SEMI-RIGID JOINTS TO TUBULAR COLUMNS
AND THEIR USE IN SEMI-CONTINUOUS
FRAME DESIGN

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Notation

- $a$: Width of RHS clear face
- $A_s$: Bolt shear cross section area
- $a_t$: Width of tension zone (Horizontal bolt spacing)
- $A_t$: Bolt tensile cross section area
- $B$: Beam stiffness factor
- $b_t$: Height of compression zone
- $C$: Column stiffness factor
- $c$: Horizontal distance from bolthole to edge of yield line
- $d$: Vertical distance from bolthole to edge of yield line (Yield line analysis); Storey or cumulative height (Frame lateral sway)
- $d_b$: Bolt diameter
- $d_{fail}$: Deformation capacity at failure
- $d_{RHS}$: RHS depth
- $d_u$: Deformation capacity at ultimate strength
- $d_y$: Deformation capacity at yield strength
- $E$: Young’s Modulus
- $e$: Edge to bolt row distance
- $E$: Bolt row to endplate edge
- $e_{1n}$: Last bolt row to edge distance
- $e_{21}$: Edge to first bolt row distance
- $e_{2n}$: Last bolt row to edge distance
- $e_e$: Material engineering strain
- $e_t$: Material true strain
- $e_u$: Material ultimate strain limit
- $e_x$: Bolt hole centre to free edge
- $F$: Capacity of a component
- $f_e$: Material engineering stress
- $F_{fail}$: Joint capacity
- $F_{G.C.}$: Capacity of RHS bolt group with circular yield line mechanism
- $F_{G.E.}$: Capacity of RHS bolt group with elliptical yield line mechanism
- $F_{G.S.}$: Capacity of RHS bolt group with straight yield line mechanism
- $F_{ps}$: Bolt punching shear capacity
- $F_{ps,nc}$: Bolt group punching shear capacity
- $F_{R.C.}$: Capacity of RHS bolt row with circular yield line mechanism
- $F_{R.E.}$: Capacity of RHS bolt row with elliptical yield line mechanism
- $F_{R.S.}$: Capacity of RHS bolt row with straight yield line mechanism
- $F_{Rd,2}$: Bolt tensile design capacity
- $F_{Rd,i}$: Joint component design capacity
- $F_{RHS,Mem.}$: Capacity of RHS face under membrane action
- $f_e$: Material true stress
- $f_s$: RHS bolt thread stripping capacity
- $f_{s,red}$: Reduced RHS bolt thread stripping capacity
- $f_u$: Material ultimate stress
- $f_y$: Material yield stress
Greek symbols

\( \gamma \) Partial safety factor
\( \delta_m \) Cross section deflection due to membrane action
\( \Delta \) Total frame deflection
\( \Delta_c \) Frame cantilever deflection
\( \Delta_s \) Frame shear racking deflection
\( \theta \) Rotational deformation
\( \mu \) Stiffness ratio
\( \tau \) Bolt pullout strength correction coefficient
\( \Phi \) Rotation of RHS face due to membrane action
\( \phi \) Non-dimensional sway index factor
\( \psi \) Beam-member stiffness factor
Abstract

Traditionally, joints are assumed to be either pinned or fully rigid, but in reality, many behave between these two extremes giving them a semi-rigid classification. By acknowledging the semi-rigidity of nominal pin joints, steel frames can be designed as semi-continuous in which the beam bending moments are partially transmitted to the column members and the need for lateral bracing is eliminated, thus reducing material and construction costs. This thesis presents the results of numerical and theoretical studies of the behaviour of bolted endplate connections to Rectangular Hollow Section (RHS) columns using flowdrill bolts and their applications in semi-continuous frame design. Such connections exhibit significant levels of initial stiffness, strength, and deformation capacity while being more cost-effective than fully welded connections. Despite this, there is limited theoretical work carried out that allows such connections to be designed using analytical methods. In addition, there are no standardised connection design tables like that for open section columns (SCI, 1995), thereby inhibiting their adoption in practice.

In this thesis, newly derived equations for initial stiffness that have a greatly improved range of validity and accuracy over existing equations are presented. Equations for bending strength of a newly derived elliptical mechanism based on yield line theory and for membrane action based on internal work principles are given. Equations for flowdrill thread stripping due to gross deformation of the RHS face are also presented thus allowing prediction of all common RHS face failure mechanisms. Equations for deformation capacity are derived thus making it possible to predict the full load-deflection behaviour of the RHS face in tension component. When combined with existing equations for bolt and endplate components, it is possible to predict the full moment-rotation behaviour of bolted endplate connections to RHS columns. Extensive parametric studies using finite element analysis (FEA) as well as validation against existing tests show that the newly derived equations can accurately predict the component-level and whole joint behaviour for a wide range of configurations.

Taking advantage of the initial stiffness properties that these connections offer, a parametric study is conducted to show that connections with relatively simple detailing can offer sufficient initial stiffness and strength to enable unbraced semi-continuous design of low-rise steel frames. A hand calculation method for SLS sway in semi-continuous frames is also presented thus allowing the designer to readily carry out scheme designs without advanced software knowledge. A systematic method for the detailing of these connections for use in unbraced frame design is presented to facilitate adoption in practice.
Declaration

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Chapter 1

Research Background

1.1 Introduction

1.1.1 Benefits of using tubular columns

The use of tubular columns in steel structures is ever increasing due to the proven structural and architectural benefits they offer from their geometric and mechanical properties. Tubular columns can also be designed as concrete-filled composite columns thereby further increasing strength properties of the column (Corus Tubes, 2002). The ductility and strength properties are also greater than other composite column types due to the concrete core being contained within the steel. They are advantageous for use as multiplanar connections in comparison to open section columns which are suitable only along their major axis. During construction, they are easier to move and erect due to their greater lateral strength.

Tubular sections include square and rectangular hollow sections (SHS/RHS), circular hollow sections (CHS), and elliptical hollow sections (EHS). When considering design and fabrication of connections, the RHS is often superior as it does not require the costly profile cutting that CHS and EHS members require.
1.1.2 Design of semi-continuous frames

Even today, the design of steel frames using tubular columns, as with designs using open section columns, are often carried out on the assumption that beam to column joints are either fully pinned or rigid. This is despite the fact that the majority of joints have properties that lie between these two extremes and thus should be designed as semi-rigid and for partial-strength. By acknowledging the semi-rigidity of joints, steel frames can be designed as semi-continuous in which beam bending moments are partially transmitted to column members. In addition, the stiffness of semi-rigid joints will contribute to the lateral resistance of semi-continuous frames. By taking advantage of semi-continuous frame design, the resulting frame can benefit in both reduced material and construction costs (Kurobane et al., 2004).

Joint semi-rigidity can be achieved using either welded or bolted connections. Although welded connections are more common, this is partly due to there being a greater understanding of welded tubular joint behaviour and thus more guidance in their design such as in CIDECT Design Guide 9 (Kurobane et al., 2004) of which there is little guidance for bolted connections. Another possible factor is that for bolted connections to tubular columns, more knowledge and guidance exists for the design of fin plate connections which do not offer beneficial initial stiffness nor strength characteristics for semi-continuous frame design. Other bolted connection types such as the endplate connection have these necessary qualities (France et al., 1999).

To make possible the application of semi-rigid joints in semi-continuous design, it is necessary to develop a full understanding of joint characteristics which are its initial stiffness, strength, and deformation capacity. Although there has been adequate research carried out in this field and the implementation of analytical methods in design codes such as the Component Method adopted by Eurocode 3 Part
1.8 (CEN, 2005), otherwise known as EN-1993-1-8, these only cover connections to open section columns. To further the understanding of these connections and to allow them to be readily adopted in design, it is necessary to develop an analytical method that allows accurate and efficient characterisation of joint properties.

1.2 Objectives of research

The objective of this research is to develop an analytical method to determine the initial stiffness, strength, and deformation capacity properties of joints to RHS columns that is both accurate and practical. Although equations for initial stiffness and strength exist, further work is necessary to improve their accuracy and practicality.

Combined with existing equations for other joint components, a method for the joint assembly will be proposed to determine the moment-rotation curve for bolted endplate connections to RHS columns. Taking advantage of the initial stiffness properties that these connections offer, it is also the aim to investigate the suitability of these semi-rigid joints in the application of unbraced frame design at the serviceability limit state (SLS) while taking into consideration their strength and ductility. The final objective is to determine a suitable hand calculation method for predicting the sway behaviour of semi-continuous frames that allows their effective design.

To support this research, finite element analysis (FEA) using ABAQUS 6.10-1 (Dassault Systèmes, 2012) is used to investigate and conduct parametric studies on the behaviour of individual joint components and combined joint behaviour. The FEA techniques used in this study are based on existing research as well as a novel method for modelling the unique behaviour of flowdrill blind bolting. Existing tests conducted by France et al. (1999) on whole joints and in CIDECT 6F-13B/96 (British Steel,
1996b) on the flowdrilled RHS face component are used in validating the FEA techniques.

1.3 Originality of research

The research presented in this thesis aims to increase the understanding of the component-level behaviour of flowdrilled endplate connections to hollow and concrete-filled RHS columns at ambient temperature. While research into the behaviour of components in bolted connections to open sections is well-established, research of bolted connections to tubular sections (including the RHS) is limited. This research forms part of a wider CIDECT programme carried out at the University of Manchester to address this gap in research. Using the testing carried out by British Steel (1996b) in CIDECT Report 6F-13B/96 and by France et al. (1999), a finite element model developed in the general finite element analysis software ABAQUS 6.10-1 (Dassault Systèmes, 2012) is validated to conduct parametric studies on the flowdrilled connections to the RHS face in tension component as well as whole joint behaviour. Analytical equations to characterise the initial stiffness of the RHS face in tension component are based on work by Jaspart et al. (2003) in CIDECT Report 5BP-6/03 but improved to consider the individual bolt row behaviour rather than bolt group behaviour. The equations derived for ultimate strength are based on the well-established yield line method and give equations identical to those given in British Steel (1996b) and Ghobarah (1996). However, a change is made to the definition of geometry to improve the accuracy of results. The equations derived for membrane action strength are based on internal work principles used in Jones et al. (2010) for fin-plate connections. The equations derived for membrane action deformation capacity are based on simple assumptions regarding geometry and material properties. For the
unbraced semi-continuous frame behaviour, an existing method of calculating lateral sway in semi-continuous frames (SCI, 1995) is compared with a new approach where a reduced stiffness coefficient developed by Wong (2007) is used in conjunction with equations for calculating lateral sway in rigid frames presented in Smith and Coull (1999) and Taranath (1997). Requirements for initial stiffness of joints in unbraced design is determined from numerical studies conducted using structural analysis software Oasys GSA 8.4 (Oasys Ltd., 2009).

The original contributions to this field of research are (1) the derivation of new component characterisation equations for the initial stiffness, strength, and deformation capacity of the RHS face in tension; (2) numerical studies to determine initial stiffness requirements of joints in unbraced semi-continuous steel frame design; and (3) development and validation of a novel hand calculation method for calculating serviceability limit state sway deflections in semi-continuous frames by combining the work of Wong (2007), Smith and Coull (1999), and Taranath (1997).

1.4 Thesis structure

This thesis contains work from the author’s three years of study and is composed of nine chapters and two appendices.

Chapter 1 presents a brief introduction on the research background, research tools, and the report structure.

Chapter 2 provides a literature review of the topics necessary to understand the work carried out in subsequent chapters including those related to semi-rigid joints and global semi-rigid frame analysis. The principles of the component-based method of joint characterisation as well as coverage of existing research are also included.
Chapter 3 presents work on the validation of FEA techniques that form a base for FEA conducted in subsequent chapters. This covers the validation of FEA techniques against existing testing and a novel method to model flowdrill connections.

Chapter 4 presents work on the derivation and validation of initial stiffness equations for the RHS face in tension component. A comparison is made against existing equations emphasising the increased simplicity and improved range of validity of the new equations. This includes a parametric study using FEM to determine accuracy and range of applicability. The joint initial stiffness calculation using the derived equations is validated against experimental testing by France et al. (1999).

Chapter 5 presents work on the derivation and validation of strength equations for the RHS face in tension component. This includes a parametric study using FEM to determine accuracy and range of applicability. The RHS face strength calculation using the derived equations is validated against experimental testing by British Steel (1996b).

Chapter 6 presents work on the derivation and validation of deformation capacity equations for the RHS face in tension component. This includes a parametric study using FEM to determine accuracy and range of applicability. The RHS face deformation capacity calculation using the derived equations is validated against experimental testing by British Steel (1996b).

Chapter 7 presents work on joint assembly using analytical equations to define the full joint moment-rotation curve. The results are validated against experimental testing by France et al. (1999) and a comparison of the moment-rotation curves are given.

Chapter 8 presents work on unbraced semi-continuous frame behaviour. This includes methods to determine the levels of joint stiffness necessary for unbraced
design as well as simplified methods to calculate semi-continuous frame deflections using hand-calculations. Requirements for joint strength and ductility are made in the context of unbraced design using semi-rigid joints. A joint design procedure for practical unbraced low-rise steel frames is presented to allow easy adoption of the research in practice.

Chapter 9 presents conclusions and recommendations for further work based on the findings of this research.
Chapter 2

Literature Review

2.1 Use of tubular columns in steel frames

2.1.1 Advantages of using tubular columns in steel frames

There is a strong argument for the use of tubular columns in steel frame design due to the proven structural and architectural benefits they offer. This comes in the form of geometric advantages such as symmetric cross sections which possess bending strength along both axes making them suitable for multiplanar connections. They also possess mechanical advantages such as lower residual stress compared with open sections that have higher residual stress due to welding. Tubular columns can also be designed as concrete-filled composite columns thereby further increasing strength properties of the column (Corus Tubes, 2002). The ductility and strength properties are also greater than other composite column types due to the concrete core being fully-encased by the steel.

The main disadvantage of using tubular columns is the difficulty in connection assembly due to the restricted access of the internal face that prevents use of traditional nut and bolt fasteners that is discussed in Section 2.1.2.
2.1.2 Flowdrilled connections to tubular columns

Due to the restricted access of the internal face of tubular columns, bolted connections using a traditional bolt and nut are impractical. Blind bolts, or purpose-designed bolts that can be installed from one side, can be used to overcome this problem. Hollo-Bolt (Lindapter) and Flowdrill are well-established blind-bolting technologies suitable for structural jointing applications (British Steel, 1997). This research is interested in Flowdrill (Flowdrill B.V., 2004) which is used in studies such as British Steel (1996b) and France et al. (1999). For comparison, descriptions of both technologies are given below.

Hollo-Bolt is a pre-assembled three or five-part fastener that is inserted into plain drilled holes. The three-part Hollo-bolt (M8, M10, and M12) consists of body, cone, and bolt. The five part Hollo-bolt (M16, M24) is similar but with the collar and the body being separated by a collapse mechanism as given in Figure 2.1. The tightening of the bolt draws the cone into the body which spreads the legs to form a secure fixing as shown in Figure 2.2. The main benefit of using Hollo-Bolt is that it uses a plain drilled hole made with standard drilling equipment. The downside is that each connection requires the Hollo-bolt fastener which is costly.
Flowdrilling consists of a thermal drilling process and tapping of threads to accept a standard threaded bolt as shown in Figure 2.3. During the thermal drilling stage, material is displaced forming a hole and internal bushing. During the thread tapping stage, the hole is tapped using a roll thread-forming tool called a Coldform Flowtap. This method can be used effectively with column faces with thicknesses as low as 6mm (British Steel, 1997) because the internal bushing strengthens the column face providing additional support against bolt pullout. This is an issue because flowdrilled holes will deform along with the column face thus losing contact with the bolt threads and causing thread stripping and bolt pullout at a reduced capacity. The main benefit of using Flowdrill is that it accepts a standard threaded bolt. The
The downside is that a flowdrill tool must be used to produce the threaded boltholes. The minimum RHS thickness to achieve full Grade 8.8 bolt tension capacity using flowdrilled connections is given in Table 2.1 by British Steel (1996a). Requirements on flowdrill detailing are given in Table 2.2 with reference to the geometry given in Figure 2.4.

![Figure 2.3. Stages of the Flowdrill Process (British Steel, 1997)](image)

**Table 2.1 Minimum RHS thickness to achieve full Grade 8.8 bolt tension capacity (British Steel, 1996b)**

<table>
<thead>
<tr>
<th>Bolt size</th>
<th>Minimum RHS thickness S275 (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M16</td>
<td>6.4</td>
</tr>
<tr>
<td>M20</td>
<td>8.0</td>
</tr>
<tr>
<td>M24</td>
<td>9.6</td>
</tr>
</tbody>
</table>

**Table 2.2 Flowdrill detailing requirements for various bolt sizes (British Steel, 1997)**

<table>
<thead>
<tr>
<th>Dimensions (mm)</th>
<th>M12</th>
<th>M16</th>
<th>M20</th>
<th>M24</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>7</td>
<td>10</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>B₁</td>
<td>13</td>
<td>17</td>
<td>22</td>
<td>25</td>
</tr>
<tr>
<td>C₁</td>
<td>18</td>
<td>20</td>
<td>26</td>
<td>29</td>
</tr>
<tr>
<td>D₁</td>
<td>Varies with overall bolt length (L_b) specified</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E₁ Min</td>
<td>(C₁/2+t_c) (for connections made to a single face or opposite faces) (B₁/2+A₁+D₁+t_c) (for connection made to adjacent faces)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Min Bolt Centres</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
</tr>
</tbody>
</table>
2.2 Joint classification

The classification and characterization of joints is important in structural design as joint behaviour can greatly affect the global behaviour of the frame. Joint stiffness can affect a wide range of design aspects from local member displacements and rotations, to the magnitude and location of bending moments and stresses.

Joints are classified in the three general categories of nominally pinned, semi-rigid, or rigid. As described in Section 5.2.2 of EN 1993-1-8 (CEN, 2005), nominally pinned joints are capable of transmitting internal forces without developing significant moments. Rigid joints have sufficient rotational stiffness to justify analysis based on full continuity. Semi-rigid joints are those that cannot be idealised as either due to their stiffness, strength, and/or ductility characteristics being between the two extremes.

The distinction between these joint types are made from the characteristics of the rotation, \( \theta \), of the joint caused by the in-plane bending moment, \( M \), as given in
Figure 2.5. The initial rotational stiffness $S_{j,ini}$ is used to distinguish between stiffness classification boundaries as given in Figure 2.6. It is necessary to note that these boundaries only apply to joints to open section columns and not the hollow columns which are the focus of this project. Boundaries for joints to hollow sections are not covered by EN 1993-1-8 or similar international design codes. As such, it is necessary to determine stiffness classification based on experimental evidence or previous performance under similar conditions.

Zone 1: Rigid, if $S_{j,ini} \geq k_b EI_b/L_b$
where $k_b=8$ for frames where the bracing system reduces the horizontal displacement by at least 80% and $k_b=25$ for other frames, provided that in every storey $K_b/K_c \geq 0.1$

Zone 2: Semi-rigid
All joints in Zone 2 should be classified as semi-rigid. Joints in Zones 1 or 3 may optionally also be treated as semi-rigid.

Zone 3: Nominally pinned, if $S_{j,ini} \leq 0.5EI_b/L_b$
For frames where $K_b/K_c < 0.1$, the joints should be classified as semi-rigid.

Key:
$K_b$ is the mean value of $EI_b/L_b$ for all the beams at the top of that storey
$K_c$ is the mean value of $EI_c/L_c$ for all the columns in that storey

Figure 2.6. EC3 1.8 Classification of joints by stiffness in EN 1993-1-8 (CEN, 2005)
In the case of rigid-plastic frame design, joint classification based on flexural resistance is also of concern. Section 5.2.3 of EN 1993-1-8 defines joints in terms of nominally-pinned, full strength, and partial-strength categories based on the design moment resistance of the connecting members.

Nominally-pinned joints are capable of transmitting internal forces without developing significant moments which adversely affect column resistance or the structure as a whole. They must be capable of accepting rotations due to design loads and the design moment resistance must be less than 0.25 times the resistance for a full-strength joint.

A full-strength joint must have design resistance greater than its connecting members. Specifically, it must have design resistance greater than beam design plastic resistance and twice the column design plastic resistance. For top-level connections, it must have design resistance greater than beam and column design plastic resistance. Joints not satisfying either of these criteria are classified partial-strength joints. The main use of this classification is to understand the potential requirement for plastic hinges to form in the joint during the global analysis when considering a partial strength joint. It is also important to note that if a partial-strength joint needs to act as a hinge when the applied bending moment is larger than the joint plastic bending resistance, it must also have sufficient ductility to allow this.

Another criterion for rigid-plastic frame design is joint ductility that is based on the plastic rotation capacity of a joint. This classification is a measure of the joint’s ability to resist premature local instability and premature brittle failure, including bolt failure. Full-ductility joints are able to develop a plastic rotation capacity equal to or greater than that of the connected member. For partial-ductility joints, the plastic rotation capacity is less than that of the connected member and therefore care needs to be taken to ensure that the required moment capacity can be achieved before this.
Although this classification is not incorporated into EN 1993-1-8 and similar modern design codes, it is still an important factor when rigid-plastic and elastic-plastic analyses are concerned. One of the reasons for this is that there is some grey area regarding the classification of joints. For example, a joint may have greater design resistance than its connecting members and thus be classified as a full-strength joint according to the criteria mentioned previously, yet only have partial-ductility and thus may fail before the design moment of the connected member is reached. The designer must be aware of complexities rather than rely on a basic interpretation of the design codes.

2.3 Methods of joint characterisation

As mentioned in Section 2.2, developing an understanding of the moment-rotation behaviour of beam-to-column joints is an integral part of semi-continuous frame analysis. Depending on factors such as necessary accuracy of results, availability of full-scale testing setups, and time, the modelling of joint behaviour is carried out using a variety of methods. These methods are generally divided into empirical models, analytical models, finite element analysis, and full scale testing.

With empirical models, the joint behaviour is represented by the geometrical and mechanical properties of the joints. Joint behaviour is derived from regression analyses of data from experimental testing and parametric analyses. Their scope is limited to the range of parameters covered in the joint configurations used for calibration of the models. Their application outside the range of tested data should be avoided as the structural failure behaviour and failure model is strongly affected by mechanical properties of the joint components and extrapolation of empirical data does not suffice for the fact that individual component behaviour is unknown. This
approach is outdated with the progress made with analytical and finite element models in the last few decades.

With analytical models, the moment rotation behaviour is derived from physical characteristics of the section such as geometric properties and component arrangements. Parametric studies can be conducted to determine the relative effects that individual components have on overall joint behaviour. Individual components can be adopted in different connection types and therefore do not have the limitation of requiring calibration for different connection types and configurations that empirical models require. Analytical models have the benefit of being simple enough to implement for practical use yet sufficiently flexible to enable different joint types to be analysed. The most commonly used of the analytical models is the Component Method, as adopted in EN 1993-1-8 that is described in Section 2.4.

With finite element analysis, the joint geometry is discretized and reduced to a large number of very small elements which allows modelling of the complex interactions between the individual components. Behaviour such as geometric/material nonlinearities, friction, slip interactions, welds, initial imperfections, and the spread of plasticity are a small selection of factors that can be incorporated in a finite element analysis. Finite element analysis has been a convenient and accurate method for joint characterization from as early on as research by Lipson and Hague (1978) and has since been used in innumerable studies to model a variety of connection types. Despite this, there is room for development of finite element modelling techniques as different connection types usually call for changes in techniques to improve accuracy and efficiency. Whilst validated finite element simulations may substitute experimental research to provide detailed and accurate behaviour of joints, the finite element simulation of joints is not a suitable approach for practical implementation. Its role is
in providing detailed understanding of joint behaviour based on which more simplified approaches may be developed for practical implementation.

Full scale testing as the name suggests involves physical testing of a joint using test apparatus. When conducted in a controlled environment, the results will always provide the real behaviour of the joint; however, it is a time-consuming and costly approach to joint characterization and is rarely conducted outside of research.

Based on the above arguments, the component based approach of quantifying joint behaviour will be pursued in this research.

2.4 Component method of joint characterisation

2.4.1 Background

Of the variety of methods that have been considered in Chapter 2.3, the component method has emerged as being most favoured because it combines the flexibility of being able to deal with any change in joint detail yet sufficiently simple for implementation in practical design. This is reflected in its adoption in EN 1993-1-8 (CEN, 2005) as well it being the platform for which the majority of research regarding joint characterisation is based on.

In the component-based method, a joint is represented by a small number of components, each representing one part of the joint due to one single action. The action of each component is characterised by a force-displacement relationship. The force-displacement relationships of all components are then assembled, based on satisfying the equilibrium conditions of the joint, to give the desired joint characteristic. Because range of applicability is defined per component, the validity of joints (or component combinations) can be quickly determined without exhaustive testing.
The first step for application of the component method is identification of the active joint components. For example, in an extended endplate to RHS column joint, Weynand et al. (2003) identifies the active components as being those given in Figure 2.7.

![Diagram of components](image)

**Compression zone**
- Column side walls in compression
- Column face in bending
- Beam flange and web in compression

**Tension zone**
- Column side walls in tension
- Column face in bending
- Bolts in tension
- Endplate in bending
- Beam web in tension

**Shear zone**
- Column web panels in shear

Figure 2.7. Components of an extended endplate to RHS column (Weynand et al., 2003)

In the second step, each component is evaluated for its mechanical characteristics such as the initial stiffness, design strength, and ductility. Stress interactions between components such as in the column which is simultaneously subject to compression/tension and shear will lead to a decrease in both strength and stiffness of individual components but does not affect the principles of the component method format. Individual components are given both a stiffness coefficient, $k_i$, and a resistance value, $F_{Rd,i}$.

In the final step, the values calculated from the individual components are combined to give the mechanical characteristics of the assembled joint. For initial stiffness, the joint stiffness, $S_{j,ini}$, is obtained from the following equation:
\[ S_{j,\text{ini}} = \frac{Ez^2}{\left( \frac{1}{\sum k_i} \right)} \]

The capacity of the joint, \( M_{j,Rd} \), is obtained from the following equation:

\[ M_{j,Rd} = \text{Min}\left(F_{Rd,1}\right) \times z \]

Although the component method has been adopted in Eurocode 3 Part 1.8 (CEN, 2005), bolted connections are limited to hot-rolled steel and composite joints to I and H sections. In addition to this, only a few connection types are covered such as welded, fin plate, and endplate connections. As the framework of the component method is generalized however, it is possible to combine existing rules with those for newly developed components. This means that a small number of components can be used to determine characteristics for a large number of different joint configurations.

Developing the component method equations for bolted endplate connections to tubular columns is one of the principal objectives of this research. Because the primary components for the characterisation of this joint type are the RHS, endplate and bolt of which the latter two are covered in the EN 1993-1-8 (for initial stiffness and strength), the focus of this work is on the components relating to the RHS of which as of yet there is an insufficient understanding. This section covers the current developments focusing on the characterisation of the RHS but also covers research on the deformation capacity of the endplate component which is not included in EN 1993-1-8.

2.4.2 Equations for initial stiffness

Jaspart et al. (2004) together with Weynand et al. (2006) are the only ones to have conducted research to develop a fully analytical method to quantify the initial
stiffness of a steel beam to RHS column joints using bolts. In their method, both the tension and compression zones of the RHS face are considered in determining whole joint behaviour as given by the following equations and with geometry given in Figure 2.8.

RHS in transverse compression and tension: Chord face failure

\[
k_{5 \text{ and } 6} = \frac{t^3_c}{14.4\beta L^{2}_{\text{stiff}}} \left(\frac{L^{2}_{\text{stiff}}}{bt_c}\right)^{1.25} \frac{c}{L_{\text{stiff}}} + \left(1 - \frac{b}{L_{\text{stiff}}}\right) \tan[\theta] \left(1 - \frac{b}{L_{\text{stiff}}}\right)^{3} + \frac{10.4 \left(k_1 - k_2 \frac{b}{L_{\text{stiff}}}\right)}{\left(\frac{L_{\text{stiff}}}{t_c}\right)^2}
\]

\[\theta = 35 - 10 \frac{b}{L_{\text{stiff}}} \text{ if } \frac{b}{L_{\text{stiff}}} < 0.7\]

\[\theta = 49 - 30 \frac{b}{L_{\text{stiff}}} \text{ if } \frac{b}{L_{\text{stiff}}} \geq 0.7\]

The formula for \(k_{5 \text{ and } 6}\) is only valid if these requirements are fulfilled:

\[10 \leq \frac{L_{\text{stiff}}}{t_c} \leq 50 \]

\[0.08 \leq \frac{b}{L_{\text{stiff}}} \leq 0.75 \]

\[0.05 \leq \frac{c}{L_{\text{stiff}}} \leq 0.20\]

RHS in transverse tension: Punching shear failure

\[k_7 = \infty\]
RHS in transverse compression: Punching shear failure

\[ k_8 = \infty \]  \hspace{1cm} [2.8]

Figure 2.8. Joint geometry for Jaspart et al. (2004) and Weynand et al. (2006) method.

However, their method has a number of shortcomings, including limited range of applicability as evidenced in Equations [2.4], [2.5], and [2.6] as well as complex equations. In Chapter 4, it is shown that there is significant error when using these equations.

Ghobarah (1996) proposes a method for calculation of the initial stiffness of the RHS face in tension which requires the use of coefficients to calculate column flange deflection and reduction factors to determine the restraining effect of the rotational springs along the longitudinal edges of the column flange. Both coefficients are calculated from FEA simulations and presented in tables for specific column
sections therefore the method is both limited in its range of applicability and its practicality due to the need to look up coefficients in tables for each joint considered.

2.4.3 Equations for strength

The following are equations by Gomes et al. (1996) for the definition of strength as used in studies such as Jaspart et al. (2004) and Weynand et al. (2006). The joint geometry is given in Figure 2.8:

RHS in transverse compression and tension: Chord face failure

\[ F_{Rd,5 \text{ and }6} = \min \{F_{pl,loc}, F_{pl,\text{glob}}\} \]  \[2.9\]

\[ F_{pl,loc} = M_p \alpha \beta \]  \[2.10\]

If \( \frac{h}{L-b} \geq 1 \), \( F_{pl,\text{glob}} = M_p \left( \frac{2b}{h} + \frac{\alpha \beta}{2} + \pi + \frac{2h}{L-b} \right) \)  \[2.11\]

If \( \frac{h}{L-b} < 1 \), \( F_{pl,\text{glob}} = M_p \left( \frac{2b}{h} + \frac{\alpha \beta}{2} + \pi + 1 \right) \)

where

\[ \alpha = \frac{4}{1 - \frac{b}{L}} \left( \pi \sqrt{1 - \frac{b}{L} + \frac{2c}{L}} \right) \]  \[2.12\]

\[ \beta = 1 \text{ if } \frac{b + c}{L} \geq 0.5 \]  \[2.13\]

\[ \beta = 0.7 + 0.6 \frac{b + c}{L} \text{ if } \frac{b + c}{L} \leq 0.5 \]

\[ M_p = \frac{1}{4} t_c^2 f_y / \gamma_M \]  \[2.14\]

\[ L = b_c - 2t_c - 1.5r \]  \[2.15\]

These equations are subject to the following rules for range of validity:
Outside this range, the equation for $F_{pl,\text{glob}}$ is not valid

RHS in transverse tension/compression: Punching shear failure

$$F_{Rd,7} = \min[F_{\text{punch,nc}}; F_{\text{punch,cp}}]$$  \hspace{1cm} [2.18]

$$F_{\text{punch,nc}} = 2(b + c)v_{pl,Rd}$$  \hspace{1cm} [2.19]

In compression:

$$F_{\text{punch,cp}} = n\pi d_m v_{pl,Rd}$$  \hspace{1cm} [2.20]

A variation of Equation [2.20] is given in Kurobane et al. (2004) given by Equation [2.31] which is specific to flowdrilled RHS faces in tension.

RHS in transverse compression: Punching shear failure

$$F_{Rd,8} = F_{\text{punch,nc}}$$  \hspace{1cm} [2.21]

$$F_{\text{punch,nc}} = 2(b + c)v_{pl,Rd}$$  \hspace{1cm} [2.22]

where

$$v_{pl,Rd} = \frac{t_c f_{yc}}{\sqrt{3} Y_{Mo}}$$  \hspace{1cm} [2.23]

British Steel (1996b) proposes an equation for yield strength derived from a simple rectangular yield line mechanism.

RHS face in transverse tension: Straight yield lines

\[ \frac{b}{L} < 0.8 \]  
\[ 0.7 < \frac{h}{L - b} \leq 10 \]  \hspace{1cm} [2.16] [2.17]
Figure 2.9. Joint geometry for British Steel (1996b) method.

This gives the same equation as presented in Ghobarah et al. (1996) (but rearranged) shown as Equation [2.25] below. Ghobarah et al. also consider a separate yield line mechanism using circular radial yield lines given by Equation [2.26]. The geometry for both mechanisms is given in Figure 2.10.

**RHS face in transverse tension: Mechanism 1 (Straight yield lines)**

\[
P_{p(cft,1)} = \frac{2\sigma_{yc}t^2_d}{(1 - \beta)} \left[ (\eta - \gamma) + 2\sqrt{(1 - \gamma)(1 - \beta)} \right] \tag{2.25}
\]

**RHS face in transverse tension: Mechanism 2 (Radial yield lines)**

\[
P_{p(cft,2)} = \sigma_{yc}t^2_d \left[ \pi \left( 1 - \frac{\gamma}{2(1 - \beta)} \right) + 2 \frac{(\beta + \eta - \gamma)}{(1 - \beta)} \right] \tag{2.26}
\]
RHS face in transverse tension

\[ p_{p\text{(cft)}} = \min(p_{p\text{(cft,1)}}, p_{p\text{(cft,2)}}) \]  \hspace{1cm} [2.27]

where

\[ \beta = \frac{X_B}{(H_O - t_O)} \]  \hspace{1cm} [2.28]

\[ \eta = \frac{Y_B}{(H_O - t_O)} \]  \hspace{1cm} [2.29]

\[ \gamma = \frac{d}{(H_O - t_O)} \]  \hspace{1cm} [2.30]

Mechanism 1 (Straight lines)  \hspace{1cm} Mechanism 2 (Radial lines)

Figure 2.10. Joint geometry for Ghobarah et al. (1996) method.

The failure due to thread stripping of a flowdrilled RHS face is considered in Kurobane et al. (2004) and is given by the following equation:

\[ F_{ts} = 0.6f_{c,y} \pi d_b (t_c + 8\text{mm}) \]  \hspace{1cm} [2.31]
The punching shear capacity of a flowdrilled RHS is given by the equation:

$$F_{ps} = 0.6f_{c,y} \pi t_c (d_b + t_c)$$  \[2.32\]

The tensile capacity of bolts is well established and given in EN 1993-1-8 by the equation:

$$F_{Rd,2} = k_2 f_{ub} A_S / \gamma_{M2}$$  \[2.33\]

where $k_2=0.9$ and $\gamma_{M2}=1.25$.

2.4.4 Equations for deformation capacity

There are no studies where a fully analytical method has been used to define the deformation capacity of the RHS face in tension component as well as deformation behaviour at the bending strength limit. Existing approaches to define the whole joint ductility consider using an estimated post-yield stiffness based on a factor of the initial stiffness determined through empirical studies. In EN 1993-1-8 (CEN,2005), this is given by the following equations:

$$S_i = \frac{Ez^2}{\mu \sum_i k_i}$$  \[2.34\]

where $\mu$ is the stiffness ratio determined from the following equation:

if $M_{j,Ed} \leq \frac{2}{3} M_{j,Rd}$

$$\mu = 1$$  \[2.35\]

if $\frac{2}{3} M_{j,Rd} < M_{j,Ed} \leq M_{j,Rd}$

$$\mu = \left(1.5 M_{j,Ed}/M_{j,Rd}\right)^\psi$$  \[2.36\]

where $\psi=2.7$ for bolted endplate connections
As bolted endplate connections are the primary focus of this research, Equation [2.36] can be rearranged for the specific value of $\psi=2.7$ to give the joint stiffness in the elastic-plastic regime:

$$S_j \approx 0.33 S_{j,\text{ini}}$$  \[2.37\]

For the endplate component, there is no method in EN 1993-1-8 (CEN, 2005) or other design codes for deformation capacity. The following describes the analytical method based on assumed hinge locations and material ultimate strain properties developed by Beg et al. (2004). The deformation capacity of the endplate is governed by one of three yield modes.

**Mode 1 (complete yielding of the flange)**

This mode uses the rotation of the plastic hinge, $\varphi$, obtained by assuming that failure occurs when the maximum material strain, $e_u$, is reached at the outer surface of the plate in bending as shown in Figure 2.11. The length of the plastic hinge $l_p$, is assumed equal to the thickness of the plate, $t_p$.

$$d_u = \varphi \times m$$  \[2.38\]

where

$$\varphi = \frac{e_u l_p}{t_p/2} = \frac{e_u t_p}{t_f/2} = 2e_u$$  \[2.39\]

Therefore,

$$d_u = 2e_u \times m$$  \[2.40\]
Figure 2.11. Mode 1 endplate deformation mechanism (Beg et al., 2004)

Mode 2 (bolt failure with yielding of the flange)

This considers the deformation shape given in Figure 2.12.

\[ d_u = \varphi_1 \times n + \varphi_2 \times m \]

\( \varphi_1 \) is obtained from the plastic deformation of bolts

\[ \varphi_1 = \frac{e_{ub} l_b}{n} \]

where \( e_{ub} \) is the ultimate strain of the bolt material and \( l_b \) is the clamping length of the bolts including the thickness of the washers.

\( \varphi_2 \) can be expressed in terms of \( \varphi_1 \) as

\[ \varphi_2 = k \varphi_1 \]

where \( k \) is an empirical factor with values between 1.0 and 5.0. Hence, 1.0 is a very conservative value and 3.0-4.0 is a typical value.

The combined expression for \( d_u \) is

\[ d_u = 0.1 l_b \left( 1 + k \frac{m}{n} \right) \] \hspace{1cm} [2.41]
Mode 3 (bolt failure)

This mode simply considers the elongation of the bolts at failure

\[ d_u = e_u b l_b \]  \hspace{2cm} [2.42]

2.5 Global analysis of semi-continuous frames

The two principal benefits of adopting semi-continuous steel frame design are:

1. using the joint bending moment capacity to reduce the in-span bending moment of the connected beam compared to assuming pin joint behaviour;
2. using the joint stiffness to provide lateral stability of the steel frame to eliminate the need for vertical bracing. Taking into consideration the former in frame design is relatively straightforward when considering beams and joints in isolation of the global frame as can be seen from the equations in Table 2.3. In contrast, including the effects of semi-rigid joint behaviour in frame analysis is more complex and is addressed in Section 2.6.

There are two general methods to adopt joint stiffness into a frame analysis. The first approach is to include additional connection elements to model the beam-to-column connections directly in frame analysis software. Each component is divided into a number of segments and in the assembly procedure, this is incorporated to form an overall stiffness matrix. One of the drawbacks of this approach is in the fact that it is difficult to obtain a physical sense of the connection member stiffness as it is
separated from the attached end connections and creating such a model is time consuming and difficult.

The second and more common approach is to model the flexibility of joints using lengthless springs between beam and column members. Eurocode 3 proposes modelling each component by an equivalent linear spring that is assembled to form a single bilinear rotational spring at the ends of beam members. Rotational springs can be defined as multilinear or with a moment-rotation curve obtained from experiments to increase the level of sophistication of the global analysis. Faella et al. (1999) also justifies the usage of this approach by concluding that this simplification is able to model semi-rigid joint behaviour with a negligible loss of accuracy when joint moment-rotation behaviour accounts for behaviour of column panel zone deformations as well as connection behaviour.

Table 2.3. BM distribution for beams with semi-rigid joints under UDL

<table>
<thead>
<tr>
<th>Load</th>
<th>BM</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image.png" alt="Diagram" /></td>
<td><img src="image.png" alt="Diagram" /></td>
</tr>
</tbody>
</table>

\[
M_1 = \frac{wL^2}{12} \left[ \frac{3r_1(2 - r_2)}{4 - r_1r_2} \right]
\]

\[
M_2 = \frac{wL^2}{12} \left[ \frac{3r_2(2 - r_1)}{4 - r_1r_2} \right]
\]

where

\[
\eta_i = \frac{1}{1 + \frac{3EI}{S_{ji}L}}
\]

For \( S_{j1} = S_{j2} \),

\[
M_1 = M_2 = \frac{wL^2}{12} - \frac{S_j}{\left( \frac{EI_b}{2L_b} - S_j \right)}
\]

\[
M_{Mid} = \frac{wL^2}{8} - M_1
\]

\( S_j \) = Joint rotational stiffness
2.6 Simplified semi-continuous analysis methods

As mentioned in Section 2.5, the most common approach to structural analysis of semi-rigid frames is the use of rotational springs at joints. In practice, the development of such models is both time-consuming and complex. In addition to this, rotational spring elements are only found in some of the more advanced structural analysis packages. Therefore, semi-rigid frame analysis using basic software or hand calculations is not possible thus acting as a barrier towards adoption of semi-continuous frame design in practice.

To overcome this problem, various efforts have been made to simplify semi-rigid frame analysis by eliminating the need for rotational springs. One method is the modified Muto method by Wong et al. (2007) which allows analysis of a semi-rigid frame as a rigid frame with reduced beam-member stiffness. Based on the analytical method for analysis of rigid frames proposed by Muto (1974) which considers the stiffness of beams and columns and the variation of the location of contraflexure in columns of multi-storey frames of different storey heights, it incorporate the effects of semi-rigid joints by means of a reduced beam-member stiffness. The beam-member stiffness reduction factor, $\psi$, is obtained from a static condensation procedure that considers the joint stiffness properties at both ends of a hybrid element with respect to both of its end nodes. The reduction factor, $\psi$, given by Equations [2.46] and [2.47], is applied to the original beam-member stiffness, $K_b$, to get the reduced beam-member stiffness, $K'_b$, given in Equations [2.48] and [2.49].

\[ R_1 = \frac{E I_b}{Lk_{si}} \]  

[2.43]
\[ R_j = \frac{E I_b}{L k_{sj}} \]  
\[ R = 1 + 4(R_i + R_j) + 12R_i R_j \]  
\[ \psi^j = \frac{2(1 + 3R_i) + 1}{3R} \]  
\[ \psi^j = \frac{2(1 + 3R_j) + 1}{3R} \]  
\[ K'^i_b = \psi^j K_b \]  
\[ K'^j_b = \psi^j K_b \]  

where

\[ K_b = \frac{E I_b}{L_b} \]  

For the specific (not uncommon) case where the joint stiffness is equal at both ends, a simple beam-stiffness reduction coefficient can be obtained by the following:

\[ \psi = \frac{k_b}{6 + k_b} \]  

where \( k_b \) is the ratio of the joint stiffness to the beam stiffness.

The modified Muto method approach is suitable for predicting the elastic behaviour of unbraced frames with semi-rigid connections under working load conditions (Wong et al., 2007) and can be used for both steel and composite frames.

To increase the practicality of using the reduced beam-member stiffness it would be advantageous to incorporate this into existing analysis methods for rigid frames. It is also useful to determine whether the reduced beam-member stiffness coefficient could be incorporated in basic rigid frame analysis software to accurately
predict the serviceability limit state (SLS) deflections of various low-rise frame types. These issues are dealt with in Chapter 8.

An alternative approach for prediction of lateral sway in semi-continuous frames is that presented in SCI P273 (Hensman and Way, 2000) but which can trace its background to Wood and Roberts (1975) for the Wind-moment Method (hereafter referred to as the SCI method). Here, the frame is replaced by a substitute beam-column frame in which the frame sway deflections are in part dependent on the stiffness distribution coefficients of the beam and column sections. These coefficients are then used to look up a non-dimensional sway-index factor ($\phi$) from a chart based on Wood and Roberts (1975) which is then used to calculate the frame sway deflection using equations given in Section 8.5.2. Because a look-up chart is used, this approach is not flexible, for example, to automate calculations using spreadsheets. A general multiplier of 1.5 is used to convert rigid frame deflections into semi-rigid frame deflections (when using standard connections). Again, this method can be modified to allow for frames with semi-rigid joints by using the reduced beam stiffness coefficient. The suitability of this approach is investigated in Chapter 8.

2.7 Summary

This chapter highlights the benefits and issues in using bolted connections to tubular columns in semi-continuous frame design. Tubular columns offer superior structural properties from their geometric and mechanical behaviour compared with open section columns. Bolted connections to tubular columns using blind bolting technologies offer a cost-effective alternative to welded joints while still offering desirable stiffness, strength and ductility properties. However, there is difficulty in effectively determining joint characteristics necessary for joint classification. The
Component Method of EN 1993-1-8 offers a suitable framework for effective characterisation of joint properties over a wide range of configurations. However, there is a need for further development of component equations specific to connections to tubular columns that are neglected in current design guides.

Semi-continuous frame design using semi-rigid joints is not widely adopted despite it offering benefits such as using the joint bending moment capacity to reduce the in-span bending moment and using the joint stiffness to provide lateral stability of the steel frame to eliminate the need for vertical bracing. One reason is that specialised structural analysis software using rotational springs is required to determine sway characteristics. It is advantageous to be able to use hand-calculations to determine sway in semi-continuous frames. A method combining a reduced beam stiffness coefficient developed by Wong et al. (2007) to be used with equations for rigid-frame sway is considered.
Chapter 3

Validation of FEM Techniques

3.1 Introduction

The development of finite element analysis (FEA) techniques for modelling flowdrilled connections to tubular columns is necessary as there are few studies carried out in this field. Research studies of a similar nature, such as those involving the modelling of regular bolted endplate connections to tubular sections and the major axis of H-section columns exist. Due to the unique characteristics of flowdrilled connections however, modelling techniques need to be refined or adapted for usage with flowdrilled connections to tubular columns.

While many of the general techniques and parameters required in FEA are well established, there is a need to confirm their suitability on a case-by-case basis to ensure the finite element model (FEM) reflects actual behaviour and failure mode of the structure by means of a validation study. For this research, the testing conducted by France et al. (1999) described in Section 3.1.1 is used to validate the finite element modelling approach for the whole joint model. The testing conducted by British Steel (1996b) is used to validate the finite element modelling approach for the flowdrilled connection.
3.1.1 Description of France et al. (1999) testing

The France et al. (1999) testing programme was conducted to determine stiffness and strength characteristics of flush, extended, and partial depth endplate connections to both filled and non-filled RHS tubular columns with flowdrill threading. 26 joint tests were carried out with 17 tests of extended endplates, 6 tests of flush endplates, and 3 tests of partial depth endplate connections. Of these, 20 tests were connections to non-filled columns and 6 tests were connections to concrete-filled columns. These tests were divided into three separate programmes with different research aims.

The first set of tests looked at the response of moment connections to steel RHS columns. Rigidity in the joints was achieved with the use of thick (25mm) endplates that ensured that the joint region could support bending moments approaching the plastic capacity of the supported beam as well as being sufficiently ductile to attain large rotational capacities.

The second set of tests looked at the response of simple connections to steel RHS columns. Here, simple connection refers to the simple detailing that is found in a typical nominally-pinned connection. However, such joints still possess a significant level of semi-rigidity. This compares with moment connections which are semi-rigid connections that are specially detailed to improve the initial stiffness behaviour. In comparison to the first set of tests, the rigidity of the joints was reduced by utilizing a standard (10mm) endplate thickness. Although semi-rigid connections with relatively simple detailing such as this possess noticeable bending strength and rotational stiffness, a key factor for their successful design is in their rotational ductility. These tests were loaded with a slow cyclic pattern which gradually increased in load after
each cycle. The monotonic behaviour was extracted by combining the peak values generated for the positive moment-rotation curve as shown in Figure 3.1.

The third set of tests looked at the response of connections to concrete-filled SHS columns. The geometry of columns, beams, and connection details were kept consistent with the second set of tests in order to provide a direct comparison with joints to unfilled steel columns. From this, a comparison can be made on the changes seen in the stiffness and strength properties of concrete-filled joints. Table 3.1 summarises the experimental parameters.

Figure 3.1. Extraction of 'monotonic' curve from cyclic loading in 2nd set of France et al. (1999) tests
Table 3.1. France et al. (1999) testing programme

<table>
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<tr>
<th>Test</th>
<th>Set</th>
<th>Endplate</th>
<th>(w_{\text{RHS}}) (mm)</th>
<th>(t_c) (mm)</th>
<th>CFT</th>
<th>Steel Grade</th>
<th>Beam Section</th>
<th>(t_p) (mm)</th>
<th>(a_i) (mm)</th>
<th>Axial Load (kN)</th>
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* Exact lever arm not given so 1.3m assumed for tests with 457x152x52 beam, 1m used for others.

The joints in the first set of tests were conducted in a typical cantilever arrangement as shown in Figure 3.2. The joints in the second and third set of tests used the cantilever arrangement as shown in Figure 3.3. The main difference between the testing rigs is that the columns in the first set were fixed whereas the columns in the second and third sets were pinned. The lengths of columns varied although they were assumed long enough not to have a detrimental effect on the strength of the tested joint.

For the first set of tests, the RHS was bolted directly to the floor thereby not allowing end rotations. There was no axial load applied to the column. For the second and third set of tests, the column was secured into the rig by two roller supports positioned above and below the column at both ends. This prevented vertical
movements but allowed horizontal displacement thus allowing the axial loading of the column. The columns were subject to an axial clamping load of 80kN in all tests except those of tests 11-13 where the effect of axial loads was investigated.

The moment was applied to the beam using a long-stroke 150kN hydraulic ram. The lever arm of the hydraulic ram used to apply the moment to the joint varied between tests and was unspecified for tests #1-13 and #18. The lever arm was varied effectively to allow the higher moments in the testing of the rigid connections to be achieved using the same hydraulic ram. The hydraulic ram was under displacement control to ensure controlled testing of the joints.

Test specimens were coated with a white emulsion prior to testing to detect surface yielding. The bolts used to connect the beams to the columns were ordinary M20 8.8 fully threaded bolts of 60mm length in all tests. Welding of the endplates to the beams was achieved with 6mm nominal fillet welds for simple connections and 12mm nominal fillet welds for moment connections both using E43 stick electrodes. Bolts were tightened to a torque of 160Nm. As the deformation behaviour under static loading was of interest, the tests were conducted under a slow displacement controlled monotonic loading regime except for the second set of tests which were loaded cyclically.
Figure 3.2. Testing rig for the first set of tests (France et al., 1999)
3.1.2 Description of British Steel (1996b) testing

The British Steel (1996b) testing programme covered a wide range of tests on the flowdrilled RHS as an isolated component and as part of an endplate connection. As the France et al. (1999) testing covered a wide range of joint configurations for use in validating the whole joint FEM in Section 3.4, the tests on the flowdrilled RHS in tension component are the most useful out of the British Steel testing programme. They are used to validate the flowdrilled connection modelling in Section 3.3 and analytical characterisation of the RHS face in tension component in Chapter 5 and Chapter 6.

The testing on the isolated component is divided into X-joints with 4 bolts in a rectangular formation, X-joints with 1 to 4 bolts in line along the RHS axis, and X-joints with 2 bolts across the RHS face. Of these, the tests on the 4 bolt rectangular
formation as given in Table 3.2 is used as it represents a combination of the bolts in line and bolts across the RHS. These also represent the typical conditions for the RHS face in tension component of a 3 bolt row joint with 2 bolts in tension. All specimens with the exception of T/12/67/16 failed by thread stripping due to gross deformation of the RHS face. British Steel (1996b) explains this as the RHS deformation causing a rotation and distortion of the flowdrilled holes causing an opening, which in turn result in a reduction in the number of threads engaging the bolt and hence thread stripping in the RHS as shown by the deformed geometry given in Figure 3.4. All tests were conducted on a standard tensile testing machine using the setup given in Figure 3.5.

Of this test set, the 15 tests utilizing the M20 bolts will be used to validate the flowdrilled connection model in Section 3.3 and analytical characterisation of the RHS face in tension component in Chapter 5 and Chapter 6. The tests with M20 bolts are chosen as they covered the greatest variation in geometric parameters and it is also the bolt size used in France et al. (1999) tests so a comparison can be drawn between these two series of tests if necessary. However, for validating the flowdrill bolt pullout mode, the whole set of M16, M20, and M24 bolt tests will be used in Section 5.3.

Figure 3.4. Cross sectional deformation after bolt pullout of T/24/50/20 test (British Steel (1996b))
Table 3.2. British Steel (1996b) X-joint 4 bolt rectangular formation testing programme

<table>
<thead>
<tr>
<th>Test</th>
<th>Bolt</th>
<th>( w_{RHS} ) (mm)</th>
<th>( d_{RHS} ) (mm)</th>
<th>( t_c ) (mm)</th>
<th>( a_t ) (mm)</th>
<th>( f_y ) (N/mm(^2))</th>
<th>Failure mode</th>
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<tbody>
<tr>
<td>T/30/27/16</td>
<td>M16</td>
<td>150</td>
<td>150</td>
<td>4.75</td>
<td>40</td>
<td>332</td>
<td>Thread stripping</td>
</tr>
<tr>
<td>T/30/67/16</td>
<td>M16</td>
<td>150</td>
<td>150</td>
<td>4.75</td>
<td>100</td>
<td>332</td>
<td>Thread stripping</td>
</tr>
<tr>
<td>T/19/27/16</td>
<td>M16</td>
<td>150</td>
<td>150</td>
<td>7.7</td>
<td>40</td>
<td>306</td>
<td>Thread stripping</td>
</tr>
<tr>
<td>T/19/67/16</td>
<td>M16</td>
<td>150</td>
<td>150</td>
<td>7.7</td>
<td>100</td>
<td>306</td>
<td>Thread stripping</td>
</tr>
<tr>
<td>T/12/27/16</td>
<td>M16</td>
<td>150</td>
<td>150</td>
<td>12.5</td>
<td>40</td>
<td>315</td>
<td>Thread stripping</td>
</tr>
<tr>
<td>T/12/67/16</td>
<td>M16</td>
<td>150</td>
<td>150</td>
<td>12.5</td>
<td>100</td>
<td>315</td>
<td>Bolt in tension</td>
</tr>
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<tr>
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<td>M20</td>
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<td>150</td>
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<td>75</td>
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<td>150</td>
<td>4.75</td>
<td>100</td>
<td>303</td>
<td>Thread stripping</td>
</tr>
<tr>
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<td>150</td>
<td>5.9</td>
<td>40</td>
<td>314</td>
<td>Thread stripping</td>
</tr>
<tr>
<td>T/24/27/20</td>
<td>M20</td>
<td>150</td>
<td>150</td>
<td>5.9</td>
<td>75</td>
<td>314</td>
<td>Thread stripping</td>
</tr>
<tr>
<td>T/24/50/20</td>
<td>M20</td>
<td>150</td>
<td>150</td>
<td>5.9</td>
<td>100</td>
<td>314</td>
<td>Thread stripping</td>
</tr>
<tr>
<td>T/19/27/20</td>
<td>M20</td>
<td>150</td>
<td>150</td>
<td>7.7</td>
<td>40</td>
<td>263</td>
<td>Thread stripping</td>
</tr>
<tr>
<td>T/19/50/20</td>
<td>M20</td>
<td>150</td>
<td>150</td>
<td>7.7</td>
<td>75</td>
<td>263</td>
<td>Thread stripping</td>
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<td>T/19/67/20</td>
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<td>100</td>
<td>263</td>
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</tr>
<tr>
<td>T/15/27/20</td>
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<td>150</td>
<td>9.6</td>
<td>40</td>
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</tr>
<tr>
<td>T/15/50/20</td>
<td>M20</td>
<td>150</td>
<td>150</td>
<td>9.6</td>
<td>75</td>
<td>293</td>
<td>Thread stripping</td>
</tr>
<tr>
<td>T/15/67/20</td>
<td>M20</td>
<td>150</td>
<td>150</td>
<td>9.6</td>
<td>100</td>
<td>293</td>
<td>Thread stripping</td>
</tr>
<tr>
<td>T/12/27/20</td>
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<td>150</td>
<td>12.5</td>
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<td>280</td>
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</tr>
<tr>
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<td>M20</td>
<td>150</td>
<td>150</td>
<td>12.5</td>
<td>75</td>
<td>280</td>
<td>Thread stripping</td>
</tr>
<tr>
<td>T/12/67/20</td>
<td>M20</td>
<td>150</td>
<td>150</td>
<td>12.5</td>
<td>100</td>
<td>280</td>
<td>Thread stripping</td>
</tr>
<tr>
<td>T/30/27/24</td>
<td>M24</td>
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<td>150</td>
<td>4.75</td>
<td>40</td>
<td>332</td>
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</tr>
<tr>
<td>T/30/67/24</td>
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<td>150</td>
<td>4.75</td>
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<tr>
<td>T/19/27/24</td>
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<tr>
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<td>150</td>
<td>12.5</td>
<td>100</td>
<td>315</td>
<td>Thread stripping</td>
</tr>
</tbody>
</table>

Figure 3.5. British Steel (1996b) X-joint 4 bolt rectangular formation test setup
3.2 Development of finite element models

The general finite element analysis (FEA) software package Abaqus FEA 6.10-1 (Dassault Systèmes, 2012) is used for the development of the finite element models (FEM) which are used throughout this research. There are a large number of variables involved with creating an efficient and effective FEM including material parameters, mesh discretization, and contact properties to name a few. Using trial and error to determine effective parameters for these variables is time consuming and ineffective so existing FEM techniques from literature is used where possible.

3.2.1 Friction properties

The effect of friction between the surfaces of the model is defined by contact controls based on an isotropic Coulomb’s friction law suitable for steel elements used in benchmark studies such as Bursi and Jaspart (1997). Friction is defined by the penalty friction formulation with a coefficient of 0.44. This value is adopted from the recommendations made in a similar study involving the FEA of steel pre-tensioned bolted end-plate connections to H-columns (Shi et al., 2008). Other studies (van der Vegte and Makino, 2003) suggest using values as high as 0.6 for steel surfaces with rust, however, there is no mention of this in the France testing so the lower value is used. Regardless, a range of values were tested with a negligible difference in results. Normal behaviour is given a hard contact pressure-overclosure with default constraint enforcement.

3.2.2 Material properties

The Young’s modulus and yield stress values given in France et al. (1999) are used to define the elastic behaviour. Although the study presented the yield and
ultimate tensile strength values from material coupon tests for the various column thicknesses given in each set of tests as given in Table 3.3, the values of strain at each stage is not given. Therefore, the typical strain properties obtained from the 50 tensile tests of S275 and S355 coupons in Byfield et al. (2005) are used to assign assumed stress-strain curves as shown in Figure 3.6. Rather than assigning a typical material ultimate strain (such as 0.2) as the limiting strain of the material, Abaqus is left to extrapolate the data forward because stress/strain can concentrate at boltholes even at lower loads causing early termination of the FEA whilst not reaching load carrying capacity. By instead looking at failure criteria, it is possible to investigate the failure behaviour more effectively without encountering artificial failure due to numerical problems in Abaqus. When analysing the results, the failure criteria checked are Tresca stress, von Mises stress, and ultimate strain against given or assumed (0.2 for ultimate strain) material properties to check if a failure mechanism has developed at a given load.

The material properties for the S275 steel used in the beam and endplate are not given so were assumed the same as for the column. The engineering stress and strains values were converted to true stress and strain values before importing into the analysis using Equations [3.1] and [3.2].

\[
\varepsilon_t = \ln(1 + \varepsilon_e) \quad [3.1]
\]

\[
f_t = f_e(1 + \varepsilon_e) \quad [3.2]
\]
Table 3.3. Summary of France et al. (1999) tensile coupon results

<table>
<thead>
<tr>
<th>SHS Section</th>
<th>Steel grade</th>
<th>E  (kN/mm²)</th>
<th>f_y (N/mm²)</th>
<th>f_u (N/mm²)</th>
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</thead>
<tbody>
<tr>
<td>200 x 200 x 6.3</td>
<td>S275</td>
<td>205</td>
<td>336</td>
<td>479</td>
</tr>
<tr>
<td>200 x 200 x 8.0</td>
<td>S275</td>
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<td>466</td>
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<tr>
<td>200 x 200 x 10.0</td>
<td>S275</td>
<td>211</td>
<td>329</td>
<td>477</td>
</tr>
<tr>
<td>200 x 200 x 10.0</td>
<td>S275</td>
<td>217</td>
<td>427</td>
<td>560</td>
</tr>
</tbody>
</table>

For the C40 concrete, the 28-day compressive strength of 43.4N/mm² for concrete is used for tests involving flush endplate joints and 50.5N/mm² for the tests involving extended endplate joints as they were cast using two separate batches of concrete. As the detailed stress-strain relationship is not given, typical load-deflection properties are assumed as given in Figure 3.7.

![Figure 3.6. Assumed engineering stress-strain curves for S275, S355, and 8.8 steel](image1)

![Figure 3.7. Assumed engineering stress-strain curves for C40 concrete](image2)
3.2.3 Concrete core

There are two common approaches for the modelling of the concrete core of concrete-filled tubular columns. One approach is to recognise the concrete core to be much stiffer than the surrounding steel and therefore assume it to be a rigid, undeformable solid. The second approach is to model the concrete core as a deformable solid. The latter approach was taken in this study to ensure that the interactions between the concrete and steel were modelled as accurately as possible although the concrete core is not expected to deform significantly.

Contact interactions are not specified between the bolt nut used in modelling the flowdrilled connection behaviour explained in Section 3.3.2 and the concrete infill to reduce the complexity of the concrete infill geometry and mesh. This means that the concrete infill will not provide additional support against pullout of the bolts which may exist to a small degree in the elastic regime where the concrete infill is in contact with the bolt threads that extend past the lobe of the flowdrilled RHS face as shown in Figure 3.8. This is a reasonable assumption as concrete offers negligible tensile strength and it is likely that there will be voids in the concrete in such small gaps.

![Figure 3.8. RHS concrete infill detailing](image-url)
3.2.4 Weld details

A study comparing 7mm, 10mm, and no fillet weld detailing between endplates and beam cross sections in finite element models of double angle connections (Hong et al., 2002) concludes that the modelling of welds affected both initial stiffness and strength values of joints by up to 35%. This is backed up by the fact that the deformation of individual components, especially the endplate, is dependent on the width of the yielding surface. An increase in weld width decreases the yieldable width and thus increases both the stiffness and strength characteristics.

With this in mind, a comparison is made with the moment-rotation behaviour of three joints with and without welding details. Due to the complex shape of the welds, they are modelled using tetrahedral mesh elements to allow effective mesh generation with minimal mesh resolution which is challenging if using a standard brick element. The welds are attached to their respective beam and endplate surfaces using tie constraints. For the no weld model, the beam cross section is tied directly to the endplate.

The three joints considered are Test 2, 6, and 19. Test 2 has 6mm fillet weld and yielding of the RHS. Test 6 has 6mm fillet weld and yielding of the endplate. Test 19 has a larger 12mm fillet weld and failure due to thread stripping of the RHS. The results are given in Figure 3.9 showing that in general, the modelling of welds does not significantly change the initial stiffness nor strength characteristics. This is attributed to the fact that the in bolted endplate connections to RHS, the joint characteristics are most dependent on the RHS component which is independent of weld detailing. The biggest difference is in Test 6 where the initial stiffness and strength is underestimated by approximately 15% in the no-weld model. This is attributed to the joint characteristics being more dependent on the endplate which is
influenced by weld detailing. It could be argued that most of the France et al. tests could be modelled sufficiently without weld details because most are reported to have significant deformations in only the RHS. However, it is necessary to include the welds detailing in all joint models to be consistent in the modelling approach.

Figure 3.9. Moment-rotation curve for joint models with and without welds

3.2.5 Mesh properties

While many studies have found success using either shell elements or solid elements for modelling bolted connections to columns, it was decided that the use of solid elements would give a better understanding of connection deformation due to the interactions between individual components as well as to allow accurate modelling of the flowdrill connection behaviour. More detail regarding the modelling of the flowdrill connections is presented in Section 3.3.

The usage of the 8-node linear brick elements C3D8, C3D8R (Reduced integration), and C3D8I (Incompatible modes) are considered for the meshing of the components. It was decided that the C3D8R element would be the most suitable due to its efficiency and accuracy as well as its widespread use and validation in similar studies involving the T-stub bending mechanism such as Bursi and Jaspart (1997) and
van der Vegte et al. (2003). Compared with other 8-node brick elements such as the C3D8, there is no locking phenomena observed or poor performance in plastic behaviour. The C3D8R may underestimate bending stiffness therefore this needs to be checked. The 16-noded quadratic brick elements C3D20 and C3D20R (Reduced integration) are also considered as they are better at capturing surface stress concentrations of which the C3D20R is an excellent general purpose element. However, both are known to cause problems in contact calculations and they are more computationally demanding making them unsuitable for modelling a large number of complex models.

The main issue with the mesh discretization is determining the optimum number of elements to use across the component thicknesses. A benchmark study on the modelling of bolted steel connections (Bursi and Jaspart, 1997) recommended the use of more than two layers to give excellent results for endplate joints to H-section columns when using linear brick elements, even in the large displacement regime in which membrane effects govern the inelastic behaviour.

A sensitivity study was carried out to verify whether these recommendations are applicable in connections to tubular columns. The sensitivity study focuses on the RHS component in tension using the flowdrill connection techniques developed in Section 3.3 because is the most sensitive component in the whole joint model.

First, the number of mesh elements required in the RHS thickness direction was investigated. The mesh size along the length and width was varied accordingly to maintain an ideal 1:1:1 cubic ratio. For this investigation, a section with typical dimensions of $a_t=10\text{mm}$, $b_t=100\text{mm}$, $w_{\text{RHS}}=200\text{mm}$, $d_{\text{RHS}}=200\text{mm}$, $t_c=10\text{mm}$, $n=2$, and $f_y=300\text{N/mm}^2$ is considered (Model #1.1.1.2 in Section 4.5). Details of the mesh are presented in Table 3.4. Figure 3.10 compares the load-deflection results using different numbers of mesh layers across the RHS cross section thickness. A comparison of the
deflection at specific loads is given in Table 3.5. Acceptable convergence is reached when four layers are used along the RHS thickness with deviations of approx. 2%. There are large deviations when three mesh layers are used (approx. 10%) and even larger deviations when only two mesh layers are used (approx. 20%). When looking at the deflection at failure, similar errors are observed with sections having two and three mesh layers giving approximately 10% and 20% error. There is convergence with the failure load and deflection when using at least four mesh layers. Based on these results, it is necessary to use at least four mesh layers to get reliable results.

Next, the mesh size along the width and length is considered by varying the aspect ratio while maintaining the four mesh layers along the depth. The mesh aspect ratio is considered rather than its absolute size so that findings can be generalised to different cases where the RHS width/depth or thickness is varied. Details of the mesh are presented in Table 3.4. Figure 3.11 compares the load-deflection results for different mesh aspect ratios along the length and width while using four mesh layers in the RHS thickness direction. A comparison of the deflection at specific loads is given in Table 3.6. Acceptable convergence is reached with a mesh aspect ratio of 1:2:2 with deviations of approx. 2%. There are large deviations when a mesh aspect ratio of 1:3:3 is used (approx. 10%). However, the mesh is not very sensitive along the length and width for the elastic regime so a mesh with 1:3:3 aspect ratio is acceptable when investigating elastic behaviour. The mesh becomes very sensitive at higher loads when membrane action begins. When looking at the failure deflection, the sections having 1:2.5:2.5 and 1:3:3 mesh aspect ratios both give approximately 10% error. There is convergence with the failure load and deflection when using a 1:2:2 mesh aspect ratio or finer. In general, a mesh aspect ratio of 1:2:2 or finer should be used with four mesh layers to get reliable results.
These recommendations are also applied to the endplate component which is reasonable because the endplate is not as sensitive in bending in comparison to the RHS component. An exception is made for very thick endplates (25mm) used in some of France et al. (1999) tests. For these endplates, six mesh layers are used to maintain a reasonable mesh size for the recommended mesh aspect ratio.

The perimeter of the bolts and boltholes in the column face and endplates are modelled with at least 24 elements to ensure reliable simulation results as recommended in van der Vegte et al. (2003). For the two mesh layer case, this was achieved by using smaller mesh elements around the boltholes.

For all models, the joint symmetry is taken advantage of by modelling half of the joint cross section and then applying a symmetry boundary condition. This allows a 50% reduction in the elements used for mesh discretization.

Table 3.4. Details of mesh size in sensitivity study

<table>
<thead>
<tr>
<th>Number of mesh elements across thickness</th>
<th>Mesh aspect ratio</th>
<th>Typical element size (mm)</th>
<th>Approximate number of total elements</th>
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<tr>
<td>2</td>
<td>1:1:1</td>
<td>5x5x5</td>
<td>10,000</td>
</tr>
<tr>
<td>3</td>
<td>1:1:1</td>
<td>3.3x3.3x3.3</td>
<td>20,000</td>
</tr>
<tr>
<td>4</td>
<td>1:1:1</td>
<td>2.5x2.5x2.5</td>
<td>35,000</td>
</tr>
<tr>
<td>5</td>
<td>1:1:1</td>
<td>2x2x2</td>
<td>60,000</td>
</tr>
<tr>
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<td>2.5x2.5x2.5</td>
<td>35,000</td>
</tr>
<tr>
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<td>1:2:2</td>
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</tr>
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<tr>
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<td>1:3:3</td>
<td>2.5x7.5x7.5</td>
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Table 3.5. Deflections when using different number of mesh layers (1:1:1 mesh aspect ratio)

<table>
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<th>100kN load</th>
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<th>250kN load</th>
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<td>Error</td>
<td>d (m)</td>
<td>Error</td>
<td>d (m)</td>
<td>Error</td>
<td>d (m)</td>
<td>Error</td>
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<td>9.01E-05</td>
<td>21.1%</td>
<td>8.13E-04</td>
<td>19.0%</td>
<td>6.17E-03</td>
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</tr>
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<td>7.3%</td>
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<td>3.4%</td>
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<td>1.85E-02</td>
<td>-3.4%</td>
</tr>
<tr>
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<td>2.0%</td>
<td>6.96E-04</td>
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<td>5.11E-03</td>
<td>2.3%</td>
<td>1.90E-02</td>
<td>-0.6%</td>
</tr>
<tr>
<td>5</td>
<td>7.44E-05</td>
<td>6.83E-04</td>
<td>5.00E-03</td>
<td>2.3%</td>
<td>1.91E-02</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The difference is relative to deflection using 5 mesh layers.
Table 3.6. Deflections for 4 layer mesh when varying mesh aspect ratios

<table>
<thead>
<tr>
<th>Mesh aspect ratio</th>
<th>10kN load</th>
<th>100kN load</th>
<th>250kN load</th>
<th>400kN load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>d (m)</td>
<td>Error</td>
<td>d (m)</td>
<td>Error</td>
</tr>
<tr>
<td>1:1:1</td>
<td>7.59E-05</td>
<td>2.0%</td>
<td>6.96E-04</td>
<td>1.8%</td>
</tr>
<tr>
<td>1:2:2</td>
<td>7.59E-05</td>
<td>2.0%</td>
<td>6.96E-04</td>
<td>1.8%</td>
</tr>
<tr>
<td>1:2.5:2.5</td>
<td>7.62E-05</td>
<td>2.3%</td>
<td>6.77E-04</td>
<td>-0.9%</td>
</tr>
<tr>
<td>1:3:3</td>
<td>7.63E-05</td>
<td>2.5%</td>
<td>6.74E-04</td>
<td>-1.3%</td>
</tr>
</tbody>
</table>

The difference is relative to deflection using 5 mesh layers in Table 3.5.

3.2.6 RHS corner radius

One issue with the modelling of the RHS column is that there is some variation in the geometric properties of the corner profile (more specifically the outer and inner radii). BS EN 10210-2:2006 (BSI, 2006), which covers section dimensions and tolerances of hot finished structural hollow sections, states that the external corner profile (R, C₁, and C₂ in Figure 3.12) can have a maximum of 3tₑ at each corner. Sectional properties are calculated using the most common outer corner radius of 1.5tₑ and inner corner radius of tₑ. However, it is common for the outer corner radius to be within the range of 1.5tₑ to 2tₑ. For the entirety of this thesis, the outer corner radius value is taken as 2tₑ because it is significantly easier to generate a suitable finite element mesh for the RHS corner. This is due to the ABAQUS medial axis algorithm for the sweep technique used which allows a suitable mesh to be generated for the corner profile when it can be partitioned into a perfect quarter-circle (i.e. using an outer radius of 2tₑ). When the smaller outer radius of 1.5tₑ is used, the partitioned corner profile contains radial and straight elements which causes difficulties when generating a suitable mesh. A comparison of the RHS corner meshes when using the two geometries is given in Figure 3.13. In theory, this should not result in a noticeable difference in load-deflection behaviour. This is because the deformable width of the RHS face remains the same at a=wₑₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐ}_{next_paragraph}
which remains constant). This assumption is investigated by checking the load-deflection behaviour of the RHS face in tension component using techniques described in Section 3.3. A comparison of the load-deflection curves for a typical case is given in Figure 3.14 showing the difference between the two geometries is small (2-3% maximum) with the difference in some part possibly due to mesh convergence issues when using \( r_o = 1.5 t_c \) as it is hard to generate a suitable mesh for this radius. Therefore, without specific corner radii data in France et al. (1999) and British Steel (1996b) testing, \( r_o = 2 t_c \) is used.

Figure 3.12. Details of RHS corner profile in BS EN 10210-2:2006 (BSI, 2006)
3.2.7 Loading details

As presented in Table 3.1, the lever arm used to apply the concentrated load to the beam varies between tests so that the same loading jack could be used to apply the wide variety of bending moments. The lever arm lengths from the tests are used to keep the bending moment and shear load ratio consistent with testing. An assumed
lever arm length as stated in Table 3.1 is used for tests where the lever arm was not specified. The bending moment is applied as a concentrated load at the end of the beam coupled to its cross section. The clamping effect was applied to the FE models as a concentrated load on a reference point on the column cross section neutral axis and coupled to the cross section of the column. The effect of bolt tension is applied to all of the bolts using the Abaqus bolt load function. The 160Nm torque used to tighten the bolts is converted into an equivalent tension load of approximately 20kN.

Each analysis was split into a 5 second contact and 10 second loading steps. During the contact step, the column clamping load and bolt loads are applied to initiate contact of the individual components. During the loading step, the main loading is applied. Bolt loads and column axial loads are ramped linearly over the step in which they were introduced and then propagated into the loading step.

3.3 Modelling of flowdrilled bolts

3.3.1 Developing an effective flowdrilled connection model

The modelling of flowdrilled connections is difficult as there are no previous studies using finite element analysis (FEA) to model the unique characteristics of this bolting system. In comparison to conventional bolting, it is expected that there will be differences in the behaviour due to: (1) the shorter deformable length of the bolt in comparison to conventional bolting because the bolt will be supported throughout the columns thickness rather than the nut, (2) loss of contact between the bolt and column face threads as the column face deforms under load, and (3) a more concentrated loading area on the internal face of the column under tensile loads where a nut and washer would be in a traditional bolt/nut system. (1) will increase the stiffness of flowdrilled bolts and its contribution can be easily incorporated by modifying the bolt
cross section properties in relation to the change in $L_b$. (2) can potentially lead to bolt pull-out before the bolt or RHS design load is reached as well as a reduced stiffness at higher loads. (3) may lead to small changes in the strength of the RHS face.

To ensure that the characteristics of flowdrilled bolts are reproduced effectively, it is necessary to develop modelling techniques that are both accurate and computationally efficient. Three different modelling approaches are considered for this purpose: regular bolt/nut, bolt with small nut, and connector section with coupling to the hole surface. The regular bolt/nut has the same diameter on both sides to reflect bolt head, nut, and washer dimensions. The bolt with small nut has a regular bolt head with a smaller nut dimensioned as having a radius 1mm larger than that of the bolt hole. The connector section with coupling to the hole surface consists of a lengthless connector section that transmits axial loads with one end coupled to the bolt shank surface and the other coupled to the RHS hole surface. The cross section details of these approaches are presented in Figure 3.15. A 0.01mm gap is applied between bolt and endplate/RHS surfaces prior to establishing contact.

Figure 3.15. Details of modelling approaches for flowdrilled connection to RHS face
3.3.2 Validation of flowdrilled connection model against British Steel (1996b) tests

The three approaches suggested in Section 3.3.1 for the modelling of flowdrilled connections to the RHS face are validated against British Steel (1996b) testing described in Section 3.1.2. By comparing the load-deflection behaviour of the isolated RHS face in tension component for the different approaches in elastic, plastic, and failure stages, it is possible to determine which is the most effective overall approach.

To test the various approaches for modelling the flowdrilled connection, it is necessary to develop a suitable FEM model that allows prediction of the isolated RHS face strength characteristics. A solid C8D8R brick mesh model based on parameters validated in Section 3.2.5 is used for this purpose. To take advantage of the model symmetry, a one-eighth model is used with appropriate symmetry boundary conditions to reduce the number of mesh elements used. The test yield and ultimate strength material properties are used with strain characteristics seen in typical mild steel stress-strain curves as shown in Figure 3.16 as strain properties are not given. Contact interactions are applied to the relevant bolt and RHS surfaces in contact. The bolt load is applied as a concentrated load to a reference point which is coupled to the bolt or connector section. The details of a typical finite element model are presented in Figure 3.17.

The comparison of different flowdrilled connection modelling approaches is conducted against two tests, T15/50/20 and T24/27/20. Comparisons of load-deflection behaviour are given in Figure 3.18 and Figure 3.19.
Figure 3.16. Engineering stress/strain curves for column sections

Figure 3.17. FE model for flowdrilled RHS face in tension component

Detail of coupled surface for connector model
In both cases, it can be seen that the regular bolt and connector approaches overestimate the stiffness of the RHS face between yield and failure stages. On the other hand, the usage of the small nut gives a consistent load-deflection prediction at all stages of the testing. Notably, both the failure strength and ductility are replicated accurately. A comparison of the deflection behaviour at the test failure load (or FEM predicted failure load for small bolt and connector models) for all approaches is given.
in Figure 3.20. With the regular bolt and nut, the bolt load is not evenly transferred to the RHS face due to the pattern of contact and instead is transferred closer to the sidewalls thereby increasing the stiffness. For the bolt with small nut, the bolt load is transferred evenly and to the edges causing the bolt holes to open up significantly mimicking the deformation of flowdrilled RHS holes as described in British Steel (1996b) where gross deformation of the RHS face causes opening up of the flowdrilled holes. For the connector with coupling to the RHS hole there is artificial restraint due to the surface of the RHS hole not being able to rotate and deform freely causing the overestimation in post-yield stiffness. From this evidence, it is clear that modelling the flowdrilled connection as a bolt with small nut is the most accurate therefore this approach is used to model the rest of the British Steel tests to further validate the FEM techniques.

![Figure 3.20. Cross section of deflected shapes for T24/27/20 at test failure load using different flowdrilled connection modelling approaches](image)

The 15 pullout tests for M20 2 bolt row RHS face in tension test set presented in Table 3.7 are considered using this bolt with small nut approach. Of these tests,
T19/67/20, T15/67/20, and T12/67/20 are excluded from this validation because the flowdrill thread is formed partially on the curved edges of the RHS which causes difficulty in discretizing the mesh in FEM and is also advised against in flowdrill literature (British Steel, 1997).

Table 3.7. British Steel (1996b) testing programme for M20 4 bolt group pullout tests

<table>
<thead>
<tr>
<th>Specimen</th>
<th>RHS Section</th>
<th>c (mm)</th>
<th>h (mm)</th>
<th>$f_y$ (N/mm$^2$)</th>
<th>$f_u$ (N/mm$^2$)</th>
<th>$e_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T/30/27</td>
<td>150×150×5</td>
<td>40</td>
<td>120</td>
<td>303</td>
<td>457</td>
<td>0.185</td>
</tr>
<tr>
<td>T/30/50</td>
<td>150×150×5</td>
<td>75</td>
<td>120</td>
<td>303</td>
<td>457</td>
<td>0.185</td>
</tr>
<tr>
<td>T/30/67</td>
<td>150×150×5</td>
<td>100</td>
<td>120</td>
<td>303</td>
<td>457</td>
<td>0.185</td>
</tr>
<tr>
<td>T/24/27</td>
<td>150×150×6.3</td>
<td>40</td>
<td>120</td>
<td>303</td>
<td>460</td>
<td>0.185</td>
</tr>
<tr>
<td>T/24/50</td>
<td>150×150×6.3</td>
<td>75</td>
<td>120</td>
<td>303</td>
<td>460</td>
<td>0.185</td>
</tr>
<tr>
<td>T/24/67</td>
<td>150×150×6.3</td>
<td>100</td>
<td>120</td>
<td>303</td>
<td>460</td>
<td>0.185</td>
</tr>
<tr>
<td>T/19/27</td>
<td>150×150×8</td>
<td>40</td>
<td>120</td>
<td>263</td>
<td>454</td>
<td>0.19</td>
</tr>
<tr>
<td>T/19/50</td>
<td>150×150×8</td>
<td>75</td>
<td>120</td>
<td>263</td>
<td>454</td>
<td>0.19</td>
</tr>
<tr>
<td>T/19/67</td>
<td>150×150×8</td>
<td>100</td>
<td>120</td>
<td>263</td>
<td>454</td>
<td>0.19</td>
</tr>
<tr>
<td>T/15/27</td>
<td>150×150×10</td>
<td>40</td>
<td>120</td>
<td>293</td>
<td>458</td>
<td>0.20</td>
</tr>
<tr>
<td>T/15/50</td>
<td>150×150×10</td>
<td>75</td>
<td>120</td>
<td>293</td>
<td>458</td>
<td>0.20</td>
</tr>
<tr>
<td>T/15/67</td>
<td>150×150×10</td>
<td>100</td>
<td>120</td>
<td>293</td>
<td>458</td>
<td>0.20</td>
</tr>
<tr>
<td>T/12/27</td>
<td>150×150×12.5</td>
<td>40</td>
<td>120</td>
<td>280</td>
<td>456</td>
<td>0.205</td>
</tr>
<tr>
<td>T/12/50</td>
<td>150×150×12.5</td>
<td>75</td>
<td>120</td>
<td>280</td>
<td>456</td>
<td>0.205</td>
</tr>
<tr>
<td>T/12/67</td>
<td>150×150×12.5</td>
<td>100</td>
<td>120</td>
<td>280</td>
<td>456</td>
<td>0.205</td>
</tr>
</tbody>
</table>

Comparisons of individual load-deflection curves are given in Appendix A.1 with a comparison of key characteristics given in Table 3.8. While there are moderate differences between the testing and FEM results, there is generally a strong correlation between the results throughout the various stages of loading. The elastic behaviour is modelled well with the exception of tests such as T30/67/20 which has a near infinite stiffness at the start. Such a discrepancy is believed to be due to lack of fit of the measurement device as evidenced by the high start load in many of the load-deflection curves or due to high pretensioning of bolts which was not considered in this model. As given in the test description, the bolts were pretensioned using the torque method which can give variations in the axial pretensioning load. Another factor that adds variability in results (especially the deformation at failure) is the variability in the material ultimate strain that would have been present in test specimens but was not
recorded and checked for using an assumed value of 0.2. The elastic strength is underestimated in many cases although not by a significant amount.

The general agreement in results suggest that the FEM techniques used are suitable for modelling the RHS face under tension for the elastic regime, the plastic regime, and at failure. This means that this model is suitable for use in the parametric study of the initial stiffness, failure strength, and failure deformation capacity of the RHS face in tension component.

Table 3.8. Comparison of key characteristics between British Steel (1999) tests and FEM

<table>
<thead>
<tr>
<th>Test #</th>
<th>Test F</th>
<th>F3 (kN)</th>
<th>Error</th>
<th>Test F</th>
<th>Failure load (kN)</th>
<th>Error</th>
<th>Deformation at failure (m)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>T/30/27/20</td>
<td>52.3</td>
<td>44.4</td>
<td>-15.1%</td>
<td>158</td>
<td>121.2</td>
<td>-23.3%</td>
<td>40.0</td>
<td>22.2</td>
</tr>
<tr>
<td>T/30/50/20</td>
<td>83.6</td>
<td>69.3</td>
<td>-17.1%</td>
<td>160</td>
<td>143.7</td>
<td>-10.2%</td>
<td>25.0</td>
<td>15.3</td>
</tr>
<tr>
<td>T/30/67/20</td>
<td>148</td>
<td>122.5</td>
<td>-17.2%</td>
<td>191</td>
<td>180.0</td>
<td>-5.8%</td>
<td>11.6</td>
<td>9.7</td>
</tr>
<tr>
<td>T/24/27/20</td>
<td>88.8</td>
<td>75.8</td>
<td>-14.7%</td>
<td>212</td>
<td>182.4</td>
<td>-14.0%</td>
<td>35.0</td>
<td>31.5</td>
</tr>
<tr>
<td>T/24/50/20</td>
<td>139</td>
<td>110.9</td>
<td>-20.2%</td>
<td>247</td>
<td>215.0</td>
<td>-12.9%</td>
<td>22.2</td>
<td>17.1</td>
</tr>
<tr>
<td>T/24/67/20</td>
<td>233</td>
<td>190.1</td>
<td>-18.4%</td>
<td>245</td>
<td>261.8</td>
<td>6.9%</td>
<td>15.0</td>
<td>12.1</td>
</tr>
<tr>
<td>T/19/27/20</td>
<td>155</td>
<td>110.8</td>
<td>-28.5%</td>
<td>310</td>
<td>269.2</td>
<td>-13.1%</td>
<td>28.0</td>
<td>26.3</td>
</tr>
<tr>
<td>T/19/50/20</td>
<td>229</td>
<td>169.7</td>
<td>-25.9%</td>
<td>330</td>
<td>332.6</td>
<td>0.8%</td>
<td>20.0</td>
<td>19.7</td>
</tr>
<tr>
<td>T/15/27/20</td>
<td>234</td>
<td>212.0</td>
<td>-9.4%</td>
<td>370</td>
<td>409.6</td>
<td>10.7%</td>
<td>25.0</td>
<td>26.3</td>
</tr>
<tr>
<td>T/15/50/20</td>
<td>367</td>
<td>327.3</td>
<td>-10.8%</td>
<td>465</td>
<td>455.7</td>
<td>-2.0%</td>
<td>16.5</td>
<td>14.9</td>
</tr>
<tr>
<td>T/12/27/20</td>
<td>355</td>
<td>324.6</td>
<td>-8.6%</td>
<td>485</td>
<td>499.1</td>
<td>2.9%</td>
<td>21.0</td>
<td>21.3</td>
</tr>
<tr>
<td>T/12/50/20</td>
<td>532</td>
<td>491.9</td>
<td>-7.5%</td>
<td>570</td>
<td>563.3</td>
<td>-1.2%</td>
<td>9.5</td>
<td>7.5</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>-16.1%</th>
<th>Mean</th>
<th>-5.1%</th>
<th>Mean</th>
<th>-15.3%</th>
</tr>
</thead>
<tbody>
<tr>
<td>St.dev</td>
<td>6.6%</td>
<td></td>
<td>St.dev</td>
<td>9.9%</td>
<td>St.dev</td>
<td>15.2%</td>
</tr>
<tr>
<td>Max</td>
<td>28.5%</td>
<td></td>
<td>Max</td>
<td>23.3%</td>
<td>Max</td>
<td>44.4%</td>
</tr>
</tbody>
</table>

3.4 Validation of whole joint FEM using France et al. (1999) tests

3.4.1 Introduction

The whole joint tests in France et al. (1999) are modelled using the FEA techniques discussed earlier in this chapter. To enable a direct comparison with test results, the displacements from the FEA full joint models are converted into moment-rotation curves. It is necessary to convert displacement into rotation as it is not
possible to directly obtain rotation of nodes when using C8D8R elements. The joint rotation is taken as the difference between the beam and column component rotations. These component rotations are calculated from points taken at locations identical to that of the inclinometers used to calculate rotation in the testing as shown in Figure 3.2 and Figure 3.3 where the column side rotation is taken at its central axis and the beam side rotation is taken 125mm from the column face. The bending moment is calculated as the concentrated load multiplied by the lever arm to the RHS face which is consistent with the definition in France et al.

3.4.2 Discussion

In general, the results show that the modelling techniques developed in this section are sufficiently accurate for predicting the moment-rotation behaviour of flowdrilled endplate connections to both unfilled and concrete-filled RHS columns as shown in the individual moment-rotation curves presented in Appendix A.2. A comparison of yield strength and the 0.03 radians joint rotation limit strength (explained in Section 8.4) is given in Table 3.9. Here, the yield strength is defined by the point at which the initial stiffness and post-yield stiffness secant lines intersect.

For the flush endplate simple connections to unfilled RHS, there is excellent agreement in the results for the initial yield behaviour as well as in plastic and membrane action stages. Figure 3.21 shows a typical comparison between test and simulation moment-rotation curves. These joints make up the second set of tests conducted by France et al. that were loaded cyclically to investigate unloading stiffness at different load levels. To investigate any fatigue caused by the cyclic loading in the plastic range, Test 4 was rerun using the same parameters but with the cyclic loading pattern used in the France et al. testing as shown in Figure 3.22. It can be seen that the cyclic moment-rotation behaviour is predicted consistently. The
'monotonic' curve extracted from the cyclic load FEM curve shows an almost exact match with the 'monotonic' curve extracted by France et al. While the monotonic FEM curve shows a slightly higher strength in the plastic range.

For the flush endplate moment connections to unfilled RHS, there is a similar agreement between test and FEM results. Elastic behaviour is predicted well with the exception of Test 26, however this is clearly due to testing errors as the test moment-rotation curve shows almost infinite initial stiffness for the first 20kNm of loading applied. Possible causes for this include lack of fit of bolts, adhesion of joint components, or high bolt pretension.

For the extended endplate moment connections to unfilled RHS, the yield and plastic behaviour is underestimated by approximately 15%. The initial stiffness of the FEM is also underestimated although the post-yield characteristics are similar. A typical moment-rotation curve is given in Figure 3.23. Besides endplate type, notable differences between these tests and the flush endplate tests are the endplate thickness and weld depth. Again, possible causes for the discrepancy in the elastic range include lack of fit of bolts, adhesion of joint components, or very high bolt pretension.

For the connections to concrete-filled RHS, the trends are similar as for the connections to hollow columns. A typical moment-rotation curve is presented in Figure 3.24. For the flush endplate connections, the moment-rotation behaviour is predicted accurately for both elastic and plastic loading stages. For joints with extended endplates, the moment-rotation behaviour is generally underestimated with the yield strength being underestimated by approximately 25% in all cases. It should be noted that due to the basic approach of modelling the concrete core, it is not possible to model the abrupt changes in moment rotation response of negative stiffness due to concrete cracking in the compression zone of the column in line with the beam flange which was detected in Tests 15 and 16.
For the flush endplate connection to unfilled RHS under varied axial loads, there is good agreement between the initial stiffness and yield strength values. The differences in the post-yield behaviour under different axial loads are predicted consistently. The tests to partial depth endplate connections were not considered as they increased the complexity of modelling while offering no benefit in joint properties.

The variability in results for all connection types is attributed to the fact that the actual stress-strain curves for the test specimens was not given and therefore the assumed stress-strain curve was used as explained in Section 3.2.2. Other possible factors include imperfections, variations in column face thickness, and variable pretensioning due to the torque method of bolt tightening being used.

Table 3.9. Comparison of key characteristics between France et al. (1999) tests and FEM

<table>
<thead>
<tr>
<th>Endplate</th>
<th>CFT</th>
<th>Test</th>
<th>FEM</th>
<th>Error</th>
<th>Test</th>
<th>FEM</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 2</td>
<td>Flush</td>
<td>57</td>
<td>55</td>
<td>-3.5%</td>
<td>67.6</td>
<td>68.5</td>
<td>1.3%</td>
</tr>
<tr>
<td>Test 4</td>
<td>Flush</td>
<td>35</td>
<td>36</td>
<td>2.9%</td>
<td>42.0</td>
<td>40.5</td>
<td>-3.5%</td>
</tr>
<tr>
<td>Test 5</td>
<td>Flush</td>
<td>36</td>
<td>30</td>
<td>-16.7%</td>
<td>36.8</td>
<td>41.2</td>
<td>12.0%</td>
</tr>
<tr>
<td>Test 6</td>
<td>Flush</td>
<td>43</td>
<td>42</td>
<td>-2.3%</td>
<td>48.7</td>
<td>50.0</td>
<td>2.6%</td>
</tr>
<tr>
<td>Test 7</td>
<td>Flush</td>
<td>23</td>
<td>20</td>
<td>-13.0%</td>
<td>25.6</td>
<td>27.0</td>
<td>5.5%</td>
</tr>
<tr>
<td>Test 8</td>
<td>Flush</td>
<td>99</td>
<td>87</td>
<td>-12.1%</td>
<td>98.3</td>
<td>108.0</td>
<td>9.8%</td>
</tr>
<tr>
<td>Test 10</td>
<td>Flush</td>
<td>19</td>
<td>21</td>
<td>10.5%</td>
<td>22.3</td>
<td>21.0</td>
<td>-5.7%</td>
</tr>
<tr>
<td>Test 18</td>
<td>Flush</td>
<td>78</td>
<td>61</td>
<td>-21.8%</td>
<td>93.3</td>
<td>101.5</td>
<td>8.8%</td>
</tr>
<tr>
<td>Test 25</td>
<td>Flush</td>
<td>48</td>
<td>39</td>
<td>-18.8%</td>
<td>57.9</td>
<td>58.4</td>
<td>0.9%</td>
</tr>
<tr>
<td>Test 26</td>
<td>Flush</td>
<td>30</td>
<td>20</td>
<td>-33.3%</td>
<td>42.3</td>
<td>50.0</td>
<td>18.3%</td>
</tr>
<tr>
<td>Test 19</td>
<td>Ext.</td>
<td>27</td>
<td>25</td>
<td>-7.4%</td>
<td>29.3</td>
<td>31.0</td>
<td>6.0%</td>
</tr>
<tr>
<td>Test 20</td>
<td>Ext.</td>
<td>35</td>
<td>34</td>
<td>-2.9%</td>
<td>36.3</td>
<td>43.6</td>
<td>20.1%</td>
</tr>
<tr>
<td>Test 21</td>
<td>Ext.</td>
<td>116</td>
<td>86</td>
<td>-25.9%</td>
<td>125.9</td>
<td>144.0</td>
<td>14.4%</td>
</tr>
<tr>
<td>Test 23</td>
<td>Ext.</td>
<td>141</td>
<td>116</td>
<td>-17.7%</td>
<td>165.2</td>
<td>186.0</td>
<td>12.6%</td>
</tr>
<tr>
<td>Test 14</td>
<td>Flush Yes</td>
<td>220</td>
<td>180</td>
<td>-18.2%</td>
<td>225.9</td>
<td>266.7</td>
<td>18.1%</td>
</tr>
<tr>
<td>Test 15</td>
<td>Flush Yes</td>
<td>160</td>
<td>175</td>
<td>9.4%</td>
<td>224.2</td>
<td>271.6</td>
<td>21.1%</td>
</tr>
<tr>
<td>Test 16</td>
<td>Flush Yes</td>
<td>182</td>
<td>147</td>
<td>-19.2%</td>
<td>189.0</td>
<td>236.5</td>
<td>25.2%</td>
</tr>
<tr>
<td>Test 17</td>
<td>Flush Yes</td>
<td>205</td>
<td>210</td>
<td>2.4%</td>
<td>251.9</td>
<td>310.5</td>
<td>23.3%</td>
</tr>
<tr>
<td>Test 22</td>
<td>Ext. Yes</td>
<td>116</td>
<td>92</td>
<td>-20.7%</td>
<td>130.6</td>
<td>113.0</td>
<td>-13.5%</td>
</tr>
<tr>
<td>Test 24</td>
<td>Ext. Yes</td>
<td>72</td>
<td>66</td>
<td>-8.3%</td>
<td>84.1</td>
<td>84.2</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

Mean -10.8%  Mean 8.9%
St.dev. 11.8%  St.dev. 10.5%
Max. 33.3%  Max. 25.2%
Figure 3.21. Typical moment-rotation curve for flush endplate simple connections to unfilled RHS (Test 6)

Figure 3.22. Test 4 moment-rotation curves when using monotonic and cyclic loading
Figure 3.23. Typical moment-rotation curve for extended endplate moment connections to unfilled RHS (Test 20)

Figure 3.24. Typical moment-rotation curve for flush endplate moment connections to concrete-filled RHS (Test 17)
In this chapter, a detailed explanation of the FEA techniques used to successfully model the joint moment-rotation behaviour of flowdrilled endplate connections to RHS columns is presented. Existing techniques for characterisation of friction and material properties as well as modelling of concrete core and weld details are confirmed. A mesh sensitivity study is conducted to determine the optimum usage of mesh elements in discretizing the RHS face in tension component. An effective method for modelling flowdrilled connections to the RHS face in tension component is presented that uses a standard bolt with a very small diameter on the column side to mimic pullout behaviour that compares favourably with British Steel (1996b) test results. The combination of existing and new techniques is applied to the modelling of France et al. (1999) whole joint tests demonstrating their suitability for modelling flowdrilled endplate connections to RHS columns. This satisfies the aim of developing

Figure 3.25. Typical moment-rotation curve for flush endplate simple connections to unfilled RHS with variable axial loading (Test 13)

3.5 Conclusions
validated FEM techniques that allows extensive parametric study of joint behaviour to be used in validating the analytical work developed in subsequent chapters.
Chapter 4

RHS Face Initial Stiffness

4.1 Introduction

Developing a method to calculate the elastic stiffness of bolted connections to Rectangular Hollow Sections (RHS) is the objective of this chapter. The flat surface of RHS allows conventional endplate connections to be used in conjunction with blind bolting systems such as Flowdrilled connections as established in Section 2.1.2 for easy construction (British Steel, 1997). This type of joint can develop significant initial stiffnesses close to those of welded joints without the associated cost of weld detailing.

As mentioned in Section 2.4.2, Jaspart et al. (2004) appear to be the only authors to have conducted research to develop a fully analytical method to quantify the initial stiffness of steel beam to RHS column joint using bolts. However, their method has a number of shortcomings, including limited range of applicability, large errors, and complex equations. Further comments will be made later regarding the error and applicability of the method of Jaspart et al. As an illustration of the limited range of applicability of this method, Figure 4.1 shows the limitations imposed by the method where joints with more than two bolt rows fall outside of the acceptable geometric
According to these limitations, 19 out of 20 tests of France et al. (1999) will be outside the ranges of this method. Clearly, these limitations are too restricting for this method to be useful as a potential design method.

The principal objective of the work in this chapter is to develop a new method of calculating the joint stiffness that is much less restricting than the currently available method from Jaspart et al. (2004), can achieve much better accuracy, and is much simpler to use.

![Figure 4.1. Example of limitations imposed by Jaspart et al. (2004) for RHS face in tension component against France et al. testing (1999)](image)

4.2 Derivation of new equations for initial stiffness of RHS face in tension

4.2.1 Unfilled RHS column

Although the method of Jaspart et al. (2004) has a number of shortcomings as explained in Section 2.4.2, it provides a good starting point for deriving the equations for the stiffness of the RHS face in transverse tension due to bolt loads. In this study,
as shown in Figure 4.2, the complex 3D column section is reduced to a 2D plate for the connecting face, supported by rotational springs of magnitude $k_r$ representing the contribution of stiffness by the RHS sidewalls. The width of the plate is the deformable width of the RHS face, $a = w_{RHS} - 4t_c$.

As shown in Figure 4.3, deflection, $w$, is obtained by considering the deflection $w_1$ of concentrated loads acting on a plate with simply supported edges and then adding a negative deflection $w_2$ due to a restraining bending moment at the sides imposed by the sidewall stiffness.

Figure 4.2. Simplification of 3D model to 2D plate model

Figure 4.3. Geometry of deflection components $w_1$ and $w_2$
The analytical solution for the deflection of a rectangular plate with simply supported edges subject to a single concentrated transverse load is derived by Timoshenko (1951) as the following:

\[
(w_1)_{y=0} = \frac{a^2P}{2D\pi^3} \sum_{m=1}^{\infty} \frac{\sin\left(\frac{m\pi c}{a}\right)\sin\left(\frac{m\pi x}{a}\right)}{m^3} \left(\tanh(\text{am}) - \frac{\text{am}}{\cosh^2(\text{am})}\right)
\]

[4.1]

where

\[\text{am} = \frac{m\pi b}{2a}\]

For an infinitely long plate, \(b \to \infty\), and therefore \(\cosh(\text{am}) \to \infty\) and \(\tanh(\text{am}) \to 1\).

Equation [4.1] reduces to:

\[
(w_1)_{y=0} = \frac{a^2P}{2D\pi^3} \sum_{m=1}^{\infty} \frac{\sin\left(\frac{m\pi c}{a}\right)\sin\left(\frac{m\pi x}{a}\right)}{m^3}
\]

[4.2]

Considering the two bolts on the same row as the joint component, the deflection at the centre of the plate, \(x = a/2\), should be used. This is the main difference between the new derivations and that of Jaspart et al. (2004). By treating each bolt row as one joint component, this allows any number of bolt rows to be used in the joint. In contrast, in Jaspart et al., it was assumed that all bolt rows in tension were considered together as one combined joint component by means of the combined height of the tension zone. The deflection at the centre of this tension zone was then calculated. Not only was this representation of joint component inflexible (e.g. not realistically able to deal with joints that have more than two bolt rows), it also made the calculation method much more complicated and more difficult to obtain simplified analytical solutions.

Assuming initial deflections occur in the elastic range, it is possible to use superposition to get the combined deflection due to two bolts (see Figure 4.3), one at
c=c_1=(a-a_t)/2 and the other c=c_2=(a+a_t)/2 with the load P shared evenly between both bolts. Modifying Equation [4.2] gives the following:

\[(w_1)_{y=0} = \frac{a^2 P}{2 D \pi^3} \sum_{m=1}^{\infty} \sin \left[ \frac{c_1 m\pi}{2a} \right] + \sin \left[ \frac{c_2 m\pi}{2a} \right] \sin \left[ \frac{m\pi x}{a} \right] \]

[4.3]

Substituting x=a/2, c_1=(a-a_t)/2, and c_2=(a+a_t)/2 into Equation [4.3], the deflection due to both bolts at the centre of the plate can be simplified as:

\[(w_1)_{x=a/2; y=0} = \frac{a^2 P}{2 D \pi^3} \sum_{m=1}^{\infty} \sin \left[ \frac{(a-a_t)mn}{2a} \right] + \sin \left[ \frac{(a+a_t)mn}{2a} \right] \sin \left[ \frac{mn}{2} \right] \]

[4.4]

As \[\frac{\sin \left[ \frac{(a-a_t)mn}{2a} \right] + \sin \left[ \frac{(a+a_t)mn}{2a} \right]}{2} \equiv \cos \left[ \frac{an\pi}{2a} \right] \sin \left[ \frac{mn}{2} \right],\]

\[(w_1)_{x=a/2; y=0} = \frac{a^2 P}{2 D \pi^3} \sum_{m=1}^{\infty} \sin \left[ \frac{mn}{2} \right] \cos \left[ \frac{an\pi}{2a} \right] \]

[4.5]

As even terms of m give \(\sin(m\pi/2)=0\), by using odd values only the above reduces to:

\[(w_1)_{x=a/2; y=0} = \frac{a^2 P}{2 D \pi^3} \sum_{m=1,3,5,...}^{\infty} \cos \left[ \frac{an\pi}{2a} \right] \]

[4.6]

where

\[D = \frac{E t_c^3}{12(1-\nu^2)}\]

[4.7]


\[(w_1)_{x=a/2; y=0} = \frac{6a^2 P(1-\nu^2)}{E \pi^3 t_c^3} \sum_{m=1,3,5,...}^{\infty} \cos \left[ \frac{an\pi}{2a} \right] \]

[4.8]
As the sum does not converge, the contribution of each term has been investigated in Section 4.2.3, and it can be shown that the sum to infinity can be replaced by a factor, \( S \), multiplied by the first term with a negligible loss in accuracy to give:

\[
(w_1)_{x=a/2; y=0} = \frac{6a^2 P(1 - \nu^2) SC\cos\left[\frac{at}{2a}\right]}{E \pi^3 t_c^3}
\]  \[4.9\]

To consider the \( w_2 \) component (the deflection due to a negative bending moment equivalent to the sidewall rotational stiffness per unit length, \( k_r \)), the solution, given by the following equation in Timoshenko (1951), is used:

\[
w_2 = \sum_{m=1}^{\infty} X_{2m} \cos\left[\frac{m\pi y}{a}\right]
\]  \[4.10\]

where

\[
X_{2m} = A_m e^{\frac{m\pi x}{a}} + B_m \frac{m\pi x}{a} e^{\frac{m\pi x}{a}} + C_m e^{-\frac{m\pi x}{a}} + D_m \frac{m\pi x}{a} e^{-\frac{m\pi x}{a}}
\]  \[4.11\]

Weynand et al. (2003) gives the following solution for the coefficients.

\[
A_m = (aA_{w1}e^{m\pi k_r}m\pi(2e^{m\pi(2D + ak_r)m\pi} + (a(-1 + e^{2m\pi})k_r + 2D(1 + e^{2m\pi})m\pi)\cos[m\pi]))
\]

\[
/\left(4D^2(-1 + e^{2m\pi})^2m^2\pi^2 - 4ak_r m\pi(1 - e^{4m\pi} + 4e^{2m\pi}m\pi)
\]

\[
+ a^2k_r^2(1 + e^{4m\pi} - 2e^{2m\pi}(1 + 2m^2\pi^2))\right)
\]

\[
B_m = -\left(\left(aA_{w1}k_r(2D(-1 + e^{2m\pi})m\pi + ak_r(1 + e^{2m\pi}(-1 + 2m\pi))
\right.
\]

\[
+ e^{m\pi\left(2D(-1 + e^{2m\pi})m\pi + ak_r(-1 + e^{2m\pi} - 2m\pi))\cos[m\pi]\right))
\]

\[
/\left(4D^2(-1 + e^{2m\pi})^2m^2\pi^2 - 4ak_r m\pi(1 - e^{4m\pi} + 4e^{2m\pi}m\pi) + a^2k_r^2(1 + e^{4m\pi}
\]

\[
- 2e^{2m\pi}(1 + 2m^2\pi^2))\right)
\]

\[
C_m = -\left(aA_{w1}e^{m\pi k_r}m\pi(2e^{m\pi}(2D + ak_r)m\pi
\right.
\]

\[
+ (a(-1 + e^{2m\pi})k_r + 2D(1 + e^{2m\pi})m\pi)\cos[m\pi])/4D^2(-1 + e^{2m\pi})^2m^2\pi^2
\]

\[
- 4ak_r m\pi(1 - e^{4m\pi} + 4e^{2m\pi}m\pi) + a^2k_r^2(1 + e^{4m\pi} - 2e^{2m\pi}(1 + 2m^2\pi^2))
\]

\[
D_m = -\left(aA_{w1}e^{m\pi k_r}(e^{m\pi(2D(-1 + e^{2m\pi})m\pi + ak_r(-1 + e^{2m\pi} - 2m\pi))
\right.
\]

\[
+ \left(2D(-1 + e^{2m\pi})m\pi + ak_r(1 + e^{2m\pi}(-1 + 2m\pi))\cos[m\pi]\right))
\]

\[
/\left(4D^2(-1 + e^{2m\pi})^2m^2\pi^2 - 4ak_r m\pi(1 - e^{4m\pi} + 4e^{2m\pi}m\pi) + a^2k_r^2(1 + e^{4m\pi}
\]

\[
- 2e^{2m\pi}(1 + 2m^2\pi^2))\right)
\]
\[ A_{w1m} = \frac{a^2 P \sin \left( \frac{cm \pi}{a} \right)}{2Dm^3 \pi^3} \]

where

\[ k_r = \frac{4EI}{d_{RHS} \left( \frac{1.5w_{RHS} + d_{RHS}}{2.0w_{RHS} + d_{RHS}} \right)} \quad [4.12] \]

This equation is derived in Section 4.3.

However, since one bolt row is a joint component, \( x = a/2 \) and \( y = 0 \). This allows considerable simplifications to be made to Equation [4.10] to give the following solution:

\[ (w_2)_{x=a/2; y=0} = \]

\[ \frac{18a^3 k_r P (-1 + v^2)^2}{E \pi^2 t_c^3} \sum_{m=1}^{\infty} \frac{e^{mn \pi} (-1 + e^{mn \pi}) (-1 + \cos(m \pi)) \sin \left( \frac{(a - a \pi)}{a} \right)}{m^2 (1 + e^{mn \pi})^2 E \pi t_c^3 - 6ak_r (-1 + e^{2mn \pi} + 2e^{mn \pi} \pi (-1 + v^2))} \quad [4.13] \]

It can be established that there is a negligible contribution from terms \( m \) larger than 1.

Therefore, by taking the first term only, Equation [4.13] can be simplified to:

\[ (w_2)_{x=a/2; y=0} = -\frac{36a^3 \pi e^{\frac{\pi}{2}} (-1 + e^{\pi}) k_r P (-1 + v^2)^2 \sin \left( \frac{(a - a \pi)}{a} \right)}{E \pi^2 t_c^3 (1 + e^{\pi})^2 E \pi t_c^3 - 6ak_r (-1 + e^{2\pi} + 2e^{\pi \pi} (-1 + v^2))} \]

\[ (w_2)_{x=a/2; y=0} = -\frac{a^3 k_r P (-1 + v^2)^2 \cos \left( \frac{a \pi}{2a} \right)}{Et_c^3 (4.7Et_c^3 - 10.5ak_r (-1 + v^2))} \quad [4.14] \]

Combining Equations [4.9] and [4.14] to get the final deflection, \( w \):

\[ (w)_{x=a/2; y=0} = \frac{6a^2 P (1 - v^2) S \cos \left( \frac{a \pi}{2a} \right)}{E \pi^3 t_c^3} \]

\[ = -\frac{a^3 k_r P (-1 + v^2)^2 \cos \left( \frac{a \pi}{2a} \right)}{Et_c^3 (4.7Et_c^3 - 10.5a k_r (-1 + v^2))} \quad [4.15] \]

To simplify further, assuming for the common case that \( v = 0.3 \) gives:
\[(w)_{x=a/2;y=0} = \frac{5.46a^2P.S.\cos\left(\frac{\alpha_{\pi}}{2a}\right)}{E\pi^3t_c^3} - \frac{a^3k_rP.\cos\left(\frac{\alpha_{\pi}}{2a}\right)}{Et_c^3(11.5ak_r+5.7Et_c^3)}\]

\[(w)_{x=a/2;y=0} = \frac{a^2P.\cos\left(\frac{\alpha_{\pi}}{2a}\right)}{Et_c^3}\left(\frac{2.024ak_rS - ak_r + EST_c^3}{11.5ak_r + 5.7Et_c^3}\right) \quad [4.16]\]

Taking the reciprocal of this value and dividing by E/P to get the stiffness in the component method format,

\[k_{RHS,unfilled} = \frac{t_c^3}{a^2\cos\left(\frac{\alpha_{\pi}}{2a}\right)}\left(\frac{11.5ak_r + 5.7Et_c^3}{2.024ak_rS - ak_r + EST_c^3}\right) \quad [4.17]\]

### 4.2.2 Concrete-filled RHS column

For the concrete-filled column case, the concrete infill prevents rotation at the sidewalls. Putting \(k_r=\infty\) into Equation [4.17] gives:

\[k_{RHS,concrete\ filled} = \frac{t_c^3}{a^2\cos\left(\frac{\alpha_{\pi}}{2a}\right)}\left(\frac{11.5}{2.024S-1}\right) \quad [4.18]\]

### 4.2.3 Determining correction factor S for convergence of summation

To quantify the ratio of sum to infinity and the first term of the sum (S in Equations [4.9] and [4.17] for calculating deflection \(w_1\)), a range of bolt hole separation to clear column face width ratios, \(a_t/a\), were plotted. As displayed in Figure 4.4, the relationship for the ratio can be represented by a simple quadratic equation with the coefficients optimised for usage within the practical joint geometry range of 0.2 < \(a_t/a\) < 0.8. This equation is:

\[S = 0.143(a_t/a)^2 - 0.306(a_t/a) + 1.076 \quad [4.19]\]
The accuracy of this simplification is investigated by plotting the error between simplified and full summation values as given in Figure 4.5. For the practical range 0.2 < \( a_t/a \) < 0.8, the error is less than 0.07% when replacing the summation of terms to infinity by a single term. This leads to a reduction of overall complexity thus making it less susceptible to calculation error in practice.
4.2.4 Comparison of full and simplified equations

A comparison of the full and simplified equations derived in the previous section is made to determine the acceptability of the simplifications made by checking the error in predictions. For the comparison, the following equations for the deflection are used where full refers to the analytically derived equation without approximations and simplified refers to the full equations which has been simplified by means of approximations and assumptions for common usage (i.e. \(v=0.3\)).

**Full equation:**

\[
\frac{6a^2P(1-v^2)}{E\pi^3t^3c^3} \sum_{m=1,3,5}^{\infty} \frac{\cos \left(\frac{am\pi}{2a}\right)}{m^3} - \frac{18a^3k_pc(-1+v^2)^2}{E\pi^2t^3c^3} \times \sum_{m=1}^{\infty} \frac{e^{m\pi}(1+e^{m\pi})(-1+\cos(m\pi))\sin\left(\frac{(a-a_t)m\pi}{a}\right)}{m^2((1+e^{m\pi})^2Em\pi t^3c^3-6ak_r(-1+e^{2m\pi}+2e^{m\pi}m\pi)(-1+v^2))}
\]  

[4.20]

**Simplified equation:**

\[
\frac{a^2PCos \left(\frac{at\pi}{2a}\right)}{Et^3c^3} \left(\frac{2.024ak_rS-a_k-pES t^3c^3}{11.5ak_r+5.7Et^3c^3}\right)
\]

[4.21]

Figure 4.6 shows the errors that arise from usage of the simplified equation in comparison to the full equation for various values of \(a_t/a\). There is up to approximately 0.6% error for very small and large values of \(a_t/a\), however, for the practical range of joint geometries \(0.2 \leq a_t/a \leq 0.8\), the simplified equations gives the solution to within 0.4% error of the full analytical equations. In general, it can be accepted that the error from simplifications and approximations is insignificant.
4.3 Derivation of $k_r$ for rotational stiffness of unfilled RHS sidewalls

This section gives the derivation of the $k_r$ coefficient for rotational stiffness contribution of RHS sidewalls that is used in the derivation of initial stiffness equations for bolted connections to RHS sections in Section 4.2.1. The $k_r$ coefficient replaces the complex 3D geometry of the RHS face in tension and replaces it with a 2D model with rotational springs at edges to replace the rotational stiffness contribution of sidewalls as shown in Figure 4.2. This is the approach adopted in Jaspart et al. (2004) and yields the same equation; however, the original work does not give the derivation of this equation so it is included here for reference.

For the derivation of this coefficient, the simplified model given Figure 4.7 is used. Rotational stiffness, $k_r$, is calculated as the ratio of the applied moment, $M$, to the rotation, $\theta$, both at Node 3 and therefore derivation requires determining the relationship between these two values.
Figure 4.7. Simplified model for calculation of RHS sidewall rotational stiffness

The following stiffness matrix is formed for this model:

**Beam 1**

\[ L_1 = \frac{w_{RHS}}{2} \]

\[
\begin{bmatrix}
P_{1x} \\
P_{1y} \\
M_1 \\
P_{2x} \\
P_{2y} \\
M_2
\end{bmatrix} =
\begin{bmatrix}
\frac{EA}{L_1} & 0 & 0 & -\frac{EA}{L_1} & 0 & 0 \\
0 & \frac{12EI}{L_1^3} & \frac{6EI}{L_1^2} & 0 & -\frac{12EI}{L_1^3} & \frac{6EI}{L_1^2} \\
0 & \frac{6EI}{L_1^2} & \frac{4EI}{L_1} & 0 & -\frac{6EI}{L_1^2} & \frac{2EI}{L_1} \\
\frac{EA}{L_1} & 0 & 0 & \frac{EA}{L_1} & 0 & 0 \\
0 & -\frac{12EI}{L_1^3} & -\frac{6EI}{L_1^2} & 0 & -\frac{12EI}{L_1^3} & -\frac{6EI}{L_1^2} \\
0 & \frac{6EI}{L_1^2} & \frac{2EI}{L_1} & 0 & -\frac{6EI}{L_1^2} & \frac{4EI}{L_1}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
y_1 \\
\theta_1 \\
x_2 \\
y_2 \\
\theta_2
\end{bmatrix}
\]

[4.22]
Beam 2

$L_2 = d_{\text{RHS}}$

\[
\begin{bmatrix}
\mathbf{P}_{2X} \\
\mathbf{P}_{2Y} \\
\mathbf{M}_2 \\
\mathbf{P}_{3X} \\
\mathbf{P}_{3Y} \\
\mathbf{M}_3
\end{bmatrix} = \begin{bmatrix}
\frac{12EI}{L_2^3} & 0 & -\frac{6EI}{L_2^2} & \frac{12EI}{L_2} & 0 & -\frac{6EI}{L_2^2} \\
0 & \frac{EA}{L_2} & 0 & 0 & -\frac{EA}{L_2} & 0 \\
\frac{6EI}{L_2^2} & 0 & \frac{4EI}{L_2} & \frac{6EI}{L_2} & 0 & \frac{2EI}{L_2} \\
-\frac{12EI}{L_2^3} & 0 & \frac{6EI}{L_2^2} & \frac{12EI}{L_2} & 0 & \frac{6EI}{L_2^2} \\
0 & \frac{EA}{L_2} & 0 & 0 & -\frac{EA}{L_2} & 0 \\
\frac{6EI}{L_2^2} & 0 & \frac{2EI}{L_2} & \frac{6EI}{L_2} & 0 & \frac{4EI}{L_2}
\end{bmatrix} \begin{bmatrix}
x_2 \\
y_2 \\
\theta_2 \\
x_3 \\
y_3 \\
\theta_3
\end{bmatrix}
\]

(Model)

$L_1 = w_{\text{RHS}}/2; L_2 = d_{\text{RHS}}$

\[
\begin{bmatrix}
\mathbf{P}_{1X} \\
\mathbf{P}_{1Y} \\
\mathbf{M}_1 \\
\mathbf{P}_{2X} \\
\mathbf{P}_{2Y} \\
\mathbf{M}_2 \\
\mathbf{P}_{3X} \\
\mathbf{P}_{3Y} \\
\mathbf{M}_3
\end{bmatrix} = \begin{bmatrix}
\frac{EA}{L_1} & 0 & 0 & -\frac{EA}{L_1} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12EI}{L_1^3} & \frac{6EI}{L_1^2} & 0 & -\frac{12EI}{L_1} & \frac{6EI}{L_1^2} & 0 & 0 & 0 \\
0 & \frac{6EI}{L_1^2} & \frac{4EI}{L_1} & 0 & -\frac{6EI}{L_1} & \frac{2EI}{L_1} & 0 & 0 & 0 \\
\frac{EA}{L_1} & 0 & 0 & \frac{EA}{L_1} + \frac{12EI}{L_1^2} & 0 & -\frac{6EI}{L_1} & \frac{12EI}{L_1} & 0 & -\frac{6EI}{L_1^2} \\
0 & \frac{12EI}{L_1^3} & \frac{6EI}{L_1^2} & 0 & \frac{EA}{L_1} + \frac{12EI}{L_1^2} & -\frac{6EI}{L_1} & 0 & \frac{EA}{L_2} & 0 \\
0 & \frac{6EI}{L_1^2} & \frac{2EI}{L_1} & 0 & -\frac{6EI}{L_1} & \frac{4EI}{L_1} + \frac{6EI}{L_1^2} & \frac{6EI}{L_1} & \frac{2EI}{L_2} & 0 \\
0 & 0 & 0 & \frac{EA}{L_2} & 0 & 0 & \frac{EA}{L_2} & 0 & 0 \\
0 & 0 & 0 & -\frac{6EI}{L_2^2} & 0 & \frac{2EI}{L_2} & \frac{6EI}{L_2} & 0 & \frac{4EI}{L_2}
\end{bmatrix} \begin{bmatrix}
x_1 \\
y_1 \\
\theta_1 \\
x_2 \\
y_2 \\
\theta_2 \\
x_3 \\
y_3 \\
\theta_3
\end{bmatrix}
\]

The following boundary conditions and loads are used to solve for $M_3$ and $\theta_3$:

\[
\begin{align*}
x_1 &= 0 \\
\theta_1 &= 0 \\
x_2 &= 0 \\
y_2 &= 0
\end{align*}
\]
\[ x_3 = 0 \]

\[ M_2 = 0 \]

\[ P_{1Y} = 0 \]

\( M_2 \) and \( P_{1Y} \) both equal zero and therefore can be used to form simultaneous equations using Eqn. [4.24] to solve for \( \theta_2 \),

\[
M_2 = \frac{2EI(w_{RHS}^2(2\theta_2 + \theta_3) + 4d_{RHS}(\theta_2 w_{RHS} + 3y_1))}{d_{RHS}w_{RHS}^2} = 0
\]

\[
P_{1Y} = \frac{24EI(\theta_2 w_{RHS} + 4y_1)}{w_{RHS}^3} = 0
\]

\[
2EI(w_{RHS}^2(2\theta_2 + \theta_3) + 4d_{RHS}(\theta_2 w_{RHS} + 3y_1)) = 24EI(\theta_2 w_{RHS} + 4y_1)
\]

\[
\theta_2 = -\frac{\theta_3 w_{RHS}}{d_{RHS} + 2w_{RHS}} \tag{4.25}
\]

Putting this value into \( M_3 \),

\[
M_3 = \frac{2EI(\theta_2 + 2\theta_3)}{d_{RHS}}
\]

\[
= \frac{2EI(\frac{\theta_3 w_{RHS}}{d_{RHS} + 2w_{RHS}} + 2\theta_3)}{d_{RHS}}
\]

\[
= \frac{2EI\theta_3(2d_{RHS} + 3w_{RHS})}{d_{RHS}(d_{RHS} + 2w_{RHS})} \tag{4.26}
\]

Dividing \( M_3 \) by \( \theta_3 \) to get \( k_r \):

\[
\frac{M_3}{\theta_3} = \frac{2EI(2d_{RHS} + 3w_{RHS})}{d_{RHS}(d_{RHS} + 2w_{RHS})}
\]

\[
\therefore k_r = \frac{4EI(d_{RHS} + 1.5w_{RHS})}{d_{RHS}(d_{RHS} + 2w_{RHS})} \tag{4.27}
\]
4.4 Validating sidewall stiffness simplification against ABAQUS modelling

As presented in Section 4.3, the derivation of the sidewall spring stiffness coefficient, \( k_r \), is conducted using a simplified 2D model that neglects the curvature of the RHS corners. It is necessary to determine the error in calculation of the \( k_r \) coefficient caused by using this simplified model compared to the actual section with curved corners. This is especially true for thick columns which have a larger root radius and therefore deviates more from the simplified model. In addition to this, the relationship in errors of \( k_r \) calculation on the overall stiffness calculation is investigated to determine if the errors from usage of the simplified model for derivation of \( k_r \) is significant.

The difference in cross section geometry is given in Figure 4.8. For the actual cross section geometry, the root radius of the RHS corners was assumed equal to twice the RHS thickness as discussed in Section 3.2.6.

![Simplified and actual RHS cross sections](image)

Figure 4.8. Simplified and actual RHS cross sections
This validation work is divided into three components. First, the analytical equations derived in Section 4.2 are validated against FEA results for the simplified section to determine if the analytical method provides an accurate calculation of the RHS sidewall stiffness. Secondly, the same FEA techniques are used for the actual cross section model and the errors in using the analytical equation based on the simplified model are determined. Finally, the total error on the prediction of the bolt row deflection caused by an error in the prediction of $k_r$ is quantified for the realistic limits of the geometric ratio $0.2 < \alpha_t/a < 0.8$.

The analytical equation is validated against FEM results obtained from a 2D beam analysis using Abaqus 6.10-1 (Dassault Systèmes, 2012). Geometry and boundary conditions are exactly as shown in Figure 4.8. The model is discretized using very small mesh elements of 2mm length to ensure results are converged. A reference bending moment of 100Nm was applied for calculation of the stiffness. The investigation was separated into three sections: variation in RHS width, variation in RHS depth, and variation in RHS thickness.

The results from this investigation are presented in Table 4.1. Firstly, it can be seen there is a negligible error in the prediction of $k_r$ between the analytical equation and the simplified FEM models. This shows that the analytical method is suitable for modelling the sidewall stiffness for the simplified cross section. When comparing the analytical equation against the FEM model with curved edges however, there is a larger error for prediction of the $k_r$ coefficient. The largest errors were found for the lower bound and upper bound variations in the RHS thickness which had errors of -4.0% (4mm thickness) and 5.2% (16mm thickness) giving a range of error of approximately ±5%. This coincides with the upper and lower bound values of the $a/t_w$ ratio of 46 and 8.5 although errors of up to -2.5% are also found in the variation of RHS depth showing that other factors also affect the calculation of $k_r$. 
The influence of this range of error can be put this into perspective by determining the effect that errors in $k_r$ has on the overall bolt row deflection calculation (as established in Section 4.2.1, this is a sum of the deflection due to the pinned condition and negative deflection due to the restraining effect of the sidewall stiffness, $k_r$). The effect of $k_r$ error on bolt row deflection error is investigated for the two realistic minimum and maximum cases of $a/a=0.2$ and 0.8 to give a generalised understanding of the error across the realistic range of geometric parameters. In all cases, the error in $k_r$ calculation does not have a significant effect on the bolt row deflection calculation. For the RHS sections investigated, the maximum error is 0.16% with the majority of sections having less than 0.1% error. It can be concluded that the simplified cross section used for derivation of the $k_r$ is acceptable for all practical section sizes.

Table 4.1. Errors from usage of simplified cross section for $k_r$ derivation

<table>
<thead>
<tr>
<th>RHS Geometry</th>
<th>Comparison of $k_r$ (kNm/rad)</th>
<th>Deflection error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{RHS}$ (mm)</td>
<td>$d_{RHS}$ (mm)</td>
<td>$t_c$ (mm)</td>
</tr>
<tr>
<td>200</td>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>300</td>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>400</td>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>200</td>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>200</td>
<td>300</td>
<td>10</td>
</tr>
<tr>
<td>200</td>
<td>400</td>
<td>10</td>
</tr>
<tr>
<td>200</td>
<td>200</td>
<td>4</td>
</tr>
<tr>
<td>200</td>
<td>200</td>
<td>5</td>
</tr>
<tr>
<td>200</td>
<td>200</td>
<td>8</td>
</tr>
<tr>
<td>200</td>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>200</td>
<td>200</td>
<td>12.5</td>
</tr>
<tr>
<td>200</td>
<td>200</td>
<td>16</td>
</tr>
</tbody>
</table>

Positive error indicates stiffness overestimation and vice versa
4.5 Parametric study of RHS initial stiffness equations using FEM

4.5.1 Introduction

The equations derived for the initial stiffness of the RHS face in tension are investigated across a variety of joint geometries to determine their validity and their range of applicability. From this it is possible to determine restrictions on their applicability to extreme cases, namely for small and large values of the ratios $a_0/a$ and $t_c/w_{RHS}$ for which there may be noticeable differences between actual and predicted behaviour. For example, thick sections with large values of $t_c/w_{RHS}$ for which actual behaviour may deviate significantly from equations based on thin plate theory.

4.5.2 Validation of ABAQUS simulation

To demonstrate that the FEM simulations conducted in this section are accurate, this section compares FEM results with test results for a T-stub assembly in tension by Bursi and Jaspart (1997) using the general finite element analysis software ABAQUS 6.7-1 (Dassault Systèmes, 2008). While the T-stub and hollow section column is not a direct comparison, there is similarity in geometrical features and the deformed shape that makes it a suitable comparison. The non preloaded T-stub connection test T1 as shown in Figure 4.9 is used as a reference model for this purpose.

Due to the validation of the initial stiffness equations being of interest in this section, the validation of the modelling focused on the accuracy of the load-displacement behaviour in the elastic range. For this reason, various simplifications were made to the model including using only elastic material properties and very small loading.
The main parameters that need validation are the type of mesh element, mesh discretization, and use of symmetry to reduce the model complexity. The 8-node linear brick with reduced integration (C3D8R) is chosen for all components of the model while the mesh is discretized with 4 mesh layers across the edge of the T-stub thickness for sections in bending as established in Section 3.2.5. The bolt and bolthole perimeters were discretized using 32 elements along their edges. The actual mesh used is given in Figure 4.10. A concentrated load, F, was applied to a reference node which was then coupled to the web section. This is the same as the use of coupling with a connector section in Section 3.3.1. To reduce model complexity the T-stub was subdivided across 3 planes and applied the relevant symmetry boundary conditions to give a one-eighth model.

A comparison of the load-displacement curves can be seen in Figure 4.11 where it is clear that there is an exact match of displacement behaviour replicated with the FEM for the elastic range where there is linear load-displacement relationship. While there is some discrepancy at higher loads, this is both expected and insignificant as convergence in the early deflection range is of interest and plastic material properties are intentionally omitted. The modelling techniques used in this section will form a basis for the parametric study in the following section.
Figure 4.9. Geometry of T-stub test T1 (Bursi and Jaspart, 1997)

Figure 4.10. One-eighth model and mesh of T-stub test T1
4.5.3 Validation of analytical equations against ABAQUS simulation

In Section 4.5.2, an analytical model for predicting the elastic load-deformation behaviour of a T-stub component was validated. This was carried out to supplement the validation work in Chapter 3 on the RHS face in tension component where only a limited comparison could be drawn for elastic load-deflection behaviour because the majority of British Steel (1996b) tests did not give a smooth initial stiffness behaviour curve. The study shows that techniques used in developing the finite element model are suitable for predicting elastic load-deflection behaviour.

To investigate the validity of the above simplified derivations for this component for a realistic RHS face (in particular the simplification of a 3-D structure into a 2-D beam and the assumption of a point load to represent bolt load), the general finite element software Abaqus FEA is used to simulate an unfilled RHS column with a single bolt row in tension. Figure 4.12 shows the FE model for the joint component under consideration. To simplify the model, the bolt loads are applied to reference nodes that are coupled to the bolt hole surface as shown in Figure 4.12 which is suitable in modelling elastic behaviour and ideal for parametric studies because it is
not computationally intensive. A column length of 1m is used with ends fully fixed and this is reduced to a quarter model with appropriate symmetry boundary conditions to reduce model complexity. The model is discretized using the 8-node cubic brick (C8D8R) element with reduced integration as described in Section 3.2.5. The relative deflection of the column face at the centre is obtained by taking the deflection of the column face at the centre and subtracting the deflection at the sidewalls.

![Figure 4.12. FE model for RHS bolt row in transverse tension component (quarter model)](image)

Figure 4.12 gives a plot of errors in prediction of deflection for varying values of $a/t/a$ across different column sections and bolthole sizes. In general, the analytical equations provide predictions of initial deflections across different column configurations with generally less than 10% error in the practical range of $a/t/a$. The ratio $a/t/a$ is physically limited to a minimum of approximately 0.1-0.15 and a maximum of 0.8-0.85 (depending on column thickness). For the two cases involving the column with $a=0.136$ and $t_w=0.016$ thus a relatively small $a/t_w$ value of 8.5, there is greater error in deflection predictions at higher values of $a/t/a$. The largest error is approximately 15% at $a/t/a=0.8$ for $a/t_w = 8.5$. It is also clear that there is a slight
deviation in predictions when different bolt hole sizes are used for a given column but
the difference is insignificant.

From this parametric study, it is possible to conclude that the analytical
equations provide a good prediction of initial deflections throughout different column
configurations with generally less than 10% error in the range of $a/a$ which is
physically possible and for the range of $a/t_w$ values for standard RHS column sections
that are commonly available.

![Graph](image_url)

Figure 4.13. Error in predictions of deflection at various levels of $a/a$ against FEM for
RHS face in transverse tension

Table 4.2 gives a summary and comparison of geometrical limits against those
given by Jaspart et al. (2004). The last restriction of Jaspart et al., $0.05 < c/L_{stiff} < 0.20$,
is particularly limiting as shown in Figure 4.1 because it limits the range of validity to
joints with only two bolt rows. As each individual bolt row is treated as one joint
component, the new derivations in this chapter have eliminated this limitation.
Table 4.2. Range of validity for new approach given by new approach - Eqn. [4.17]

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalent yield width / thickness</td>
<td>12&lt;(a/t_c)&lt;27.7* (&lt;10% error) 8.5&lt;(a/t_c)&lt;27.7* (&lt;15% error)</td>
<td>10&lt;(L_{stiff}/t_c)&lt;50</td>
</tr>
<tr>
<td>Bolt hole separation / Equivalent yield width</td>
<td>0.2&lt;(a_a/a&lt;0.7) (&lt;10% error) 0.1&lt;(a_a/a&lt;0.8) (&lt;15% error)</td>
<td>0.08&lt;(b/L_{stiff}&lt;0.75)</td>
</tr>
<tr>
<td>Tension zone height / Equivalent yield width</td>
<td>N/A</td>
<td>0.05&lt;(c/L_{stiff}&lt;0.20)</td>
</tr>
</tbody>
</table>

*Upper bound may be increased with further study but little practical usage

4.6 Validation of stiffness equations against France testing (1999)

4.6.1 Calculation of joint stiffness

France et al. (1999) carried out tests on bolted endplate connections to hollow and concrete-filled rectangular hollow sections using Flowdrill bolts as described in Section 3.1.1. In this section, these tests are used to assess the accuracy of the new approach given by Eqn. [4.17] for calculating the component stiffness for the RHS face under transverse tension.

One of the assumptions made is that the RHS face in transverse compression component is assumed to be near infinite for initial stiffness and is therefore not included as an effective component in the approach proposed in this research. This is believed to apply for all practical joints in which a high level of stiffness is desired, as the width of the compression zone will typically be equal to or greater than the deformable width of the RHS yield area as shown in Figure 4.14. This means that the majority of the loading will be transferred directly to the RHS sidewalls that have near infinite stiffness. This effect is even stronger when the endplate thickness is greater than the RHS thickness which will often be the case.
Another assumption made in the derivation of the RHS stiffness equations is that deflections are taken at the centre of the RHS section as opposed to the centre of the boltholes. It can also be argued that the endplate will have a constricting effect on this RHS face and thus deflections will be limited to that of the boltholes. An investigation of both approaches is conducted in Section 4.7.

For the unfilled columns, all bolt rows except for the row closest to the flange on the compression side of the beam are assumed to be in tension as given by EN 1993-1-8 (CEN, 2005). For the concrete-filled column, all bolt rows above the beam flange in compression are considered to be in tension for the calculation of initial stiffness. This is due to the concrete filling preventing deflections in the compression zone and thus lowering the centre of rotation to that of the beam flange in compression.

Figure 4.15 shows how the joints tested by France et al. (1999) is represented by the component based method if using the new derivations of this chapter. Each individual bolt row is treated as a tension component. As has been explained, this approach is different to that of Jaspart et al. (2004) who treated the entire bolt group as one combined column tension zone. Because of this change, all tests by France et al. can be assessed using the new derivations as opposed to the Jaspart et al. equations which practically limits the applicability to joints with two bolt rows (i.e. one bolt row...
in tension) of which there is only one in the France tests. The RHS column in compression component is considered insignificant for initial stiffness calculation and is therefore not included in the approach proposed in this study.

Thus, incorporating equations already existing in EN 1993-1-8 for joint components related to the endplate and bolts, the effective components of bolted endplate joints to RHS columns for joint initial stiffness calculation are given in Table 4.3. The equations used in assembling the joint initial stiffness from individual components are given in Table 4.4.

Figure 4.15. Differences in approach for component based method of bolted endplate joints to RHS columns.
Table 4.3. Summary of effective components for calculating initial stiffness

<table>
<thead>
<tr>
<th>Component Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Bolts in tension (EN 1993-1-8)</td>
<td>( k_z = 1.6 A_p / L_p )</td>
</tr>
<tr>
<td>(2) Endplate in bending (EN 1993-1-8)</td>
<td>( k_z = 0.9 l_{eff} t_p^3 / m^3 )</td>
</tr>
<tr>
<td>(3) RHS in transverse tension</td>
<td>( k_{RHS} = \frac{f_t t_c^3}{a^2 S \cos \left[ \frac{2a}{2a} \right]} )</td>
</tr>
</tbody>
</table>

For hollow sections: \( f_1 = \left( \frac{11.5 a k_r + 5.7 E t_c^2}{2.024 a k_s S - a k_r + E S t_c^2} \right) \)

For concrete-filled sections: \( f_1 = \left( \frac{11.5}{2.025 S - 1} \right) \)

\[ k_r = \frac{4E I}{d_{RHS}} \left( \frac{1.5 w_{RHS} + d_{RHS}}{2.0 w_{RHS} + d_{RHS}} \right) \]

\[ S = 0.143 (a_t / a)^2 - 0.306 (a_t / a) + 1.076 \]

Table 4.4. Summary of equations for calculating joint initial stiffness from components

Joint initial stiffness (EN 1993-1-8)

\[ S_{j, ini} = \frac{EZ^2}{\sum k_{eq}} \]

where

\[ k_{eq} = \sum k h_r / z_{eq} \]

\[ z_{eq} = \frac{\sum r k_{eff,r} h_r^2}{\sum r k_{eff,r} h_r} \]

See Figure 6.15 of EN 1993-1-8 for definition of lever arm, \( z \).

4.6.2 Results

Table 4.5 gives a summary of the results obtained from application of the above equations to all joint tests conducted by France et al. (1999). Joint initial stiffness is obtained from the secant stiffness of the origin and first data point of the moment-rotation curves, as shown in Figure 4.16.
As shown in Table 4.5, the prediction of joint initial stiffness when applying the proposed component method equations gives an average error of -1.8% and a maximum of 21.4%. This level of error is similar in magnitude to that found in the parametric study of the RHS face in transverse tension component conducted with FEA in Section 4.5. In addition to this, some variability in test results can be expected due to imperfections, minor variations in column face thickness, and the effects of variable pretensioning due to the torque method of bolt tightening being used. This validation study suggests that the equations derived for the RHS face in transverse tension component for determining initial joint stiffness and the selection of effective components are suitable for application to both flush/extended endplates to unfilled/concrete-filled columns covering a range of geometries. For comparison, although the Jaspart et al. equations are outside the range of validity for the $c/L_{\text{stiff}}$ ratio for most of the France et al. tests, calculations were carried out as if the Jaspart et al. equations could be applied to all 20 France tests. These results give an average of approximately 40% and maximum 80% error in prediction.
Table 4.5. Comparison of initial stiffness predictions and values from France et al. (1999) testing when using different equations for RHS component

<table>
<thead>
<tr>
<th>Test</th>
<th>Endplate type</th>
<th>With</th>
<th>Test result (kNm/rad)</th>
<th>Jaspart et al. Eqn. [2.3] (kNm/rad)</th>
<th>Error(%)</th>
<th>New method Eqn. [4.17] (kNm/rad)</th>
<th>Error(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Flush</td>
<td>-</td>
<td>12500</td>
<td>7247*</td>
<td>42.1%</td>
<td>14358</td>
<td>14.9%</td>
</tr>
<tr>
<td>4</td>
<td>Flush</td>
<td>-</td>
<td>5200</td>
<td>3342*</td>
<td>35.8%</td>
<td>4942</td>
<td>5.0%</td>
</tr>
<tr>
<td>5</td>
<td>Flush</td>
<td>-</td>
<td>3700</td>
<td>2646*</td>
<td>28.6%</td>
<td>4179</td>
<td>-12.9%</td>
</tr>
<tr>
<td>6</td>
<td>Flush</td>
<td>-</td>
<td>6200</td>
<td>4883*</td>
<td>21.4%</td>
<td>5793</td>
<td>6.6%</td>
</tr>
<tr>
<td>7</td>
<td>Flush</td>
<td>-</td>
<td>2120</td>
<td>2388*</td>
<td>-12.5%</td>
<td>2173</td>
<td>-2.5%</td>
</tr>
<tr>
<td>8</td>
<td>Flush</td>
<td>-</td>
<td>23000</td>
<td>6222*</td>
<td>73.1%</td>
<td>26227</td>
<td>-14.0%</td>
</tr>
<tr>
<td>10</td>
<td>Flush</td>
<td>-</td>
<td>1960</td>
<td>1685</td>
<td>14.2%</td>
<td>1648</td>
<td>15.9%</td>
</tr>
<tr>
<td>18</td>
<td>Flush</td>
<td>-</td>
<td>5350</td>
<td>3538*</td>
<td>34.0%</td>
<td>5417</td>
<td>-1.3%</td>
</tr>
<tr>
<td>25</td>
<td>Flush</td>
<td>-</td>
<td>17500</td>
<td>7911*</td>
<td>55.1%</td>
<td>16796</td>
<td>4.0%</td>
</tr>
<tr>
<td>26</td>
<td>Flush</td>
<td>-</td>
<td>7700 [*1]</td>
<td>6119*</td>
<td>21.0%</td>
<td>8038</td>
<td>-4.4%</td>
</tr>
<tr>
<td>19</td>
<td>Extended</td>
<td>-</td>
<td>31000 [*2]</td>
<td>16061*</td>
<td>32.2%</td>
<td>25793</td>
<td>16.8%</td>
</tr>
<tr>
<td>20</td>
<td>Extended</td>
<td>-</td>
<td>60000</td>
<td>20827*</td>
<td>65.6%</td>
<td>53873</td>
<td>10.2%</td>
</tr>
<tr>
<td>21</td>
<td>Extended</td>
<td>-</td>
<td>125000</td>
<td>25922*</td>
<td>79.5%</td>
<td>146478</td>
<td>-17.2%</td>
</tr>
<tr>
<td>23</td>
<td>Extended</td>
<td>-</td>
<td>65000</td>
<td>20825*</td>
<td>68.0%</td>
<td>55392</td>
<td>14.8%</td>
</tr>
<tr>
<td>14</td>
<td>Flush</td>
<td>Yes</td>
<td>25000</td>
<td></td>
<td></td>
<td>26019</td>
<td>-4.1%</td>
</tr>
<tr>
<td>15</td>
<td>Flush</td>
<td>Yes</td>
<td>9700</td>
<td></td>
<td></td>
<td>10141</td>
<td>-4.5%</td>
</tr>
<tr>
<td>16</td>
<td>Flush</td>
<td>Yes</td>
<td>4500 [*3]</td>
<td></td>
<td></td>
<td>4697</td>
<td>-4.4%</td>
</tr>
<tr>
<td>17</td>
<td>Flush</td>
<td>Yes</td>
<td>3700</td>
<td></td>
<td></td>
<td>4492</td>
<td>21.4%</td>
</tr>
<tr>
<td>22</td>
<td>Extended</td>
<td>Yes</td>
<td>100000</td>
<td></td>
<td></td>
<td>104185</td>
<td>-4.2%</td>
</tr>
<tr>
<td>24</td>
<td>Extended</td>
<td>Yes</td>
<td>100000</td>
<td></td>
<td></td>
<td>104119</td>
<td>-4.1%</td>
</tr>
</tbody>
</table>

*Outside range of validity
Positive error indicates underestimation of stiffness and vice versa.

*1 1st data point gives near infinite stiffness. In addition, 2nd-4th data points give 27,000kNm/rad which is much larger than in Test 25 which is identical except for a thicker column section. 4th-5th data points give 7700kNm/rad so use this value.

*2 First 3 data points give near infinite stiffness so use 4th-5th data points which gives 31,000kNm/rad.

*3 1st data point gives 11000 kNm/rad. Initial stiffness should be much less than in Test 15 so use 2nd-3rd data points which gives 4500kNm/rad.

4.7 Alternative joint stiffness formulations

This section investigates the applicability of the assumptions given in Section 4.2 and Section 4.6 regarding the formulation of the joint stiffness equations. One assumption is regarding whether deflections should be taken at the centre of the RHS face or at the boltholes. The other assumption is regarding the RHS face in transverse compression component and whether it should be considered even when the compression zone is generally equal to or larger than the RHS yield width in practical
joints and therefore is likely to have an insignificant effect contribution to joint initial stiffness.

Four approaches are considered for the joint stiffness formulations. These are (a) taking deflections at centre of RHS face and using infinite compression zone stiffness (the recommended approach), (b) taking deflections at centre of RHS face and considering compression zone stiffness, (c) taking deflections at bolt centres and using infinite compression zone stiffness, and (d) taking deflections at bolt centres and considering compression zone stiffness. For the joints with concrete-filled RHS, all cases consider an infinite compression zone stiffness as explained in Section 4.2.2.

4.7.1 RHS in transverse tension with deflections at bolt holes

This is obtained by using $x=(a-a_t)/2$ in Equations [4.3] and [4.11] of Section 4.2.1.

$$\left(\begin{array}{c} \frac{18a^2e^{(a-a_t)/2a}}{2a}\left(-a_t(1+e^{\pi})(-1+e^{\frac{a_t\pi}{3}})+a(-1+e^{\pi})(1+e^{\frac{a_t\pi}{3}}))k_p(-1+e^{a^2})^2\cos\left(\frac{(a-a_t)\pi m}{2a}\right)\right) \\
\end{array}\right)$$

where

$$w(x) = \frac{6a^2(1-e^2)}{E \pi^3 t_c^3} \sum_{m=1,3,5,..}^{\infty} \frac{\cos\left(\frac{(a-a_t)\pi m}{4a}\right)\cos\left(\frac{(a-a_t)\pi m}{2a}\right)}{m^3}$$

4.7.2 RHS in transverse compression with deflections at centre of RHS

This is derived by taking deflections of a uniform distributed load with width and height equal to the compression zone dimensions with simply supported edges as in Figure 4.17. The contribution of the sidewall stiffness given by Equation [4.14] is then applied as in Section 4.2.1 for the unfilled column. The width of the simply supported plate is the deformable width of the RHS face, $a=w_{RHS}-4t_c$. Where the compression zone width is larger than the RHS yield width, the compression zone width is taken as equal to the RHS yield width.
The analytical solution for the deflection of a rectangular plate with simply supported edges subject to a uniform distributed load is derived in Timoshenko (1940) as the following:

\[
\begin{align*}
    w(x) &= \frac{4qa^4}{D\pi^5} \sum_{m=1,3,5,\ldots}^{\infty} \left( (-1)^{\frac{m-1}{2}} \frac{m\pi u}{m^5} \sin \left( \frac{m\pi u}{2a} \right) \left( 1 - \frac{\cosh \left[ \frac{mn\pi y}{a} \right]}{\cosh[a_m]} \right) \right. \\
    & \quad \times \left( \cosh[a_m - 2y_m] + y_m \sinh[a_m - 2y_m] + a_m \frac{\sinh[2y_m]}{2\cosh[a_m]} \right) \\
    & \quad + \left. \frac{\cosh[a_m - 2y_m]}{2\cosh[a_m]} \frac{m\pi y}{a} \sinh \left( \frac{m\pi y}{a} \right) \sin \left( \frac{m\pi x}{a} \right) \right) \\
\end{align*}
\]

where

\[
    a_m = \frac{m\pi b}{2a} \\
    y_m = \frac{m\pi v}{4a}
\]

Note: \(v\) = height of compression zone, \(v\) = shear modulus

For an infinitely long column (i.e. \(b=\infty\)) and taking \(x=a/2, y=0\), \(q=P/(u\times v)\) gives:

\[
(w_1)_{x=a/2; y=0} = \frac{48a^4P(1-v^2)}{E\pi^5c^2uv} \sum_{m=1,3,5,\ldots}^{\infty} \left( (-1)^{\frac{m-1}{2}} \frac{m\pi u}{m^5} \sin \left( \frac{m\pi u}{2a} \right) \frac{m\pi v}{4a} \right)
\]
Adding the contribution of the sidewall stiffness, $w_2$, from Equation [4.14],

$$w_{x=a/2; y=0} = \left( \frac{48a^4P(1-v^2)}{E\pi^5t_c^3u v} \right) \sum_{m=1,3,5,\ldots}^{\infty} \frac{(-1)^{(m-1)/2}}{m^5} \frac{\sin \left( \frac{m\pi u}{2a} \right)}{4a} \frac{\sin \left( \frac{m\pi(v-a_t)}{2a} \right)}{4a} \frac{a^3k_rP(-1+v^2)^2 \cos \left( \frac{a_t\pi}{2a} \right)}{E_t^3(4.7E_t^3-10.5ak_r(-1+v^2))}$$

4.7.3 RHS in transverse compression with deflections at bolt holes

This is derived in a similar manner to above taking the deflection of a rectangular plate with simply supported edges subject to a uniform distributed load at $x=(a-a_t)/2$ and then adding the contribution of the sidewall stiffness at $x=(a-a_t)/2$ in Equation [4.14].

$$w_{x=(a-a_t)/2; y=0} = \frac{48a^4P(1-v^2)}{E\pi^5t_c^3u v} \sum_{m=1,3,5,\ldots}^{\infty} \frac{(-1)^{(m-1)/2}}{m^5} \frac{\sin \left( \frac{m\pi u}{2a} \right)}{4a} \frac{\sin \left( \frac{m\pi(a-a_t)}{2a} \right)}{4a} - \frac{18a^2e^{\frac{(a-a_t)\pi}{2a}}(-a_t(1 + e^{\pi})(1 - 1 + e^{\frac{\pi}{2}}) + a(-1 + e^{\frac{\pi}{2}})(1 + e^{\frac{\pi}{2}}))k_rP(-1 + v^2)^2 \cos \left( \frac{a_t\pi}{2a} \right)}{E_t^2t_c^3(1 + e^{\pi})^2E\pi t_c^3-6ak_r(-1 + e^{2\pi} + 2e^{\pi})(-1 + v^2))}$$

4.7.4 Comparison of different formulations

The different formulations are used to calculate joint stiffness in the joints tested by France et al. (1999) to determine the validity in the assumptions used. The results are presented in Table 4.6.

There is a clear distinction between the accuracy of approaches used. Approach (a) which takes deflections at the centre of the RHS face and infinite compression zone stiffness is the most accurate and consistent with the lowest average and maximum errors. Approach (c) which takes deflections at the bolt holes show significant stiffness overestimation in all cases while Approaches (b) and (d) which incorporate the compression zone stiffness significantly underestimate stiffness in all cases. For this
reason, it is believed that Approach (a) which is the basis for the assumptions given in
the previous section, is appropriate.

Table 4.6. Comparison of initial stiffness predictions using different approaches

<table>
<thead>
<tr>
<th>Test</th>
<th>With concrete infill</th>
<th>France test result (kNm.rad)</th>
<th>(a) Error</th>
<th>(b) Error</th>
<th>(c) Error</th>
<th>(d) Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 2</td>
<td>-</td>
<td>12500</td>
<td>16.9%</td>
<td>50.2%</td>
<td>-31.9%</td>
<td>68.2%</td>
</tr>
<tr>
<td>Test 4</td>
<td>-</td>
<td>5200</td>
<td>5.0%</td>
<td>39.2%</td>
<td>-51.6%</td>
<td>52.7%</td>
</tr>
<tr>
<td>Test 5</td>
<td>-</td>
<td>3700</td>
<td>-12.9%</td>
<td>22.5%</td>
<td>-82.9%</td>
<td>44.4%</td>
</tr>
<tr>
<td>Test 6</td>
<td>-</td>
<td>6200</td>
<td>6.6%</td>
<td>43.7%</td>
<td>-43.5%</td>
<td>48.2%</td>
</tr>
<tr>
<td>Test 7</td>
<td>-</td>
<td>2120</td>
<td>-2.5%</td>
<td>35.8%</td>
<td>-70.1%</td>
<td>52.4%</td>
</tr>
<tr>
<td>Test 8</td>
<td>-</td>
<td>23000</td>
<td>-14.0%</td>
<td>29.0%</td>
<td>-78.7%</td>
<td>38.9%</td>
</tr>
<tr>
<td>Test 10</td>
<td>-</td>
<td>1960</td>
<td>15.9%</td>
<td>37.7%</td>
<td>-36.0%</td>
<td>38.1%</td>
</tr>
<tr>
<td>Test 18</td>
<td>-</td>
<td>5350</td>
<td>-1.3%</td>
<td>37.4%</td>
<td>-71.4%</td>
<td>52.0%</td>
</tr>
<tr>
<td>Test 25</td>
<td>-</td>
<td>17500</td>
<td>4.0%</td>
<td>51.3%</td>
<td>-84.5%</td>
<td>52.9%</td>
</tr>
<tr>
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<td>-</td>
<td>7700</td>
<td>-4.4%</td>
<td>44.9%</td>
<td>-98.0%</td>
<td>50.1%</td>
</tr>
<tr>
<td>Test 19</td>
<td>-</td>
<td>31000</td>
<td>16.8%</td>
<td>55.5%</td>
<td>-57.7%</td>
<td>70.2%</td>
</tr>
<tr>
<td>Test 20</td>
<td>-</td>
<td>60000</td>
<td>10.2%</td>
<td>53.8%</td>
<td>-72.4%</td>
<td>66.9%</td>
</tr>
<tr>
<td>Test 21</td>
<td>-</td>
<td>125000</td>
<td>-17.2%</td>
<td>43.6%</td>
<td>-122.4%</td>
<td>52.0%</td>
</tr>
<tr>
<td>Test 23</td>
<td>-</td>
<td>65000</td>
<td>14.8%</td>
<td>56.2%</td>
<td>-63.6%</td>
<td>68.6%</td>
</tr>
<tr>
<td>Test 14</td>
<td>Yes</td>
<td>25000</td>
<td>-4.1%</td>
<td>-4.1%</td>
<td>-4.6%</td>
<td>-4.6%</td>
</tr>
<tr>
<td>Test 15</td>
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<td>9700</td>
<td>-4.5%</td>
<td>-4.5%</td>
<td>-5.1%</td>
<td>-5.1%</td>
</tr>
<tr>
<td>Test 16</td>
<td>Yes</td>
<td>4500</td>
<td>-4.4%</td>
<td>-4.4%</td>
<td>-4.5%</td>
<td>-4.5%</td>
</tr>
<tr>
<td>Test 17</td>
<td>Yes</td>
<td>3700</td>
<td>-21.4%</td>
<td>-21.4%</td>
<td>-22.1%</td>
<td>-22.1%</td>
</tr>
<tr>
<td>Test 22</td>
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<td>-4.2%</td>
<td>-4.2%</td>
<td>-18.0%</td>
<td>-18.0%</td>
</tr>
<tr>
<td>Test 24</td>
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<td>100000</td>
<td>-4.1%</td>
<td>-4.1%</td>
<td>-17.9%</td>
<td>-17.9%</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td>-0.2%</td>
<td>27.9%</td>
<td>-51.8%</td>
<td>34.2%</td>
</tr>
<tr>
<td>St. dev.</td>
<td></td>
<td></td>
<td>11.4%</td>
<td>25.2%</td>
<td>33.8%</td>
<td>32.5%</td>
</tr>
<tr>
<td>Maximum</td>
<td></td>
<td></td>
<td>-21.4%</td>
<td>56.2%</td>
<td>122.4%</td>
<td>70.2%</td>
</tr>
</tbody>
</table>

Positive error indicates underestimation of stiffness and vice versa.
See Table 3.9 for notes regarding France test results.

4.8 Conclusion

In this chapter, a new simplified derivation of the RHS face in transverse
tension component is presented. Although partly based on work in Jaspart et al. (2004)
and Weynand et al. (2003), the approach used in this study of calculating stiffness on a
per bolt row basis overcomes the shortcomings of equations presented in Jaspart et al.
(2004) which calculates stiffness as a group of bolt rows while severely limiting the
range of applicability to joints with two bolt rows.
Various simplifications are applied to the derived equations and then checked for consistency against the results of the full equations. It was found that there were errors of less than 0.4% between the simplified and full equations in the realistic range of joint geometries while greatly improving the simplicity of the equations and thus the likelihood of calculation errors during the design process. Checks are also conducted on the geometrical simplifications used in Weynand et al. (2003) on the derivation of the sidewall stiffness coefficient. It was found that errors of up to approximately 5% exist by using the simplified cross section for sidewall stiffness; however, such errors have a negligible effect on the overall error when calculating the RHS stiffness.

For calculating the joint rotational stiffness, evidence suggests that the RHS tension zone deflection should be calculated at the centre between the bolts and that the compression zone should be assumed to possess infinite stiffness. Combining this with existing equations for bolts in tension and endplate in bending given in EN 1993-1-8 (CEN, 2005), it is possible to predict the initial stiffness of bolted endplate connections to RHS column sections. Improvements over the existing equations of Jaspart et al. (2004) include greatly increased range of validity, accuracy of predictions and simplicity in implementation. The validation study using experimental testing by France et al. (1999) shows that the proposed approach gives predictions with a mean error of -0.2% and maximum of 21.4% that is acceptable for practical usage. In the context of serviceability limit state design of semi-continuous frames, errors of this size are acceptable because the frame sway deflection is not particularly sensitive to this level of change in joint rotational stiffness, as will be demonstrated in Chapter 8.
Chapter 5

RHS Face Strength

5.1 Introduction

This chapter looks at the characterisation of strength for the RHS face in tension component at yield and failure stages. The derivation of yield strength uses the well-established yield line method based on the Upper bound theorem of the Theory of Plasticity. For any assumed collapse mechanism, the collapse load is calculated by equating the energy dissipation at the plastic hinges to the work done by the external load. According to the theorem, this calculated load is equal to or greater than the true collapse load.

The failure strength of the RHS face in tension component is the weakest of the following mechanisms: (1) punching shear of the bolt group and (2) bolt pullout due to thread stripping of flowdrilled RHS faces, and (3) membrane action. (1) and (2) are well-established and included in design guides such as CIDECT Guide 9 (Kurobane et al., 2004) and are given by Equations [2.18] and [2.31] respectively. However, the existing equation for thread stripping neglects the effect of gross deformation of the flowdrilled RHS which can greatly reduce the thread stripping capacity. A modified version of this equation is considered in Section 5.3 that considers the reduced contact
of bolt and RHS face threads in relation to RHS face deformation. (3) is significant for RHS faces due to the considerable support of the sidewalls that enable tensile membrane action to develop past its bending strength. New equations are derived for the membrane action of the RHS face in tension in Section 5.4.

5.2 Derivation of yield strength for RHS face in tension

This section gives the derivation of both existing and new equations for the yield strength of the RHS face in tension component. Derivations of existing equations using a straight yield line mechanism and circular yield line mechanism are given in Section 5.2.1 and 5.2.2 respectively. These yield the same results as equations in British Steel (1996b) given by Equation [2.24] and in Ghobarah et al. (1996) given by Equations [2.25] and [2.26]. However, a modification is made to the definition of the deformable width of the RHS face. These derivations are conducted to confirm the assumptions and methods used by other researchers. Derivation of a new equation using an elliptical yield line mechanism is presented in Section 5.2.3 which gives the minimum yield line solution for all geometries and therefore is expected to give a better prediction of yield strength of the RHS face in tension component.

For the derivation of equations based on the virtual work method using assumed yield line patterns, various assumptions are used. The plate thickness is assumed small compared to dimensions of plane faces. Loads acting on the plate are assumed to remain normal to its surface. Vertical displacement of the middle surface is assumed very small in comparison to the plate thickness. Because of the restricting effect of the RHS sidewalls, the RHS face is assumed to have fixed edges allowing both positive and negative moments to develop.
5.2.1 Straight yield line pattern in bending

The simple case of a bolt group with a straight yield line pattern and fixed edges is given in Figure 5.1. The components of the internal virtual work are the vertical, horizontal, and diagonal positive and negative yield lines. Ignoring the boltholes, the internal virtual work is:

\[
I_{W(\text{no holes})} = M \left( \frac{4a_t}{d} + \frac{4b_t}{c} + \frac{8c}{d} + \frac{8d}{c} \right) \tag{5.1}
\]

In addition to this, it is necessary to consider the effect of the bolthole openings along the yield lines that reduce the strength of the plate. The length of material removed along the yield lines is subtracted from Equation [5.1] to give:

\[
I_{W(\text{with holes})} = M \left( \frac{4a_t - 2d_b}{d} + \frac{4b_t - 2(n - 1)d_b}{c} + \frac{8c - 2d_b}{d} + \frac{8d - 2d_b}{c} \right) = 4M \left( \frac{a_t + 2c - d_b}{d} + \frac{b_t + 2d - 0.5nd_b}{c} \right) \tag{5.2}
\]

The external work is equal to the work done by the joint load \( F \) on unit displacement.

\[ E_W = F \]

Balancing the internal work given by Equation [5.1] with the external work gives the failure load \( F \):

\[
I_W = E_W
\]

\[
F_{G.S} = 4M_p \left( \frac{a_t + 2c - d_b}{d} + \frac{b_t + 2d - 0.5nd_b}{c} \right) \tag{5.3}
\]

The length \( d \) is unknown and can be obtained when the yield line solution is a minimum because yield lines will always follow the path of least resistance. As the equation for \( F \) yields a simple parabolic curve for possible values of \( d \) (i.e. \( d \geq 0 \)), the minimum can be found by using the first derivative of Equation [5.3]:
\[
\frac{\partial F}{\partial d} = 4M_p \left( -\frac{a_t + 2c - d_b}{d^2} + \frac{2}{c} \right)
\]

With \( \frac{\partial F}{\partial d} = 0 \),
\[
d = \sqrt{\frac{c}{2}} \left( a_t + 2c - d_b \right)
\]  \hspace{1cm} [5.4]

To consider the strength due to a combination of bolt row mechanisms with the same yield line pattern as shown in Figure 5.1, Equation [5.3] can be used by setting \( b_t = 0 \). To obtain the strength of the combined bolt row mechanisms, the equation is multiplied by the number of rows in tension to give:
\[
F_{R,S} = 4M_p \left( \frac{a_t + 2c - d_b}{d} + \frac{2d - 0.5d_b}{c} \right) \times n
\]  \hspace{1cm} [5.5]

Although presented differently, these equations yield the same equations as used in British Steel (1996b) given by Equation [2.24] and by Ghobarah (1996) given by Equation [2.25]. The only difference is that both approaches define the deformable width of the face as \( a = w_{RHS} - t_c \) whereas here it is \( a = w_{RHS} - 4t_c \), with the latter giving a higher value of load carrying capacity. This assumes that the yield line forms at the edge of clear width of the RHS face due to the higher stiffness of the sidewalls. This is the same definition of RHS face width as was made for the derivation of initial stiffness equations in Chapter 4. Other definitions of the yieldable width of the RHS face between \( w_{RHS} - t_c \) and \( w_{RHS} - 4t_c \) will be investigated in the parametric study of Section 5.5. In comparison, Gomes et al. (1996) uses an even smaller value of \( a = w_{RHS} - 2t_c - 1.5r \). Assuming typical values of \( r = 1.5t_c \) and \( r = 2t_c \), this gives \( a = w_{RHS} - 4.25t_c \) and \( w_{RHS} - 5t_c \) respectively. Because yield lines must form at the edge of the plate in consideration, it is invalid to use a value smaller than the clear face width (\( w_{RHS} - 4t_c \))
for common sections where \( r_0 \leq 2t_c \) as discussed in Section 3.2.6). Therefore, this definition is not considered.

![Figure 5.1. Geometry of yield line patterns using straight yield lines](image)

**5.2.2 Circular yield line pattern in bending**

The case of a bolt group with a circular yield line pattern is considered, as given in Figure 5.3. To simplify the derivation, the components of the yield line mechanism are considered to be the sum of two parts: (1) the straight yield lines (horizontal/vertical) whose derivations are given in Section 5.2.1, and (2) the 4 circular radial yield segments which when combined, form a complete circle with radius equal to \( c \). The virtual work for the yield line pattern neglecting the bolt holes is given by:

\[
I_{W(\text{no holes})} = M \left( \frac{4a_t + 4b_t}{c} + 4\pi \right) \quad [5.6]
\]

In addition to this, it is necessary to consider the effect of the bolt hole openings along the yield lines that reduce the strength of the plate. Determining this effect for

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regions bound by straight yield lines is straightforward. The reduction of internal virtual work due to bolthole openings for the circular portion of the yield line mechanism is obtained by considering the positive radial yield lines as shown in Figure 5.2. The boltholes do not affect the negative circumferential yield lines.

Figure 5.2. Geometry of radial yield line with hole

For the whole circle with angle $2\pi$ radians, the bolthole reduces the internal virtual work of the radial yield lines by the following:

$$I_{W(circular\ region\ holes)} = M \left( \frac{2\pi \frac{d_b}{2}}{c} \right)$$

$$= M \left( \frac{\pi d_b}{c} \right) \tag{5.7}$$

The material removed along the yield lines is subtracted from Equation [5.12] to give the internal virtual work considering the effect of boltholes:

$$I_{W(with\ holes)} = 4M \left( \frac{a_t + b_t - 0.5(n - 1)d_b}{c} + \pi - \frac{\pi d_b}{4c} \right) \tag{5.8}$$

By balancing the internal and external work ($E_w = F$), the failure load $F$ can be obtained as:

$$I_W = E_W$$
\[ F_{G,c} = 4M_p \left( \frac{a_t + b_t - 0.5(n-1)d_b}{c} + \pi - \frac{\pi d_b}{4c} \right) \]  

\[ 5.9 \]

It is also necessary to consider the strength due to a combination of bolt row mechanisms as shown in Figure 5.3. To obtain the strength for a single bolt row from Equation [5.9], terms relating to \( b_t \) are removed. To obtain the strength of the combined bolt row mechanisms, the equation is multiplied by the number of rows in tension:

\[ F_{R,c} = 4M_p \left( \frac{a_t - 0.5d_b}{c} + \pi - \frac{\pi d_b}{4c} \right) \times n \]

\[ 5.10 \]

Although presented differently, these equations yield the same equation as derived by Ghobarah (1996) given by Equation [2.26]. Again, the only difference is that Ghobarah defines the deformable width of the RHS face \( a = w_{RHS} - t_c \) whereas here, as in the previous section, it is \( a = w_{RHS} - 4t_c \) based on the assumption that the yield line forms at the edge of clear width of the RHS face due to the higher stiffness of the sidewalls. Again, other definitions of the yieldable width of the RHS face between \( w_{RHS} - t_c \) and \( w_{RHS} - 4t_c \), will be investigated in the parametric study of Section 5.5. As explained in Section 5.2.1, the definition of the deformable width of the RHS face used in Gomes et al. is not considered because it assumes that the yield lines form at locations within the clear face width.
5.2.3 Elliptical yield line pattern in bending

The circular yield line patterns in Figure 5.3 are a special case of elliptical yield line pattern in which the length of yield lines along the tube length is the same as in the perpendicular direction. This section derives equations for the general case when these two lengths are not the same. The yield line pattern is shown in Figure 5.5.

The virtual work due to a radial yield line can be evaluated by taking an infinite number of infinitely small slices with an angle dθ and then evaluating the integral across the total angle covered by the yield line as shown in Figure 5.4.
Chakrabarty (2010) provides a derivation for the complex mechanism of an elliptical plate with a concentrated load at the centre which yields the following solution:

\[ I_{W, Ellipse} = M\pi \left( \frac{c}{d} + \frac{d}{c} \right) \]  \[ (5.11) \]

Including the virtual work for the straight yield lines, the total virtual work for the full pattern incorporating the elliptical portion given by Equation [5.11] but neglecting boltholes is:

\[ I_{W(\text{no holes})} = M \left( \frac{4a_\ell}{d} + \frac{4b_\ell}{c} + 2\pi \left( \frac{c}{d} + \frac{d}{c} \right) \right) \]  \[ (5.12) \]

For the bolthole openings on radial yield lines of the elliptical region, the reduction of internal virtual work can be determined in a similar manner to that used for radial yield lines of the circular region given in Equation [5.7]. For the circular region, the reduction of internal virtual work is essentially obtained by dividing the circumference of the bolthole (\( \pi d_b \)) by the lever to the radial yield line, c. For this elliptical region, the reduction of internal virtual work can be obtained by dividing the
circumference of the bolthole \((\pi d_b)\) by the lever to the radial yield line. Because the lever varies in the ellipse due to the variation in major/minor axis characteristics, an equivalent characteristic length must be used. This can be determined by dividing the internal virtual work of the ellipse (Equation [5.11]) by the perimeter of the ellipse. Because the exact perimeter of an ellipse is given by an infinite series, it is possible to use the following simplification that approximates the perimeter within 5% error for \(1/3<r/c<3\):

\[
p_{\text{ellipse}} = 2\pi \sqrt{\frac{c^2 + d^2}{2}}
\]  

[5.13]

Dividing Equation [5.13] by Equation [5.11] (leaving out \(M\)) gives the characteristic distance from the elliptical yield line:

\[
I_{\text{edge}} = \frac{2\pi \sqrt{\frac{c^2 + d^2}{2}}}{\pi \left( \frac{c}{a} + \frac{d}{c} \right)}
\]  

[5.14]

\[
= \frac{\sqrt{2}cd}{\sqrt{c^2 + d^2}}
\]

The internal virtual work of the boltholes in the elliptical yield line region is thus:

\[
I_{\text{W(elliptical holes)}} = M \left( \frac{2\pi \frac{d_b}{2}}{I_{\text{edge}}} \right)
\]  

[5.15]

\[
= M \left( \frac{\pi d_b \sqrt{c^2 + d^2}}{\sqrt{2}cd} \right)
\]

The effect of the boltholes on the straight yield line region and the elliptical region given by Equation [5.15] is subtracted from Equation [5.12] to give the internal virtual work when considering the combined effect of boltholes:
By balancing the internal and external work done by the force $F$ over a unit displacement, the failure load $F$ is obtained as:

$$I_{W(\text{with holes})} = M \left( \frac{4a_t - 2d_b}{d} + \frac{4b_t - 2(n - 1)d_b}{c} + 2\pi \left( \frac{c}{d} + \frac{d}{c} \right) - \pi d_b \frac{\sqrt{c^2 + d^2}}{\sqrt{2}cd} \right)$$

[5.16]

As in the previous example, the first derivative can be used to determine the unknown length $d$.

$$\frac{\partial F}{\partial d} = M_p \left( - \frac{4a_t - 2d_b + 2\pi c}{d^2} + \frac{2\pi}{c} + \frac{2\pi d_b}{c\sqrt{c^2 + d^2}} - \pi d_b \frac{\sqrt{c^2 + d^2}}{\sqrt{2}cd^2} \right)$$

Due to the complexity of the equation however, it is only possible to represent $d$ as a complex equation which makes it unsuitable for hand-calculation. For a computer-based application, $d$ can be obtained through an iterative process to find the minimum $F$.

An alternative using a simplification for the reduction of strength due to bolt holes on the elliptical section can be considered to provide an approximate solution for $d$. A slightly conservative value for the reduction of strength due to bolt holes can be used by taking that for the circular yield line given by Equation [5.7] but using the major radius $d$. This is conservative because the major radius $d$ is always larger than the minor radius $c$. Using this simplification, the combined internal virtual work is:
\[ F_{G,E} = M_p \left( \frac{4a_t - 2d_b + 2\pi c - \pi d_b}{d} + \frac{4b_t - 2(n - 1)d_b + 2\pi d}{c} \right) \]  \[ [5.18] \]

This gives the following first derivative:

\[ \frac{\partial F}{\partial d} = M_p \left( -\frac{4a_t - 2d_b + 2c\pi - \pi d_b}{d^2} + \frac{2\pi}{c} \right) \]

With \( \frac{\partial F}{\partial d} = 0 \),

\[ d = \frac{c}{\sqrt{2\pi}} \frac{2\pi c - \pi d_b}{4a_t - 2d_b + 2\pi c - \pi d_b} \]  \[ [5.19] \]

It is also necessary to consider the strength due to a combination of bolt row mechanisms with a yield line pattern as shown in Figure 5.5. To obtain the strength for a single bolt row from Equation [5.17], terms relating to \( b_t \) are removed. To obtain the strength of the combined bolt row mechanisms, the equation is multiplied by the number of rows, \( n \):

\[ F_{R,E} = M_p \left( \frac{4a_t - 2d_b + 2\pi c - \pi d_b}{d} + \frac{2\pi d}{c} \right) \times n \]  \[ [5.20] \]
5.2.4 Summary of RHS bending strength equations

This section has covered the derivation of the existing rectangular straight yield line mechanism and circular straight yield line mechanism as given in British Steel (1996b) and Ghobarah et al. (1996) with a notable difference in the definition of the deformable width of the RHS face. A newly derived elliptical radial yield line mechanism is also presented. Due to the large number of variables involved, it is not possible to provide a conclusive proof of the relationship between these three mechanisms. In lieu of this, a comparison of the bending strength for realistic geometries is given when using the various equations. Figure 5.6 compares the bending strength (minimum of bolt group and bolt rows) when using straight, circular, and elliptical yield line mechanisms with varying horizontal bolthole spacing, $a_t$. Figure 5.7 compares the bending strength when using the elliptical equations for bolt group and bolt rows with different vertical bolthole spacing, $b_t$. 

Figure 5.5. Geometry of yield line patterns using radial yield lines

Bolt group

Bolt rows

\[ a = w_{RHS} - 4t_c \]
The comparison of bending strength when using the straight, circular, and elliptical yield line mechanisms shows that the elliptical yield line mechanism always gives the minimum (least conservative) strength. For low values of $a_t$, the circular and elliptical yield lines give identical minimum solutions. For high values of $a_t$, the straight and elliptical yield lines gives identical minimum solutions. This shows that the Ghobarah et al. (1996) method of using the minimum of straight and circular yield line mechanisms is accurate for low and high values of $a_t$ (relative to $a$), but not at the crossover point when these two mechanisms are equal. The newly derived elliptical yield line mechanism fills the gap by providing the minimum strength for all values of $a_t$. At the crossover point where straight and circular yield line mechanisms give equal strength, the elliptical yield line mechanisms provides an approximate 8% improvement in both cases given in Figure 5.6.

Figure 5.6. Bending strength when using different strength equations with variations in horizontal bolthole spacing, $a_t$
It has been established that the minimum solution is always given by the elliptical yield line equations. Thus, the plastic strength of the RHS face in tension can be obtained by taking the minimum of elliptical bolt group and elliptical bolt row equations as follows:

\[
F_P = \text{Min}(F_{G,E}, F_{R,E})
\]  \[5.21\]

A comparison of bending strengths when using the elliptical yield line mechanisms for bolt group and bolt rows shows that the critical failure mode is dependent on the vertical bolthole separation, \(b_t\). For smaller values of \(b_t\), the bolt group mechanism is dominant. For larger values, the isolated bolt row mechanism is dominant. For most cases, the isolated bolt row mechanism will not occur because a relatively high value of \(b_t\) is necessary. For the two cases considered in Figure 5.7, the isolated bolt row mechanism becomes dominant when \(b_t\) is approximately equal to the RHS width.

Figure 5.7. Bending strength when using different strength equations with variations in vertical bolthole spacing, \(b_t\)
In all equations, \( M_p \) is the plastic moment resistance per unit width used to define the plastic strength of the component which is based on assumptions about the distribution of stress/strain in the cross section as shown in Figure 5.8. In the elastic range, the stresses are distributed linearly with the outermost material reaching stresses of magnitude \( f_y \) at the limit of the elastic range. In the plastic range, the stress \( f_y \) extends to the midpoint of the cross section. The plastic bending moment capacity is:

\[
M_p = \frac{t_c^2 f_y}{4}
\]  \[5.22\]

Using \( M_y \) with yield line equations gives the elastic yield strength of a component to define the limit of fully-elastic behaviour used in Section 6.2 to define the full load-deflection curve. The elastic bending moment capacity is:

\[
M_y = \frac{t_c^2 f_y}{6}
\]  \[5.23\]

Figure 5.8. Cross section stress distribution to calculate moment resistance per unit width

A summary of yield line mechanism equations is given in Table 5.1.
Table 5.1. Summary of yield line mechanism equations

<table>
<thead>
<tr>
<th></th>
<th>Straight yield lines</th>
<th>Circular yield lines</th>
<th>Elliptical yield lines</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bolt group</td>
<td>Bolt group</td>
<td>Bolt group</td>
</tr>
<tr>
<td></td>
<td>$F_{G,S} = 4M_p \left( \frac{a_t + 2c - d_b}{d} + \frac{b_t + 2d - 0.5nd_b}{c} \right)$</td>
<td>$F_{G,C} = 4M_p \left( \frac{a_t + b_t - 0.5nd_b}{c} + \frac{\pi - \pi d_b}{4c} \right)$</td>
<td>$F_{G,E} = M_p \left( \frac{4a_t - 2d_b + 2\pi c - \pi d_b}{d} + \frac{4b_t - 2(n-1)d_b + 2\pi d}{c} \right)$</td>
</tr>
<tr>
<td></td>
<td>Bolt rows</td>
<td>Bolt rows</td>
<td>Bolt rows</td>
</tr>
<tr>
<td></td>
<td>$F_{R,S} = 4M_p \left( \frac{a_t + 2c - d_b}{d} + \frac{2d - 0.5 d_b}{c} \right) \times n$</td>
<td>$F_{R,C} = 4M_p \left( \frac{a_t - 0.5d_b}{c} + \frac{\pi - \pi d_b}{4c} \right) \times n$</td>
<td>$F_{R,E} = M_p \left( \frac{4a_t - 2d_b + 2\pi c - \pi d_b}{d} + \frac{2\pi d}{c} \right) \times n$</td>
</tr>
<tr>
<td></td>
<td>where</td>
<td>where</td>
<td>where</td>
</tr>
<tr>
<td></td>
<td>$d = \sqrt{\frac{c}{2} (a_t + 2c - d_b)}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.3 Prediction of flowdrill bolt pullout

5.3.1 Assumed deformed shape

The equations covered in Section 5.2 represent the strength of the RHS face as an isolated component where the bolt load is transferred directly to the RHS face. For flowdrilled RHS faces it is also necessary to consider thread stripping of the RHS threads under two conditions: (1) pure thread stripping capacity when the RHS and bolt threads are in full contact and (2) thread stripping or pullout at a lower capacity due to loss of contact with thread due to opening of the threaded holes at large deformations which causes the bolt forces to be transferred to a small number of threads. The former is a strength limit that is addressed by Kurobane et al. (2004) and given by Equation [2.31]. The latter is controlled by the opening of the threaded hole...
which is dependent on the RHS geometry and is necessary to quantify when considering the whole joint load-deflection characteristics as it may induce a premature and sudden failure of the joint. The following approach is proposed as an approximate method of predicting bolt pullout using a reduction factor based on the proportion of threads that are expected to be in contact with the threads of the RHS face.

Consider the deformed shape of the RHS with flowdrilled face in tension given in Figure 5.9. Assume that the lobe created during the flowdrill process at Point B deforms under load such that it offers negligible strength against pullout. The strength against pullout is proportional to the number of threads of the bolt and RHS face that are in contact with one another. As can be seen in the deformed shape, the contact between threads can be characterised as being linearly proportional to the RHS face thickness. As such, it is proposed that the reduction in bolt thread stripping is proportional to the RHS face thickness and the following reduction factor is applied:

\[ R = \tau \times t_c, \text{ but } R \leq 1 \]  

where \( \tau \) is a coefficient determined from experimental testing in Section 5.3.2 (\( \tau = 60 \))

This reduction factor is applied to the full flowdrill thread stripping capacity, \( F_{ts} \), given by Equation [2.31] to predict thread stripping due to gross deformation of the RHS face in bending as follows:

\[ F_{ts, red} = R \times F_{ts} \]
5.3.2 Validation against British Steel testing (1996b)

The testing on the flowdrilled RHS face in tension described in Section 3.1.2 is used to validate the equation proposed in Section 5.3.1 and determine the correction coefficient $\tau$ for predicting thread stripping bolt pullout. In British Steel (1996b), it is stated that all specimens with 4 M20 bolts in a rectangular formation failed due to thread stripping action with the exception of test T/12/67/16 with bolt failure. Therefore, all tests except this are used to determine the value of $\tau$ using a regression method. The value obtained for $\tau$, based on minimum mean error, is $\tau=61.2$ (mean error=0%, st.dev=14.2%, max error=24.9%), rounded to $\tau=60$. A comparison of test values and analytical values using the reduced thread stripping strength based on $\tau=60$ is presented in Table 5.2. A comparison of the test pullout strength divided by the predicted full flowdrill thread stripping strength for different RHS thicknesses is given in Figure 5.10 indicating the actual reduction factor, $R$, for individual tests.
### Table 5.2 Analytical and test thread stripping strength (τ=60)

<table>
<thead>
<tr>
<th>Test</th>
<th>Bolt</th>
<th>( t_c ) (m)</th>
<th>( f_y ) (N/mm(^2))</th>
<th>( F_{tu} ) (kN)</th>
<th>( R ) Eqn. [2.31]</th>
<th>( F_{tu,red} ) (kN) Eqn. [5.24]</th>
<th>Test ( F_{fail} ) (kN)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T/30/27/16</td>
<td>M16</td>
<td>0.00483</td>
<td>332</td>
<td>513.9</td>
<td>0.29</td>
<td>148.9</td>
<td>155.0</td>
<td>-3.9%</td>
</tr>
<tr>
<td>T/30/67/16</td>
<td>M16</td>
<td>0.00483</td>
<td>332</td>
<td>513.9</td>
<td>0.29</td>
<td>148.9</td>
<td>186.0</td>
<td>-19.9%</td>
</tr>
<tr>
<td>T/19/27/16</td>
<td>M16</td>
<td>0.00772</td>
<td>306</td>
<td>580.3</td>
<td>0.46</td>
<td>268.8</td>
<td>290.0</td>
<td>-7.3%</td>
</tr>
<tr>
<td>T/19/67/16</td>
<td>M16</td>
<td>0.00772</td>
<td>306</td>
<td>580.3</td>
<td>0.46</td>
<td>268.8</td>
<td>361.0</td>
<td>-25.5%</td>
</tr>
<tr>
<td>T/12/27/16</td>
<td>M16</td>
<td>0.0121</td>
<td>315</td>
<td>763.8</td>
<td>0.73</td>
<td>554.5</td>
<td>498.0</td>
<td>11.4%</td>
</tr>
<tr>
<td>T/30/27/20</td>
<td>M20</td>
<td>0.00475</td>
<td>303</td>
<td>582.6</td>
<td>0.29</td>
<td>166.0</td>
<td>172.0</td>
<td>-3.5%</td>
</tr>
<tr>
<td>T/30/50/20</td>
<td>M20</td>
<td>0.00475</td>
<td>303</td>
<td>582.6</td>
<td>0.29</td>
<td>166.0</td>
<td>198.0</td>
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<td>303</td>
<td>582.6</td>
<td>0.29</td>
<td>166.0</td>
<td>198.0</td>
<td>-16.1%</td>
</tr>
<tr>
<td>T/24/67/20</td>
<td>M20</td>
<td>0.0059</td>
<td>314</td>
<td>658.2</td>
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</tr>
<tr>
<td>T/24/50/20</td>
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<td>314</td>
<td>658.2</td>
<td>0.35</td>
<td>233.0</td>
<td>276.0</td>
<td>-15.6%</td>
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<td>T/24/67/20</td>
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<td>314</td>
<td>658.2</td>
<td>0.35</td>
<td>233.0</td>
<td>276.0</td>
<td>-15.6%</td>
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<tr>
<td>T/19/27/20</td>
<td>M20</td>
<td>0.0077</td>
<td>263</td>
<td>622.7</td>
<td>0.46</td>
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<td>263</td>
<td>622.7</td>
<td>0.46</td>
<td>287.7</td>
<td>337.0</td>
<td>-14.6%</td>
</tr>
<tr>
<td>T/19/67/20*</td>
<td>M20</td>
<td>0.0077</td>
<td>263</td>
<td>622.7</td>
<td>0.46</td>
<td>287.7</td>
<td>384.0</td>
<td>-25.1%</td>
</tr>
<tr>
<td>T/15/27/20</td>
<td>M20</td>
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<td>293</td>
<td>777.6</td>
<td>0.58</td>
<td>447.9</td>
<td>390.0</td>
<td>14.8%</td>
</tr>
<tr>
<td>T/15/50/20</td>
<td>M20</td>
<td>0.0096</td>
<td>293</td>
<td>777.6</td>
<td>0.58</td>
<td>447.9</td>
<td>472.0</td>
<td>-5.1%</td>
</tr>
<tr>
<td>T/15/67/20*</td>
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<td>293</td>
<td>777.6</td>
<td>0.58</td>
<td>447.9</td>
<td>511.0</td>
<td>-12.3%</td>
</tr>
<tr>
<td>T/12/27/20</td>
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<td>0.0119</td>
<td>280</td>
<td>840.2</td>
<td>0.71</td>
<td>599.9</td>
<td>490.0</td>
<td>22.4%</td>
</tr>
<tr>
<td>T/12/50/20</td>
<td>M20</td>
<td>0.0119</td>
<td>280</td>
<td>840.2</td>
<td>0.71</td>
<td>599.9</td>
<td>575.0</td>
<td>4.3%</td>
</tr>
<tr>
<td>T/12/67/20*</td>
<td>M20</td>
<td>0.0119</td>
<td>280</td>
<td>840.2</td>
<td>0.71</td>
<td>599.9</td>
<td>670.0</td>
<td>-10.5%</td>
</tr>
<tr>
<td>T/30/27/24</td>
<td>M24</td>
<td>0.00483</td>
<td>332</td>
<td>770.8</td>
<td>0.29</td>
<td>223.4</td>
<td>190.0</td>
<td>17.6%</td>
</tr>
<tr>
<td>T/30/67/24</td>
<td>M24</td>
<td>0.00483</td>
<td>332</td>
<td>770.8</td>
<td>0.29</td>
<td>223.4</td>
<td>233.0</td>
<td>-4.1%</td>
</tr>
<tr>
<td>T/19/27/24</td>
<td>M24</td>
<td>0.00772</td>
<td>306</td>
<td>870.5</td>
<td>0.46</td>
<td>403.2</td>
<td>373.0</td>
<td>8.1%</td>
</tr>
<tr>
<td>T/19/67/24*</td>
<td>M24</td>
<td>0.00772</td>
<td>306</td>
<td>870.5</td>
<td>0.46</td>
<td>403.2</td>
<td>405.0</td>
<td>-0.4%</td>
</tr>
<tr>
<td>T/12/27/24</td>
<td>M24</td>
<td>0.0121</td>
<td>315</td>
<td>1145.7</td>
<td>0.73</td>
<td>831.8</td>
<td>681.0</td>
<td>22.1%</td>
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<tr>
<td>T/12/67/24*</td>
<td>M24</td>
<td>0.0121</td>
<td>315</td>
<td>1145.7</td>
<td>0.73</td>
<td>831.8</td>
<td>714.0</td>
<td>16.5%</td>
</tr>
</tbody>
</table>

Mean -2.0%  
St.dev 13.9%  
Max 25.5%

*These tests have flowdrill threads formed on the RHS curved edge.

![Graph showing comparison of actual reduction factor, R, for different RHS thicknesses](image)

Figure 5.10. Comparison of actual reduction factor, R, for different RHS thicknesses
This comparison suggests that the linear relationship proposed in Section 5.3.1 for thread stripping is suitable in predicting the failure strength of the flowdrilled RHS face component when used with a correction factor \( \tau = 60 \) as it gives acceptable mean error, standard deviation, and maximum errors. The variability in results can partly be attributed to imperfections and minor variations in column face thickness. The range of tests sufficiently covers the variables of bolt hole dimension, horizontal bolthole spacing, vertical bolthole spacing, and RHS face thickness as well as cases where the flowdrill thread is formed partially on the curved portion of the RHS radius. This suggests that this equation is suitable in predicting failure strength regardless of geometric range. This is a significant improvement over the original equation in Kurobane et al. and provides insight into the capacity of flowdrill connections in bending. This is important because bolt pullout is a sudden mode of failure that must be avoided. Until now, the only method of estimating flowdrilled bolt pullout capacity is by full-scale testing.

With the new equations to predict the capacity of thread stripping due to gross deformation of the RHS face, it is possible to give guidelines on the minimum RHS thickness requirements to achieve full grade 8.8 bolt tension capacity for cases with bending. This compares with Table 5.3 from CIDECT Report 6F-13A/96 (British Steel, 1996b) which gives similar guidelines based on tensile pullout testing of flowdrilled threads with no bending as given in Figure 5.11. Because no bending was involved, the bolt and RHS face threads were in full contact until failure therefore giving the upper bound of thread stripping capacity given by Equation [2.31]. This differs to cases with bending where there is deformation of the boltholes and thus loss of contact between bolt and RHS face threads. The revised minimum RHS thickness requirements are obtained by equating Equation [2.33] for the tensile capacity of a bolt with Equation [5.24] for the thread stripping capacity due to gross deformation of the
RHS face and solving for RHS thickness. The same safety factor is considered for both bolt tensile capacity and thread stripping capacity equations thus cancelling them out. The revised recommendations are presented in Table 5.4. There is some difference in the minimum thickness for the standard case with no bending for S275 steel, possibly due to the British Steel values being based on the weakest test results normalised to nominal S275 yield stress. However, it is clear that when considering thread stripping due to bending of the RHS face and boltholes, a much greater RHS thickness is required. The increase in RHS thickness requirement for cases with bending is not explicitly stated in British Steel (1996a).

Table 5.3 Minimum RHS thickness to achieve full Grade 8.8 bolt tension capacity (British Steel, 1996b)

<table>
<thead>
<tr>
<th>Bolt size</th>
<th>Minimum RHS thickness S275 (mm)</th>
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<tr>
<td>M16</td>
<td>6.4</td>
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<tr>
<td>M20</td>
<td>8.0</td>
</tr>
<tr>
<td>M24</td>
<td>9.6</td>
</tr>
</tbody>
</table>

Table 5.4 Modified guidelines for minimum RHS thickness to achieve full Grade 8.8 bolt tension capacity for no-bending and bending cases based on analytical equations

<table>
<thead>
<tr>
<th>Bolt size</th>
<th>( A_t ) (mm(^2))</th>
<th>( d_b ) (mm)</th>
<th>No bending (No plate deformations)</th>
<th>With bending (With plate deformations)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>S275</td>
<td>S355</td>
</tr>
<tr>
<td>M16</td>
<td>156</td>
<td>16</td>
<td>5.6</td>
<td>2.5</td>
</tr>
<tr>
<td>M20</td>
<td>245</td>
<td>20</td>
<td>9.1</td>
<td>5.2</td>
</tr>
<tr>
<td>M24</td>
<td>352</td>
<td>24</td>
<td>12.4</td>
<td>7.8</td>
</tr>
</tbody>
</table>

Figure 5.11. British Steel (1996a) pullout test setup
5.4 Membrane action strength

5.4.1 Derivation of membrane action strength equations

With the considerable support of the sidewalls, the RHS face in tension is able to develop tensile membrane action that increases the load capacity past its bending strength (determined from yield line mechanisms). By using principles of virtual work, it is possible to relate the failure load of the RHS face to its deflection. Jones et al. (2007) use this approach to develop equations to predict the membrane action capacity of the RHS face for a fin-plate connection. By extending the general method to the geometry of a bolted connection, it is possible to predict the membrane action capacity of the RHS face in tension for bolted connections.

For simplification, the column face deflection profile is assumed to be based on the straight yield line pattern used in Section 5.2.1. The deflection profile is assumed to be linear with a maximum deflection at boltholes (Point B) and zero deflection at the corners (Point C) as shown in Figure 5.12.

![Figure 5.12. Geometry of bolt group under membrane action and detail of a diagonal ridge](image)
First, establish the equilibrium condition for the cross section shown in Figure 5.13 when the applied load $F$ resisted by pure catenary action force $F_T$:

$$F_T = \frac{F}{2\sin[\phi]} \quad [5.26]$$

For small deflections,

$$\sin[\phi] = \frac{\delta_m}{c} \quad [5.27]$$

Therefore,

$$F_T\delta_m = \frac{F \times c}{2} \quad [5.28]$$

![Figure 5.13. Axially restrained cross section under lateral load](image)

The total internal virtual work is obtained as the sum of the virtual work along the direction $a_t$, the direction $b_t$, and the diagonal ridges:

$$IVW_{total} = IVW_{at} + IVW_{bt} + IVW_{diag} \quad [5.29]$$

As the deflection $\delta_m$ increases, Line A.B. will stretch beyond its original value of $c$, uniformly along the yield line B.B' as given in Figure 5.12. This region can be characterised as the cross section under tension given by Equation [5.28]. The change in length across this yield line (ignoring small terms) is given by:
Now, considering the diagonal deformation profile given in Figure 5.14, at the maximum deflection $\delta_m$, the maximum change in the length $c$ is:

$$\delta c = \frac{\delta_m^2}{2c}$$  \[5.30\]

The internal virtual work done for yield line B.B’ is:

$$IVW_{B,B'} = \frac{F_T\delta_m \partial (\delta_m)}{c}$$  \[5.32\]

The combined internal virtual work along vertical yield lines is obtained by doubling Equation [5.32] to consider both yield lines and substituting $F_T = f_y \times A$:

$$IVW_{bt} = \frac{2f_y t c \delta_m \partial (\delta_m)}{c} b_t$$  \[5.33\]

The combined internal virtual work along horizontal yield lines is obtained from the same mechanism but by replacing $c$ with $d$ and $b_t$ with $a_t$:

$$IVW_{at} = \frac{2f_y t c \delta_m \partial (\delta_m)}{d} a_t$$  \[5.34\]

When incorporating the effect of the boltholes, this gives the following:

$$IVW_{at(\text{with holes})} = \frac{2f_y t w \delta_m \partial (\delta_m)}{d} (a_t - d_b)$$  \[5.35\]

$$IVW_{bt(\text{with holes})} = \frac{2f_y t w \delta_m \partial (\delta_m)}{c} (b_t - (n - 1)d_b)$$  \[5.36\]
Figure 5.14. Geometry of diagonal deformation profile

Figure 5.15 shows the assumed force distribution along the ridges where equivalent yield lines are assumed between the rigid plates and their associated deformations. The total opening at any point along the ridge is linearly proportional to the maximum opening at point B given by Equation [5.31].
Figure 5.15. Internal force distribution along equivalent yield lines

The total change in length perpendicular to yield line B.C. can be expressed as:

\[ \Delta L_{\text{Diag}} = \frac{\delta_m^2}{2L_{\text{Diag}}} \]

[5.37]

where

\[ L_{\text{Diag}} = \sqrt{c^2 + d^2} \]

[5.38]

Assuming that \( \Delta L_{\text{Diag}} \) varies linearly along the diagonal length \( L_{\text{Diag}} \), the distance along length \( S \) as shown in Figure 5.15 from zero at C to a maximum at B, then:

\[ \Delta L_{\text{Diag}} = \Delta L_{\text{Diag}} \frac{S}{L_{\text{Diag}}} \]

\[ \Delta L_{\text{Diag}} = \frac{\delta_m^2}{2L_{\text{Diag}}} \frac{S}{L_{\text{Diag}}} \]

[5.39]

Therefore,

\[ \partial(\Delta L_{\text{Diag}}) = \frac{\delta_m \partial(\delta_m)}{L_{\text{Diag}}} \frac{S}{L_{\text{Diag}}} \]

[5.40]
Assuming a linear force distribution along the diagonal B.C., the internal virtual work along a single diagonal yield line is:

$$IVW_{\text{Diag, single}} = \int_{0}^{L_{\text{Diag}}} t_{c}f_{y} \frac{S}{L_{\text{Diag}}} \varphi(\Delta L_{\text{Diag}}) \, dS$$  \[5.41\]

When considering the effect of the boltholes, this becomes the following:

$$IVW_{\text{Diag, single, with holes}} = \int_{d_{b}/2}^{L_{\text{Diag}}} t_{c}f_{y} \frac{S}{L_{\text{Diag}}} \varphi(\Delta L_{\text{Diag}}) \, dS$$  \[5.42\]

Substituting Equation \[5.40\] into Equation \[5.42\] gives:

$$IVW_{\text{Diag, single, with holes}} = \int_{d_{b}/2}^{L_{\text{Diag}}} t_{c}f_{y} \frac{\delta_{m} \varphi(\delta_{m}) (\frac{S}{L_{\text{Diag}}})^{2}}{3L_{\text{Diag}}^{2}} \, dS$$

$$= t_{c}f_{y} \frac{\delta_{m} \varphi(\delta_{m}) (L_{\text{Diag}}^{2} - \frac{d_{b}^{3}}{24L_{\text{Diag}}^{2}})}{3}$$  \[5.43\]

Given that \(\delta_{m}\) is relatively small, \(L_{\text{Diag}} \approx L_{\text{Diag}}\), giving:

$$IVW_{\text{Diag, single, with holes}} = t_{c}f_{y} \delta_{m} \varphi(\delta_{m}) \left(\frac{1}{3} - \frac{d_{b}^{3}}{24L_{\text{Diag}}^{3}}\right)$$  \[5.44\]

Therefore the total IVW for all four diagonal yield lines at the failure deflection \(\delta_{m}\) is:

$$IVW_{\text{Diag}} = 4t_{c}f_{y} \delta_{m} \varphi(\delta_{m}) \left(\frac{1}{3} - \frac{d_{b}^{3}}{24L_{\text{Diag}}^{3}}\right)$$  \[5.45\]

The total internal virtual work along the whole yield mechanism is given by the sum of internal virtual work of the horizontal, vertical, and diagonal components:
\[
IVW_{\text{Total}} = \frac{2f_y t_c \delta_m \partial(\delta_m)}{d} (a_t - d_b) + \frac{2f_y t_c \delta_m \partial(\delta_m)}{c} (b_t - (n-1)d_b) \\
+ 4t_c f_y \delta_m \partial(\delta_m) \left( \frac{1}{3} - \frac{d_b^3}{24L_{\text{Diag}}^3} \right) \\
= 2f_y t_c \delta_m \partial(\delta_m) \left( \frac{a_t - d_b}{d} + \frac{b_t - (n-1)d_b}{c} + 2 - \frac{d_b^3}{12L_{\text{Diag}}^3} \right) 
\]  

[5.46]

Equating this to the external work, \( P \cdot d(\delta_m) \), and rearranging to find \( F \) gives:

\[
F_{\text{Mem.,Group}} = 2f_y t_c \delta_m \partial(\delta_m) \left( \frac{a_t - d_b}{d} + \frac{b_t - (n-1)d_b}{c} + 2 - \frac{d_b^3}{12L_{\text{Diag}}^3} \right) 
\]  

[5.47]

It is also necessary to consider the strength under a combination of bolt row mechanisms. To obtain the strength of a single bolt row from Equation [5.47], terms relating to \( b_t \) are removed. To obtain the strength of the combined bolt row mechanisms, the equation is multiplied by the number of rows in tension:

\[
F_{\text{Mem.,Rows}} = 2f_y t_c \delta_m \partial(\delta_m) \left( \frac{a_t - d_b}{d} + 2 - \frac{d_b^3}{12L_{\text{Diag}}^3} \right) \times n 
\]  

[5.48]

To use Equations [5.47] and [5.48], a prediction of the deflection at the membrane action limit, \( \delta_m \), is necessary. The following equation can be used to estimate \( \delta_m \) by relating the undeformed length, \( c \), to the deformed length, \( c' \), assumed to have reached the material ultimate strain limit, \( e_u \). This gives the following relationship:

\[
c'^2 = c^2 + \delta_m^2 
\]  

[5.49]

where

\[ c' = c(1+e_u) \]

Solving for \( \delta_m \), 

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\[ \delta_m = \sqrt{c'^2 - c^2} \]  \hfill [5.50]

5.4.2 Contribution of membrane action to failure mechanism

For the concrete-filled RHS face, the fully restraining effect of the sidewalls allows the full membrane action to develop. Therefore, the membrane action strength given by Equations [5.47] and [5.48] can simply be used to define the failure strength.

For the hollow RHS face, the sidewalls do not provide a full restraint therefore the failure mechanism is a combination of yield line bending and membrane action strengths. The following relationship, based on taking equal parts yield line bending and membrane action strengths, is proposed to predict the failure strength of the hollow RHS face. This is a simplified version of the relationship proposed in Jones et al. (2007) for fin-plate connections to RHS where a value ranging from half bending and half membrane action to full membrane action is suggested based on the cross section slenderness. This simplification is validated against the FEM parametric study in Section 5.5. The equations for membrane strength of hollow and filled RHS faces in tension are thus:

Hollow RHS membrane strength
\[ F_{HollowRHS,Mem.} = 0.5 \times F_p + 0.5 \times \text{Min}[F_{\text{Mem.,Group}}, F_{\text{Mem.,Rows}}] \]  \hfill [5.51]

Filled RHS membrane strength
\[ F_{FilledRHS,Mem.} = \text{Min}[F_{\text{Mem.,Group}}, F_{\text{Mem.,Rows}}] \]  \hfill [5.52]

In both cases, the membrane action contributions begin at the initial yield determined from the yield line bending of Section 5.2.
5.5 Parametric study of RHS strength equations using FEM

5.5.1 Details of parametric study

To conduct a parametric study for the strength of the RHS face in transverse tension component, the finite element model developed and validated for the hollow RHS in Section 3.3.2 is used. For the concrete-filled column model, a solid concrete infill with contact interactions is specified using parameters specified in Section 3.2. The column length modelled is 4m to ensure that the deformation mechanism can fully develop without the constraints of the column ends.

To determine the validity of strength equations and their range of applicability, it was necessary to design a suitable parametric study where all relevant variables are tested. These variables are those relating to geometry: RHS thickness ($t_w$), bolt hole separation to deformable yield width ratio ($a_t/a$), bolt pitch ($b_t$), RHS width ($w$); as well as material yield stress ($f_y$). Although it is ideal to test every combination of variables within realistic ranges of values, this is impractical due to the number of variables considered and subsequent computation time. The parametric study is reduced to the models given in Table 5.5 to provide enough variation in variable combinations to determine range of applicability issues whilst remaining practical. These parameters are investigated for both hollow and concrete-filled RHS columns. Model #1.1.10.2 represents the standard case with geometry that is typical of semi-rigid joints to be used in semi-continuous frame design. It also coincides with the geometry found in a few tests conducted by France et al. (1999) allowing for further comparison of results against experimental data as well as providing insight into any errors found in the joint assembly process of Chapter 7 which is also applied to this testing.
The naming convention for finite element models reflects the key geometric parameters. The first number reflects the horizontal bolt hole separation, \(a_t\); the second number reflects the vertical bolt hole separation, \(b_t\); the third number reflects the RHS thickness, \(t_c\); the fourth number reflects the RHS width, \(w_{RHS}\). Suffices denote variation in material or number of bolt rows.

The first set of tests looks at variation in \(t_c\); the second set, \(a_t\); the third set, \(b_t\); the fourth set, \(w_{RHS}\); and the fifth set, material yield stress. In each set of tests, useful low-end and high-end values of the variables centred on the base case are considered.

A fixed value of 210GPa is used for the Young’s Modulus as differences in the elastic behaviour of the material have no effect on the ultimate strength and deformation capacity. Details of the material properties are given in Table 5.6 and Figure 5.16.

Table 5.5 Summary of geometry and material variables in parametric study

<table>
<thead>
<tr>
<th>Model #</th>
<th>(a_t) (m)</th>
<th>(b_t) (m)</th>
<th>(t_c) (m)</th>
<th>(w_{RHS}) (m)</th>
<th>(d_{RHS}) (m)</th>
<th>(d_b) (m)</th>
<th>(c) (m)</th>
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<td>0.04</td>
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Table 5.6 Material properties used in parametric study

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<tr>
<th>Material</th>
<th>(f_y) (N/mm(^2))</th>
<th>(f_u) (N/mm(^2))</th>
<th>(E) (N/m(^2))</th>
<th>Notes</th>
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<td>450</td>
<td>210E9</td>
<td>Standard</td>
</tr>
<tr>
<td>B</td>
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</tr>
<tr>
<td>C</td>
<td>400</td>
<td>600</td>
<td>210E9</td>
<td>High strength</td>
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</table>
5.5.2 Summary of strength equations

The analytical equations used to define the different strength states are given in Table 5.7. The failure strength of the RHS face is dependent on thread stripping in bending \( (F_{ts,\text{red}}) \), punching shear \( (F_{\text{punch,nc}}) \), bolt punching shear \( (F_{ps}) \), bolts in tension \( (F_{Rd,2}) \), or membrane action \( (F_{RHS,\text{Mem.}}) \) and is given by the following equation:

\[
F_{\text{Fail}} = \text{Min}(F_{ts,\text{red}}, F_{\text{punch,nc}}, F_{ps}, F_{Rd,2}, F_{RHS,\text{Mem.}})
\]

[5.53]
Table 5.7. Summary of strength equations for RHS face in transverse tension

<table>
<thead>
<tr>
<th>Yield line mechanisms</th>
<th>Bolt group, Elliptical yield lines - Equation [5.18]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F_{G,E} = M_p \left( \frac{4a_t - 2d_b + 2\pi c - \pi d_b + 4b_t - n d_b + 2\pi d}{c} \right)$</td>
</tr>
<tr>
<td></td>
<td>Bolt rows, Elliptical yield lines - Equation [5.20]</td>
</tr>
<tr>
<td></td>
<td>$F_{R,E} = M_p \left( \frac{4a_t - 2d_b + 2\pi c - \pi d_b + 2\pi d}{c} \right) \times n$</td>
</tr>
<tr>
<td>where</td>
<td></td>
</tr>
<tr>
<td>$d = \sqrt{4\pi} (4a_t - 2d_b + 2\pi c - \pi d_b)$</td>
<td>Equation [5.19]</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Thread stripping capacity (in bending)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$F_{ts,red} = R \times F_{ts}$</td>
</tr>
<tr>
<td></td>
<td>where</td>
</tr>
<tr>
<td></td>
<td>$R = \tau \times t_c$, but $R \leq 1$</td>
</tr>
<tr>
<td></td>
<td>$\tau = 60$</td>
</tr>
<tr>
<td></td>
<td>$F_{ts} = 0.6f_{Cy}\pi d_b (t_c + 8\text{mm})$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Membrane action</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$F_{Mem,\text{Group}} = 2f_y t_w \delta_b \left( \frac{a_t - d_b}{d} + \frac{b_t - (n-1)d_b}{c} + \frac{2}{3} - \frac{d_b^3}{12L_{\text{Diag}}} \right)$</td>
</tr>
<tr>
<td></td>
<td>$F_{Mem,\text{Rows}} = 2f_y t_w \delta_b \left( \frac{a_t - d_b}{d} + \frac{2}{3} - \frac{d_b^3}{12L_{\text{Diag}}} \right) \times n$</td>
</tr>
<tr>
<td></td>
<td>$F_{\text{HollowRHS,Mem}} = 0.5 \times F_B + 0.5 \times \text{Min} \left[ F_{\text{Mem,Group}}, F_{\text{Mem,Rows}} \right]$</td>
</tr>
<tr>
<td></td>
<td>$F_{\text{FilledRHS,Mem}} = \text{Min} \left[ F_{\text{Mem,Group}}, F_{\text{Mem,Rows}} \right]$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Bolt group punching shear capacity</td>
<td>$F_{punch,nc} = 2(b + c) v_{pl,rd}$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Bolt punching shear capacity</td>
<td>$F_{ps} = 0.6f_y \pi t_c (d_b + t_c)$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Bolt tensile capacity</td>
<td>$F_{Rd,2} = k_2 f_{ub} A_S / \gamma_{M2}$</td>
</tr>
<tr>
<td></td>
<td>where $k_2 = 0.9$ and $\gamma_{M2} = 1.25.$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Failure strength</td>
<td>$F_{fail} = \text{Min} \left( F_{ts,red}, F_{punch,nc}, F_{ps}, F_{Rd,2}, F_{\text{Mem}} \right)$</td>
</tr>
</tbody>
</table>

5.5.3 Results for 3% $w_{RHS}$ limit capacity

A comparison of results for the capacity of the hollow RHS at the 3% $w_{RHS}$ serviceability limit, $F_3$ (explained in Chapter 6), using different definitions of $a$ is given in Table 5.8. As explained in Section 6.2, $F_3$ is determined by interpolation from elliptical yield line bending and membrane action states and is given by Equation [6.4]. The definition of $a$, discussed in Section 4.2, is the characterisation of the deformable width of the RHS face. For a standard plate, the hinges form at the edges. For the RHS face however, the curved edges provide an increased level of support so it is debatable whether the hinges form at the far edge ($a=w_{RHS-t_c}$), at the edge of the clear face...
or at an intermediate location. Studies such as British Steel (1996b) and Ghobarah et al. (1999) indicate that a value of \( a = w_{RHS} \cdot t_c \) is appropriate. Results from the parametric study suggest this formulation consistently underestimates the strength with a mean error of -25.4\%. Moreover, it does not reflect the yield line patterns obtained in the FEM as given in Appendix B.2. Instead, \( a = w_{RHS} \cdot 4t_c \), as used in the calculation of initial stiffness characteristics, is used because it better reflects the yield lines pattern obtained using FEM. Using this value gives the best predictions with a mean error of -10.7\%. Some variability in results is attributed to the fact that the prediction of \( F_3 \) is determined by interpolation from bending and membrane action states and therefore any errors in prediction can compound or cancel out. Gomes et al. (1996) assigns a value of \( a = w_{RHS} \cdot 2t_c \cdot 1.5r \), which for the typical values of \( r = 1.5t_c \) and \( r = 2t_c \), gives \( a = w_{RHS} \cdot 4.25t_c \) and \( w_{RHS} \cdot 5t_c \) respectively. Because yield lines must form at the edge of the plate in consideration, it is invalid to use a value smaller than the clear face width \( (w_{RHS} \cdot 4t_c \) for common sections where \( r_o \leq 2t_c \) as discussed in Section 3.2.6). Therefore, values of \( a \) in this range are not considered even though using a smaller value of \( a \) would increase the predicted strength to give better predictions.

A detailed summary of yield strength when using the elliptical yield line equation with \( a = w_{RHS} \cdot 4t_c \) is given in Table 5.9. It can be seen that all simulations excluding #1.20.1.2 yield due to the bolt group mechanism. #1.20.1.2 yields due to the bolt row mechanism that is explained by the fact that the vertical bolt spacing, \( b_v \), is disproportionately large. This is observed in the yield line plots given in Appendix B.2. Another simulation, #1.15.1.2 yields due to the bolt group mechanism. However, it can be seen that the bolt group yield lines are not as pronounced as with other geometries. This is attributed to the fact that bolt group and individual bolt row mechanisms yield at similar capacities (194.2kN and 201.7kN) therefore the
characteristics of the horizontal yield lines, as is dominant for bolt row yield lines, are more pronounced in comparison to the vertical yield lines.

A comparison of the 3\% \( w_{\text{RHS}} \) limit strength of the hollow section obtained from using alternative equations by Gomes et al. (1996) as used in Weynand et al. (2003), Ghobarah et al. (1996), and British Steel (1996b) is given in Table 5.10. As explained in Section 2.4.3, Ghobarah et al. considers the minimum of straight and circular yield lines whereas British Steel considers only the straight yield line effectively making it a comparison of circular and straight yield line equations. With all methods other than the one developed in this research, the effect of membrane action is not considered. For the equations that only consider the yield line bending mechanism, the British Steel equations using \( a = w_{\text{RHS}} - t_c \) gives the best agreement. However, the new method using Equation [6.4] that considers bending and membrane action effects using \( a = w_{\text{RHS}} - 4t_c \) is superior.

A comparison of the 3\% \( w_{\text{RHS}} \) limit strength for the concrete-filled RHS is given in Table 5.11. Results indicate that the new method gives good agreement with results with a mean error of -4.6\% and maximum of 17.7\%. Again, the comparison of yield line plots from the FEM given in Appendix B.3 shows that the assumed yield line mechanisms reflects the FEM behaviour consistently.

A comparison of the load-deflection behaviour for the analytical approach (using equations in Section 6.2) and FEA for individual tests is given in Appendix A.3 for the hollow RHS and in A.4 for the concrete-filled RHS.
Table 5.8. Analytical and FEM 3% $w_{RHS}$ limit strength for different values of $a$ using elliptical pattern (hollow RHS)

<table>
<thead>
<tr>
<th>Model #</th>
<th>FEM (kN)</th>
<th>$F_3$ - Eqn. [6.4] $(a=w_{RHS}-t_c)$ Error</th>
<th>$F_3$ - Eqn. [6.4] $(a=w_{RHS}-2t_c)$ Error</th>
<th>$F_3$ - Eqn. [6.4] $(a=w_{RHS}-4t_c)$ Error</th>
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</thead>
<tbody>
<tr>
<td>1.1.5.2</td>
<td>59.5</td>
<td>53.9 -9.4%</td>
<td>55.4 -6.8%</td>
<td>59.0 -0.9%</td>
</tr>
<tr>
<td>1.1.8.2</td>
<td>160.4</td>
<td>124.6 -22.3%</td>
<td>130.8 -18.4%</td>
<td>146.1 -8.9%</td>
</tr>
<tr>
<td>1.1.10.2</td>
<td>262.4</td>
<td>185.2 -29.4%</td>
<td>197.3 -24.8%</td>
<td>228.0 -13.1%</td>
</tr>
<tr>
<td>1.1.125.2</td>
<td>417.4</td>
<td>276.8 -33.7%</td>
<td>300.6 -28.0%</td>
<td>367.3 -12.0%</td>
</tr>
<tr>
<td>1.1.16.2</td>
<td>846.7</td>
<td>436.2 -35.9%</td>
<td>485.6 -28.6%</td>
<td>668.6 -1.8%</td>
</tr>
<tr>
<td>6.1.1.2</td>
<td>181.1</td>
<td>135.1 -25.4%</td>
<td>138.9 -23.3%</td>
<td>148.7 -17.9%</td>
</tr>
<tr>
<td>8.1.1.2</td>
<td>214.5</td>
<td>156.7 -26.9%</td>
<td>164.3 -23.4%</td>
<td>181.2 -15.5%</td>
</tr>
<tr>
<td>12.1.1.2</td>
<td>332.9</td>
<td>226.0 -32.1%</td>
<td>247.1 -25.8%</td>
<td>304.5 -8.5%</td>
</tr>
<tr>
<td>1.1.5.2</td>
<td>34.5</td>
<td>46.4 34.5 Group</td>
<td>46.4 34.5 Group</td>
<td>100.9 40.9 Group</td>
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<tr>
<td>1.1.8.2</td>
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<td>124.4 96.1 Group</td>
<td>124.4 96.1 Group</td>
<td>124.4 96.1 Group</td>
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<td>201.7 160.9 Group</td>
<td>201.7 160.9 Group</td>
<td>124.4 96.1 Group</td>
</tr>
<tr>
<td>1.1.125.2</td>
<td>279.5</td>
<td>333.9 279.5 Group</td>
<td>333.9 279.5 Group</td>
<td>124.4 96.1 Group</td>
</tr>
<tr>
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<td>562.4</td>
<td>612.7 562.4 Group</td>
<td>612.7 562.4 Group</td>
<td>124.4 96.1 Group</td>
</tr>
<tr>
<td>6.1.1.2</td>
<td>111.3</td>
<td>150.6 111.3 Group</td>
<td>150.6 111.3 Group</td>
<td>124.4 96.1 Group</td>
</tr>
<tr>
<td>8.1.1.2</td>
<td>130.8</td>
<td>171.6 130.8 Group</td>
<td>171.6 130.8 Group</td>
<td>124.4 96.1 Group</td>
</tr>
<tr>
<td>12.1.1.2</td>
<td>215.7</td>
<td>251.4 215.7 Group</td>
<td>251.4 215.7 Group</td>
<td>124.4 96.1 Group</td>
</tr>
<tr>
<td>1.1.5.2</td>
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<td>201.7 127.5 Group</td>
<td>201.7 127.5 Group</td>
<td>124.4 96.1 Group</td>
</tr>
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<td>1.1.1.2</td>
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<td>201.7 194.2 Group</td>
<td>201.7 194.2 Group</td>
<td>124.4 96.1 Group</td>
</tr>
<tr>
<td>1.20.1.2</td>
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<td>201.7 201.7 Group</td>
<td>201.7 201.7 Group</td>
<td>124.4 96.1 Group</td>
</tr>
<tr>
<td>1.1.1.3</td>
<td>101.8</td>
<td>158.5 101.8 Group</td>
<td>158.5 101.8 Group</td>
<td>124.4 96.1 Group</td>
</tr>
<tr>
<td>1.1.1.4</td>
<td>87.2</td>
<td>146.8 87.2 Group</td>
<td>146.8 87.2 Group</td>
<td>124.4 96.1 Group</td>
</tr>
<tr>
<td>1.1.1.2B</td>
<td>107.2</td>
<td>134.5 107.2 Group</td>
<td>134.5 107.2 Group</td>
<td>124.4 96.1 Group</td>
</tr>
<tr>
<td>1.1.1.2C</td>
<td>214.5</td>
<td>268.9 214.5 Group</td>
<td>268.9 214.5 Group</td>
<td>124.4 96.1 Group</td>
</tr>
<tr>
<td>1.0.1.2R1</td>
<td>100.9</td>
<td>100.9 100.9 Group</td>
<td>100.9 100.9 Group</td>
<td>124.4 96.1 Group</td>
</tr>
<tr>
<td>1.3.1.2R4</td>
<td>280.9</td>
<td>403.4 280.9 Group</td>
<td>403.4 280.9 Group</td>
<td>124.4 96.1 Group</td>
</tr>
</tbody>
</table>

Table 5.9. Analytical and FEM yield strength using elliptical yield lines when $a=w_{RHS}-4t_c$ (Hollow RHS)

<table>
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<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
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<td>46.4</td>
<td>34.5</td>
<td>Group</td>
</tr>
<tr>
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<td>96.1</td>
<td>124.4</td>
<td>96.1</td>
<td>Group</td>
</tr>
<tr>
<td>1.1.10.2</td>
<td>160.9</td>
<td>201.7</td>
<td>160.9</td>
<td>Group</td>
</tr>
<tr>
<td>1.1.125.2</td>
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<td>333.9</td>
<td>279.5</td>
<td>Group</td>
</tr>
<tr>
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<td>612.7</td>
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<td>Group</td>
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<td>150.6</td>
<td>111.3</td>
<td>Group</td>
</tr>
<tr>
<td>8.1.1.2</td>
<td>130.8</td>
<td>171.6</td>
<td>130.8</td>
<td>Group</td>
</tr>
<tr>
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<td>251.4</td>
<td>215.7</td>
<td>Group</td>
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<td>1.5.1.2</td>
<td>127.5</td>
<td>201.7</td>
<td>127.5</td>
<td>Group</td>
</tr>
<tr>
<td>1.1.5.2</td>
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<td>158.5</td>
<td>101.8</td>
<td>Group</td>
</tr>
<tr>
<td>1.1.1.4</td>
<td>87.2</td>
<td>146.8</td>
<td>87.2</td>
<td>Group</td>
</tr>
<tr>
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<td>107.2</td>
<td>134.5</td>
<td>107.2</td>
<td>Group</td>
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<td>Group</td>
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<td>280.9</td>
<td>Group</td>
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</table>
Table 5.10 Analytical and FEM 3% \(w_{\text{RHS}}\) limit strength using alternative equations (hollow RHS)

<table>
<thead>
<tr>
<th>Model #</th>
<th>FEM (kN)</th>
<th>Gomes ([2.9])</th>
<th>Error</th>
<th>Ghobarah ([2.27])</th>
<th>Error</th>
<th>British Steel ([2.24])</th>
<th>Error</th>
<th>New method ([6.4])</th>
<th>Error</th>
</tr>
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<tr>
<td>1.1.5.2</td>
<td>59.5</td>
<td>61.4</td>
<td>3.2%</td>
<td>49.5</td>
<td>-16.8%</td>
<td>53.3</td>
<td>-10.3%</td>
<td>59.0</td>
<td>-0.9%</td>
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<tr>
<td>1.1.8.2</td>
<td>160.4</td>
<td>178.9</td>
<td>11.5%</td>
<td>128.9</td>
<td>-19.6%</td>
<td>138.4</td>
<td>-13.7%</td>
<td>146.1</td>
<td>-8.9%</td>
</tr>
<tr>
<td>1.1.10.2</td>
<td>262.4</td>
<td>317.1</td>
<td>20.8%</td>
<td>203.8</td>
<td>-22.3%</td>
<td>218.3</td>
<td>-16.8%</td>
<td>228.0</td>
<td>-13.1%</td>
</tr>
<tr>
<td>1.1.125.2</td>
<td>417.4</td>
<td>323.3</td>
<td>*</td>
<td>345.1</td>
<td>-17.3%</td>
<td>367.3</td>
<td>-12.0%</td>
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<td></td>
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<td>575.5</td>
<td>-15.4%</td>
<td>618.6</td>
<td>-9.1%</td>
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<td>151.6</td>
<td>-16.3%</td>
<td>174.1</td>
<td>-3.8%</td>
<td>183.2</td>
<td>-7.9%</td>
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<td>192.8</td>
<td>-10.1%</td>
<td>202.0</td>
<td>-15.5%</td>
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<td>332.9</td>
<td>252.2</td>
<td>-24.2%</td>
<td>285.6</td>
<td>-23.2%</td>
<td>304.5</td>
<td>-10.7%</td>
<td>313.3</td>
<td>-12.3%</td>
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<td>170.4</td>
<td>-22.0%</td>
<td>184.9</td>
<td>-15.3%</td>
<td>199.1</td>
<td>-19.8%</td>
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<td>9.9%</td>
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<td>-17.8%</td>
<td>251.6</td>
<td>-12.8%</td>
<td>265.7</td>
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<td>-7.4%</td>
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<td>146.1</td>
<td>-8.3%</td>
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<td>175.6</td>
<td>5.5%</td>
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<td>15.5%</td>
<td>128.2</td>
<td>-1.2%</td>
<td>152.1</td>
<td>17.2%</td>
<td>160.0</td>
<td>14.2%</td>
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<td>211.4</td>
<td>16.7%</td>
<td>135.9</td>
<td>-25.0%</td>
<td>145.5</td>
<td>-19.7%</td>
<td>153.7</td>
<td>-15.1%</td>
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<tr>
<td>1.1.1.2C</td>
<td>334.7</td>
<td>422.8</td>
<td>26.3%</td>
<td>271.7</td>
<td>-18.8%</td>
<td>291.0</td>
<td>13.0%</td>
<td>300.4</td>
<td>13.0%</td>
</tr>
<tr>
<td>1.0.1.2R1</td>
<td>162.7</td>
<td>158.5</td>
<td>-2.5%</td>
<td>137.1</td>
<td>-15.7%</td>
<td>151.6</td>
<td>-6.8%</td>
<td>164.4</td>
<td>-7.4%</td>
</tr>
<tr>
<td>1.3.1.2R4</td>
<td>450.7</td>
<td>634.1</td>
<td>40.7%</td>
<td>337.1</td>
<td>-25.2%</td>
<td>351.6</td>
<td>-22.0%</td>
<td>405.6</td>
<td>-10.0%</td>
</tr>
</tbody>
</table>

| Mean     | 16.4%   | Mean          | -18.1% | Mean          | -10.9% | Mean          | -10.7% |               |        |
| St.dev.  | 13.5%   | St.dev        | 6.3%   | St.dev        | 10.1%  | St.dev        | 5.9%   |               |        |
| Max      | 45.2%   | Max           | 25.2%  | Max           | 23.2%  | Max           | 19.8%  |               |        |

*Outside range of validity constraint \(b/L < 0.8\)

Table 5.11 Analytical vs. FEM 3% \(w_{\text{RHS}}\) limit strength for concrete-filled RHS

<table>
<thead>
<tr>
<th>Test #</th>
<th>FEM (kN)</th>
<th>(F_3) - Eqn. ([6.4]) (kN)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1.5.2</td>
<td>90.7</td>
<td>91.8</td>
<td>1.3%</td>
</tr>
<tr>
<td>1.1.8.2</td>
<td>217.3</td>
<td>212.3</td>
<td>-2.3%</td>
</tr>
<tr>
<td>1.1.10.2</td>
<td>347.4</td>
<td>318.0</td>
<td>-8.5%</td>
</tr>
<tr>
<td>1.1.125.2</td>
<td>535.5</td>
<td>486.6</td>
<td>-9.1%</td>
</tr>
<tr>
<td>1.1.16.2</td>
<td>828.9</td>
<td>809.2</td>
<td>-2.4%</td>
</tr>
<tr>
<td>6.1.1.2</td>
<td>236.6</td>
<td>208.7</td>
<td>-11.8%</td>
</tr>
<tr>
<td>8.1.1.2</td>
<td>284.1</td>
<td>253.3</td>
<td>-10.8%</td>
</tr>
<tr>
<td>12.1.1.2</td>
<td>436.2</td>
<td>422.5</td>
<td>-3.1%</td>
</tr>
<tr>
<td>1.5.1.2</td>
<td>283.7</td>
<td>242.2</td>
<td>-14.6%</td>
</tr>
<tr>
<td>1.15.1.2</td>
<td>372.7</td>
<td>391.6</td>
<td>5.1%</td>
</tr>
<tr>
<td>1.20.1.2</td>
<td>381.3</td>
<td>448.7</td>
<td>17.7%</td>
</tr>
<tr>
<td>1.1.1.3</td>
<td>209.9</td>
<td>212.4</td>
<td>1.2%</td>
</tr>
<tr>
<td>1.1.1.4</td>
<td>178.8</td>
<td>180.8</td>
<td>1.1%</td>
</tr>
<tr>
<td>1.1.1.2B</td>
<td>242.5</td>
<td>215.9</td>
<td>-11.0%</td>
</tr>
<tr>
<td>1.1.1.2C</td>
<td>450.2</td>
<td>416.0</td>
<td>-7.6%</td>
</tr>
<tr>
<td>1.0.1.2R1</td>
<td>193.1</td>
<td>164.3</td>
<td>-14.9%</td>
</tr>
<tr>
<td>1.3.1.2R4</td>
<td>655.1</td>
<td>598.9</td>
<td>-8.6%</td>
</tr>
</tbody>
</table>

| Mean     | -4.6%   | St.dev.                     | 8.3%   |
| Max      | 17.7%   |                           |        |
5.5.4 Results for failure strength

A comparison of analytical and FEA hollow RHS failure strength is given in Table 5.12. Results show that the analytical approach is suitable for predicting the failure strength with mean error of -5.3% and maximum error of 18.2%. The analytical approach does not show trends in errors associated with geometry and therefore rules on the range of validity are not applied. All sections showed either thread stripping due to bending or membrane action failure (combined bending and membrane action given by Equation [5.51]).

A comparison for the failure strength for concrete-filled RHS sections is given in Table 5.13 showing that the analytical approach of estimating failure strength is consistent with FEM results with a mean error of -2.3% and maximum of 20.2%. Owing to the pure membrane action strength (Equation [5.52]) achieved for the filled RHS, all sections fail due to thread stripping.

In both hollow and concrete-filled sections, some variability in test results can be expected due to variations in column face thickness and imperfections which can strongly affect the formation of a fracture (failure) mechanism.
Table 5.12 Analytical and FEM failure strength comparison for hollow RHS

<table>
<thead>
<tr>
<th>Model #</th>
<th>$F_{p,s}$</th>
<th>$F_{s,red}$</th>
<th>$F_{Mem.}$</th>
<th>FEM</th>
<th>Failure mode</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[kN]</td>
<td>[kN]</td>
<td>[kN]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1.5.2</td>
<td>346.4</td>
<td>176.4</td>
<td>188.2</td>
<td>176.4</td>
<td>T.Strip</td>
<td>149.3</td>
</tr>
<tr>
<td>1.1.8.2</td>
<td>554.3</td>
<td>347.4</td>
<td>316.5</td>
<td>316.5</td>
<td>Mem.</td>
<td>337.5</td>
</tr>
<tr>
<td>1.1.10.2</td>
<td>692.8</td>
<td>488.6</td>
<td>408.2</td>
<td>408.2</td>
<td>Mem.</td>
<td>465.6</td>
</tr>
<tr>
<td>1.1.125.2</td>
<td>866.0</td>
<td>695.5</td>
<td>538.6</td>
<td>538.6</td>
<td>Mem.</td>
<td>627.5</td>
</tr>
<tr>
<td>1.1.16.2</td>
<td>1108.5</td>
<td>1042.3</td>
<td>780.5</td>
<td>780.5</td>
<td>Mem.</td>
<td>847.0</td>
</tr>
<tr>
<td>6.1.1.2</td>
<td>554.3</td>
<td>488.6</td>
<td>350.4</td>
<td>350.4</td>
<td>Mem.</td>
<td>335.8</td>
</tr>
<tr>
<td>8.1.1.2</td>
<td>623.5</td>
<td>488.6</td>
<td>384.4</td>
<td>384.4</td>
<td>Mem.</td>
<td>443.1</td>
</tr>
<tr>
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<td>762.1</td>
<td>488.6</td>
<td>426.9</td>
<td>426.9</td>
<td>Mem.</td>
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<td>488.6</td>
<td>299.1</td>
<td>299.1</td>
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<td>479.8</td>
</tr>
<tr>
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<td>488.6</td>
<td>433.2</td>
<td>433.2</td>
<td>Mem.</td>
<td>486.6</td>
</tr>
<tr>
<td>1.1.1.3</td>
<td>692.8</td>
<td>488.6</td>
<td>471.6</td>
<td>471.6</td>
<td>Mem.</td>
<td>449.3</td>
</tr>
<tr>
<td>1.1.1.4</td>
<td>692.8</td>
<td>488.6</td>
<td>536.3</td>
<td>536.3</td>
<td>Mem.</td>
<td>486.6</td>
</tr>
<tr>
<td>1.11.1.2B</td>
<td>461.9</td>
<td>325.7</td>
<td>272.1</td>
<td>272.1</td>
<td>Mem.</td>
<td>324.7</td>
</tr>
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<td>544.3</td>
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<td>Mem.</td>
<td>583.9</td>
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<td>219.4</td>
<td>Mem.</td>
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<td>977.2</td>
<td>785.7</td>
<td>785.7</td>
<td>Mem.</td>
<td>799.7</td>
</tr>
</tbody>
</table>

| Mean        | -5.3%     |
| St.dev      | 10.2%     |
| Max         | 18.2%     |

Table 5.13 Analytical and FEM failure strength for concrete-filled RHS

<table>
<thead>
<tr>
<th>Model #</th>
<th>$F_{p,s}$</th>
<th>$F_{s,red}$</th>
<th>$F_{Mem.}$</th>
<th>FEM</th>
<th>Failure mode</th>
<th>Error</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>[kN]</td>
<td>[kN]</td>
<td>[kN]</td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>1.1.5.2</td>
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<td>176.4</td>
<td>392.8</td>
<td>176.4</td>
<td>T.Strip</td>
<td>209.2</td>
</tr>
<tr>
<td>1.1.8.2</td>
<td>554.3</td>
<td>347.4</td>
<td>606.4</td>
<td>347.4</td>
<td>T.Strip</td>
<td>383.4</td>
</tr>
<tr>
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<td>692.8</td>
<td>488.6</td>
<td>738.1</td>
<td>488.6</td>
<td>T.Strip</td>
<td>517.1</td>
</tr>
<tr>
<td>1.1.125.2</td>
<td>866.0</td>
<td>695.5</td>
<td>888.9</td>
<td>695.5</td>
<td>T.Strip</td>
<td>676.6</td>
</tr>
<tr>
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<td>1042.3</td>
<td>1068.0</td>
<td>1068.0</td>
<td>T.Strip</td>
<td>866.9</td>
</tr>
<tr>
<td>6.1.1.2</td>
<td>554.3</td>
<td>488.6</td>
<td>732.5</td>
<td>488.6</td>
<td>T.Strip</td>
<td>470.2</td>
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<td>744.8</td>
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<td>488.6</td>
<td>T.Strip</td>
<td>540.3</td>
</tr>
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<td>488.6</td>
<td>539.1</td>
<td>488.6</td>
<td>T.Strip</td>
<td>456.8</td>
</tr>
<tr>
<td>1.15.1.2</td>
<td>866.0</td>
<td>488.6</td>
<td>937.1</td>
<td>488.6</td>
<td>T.Strip</td>
<td>525.4</td>
</tr>
<tr>
<td>1.20.1.2</td>
<td>1039.2</td>
<td>488.6</td>
<td>1136.1</td>
<td>488.6</td>
<td>T.Strip</td>
<td>527.5</td>
</tr>
<tr>
<td>1.1.1.3</td>
<td>692.8</td>
<td>488.6</td>
<td>935.2</td>
<td>488.6</td>
<td>T.Strip</td>
<td>476.0</td>
</tr>
<tr>
<td>1.1.1.4</td>
<td>692.8</td>
<td>488.6</td>
<td>1091.0</td>
<td>488.6</td>
<td>T.Strip</td>
<td>457.4</td>
</tr>
<tr>
<td>1.1.1.2B</td>
<td>461.9</td>
<td>325.7</td>
<td>492.1</td>
<td>325.7</td>
<td>T.Strip</td>
<td>376.9</td>
</tr>
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<td>923.8</td>
<td>651.4</td>
<td>984.2</td>
<td>651.4</td>
<td>T.Strip</td>
<td>647.8</td>
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<tr>
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<td>244.3</td>
<td>340.1</td>
<td>244.3</td>
<td>T.Strip</td>
<td>268.2</td>
</tr>
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<td>977.2</td>
<td>1534.1</td>
<td>977.2</td>
<td>T.Strip</td>
<td>1011.8</td>
</tr>
</tbody>
</table>

| Mean        | -2.3%     |
| St.dev      | 9.0%      |
| Max         | 20.2%     |
5.6 Validation of ultimate strength equations British Steel (1996b) testing

The testing conducted by British Steel (1996b) as described in Section 3.1.2 is used to determine the validity of ultimate strength equations proposed in Section 5.2. In doing so, it is possible to reinforce the findings of the parametric study of Section 5.5.

A comparison of analytical and testing strength values for the 3% $w_{RHS}$ deflection limit from British Steel (1996b) is given in Table 5.15. The analytical strength obtained when using bending strength equations by Ghobarah et al., British Steel and Jaspart et al. are included for comparison. From these results, the same conclusion as for the parametric study is drawn in that the new elliptical yield line equations with the contribution of membrane action using $a=w_{RHS}-4t_c$ gives the most consistent and accurate predictions of the 3% RHS width limit strength of the hollow RHS. There is a mean error of -21.7% and maximum error of 36.8%. The Gomes et al. (1996) equation gives a marginally better mean and maximum error, however, there are too many cases where the range of validity prevents its usage for it to be a practical method. A detailed comparison when using Gomes et al. equations is given in Table 5.16. This shows that when ignoring the rules for range of validity, the error in some cases is significant. In one case, usage of the equations returns a numerical error due to the width of the tension zone being larger than the deformable width (defined by Gomes as $a=w_{RHS}-2t_c+1.5r$ or assuming $r=2t_c$, $a=w_{RHS}-5t_c$).

In general, all approaches underestimate the 3% $w_{RHS}$ limit strength of the British Steel tests by a noticeable amount. This is believed to be due to either lack of fit of components or due to the effects of bolt pretension. As mentioned earlier, in the British Steel tests, bolt pretension is applied using the torque method which can lead to
variations in the actual pretension applied. If the pretension is very high, then it may change the load transfer mechanism of components, for example, on the way the bolt loads are transferred to the RHS face. This may explain why the elastic and early plastic behaviour deviates from the analytical solutions but not the membrane action and failure behaviour for which bolt pretensioning does not affect.

A comparison of the load-deflection behaviour for the analytical approach (using equations in Section 6.2) and FEM for individual tests are given in Appendix A.1. A comparison of the failure strength is given in Section 5.3.2.

Table 5.14 Analytical and testing 3% \( w_{\text{RHS}} \) limit strength, \( F_3 \), comparison for 4 bolt group

<table>
<thead>
<tr>
<th>Model #</th>
<th>( F_3 ) based on ( F_{\text{GE}}/{5.18} ) (kN)</th>
<th>( F_3 ) based on ( F_{\text{RE}}/{5.20} ) (kN)</th>
<th>Failure Mode</th>
<th>Testing (kN)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T/30/27/20</td>
<td>40.2</td>
<td>47.2</td>
<td>Group</td>
<td>52.3</td>
<td>-23.2%</td>
</tr>
<tr>
<td>T/30/50/20</td>
<td>60.4</td>
<td>67.0</td>
<td>Group</td>
<td>83.6</td>
<td>-27.8%</td>
</tr>
<tr>
<td>T/30/67/20</td>
<td>102.8</td>
<td>108.5</td>
<td>Group</td>
<td>148.0</td>
<td>-30.6%</td>
</tr>
<tr>
<td>T/24/27/20</td>
<td>65.8</td>
<td>75.9</td>
<td>Group</td>
<td>88.8</td>
<td>-25.9%</td>
</tr>
<tr>
<td>T/24/50/20</td>
<td>102.1</td>
<td>110.6</td>
<td>Group</td>
<td>139.0</td>
<td>-26.6%</td>
</tr>
<tr>
<td>T/24/67/20</td>
<td>186.9</td>
<td>191.7</td>
<td>Group</td>
<td>233.0</td>
<td>-19.8%</td>
</tr>
<tr>
<td>T/19/27/20</td>
<td>97.9</td>
<td>109.2</td>
<td>Group</td>
<td>155.0</td>
<td>-36.8%</td>
</tr>
<tr>
<td>T/19/50/20</td>
<td>161.4</td>
<td>167.5</td>
<td>Group</td>
<td>229.0</td>
<td>-29.5%</td>
</tr>
<tr>
<td>T/15/27/20</td>
<td>178.6</td>
<td>191.2</td>
<td>Group</td>
<td>234.0</td>
<td>-23.7%</td>
</tr>
<tr>
<td>T/15/50/20</td>
<td>319.9</td>
<td>315.1</td>
<td>Group</td>
<td>367.0</td>
<td>-14.1%</td>
</tr>
<tr>
<td>T/12/27/20</td>
<td>282.5</td>
<td>285.5</td>
<td>Group</td>
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<td>-20.4%</td>
</tr>
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<td>585.6</td>
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<td>0.5%</td>
</tr>
</tbody>
</table>

Mean: -23.2%
St.dev: 9.4%
Max: 36.8%
Table 5.15 Analytical and testing 3% wRHS limit strength, F₃, comparison for 4 bolt group

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>T/30/27/20</td>
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<td>43.8 -16.3%</td>
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<td></td>
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<tr>
<td>T/30/50/20</td>
<td>83.6</td>
<td>65.7 -21.4%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T/30/67/20</td>
<td>148</td>
<td>*</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>T/24/27/20</td>
<td>233</td>
<td>*</td>
<td></td>
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</tr>
<tr>
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<td>*</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>T/19/27/20</td>
<td>155</td>
<td>108.3 -30.1%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T/19/50/20</td>
<td>229</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T/15/27/20</td>
<td>234</td>
<td>201.7 -13.8%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T/15/50/20</td>
<td>367</td>
<td>*</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T/12/27/20</td>
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<td>335.7 -5.4%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T/12/50/20</td>
<td>532</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mean: -17.6%  St.dev: 7.5%  Max: 30.1%

Table 5.16 Detailed comparison of 3% wRHS limit strength, F₃, of British Steel (1996b) tests using Gomes et al. (1996) equations

<table>
<thead>
<tr>
<th>Test #</th>
<th>Check (b+c)/L&lt;0.8</th>
<th>Gomes equations [2.9] (within range)</th>
<th>Error</th>
<th>Gomes equations [2.9] (ignoring range)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>T/30/27/20</td>
<td>Yes</td>
<td>43.8</td>
<td>-16.3%</td>
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<td>-16.3%</td>
</tr>
<tr>
<td>T/30/50/20</td>
<td>Yes</td>
<td>65.7</td>
<td>-24.1%</td>
<td>65.7</td>
<td>-24.1%</td>
</tr>
<tr>
<td>T/30/67/20</td>
<td>No</td>
<td>72.0</td>
<td>-19.0%</td>
<td>72.0</td>
<td>-19.0%</td>
</tr>
<tr>
<td>T/24/27/20</td>
<td>Yes</td>
<td>114.9</td>
<td>-17.3%</td>
<td>114.9</td>
<td>-17.3%</td>
</tr>
<tr>
<td>T/24/50/20</td>
<td>No</td>
<td>229.1</td>
<td>-41.0%</td>
<td>229.1</td>
<td>-41.0%</td>
</tr>
<tr>
<td>T/24/67/20</td>
<td>No</td>
<td>342.7</td>
<td>-39.0%</td>
<td>342.7</td>
<td>-39.0%</td>
</tr>
<tr>
<td>T/19/27/20</td>
<td>Yes</td>
<td>108.3</td>
<td>-30.1%</td>
<td>108.3</td>
<td>-30.1%</td>
</tr>
<tr>
<td>T/19/50/20</td>
<td>No</td>
<td>105.8</td>
<td>-30.1%</td>
<td>105.8</td>
<td>-30.1%</td>
</tr>
<tr>
<td>T/15/27/20</td>
<td>Yes</td>
<td>201.7</td>
<td>-13.8%</td>
<td>201.7</td>
<td>-13.8%</td>
</tr>
<tr>
<td>T/15/50/20</td>
<td>No</td>
<td>335.7</td>
<td>-5.4%</td>
<td>335.7</td>
<td>-5.4%</td>
</tr>
<tr>
<td>T/12/27/20</td>
<td>Yes</td>
<td>335.7</td>
<td>-5.4%</td>
<td>335.7</td>
<td>-5.4%</td>
</tr>
<tr>
<td>T/12/50/20</td>
<td>No</td>
<td>342.7</td>
<td>-39.0%</td>
<td>342.7</td>
<td>-39.0%</td>
</tr>
</tbody>
</table>

Mean: -17.6%  St.dev: 7.5%  Max: 30.1%

* Returns numerical error

5.7 Conclusions

In this chapter, analytical equations to define the bending and membrane action strength of the RHS face in tension component are presented. Bending strength equations are derived using yield line theory for a new elliptical yield line mechanism.
that gives the minimum solution for all geometries with up to 8% improvement over existing straight and circular yield line equations given by Ghobarah et al. (1996) and British Steel (1996b). A different definition of the deformable width of the RHS face compared with these studies is used to reflect on actual hinge locations. The equations are consistent when compared against a parametric study using FEM for both hollow and filled sections with mean errors of approx. 5% and maximum of approx. 20%. A comparison of the yield line plots obtained in the parametric study show that the elliptical yield line pattern replicates the yield line mechanism accurately and that the revised definition of the deformable width of the RHS face should be used. The equations are validated against existing results from testing (British Steel, 1996) for the hollow section showing reasonable agreement.

The membrane action strength is derived using internal work principles and assumes that membrane action deformation can be determined from a simple relationship between cross section geometry and material failure strain. The contribution of membrane action to the failure mechanism for hollow sections is assumed to be half bending and half membrane action owing to the inward deflection of RHS sidewalls. For concrete-filled sections, the failure mechanism is assumed to be due to pure membrane action because of the restraining effect of the concrete infill that allows full membrane action to develop.

In addition, equations for the thread stripping capacity of flowdrilled connections due to gross deformation of the RHS face are newly derived. This is important because the contact between the threads of the bolt and RHS face reduces in proportion to deformation of the RHS face thus causing thread stripping at a much lower load than under full contact conditions. The reduction in contact is related to the RHS thickness and is given in the form of a reduction factor to be applied to existing equations for the full thread stripping capacity. A validation study against British Steel
(1996b) tests shows that the thread stripping capacity is predicted consistently across a wide range of geometries with a mean error of 2% and maximum of approximately 25%.
Chapter 6

RHS Face Deformation Capacity

6.1 Introduction

It is necessary to develop an understanding of the deformation capacity of the RHS face in transverse tension to (1) check that the characteristic bending strength is attained within a practical level of joint deformation, (2) ensure that the combined joint behaviour gives a ductile joint, and (3) to estimate the deformation capacity characteristics of the RHS face at failure due to membrane action behaviour to predict structural robustness.

As mentioned in Section 3.1.2, the only tests in literature covering the isolated load-deformation behaviour of the RHS face in transverse tension for flowdrilled connections is that by British Steel (1996b). These tests cover a variety of arrangements including a 2 bolt row group in tension. To expand on the understanding of this component, it was necessary to investigate the deformation capacity through a series of parametric tests using Abaqus FEA. The parametric study used in Chapter 5 for strength using techniques validated in Section 3.3.2 is used for this purpose.
6.2 Deformation capacity characteristics

Due to the influence of membrane action on the RHS face in tension, it has been established in Chapter 5 that the strength of the RHS face in tension is best characterised by the initial yield (bending) and membrane action ultimate strength (combination of bending and membrane action for hollow sections, pure membrane action for filled sections). The elastic yield deformation capacity can be determined using the initial stiffness equations derived in Chapter 4. The membrane action deformation capacity is given by the assumed deflection for calculation of membrane strength as given in Section 5.4.1. The characteristic load-deformation equations for the RHS face in tension component are given by the following equations.

\[ \delta_y = \frac{F_Y}{E k_{RHS,eq}} \]  \[ \text{[6.1]} \]

\[ \delta_m = \sqrt{c'^2 - c^2} \]  \[ \text{[6.2]} \]

where \( c' = c(1+e_u) \)

In many cases however, the RHS face in tension component will fail before the membrane action strength is reached. The failure strength, \( F_{\text{Fail}} \), is dependent on the weakest of thread stripping in bending (\( F_{\text{ts,red}} \)), punching shear (\( F_{\text{punch,nc}} \)), bolt punching shear (\( F_{\text{ps}} \)), or bolts in tension (\( F_{\text{Rd,2}} \)) as given by Equation [5.53]. The deformation capacity at failure, \( d_{\text{Fail}} \), is approximated using linear interpolation from elastic yield and membrane action states given by the following equation:

\[ \delta_{\text{Fail}} = \left( \frac{\delta_m - \delta_y}{F_{\text{Mem}} - F_Y} \right) (F_{\text{Fail}} - F_Y) + \delta_y \]  \[ \text{[6.3]} \]
For comparison purposes, it is useful to know the capacity at given deformation limits such as the well-established 3% RHS width limit proposed by Lu (1997). This limit serves two purposes: (1) as an ultimate deformation limit state in joints that do not exhibit a pronounced peak load and for which there is good agreement in cases that do exhibit a pronounced peak load; and (2) limiting the strength of the joint to the load at this deflection limit to ensure that the behaviour of the joint is within the elastic range under serviceability loading. The capacity at the 3% RHS width limit, $F_3$, is given by the following equation.

$$F_3 = \frac{(F_{\text{Mem}} - F_Y)}{(\delta_m - \delta_y)}(0.03w_{\text{RHS}} - d_y) + F_Y$$  \[6.4\]

where $F_{\text{Mem}}$ is given by Equation [5.51] or [5.52] depending on the section.

![Figure 6.1. Load deflection relationship for RHS face in tension](image)

6.3 Parametric study of RHS face deformation capacity

The parametric study described in Section 5.5.1 for the validation of strength equations is used to validate the deformation capacity at failure of the hollow and filled RHS face in tension component using equations given in Section 6.2. A
A comparison of FEM and analytical failure ductility values for the hollow RHS is given in Table 6.1. A comparison of FEM and analytical failure ductility values for the filled RHS is given in Table 6.2. Comparisons of analytical and FEM load-deflection curves are given in Appendix A.3 for the hollow RHS and in Appendix A.4 for the filled RHS. A selection of load-deflection curves is given in Figure 6.2 and Figure 6.3 for the hollow and filled RHS respectively.

The results indicate that the deformation capacity at failure of the hollow RHS face is generally predicted conservatively with an average error of -11.9% and maximum error of -34.5%. The deformation capacity at failure for the filled section is predicted with a mean error of -2.0% and maximum error of 43.2%. The load-displacement curves for both hollow and filled cases show that the deformation behaviour at the elastic and membrane action failure stages are replicated accurately while the intermediate bending/membrane stages is underestimated. This is attributed to the fact that the analytical approach simplifies the complex non-linear behaviour to the simplified bilinear characteristic of elastic and failure stages. The relatively high maximum error for both cases is attributed to the fact that the deformation capacity is calculated using Eqn. [6.3] that considers the deformation due to membrane action using the simplified deflection shape explained in Section 5.4.1. Test #12.1.1.2 and #1.1.16.2(C) which have high errors have cross sections with relatively low slenderness that may not develop sufficient curvature to match the simplified deflection shape.

For practical usage, errors of this magnitude are not a significant issue as they are generally conservative. As mentioned in Section 6.1, there are three main issues relating to the ductility of joints. These are to (1) check that the characteristic bending strength is attained within a practical level of joint deformation, (2) ensure that the combined joint behaviour gives a ductile joint, and (3) to estimate the ductility
characteristics of the RHS face at failure due to membrane action behaviour to predict structural robustness. The prediction of failure ductility only concerns (2) and (3) for which a conservative estimate of ductility has no negative implications.

Table 6.1. Analytical and FEM failure ductility for hollow RHS

<table>
<thead>
<tr>
<th>Test #</th>
<th>FEM (mm)</th>
<th>Analytical - Eqn. [6.3] (mm)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
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<td>1.1.5.2</td>
<td>23.7</td>
<td>24.7</td>
<td>4.2%</td>
</tr>
<tr>
<td>1.1.8.2</td>
<td>27.3</td>
<td>22.6</td>
<td>-17.4%</td>
</tr>
<tr>
<td>1.1.10.2</td>
<td>26.2</td>
<td>19.9</td>
<td>-23.9%</td>
</tr>
<tr>
<td>1.1.125.2</td>
<td>23.2</td>
<td>16.6</td>
<td>-28.6%</td>
</tr>
<tr>
<td>1.1.16.2</td>
<td>14.3</td>
<td>11.9</td>
<td>-16.8%</td>
</tr>
<tr>
<td>6.1.1.2</td>
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</tr>
<tr>
<td>1.5.1.2</td>
<td>16.5</td>
<td>19.9</td>
<td>20.8%</td>
</tr>
<tr>
<td>1.15.1.2</td>
<td>24.8</td>
<td>19.9</td>
<td>-19.6%</td>
</tr>
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<td>53.1</td>
<td>4.6%</td>
</tr>
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<td>1.1.1.4</td>
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<td>77.5</td>
<td>18.8%</td>
</tr>
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<td>19.9</td>
<td>-21.4%</td>
</tr>
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<tr>
<td></td>
<td>Mean</td>
<td>-11.9%</td>
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<tr>
<td></td>
<td>St.dev</td>
<td>15.8%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>-34.5%</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.2. FEM and analytical load-deflection relationship for typical hollow sections
### Table 6.2 Analytical and FEM failure ductility for filled RHS

<table>
<thead>
<tr>
<th>Test #</th>
<th>FEM (mm)</th>
<th>Analytical - Eqn. [6.3] (mm)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1.5.2</td>
<td>17.1</td>
<td>14.0</td>
<td>-17.8%</td>
</tr>
<tr>
<td>1.1.8.2</td>
<td>18.9</td>
<td>14.8</td>
<td>-21.7%</td>
</tr>
<tr>
<td>1.1.10.2</td>
<td>19.1</td>
<td>15.9</td>
<td>-16.7%</td>
</tr>
<tr>
<td>1.1.125.2</td>
<td>14.5</td>
<td>16.6</td>
<td>14.1%</td>
</tr>
<tr>
<td>1.1.16.2</td>
<td>8.3</td>
<td>11.9</td>
<td>43.2%</td>
</tr>
<tr>
<td>6.1.1.2</td>
<td>24.2</td>
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<td>23.0%</td>
</tr>
<tr>
<td>8.1.1.2</td>
<td>21.8</td>
<td>21.7</td>
<td>-0.8%</td>
</tr>
<tr>
<td>12.1.1.2</td>
<td>13.8</td>
<td>11.6</td>
<td>-16.0%</td>
</tr>
<tr>
<td>1.5.1.2</td>
<td>21.0</td>
<td>19.9</td>
<td>-5.1%</td>
</tr>
<tr>
<td>1.15.1.2</td>
<td>18.7</td>
<td>15.6</td>
<td>-16.6%</td>
</tr>
<tr>
<td>1.20.1.2</td>
<td>17.3</td>
<td>15.5</td>
<td>-10.2%</td>
</tr>
<tr>
<td>1.1.1.3</td>
<td>27.6</td>
<td>30.8</td>
<td>11.6%</td>
</tr>
<tr>
<td>1.1.1.4</td>
<td>34.5</td>
<td>42.8</td>
<td>24.0%</td>
</tr>
<tr>
<td>1.1.1.2B</td>
<td>17.7</td>
<td>15.9</td>
<td>-10.4%</td>
</tr>
<tr>
<td>1.1.1.2C</td>
<td>16.9</td>
<td>16.0</td>
<td>-5.5%</td>
</tr>
<tr>
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<td>17.2</td>
<td>15.6</td>
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<tr>
<td>1.3.1.2R4</td>
<td>19.9</td>
<td>16.2</td>
<td>-18.9%</td>
</tr>
</tbody>
</table>

Mean: -2.0%  
St.dev: 18.6%  
Max: 43.2%

![FEM and Analytical Load-Deflection Relationship](image)

**Figure 6.3.** FEM and analytical load-deflection relationship for typical filled sections

### 6.4 Validation of hollow RHS deformation capacity against British Steel (1996b) testing

The testing in British Steel (1996b) described in Section 3.1.2 is used to validate the analytical equations for the failure ductility of the hollow RHS face in transverse tension. As the deflection characteristics of these tests included that of the bolts, it is necessary to consider the contribution of the bolts to the ductility in cases where the overall load was relatively high and thus the bolt deformations were
noticeable. This is considered by assigning a bi-linear characterisation of the bolt load-deflection as explained in Section 7.2 based on the bolt tensile stresses. By comparing the bolt stresses due to the applied load with its material properties, it was determined that only T12/50/20 had bolts that had exceeded its yield stress. The other tests had only elastic bolt stresses therefore their deformation was neglected. A comparison between the analytical failure deformation capacity and British Steel testing deformation capacity at the failure strength is given in Table 6.3. A comparison of analytical and FEM load-deflection plots for individual tests are presented in Appendix A.1.

The numerical comparison shows that the failure deformation capacity is generally underestimated with a mean error of -5.7% and maximum of approximately 20.8%. There is no distinct trend between geometric ratios of specimens and errors in the prediction of failure deformation capacity. The predicted failure mode for all cases is thread stripping (\(F_{\text{ts,red}}\)) as observed in the tests. Therefore, the analytical failure deformation capacity, \(d_{\text{Fail}}\), was determined using linear interpolation between yield and membrane action deformation capacities for the failure capacity, \(F_{\text{Fail}}\), using Equation [6.3]. Any error in the prediction of \(F_{\text{Fail}}\) can compound or cancel out the error in the prediction of \(d_{\text{Fail}}\).
Table 6.3. Comparison of analytical and test deformation capacity at failure for hollow RHS

<table>
<thead>
<tr>
<th>Test (mm)</th>
<th>Analytical RHS Eqn. [6.3] (mm)</th>
<th>Analytical bolt (mm)</th>
<th>Analytical combined (mm)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>T/30/27/20</td>
<td>30.0</td>
<td>29.8</td>
<td>29.8</td>
<td>-0.5%</td>
</tr>
<tr>
<td>T/30/50/20</td>
<td>17.0</td>
<td>18.2</td>
<td>18.2</td>
<td>7.3%</td>
</tr>
<tr>
<td>T/30/67/20</td>
<td>11.0</td>
<td>9.9</td>
<td>9.9</td>
<td>-9.5%</td>
</tr>
<tr>
<td>T/24/27/20</td>
<td>30.0</td>
<td>28.1</td>
<td>28.1</td>
<td>-6.3%</td>
</tr>
<tr>
<td>T/24/50/20</td>
<td>17.0</td>
<td>16.5</td>
<td>16.5</td>
<td>-2.8%</td>
</tr>
<tr>
<td>T/24/67/20</td>
<td>9.0</td>
<td>8.2</td>
<td>8.2</td>
<td>-8.6%</td>
</tr>
<tr>
<td>T/19/27/20</td>
<td>28.0</td>
<td>25.9</td>
<td>25.9</td>
<td>-7.6%</td>
</tr>
<tr>
<td>T/19/50/20</td>
<td>18.0</td>
<td>14.3</td>
<td>14.3</td>
<td>-20.8%</td>
</tr>
<tr>
<td>T/15/27/20</td>
<td>24.0</td>
<td>23.2</td>
<td>23.2</td>
<td>-3.3%</td>
</tr>
<tr>
<td>T/15/50/20</td>
<td>13.0</td>
<td>11.6</td>
<td>11.6</td>
<td>-10.7%</td>
</tr>
<tr>
<td>T/12/27/20</td>
<td>20.0</td>
<td>19.9</td>
<td>19.9</td>
<td>-0.5%</td>
</tr>
<tr>
<td>T/12/50/20</td>
<td>10.0</td>
<td>8.9</td>
<td>8.9</td>
<td>-5.0%</td>
</tr>
</tbody>
</table>

Mean -5.7%
St.dev 6.9%
Max 20.8%

6.5 Conclusions

The equations derived in this chapter for the deformation capacity of the RHS face in tension component are suitable regardless of the failure mode. This covers the hollow RHS face when compared with testing (British Steel, 1996b) as well as the hollow and filled RHS face when validated against the FEM parametric study.

For the failure due to membrane action, a simple relationship based on the cross section geometry and material ultimate strain properties is used. For failure mechanisms that occur before membrane action capacity, linear interpolation between yield and membrane action states is used to define the deformation capacity. Although the failure deformation is predicted adequately in comparison to the FEM parametric study, it is generally conservative for both the hollow and filled RHS face components with mean errors of -11.9% and -2.0% respectively. Similarly, the validation study against British Steel testing for the hollow RHS shows that equations are conservative with a mean error of -5.7%. This is attributed to the additional displacement due to opening of the flowdrilled RHS face that is not captured in the simple assumptions.
used in defining the membrane action deformation. For practical usage, the errors in
the prediction of the failure ductility are conservative therefore there are no serious
negative implications as typically the large failure deflections associated with the RHS
face in tension will only be used for assessing membrane/catenary action behaviour for
structural robustness calculations.
Chapter 7

Joint assembly using analytical equations

7.1 Introduction

With the development of equations to describe the full load-deflection behaviour of the RHS face component and existing equations for the bolt and endplate components, it is now possible to analytically predict the full moment-rotation curve of flowdrilled endplate connections to RHS columns. This is possible with existing equations for bolt strength, bolt ductility and endplate strength given in EN 1993-1-8 (CEN, 2005) as well as endplate ductility given in Beg et al. (2004). An understanding of the full-moment rotation curve characteristics allows characterisation of the joint into EN 1993-1-8 joint categories (e.g. full-strength/partial-strength) as well as providing an additional level of understanding on the suitability of joints for different requirements. For example, the actual failure behaviour can be used in designing against partial collapse where joint characteristics outside the typical working range of ductility are required.
7.2 Rules for calculation of the moment-rotation curve

7.2.1 Proposed method of joint assembly

The joint assembly method used for obtaining the full moment-rotation curve is a simplified mechanical model based on characterising individual component behaviour as a non-linear spring with a bilinear load-deformation relationship and finding their relative contribution towards overall joint rotation at characteristic strength values. It is the same concept as the component method and is adopted in work such as Beg et al. (2004). As shown in Figure 7.1, the contributions of deformations from individual components are determined in relation to the capacity of the weakest component. The weakest component contributes its full deformation capacity and other components contribute ductility determined by linear interpolation from its load-deflection curve.

\[ M_{\text{Fail},j} = M_{\text{Fail},A} \]
\[ \theta_{\text{Fail},j} = \theta_{\text{Fail},A} + \theta_B + \theta_C \]

![Whole joint and joint components diagram](image)

**Figure 7.1.** Component ductility contribution to joint moment-rotation
Although the individual components are characterised by a bilinear curve, by adopting all load definition points used to define the components into the joint characterisation, the resulting joint moment-rotation curve will be multi-linear. An example of the component moment-rotation characteristics and the assembled joint characteristics is given in Figure 7.2 and Table 7.1 for Test 4 of France et al. (1999). This shows how the combination of bilinear component moment-rotation curves gives a joint moment-rotation curve defined by four points.

Figure 7.2. Example of component contribution to whole joint behaviour (Test 4)

Table 7.1. Individual component and joint moment-rotation definition (Test 4)

<table>
<thead>
<tr>
<th>Components</th>
<th>Moment (kNm)</th>
<th>Rotation (radians)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>RHS</td>
<td>0.0</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>26.0</strong></td>
<td><strong>0.005</strong></td>
<td>Yield strength</td>
</tr>
<tr>
<td></td>
<td>91.5</td>
<td>0.064</td>
<td>Ultimate strength (Membrane action)</td>
</tr>
<tr>
<td>Bolts</td>
<td>0.0</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>132.7</td>
<td>0.000</td>
<td>Yield strength</td>
</tr>
<tr>
<td></td>
<td>165.9</td>
<td>0.011</td>
<td>Ultimate strength</td>
</tr>
<tr>
<td>Flush endplate</td>
<td>0.0</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>34.0</strong></td>
<td><strong>0.001</strong></td>
<td>Yield strength</td>
</tr>
<tr>
<td></td>
<td><strong>51.0</strong></td>
<td><strong>0.034</strong></td>
<td>Ultimate strength</td>
</tr>
<tr>
<td>Whole joint</td>
<td>0.0</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>26.0</td>
<td>0.005</td>
<td>Yield strength</td>
</tr>
<tr>
<td></td>
<td>34.0</td>
<td>0.013</td>
<td></td>
</tr>
<tr>
<td></td>
<td>51.0</td>
<td>0.062</td>
<td>Ultimate strength</td>
</tr>
</tbody>
</table>

*Bold text denotes component moment-rotation stages that are reached in whole joint moment-rotation*
7.2.2 Rules for component moment-rotation characterisation

The characterisation of the RHS face in tension load-deflection behaviour is explained in Chapter 5 and Chapter 6 and equations to define the load-deflection curve are given in Section 5.2.4 and 6.2.

For the endplate, the equations given in Beg et al. (2004) for ductility as explained in Section 2.4.4 are used in conjunction with initial stiffness and strength equations given by EN 1993-1-8 (CEN, 2005). For comparison and checking purposes, the unfactored strength (i.e. $\gamma_{M0}=1$) is used to compare actual strength values rather than the factored design strength. A summary of the load-deflection definition points are given in Table 7.2. The load-deflection of the bolts uses equations from EN 1993-1-8 as presented in Table 7.3.

### Table 7.2 Load-deflection definition for endplate component

<table>
<thead>
<tr>
<th>Point #</th>
<th>Load</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1 - Yield strength</td>
<td>$F_{y,\text{Endplate}}$ (Unfactored)</td>
<td>$d_y = \frac{F_{y,\text{Endplate}}}{E_k\text{Endplate,eq}}$</td>
</tr>
</tbody>
</table>
| 2 - Ultimate strength | 1.5×$F_{y,\text{Endplate}}$ | Mode 1  
$du = 2e_u\times m$  
Mode 2  
$du = 0.1l_b \left(1 + k \frac{m}{n}\right)$  
Mode 3  
See bolt ultimate deformation capacity |

### Table 7.3 Load-deflection definition for bolt component

<table>
<thead>
<tr>
<th>Point #</th>
<th>Load</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1 - Yield strength</td>
<td>$F_{y,\text{bolt}}$</td>
<td>$d_y = \frac{F_{y,\text{bolt}}}{E_k\text{Bolt,eq}}$</td>
</tr>
<tr>
<td>2 - Ultimate strength</td>
<td>1.5×$F_{y,\text{bolt}}$</td>
<td>$du = e_u\times l_b$</td>
</tr>
</tbody>
</table>

The component load-deflection is converted into moment-rotation using the following equations:

$$M = F \times z$$
\[ \theta = \frac{d}{z} \]  

where lever arm \( z \) is obtained from Figure 6.15 of EN 1993-1.8

7.2.3 Rules for joint moment-rotation characterisation

This research is focused on the joint assembly of bolted endplate connections to RHS sections for which the key components and strength dependencies are shown in Figure 7.3. Although these are the key components, other joint components such as those relating to the beam should still be checked using existing equations in EN 1993-1-8.

For flush endplate connections, the failure strength of the connection is dependent on the weakest of the RHS, endplate, and bolt components. For extended endplate connections, the capacity is determined from the sum of flush and extended endplate portions. As with the calculation of initial stiffness, it is assumed that the compression zone of the RHS face has a negligible contribution on the behaviour of the whole joint. As reported in the British Steel (1996b), for flowdrilled endplate connections to RHS columns, a certain amount of deformation takes place in the compression zone of the RHS face, however this is always negligible in comparison to that in the tension area.

For flush endplate joints, the failure strength is defined by:

\[ M_{\text{Fail}} = \min[M_{\text{RHS,Fail}}, M_{\text{U,F,Endplate}}, M_{\text{U,Bolts}}] \]  

The ductility at the \( n \)th definition point of the moment-rotation curve for a joint with flush endplate is given by:

\[ \theta_n = \theta_{\text{RHS,n}} + \theta_{\text{F,Endplate,n}} + \theta_{\text{Bolts,n}} \]  

For extended endplate joints, the failure strength is defined by:
The ductility at the \( n^{th} \) definition point of the moment-rotation curve for a joint with extended endplate is given by:

\[
\theta_n = \theta_{\text{RHS},n} + \\text{Min}(\theta_{\text{Endplate,nu}} \theta_{\text{Ext.Endplate},n}) + \theta_{\text{Bolts},n}
\]  

[7.6]

Figure 7.3. Key components for bolted endplate connections to RHS sections

7.3 Validation against testing

The joint assembly approach described in Section 7.2 for predicting whole joint moment-rotation behaviour is validated against testing by France et al. (1999) for bolted flush and extended endplate connections to RHS columns as described in Section 3.1.1.

Comparisons of the individual moment-rotation curves are presented in Appendix A.2. Due to the difficulties in comparing the accuracy of the various characteristics of the moment-rotation curves given by the analytical approach, a comparison of strength characteristics at the 0.03radian joint rotation limit is
considered. This is the often quoted minimum joint ductility requirement under the LRFD connection classification of AISC for a ductile joint and the upper bound expected ductility requirement in practical frame design according to SCI both discussed in Section 8.4. Comparing strength characteristics at this joint rotation will give insight into the accuracy of the joint characteristics at the limits that a typical frame will require. A comparison is given in Table 7.4 showing good agreement with strength characteristics at the 0.03radian joint rotation limit with a mean error of -1.9% and maximum of 33.3%. The variation in results is attributed to the fact that the analytical strength at 0.03 radians is obtained by interpolation from the elastic and membrane action states and thus any errors in the prediction of these values may compound. As the membrane action strength was predicted using an assumed material ultimate strain of 0.2, additional error would have occurred due to actual variation in the material ultimate strain.

Because the majority of France et al. tests were not carried out to failure, it is not possible to validate the failure behaviour against all tests. However, Tests 19, 20, 22, 23, and 24 are recorded as having bolt pullout failure due to thread stripping. These tests are used to check the reduced strength thread stripping equation validated in Section 5.3.2. A comparison between analytical and test failure strength is given in Table 7.5. The analytical approach predicts that Test 19, 20, and 22 fail due to thread stripping of the RHS face while Test 23 and 24 are predicted to have bolt failure. This is attributed to the fact that bolt failure is predicted to occur marginally below the thread stripping strength. Overall, the failure strength is overestimated with a mean error of 23.7% and maximum error of 31.5%. The greater level of error in comparison to the prediction of bolt pullout against British Steel (1996b) tests in Section 5.3.2 is attributed to minor differences in the contact behaviour between the bolt threads and RHS face due to the different loading conditions (tensile load vs. bending moment).
The same comparison is conducted but with a modified value of \( \tau = 50 \). As explained in Section 5.3.2, this coefficient is determined from British Steel (1996b) testing that relates the reduction in thread stripping capacity due to bending deformation of the RHS face to its thickness. As given in Table 7.6, a modified value of \( \tau = 50 \) greatly improves the correlation with France et al. tests results giving a mean error of 3.8% and maximum of 11.4%. The failure mode is also predicted to be thread stripping for all five tests. In either case however, the prediction of the failure deformation capacity is poor. This is explained by the fact that deformation capacity is largely dependent on the prediction of the RHS face deformation capacity at the thread stripping strength which is determined from interpolation of the yield and membrane strength deformations as explained in Section 6.2. Because the deformation is sensitive to the load in this region (i.e. a large increase in deformation occurs under a small increase in load), the errors in prediction of deformation are significantly amplified. A comparison of individual moment-rotation curves shows the key characteristics are predicted accurately.
Table 7.4. Analytical and test strength at 0.03 radians

<table>
<thead>
<tr>
<th>Test #</th>
<th>Analytical - Eqn. [6.4] (kNm)</th>
<th>Test (kNm)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>79.6</td>
<td>68.5</td>
<td>16.2%</td>
</tr>
<tr>
<td>4</td>
<td>40.0</td>
<td>40.5</td>
<td>-1.3%</td>
</tr>
<tr>
<td>5</td>
<td>42.7</td>
<td>41.2</td>
<td>3.6%</td>
</tr>
<tr>
<td>6</td>
<td>36.5</td>
<td>50.0</td>
<td>-27.0%</td>
</tr>
<tr>
<td>7</td>
<td>34.9</td>
<td>27.0</td>
<td>29.1%</td>
</tr>
<tr>
<td>8</td>
<td>89.6</td>
<td>108.0</td>
<td>-17.0%</td>
</tr>
<tr>
<td>10</td>
<td>17.9</td>
<td>21.0</td>
<td>-14.9%</td>
</tr>
<tr>
<td>14</td>
<td>109.1</td>
<td>101.5</td>
<td>7.5%</td>
</tr>
<tr>
<td>15</td>
<td>52.3</td>
<td>58.4</td>
<td>-10.5%</td>
</tr>
<tr>
<td>16</td>
<td>45.8</td>
<td>50.0</td>
<td>-8.4%</td>
</tr>
<tr>
<td>17</td>
<td>20.7</td>
<td>31.0</td>
<td>-33.3%</td>
</tr>
<tr>
<td>18</td>
<td>53.4</td>
<td>43.6</td>
<td>22.6%</td>
</tr>
<tr>
<td>19</td>
<td>176.8</td>
<td>144.0</td>
<td>22.8%</td>
</tr>
<tr>
<td>20</td>
<td>223.0</td>
<td>186.0</td>
<td>19.9%</td>
</tr>
<tr>
<td>21</td>
<td>293.9</td>
<td>266.7</td>
<td>10.2%</td>
</tr>
<tr>
<td>22</td>
<td>267.2</td>
<td>271.6</td>
<td>-1.6%</td>
</tr>
<tr>
<td>23</td>
<td>211.1</td>
<td>236.5</td>
<td>-10.7%</td>
</tr>
<tr>
<td>24</td>
<td>259.8</td>
<td>310.5</td>
<td>-16.3%</td>
</tr>
<tr>
<td>25</td>
<td>94.8</td>
<td>113.0</td>
<td>-16.1%</td>
</tr>
<tr>
<td>26</td>
<td>74.2</td>
<td>84.2</td>
<td>-11.9%</td>
</tr>
</tbody>
</table>

Average -1.9%
St.dev. 17.7%
Max 33.3%

Table 7.5. Failure strength and deformation capacity for France et al. (1999) tests with thread stripping failure

<table>
<thead>
<tr>
<th>Test #</th>
<th>Failure mode</th>
<th>Failure strength</th>
<th>Failure deformation capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Analytical Test (kNm)</td>
<td>Analytical Eqn. [7.5] (kNm)</td>
</tr>
<tr>
<td>19</td>
<td>T. Strip</td>
<td>163.0</td>
<td>199.5</td>
</tr>
<tr>
<td>20</td>
<td>T. Strip</td>
<td>209.0</td>
<td>256.3</td>
</tr>
<tr>
<td>22</td>
<td>T. Strip</td>
<td>288.0</td>
<td>319.4</td>
</tr>
<tr>
<td>23</td>
<td>Bolt&lt;sup&gt;1&lt;/sup&gt;</td>
<td>253.0</td>
<td>332.3</td>
</tr>
<tr>
<td>24</td>
<td>Bolt&lt;sup&gt;2&lt;/sup&gt;</td>
<td>315.0</td>
<td>414.2</td>
</tr>
</tbody>
</table>

Average 23.7%
St.dev. 8.5%
Max 31.5%

Average 66.3%
St.dev. 75.3%
Max 177.7%

<sup>1</sup> Bolt fails at 332.3kNm, RHS thread stripping predicted at 337.8kNm.

<sup>2</sup> Bolt fails at 414.2kNm, RHS thread stripping predicted at 421.0kNm.
Table 7.6. Failure strength and deformation capacity for France et al. (1999) tests with thread stripping failure (If $\tau=50$)

<table>
<thead>
<tr>
<th>Test #</th>
<th>Failure mode</th>
<th>Failure strength</th>
<th>Failure deformation capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Analytical Eqn. [7.5] (kNm)</td>
<td>Error</td>
</tr>
<tr>
<td>19</td>
<td>T. Strip</td>
<td>163.0</td>
<td>166.2</td>
</tr>
<tr>
<td>20</td>
<td>T. Strip</td>
<td>209.0</td>
<td>213.6</td>
</tr>
<tr>
<td>22</td>
<td>T. Strip</td>
<td>288.0</td>
<td>266.2</td>
</tr>
<tr>
<td>23</td>
<td>T. Strip</td>
<td>253.0</td>
<td>281.5</td>
</tr>
<tr>
<td>24</td>
<td>T. Strip</td>
<td>315.0</td>
<td>350.8</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>3.8%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>St.dev.</td>
<td>7.9%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>11.4%</td>
<td></td>
</tr>
</tbody>
</table>

7.4 Conclusions

This chapter presents an analytical approach for joint assembly using newly derived equations for the RHS face in tension strength and deformation capacity combined with existing equations for the bolt and endplate given in EN 1993-1-8 (CEN, 2005) and Beg et al. (2004). The joint is assembled using a simplified mechanical model based on characterising individual component behaviour as a non-linear spring with a bilinear load-deformation relationship and finding their relative contribution towards overall joint rotation at characteristic strength values.

The validation study against France et al. testing shows that moment-rotation in elastic and plastic regimes is predicted accurately and to a lesser degree, the failure behaviour. A comparison of the 0.03radian joint rotation limit strength gives a mean error of -1.9% and maximum of 33.3%. In all tests, the general characteristics of the moment-rotation curves are predicted well. Overall, the analytical approach gives a consistent prediction of the testing moment-rotation behaviour with similar accuracy to the FEM approach of Section 3.4 and at a significantly lower computational cost. By using the analytical approach, it is possible to conduct effective parametric studies to optimise joint design parameters and gain a detailed understanding about
component contributions to joint behaviour which is difficult or costly when using full-scale testing or FEM approaches.
Chapter 8

Unbraced semi-continuous frame behaviour

8.1 Introduction

The advantages of allowing for semi-rigid connection behaviour in steel frame design have been very well rehearsed based on active research studies going back more than 30 years. Whilst much of the knowledge gained can be directly applied to steel frames using Hollow Steel Sections (HSS) as columns, there is surprisingly a lack of direct research to apply the semi-rigid design philosophy to HSS structures. The use of HSS columns can be highly beneficial owing to their superior geometrical properties and mechanical performance over traditional open sections. When used with blindbolts and relatively simple endplate detailing, connections can be made which possess substantial levels of rotational stiffness. For low-rise steel frames, the rotational stiffness may be sufficient to control the frame sway deflection below that required for Serviceability Limit State (SLS) design without using lateral bracing or expensive welded fully rigid connections.

This chapter applies the semi-rigid design method to low-rise steel frames using HSS as columns and blindbolted endplate connections. It will address two issues: (1) lateral deflection calculation for steel frames with semi-rigid connections,
and (2) connection detailing for blindbolted endplate connections to HSS columns. For the former, although a design method already exists (SCI, 1995) this method refers to charts that are difficult to use when carrying out rapid calculations and uses a general multiplier of 1.5 to convert rigid frame deflections into semi-rigid frame deflections using standard connections. It is necessary to assess the accuracy of the existing method and to propose a direct calculation method. For the latter, this research will use the proposed direct calculation method to derive the level of required connection rotational stiffness to control the frame sway deflection to be within the SLS limits. Afterwards, this method will show how to determine the connection detailing, using the connection component stiffness equations derived in Section 4.2 for the RHS component and the existing equations in EN 1993-1-8 (CEN, 2005) for the endplate and bolt component stiffnesses using the Component Method.

8.2 Joint initial stiffness requirements

In EN 1993-1-8, the fully rigid classification of joints can be determined from its moment rotation characteristics when its relationship with the connecting beam-members satisfies the following:

\[ S_{j,ini} \geq k_b E I_b / L_b \]  \[8.1\]

where \( k_b = 8 \) for frames where the bracing system reduces the horizontal displacement by at least 80%; and 25 for unbraced frames provided that in every storey \( K_b / K_c \geq 0.1 \)

This research is only interested in unbraced frames. The coefficient of 25 is to allow unbraced frames to be analysed as fully rigid (Ivanyi and Baniotopoulos, 2000). For low-rise frames where the horizontal loading is not as significant as in high-rise frames, a lower level of joint rotational stiffness may be sufficient to achieve the SLS design requirement of controlling the frame lateral movements within the design limits.
To investigate this claim, a variety of low-rise frame geometries are considered with variations in the number of storeys, the bay width, and the number of bays to reflect some realistic variations in practice using the structural analysis software Oasys GSA 8.4 (Oasys Ltd., 2009). Member specifications are based on a minimum weight ULS design of the basic 1.35 Dead + 1.5 Live load case and are representative of a typical low-rise office building designed to Eurocode 3. The SLS loading is assessed with a 1.0 Dead + 1.0 Wind load case as this has the greatest potential for causing lateral sway. Figure 8.1, Figure 8.2, Figure 8.3, and Figure 8.4 show the characteristic loads and dimensions for the following frame configurations.

A. 4 storey frame with 6m wide bays, 3 bays wide, and 3 bays deep.
B. 4 storey frame with 6m wide bays, 6 bays wide, and 6 bays deep.
C. 4 storey frame with 8m wide bays, 3 bays wide, and 3 bays deep.
D. 6 storey frame with 6m wide bays, 3 bays wide, and 3 bays deep.

Frame A has a realistic and efficient design in which the edge columns are smaller than the inner columns and the roof beams are smaller than the inner floor beams. For simplicity, Frames B, C, and D utilize the same beam and column sections throughout the frame.

The effect of lateral stiffness provided by the semi-rigid joints can be assessed using a semi-continuous frame analysis by implementing a simple hybrid beam element with rotational springs at edges as discussed in Section 2.5. The initial stiffness of the joints is varied as a ratio of the connecting beam-member stiffness, $\frac{E I_b}{L_b}$, so that generalized comments can be made on the joint rotational stiffness required to control frame sway deflections.

In Figure 8.5, Figure 8.6, Figure 8.7, and Figure 8.8, the lateral sway characteristics of the frames are plotted for different levels of joint stiffness in
proportion to the connecting beam stiffne$$\text{ss} \frac{E_\text{b}L_\text{b}}{b}$. The $d=h/300$ represents the acceptable lateral sway and this is used to estimate the magnitude of joint rotational stiffness required to provide sufficient lateral support in the unbraced frame. From the four cases considered, it is possible to see that each case gives a different value for joint stiffness that is necessary to provide lateral stiffness suggesting that relying on a single relative beam stiffness coefficient alone, such as with the value of 25 as in Eurocode 3, is not a conclusive check to determining the case-by-case feasibility of unbraced frame design. In addition, it is important to note that in all cases this value is considerably smaller than the fully rigid value of 25 required in Eurocode 3, confirming that semi-rigid joints with relatively low rotational stiffness may be suitable for providing sufficient lateral stiffness in low-rise unbraced frame design.

With regards to the aforementioned use of the general multiplier of 1.5 to convert rigid frame deflections into semi-rigid frame deflections in SCI P263 (Hensman and Way, 2000), it is clear that this approach may give largely underestimated predictions of lateral sway in semi-rigid frames when joints with very low stiffnesses are used. However, Couchman (2012) states this general multiplier should only be used for the standardised joint designs given in SCI P263.

Another observation is that the initial stiffness of the joint is not directly proportional to the variations in the lateral sway of the global frame. For example, comparing the cumulative lateral sway at the roof of Frame A between joint stiffness levels of 5 $\frac{E_\text{b}L_\text{b}}{b}$ and 10 $\frac{E_\text{b}L_\text{b}}{b}$, it is apparent that doubling the joint stiffness only reduces the sway deflection by approximately 15%. This implies that if the required joint stiffness is calculated analytically, relatively large errors in estimating the required joint stiffness may be tolerated.

The above analysis specifies the required joint stiffness. Owing to the non-linearity of joint moment-rotational characteristics, particularly joints to hollow
section columns, it is necessary to make proper allowance when relating the above joint stiffness requirements to the joint initial stiffness. van Keulen et al. (2003) suggests that at SLS level, the joint secant stiffness can be conservatively estimated as being half of the joint initial stiffness. This means that the required level of joint initial stiffness is approximately double the guideline joint stiffness obtained from the above frame analysis. Another important consideration in the serviceability limit state design of joints is ensuring that the applied moment is less than the yield strength of the joint. As will be explained in Section 8.3, this is to ensure permanent rotations are not accumulated thus lowering the ultimate strength of the joint.

**Frame A**

![Frame A diagram](image)

Columns: SHS 220×220×14.2 S355 (Inner)  
SHS 220×220×6.3 S355 (Edge)  
Beams: UB 406×178×74 S275 (Inner storey)  
UB 203×133×25 S275 (Roof)

Figure 8.1. Frame geometry, member specification, and details of loading for Frame A

**Frame B**

Columns: SHS 250×250×12.5 S355  
Beams: UB 457×191×74

![Frame B diagram](image)

Figure 8.2. Frame geometry, member specification, and details of loading for Frame B
Figure 8.3. Frame geometry, member specification, and details of loading for Frame A

**Frame C**

Y(m)  
16  17.4 kN  
12  17.4 kN  
8  17.4 kN  
4  17.4 kN  
0  0  8  16  24  X(m)

Columns: SHS 300×300×12.5 S355  
Beams: UB 533×312×150

Figure 8.4. Frame geometry, member specification, and details of loading for Frame D

**Frame D**

Y(m)  
24  21.5 kN  
20  20.1 kN  
16  18.6 kN  
12  18.6 kN  
8  18.6 kN  
4  18.6 kN  
0  0  6  12  18  X(m)

Columns: SHS 300×300×10 S355  
Beams: UB 457×191×74
Figure 8.5. Cumulative lateral sway for varying levels of joint stiffness in Frame A

Figure 8.6. Cumulative lateral sway for varying levels of joint stiffness in Frame B
Figure 8.7. Cumulative lateral sway for varying levels of joint stiffness in Frame C

Figure 8.8. Cumulative lateral sway for varying levels of joint stiffness in Frame D

8.3 Joint strength requirements

While Section 8.2 establishes the importance of joint initial stiffness in controlling the SLS deflections in unbraced semi-continuous frame design, it is also vital to consider the joints strength characteristics. As mentioned in Section 2.2, joints
are categorised based on their strength as being nominally-pinned, full strength, or partial-strength. Semi-rigid joints can be either full strength or partial strength. Full strength joints can carry loads equal to or greater than that of the connecting beam-member and thus will not have problems maintaining their initial stiffness under SLS loading. Partial strength joints carry loads that are less than that of the connecting beam-member. As semi-rigid joints will typically be partial strength, it is important to check that the SLS loads imposed on the joint are less than the elastic strength limit of the joint to ensure the levels of initial stiffness mentioned in Section 8.2 are attained. In addition to this, partial strength connections must ensure that the designed failure mode permits ductile yielding (covered in Section 8.4).

Equation [8.2] determines whether a partial strength semi-rigid joint can maintain elastic behaviour and thus have the necessary initial stiffness for unbraced frame design. If this equation is not satisfied, the joint will experience plastic deformations thus have significantly less stiffness that not only has implications in SLS deformations but also for permanent deformation issues and P-delta effects which may induce collapse.

\[ M_{Rd,SLS} \leq M_{Y,Joint} \]  \[ \text{[8.2]} \]

8.4 Joint ductility requirements

An understanding of joint ductility is important when the deformations are concentrated in the connection elements, as is typical in partial strength connections (Chen, 2000). Ductility requirements are not as established in comparison to initial stiffness and strength requirements. For example, EN 1993-1-8 does not specify ductility requirements and instead leaves this to the judgement of the designer. An
often quoted requirement is that of the LRFD connection classification of AISC which suggests in Bjorhovde (1997) the general case of a joint being ductile if \( \theta_u^* \geq 0.03 \) radians (where \( \theta_u^* \) is the joint rotation when the moment has dropped to 80% of its maximum value as in Figure 8.9). AISC (1997) also makes a distinction with ductility requirements according to the applications. For example, a braced frame in a non-seismic area will require significantly less ductility than an unbraced frame in a high seismic area. Alternative criteria are suggested in AISC (1997) for ductility including \( \theta_u^* \geq 0.04 \) radians for special moment frames (SMF) used to resist strong earthquake shaking with substantial inelastic behaviour and \( \theta_u^* \geq 0.02 \) radians for intermediate moment frames (IMF) with less stringent requirements.

![Figure 8.9. Typical semi-rigid joint moment-rotation curve for AISC LRFD classification](image)

For wind-moment frames, SCI (1995) states that the actual rotation capacity needed is usually expected not to exceed 0.02 to 0.03 radians. It suggests that endplate connections (to H-sections) are suitable for this purpose because the endplate typically has sufficient ductility providing that the endplate is thin enough to be the weak link relative to bolts (mode 1 failure). In the context of connections to RHS sections, the
RHS face offers even greater levels of ductility giving the designer more options for creating and controlling a ductile failure mechanism.

Another consideration for joint ductility is in the assessment of structural robustness for which connection performance plays an important part in resisting disproportionate collapse of steel framed structures (Wang and Orton, 2006). Having an estimate of the failure rotation capacity, $\theta_u^*$, can provide insight into suitable detailing to improve structural performance against this collapse. From the testing by France (1999) and from Chapter 7 it is clear that the bolted endplate connections to RHS which are covered in this project offer levels of rotation capacity greater than the 0.03 radians criterion. By taking advantage of the equations proposed in Chapter 6, it is possible to optimize the joint detailing to fulfil design requirements.

8.5 Simplified analytical model for steel frames with semi-rigid joints

In Section 8.2, the feasibility of using semi-rigid joints to limit unbraced frame lateral sway deflections is established. However, for determining the necessary level of joint stiffness for unbraced frames, it would be desirable to have a quick analytical method, rather than using a semi-continuous frame analysis with the semi-rigidity of the joints being represented as rotational springs. The hand calculation method will require two simplifications. These are (1) to convert a frame with semi-rigid joints to one with rigid joints by using reduced beam stiffness and (2) to calculate the lateral deformation of the rigid frame. Developing hand calculation methods has received considerable attention from researchers; therefore, this section will only assess the accuracy of two related analytical methods.
8.5.1 Rigid frame sway deflection calculation method (Basic Method)

For calculating the sway deflection of rigid frames, two hand calculation methods may be used. The first method, presented by Smith and Coull (1999) and Taranath (1997), is based on decomposing the frame sway deflection into three components: beam rotations assuming rigid columns, column rotations assuming rigid beams, and cantilever deflection of the entire frame. The sum of the first two is referred to as the shear racking deflection. For low-rise frames, the shear racking deflection contributes the majority of the total frame deflection. As a simplification, the points of contraflexure are assumed to occur at the column mid-heights and at the beam mid-spans. The shear racking deflection ($\Delta_s$), cantilever deflection ($\Delta_c$), and the total frame deflection ($\Delta$) are calculated from the following equations:

\[ \Delta_s = \frac{Vh^2}{12E} \left( \frac{1}{B} + \frac{1}{C} \right) \]  \[8.3\]

\[ \Delta_c = \frac{Vh}{GA_v} \]  \[8.4\]

\[ \Delta = \Delta_s + \Delta_c \]  \[8.5\]

where

\[ B = \sum \left( \frac{l_{\text{beam}}}{L} \right) \]  \[8.6\]

\[ C = \sum \left( \frac{l_{\text{column}}}{h} \right) \]  \[8.7\]

\[ G = 0.4E \]  \[8.8\]
The assumption of column mid-height point of contraflexure is not valid for the ground floor due to differences in the base support conditions. This can be taken into account by using the following equations:

For fixed base supports:

\[
\Delta_s = \frac{Vh^2}{12E} \left( \frac{2}{3B} + \frac{1}{C} \right) \frac{1}{\left( 1 + \frac{C}{6B} \right)}
\]

For pinned base supports:

\[
\Delta_s = \frac{Vh^2}{12E} \left( \frac{3}{2B} + \frac{4}{C} \right)
\]

The above equations are for frames with rigid joints. For frames with semi-rigid joints, all that is required is to use the reduced beam stiffness given in Section 2.6, and using it in Equation [8.6] to get the following:

\[
B = \sum \psi (I_{beam}/L)
\]

8.5.2 SCI Wind moment design method

The second approach is presented in the SCI wind-moment design method (Hensman and Way, 2000) but can trace its background to Wood and Roberts (1975). Similar to the previous method, the frame is replaced by a substitute beam-column frame in which the frame sway deflections are in part dependent on the stiffness distribution coefficients of the beam and column sections. These coefficients are then used to look up a non-dimensional sway-index factor (\(\phi\)) from a chart based on Wood
and Roberts (1975). Because a look up chart is used, this approach is not flexible, for example, to automate calculations using spreadsheets. Having found this sway-index factor, total frame shear racking deflection $\Delta_s$, is calculated per storey from the following equation:

$$\Delta_s = \frac{Vh^2\phi}{12EK_c}$$

(8.13)

$\phi$ is obtained from Figure D.8 in Hensman and Way (2000) based on the following values of $K_r$ and $K_u$.

$$K_r = \frac{K_c + K_u}{K_c + K_u + K_{bt}}$$

(8.14)

$$K_b = \frac{K_c + K_t}{K_c + K_t + K_{bb}}$$

(8.15)

Again, this method can be modified to allow for frames with semi-rigid joints by using the reduced beam stiffness coefficient, given in Section 2.6.

8.5.3 Comparison of rigid frame and SCI wind moment design methods

Table 8.1 compares the predicted lateral deflections for the frames presented in the previous section but with rigid joints using the two hand calculation approaches against those obtained using the computer-based frame analysis.

The results indicate that both simplified frame analysis methods give suitable estimations for predicting lateral sway deflections of rigid frames. The first method (hereafter referred to as the Basic method) gives errors with an average of 4.7%, maximum of 11.1%, and a standard deviation of 5.6% which is excellent for an approximate method. The SCI wind moment method gives errors with an average of 9.8%, maximum 28.8%, and standard deviation of 6.6%. While the errors from using
the SCI wind moment method are slightly larger, as an approximate method, these errors may be considered acceptable. However, given that the SCI wind moment method requires looking up a chart for each calculation thereby making it impossible to automate the calculation, it is proposed to use the first method.

To check the accuracy of the Wong and Chan (2007) method for reducing the beam stiffness to allow for semi-rigidity of the joints, Table 8.2 compares the predicted per-storey lateral sway deflections of the four frames in Section 8.2 for varying levels of initial joint stiffness using the two rigid-frame deflection calculation methods against those from computer-based frame analysis using rotational springs. The results indicate that if the joint stiffness is greater than $2.0K_b$, the accuracy of the analytical methods is similar as that as shown in Table 8.1 for rigid frames. This confirms that the Wong and Chan (2007) method of calculating the reduced beam stiffness is accurate in this context. Because they are approximate methods, the hand calculation methods became less accurate for lower levels of joint stiffness. Nevertheless, as approximate predictions in scheme design, the hand calculation methods, particularly the basic method, may still be considered usable.
Table 8.1. Comparison of predicted rigid frame lateral deflections

<table>
<thead>
<tr>
<th>Frame</th>
<th>Storey</th>
<th>FEM (mm)</th>
<th>Basic [8.3] (mm)</th>
<th>Error</th>
<th>Wind moment [8.13] (mm)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>4.58</td>
<td>5.09</td>
<td>11.1%</td>
<td>4.03</td>
<td>-12.0%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5.66</td>
<td>5.76</td>
<td>1.9%</td>
<td>5.31</td>
<td>-6.0%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8.26</td>
<td>8.41</td>
<td>1.8%</td>
<td>7.97</td>
<td>-3.4%</td>
</tr>
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<td></td>
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<td>9.04</td>
<td>9.53</td>
<td>5.5%</td>
<td>8.80</td>
<td>-2.7%</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>1.06</td>
<td>1.12</td>
<td>5.8%</td>
<td>0.87</td>
<td>-17.6%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2.07</td>
<td>2.07</td>
<td>0.0%</td>
<td>1.93</td>
<td>-7.0%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3.06</td>
<td>3.02</td>
<td>-1.2%</td>
<td>2.89</td>
<td>-5.6%</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3.22</td>
<td>3.33</td>
<td>3.3%</td>
<td>3.11</td>
<td>-3.7%</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
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<td>1.05</td>
<td>2.4%</td>
<td>0.84</td>
<td>-17.8%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2.00</td>
<td>1.94</td>
<td>-2.8%</td>
<td>1.87</td>
<td>-6.6%</td>
</tr>
<tr>
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<td>2</td>
<td>2.95</td>
<td>2.84</td>
<td>-3.9%</td>
<td>2.80</td>
<td>-5.2%</td>
</tr>
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<td>3.18</td>
<td>1.1%</td>
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<td>-4.2%</td>
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<tr>
<td>D</td>
<td>6</td>
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<td>1.50</td>
<td>-28.8%</td>
</tr>
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<td>5</td>
<td>3.78</td>
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<td>-7.8%</td>
<td>3.28</td>
<td>-13.3%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>5.43</td>
<td>4.97</td>
<td>-8.6%</td>
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<td>-12.7%</td>
</tr>
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<td>3</td>
<td>7.02</td>
<td>6.43</td>
<td>-8.4%</td>
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<td>-11.6%</td>
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<td>8.39</td>
<td>7.90</td>
<td>-5.8%</td>
<td>7.67</td>
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</tr>
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<td>1</td>
<td>7.32</td>
<td>6.94</td>
<td>-5.1%</td>
<td>6.64</td>
<td>-9.3%</td>
</tr>
</tbody>
</table>

*Mean 4.7%  St.dev. 5.6%  Max 11.1%*

**Positive values indicate overestimation and vice versa.**

Table 8.2. Percent errors in semi-rigid frame lateral deflection predictions using the Basic method compared to computer based frame analysis with rotational springs

<table>
<thead>
<tr>
<th>Frame</th>
<th>Storey</th>
<th>Rigid 5.0Kb</th>
<th>2.0 Kb</th>
<th>1.0 Kb</th>
<th>0.75 Kb</th>
<th>0.5 Kb</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>11.1</td>
<td>-8.1</td>
<td>-26.5</td>
<td>-43.3</td>
<td>-51.5</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.9</td>
<td>-0.1</td>
<td>-2.2</td>
<td>-3.8</td>
<td>-4.3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.8</td>
<td>3.7</td>
<td>7.1</td>
<td>12.9</td>
<td>17.7</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>5.5</td>
<td>5.6</td>
<td>4.4</td>
<td>1.1</td>
<td>-2.0</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>5.8</td>
<td>2.8</td>
<td>-1.2</td>
<td>-5.7</td>
<td>-8.1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0</td>
<td>0.3</td>
<td>1.4</td>
<td>4.0</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-1.2</td>
<td>2.0</td>
<td>7.2</td>
<td>16.0</td>
<td>21.3</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3.3</td>
<td>3.4</td>
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<td>-3.9</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
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<td>-0.3</td>
<td>-3.1</td>
<td>-7.1</td>
<td>-9.1</td>
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<td>-0.8</td>
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<tr>
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<td>-3.9</td>
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<td>17.8</td>
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<tr>
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<td>1.4</td>
<td>0.7</td>
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<td>-4.0</td>
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<tr>
<td>D</td>
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<td>-10.5</td>
<td>-15.0</td>
<td>-18.9</td>
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</tr>
</tbody>
</table>

*Mean 4.7%  St.dev. 5.6%  Max 11.1%*  

**Positive values indicate overestimation, and vice versa.**
Table 8.3. Percent errors in semi-rigid frame lateral deflection predictions using the SCI method compared with computer-based frame analysis with rotational springs

<table>
<thead>
<tr>
<th>Frame</th>
<th>Storey</th>
<th>Rigid</th>
<th>5.0 $K_b$</th>
<th>2.0 $K_b$</th>
<th>1.0 $K_b$</th>
<th>0.75 $K_b$</th>
<th>0.5 $K_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>-12.0</td>
<td>-10.1</td>
<td>-11.4</td>
<td>-5.9</td>
<td>-8.3</td>
<td>-14.7</td>
</tr>
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<td>3</td>
<td>-6.0</td>
<td>-5.3</td>
<td>-6.1</td>
<td>-9.6</td>
<td>-6.2</td>
<td>-3.3</td>
</tr>
<tr>
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<td>2</td>
<td>-3.4</td>
<td>0.5</td>
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<td>-0.3</td>
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<td>-10.4</td>
</tr>
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<td>-11.8</td>
<td>-18.4</td>
<td>-22.5</td>
<td>-28.1</td>
</tr>
</tbody>
</table>

Mean: 9.8 7.8 9.1 13.6 15.2 26.4
St. dev.: 6.6 8.8 12.1 17.9 19.3 31.8
Max.: 28.8 31.9 36.5 39.1 39.3 74.7

*Positive values indicate overestimation.

8.6 Joint design procedure for unbraced low-rise steel frames using tubular columns

In the previous sections, it has been shown that not only is the design of low-rise unbraced semi-rigid frames feasible, it can be done with simple hand calculation methods. To develop a method that can be followed in practice, it is necessary to develop a design procedure to assist the designer in selecting suitable joint details. Such a procedure is proposed in Figure 8.10 for steel frames using flowdrilled endplate connections to RHS columns. This procedure is based on selecting joint details to give the joint sufficient rotational stiffness to control the frame sway deflection. The equations for prediction of the joint initial stiffness are from those derived in Section 4.2 for the RHS face in transverse tension and in EN 1993-1-8 (CEN, 2005) for the endplate and bolt components. The initial stiffnesses of various
joint components using bolted endplate to RHS sections with or without concrete infill are listed in Table 8.4. Calculation of the required levels of joint stiffness is based on the Basic method described in Section 8.5.1 and validated in Section 8.5.3.

It is important to consider the relative influences of the various geometrical parameters of a joint that dictate the initial joint stiffness of bolted endplate joints to RHS sections so that the determination of connection parameters can be done as quickly as possible with the minimum increase in construction cost. In a practical situation, many of these parameters would be fixed for purposes of economy. This includes the RHS thickness that the initial joint stiffness is most sensitive to. Fortunately, other parameters can be modified to improve the initial stiffness characteristics of such joints with little or no effect on the cost of construction. The following describes a step-by-step procedure.

Firstly, modifications that require little or no additional construction cost should be considered. The easiest and most cost-effective modification is to increase the horizontal bolt spacing while ensuring that the bolts are not placed too close to the RHS sidewalls. The connection lever arm can also be increased by increasing the vertical bolt spacing. Secondly, modifications that require a minor increase in construction costs can be considered should the above adjustment not yield the required joint stiffness. In increasing order of costs, this includes increasing the endplate thickness, the endplate width, and increasing the lever arm by means of increasing the number of bolt rows and/or conversion from flush to extended endplate connection. Alternatively, concrete filling an RHS can give a significant increase in the joint initial stiffness. Particularly, concrete filling an HSS can bring about significant cost reduction in itself. Modifications to the bolt dimensions typically have an insignificant effect on the joint initial stiffness. Example joint designs are presented
in Appendix C to provide a step-by-step guide on the usage of the joint design flowchart for unbraced frame design.

<table>
<thead>
<tr>
<th>Step 1. Preliminary sizing of beam/column sections based on simple construction.</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Step 2. Determine the required $S_{j,ini}$ using the hand-calculation method using the reduced beam-member stiffness coefficient.</th>
</tr>
</thead>
</table>

| Step 3. Check if the joint details provide sufficient initial stiffness.  
Start with a standard bolt pitch (i.e. 80mm), $t_p$ equal to $t_c$, and number of bolt rows based on shear resistance requirements. If there is sufficient stiffness, skip to Step 5. |
|-------------------------------|

| Step 4. Improve joint detailing in the following order:  
1. Increase bolt pitch  
2. Increase vertical spacing between bolt rows  
3. Increase endplate thickness/width  
4. Increase number of bolt rows and/or change to extended endplate.  
5. Use concrete filled column (Repeat Step 2)  
Check if the joint provides sufficient initial stiffness. If sufficient, proceed to next step.  
If insufficient, repeat Step 4 or determine that unbraced design is not feasible. |
|-------------------------------|

| Step 5. Check joint strength to ensure that SLS loads remain in the elastic range.  
Check for sufficient joint ductility. If both these conditions are met, the joint is suitable for unbraced frame design. |
|-------------------------------|

Figure 8.10. Joint design flowchart for usage with unbraced frame design.
Table 8.4. Summary of initial stiffnesses of joint components for bolted endplate connections to RHS columns

<table>
<thead>
<tr>
<th>Component</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolts in tension (EN 1993-1-8)</td>
<td>$k_{2,indv} = 1.6 A_s / L_{bott}$</td>
</tr>
<tr>
<td>Endplate in bending (EN 1993-1-8)</td>
<td>$k_{3,indv} = 0.9 t_p / m^3$</td>
</tr>
<tr>
<td>RHS in transverse tension (Chapter 4)</td>
<td>$k_{RHS,indv} = \frac{f_1 t_c^3}{a^2 S \cos \left( \frac{a_1 \pi}{2a} \right)}$</td>
</tr>
</tbody>
</table>

where

$f_1 = \left( \frac{11.5 a_k S}{a_k + S} \right)$ for unfilled or $(\frac{11.5}{2.025 S})$ for concrete-filled columns

$S = 0.143(a_1/a)^2 - 0.306(a_1/a) + 1.076$

$k_r = \frac{4 E I}{a_{RHS} \left( \frac{1.5 w_{RHS} + d_{RHS}}{2.0 w_{RHS} + d_{RHS}} \right)}$

Equivalent component stiffness from individual bolt row stiffness (EN 1993-1-8)

$k_{eq} = \sum k_{indv} h_r / z_{eq}$

where

$z_{eq} = \frac{\sum_i k_{eff,i} h_r^2}{\sum_i k_{eff,i} h_r}$

Joint initial stiffness (EN 1993-1-8)

$S_{j,ini} = \frac{E z^2}{\sum k_{eq}}$

Where lever arm, $z$, is as defined in Figure 6.15 of EN 1993-1.8.

8.7 Conclusion

This chapter investigates the feasibility of using semi-continuous frame design for unbraced low-rise steel frames using RHS columns with or without concrete infill. Using a set of four low-rise steel frames as examples, it concludes that the required joint stiffness is modest to control the frame sway deflection within SLS limits.

The main emphasis of this chapter is to develop a hand calculation method to allow the designer to quickly establish the necessary joint details to achieve the required level of joint initial stiffness. It addressed the following three aspects: (1) an approximate method to calculate the sway deflection of unbraced frames with rigid joints; (2) an approximate method to incorporate the effects of semi-rigid joint
stiffness; (3) a procedure to determine the details of joints to RHS columns to reach the required joint stiffness.

For calculating the lateral sway in rigid frames under serviceability loads, two methods were compared for their ease of use and accuracy. It was found that the method of Smith and Coull (1999) and Taranath (1997) would be more preferable than the SCI wind moment method (Hensman and Way, 2000) because the former gave more accurate results and is completely analytical but the SCI wind moment method requires looking up charts which would not be suitable for fast automated applications. To incorporate the effects of semi-rigid joint stiffness, this study confirmed that the method of Wong and Chan (2007) would be suitable to calculate the equivalent reduced beam stiffness to be used with rigid frame analysis.

Finally, a few examples were used to demonstrate how to change the details of bolted joints to tubular columns in a systematic procedure to achieve the required joint stiffness to control the frame lateral sway deflection. Using concrete infill is a particularly useful way of increasing the joint stiffness.
Chapter 9

Conclusions and recommendations for future work

9.1 Summary of presented work and conclusions

The main objectives of this research are to (1) develop an effective analytical method to characterise the load-deflection behaviour of the RHS face in tension component in addition to the moment-rotation behaviour of the whole joint and (2) determine an effective hand-calculation method for predicting serviceability limit state sway behaviour in unbraced semi-continuous frames. The combination of joint and frame hand calculation methods allows quick calculations to be made so that feasibility studies can be carried out. The generous levels of initial stiffness offered by flowdrilled endplate connections to RHS columns is used to design low-rise unbraced semi-continuous frames at the serviceability limit state. These objectives are satisfied in the following chapters:

Chapter 1 introduces the subject covering the benefits of using tubular columns and design of semi-continuous frames. The objectives and originality of the research are presented with a description of thesis structure.
Chapter 2 gives a literature review of the subject covering joint classification, methods of joint characterisation, the Eurocode 3 component method with existing equations, and the global analysis of semi-continuous frames.

Chapter 3 gives whole joint and flowdrilled connection FEM techniques validated against testing by France et al. (1999) and British Steel (1996b). Existing techniques for characterisation of friction and material properties as well as modelling of concrete core and weld details are confirmed. A mesh sensitivity study is conducted to determine the optimum usage of mesh elements in discretizing the RHS face in tension component. A new approach for modelling flowdrilled connections to the RHS face is developed using a standard bolt with a very small diameter on the column side to mimic pullout behaviour that compares favourably with British Steel (1996b) test results. The combination of existing and new techniques is applied to the modelling of France et al. (1999) whole joint tests demonstrating their suitability for modelling flowdrilled endplate connections to RHS columns.

Chapter 4 looks at the initial stiffness characterisation of the RHS face in tension component. The derivation of new equations based on existing equations by Jaspart et al. (2004) is given which has a significantly improved range of validity and accuracy. This is because equations are derived on a per bolt row basis thus overcoming the shortcomings of existing equations derived for a group of bolt rows which severely limits the range of applicability to joints with two bolt rows. Numerous simplifications are made to the equations to make them more practical for use in hand calculations including replacing an infinite series with a cubic equation. A parametric study using FEM shows that the new equations predict the initial stiffness with less than 15% error for all realistic geometries with 8.5<\(a/t_c\)<27.7 and 0.1<\(a/t\)<0.8. More importantly, there is no range of validity restrictions for \(b/t\) as imposed by Jaspart et al.(2004) therefore the equations can be used for virtually any joint. The validation
study using France et al. (1999) testing shows that using the new equations, the joint initial stiffness is predicted with a mean error of 1.8% and maximum of 21.4%.

Chapter 5 covers the strength characterisation of the RHS face in tension component. Bending strength equations are derived using yield line theory for a new elliptical yield line mechanism that gives the minimum solution for all geometries with up to 8% improvement over existing straight and circular yield line equations given by Ghobarah et al. (1996) and British Steel (1996b). A revised definition of the deformable width of the RHS face is used to reflect on hinge locations observed in the FEM. The equations are consistent when compared against a parametric study for both hollow and filled sections with mean errors of approx. 5% and maximum of approx. 20%. A comparison of the yield line plots obtained in the parametric study show that the elliptical yield line pattern replicates the yield line mechanism accurately and that the revised definition of the deformable width of the RHS face should be used. The equations are validated against existing results from testing (British Steel, 1996) for the hollow section showing reasonable agreement. The membrane action strength is derived using internal work principles and assumes that membrane action deformation can be determined from a simple relationship between cross section geometry and material failure strain. In addition, equations for the thread stripping capacity of flowdrilled connections due to gross deformation of the RHS face are newly derived. The reduction in contact is related to the RHS thickness and is given in the form of a reduction factor to be applied to existing equations for the full thread stripping capacity. A validation study against British Steel (1996b) tests shows that the thread stripping capacity is predicted consistently across a wide range of geometries with a mean error of 2% and maximum of approx. 25%.

Chapter 6 covers the deformation capacity characterisation of the RHS face in tension component for membrane action failure based on simple assumptions
regarding cross section geometry and material ultimate strain. Deformation capacity for weaker failure mechanisms are predicted using linear interpolation methods. The parametric study for strength is used to check the equations showing that deformation capacity is predicted consistently with a mean error of -11.9% and -2.0% for hollow and concrete-filled sections respectively. Validation against British Steel (1996b) testing shows that the equations are suitable for predicting the failure deformation with a mean error of -12.0% and maximum of 35.0%. In both cases, the deformation capacity is predicted conservatively owing to the fact that there is additional displacement due to opening of the flowdrilled RHS face that is not captured in the simple assumptions used in defining the membrane action deformation.

Chapter 7 looks at the joint assembly using the newly derived equations for the RHS face in tension combined with existing equations for the endplate and bolts by Beg et al. (2004) and EN 1993-1-8 (CEN, 2005). The joint is assembled using a simplified mechanical model based on characterising individual component behaviour as a non-linear spring with a bilinear load-deformation relationship and finding their relative contribution towards overall joint rotation at characteristic strength value. The approach is validated against France et al. (1999) testing. The analytical method predicted the 0.03 radian characteristic strength with a mean error of -1.9% and maximum of 33.3%. The strength of tests with thread stripping failure is predicted with a mean error of 23.7% and maximum of 31.5%. When using a lower thread stripping reduction factor than for British Steel (1996b) tests, the failure strength is predicted with a mean error of 3.8% and maximum of 11.4%. In all tests, the curvature of the moment-rotation curve is predicted well. Overall, the analytical approach gives a consistent prediction of the testing moment-rotation behaviour using hand calculations. By using the analytical approach, it is possible to conduct effective
parametric studies to optimise joint design parameters and gain a detailed understanding about component contributions to joint behaviour.

Chapter 8 covers joint initial stiffness, strength, and ductility requirements for unbraced frame design as well as developing a simplified hand calculation method to quickly establish the necessary joint details to achieve the required level of joint initial stiffness. For calculating the lateral sway in rigid frames under serviceability loads, the method of Smith and Coull (1999) and Taranath (1997) was more preferable than the SCI wind moment method (Hensman and Way, 2000) because the former gave more accurate results and is completely analytical. To incorporate the effects of semi-rigid joint stiffness, this study confirmed that the method of Wong and Chan (2007) would be suitable to calculate the equivalent reduced beam stiffness to be used with rigid frame analysis. A systematic joint design procedure for unbraced low-rise steel frames using tubular columns is presented with worked examples to demonstrate their usage.

9.2 Recommendations for future research

The following recommendations based on directing findings and related areas of this research and suggested:

(1) The comparison of the monotonic and cyclic loading for whole joints given in Section 3.4.2 touches on the fact that repeated loading and unloading in the elastoplastic regime can potentially affect the ultimate and failure strength behaviour due to fatigue. It would be useful to be able to analytically quantify the reduction in strength due to fatigue for bolted connections to tubular columns. This research considers that normal loading and unloading of joints in real frame behaviour occurs in the elastic range and therefore fatigue is not an issue that requires immediate attention.
(2) The characterisation of the whole joint properties recognises that the RHS face in tension, endplate in tension, and bolts in tension components are the three main components that define the moment-rotation behaviour for joints where the compression zone width is roughly equal to or larger than the deformable width of the RHS face. The RHS face in compression component is not included as it has a negligible effect on ductility under these conditions. For cases where the compression zone width is much smaller, it is necessary to characterise the RHS face in compression component. To make use of the beneficial properties (initial stiffness/strength) that bolted endplate connections to RHS columns offer however, the compression zone width should be large so this can be ignored by specifying a sufficiently wide endplate.

(3) Although the equation for thread stripping due to gross deformation of flowdrilled holes of the RHS face is adequately validated as an individual component against British Steel (1996b) testing, there is limited validation done for this component as part of a whole joint. This is due to the large ductility that these joints offer before failure that the testing rig in France et al. (1999) could not handle. The validation conducted when using the few tests of France et al. that exhibited thread stripping failure show that \( \tau = 60 \) is suitable but \( \tau = 50 \) gives a better prediction. Further joint tests with thread stripping failure as the failure mode must be carried out to confirm the component equation is valid when assembled as part of a joint. Difficulties in testing joints with large ductility may be avoided for example by shortening the lever arm of the testing rig.

(4) All sections investigated in the parametric study of the RHS face in tension component show that membrane action occurs after the limits of bending strength. This research was interested in flowdrilled connections that almost always failed due to thread stripping caused by gross deformation of the RHS face (one specimen in
British Steel testing experienced bolt tensile failure but this was predicted by bolt equations). Therefore, validation of the full membrane action capacity was not specifically conducted. Although usage of flowdrilled connections is perfectly adequate for structural applications, if a more robust connection is used, it would be useful to investigate the validity of the full membrane action capacity (and deformation capacity) of the RHS face in tension.

(5) With the derivation of equations to give the full load-deflection behaviour of the key components in bolted endplate to RHS column connections using flowdrill bolts, they can be used to investigate structural robustness. Because the equations are fully analytical, they can be used to conduct rapid calculations according to various design scenarios which was previously not possible because full joint load-deflection behaviour had to be obtained from full-scale testing or numerical models which is time-consuming.

(6) This research focused on the joint characterisation of bolted endplate connections to RHS columns under ambient temperatures. To further the understanding and uses of this connection type, it is necessary to extend the analytical method to performance under fire conditions.
References


Jaspart, J., Weynand, K., & Klinkhammer, R. (2003). *Development of a full consistent design approach for bolted and welded joints in building frames and trusses between steel members made of hollow and/or open sections - Application of the component method* - CIDECT Report 5BP-6/03. Université de Liégé.


Publications


Appendix A Load-deflection curves

A.1. British Steel (1996b) RHS face in tension component tests
A.2. France et al. (1999) joint tests
Test 15c

Test 16c

Test 17c
A.3. FEM and analytical load-deflection curves (Hollow RHS)

Variation in thickness, $t$.
Variation in horizontal bolt spacing, $a_i$
Variation in vertical bolt spacing, $b_v$

# 1.5.1.2

Load (kN) vs. Deflection (m)

- FEM
- Analytical

# 1.15.1.2

Load (kN) vs. Deflection (m)

- FEM
- Analytical

# 1.20.1.2

Load (kN) vs. Deflection (m)

- FEM
- Analytical
Variation in RHS width, wRHS and depth, dRHS

#1.1.1.3

//1.1.1.4
Variation in material yield stress, $f_y$
Variation in bolt rows, n
A.4. FEM and analytical load-deflection curves (Filled RHS)

Variation in thickness, $t_c$
Variation in horizontal bolt spacing, \( a_b \).
Variation in vertical bolt spacing, $b$.
Variation in RHS width, $w_{RHS}$ and depth, $d_{RHS}$

Variation in material yield stress, $f_y$

253
Variation in bolt rows, n
Appendix B Yield line plots

B.1. Introduction

In this section, a selection of yield line plots from the finite element analysis of the RHS face in tension component parametric study are given. These are compared with the analytical yield line patterns for the elliptical yield line mechanism explained in Section 5.2.3 that give the minimum solution in all cases.

The yield line plots are obtained by plotting the maximum principal strain component at the bending strength capacity determined from analytical equations. The maximum limits to define the contours are modified slightly in each case to better display the yield lines and this is reflected in the legend given for each plot. The yield lines are plotted on the undeformed shape to allow easy comparison with the geometry.

The sections compared show that the assumed elliptical yield lines are suitable to reflect the actual yield line formation in the FEA. This is regardless of variations in geometric parameters such as a, b, or t.
B.2. Hollow RHS yield line plots

#6.1.1.2

\[ a_{1/2} = 30\text{mm} \quad c = 50\text{mm} \quad 2t_c = 20\text{mm} \]

\[ b_{1/2} = 50\text{mm} \quad d = 60\text{mm} \quad w_{1/2} = 100\text{mm} \quad a_{1/2} = 80\text{mm} \]

\[ \therefore a = w_{\text{RHS}} - 4t_c = 160\text{mm} \]
#1.1.1.2

Negative rotation yield
Positive rotation yield line

d=48mm

b/2=50mm

a/2=50mm  c=30mm  2t_c=20mm

w/2=100mm

a/2=80mm

∴ a=w_{RHS}-4t_c=160mm
\[ a = \frac{w_{RHS} - 4t_c}{2} = 160\text{mm} \]

\[ a/2 = 80\text{mm} \]

\[ w/2 = 100\text{mm} \]

\[ c = 30\text{mm} \]

\[ 2t_c = 20\text{mm} \]

\[ d = 48\text{mm} \]

\[ b/2 = 75\text{mm} \]

\[ \therefore a = w_{RHS} - 4t_c = 160\text{mm} \]


#1.20.1.2

\[
\begin{align*}
2t & = c = 50 \text{mm} \\
\therefore a & = w_{\text{RHS}} - 4t_c = 160 \text{mm}
\end{align*}
\]
#1.1.5.2

\[ a = w_{\text{RHS}} - 4t_c = 180\text{mm} \]
#1.1.16.2

\[ a/2 = 50\text{mm} \quad c = 18\text{mm} \quad 2t_c = 32\text{mm} \]

\[ \therefore a = w_{\text{RHS}} - 4t_c = 136\text{mm} \]
B.3. Concrete-filled RHS yield line plots

#6.1.1.2C

\[
\begin{align*}
& \text{Negative rotation yield} \\
& \text{Positive rotation yield line}
\end{align*}
\]

\[
\begin{align*}
& a/2 = 30\text{mm} \\
& c = 50\text{mm} \\
& 2t_c = 20\text{mm} \\
& w/2 = 100\text{mm} \\
& a/2 = 80\text{mm} \\
& \therefore a = w_{RHS} - 4t_c = 160\text{mm}
\end{align*}
\]

d = 60\text{mm} \\
b/2 = 50\text{mm}
#1.1.1.2C

\[ a = w_{RHS} - 4t_c = 160\text{mm} \]
\[ a = w_{RHS} - 4t_c = 160\text{mm} \]
\[
\begin{align*}
2t_c &= 20\text{mm} \\
50\text{mm} &= a \\
a/2 &= 80\text{mm} \\
\therefore a &= w_{\text{RHS}} - 4t_c = 160\text{mm}
\end{align*}
\]
\#1.1.5.2C

\[ a = w_{\text{RHS}} - 4t_c = 180\text{mm} \]
\[a \approx w_{RHS} - 4t_c = 136 \text{mm}\]
Appendix C

Example joint designs for specific frame configurations

C.1. Introduction

In this section, three design examples based on the design flowchart in Figure 8.10 are presented for unbraced frames A and B investigated in Section 8.2. This will give insight into the different approaches to changing joint detailing where the initial design is insufficient for unbraced frame design. It will also cover the checks necessary to ensure adequate strength to ensure the joint remains in its elastic state and that the joint has sufficient ductility. Joints are designed symmetrically to ensure the stiffness level is maintained on load reversal (i.e. due to wind loads).
C.2. Example 1a: Frame A – Inner column joint

Step 1. Preliminary sizing of beam/column sections based on simple construction

Load case 6m bays in both directions. 4 storey frame.
30kN/m Dead loading (5kN/m² over 6m span)
30kN/m Live loading (5kN/m² over 6m span)
SLS loading of 1.0 Dead + 1.0 Wind
ULS loading of 1.5 Live + 1.35 Dead

Columns
SHS 220×220×14.2 S355 (Inner)
SHS 220×220×6.3 S355 (Edge)

Beams
UB 406×178×74 S275 (Inner floors)
UB 203×133×25 S275 (Roof)

Bolts
Grade 8.8 M20 bolts, Non-preloaded (f_{ub} = 800N/mm²)
For all sections, use E=210×10^8 N/m²

Step 2. Determine required joint initial stiffness

Use the basic hand calculation method with reduced beam-member stiffness coefficient and assume that the required joint initial stiffness is twice the secant stiffness calculated to limit the frame sway deflection. The minimum joint secant stiffness from the basic hand calculation method gives 3.0 K_b as given in Table A.1 so the required joint initial stiffness is 6.0K_b.

\[ \therefore \text{Required joint initial stiffness: } 6.0 \text{K}_b \]

Table A.1. SLS check for 3.0.K_b joint secant stiffness

<table>
<thead>
<tr>
<th>Storey</th>
<th>Storey height (m)</th>
<th>Storey deflection (mm)</th>
<th>d=h/300 (mm)</th>
<th>Within limit?</th>
<th>Cumulative height (m)</th>
<th>Cumulative deflection (mm)</th>
<th>d=h/300 (mm)</th>
<th>Within limit?</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4</td>
<td>7.04</td>
<td>13.33</td>
<td>Yes</td>
<td>16</td>
<td>41.40</td>
<td>53.33</td>
<td>Yes</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>9.05</td>
<td>13.33</td>
<td>Yes</td>
<td>12</td>
<td>34.37</td>
<td>40.00</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>13.31</td>
<td>13.33</td>
<td>Yes</td>
<td>8</td>
<td>25.32</td>
<td>26.67</td>
<td>Yes</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>12.01</td>
<td>13.33</td>
<td>Yes</td>
<td>4</td>
<td>12.01</td>
<td>13.33</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Step 3. Check if the joint provides sufficient stiffness

Start with a flush endplate with standard bolt pitch (80mm), endplate thickness equal to RHS thickness (14.2mm rounded up to 15mm), top and bottom bolt rows 50mm from beam flange, and 6mm fillet welds.

Determine the number of bolt rows needed for shear resistance (ULS)

Shear load, \( N_{v,Ed} = 0.5 \times 6m \times (1.5 \times 30kN/m + 1.35 \times 30kN/m) = 256.5kN \)

Shear resistance of bolts

\[
F_{v,Rd,bolt} = \frac{\alpha_v f_{ub} A_S}{\gamma_{M2}}
\]

where the shear plane passes through the threaded portion of the bolt

for Grade 8.8 bolt, \( \alpha_v = 0.6 \)

\[
F_{v,Rd,bolt} = 0.6 \times 800N/mm^2 \times 245mm^2 / 1.25 = 94.1kN
\]

Number of bolts required for shear resistance

\[
\frac{N_{v,Ed}}{F_{v,Rd,bolt}} = \frac{256.5}{94.1} = 2.72
\]

∴ Use 2 rows of 2 bolts

<table>
<thead>
<tr>
<th>Beam</th>
<th>Column</th>
<th>Bolts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>Column</td>
<td>Bolts</td>
</tr>
<tr>
<td>UB 406×178×74</td>
<td>SHS 220×220×14.2</td>
<td>M20</td>
</tr>
<tr>
<td>Endplate thickness (m)</td>
<td>0.015</td>
<td></td>
</tr>
<tr>
<td>Endplate width (m)</td>
<td>0.18</td>
<td></td>
</tr>
<tr>
<td>Top edge to top bolt row distance (m)</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Lower edge to lower bolt row distance (m)</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Bolt hole separation, ( a_t ) (m)</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>( f ) (GPa)</td>
<td>0.3</td>
<td>210</td>
</tr>
<tr>
<td>( h_t ) (m) Row 1</td>
<td>0.342</td>
<td></td>
</tr>
<tr>
<td>Row 2</td>
<td>0.042</td>
<td></td>
</tr>
<tr>
<td>( z ) (m) lever arm</td>
<td>0.334</td>
<td></td>
</tr>
<tr>
<td>Bolts in tension, ( k_{eq} )</td>
<td>2.74E-02</td>
<td></td>
</tr>
<tr>
<td>Endplate in bending, ( k_{eq} )</td>
<td>1.42E-02</td>
<td></td>
</tr>
<tr>
<td>RHS in transverse tension, ( k_{eq} )</td>
<td>1.09E-03</td>
<td></td>
</tr>
<tr>
<td>Joint stiffness (kNm/rad)</td>
<td>22857</td>
<td></td>
</tr>
<tr>
<td>( K_b ) (for ( L_b=6m ))</td>
<td>2.39</td>
<td></td>
</tr>
</tbody>
</table>

2.39 \( K_b < 6.0K_b \) ∴ Insufficient stiffness, proceed to Step 4.
Step 4. Improve joint detailing

Begin by increasing the bolt pitch to 120mm.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Column</th>
<th>UB 406×178×74</th>
<th>SHS 220×220×14.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolts</td>
<td>M20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Endplate thickness (m)</td>
<td>0.015</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Endplate width (m)</td>
<td>0.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top edge to top bolt row distance (m)</td>
<td>0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lower edge to lower bolt row distance (m)</td>
<td>0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bolthole separation, $a_t$ (m)</td>
<td>0.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v$ (GPa)</td>
<td>0.3</td>
<td>210</td>
<td></td>
</tr>
<tr>
<td>$E$ (GPa)</td>
<td>210</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h_t$ (m) Row 1</td>
<td>0.342</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Row 2</td>
<td>0.042</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$z$ (m) lever arm</td>
<td>0.334</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bolts in tension, $k_{eq}$</td>
<td>2.74E-02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Endplate in bending, $k_{eq}$</td>
<td>3.79E-03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RHS in transverse tension, $k_{eq}$</td>
<td>2.09E-03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint stiffness (kNm/rad)</td>
<td>30042</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_b$ (for $L_b=6m$)</td>
<td>3.14</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$3.14 \, K_b < 6.0K_b \quad \therefore \text{Insufficient stiffness, modify further.}$

Next, increase endplate thickness to 20mm.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Column</th>
<th>UB 406×178×74</th>
<th>SHS 220×220×14.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolts</td>
<td>M20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Endplate thickness (m)</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Endplate width (m)</td>
<td>0.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top edge to top bolt row distance (m)</td>
<td>0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lower edge to lower bolt row distance (m)</td>
<td>0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bolthole separation, $a_t$ (m)</td>
<td>0.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v$ (GPa)</td>
<td>0.3</td>
<td>210</td>
<td></td>
</tr>
<tr>
<td>$E$ (GPa)</td>
<td>210</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h_t$ (m) Row 1</td>
<td>0.342</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Row 2</td>
<td>0.042</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$z$ (m) lever arm</td>
<td>0.334</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bolts in tension, $k_{eq}$</td>
<td>2.33E-02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Endplate in bending, $k_{eq}$</td>
<td>8.99E-03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RHS in transverse tension, $k_{eq}$</td>
<td>2.09E-03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint stiffness (kNm/rad)</td>
<td>36970</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_b$ (for $L_b=6m$)</td>
<td>3.87</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$3.87 \, K_b < 6.0K_b \quad \therefore \text{Insufficient stiffness, modify further.}$
Next, increase to four rows of bolts.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Column</th>
<th>Endplate thickness (m)</th>
<th>Endplate width (m)</th>
<th>Top edge to top bolt row distance (m)</th>
<th>Lower edge to lower bolt row distance (m)</th>
<th>Bolthole separation, a_*(m)</th>
<th>ν</th>
<th>E (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column SHS 220×220×14.2 M20</td>
<td></td>
<td>0.02</td>
<td>0.18</td>
<td>0.05</td>
<td>0.05</td>
<td>0.12</td>
<td>0.3</td>
<td>210</td>
</tr>
<tr>
<td>Bolts M20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| h_*(m) Row 1              |                             | 0.342                  |                    |                                       |                                           |                             |     |         |
| Row 2                     |                             | 0.242                  |                    |                                       |                                           |                             |     |         |
| Row 3                     |                             | 0.142                  |                    |                                       |                                           |                             |     |         |
| Row 4                     |                             | 0.042                  |                    |                                       |                                           |                             |     |         |
| z (m) lever arm            |                             | 0.284                  |                    |                                       |                                           |                             |     |         |

| Bolts in tension, k_eq   |                             | 6.28E-02               |                    |                                       |                                           |                             |     |         |
| Endplate in bending, k_eq |                             | 2.70E-02               |                    |                                       |                                           |                             |     |         |
| RHS in transverse tension, k_eq |                   | 5.62E-03               |                    |                                       |                                           |                             |     |         |
| Joint stiffness (kNm/rad) |                             | 73322                  |                    |                                       |                                           |                             |     |         |
| K_b,(for L_b=6m)          |                             | 7.67                   |                    |                                       |                                           |                             |     |         |

7.67 K_b > 6.0 K_b ∴ The joint initial stiffness is sufficient for unbraced frame design.

Step 5. Check joint strength and ductility

Using the joint load-deflection characteristics from the equations proposed in Chapter 7 gives the moment-rotation curve in Figure A.1 with a yield strength of 181.4kNm and ultimate strength of 272.2kNm using nominal material properties of f_y = 275N/mm² for S275 steel and f_y = 355N/mm² for S355 steel. In comparison, the moment resistance of the beam is 412.5kNm which classifies this connection as partial strength. Using the equation in Table 2.3 for semi-rigid joints, the joint load can be estimated assuming that the joint remains in its elastic state and therefore S_j = S_j,ini. This gives a load of approximately 71.4kNm for the SLS condition.

**184.8 kNm > 71.4kNm**

The joint will remain in its elastic state in the SLS condition and therefore it is safe to assume that it will not accumulate permanent rotations. Based on the 0.03
radian ductility criterion, the joint has adequate ductility. The predicted failure mode is
endplate bending failure at 272.2 kNm and a rotation of 0.072 radians.

\[ \text{The joint is suitable for unbraced frame design.} \]

![Moment-rotation curve for Example Joint 1a](image)

Figure A.1. Moment-rotation curve for Example Joint 1a

C.3. Example 1b: Frame A – Inner column joint (Concrete-filled RHS design)

As an alternative design, the joint given in Example 1a is redesigned using a
concrete-filled RHS section. While this modification would mainly be considered to
increase column resistance and thus reduce column section size, this example
highlights how there is also a generous increase in the joint initial stiffness. For
simplicity, the same RHS section size is used with a typical C30 concrete in-fill.

Step 1. Preliminary sizing of beam/column sections based on simple construction

Same as previous example.
Step 2. Determine required joint initial stiffness

The stiffness of the composite columns is assumed the sum of the contribution of the column steel and concrete components. Use the basic hand calculation method with reduced beam-member stiffness coefficient, and assume that the required joint initial stiffness is twice the secant stiffness calculated to limit the frame sway deflection. The minimum joint secant stiffness required from the basic hand calculation method gives 2.3 $K_b$ as given in Table A.2 so the required joint initial stiffness is 4.6 $K_b$.

∴ Required joint initial stiffness: 4.6 $K_b$

Table A.2. SLS check for 2.3 $K_b$ joint secant stiffness

<table>
<thead>
<tr>
<th>Storey</th>
<th>Storey height (m)</th>
<th>Storey deflection (mm)</th>
<th>d=h/300 limit (mm)</th>
<th>Within limit?</th>
<th>Cumulative height (m)</th>
<th>Cumulative deflection (mm)</th>
<th>d=h/300 limit (mm)</th>
<th>Within limit?</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4</td>
<td>5.87</td>
<td>13.33</td>
<td>Yes</td>
<td>16</td>
<td>38.49</td>
<td>53.33</td>
<td>Yes</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>9.00</td>
<td>13.33</td>
<td>Yes</td>
<td>12</td>
<td>32.62</td>
<td>40.00</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>13.29</td>
<td>13.33</td>
<td>Yes</td>
<td>8</td>
<td>23.62</td>
<td>26.67</td>
<td>Yes</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>10.33</td>
<td>13.33</td>
<td>Yes</td>
<td>4</td>
<td>10.33</td>
<td>13.33</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Step 3. Check if the joint provides sufficient stiffness

Start with a flush endplate with standard bolt pitch (80mm), endplate thickness equal to RHS thickness (14.2mm rounded up to 15mm), top bolt row 50mm from beam flange. 6mm fillet welds. In this example, also start with a concrete in-fill. From the previous example it was determined that 2 bolt rows give sufficient shear resistance.
Step 4. Improve joint detailing

Skip to Step 5.

Step 5. Check joint strength and ductility

Using the joint load-deflection characteristics from the method described in Chapter 7 gives the moment-rotation curve in Figure A.2 with a yield strength of 118.5kNm and ultimate strength of 206.0kNm using nominal material properties of \( f_y = 275\text{N/mm}^2 \) for S275 steel and \( f_y = 355\text{N/mm}^2 \) for S355 steel. In comparison, the moment resistance of the beam is 412.5kNm which classifies this connection as partial strength. Using the equation in Table 2.3 for semi-rigid joints, the joint load can be estimated assuming that the joint remains in its elastic state and therefore \( S_J = S_{J,\text{ini}} \). This gives a load of approximately 63.7kNm for the SLS condition. Comparing the predicted joint yield strength with the SLS joint load:

\[
118.5\text{kNm} > 63.7\text{kNm}
\]

4.84 \( K_b > 4.6 \) \( K_b \) \( \therefore \) The joint initial stiffness is sufficient for unbraced frame design.
The joint will remain in its elastic state in the SLS condition and therefore not accumulate permanent rotations. Based on the 0.03 radian ductility criterion, the joint has adequate ductility. The predicted failure mode is tensile bolt failure at 206.0kNm and a rotation of 0.065 radians.

**The joint is suitable for unbraced frame design.**

![Moment-rotation curve for Example Joint 1b](image)

**Figure A.2. Moment-rotation curve for Example Joint 1b**

### C.4. Example 2: Frame B – Outer column joint

#### Step 1. Preliminary sizing of beam/column sections based on simple construction

- **Load case**: 6m bays in both directions, 4 storey frame. 30kN/m Dead loading (5kN/m² over 6m span) 30kN/m Live loading (5kN/m² over 6m span) SLS loading of 1.0 Dead + 1.0 Wind ULS loading of 1.5 Live + 1.35 Dead
- **Columns**: SHS 250×250×12.5 S355
- **Beams**: UB 457×191×74 S275
- **Bolts**: Grade 8.8 M20 bolts, Non-preloaded ($f_{ub} = 800\text{N/mm}^2$)
  
  For all sections, use $E=210\times10^8 \text{N/m}^2$

#### Step 2. Determine required joint initial stiffness

Use the basic hand calculation method with reduced beam-member stiffness coefficient and assume that the required joint initial stiffness is twice the secant...
stiffness calculated to limit the frame sway deflection. The minimum secant stiffness required from the basic hand calculation method gives $0.59 \, K_b$ as given in Table A.3 so the required joint initial stiffness is $1.18 \, K_b$. While use of a safety factor should be considered to account for errors in the prediction of deflections for joints with low secant stiffnesses (i.e. lower than $2.0K_b$), in this instance the deflections for Storey 2 in Frame B as shown in Table 8.2 are overestimated so the subsequent prediction of required stiffness is assumed to be conservative.

\[\therefore \] Required joint initial stiffness: $1.18 \, K_b$

Table A.3. SLS check for $0.59K_b$ joint stiffness

<table>
<thead>
<tr>
<th>Storey</th>
<th>Storey height (m)</th>
<th>Storey deflection (mm)</th>
<th>$d=h/300$ limit (mm)</th>
<th>Within limit?</th>
<th>Cumulative height (m)</th>
<th>Cumulative deflection (mm)</th>
<th>$d=h/300$ limit (mm)</th>
<th>Within limit?</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4</td>
<td>4.58</td>
<td>13.33</td>
<td>Yes</td>
<td>16</td>
<td>33.19</td>
<td>53.33</td>
<td>Yes</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>8.91</td>
<td>13.33</td>
<td>Yes</td>
<td>12</td>
<td>28.60</td>
<td>40.00</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>13.23</td>
<td>13.33</td>
<td>Yes</td>
<td>8</td>
<td>19.70</td>
<td>26.67</td>
<td>Yes</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>6.46</td>
<td>13.33</td>
<td>Yes</td>
<td>4</td>
<td>6.46</td>
<td>13.33</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Step 4. Check if joint provides sufficient stiffness

Start with a flush endplate with standard bolt pitch (80mm), endplate thickness equal to RHS thickness, top bolt row 50mm from beam flange, 6mm fillet welds.

Determine number of bolt rows needed for shear resistance (ULS)

Shear load $N_{v,Ed} = 0.5 \times 6m \times (1.5 \times 30kN/m + 1.35 \times 30kN/m) = 256.5kN$

Number of bolts required for shear resistance

\[
\frac{N_{v,Ed}}{F_{v,Rd,bolt}} = \frac{256.5}{94.1} = 2.72
\]

\[\therefore \text{Use 2 rows of 2 bolts}\]
<table>
<thead>
<tr>
<th>Beam</th>
<th>Column</th>
<th>Bolts</th>
<th>UB 457×191×74</th>
<th>SHS 250×250×12.5</th>
<th>M20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endplate thickness (m)</td>
<td>0.0125</td>
<td>Endplate width (m)</td>
<td>0.19</td>
<td>Top edge to top bolt row distance (m)</td>
<td>0.05</td>
</tr>
<tr>
<td>$v$ (GPa)</td>
<td>0.3</td>
<td>$E$ (GPa)</td>
<td>210</td>
<td>$h_r$ (m) Row 1</td>
<td>0.393</td>
</tr>
<tr>
<td>Bolts in tension, $k_{eq}$</td>
<td>3.10E-02</td>
<td>Endplate in bending, $k_{eq}$</td>
<td>8.35E-03</td>
<td>RHS in transverse tension, $k_{eq}$</td>
<td>4.28E-04</td>
</tr>
<tr>
<td>$K_b$ (for $L_b=6m$)</td>
<td>1.07</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1.07 $K_b < 1.18K_b \therefore$ Insufficient stiffness, proceed to Step 5.

**Step 5. Change joint detailing in order of increase in cost**

Begin by increasing bolt pitch to 120mm.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Column</th>
<th>Bolts</th>
<th>UB 457×191×74</th>
<th>SHS 250×250×12.5</th>
<th>M20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endplate thickness (m)</td>
<td>0.0125</td>
<td>Endplate width (m)</td>
<td>0.19</td>
<td>Top edge to top bolt row distance (m)</td>
<td>0.05</td>
</tr>
<tr>
<td>$v$ (GPa)</td>
<td>0.3</td>
<td>$E$ (GPa)</td>
<td>210</td>
<td>$h_r$ (m) Row 1</td>
<td>0.393</td>
</tr>
<tr>
<td>Bolts in tension, $k_{eq}$</td>
<td>3.10E-02</td>
<td>Endplate in bending, $k_{eq}$</td>
<td>2.34E-03</td>
<td>RHS in transverse tension, $k_{eq}$</td>
<td>6.34E-04</td>
</tr>
<tr>
<td>$K_b$ (for $L_b=6m$)</td>
<td>1.31</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1.31 $K_b > 1.18K_b \therefore$ The joint initial stiffness is sufficient for unbraced frame design.

**Step 5. Check joint strength and ductility**

Using the joint load-deflection characteristics from the equations proposed in Chapter 7 gives the moment-rotation curve in Figure A.3 with a yield strength of...
72.9kNm and ultimate strength of 134.7kNm using nominal material properties of $f_y=275\text{N/mm}^2$ for S275 steel and $f_y=355\text{N/mm}^2$ for S355 steel. In comparison, the moment resistance of the beam is 453.8kNm, which classifies this connection as partial strength. Using the equation in Table 2.3 for semi-rigid joints, the joint load can be estimated assuming that the joint remains in its elastic state and therefore $S_j=S_{j,\text{ini}}$. This gives a load of approximately 59.2kNm for the SLS condition. Comparing the predicted joint yield strength with the SLS joint load:

72.9kNm > 59.2kNm

The joint will remain in its elastic state in the SLS condition and therefore not accumulate permanent rotations. Based on the 0.03 radian ductility criterion, the joint has adequate ductility. The predicted failure mode is thread stripping due to gross deformation of the RHS face at 134.7kNm and a rotation of 0.038 radians. Although bolt thread stripping is a sudden mode of failure, with the generous ductility provided by the RHS and endplate components this is not an issue. For special deformation capacity requirements, the designer may choose to further optimize the design.

∴ The joint is suitable for unbraced frame design.

Figure A.3. Moment-rotation curve for Example Joint 2