LINK ENHANCEMENT
TECHNIQUES FOR FUTURE
MULTICARRIER SYSTEMS

A thesis submitted to the University of Manchester
for the degree of Doctor of Philosophy
in the Faculty of Engineering and Physical Sciences

2012

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School of Electrical and Electronic Engineering
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Abstract

Orthogonal Frequency Division Multiplexing (OFDM) is very effective in combating the distortive effects of wireless channel and promises high data rate capabilities with reasonable complexity and accuracy. Other advantages of OFDM include significantly simple equalizer requirement, high spectral efficiency, computationally inexpensive implementation, increased immunity to impulse noise, ability to support adaptive modulation schemes and high flexibility in resource allocation.

This thesis investigates two vital issues regarding the OFDM system design requirements, namely, Channel Estimation (CE) and Carrier Frequency Offset Estimation (CFOE). The accuracy of these two estimators plays a crucial role in the overall performance of OFDM systems. Whether it is a single antenna or a multi-antenna OFDM system, accurate channel estimation (CE) is required for coherent reception. The channel estimation requirement is further exacerbated in the case of OFDM systems with multiple transmit and/or receive antennas as the signals are simultaneously transmitted / received and consequently arrive at the receiver through many channels. CE techniques for OFDM systems are broadly classified into pilot-aided and blind techniques. As compared to blind algorithms, pilot-aided CE algorithms are more robust to high Doppler frequency, and hence, are useful for high mobility applications.

One of the major drawbacks of OFDM is its sensitivity to time and frequency synchronization errors. Owing to its inherent cyclic symbol structure, the time synchronization requirements are somewhat relaxed for OFDM systems. Conversely, the frequency synchronization requirements are more stringent because of its tightly packed subcarriers. The frequency offset results in loss of orthogonality of subcarriers which subsequently causes significant performance degradation. Therefore, it is imperative to estimate the CFO and thereafter eliminate or minimize its effects.

This thesis proposes a new set of techniques for pilot-aided CE namely “undersampled channel estimation” for OFDM systems. In such techniques, the number of pilots used to sample the channel are less than those allowed by Nyquist sampling theorem. Virtually blind (VB) CE uses only one pilot to estimate the channel frequency response (CFR). The performance of VB CE is hindered by the occurrence of CFR inversion (CFRI). Uniformly spaced fixed additional pilots and dynamically assigned additional pilots were then augmented with the only
pilot in order to take more samples of channel and to stop propagating CFRI
effect further. For joint blind channel and control signal estimation for OFDM
systems, the detectability of control information dependent (CID) pilot sequences
is highly dependent on the type of sequences used. An algorithm to design a new
set of pilot sequences with better detectability is proposed in this thesis.
Declaration

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Acknowledgements

I would like to express my gratitude to my supervisor Dr. Emad Alsusa for his continual advice and encouragement throughout this project. He has been a source of constant support and encouragement, mentoring and friendship.

I wish to express my thanks to all my postgraduate colleagues especially, Mr. Ahsan Ali, Mr. Imran Rashid, Mr. Sarmad Sohaib, Mr. Junaid Ahmed, Mr. Rashid Saleem and Mr. Inam-ul-Hassan Sheikh for their support, motivation and fruitful discussions.

I am always indebted to my parents and my sisters for their selfless support and prayers. Special thanks go to my wife and my sons, Ahmed and Saad.

The University of Engineering and Technology, Taxila, Pakistan and Higher Education Commission, Pakistan are acknowledged for their support and funding for this project.
Dedication

To my family and my teachers.
### List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>3GPP</td>
<td>Third Generation Partnership Project</td>
</tr>
<tr>
<td>ADC</td>
<td>Analog-to-Digital Converter</td>
</tr>
<tr>
<td>ADSL</td>
<td>Asymmetric Digital Subscriber Line</td>
</tr>
<tr>
<td>AMPS</td>
<td>Advanced Mobile Phone Services</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
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<tr>
<td>CE</td>
<td>Channel Estimation</td>
</tr>
<tr>
<td>CFO</td>
<td>Carrier Frequency Offset</td>
</tr>
<tr>
<td>CFOE</td>
<td>Carrier Frequency Offset Estimation</td>
</tr>
<tr>
<td>CFR</td>
<td>Channel Frequency Response</td>
</tr>
<tr>
<td>CID</td>
<td>Control Information Dependent</td>
</tr>
<tr>
<td>CIR</td>
<td>Channel Impulse Response</td>
</tr>
<tr>
<td>CP</td>
<td>Cyclic prefix</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>DAB</td>
<td>Digital Audio Broadcasting</td>
</tr>
<tr>
<td>DAC</td>
<td>Digital-to-Analog Converter</td>
</tr>
<tr>
<td>dB</td>
<td>Decibels</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>DVB</td>
<td>Digital Video Broadcasting</td>
</tr>
<tr>
<td>DVB-T</td>
<td>DVB standard for Terrestrial broadcasting</td>
</tr>
<tr>
<td>EDGE</td>
<td>Enhanced Data Rates for GSM Evolution</td>
</tr>
<tr>
<td>EM</td>
<td>Electromagnetic</td>
</tr>
<tr>
<td>ETSI</td>
<td>European Telecommunication Standard Institute</td>
</tr>
<tr>
<td>EV-DO</td>
<td>Evolution for Data-Only Systems</td>
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<tr>
<td>Abbreviation</td>
<td>Full Form</td>
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<tr>
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<tr>
<td>EV-DV</td>
<td>Evolution for Data and Voice Systems</td>
</tr>
<tr>
<td>FDMA</td>
<td>Frequency Division Multiple Access</td>
</tr>
<tr>
<td>FEC</td>
<td>Forward Error Correction</td>
</tr>
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<td>FFT</td>
<td>Fast Fourier Transform</td>
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<tr>
<td>FOMA</td>
<td>Freedom of Mobile Multimedia Access</td>
</tr>
<tr>
<td>FSPL</td>
<td>Free Space Path Loss</td>
</tr>
<tr>
<td>GI</td>
<td>Guard Interval</td>
</tr>
<tr>
<td>GPRS</td>
<td>General Packet Radio Service</td>
</tr>
<tr>
<td>GSM</td>
<td>Global System for Mobile Communications</td>
</tr>
<tr>
<td>HIPERLAN</td>
<td>High Performance Radio LAN</td>
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<tr>
<td>HIPERMAN</td>
<td>High Performance Radio MAN</td>
</tr>
<tr>
<td>IDFT</td>
<td>Inverse Discrete Fourier Transform</td>
</tr>
<tr>
<td>ICI</td>
<td>Inter-Channel Interference</td>
</tr>
<tr>
<td>IFFT</td>
<td>Inverse Fast Fourier Transform</td>
</tr>
<tr>
<td>iid</td>
<td>Independent, Identically Distributed (Random Variable)</td>
</tr>
<tr>
<td>IMT-2000</td>
<td>International Mobile Telecommunications - 2000</td>
</tr>
<tr>
<td>IPP</td>
<td>Independent Pilot Pattern</td>
</tr>
<tr>
<td>IS-95</td>
<td>Interim Standard 95</td>
</tr>
<tr>
<td>ISI</td>
<td>Inter symbol interference</td>
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<tr>
<td>ITU-R</td>
<td>International Telecommunication Union - Radiocommunication Sector</td>
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<tr>
<td>LNA</td>
<td>Low-Noise Amplifier</td>
</tr>
<tr>
<td>LO</td>
<td>Local Oscillator</td>
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<tr>
<td>LOS</td>
<td>Line-of-Sight</td>
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<td>LSE</td>
<td>Least Squares Estimator</td>
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<tr>
<td>LPF</td>
<td>Low-Pass Filter</td>
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<tr>
<td>LTE</td>
<td>Long Term Evolution</td>
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<tr>
<td>MC</td>
<td>Multicarrier</td>
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<tr>
<td>MC-CDMA</td>
<td>Multi-Carrier Code Division Multiple Access</td>
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<tr>
<td>MIMO</td>
<td>Multiple-Input Multiple-Output</td>
</tr>
<tr>
<td>ML</td>
<td>Maximum Likelihood</td>
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<td>MMSE</td>
<td>Minimum mean square error</td>
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<td>Description</td>
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<tr>
<td>MSE</td>
<td>Mean Square Error</td>
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<td>MU</td>
<td>Mobile Unit</td>
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<td>N-LOS</td>
<td>Non-Line-of-Sight</td>
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<td>NMT</td>
<td>Nordic Mobile Telephone</td>
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<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
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<tr>
<td>OFDMA</td>
<td>Orthogonal Frequency Division Multiple Access</td>
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<tr>
<td>OPP</td>
<td>Orthogonal Pilot Pattern</td>
</tr>
<tr>
<td>PA</td>
<td>Power Amplifier</td>
</tr>
<tr>
<td>PAPR</td>
<td>Peak-to-Average Power Ratio</td>
</tr>
<tr>
<td>PDC</td>
<td>Personal Digital Cellular</td>
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<tr>
<td>PDF</td>
<td>Probability Density Function</td>
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<tr>
<td>PDP</td>
<td>Power Delay Profile</td>
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<tr>
<td>PER</td>
<td>Pilot Sequence Error Rate</td>
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<tr>
<td>PPM</td>
<td>Parts per Million</td>
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<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
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<tr>
<td>QPSK</td>
<td>Quadratic Phase Shift Keying</td>
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<td>RF</td>
<td>Radio Frequency</td>
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<td>RMS</td>
<td>Root Mean Square</td>
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<td>SC(s)</td>
<td>Subcarriers(s)</td>
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<td>SCM</td>
<td>Single Carrier Modulation</td>
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<td>SER</td>
<td>Symbol Error Rate</td>
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<td>SMS</td>
<td>Short Message Service</td>
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<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
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<td>SPP</td>
<td>Scattered Pilot Pattern</td>
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<td>STFT</td>
<td>Short-Time Fourier Transform</td>
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<td>TACS</td>
<td>Total Access Communications Systems</td>
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<td>TDMA</td>
<td>Time Division Multiple Access</td>
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<td>UMTS</td>
<td>Universal Mobile Telecommunication Systems</td>
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<td>VB</td>
<td>Virtually Blind</td>
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<tr>
<td>WCDMA</td>
<td>Wideband Code Division Multiple Access</td>
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<td>WiMAX</td>
<td>Worldwide Inter-operability for Microwave Access</td>
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<td>ZF</td>
<td>Zero Forcing</td>
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List of Variables

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<td>Peak Amplitude of LOS</td>
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<td>BC</td>
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<td>d</td>
<td>Distance for Path Loss</td>
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<td>do</td>
<td>Reference Distance for Path Loss</td>
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<td>fd</td>
<td>Maximum Doppler Shift</td>
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<td>Subcarrier Spacing</td>
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<td>fs</td>
<td>Sampling Frequency</td>
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<td>Gr</td>
<td>Receive Antenna Gain</td>
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<td>Channel Frequency Response</td>
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<td>Hn</td>
<td>Estimated Channel Frequency Response</td>
</tr>
<tr>
<td>H[k_p]</td>
<td>Initial Channel Estimates at Pilot Locations only</td>
</tr>
<tr>
<td>H</td>
<td>Channel Mixing Matrix for MIMO Channel</td>
</tr>
<tr>
<td>k</td>
<td>Ricean Factor</td>
</tr>
<tr>
<td>lg</td>
<td>Length of Guard Interval</td>
</tr>
<tr>
<td>Mr</td>
<td>Number of Receive Antennas</td>
</tr>
<tr>
<td>Mt</td>
<td>Number of Transmit Antennas</td>
</tr>
<tr>
<td>Nc</td>
<td>Total Number of Subcarriers in an OFDM Symbol</td>
</tr>
<tr>
<td>Nh</td>
<td>Length of Multipath Dispersive Channel</td>
</tr>
<tr>
<td>No</td>
<td>Power Spectral Density of AWGN</td>
</tr>
<tr>
<td>Np</td>
<td>Number of Pilot Subcarriers in an OFDM Symbol</td>
</tr>
<tr>
<td>PL</td>
<td>Path Loss</td>
</tr>
</tbody>
</table>
\[ PL_o \] Path Loss at reference distance \( d_o \)
\[ P_r \] Received Power
\[ P_t \] Transmit power
\[ T_C \] Coherence Time
\[ T_g \] CP Duration
\[ T_s \] Duration of OFDM Symbol
\[ \hat{T}_s \] Duration of OFDM Symbol with CP
\[ v \] Path loss exponent
\[ X \] Transmitted Signal
\[ Y \] Received Signal
\[ \epsilon \] Normalized Carrier Frequency Offset
\[ \delta \] Timing Offset
\[ \lambda \] Wavelength
\[ \sigma_d \] RMS Delay Spread of CIR
\[ \langle \tau \rangle \] Average Delay of CIR
\[ \tau_{max} \] Duration of CIR
List of Mathematical Notations

$(\cdot)^H$ Hermitian Transpose of a matrix
$(\cdot)^T$ Transpose of a matrix
$(\cdot)^\dagger$ Pseudo-Inverse of a matrix
$(\cdot)^*$ Complex Conjugate
$\det(\cdot)$ Determinant of a matrix
diag$(\cdot)$ A vector containing diagonal elements of argument matrix
$\mathbb{E}(\cdot)$ Expectation operator of a random variable
$f(\cdot)$ Probability Density Function
$I_0$ Modified Bessel Function of the First Kind and Zeroth Order
$I_M$ $M \times M$ Identity matrix
$\Im$ Imaginary Part of a Complex Number (or Signal)
$\log_{10}(\cdot)$ Common (Base-10) Logarithm
$\log_e(\cdot)$ Natural Logarithm
$\Re$ Real Part of a Complex Number (or Signal)
$\mathbb{Z}^+$ Set of Positive Integers
$\|\cdot\|$ Euclidean Norm
$\approx$ Approximately equal to
$\cdot \ast$ Element-by-element multiplication
$\ast$ Convolution
$\circledast$ Circular convolution
$\angle$ Angle or Argument of a complex number
$\lceil \cdot \rceil$ Ceiling operator
$\langle, \cdot \rangle$ Right-Open Interval
$\triangleq$ Defined as
Chapter 1

Introduction

Wireless communication is a broad, dynamic and fastest growing sector of communication industry. Its influence is all-pervasive and has changed the lifestyle and working habits of people. The exponential increase in the number of cellular subscribers, proliferation of hand-held computers and wireless networks emphasize the importance of wireless communication [1]. The aim of this chapter is to provide an overview of wireless communications, including its history and current trends.

1.1 History of Wireless Communication Systems

The evolution of the wireless communication systems actually started with the electromagnetic (EM) waves theory formulated by James Clark Maxwell in 1873. Heinrich Hertz then demonstrated the existence of EM waves for the first time in 1887. Soon after the groundbreaking work of Maxwell and Hertz, Nikola Tesla showed the transmission of information through EM waves. In 1898, Guglielmo Marconi performed first well-publicized experiment of wireless communication from a boat in the English Channel to the Isle of Wight [2]. Since then wireless communication became widespread throughout the world. Inventions like radio, TV and mobile phones etc could only be possible due to the success in wireless
technology.

Almost every ten years, the world witnesses significant changes in the telecommunication industry. The first generation (1G) systems introduced in the 1980s were based on analog transmission technology and were focused primarily on voice transmission. The examples of 1G systems include NMT, AMPS and TACS etc. The second generation (2G) systems introduced in the 1990s (e.g., GSM, PDC, and IS-95 a.k.a. cdmaOne) were based on digital radio technologies. The 2G systems offered higher network capacity as well as lower cost and better performance of digital hardware than analog circuitry. In addition to voice, the 2G systems introduced low-rate data services like e-mail and short messaging upto 14.4 Kbps [2]. The third generation (3G / IMT-2000) systems were also based on digital technologies and were characterized by internet connectivity and multimedia applications. The first 3G network was introduced in 2001 in Japan under the name of FOMA. Major 3G standards include CDMA2000, UMTS, and EDGE etc. The 3G systems offered data rates of 144 - 384 Kbps for fast moving users and up to 2 Mbps for stationary or slowly moving users [2–4].

With the advent of numerous internet-based multimedia applications and proliferation of hand-held computers and smart phones, the demand for high-rate wireless communication services still continued to grow. Therefore, 3GPP Long Term Evolution (LTE) standard with significantly higher data rates than in 3G systems was evolved. In order to achieve higher data rates, 3GPP LTE specifications defined a new physical air interface [4]. The key difference of LTE from its predecessors (WCDMA and HSPA) is the usage of OFDM at the downlink and single-carrier FDMA at the uplink to conserve power. The original target data rates for LTE were 100 Mbps in the downlink and 50 Mbps in the uplink. In order to achieve such higher data rates, LTE makes use of enhanced channel-dependent scheduling and rate adaptation, spatial multiplexing with multiple-input multiple-output (MIMO) antennas and large channel bandwidths of upto 20 MHz [4]. Peak data rates actually supported by LTE are higher than the
CHAPTER 1. INTRODUCTION

IMT-Advanced or fourth generation (4G) of wireless systems is the complete evolution expected in wireless technology. According to ITU-R requirements, 4G is all IP-based solution capable of providing voice, data and streamed multimedia to subscribers on an “Anytime, Anywhere” basis. The new physical air interfaces is aimed at providing data rates of upto 100 Mbps for vehicular subscribers and upto 1Gbps for pedestrian subscribers [4].

1.2 4G, OFDM and MIMO

Long term evolution – commonly termed as “4G” – is the step towards future wireless technologies with the aim of higher performance at reduce cost for radio access. At the backbone of LTE are OFDM and MIMO, which are the key technologies responsible for providing higher performance [5, 6].

Wireless transmission is primarily impaired by multi-path fading. Additional constraints of limited power and scarce bandwidth make the task of designing fast wireless systems further demanding [7]. OFDM transforms frequency selective channel into a set of parallel flat fading channels and consequently is robust against multipath channel distortions and narrowband interference. Additional advantages of OFDM include higher spectral efficiency, ability to support adaptive modulation schemes and high flexibility in resource allocation [2, 4].

MIMO is well known for capacity enhancement and increased coverage range of the wireless communication systems. However, at high data rate communication, MIMO multipath channels are frequency selective and hence complex equalization is required. MIMO-OFDM combination offers solutions to the problems of constrained resources and multipath fading. OFDM combined with MIMO provides high spectral efficiency and increased data throughput, and has become a most promising broadband wireless access scheme [8].

Next generation cellular systems, as well as WLAN and broadcasting standard
are all based on MIMO-OFDM. For example, the physical layer of 3GPP-LTE, Mobile WiMAX, IMT-Advanced, IEEE 802.11a, IEEE 802.11n, DAB and DVB are based on MIMO-OFDM [4,9].

1.3 Motivation

Most of the current wireless standards offering high data rates are based on OFDM. OFDM transforms frequency selective channel into a set of parallel flat fading channels and consequently the wireless channel distorts the amplitude and phase of each subcarrier individually and this distortion effect can be represented by a single complex-valued coefficient. For coherent detection of transmitted information, this multiplicative distortion needs to be compensated for. This compensation process is called channel equalization and requires the estimates of channel impulse response [2]. Although differential PSK could be used in OFDM systems without requiring channel estimates, it limits the number of bits per symbol and incurs about 3 dB loss in SNR as compared to coherent detection [10,11]. Moreover, new standards are based on higher order QAM modulation schemes and thus the channel estimation becomes indispensable for OFDM-based systems.

Channel estimation (CE) is a process of characterizing the effect of the channel on the transmitted data and allows the receiver to approximate how the channel distorted the input data [12]. CE is essential for channel equalization as well as for diversity combining and spatial interference suppression in MIMO systems [13]. CE can be done either by exploiting known training symbols (pilot-aided CE) or by exploiting the inherent redundancy of OFDM signal (blind CE). Blind CE techniques save the bandwidth needed for pilots but generally require iterations and hence introduce high latency; and generally offer poor performance. Pilot-aided CE methods are bandwidth-inefficient but offer low latency and better performance, especially for time varying channels where CE has to be performed
periodically or even continuously. Therefore, a good CE algorithm requiring least number of pilot symbols is highly desirable.

In many cases, multicarrier systems require some side information (SI) to work properly. In such transmission schemes, SI is mandatory as it informs the receiver about what has been done with transmitted signal in the transmitter. Examples of cases requiring side information include various popular peak-to-average power ratio reduction techniques (SLM, PTS etc) and adaptive beamforming algorithms. Side information is the overhead data which needs to be conveyed to receiver. The best choice for SI transmission is to embed it within the pilot tones intended for channel estimation [14]. Therefore, the pilots perform a dual function – they carry the side information and simultaneously “sample” the channel transfer function. Since no extra pilots are used, this techniques conserves bandwidth and transmission power. The performance of the correlation-based receiver proposed in [14] has shown to be highly dependent on pilot sequence type. Therefore, a suitable pilot sequence needs to be investigated which works optimally with the receiver in order to yield best performance.

To fully reap the benefits of OFDM system, the orthogonality among the various subcarriers must be maintained. However, misalignment between transmitter and receiver RF local oscillators (LOs) and/or Doppler spread caused by the relative motion of transmitter and receiver induces carrier frequency offset (CFO). This CFO in OFDM systems causes loss of orthogonality among various subcarriers which subsequently results in significant performance degradation. Therefore, it is imperative to estimate CFO and hence thereafter compensate for.

1.4 Contribution

The contribution of this thesis can be summarized as follow.

- A new method of channel estimation for OFDM systems namely “Under-sampled channel estimation” is proposed. This technique aims at providing
good performance while requiring least number of pilot subcarriers. In its most primitive form this technique uses only one pilot symbol and is referred to as “Virtually-Blind (VB) CE” scheme.

- The problem of channel frequency response inversion (CFRI) inherently related to the VB CE is analyzed. The reasons behind CFRI are investigated and an exact mathematical expression for noise-free case is derived. STFT-based and 2nd Derivative-based two solutions for locating CFRI are also proposed.

- The viability of under-sampled CE methods is also verified for MIMO-OFDM case.

- A new sequence with enhanced detectability is proposed along with its design algorithm. Such sequences are found to be useful for joint estimation of side and channel information.

### 1.5 Thesis Organization

This thesis addresses the issues related to channel estimation methods for OFDM and MIMO systems, carrier frequency offset estimation and joint estimation of side and channel information for OFDM systems. The rest of the thesis is organized as follow.

Chapter 2 provides an overview of some fundamental building blocks of future communication systems. Fading in wireless environment and basic principles of OFDM are explained to some extent. Some critical issues regarding OFDM including digital implementation of OFDM systems, cyclic prefix, timing and frequency synchronization, and peak-to-average power ratio (PAPR) are discussed in detail. The need and benefits of MIMO wireless channels are also discussed in this chapter. Chapter 3 discuss origin and effects of CFO on OFDM systems. Different time domain and frequency domain methods for CFO estimation are
analyzed. The chapter concludes with simulation results.

Chapter 4 is about training-based channel estimation. It starts with some basic pilot structures and interpolation techniques. Next, popular 1D CE algorithms including LS, MMSE and ML are discussed. Then three important pilot schemes for MIMO-OFDM systems are presented and CE algorithms using those pilot schemes are discussed as well. The chapter concludes with simulation results for SISO- and MIMO-OFDM systems. Chapter 5 proposes new techniques for CE. Under-sampled CE methods aim at providing good performance while utilizing least number of pilot subcarriers. In its most primitive form – known as virtually blind CE – this technique uses only one pilot symbol but suffers from CFRI. The chapter then investigates the reason behind CFRI and proposes a couple of solutions. Uniformly spaced and dynamically assigned additional pilot methods are then discussed. The viability of under-sampled CE methods for MIMO-OFDM systems is also verified. The chapter concludes with simulation results.

Chapter 6 is about joint estimation of control and channel information. It explains why control information is necessary in certain circumstances and how it can be transmitted. Different pilot placement methods are analyzed. The performance of this technique is highly dependent on type of pilot sequence used. Then an algorithm to generate pilot sequences with enhanced detectability is proposed in this chapter. Chapter 7 concludes the thesis and suggests some directions for future research.

1.6 List of Publications


Technique”, IET Electronics Letters (Submitted).


Chapter 2

Fundamentals Of Wireless Communication

2.1 Introduction

The performance of a wireless communication system is highly dependent on wireless channel environment and its characteristics are quite different from those of wired channel. The physical layer of many advanced systems is based on OFDM. Exploiting multiple antennas at the receiver and transmitter provides higher data rates and longer range without additional power. The objective of this chapter is to present the basics of wireless communication channel, OFDM and MIMO systems.

2.2 Characteristics of Wireless Channel

The characteristics of wireless channel are quite different from those of wired channel. Unlike wired channel, the mobile radio channel is strongly time-varying and unpredictable. When there is no hindrance in between transmitter (TX) and receiver (RX), the propagation mechanism is commonly known as “line of sight” (LOS) propagation. However, in urban environments, the path between
TX and RX is usually blocked by many obstacles and the signal reaches the RX through the processes of reflection, diffraction, and scattering from obstacles in the path. This indirect signal transmission is an example of “non-line-of-sight” (NLOS) radio propagation [15].

Reflection occurs when electromagnetic wave impinges on a smooth object with dimensions much larger than its wavelength e.g., surface of earth and walls of a building [15, 16]. It causes the transmit signal power to be reflected back towards its origin. When reflection happens, the wave may be partially refracted as well. The coefficients of reflection and refraction depend on material properties, wave polarization, angle of incidence and frequency of the EM wave [16, 17]. Diffraction is the phenomenon by which EM waves bend or deviate in the vicinity of an obstruction with sharp edges. The resulting secondary waves are present throughout the space, even behind the obstruction. The diffraction is much more pronounced when the wavelength of EM wave is of the order of the diffracting objects. Moreover, low frequency signals diffract more significantly than higher frequency signals [18]. The main sources of diffraction include irregular terrain and building edges etc [1]. Scattering occurs when EM signal strikes a rough surface or a very small object as compared to the signal’s wavelength, and where the number of obstacles per unit volume is large [16]. Scattering causes the signal energy to spread out in all directions and consequently the EM signal is scattered into several weaker signals [15, 19]. In practice, foliage, street signs, lamp posts and stairs within buildings cause scattering in mobile communication systems [16, 17].

2.2.1 Attenuation and Fading

Under LOS propagation conditions, the received signal power, \( P_r \), is a function of distance \( d \) between TX and RX, and follows the inverse square law [15] i.e., \( P_r \propto d^{-2} \). The received power \( P_r(d) \) is usually stated in terms of free space path
loss (FSPL) model as [15]

\[ P_r(d) = \frac{P_t G_r G_t}{L} \left( \frac{\lambda}{4\pi d} \right)^2, \quad d > 0 \] (2.1)

where \( P_t \) is the transmit power, \( L (\geq 1) \) is the hardware losses, \( G_r \) is the receiving antenna gain and \( G_t \) is the gain of transmit antenna. When there are obstructions between TX and RX, the received power, \( P_r \), reduces even more quickly and follows the following exponential law [15]:

\[ P_r \propto d^{-v} \] (2.2)

where \( v \) is the propagation loss parameter. The case \( v = 2 \) refers to propagation under free space conditions, while \( v > 2 \) corresponds to obstructed paths. The higher values of \( v \) correspond to urban surroundings while lower values of \( v \) characterize suburban or rural areas [15]. For most mobile channels, the value of \( v \) ranges from 2 to 5. Typically, \( 2 \leq v \leq 4 \) corresponds to indoor conditions while \( 3.5 \leq v \leq 5 \) corresponds to outdoor environments [20].

Attenuation of power is also accompanied by fluctuations around its mean value. Many replicas of the EM signal coming through different paths and hence experiencing different delays add up either constructively or destructively at the RX. As a result the RX receives a very strong signal at one location and possibly very weak signal at a nearby location. Moreover, when a RX moves, the phase relation between incoming replicas also change. The term “fading” is used to refer to such substantial amplitude and phase fluctuations [19].

When we observe the received power over a distance of several kilometers, there is steady decrease in signal strength. This is known as attenuation and is shown in lower part of Figure 2.1. When we examine the signal power over a couple of kilometers, the signal strength fluctuating around the mean value is noticeable and is depicted in middle part of Figure 2.1. Such fluctuations are referred to as long-term fading or large-scale fading. Long-term fading can be
expressed in terms of Lognormal distribution [15]. For shorter distances (few meters), the signal power fluctuates more rapidly. Such fluctuations are known as short-term fading or small-scale fading. Short-term fading is caused by local multipath [19] and is depicted in top portion of Figure 2.1. Short-term fading occurs over a distance of about half a wavelength [19]; and can be expressed in terms of Rayleigh distribution [15].

2.2.2 Multipath and Flat vs. Frequency Selective Fading

As clear from its name, multipath fading arises from the existence of various (multiple) paths between the TX and the RX. Assuming that different signal components scattered by different objects (known as scatterers) arrive at the RX antenna independent of each other, the received signal can be expressed as the vector sum of all such components [15]. Mathematically, the multipath channel can be presented by its impulse response $h(t)$. The channel impulse response

Figure 2.1: Attenuation and Fading in Wireless Channel [15]
(CIR) for \( N \)-path channel can be written as :

\[
h(t) = \sum_{i=1}^{N} p_i e^{j\phi_i} \delta(t - \tau_i)
\]  

(2.3)

where \( N \) is the number of paths, \( \tau_i \) is the time delay of the \( i^{th} \) path and \( p_i e^{j\phi_i} \) is the complex amplitude of the pulse received through \( i^{th} \) path. The consequence of multipath effect is the broadening of transmitted pulse which further leads to inter-symbol interference (ISI) [15]. The mean excess delay, \( \langle \tau \rangle \), and root mean square (rms) delay spread, \( \sigma_d \), of the multipath channel’s power delay profile (PDP) can be expressed as [16, 21]

\[
\langle \tau \rangle = \frac{\sum_{i=1}^{N} p_i \tau_i}{\sum_{i=1}^{N} p_i}
\]  

(2.4)

\[
\sigma_d = \sqrt{\langle \tau^2 \rangle - \langle \tau \rangle^2}
\]  

(2.5)

The mean excess delay is the first moment of PDP while rms delay spread is the square root of the second central moment of the PDP [21]. The typical values of \( \sigma_d \) are few microseconds in outdoor environments and few hundred nanoseconds in indoor radio channels [16]. The quantity \( \sigma_d \) indicates the severity of multipath effect and hence its high values are usually undesirable as they cause considerable pulse broadening. Hence rms delay spread, \( \sigma_d \), can be used to quantify the pulse broadening effect by defining a new variable namely “coherence bandwidth”, \( B_C \), in terms of \( \sigma_d \). Coherence bandwidth is the range of frequencies over which the channel passes all spectral components with approximately equal gain and linear phase. If we define the coherence bandwidth as the range of frequencies over which the frequency correlation function is above 0.9, then coherence bandwidth is approximated as [16]

\[
B_C = \frac{1}{50\sigma_d}
\]  

(2.6)
For a narrowband transmitted signal with bandwidth $B_s \ll B_C$, the fading over the entire signal bandwidth is almost same; where $B_s$ and $B_C$ denote bandwidth of transmit signal and coherence bandwidth respectively. This case is referred to as flat fading. Conversely, if $B_s \gg B_C$ (wideband signal), the channel amplitude varies considerably over a separation greater than $B_C$, and hence the message signal will observe different attenuations over different frequencies. Such a channel is known as frequency-selective channel and resultant fading is called frequency-selective fading [1]. Figure 2.2 distinguishes the two cases. Most rural areas are characterized as nearly “flat channels” since the probable values of $\sigma_d$ are very small [15].

![Figure 2.2: Flat Fading vs. Frequency Selective Fading](image)

For linearly modulated signals, the signal bandwidth is inversely proportional to the signal duration, $T_s$, so the flat fading case corresponds to the condition $T_s(\approx \frac{1}{B_s}) \gg \sigma_d(\approx \frac{1}{B_C})$ [1]. Therefore flat fading corresponds to the case when signal experiences negligible ISI. Conversely, frequency-selective fading corresponds to the case when the signal experiences a significant ISI i.e., the case when $T_s \ll \sigma_d$.

### 2.2.3 Rayleigh and Rician Fading

Consider a signal consisting of single frequency, $f_0$, is transmitted over an $N$-path wireless channel. The received signal for a stationary receiver in the absence of
any direct path can be expressed as [15]

\[
y(t) = \sum_{i=1}^{N} a_i \cos(2\pi f_0 t + \phi_i)
\]

\[
= \cos(2\pi f_0 t) \sum_{i=1}^{N} a_i \cos(\phi_i) - \sin(2\pi f_0 t) \sum_{i=1}^{N} a_i \sin(\phi_i)
\]

\[
= X \cos(2\pi f_0 t) - Y \sin(2\pi f_0 t)
\]

(2.7)

where \(a_i\) and \(\phi_i\) denote the amplitude and phase of \(i^{th}\) component, respectively; and \(X = \sum_{i=1}^{N} a_i \cos(\phi_i)\) and \(Y = \sum_{i=1}^{N} a_i \sin(\phi_i)\) are the in-phase and quadrature terms respectively. In the absence of any LOS component, no dominant multipath component exists and the phase is uniformly distributed over the range \((0, 2\pi)\) [2]. Moreover, when \(N\) is sufficiently large, according to the central limit theorem (CLT), \(X\) and \(Y\) (each being a summation of \(N\) random variables) will become independent, identically distributed (iid) Gaussian random variables (RVs). Under such conditions, the channel output (received signal) can be represented as a complex Gaussian process and its real and imaginary parts both have Gaussian PDFs [22]. Therefore, the envelope \(R\) of the received signal is given by \(R = \sqrt{X^2 + Y^2}\) and has Rayleigh PDF of the form [15, 22]

\[
f_R(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}, \quad r > 0
\]

(2.8)

where \(r\) denotes amplitude fluctuations of the received signal and \(\sigma^2\) is the variance of the RV. In the presence of LOS non-faded path (a.k.a. specular component) between TX and RX, the Gaussian process has non-zero mean. The LOS signal adds a deterministic component to the multipath signal and consequently the envelope follows Rician distribution of the form [15, 22]

\[
f_R(r) = \frac{r}{\sigma^2} I_0 \left( \frac{rA}{\sigma^2} \right) e^{-\frac{r^2+A^2}{2\sigma^2}}
\]

(2.9)
where \( I_0 \) is the modified Bessel function of first kind and zero-order, and \( A \) is the non-zero mean of the received signal amplitude. The Rician factor – defined as the ratio \( k = \frac{A^2}{2\sigma^2} \) – characterizes the relative power of unfaded and faded components. In terms of dB, it is given by \([23]\)

\[
k(dB) = 10 \log_{10} \left( \frac{A^2}{2\sigma^2} \right)
\]

(2.10)

The case \( A = 0 \) means absence of LOS component and reverts the Rician PDF back to Rayleigh PDF.

### 2.2.4 Slow vs. Flat Fading

The relative motion between transmitter and receiver results in a shift in the frequency of the received signal. This phenomenon is known as Doppler Shift. Considering all possible angles of arrival (cf. Figure 2.3), the received instantaneous frequency can be expressed as \([1]\)

\[
f_{in} = f_0 + \frac{v}{\lambda} \cos(\theta_i)
\]

(2.11)

where \( \theta_i \) is the angle of arrival of received signal with respect to direction of motion, \( v \) is the receiver speed toward transmitter and \( \lambda = \frac{c}{f_0} \) is the wavelength of received signal. Therefore, the maximum Doppler shift (maximum shift in carrier frequency), \( f_d \), occurs at \( \theta_i = 0 \) and can be expressed as \([24]\):

\[
f_d = \frac{v}{\lambda}
\]

(2.12)

Therefore, the Doppler spread causes a frequency dispersion i.e., a transmitted frequency \( f_0 \) can be received anywhere in the range \([f_0 - f_d, f_0 + f_d] \) \([24, 25]\).

Therefore, the relative motion between the TX and RX brings in changes in the channel at a rate of \( f_d \) Hz. When the pulse duration is very small i.e., \( B_s \gg f_d \), the channel response will vary slowly and is called “slow-fading” channel. In this
case, the motion causes a very little or no impact on the pulse. Conversely, if the pulse duration is large i.e., $B_s \ll f_d$, motion-induced changes in the channel response will be “fast” and will considerably affect the pulse transmission. Such a channel is known as “fast-fading” channel [15, 25].

The distinction between slow and fast fading can be made in terms of “coherence time”. Assuming Clark’s Model, 50% coherence time can be defined as [16]:

$$T_C = \frac{9}{16\pi f_d}$$

If the pulse duration $T_s$ is much shorter than $T_C$, the pulse does not suffer distortion and this case is referred to as slow-fading. On the other hand, the transmitted signal gets distorted if $T_s > T_C$ and this case is referred to as fast fading [25].

### 2.3 Principles of Orthogonal Frequency Division Multiplexing

Orthogonal Frequency Division Multiplexing (OFDM) is a key technology for future cellular communication, wireless local area networks, and broadcasting as well [9]. OFDM – a special case of multicarrier modulation (MCM) – is based on parallel data transmission scheme. OFDM reduces the multipath effect by transmitting a data stream over a number of lower rate subcarriers, and thus makes complex equalization avoidable [26]. Unlike conventional multicarrier schemes, OFDM is based on overlapping subcarriers.

Assume there is no guard interval in between and the two successive subcarriers in a conventional MC system are contiguous to each other (cf. Figure 2.4). In
orthogonal MC systems, each subcarrier has the maximum value at its own centre frequency and zero at the centre frequency of all other subcarriers. Hence, the use of such overlapping orthogonal subcarriers renders a saving of almost 50% of bandwidth as compared to conventional MC systems without any guard interval. This is achieved through “orthogonality” of the subcarriers.

![Conventional MC and Orthogonal MC Modulation Techniques](image)

Figure 2.4: Conventional MC and Orthogonal MC Modulation Techniques

The word “orthogonal” implies a precise mathematical relationship among different subcarrier frequencies. In time domain, orthogonality means each subcarrier is periodic with an integer number of cycles within a fixed interval (FFT interval). In frequency domain, orthogonality implies that each subcarrier having the maximum value at its own centre frequency and zero at the centre frequency of all other (orthogonal) subcarrier [27, 28].

### 2.3.1 Implementation of OFDM Transceiver

Suppose we want to transmit $N_c$ complex-valued source symbols $X[k]$, with $k = 0, 1, \ldots, N_c - 1$. The serial data is first converted into parallel and then modulated to $N_c$ sub-carriers. Initially, the source symbol duration is $T_d$, which after serial-to-parallel (S/P) conversion becomes [4]:

$$T_s = N_c T_d \quad (2.14)$$
CHAPTER 2. FUNDAMENTALS OF WIRELESS COMMUNICATION

To maintain the orthogonality among $N_c$ sub-carriers, the subcarriers use a frequency spacing of $F_s = \frac{1}{T_s}$. Therefore, sub-carrier frequencies are $f_k = k \cdot F_s = \frac{k}{T_s}$ with $k = 0, 1, \ldots, N_c - 1$. The $N_c$ parallel modulated source symbols $X[k]$ are considered to be one OFDM symbol. The $l^{th}$ OFDM symbol with rectangular pulse shaping can be written in term of its complex envelope form [4, 9] as:

$$x_l(t) = \frac{1}{N_c} \sum_{k=0}^{N_c-1} X_l[k] e^{j2\pi f_k t}, \quad (l-1)T_s \leq t < lT_s \quad (2.15)$$

By sampling the continuous time at $t = nT_d = n \frac{T_s}{N_c}$ and $f_k = kF_s$, the corresponding discrete time OFDM symbol can be written [4, 9] as

$$x_l[n] = \frac{1}{N_c} \sum_{k=0}^{N_c-1} X_l[k] e^{j2\pi kn/N_c}, \quad l=0,1,\ldots,\infty \quad n=0,1,\ldots,N_c-1 \quad (2.16)$$

Note that $x_l[n]$ is the $N_c$-point inverse DFT of data symbols $X_l[k]_{k=0}^{N_c-1}$. Let $y_l[n]_{n=0}^{N_c-1}$ is the discrete time $l^{th}$ received OFDM symbol, then the demodulated symbol (ignoring the noise and channel effects) is

$$Y_l[k] = \sum_{n=0}^{N_c-1} y_l[n] e^{-j2\pi kn/N_c}$$

$$= \sum_{n=0}^{N_c-1} \left\{ \frac{1}{N_c} \sum_{i=0}^{N_c-1} X_l[i] e^{j2\pi in/N_c} \right\} e^{-j2\pi kn/N_c}$$

$$= \frac{1}{N_c} \sum_{n=0}^{N_c-1} \sum_{i=0}^{N_c-1} X_l[i] e^{j2\pi (i-k)n/N_c}$$

$$= X_l[k] \quad (2.17)$$

Note that the demodulated symbol $Y_l[k]$ is the $N_c$-point DFT of received data symbols $y_l[n]_{n=0}^{N_c-1}$. This IDFT / DFT based digital implementation is the key advantage of OFDM systems. OFDM modem based on IDFT and DFT is shown in Figure 2.5.

The use of IDFT/DFT absolutely eradicates the need of multiple oscillators to
transmit / receive an OFDM signal. Additionally, if \( N_c = 2^n \), the DFT can be replaced by computationally more efficient Fast Fourier Transform (FFT) [3]. Moreover, inter-carrier interference (ICI) can be eliminated with the use of rectangular DFT window at the receiver side [4].

Figure 2.6 shows the normalized power spectral density (PSD) versus normalized frequency for an OFDM symbol with \( N_c = 16 \) subcarriers. Since subcarriers only near the band edges contribute to the out-of-band radiation, thus, for large \( N_c \), the PSD becomes flat and approaches that of single-carrier modulation with ideal Nyquist filtering [4].

### 2.3.2 Insertion of Guard Interval

Since \( T_s = N_c \times T_d \), therefore increase in the number of parallel sub-channels reduces the data rate for each sub-channel. Effectively, it is equivalent to lengthening the OFDM symbol duration with respect to duration of CIR, \( \tau_{\text{max}} \), which corresponds to reduction in ISI. In order to completely avoid the effects of ISI and ICI, a guard interval is inserted between adjacent OFDM symbols. The guard interval is also known as Cyclic Prefix (CP) because it cyclically extends the original OFDM symbol (cf. Figure 2.7). This means a copy of length \( T_g \) from the
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43

end of the OFDM symbol is inserted in front of the symbol.

Figure 2.6: OFDM Spectrum with $N_c = 16$ Sub-channels [4]

In order to completely avoid ISI and ICI, the length of the guard interval (cyclic prefix) must be at least equal to duration of CIR, $\tau_{max}$, [4] i.e., $T_g \geq \tau_{max}$. Thus, cyclic prefix extends the duration of the OFDM symbol to $T'_s = T_s + T_g$. Correspondingly, the discrete length in number of samples of the guard interval has to be

$$L_g \geq \left\lceil \frac{\tau_{max} N_c}{T_s} \right\rceil \quad (2.18)$$

where $\lceil \cdot \rceil$ denotes the rounding towards positive infinity. The CP is an overhead and results in power and bandwidth wastage. However, CP can be used for
time and frequency synchronization in the receiver, since CP consists of repeated
symbols at a known sample spacing [29]. The discrete time \( l \)th OFDM symbol
after after cyclic extension becomes :

\[
x_l[n] = \frac{1}{N_c} \sum_{k=0}^{N_c-1} X_l[k] e^{j2\pi kn/N_c}, \quad n = -L_g, -L_g + 1, \ldots, N_c - 1
\] (2.19)

Digital-to-Analog converter (DAC) converts this sequence into the signal waveform \( x(t) \) whose duration is \( \hat{T}_s = T_s + T_g \). After up-conversion, the RF signal is
transmitted over the channel. The received signal after RF down-conversion can
be written as:

\[
y(t) = x(t) * h(t, \tau) + z(t)
\] (2.20)

where \( y(t) \), \( x(t) \), \( h(t, \tau) \) and \( z(t) \) denote the received signal, transmitted signal,
channel impulse response and additive noise, respectively. Analog-to-digital con-
verter (ADC) converts the received signal \( y(t) \) into the received sequence \( y_l[n] \)
with \( n = -L_g, ..., N_c - 1 \). Note that \( y_l[n] \) is the sampled version of \( y(t) \) sampled
at \( t = nT_d \). From Equation (2.19), we can directly write:

\[
y_l[n] = \frac{1}{N_c} \sum_{k=0}^{N_c-1} H_l[k] X_l[k] e^{j2\pi kn/N_c} + z_l[n], \quad n = -L_g, -L_g + 1, \ldots, N_c - 1
\] (2.21)

where \( H_l[k] \) and \( z_l[n] = \text{IDFT}\{Z_l[k]\} \) denote the channel response at \( k \)th subcar-
rrier and additive noise at \( n \)th sample, respectively. A key role imparted by CP is
the transformation of linear convolution into circular convolution [13]. Moreover,
circular convolution in time domain is equivalent to point-wise multiplication of
DFT samples [30]. Before multi-carrier demodulation, we remove cyclic prefix
i.e. first \( L_g \) samples of the received sequence. The remaining signal segment with
index \( n = 0, 1, \ldots, N_c - 1 \) is free from ISI and is demodulated by inverse OFDM
utilizing DFT (cf. Equation (2.17)).

The cyclic prefix with \( T_g \geq \tau_{\text{max}} \) completely removes the ICI and therefore,
CHAPTER 2. FUNDAMENTALS OF WIRELESS COMMUNICATION

Every subcarrier is free from interference from other subcarriers. This fact enables us to consider each sub-channel separately and independently. Moreover, assuming each sub-channel is flat fading and ISI-free, the received symbols can be expressed in the frequency domain as:

\[ Y[k] = H[k]X[k] + Z[k], \quad k = 0, 1, \ldots, N_c - 1 \] (2.22)

where \( H[k] \) and \( Z[k] \) denote the flat-fading channel gain and additive noise for the \( k^{th} \) subcarrier, respectively.

2.3.3 Windowing of OFDM Symbol

OFDM maintains orthogonality between subcarriers through the insertion of CP when transmitted over a multipath channel. Addition of CP to the transmitted OFDM symbol creates a signal that appears to be periodic, and consequently, the effective part of the received signal can be seen as the cyclic convolution of the transmitted OFDM symbol and the CIR [13, 31]. A rectangular pulse has infinite bandwidth because its Fourier transform is a sinc function with infinite sidelobes. To restrict these sidelobes and consequently to reduce the out-of-band radiation, windowing technique is used. Instead of rectangular pulse shaping, cyclically extended parts of the OFDM symbol are pulse shaped so that the applied window may not influence the signal during its effective period [31].

As shown in Figure 2.8, pulse shaping window of length \( T_{\text{win}} \) can be considered as an additional cyclic prefix as it extends the guard interval further. Hence the robustness of OFDM signal against delay spread is further improved at the cost of further decrease in efficiency. The orthogonality of the OFDM subcarriers is restored at the receiver by the DFT rectangular window. The transmitter pulse...
shape \( w(t) \) for an OFDM symbol starting at \( t = 0 \) is defined as [31]

\[
\begin{align*}
w(t) = \begin{cases} 
\frac{1}{2} \left[ 1 - \cos\left(\pi \left( t + T_{\text{win}} + T_g \right) / T_{\text{win}} \right) \right] & -T_{\text{win}} - T_g \leq t < -T_g \\
1 & -T_g \leq t \leq T_s \\
\frac{1}{2} \left[ 1 + \cos\left(\pi \left( t - T_s \right) / T_{\text{win}} \right) \right] & T_s < t \leq T_s + T_{\text{win}}
\end{cases}
\end{align*}
\]  

(2.23)

where \( T_{\text{win}} \) is the duration of windowed prefix/postfix for spectral shaping, \( T_g \) is the duration of cyclic prefix, and \( T_{\text{FFT}} = T_s \) is the effective part of the OFDM symbol (FFT interval).

### 2.3.4 Peak-to-Average Power Ratio

Multicarrier systems inherently possess high dynamic range which means that the signal has large variations between the average signal power and the maximum signal power. Since each subcarrier is essentially independent, subcarriers can add constructively (resulting a very large value) or destructively (giving a very small value), which may result in large amplitude variations (and hence large
power fluctuations). This high dynamic range imposes a need for transmitter and receiver which are able to accommodate a large range of signal power with minimum distortion [32]. The large dynamic range of OFDM signal is often described in terms of peak-to-average power ratio (PAPR), which is given by [4]

\[
PAPR = \frac{\text{Peak Power}}{\text{Average Power}} = \max_{n=0}^{N_c-1} \frac{|x[n]|^2}{\frac{1}{N_c} \sum_{n=0}^{N_c-1} |x[n]|^2}
\]  

(2.24)

where \( n = 0, 1, \ldots, N_c - 1 \) are the time samples of an OFDM symbol. An alternative measure of amplitude variations is the crest factor (CF), which is defined as [4]

\[
CF = \frac{\text{Peak Amplitude}}{\text{RMS Amplitude}} = \sqrt{\text{PAPR}}
\]  

(2.25)

A reduction of the PAPR is highly desirable. The higher PAPR lowers the efficiency of circuits like power amplifiers (PA), ADCs, and low-noise amplifiers (LNA). The large signals drive the PA to saturation mode, which results in distortion. To minimize the amount of distortion, OFDM signal values should be such that the operation of a PA is limited to the linear amplification region. Since OFDM inherently possesses high dynamic range, this condition can be achieved only if OFDM keeps its average power well below the nonlinear region of the PA so that high dynamic range is achieved. However, the average power cannot be lowered without bound since lower average power reduces the efficiency and consequently the range. The reason behind this is that lowering the average power corresponds to a lower output power for the most of the signal in order to accommodate the sporadic peaks. Hence a careful trade-off has to be made between allowable distortion and required output power [32].

Several techniques are used to reduce PAPR [31]. Signal distortion techniques simply reduce the peak amplitudes through nonlinear distortion of OFDM signal e.g., peak windowing. Coding techniques use special forward error correction (FEC) code set to eliminate the OFDM symbols with large PAPR. Scrambling techniques scramble the OFDM symbol with different sequences and choose the
sequence which gives least PAPR.

### 2.3.5 Coherent vs. Differential Detection

Phase Shift Keying (PSK) is most commonly used modulation scheme for fading channels as it is insensitive to channel amplitude variations [29]. Two demodulation techniques for PSK include coherent demodulation and differential demodulation. Coherent detection schemes estimate the channel to get an absolute reference phase and amplitude for each subcarrier in each OFDM symbol. Contrary to this, instead of using an absolute reference, the differential detection schemes compare each subcarrier with another subcarrier [28]. Differential phase shift keying is significantly simpler to implement than ordinary PSK but it produces more erroneous demodulations since the reference signal for demodulation is not fixed. More specifically, there are two noise terms instead of one and hence the noise power is double than that of coherent case. This causes the performance of differential modulation to be roughly 3-dB worse than that of coherent modulation [1].

Although coherent detection provides a 3-dB SNR gain over differential detection; however channel state information is required for coherent detection [10]. Block diagram representation of a coherent and differential OFDM receiver is shown in Figure 2.9. $N_c$ subcarriers of OFDM signal are demodulated by FFT block. The FFT output contains PSK values with random phase shifts and amplitude variations caused by the channel impulse response, carrier frequency offset, and timing offset [28]. For coherent receiver, channel estimation block is used for estimating the reference phases and amplitudes for all subcarriers, so that PSK symbols can be decoded. The main problem with coherent detection is to estimate the channel to find an absolute reference phase and amplitude for each subcarrier in each OFDM symbol without introducing too much training overhead. In contrast to coherent detection, differential detection doesn’t need channel estimation and hence saves both complexity and bandwidth. The cost to be paid for this
is reduced SNR performance. Instead of using an absolute reference, differential
detection compares each subcarrier with another subcarrier.

![OFDM Receiver with Coherent and Differential Detection](image)

Figure 2.9: OFDM Receiver with Coherent and Differential Detection [28]

### 2.4 Synchronization For OFDM

Receiver synchronization is one of the vital issues in multicarrier communication systems. Before the OFDM receiver starts demodulation, it has to perform two synchronization tasks. The first one is referred to as timing synchronization which involves the “decision making” about timing instants of OFDM symbols. The second task is known as frequency synchronization in which the receiver tries to align its carrier frequency with that of the transmitter. Owing to its inherent symbol structure, the time synchronization requirements are somewhat relaxed for OFDM systems (because of longer OFDM symbol period and cyclic prefix). Conversely, the frequency synchronization requirements are more stringent because of extremely tight packing of OFDM subcarriers (as compared to conventional modulation schemes). The CFO results in loss of orthogonality of subcarriers which subsequently causes high performance degradation [13, 29].
2.4.1 Effects of Timing Offset

An OFDM symbol is a combination of many subcarriers and each of them has a frequency response of "sinc" function when rectangular window is used. If the timing window is moved even slightly to the left or the right of correct position, a unique phase shift will be introduced to each subcarrier. Moreover, the introduced phase offset affects all subcarriers linearly [13, 30].

Let $\delta$ denote the symbol time offset (STO), the $l^{th}$ received baseband OFDM symbol in the presence of timing offset then can be expressed as [9]

$$y_l[n] = \frac{1}{N_c} \sum_{k=0}^{N_c-1} H_l[k] X_l[k] e^{j2\pi k(n+\delta)/N_c} + z_l[n] \quad (2.26)$$

where $H_l[k]$ is the channel response at $k^{th}$ subcarrier. Ignoring the noise and channel effects (cf. Equation (2.17)), the received signal in frequency domain becomes

$$Y_l[k] = X_l[k] e^{j2\pi k\delta/N_c} \quad (2.27)$$

and corresponding received signal in time domain is

$$y_l[n] = x_l[n + \delta] \quad (2.28)$$

According to Equations (2.27) and (2.28), a timing offset of $\delta$ samples in time domain introduces a phase shift of $2\pi k\delta/N_c$ in frequency domain to all subcarriers which is directly proportional to subcarrier index $k$ and timing offset $\delta$ [9]. This phase offset rotates the signal constellation of the received signal around the origin.

There is another impairment caused by timing offset error. This is additive interference from neighboring symbols and depends upon the location of estimated starting point of OFDM symbol [9]. If the estimated starting point is exact or little earlier than the exact starting point of OFDM symbol, the orthogonality
among subcarriers is not destroyed and only the phase of received signal is rotated. However, if the estimated starting point is too earlier or little later than the exact starting point of OFDM symbol, the orthogonality among subcarriers destroys and interference from neighboring symbols also affects the received signal in addition to phase rotation.

Left part of Figure 2.10 shows the effect of timing offset when the estimated starting point is a little earlier than exact starting point with $\delta = -3$. In this case, the orthogonality of subcarriers is maintained and the only effect of timing offset is phase rotation of the received signal. Right part of Figure 2.10 shows the effect of timing offset when the estimated starting point is later than exact starting point with $\delta = 20$. In this case, the orthogonality of subcarriers is destroyed and the distortion is caused by phase rotation as well as interference from neighboring symbols [9].

![Figure 2.10: Constellation Rotation Caused by Timing Offset][9]

### 2.4.2 Effects of Frequency Offset

OFDM modulates source symbols onto orthogonal subcarriers. To maintain orthogonality among $N_c$ subcarrier, the subcarriers are separated by a frequency spacing of $F_s = \frac{1}{T_s}$ where $T_s$ is the OFDM symbol duration. However, in practical
systems, the subcarriers’ frequency spacing is not exactly $\frac{1}{T_s}$ [1]. This is caused by the difference between frequency of the local oscillator and the received carrier frequency. This difference is known as CFO and can be expressed mathematically as

$$f_{\text{offset}} = f_c - f_{LO} = f_{TX} - f_{RX}$$ (2.29)

Generally, the CFO is expressed as a fraction of subcarrier spacing. The CFO normalized by subcarrier spacing is called normalized CFO and is defined as $\epsilon = \frac{f_{\text{offset}}}{F_s}$. In the presence of carrier frequency offset, the received signal in time domain can be expressed as:

$$y_l[n] = \frac{1}{N_c} \sum_{k=0}^{N_c-1} H_l[k] \cdot X_l[k] \cdot e^{j2\pi(k+\epsilon)n/N_c} + z_l[n]$$ (2.30)

Ignoring the effect of channel and noise, the received signal in the presence of CFO in time and frequency domain can be written as:

$$y_l[n] = x_l[n] e^{j2\pi n \epsilon/N_c}$$ (2.31)

$$Y_l[k] = X_l[k - \epsilon]$$ (2.32)

Therefore, in time domain, the CFO can be mathematically modeled as a complex multiplicative distortion of received signal. According to Equation (2.31), a CFO of $\epsilon$ induces a phase shift of $\frac{2\pi n \epsilon}{N_c}$ to a time-domain signal $x_l[n]$ [9]. This phase shift is proportional to CFO $\epsilon$ and time index $n$. The Equation (2.32) shows the effect of CFO on frequency domain signal. According to this Equation, CFO causes the frequency signal $X_l[k]$ to be frequency translated by $\epsilon$. Obviously, all the subcarriers observe the same frequency shift of $\epsilon$. This topic is dealt in more detail in Chapter 3.
2.5 Multiple Antenna Systems

Multiple input multiple output (MIMO) systems use multiple antennas at both ends of the communication link. Exploiting multiple antennas at the receiver and transmitter provides higher data rates and longer range without additional power.

2.5.1 Need For MIMO

The scarcity of frequency spectrum and continuous increase in wireless applications impose a need of wireless systems with greater capacity, range, and reliability which can accommodate all applications. Poor reliability associated with higher-order modulation techniques forbids their use as a potential solution to this problem. The most effective solution to achieve reliable communication over a wireless channel is diversity [33]. There are many types of diversity including, frequency diversity, time diversity, antenna diversity, etc. Antenna diversity (a.k.a. spatial diversity) is a method to mitigate the effects of fading. MIMO exploits the space dimension to improve wireless systems capacity, range and reliability. MIMO channels work best in highly scattering transmission environment, where multiple paths exist between transmitters and receivers. The main advantages of MIMO systems – increase in channel capacity and enhancement of transmission reliability – are achievable without any expansion in the required bandwidth or increase in the transmitted power [33].

2.5.2 Benefits of MIMO

Wireless transmission is primarily impaired by multi-path fading. Additional constraints of limited power and scarce bandwidth make the task of designing fast wireless systems further demanding. MIMO technology offers solutions to the problems of constrained resources and multipath fading. The benefits exhibited by MIMO technology are briefly stated below.
Spatial Diversity Gain

Spatial diversity (SD) means transmitting independent multiple copies of the transmitted signal from several antennas to the receiver in a hope that at least one copy will reach the receiver without deep fading. The number of copies is generally referred to as the diversity order. With an increase in diversity order, the probability that at least one of the copies is not experiencing a deep fade at any given instant increases [34]. Therefore, the quality and reliability of reception improves with increasing diversity order. SD improves the quality and reliability of reception through fading mitigation. An $M_T \times M_R$ MIMO channel potentially offers $M_T \times M_R$ independent fading links, and hence offers a spatial diversity order of $M_T \times M_R [7]$.

Spatial Multiplexing Gain

Simultaneous transmission of multiple independent data streams in the same frequency band from each of transmit antennas is known as spatial multiplexing [34]. In a multipath propagation environment with rich scattering, the receiver is able to separate the data streams. Each data stream experiences at least the same channel quality that would be experienced by a single-input single-output (SISO) system. This effect enhances the channel capacity by a factor equal to the number of data streams [7]. Generally, an $M_T \times M_R$ MIMO channel reliably supports $\min(M_T, M_R)$ data streams and consequently increases the capacity by the same factor.

Interference Reduction and Avoidance

Interference from multiple users can be mitigated in MIMO systems by exploiting the spatial dimension to increase the separation between users. For instance, in the presence of interference, power gain increases the tolerance to noise and interference power, and hence the signal-to-noise-plus-interference ratio (SINR) is improved [7]. Moreover, spatial dimension is exploited for interference avoidance
e.g., by directing the signal energy towards specific user and minimizing interference to other users. Both interference reduction and avoidance improve the coverage and range of a wireless system.

2.5.3 MIMO Channel Model

Consider the case of $2 \times 2$ MIMO transmission which means that two antennas are used at TX and two antennas are used at RX. In spatial multiplexing technique, transmitters TX	extsubscript{1} and TX	extsubscript{2} send OFDM symbols $X_1$ and $X_2$ simultaneously at time $t_1$. At receiver side, receivers RX	extsubscript{1} and RX	extsubscript{2} receive $Y_1$ and $Y_2$ simultaneously at time $t_2$.

![Figure 2.11: A 2 \times 2 MIMO Channel](image)

The received OFDM symbols at two receive antennas can be written as:

\[
Y_1 = X_1 H_{11} + X_2 H_{21} + N_1 \tag{2.33}
\]
\[
Y_2 = X_1 H_{12} + X_2 H_{22} + N_2 \tag{2.34}
\]

where $N_1$ and $N_2$ represent AWGN noise vectors. In a more compact form, it can be written as:

\[
\begin{bmatrix}
Y_1 \\
Y_2
\end{bmatrix} =
\begin{bmatrix}
X_1 & X_2 \\
X_1 & X_2
\end{bmatrix} \ast
\begin{bmatrix}
H_{11} & H_{21} \\
H_{12} & H_{22}
\end{bmatrix} +
\begin{bmatrix}
N_1 \\
N_2
\end{bmatrix} \tag{2.35}
\]

where

\[
H =
\begin{bmatrix}
H_{11} & H_{21} \\
H_{12} & H_{22}
\end{bmatrix} \tag{2.36}
\]
is the channel mixing matrix.

### 2.5.4 Detection of Spatially Multiplexed Signals

All the MIMO techniques including spatial multiplexing, space block code and precoding require MIMO detection – a process which entails quite enormous computations. MIMO-OFDM baseband decoder performs MIMO detection with the acquired channel estimates, and the detector output is fed to the digital demodulator (cf. Figure 2.13). The MIMO detection block plays the same role as the equalization block in SISO-OFDM systems [35]. Detection techniques for spatially multiplexed MIMO signals include zero forcing (ZF) detection, minimum mean squared error (MMSE) detection, ordered successive interference cancellation (OSIC); and sphere decoding (SD). This section presents zero forcing detector for spatially multiplexed MIMO signals which is also known as the decorrelator. By ignoring the noise terms in equations (2.33) and (2.34), these equations can be rewritten as

\[ Y_1 = X_1 H_{11} + X_2 H_{21} \]  
\[ Y_2 = X_1 H_{12} + X_2 H_{22} \]

Therefore assuming that channel knowledge is available at receiver, we have two equations with two unknowns i.e., \( X_1 \) and \( X_2 \). Using substitution method, the solution of these linear simultaneous equations can be found. By substituting the value of \( X_2 \) from (2.38) into (2.37), we get

\[ Y_1 = X_1 H_{11} + \left( \frac{Y_2 - X_1 H_{12}}{H_{22}} \right) H_{21} \]

which after simplification leads to the following solution:

\[ \hat{X}_1 = \frac{(Y_1 H_{22} - Y_2 H_{21})}{|H|} \]

By substituting this into one of the original equations yields the other solution.
as:
\[ \hat{X}_2 = \frac{(Y_2 H_{11} - Y_1 H_{12})}{|H|} \] (2.41)

where \(|H|\) is the determinant of channel mixing matrix defined as
\[ |H| \triangleq H_{11} H_{22} - H_{12} H_{21} \] (2.42)

### 2.5.5 MIMO-OFDM System Model

MIMO is well known for capacity enhancement of the wireless communication systems. However, at high data rate communication, MIMO multipath channels are frequency selective and hence complex equalization is required. OFDM has the capability of transforming frequency selective channel into a set of parallel flat fading channels and consequently requiring very simple equalizers. OFDM combined with MIMO provides high spectral efficiency and increased data throughput, and has become a most promising broadband wireless access scheme [8].

The MIMO-OFDM transmitter simultaneously transmits independent OFDM modulated data from multiple transmit antennas as shown in Figure 2.12. The source bit stream is initially encoded by a channel encoder whose output is fed into a digital modulator for constellation mapping. The complex constellation symbols are then encoded by a MIMO encoder which generates parallel streams for all transmit antennas. Each antenna stream follows the same steps. After pilot insertion, the symbol sequence is passed through an OFDM modulator. The CP is then added to every OFDM symbol and a preamble is inserted in every slot for timing synchronization. The OFDM frame is then sent to IF/RF components for transmission [8].

A simplified MIMO-OFDM receiver is shown in Figure 2.13. The received streams are first synchronized over all RX antennas. After the extraction of CP and preamble, the received streams are passed through OFDM demodulator. Then frequency domain pilot symbols are extracted which are used for channel estimation. Next, MIMO decoder decodes the OFDM symbols using estimated
channel frequency response obtained from channel estimator. The digital demodulator then demodulates and decodes the estimated transmit symbols obtained from MIMO decoder and decoded bit streams are fed to the data sink [8].
Chapter 3

Carrier Frequency Offset Estimation

3.1 Introduction

Like any other digital communication system, receiver synchronization is one of the vital issues in OFDM systems. However, being a multicarrier modulation (MCM) scheme, its synchronization requirements are quite different from those of single carrier modulation (SCM) systems. For example, OFDM systems can tolerate relatively higher timing offset errors than single carrier systems because of its longer symbol period and cyclic prefix. Conversely, the frequency synchronization requirements are more stringent in OFDM systems than single carrier systems because of its narrow subcarriers [29].

To optimize the performance of an OFDM system, the orthogonality among subcarriers must not be lost. However, misalignment between transmitter and receiver RF local oscillators and/or Doppler spread caused by the relative motion of transmitter and receiver induces carrier frequency offset (CFO). This CFO in OFDM systems causes loss of orthogonality among subcarriers and subsequently leads to significant performance degradation. Therefore, it is imperative to estimate CFO and hence thereafter compensate for [36].
3.2 Origin and Effects of CFO in OFDM Systems

OFDM modulates source symbols onto orthogonal subcarriers. To maintain orthogonality among $N_c$ subcarrier, the subcarriers are separated by a frequency spacing of $F_s = \frac{1}{T_s}$ where $T_s$ is the OFDM symbol duration. However, in practical systems, the subcarriers’ frequency spacing is not exactly $\frac{1}{T_s}$ [1]. This is caused by the difference between frequency of the local oscillator (LO) and the received carrier frequency. This difference is known as CFO and can be expressed mathematically as in Equation (2.29).

Generally, the CFO is expressed as a fraction of subcarrier spacing. The CFO normalized by subcarrier spacing is called normalized CFO and is defined as

$$\epsilon = \frac{f_{\text{offset}}}{F_s}$$ (3.1)

The origin of CFO is the discrepancy or misalignment between transmitter and receiver local RF oscillators and/or Doppler shifts. Normally, the base station (BS) uses very precise Rubidium clocks and Global Positioning System (GPS) while user equipments have low cost quartz oscillators with much lower precision [37]. Secondly, Doppler shifts caused by the relative motion of transmitter and receiver also induce CFO. Moreover, due to timing synchronization errors, the DFT window would be placed at wrong position. Therefore, timing synchronization error contributes to CFO as well.

The accuracy of an oscillator is measured in terms of parts per million (PPM). A typical oscillator drift is of the order of 10 $\sim$ 20 PPM. Assuming a carrier frequency of 5GHz and an oscillator with 0.1 PPM accuracy, the resultant frequency offset is $5 \times 10^9 \times 0.1 \times 10^{-6} = 500Hz$. 

\[\epsilon = \frac{f_{\text{offset}}}{F_s}\] (3.1)
3.2.1 Inter Carrier Interference

When the subcarriers frequency spacing $F_s$ is different from $\frac{1}{T_s}$, then the orthogonality among various subcarriers is not maintained. Loss of orthogonality in time domain means that sinusoids (subcarriers) with an integer number of cycles within a fixed interval (FFT interval) are no longer there. Loss of orthogonality in frequency domain implies that the individual subcarriers now don’t have zero value at the centre frequency of other subcarriers. This situation is depicted in Figure 3.1.

![Figure 3.1: InterCarrier Interference caused by CFO](image)

As shown in Figure 3.1, a frequency translation of $\epsilon$ causes an interference to a subcarriers from all other subcarriers. This intercarrier interference destroys orthogonality of subcarriers and results in considerable degradation of system performance.

The $i^{th}$ subcarrier signal (ignoring the data symbol and carrier frequency) can be expressed as [1] :

\[ x_i(t) = e^{j2\pi t/T_s} \] (3.2)
An \((i+k)^{th}\) interfering subcarrier can be expressed as:

\[
x_{i+k}(t) = e^{j2\pi(i+k)t/T_s}
\]  

(3.3)

The demodulation process is affected by the frequency offset, \(\epsilon\), and thus the interference becomes

\[
x_{i+k}(t) = e^{j2\pi(i+k+\epsilon)t/T_s}
\]  

(3.4)

The ICI between \(i^{th}\) and \((i+k)^{th}\) subcarriers can be found by their inner product

\[
I_k = \int_0^{T_s} x_i(t) x^*_i+k(t) \, dt
= T_s(1 - e^{-j2\pi(k+\epsilon)}) / j2\pi(k + \epsilon)
\]  

(3.5)

The total interference power on \(i^{th}\) subcarriers is the sum of interference from all other subcarriers and can be expressed as [1,32] :

\[
ICI_i = \sum_{k \neq i} |I_k|^2
\approx C_0(T_s \epsilon)^2
\]  

(3.6)

where \(C_0\) is some constant. According to Equation (3.6), the ICI increases quadratically with frequency offset \(\epsilon\) as well as with OFDM symbol duration \(T_s\). This is quite intuitive, as higher \(T_s\) value means smaller \(F_s\) value or equivalently tightly packed subcarriers which results in higher ICI.

### 3.2.2 CFO Problem Modeling

As mentioned earlier, discrete-time baseband OFDM signal can be expressed as:

\[
x_i[n] = \sum_{k=0}^{N_c-1} X_i[k] e^{j2\pi kn/N_c}, \quad n = 0, 1, \ldots N_c - 1
\]  

(3.7)

where \(X_i[k]\) denotes \(l^{th}\) symbol transmitted over \(k^{th}\) subcarrier and \(N_c\) denotes
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number of subcarriers. If $h_l(t)$ is the channel impulse response and $z_l(t)$ is the additive white Gaussian noise, then the received discrete-time signal can be expressed as:

$$y_l[n] = x_l[n] * h_l[n] + z_l[n] \quad , \quad n = 0, 1, \ldots, N_c - 1 \quad (3.8)$$

The OFDM receiver performs FFT of the received samples to yield:

$$Y_l[k] = X_l[k] \cdot H_l[k] + Z_l[k] \quad , \quad k = 0, 1, \ldots, N_c - 1 \quad (3.9)$$

where $Y_l[k]$, $X_l[k]$, $H_l[k]$, and $Z_l[k]$ denote the $k$th subcarrier component of the $l$th received symbol, transmitted symbol, channel frequency response, and noise in frequency domain, respectively.

In the presence of carrier frequency offset, the received signal in time domain can be expressed as:

$$y_l[n] = \frac{1}{N_c} \sum_{k=0}^{N_c-1} H_l[k] \cdot X_l[k] \cdot e^{j2\pi(k+\epsilon)n/N_c} + z_l[n] \quad (3.10)$$

Ignoring the effect of channel and noise, the received signal in the presence of CFO in time and frequency domain can be written as:

$$y[n] = x[n] e^{j2\pi n\epsilon/N_c} \quad (3.11)$$

$$Y[k] = X[k - \epsilon] \quad (3.12)$$

Therefore, in time domain, the CFO can be mathematically modeled as a complex multiplicative distortion of received signal. According to Equation (3.11), a CFO of $\epsilon$ induces a phase shift of $\frac{2\pi n\epsilon}{N_c}$ to a time-domain signal $x[n]$ [9]. This phase shift is proportional to CFO $\epsilon$ and time index $n$. Hence, CFO problem in OFDM systems can be modeled as shown in Figure 3.2.

The Equation (3.12) shows the effect of CFO on frequency domain signal.
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According to this Equation, CFO causes the frequency signal \( X[k] \) to be frequency translated by \( \epsilon \). Obviously, all the subcarriers observe the same frequency shift of \( \epsilon \).

3.2.3 Phase Rotation

As mentioned earlier, CFO is modeled as a complex multiplicative distortion of received signal by a factor of \( e^{j2\pi \epsilon n/N_c} \). This means that the signal \( x[n] \) is rotated by a phase angle of \( 2\pi \epsilon [N_c + L_g]/N_c \) between two consecutive OFDM symbols with time indices \( l \) and \( l + 1 \) [38,39]. Note that \( N_c + L_g \) is the number of samples in an OFDM symbol with CP. Therefore, in the absence of channel and noise effects, we can express the \( l \)th received OFDM symbols as

\[
y[n, l] = x[n, l]e^{j\frac{2\pi \epsilon}{N_c}[n+(N_c+L_g)(l-1)]}
\]  

(3.13)

where \( l \) denotes the OFDM symbol index. Obviously, for first OFDM symbols, \( l = 1 \) and the received signal becomes \( y[n, 1] = x[n, 1]e^{j\frac{2\pi \epsilon}{N_c}} \). This phase rotation brought about by CFO is depicted in the following Figure.
Figure 3.3 shows the constellation of five consecutively received 16-QAM OFDM symbols with $N_c = 64$ subcarriers. This figure reveals purely the effect of CFO on signal constellation in the absence of any channel and noise impairments.

### 3.2.4 Integer and Fractional CFO

The normalized CFO $\epsilon$ can be expressed as a sum of its integer and fractional parts as [9]:

$$\epsilon = \epsilon_i + \epsilon_f$$  \hspace{1cm} (3.14)

where $\epsilon_i$ is the largest integer less than or equal to $\epsilon$. When $\epsilon_f = 0$, then $f_{\text{offset}} = n \cdot F_s$, where $n$ is a positive integer. The CFO in this case is referred to as integer CFO (IFO). In such case, the received signal (without channel and noise effects) can be expressed in time and frequency domain as:

$$y[n] = x[n]e^{j2\pi \epsilon_i n / N_c}$$  \hspace{1cm} (3.15)  

$$Y[k] = X[k - \epsilon_i]$$  \hspace{1cm} (3.16)
As evident from Equation (3.16), the frequency domain received signal is the circular shifted version of transmit signal $X[k]$. Since all the subcarriers are translated by an integer multiple of subcarrier spacing i.e., $n \cdot F_s$, the orthogonality among the subcarriers is not destroyed and hence, there is no ICI. This frequency translation, however, causes a considerable loss in BER performance [9].

When $\epsilon = \epsilon_i + \epsilon_f$ with $\epsilon_f \neq 0$, the CFO is referred to as fractional CFO (FFO). In that case, all the subcarriers are shifted by an integer multiple of subcarrier spacing $n \cdot F_s$ (depending upon the $\epsilon_i$ value) plus a fraction of subcarrier spacing (depending upon the $\epsilon_f$ value). This FFO induced fractional translation of subcarriers in frequency domain destroys the orthogonality of subcarriers and brings about ICI. In addition to ICI from other subcarriers, FFO causes amplitude and phase distortion to the desired subcarrier as well [9, 31]. In Figure 3.1, the amplitude distortion is shown by a “◦” and ICI by adjacent subcarriers is denoted by “•”.

### 3.3 CFO Estimation Techniques

This section first explains why CFO estimation is crucial in OFDM systems. Next, CFO estimation techniques for OFDM systems are divided into different categories.

#### 3.3.1 Need For CFO Estimation

The overall impact of CFO in multicarrier systems is the severe performance degradation. This impact is generally expressed in terms of SNR loss, which is defined as [2]

$$\gamma(\epsilon) = \frac{SNR_{\text{ideal}}}{SNR_{\text{real}}}$$

(3.17)

where $SNR_{\text{ideal}}$ is the SNR under perfect frequency synchronization conditions while $SNR_{\text{real}}$ is the SNR in the presence of CFO, $\epsilon$. For OFDM systems with
small values of $\epsilon$, this SNR loss can be approximated as [2]

$$\gamma(\epsilon) \approx 1 + \frac{1}{3} \cdot \frac{E_s}{N_0} \cdot (\pi \epsilon)^2$$

(3.18)

From Equation (3.18), it is quite obvious that SNR degradation is almost proportional with the square of the normalized CFO. Moreover, SNR degradation is proportional with $E_s/N_0$. This is due to the fact that at lower $E_S/N_0$ values, the system performance is mostly limited by the thermal noise and hence, the impact of synchronization errors are not manifested entirely.

Equation (3.18) is plotted in Figure 3.4 as a function of normalized CFO, $\epsilon$, and for three different values of $E_S/N_0$. The SNR degradation values at 5% of subcarrier spacing are highlighted with “o” marks. It is quite obvious that, in order to avoid serious performance degradation, the frequency offset must be kept less than 5% of subcarrier spacing. For example, consider an OFDM system with a subcarrier spacing of 10 KHz. The oscillator accuracy, in this case, is required to be 500 Hz (= 5% of 10 KHz). Assuming a carrier frequency of 5 GHz, this requires an oscillator with 0.1 PPM accuracy [2, 28]. As mentioned earlier, user end equipments use low-cost, low-precision oscillators, and hence are unable to meet this requirement. Therefore, it is imperative to estimate the CFO and thereafter eliminate or minimize its effects [2, 28].

### 3.3.2 Types of CFO Estimation Techniques

Many CFO estimation techniques for OFDM systems have been proposed and analyzed in literature. We can divide the CFO estimation (CFOE) techniques into two broad categories [9, 40]:

- Time-Domain CFOE (TD-CFOE) Techniques, and
- Frequency-Domain CFOE (FD-CFOE) Techniques

The basic idea behind the time-domain CFOE methods is to utilize the autocorrelation of a certain recurring segment of the OFDM symbol in time domain.
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This recurring segment of OFDM symbol could be either in the form of cyclic prefix (Blind CFO estimation), or in the form of specially designed repetitive training symbols (Pilot-aided CFO estimation) [9, 40]. Obviously, pilot-aided methods are more accurate and reliable. On the other hand, blind techniques are more bandwidth efficient since no extra overhead is needed.

Frequency-domain CFOE techniques exploit the CFO-induced phase rotation in subcarriers in successive OFDM symbols. These techniques are based on the fact that CFO-induced phase rotation affects all the subcarriers equally and hence, all the subcarriers are translated by the same amount. Therefore, FD-CFOE techniques use the phase rotation of known pilots to compute estimated CFO, $\hat{\epsilon}$ [2].

The distinction between TD-CFOE and FD-CFOE algorithms is that the former one computes $\hat{\epsilon}$ from the input signal being fed to the FFT device, while the latter one computes $\hat{\epsilon}$ from the output signal of the FFT device. For this very reason, TD-CFOE and FD-CFOE techniques are also referred to as pre-FFT and post-FFT techniques, respectively [2, 40].

Figure 3.4: SNR Degradation caused by CFO
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Generally, CFO estimation in OFDM systems is done in two phases, viz. acquisition phase and tracking phase. In acquisition mode, integer CFO ($\epsilon_i$) is estimated and is also referred to as coarse CFO estimation. Tracking mode estimates fractional CFO ($\epsilon_f$) which is also known as fine CFO estimation. In acquisition mode, estimation range is of primary importance; while accuracy is the major design criterion for tracking stage [29, 39].

3.4 CFO Estimation in Time-Domain

Time-domain CFO estimation techniques exploit the autocorrelation of the received signal samples in the time domain. As the autocorrelation of the received signal samples must contain CFO related information, CP or special training symbols are utilized in such techniques.

3.4.1 CFO Estimation Using Cyclic Prefix

CP is an identical copy of the last $N_g$ samples of every OFDM symbol which is appended in front of the symbol. Although CP is an overhead and results in power and bandwidth inefficiencies, it can be used for CFO estimation. Such CFO estimators exploit the correlation between $N_g$ samples of CP and the corresponding rearmost chunk of an OFDM symbol [9, 28]. The correlation between two received samples at a distance of $N_c$ can be expressed as [41]

$$ r_{yy}[\tau] = y_l[n]y_l[n + N_c] = y_l[n]y_l[n + N_c] \quad \tau = N_c, \quad n \in \{-1, -2, \cdots, -N_g\} \quad (3.19) $$

In AWGN scenario, when there is no CFO i.e., $\epsilon = 0$, $y_l[n] = x_l[n] + n$ and $y_l[n + N_c] = x_l[n + N_c] + n_{n+\tau}$. Therefore, the correlation product is [42]

$$ y_l^*[n]y_l[n + N_c] \approx |x_l[n]|^2 + noise $$

However, in the presence of CFO i.e., when $\epsilon \neq 0$, the samples $y_l[n]$ and $y_l[n + N_c]$
observe different phase rotation which is proportional to the time index $n$. In this case, $y_l[n] = x_l[n]e^{j2\pi \epsilon n/N_c} + n_n$ and $y_l[n + N_c] = x_l[n + N_c]e^{j2\pi \epsilon (n + N_c)/N_c} + n_{n+\tau}$.

Since $x_l[n] = x_l[n + N_c]$, the correlation product becomes [29]

$$y_l^*[n]y_l[n + N_c] = |x_l[n]|^2 \cdot e^{j2\pi \epsilon N_c/N_c} + \text{noise}$$

In other words, the phase of the correlation product $y_l^*[n]y_l[n + N_c]$ contains information about CFO, $\epsilon$ [29, 42]. This CFO related information is exploited to estimate CFO. Under negligible channel effect, the phase of the above correlation is the phase difference between $N_c$-apart samples i.e., $\frac{2\pi \epsilon N_c}{N_c} = 2\pi \epsilon$. Therefore, it is straightforward to estimate CFO simply by dividing the correlation phase with $2\pi$ [28]. This can be expressed mathematically as [29]

$$\hat{\epsilon} = \frac{1}{2\pi} \arg(y_l^*[n]y_l[n + N_c]) , \quad n \in -1, -2, \cdots, -N_g$$

(3.20)

By averaging the correlation product over an interval equal to CP i.e., over $N_g$ samples, the effect of noise can be reduced. Hence [9, 29]

$$\hat{\epsilon} = \frac{1}{2\pi} \arg \left( \sum_{n=-N_g}^{-1} y_l^*[n]y_l[n + N_c] \right)$$

(3.21)

The $\arg(.)$ function gives values in the range $[-\pi, \pi)$, and hence, the CFO estimation range of Equation (3.21) is $[-0.5, +0.5)$, which is less than one half of the subcarrier spacing. For this reason, this technique is unable to estimate integral CFO. Generally, non-pilot-aided schemes are used for fine CFO estimation only [29].

Although the CFO estimator described in Equation (3.21) was originally derived for AWGN scenario, it can be used under multipath environment. Nevertheless, estimator performance in multipath dispersive channel will be poorer. This is due to fact that the multipath-induced ISI and ICI can be considered as an additional noise in $y_l[n]$ for $n = [-N_g, -N_g + 1, \cdots, -N_g + N_h]$ where $N_h$ is
the dispersive channel length in number of samples [29].

3.4.2 CFO Estimation Using Training Symbol

As described in Section 3.4.1, the estimation range of CP-based CFOE technique is less than one half of the subcarrier spacing i.e., \(|\hat{\epsilon}| < 0.5\). In other words, CP-based method is suitable for tracking mode and cannot estimate integral CFO. In acquisition mode, the CFO may be of the order of multiples of subcarrier spacing. Therefore, we need an estimation technique with wider estimation range.

Assume that the same symbol \(x\) is transmitted at time index \(n\) and \(n + \tau\), where \(\tau \neq N_c\). In this case, the two received symbols will be

\[
y_l[n] = x_l[n]e^{j2\pi\epsilon n/N_c} + n_n
\]
\[
y_l[n + \tau] = x_l[n]e^{j2\pi(\epsilon n + \tau)/N_c} + n_{n+\tau}
\] (3.22)

Therefore, the correlation product is

\[
y_l^*[n]y_l[n + \tau] \approx |x_l[n]|^2 \cdot e^{j2\pi\epsilon \tau/N_c} + noise
\] (3.23)

The estimation range of Equation (3.23) is \(\frac{\pm \pi}{2\pi(\tau/N_c)}\) i.e.,

\[
|\hat{\epsilon}| \leq \frac{N_c}{\tau}
\] (3.24)

The Equation (3.24) implies that higher the separation, \(\tau\), between two samples used for correlation, lower the estimation range is. Therefore, it can be inferred that coarse estimation requires correlation between two symbols with smaller separation while fine frequency estimation requires correlation between two symbols with larger separation [29].

Now keeping this in mind, the CFO estimation range can be widened by reducing the distance between two identical blocks of data used for correlation. This is done by designing special training symbols which are periodic with smaller
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period. We start by assuming \( Q = \frac{N_c}{\tau} \) with \( Q \in \mathbb{Z}^+ \) i.e., \( Q \) is a positive integer representing the ratio of OFDM symbol duration to the primitive period of the desired periodic training sequence. Our objective is to design a reference block made up of \( Q \geq 2 \) repetitive patterns, each consisting of \( \tau = \frac{N_c}{Q} \) time-domain samples. Such a time-domain periodic pilot sequence can be generated by taking IDFT of a frequency-domain sampling (comb) function of the form [9]

\[
X_l[k] = \begin{cases} 
    a_k & k = Q \cdot i ; \ i = [0, 1, \cdots, \tau - 1] \\
    0 & \text{otherwise}
\end{cases}
\]

(3.25)

where \( a_k = e^{j\pi k^2/\tau} \) are unity modulus complex numbers [43] and \( \tau = \frac{N_c}{Q} \) is the period of time-domain training signal. Figure 3.5 shows the training symbol generated according to Equation (3.25) with \( N_c = 128 \) total subcarriers, \( \tau = 32 \) samples in one period, and \( Q = 4 \) repetitive patterns in time-domain.

![Frequency-Domain Comb Signal](image1)

![Real Part of Time-Domain Training Signal](image2)

![Imaginary Part of Time-Domain Training Signal](image3)

Figure 3.5: Training Symbol with 4 Repetitive Patterns

Since \( x_l[n] \) and \( x_l[n + \tau] \) are identical samples, the correlation product of corresponding received samples is \( y_l[n]y_l[n + \tau] \approx |x_l[n]|^2 \cdot e^{j2\pi \tau / N_c} + \text{noise} \) (same as given in Equation (3.23)). Obviously, the phase of this correlation product
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contains information about CFO. By averaging the correlation product over an interval equal to one period of training symbol i.e., over $\tau$ samples, the effect of noise can be reduced. Hence

$$\hat{\epsilon} = \frac{N_c}{\tau} \arg \left( \sum_{n=0}^{(N_c/Q)-1} y_l^*[n]y_l[n+\tau] \right)$$

$$= \frac{Q}{2\pi} \arg \left( \sum_{n=0}^{\tau-1} y_l^*[n]y_l[n+\tau] \right)$$

(3.26)

The CFO estimation range of Equation (3.26) is $|\hat{\epsilon}| \leq Q/2$, which shows that the estimation range is directly proportional to $Q$, the number of repetitive patterns. However, if the value of $Q$ is increased to broaden the CFO estimation range, $\tau$, the primitive period of training symbol is decreased (as $\tau = \frac{N_c}{Q}$). The smaller value of $\tau$ means the lesser number of samples being used for correlation in Equation (3.26), and hence MSE performance is degraded. Therefore, we can conclude that there exists a trade off between CFO estimation range and MSE performance.

By averaging the correlation product over $Q$ repetitive patterns, the noise effects can be further reduced and we can get performance enhancement without lowering the estimation range [9]. Hence

$$\hat{\epsilon} = \frac{Q}{2\pi} \arg \left( \sum_{q=0}^{Q-1} \sum_{n=0}^{\tau-1} y_l^*[n+q\tau]y_l[n+(q+1)\tau] \right)$$

(3.27)

3.5 CFO Estimation in Frequency-Domain

As shown in Figure 3.3, CFO rotates the constellation diagram of the transmitted signal. Frequency-domain CFO estimation methods exploit the phase rotation of known pilot subcarriers to estimate the frequency offset. Contrary to time-domain techniques, FD CFOE techniques compute $\hat{\epsilon}$ from the demodulated OFDM signal.
3.5.1 CFO Estimation Using Double OFDM Symbol

As mentioned before, the reference training symbols have \( Q \geq 2 \) identical repetitive patterns; and CFO is estimated by measuring the phase shift. One such technique is described in [44], where two identical training symbols are transmitted consecutively. However, this technique estimates CFO by the phase shift between these two identical symbols in the frequency domain.

Equation (3.10) shows the \( l \)th time-domain received OFDM symbol in the presence of CFO. This equation is reproduced in the following as

\[
y_l[n] = \frac{1}{N_c} \sum_{k=0}^{N_c-1} H_l[k] \cdot X_l[k] \cdot e^{j2\pi(k+\epsilon)n/N_c} + z_l[n] \quad n = 0, 1, \ldots, N_c
\]  

When two identical OFDM symbols are transmitted consecutively (without any guard interval in between), it is called Moose’s double OFDM symbol or super OFDM symbol [29]. The corresponding received signal in time domain can be expressed as

\[
y[n] = \frac{1}{N_c} \sum_{k=0}^{N_c-1} H[k] \cdot X[k] \cdot e^{j2\pi(k+\epsilon)n/N_c} + z[n]; \quad n = 0, 1, \ldots, 2N_c - 1
\]  

Note that we have dropped the subcarrier \( l \) and instead, the time index \( n \) is extended to \( 2N_c - 1 \). Now the \( k \)th element after \( N_c \)-point FFT of the first \( N_c \) points can be written as

\[
Y_1[k] = \sum_{n=0}^{N_c-1} y[n]e^{-j2\pi kn/N_c} + Z_1[k]; \quad k = 0, 1, \ldots, N_c - 1
\]  

Similarly, the \( k \)th element after \( N_c \)-point FFT of the last \( N_c \) points can be written as

\[
Y_2[k] = \sum_{n=0}^{N_c-1} y[n]e^{-j2\pi kn/N_c} + Z_2[k]; \quad k = 0, 1, \ldots, N_c - 1
\]
as [44]

\[ Y_2[k] = \sum_{n=N_c}^{2N_c-1} y[n] e^{-j2\pi kn/N_c} + Z_2[k] \]

\[ = \sum_{n=0}^{N_c-1} y[n + N_c] e^{-j2\pi kn/N_c} + Z_2[k] \]

\[ = \sum_{n=0}^{N_c-1} (y[n] e^{j2\pi \epsilon}) e^{-j2\pi kn/N_c} + Z_2[k] \]

\[ = Y_1[k] e^{j2\pi \epsilon} + Z_2[k] ; \quad k = 0, 1, \ldots, N_c - 1 \quad (3.30) \]

Note that both the ICI and the desired signal components are phase rotated equally by the CFO, from first DFT to the second one. Therefore, Equation (3.30) can be used to estimate the CFO accurately even if the offset is too large to demodulate the data satisfactorily [44]. From Equations (3.30), we can write that

\[ Y_2[k] \cdot Y_1^*[k] = (Y_1[k] e^{j2\pi \epsilon} + Z_2[k]) \cdot Y_1^*[k] \]

\[ \approx |Y_1[k]|^2 e^{j2\pi \epsilon} \quad k \in [0, 1, 2, \ldots, N_c - 1] \quad (3.31) \]

Equation (3.31) shows that the phase of the correlation between corresponding subcarriers in each training symbol i.e., \( Y_2[k] \cdot Y_1^*[k] \) contains information about CFO, \( \epsilon \). Therefore, using all the subcarriers in both training symbols, we can estimate CFO as [44]

\[ \hat{\epsilon} = \frac{1}{2\pi} \tan^{-1} \left\{ \frac{\sum_{k=0}^{N_c-1} \text{Im}(Y_2[k] \cdot Y_1^*[k])}{\sum_{k=0}^{N_c-1} \text{Re}(Y_2[k] \cdot Y_1^*[k])} \right\} \quad (3.32) \]

It is shown in [44] that the above estimator is maximum likelihood estimator of \( \epsilon \). Moreover, the variance of estimation error for above estimator is derived
in [44] as
\[
Var[\hat{\epsilon} | \epsilon, \{Y[k]\}] = \frac{N_0}{(2\pi)^2 T_s N_c \sum_{n=0}^{N_c-1} |x[n]|^2}
\] (3.33)
which shows that as the number of subcarriers increases, the variance gets smaller which is an indication of performance improvement.

Next, consider the multipath channel which remain static during the two training symbols. Moreover, the training symbol pair is received over the dispersive channel preceded by a cyclic prefix of length \(L_g \geq N_h\), where \(N_h\) is the length of channel impulse response. Since the modulation phase values are repeated, the phase shift of all the subcarriers between consecutive training symbols is cause by CFO only and hence remains unchanged. Therefore, no guard interval is needed in between the two identical training symbols and the Equation (3.32) can be used for multipath environment as well [44].

The estimation range of Equation (3.32) is \([-\pi/2, \pi/2]\) i.e., \(|\hat{\epsilon}| < 0.5\). Therefore, this technique is not suitable for acquisition mode where estimation range is of primary focus. A technique to increase this estimation range is also described in [44] by shortening the training symbols. However the performance of estimator gets worse as the estimation range increases. The reason for this performance degradation is that as the training symbols get shorter, fewer samples are available for correlation average. Another technique to increase the frequency acquisition range is proposed in [45].

3.5.2 Data Aided CFO Estimation
As discussed in Section 3.5.1, Moose’s double training symbol is transmitted as a preamble. Such preamble based CFO estimation methods are, in general, suitable for initial acquisition. During this preamble period, no data symbols are permitted. This particular mode of transmission is called non-data aided (NDA) mode. Such NDA schemes do not make use of known symbols. A CFO
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estimation method for data aided (DA) mode is presented in [46]. In this method, known pilot symbols are spread uniformly in the frequency domain. The pilot subcarriers – known as sync-subchannels – can be transmitted in every OFDM symbol for CFO tracking.

CFO estimation scheme for DA mode proposed in [46] is performed in two stages: an acquisition stage and a tracking stage. During the acquisition stage, large frequency offsets (multiples of subcarrier spacing) are estimated whereas the tracking stage deals with small fractional frequency offset. Figure 3.6 depicts this two-stage synchronization structure.

After time synchronization, $N_c$ samples of two OFDM symbols $y_i[n]$ and $y_{i+D}[n]$ are saved in the memory. The FFT unit transforms these symbols into frequency domain (FD) signals $Y_i[k]$ and $Y_{i+D}[k]$ with $k = 0, 1, \ldots, N_c - 1$. Then pilots at known locations are extracted from these FD signals from $n^{th}$ and $(n+D)^{th}$ time slots. These pilot subcarriers are then utilized for CFO estimation in frequency domain and subsequently, the frequency correction is done in time domain (cf Figure 3.6).

The function of stage 1 (cf Figure 3.6) is to obtain a coarse CFO estimate as quickly as possible. Stage 2 uses coarse estimate from stage 1 and performs
the tracking. Splitting the overall estimation problem into two stages allows us
to tailor the two algorithms independently. Therefore, stage 1 can be optimized
for higher estimation range and speed while stage 2 can be optimized for higher
accuracy [46].

**Tracking Mode Algorithm**

After the integral CFO estimation and correction have been done by the acquisi-
tion stage, the remaining CFO is substantially smaller than one half of the
subcarrier spacing. Fine CFO can be estimated from the phase shift between two
successive subcarriers i.e., \( Y_l[k] \) and \( Y_{l+1}[k] \). Hence, we can write that

\[
\hat{\epsilon}_f = \frac{1}{2\pi} \cdot \frac{1}{T_s} \cdot \arg \left\{ \sum_{j=0}^{L-1} \left( Y_{l+1}[p(j), \hat{\epsilon}_{acq}] \cdot Y^*_l[p(j), \hat{\epsilon}_{acq}] \right) \cdot \left( X^*_{l+1}[p(j) \cdot X_l[p(j)] \right) \right\}
\]

(3.34)

where \( p(j) \) denotes the location of \( j \)th pilot tone, \( X_l[p(j)] \) denotes the pilot tone at
location \( p(j) \) in frequency domain at the \( l \)th index, and \( L \) denotes the number of
pilots. Moreover, the complex conjugate of known pilot tones \( X_l[k] \) and \( X_{l+1}[k] \)
is used to undo the effect of modulation at pilot tones. Since the frequency
correction by counter rotating the received samples in time domain by multiplying
with \( e^{-j2\pi\hat{\epsilon}_{acq}/N_c} \) (cf Figure 3.6) is done before FFT unit, hence the variable \( \hat{\epsilon}_{acq} \)
is also shown as an argument of the FFT output.

In practice, pilot tones are transmitted over a subset of available subcarriers.
Let \( \hat{L} \) uniformly spaced subcarriers are reserved for pilot tones, then a more
generalized fine CFO estimator can be expressed mathematically as [46]

\[
\hat{\epsilon}_f = \frac{1}{2\pi} \cdot \frac{1}{DT_s} \cdot \arg \left\{ \sum_{j=0}^{\hat{L}-1} \left( Y_{l+D}[p(j), \hat{\epsilon}_{acq}] \cdot Y^*_l[p(j), \hat{\epsilon}_{acq}] \right) \cdot \left( X^*_{l+D}[p(j) \cdot X_l[p(j)] \right) \right\}
\]

(3.35)

where integer \( D = n + D - n \) represents the difference between time index of two
symbols used for estimation and \( \hat{L} \) represents the known pilot symbol pairs in \( n^{th} \) and \( (n + D)^{th} \) time slots.

**Acquisition Mode Algorithm**

As mentioned earlier, the acquisition task should be done as quickly as possible and the estimation range should be higher as well. Moreover, Accuracy is not the prime objective of this stage, as it is followed by a more accurate tracking stage. The acquisition algorithm proposed in [46] involves a search operation for the training symbols transmitted over \( \hat{L} \) pilot subcarriers.

Note that the magnitude of the argument of the \( \text{arg}(.) \) function of Equation (3.35) is maximum when \( \hat{\epsilon}_{\text{acq}} \) coincides with true value of CFO i.e., \( \epsilon \). Therefore, the coarse CFO estimation can be calculated as [46]

\[
\hat{\epsilon}_{\text{acq}} = \frac{1}{2\pi} \cdot \frac{1}{T_s} \cdot \max_{\epsilon_{\text{trial}}} \left\{ \left| \sum_{j=0}^{\hat{L}-1} \left( Y_{i+D}[p(j), \hat{\epsilon}_{\text{trial}}] \cdot Y_{i}^*[p(j), \hat{\epsilon}_{\text{trial}}] \right) \right| \cdot \left( X_{i+D}^*[p(j)] \cdot X_{i}[p(j)] \right) \right\}
\]

(3.36)

where \( \epsilon_{\text{trial}} \) is the trial frequency values and \( Y_{i}^*[p(j), \hat{\epsilon}_{\text{trial}}] \) is the FFT output with \( \epsilon_{\text{trial}} \)-offset-corrected input. In practice, \( 0.1 \times F_s \) spaced trial values are sufficient, where \( F_s \) is the subcarrier spacing [46]. In tracking stage, only fine CFO, \( \hat{\epsilon}_f \), is estimated and corrected. Whereas in acquisition mode, both coarse and fine CFOs are estimated (cf Figure 3.6) and overall correction is done according to

\( \hat{\epsilon} = \hat{\epsilon}_{\text{acq}} + \hat{\epsilon}_{\text{trac}} \).

**3.6 Joint ML Time and CFO Estimation**

As mentioned in Section 3.4.1, CP – in spite of being an overhead – can be used for CFO estimation. As one step further, the redundant information contained in CP can be used for joint symbol-time and carrier-frequency offset estimation without inserting additional pilots. A CP-based joint maximum likelihood (ML)
time and CFO estimator is proposed in [47].

We will treat the two problem in the OFDM receiver jointly. First, a timing offset introduces an uncertainty about the arrival time of OFDM symbol. This time offset causes a rotation of the data symbols and can be modeled mathematically as a delay in the CIR. Secondly, the CFO introduces an uncertainty regarding the carrier frequency and translates all subcarriers. Carrier frequency offset is mathematically modeled as a complex multiplicative distortion of the received signal. The received signal, in the presence of these two anomalies, can therefore be written as:

\[ y_l[n] = x_l[n - \theta]e^{j2\pi \epsilon n/N_c} + z[n] \]  

(3.37)

where \( \theta \) is the integer-valued time offset, \( \epsilon \) is the normalized CFO, and \( z[n] \) is the white Gaussian noise.

The transmitted signal \( x_l[n] \) is the IDFT of the data symbols \( X_l[k] \) which are supposed to be independent of each other. Therefore, \( x_l[n] \) can be considered as a linear combination of iid random variables. Central limit theorem – provided the number of subcarriers is sufficiently large – can be used to approximate \( x_l[n] \) as a complex Gaussian process with real and imaginary parts being independent. In the absence of CP, this process is white as well. However, \( x_l[n] \) becomes non-white Gaussian process when it is cyclically extended, as CP introduces a correlation between \( N_c \)-apart data samples. Correspondingly, \( y_l[n] \) is also a non-white process, however it contains vital information about time offset and carrier frequency offset [47].

Consider the structure of received OFDM signal as depicted in Figure 3.7 [47]. We have an observation window of \( 2N_c + N_g \) samples of \( y[n] \). Obviously, within this window, there will definitely be a complete OFDM symbol of \( N_c+N_g \) samples. However, the exact starting point of an OFDM symbol is uncertain due to timing offset \( \theta \).

Let we denote the time indices of the last \( N_g \) samples of the complete OFDM
symbol within the observation window by $\hat{I}$ and indices of its identical copy by $I$ (cf Figure 3.7). These two set of indices can be written in the form of equations as

$$\hat{I} = \{\theta + N_c, \theta + N_c + 1, \ldots, \theta + N_c + N_g - 1\}$$

$$I = \{\theta, \theta + 1, \ldots, \theta + N_g - 1\} \quad (3.38)$$

The received signal within the observation window can be expressed as

$$y[n] = [y(1) \; y(2) \; \ldots \; y(2N_c + N_g)]^T \quad (3.39)$$

Using the fact that CP samples and their original matching samples are pair-wise (being same) and all other samples are mutually uncorrelated, we can write the received signal correlation as [47]

$$E\{y[n]y^*[n + m]\} = \begin{cases} 
\sigma_s^2 + \sigma_n^2 & m = 0 \\
\sigma_s^2 \cdot e^{-j2\pi\epsilon} & m = N_c \\
0 & \text{otherwise}
\end{cases} \quad \forall n \in I \quad (3.40)$$

Equation 3.40 is intuitively satisfying. For $m = 0$, the signal correlation equals to sum of signal energy and noise energy; and for $m = N_c$, the correlation equals to
the signal energy times CFO-induced phase rotation. At all other lag values, the samples are uncorrelated. Moreover, For all other values, \( \forall n \notin I \), \( y[n] \) samples are mutually uncorrelated.

Recalling that maximum likelihood estimate, \( \hat{\Theta}_{ML} \), of a fixed (non-random) but unknown parameter \( \theta \) is that value of \( \hat{\Theta}_{ML} \) which maximizes the probability density function (pdf) \( f_X(x|\hat{\Theta}_{ML}) \). For an iid data sequence \( \{x(0), x(1), ..., x(N-1)\} \), the corresponding likelihood function (LF) and log likelihood function (LLF) can be written as [48]:

\[
L = \prod_{i=0}^{N-1} f_{x(i)}(x(i)|\theta) \tag{3.41}
\]

\[
\log[L] = \sum_{i=0}^{N-1} \log[f_{x(i)}(x(i)|\theta)] \tag{3.42}
\]

where \( f(.) \) denotes the probability density function of its argument. Maximum likelihood estimate is then computed by differentiating \( L \), or equivalently, \( \log[L] \) with respect to \( \theta \) and setting the derivative equal to zero i.e., \( \frac{\partial L}{\partial \theta} = 0 \) or \( \log[\frac{\partial L}{\partial \theta}] = 0 \).

Therefore, the LLF of time offset \( \theta \) and frequency offset \( \epsilon \) is the logarithm of the likelihood function of the \( 2N_c + N_g \) samples received within the observation window. The LLF can, therefore, be written as [47]

\[
\Lambda(\theta, \epsilon) = \log \left( f(y|\theta, \epsilon) \right) = \log \left( \prod_{n \in I} f(y[n], y[n + N_c]) \cdot \prod_{n \notin (I \cup I')} f(y[n]) \right) = \log \left( \prod_{n \in I} f(y[n], y[n + N_c]) \cdot \prod_{n} f(y[n]) \right) \tag{3.43}
\]

Since the term \( \prod f(y[n]) \) is independent of \( \theta \) and \( \epsilon \), it can be omitted from the argument maximizing \( \Lambda(\theta, \epsilon) \). Under the assumption that \( y \) is a jointly Gaussian
vector, the LLF $\Lambda(\theta, \epsilon)$ is shown in [47] to be equal to

$$\Lambda(\theta, \epsilon) = |\gamma(\theta)| \cdot \cos(2\pi\epsilon + \angle \gamma(\theta)) - \rho \Phi(\theta)$$  \hspace{1cm} (3.44)$$

$\angle$ denotes the argument of a complex number. The functions $\gamma(\theta)$ and $\Phi(\theta)$ are defined as follow:

$$\gamma(m) \triangleq \sum_{n=m}^{m+N_g-1} y[n]y^*[n + N_c]$$  \hspace{1cm} (3.45)$$

$$\Phi(m) \triangleq \frac{1}{2} \sum_{n=m}^{m+N_g-1} |y[n]|^2 + |y[n + N_c]|^2$$  \hspace{1cm} (3.46)$$

The variable $\rho$ is the magnitude of the correlation coefficient between $y[n]$ and $y[n + N_c]$, and is defined as

$$\rho \triangleq \left| \frac{E\{y[n]y^*[n + N_c]\}}{\sqrt{E\{|y[n]|^2\}E\{|y[n + N_c]|^2\}}} \right|$$

$$= \frac{\sigma^2_y}{\sigma^2_y + \sigma^2_n}$$

$$= \frac{SNR}{SNR + 1}$$  \hspace{1cm} (3.47)$$

The first term in the Equation (3.44) is the weighted magnitude of $\gamma(\theta)$, which itself is a sum of $N_g$ consecutive correlations between $N_C$-apart sample pairs as defined in Equation (3.45). Moreover, the weighting factor is a function of frequency offset, $\epsilon$. The function $\Phi(\theta)$, as defined in Equation (3.46), is independent of frequency offset, $\epsilon$, and is known as energy term. This energy term is scaled by a constant $\rho$ (cf Equation (3.44)) which depends upon SNR.

The maximization of the LLF function is performed in two steps [47]

$$\max_{(\theta, \epsilon)} \Lambda(\theta, \epsilon) = \max_\theta \max_\epsilon \Lambda(\theta, \epsilon)$$

$$= \max_\theta \Lambda(\theta, \hat{\epsilon}_{ML}(\theta))$$  \hspace{1cm} (3.48)$$
The maximum with respect to $\epsilon$ can be obtained by equating the cosine term in Equation (3.44) to one. Therefore

$$\hat{\epsilon}_{ML}(\theta) = -\frac{1}{2\pi} \angle \gamma(\theta) + n$$  \hspace{1cm} (3.49)

where $n$ is any integer. Assuming that during the acquisition mode, a coarse CFO estimate $\hat{\epsilon}_{acq}$ has been obtained and $\epsilon \leq 1/2$; thus we can take $n = 0$. Therefore by taking cosine term in Equation (3.44) as unity, the LLF of $\theta$ becomes

$$\Lambda(\theta, \hat{\epsilon}_{ML}(\theta)) = |\gamma(\theta)| - \rho \Phi(\theta)$$  \hspace{1cm} (3.50)

Consequently, the joint ML estimation of $\theta$ and $\epsilon$ becomes

$$\hat{\theta}_{ML} = \arg \max_\theta \{ |\gamma(\theta)| - \rho \Phi(\theta) \}$$ \hspace{1cm} (3.51)

$$\hat{\epsilon}_{ML} = -\frac{1}{2\pi} \angle \gamma(\hat{\theta}_{ML})$$  \hspace{1cm} (3.52)

This joint ML time and frequency offset estimator can be viewed in the form of a block diagram as given in Figure 3.8.

![Figure 3.8: Structure of the Joint ML Time and CFO Estimator](image-url)
The quantities $|\gamma(\theta)| - \rho\Phi(\theta)$ and $-\frac{1}{2\pi}\angle\gamma(\theta)$ play a very important role in estimating the time and frequency offsets. These two quantities are shown in graphical form in Figure 3.9 for an OFDM system with $N_c = 1024$, $N_g = 128$, $\epsilon = 0.25$ and $\theta = 150$ at SNR = 15 dB. The quantity $|\gamma(\theta)| - \rho\Phi(\theta)$ yields peaks at time instants $\hat{\theta}_{ML}$ hence, the maximizing indices of this quantity generate timing offset estimates. Corresponding to these time instants, the phase of $\gamma(\theta)$ generates the CFO estimates i.e, $\hat{\epsilon}_{ML}$ (cf Figure 3.9).

### 3.7 Simulation Results and Performance Analysis

Figures 3.10 and 3.11 show the performance of CP-based time-domain CFO estimator as outlined in Section 3.4.1. These simulation results are based on 1000 iterations with an OFDM system of 128 subcarriers. Figure 3.10 represents the MSE performance in AWGN channel environment with different values of CP expressed as a percentage of OFDM symbol duration ($N_c$). As evident from these
figures, the performance of CP-based estimation technique improves with the size of CP. A higher $\frac{N_c}{N}$ ratio means a bigger fraction of an OFDM symbol is used for correlation and hence performance improvement.

Figure 3.10: CP-Based Time-Domain CFO Estimation in AWGN Channel

Figure 3.11: CP-Based Time-Domain CFO Estimation in Rayleigh Channel

Figure 3.11 shows the MSE performance for a 2-tap Rayleigh channel with
different values of CP. As CP length increases, the performance becomes closer and closer to that for AWGN case. It is because, as the ratio $\frac{N_g}{N_h}$ increases with increasing $N_g$ values (CP size), the noise due to ICI and ISI in $n = [-N_g, -N_g + 1, \cdots, -N_g + N_h]$ samples corrupts smaller percentage of samples within the correlation window.

Next, the simulation results for CFO estimation in time-domain using special training symbol are presented. The simulation results are based on 128-point OFDM system in AWGN environment at a fixed value of SNR = 8. Figure 3.12 and Figure 3.13 show the trade-off between CFO estimation range versus MSE performance according to Equation (3.26) and (3.27), respectively. As evident in Figure 3.12, the estimation range increases with increasing $Q$, the number of repetitive patterns in training symbol. However, this broader estimation range comes at the cost of MSE performance degradation as depicted in Figure 3.12.

Figure 3.14 and 3.15 show the MSE performance of CFO estimators based on repetitive time-domain training symbol according to Equation (3.26) and (3.27), respectively. Figure 3.14 clearly reveals that with increasing $Q$, the performance degrades, at the benefit of increased estimation range. However, after averaging over all repetitive patterns, there is a huge performance improvement while still achieving wider estimation ranges as shown in Figure 3.15.
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Figure 3.12: Trade-off between CFO estimation range and MSE performance

Figure 3.13: Performance improvement after averaging over all repetitive patterns
Figure 3.14: Periodic Training Symbol Based Time-Domain CFO Estimation

Figure 3.15: Periodic Training Symbol Based Time-Domain CFO Estimation
Next, the simulation results for CFO estimation in frequency-domain using Moose’s double training symbol are presented. The simulation results are based on an OFDM system with 128 subcarriers. Figure 3.16 shows the MSE performance for AWGN and Rayleigh channel. As explained in Section 3.5.1, the CFO estimation technique based on double training symbol in frequency domain works equally well in AWGN as well as Rayleigh environment as long as the CP length $N_g$ is longer than the length of channel impulse response $N_h$.

![Figure 3.16: MSE Performance Using Double Training Symbol](image)

Figure 3.17 shows the MSE performance as a function of DFT size at a fixed $SNR$ value of 5 dB. According to Equation (3.33), the variance of estimation error reduces as the number of subcarriers (DFT size) increases. Therefore, the performance improves with the number of subcarriers, as more samples are now available for correlation.

Figure 3.18 shows the tracking performance for Data Aided CFO estimation technique as explained in Section 3.5.2. The simulation results are are based on an OFDM system with 128 subcarriers with different number of pilot tones. Pilot subcarriers in two successive OFDM symbols are used for correlation i.e., $D = 1$. Intuitively, the performance improves with increasing number of pilots.
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Figure 3.17: MSE vs DFT Size

Figure 3.18: Tracking Performance of DA CFO Estimator in AWGN
Next, the simulation results for joint ML time and frequency offset estimator are presented for an OFDM system with $N_c = 128$. As discussed in Section 3.6, the quantities influencing the log-likelihood function and consequently, the performance are number of samples in CP i.e., $N_g$, and the correlation coefficient $\rho$ which in turn depends on SNR. Figures 3.19 and 3.20 show the performance of time offset estimator and frequency offset estimator as a function of CP length at different SNR values in AWGN environment.

![Figure 3.19: Performance of Time Estimator vs CP Length](image)

According to these results, the performance of time offset estimator is asymptotically independent of CP length after a certain threshold value. This threshold value is a function of SNR. There is no such threshold for frequency offset estimator as its performance continues to improve.

Figures 3.21 and 3.22 present the time and frequency estimator performance as a function of SNR at different lengths of cyclic prefix in AWGN environment. According to these results, CFO estimator performs better than time estimator. The reason behind the better performance of CFO estimator is its implicit averaging according to Equation (3.45). This Equation also explains why the performance improves as the length of cyclic prefix extends.
Figure 3.20: Performance of CFO Estimator vs CP Length

Figure 3.21: Performance of Time Estimator vs SNR
3.8 Conclusion

This chapter dealt with one of the most important functions performed by an OFDM receiver namely CFO estimation. Owing to its tightly packed subcarriers structure, the frequency synchronization requirements of OFDM systems are more demanding than single carrier systems. Basically, the carrier frequency offset is originated by the misalignment between transmitter and receiver RF local oscillators and/or Doppler spread. This CFO in OFDM systems destroys the orthogonality among subcarriers and leads to significant performance degradation. Various time-domain (pre-FFT) and frequency-domain (post-FFT) techniques for CFO estimators in OFDM systems have been discussed in this chapter. Through simulations, the performance of different techniques as a function of relevant parameters were presented as well. Finally, the joint ML-base time and frequency offset estimator was discussed. According to simulation results, the time estimator was suitable for acquisition mode while frequency estimator may be used in tracking mode.
Chapter 4

Pilot-Aided Channel Estimation

OFDM overcomes one of the major adversities of wireless channel namely ISI and consequently requires a simple equalizer. In OFDM, the wireless channel distorts the amplitude and phase of each subcarrier individually and this distortion effect can be represented by a single complex-valued coefficient. For coherent detection of transmitted information, this multiplicative distortion needs to be compensated for. This compensation process is called channel equalization and requires the estimates of channel impulse response [2]. Although differential PSK could be used in OFDM systems without requiring channel estimates, but it limits the number of bits per symbol and incurs about 3 dB loss in SNR as compared to coherent detection [10,11]. Moreover, new standards are based on QAM and thus need channel estimates.

4.1 Introduction

Channel estimation (CE) is a process of characterizing the effect of the channel on the transmitted data and allows the receiver to approximate how the channel has been distorted the input data [12]. CE is essential for channel equalization as well as for diversity combining and spatial interference suppression in MIMO systems [13].
Channel estimation for OFDM systems has been thoroughly researched and can be broadly classified into following three categories:

- Pilot-Aided Techniques,
- Blind Techniques, and
- Semi-Blind Techniques

Pilot-aided algorithms exploit periodically inserted training sequences known as Pilots [30], while blind or pilot-less techniques exploit the inherent redundancy present in the transmitted signal to get channel state information (CSI). Semi-blind algorithms try to improve the performance of blind algorithms by exploiting the knowledge of both known pilot symbols and properties of the transmitted signals at the same time.

Pilot-aided CE techniques offer good performance and low latency, but at the same time, are bandwidth-inefficient because of pilot overhead. Pilot-based techniques also reduce the effective SNR that is available for data symbols [28]. Blind techniques don’t waste the bandwidth needed for pilots but require iterations and hence introduce high latency; and generally offer poor performance. The aim of semi-blind channel estimation algorithms is to improve the performance offered by blind algorithms while requiring fewer known symbols than training based channel estimation algorithms to save bandwidth [2]. The main focus of this chapter is on pilot-aided channel estimation.

### 4.2 Pilot Structures

Pilot symbols known to the receiver are inserted into the OFDM symbol over a subset of available subcarriers. Such pilot symbols are used to “sample” the channel and measure the channel distortion. These pilots can be arranged in different fashions as shown in Figure 4.1.
4.2.1 Block-Type Pilots

In this pilot arrangement scheme, pilots are inserted at all subcarriers of some OFDM symbols. Such OFDM symbols (containing only pilots subcarriers) are referred to as pilot symbols or reference symbols. Reference OFDM symbols are then transmitted periodically with a specific time period [30]. Time-domain interpolation is then used to estimate the channel for intermediate data OFDM symbols. Block-type pilots are suitable for frequency selective channels because pilots are inserted at all subcarriers of reference symbols with a period in time.

4.2.2 Comb-Type Pilots

When the channel changes from one OFDM symbol to the next one, block-type pilot arrangement incurs too much pilot overhead and is not feasible. For fast fading channel conditions, comb-type pilot arrangement is used. In this scheme the pilots are inserted with a specific frequency period into subcarriers of each OFDM symbol [30]. The values of channel gains at remaining (data) subcarriers
are then obtained through frequency-domain interpolation. In order to keep track of the frequency selective channel, the pilots must be placed at least as frequently as coherent bandwidth. Contrary to block-type arrangement, the comb-type pilot scheme is suitable for fast-fading channels as pilots are inserted at all time instances. For frequency selective channel conditions, comb-type pilot arrangement is not suitable as it incurs too much pilot overhead. Moreover, comb-type pilots are badly affected throughout the entire OFDM frame at subcarriers where a deep fade hits the channel.

### 4.2.3 Lattice-Type Pilots

Comb-type pilot scheme is simpler to implement as positions occupied by pilot tones are fixed. However, such pilots suffer badly in case of a deep fade that hits some of these pilot subcarriers for entire duration of OFDM frame. This problem can be remedied by placing pilots at different subcarriers in different OFDM symbols. In other words, the pilots are inserted along both the time and frequency axes. Such pilot tones scattered in both time and frequency axes provide better channel tracking capabilities because they are more robust against deep fades [2].

### 4.2.4 Pilot Spacing

Pilot spacing in time and frequency dimensions depends upon coherence time and coherence bandwidth of the channel, respectively [49]. For block-type pilot arrangement, the maximum spacing between pilot tones must not exceed coherence time in order to keep track to time varying channel characteristics. Similarly for comb-type pilot arrangement, the maximum spacing between pilot tones must not exceed coherence bandwidth in order to keep track of frequency selective channel characteristics [9]. For lattice-type pilot arrangement, the maximum pilot spacing must not exceed coherence time as well as coherence bandwidth of the channel in order to keep track of the time varying and frequency selective
Let pilot tones are placed $N_f$ subcarriers apart in frequency dimension and $N_t$ OFDM symbols apart in time dimension. According to sampling theorem, pilot spacing in frequency dimension (cf. Figure 4.3) should be [39] 

$$N_f \times F_s < \frac{1}{\tau_{\text{max}}}$$

$$\Rightarrow \quad N_f < \frac{1}{\tau_{\text{max}} F_s} \quad (4.1)$$

According to sampling theorem, pilot spacing in time dimension (cf. Figure 4.3) should be [39] 

$$\frac{1}{N_t \times \hat{T}_s} > 2f_d$$

$$\Rightarrow \quad N_t < \frac{1}{2f_d \hat{T}_s} \quad (4.2)$$

where $\tau_{\text{max}}$ denotes maximum delay spread of channel, $F_s$ denotes subcarrier spacing, $f_d$ denotes maximum Doppler shift and $\hat{T}_s$ denotes the duration of CP extended OFDM symbol.

Pilot spacings specified by Equations (4.1) and (4.2) are the maximum allowable values. In practice however, $N_f$ and $N_t$ are fixed roughly to one-half of their maximum values. This corresponds to an oversampling by a factor of 2 and helps to relax the constraints of interpolation filters [2].

### 4.3 One Dimensional Channel Estimation

As stated in chapter 2, when ICI and ISI are assumed to be completely removed through cyclic prefix, each sub-channel can be considered as a flat fading channel. Consequently, the received OFDM symbol can be expressed as:

$$Y[k] = X[k] \cdot H[k] + Z[k] \quad , \quad k = 0, 1, \ldots N_c - 1 \quad (4.3)$$
where $Y[k]$, $X[k]$, $H[k]$, and $Z[k]$ denote the $k^{th}$ subcarrier component of the received symbol, transmitted symbol, channel frequency response, and noise in frequency domain, respectively.

In one dimensional (1D) channel estimation, pilot symbols are uniformly inserted across the OFDM symbol with period $N_f$ as shown in Figure 4.2. Let the first pilot is positioned at 1st subcarrier, then equally spaced pilots are positioned at subcarrier indices

$$k_p = pN_f, \quad p = 0, 1, \ldots, \left\lfloor \frac{N_c}{N_f} \right\rfloor - 1$$ (4.4)

where $\left\lfloor \cdot \right\rfloor$ denotes the rounding towards positive infinity. Thus total number of pilot symbols in an OFDM symbol becomes

$$N_{\text{pilot}} = \left\lfloor \frac{N_c}{N_f} \right\rfloor$$ (4.5)

Pilot-aided channel estimation operates in two steps [4]. In the first step, the initial channel estimates at pilot subcarrier indices, $\hat{H}[k_p]$, are found from received pilot symbols as

$$\hat{H}[k_p] = \frac{Y[k_p]}{X[k_p]} = H[k_p] + \frac{Z[k_p]}{X[k_p]}$$ (4.6)

where $Y[k_p]$ and $X[k_p]$ are the received symbols at pilot subcarriers and originally transmitted pilot symbols, respectively. In second step, the final estimates of the
complete CFR, $\hat{H}[k]$, are obtained from the initial estimates, $\tilde{H}[k_p]$, by a well known process of interpolation (See section 4.5).

### 4.4 Two Dimensional Channel Estimation

Channel estimation can be done in two dimensions by inserting pilot symbols on frequency time grid with the objective to “sample” the channel along two dimensions i.e., frequency and time. If we choose the pilot spacing in time and frequency direction sufficiently smaller than the channel coherence time and coherent bandwidth, final estimates of the CFR can be obtained by interpolation [4].

![2-D Channel Estimation](image)

Figure 4.3: Rectangular Arrangement of Pilots for 2D CE [4]

2D channel estimation works on OFDM frames and tries to estimate the 2D CFR, $H(f,t)$. The discrete channel response is denoted by $H_l[k]$ where $k = 0, 1, \ldots, N_c - 1$ represents subcarrier index and $l = 0, 1, \ldots, N_s - 1$ represents OFDM symbol index. $N_c$ is the total number of subcarriers per OFDM symbol and $N_s$ is the total number of OFDM symbols per OFDM frame. Figure 4.3 shows an exemplary OFDM frame made of 13 OFDM symbols while each symbol is comprised of 11 subcarriers. On the shown rectangular grid, the discrete pilot spacing in frequency and time direction is $N_f$ and $N_t$ respectively.

The received OFDM signal can be expressed as:

$$Y_l[k] = X_l[k] \cdot H_l[k] + Z_l[k], \quad k=0,1,\ldots,N_c-1, \quad l=0,1,\ldots,N_s-1 \quad (4.7)$$
where \( Y_l[k] \), \( X_l[k] \), \( H_l[k] \), and \( Z_l[k] \) denote the \( k^{th} \) subcarrier component of the \( l^{th} \) received symbol, transmitted symbol, channel frequency response, and noise in frequency domain, respectively. If \( X_{lp}[k_p] \) represents the pilot symbols where \( k_p \) and \( l_p \) specify the pilot symbol locations on frequency and time indices respectively. Hence, for uniformly space pilot tones, we have

\[
k_p = pN_f, \quad p = 0, 1, \ldots, \left\lfloor \frac{N_c}{N_f} \right\rfloor - 1
\]

\[
l_p = qN_t, \quad q = 0, 1, \ldots, \left\lfloor \frac{N_s}{N_t} \right\rfloor - 1
\]

When the first pilot is assumed at the first subcarrier of the first OFDM symbol, the total number of pilots in an OFDM frame becomes [4]

\[
N_{\text{pilot}} = \left\lfloor \frac{N_c}{N_f} \right\rfloor \cdot \left\lfloor \frac{N_s}{N_t} \right\rfloor
\]

(4.10)

Similar to 1D case, pilot-aided 2D channel estimation works in two steps. In the first step, initial channel estimates, \( \hat{H}_{lp}[k_p] \) at pilot locations are computed from pilot symbols as:

\[
\hat{H}_{lp}[k_p] = \frac{Y_{lp}[k_p]}{X_{lp}[k_p]}
\]

\[
= H_{lp}[k_p] + \frac{Z_{lp}[k_p]}{X_{lp}[k_p]}
\]

(4.11)

where \( Y_{lp}[k_p] \) and \( X_{lp}[k_p] \) are the received symbols at pilot subcarriers and originally transmitted pilot symbols, respectively. In second step, the final estimates of the complete CFR, \( \hat{H}_l[k] \), are obtained from the initial estimates, \( \hat{H}_{lp}[k_p] \), by a well known process of interpolation.
4.5 Interpolation Techniques For Pilot-Aided Channel Estimation

As mentioned earlier, the second step of pilot-aided channel estimation is to interpolate the initial estimates to all data subcarriers. 2D interpolation filters are usually too complex for practical implementations \[11\] and generally are implemented by cascading two 1D interpolators. Some important 1D interpolation techniques are described in the following.

4.5.1 Linear Interpolation

In linear interpolation, the intermediate values (channel estimates at data subcarriers) are based on a straight line connecting neighbouring pairs of data points (channel estimates at at pilot subcarriers). If the coordinates of two known data points are \((x_0, y_0)\) and \((x_1, y_1)\), then the straight line between these two points is known as “linear interpolant”. Then the interpolated value \(y\) (ordinate) at \(x\) (abscissa), when \(x\) lies in the range \((x_0, x_1)\) is \[50\]:

\[
y = y_0 + \frac{y_1 - y_0}{x_1 - x_0} (x - x_0) \tag{4.12}
\]

Linear interpolation on a set of data points \((x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)\) is defined as the concatenation of linear interpolants between each pair of data points. This function, denoted by \(\prod_1^H f\), is called “piecewise linear interpolation polynomial” of \(f\) and is expressed as \[51\]

\[
\prod_1^H f(x) = \prod_1^H y = y_i + \frac{y_{i+1} - y_i}{x_{i+1} - x_i} (x - x_i), \quad x \in I_i, \quad i=0,1,\ldots,n \tag{4.13}
\]

where \(H\) is the maximum length of the intervals \(I_i\). Hence, the estimated CFR
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obtained by piecewise linear interpolation polynomial becomes

\[ \hat{H}[k] = \hat{H}[k_p(i)] + \frac{\hat{H}[k_p(i+1)] - \hat{H}[k_p(i)]}{k_p(i+1) - k_p(i)}(k - k_p(i)), \forall k \in \left[ k_p(i), k_p(i+1) \right] \]  (4.14)

where \( k_p(i) \) is the \( i \)th value of \( k_p \) and \( \hat{H}[k_p(i)] \) is the initial CFR estimate at the \( i \)th value of \( k_p \). This method is feasible when the pilot tones are sufficiently closed.

4.5.2 Cubic-Spline Interpolation

In cubic-spline interpolation, an individual third order polynomial is assigned to each interval between data points and a piecewise cubic curve is produced [52]. The polynomial coefficients are determined by using additional data point adjacent to the two data points. The interpolation curve produced by this technique is continuous and smoothly fits to given data points. The interpolated spline between the data points \((x_i, f(x_i))\) and \((x_{i+1}, f(x_{i+1}))\) evenly spaced by \( h \) can be expressed mathematically as [50]:

\[ \hat{f}_i(x) = a_{3i} \left( \frac{x - x_i}{h} \right)^3 + a_{2i} \left( \frac{x - x_i}{h} \right)^2 + a_{1i} \left( \frac{x - x_i}{h} \right) + a_{0i} \]  (4.15)

where

\[ a_{3i} = \frac{g_{i+1}}{6} - \frac{g_i}{6} \]
\[ a_{2i} = \frac{g_i}{2} \]
\[ a_{1i} = f_{i+1} - f_i - \frac{g_i}{3} - \frac{g_{i+1}}{6} \]
\[ a_{0i} = f_i \]

The \( g \) terms are directly proportional to the second derivatives of the spline function at data points. The cubic-spline interpolation is computationally very extensive but the final interpolation curve is very smooth [50].
4.5.3 DFT-Based Interpolation

If, before taking the DFT, a time-domain signal is padded with zeros, the period of the signal extends. When such a zero-padded signal is passed through a DFT operation, the frequency resolution of the output frequency domain signal is increased. Hence, by padding the CIR with zeros, higher resolution in CFR can be obtained. Therefore, zero-padding in time domain results in interpolation in frequency domain [53].

Let initial CFR estimates are denoted by \( \hat{H}[k_p] \). The total number of initial estimates is same as total number of pilots i.e., \( N_{pilot} = \left\lfloor \frac{N_c}{N_f} \right\rfloor \). Hence, the vector containing initial channel estimates at pilot subcarriers can be expressed as \( \mathbf{p} = [ \hat{H}[0] \hat{H}[N_f] \hat{H}[2N_f] \ldots \hat{H}[(\left\lfloor \frac{N_c}{N_f} \right\rfloor - 1)N_f] ] \). Now taking inverse DFT of the vector of initial estimates as

\[
\hat{h}_i = \text{IDFT}\{\mathbf{p}\}
\] (4.16)

The length of time domain vector \( \hat{h}_i \) is \( N_{pilot} = \left\lfloor \frac{N_c}{N_f} \right\rfloor \) as well. Thereafter, this time domain signal is zero padded with \( N_c - N_{pilot} \) zeros in order to get a vector of length \( N_c \) as:

\[
\hat{h} = [ \hat{h}_i \ 0_{1 \times (N_c-N_{pilot})} ]
\] (4.17)

The estimated CFR vector of length \( N_c \) is then obtained by taking DFT of above vector i.e.,

\[
\hat{H}[k] = \text{DFT}\{\hat{h}\}
\] (4.18)

This vector contains channel estimates at all \( N_c \) subcarriers. Figure 4.4 summarizes the structure of DFT-based interpolator.

4.6 Pilot-Aided Channel Estimation Techniques

This section presents some widely used pilot-aided channel estimation schemes.
4.6.1 Least Squares CE

Let the received OFDM symbol at pilot subcarriers be expressed as:

\[ Y[k_p] = X[k_p] \cdot H[k_p] + Z[k_p], \quad k_p = 0, N_f, 2N_f, \ldots, N_{\text{pilot}} - 1 \]  \hfill (4.19)

where \( Y[k_p], X[k_p], H[k_p], \) and \( Z[k_p] \) denote the pilot subcarrier components of the received symbol, transmitted symbol, channel frequency response, and noise in frequency domain, respectively. Let \( \hat{H}[k_p] \) denotes the estimated CFR samples at pilot indices.

The objective of a channel estimation algorithm is to recover the channel matrix based on the knowledge of pilot and received data. Given an \( N_{\text{pilot}} \times 1 \) column vector \( \mathbf{Y} \), and an \( N_{\text{pilot}} \times N_{\text{pilot}} \) square matrix \( \mathbf{X} \), the least-squares problem seeks an \( N_{\text{pilot}} \times 1 \) column vector \( \hat{\mathbf{H}} \) by minimizing the following cost function [54]

\[ J_{LSE}(\hat{\mathbf{H}}) = \| \mathbf{Y} - \mathbf{X}\hat{\mathbf{H}} \|^2 \]  \hfill (4.20)

where \( \| \cdot \|^2 \) denotes the squared Euclidean norm of its argument e.g., for a column vector \( x \), \( \| x \|^2 = x^*x \). Least square estimator (LSE) tries to minimize the sum of squared errors (SSE) of prediction [55]. In other words, LSE minimizes the squared difference between the observation \( \mathbf{Y} \) (signal plus noise) and the assumed
signal data $XH$ [56].

Note that the column vector $Y$ is made up of entries from $Y[k_p]$, a square diagonal matrix $X$ is made up of diagonal entries from $X[k_p]$, and the column vector $\hat{H}$ is made up of entries from $\hat{H}[k_p]$. Now, expanding the expression of LSE criterion as

$$J_{LSE}(\hat{H}) = \left\| Y - X\hat{H} \right\|^2$$

$$= (Y - X\hat{H})^H(Y - X\hat{H})$$

$$= \left[ Y^H - (X\hat{H})^H \right] \left[ Y - X\hat{H} \right]$$

$$= \left[ Y^H - \hat{H}^HX^H \right] \left[ Y - X\hat{H} \right]$$

$$= Y^HY - Y^HX\hat{H} - \hat{H}^HX^HY + \hat{H}^HX^HX\hat{H} \quad (4.21)$$

where $(\cdot)^H$ denotes the Hermitian (conjugate) Transpose of its argument. Moreover, in deriving the Equation (4.21), we used the identities $(A+B)^H = A^H + B^H$ and $(AB)^H = B^HA^H$. The middle two terms of Equation (4.21) are Hermitian of each other and hence have same magnitude. Since, the LSE cost function depends upon $\| \cdot \|^2$, hence

$$J_{LSE}(\hat{H}) = Y^HY - 2Y^HX\hat{H} + \hat{H}^HX^HX\hat{H} \quad (4.22)$$

If $m$ and $b$ are two $n \times 1$ non-zero vectors and $A$ is an $n \times n$ symmetric matrix, then from vector calculus [57]

$$\frac{d}{dm}(b^Hm) = m \quad (4.23)$$

$$\frac{d}{dm}(m^HA\!m) = 2Am \quad (4.24)$$

Utilizing these two identities, the vector derivative of the LSE cost function with
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respect to $\hat{H}$ becomes

$$\frac{\partial J_{LSE}(\hat{H})}{\partial \hat{H}} = 0 - 2X^H Y + 2X^H X \hat{H}$$  \hspace{1cm} (4.25)$$

The next step is to set this partial derivative to zero and solving for the matrix $\hat{H}$ as \[57\]

$$- 2X^H Y + 2X^H X \hat{H} = 0$$

$$\Rightarrow \quad X^H Y = X^H X \hat{H}$$  \hspace{1cm} (4.26)$$

By pre-multiplying both side with $(X^H X)^{-1}$, we get

$$\hat{H}_{LSE} = (X^H X)^{-1} X^H Y = X^{-1} Y$$  \hspace{1cm} (4.27)$$

Therefore, least square channel estimates at pilot locations can be expressed as

$$\hat{H}_{LSE}[k_p] = \frac{Y[k_p]}{X[k_p]}, \quad k_p = 0, N_f, 2N_f, \ldots, N_{\text{pilot}} - 1$$  \hspace{1cm} (4.28)$$

LSE is a widely used CE method as it has low complexity and yields reasonable channel estimates. The LSE is also referred to as zero-forcing (ZF) estimator \[58\].

4.6.2 Minimum Mean Square Error CE

LSE utilizes received signal and known pilot symbols to estimate the channel. These channel estimates can be improved by exploiting the frequency correlation of fading channel. Exploiting the correlation among different CFR samples, we define a better estimate as \[9, 29\]

$$\hat{H} \triangleq W \hat{H}_{LSE}$$  \hspace{1cm} (4.29)$$
where $W$ is a weight coefficient matrix. The matrix $W$ can be determined by minimizing the following cost function \[29\] :

$$J_{MMSE}(\hat{H}) = E\{\|H - \hat{H}\|^2\}$$ \hspace{1cm} \text{(4.30)}$$

Note that this cost function is a function of weighting coefficient matrix $W$. Using the orthogonality principle (or projection theorem), the matrix $W$ can be expressed as \[59\]

$$W = R_{HHLSE} (R_{HLSE} LHSE)^{-1}$$ \hspace{1cm} \text{(4.31)}$$

where matrix $R_{HLSE} LHSE$ is the autocorrelation matrix of LSE estimates and matrix $R_{HHLSE}$ is the cross-correlation matrix between true channel gains and the LSE estimates. Assuming AWGN with variance $\sigma_z^2$ on each subcarrier, these correlation matrices can be expressed as \[9, 28, 37\] :

$$R_{HLSE} LHSE = E\{\hat{H}_{LSE}^H \hat{H}_{LSE}\} = R_{HH} + \frac{1}{\text{SNR}} I$$ \hspace{1cm} \text{(4.32)}$$

$$R_{HHLSE} = R_{HH}$$ \hspace{1cm} \text{(4.33)}$$

where SNR is the signal to noise ratio per pilot, $(\cdot)^H$ denotes Hermitian of its argument, and $I$ denotes the identity matrix. Hence, the weighting coefficient matrix $W$ becomes :

$$W = R_{HH} \left( R_{HH} + \frac{1}{\text{SNR}} I \right)^{-1}$$ \hspace{1cm} \text{(4.34)}$$

and consequently, the MMSE estimator can be expressed as

$$\hat{H}_{MMSE} = R_{HH} \left( R_{HH} + \frac{1}{\text{SNR}} I \right)^{-1} \hat{H}_{LSE}$$ \hspace{1cm} \text{(4.35)}$$

Hence, in order to get frequency domain MMSE channel estimates, CFR correlation at different frequencies and SNR is required. MMSE estimator generates
much better channel estimates than LSE but MMSE estimator is computationally very complex because a matrix inversion is involved every time the data $X$ changes [60].

4.6.3 Maximum Likelihood CE

Let the observed data $Y$ is a linear combination of vector $H$ and is corrupted by additive noise i.e.,

$$Y = XH + Z$$  \hspace{1cm} (4.36)

where $X$ is known $N_{pilot} \times N_{pilot}$ diagonal matrix, $Z$ is zero-mean $N_{pilot} \times 1$ random vector with covariance matrix $C_Z$. Moreover, $H$ and $Z$ are assumed to be uncorrelated. Under these conditions, the likelihood function is [59]

$$f(Y; H) = \frac{1}{(2\pi)^{N_{pilot}/2} \det(C_Z)} \exp \left[-\frac{1}{2}(Y - XH)^T C_Z^{-1} (Y - XH) \right]$$  \hspace{1cm} (4.37)

This likelihood function can be maximized by minimizing the following function [59]

$$J_{ML}(H) = (Y - XH)^T C_Z^{-1} (Y - XH)$$  \hspace{1cm} (4.38)

By differentiating this function with respect to $H$ and setting the derivative to zero, we get the maximum likelihood estimate of $H$ as [59, 61]

$$\hat{H}_{ML} = (X^T C_Z^{-1} X)^{-1} X^T C_Z^{-1} Y$$  \hspace{1cm} (4.39)

If noise is assumed to be white, then $C_Z = \sigma_Z^2 I_{N_{pilot}}$ [10], then maximum likelihood estimates can be simplified as [61]

$$\hat{H}_{ML} = (X^T X)^{-1} X^T Y$$  \hspace{1cm} (4.40)

where $(X^T X)^{-1} X^T$ is the pseudo-inverse of $X$. As ML estimator does not require channel statistics, it is simpler to implement.
4.7 Pilot-Aided CE for MIMO-OFDM Systems

MIMO technology is used to increase the capacity and performance of wireless communication systems. For wideband channels, OFDM is a good combination with MIMO for ISI mitigation and capacity improvement [29]. For MIMO-OFDM systems, channel information is vital for coherent demodulation as well as for diversity combining and spatial interference suppression [13]. The channel estimation requirement is further exacerbated in the case of OFDM systems with multiple transmit and/or receive antennas as the signals are simultaneously transmitted / received and consequently arrive at the receiver through many channels.

4.7.1 Pilot Schemes For MIMO-OFDM CE

In MIMO systems, the signal received at each antenna is the superposition of signals sent from multiple transmit antennas. Thus, the pilot sequences sent from each transmit antenna must be free from interference from other transmit antennas. Most commonly used MIMO-OFDM pilot patterns that are interference-free include Independent Pilot Pattern (IPP), Scattered Pilot Pattern (SPP), and Orthogonal Pilot Pattern (OPP) [13]. Independent pilot (time orthogonal) scheme assumes that channel remains constant over $M_T$ consecutive OFDM symbols, where $M_T$ is the number of transmit antennas. Scattered pilot (frequency orthogonal) scheme assumes the channel to be constant over just one OFDM symbol. Orthogonal pilot (code orthogonal) scheme assumes that channel remains constant over two consecutive OFDM symbols.

Figure 4.5 shows the independent and scattered pilot scheme for MIMO Systems. In IPP, only one transmit antenna sends pilot signal at a time while all other transmit antennas are switched off. Obviously this pilot scheme maintains orthogonality among pilots in time domain. Moreover, for an $M_T \times M_R$ MIMO channel, $M_T$ training signal times are required [13]. SPP preserves orthogonality among pilots in frequency domain i.e. each antenna sends pilot symbols on
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Figure 4.5: Independent and Scattered Pilots Schemes for MIMO-OFDM CE

Figure 4.6: Orthogonal Pilots Schemes for MIMO-OFDM CE
different subcarriers, whilst all other antennas send zeros on those subcarriers. OPP is made up of mathematically orthogonal pilots. As shown in Figure 4.6, both antennas simultaneously send pilots which are orthogonal in space and time. OPP are orthogonal in code.

4.7.2 MIMO-OFDM CE Using IPP

Consider the case of $2 \times 2$ MIMO transmission as shown in Figure 4.7. Transmitter $TX_1$ sends OFDM symbol $X_1$ at time $t_1$ while transmitter $TX_2$ sends OFDM symbol $X_2$ at that time. Then at time $t_2$, $TX_1$ transmits $X_3$ while transmitter $TX_2$ sends $X_4$. At receiver side, $RX_1$ and $RX_2$ receive $Y_1$ and $Y_2$ simultaneously at time $t_1$ and then $Y_3$ and $Y_4$ simultaneously at time $t_2$.

$$Y_1 = X_1H_{11} + X_2H_{21} + N_1$$  \hspace{1cm} (4.41a)  \\
$$Y_2 = X_1H_{12} + X_2H_{22} + N_2$$ \hspace{1cm} (4.41b)  \\
$$Y_3 = X_3H_{11} + X_4H_{21} + N_3$$ \hspace{1cm} (4.41c)  \\
$$Y_4 = X_3H_{12} + X_4H_{22} + N_4$$ \hspace{1cm} (4.41d)

where $N_1$, $N_2$, $N_3$, and $N_4$ represent AWGN noise vectors. In a more compact
form, the above equations can be written as:

\[
\begin{bmatrix}
Y_1 \\
Y_2 \\
Y_3 \\
Y_4
\end{bmatrix} =
\begin{bmatrix}
X_1 & X_2 \\
X_1 & X_2 \\
X_3 & X_4 \\
X_3 & X_4
\end{bmatrix} * \begin{bmatrix}
H_{11} & H_{21} \\
H_{12} & H_{22}
\end{bmatrix} + \begin{bmatrix}
N_1 \\
N_2 \\
N_3 \\
N_4
\end{bmatrix}
\] (4.42)

\[
\begin{bmatrix}
Y_1 \\
Y_2 \\
Y_3 \\
Y_4
\end{bmatrix} =
\begin{bmatrix}
X_3 & X_4 \\
X_3 & X_4
\end{bmatrix} * \begin{bmatrix}
H_{11} & H_{21} \\
H_{12} & H_{22}
\end{bmatrix} + \begin{bmatrix}
N_3 \\
N_4
\end{bmatrix}
\] (4.43)

where

\[
\mathbf{H} = \begin{bmatrix}
H_{11} & H_{21} \\
H_{12} & H_{22}
\end{bmatrix}
\] (4.44)

is the channel mixing matrix.

**Channel Estimation**

Sampling the received symbols \(Y_1, Y_2, Y_3, \) and \(Y_4\) at pilot locations readily gives the LS initial estimates of \(H_{11}, H_{12}, H_{21}\) and \(H_{22}\), respectively. The initial estimates \(\hat{H}_{11}[k_p], \hat{H}_{12}[k_p], \hat{H}_{21}[k_p]\) and \(\hat{H}_{22}[k_p]\) can then be used to get estimates at data subcarriers \(\hat{H}_{11}[k], \hat{H}_{12}[k], \hat{H}_{21}[k],\) and \(\hat{H}_{22}[k]\) through the process of interpolation, where \(k_p\) represents a subset of \(k = [0, 1, \ldots, N_{\text{pilot}} - 1]\) denoting the pilot location indices with \(N_{\text{pilot}}\) being total number of pilots.

**The Decorrelator Detector**

Using the estimated CFRs \(\hat{H}_{11}, \hat{H}_{12}, \hat{H}_{21}\) and \(\hat{H}_{22}\), ZF solution of Equations (4.42) and (4.43) gives the demodulated OFDM symbols as \([62]\)

\[
\hat{X}_1 = \frac{(Y_1 \hat{H}_{22} - Y_2 \hat{H}_{21})}{|\mathbf{H}|}
\] (4.45a)

\[
\hat{X}_2 = \frac{(Y_2 \hat{H}_{11} - Y_1 \hat{H}_{12})}{|\mathbf{H}|}
\] (4.45b)

\[
\hat{X}_3 = \frac{(Y_3 \hat{H}_{22} - Y_4 \hat{H}_{21})}{|\mathbf{H}|}
\] (4.45c)

\[
\hat{X}_4 = \frac{(Y_4 \hat{H}_{11} - Y_3 \hat{H}_{12})}{|\mathbf{H}|}
\] (4.45d)
where $|\hat{H}|$ is the determinant of estimated channel mixing matrix and is defined as:

$$|\hat{H}| \triangleq \hat{H}_{11}\hat{H}_{22} - \hat{H}_{12}\hat{H}_{21} \quad (4.46)$$

### 4.7.3 MIMO-OFDM CE Using SPP

Consider the case of $2 \times 2$ MIMO transmission as shown in Figure 4.8. Transmitters $\text{TX}_1$ and $\text{TX}_2$ send OFDM symbols $X_1$ and $X_2$ simultaneously. At receiver side, receivers $\text{RX}_1$ and $\text{RX}_2$ receive $Y_1$ and $Y_2$ simultaneously.

![Figure 4.8: 2 × 2 MIMO-OFDM CE Employing SPP](image)

The received OFDM symbols at two receive antennas can be written as:

$$Y_1 = X_1H_{11} + X_2H_{21} + N_1 \quad (4.47)$$

$$Y_2 = X_1H_{12} + X_2H_{22} + N_2 \quad (4.48)$$

where $N_1$ and $N_2$ represent AWGN noise vectors. In a more compact form, it can be written as:

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} X_1 & X_2 \end{bmatrix} \ast \begin{bmatrix} H_{11} & H_{21} \\ H_{12} & H_{22} \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \quad (4.49)$$

where $\mathbf{H} = \begin{bmatrix} H_{11} & H_{21} \\ H_{12} & H_{22} \end{bmatrix}$ is the channel mixing matrix.
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Channel Estimation

Keeping in mind the SPP scheme (cf. Figure 4.5), typical OFDM symbols employing SPP are as follow [63]

\[
X_1 = [P \ Z \ D \ D \ P \ Z \ D \ D \ldots] \quad (4.50)
\]
\[
X_2 = [Z \ P \ D \ D \ Z \ P \ D \ D \ldots] \quad (4.51)
\]

where \(X_1\) and \(X_2\) are simultaneously transmitted OFDM symbols; \(P\), \(Z\), and \(D\) indicate the location indices of pilot, zero, and data subcarriers in two OFDM symbols, respectively. Then \(Y_1\) samples at \(P\) locations readily yield initial estimates for \(H_{11}\) and \(Y_1\) samples at \(Z\) locations readily yield initial estimates for \(H_{21}\). Similarly, \(Y_2\) samples at \(Z\) locations readily yield initial estimates for \(H_{12}\) and \(Y_2\) samples at \(P\) locations readily yield initial estimates for \(H_{22}\). The initial estimates \(\hat{H}_{11}[k_p], \hat{H}_{12}[k_p], \hat{H}_{21}[k_p]\) and \(\hat{H}_{22}[k_p]\) can then be used to get estimates at data subcarriers \(\hat{H}_{11}[k], \hat{H}_{12}[k], \hat{H}_{21}[k],\) and \(\hat{H}_{22}[k]\) through the process of interpolation.

The Decorrelator Detector

Using the estimated CFRs \(\hat{H}_{11}, \hat{H}_{12}, \hat{H}_{21}\) and \(\hat{H}_{22}\), ZF solution of Equation (4.49) gives the demodulated OFDM symbols as [62]

\[
\hat{X}_1 = \frac{(Y_1\hat{H}_{22} - Y_2\hat{H}_{21})}{|H|} \quad (4.52)
\]
\[
\hat{X}_2 = \frac{(Y_2\hat{H}_{12} - Y_1\hat{H}_{11})}{|H|} \quad (4.53)
\]

where \(|H|\) is the determinant of estimated channel mixing matrix.
4.7.4 MIMO-OFDM CE Using OPP

Considering the same $2 \times 2$ MIMO transmission scheme shown in Figure 4.7, the received OFDM symbols at two receive antennas can be written as:

$$
\begin{align*}
Y_1 &= X_1 H_{11} + X_2 H_{21} + N_1 \\
Y_2 &= X_1 H_{12} + X_2 H_{22} + N_2 \\
Y_3 &= X_3 H_{11} + X_4 H_{21} + N_3 \\
Y_4 &= X_3 H_{12} + X_4 H_{22} + N_4
\end{align*}
$$

Channel Estimation

Hadamard matrices can be used to generate orthogonal pilot patterns that are orthogonal in time and space as [63]

$$
P = \begin{bmatrix} P_k & P_k \\ P_k & -P_k \end{bmatrix}
$$

where $P_k$ is the known pilot symbol at $k^{th}$ subcarrier. The orthogonal pilot sequence embedded in four OFDM symbols for $2 \times 2$ system can be depicted on a time-space grid as

$$
\begin{align*}
X_1 &= [+P \ D \ldots \ D \ +P \ D \ldots \ D] \\
X_2 &= [+P \ D \ldots \ D \ +P \ D \ldots \ D] \\
X_3 &= [+P \ D \ldots \ D \ +P \ D \ldots \ D] \\
X_4 &= [-P \ D \ldots \ D \ -P \ D \ldots \ D]
\end{align*}
$$
where horizontal axis is the time dimension while vertical axis is the space dimension. Now considering the received symbols at pilot subcarriers as

\[
Y_1[k_p] = (+P)H_{11}[k_p] + (+P)H_{21}[k_p] + N_1[k_p]
\]
\[
Y_2[k_p] = (+P)H_{12}[k_p] + (+P)H_{22}[k_p] + N_2[k_p]
\]
\[
Y_3[k_p] = (+P)H_{11}[k_p] + (-P)H_{21}[k_p] + N_3[k_p]
\]
\[
Y_4[k_p] = (+P)H_{12}[k_p] + (-P)H_{22}[k_p] + N_4[k_p]
\]

(4.57)

The orthogonality of pilots helps in simplifying the above set of equations and the following initial channel estimates are hence obtained.

\[
H_{11}[k_p] \approx \frac{Y_1[k_p] + Y_3[k_p]}{2P} = \hat{H}_{11}[k_p]
\]
\[
H_{12}[k_p] \approx \frac{Y_2[k_p] + Y_4[k_p]}{2P} = \hat{H}_{12}[k_p]
\]
\[
H_{21}[k_p] \approx \frac{Y_1[k_p] - Y_3[k_p]}{2P} = \hat{H}_{21}[k_p]
\]
\[
H_{22}[k_p] \approx \frac{Y_2[k_p] - Y_4[k_p]}{2P} = \hat{H}_{22}[k_p]
\]

(4.58)

These initial estimates at pilot locations \(\hat{H}_{11}[k_p], \hat{H}_{12}[k_p], \hat{H}_{21}[k_p]\) and \(\hat{H}_{22}[k_p]\) are used to get estimated CFRs at all subcarriers \(\hat{H}_{11}, \hat{H}_{12}, \hat{H}_{21}\) and \(\hat{H}_{22}\) through interpolation.
The Decorrelator Detector

Using the estimated CFRs $\hat{H}_{11}$, $\hat{H}_{12}$, $\hat{H}_{21}$ and $\hat{H}_{22}$, the demodulated OFDM symbols are

\[
\hat{X}_1 = \frac{(Y_1 \hat{H}_{22} - Y_2 \hat{H}_{21})}{|\hat{H}|} \quad (4.59a)
\]
\[
\hat{X}_2 = \frac{(Y_2 \hat{H}_{11} - Y_1 \hat{H}_{12})}{|\hat{H}|} \quad (4.59b)
\]
\[
\hat{X}_3 = \frac{(Y_3 \hat{H}_{22} - Y_4 \hat{H}_{21})}{|\hat{H}|} \quad (4.59c)
\]
\[
\hat{X}_4 = \frac{(Y_4 \hat{H}_{11} - Y_3 \hat{H}_{12})}{|\hat{H}|} \quad (4.59d)
\]

where $|\hat{H}| \triangleq \hat{H}_{11}\hat{H}_{22} - \hat{H}_{12}\hat{H}_{21}$ is the determinant of estimated channel mixing matrix.

4.8 Simulation Results and Performance Analysis

This section first presents the performance of LSE with different kinds of interpolation techniques. Next, the performance comparison of LSE and MMSE channel estimators is given. The performance of ML channel estimator is then presented with different number of pilot tones. Finally, the performance comparison of different pilot structure for indoor and outdoor MIMO channel scenarios is provided.

Figure 4.9 shows the performance of LSE with different interpolation methods. The simulation results are based on an OFDM systems with $N_c = 32$ total subcarriers and $N_{pilot} = 8$ pilot subcarriers in a multipath Rayleigh fading channel. According to the results shown, all the three interpolation methods almost give similar performance at low SNR values. However at higher SNR values, the DFT-based interpolation outperform the other two methods.
Figure 4.9 shows the performance comparison between LSE and MMSE channel estimators. The simulation results are based on the same OFDM system as mentioned above. According to the results shown, MMSE estimator’s performance is far better than LSE estimator especially at lower SNR values. The
price paid for better performance of MMSE estimator is the increased complexity.

Figures 4.11 shows the performance of maximum likelihood estimator for different number of pilot subcarriers. The simulation results are based on an OFDM systems with $N_c = 128$ total subcarriers in a multipath Rayleigh fading channel. According to the results shown, the performance of ML estimator is better than LS and MMSE estimators especially at lower SNR values.

![Figure 4.11: Performance of ML Channel Estimator](image)

Figures 4.12 and 4.13 show the performance comparison of different pilot structures for indoor and outdoor MIMO channel conditions. A $2 \times 2$ MIMO configuration is used along with $N_c = 128$ OFDM system. DFT-based interpolation is used for estimation of MIMO CFRs at data subcarriers. Zero-forcing (the decorrelator) receiver is used for the detection of spatially multiplexed OFDM data streams.

It is found that different pilot patterns offer different performances in different channel scenarios. For this purpose two environments were considered. Indoor environment is a channel with maximum delay spread equal to 5% of OFDM symbol duration and a maximum mobile velocity of 3km/h, resulting in a maximum
Doppler frequency of 5.55Hz when operating in the 2GHz band. The average user speed of 3 km/h characterizes a typical indoor office environment in a three floors office building where users are moving between an office room to the corridor or
vice versa [64]. Outdoor environment is a typical outdoor vehicular channel with maximum delay spread equal to 15% of OFDM symbol duration. Assuming high speed trains or vehicles on a motorway, the maximum mobile velocity is taken as 120km/h, which results in a maximum Doppler frequency of 222Hz.

As evident from the results, IPP and OPP schemes give almost similar performance and both are better than SPP scheme in a typical indoor environment. In a typical outdoor vehicular environment, SPP scheme outperforms IPP and OPP schemes. The reason is that both IPP and OPP schemes assume that the channel remains constant over two consecutive OFDM symbols. For high Doppler spreads (outdoor environment) this is not the case and hence performance is degraded.

### 4.9 Conclusion

Pilot-aided CE is most popular method among CE techniques and is computationally less intensive and hence easy to implement on real time systems. Moreover, Pilot-aided CE algorithms are more robust to high Doppler frequency, and hence, are useful for high mobility applications.

The MMSE estimator provides better performance than LSE at the cost of high complexity. The MMSE estimator requires knowledge of noise variance and channel auto-covariance. The LSE has low complexity than MMSE estimator as it does not use channel statistics, and has relatively high mean-square error (MSE). LSE is a very popular CE method because of its simplicity. ML estimator also requires no information about channel statistics and operating SNR.

Different pilot patterns have been tested for MIMO-OFDM systems. The results show that different pilot patterns offer different performances in indoor and outdoor environments. It is found that independent scheme along with FFT based interpolation gives best results for indoor channels. For vehicular channels (high Doppler spreads), scattered pilots along with FFT based interpolation offers the best performance.
Chapter 5

Under-Sampled Channel Estimation

5.1 Introduction

CE techniques are broadly classified into training-based, blind and semi-blind techniques. Training-based methods exploit known pilots whilst blind (pilot-less) schemes exploit the inherent redundancy of the transmitted signal to obtain channel state information. The former techniques offer good performance and low latency at the cost of bandwidth wastage while the latter ones are bandwidth efficient but offer poor performance and high latency. Semi-blind schemes aim at performance improvement over blind schemes while needing a fewer pilots than training-based algorithms [2, 28, 30].

Although the pilot symbols simplify the channel estimation task significantly, but inevitably reduces the spectral efficiency of the system. This fact has stimulated the interest in blind and semi-blind CE techniques which require fewer pilots than training-based methods. Blind and semi-blind CE techniques can be further subdivided into subspace based and decision directed methods [2]. Subspace based methods exploit the redundancy introduced by cyclic prefix or by virtual carriers to estimate the channel state information. A good sample of
the results obtained in this area can be found in [65–68] and references therein. In decision directed (DD) methods, previous data estimates (decisions) are exploited along with few pilots to improve the accuracy of channel estimates. The references [69–71] provide a good sample of decision directed CE techniques.

To reduce the pilot overhead used for training-based channel estimation algorithms, under-sampled CE methods for OFDM systems are proposed in this chapter. As indicated by their name, such methods use fewer number of pilots to sample the channel than those allowed by Nyquist sampling theorem. Contrary to subspace methods, the proposed techniques don’t require any cyclic structures caused by cyclic prefix or virtual carriers. Compared to decision directed methods, the proposed techniques don’t work in an iterative mode. Hence, the proposed techniques are computationally more efficient than decision directed methods as well as introduce lesser latency.

## 5.2 Virtually Blind Channel Estimation

The most primitive form of under-sampled CE methods is virtually blind (VB) CE [72]. In this scheme, only one pilot symbol is inserted at the start of each OFDM symbol as shown in Figure 5.1. Using this single pilot, CFR at first subcarrier is estimated using a simple LS algorithm. If the channel is slowly varying in frequency such that there is a high correlation between adjacent CFR samples, then channel estimate at first subcarrier can be used to decode the next data symbol at the second subcarrier. This data decision acts as a pilot, and in turn, is used to estimate the CFR at second subcarrier [73]. Hence, in general, CFR at $n^{th}$ subcarrier is estimated using the $n^{th}$ received symbol and data decision at $n^{th}$ subcarrier which in turn was calculated using the estimated CFR at $(n-1)^{th}$ subcarrier. Therefore, the virtually blind CE estimates the CFR in a successive manner based on the estimated CFR at previous location.

Using the only available pilot, the LS estimate of CFR at first subcarrier (pilot)
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Figure 5.1: Virtually Blind CE using Single Pilot

location is:

\[ \hat{H}_1 = \frac{Y_1}{X_1} \quad (5.1) \]

where \( Y_1 \) is the received symbol over first subcarrier and \( X_1 \) is the known signal transmitted at first subcarrier i.e., the only pilot symbol. Assuming \( H_2 \approx H_1 \), the estimated CFR at first subcarrier, \( \hat{H}_1 \), can be used to detect the transmitted data symbol at second subcarrier. For BPSK case, it can be expressed as

\[ \hat{X}_2 = \text{sign} \left( \Re \left( \frac{Y_2}{\hat{H}_1} \right) \right) \quad (5.2) \]

where zero forcing equalization has been followed by hard decision decoding (\( \text{sign}[\cdot] \) operation). \( \Re \) denotes the real part of complex number. This symbol decision, \( \hat{X}_2 \), is used, in turn, to estimate the CFR at second subcarriers as

\[ \hat{H}_2 = \frac{Y_2}{X_2} = \frac{X_2H_2}{X_2} \approx H_2 \quad (5.3) \]

In a similar fashion, \( \hat{H}_3 \) can be approximated by exploiting \( \hat{H}_2 \), and so on. In general [72]

\[ \hat{H}_n = \frac{Y_n}{X_n} \]

\[ = \frac{Y_n}{\text{sign} \left( \Re \left( \frac{Y_n}{H_{n-1}} \right) \right)} \]

\[ \approx H_n \quad (5.4) \]
5.2.1 CFR Inversion

The performance of VB channel estimator for an OFDM system with 256 sub-carriers in Rayleigh channel based on Equation (5.4) is shown in Figure 5.2. Obviously, the performance of VB estimator is quite unsatisfactory especially at higher SNR values. The BER performance curve saturates after $SNR = 40dB$ with an error floor just above $10^{-2}$. The source of this poor performance is the occasional occurrence of CFR inversion as depicted in Figure 5.3. In this figure, the CFR inversion hits the channel at about 65th subcarrier after which the estimated CFR is the reflected (sign reversed) approximation of true CFR.

![Figure 5.2: Performance of VB CE in Rayleigh Fading Environment](image)

CFR inversion is the sign reversal of estimated CFR as compared to actual CFR and occurs at the instants when the CFR gain values are very small i.e., when the power of actual CFR is in the vicinity of zero. When the power of actual CFR is very low, the received symbol’s power is also very low (being a product of input symbol and CFR sample). Under such circumstances, the decoding decision occasionally becomes incorrect because of either additive white noise or zero crossing of CFR values. Consequently the estimated CFR sample
gets inverted. From Equation (5.3); if $\hat{X}_2 = -X_2$ then $\hat{H}_2 \approx -H_2$ and this sign reversal of $\hat{H}_2$ from $H_2$ causes CFR inversion. Even in the absence of additive noise, the CFR inversion may occur as shown in Figure 5.4. This explains the reason why there is no improvement in the BER performance after SNR = 40 dB in Figure 5.2. No matter how strong is the signal as compared to noise, the CFR inversion still may hit the channel and hence there is an error floor of just above $10^{-2}$, as evident in Figure 5.2.

The noise-free CFR inversion occurs when the two consecutive CFR samples are of opposite sign i.e., when the CFR amplitude crosses the zero axis; provided the CFR power is very small. In that case, the assumption $H_{n+1} \approx H_n$ is no
longer valid, and the estimated CFR at $n^{th}$ subcarrier can be expressed as

$$
\hat{H}_n = \frac{Y_n}{\text{sign} \left[ \Re \left( \frac{Y_n}{H_{n-1}} \right) \right]}
= \frac{Y_n}{\text{sign} \left[ \Re \left( \frac{X_n H_n}{H_{n-1}} \right) \right]}
= \frac{Y_n}{\text{sign} \left[ \Re \left( \frac{X_n H_n}{H_{n-1}} \right) \right]}
\approx \frac{Y_n}{\text{sign} \left[ \Re (X_n \times -1) \right]}
\approx -H_n
$$

This shows that the CFR zero crossing causes wrong decoding decision and subsequently leads to CFR inversion. The term “sign $\left[ \Re \left( \frac{H_n}{H_{n-1}} \right) \right]$” needs further investigation. Let $a$ and $c$ be the real parts; and $b$ and $d$ the imaginary parts of two consecutive CFR samples $H_n$ and $H_{n-1}$, respectively. Then

$$
\text{sign} \left[ \Re \left( \frac{H_n}{H_{n-1}} \right) \right] = \text{sign} \left[ \Re \left( \frac{a + jb}{c + jd} \right) \right] = \text{sign} \left[ \Re \left( \frac{ac + bd}{c^2 + d^2} \right) \right] = \text{sign} \left( \frac{ac + bd}{c^2 + d^2} \right)
$$

When the Equation (5.6) results in “$-1$” (cf. Equation (5.5)), the receiver makes an erroneous decision and the VB estimator suffers from CFR inversion. Therefore, it can be concluded that the CFR inversion, even in the absence of additive noise, occurs when [72]:

- Both real and imaginary CFR amplitudes cross zero axis (equivalently, both product terms $ac$ and $bd$ in Equation (5.6) are negative).
- One of the real or imaginary CFR amplitudes crosses zero axis (equivalently,
either $ac$ or $bd$ term is negative and its magnitude is higher than that of positive product).

The left side of the Figure 5.5 shows the case when CFR inversion is caused by the simultaneous zero axis crossing of real and imaginary part of original CFR. The right side of Figure 5.5 depicts the situation when only the zero crossing of the real part of CFR is the cause of CFR inversion. Obviously, in both cases, the CFR inversion occurs when the original CFR power is very small.

![Figure 5.5: Cause of CFR Inversion in the Absence of Noise](image)

Regardless of the original cause of CFR inversion, the CFR inversion ripples forward along with wrong decoding decisions at all subsequent subcarriers. This is because VB channel estimate at current subcarrier location are based on the previous estimate. The situation prevails until and unless a second CFR inversion hits the channel, as shown in Figure 5.6. In the event of second CFR inversion, the re-inverted CFR automatically gets rectified. In Figure 5.6, the first inversion hits at about 60th subcarrier while the second inversion occurs around 140th subcarrier after which the estimated CFR automatically gets corrected.

The CFR inversion examples given in Figures 5.3 - 5.6 are all based on an OFDM systems with 256 subcarriers under 4-tap Rayleigh channel with exponentially decaying PDP.
5.3 Probabilistic Analysis of CFR Inversion

CFR inversion is the factor accountable for poor performance of virtually blind channel estimator. If the location of CFR inversion is found by any means, then it can be easily undone and consequently VB channel estimator can become a useful technique. In order to do so, the information about the likelihood of CFR inversion is very important.

As found in previous section, the CFR inversion occurs when either of the following channel conditions prevail:

- When real and imaginary parts of CFR cross the zero axis simultaneously.
- When one of them crosses zero-axis and the other one is close to zero axis.

In short, when the CFR amplitude values lie close to zero, the chances of CFR inversion increase. This fact is quite evident from Figure 5.7. Therefore, when the amplitude of CFR goes below a certain threshold level, the likelihood of occurrence of CFR inversion increases.

Figure 5.8 depicts the likelihood of occurrence of CFR inversion as a function of threshold level for different values of SNR. The results shown provide us some insight regarding the behavior of occurrence of CFR inversion and may help us to predict the location of CFR inversion.

Figure 5.8 provides two important bits of information. First, for a fixed value
Figure 5.7: CFR Amplitudes at CFR Inversion Locations (Without Noise)

Figure 5.8: Probability of Occurrence of CFR Inversion
of SNR, the probability of CFR inversion decreases as the threshold level increases. For example, at SNR = 20 dB, the probability of inversion goes below $10^{-3}$ as the threshold goes above 0.14 V. In other words, the probability of CFR inversion is inversely proportional to threshold level. Secondly, for a fixed threshold, the probability of CFR inversion at higher SNR values is considerably less than that at lower SNR values. For example, at a threshold level of 0.1 V, the probability of inversion is about $2 \times 10^{-4}$ at 30 dB while probability of inversion at 5 dB is $4 \times 10^{-2}$. Both of these results are intuitively satisfying as well.

Since probability of CFR inversion is inversely proportional to threshold level, hence BER decreases with the increase in threshold. Moreover, since probability of CFR inversion introduces errors in the estimated CFR, hence BER is directly proportional to probability of inversion. These two facts are depicted in Figure 5.9 and 5.10. All the simulation results in Figures 5.7 - 5.10 are based on an OFDM systems with 256 subcarriers under 4-tap Rayleigh channel with exponentially decaying PDP.
5.4 Locating CFR Inversion (CFRI)

As mentioned earlier, if we are able to find the exact location of CFR inversion (CFRI), then it is quite straightforward to reverse the effects of CFRI. This section explores a couple of techniques to locate CFRI.

5.4.1 Locating CFRI Using Spectrogram

In order to realize how spectrogram can be used to find CFRI location, it is important to understand the difference between ordinary Fourier Transform and Short-Time Fourier Transform.

Short-Time Fourier Transform and Spectrogram

The ordinary Fourier Transform decomposes a signal into complex exponential functions of different frequencies. The way it does this is defined by the following equation [74]:

\[
F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt
\]  

(5.7)

The signal \(f(t)\) is multiplied with an exponential term with frequency \(\omega\) and then integrated over all times. The information provided by the integral corresponds to all time instances, since the integration is done from \(-\infty\) to \(+\infty\) over time. Therefore, no matter where in time the component with frequency \(\omega\) appears, it will affect the result of the integration. Because of this reason, Fourier transform is not suitable for non-stationary signals (signals having time varying frequencies). Fourier transform only provides information regarding the presence of different spectral components but tells nothing about their location. When the time localization of spectral components is needed, a transform giving time-frequency representation of the signal is needed.
The Short-Time Fourier Transform (STFT) is used to find the spectral composition of finite local segments of non-stationary time-varying signals. The function to be transformed is multiplied with a window function. Then the (ordinary) Fourier Transform of the resultant signal is computed while the window is continuously slid along the time axis. Consequently we get a two dimensional time-frequency representation of the original signal. Mathematically [75]:

$$STFT\{f(t)\} = F(\tau, \omega) = \int_{-\infty}^{\infty} f(t)w(t - \tau)e^{-j\omega t}dt$$ (5.8)

where \(w(t)\) is the window function. The spectrogram is usually given as the squared magnitude of the STFT [41] as

$$Spectrogram\{f(t)\} = |F(\tau, \omega)|^2 = \left| \int_{-\infty}^{\infty} f(t)w(t - \tau)e^{-j\omega t}dt \right|^2$$ (5.9)

A spectrogram is a visual representation (an image) that displays the time variations of short-time magnitude spectrum of a time-varying signal [76]. A spectrogram can easily be created using STFT in which a discrete time signal is segmented into overlapping chunks, and magnitude frequency spectrum of each chunk is calculated using Fourier Transform. The magnitude frequency spectrum of each chunk then corresponds to a vertical line in the spectrogram. Therefore, spectrogram is an intensity plot of the Short-Time Fourier Transform. This process is outlined in Figure 5.11 in the form of a block diagram.

![Figure 5.11: Block Diagram Representation of Spectrogram](image-url)
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Spectrogram of VB-Estimated CFR in the Absence Of Noise

Figure 5.13 represent the spectrogram for VB-estimated CFR for the CFR inversion shown in Figure 5.12. As evident from Figure 5.13, the spectrogram (or equivalently STFT) of estimated CFR exactly indicates the location of CFR inversion by easily distinguishable vertical bar at CFR inversion location [73].

Figure 5.12: CFR Inversion around 40th Subcarriers

Figure 5.13: Spectrogram of VB-Estimated CFR

Spectrogram of VB-Estimated CFR in the Presence Of Noise

Figure 5.15 represents the spectrogram for VB-estimated CFR before and after smoothing against the CFR inversion case shown in Figure 5.14. This time CFR inversion occurs around 50th Subcarrier in the presence of additive noise with SNR = 20dB. Smoothing is done with an 11-point weighted moving average
filter. As evident from Figure 5.15, the spectrogram of estimated CFR after smoothing indicates the location of CFR inversion by a vertical bar at CFR inversion locations. However, because of the additive noise, this bar is not very sharp.

![Figure 5.14: CFR Inversion around 50th Subcarrier at SNR = 20dB](image1)

![Figure 5.15: Spectrogram of Estimated CFR before after Smoothing at SNR = 20dB](image2)

### 5.4.2 Locating CFRI Using Second Derivative

Since the amplitude of VB-estimated CFR abruptly changes its direction at the instance of CFR inversion, the derivative of estimated CFR can help us to locate CFR inversion position [72].
Second Derivative In The Absence Of Noise

At the point of CFR inversion, the VB-estimated CFR abruptly changes its slope. Therefore, the derivative at that location has an abrupt change in its sign. This situation is depicted in Figure 5.17 where the gradients of real and imaginary parts take a sudden jump from positive values to negative values at CFR inversion location. This figure corresponds to the CFR inversion case shown in Figure 5.16.

The second derivative of estimated CFR (cf. Figure 5.18) more clearly highlights the location of CFR inversion with a spike. If we add the absolute values of second derivatives for real and imaginary parts, it gives us a spike with higher peak to average ratio [72]. This absolute sum of 2nd derivatives is shown in Figure 5.19. Figure 5.20 shows the absolute amplitude of estimated CFR. As highlighted by a □, the absolute amplitude has a v-shaped notch at CFR inversion instance which confirms that inversion takes place when the CFR amplitude is in the vicinity of zero.

The BER for noise-free case for 256 subcarriers OFDM systems in 4-tap Rayleigh fading channel was found to be $1.96 \times 10^{-2}$. After finding the CFR location with the help of second derivative and consequently rectifying the estimated CFR, the BER for noise-free case becomes $8.54 \times 10^{-5}$.

![Figure 5.16: CFR Inversion around 40th Subcarrier](image-url)
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Figure 5.17: First Derivative of Estimated CFR

Figure 5.18: Second Derivative of Estimated CFR

Figure 5.19: Sum of Real and Imaginary 2nd Derivatives

Figure 5.20: Absolute Values of Estimated CFR Amplitudes
Second Derivative In The Presence Of Noise

The location finding of CFR inversion is more challenging in the presence of additive noise. It is important to reduce the effects of noise before taking a derivative. One way to do this is to use a moving average filter which removes the noise and smooths a noisy signal [72]. However, as the number of points in the filter is increased to reduce noise further, the signal becomes more and more smooth and the sharp features of the signal itself start losing their sharpness [77, 78]. It means that the noise reduction process also eliminates the abrupt amplitude changes at CFR inversion. Therefore, there exists a trade-off between noise reduction and locating CFR inversion.

After reducing noise, the abrupt change in estimated CFR’s amplitude is measured by taking its second derivative. The peaks of the second derivative indicate the potential location(s) of CFR inversion. Figure 5.21 shows the use of second derivative to locate CFR inversion in the presence of additive noise at SNR = 20dB. There is a sharp spike around 60th subcarriers in the 2nd derivative (bottom left plot) at the CFR inversion. CFR inversion location is further confirmed by a v-shaped notch in absolute amplitude of estimated CFR indicating a very low power at CFRI (bottom right plot).

Sometimes, the additive noise generates spurious spikes in the 2nd derivative. Figure 5.22 shows an example of a spurious spike around 20th subcarriers in the 2nd derivative (bottom left plot) indicating a false CFRI. Generally, such spurious spikes are major hindrance in locating CFRI. Fortunately, sometimes such spurious spikes can be eliminated by looking at the absence of v-shaped notch at their corresponding subcarrier locations in absolute amplitude of estimated CFR (e.g., bottom right plot of Figure 5.22). Moreover if the CFR inversion happens just at the beginning, the 2nd derivative fails to locate CFRI. The reason behind this inability is the special beginning- and end-points treatment of moving average filter. This situation is depicted in Figure 5.23 where CFR inversion occurs in the very beginning but the 2nd derivative tells nothing about it.
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Figure 5.21: 2nd Derivative in the Presence of Noise at SNR = 20dB

Figure 5.22: Spurious Spikes in 2nd Derivative Caused by Noise
5.5 Uniformly Spaced Additional Pilots

Keeping in view the mild performance offered by virtually blind channel estimator (cf. Figure 5.27) and the fact that we can predict the occurrence of CFR inversion with a certain level of accuracy, the performance offered by VB CE technique can be improved by augmenting the only pilot with few more pilots [79].

As described in Chapter 4, the pilot spacing in frequency dimension according to sampling theorem should be [39]

\[
N_f < \frac{1}{\tau_{\text{max}} \times F_s}
\] (5.10)

where \(\tau_{\text{max}}\) and \(F_s\) denote maximum delay spread of channel and subcarrier spacing, respectively. Practically, the channel is usually over-sampled by a factor of 2 in order to relax the constraints of interpolation filters [2]. In other words, the maximum pilot spacing allowed by Equation (5.10) is reduced to its half. As an
example, for a DAB system in a typical urban channel with $\tau_{max} = 5\mu sec$ and $F_s = 992Hz$; the pilot distance calculated according to Equation (5.10) comes to be 201. However, the actual pilot spacing as described in DAB specifications is 12 [2] which corresponds to over-sampling by a factor of 16.

In multicarrier communication, the transmission data is usually arranged in the form of frames, each frame comprising some fixed number of OFDM symbols. In environments characterized by low mobility such as WLAN systems, the channel remains static over the entire OFDM frame duration. Consequently the channel coherence time is expected to be greater than the total frame duration [2]. Under such channel environments, if we uniformly scatter some additional pilots over time-frequency grid in such a way that there is only one pilot per OFDM symbol, then a better channel sampling can be achieved. We store an entire OFDM frame on receiver side before we get the full estimated CFR.

![Uniformly Spaced Pilot Tones in an OFDM Frame](image)

In Figure 5.24, the horizontal axis specifies the frequency axis and represents the subcarrier index in a given OFDM symbol. The vertical axis specifies the time axis and represents OFDM symbol index in an OFDM frame. Depending upon the tolerable latency of the system, we can insert the same number of pilots in whole OFDM frame as the number of OFDM symbols to be stored in receiver memory. Typically, the number of pilot subcarriers needed for LS and MMSE channel estimators is 12.5% of total subcarriers [11]. On the other hand, for under-sampled channel estimator, the number of required uniformly spaced pilot subcarriers is typically 6.25%.
By increasing the number of pilots, we actually increase the robustness of channel estimator against deep fades where the likelihood of occurrence of CFR inversion is high. Since every new pilot gets a new channel sample from a distant subcarrier location than the previous pilot, hence the effect of previous CFR inversion, if any, doesn’t propagate beyond this point.

5.6 Dynamic Allocation of Additional Pilots

Instead of inserting additional pilots uniformly over the time-frequency grid, we can utilize the knowledge of probability of CFR inversion to dynamically assign additional pilots. In this closed-loop scheme, the channel estimation procedure is divided into an acquisition step followed by a tracking phase as shown in Figure 5.25.

![Figure 5.25: Dynamically Assigned Pilot Tones in an OFDM Frame](image)

At 1st subcarrier of first OFDM symbol of every frame, an initial acquisition pilot is sent. The receiver then performs channel estimation using this only pilot according to Equation (5.4) and decides about the potential locations of CFRI as well. The receiver does this by utilizing the probability of inversion, \( P(I) \), information. This information regarding the potential CFRI locations is then
sent back to the transmitter through a feedback channel. The transmitter then responds to this information by sending pilots in the next OFDM symbol at the subcarriers pointed out by the receiver.

5.7 Under-Sampled CE For MIMO-OFDM Systems

Figure 5.26 shows a simple schematic for $2 \times 2$ MIMO-OFDM system. We use spatial multiplexing technique in order to achieve higher data rates. This technique involves the transmission of multiple independent data streams from different transmit antennas. Furthermore, keeping in view the basic idea of VB CE method, it becomes clear that for $2 \times 2$ MIMO scheme we will need 2 null OFDM symbols from each of the two transmitting antennas. When one of the transmit antenna transmits either initial acquisition pilot or dynamically requested pilots, the other transmit antenna must be silent at that time. Moreover, zero-forcing (the decorrelator) receiver is used for the detection of spatially multiplexed OFDM data streams.

$x_{mn}$ and $y_{mn}$ represent the transmitted and received OFDM symbols with the first subscript, $m$, representing (transmit or receive) antenna index and the second subscript, $n$, representing the time index. For example, $x_{12}$ is the 2nd OFDM symbol transmitted from 1st transmit antenna and $y_{24}$ is the 4th OFDM symbol received at 2nd receive antenna. The first two OFDM symbols received at both antennas can be expressed in terms of CFRs as:

$$y_{11} = x_{11}h_{11} + x_{21}h_{21} + n_1$$  \hspace{1cm} (5.11a)
$$y_{21} = x_{11}h_{12} + x_{21}h_{22} + n_2$$  \hspace{1cm} (5.11b)
$$y_{12} = x_{12}h_{11} + x_{22}h_{21} + n_3$$  \hspace{1cm} (5.11c)
$$y_{22} = x_{12}h_{12} + x_{22}h_{22} + n_4$$  \hspace{1cm} (5.11d)
where $N_1$, $N_2$, $N_3$, and $N_4$ represent AWGN noise vectors. The four received OFDM symbols $Y_{11}$, $Y_{21}$, $Y_{12}$ and $Y_{22}$ are used to obtain the initial versions of four channel responses i.e., $\hat{H}_{11}$, $\hat{H}_{12}$, $\hat{H}_{21}$, and $\hat{H}_{22}$ respectively. These four estimates are obtained using only two acquisition pilots with the aid of two null OFDM symbols. These initial estimates $\hat{H}_{11}$, $\hat{H}_{12}$, $\hat{H}_{21}$, and $\hat{H}_{22}$ are used to estimate the potential CFR inversion locations. The potential locations are then feedback to transmitters which consequently respond by sending additional pilots at requested subcarriers in the next OFDM symbols. The receivers receive the additional dynamic pilots in the following four OFDM symbols:

$$Y_{15} = X_{15}H_{11} + X_{25}H_{21} + N_5 \quad (5.12a)$$
$$Y_{25} = X_{15}H_{12} + X_{25}H_{22} + N_6 \quad (5.12b)$$
$$Y_{16} = X_{16}H_{11} + X_{26}H_{21} + N_7 \quad (5.12c)$$
$$Y_{26} = X_{16}H_{12} + X_{26}H_{22} + N_8 \quad (5.12d)$$
Zero-Forcing Receiver – The Decorrelator

The receivers then reconstruct the final CFRs $\hat{H}_{11}$, $\hat{H}_{12}$, $\hat{H}_{21}$ and $\hat{H}_{22}$ using the additional dynamic pilots in order to rectify the CFR inversion(s). The estimated CFRs $\hat{H}_{11}$, $\hat{H}_{12}$, $\hat{H}_{21}$ and $\hat{H}_{22}$ are then used to demodulate the OFDM symbols using the decorrelator receiver as follow

$$\hat{X}_{11} = \frac{(Y_{11}\hat{H}_{22} - Y_{21}\hat{H}_{21})}{|\hat{H}|} (5.13a)$$
$$\hat{X}_{12} = \frac{(Y_{12}\hat{H}_{22} - Y_{22}\hat{H}_{21})}{|\hat{H}|} (5.13b)$$
$$\hat{X}_{21} = \frac{(Y_{21}\hat{H}_{11} - Y_{11}\hat{H}_{12})}{|\hat{H}|} (5.13c)$$
$$\hat{X}_{22} = \frac{(Y_{22}\hat{H}_{11} - Y_{12}\hat{H}_{12})}{|\hat{H}|} (5.13d)$$

where $|\hat{H}|$ is the determinant of estimated channel mixing matrix and is defined as:

$$|\hat{H}| \triangleq \hat{H}_{11}\hat{H}_{22} - \hat{H}_{12}\hat{H}_{21} (5.14)$$

The equations (5.13a) - (5.13d) are the ZF solutions of equations (5.11a) - (5.11d).

5.8 Simulation Results and Performance Analysis

This section first presents the performance of virtually blind channel estimator with and without 2nd derivative based correction. Next, the performance of under-sampled channel estimator with statically as well as dynamically assigned additional pilots is given. The average number of dynamic pilots per OFDM frame as a function of SNR is also presented. Finally, the performance of under-sampled estimator with dynamic pilots for MIMO-OFDM systems is presented.

The results shown in this section are based on an OFDM system with $N_c =$
256 and/or $N_c = 512$ in Rayleigh fading channel with exponentially decaying power delay profile and $\tau_{\text{max}} = 6.25\%$ of OFDM duration. In this case, the Nyquist sampling theorem governs the pilot spacing to be $N_f = 16$ according to Equation (5.10). Hence the number of pilots less than $N_c/N_f = 16$ correspond to under-sampled case. The operating frequency is 2 GHz; which means that the proposed methods are suitable for applications operating in low microwave frequency range or more specifically in ultra high frequency (UHF) band.

Figure 5.27 shows the performance of VB estimator after 2nd derivative-based correction in Rayleigh fading environment. Because of the problems associated with moving average filter’s behavior at endpoints and spurious spikes caused by noise, the performance improvement after CFR correction is not very impressive.

![Figure 5.27: Performance of VB CE After 2nd Derivative-Based Correction](image)

Figure 5.28 shows the BER performance for various numbers of uniformly spaced pilots scattered over time-frequency grid. As evident from this figure, every time we double the number of pilots, we get improvement in BER especially at higher SNR values. Moreover, inserting more pilots in one OFDM frame means we have to store more OFDM symbols at the receiver to get the full estimate of CFR, so the cost paid for better BER performance is the more latency introduced.
Figure 5.28: Performance of Under-Sampled CE with Uniformly Scattered Pilots in the system. The improvement in BER vs latency at SNR = 20 dB is shown in Figure 5.29.

Figure 5.30 shows the performance with dynamically assigned pilots for an OFDM system with 256 subcarriers in 4-tap Rayleigh fading channel. Figure 5.31 shows the same results for an OFDM system with 512 subcarriers in 4-tap Rayleigh fading channel. As compared to single pilot based virtually blind CE (shown dotted), this scheme offers substantial improvement in BER performance. The cost paid for this improvement in BER performance is the increased number of pilots. Fortunately, the required number of pilots decreases rapidly with increasing SNR value.

Figures 5.32 and 5.33 show the average number of tracking pilots per OFDM frame as a function of SNR. The overall average number of dynamic pilots for 256 subcarrier OFDM system is 4.9 while for 512 subcarrier OFDM system is 7.8. For both of these cases, the number of pilots used to sample the channel are fewer than those allowed by Nyquist sampling theorem.

Figure 5.34 shows the BER performance comparison between uniformly spaced fixed pilots and dynamically assigned pilots for an OFDM system with $N_c = 256$. 
Figure 5.29: BER Improvement Vs Latency at SNR = 20 dB

Figure 5.30: BER Performance with Dynamically Assigned Pilots with $N_c = 256$
CHAPTER 5. UNDER-SAMPLED CHANNEL ESTIMATION

Figure 5.31: BER Performance with Dynamically Assigned Pilots with $N_c = 512$

Figure 5.32: Average No. of Dynamic Pilots per OFDM Frame for $N_c = 256$

Figure 5.33: Average No. of Dynamic Pilots per OFDM Frame for $N_c = 512$
The dynamic pilots with an average of 4.9 pilots perform almost same as 16 uniformly space fixed pilots.

![Performance Comparison between Fixed and Dynamically Assigned Pilot Schemes with $N_c = 256$](image)

The fixed tracking pilot scheme scatters the pilots over whole OFDM frame so the receiver has to wait for full frame to receive in order to get full CFR estimate. On the other hand, the dynamic scheme can estimate the full CFR as soon as it receives the dynamically assigned tracking pilots. Therefore, the dynamic scheme introduces lesser latency and can be used for relatively fast varying channel environments. Moreover, the dynamic scheme is more flexible in terms of number of tracking pilots used, as we can control the number of tracking pilots depending upon the prevailing SNR conditions.

The disadvantage of dynamic scheme is that it is a closed loop system and hence needs a feedback channel in order to convey the potential locations of CFR inversion to the transmitter. This feedback channel may introduce errors and hence the overall performance of the system may degrade. Moreover, the dynamic scheme is computationally more extensive as it has to compute the locations of potential CFR inversions from the initial version of estimated CFR along with
CHAPTER 5. UNDER-SAMPLED CHANNEL ESTIMATION

the information of probability of inversion and then has to reconstruct the CFR again.

Figure 5.35 shows the BER performance of under-sampled CE with dynamic additional pilots for $2 \times 2$ MIMO-OFDM system. As compared to SISO case, the BER performance offered by this $2 \times 2$ MIMO system is considerably better. The reason for this improvement is that each MIMO receiver requests additional pilots for two different CFRs to the same transmitter. Hence, the received additional pilots are combination of request for two CFRs. Therefore, the improved performance is achieved at the cost of additional pilot overhead.

![Figure 5.35: Performance of Under-Sampled CE with Dynamic Pilots for 2 \times 2 MIMO-OFDM system with \(N_c = 512\)](image)

The proposed techniques in this chapter are computationally more efficient than training based MMSE channel estimator. For an OFDM system with $N_c = 256$ subcarriers, the typical execution running time for MMSE estimator with 64 pilot subcarriers on an AMD Turion 64 X2 1.90 GHz PC is $7 \times 10^{-3}$ sec. For the same OFDM system, the typical execution time for primitive VB channel estimator is $1.63 \times 10^{-4}$ sec. For statically and dynamically pilot schemes, the typical execution time values are $2.37 \times 10^{-4}$ sec and $8.8 \times 10^{-4}$ sec respectively.
This computational efficiency is owing to the fact that unlike MMSE estimator, the proposed techniques neither require computation of correlation matrices nor matrix inversion is involved.

5.9 Conclusion

This chapter presented under-sampled channel estimation techniques for OFDM systems. In such techniques, the number of pilots used to sample the channel are less than those allowed by Nyquist sampling theorem. Virtually blind CE uses only one pilot to estimate the CFR. The performance of VB CE is severely damaged by the occurrence of CFRI. A couple of methods were discussed to locate the CFRI. Uniformly spaced fixed additional pilots and dynamically assigned additional pilots were then augmented with the only pilot in order to take more samples of channel and to stop propagating CFRI effect further to remaining subcarriers. The under-sampled CE, according to simulation results, are also useful for MIMO-OFDM systems.

For static and quasi-static channel scenarios like WLAN, the proposed under-sampled CE methods can be effectively used. Unlike some popular CE techniques, under-sampled methods are independent of channel statistics and operating SNR i.e., they don’t require the knowledge of channel parameters like noise variance and channel covariance matrix; and operating SNR. Therefore, this method is simpler to implement.
Chapter 6

Joint Blind Channel and Control Signal Estimation

6.1 Introduction

This chapter presents a technique for joint estimation of channel and control information for OFDM systems. In this technique, the control signal information is embedded into the pilot sequence intended for channel estimation. A blind estimation technique based on channel correlation between neighboring subcarriers in the frequency domain [14] is used for pilot sequence recovery at receiver. In this way, the estimation of control information dependent pilot sequence at the receiver side reveal control signal information in addition to channel response.

The results show that the proposed method works satisfactorily; and accurate channel and control signal can be achieved in the practical range of SNR values. Different pilot sequences have different detectability capabilities in different channel scenarios. Therefore, performance of various pilot sequence types under different channel environments is investigated as well. At the end, an algorithm for generating a set of pilot sequences with enhanced detectability is proposed. Subsequently, the desirable properties of pilot sequence with respect of detectability are discussed.
6.2 Need Of Side Information in Various Multi-carrier Systems

This section introduces some multicarrier transmission schemes where the insertion of side information (SI) plays a crucial role. In such transmission schemes, SI is mandatory in a sense that it informs the receiver about what has been done with transmitted signal in the transmitter [80].

One of the main disadvantages of OFDM scheme is high PAPR. Higher PAPR lowers the efficiency of circuits like power amplifiers (PA), ADCs, and low-noise amplifiers (LNA) etc [32]. Hence, a reduction in PAPR is highly desirable. Several PAPR reduction techniques have been presented in literature and many of them are reviewed in [2] and [5]. Many popular PAPR reduction techniques including selective mapping (SLM), partial transmit sequence (PTS) and interleaving method utilize some redundant information known as SI. The receiver must know this SI in order to reverse what has been done at transmitter and to accurately retrieve the transmitted data [14, 80].

Another scenario which entails side information is MIMO precoding systems or adaptive beamforming algorithms [14,81]. For example, as discussed in [82], an essential requirement of the adaptive beamforming algorithm is the transmission of side information in every MC-CDMA (or OFDM-CDMA) symbol.

6.2.1 Transmission of Side Information

Side information is the overhead data which needs to be conveyed to receiver. Naturally, there are two obvious choices regarding transmission of SI. The first choice is to transmit side information over a separate channel than the data channel. Obviously, this approach results in a waste of bandwidth and higher power consumption. The second choice is to embed side information within the data, but it results in loss of data rate. Moreover, if the SI is received in error, it will considerably affect the performance of the system. In some cases, even
the entire data block is lost if the SI is not correctly detected. To overcome this problem, generally a strong channel code is used as a protection of SI which results in a further loss of data rate [14, 80].

6.3 Control Information Dependent (CID) Pilots Transmission Scheme

To overcome the aforementioned problems associated with SI transmission, a technique to embed the SI within pilot tones intended for channel estimation is proposed in [14]. In this technique, all the pilots carry the side information and simultaneously “sample” the channel transfer function. Since no extra pilots are being used, this techniques conserves bandwidth and transmission power. It is important to indicate that the technique described in [14] is quite different from that described in [83]. In the latter technique, half of the pilot subcarriers are replaced by side information and an iterative channel estimation scheme is used for retrieving the channel information. On the other hand, in the former technique, a simple LS channel estimator followed by interpolation can be used for CE after extracting the control information from the pilots.

As mentioned earlier, many popular OFDM standards reserve some percentage of subcarriers for pilots in every OFDM symbol. These pilots act as reference signals and are primarily used for channel estimation. This section explains how we can use the same pilots (intended for CE) for the purpose of transmission of side information as proposed in [14].

Let us consider an OFDM system where every OFDM symbol has total $N$ subcarriers, $N_p$ pilot subcarriers and $N - N_p$ data subcarriers. Therefore, the transmitted OFDM symbol $X[k]$ can be expressed as

$$X[k] = \begin{cases} 
P[k] & k \in \varphi \\
D[k] & \text{otherwise} 
\end{cases}$$  (6.1)
where \( P[k] \) represents pilot symbols transmitted over \( k^{th} \) subcarriers; \( D[k] \) represents data symbols transmitted over \( k^{th} \) subcarriers; and \( \varphi \) is a subset of \( N \) subcarriers consisting of \( N_p \) pilot subcarriers. The received corresponding OFDM symbol at the receiver is

\[
Y[k] = X[k] \cdot H[k] + N[k]
\]

where \( Y[k] \) is the received OFDM symbol, \( X[k] \) is the transmitted OFDM symbol, \( H[k] \) is the CFR vector and \( N[k] \) is the AWGN vector.

### 6.4 Placement and Detection Techniques for CID Pilot Sequences

This section describes different techniques for placement locations of control information dependent pilots. Moreover, pilot sequence selection method at transmitter and pilot sequence detection method at receiver is explained in detail.

#### 6.4.1 Scattered Pilot Tones at Fixed Locations

As apparent from name, the pilot tones are scattered uniformly along frequency dimension, and are always placed at specific predetermined subcarriers. This placement technique is depicted in Figure 6.1.

![Figure 6.1: Placement of Scattered Pilot Tones at Fixed Locations](image)

For this scheme to work, the transmitter has a finite pool of \( L \) pilot sequences, where \( L \) depends upon the size of the control information (CI) to be transmitted. A pilot sequence \( P_m[k] \) is then picked from the pool of sequence in accordance with the intended control information. The transmitter does this by consulting
a control information – pilot sequence mapping table. Table 6.1 shows such a mapping table for mapping a 3-bit control signal onto \( L = 8 \) pilot sequences.

<table>
<thead>
<tr>
<th>Control Information</th>
<th>Pilot Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0</td>
<td>( P_1 )</td>
</tr>
<tr>
<td>0 0 1</td>
<td>( P_2 )</td>
</tr>
<tr>
<td>0 1 0</td>
<td>( P_3 )</td>
</tr>
<tr>
<td>0 1 1</td>
<td>( P_4 )</td>
</tr>
<tr>
<td>1 0 0</td>
<td>( P_5 )</td>
</tr>
<tr>
<td>1 0 1</td>
<td>( P_6 )</td>
</tr>
<tr>
<td>1 1 0</td>
<td>( P_7 )</td>
</tr>
<tr>
<td>1 1 1</td>
<td>( P_8 )</td>
</tr>
</tbody>
</table>

Table 6.1: Fixed Location Pilot Sequence Mapping Table [14]

The transmitted OFDM symbol thus becomes

\[
X[k] = \begin{cases} 
  P_m[k] & k \in \varphi \\
  D[k] & \text{otherwise} 
\end{cases} \quad (6.2)
\]

The receiver also has knowledge of all possibilities of transmitted pilot sequence and their positions (fixed, in this case). The first step towards detection is to have a normalized received signal \( \hat{Y}_i[k] \) by dividing the received signal at pilot subcarriers with each possible pilot sequence.

\[
\hat{Y}_i[k] = \frac{Y[k]}{P_i[k]} \quad k \in \varphi, i = 1, 2, \ldots, L \quad (6.3)
\]

When \( i = m \), the corresponding normalized received signal vector \( \hat{Y}_m[k] \) will be

\[
\hat{Y}_m[k] = \frac{P_m[k] \cdot H[k] + N[k]}{P_m[k]} \quad k \in \varphi, \quad i = m
\]

\[
= H[k] + \frac{N[k]}{P_m[k]} \quad (6.4)
\]

This normalized signal \( \hat{Y}_m[k] \) is a somewhat noisy version of CFR at pilot locations \( H[k]_{k \in \varphi} \); and its elements will be most correlated than those of all other normalized signal vectors \( \hat{Y}_{i \neq m}[k] \). Therefore, it is now quite obvious that the
correlation between neighboring subcarriers of CFR can be exploited to detect \( P_m[k] \) [14].

Therefore, the next step towards detection is to find the correlation between the neighboring elements of all the normalized received signals \( \hat{Y}_i[k] \). It is done by using a differentiator followed by an integrator as shown below

\[
g(i) = \sum_{N_p} [\text{diff}(\hat{Y}_i[k])] \quad \text{for } k \in \varphi, i = 1, 2, \ldots, L \tag{6.5}
\]

where \( g(i) \) represents the correlation between the neighboring elements of \( \hat{Y}_i[k] \). Obviously, the highest correlated \( \hat{Y}_i[k] \) will have the lowest \( g \) value. Hence, the transmitted pilot sequence index is

\[
l = \text{min}[g(i)] \tag{6.6}
\]

The index \( l \) and control information - pilot sequence mapping table is then used to retrieve the control signal corresponding to \( P_l[k] \). After deciding about the control information, the least square channel estimates at pilot locations could be calculated as

\[
\hat{H}[k] = \frac{Y[k]}{P_l[k]} \quad k \in \varphi \tag{6.7}
\]

where \( \hat{H}[k] \) are the LS estimates of CFR at \( N_p \) pilot subcarriers. In order to estimate the CFR at remaining \( (N - N_p) \) data subcarriers, DFT-based interpolation could be used. In this process, the vector of initial channel estimates is transformed to time domain using IDFT and a zero-padding is accomplished before retransforming this time domain signal back to frequency domain using DFT.

### 6.4.2 Scattered Pilot Tones at Fixed Locations With Feedback

Utilizing the fact that pilot tone locations are fixed and are known to receiver, we can introduce feedback from previous symbol’s channel estimates \( \hat{H}[k](t - 1) \)
to get better results [14]. In other words, instead of using correlation among neighboring subcarriers of CFR, correlation between two successive estimates of CFR (i.e., at time $t$ and $t + 1$) at fixed pilot subcarrier locations is utilized. This will obviously result in better detection of pilot sequence because it is based on correlation between successive time samples of channel response.

The first step towards detection is similar to the previous technique i.e., to have a normalized received signal $\tilde{Y}_i[k]$ by dividing the received signal at pilot subcarriers with each possible pilot sequence successively:

$$\tilde{Y}_i[k] = \frac{Y[k]}{P_i[k]} \quad k \in \varphi_{i = 1, 2, \ldots, L} \tag{6.8}$$

When $i = m$,

$$\tilde{Y}_i[k] = \tilde{Y}_m[k] \quad k \in \varphi_{i = 1, 2, \ldots, L}$$

$$= H[k] + \frac{N[k]}{P_m[k]}$$

$$\approx H[k]\big|_{k \in \varphi} \tag{6.9}$$

Hence, the correlation between elements of $\tilde{Y}_m[k]$ and the corresponding subcarriers in $\hat{H}[k](t-1)$ will be the correlation between successive time samples of the channel response; and hence will be very high.

Therefore, the next obvious step in detection process is to calculate the correlation between normalized received signal $\tilde{Y}_i[k]$ and corresponding subcarriers in the previous symbol’s channel response estimate $\hat{H}[k](t - 1)|_{k \in \varphi}$ as

$$g(i) = \sum_{N_p} \left[ \tilde{Y}_i[k](t) - \hat{H}[k](t - 1) \right] \quad k \in \varphi_{i = 1, 2, \ldots, L} \tag{6.10}$$

Finally, the transmitted pilot sequence index $l$ is

$$l = \min[g(i)] \tag{6.11}$$
6.4.3 Scattered Pilot Tones at Variable Locations

The performance of correlation-based detector, as described in previous section, depends upon the pool size of pilot sequences. The higher number of pilot sequences elevates the chances of errors in detection. Therefore, in order to reduce the number of pilot sequences in the pool and simultaneously to maintain the same control information transmission rate, we can introduce variable set of pilot tones locations [14]. Figure 6.2 depicts CID pilot placement scheme with variable locations.

![Figure 6.2: Placement of Scattered Pilot Tones at Variable Locations](image)

Let us assume, we introduce $N_s$ different set of locations for pilot subcarriers. Hence, we can maintain the same control information rate with smaller pool size of $\hat{L} = \frac{L}{N_s}$ pilot sequences. The control information is mapped onto $P_m[k]$ (one of the $\hat{L}$ possible pilot sequences) at subcarrier locations $\varphi_p$ (one of the $N_s$ possible location sets). In other words, the control information becomes a function of pilot sequence index as well as pilot tones location index. Hence, the transmitter needs to refer to a three dimensional (3D) control information - pilot sequence mapping table (See Table 6.2).

Similar to the fixed location method without feedback, the detection process for variable pilot tones locations starts with the calculation of normalized received signal vectors. However, in this case, the normalized received signal vectors $\hat{Y}_{i,j}[k]$ are obtained for each possible pilot sequence in the pool as well as
at each possible pilot locations.

\[ \hat{Y}_{i,j}[k] = \frac{Y[k]}{P_i[k]} = \sum_{k \in \varphi_j} P_i[k] = 1, \ldots, L \]

(6.12)

where \( \varphi_j \) denotes one of the \( N_s \) different sets of pilot tones locations.

When \( i = m \) and \( j = p \), the corresponding normalized received signal vector \( \hat{Y}_{m,p}[k] \) becomes

\[
\hat{Y}_{m,p}[k] = \frac{P_m[\varphi_p]}{P_i[\varphi_p]} \cdot H[\varphi_p] + N[\varphi_p] \\
= H[\varphi_p] + \frac{N[\varphi_p]}{P_m[\varphi_p]} \\
\approx H[\varphi_p] 
\]

(6.13)

This means that the normalized signal \( \hat{Y}_{m,p}[k] \) is a somewhat noisy version of CFR at pilot locations \( H[k]_{k \in \varphi_p} \); and its elements will be most correlated than those of all other normalized signal vectors \( \hat{Y}_{i \neq m, j \neq p}[k] \). Therefore, it is now quite obvious that the correlation between neighboring subcarriers of CFR can be exploited to detect the correct pilot sequence and the correct location set i.e., \( P_m[\varphi_p] \) [14].

Hence, the next step towards detection is to find the correlation between the neighboring elements of all the normalized received signals \( \hat{Y}_{i,j}[k] \). Again, it is
done by using a differentiator followed by an integrator as shown below

\[ g(i, j) = \sum_{N_p \in \varphi_j} \left[ \text{diff}(\hat{Y}_{i,j}[k]) \right]_{k \in \varphi_j} \]

\[ i = 1, 2, \ldots, L \]

\[ j = 1, 2, \ldots, N_s \]

(6.14)

where \( g(i, j) \) represents the correlation between the neighboring elements of \( \hat{Y}_{i,j}[k] \).

Obviously, the highest correlated \( \hat{Y}_{i,j}[k] \) will have the lowest \( g \) value. Hence, the transmitted pilot sequence index \( l_r \) and the pilot location index \( l_c \) are

\[ [l_r, l_c] = \min[g(i, j)] \]

(6.15)

These two indices \( l_r \) and \( l_c \) are then used to retrieve the control information by referring back to the control information - pilot sequence mapping table. After extracting the control information, the least square channel estimates at pilot locations becomes a straightforward procedure

\[ \hat{H}[k] = \frac{Y[k]}{P_{l_r,k}} \quad k \in \varphi_{l_c} \]

(6.16)

where \( \hat{H}[k] \) are the LS estimates of CFR at \( N_p \) pilot subcarriers. In order to estimate the CFR at remaining \( (N - N_p) \) data subcarriers, DFT-based interpolation could be used.

6.4.4 Block-Type Pilot Tones at Fixed Locations

Similar to the original scattered pilot scheme at fixed locations, the pilots in this scheme are also placed at predetermined fixed locations. However, more than one pilots at consecutive subcarriers are lumped together to form a block of pilots; and then pilot blocks are scattered uniformly at fixed subcarrier locations. Figure 6.3 clarifies this placement scheme.

The transmission and detection procedure is almost similar to the original scattered pilot technique at fixed locations. The transmitter picks a particular pilot sequence \( P_m[k] \) from the pool of available pilot sequences by consulting a
control information - pilot sequence mapping table. The receiver first finds normalized signal vectors $\hat{Y}_i[k]$ by dividing the received signal at fixed pilot locations with each possible pilot sequence successively. Next, the correlation $g(i)$ between neighboring elements of all the normalized signals $\hat{Y}_i[k]$ is calculated. The transmitted pilot sequence $P_i[k]$ is then decided according to $l = \min [g(i)]$ i.e., the most correlated of all normalized sequences. The index $l$ and mapping table are finally used to retrieve the control sequence corresponding to $P_i[k]$.

Again similar to the original scattered pilot scheme at fixed locations, block-type pilot scheme can utilize the fact that pilot tones occupy fixed known locations. Hence feedback can be introduced from previous symbol’s channel estimates $\hat{H}[k](t-1)$ and better performance can be achieved.

Clustering of pilot tones at consecutive subcarriers will obviously increase the correlation among the neighboring elements of normalized signal vectors. Hence, an improvement in the pilot sequence detection is anticipated. However, it will degrade the channel estimator performance because the channel is now being “sampled” less frequently or more sparsely.

6.5 Simulation Results And Performance Analysis

This section first describes the OFDM system model and wireless channel model used for simulations. Next simulation results in terms of pilot sequence error rate (PER) are presented and compared for all the CID pilot placement techniques.
The used OFDM systems have $N = 256$ subcarriers in total; and a subset $\varphi$ of $N_p = 64$ subcarriers are reserved for pilot sequence transmission in all placement techniques. Therefore, the spacing between two consecutive pilots is 4. The CIR used in simulations is

$$h(t) = \sum_{v=1}^{V} (\alpha_v + j\beta_v)\delta(t - \tau_v)$$

(6.17)

where $V$ is the number of multipaths, $\alpha_v$ and $\beta_v$ are path gains for $v^{th}$ path and are iid Gaussian random variables, and $\tau_v$ is the delay of $v^{th}$ multipath. The number of multipaths, $V$, is 4; the maximum delay spread of CIR, $\tau_{\text{max}}$, is 12.5% of OFDM symbol length; and the maximum Doppler shift, $f_d$, is taken as 500 Hz. Moreover, the block-fading model is assumed i.e., the channel impulse response remains constant during the entire OFDM symbol and changes randomly from one OFDM symbol to the next one.

Figure 6.4 shows the performance of scattered pilot tones at fixed locations in Rayleigh channel. A pool size of $L = 32$ pilot sequences is used where each pilot sequence is consisted of 64 subcarriers. This means that we can transmit 5-bit control information on the same pilots intended for channel estimation. As shown in Figure 6.4, the 4-PAM pilot sequences achieve the best results than rest of the investigated sequence types. On the other hand, Walsh Hadamard (WH) and Convolutional Codes (CC) pilot sequences yield worst performances.

Figure 6.5 shows the performance of scattered pilot tones at fixed locations with feedback in Rayleigh channel environment. The results exhibit an improvement in performance which again is predictable and can be explained in terms of detector’s structure. As explained in section 6.4.2, this type of detector exploits the correlation between successive time samples of channel response rather than correlation between successive subcarriers in frequency domain.

The performance of scattered pilots at variable locations in Rayleigh channel is presented in figure 6.6. The results shown are for $N_s = 4$ possible sets of
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Figure 6.4: Performance Of Scattered Pilot Tones at Fixed Locations in Rayleigh Channel [14]

Figure 6.5: Performance Of Scattered Pilot Tones at Fixed Locations With Feedback
pilot locations. Therefore, in order to transmit 5-bit control information, a pool size of $L = 8$ pilot sequences is used where each pilot sequence is consisted of 64 subcarriers. The simulation results show that the performance improvement is not upto expectation. Although a reduction in the total number of different sequences in the pool reduces the chances of erroneous detection, but at the same, the uncertainty about the exact location of pilot sequence increases the chances of incorrect detection.

![Figure 6.6: Performance Of Scattered Pilot Tones at Variable Locations in Rayleigh Environment](image)

Figures 6.7 and 6.8 show the performance of block-type pilots at fixed locations. A block size of 3 pilots is used, and hence the spacing between two consecutive pilot blocks is 12. As shown in figure 6.7, there is a tremendous performance improvement for all four type of pilot sequences under Rayleigh environment. Both 4PAM and QPSK sequences perform equally well without having an error floor. WH and CC sequences also improve considerably and approach an PER of $10^{-2}$ at higher SRN values.

Figure 6.8 show that clustering of pilots at consecutive subcarriers does not provide improvement when used with feedback technique and perform almost
Figure 6.7: Performance Of Block-Type Pilot Tones at Fixed Locations in Rayleigh Environment

Figure 6.8: Performance Of Block-Type Pilot Tones at Fixed Locations With Feedback
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Figure 6.9: Performance Comparison of CE based on Scattered and Block-Type Pilot Tones at Fixed Locations

similar to scattered pilots at fixed locations with feedback. This is due to the fact that clustering of pilots at consecutive SCs boosts the frequency domain correlation while feedback technique exploits the correlation between successive time samples of channel response.

As suggested earlier, the performance improvement of block-type pilots scheme is owed to the fact that clustering of pilot tones at consecutive subcarriers increases the correlation among the neighboring elements of normalized signal vectors. However, since the pilot spacing has increased simultaneously from 4 to 12, the channel estimator performance is lowered because of the fact that the channel is being “sampled” less frequently. The performance degradation of block-type pilots CE as compared with scattered pilots CE is shown in figure 6.9 in terms of mean squared error (MSE)

\[ MSE = \frac{1}{N} \sum_{n} |H - \hat{H}|^2 \]

Figure 6.9 shows that at lower SNR values, both estimators almost perform
equally. However, as the SNR value increases, the block-type channel estimator’s performance becomes inferior.

6.6 Designing Pilot Sequences With Enhanced Detectability

As evident from the results shown in previous section, the performance of joint channel and control information estimator for OFDM systems is strongly dependent on the properties of the pilot sequences used. The desirable properties of pilot sequences which enhance the sequence detectability have not yet been investigated in current literature. This section thoroughly discusses such properties as well as presents a practical algorithm for generating near-optimal sequences which can achieve excellent performance even in the low SNR region.

6.6.1 Receiver Structure

Keeping in mind the structure of correlation-based detector as explained in section 6.4.1, the detector utilizes a series of difference and summation operations. The operation of detector can be summarized in the following steps:

1. $L$ normalized received signal vectors are obtained by dividing the received signal with each possible pilot sequence at pilot locations.

2. The correlation factor between neighboring subcarriers of CFR at pilot locations is computed for all $L$ normalized vectors, using a “Summation of Difference” (SoD) operation.

3. The transmitted sequence is then detected corresponding to the minimum correlation factor of all $L$ values.
6.6.2 Desirable Properties of Pilot Sequences

According to step 1 outlined in section 6.6.1, any two sequences of the sequence set should produce a “Maximum Summation of Difference after Division” (MSDAD) [84]. This is to get a larger correlation factor in case of “wrong” sequence and smaller correlation factor for “correct” sequence. This is explained below with the help of a hypothetical pilot sequence set of \( L = 2 \) sequences.

Let us assume that \( seq1 = [s_{11} \ s_{12} \ s_{13} \ s_{14}] \) and \( seq2 = [s_{21} \ s_{22} \ s_{23} \ s_{24}] \) are two pilot sequences. When \( seq1 \) is transmitted at pilot locations, the frequency domain received signal in an ideal noiseless case is \( [s_{11} \cdot H_1 \ s_{12} \cdot H_2 \ s_{13} \cdot H_3 \ s_{14} \cdot H_4] \). The two normalized received sequences, in this case, will be

\[
\tilde{Y}_1[k] = \begin{bmatrix} s_{11} \cdot H_1 & s_{12} \cdot H_2 & s_{13} \cdot H_3 & s_{14} \cdot H_4 \end{bmatrix} \quad k \in \varphi
\]

and

\[
\tilde{Y}_2[k] = \begin{bmatrix} s_{11} \cdot H_1 & s_{12} \cdot H_2 & s_{13} \cdot H_3 & s_{14} \cdot H_4 \end{bmatrix} \quad k \in \varphi
\]

These can be achieved if the alphabets of the pilot sequences are arranged in such a fashion that they produce MSDAD.

Therefore, in order to avoid false detection, or equivalently, to have MSDAD distance, every parenthesis in equation (6.20) should produce a maximum possible value.

\[
\sum \left[ \left( \frac{s_{12}}{s_{22}} - \frac{s_{11}}{s_{21}} \right) + \left( \frac{s_{13}}{s_{23}} - \frac{s_{12}}{s_{22}} \right) + \left( \frac{s_{14}}{s_{24}} - \frac{s_{13}}{s_{23}} \right) \right]
\]

This can be achieved if first fraction in every parenthesis returns a maximum possible number and second fraction yields a minimum possible number. Therefore, the alphabets of the pilot sequences should be arranged in such a fashion that they produce MSDAD.
Hence, the very first property of sequences with enhanced or robust detectability is that every two members of the sequence set should have maximum possible SDAD values. The second desirable property is the “impulse-like” autocorrelation function (or equivalently flat power spectral density) of every member sequence. This means that every member sequence should be spectrally “white”, or in other words, should have consecutive samples completely uncorrelated. This is desired for having a higher correlation factor in case of false detection. Otherwise any sort of periodicity or regularity (non-randomness) within the sequence will generate a smaller correlation factor and can lead to a false detection.

The third desirable property is that every two members of the sequence set should have zero or minimum cross-correlation coefficient. This means that every two sequences must be independent or mathematical dissimilar to each other. This is obviously desired to maintain maximum dissimilarity among the different sequences and consequently enhances the correct detectability capability of sequence set.

6.6.3 Algorithm For Generating Proposed MSDAD Sequences

The algorithm for generating proposed “Maximum Summation of Difference After Division” (MSDAD) sequences is as follows [84]:

1. Alphabets or primitive constituent components of the sequence are chosen. Alphabets may be real or complex.

2. Calculate all possible permutations (rearrangements) of chosen alphabets. Note that, the number of permutations of alphabets set of \( n \) elements will be \( n! \) (\( n \) factorial).

3. “Summation of Difference After Division” (SDAD) distance is calculated for every permutation with respect to all other permutations.
4. Permutations with maximum SDAD distance are selected and then arranged in such a manner that every sequence formed have “impulse-like” auto-correlation function and simultaneously every two sequences have minimum cross-correlation coefficient.

### 6.6.4 Simulation Results and Performance Analysis

Figures 6.10 and 6.11 show the performance comparison of 4PAM and MSDAD sequences for different values of pool size, \( L \). It is quite evident from these figures that for all values of pool size, MSDAD sequences outperform 4PAM sequences which have already proven to be superior than rest of the sequence types.

Initially, 4PAM alphabet set was chosen to design MSDAD sequences i.e.,

\[
\Omega_{4PAM} = \{\pm 1, \pm 3\}
\]  

(6.21)

Then \(4! = 24\) possible permutations of alphabet set \(\Omega_{4PAM}\) were computed. Then SDAD distances were calculated for every permutation with respect to all other permutations. The MSDAD distance in this case was found to be ‘10’. For example, for the following two permutations

\[
\{ +1 , +3 , -1 , -3 \}
\]

and

\[
\{ -3 , +1 , +3 , -1 \}
\]

the SDAD distance calculated according to (6.20) is found to be 10 and is the MSDAD value.

Next, permutations with this MSDAD distance were then selected and rearranged in such a manner that every sequence formed had “impulse-like” auto-correlation function and simultaneously every two sequences had minimum cross-correlation coefficient. The average value of cross-correlation coefficient of two sequences was found to be 0.0667 i.e., 6.67% average similarity for \(4 \times 64\) MSDAD sequences. An example autocorrelation function for one of the \(4 \times 64\) MSDAD
Figure 6.10: Performance of 4PAM scattered pilot sequences at fixed locations with different pool sizes in Rayleigh channel.

Figure 6.11: Performance of MSDAD scattered pilot sequences at fixed locations with different pool sizes in Rayleigh channel.
sequences is shown in figure 6.12.

![Autocorrelation Function for an example MSDAD sequence](image)

Figure 6.12: Autocorrelation Function for an example MSDAD sequence

### 6.7 Conclusion

This chapter presented the joint blind channel and control signal estimation for OFDM systems as described in [14]. The basic idea was to exploit the pilot intended for CE for the transmission of control signal without extra overhead. Several CID pilot placement schemes were analyzed under Rayleigh fading scenario. The variable location pilot technique allows to use a smaller pool size of pilot sequence at the cost of uncertainty about pilot locations. Variable location pilot scheme performs slightly better than fixed location scheme. Clustered pilot scheme further improve the PER performance at the cost performance degradation of channel estimator. Finally, an algorithm to design a new set of pilot sequences with better detectability was presented. It was shown that the desirable properties of the new pilot sequences must include MSDAD distance, “impulse like” autocorrelation function, and minimum cross correlation coefficients.
Chapter 7

Conclusion and Future Work

7.1 Conclusion

In this thesis, ODFM wireless communication systems have been studied. The major advantages of OFDM systems are simple equalizer requirement, spectral efficiency, ability to support adaptive modulation schemes and high flexibility in resource allocation. This thesis dealt with two vital issues regarding the OFDM system design requirements, namely, channel estimation (CE) and carrier frequency offset estimation (CFOE). The accuracy of these two estimators play very crucial role in the overall performance of OFDM systems.

As compared to blind algorithms, pilot-aided CE algorithms are more robust to high Doppler frequency, and hence, are useful for high mobility applications. The MMSE channel estimator provides better performance than LS estimator at the cost of high complexity. For MIMO-OFDM channel estimation, orthogonal pilot pattern have been shown to perform better than scattered pilot pattern for indoor channels while for vehicular channels, the scattered scheme outperforms the orthogonal one. Carrier frequency offset (CFO) estimation techniques for OFDM systems have also been studied in this thesis. Various time-domain (pre-FFT) and frequency-domain (post-FFT) techniques for CFO estimators in OFDM systems were discussed.
A new set of techniques for pilot-aided CE is proposed in this thesis namely under-sampled channel estimation for OFDM systems. In such techniques, the number of pilots used to sample the channel are less than those allowed by Nyquist sampling theorem. Virtually blind (VB) CE uses only one pilot to estimate the CFR. The performance of VB CE is severely damaged by the occurrence of CFR inversion (CFRI). A couple of methods were also proposed to locate the CFRI. Uniformly spaced fixed additional pilots and dynamically assigned additional pilots were then augmented with the only pilot in order to take more samples of channel and to stop propagating CFRI effect further to remaining subcarriers. The under-sampled CE have also been shown useful for MIMO-OFDM systems. For static and quasi-static channel scenarios like WLAN, the proposed under-sampled CE methods can be effectively used. Unlike some popular CE techniques, under-sampled methods don’t require the knowledge of channel parameters like noise variance and channel covariance matrix; and operating SNR. Therefore, this method is simpler to implement.

Joint blind channel and control signal estimation for OFDM systems has been studied where the pilots intended for CE were exploited for the transmission of control signal without extra overhead. Such pilots were given the name control information dependent (CID) pilots. The detectability of such CID pilot sequences is highly dependent of what type of sequences are being used. An algorithm to design a new set of pilot sequences with better detectability was proposed. Furthermore, it was shown that the desirable properties of the new pilot sequences must include “Maximum Summation of Difference after Division” (MSDAD) distance, “impulse like” autocorrelation function, and minimum cross correlation coefficients.

7.2 Future Work

We list in the following several possible research directions in this research area.
• The main issue concerned with proposed virtually blind channel estimation technique is that the noise reduction process also eliminates the sharp features of the signal which are potentially caused by CFR inversion. Therefore, a special type of filtering needs to be researched which removes only noise while preserving the sharp features of signal itself.

• The under-sampled channel estimation techniques can be investigated for higher order modulation schemes like 16-QAM etc. Moreover, discrete wavelet transform (DWT) has the capability of splitting a signal into detail and approximation, and can be investigated for noise removal while retaining the sharp features of signal itself.

• The potential ideas from “Compressive Sensing” technique can be investigated and incorporated to under-sampled CE. Another idea is to exploit the cyclic prefix information to aid the under-sampled CE process. Moreover, joint estimation of channel and carrier frequency offset using under-sampled techniques can be investigated.

• For joint blind channel and control signal estimation for OFDM systems, CID pilot sequences with better detectability must have “Maximum Summation of Difference after Division” (MSDAD) distance. This MSDAD distance is highly dependent on alphabet type and size. Longer alphabets have more potential of having larger MSDAD distances than shorter ones, yet they require more powerful computers with larger memory to be tested.

• The idea of joint estimation of channel and control signal can be further extended to jointly estimate a third parameter say, frequency offset. After successfully extracting the correct pilot sequence, the phase difference between fixed location pilots can be exploited to estimate CFO as well. Moreover, this technique can be extended to MIMO-OFDM systems.
Bibliography


[27] http://www.mimo.ucla.edu/summaries/INTRO_MIMO&OFDM.


