnes: (A) transverse displacement; (B) axial displacement
Chapter Seven: Validation of the Simplified Analytical Method

7.2.2. Critical impact velocity-axial load interaction curves

Figs. 7.4 to 7.6 compare the critical impact velocity-axial force curves between the analytical method and the numerical simulation results for different column sections (UC 356×406×340 and UC 305×305×118), two different boundary conditions (simply supported and fix-ended) and two impact masses (3 tonnes and 6 tonnes). For the simply supported columns (Figs. 7.4(A), 7.5 (impact locations=2m and 4m) and 7.6 (impact locations=2m and 4m)), very good correlation is obtained. As the impact location moves closer to the column base, the difference between the analytical and the simulation results increases. This reflects the increasing error of assuming the
intermediate plastic hinge location (mid-height for simply supported and fix-ended boundaries, 0.6L from the column base for a propped cantilever column) to be at a different location from the impact position. This assumption particularly affects the accuracy of the analytical solution for the propped cantilever column (Fig. 7.4(B)) and low impact locations (1m, Figs. 7.5 and 7.6(B))

Figure 7.4: Comparison between analytical and ABAQUS predictions of critical impact velocity - axial force curve at different levels of axial force for column UC 305×305×118, L=4, Impact mass = 3tonnes: (A) Simply supported column; (B) Propped cantilever

Figure 7.5: Comparison between analytical and ABAQUS predictions of critical impact velocity - axial force curve at different levels of axial force for simply supported UC 356×406×340, L=8m, Impact mass = 3tonnes
Chapter Seven: Validation of the Simplified Analytical Method

Figure 7.6: Comparison between analytical and ABAQU S predictions of critical impact velocity - axial force curve at different levels of axial force for a simply supported column at impact mass = 6 tonnes: (A) UC 305 × 305 × 118, L = 4; (B) UC 356 × 406 × 340, L = 8m

Another factor that affects the accuracy of the analytical method is the energy absorbed by the column through distortion at the impact zone, in particular when the impact is near the column base. This energy absorption is not included in the analytical model, thus resulting in a lower critical impact velocity predicted by the analytical method. For example, Fig. 7.7 shows the deformed shape of two simply supported columns when the impact is close to the column base (0.125L). It can be clearly seen from this figure that the deformed shape of the columns significantly differs from that assumed in the derivation (the columns deformed into two straight segments). The distortion in the column near the impact zone enables the column to absorb a considerable amount of plastic strain energy thus leading to increased column resistance. This effect was not included in the analytical model, explaining the predicted lower critical velocity when using the analytical model than when using the FE simulation results. Nevertheless, the analytical model predicts sufficiently accurate results in most cases and errs on the safe side.
Chapter Seven: Validation of the Simplified Analytical Method

7.2.3. Energy quantities

During the derivation of the proposed analytical method, many assumptions have been made. Each will contribute to the discrepancy between the analytical and the simulation results. These assumptions affect the different energy terms in the energy balance equation. To check where the critical source of discrepancy is, Figs. 7.8 to 7.10 provide comparisons of the different energy quantities, corresponding to the cases in Figs 7.4 and 7.5. The correlation between the analytical and numerical outputs for the different energy quantities is similar to that for the critical impact velocity.

Figure 7.7: Deformation shapes of two simply supported steel columns, (cf Fig. 7.5, section UC 356×406×340, L=8m, impact mass = 3tonness, Impact location=1m)
Figure 7.8: Comparison between analytical and ABAQUS results for external work and plastic dissipation energy for simply supported columns, (UC 305×305×118, L=4m, impact mass= 3tonnes, different impact locations); cf. Fig. 7.4(A) for axial load – critical velocity relationships

The external work undertaken by the column axial force moving through the column shortening increases as the applied axial force decreases. This is due to the increased lateral deformation that the column can undergo before failure and that the external work done is related to the square of the column lateral deformation (Eq. 6. 53 in
chapter six). As expected, the plastic dissipation energy also increases as the axial load in the column decreases, confirming that the column can deform more before buckling failure if the axial load is small.

In all cases, since the external work is small compared to the plastic dissipation energy, the level of inaccuracy in the external work figures may be considered acceptable.

Therefore, the level of accuracy in the calculated plastic dissipation energy determines the overall accuracy of the critical velocity - axial load relationship. When the agreement is very good (Figs 7.8 and 7.9), the agreement between the analytical and the ABAQUS results for the column critical velocity - axial force relationship is very good (Fig. 7.4). Otherwise, the discrepancy is relatively high. Nevertheless, the analytical model gives critical velocities on the safe side.
Figure 7.9: Comparison between analytical and ABAQUS results for external work and plastic dissipation energy for a propped cantilever column (UC 305×305×118, L=4m, impact mass =3 tonnes, different impact locations) cf. Fig. 7.4(B) for axial load - critical velocity relationships
Figure 7.10: Comparison between analytical and ABAQUS results for external work and plastic dissipation energy for a simply supported column (UC 356×406×340, L=8m, impact mass = 3 tonnes, different impact locations), cf. Fig. 7.5 for axial load - critical velocity relationships.

In summary, the proposed analytical method provides reasonably accurate predictions of the critical impact velocities of axially preloaded steel columns under transverse rigid impact. Comparisons of the different energy quantities indicate good agreement.
between the numerical and analytical results. This validation suggests that column behaviour is predicted with good accuracy.

### 7.3. Validation of the analytical method against vehicle impact

According to the procedure described in chapter six, determination of vehicle deformation at column global failure requires the input of the column’s resistance to a static force at the position of impact. This resistance may be obtained using a static finite element simulation. Fig. 7.11 shows an example of such simulation results. As explained in section 6.2.3 in chapter six, this resistance may also be determined analytically, using the method in Eurocode 3 or the newly developed proposal within this research. In this validation research, the ABAQUS simulation results are first used to ensure any error from estimating the column transverse resistance is minimized. Afterwards, in section 7.3.4, the effects of using different methods to predict column transverse resistance are evaluated.

![Graph](image)

**Figure 7.11:** Numerical simulations’ results for the transverse force - transverse displacement relationship of a simply supported column UC 305×305×118 under different axial load ratios.

The maximum vehicle deformation is determined by substituting the column transverse resistance into Eq. 6.35 in chapter six. Tables 7.2 to 7.4 compare the calculated values of vehicle maximum deformation with the numerical simulation results. The values of the vehicle frontal stiffness \((K_f)\) are 510kN/m, 463kN/m and 546 kN/m as calculated in section 5.3.1 in chapter five representing the stiffness of the Chevrolet pick-up 1994 vehicle impacting on steel columns with section sizes of UC 305×305×118, UC 254×254×89 and UC 356×368×202 respectively. Good agreement is achieved between
the two sets of results. In the case of vehicle deformation before touching the vehicle engine, the numerical and analytical values of vehicle deformation are very close. When the vehicle deformation reaches the engine position, it is assumed that the vehicle acts as a rigid impactor and does not consume any impact energy. This is indicated in Tables 7.2 to 7.4 by the symbol > $C_{\text{max}}$. It is clear that the analytical method has correctly predicted this in all cases.

Table 7.2: The calculated values of vehicle maximum deformation as compared with the numerical simulation results for a steel column section UC 305×305×118, $K_1$=510kN/m

<table>
<thead>
<tr>
<th>P/P_{Design}</th>
<th>Simply supported column</th>
<th>Propped cantilever column</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Crush displacement, C(m)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Analytical</td>
<td>ABAQUS</td>
</tr>
<tr>
<td>1</td>
<td>0.233</td>
<td>0.247</td>
</tr>
<tr>
<td>0.85</td>
<td>0.401</td>
<td>0.424</td>
</tr>
<tr>
<td>0.65</td>
<td>&gt; $C_{\text{max}}$</td>
<td>&gt; $C_{\text{max}}$</td>
</tr>
<tr>
<td>0.43</td>
<td>&gt; $C_{\text{max}}$</td>
<td>&gt; $C_{\text{max}}$</td>
</tr>
<tr>
<td>0.25</td>
<td>&gt; $C_{\text{max}}$</td>
<td>&gt; $C_{\text{max}}$</td>
</tr>
</tbody>
</table>

Table 7.3: The calculated values of vehicle maximum deformation as compared with the numerical simulation results for a steel column section UC 254×254×89, $K_1$=463kN/m

<table>
<thead>
<tr>
<th>P/P_{Design}</th>
<th>Simply supported column</th>
<th>Propped cantilever column</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Crush displacement, C(m)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Analytical</td>
<td>ABAQUS</td>
</tr>
<tr>
<td>1</td>
<td>0.169</td>
<td>0.181</td>
</tr>
<tr>
<td>0.85</td>
<td>0.261</td>
<td>0.273</td>
</tr>
<tr>
<td>0.65</td>
<td>0.44</td>
<td>0.456</td>
</tr>
<tr>
<td>0.43</td>
<td>0.6</td>
<td>0.613</td>
</tr>
<tr>
<td>0.25</td>
<td>&gt; $C_{\text{max}}$</td>
<td>&gt; $C_{\text{max}}$</td>
</tr>
</tbody>
</table>

Table 7.4: The calculated values of vehicle maximum deformation as compared with the numerical simulation results for a steel column section UC 356×368×202, $K_1$=546kN/m

<table>
<thead>
<tr>
<th>P/P_{Design}</th>
<th>Simply supported column</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Crush displacement, C(m)</td>
</tr>
<tr>
<td></td>
<td>Analytical</td>
</tr>
<tr>
<td>1</td>
<td>0.622</td>
</tr>
<tr>
<td>0.85</td>
<td>&gt; $C_{\text{max}}$</td>
</tr>
<tr>
<td>0.65</td>
<td>&gt; $C_{\text{max}}$</td>
</tr>
<tr>
<td>0.43</td>
<td>&gt; $C_{\text{max}}$</td>
</tr>
<tr>
<td>0.25</td>
<td>&gt; $C_{\text{max}}$</td>
</tr>
</tbody>
</table>
7.3.1. Critical impact velocity- axial load interaction curves

Having determined the vehicle deformation, the energy balance equations derived in chapter six can be used now to determine the critical impact velocity of the vehicle that can cause the failure of steel columns. Figs. 7.12 to 7.14 compare the numerical and analytical results. Good agreement can be seen between the two sets of results, especially for simply supported columns and for columns with high axial load ratios. Including the effects of vehicle stiffness gives much higher critical impact velocities than assuming a rigid impact mass. This considerable difference in column critical impact velocity highlights the importance of including the energy absorbed by the vehicle in design.

![Graph A](image1.png)

![Graph B](image2.png)

Figure 7.12: Comparison between analytical and ABAQUS predictions of critical impact velocity - axial force curve at different levels of axial force for the steel column section UC 254×254×89: (A) simply supported column (B) propped cantilever column.
Figure 7.13: Comparison between analytical and ABAQUS predictions of critical impact velocity - axial force curve at different levels of axial force for the steel column section UC 305×305×118: (A) simply supported column; (B) propped cantilever column.

Figure 7.14: Comparison between analytical and ABAQUS predictions of critical impact velocity - axial force curve at different levels of axial force for the simply supported steel column section UC 356×368×202.

The change in accuracy at different column axial load levels (that is, being better at higher axial loads) may be explained by the role played by the vehicle. Figs. 7.15 and 7.16 compare the analytical and the simulation values of energy absorption by the vehicle. The agreement is excellent, confirming the accuracy of the vehicle model. At high column axial loads, as shown in Tables 7.2 to 7.4, column failure was before the vehicle had deformed to the engine position. The accuracy of the analytical method
depends largely on the accuracy of predicting vehicle behaviour, hence the good accuracy achieved for the vehicle energy absorption is reflected in the good accuracy of the overall prediction of the critical impact velocity. When column failure occurs after contacting the vehicle engine (Fig. 7.13(B) and Fig. 7.14), the value of the energy absorbed by the vehicle was constant and equal to the maximum energy absorption capacity of the vehicle (Eq. 6.36 in chapter six). Therefore, in this case, the accuracy of predicting the critical impact velocity depends solely upon the accuracy of predicting the energy absorbed by the column. As shown in section 7.2 which discussed the rigid body impact, there was high inaccuracy under some conditions.

Figure 7.15: Comparison between analytical and ABAQUS results for energy absorbed by vehicle for the steel column section UC 305×305×118: (A) simply supported column; (B) propped cantilever column.

Figure 7.16: Comparison between analytical and ABAQUS results for energy absorbed by vehicle for the steel column section UC 254×254×89: (A) simply supported column; (B) propped cantilever column.
The difference in the analytical and numerical solutions for column energy and work done may be explained by the differences in the column lateral displacement and axial deformation, which are shown in Figs. 7.17 and 7.18. Reasonable agreement is shown between the analytical and simulation results for transverse displacements. The differences in axial displacements are high, indicating that the column deformation shape may be revised to improve the accuracy of the analytical solution. Nevertheless, since the analytical solution of the critical impact velocity is on the safe side and the difference between the analytical and numerical simulation results is within an acceptable range, this refinement has not been attempted in this research.

Figure 7.17: Comparison between analytical and ABAQUS results for axial and transverse displacements of the steel column section UC 305×305×118: (A) simply supported column; (B) propped cantilever.
Chapter Seven: Validation of the Simplified Analytical Method

Figure 7.18: Comparison between analytical and ABAQUS results for axial and transverse displacements of the steel column section UC 254×254×89: (A) simply supported column; (B) propped cantilever

7.3.2. Energy quantities

The discrepancy between the analytical and numerical simulation results for the critical impact velocity may also be explained by comparing the values of plastic energy and the external work used in the energy balance equation. The results are shown in Fig. 7.19(A and B). The accuracy for predicting column energy absorption and work done is similar for different axial load ratios. However, at high axial load level (load ratio > 0.6), the column energy absorption and external work are small compared to the energy absorption by the vehicle. Therefore, the analytical solution for the critical velocity is still good because the analytical results for predicting vehicle energy absorption is very accurate. At lower load ratios, since the energy absorbed by the vehicle is fixed and...
smaller than the energy absorption by the column, the inaccuracy in predicting column energy absorption is reflected in the critical velocity results. However, compared to the prediction results of the critical impact velocity for a rigid mass impact in the previous section, including the realistic condition of vehicle impact reduces the difference between the analytical and numerical simulation results by reducing the influence of the assumptions made to column behaviour.

Figure 7.19: Comparison of energy components for vehicle impact on (A) Steel column section UC 305×305×118; (B) Steel column section UC 254×254×89
7.3.3. Validation of the simplified analytical method against different values of $K_1$ and $C_{\text{max}}$

In the previous sub-sections, the proposed simplified analytical method was validated against different cases of column sections and boundary conditions. However, changing either the vehicle stiffness $K_1$ or the vehicle crush distance $C_{\text{max}}$ will change the energy absorption capacity of the vehicle. In order to ensure that the analytical method is generally applicable, comparisons were made between the analytical and simulation results for different values of $K_1$ and $C_{\text{max}}$. In this study, the vehicle frontal stiffness and the vehicle maximum crush distance were separately changed to be 50% or 2 times the values used in the previous sections. Fig. 7.20 (A and B) shows the assumed vehicle load-deformation relationships. Changing one of these two factors alone would change the maximum vehicle energy absorption proportionally.

![Figure 7.20: Values of vehicle stiffness and vehicle maximum crush distance used in the validation study](image)

Figs. 7.21 and 7.22 compare the simulation and analytical results for two simply supported steel column sections. The good agreement shown in these figures confirms the general applicability of the analytical method for different vehicle characteristics.

The results in Figs. 7.21(B) and 7.22(B) clearly indicate that, because changing the vehicle maximum crush displacement ($C_{\text{max}}$) only proportionally changes the vehicle energy absorption capacity, the critical impact velocity increases as $C_{\text{max}}$ is increased. In contrast, when only the vehicle frontal stiffness is increased, the results in Fig. 7.21(A) and 7.22(A) show cross-over. This is because increasing this value may be either
beneficial or detrimental to the column. Before the vehicle has deformed to the engine position, a more rigid vehicle (higher $K_1$) is detrimental to the column. However, after the vehicle has deformed to the engine position, a higher value of $K_1$ (while keeping $C_{\text{max}}$ the same) means more vehicle energy absorption, so being beneficial to the column. This cross-over behaviour was accurately predicted by the analytical model.

Figure 7.21: Simply supported column section UC 305\times305\times118: (A) using three values of vehicle stiffness, and (B) using three values of vehicle crush distance.
7.3.4. Using alternative values for the maximum static transverse resistance of steel columns

The comparisons in the previous sections were based on calculating the column transverse resistance using the finite element method; the purpose of these comparisons was to minimize errors in estimating the column transverse resistance. This section assesses the accuracy of using the two analytical methods in section 6.2.3 to calculate the column transverse resistance. For the simulated cases, Fig. 7.23 presents the column axial load - transverse resistance relationships, calculated using the two analytical methods and compares them with the ABAQUS static simulation results. The Eurocode 3 results are very safe and the alternative calculation results are in good agreement with the ABAQUS static simulation results. In particular, when the applied axial load ratio is high (the axial load ratio being defined as the ratio of the applied axial load to the column compression resistance calculated using Eurocode 3), the Eurocode 3 gives very low values of column transverse resistance.
Figure 7.23: Comparison of the maximum transverse resistance of the steel column between the numerical simulation using ABAQUS, the proposed equation (Eq. 6.45) and EC3 (Eq. 6.37): (A) steel column section UC 254×254×89; (B) steel column section UC 305×305×118; (C) steel column section UC 356×368×202
Figs. 7.24 and 7.25 compare the critical velocity results by using the column transverse resistance in Fig. 7.23. Not surprisingly, using the column transverse resistance from the proposed method of this research (section 6.2.3) gives column critical velocities very close to those when using the ABAQUS column transverse resistance. The low values of column transverse resistance calculated by Eurocode 3 are directly reflected in the critical velocity results. This is especially the case when the applied axial loads on the columns were high and in the cases before the vehicle deformation reached the vehicle engine. When the vehicle deformation has reached the engine position, the column transverse resistance had no effect because the vehicle energy absorption became the same and equal to the maximum value (Eq. 6.36). This trend is clearly shown in Figs. 7.24 and 7.25.

![Graph showing critical impact velocity vs. axial force for different types of columns](image)

Figure 7.24: Comparison between using ABAQUS, EC3 and the proposed equation to predict the critical impact velocity - axial force curve at different levels of axial force for the steel column section UC 305×305×118.
Chapter Seven: Validation of the Simplified Analytical Method

Figure 7.25: Comparison between using ABAQUS, EC3 and the proposed equation to predict the critical impact velocity - axial force curve at different levels of axial force for the steel column section UC 254×254×89.

7.4. The strain hardening effect

Section 6.2.6 in chapter six has explained how the strain hardening effect of steel may be incorporated in the analytical method which assumes a linear elastic-perfectly plastic stress-strain curve for steel; the equivalent yield stress in the analytical method is the average of the real yield stress and the ultimate tensile stress. Fig. 7.26(A and B) compares the analytical values of the critical impact velocity with the ABAQUS simulation results with, and without, the inclusion of the effects of strain hardening. For simplicity, rigid impact was assumed. As explained in section 7.3, including the vehicle effect would improve the accuracy of the analytical solutions. As can be seen from this figure, using the equivalent yield stress in the analytical approach increases the critical impact velocity and is suitable for predicting the critical impact velocity if the strain hardening effect is to be included.
Chapter Seven: Validation of the Simplified Analytical Method

Figure 7.26: The effect of strain hardening on the critical impact velocity of simply supported steel columns subjected to transverse rigid impact: (A) section UC 305×305×118, L=4 m, impact mass = 6 tonnes, impact location =1.5m; (B) section UC356×406×340, L=8m, impact mass = 6 tonnes, impact location =2m.

7.5. Summary

In this chapter, the accuracy of the proposed analytical method was checked against a range of ABAQUS simulation results, by comparing the maximum column displacements-axial force curves, the critical impact velocity-axial force curves, the external work-axial load curves and the plastic dissipation energy-axial load curves. In most cases, the agreement between the analytical method results and the ABAQUS simulation results was very good. Moderately large inaccuracy occurred for propped cantilever columns and when the impact location was close to the column base (when
the impact location \(<.25L\) measured from the column base). Under this circumstance, the deformation shape of the column at failure was not as assumed in the derivations and the column distortion near the impact location became an important factor but it is not included in the simplified analytical method. Nevertheless, the simplified analytical method gave critical impact velocities on the safe side.

Assuming a rigid impact gave a lower critical impact velocity than in the case of a flexible vehicle impact, so the results of rigid impact were on the safe side. However, an accurate prediction of column behaviour and critical impact velocity should include the energy absorption of the flexible vehicle. In fact, by including a vehicle model in the analytical method, which makes the analytical method more realistic, the accuracy of the analytical method was improved. This was because including the vehicle energy absorption reduces the effects of inaccurate assumptions on column behaviour. This research demonstrated that vehicle behaviour was modelled accurately.

The results of validation examples using different vehicle characteristics (vehicle stiffness and vehicle maximum crush distance) confirmed that the proposed analytical method was generally applicable for vehicles with different load-deformation characteristics.

For accurate prediction of critical impact velocity, the correct maximum vehicle deformation should be used. Since this value is calculated based on the column static transverse resistance, the column resistance should be calculated accurately. The results of this chapter indicate that the Eurocode 3 results for column transverse resistance are safe, but with a high margin of safety. The critical velocity results using the proposed method of calculating column transverse resistance were in much better agreement with the results using the ABAQUS static simulation of column transverse resistance.

Without including the effect of strain hardening in modelling column behaviour would reduce the critical impact velocity, thereby making the analytical results on the safe side. Should strain hardening of steel be included, then it is acceptable to use an effective yield stress and this effective yield stress is the average of the actual yield stress and ultimate tensile stress of steel.
8.1. Introduction

In EN 1991 (Eurocode 1) provisions are made for the analysis and design of structures to resist vehicle impact. The effect of vehicle impact is treated either as an equivalent static force on the structure as in EN 1991-1-1 (Eurocode1, 2002) or as an approximate dynamic action as in EN 1991-1-7 (Eurocode1, 2006). This chapter assesses these requirements and evaluates the applicability of these requirements. In addition, a possible approach is proposed to improve the accuracy of the design method.

8.2. The equivalent static force approach

The equivalent static force is either calculated using a simple equation (Eurocode1, 2002) or taken as constant values (Eurocode1, 2006, Eurocode1, 2003). Whilst the basis for the calculation equation is clear, the problem with using this equation is that it is necessary to obtain the structural and vehicle deformations from other means. This is not really feasible in design calculations. Otherwise, this design method becomes redundant. Therefore, this research will not consider this approach any further and will assess the constant impact forces approach. In section 4.3 of EN 1991-1-7, the suggested equivalent static forces are 500kN, 700kN and 1000kN respectively for structural members subjected to transverse vehicle impact in the direction of normal travel in urban areas, rural areas and on national roads respectively. In addition, sections 4.7.2.1 and 5.6.2.1 of Eurocode 1: Part 2 (Eurocode1, 2003) also gives constant design values for bridges due to vehicle collision, being 1000kN and 500kN in the parallel and perpendicular directions of vehicle travel respectively.

To investigate the validity of the equivalent static force assumption in the design of steel columns subjected to vehicle impact, comparisons between numerical simulation results and design values were carried out for the four steel column sections shown in
Table 8.1. The stress-strain curve of the steel (S355) is assumed to be elastic-perfectly plastic with a modulus of elasticity of $206 \times 10^9$ N/m$^2$ and a yield stress of $355 \times 10^6$ N/m$^2$.

The validation procedure is as follows:

a. The axial force in the column at which the column fails under vehicle impact at a certain speed was determined. The simplified numerical vehicle model described in chapter five was used here to simulate the effect of vehicle impact. Three impact velocities, 50 km/h, 80 km/h and 120 km/h, were considered, representing speed limits in urban areas, in rural areas and on national roads respectively.

b. Under the same column axial load from (a), the equivalent transverse static force to cause the column to fail was obtained from a nonlinear static analysis using ABAQUS. This force was considered to represent the equivalent static design force corresponding to that of the vehicle impact speed.

Table 8.1: Steel column properties used in the numerical simulations

<table>
<thead>
<tr>
<th>Column length (m)</th>
<th>Column section</th>
<th>Impact direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>UC 254×254×89</td>
<td>Weak axis</td>
</tr>
<tr>
<td>4</td>
<td>UC 305×305×118</td>
<td>Weak axis</td>
</tr>
<tr>
<td>4</td>
<td>UC U356×368×202</td>
<td>Weak axis</td>
</tr>
<tr>
<td>4</td>
<td>UC U356×406×340</td>
<td>Weak axis</td>
</tr>
</tbody>
</table>

In the static analysis (step b), the lateral force can be applied on the steel column as a pressure load with the area of application being taken as 250mm×column width for car collision and 500mm×column width for lorry collisions as suggested in EN 1991-1-7 (Eurocode1, 2002, Ferrer et al., 2010), see Fig. 8.1. The location of the lateral load application was suggested in the code to be 0.5 m for car collision and 1.5m for lorry collision. In this research, the vehicle considered was a Chevrolet 1994 Pick-up vehicle. Since this does not completely match any of the above two vehicle types, the area of pressure load application was taken as 350mm×column width and the location of impact was taken as the same as the actual impact location measured in the numerical simulation model (section 5.3.1.2 in chapter five), which was 0.811m.

Fig. 8.2 compares the equivalent static forces of this research with the design equivalent static forces suggested by EN 1991-1-7.
Figure 8.1: Area of application of the equivalent lateral static load according to EN 1991-1-7 (Eurocode1, 2006)

Figure 8.2: Comparison of equivalent static forces between this research and EN 1991-1-7 design values.

It can be seen from Fig. 8.2 that, at the same impact speed, the equivalent static force values should depend on the column, as represented by the column size. The force increases as the column size increases. Using a constant value for all column sizes, as in the current Eurocode 1, is not accurate.

The results in Fig. 8.2 suggest that the values suggested by Eurocode 1 may be considered acceptable for small and medium sized columns that are commonly used in buildings of a few storeys high, as represented by the two smaller columns in the figure.
For large columns, the Eurocode 1 design values are much lower than the true equivalent lateral static load; therefore, using the Eurocode 1 values for such columns may lead to an unsafe design. When designing for columns under vehicle impact, one design objective would be to calculate the column axial compression resistance under the equivalent static load for vehicle impact. Using the Eurocode 1 values for the larger columns would give higher column resistances than the true values. Table 8.2 presents a comparison of the axial load capacities (as load ratios) obtained from using the Eurocode 1 static values and the ABAQUS equivalent static values. It can be seen from this table that using the Eurocode 1 equivalent static design forces gives considerably higher axial load values than the true column axial resistances under vehicle impact velocity at the high velocities of 80km/h and 120km/h. Using the Eurocode 1 equivalent static loads gave column axial compression resistances of 3.86 and 2.33 times the column axial resistances when using the ABAQUS equivalent static loads for an impact velocity of 120 km/h. The differences are smaller at lower impact velocities, but they are still considerable for an impact velocity of 80 km/h. For the low impact velocity of 50km/h, the Eurocode 1 equivalent static loads may be considered usable because the overestimation by using the Eurocode 1 values is around 20%.

<table>
<thead>
<tr>
<th>Impact velocity km/h</th>
<th>Column axial compression resistance (as ratio to column resistance without impact)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UC U356×368×202</td>
</tr>
<tr>
<td></td>
<td>Using EC1</td>
</tr>
<tr>
<td>50 (Urban areas)</td>
<td>0.95</td>
</tr>
<tr>
<td>80 (Rural areas)</td>
<td>0.892</td>
</tr>
<tr>
<td>120 (National roads)</td>
<td>0.772</td>
</tr>
</tbody>
</table>

8.3. The dynamic impulse approach

According to Annex C of Eurocode 1 (Eurocode1, 2006), an alternative dynamic approach can be used to address the effect of vehicle impact on structures. In this approach, vehicle impact can be treated as a dynamic impulse. The maximum value of the force and the time duration of the impulse can be calculated using Eqs. 2.2 and 2.3 in chapter two. In these two equations, the stiffness $k_e$ is suggested to be the equivalent
elastic stiffness of the impacting object (vehicle) for the case of hard impact or elastic stiffness of the impacted structure (column) for the case of soft impact. However, the design code gives no guidance on how to define hard or soft impacts. In fact, results of the numerical simulations presented in chapters five and seven of this thesis have shown that, in most cases, both the vehicle and the steel column may experience considerable deformations during the impact. Therefore, both stiffness terms should be included. This section will first assess the effects of using either the column or vehicle elastic stiffness in Eqs. 2.2 and 2.3. Afterwards, it will propose a method of including both stiffness’ values and assess its accuracy.

A. Using vehicle stiffness

Vehicle stiffness corresponding to each column size can be obtained from the numerical simulations in chapter five. However, it can also be calculated analytically using the proposed equation in the same chapter (Eq. 5.14) provided that the required information about the vehicle crush test on a rigid barrier is available. For the Chevrolet 1994 Pick-up impacting on a steel column with section size UC $305 \times 305 \times 118$, the vehicle stiffness was 510kN/m (refer to section 5.3.1.2 in chapter five). Therefore, the equivalent impact impulse can be calculated as follows:

$$F = v, \sqrt{K_1 \cdot M} = v, \sqrt{510000 \times 1840} = v, \times 30633.3 \text{kg/sec}.$$  
$$\Delta t = M / K_1 = \sqrt{1840 / 510000} = 0.06 \text{sec}$$

B. Using column stiffness

To account for the effect of axial compressive force on the column elastic lateral stiffness, the column stiffness values were calculated from the lateral load-deformation curves extracted from the nonlinear static simulations conducted in chapter seven (Fig. 7.11).

Alternatively, Eq. 6.42 in chapter six can be used to determine the general equation for column elastic stiffness taking into account the axial compression load and the location of the lateral load as follows:

$$k_{col} = \frac{F}{W}$$
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\[
F = \frac{\frac{\pi^2 EI}{k^2L^2} - P)W \times L}{(L - x')\bar{x}} \quad \text{(Eq. 6.42 in chapter six for } \bar{x} \leq x')
\]

\[
k_{col} = \frac{\frac{\pi^2 EI}{k^2L^2} - P)W \times L}{(L - x')\bar{x}} = \frac{\pi^2 EI}{k^2L^2} - P)\times L
\]

\[
\text{where } k_{col} \text{ is the lateral stiffness of the steel column under axial load. Fig. 8.3 compares the calculation results using Eq. 8.1 with the numerical simulation results.}
\]

![Figure 8.3: Comparison of column elastic stiffness between using Eq. 8.1 and ABAQUS simulation results for the simply supported steel column UC 305×305×118](image)

For example, for a Chevrolet 1994 Pick-up vehicle impact on a simply supported steel column with a section size of UC 305×305×118 under an axial load ratio \(P/P_{\text{Design}}\) of 0.425, the equivalent dynamic impulse can be calculated as follows:

\[
F = v_r \sqrt{k_{col} \cdot M} = v_r \sqrt{22937710 \times 1840} = v_r \times 205439.5 \text{ kg/sec.}
\]

\[
\Delta t = \sqrt{\frac{M}{k_{col}}} = \sqrt{\frac{1840}{22937710}} = 0.00895 \text{ sec}
\]

Fig. 8.4 compares the dynamic impulse force-time relationships for the same case to illustrate the vastly different characteristics that may be obtained by using only the vehicle or the column stiffness value.
Figure 8.4: Comparison of idealised dynamic impulses by using only the vehicle stiffness or the column stiffness for column size UC 305×305×118 under impact by a Chevrolet 1994 Pick-up.

The dynamic impulse was applied on the steel column using the AMPLITUDE option in ABAQUS to determine the critical impact velocity to cause column failure. Figs. 8.5 and 8.6 compare the critical impact velocities from using the dynamic impulses and from using the vehicle impact simulations in chapter seven.
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Figure 8.5: Comparison of critical impact velocity-axial load curves between using dynamic impulse simulation (EC1) and vehicle simulation for steel column section UC 305×305×118: (A) simply supported column; (B) propped cantilever column
Figure 8.6: Comparison of critical impact velocity-axial load curves between using dynamic impulse simulation (EC1) and vehicle simulation for steel column section UC 254×254×89: (A) simply supported column; (B) propped cantilever column.

For the cases considered, the vehicle was more flexible than the columns. Therefore, a large proportion of the impact energy was absorbed by the vehicle. The results in Figs. 8.5 and 8.6 show that, by using the column stiffness only, meaning ignoring the energy absorbed by the vehicle, the critical impact velocities were underestimated significantly. If using the vehicle stiffness only, the impulse analysis method can give close results to the vehicle impact simulation method at higher axial load ratios, but overestimate the critical impact velocities at lower axial load ratios. This is mainly because it is not appropriate to use the vehicle elastic stiffness after the vehicle deformation has reached the engine box.
After the vehicle deformation has reached the vehicle engine position, the vehicle becomes rigid and cannot absorb any more impact energy. The following section will present a modification to take this into consideration.

### 8.3.1. A proposed modification

According to the assessment results presented in the previous sub-section, it is possible to use the impulse dynamic analysis method to simulate vehicle impact on columns, but two modifications should be made. These are explained below.

1. Both the column and the vehicle stiffness should be included when calculating the impact force. According to energy absorption:

\[
\frac{F^2}{2K_1} + \frac{F^2}{2k_{col.}} = \frac{v_r^2}{2} M
\]

2. The vehicle energy absorption value reaches the maximum when the vehicle deformation has reached the engine position. This gives:

\[
\frac{K_1 C^2_{\text{max}}}{2} + \frac{F^2}{2k_{col.}} = \frac{v_r^2}{2} M
\]

Where \( C_{\text{max}} \) is the maximum vehicle deformation before the engine box.

Based on these two modifications, the new impulse can be estimated as in the following procedure:

(i) The equivalent impact force is first calculated using the equation below (based on Eq. 8.2):

\[
F = v_r \sqrt{\frac{M \cdot K_1 \cdot k_{col.}}{K_1 + k_{col.}}}
\]

(ii) This gives the impulse duration as:

\[
\Delta t = \sqrt{\frac{M \cdot (K_1 + k_{col.})}{K_1 \cdot k_{col.}}}
\]

(iii) The vehicle deformation, \( C \), can be calculated as follows:
$C = \frac{F}{K_i}$

Based on the calculated vehicle deformation value, two cases may be considered:

**A.** $C \leq C_{\text{max}}$

This represents vehicle deformation before the column contacts the engine. In this case, the impulse is given by the $F$ and $\Delta t$ values as calculated in Eqs. 8.4 and 8.5.

**B.** $C > C_{\text{max}}$

This represents vehicle deformation after the column contacts the engine. In this case, the equivalent impact force is calculated using the following equation (based on Eq. 8.3):

$$F = \sqrt{(v_r^2 \cdot M - K_1 \cdot C_{\text{max}}^2) \cdot k_{\text{col}}}$$ .................................................. 8.6

Hence, the impulse duration can be calculated from:

$$\Delta t = \frac{M \cdot v_r}{F}$$ .................................................. 8.7

Fig. 8.7 compares the equivalent impact force-velocity relationships determined using Eq. 2.2 (using either vehicle or column stiffness) and using Eqs. 8.4 and 8.6. In these calculations, the vehicle stiffness was 510kN/m and the column stiffness was 22937710N/m for the column with section UC 305×305×118 under an axial load ratio ($P/P_{\text{Design}}$) of 0.425.

![Figure 8.7: Comparison of the equivalent impact force-impact velocity curves between using column stiffness only, vehicle stiffness only and combined stiffness for a simply supported steel column section UC 305×305×118.](image)
Figs. 8.5 and 8.6 compare the different critical impact velocity - axial load curves obtained from the dynamic analyses using the modified impulse calculation method with using either the vehicle stiffness only or the column stiffness only, and with ABAQUS vehicle impact analysis.

Because the elastic column stiffness was many times (>30) greater than the vehicle stiffness, the modified impulse was almost the same as that calculated using the vehicle stiffness only. Hence, the results in Figs. 8.5 and 8.6 show that when column failure occurred before the vehicle deformation reached the engine position, the proposed modification for calculating the impulse gave almost identical results compared with using the vehicle stiffness only and much better results when compared with using the column stiffness only.

However, when column failure occurred after the column had made contact with the vehicle engine and the vehicle should be treated as a rigid body, continuing to use the vehicle elastic stiffness only for calculating the impulse would mean continued energy absorption by the vehicle. This would result in overestimation (unsafe) of the critical impact velocity. Using the modified impulse correctly limited the energy absorption by the vehicle and produced critical impact velocities that are reasonably close to, and provide a safe estimation of, the ABAQUS vehicle impact simulation results.

Figs. 8.5 and 8.6 show that the impulse analysis results are not sensitive to the axial load ratio, particularly when the axial load ratio is low. This is because the impact force was high and the analysis results were not sensitive to the time duration of the impulse (Thilakarathna et al., 2010). Nevertheless, using the proposed modification for impulse made a huge improvement to the critical velocity results compared with using only either the vehicle or column stiffness.

In summary, using the proposed method of calculating the dynamic impulse eliminated the gross errors of using either the column stiffness only or the vehicle stiffness only. Although using the impulse-based dynamic analysis still resulted in some errors, particularly when the applied axial loads were low, the dynamic analysis results may be considered acceptable.
8.4. Summary

This chapter has presented a detailed assessment of the design requirements suggested by Eurocode 1 regarding the design of steel columns to resist vehicle impact. Both the static and dynamic approaches in Eurocode 1 have been evaluated. From the detailed comparisons between the design results and the ABAQUS impact simulation results, the following conclusions may be drawn:

(1) the equivalent static design forces may be considered acceptable for column design if the column section sizes are moderate, as used in typical multi-storey buildings of no more than 10 storeys. If the column sizes are greater, using the Eurocode 1 equivalent static forces will overestimate the axial compression resistance of the columns especially when the columns are used in structures located in rural areas or near national roads with high vehicle speeds. However, if the vehicle velocities are low (<50 km/h), the Eurocode 1 equivalent static forces may still be used.

(2) Using a dynamic impulse to represent the dynamic action of vehicle impact is a reasonable approximation. However, when calculating the force, it is not appropriate to use only either the column elastic stiffness or the vehicle elastic stiffness, as recommended by Eurocode 1. It is necessary to include both the column and vehicle stiffness values. Furthermore, vehicle behaviour should be divided into two stages, before the column is in contact with the vehicle engine and after contact. After the column is in contact with the vehicle engine, the energy absorbed by the vehicle should be limited by the maximum deformation of the vehicle. In this chapter, a modification has been suggested for calculating the dynamic impulse of vehicle impact. Using this modified dynamic impulse in dynamic analysis was able to eliminate the gross errors caused by using only the vehicle stiffness or the column stiffness and produced results of critical vehicle velocity that are close to the ABAQUS vehicle impact simulation results.
Chapter Nine

Conclusions and Recommendations for Future Research Studies

9.1. Introduction

This thesis has presented, in detail, the work undertaken by the author in a PhD research programme to investigate the behaviour of axially compressed steel columns under vehicle impact. The ultimate aim of the research was to gain a thorough understanding of the aforementioned problem and to develop an efficient, accurate and robust design approach. This research included the following research work packages: numerical modelling using ABAQUS/Explicit, a parametric study of steel column behaviour under transverse impact, development of a simplified vehicle model, development of an analytical method for column response under transverse impact and an assessment of the current design provisions.

This chapter will summarise the main findings of the study in each of these packages and provides recommendations for future research studies to improve or extend the current work.

9.2. Conclusions of this research

The following conclusions can be extracted from the following research tasks:

9.2.1. Numerical modelling of steel column behaviour under transverse impact

a. The results presented in this thesis can be considered to have provided an extensive body of evidence that the finite element code ABAQUS/Explicit is capable of modelling axially loaded steel columns under transverse impact provided that the associated geometrical, material and contact modelling parameters are selected and implemented correctly as presented in this study. Based on the validation results presented in this thesis, using first order (linear) three dimensional solid or shell
elements with reduced integration and hourglass control is suitable. An isotropic classical metal plasticity model in connection with the progressive damage and failure model available in ABAQUS/Explicit can be used to simulate the behavior and all possible failure modes of the steel columns subjected to transverse impact. Contact interaction between the impacting body and the steel column can be simulated using either the contact pair algorithm or the general contact algorithm available in ABAQUS/Explicit, depending on the type of surfaces involved in the contact, with hard and penalty friction formulations to describe the mechanical properties in the normal and tangential directions respectively.

b. It has been proven numerically that damping has only a minor effect on the response and contact force of pre-compressed columns subjected to transverse impact load.

9.2.2. Behaviour and failure modes of steel columns under transverse impact

a. Global buckling is the predominant failure mode for axially unrestrained compressed steel columns under transverse impact. Some column failure may involve large local flange distortion at, and around, the impact area. However, detailed inspections of the column behaviour at failure simulated in this study have revealed that the flange local distortion is a result, not the cause, of column global failure.

b. The value of the kinetic energy of the impact is the key factor in determining the column’s global failure. At the same impact kinetic energy, different values of impacting mass and velocity have a minor effect on column failure.

c. Except at very low levels of axial compressive loads (<25% design resistance), the formation of the plastic hinge within the column length is almost independent of the impact position, with the plastic hinge location being close to the centre of the column.

d. The most critical direction of impact is that which causes bending of the column about the minor axis.

e. Damping can be neglected when calculating the critical impact velocity because of its minor effects on column behaviour and failure.
f. Both strain hardening and strain rate have beneficial effects on column critical impact velocity. While it is important to include the effect of strain hardening in the development of a simplified method, the effect of strain rate can be discarded safely because of the relatively low influence of this effect.

9.2.3. Simplified vehicle model

a. It is not appropriate to assume vehicles as rigid impactors when studying the behaviour of columns under vehicle impact.

b. The impacting vehicle can be simplified as a spring mass system with the linear spring representing the stiffness characteristics of the vehicle. The spring force-deformation relationship is assumed to be bilinear, with the first part representing the vehicle deformation behaviour up to the engine box and the second part representing the stiffness of the engine box, which can be assumed to be rigid.

c. The current study has presented a method to obtain the stiffness value of a vehicle before reaching the engine box. This method is based on using the method originally suggested by Campbell to obtain the vehicle force-deformation relationship per unit width and integrating this force-deformation relationship over the deformation profile of the vehicle after impact on a column with a finite width. Comparison between the vehicle stiffness values derived in such a way and those extracted from the corresponding numerical simulations indicates that the difference is less than 25% for different column section sizes.

9.2.4. Development of an analytical method for column behaviour under transverse impact.

a. A quasi-static response for an impacted steel column can be assumed when conducting an energy balance analysis of the vehicle-column system. The energy absorbed by the column is by elastic deformation and plastic hinge rotation at the plastic hinge locations. It is important to include the work undertaken by the axial force in the column due to column shortening when undergoing lateral deformation. Column global plastic buckling occurs when the column plastic hinge mechanism loses equilibrium under an axial compressive force in a deformed state. The final column equilibrium position determines the maximum plastic hinge rotations.
b. To calculate the energy absorbed by the vehicle, the maximum deformation of the vehicle frontal structure is determined based on the maximum transverse static resistance of the column. The Eurocode 3 method to obtain the column transverse static resistance under the influence of an axial load is overly conservative. An alternative column transverse static resistance – axial force relationship has been proposed.

c. To include the effect of strain hardening, the average of the steel yield stress and ultimate tensile stress can be approximately used in the elastic-perfectly-plastic representation of the steel stress-strain relationship.

**9.2.5. Assessment of current design methods in Eurocode 1**

a. The equivalent static design force approach can be used in the design of steel columns with moderate sizes which are typically used in low multi-storey buildings of no more than 10 storeys. For bigger sizes of columns, this study has shown that it is unsafe to use the Eurocode 1 equivalent static forces especially when the columns are used in structures located in rural areas or near national roads when the vehicle velocity exceeds 80 km/h.

b. It is acceptable to use dynamic impulse to represent the dynamic action of vehicle impact in a dynamic analysis. However, it is not appropriate to use only either the column elastic stiffness or the vehicle elastic stiffness to calculate the equivalent impulse force as recommended by Eurocode 1. Instead, it is necessary to include both the column and vehicle stiffness values with the vehicle behaviour divided into two stages: before, and after, the column is in contact with the vehicle engine.

**9.3. Recommendations for future research works**

Since this research is amongst the first in the numerical and analytical studies of the behaviour of axially loaded steel columns subjected to vehicle impact, it was not possible to cover all aspects of the problem. A number of further research studies can be pursued to improve the knowledge and understanding in this area:

1. The present study has focused on the behaviour of H section steel columns under vehicle impact. The behaviour of other section shapes should be investigated. In addition, the study can also be developed to consider other types of columns including
reinforced concrete columns, pre-stressed concrete columns and concrete filled steel columns.

2. The numerical simulations carried out in this study have utilized solid elements to model the steel column. Although using solid elements gives more accurate results, it requires a large amount of computation time. Using line (beam) elements will be much more efficient, particularly as a tool for practical design. It would, however, be necessary to check the accuracy of using more simplified simulation methods.

3. The current study has focused on vehicle impact in the direction that would cause column bending about the weak axis of the column, this being considered as the most critical direction of impact. Further research studies could investigate in more detail the behaviour of column behaviour under impact in other directions.

4. In this study, the steel column is subjected to pure axial compression without any bending moment. The effects of combined axial compression and bending moment should be investigated.

5. In the present study, the effect of vehicle impact on the behaviour and failure modes of steel columns has been studied by investigating the impact behaviour of an isolated column. Since the context of this research is structural robustness under extreme loading, further research studies should be carried out to investigate the effect of whole structures.

6. Because of its availability, the presented study has used the numerical model of a Chevrolet C2500 1994 Pick-up vehicle to validate the proposed simplified numerical vehicle model. Further research studies can be carried out to investigate whether the proposed simplified vehicle model can be applied to other types of vehicles.

7. In the simplified analytical method, the moment-rotation curve of the plastic hinge formed in the column is assumed to be elastic-perfectly plastic. For refinement of the analytical model, more realistic moment-rotation curves (considering elastic-plastic behaviour and strain hardening behaviour) should be investigated.

8. The analytical model has been developed based on the assumption of column failure by global buckling. The effects of energy absorption in the column through local
deformations in the impact zone should be investigated. In addition, the energy absorbed by shear deformation in the column may have some effect, particularly when the impact location is very close to the column base. This should be researched.

9. It is expected that the conclusions of this research can be applied to columns under impact from other sources, such as cranes, rocks and flying debris. Further research studies may be undertaken to assess the applicability of the proposed method of analysis in such cases.
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References


Appendix A

Publications

The following papers were extracted from this study:

A. Published papers:


B. Submitted papers:

