ELECTRO-MECHANICAL INTERACTION IN GAS TURBINE-GENERATOR SYSTEMS FOR MORE-ELECTRIC AIRCRAFT

A thesis submitted to the University of Manchester for the degree of Doctor of Philosophy in the Faculty of Engineering and Physical Sciences

2012

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SCHOOL OF ELECTRICAL AND ELECTRONIC ENGINEERING
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Abstract

Name of University: The University of Manchester
Candidate's Name: Thomas Feehally
Degree Title: Doctor of Philosophy (PhD)
Thesis Title: Electro-Mechanical Interaction in Aero Gas Turbine-Generator Systems for More-Electric Aircraft
Date: March 2012

Modern 'more-electric' aircraft demand increased levels of electrical power as non-propulsive power systems are replaced with electrical equivalents. This electrical power is provided by electrical generators, driven via a mechanical transmission system, from a rotating spool in the gas turbine core. A wide range of electrical loads exist throughout the aircraft, which may be pulsating and high powered, and this electrical power demand is transferred though the generators to produce a torque load on the drivetrain. The mechanical components of the drivetrain are designed for minimum mass and so are susceptible to fatigue, therefore the electrical loading existing on modern airframes may induce fatigue in key mechanical components and excite system resonances in both mechanical and electrical domains. This electro-mechanical interaction could lead to a reduced lifespan for mechanical components and electrical network instability.

This project investigates electro-mechanical interaction in the electrical power offtake from large diameter aero gas turbines. High fidelity modelling of the drivetrain, and generator, allow the prediction of system resonances for a generic gas turbine-generator system. A Doubly-Fed Induction Generator (DFIG) is considered and modelled. DFIGs offer opportunities due to their fast dynamics and their ability to decouple electrical and mechanical frequencies (e.g. enabling a constant frequency electrical system with a variable speed mechanical drive). A test platform is produced which is representative of a large diameter gas turbine and reproduces the electro-mechanical system behaviour. The test platform is scaled with respect to speed and power but maintains realistic sizing between component dimensions which include: a gas turbine mechanical spool emulation, transmission driveshafts and gearbox, and accessory loads such as a generator. This test platform is used to validate theoretical understanding and suggest alternative mechanical configurations, and generator control schemes, for the mitigation of electro-mechanical interaction.

The novel use of a DFIG and an understanding of electro-mechanical interaction allow future aircraft designs to benefit from the increased electrification of systems by ensuring that sufficient electrical power can be provided by a robust gas turbine-generator system.
Declaration

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Acknowledgement

I wish to thank my supervisor, Judith Apsley, for her guidance and tireless support throughout the project.

The good humoured assistance of other members of the Power Conversion Group has also been gratefully received.

A special mention must be made to the Mechanical Workshop, in particular Paul Shaw, who has made a significant contribution to the design and construction of the test rig.

I would like to acknowledge the support of the industrial sponsor, Rolls-Royce plc, notably the individuals who have offered impartial technical insight and advice during my research.

Finally, to my family who have made this possible, and my friends who have always offered their opinions, thank you for your encouragement and wisdom.

______________________________
### Table of Abbreviations

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<tr>
<td>ADS</td>
<td>Angular Driveshaft</td>
</tr>
<tr>
<td>AEA</td>
<td>All-Electric Aircraft</td>
</tr>
<tr>
<td>AEE</td>
<td>All-Electric Engine</td>
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<tr>
<td>AGB</td>
<td>Accessory Gearbox</td>
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<tr>
<td>APU</td>
<td>Auxiliary Power Unit</td>
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<tr>
<td>BDFM</td>
<td>Brushless Doubly-Fed (Induction) Machine</td>
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<td>Carbon Dioxide</td>
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<td>CSD</td>
<td>Constant Speed Drive</td>
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<td>CSV</td>
<td>Comma Separated Variable</td>
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<tr>
<td>DAQ</td>
<td>Data Acquisition</td>
</tr>
<tr>
<td>DFIG</td>
<td>Doubly-Fed Induction Generator</td>
</tr>
<tr>
<td>DHA</td>
<td>Distributed Hydraulic Actuator</td>
</tr>
<tr>
<td>ECS</td>
<td>Environmental Control System</td>
</tr>
<tr>
<td>EHA</td>
<td>Electro Hydrostatic Actuator</td>
</tr>
<tr>
<td>EIS</td>
<td>Entry Into Service</td>
</tr>
<tr>
<td>EMA</td>
<td>Electro Mechanical Actuator</td>
</tr>
<tr>
<td>EV</td>
<td>Electric Vehicle</td>
</tr>
<tr>
<td>FADEC</td>
<td>Full Authority Digital Engine Control</td>
</tr>
<tr>
<td>FCE</td>
<td>Flight Control Electronics</td>
</tr>
<tr>
<td>FOC</td>
<td>Field Orientated Control</td>
</tr>
<tr>
<td>GCU</td>
<td>Generator Control Unit</td>
</tr>
<tr>
<td>GHG</td>
<td>‘Green House’ Gas</td>
</tr>
<tr>
<td>GT</td>
<td>Gas Turbine</td>
</tr>
<tr>
<td>HEV</td>
<td>Hybrid Electric Vehicle</td>
</tr>
<tr>
<td>HP</td>
<td>High Pressure (GT spool)</td>
</tr>
<tr>
<td>ICE</td>
<td>Internal Combustion Engine</td>
</tr>
<tr>
<td>IDG</td>
<td>Integrated Drive Generator</td>
</tr>
<tr>
<td>IFE</td>
<td>In Flight Entertainment</td>
</tr>
<tr>
<td>IFSD</td>
<td>In Flight Shut Down</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Definition</td>
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<td>--------------</td>
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</tr>
<tr>
<td>IGB</td>
<td>Internal Gearbox</td>
</tr>
<tr>
<td>IP</td>
<td>Intermediate Pressure (GT spool)</td>
</tr>
<tr>
<td>LP</td>
<td>Low Pressure (GT spool)</td>
</tr>
<tr>
<td>MEA</td>
<td>More-Electric Aircraft</td>
</tr>
<tr>
<td>MEE</td>
<td>More-Electric Engine</td>
</tr>
<tr>
<td>N2</td>
<td>IP spool angular velocity</td>
</tr>
<tr>
<td>NOx</td>
<td>Nitrous Oxides</td>
</tr>
<tr>
<td>PCD</td>
<td>Pitch Circle Diameter (gears)</td>
</tr>
<tr>
<td>PEC</td>
<td>Power Electronic Converter</td>
</tr>
<tr>
<td>PM</td>
<td>Permanent Magnet (machine)</td>
</tr>
<tr>
<td>PMA</td>
<td>Permanent Magnet Alternator</td>
</tr>
<tr>
<td>PMG</td>
<td>Permanent Magnet Generator</td>
</tr>
<tr>
<td>pu</td>
<td>per unit</td>
</tr>
<tr>
<td>RDS</td>
<td>Radial Driveshaft</td>
</tr>
<tr>
<td>RR</td>
<td>Rolls-Royce plc</td>
</tr>
<tr>
<td>SAGB</td>
<td>Step-Aside Gearbox</td>
</tr>
<tr>
<td>SFC</td>
<td>Specific Fuel Consumption</td>
</tr>
<tr>
<td>SG</td>
<td>Synchronous Generator</td>
</tr>
<tr>
<td>SHA</td>
<td>Smart Hydraulic Actuator</td>
</tr>
<tr>
<td>SPC</td>
<td>Specific Fuel Consumption</td>
</tr>
<tr>
<td>SR</td>
<td>Switched Reluctance (machine)</td>
</tr>
<tr>
<td>TGB</td>
<td>Transfer Gearbox</td>
</tr>
<tr>
<td>TRU</td>
<td>Transformer Rectifier Unit</td>
</tr>
<tr>
<td>VF</td>
<td>Variable Frequency</td>
</tr>
<tr>
<td>VFSG</td>
<td>Variable Frequency Starter Generator</td>
</tr>
<tr>
<td>VSCF</td>
<td>Variable Speed Constant Frequency</td>
</tr>
<tr>
<td>WAI</td>
<td>Wing Anti Icing</td>
</tr>
</tbody>
</table>
### Table of Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>Torsional linear damping factor (Nm.s.rad$^{-1}$)</td>
</tr>
<tr>
<td>$F$</td>
<td>Force (Nm)</td>
</tr>
<tr>
<td>$G$</td>
<td>Material shear modulus (N.m$^{-2}$, Pa)</td>
</tr>
<tr>
<td>$I_A$</td>
<td>Second moment of area (m$^4$)</td>
</tr>
<tr>
<td>$I$</td>
<td>Current (A)</td>
</tr>
<tr>
<td>$J$</td>
<td>Mass moment of inertia (kg.m$^2$)</td>
</tr>
<tr>
<td>$J_p$</td>
<td>Polar second moment of area (m$^4$)</td>
</tr>
<tr>
<td>$k$</td>
<td>Torsional stiffness (Nm.rad$^{-1}$)</td>
</tr>
<tr>
<td>$L$</td>
<td>Length (m)</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass (kg)</td>
</tr>
<tr>
<td>$P$</td>
<td>Power (W)</td>
</tr>
<tr>
<td>pp</td>
<td>Pole pairs</td>
</tr>
<tr>
<td>$r_o$, $r_i$</td>
<td>Radius, outer and inner (m)</td>
</tr>
<tr>
<td>$R$</td>
<td>Resistance (Ω)</td>
</tr>
<tr>
<td>$s$</td>
<td>Slip</td>
</tr>
<tr>
<td>$T$</td>
<td>Torque (Nm)</td>
</tr>
<tr>
<td>$V$</td>
<td>Voltage (V)</td>
</tr>
<tr>
<td>$X$</td>
<td>Reactance (Ω)</td>
</tr>
<tr>
<td>$z$</td>
<td>Impedance (Ω)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angular position (rad)</td>
</tr>
<tr>
<td>$\dot{\theta}$</td>
<td>Angular velocity (rad.s$^{-1}$)</td>
</tr>
<tr>
<td>$\ddot{\theta}$</td>
<td>Angular acceleration (rad.s$^{-2}$)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Material mass density (kg.m$^{-3}$)</td>
</tr>
<tr>
<td>$\sigma_t$, $\sigma_r$</td>
<td>Tangential / radial stress (Pa)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>shear stress (N.m$^{-2}$ or Pa)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson's ratio</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Magnetic flux (Wb)</td>
</tr>
<tr>
<td>$\omega_m$</td>
<td>Mechanical angular velocity (rad.s$^{-1}$)</td>
</tr>
</tbody>
</table>
Abbreviation and Notation

\( \omega_n \) | Natural frequency (rad.s\(^{-1}\))
\( \omega_r \) | Rotor frequency (rad.s\(^{-1}\))
\( \omega_s \) | Synchronous frequency (rad.s\(^{-1}\)) [note: not stator]

Subscript:

- \( A_s \) | Stator
- \( A_r \) | Rotor
- \( A_m \) | Magnetising
- \( A_d \) | Direct
- \( A_q \) | Quadrature

Superscript:

- \( A' \) | Control set point
- \( \hat{A} \) | Control error
- \( \hat{A} \) | Referred value
- \( \tilde{A} \) | Complex flux
- \( |A| \) | Magnitude
- \( \dot{A} \) | Derivative
- \( \ddot{A} \) | Second derivative
Chapter 1  Introduction

1.1 Background

The growing civil aviation market has two major concerns, environmental costs and operating costs. Aircraft burn fossil fuels releasing gases into the environment; where once these fuels were cheap, prices have now risen forming a substantial portion of aircraft operating costs. Concern for the environment and desires to reduce aircraft operating costs are now linked by a common objective - aircraft efficiency.

1.1.1 Environmental Considerations

It is agreed within the scientific community that changes in global climate can be attributed to the human based combustion of fossil fuels which results in the emission of gases including carbon dioxide (CO₂), methane, and nitrous oxides (NOₓ). These emissions are known collectively as ‘green house’ gases (GHGs) [1].

Governmental and public pressure is increasingly demanding a reduction in GHG emissions from industries, for example the United Nations Framework Convention on Climate Change [2], ratified in 2005, formalises an agreement between leading industrial nations to reduce CO₂ emission by 12% by 2012. From a commercial point of view, demonstrating an effort to reduce GHG emissions by reducing fuel burn can lead to positive press as well as financial savings.

1.1.1.1 Aviation and Climate Change

Aircraft CO₂ emissions account for 12% of the total GHG emissions from all transport sources [1], which is 2% of total man-made emissions [3]. The impact of these emissions on the atmosphere is more acute as a result of the high altitude operation of aircraft [4] [5], and with the aviation industry predicted to grow by 5% per annum over the next 10 years [1] [6], the influence of the aviation industry on global climate change can be considered significant.

Although GHG emissions can be reduced by a range of clean burn options [7] GHG emissions are most simply reduced by lowering fuel burn. Research carried out by the
Advisory Council for Aeronautics Research in Europe (ACARE) has led the European aviation community to target a 50% reduction in aircraft CO₂ emissions (per passenger kilometre) by 2020 [8].

Figure 1.1 shows the average fuel burn for new civil aircraft (greater than 30 seat) introduced between 1960 and 2008.

Aircraft fuel burn per passenger seat per km has approximately halved since the 1960s. It is noted that periods of lower gains in fuel burn reduction are linked to low fuel prices [9], and that the reduction in fuel burn has been very limited over the last 20 years perhaps due to the technological maturity of their designs.

1.1.2 Aircraft operating costs

Reduced operating costs have always been attractive for aircraft operators, and they may wish to pass these savings on to their customers. These operating costs are now also felt by the engine manufacturer through service contracts, for example Rolls-Royce plc (RR) now maintains 76% of its large civil engines on some form of service contract, with total service revenues £5.5 billion in 2010 [10].

Figure 1.2 shows the average operating costs for a number of civil aircraft.
All aircraft require regular maintenance to sustain safety and performance which costs the operator directly for the labour and parts but also in lost revenue from the aircraft. Simplifying maintenance requirements for the aircraft will offer a significant financial saving to the operator.

Fuel and maintenance account for nearly half of the total aircraft running costs. Reducing aircraft fuel burn will reduce aircraft operating costs as well as GHG emissions.

1.1.3 Reducing fuel burn and maintenance requirements

Fuel is burnt within the aircraft engines to achieve the work of carrying passengers (or cargos) to their destination. A reduction in fuel burn is possible by increasing the efficiency of the aircraft; that is making more efficient use of the chemical energy within the fuel to reduce the volume burned per passenger mile flown.

One strategy to increase aircraft efficiency and reduce maintenance requirements is to increase the use of electrical power systems. Utilising electrical systems allows the engine and airframe to be simplified making flight cheaper and reducing GHG emissions. Aircraft electrification can be achieved using existing technologies. However the concept is radical for an industry with design conservatism, due to strong safety requirements, and does pose significant design challenges.
The increased use of electrical systems on aircraft requires a robust and reliable means of providing electrical power from the engine. Rather than utilising separate prime mover / generator sets, energy is extracted from the existing engines to generate electrical power, thus minimising weight increase. Where once engines were required to produce thrust only, now they must provide significant electrical power to the airframe.

Energy to drive the electrical generators is extracted mechanically from the engine core (typically a gas turbine) and transferred via a drivetrain. This gas turbine-generator system is both critical to the operation of the aircraft and vulnerable to failure (as a result of its lightweight construction and exposed nature). The electrical loads on aircraft are highly transient, and as a result of the uprated generation these transients are also passed through to the engine's mechanical components. Electrical systems are therefore able to disturb the operation of mechanical components and likewise mechanical disturbances can disrupt the electrical system. This electro-mechanical interaction can lead to a reduced lifespan for mechanical components and electrical network instability.

An understanding of the phenomena of electro-mechanical interaction in aero gas turbine-generator systems is essential in order to facilitate the electrification of aircraft power systems which offers the potential to reduce GHG emissions and operating costs.

1.2 Aims and Objectives of research

The aims of this research are:

- To develop methods to understand and predict electro-mechanical interaction in aero gas turbine-generator systems for the purpose of mitigation.
- To evaluate the performance of a doubly-fed machine as an aero generator.

A multi-domain electro-mechanical model is produced as well as a hardware test platform. The breakdown of project objectives, to meet the research aims, is shown below:

- Investigate gas turbine drivetrain torsional dynamics
Chapter 1 - Introduction

- Model gas turbine drivetrain
- Determine resonant modes
- Identify components and properties key to system behaviour

- Doubly-fed machine
  - Characterise and model real machine
  - Select and assemble control scheme
  - Consider methods of integration with gas turbine

- Design and commission electro-mechanical test platform
  - Simulate electro-mechanical system
  - Define behaviours and ratings
  - Design, manufacture and assemble system
  - Validate modelling (drivetrain and generator)

1.3 Significant contribution

This work develops an electro-mechanical model of an aero gas turbine-generator system. Unlike existing research, this model represents both the mechanical drivetrain and the electrical generator at a high fidelity, allowing the key components to be extracted in a reduced-order model, and enabling the design of an integrated electro-mechanical test system. The issue of electro-mechanical interaction, already identified by others in non-aero systems, is extended for the specific requirements of aero gas turbine-generator systems. Several methods for the mitigation of the electro-mechanical interaction are proposed.

Contrary to existing aircraft designs, a doubly-fed machine is evaluated as an aero generator and its performance and integration with a gas turbine is analysed. Both the electro-mechanical model and generator analysis are validated with practical results from a hardware test platform.

1.4 Overview of thesis

Chapter 2 provides a literature review with relevance to the research carried out. The operation of the aero gas turbine is considered, and the electrification of airframe ancillaries identified as a means of increasing aircraft efficiency. Attention is paid to the Boeing 787 and its engines as a realisation of a more-electric aircraft. The system for
Chapter 1 - Introduction

electrical generation from aero gas turbines is detailed, and its vulnerabilities highlighted. Interaction between mechanical drivetrain and electrical loading is compared with other non-aero systems and their methods for vibration mitigation are considered. The Doubly-Fed Induction Generator is identified as a means of providing a frequency controlled AC network for the aircraft with benefits over alternative technologies. The prior use of such a generator is studied as well as suitable methods for its control as an aero generator.

Chapter 3 details the computer modelling of a gas turbine mechanical drivetrain. Methods for the modelling of components within the drivetrain (driveshaft, gearbox, loads) are compared, and the most suitable identified, allowing a high fidelity model to be produced. Simulation is carried out using the model and the key behaviours of the drivetrain identified, namely the significant resonances. This allows a reduced order model to be derived which accurately represents the behaviour of the gas turbine drivetrain but with a greatly reduced number of active components. Drivetrain designs are considered with the aim of reducing and altering resonance modes.

Chapter 4 contains the modelling and simulation of a real Doubly-Fed Induction Generator. Machine equations are implemented in Simulink and a laboratory machine characterised to provide real parameters. Simulation of the model is used to understand the performance of the machine as an aero generator operating in variable speed standalone conditions. A field orientated control scheme is selected to provide voltage magnitude and frequency regulation, the controller is developed and is applied to the machine model and tuned.

Chapter 5 documents the design and construction of a test platform able to replicate an aero electro-mechanical system. The platform is specified to replicate the behaviour of a large civil aircraft such as the Boeing 787 within a laboratory environment. The mechanical drivetrain is based on the reduced order model detailed in chapter 3 with resonances tuned to be similar to that of a real aircraft. The generator control scheme assembled in chapter 4 is implemented on a real machine using power electronic systems, to supply a load bank. Mechanical components are designed for machining and assembly and a thorough safety analysis carried out. Control and data acquisition is
implemented and the platform assessed (by testing) in comparison to its design specifications.

Chapter 6 provides the results from the test platform. Data from the test platform is analysed to provide validation of the modelling work, most significantly the drivetrain analysis and simplification work. The behaviour of the generator control scheme is also appraised and its impact on electro-mechanical interaction recorded.

Chapter 7 presents the conclusions, as well as discussions of the research work. Potential enhancements in the design of aero electro-mechanical system are discussed as well as the opportunities which the Doubly-Fed Induction Generator offers for use in an aero system.
Chapter 2  Literature review

The Aero Gas turbine (GT) is a relatively mature technology making improvements in efficiency hard to come by. One possible area for improvement is in the optimisation of the energy used to supply airframe auxiliary loads, which is extracted from the GT. These aircraft secondary power systems are discussed and their replacement with electrical equivalents is proposed. Such an electrification of aircraft systems requires an up rated and robust electrical generation system and so several generation schemes are compared. The Doubly-Fed Induction Generator (DFIG) is suggested as an electrical generation solution to replace traditional systems whilst offering beneficial performance. The issue of electro-mechanical interaction is well documented in other, non-aero, systems and these are compared to the aero system in question.

2.1 The Aero Gas Turbine

Since their introduction in the 1940s GTs have proven to be a popular means of aircraft propulsion due to their high power to weight ratio [12] and high thrust speeds. GT power plants extract energy, developed through pressure and temperature rises, from gas flow through the engine. This processes is described mathematically by the Brayton Cycle [13] [14]. Modern GTs make use of axial compressors and axial turbines allowing a thin tube like structure, as shown in Figure 2.1.

![Figure 2.1: GT Structure [15]](image)

Air enters the GT and is compressed by multiple compressor stages, this rise in pressure is known as the pressure ratio. Fuel is burned in the combustion chamber, expanding the compressed air, which forces it way past the turbine. Power extracted from by the air by the turbine, drives the compressors via the connecting shaft, the interconnected compressor and turbine are together known as a spool. After passing through the
turbines, excess pressure accelerates air out of the exhaust nozzle to create thrust. The GT and its supporting ancillaries, which allow it to operate, is sometimes referred to as an engine.

2.1.1 The Turbofan

To achieve higher efficiencies for the relatively low-speed, civil, aero application a turbofan arrangement can be used. This utilises a GT core (as seen in Figure 2.1) to produce high-pressure exhaust gas, but adds a further turbine which is used to drive a large diameter fan. This arrangement is shown in Figure 2.2 for the example of a RR Trent1000.

![Figure 2.2: RR Trent1000 turbofan GT cutaway [16]](image)

Exhaust gas from the GT core is used to drive one or more additional spools. For the Trent1000, the Intermediate Pressure (IP) spool powers an additional compressor and a Low Pressure (LP) spool powers the fan driving air around the core of the GT. At the core of the GT is the High Pressure (HP) spool. The spools are not linked to each other so are free to move independently; however, in reality their behaviour is coupled by the air flow within the engine. Spool speed is related to thrust generated by the engine and so will vary during operation, HP spool speed range is typically around 1.7:1 and on a three-spool system, the IP spool speed range is approximately 2.0:1. A three-spool gas turbine structure is shown in Figure 2.3.
Number of spools
Some manufactures prefer a two spool configuration in which the IP spool is not included. This creates a simpler, cheaper, engine which is easier to maintain [15]. However, a two spool configuration requires additional compressor stages (to achieve the pressure ratio) which are mounted on the LP shaft which are therefore driven at a lower speed, reducing efficiency and making the engine larger. By way of comparison the GE Aviation GEnx-1B and RR Trent1000 are engines designed to identical airframe specifications, the GEnx is a two spool arrangement with an overall pressure ratio of 43:1 and a length of 4.9 m, the Trent1000 engine has three spools enabling it to achieve a pressure ratio of 50:1 [17] and a length of 4.7 m. Typically three-spool systems are preferred for long haul flight, where fuel burn is a greater fraction of cost, and two spool systems preferred for short haul operation [15].

Bypass
The fan diameter is larger than that of the GT core and so a large amount of air passes around the core, the ratio of air which passes round the core to that entering it is referred to as the bypass ratio. On a modern civil aircraft turbofan the bypass air driven by the fan creates approximately 80% of the thrust [15].

Engine performance is measured in terms of Specific Fuel Consumption (SFC) [13] [12], calculated as fuel burned for thrust developed. Higher bypass ratio is linked to
increased SFC and decreased noise [7]. The Trent1000 and Genx have bypass ratios of 10:1 [18] and 9.6:1 respectively.

2.1.2 Improving efficiency of GT powered aircraft

The drive to improve efficiency is motivated by both financial and environmental considerations but essentially requires a reduction in fuel burn from the GT. The GT is a relatively mature technology and improvements in efficiency tend to be low, but over a long service life significant fuel burn savings are obtainable. A number of possibilities are available to improve the efficiency of the GT, some of these are summarised below.

2.1.2.1 Pressure ratio

Increasing the pressure ratio of the GT increases efficiency. One option is to introduce intercooling using the bypass air to increase the density of the air before it enters the combustion chamber, an affect which is equivalent to increasing the pressure ratio. Research by New Aero Engine Core Concepts (NEWAC), being led by RR, aims to demonstrate a 0.8% improvement in efficiency [19] using engine core technologies including intercooling.

2.1.2.2 Geared GT

A geared turbofan uses a step down gearbox to reduce the fan speed, allowing it to operate at a more optimal, slower, speed while the driving turbine still operates at the preferred higher speed. This is especially beneficial in a two core architecture where turbine and compressor blade speeds can be less well optimised and there is a compressor (booster) is required on the fan shaft [20]. Pratt & Whitney claim their geared turbofan offers a 12% reduction in fuel burn compared to their current (two spool) engines, and that this could rise to 19% by 2020 [21].

2.1.2.3 Open rotor GT

Increasing the air mass flow rate by increasing fan diameter increases turbo fan efficiency. Eventually a point is reached where the increased weight and drag of the expanded fan casing offsets the benefit of increased diameter. It is then beneficial to consider an open rotor engine, which does not have a fan casing and so the diameter of the rotor is only constrained by its integration onto the airframe. The rotor blades may be positioned forward (puller configuration) or aft (pusher configuration) of the GT core
[20] and can either be connected directly to a turbine or via a reduction gearbox to reduce blade speed and noise. A significant benefit of an open rotor design is the ability to vary the pitch of the blades, which is done using power taken from a low pressure turbine, this allows the GT core to operate over a much narrower and more efficient speed range, perhaps 1:1.3.

RR predicts a 25-30% reduction in SFC compared to an equivalent turbofan [20]. However there are drawbacks, for example increased noise. Also open rotor engines have optimal efficiency at speeds lower than current aircraft cruise speeds, this would create an increase in journey time of 4% to 8%. Open rotor engines are expected to be commercially available from 2015 onwards.

2.1.2.4 Airframe design
Thrust (and so fuel burn) requirement is dependent on drag, which itself is a result of the aircraft cross sectional area and lift generation from the wings. Aircraft designs show little or no variation from the standard 'cigar' shape due in part to the size and layout requirements of the many systems, for example the engines and pneumatic ducting. A large-scale reduction in drag is only possible with an entirely different aircraft design, in the future a blended wing body aircraft could offer reduced drag [7], however this will not be realised in the near future.

Reducing aircraft weight reduces lift requirements and would also reduced fuel burn. Secondary power systems account for a significant portion of the aircraft's weight, and can also dictate airframe layout. Considering these factors in the selection and design of secondary power systems offers a means of reducing fuel burn.

2.1.2.5 Secondary power systems
Secondary power is generated and distributed throughout the airframe in several forms; the means of power transfer for each system is typically based on the system's function arising from historical technological capabilities. For example, hydraulic power is utilised for actuation, pneumatic power drives the cabin ventilation, and electrical power supplies low power ancillaries. While each of these systems is individually optimised for the work it is carrying out, the use of multiple power systems is complex, and does not make use of advancements in technology capabilities (particularly for power
electronic systems), meaning the secondary power system is collectively sub-optimised [6]. It is estimated that the combined pneumatic, hydraulic and electrical secondary power systems of an aircraft have a power density of 0.35 kW/kg which is very low [22].

GT design can be simplified, and efficiency increased, if only a single means of generating secondary power is required. Similarly airframe design is more flexible if routing is necessary for a single power distribution system rather than the three currently used.

2.1.2.6 Summary of methods to improve GT power aircraft efficiency
The operation of the GT is optimised towards generating thrust for the aircraft, and as a mature technology the advancements in efficiency are relatively hard to come by. The wholesale changes in engine and airframe design required to significantly improve efficiency are unlikely to be introduced in the near future. However, secondary power systems, which use power from the GT, are less thoroughly optimised and provide a means of increasing GT efficiency without significant changes to its structure. These changes can be readily implemented on current and near-future aircraft designs.

2.2 Aircraft Secondary Power Systems
Aero GTs are fundamentally required to provide thrust to propel the aircraft, but they must also provide power to the airframe for a number of auxiliary systems. As aircraft design has developed over the years, the scale and number of secondary power distribution networks throughout the airframe has risen. It is now practical to consider the engine as a power system delivering thrust as well as considerable secondary power to the airframe. The provision of this power must be sufficient and reliable. Consideration is required at a system level, as the means of power distribution must be weighed up against the associated means of generation and loading.

2.2.1 Secondary power offtake
Energy is extracted mechanically and pneumatically from the GT core. The energy conversion process of a traditional GT system is shown in Figure 2.4.
The traditional GT power plant is required to provide for pneumatic, hydraulic and electrical power systems as well as developing thrust. The extraction of power from regions within the engine core impairs efficiency and disturbs GT performance, but it is essential for airframe operation.

Mechanical
A mechanical offtake from the spinning engine spool is used to drive various auxiliary loads which include a hydraulic compressor and an electrical generator. Although not the sole source of power for the airframe, the power mechanically extracted from the GT spools has risen as auxiliary loading has increased. A large three-spool civil turbofan may have a total auxiliary load of up to 400 kW [7], which is driven in part by high levels of electrical power needed on modern aircraft as shown in Figure 2.5.
Pneumatic

Pneumatic power is bled off the high pressure regions of the engine core. Although the power is extracted from several points (usually around the IP and HP compressors on a three-spool GT) so that the pressure can be varied, the volume of air extracted cannot be controlled. Air bleed is also used at lower levels to maintain engine stability, for example to prevent surge.

2.2.2 Comparison of Secondary Power Distribution Systems

In recent times electricity has risen in popularity to become the preferred power distribution system. Aircraft secondary power systems are discussed here with comparison to electrical equivalents.

2.2.2.1 Hydraulic

Hydraulic power is distributed via duplicated networks throughout the aircraft. Loads include primary (elevators, rudders, and ailerons) and secondary (flaps, slats, spoilers, and airbrakes) flight control surfaces [25], landing gear actuation and braking, and engine component actuation. Hydraulic actuators have a high force to weight ratio, however their distribution system is weighty (the hydraulic pipe work alone weighs

Figure 2.5: Historical electrical power demand for civil aircraft [23]
800kg on the Airbus A300-600 [26]). Maintenance is costly as leak prevention is essential and the fluids used are flammable and corrosive [27]. A significant proportion of hydraulic piping travels along the length of the wings, so a reduction in this weight greatly reduces the stress design requirements for the wing which can be used to reduce their material weight. For example it is stated that the removal of hydraulic systems of this nature on the Lockheed L1011 TriStar reduced the wing loading sufficiently to add a further 1m length of lift surface to each wing without increasing wing stress [28], which would increase lift capability.

Hydraulic pumps are driven mechanically by an offtake from the variable speed GT spool, the pump is required to provide hydraulic pressure across all engine speed ranges, and will draw energy from the engine even when actuation is not required [29].

Although an electrical power distribution network is lighter than the equivalent hydraulic system, the power to weight ratio of an electrical actuator is lower than that of an hydraulic actuator [25]. With increased hydraulic pressure on modern airframes (5000 psi on the Boeing 787 [25]) and efficiencies peaking at 85% [28], hydraulic systems still offer an advantageous power density over an equivalent electrical system.

**Types of actuator**

Traditional hydraulic systems are controlled by the switching of a hydraulic valve which controls the flow of hydraulic fluid driven by a central, constantly driven, hydraulic pump. This type of actuator is known as a Conventional Linear Actuator (CLA). A Smart Hydraulic Actuator (SHA) includes electronic position sensing and control of the hydraulic valve to improve performance.

The Distributed Hydraulic Actuator (DHA), shown in Figure 2.6, allows hydraulic actuators to be placed throughout the airframe with reduced hydraulic piping, using a local hydraulic reservoir and electrically powered pumps. With the hydraulic pumps distributed throughout the airframe reliability can be improved by an order of magnitude [28].
The DHA system as shown in Figure 2.6 is identical to the SHA at the point of actuation, however power is distributed to the system electrically with a localised AC machine driving a variable displacement pump. A hydraulic valve is used to control the position of the actuator. A DHA may include a hydraulic accumulator by way of a power buffer, thus reducing the size of the AC machine and pump, however the addition of hydraulic components reduces reliability and increases maintenance requirements. The DHA is driven hydraulically by a variable displacement pump which, as it is driving a single load only, has a low efficiency. Weight saving occurs as electrical power distribution is lighter than hydraulic power distribution while still making use of the higher power density hydraulic actuators.

Building on the DHA, the Electro Hydrostatic Actuator (EHA) uses an electrically driven, bi-directional, fixed displacement pump. The hydraulic flow rate from a fixed displacement hydraulic pump is directly proportional to the input mechanical speed, and hence the movement of the actuator can be precisely controlled by controlling the driving machine. A DC machine is used with a Power Electronic Converter (PEC). Using an EHA, as shown in Figure 2.7, enables precise control of actuator position as well has rate of change of position. The efficiency is high as the pump is only driven when actuation is required. EHAs are in use in a range of aircraft including flight control surface actuation of the Airbus A380 and Lockheed Martin F-35.
Chapter 2 - Literature Review

The EHA has only a small, localised, hydraulic system, an Electro Mechanical Actuator (EMA) dispenses with hydraulics completely. Power is distributed electrically, with linear motion created by a mechanically driven ball screw system. The ball screw actuator is driven, via a step down gearbox, from a PEC controlled DC machine. As with an EHA a high level of control is afforded over both the position and rate of change of position of the actuator. However the EMA is lighter and smaller than an EHA (but both are still less power dense than the CLA or SHA), and having no hydraulic fluid, maintenance requirements are lower, and pump windage losses do not exist so efficiency is higher [29]. From a control view the lack of hydraulic fluid increases the system stiffness so improving the operational dynamics [29]. EMAs also offer the ability to regenerate onto the electrical bus, although it is not currently permitted [30]. Figure 2.8 shows an EMA system.

The drawback of EMAs is that the ball screw can jam during failure, however advanced ball screw actuator systems are being developed by Goodrich [31] which include a redundant secondary ballscrew systems to achieve fail safe operation.

Figure 2.7: Electro Hydrostatic Actuator system diagram

Figure 2.8: Electro-Mechanical Actuator system diagram
A weight saving cannot be achieved through the like for like introduction of EMAs, however maintenance reductions and improved controllability can be expected.

Some military engines (such as the Turbo Union RB199 which powers the Panavia Tornado) use fuel/draulic powered systems for the actuation of engine components [32]. These systems make use of the pressurised fuel flow rather than a dedicated hydraulic fluid, offering a weight saving as additional compressors and fluid reservoirs are not required.

2.2.2.2 Pneumatic

Pneumatic power is used to drive the Environmental Control System (ECS) and Wing Anti Icing (WAI). Pneumatic systems require physically large distribution networks, in some cases ducting at 200 mm diameter is required for ECS supply [25], and so they inevitably dictate airframe design. Pneumatic systems typically offer low efficiency and require time-consuming maintenance [27], for example leak detection is not a simple process.

Air pressure and temperature from the GT core varies depending on operating conditions (pressures between 700 kPa and 2000 kPa and temperatures from 40°C to 300°C) this is compared to a ECS input requirement of 70kPa at 5°C [28] to deliver an cabin temperature between 18°C and 29°C [33]. Significant additional effort is therefore required, by a precooler heat exchanger with fan air on the heatsink [34] to convert pneumatic offtake from the engine into useful power for the ECS. Excess energy (sometimes as much as 50% of the offtake [33]) is exhausted as waste heat. Varying the extraction point on the GT core (depending on operating conditions) has some benefits but is not very controllable. Extracting pneumatic energy from the GT core is inefficient and detrimental to the finely tuned performance of the GT.

2.2.2.3 Electrical

Electrical power is typically confined to lower rated loads and control systems. Traditional airframes utilise 115 V\text{AC} generation, at 400 Hz, to supply the larger loads with Transformer Rectifier Units (TRUs) used to provide a 28 V\text{DC} bus for smaller loads.
Generation

Electrical generation is carried out on the majority of civil aircraft by wound field synchronous machines due to their high power density and robust design [35]. Variable mechanical drive speed is regulated by a Constant Speed Drive (CSD) [25] to produce a constant generator shaft speed and hence constant electrical frequency. The combination of generator and CSD is referred to as an Integrated Drive Generator (IDG)

Distribution

Distribution is through copper cables, which are smaller and require less maintenance than hydraulic and pneumatic distribution. The cable mass can be reduced by increasing the bus voltage, however, up-rated cable insulation is required and the impact of partial discharge must be taken into account [36]. With higher powered electrical distribution circuit breakers must be designed to cope accordingly, this is often at the expense of size and weight, which when large aircraft require in the region of 600 breakers [25], is a considerable additional mass.

Loads

A wide range of aircraft systems are powered electrically, these include flight control avionics, lighting, galley ovens and In Flight Entertainment (IFE). In-flight electrical power requirements for the 300 seat Boeing 777 are given in Table 2.1, for this aircraft a total of 313.6 kVA of load is supplied by 240 kVA [37] of GT mounted generation and a supply from the Auxiliary Power Unit (APU).
<table>
<thead>
<tr>
<th>Load</th>
<th>Rating (kVA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IFE</td>
<td>28.0*</td>
</tr>
<tr>
<td>Galley ovens</td>
<td>140.0*</td>
</tr>
<tr>
<td>Ventilation (toilet / galley)</td>
<td>23.4</td>
</tr>
<tr>
<td>Miscellaneous DC loads</td>
<td>11.6</td>
</tr>
<tr>
<td>Fuel pumps</td>
<td>16.0</td>
</tr>
<tr>
<td>Hydraulic pumps</td>
<td>35.0</td>
</tr>
<tr>
<td>Ice and Rain protection</td>
<td>9.0</td>
</tr>
<tr>
<td>Cabin management system</td>
<td>12.0</td>
</tr>
<tr>
<td>Lights</td>
<td>6.3</td>
</tr>
<tr>
<td>Fuel jettison pumps</td>
<td>14.7</td>
</tr>
<tr>
<td>Avionics</td>
<td>6.0</td>
</tr>
<tr>
<td>Equipment cooling</td>
<td>11.6</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>313.6</strong></td>
</tr>
</tbody>
</table>

* Approximated based on 280 seat aircraft.

The low power rating of the majority of electrical loads can be seen. The galley ovens have the highest rating at 140.0 kVA, however they are not in use continually and it is not likely for all ovens to be in use at the same time.

Summary Aircraft Electrical Systems

Electrical power demands are rising on the latest aircraft as the rating of existing loads is increased. Furthermore, modern power electronic systems and drives have increased the possibility of electrically driven actuation systems which can operate more efficiently than equivalent hydraulic systems, drawing energy from the GT only during actuation [25]. Electro-mechanical actuators also offer the ability to regenerate energy to the bus [30] although this is currently not allowed by specification. Electrically powered actuators rated at 50kW [40] are now being utilised on modern aircraft. The electrical nature of these systems is non-linear, drawing non-sinusoidal currents [40], and hence causing significant harmonic pollution to the distribution network. This issue is exacerbated by the use of basic 6-pulse rectifiers to supply DC loads from the AC bus [40]. Due to the low duty cycle of the actuators it may be acceptable to have a loss of power quality during their brief activation [40]. However as the number of electrical actuators and their power ratings increases the substantial harmonics they create will no longer be tolerable.

Electro-mechanical actuators are not currently fail-safe, as hydraulic actuators are, but advanced systems are under development which have the potential to offer fail safe
operation [31]. Electro-mechanical actuators require lower maintenance than hydraulic systems [29].

### 2.2.2.4 Summary of secondary power systems on traditional aircraft

On a traditional aircraft pneumatic, electrical and hydraulic power systems are used in parallel, with mechanical systems supplying electrical and hydraulic generation. The share of secondary load between these three systems is shown in Figure 2.9.

![Figure 2.9: Make up of secondary power system on conventional aircraft [41]](image)

The pneumatic system has the highest rating as the ECS is the largest single load on the aircraft [42]. It is not ideal to maintain three different generation and distribution systems throughout the airframe.

The four means of secondary power distribution are summarised in Table 2.2.

<table>
<thead>
<tr>
<th>System</th>
<th>Complexity</th>
<th>Maintenance</th>
<th>Technological Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical</td>
<td>Complex</td>
<td>Simple</td>
<td>System (mature) New technologies (immature)</td>
</tr>
<tr>
<td>Hydraulic</td>
<td>Simple</td>
<td>Complex &amp; hazardous</td>
<td>Mature</td>
</tr>
<tr>
<td>Mechanical</td>
<td>Very complex</td>
<td>Frequent &amp; slow</td>
<td>Very Mature</td>
</tr>
<tr>
<td>Pneumatic</td>
<td>Simple</td>
<td>Complex</td>
<td>Very Mature</td>
</tr>
</tbody>
</table>

It can be seen that hydraulic and pneumatic systems are both mature and relatively simple, however maintenance is considerable when compared to electrical systems, which while being complex in design, are more simply maintained.
Preference for a single, electrical, power system

A single secondary power system is preferable to the three parallel systems utilised in traditional aircraft. Electrical power is more versatile than hydraulic and pneumatic systems and so capable of replacing their functionality. Although electrical systems have a lower force to weight ratio than equivalent hydraulic systems, and electrical power is not readily available from the GT as pneumatic power is, electrical distribution is more robust than hydraulic distribution (requiring less maintenance) and smaller than pneumatic distribution. Additional electrical usage requires increased generation and distribution capacity but these are more controllable than hydraulic or pneumatic systems allowing offtake power to be used more efficiently.

The use of a single secondary power system, rather than three separate systems, offers greater optimisation of power usage and controllability which increases efficiency at the aircraft system level.

2.2.3 Electrification of Aircraft Secondary Power Systems

The rise in aircraft electrical systems was originally motivated by the inclusion of electronic instrumentation. The advent of power electronic systems has opened up the possibility of replacing existing hydraulic and pneumatic power systems with electrical equivalents. This section provides practical details of the electrification of aircraft power systems.

2.2.3.1 History of Aircraft Electrification

Even before the introduction to civil aviation of electrically-powered fly-by-wire systems (on the A320 in the 1980’s [25] [43]), discussion had taken place as to what was the best medium to distribute power throughout an aircraft; results being published in 1945 [44]. During the 1950s the aircraft of the British V-bomber force all made extensive use of electrical power distribution because of their high-powered electronic-warfare equipment. Most notably, the Vickers Valiant was highly electric [26] [25] and the Vickers VC-10 used electrical power for most of its flight control surfaces [25]. Despite a number of studies in the 1970s and 1980s by leading powers within the industry (including the NASA led Integrated Digital/Electric Airplane (IDEA) study,
completed 1985, in which both Boeing and Lockheed Martin were contracted) outlining the positives of electrical power distribution, its widespread usage did not take off.

### 2.2.3.2 The Advantages and Disadvantages of Aircraft Electrification

Boeing’s IDEA study predicted a 3% fuel consumption reduction for their 767 airframe and Lockheed’s IDEA study predicted a 10-13% fuel consumption reduction for their L-1011 TriStar when considering the removal and replacement of pneumatic offtakes. These predicted fuel savings were significant, however they were partially discredited by further British studies [26].

Studies led by Cranfield University entitled ‘zero-bleed secondary power systems’ incorporated work from British Aerospace, Lucas Aerospace, Royal Academy of Engineering and RR. This study used an Airbus A300-600 powered by RR RB211-534D as a benchmark and considered the theoretical replacement of a pneumatically powered ECS with an electrically powered system. Results published in 1987 predicted a fuel burn reduction of less than 1%, substantially lower than the findings of the IDEA study. This was explained by several anomalies with the IDEA research. Firstly that the baseline GT used in the IDEA study was optimised for shaft power and not pneumatic offtake making the baseline model excessively inefficient to begin with. Also the IDEA study considered advanced computerised fly-by-wire systems only on their All-Electric Aircraft (AEA) design (rather than standard hydraulic systems), the computerised system alone was claimed to provide a 10-13% fuel saving, making the comparison unfair.

Figure 2.10 shows the efficiency (fuel burn per seat) of historical large civil aircraft.
Figure 2.10: Fuel burn reduction on large civil aircraft (data from [45])

Improvements in GT efficiency have reached somewhat of a plateau and so it is necessary to look at a step change in system design to improve this. Electrification is once again considered as a means of reducing in-flight fuel burn, as shown in Figure 2.11.

Figure 2.11: Enhanced aero technologies, fuel savings against risk of adoption considered on an A320 (data from [26])
Fuel savings are predicted for the electrification of auxiliary systems, however these improvements are only small when considered on a traditional or more-electric architecture. These fuel savings are significant when full aircraft electrification is carried out, creating an AEA. The adoption of electrical systems increases aircraft mass, however if the AEA is adopted, the complete removal of hydraulic and pneumatic systems could lead to a reduction in aircraft weight of 0.5% (considering an Airbus A330 or Boeing 767) [46].

Analysis of aircraft electrification has tended not to include the latest advances in other non-electric power systems. For example, the development of more power dense high pressure hydraulic systems (now typically 20.7kPa (3,000psi) [25]) has further improved the power density comparison against electrical actuation in favour of hydraulic systems.

2.2.3.3 Electrification of Hydraulic and Pneumatic Systems

Electrical equivalents are available for both hydraulic and pneumatic systems on aircraft, however it is rare that they can be directly replaced, and usually require additional system (distribution / generation) modifications.

Hydraulic

Hydraulic actuators (like the SHA) are directly replaced with electrically powered equivalents (such as the EMA). Hydraulic distribution can therefore be removed and replaced with electrical cabling connecting power from the main bus. The inefficient hydraulic pump is removed with the replacement electrical power supplied by an uprated generator.

Electrical power distribution is lighter than equivalent hydraulic systems, however electrical generation and actuation is less power dense [47]. Maintenance is less costly for electrical systems, and the power extracted from the GT can be better controlled.
Pneumatic

Pneumatic loading primarily consists of the ECS and WAI. If both these systems are electrified pneumatic power offtake can be removed, increasing GT efficiency, as core pressure has been increased on modern engines, the offtake has a greater detrimental effect on GT performance [25]. Power will instead be supplied from electrical generators which must be uprated accordingly. Pneumatic ducting is replaced with smaller and more flexible electrical cables, reducing weight and improving airframe design. Electrically driven compressors are required to power the ECS although the work they carry out is less than equivalent pneumatic systems as there is a closer match between external source air and target conditions. Lower levels of air filtering are required, potentially improving air quality. The electrification of the ECS adds additional energy conversion stages to the system which means that efficiency benefits are still not definitively proven, and may well depend on individual mission profile [34]. The removal of pneumatic ducting and air temperature regulators reduces airframe weight (for an Airbus A330 or Boeing 767) by approximately 600 kg, however another 550 kg of electrically driven compressors must be added, as well as 150 kg on the generators to increase their rating. This gives a net increase of approximately 100 kg when the ECS is electrified [46].

Electrical WAI system require the placement of resistive heating elements on the surfaces, these are both efficient and lightweight. Their electrical load is usually pulsed to dislodge built up ice.

It has not conclusively been shown that electrical systems offer weight saving or efficiency benefits over pneumatic and hydraulic systems. However, electrical systems can be better regulated / controlled and do require consistently lower levels of maintenance.

2.2.3.4 Summary Aircraft Electrification

Research into the electrification of aero power systems began in 1970s, driven by the fuel crisis of the time. Advancements in electrical generators and actuators as well as power silicon systems have made an AEA technically feasible and there is a renewed
interest in the subject with the objective of reducing airframe operational costs as well as maintenance costs.

Benefits in in-flight efficiency through electrification are for the most part unproven, this is demonstrated by the reluctance of the aero industry to commit to the AEA design. While the Boeing 787 discarded pneumatic power offtake (in favour of an electrically driven system) the most recent similar design, the Airbus A350 XWB, scheduled for Entry Into Service (EIS) in 2014, maintains pneumatic offtake for the ECS and WAI [48]. Maintenance costs are lower and controllability greater for electrical systems which has led the Airbus A380 to make greater use of these systems alongside pneumatic and hydraulic loads.

2.2.4 All-Electric Aircraft

To best benefit from aircraft electrification the AEA is proposed. Traditional power systems (pneumatic, hydraulic and electrical) are replaced with a single, electrical, power distribution network. An uprated electrical generator is required to supply these loads. The simplified energy conversion process of the All-Electric Engine (AEE) is shown in Figure 2.12.

![Figure 2.12: All-Electric Aircraft GT energy conversion (GT image from [24])]({})

With only a single, electrical, power offtake, engine design is simplified and efficiency is increased. However larger, heavier, electrical generators are required.
Despite numerous advantages in terms of design (airframe and engine) and control, and the possibility of operational cost savings, the AEA has yet to be fully realised. Implementing such a radical new architecture offers a substantial financial risk for engine and airframe manufacturers [49]. High certification costs would also be felt as both the engine and airframe have a new architecture.

### 2.2.5 More-Electric Aircraft

The More-Electric Aircraft (MEA) offers an increase in electrical power systems without the complete removal of trusted traditional systems. Its introduction has a far lower financial and commercial risk while achieving some operational benefits such as higher controllability and lower maintenance.

MEA have reduced pneumatic and hydraulic power networks. This is achieved by using an electrical system which is the dominant, although not sole, power network. Likewise the More-Electric Engine (MEE) is only required to provide negligible pneumatic power and limited hydraulic power, but must be able to deliver substantial electrical power. Figure 2.13 shows the MEA energy conversion process.

![More-Electric Aircraft GT energy conversion](image)

**Figure 2.13: More-Electric Aircraft GT energy conversion (GT image from [24])**

No pneumatic power is extracted, however this leads to an increased mechanical offtake, mainly to supply the high electrical generation demand as a result of the
electrically powered ECS. Hydraulic power is available so the high power density of these loads can still be utilised, although hydraulic generation is greatly reduced by the use of electrically powered actuation systems on non safety-critical surfaces.

2.2.5.1 Boeing 787 MEA

The Boeing 787 ‘Dreamliner’ is to date the only civil MEA in service, achieving EIS in October 2011. Although the other recent large civil aircraft design, the Airbus A380, has many more electrical features, particularly for actuation purposes, it maintains a pneumatically powered ECS. The Boeing 787 is therefore used as the main design basis throughout this thesis. It is a mid-sized, wide body, aircraft carrying approximately 250 passengers [50] which is powered by two GT engines [50]. The launch engine is the RR Trent1000 GT, although airlines can also opt for the GE Aviation GEnx-1B. Both engine options offer identical power facilities to the airframe through a common interface.

2.2.5.2 More-Electric Engine

Work undertaken throughout this study is based on a large diameter civil GT powering the Boeing 787, of the two options, the Trent1000 is considered in most detail.

Rolls-Royce Trent1000

A cutaway of the Trent1000 is shown in Figure 2.14

![Figure 2.14: RR Trent1000 cutaway [24]](image-url)
The RR Trent1000 MEE has a three-spool configuration with the mechanical power offtake coupled to the IP spool. The engine has a bypass ratio of up to 10:1, and can produce 330 kN (75,000 lbs) of thrust at maximum power. A pressure ratio of 50:1 is achieved by the fan, 8 IP stages and 6 HP stages. A single HP turbine, single IP turbine and 6 LP turbines power the spools [18]. The Trent1000 uses a Variable Frequency Starter Generator (VFSG) driven by the fan case mounted Accessory Gearbox (AGB). As a synchronous machine, this generates variable frequency AC for the airframe, but is also used to provide starting torque for the GT and during this mode of operation a clutch coupling connects drive to the HP spool as well as the IP spool.

**GE Aviation GEnx-1B**

The GEnx is a two spool system, with the mechanical offtake taken from the HP spool, a cutaway is shown in Figure 2.15.

![GE Aviation GEnx cutaway](image)

**Figure 2.15: GE Aviation GEnx cutaway [51]**

The GEnx has a bypass ratio of 9.6:1 and can produce 330 kN (75,000 lbs) thrust. A pressure ratio 43:1 is achieved through 4 fan mounted booster stages and 6 HP stages. Power is extracted from the fluid flow by 2 HP and 7 LP turbines. Note the core case mounted AGB [52], this allows the transmission driveshafts to be physically shorter that that on the Trent1000, potentially having lower compliance. It does, however, place the Accessory Gearbox in a more hazardous environment and further disrupts bypass
airflow. Other differences with the Trent1000 are the composite fan casing, composite (titanium tipped) fan blades, and a longer length due to the two spool architecture.

2.2.6 Aero GT Electrical-Mechanical System
This section describes the method of generating and distributing electrical power on the airframe which is common to the majority of modern civil aircraft.

2.2.6.1 Mechanical offtake and drivetrain
Power is extracted mechanically from variable speed spool at the core of the GT and used to drive electrical generators mounted on the fan casing (seen in Figure 2.14). The spool and AGB are connected by a transmission systems consisting of fixed ratio gearboxes and driveshafts which pass through both the GT core and bypass areas of the engine.

Driveshaft compliance
The transmission driveshafts are required to transfer power from the very core of the GT to the outer edge of the fan casing while minimising their impact on gas flow through the engine. They therefore have low diameters and are driven at high speeds to ensure power transfer. This length and narrow cross section produces driveshafts with high levels of torsional compliance.

Accessory load inertia
The AGB drives a number of auxiliary systems which include, alongside the electrical generators, a hydraulic pump, fuel pump and oil pump. These are collectively referred to as the accessory load. Despite being designed for high speed operation they still have a relatively high inertia. The AGB, and the accessory loads it drives, are critical to the operation of the engine and airframe, its failure or a loss of drive will result in engine shutdown and airframe secondary power must be sourced from other engines.

The mounting of accessory load on the outer casing of the gas turbine distances these systems from the hostile environment within the engine core. This design requires a transmission system to provide a mechanical connection between the accessory load and the spool offtake. The lightweight, high length and low diameter of the transmission
shafts makes them vulnerable to cyclic fatigue. Furthermore, the high compliance of the transmission and high inertia of the accessory load produce a mechanical drivetrain which is prone to resonance.

### 2.2.6.2 Electrical generation and distribution

**Generation**

For redundancy reasons the Boeing 787 requires two equally sized generators mounted on each engine. These are synchronous generators, which deliver Variable Frequency (VF) AC to the airframe within the range of 360 Hz to 800 Hz as per airframer specification [53]. The 3-phase VFSGs are nominally rated at 250 kVA [25] this provides the Boeing 787 with 1 MVA generation capacity, but they also have an additional, short term and failure, overload rating. Voltage regulation is provided by a Generator Control Unit (GCU) which adjusts rotor field magnitude to maintain 230 V\_phase [25].

**Distribution**

Electrical power is distributed throughout the airframe by four separate buses [25]. Buses are only interconnected in the case of failure and in this circumstance the original generation source is disconnected before the new supply is connected. It is common for airframe electrical wiring to follow different paths throughout the airframe so that a single incident is less likely to sever multiple buses.

**Loads**

A wide range of loads appear on modern aircraft, from high-powered continual loads (ECS), to constant power loads (fuel pumps), transient loads (actuators), and pulsating loads (radar). As well as loads of the type detailed in Table 2.1, the Boeing 787 also includes several larger loads as a result of its more-electric architecture, these additional loads are given in Table 2.3.
A wide number and type of electrical loads are supplied from the generators. Power from these generators is not only used to provide passenger comfort, but also critical flight control and actuation systems as well as life support. Redundancy is assured by multiple, isolated, electrical systems so a failure (such as a short circuit) in one does not propagate to the others, allowing electrical supply to the airframe to continue. Furthermore, no energy storage is utilised, so power must be available instantaneously from the generators for all loads, including non-continuous/transient systems.

2.2.6.3 Electro-mechanical system

An electro-mechanical network exists within the modern GT, this system can be described as shown in Figure 2.16.

![Electro-mechanical system schematic](image)

Figure 2.16: Aero electro-mechanical system schematic

The GT core (spool) is coupled through a mechanical transmission system to an electrical generator which provides the point of interface to an airframe electrical network with a wide range of high powered and transient electrical loads. Control exists for the GT, generator, and often at individual electrical loads, however, the GT has a slow response time due to its high inertia, and individual electrical loads are relatively low powered making the GCU most significant.

As electrical supply becomes an increasing fraction of the engine’s total energy production (estimated to be around 1% on the latest aircraft designs), the electrical and
mechanical networks can no longer be considered as isolated systems. Instead the actions of the whole electro-mechanical network must be understood, with the generator acting as the coupling (energy conversion mechanism) between the two networks.

Summary of aircraft electro-mechanical system
Power is taken from the very heart of the GT, where lightweight mechanical components provide safety-critical power to the aircraft’s auxiliary loads. Airframe architecture means that electrical generators are individually required to provide a stable electrical network for a number of high-powered and highly-variable electrical loads. As a result of the significant sizing of modern generators, electrical transients are passed through to the mechanical components of the transmission. Interaction within this electro-mechanical system could potentially lead to electrical network instability and reduced lifespan of key mechanical components.

2.2.6.4 Impact of GT structure
While the same electro-mechanical system is present on all MEA, the implications differ depending on GT structure. Small diameter military GTs have a much shorter drivetrain system which more rigidly couples the GT and electrical generator. The GT spools have a smaller diameter and inertia than a civil GT, and electrical loading is proportionally larger, due in part to electronic warfare type systems.

Figure 2.17 compares the electro-mechanical network on large diameter civil engines and small diameter military engines.
The drivetrain of a military GT has a lower compliance and so couples transients from the electrically network more rigidly to the spool. The low spool inertia and high electrical loading means that electrical power demand is more likely to directly influence spool speed. Engine control systems can be modified to adjust fuel burn according to electrical loading and so compensate to maintain spool speed.

In a large diameter, civil, GTs electrical loading has little influence on spool speed due to the high spool inertia. Transients are instead absorbed by a more compliant drivetrain potentially leading to mechanical fatigue.

2.2.6.5 Design Challenges Associated with the More-Electric Aircraft
The electrification of the airframe and GT systems requires not only an increased electrical generation capacity but also stronger distribution and regulation. The transfer of energy between the GT and electrical loads must be considered at a system level.

Electrical Power Demand
With the introduction of the MEA architecture, the airframe electrical power requirements are increased as both mechanical and fluid non-propulsive power systems are replaced with electrical alternatives. Electrical power generation is now not only the
dominant secondary power system, but the energy used is a significant fraction of the GTs total energy process.

The removal of pneumatic offtake increases the power required from electrical generators, which is itself derived from the mechanical offtake from the spool, and so the direct result of removing the pneumatic offtake is an increased power transfer through the drivetrain and generators. The replacement of some hydraulic systems with electrical equivalents does increase the electrical generation demand, however as the hydraulic pumps and electrical generators are both mounted on the AGB; there is little change in the original mechanical energy extraction. There is however an increase in the dominance of the generators amongst the other accessory loads on the AGB due to their sizing. Generator systems and mechanical components must be uprated to cope with this increased loading, adding additional weight to the aircraft.

Mechanical Resonance
The mechanical drivetrain, consisting of high inertia loads coupled by driveshafts of low torsional stiffness and low damping, is prone to resonance. High-rated electrical loads cause large transients on the electrical network which are transferred through the generator to the mechanical network, potentially exciting drivetrain resonances. Vibration may lead to increased fatigue in mechanical components.

Electrical network regulation
The quantity and high rating of electrical loads, coupled with the presence of switched power electronics means that high levels of harmonic content are injected onto the network. It is therefore challenging to ensure that the network is regulated within the airframe specifications, [30]. This is more complex than other electrical generation systems (ground based, marine) because the architecture dictates only a single, standalone, generator per bus.

Energy Storage
Two forms of energy storage are considered. Inherent energy storage that exists due to system structure, and energy storage added simply for its performance benefit.
Inherent Energy Storage

The GT architecture includes several energy storage mechanisms. The spool and, to a lesser extent, the electrical generators store energy kinetically due to their inertia. Energy is stored torsionally as a result of the transmission compliance. These energy storage elements create a system with resonance. It is attractive to minimise inertial energy storage as this reduces mass.

Additional Energy Storage

It is common for transport applications to utilise energy storage systems to act as a power buffer for supply smoothing, this is also desirable on aircraft systems. Mechanical energy storage may be used to smooth mechanical transients on an Internal Combustion Engine (ICE). However, due to their operating conditions many onboard energy storage mechanisms are undesirable (electrolytic capacitors are susceptible to temperature and pressure variations associated with aircraft operation, and inductances and inertias are often very heavy). Although it would be beneficial given the extreme electrical loading transients, providing large-scale energy storage on the electro-mechanical network is unattractive because of the operational requirements of the aircraft.

Summary of More-Electric Aircraft design challenges

The high rated electrical generator forms a strong coupling between the mechanical and electrical domains, meaning that these systems influence each other. GT architecture produces a vulnerable mechanical drivetrain which is prone to resonance, while electrical load variation can induce these resonances leading to mechanical vibrations which in turn disrupt electrical network regulation. High levels of electrical loading and a weak network make electrical network regulation challenging, this problem must be overcome without the luxury of adding a power buffer.

2.2.7 Summary of electrical secondary power systems

Powerful hydraulic, pneumatic, and electrical networks exist to supply auxiliary loads within modern aircraft. Electrical devices are capable of replacing the functionality of pneumatic and hydraulic systems so that a single generation and distributions system can be used. A single, electrical, power system offers greater control of the power
extracted from the GT, hence increasing aircraft efficiency, furthermore reducing maintenance costs. The AEA has yet to be realised, however the MEA has been introduced to service, this utilises increased electrification but still maintains some hydraulic systems to benefit from their lightweight and failsafe nature.

Increased electrical power is now expected from the engine, and this requires an uprated and robust electrical generation system. The strong coupling between vulnerable and compliant components of the mechanical drivetrain and high powered, non-linear, electrical loads creates an electro-mechanical system which is prone to interaction.

2.3 Electro-Mechanical Interaction

The modern airframe electrical network is large in both size and rating, making up the majority of secondary power extracted from the GT. The similarity in power rating of the electrical network and mechanical network which supplies it, and the rigidity of coupling between the two domains via the generator, means that two-way interaction is possible within this electro-mechanical system.

Interaction in electro-mechanical systems is not unknown. This section reviews existing research and strategies for modelling and control in other applications.

2.3.1 Land based generation

The phenomenon of interaction between electrical and mechanical elements is well documented in land-based generation where sub-synchronous armature currents induce torque disturbances. The interaction is described as sub-synchronous resonance [56]. The process is identified on a real system in [57] and analysis of the behaviour validated practically. Shaft torques produced by sub-synchronous resonance are predicted in [58] using a state-space model, although no practical validation is carried out. Modelling to predict torsional modes is carried out in [59], and also goes further to demonstrate a link between electrical disturbance and mechanical fatigue life expenditure.

Land based generation systems, such as that shown in Figure 2.18, have the luxury of a strong power network, with multiple generator sources and multiple loads, ensuring a low variation in load relative to network rating.
On a fixed-frequency electrical network the mechanical drive has a very narrow speed range. To combat sub-synchronous resonance, mechanical systems can be augmented which also has the effect of altering their resonant modes, reducing resultant damage and prevent excitation. This is possible in land based systems as weight is not a pressing restriction, and the speed range is small.

### 2.3.2 Industrial automation

Rolling mills also suffer from resonance problems. In what is a common industrial process, an electrical motor is used to drive a load of high inertia through a gearbox and drivetrain system, as shown in Figure 2.19.

Loading on the roller varies as the control scheme modulates the roll height to regulate output. [60] finds that this interaction can be reduced by, amongst other things, minimising backlash (from gearboxes and couplings), and designing controllers without overshoot. Interaction can therefore be managed by system design and fast-acting control. Electrical power is supplied by a strong mains network, usually via a converter.
with DC link capacitance, and so unlike a GT system electrical loading does not affect supply regulation. Other industrial electro-mechanical systems are discussed in [61] and sources of excitation are identified from the electrical (transients) and mechanical (imbalance) domains.

2.3.2.1 Vibration mitigation by control

The concept of fast acting control to suppress vibration is the subject of a number of studies. Vibration on an electro-mechanical servo drive system is investigated in [62], a torsional mechanical model is derived which allows a notch filter to be applied to the controller so as to avoid torsional resonant modes. A similar scheme is carried out in [63] to identify and mitigate vibration in industrial drives, and a control scheme is devised in [64] and [65] to suppress vibration on a simple torsional system. A laboratory electro-mechanical system is considered in [66] and incorporates both a dynamic machine model and dynamic gearbox model, the mechanical system is relatively simple but key resonances are detectable using machine signals. Vibration suppression through control is carried out in [67] on a non-torsional robotic system. Even on the most basic systems, with a drive connected directly to a load, interaction can lead to reduced lifespan of mechanical components; a case study, [68], recommends that torsional natural frequencies are kept separate from electrical frequencies to reduce interaction. [69] identifies the issue of cumulative fatigue induced in mechanical components as a result of periodic torque variations from variable speed drives, and outlines methods to model the mechanical system.

These studies investigate electro-mechanical systems which have a much lower level of complexity (both in electrical loading and mechanical drive) than aero systems, however the use of control to suppress vibration is applicable to aero system.

2.3.3 Automotive

Automotive systems generate electrical power from an 'alternator' which, conventionally, is coupled to the prime mover via a belt drive. This system is shown in Figure 2.20.
This belt drive will have greater levels of damping than equivalent GT transmission system. The absence of a complex mechanical transmission and the presence of a battery, which provides an electrical power buffer, make automotive electrical power systems far less susceptible to interaction than the aero system described. The ICE provides variable speed drive to the generator, as per the GT system, but this torque has considerable fluctuation because of the piston action [70]. Belt drives are unsuitable for aero systems as they require greater levels of maintenance and perform badly in the extreme temperature conditions compared to a shaft / geared transmissions.

2.3.3.1 Drivetrain dynamics
Research into drivetrain dynamics [70] [71] [72], where disturbances (e.g. fluctuating prime mover torque, gear meshing, wheel loading) are exerted onto a complex network of shafts and gears, are highly relevant to the mechanical drivetrain within a GT.

[70] utilises modelling to predict the dynamic behaviour of drivetrains, mechanical components, such as gears are modelled, and backlash is covered in detail. Modelling assumptions include a lack of gear chatter. The approach of comparing practical and simulation in the frequency domain is used. [71], again, discusses the modelling of non-linear drivetrain elements such as backlash. Modelling strategies for drivetrain components and various gearing configurations are described. Simulation methodology is detailed such as means of overcoming numerical issues to produce holistic drivetrain models through the combination of component models. These papers highlight the importance of model accuracy at a component level and how non-linear affects are not always considered. Models in both of these papers are different to a GT system, however, as no electrical system is considered. A multi-mass modelling approach is
carried out in [72] with less detail of individual drivetrain components and the sources of drivetrain excitation are identified.

An automotive drivetrain is shown in Figure 2.21 with an ICE providing variable speed power to two of the vehicles wheels, the drivetrain structure is similar to that of a GT.

![Figure 2.21: Example automotive drivetrain](image)

The scenario is somewhat different for Hybrid Electric Vehicles (HEVs) and Electric Vehicles (EVs) which may have either a mechanical drivetrain or wheel-mounted motors. The higher rated generator makes interaction more of a concern; however a power buffer will almost certainly exist and can alleviate the issue. An attempt to consider these interactions is made briefly in [73] with equivalent circuit modelling used to replicate mechanical systems to form a electro-mechanical model.

### 2.3.4 Marine

Electro-mechanical interaction in marine systems has also been considered [74]. The main electro-mechanical issues arise between the drive motor and propeller for the system shown in Figure 2.22.
2.3.5 Wind power generation

Wind turbine systems are similar to aero systems in that they have a high inertia, variable speed, prime mover which drives an electrical generator through a drivetrain of shafts and gearboxes. Although electrical network stability is ensured by the presence of multiple generators, and only a single prime mover drives a single generator, mechanical component fatigue is caused by variable loading from the wind (despite blade pitch control). The wind power generation electro-mechanical system is shown in Figure 2.23.
The electro-mechanical system of a wind turbine is considered in [76], through a multi-domain simulation and analysis of the mechanical drivetrain. A control scheme is introduced which successfully reduces mechanical stress by applying a bandpass filter to the grid interface converter.

### 2.3.6 Aero

The GT produces a variable speed drive which transfers through a mechanical drivetrain to drive two generators supplying isolated networks, this arrangement is shown in Figure 2.24.

![Figure 2.24: Aero electro-mechanical system](image)

Investigations into electro-mechanical interaction within aero systems have so far focused on a systems level approach [77], considering powerflow and stability between the GT spool and the electrical network [78], this is perhaps the effect mostly likely to occur on a low diameter GT. More detailed research is carried out on power quality for an aero electrical network with single generator [79], although no drivetrain dynamics are considered. Acknowledging that a full systems design approach must be adopted in order to fully exploit the electrification of loads a full aircraft electrical system test facility is developed in [80]. This allows power management consideration but this does not include any GT dynamics.
Consideration has thus far not been given in the literature to the behaviour of all individual elements within a large scale, aero, electro-mechanical system. This thesis considers, at a high level of fidelity, the interaction between mechanical components, electrical machines, and electrical loads within an aero GT system.

Anecdotal evidence suggests that incidents of suspected electro-mechanical interaction have been recorded within the aero industry. Mechanical issues have ranged from a component lifespan reduction to extreme failure leading to In Flight Shut Down (IFSD). Power surges in the electrical network and an inability to regulate the network have been observed. The preferred option in these cases is to adjust the GCU or place limits on electrical loading rather than redesigning the whole GT architecture. However due to the commercially sensitive nature of these events, information relating to these is not readily available.

2.3.7 Summary of electro-mechanical interaction review

Electro-mechanical interaction in land-based generation systems is documented, but these systems operate at fixed frequency (and mechanical speed) and have a relatively simple drivetrain, designed without weight restriction. Land-based generation, marine systems, and wind turbine systems have strong electrical networks supplied by multiple generators; only the wind turbine systems have a varying mechanical speed. Industrial processes are motor-driven with relatively simple drivetrains and are generally decoupled from electrical supply through a DC link capacitor. Automotive systems have higher levels of mechanical damping, and a large electrical power buffer so preventing the occurrence of interaction.

Aircraft generators must operate with a prime mover designed primarily for propulsion and operating at variable speed. The weak electrical network has only a single generator, and load fluctuations are large. The drivetrain is complex and vulnerable due to its low weight design. Existing investigations into aircraft electro-mechanical interaction take a systems-level approach, neglecting the mechanical coupling. Electro-mechanical interaction has been observed in many circumstances, however the aero system offers some unique challenges due to its mechanical complexity, weight limitations, and variable speed operation.
The literature survey has not identified any previous work relating to aircraft electrical power generation that includes the drivetrain dynamics.

2.4 Aero Electrical generation and distribution

This section covers a range of electrical generation solutions for aircraft. Distribution standards and electrical generators are discussed, including the doubly-fed machine being considered for this study and how it may best be integrated into an aero system.

2.4.1 Aero electrical generation system

With similarity to a wind turbine system, mechanical drive speed, taken from a GT spool varies to meet the aircraft propulsion requirements, and so unless some form of regulation exists the electrical output will vary with spool speed. The general scheme of electrical generation from GT shaft power is shown in Figure 2.25.

![Figure 2.25: GT system for electrical generation](image)

Figure 2.25 shows a VF AC electrical generation scheme (with electrical frequency $f$ proportional to spool speed, $\omega_{\text{mech}}$), however DC schemes are used in some aircraft. Voltage, and potentially frequency, regulation is required but this is more challenging in aero systems than for a wind turbine as there is no rigid network with which to interface as, for redundancy reasons, distribution networks are not usually interconnected. Each generator must therefore be controlled to operate in a standalone condition.
2.4.1.1 Aero distribution solutions

The electrical supply regulations are dictated by the airframe manufacturer and so the choice of generation strategy is generally decided by choice of distribution standard. Power electronic interfaces and conditioning electronics have to a certain extent removed this relationship in recent years however. Electrical distribution may be carried out by either an AC or a DC network. There are three main configurations for aircraft electrical power distribution, DC, VF AC, and Constant Frequency (CF) AC, which are compared below.

AC Distribution

The various options for AC generation and the associated distribution strategies are shown in Figure 2.26.

![Diagram](image)

**Figure 2.26: Aero AC Electrical Power Generation Types (using data from [37])**

a) Integrated Drive Generator

IDG systems utilise a CSD to maintain a constant mechanical speed input to the generator. CSDs are hydro-mechanical systems requiring regular maintenance [81] and
adding significant size and weight to the system [82] (typically having four times the mass of the generator they are coupled to [83]), as well as cost [82]. Mechanical speed is regulated and so a CF electrical output is therefore achieved from a synchronous machine.

b) Variable Speed Constant Frequency
The mechanical speed at the input to the generator is allowed to fluctuate while electrical conditioning is used to produce a CF electrical output. This configuration is investigated in detail in [82], and with a hardware rig in [84]. Typically a cycloconverter [25] is used to produce the required bus frequency from a higher generated frequency. Alternatively a back-to-back converter with DC link may be used to synthesis the desired bus frequency and voltage. This converter must be fully rated.

Various implementations which produce constant electrical frequency from a variable speed mechanical system, such as that of a wind turbine, are referred to as Variable Speed Constant Frequency (VSCF).

c) Variable Speed Constant Frequency DC generation
A DC generator is used with an inverter to synthesise an AC waveform at the required voltage and frequency. The use of a DC generator simplifies the process of paralleling generators, which may be required under fault conditions. It should be noted that DC generation is only realistically possible with the use of commutators which are undesirable for aero applications.

d) Variable Frequency
VF distribution greatly simplifies the generation as a synchronous generator is directly coupled to the mechanical network and no frequency regulation is required. This type of distribution is sometimes referred to as 'frequency wild'.

DC Distribution
Methods for generation and DC distribution strategies are shown in Figure 2.27.
a) Direct DC
DC generation can be used to provide for a DC bus. Again, commutators must be used which is not ideal for an aero system.

b) DC, AC link
An AC generator is used to power an AC link. A rectifier is then used to deliver the required DC output voltages. This topology allows numerous generation sources to be coupled to the same DC bus via power electronic converters and requires a fully rated converter.

Control / Regulation
Voltage wild networks are not common and so all generation and distribution schemes require control to provide voltage regulation. Typically a GCU carries out this function at the generation source (the machine) but on systems with an inline converter, regulation can be carried out by the converter which can remove the need for a GCU or require it to be integrated with the converter control systems. Similarly frequency control is required for fixed frequency systems.

Distribution Strategy Selection
There are pros and cons to each distribution configuration and the associated provision of electrical power. The chosen strategy depends on the exact operational requirements of the airframe.
Although parallel generation is not used in normal circumstances on airframe, DC systems are easier to parallel, potentially making a more robust, multi-generator, system. However, DC networks are less appealing as current must be limited to around 400 A to keep the size of switchgear down [25]. Therefore for higher rated systems, such as civil aircraft like the Boeing 787, AC distribution is considered lighter, however, depending on the frequency (and possible variation) of the distribution frequency sensitive loads (such as motors) may require additional power electronics [37]. CF AC is considered preferable as it simplifies and reduces the power electronics required for frequency sensitive loads. Mechanical speed regulation (Figure 2.26a) requires the use of CSDs which are heavy and need regular maintenance, and electrical frequency regulation (Figure 2.26b and Figure 2.26c) requires a fully rated power converter adding mass and cost. As a means of providing a CF system the use of a CSD compares favourably in terms of mass against systems with electrical regulation using either a cyclo-converter or DC link [83], although this study is dated and for a specific aircraft only. A more recent mass comparison, for a CF and VF system on a more-electric Airbus A330 or Boeing 767 predicts a aircraft weight saving of around 220 kg for a VF system [46], although the VF scheme considered is also voltage wild.

Although CF is preferable for the electrical loads, because of the additional systems required for mechanical or electrical frequency regulation, airframe manufacturers have rejected CF in favour of VF for their latest designs.

Distribution standards
Traditionally airframes used both 115 V 400Hz CF AC and 28 V DC (predominantly for emergency / flight critical systems) distribution busses [23] [85]. The choice of 400Hz AC frequency is mentioned as far back as 1945 [44] as being most suitable for the supply of aircraft instrumentation and equipment at the time, this relatively high frequency increases the power density in transformer systems [82] and can reduce the size of converters [86]. Higher power distribution systems are now a must on modern aircraft and so voltages have been increased giving distribution systems of 230 V AC [25] for civil systems and 270 V DC [37] for military systems.
The most recent airframe design, the Boeing 787, specifies an electrical generation of 360-800 Hz regulated at 230 V. Boeing’s power quality specifications for electrical loads, dictates harmonic limits for propagation onto the network and the accurate operation of loads, [30].

2.4.1.2 Aero Generation Solutions

The choice of electrical generator is mainly determined by the selection of electrical power distribution strategy. There are however often several generation options available for each distribution strategy each with associated benefits and drawbacks.

Wound Field Synchronous Generator

Synchronous Generators (SGs) offer reliable, robust and technologically mature generation solutions. Electrical frequency is proportional to mechanical drive frequency, and with associated mechanical conditioning they have traditionally been used in CF IDG systems but are also suitable for VSCF, VF and DC, AC link distribution.

Switched Reluctance Generator

Switched Reluctance (SR) systems offer high speed generation and good power density, they are highly robust in extreme environments, for example high temperatures, and offer good fault tolerance [82]. They do however require a fully-rated back-to-back converter, to operate and condition the pulsating electrical power output.

Permanent Magnet Generator

Permanent Magnet Generators (PMGs) offer high power density, and as such they are usually used for military airframes and UAVs applications or for backup supply on civil systems. A rectified DC voltage output may be used but this requires a fully rated back-to-back converter to regulate AC voltage and frequency. The electro-magnetic coupling cannot be deactivated during electrical or mechanical faults and so fault tolerance is considered to be poor. PM machines are also not particularly robust as the containment of magnetic components is challenging and demagnetisation is possible in environments with high levels of vibration and temperature also the machine cost is likely to be high due to the necessary materials.
Doubly-Fed Induction Generators

DFIG systems offer the benefits of a wound field synchronous generator but with the ability to create a CF system using only a fractionally rated converter. The machine itself is, however, typically considered less power dense than an equivalent synchronous machine. Several doubly-fed arrangements are possible which may or may utilise use slip-rings.

Embedded Generation

Electrical generation schemes considered so far have assumed a fan case mounted generator with drivetrain coupling to the spool as shown in Figure 2.25. This mechanical drivetrain is another potential source of failure for the GT, it disrupts gas flow through the GT core, and is not lossless. The drivetrain is no longer required if electrical generators are 'embedded' within the core of the GT with a spool acting as the machine rotor [7]. Any embedded generator has to be able to operate within the hazardous core environment, experiencing high levels of temperature and vibration and as such only the most robust machines can be used [87]. Power generated by embedded machines must still be transferred through the core and bypass regions of the engine but this can be done more conveniently than with mechanical driveshafts.

Embedded generation on multiple spools allows power to be extracted optimally from each spool or even to be transferred between spools to aid with engine control.

2.4.1.2 Summary of aircraft electrical generation and distribution methods

Aircraft have several, isolated, electrical networks which are each supplied by single generator. These generators must be controlled to provide standalone network regulation. Electrical generators are driven at a varying mechanical speed by the GT. CF AC is desirable for high power electrical distribution, however current technologies to produce this supply are not adequate and so VF is utilised on the latest aircraft. DFIGs have the ability to provide a CF network without the need for a CSD or fully rated converter.
2.5 The Doubly-Fed Induction Generator

This section covers in detail the doubly-fed induction generator, its existing uses and prior modelling and control.

2.5.1 The DFIG system

Throughout this thesis the term 'doubly-fed' is used to describe a machine in which there are two electrical power connections. The stator side connection is directly connected to the electrical network, and acts as a generator, the rotor is supplied through slip-rings from a power converter, which may be required to source or sink power.

The DFIG is similar in structure to a standard induction machine; however it has a wound rotor rather than a shorted cage. The rotor is wound to have the same number of phases as the stator but with a different number of slots so as to avoid slotting effects.

An overview of the electrical connections for the three phase DFIG being considered is shown in Figure 2.28.

Control of the frequency and phase of supply to the rotor allows the rotor currents to be directly controlled. Unlike a standard induction machine, slip is controlled independently of mechanical drive speed and hence allows mechanical drive frequency and electrical generation frequency to be decoupled [88]. Rotor magnetising current amplitude dictates the field magnitude and induced stator voltage, allowing the generator output voltage to be regulated directly, and hence at a high bandwidth.
2.5.2 Wind turbine applications

DFIG technologies are most advanced in wind turbine systems. Energy is extracted from the wind by a series of turbine blades mounted to produce shaft torque which drives an electrical generator feeding electrical power to the supply network. This arrangement is shown in Figure 2.29

Interfacing with the electrical supply network requires the frequency and, to a lesser extent, magnitude of the voltage to be strictly regulated despite fluctuations in available wind speed. By varying the slip frequency the turbine blades can be kept at the optimum speed for maximum power extraction, whilst the stator is held at the network frequency. Mechanical speed regulation by turbine blade pitch variation or variable ratio gearbox, combined with a cage induction generator offers another approach, however it is relatively slow in response.

Similarities can be seen with the GT electrical generation system shown in Figure 2.25. Both systems have a variable speed mechanical system, and CF generation is preferred from the aero system.

2.5.3 DFIG operation

The use of a DFIG, with rotor side converter, provides electrical frequency regulation at the electro-magnetic stage of the power train. This removes the need for complex
mechanical speed regulation (via blade pitch control or a variable ratio gearbox), blade pitch control is often still used, however, to ensure optimal energy extraction from the wind. The scaling between rotor and stator windings means that converter rating (and so costs) are reduced compared to the full rated power of the system [89]. Rotor side converter rating is scaled according to the mechanical speed range extension around the synchronous speed [90], and is approximately proportional to slip [91]. The rotor side converter is usually designed with a rating between 25% [92] and 33% [93] of the total system power. Given the reduced power flowing through the power electronics, compared to a synchronous machine with fully-rated stator-side converter, system efficiency can be increased by 2-3% [92]. Analysis of various VSCF wind systems, [94], shows a DFIG to be both cheaper and more efficient than an equivalent cage induction machine system.

A review of VSCF wind generation systems, [95], finds the DFIG to superior to other options based on its low converter cost, and higher energy output by better utilisation of generator machine (compared to a cage rotor induction machine). [96] identifies reduced converter and filter costs and improved efficiency as advantages of a DFIG over other VSCF options.

2.5.4 As an aero generator
Consideration is given to a range of technologies for generating electrical power on a VSCF aero system [97], and the use of a DFIG in such a scheme is mentioned conceptually, but not investigated in detail. Aero DFIG systems pose additional challenges as drive speeds are nearer 20,000 rpm compared to 1,000 rpm of wind generator systems. Also, unlike a conventional CSD synchronous machine VSCF, the rotor side converter will introduce harmonic distortion on the electrical power network [97]. A detailed investigate of aero VSCF generation through the use of a synchronous machine and back-to-back converter is carried out in [82] (and with a hardware test platform detailed in [84]), the DFIG is mentioned, conceptually only, as an alternative. A thorough overview of DFIG systems is provided in [96], and again the aero application is put forward as a example of a VSCF system, but no consideration is given to the actual integration of such a system.
Technical consideration is given to the performance and control of a cascaded DFIG in [98], both grid connected and isolated conditions are simulated showing good voltage regulation over a speed range of ±33% for both linear and non-linear loads. Although aero is discussed as an application due to its VSCF requirement, it is not considered further. [99], by the same author, considers a DFIG with standalone control and this time with experimental results on a 4.5 kW machine. The work in both these papers is general to all VSCF systems with aero only mentioned as one such system. Control is designed for a cascaded DFIG in [100], and successfully simulated, but while the aero application is the main driver, the actual realisation of such a system is not discussed.

A DFIG specifically for aero VSCF is discussed in [101] with a shaft mounted PM machine used to supply the rotor side converter for true standalone operation. Vector control is implemented and simulated for a 50 kVA system (115 V, 400 Hz), but integration into the aircraft system (ie matching generation with spool speed range) is not considered. Further work is carried out by the same author to consider the impact of non-linear loads on the VSCF DFIG fed bus in [81], the DFIG is selected as an alternative to a CSD (higher maintenance) or synchronous generator with back-to-back converter (more costly converter) system. An LC filter is designed to improved power quality and simulated.

This literature search has not identified any previous work detailing the integration of DFIGs into an aircraft system. No experimentally verified aero DFIG control schemes have been identified, nor any that consider drivetrain dynamics.

2.5.5 Field Orientated Control
Variable speed control is more complex for an AC machine than for an equivalent DC machine due to the multiple interacting variables that control the performance of an induction machine. Field Orientated Control (FOC) is designed to allow an AC machine to be controlled as if it were a separately excited DC machine [102]. Current phase angle as well as magnitude must be controlled [103] hence the method is also referred to as Vector Control [104].
FOC allows flux and torque to be controlled independently by the direct and quadrature currents respectively. For the DFIG the direct and quadrature rotor currents are controlled to control the stator flux and torque.

### 2.5.5.1 Direct and indirect FOC

FOC requires the flux angle to be known or estimated. Indirect FOC uses feedforward reference currents to determine the rotor flux position, while direct FOC uses values determined by flux measurement or derived from the terminal measurements [104].

### 2.5.6 Review of DFIG control schemes

DFIG systems are normally considered for a speed ranges in the region of ±30% [92] [93] [90] around synchronous speed. This speed range is limited by the rotor side converter rating. A lower rated converter can be used if a limited variation in electrical frequency is allowed. The DFIG system in this research is designed to function over a speed range of ±40%, making the system more stable at initial startup speeds, and more closely matching the mechanical speed range of a real GT.

### 2.5.6.1 Control with rigid grid connection

In wind generation systems, constant frequency output is required since grid connection can only be approved when the voltage, frequency and phase of the stator terminals are matched to the grid. The rigid grid defines the system voltage magnitude, frequency and phase, so the inner current control loops control torque and flux, but outer loops control real and reactive power with the objective of maximising energy transfer, and meeting reactive power requirements.

A FOC scheme for a grid-connected DFIG was first suggested in [105], and the performance of a FOC scheme analysed in [106]. The use of FOC is considered with a method of synchronisation and disconnection from the grid in [107]. Conventional induction machines use rotor flux orientation because they are controlled from the stator side, [108] identifies stator flux orientated control as the best option for a DFIG as it is controlled through a rotor connected converter. A variation on this control scheme, for the purpose of power factor stabilisation and torque regulation, is given in [109]. [110] presents FOC with an outer loop sliding mode control scheme for a grid connected wind
power driven DFIG to provide improve energy conversion efficiency and damp torque oscillations.

2.5.6.2 Stand-alone control
The stator flux orientated control scheme is suggested by a number of authors as a means of achieving voltage and frequency regulation from a variable-speed stand-alone DFIG system [88] [89] [111] [112] [113]. The choice of stator flux orientation provides a means of readily dictating the stator field angle, and hence generated frequency. Predictive control is added to FOC for a standalone aero generator in [99]. Without a rigid grid connection, the aero DFIG operates as a stand-alone generator meaning that the controller has to regulate voltage magnitude and frequency. Stand-alone FOC requires the same inner current control loops as the grid connected system but the outer control loops control voltage magnitude (instead of reactive power) and slip, hence frequency (instead of real power).

2.5.7 Types of machine arrangement
The doubly-fed machine configuration can be practically implemented in a number of arrangements. The use of slip-rings on the DFIG being considered in this research, provides a direct electrical connection for the rotor and therefore offers the simplest, most controllable machine. However, it does not offer a practical implementation for an aero system due to the maintenance requirements (especially at the high drive speeds of aero generators) and sparking risks associated with slip-rings.

This research is carried out on a slip-ring fed DFIG, but alternative doubly-fed schemes are considered for the realisation on an actual aero system.

2.5.7.1 Brushless Doubly-Fed Induction Machine
The Brushless Doubly-Fed Induction Machine (BDFM), [114], [115] (with experimental validation) and [97] (within an aero setting), has a secondary, or auxiliary, winding on the stator which is designed so it does not interact with the main stator winding. It has a complicated interleaved rotor cage design such that the rotor currents induced by the secondary stator winding can be used to control the main winding. Control complexity is increased compared to a slip-ring DFIG and also the rotor cage and stator winding design and construction are more challenging.
2.5.7.2 Cascaded induction machine

The doubly-fed effect can be achieved by a cascade of back-to-back induction machines [116]. [98] and [100] also discuss a cascaded machine with aircraft mentioned as a possible application. An exciter machine and power machine are mounted on a common shaft with electrical connection between the two sets of rotor windings. It should be noted that this method makes it impossible to induce a DC field in the rotor hence the machine control must be designed to operate in a non-synchronous condition.

A potential realisation for a cascaded induction machine, including a PMG for standalone operation, is shown in Figure 2.30.

![Figure 2.30: Cross section of cascaded induction machine](image)

Power electronics are used to control the flow of power to and from the power stage rotor via the exciter stage, control of the power stage rotor field is therefore not direct and subject to the bandwidth limitations of the exciter stage. Initial power is generated by the PMG. The bus is connected to the stator side of the power stage and at 'subsynchronous' speeds power is fed into the exciter and through to the power stage rotor. In 'supersynchronous' conditions power is recovered from the rotor side power stage and transferred through the exciter and power electronics to the bus.
Design of cascaded induction machine

The cascaded arrangement makes use of a number of standard machine designs. However, balancing the design of each sub system to achieve the desired power flows is not simple as the exciter stage rotor must be rated for the full voltage and current of the main machine rotor. The rating of the converter is dependent on the ratio of exciter poles and main machine poles as well as slip, if the pole numbers are not identical cascaded synchronous speed must be considered [116] which corresponds to zero frequency (ie DC) on the exciter stator. The direction of power flow is determined by the cascaded synchronous speed [116]. Generally, control is more complex due to the indirect connection between the power electronics and the rotor current at the power stage.

### 2.5.7.3 Summary of DFIG systems

DFIGs may offer a cost and weight savings over other aero VSCF options. Also, the DFIG performance is well suited to a role in limiting interaction within the electromechanical system. DFIGs have most notably been considered for VSCF wind generation systems, however these benefit connection to a rigid grid. Adaption of these FOC schemes make the DFIG suitable for standalone operation.

### 2.6 Summary of Literature Review

The operation of the aero GT has been considered and the secondary power offtake identified as a potential area, which if better optimised (in terms of weight, efficiency, control of energy usage) could increase aircraft efficiency. The electrification of this system has not conclusively been shown to increase efficiency or reduce weight per element, however energy usage can be better controlled, increasing the overall system efficiency. Maintenance requirements are reduced when systems are electrified which provides an additional operational cost saving.

In order to benefit from an increased electrification the aircraft requires an uprated electrical generation system. The structure of the GT places design restrictions on the mechanical components of the drivetrain which link the electrical generators to the GT spool making these components vulnerable to fatigue. Electrical network regulation
must be achieved by a single generator, without a power buffer, while providing suitable dispatchable power for a number of dynamic loads. A number of electrical generation and distribution schemes have been detailed, of which the most beneficial, from the design of electrical loads, is the CF AC system. However, this is currently unattractive as it requires complex drive coupling to regulate mechanical speed. The DFIG is identified as a generator which is capable of producing a CF network without mechanical regulation. DFIGs are most commonly used for wind generation systems where they are cheaper to install than alternative generation systems, due to the fractionally rated converter, and can have higher levels of efficiency. Where wind generations systems connect to a strong network, aero generators are required to operate in a standalone condition, and the control schemes necessary to achieve this are identified.

Tight regulation of the electrical network, and the high rating of the electrical generator, creates a rigidly coupled electro-mechanical system. The mechanical drivetrain is prone to resonance which may be excited by variations in electrical loading leading to reduced mechanical component lifespan and electrical network instability. Compared to other electro-mechanical systems investigated the aero system has a unique combination of a complex mechanical drivetrain powering a single generator, with no power buffer / energy storage, and stringent weight restrictions.

Previous investigation of an aero electro-mechanical system, at a high fidelity, has not been identified in the literature. While mentioned conceptually, the implementation, and practical validation, of a DFIG aero generator has also not been identified.
Chapter 3  Modelling and Analysis of Mechanical Systems

This chapter describes modelling and analysis work carried out to produce a model of a generic gas turbine mechanical drivetrain which is suitable for integration into a wider gas turbine electro-mechanical system model and may be adapted to represent other GT architectures. Chapter 4 discusses the modelling of the electrical system, the generator, and its control.

An assessment of modelling strategies for each key element of the drivetrain is undertaken considering both simulation complexity and output fidelity. Using this knowledge a full definition model is produced of a gas turbine drivetrain which is validated against real gas turbine data. The validated drivetrain model is used to develop an understanding of the gas turbine’s mechanical drivetrain behaviour and the impact of system architecture and individual element design on this behaviour. The most significant drivetrain elements are identified and incorporated into a reduced parameter model, which while still being representative of the full drivetrain, offers a reduction in the number of degrees of freedom and hence simulation complexity. The reduced parameter model forms a basis for the design of the electro-mechanical test rig described in Chapter 5.

3.1 Model specification

Analysis of gas turbine electro-mechanical interaction requires a model of the full engine system which includes the gas turbine mechanics, mechanical drivetrain, electrical generator, and electrical loads. The electro-mechanical system is shown in Figure 3.1 (given previously as Figure 2.16).

Figure 3.1: Aero electro-mechanical system schematic
The drivetrain links the gas turbine core to the electrical generator, transmitting power between the two torsionally, through mechanically rotating elements, where the power transmitted is the product of torque and angular velocity as shown in (3.1), [117]. Note: symbols are defined in the table of notation.

\[ P = T \dot{\theta} \]  

(3.1)

3.1.1 Requirements

As a sub-system within a wider electro-mechanical model the drivetrain model must provide bidirectional interfaces with adjacent models; a gas turbine model and an electrical machine model. While predominantly power is transmitted from the gas turbine to the electrical generator, disturbances on this generator are also transmitted through the drivetrain to the gas turbine. The mechanical model must also have sufficiently high bandwidth so as to accurately respond to load transients which could potentially be up to 10 kHz from converter electrical switching. However, torque bandwidth for the electrical generator is dictated by its current bandwidth which itself is limited by the winding inductance. Higher electrical frequencies, such as switching transients, will not be apparent to the mechanical system so the model bandwidth does not need to consider these.

Principally the mechanical model is used to represent resonant modes within the mechanical system. While the characteristic frequencies can be identified with a frequency domain model, information on the rate of decay of each oscillation would not be obtainable. Repeated cycles of load stress induce fatigue in components and as the number of cycles accumulates the failure stress of the component is reduced [118]. Hence the lifespan of a component is a product of both the stress amplitude and the number of cycles for which it is applied. This makes the rate of decay of oscillations within the drivetrain a significant consideration. A time domain model is therefore developed which is used to extract the frequency response data.

3.1.2 Modelling package

Modelling and simulation work carried throughout this project is done within the MathWorks Simulink environment. Simulink’s graphical interface enables easy
visualisation of complex mathematical models and is built upon the MATLAB numerical computing platform allowing for powerful analysis of results obtained through model simulation. Importantly the environment enables multi-domain simulation, this is simplified by the availability of ‘blocksets’ which contain prebuilt libraries of components for various domains. Multi-domain models are constructed by interfacing prebuilt blocks applicable to each domain.

Torsional mechanical and electrical components can be readily modelled using the SimDriveline (version 1.5.2) and SimPowerSystems (version 5.1) ‘blocksets’ respectively, core mathematics and control, as well as basic electrical loads, are modelled directly in Simulink (version 7.3). Figure 3.2 shows the electro-mechanical model being constructed and the Simulink ‘blocksets’ used to represent each system.

![Figure 3.2: Aero Electro-mechanical system schematic and selected Simulink ‘blocksets’](image)

3.1.3 Modelling approach

The suitable level of modelling fidelity must be considered at both a system level and component level to ensure that the model produced is both sufficiently accurate and may be solved within an acceptable time frame. A model may incorporate high levels of detail across all components, however the usability of the model will be severely diminished due to the vast computational power required to solve it. Conversely, a model with little detail will solve quickly, and be readily accessible, but may not provide sufficient accuracy in results. The challenge, as with all models, is to gain an understanding of the requirements in terms of output resolution and solving time. This must also include an understanding of the impact that each subsystem has on the wider mechanical systems response.
By selecting an optimised modelling approach for each subcomponent, the accuracy of the overall model can be assured while minimising simulation run time. In reality many of the more detailed analyses may be neglected if it can be demonstrated that these have a negligible impact on the response of the component in question and the wider drivetrain.

### 3.1.4 Drivetrain Modelling Assumptions

The drivetrain is modelled as a purely torsional system. Within a real engine drivetrain rotating components are supported well and have a relatively high lateral stiffness. As a result sag is minimal and so vibration effects such as whirl need not be considered. High precision manufacture means that axial displacement, misalignment and unbalance are low and hence not considered. Similarly lateral vibrations need not be considered as bearing support is sufficiently precise that they are negligible.

Engines include various roller and ball bearings to support radial and axial loads [7] these components have minimal losses and very long lifespan. Due to their negligible losses, all bearings are modelled as being ideal. It is assumed that they hold the components exactly in all ranges of motion, with no imbalance, and have no damping losses. This idealisation does remove the ability to investigate the impact of electrically induced torque excitation on bearing vibrations and it is possible that bearing resonant frequencies are within a range that could be excited by electrical systems. However due to the small mass of the bearing components these vibrations are likely to be localised without extending to the surrounding drivetrain. The likely outcome of bearing vibration is increased wear and decreased lifespan [119]. The extent to which electrical harmonics excite this resonance could be investigated independently.

All vibrations are considered to be mathematically linear, non-linear effects such as gear backlash are neglected during the model development.

### 3.2 Gas turbine mechanical drivetrain

This research considers a large diameter civil MEE, a generic 3 spool gas turbine is used for drivetrain analysis.
Power is extracted mechanically from a rotating spool in the GT core, it is transmitted via a series of driveshafts to supply accessory loads which include the electrical generators. Figure 3.3 shows the mechanical drivetrain on a GT.

Electrical power from the generators is distributed throughout the airframe on isolated buses to supply a wide range of electrical loads.

### 3.2.1 Spool coupling

Power is extracted mechanically from a variable speed spool at the core of the GT. In Figure 3.3 this is from the IP pressure spool, but on other two spool type systems this occurs at the HP spool. The coupling is via a fixed-ratio bevel gear situated behind the final compressor stage, which means that the point of offtake is separated from the source of torque (the turbine) by the IP spool shaft, although this has a high level of stiffness.

### 3.2.2 Transmission

From the spool coupling, power is transferred mechanically through both the GT core and bypass area to the outer fan casing. The Radial Driveshaft (RDS) connects the Internal Gearbox (IGB) to the Step-Aside Gearbox (SAGB) at the core casing. The
Angular Driveshaft (ADS) links the SAGB to the Transfer Gearbox (TGB) on the outer fan casing. The relative high drivespeeds of both the RDS and ADS can be seen in Table 3.1, data provided by the engine manufacturer.

<table>
<thead>
<tr>
<th>Component</th>
<th>Speed (normalised)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IP Spool</td>
<td>1.00</td>
</tr>
<tr>
<td>RDS</td>
<td>3.55</td>
</tr>
<tr>
<td>ADS</td>
<td>1.70</td>
</tr>
<tr>
<td>TGB</td>
<td>1.05</td>
</tr>
</tbody>
</table>

The shafts pass through both the central core of the gas turbine and the bypass region, therefore a low cross section is desirable so as to minimise disruption to the airflow [7]. This low shaft diameter reduces the torque rating of the shaft and so high angular velocity is necessary in order to transmit the required power. As a result of the long length and generally narrow diameter of components, the transmission system as a whole has a high level of torsional compliance compared to surrounding systems.

3.2.3 Accessory Load

The Accessory Gearbox (AGB) is mounted on the underside of the fan casing and is driven via the TGB. All auxiliary loading for the airframe and GT are powered from the AGB which is a parallel alignment of straight cut spur gears [7]. This includes electrical generators and hydraulic pumps for airframe secondary power systems as well as ancillaries for the GT itself such as the oil pump (and de oiler) and fuel pump. A Permanent Magnet Alternator (PMA) provides low power electrical backup. Normal auxiliary load speeds are in the region of 15,000 rpm and possibly as high as 20,000 rpm for centrifugal breathers [7], typical accessory load drive speeds are shown in Table 3.2, data provided by the engine manufacturer.

<table>
<thead>
<tr>
<th>Component</th>
<th>Speed (normalised)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IP Spool</td>
<td>1.00</td>
</tr>
<tr>
<td>Hydraulic Pump</td>
<td>0.50</td>
</tr>
<tr>
<td>Fuel Pump</td>
<td>0.90</td>
</tr>
<tr>
<td>De Oiler</td>
<td>0.69</td>
</tr>
<tr>
<td>Generator</td>
<td>1.79</td>
</tr>
<tr>
<td>Oil Pump</td>
<td>0.62</td>
</tr>
</tbody>
</table>
An example AGB configuration is shown as part of the complete drivetrain in Figure 3.4.

The arrangement of the accessory load on the parallel axis spur gearbox can be seen, the work carried out by the PMA and the De Oiler is low compared to the other systems and so they are not considered further. Accessory loads driven from the AGB are, like all aero systems, designed to have minimum mass, however the role they carry out is both critical and significant, so these systems are inevitably sizable and in particular have substantial inertia. The referred inertia (referred to IP spool speed) of the accessory loads are given in Table 3.3, the dominant size of the electrical generators can be seen.

Figure 3.4: Gas turbine drivetrain schematic
Table 3.3: Accessory load referred inertia

<table>
<thead>
<tr>
<th>Component</th>
<th>Inertia, referred (normalised)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydraulic Pump</td>
<td>5.65x10^4</td>
</tr>
<tr>
<td>Fuel Pump</td>
<td>3.46x10^2</td>
</tr>
<tr>
<td>Generator</td>
<td>1.00</td>
</tr>
<tr>
<td>Oil Pump</td>
<td>4.01x10^4</td>
</tr>
</tbody>
</table>

The AGB, and the accessory loads it drives, are critical to the operation of the GT and airframe, its failure or a loss of drive will result in engine shutdown while airframe secondary power must be sourced from other engines.

3.2.3.1 Embedded Generation

Conceptually it is possible to replace the externally mounted accessory load with an electrical system powered by an electrical generator, mounted directly on a spool shaft and ‘embedded’ within the gas turbine core [87]. This simplifies gas turbine construction and maintenance by removing the mechanical drivetrain. However, this removes the damping which may be present in the drivetrain and so electro-magnetic torque from the generator will be applied directly to the GT spool. Potentially increasing vibration, and so fatigue, on components within the GT core.

3.2.4 Drivetrain overview

The drivetrain consists of three main subsystems, a spool coupling which links it to the varying speed spool in the gas turbine core, the transmission consisting of two coupled high compliance driveshafts, and the AGB driving (via their own driveshafts) the high inertia loads which are critical to aircraft operation.

3.2 Modelling torsional systems

This section outlines the basic mathematical descriptions used to represent torsional elements during this research.

3.2.1 Inertia

The behaviour of torsional systems is mathematically described by a torsional interpretation of Newton’s Second Law of Motion. The angular acceleration of a
component with constant mass moment of inertia is proportional to the torque applied (3.2), [117].

\[ T_J = J \dot{\theta} \]  

(3.2)

An object's mass moment of inertia, \( J \), acts to oppose its angular acceleration, producing a reaction torque. The level of inertia is dependent on an object's mass and its dimensions around the axis of rotation, or polar moment of area. The polar moment of area of a cylinder and that of a tube, rotating around an axis in line with its length and centered within its circular cross section, are given in (3.3) and (3.4) respectively, [120].

Cylinder: \( J_p = \frac{\pi}{2} r^4 \)  

(3.3)

Tube: \( J_p = \frac{\pi}{2} \left( r_o^4 - r_i^4 \right) \)  

(3.4)

From this point the term inertia is used to describe mass moment of inertia. Inertias of a cylinder and that of a tube are given in (3.5) and (3.6) respectively, [121].

\[ J = \frac{1}{2} M r^2 \]  

(3.5)

\[ J = \frac{1}{2} M \left( r_i^2 + r_o^2 \right) \]  

(3.6)

3.2.2 Torsional stiffness

A component's torsional compliance describes the angular displacement between two separated points along its axis of rotation when a torque is applied. Stiffness is the opposition to this angular displacement and results in an opposing reaction torque, (3.7).

\[ T_k = k \left( \theta_i - \theta_o \right) \]  

(3.7)
Where $\theta_0$ and $\theta_1$ are the angular positions of two points on the shaft,

An element’s torsional stiffness is determined by its physical structure and the density and shear modulus of the material of which it is composed, this model is shown in (3.8), [120].

$$k = \frac{GJ}{L}$$

(3.8)

The mathematical model (3.8) assumes a constant shear modulus over the range of angular displacement applied for all components, the full failure modes of components are therefore not considered, only the elastic region is modelled.

### 3.2.3 Damping

Damping describes the energy losses in the system and is notoriously difficult to accurately quantify. A calculation of the energy lost per vibration cycle may be used to determine a value for damping, however this is a complex process for a large network and is challenging to achieve at an early design stage. In reality damping values are often derived experimentally, rather than through calculation, and specified to match the real system performance.

Damping may be the result of many influences throughout a mechanical network, and hence there are a wide range of strategies to mathematically describe their impact. A non exhaustive list is summarised in Table 3.4.

<table>
<thead>
<tr>
<th>Name</th>
<th>Damping torque</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear viscous damping</td>
<td>$c\dot{\theta}$</td>
<td>Movement through slow fluid (e.g. oil)</td>
</tr>
<tr>
<td>Air damping</td>
<td>$a.\text{sgn}(\dot{\theta})\theta^2$</td>
<td>Movement through fast fluid (e.g. air)</td>
</tr>
<tr>
<td>Coulomb damping</td>
<td>$\beta.\text{sgn}(\dot{\theta})$</td>
<td>Friction</td>
</tr>
<tr>
<td>Solid / Structural / Hysteretic damping</td>
<td>$b.\text{sgn}(\dot{\theta})</td>
<td>\theta</td>
</tr>
</tbody>
</table>

Table 3.4: Mathematical damping models (data from [122])
Where: $c$, $a$, $\beta$ and $b$ are fixed terms notating the amplitude of each damping model, and $\text{sgn}$ is the sign function.

The mathematical damping models shown in Table 3.4 produce a torque which is dependent on combinations of angular velocity and angular acceleration. The responses of each model are shown, with respect to angular velocity, in Figure 3.5.

![Figure 3.5: Comparison of damping models](image)

The most basic description of damping is as a linear function which is proportional to speed, linear viscous damping best describes a slow fluid damper such as a dashpot. Should the fluid be faster than oil found in a dashpot, such as air, then the damping should be related to the speed squared. Displacement-squared damping represents the vibration of a material within a fluid. When a component is stressed and released, energy is lost at a molecular level, this energy loss produces a hysteretic damping effect on the component.

Further damping models also exist which describe damping dependent on frequency, so called 'modal' damping is challenging to accurately determine and cannot be implemented readily within time domain models.
3.2.3.1 Selected modelling strategy

Damping can impact the system from one or more of a wide range of sources. In reality as damping is difficult to mathematically model, and difficult to measure, it is approximated as a linear function proportional to speed. The linear viscous damping approximation is used throughout this study, producing the response given in (3.9).

\[ T_D = c \dot{\theta} \]  

(3.9)

3.2.4 Holistic torsional element

For any component the applied torque and reaction torques must be in equilibrium. The torsional response of a component can therefore be modelled by the summation of inertia, damping and stiffness models, as shown in (3.10) and (3.11).

\[ T = T_J + T_D + T_k \]  

(3.10)

\[ T = J \ddot{\theta} + c \dot{\theta} + k \theta \]  

(3.11)

3.3 Assessment of modelling strategies

The drivetrain is an interconnection of three types of element; this section outlines the selection of modelling techniques for driveshafts, accessory loads and gears.

Due to their length and narrow diameter driveshafts provide the majority of torsional compliance within the network, energy lost during driveshaft stressing provides damping. The accessory load provides the majority of the drivetrain inertia, combined with driveshaft compliance they dictate the resonance modes of the network. Meshing between gears introduces a non-linearity to the network.

3.3.1 Driveshafts

The purpose of a driveshaft within a mechanical network is to transmit power (by way of angular rotation and torque) from the coordinate at one end to the shaft to the coordinate at a distant end. This basic driveshaft form is shown in Figure 3.6.
The driveshaft itself is not in infinitely stiff and so an angular displacement occurs between the two ends which produces an associated energy loss, these affects are represented mathematically by stiffness, k, and damping, c, terms. Under true steady-state conditions the angular velocities, $\dot{\theta}_1$ and $\dot{\theta}_2$, remain identical resulting in no energy loss on the viscous damping model being considered. Power demand variations and induced harmonics ensure that steady-state conditions will rarely be met and so energy will be lost during power transfer along the driveshaft.

Bearings are assumed to be ideal so that driveshaft is free to rotate and experiences no losses, and linear motion does not occur. A torque, $T_1$, applied at the driving end of the shaft leads to a positional change, $\theta_1$, at the same end and results in a positional change at the far end, $\theta_n$, and a resultant torque, $T_n$. Load torque, $T_L$, acts at the far end of the shaft, opposing the resultant drive torque, for simplification it is assumed to be zero for this analysis. It is the dynamics between the application of the applied torque, $T_1$, and the resultant torque, $T_n$, which is of interest for the purpose of creating an accurate model.

Displacement due to an applied torque is a direct result of the stiffness and damping of the shaft. An applied torque, $T_1$, results in a twist in the shaft ($\theta_1 - \theta_n$), energy is stored within the shaft stiffness and produces a response torque applied equally to either end of the shaft, causing this displacement at the far end, $\theta_n$.

In this section several driveshaft models are considered and compared for the modelling of a generic driveshaft.
### 3.3.1.1 Generic driveshaft

For the purpose of evaluating modelling strategies, a hollow cylindrical driveshaft is defined as given in Table 3.5.

**Table 3.5: Generic driveshaft physical parameters**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>8.00x10^1 m</td>
</tr>
<tr>
<td>R_i</td>
<td>5.10x10^-2 m</td>
</tr>
<tr>
<td>R_o</td>
<td>5.25x10^-2 m</td>
</tr>
<tr>
<td>ρ</td>
<td>9.75x10^57 kg.m^-3</td>
</tr>
<tr>
<td>G</td>
<td>1.20x10^10 Pa</td>
</tr>
</tbody>
</table>

The generic driveshaft is based on a 3 spool GT ADS, having similar physical dimensions. For the purpose of modelling density and shear modulus is chosen arbitrarily to produce a low frequency 1st mode hence keeping the frequency range which must be considered low. Using the derivations described previously the torsional properties associated with the shaft are calculated as shown in Table 3.6. The damping value, c, chosen for the generic driveshaft is not derived through calculation, it is instead chosen as being representative of the performance of the 3 spool GT ADS.

**Table 3.6: Generic driveshaft torsional properties**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>1.02 kg.m^2</td>
</tr>
<tr>
<td>k</td>
<td>1.67x10^7 Nm.rad^-1</td>
</tr>
<tr>
<td>c</td>
<td>3.26 Nm.s.rad^-1</td>
</tr>
</tbody>
</table>

### 3.3.1.2 Single lumped inertia

The most basic driveshaft model may be achieved by rearrangement of the basic torsional equations described in (3.11). Torque applied to the driveshaft inertia is opposed by the damping and stiffness of the shaft in providing a resultant torque at the opposite end. Taking a Laplace Transform and rearranging gives the transfer function shown in (3.12).

\[
\frac{\theta_i(s)}{T_i(s)-T_L(s)} = \frac{1}{Js^2 + cs + k}
\]  

(3.12)
Input torque, $T_1$, results in an angular displacement, $\theta_1$, and hence, as a result of shaft shaft stiffness, $k$, an output torque, $T_n$. The load torque, $T_L$, is assumed to be zero in this instance. Figure 3.7 shows a diagram of the shaft which is modelled.

![Figure 3.7: Lumped inertia driveshaft model diagram](image)

The transfer function is modelled in Simulink as shown Figure 3.8.

![Figure 3.8: Lumped inertia Simulink driveshaft model](image)

Frequency response data is extracted from the model by way of a frequency sweep; a sinusoidal excitation is applied to the model and peak response (after a settling time) is recorded. This process is carried out separately for a number of excitation frequencies over the desired range. A frequency sweep, determining an amplitude, $\frac{\theta}{T}$, is used to derive the frequency response shown in Figure 3.9 with a maximum frequency of 2,000 rad.s$^{-1}$ and a resolution of 1 rad.s$^{-1}$. 
Chapter 3 - Mechanical Systems

The lumped inertia model has a single mode frequency response with peak amplitude at 128 rad.s\(^{-1}\); response is minimal at all other frequencies. This response matches that predicted by equation (3.13), [123], which derives the resonant frequency of a single inertia tethered with stiffness and damping.

\[
\omega_n = \sqrt{\frac{k}{J}}
\]  

(3.13)

The single lumped inertia modelling strategy represents a stiffness and damping acting from a fixed point on the inertia body. It does not model a driveshaft with distributed inertia nor one which exhibits flexibility and energy loss along the length of the shaft. With the single lumped inertia model it is impossible to demonstrate and angular displacement between ends (\(\theta_n = \theta_1\)) and so \(T_n\) is simply identical to \(T_1\).

3.3.1.3 Split inertia

Splitting the driveshaft inertia into two equally sized inertia 'lumps' allows stiffness and damping to be considered between the two lumps providing a more realistic torsional response. This arrangement for a driveshaft model is shown in Figure 3.10.
The load torque, $T_L$, is assumed to be zero for the purposes of analysis. An applied torque, $T_1$, causes an angular displacement in inertia lump 1, $\theta_1$. The resultant angular separation between inertia lump 1 and 2 produces a torque, which is exerted equally on both inertia lumps with equal magnitude but with opposite vectors, opposing the angular displacement of lump 1 and driving the angular displacement of lump 2. Energy is stored within the driveshaft due to its stiffness, $k$, and losses associated with energy storage in the driveshaft are represented by linear viscous torsional damper, $c$. The inertial lumps are assumed to be ideal point inertias, having no torsional flexibility themselves.

An additional viscous damper, $D$, acting between inertia lump 1 and ground should be noted in Figure 3.10. Its position suggests a continual loss during driveshaft rotation, and would represent bearing losses, however as has already been stated, detailed models of bearing losses are neglected throughout this study. Its presence is not required to model the driveshaft, however, it exists to improve model solving. The additional damping term, $D$, reduces simulation run time, by removing the steady-state error from the simulation, stabilising the output angular positions, and ensuring convergence with the initial conditions. This term is sufficiently low compared to other properties to have minimal impact on the model's torsional response.

A mathematical model is derived from the systems equations of motion which are combined and implemented as a differential equation in Simulink. A schematic of this is shown in Figure 3.11. This arrangement produces a time domain simulation where it is
possible to access angular positions and torques for each end of the driveshaft during simulation.

\[
\begin{align*}
T_1 & \quad \theta_1 \\
T_n & \quad \theta_n \\
T_L &
\end{align*}
\]

Figure 3.11: Split inertia Simulink driveshaft model

The split inertia driveshaft model is simulated using the generic driveshaft parameters described previously.

The modelled torsional compliance of the driveshaft can be demonstrated by observing the difference in angular position between the two inertia lumps. Figure 3.12 shows the difference in angular position between the two inertias in response to a 1 Nm torque step applied at time = 0.2 s.
A single mode is identified at 256 rad.s\(^{-1}\), with the oscillations present for approximately 0.8 s. The mathematical derivation of the frequency response comes from calculation of the effective inertia of the split driveshaft model. The resonant frequency is determined using the classical mechanical resonance equation for a free-free system [124], shown in (3.14).

\[
\omega_n = \sqrt{k\left(\frac{1}{J_1} + \frac{1}{J_2}\right)}
\]  

(3.14)

In this instance, the interconnecting stiffness, k, represents the stiffness of the driveshaft and the two inertias, J\(_1\) and J\(_2\), are identical at half the total driveshaft inertia.

The split inertia driveshaft model provides a realistic model of driveshaft dynamics by replicating the angular displacement between either shaft end. A true driveshaft, however, also exhibits a linear variation in angular position along its axial length. This can only be achieved by increasing the number of inertia lumps into which the shaft is divided.
3.3.1.4 n-inertia

A real driveshaft has an infinite number of small inertia elements that are displaced angularly when torque is applied along the length of the shaft. A representation of this is achieved by increasing the number of inertia lumps which are considered. Figure 3.13 shows the driveshaft model with n equal inertia 'lumps' each coupled with a representative stiffness and damping.

![n-inertia driveshaft model diagram](image)

As per split inertia model, load torque, $T_L$, is considered to be zero, and linear damping, $D$, is added to steady the simulation. The stiffness and damping coupling each inertia lump are scaled from the total shaft parameters according equations (3.15) and (3.16) [123].

$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2} \quad (3.15)$$

$$c_{eq} = c_1 + c_2 \quad (3.16)$$

Where: $k_{eq}$ and $c_{eq}$ are equivalent parameters for elements $k_1$, $k_2$ and $c_1$, $c_2$ combined in series.

Equations of motion are determined for each inertia 'lump' and combined to produce a transfer function which is implemented in Simulink. A frequency sweep is used to identify the frequency response of the n-inertia driveshaft model, this is carried out for range of values of n (the number of inertia lumps), the data is shown in Figure 3.14.
Each peak on the trace represents a resonant mode, with a single mode appearing for each energy storage (torsional stiffness) element within the model. For example the 7 inertia model has 6 stiffness elements and so 6 modes of resonance. The resonance modes of all models are similar and tend towards a common value as the number of inertia lumps, n, is increased. 1st mode resonances are seen to tend towards approximately 400 rad.s\(^{-1}\) while 2nd mode resonances tend towards approximately 800 rad.s\(^{-1}\). High frequency noise is apparent above the highest existing peak for each trace, this is a result of the time-stepping simulation solver.

The trend of the 1st mode is more clearly shown in Figure 3.15, showing only the 1st mode resonance for a range of models from single inertia, to 30 inertia lumps. It can be seen that the basic lumped inertia model (n=1) does not match the trend of the other models, and therefore should be discounted as an inadequate modelling technique.
The n-inertial modelling has demonstrated that as the number of inertias considered is increased the frequency position of resonant modes increases, tending towards a maximum frequency.

Model complexity
As the number of inertias considered, and so the number of spring damper networks, is increased the model computation time increases exponentially, eventually reaching a point where it is impractical to model even a single driveshaft component, with the challenge only exacerbated when considering many components in the wider drivetrain model. The accuracy of the driveshaft model is limited by the computation solving power available, the number of inertias selected for the n-inertia model therefore forms a tradeoff between accuracy and simulation complexity.

Transfer function realisation of state-space model
The n-inertia models require a nth-order differential equation to be solved at each frequency to produce a frequency response plot. It is possible to simplify these equations in the s-domain to produce a transfer function which can be more readily solved.
The driveshift was first modelled as a state-space system, producing a number of coupled first order systems which may be converted to a transfer function using a computer solver, allowing driveshift models which consider a high number of inertia lumps to be simulated in a reduced time frame.

Considering the basic two inertia driveshift model (again with the additional damping term, D), as shown in Figure 3.11, the state-space representation, (3.17), is derived.

\[
\frac{d}{dt} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \frac{-D+c}{J} & \frac{c}{J} & \frac{-k}{J} & \frac{k}{J} \\ \frac{c}{J} & \frac{-c}{J} & \frac{k}{J} & \frac{-k}{J} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \theta_1 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} T \\ 0 \\ 0 \\ 0 \end{bmatrix}
\]

(3.17)

The state-space model has a visibly repetitive structure, and so the process of deriving the state-space equation for a system with a higher number of inertia lumps is automated. A MATLAB function is derived which produces the state-space model for a driveshift with n-inertia lumps, a bode plot is then used to determine the frequency and phase response. The code for this function is provided in Appendix 3.A.

The bode diagram for a 7 inertia (n=7) driveshift model is shown in Figure 3.16.
The bode plot of the state-space model shows resonances modes which match that of the same Simulink model. A phase change, between 90° leading and 90° lagging, occurs at each resonant and anti-resonant frequency.

3.3.1.5 Continuous
Driveshaft models considered thus far have divided the driveshaft into a finite number of inertia lumps so as to replicate the variation in behaviour along the shaft length. The true shaft behaviour is that of an infinite number of inertia elements interacting along the shaft length, and so the true response the driveshaft must be determined by considering it as a continual system. A continuous driveshaft model is mathematically derived in [125] and [126], and by solving for given boundary conditions, a frequency domain response is obtained showing an infinite number of resonant modes. Figure 3.17 shows the frequency response for the generic driveshaft being considered.
The resonance modes of the continuous model match the limit as \( n \) tends towards infinity for the \( n \)-inertia lump model shown in Figure 3.14. The amplitudes are lower within the continuous model response as damping is not represented in a comparable way.

### 3.3.1.6 Comparison of strategies

The continuous driveshaft model requires prior knowledge of boundary conditions and, as implemented in Figure 3.17, may only be solved in the frequency domain, making it impractical for implementation within a time domain drivetrain model. However it does provide a true response to which other driveshaft modelling strategies can be compared.

The state-space representation provides a more easily solvable way of evaluating high order \( n \)-inertia lump models, it cannot be readily incorporated into a time domain system model but is used here to compare the various modelling strategies. Figure 3.18 shows the frequency response (only the 1st mode is shown) of all the driveshaft models considered.
Figure 3.18: Comparison of 1st mode response for driveshaft models

It is immediately apparent that the response of the lumped inertia (n=1) model does not match the trend of the others and is therefore inadequate. By considering the continuous model as a benchmark it is shown that as the number of lumps, n, considered is increased (from two with the split inertia model and upward with the n-inertia model), the model becomes more representative with the resonance mode tending towards that predicted by the continuous model.

Table 3.7 details the resonances identified by each drivetrain model up to the 4th mode.

Table 3.7: Comparison of resonance modes for various driveshaft models

<table>
<thead>
<tr>
<th>Model</th>
<th>1st mode (rad.s(^{-1}))</th>
<th>2nd mode (rad.s(^{-1}))</th>
<th>3rd mode (rad.s(^{-1}))</th>
<th>4th mode (rad.s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lumped inertia</td>
<td>128</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Split inertia (n=2)</td>
<td>256</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Split inertia (n=3)</td>
<td>314</td>
<td>543</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Split inertia (n=4)</td>
<td>340</td>
<td>627</td>
<td>819</td>
<td>-</td>
</tr>
<tr>
<td>Split inertia (n=5)</td>
<td>354</td>
<td>673</td>
<td>927</td>
<td>1,089</td>
</tr>
<tr>
<td>Split inertia (n=6)</td>
<td>363</td>
<td>702</td>
<td>992</td>
<td>1,215</td>
</tr>
<tr>
<td>Split inertia (n=7)</td>
<td>369</td>
<td>720</td>
<td>1,035</td>
<td>1,298</td>
</tr>
<tr>
<td>Split inertia (n=8)</td>
<td>374</td>
<td>734</td>
<td>1,065</td>
<td>1,356</td>
</tr>
<tr>
<td>Split inertia (n=10)</td>
<td>380</td>
<td>751</td>
<td>1,103</td>
<td>1,429</td>
</tr>
<tr>
<td>Split inertia (n=15)</td>
<td>388</td>
<td>772</td>
<td>1,147</td>
<td>1,510</td>
</tr>
<tr>
<td>Split inertia (n=30)</td>
<td>396</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Continuous</td>
<td>402</td>
<td>805</td>
<td>1,207</td>
<td>1,610</td>
</tr>
</tbody>
</table>
Note: only the 1st mode was calculated for the 30 lump model because computation time was high.

Higher resonance modes cannot be detected by the low order models. The frequency of each resonance tends towards that of the continuous model as the number of inertia lumps modelled is increased, however this substantially increases model complexity and achieves diminishing returns in terms of accuracy. Figure 3.19 shows the 1st mode resonance frequency against the number of inertia ‘lumps’ considered within the model.

![Figure 3.19: State Space driveshaft model - 1st mode frequency against number of inertias](image)

While increasing the number of inertia lumps increases the accuracy of the model towards that of the continuous model only limited gains are achieved above the knee of the trend, at \( n=7 \).

### 3.3.1.7 Selection of modelling strategy

The continuous model provides the most accurate means of mathematically representing driveshaft behaviour; however it cannot be implemented in a wider drivetrain model. A realistic time domain model can be produced by considering the driveshaft as a finite number of inertia lumps but as this number is increased it soon becomes excessively complex.
While as a single element the true driveshaft behaviour is important, when combined with other systems in the drivetrain the driveshafts properties should be considered to determine the level of representation required. Compared to other elements within the drivetrain, driveshafts typically have low inertia and low torsional stiffness, and interconnect with elements with high inertia and high torsional stiffness. As a result driveshaft inertia is not significant but stiffness is. Therefore the driveshaft model included must represent end to end shaft compliance accurately but is not required to accurately replicate its stand-alone frequency response, the split inertia (n=2) model is selected as it achieves this with minimal model complexity.

For systems where surrounding inertias are not significantly larger than driveshaft inertia a model with distributed inertia, such as the n-inertia model with n=7, would be better suited.

### 3.3.2 Auxiliary loads

Auxiliary loads are characterised by their high inertia for example on a 3 spool GT the oil pump, hydraulic pump, fuel pump and two electrical generators represent 67% of the total drivetrain inertia, this rises further to nearly 80% when referred to speed. Loads are coupled to the AGB by a shaft exhibiting compliance, this is most notable in the case of electrical generators where an extended driveshaft is used to offer damping for transient electro-magnetic torque. Figure 3.20 shows a high inertia auxiliary load and driveshaft coupling.
Power extracted from the AGB is transmitted along the loads driveshaft to the inertia of the load itself. The normal inertia of the auxiliary loads is dependant on its behaviour as well as mechanical properties. For example the inertia of a fluid pump also includes the inertia of the fluid which may vary with pneumatic and hydraulic loading. Torque loading applied to an auxiliary load is typically non-constant, as with an electrical generator, but more consistent loads such as an oil scavenger do exist. Torque transmitted along the loads driveshaft and applied to the AGB is modified by the inertia of the load and compliance of the driveshaft which acts as a low pass filter to any torque transients.

For the purpose of modelling, auxiliary loads are assumed to be ideal, having no bearing losses or non-linear couplings and load inertia is assumed to be non variable.

### 3.3.2.1 Lumped inertia

An auxiliary load is best considered as two elements, the driveshaft and load inertia, hence models for these two individual subsystems can be combined to produce an accurate auxiliary load model.

The auxiliary load driveshaft model has the same requirements as the driveshaft model discusses previously and so the same lumped inertia driveshaft model is used. Modelling of the load's inertia uses (3.2). The two models are combined, as shown in...
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Figure 3.21, by adding the load-end driveshaft inertia lump with the additional load inertia.

![Figure 3.21: Auxiliary load lumped inertia model diagram](image)

The model represents both the driveshaft shaft stiffness, $k$, and load inertia, $J_A$, of the auxiliary load. In reality the load inertia will be much greater than the shaft inertia.

### 3.3.2.2 Torque profile

A more accurate auxiliary load model should also include a loading torque, $T_L$, which is related to the loads state of operation, this is essential in the case of the generator model as it allows interfacing with the generator electro-magnetic model. Representation of auxiliary loads this way requires knowledge of the loads torque / speed profile which are currently unavailable to the project, simplified nominal loads are used for each of the auxiliary loads.

### 3.3.2.3 Selection of modelling strategy

The generator auxiliary load model requires an input for the varying load torque as derived from the generator electro-magnetic model. Other auxiliary loads can be modelled using a constant normal load torque and their torsional behaviour. The torque load profile modelling strategy is selected, allowing all auxiliary loads to be modelled in an identical fashion. This strategy makes provision for the input of load profile data for all auxiliary loads should it become available.
3.3.3 Gearbox

Gears provide a means of torsionally coupling the drive of two or more mechanical elements. Torque is transmitted through the interaction of gear teeth in a meshing pair of gears, the ratio of teeth on the two intermeshing gears \((n_1/n_2)\) dictates the relative drive speed of each gear. Such a gear pair is shown in Figure 3.22.

![Gear meshing diagram](image)

**Figure 3.22: Parallel axis gear meshing diagram**

Gears themselves have an inertia, and the gear teeth in particular have compliance and an associated damping. These parameters must be considered alongside the functional behaviour of the gearbox when considering a modelling strategy. Methods for modelling a spur gear pair are discussed in [127] and [128] for frequency and time domains respectively and consider the gear inertia as well as introducing stiffness and damping function for the mesh.

3.3.3.1 Power Transfer

Mechanical power remains constant through a gearbox (3.18), neglecting losses (typically less than 1% in for parallel axis gears [129]), and so a rise in speed will reduce torque output, as shown in (3.19) and (3.20).

\[
T_2 \omega_2 = T_1 \omega_1 \quad (3.18)
\]

\[
T_2 = \left(\frac{n_2}{n_1}\right) T_1 \quad (3.19)
\]
The most basic gear model considers the gears purely by their work carried out using the mathematical functions (3.19) and (3.20), assuming an ideal, lossless, power transmission.

### 3.3.3.2 Dynamic Properties

On top of the functional behaviour of the gearbox, the dynamic behaviour of the gears can also be considered, in terms of the gear inertia and gear tooth compliance, as well as a disturbance induced by the teeth meshing process. As a gear pair turn, the teeth engage and disengage with the teeth of an adjacent gear. This creates a frequency which is a multiple of the rotational frequency of the gear and its teeth number \([130]\), as described by (3.21).

\[
GTM \text{ mech } f = GTM \text{ mech } f_n \text{ mech } \quad (3.21)
\]

Where:

\[
GTM \text{ mech } f = \text{ Gear tooth mesh frequency (Hz)},
\]

\[
mech \text{ mech } f = \text{ Gear rotational frequency (Hz)},
\]

\[
n = \text{ Gear teeth number}
\]

Imbalance between the gear teeth, perhaps due to manufacturing tolerances or gear wear, can be observed as sidebands of this frequency. It should be noted that gear tooth frequency does not represent a resonance which may be excited, but the meshing frequencies inject a disturbance into the drivetrain which may excite resonances across the wider electro-mechanical system. Due to the multiplication factor with respect to shaft speed, meshing frequencies are typically the highest mechanical frequencies present.

A basic spur gear pair is shown in Figure 3.23. The gear teeth mesh point has both a stiffness, \(k_m\), and damping, \(c_m\). The process of gear meshing introduces a disturbance torque, \(m\).
The consideration of meshing frequency disturbance and gear torsional behaviour increases the order of the system model. While providing an accurate representation of gearbox behaviour it proves too processor-intensive to implement in the wider drivetrain model. Furthermore high gear teeth stiffness producing high frequency resonances and high gear-mesh frequencies are expected to be well separated from fundamental system resonances.

### 3.3.3.3 Backlash

Clearance is required between any inter-meshing gear pair to avoid interference, which may lead to excessive heat / wear. Clearance creates a difference between the gear tooth thickness and the gear tooth space at the engagement operating point (or the pitch circle [129]) which is referred to as backlash, see Figure 3.24. This clearance is defined to allow lubrication space and take into account machining tolerances and adds space between the Pitch Circle Diameter (PCD) of each gear.

![Figure 3.23: Torsional gear model diagram](image)
Backlash is a physical parameter of gear design and alignment which manifests itself as a non-linearity in the relationship between angular position of engaged gears and torque transfer. Figure 3.25 shows the impact of backlash on the apparent stiffness of the gear mesh.

Backlash has a minimal impact on the behaviour of gears with a constant torque, however torque fluctuation or full reversal will induce this non-linear behaviour in the gearbox. The transient loading expected would make gear backlash likely and so is a significant property for consideration. Low amplitude torque oscillation may not induce full gear reversal but instead lead to gear chatter within the backlash region where gear
teeth vibrate against each other. Anecdotally, the introduction of backlash or chatter is considered to have the effect of increasing the stress on mechanical elements two fold.

While the impact of backlash is significant, the aero drivetrains are designed to have minimal backlash. High machining tolerances allows pitch centre clearances to be kept low, and high gear teeth numbers (creating smaller teeth) further reduces the backlash region. An encapsulating lubrication system is provided which, as well as preventing overheating, provides damping within the backlash region so partially negating its impact.

3.3.4 Selection of modelling strategy

Gears have relatively low inertia and energy losses, and high stiffness compared to other components within the drivetrain. Therefore their meshing behaviour has little impact on the wider behaviour of the drivetrain. To maintain a drivetrain model with low computational requirements, the modelling of gear meshing behaviour and backlash are neglected. The key importance which cannot be neglected is the ratio of gear teeth between the meshing gears as this dictates the torque transmitted throughout the network. Therefore gears are modelled as ideal, with no losses or stiffness within their meshing, purely as a ratio. Furthermore given that the gear inertia is significantly lower than that of surrounding elements, gear inertia is neglected.

Further to the simplified gear model, the option exists to inject a synthesised gear meshing torque into a complete drivetrain model. The extent to which gear meshing frequencies propagate throughout the drivetrain could then be identified. The amplitude of such a disturbance torque would have to be evaluated through measurement of a real system as this is an unknown parameter.

3.4 Full definition model

This section details the integration of a full definition mechanical drivetrain using the modelling strategies identified previously. The behaviour of the drivetrain model in both the frequency and time domain is analysed and this is compared with the measured response from engine test data.
3.4.1 Model implementation

Mathematical models have been derived for driveshafts, auxiliary loads and gears. A variety of time-domain and frequency-domain modelling approaches have been used to assess the model responses. These individual models must be combined and interfaced to produce a full drivetrain model.

The Simulink environment is a graphical extension to MATLAB which allows large models to be constructed visually using sub-systems containing the individual mathematical model, Simulink automatically handles the bi-directional interfacing of each sub-system model, and a time-stepping solver produces the time-domain response. Alternatively the drivetrain model could be coded directly in MATLAB, this offers reduced simulation runtime for the full drivetrain model as it does not have to be compiled as a Simulink model would. However the process is far more complex and does requires greater development in order to produce accurate interfacing between sub-systems, this interfacing can be carried out using a receptance modelling approach as described in [131]. Should a frequency-domain model be required, receptance modelling would be preferable rather than using a frequency sweep of a Simulink model.

The Simulink modelling approach is selected as it is more user friendly and produces models which can be more easily adapted than a MATLAB coded model, for example altering the gearbox configuration to represent different engine structures. The addition of gear meshing torque disturbances and non-linearities, such as gear backlash, is possible in Simulink but cannot be readily added to a receptance model coded directly in MATLAB.

A receptance based modelling approach provides a means of mathematical model interfacing which Simulink carries out automatically. Although not being used, an understanding of the receptance modelling strategy ensures a more comprehensive appreciation of the work being undertaken by Simulink.
3.4.1.1 Receptance modelling

The torsional behaviour of any drivetrain component can be fully described by four sub-actions. These are listed below with reference to the arbitrary torsional component (shown in Figure 3.26):

- Deflection at far end ($\theta_2$) as a result of torque ($T_1$) applied at driving end
- Deflection at driving end ($\theta_1$) as a result of torque ($T_1$) applied at driving end
- Deflection at far end ($\theta_2$) as a result of torque ($T_2$) applied at far end
- Deflection at driving end ($\theta_1$) as a result of torque ($T_2$) applied at far end

By describing these actions as the ratio of response amplitude and phase to excitation amplitude and phase the system's behaviour is completely described by four mathematical functions. These mathematical functions describe the receptance of a component.

Receptance equations are typically arranged as a two-by-two matrices, here denoted $\alpha$, with individual receptances given the symbol $\alpha_{mn}$ where $m$ is the coordinate of the response amplitude, and $n$ is the coordinate of the excitation amplitude, as shown in (3.22). The arrangement of these receptance equations is shown in Table 3.8.

\[
\alpha = \begin{bmatrix}
\alpha_{11} & \alpha_{21} \\
\alpha_{12} & \alpha_{22}
\end{bmatrix}
\]

(3.22)
### Table 3.8: Torsional receptances

<table>
<thead>
<tr>
<th>Applied Torque</th>
<th>Response</th>
<th>0\textsubscript{1}</th>
<th>0\textsubscript{2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_1 )</td>
<td>( \alpha_{11} = \frac{\theta_1}{T_1} )</td>
<td>( \alpha_{21} = \frac{\theta_2}{T_1} )</td>
<td></td>
</tr>
<tr>
<td>( T_2 )</td>
<td>( \alpha_{12} = \frac{\theta_1}{T_2} )</td>
<td>( \alpha_{22} = \frac{\theta_2}{T_2} )</td>
<td></td>
</tr>
</tbody>
</table>

Note: \( \alpha \) may be complex.

Receptances describing an excitation and a response at the same point are referred to as direct receptances (\( \alpha_{11} \) and \( \alpha_{22} \)) while a cross receptance describes a response at a different point to the excitation (\( \alpha_{12} \) and \( \alpha_{21} \)) [131]. It is usual for receptances to form two common pairs. For the majority systems both cross receptances are the same, and both direct receptances are the same. The transmission of a driveshaft is identical in both directions for example, (3.23) and (3.24).

\[
\begin{align*}
\alpha_{11} &= \alpha_{22} \quad (3.23) \\
\alpha_{12} &= \alpha_{21} \quad (3.24)
\end{align*}
\]

The receptance modelling strategy can be used to model a system of mechanical components in either the frequency domain or time domain as described in [127] and [128] respectively. A receptance derivation for a driveshaft as well as a method for combining individual receptance models is shown in Appendix 3.B.

**3.4.1.2 Model parameters**

Specifications for all components within the drivetrain are provided by the manufacturer. These have been interpreted to provide values of inertia, torsional stiffness and torsional damping as well as gear ratios. Accessory load data is also collated enabling a nominal torque to be identified for each of the auxiliary loads and is included in the model.
3.4.2 Real system analysis

In-depth analyses of torsional modelling methods have allowed a full definition drivetrain model to be produced. While the accuracy of each sub-component model has been assessed, full model validation must be carried out by comparison of results with data from a real system.

The drivetrain model can be validated by comparison with data from a real engine test run. The engine test in question maintains the gas turbine at a steady 'low idle' speed while using a resistive load bank to provide a load on the electrical generators. The electrical load on each generator is increased from 50 kW to 100 kW (from 20% to 40% rated electrical load), this creates an electro-magnetic torque step onto the mechanical network.

In the absence of position and torque data for individual components, shaft speed data for both of the electrical generators is obtained from the generator shaft mounted Permanent Magnet Generator (PMG). Each main generator incorporates a 3-phase, 18-pole, PMG which ensures a robust electrical supply to the safety critical Flight Control Electronics (FCE). Shaft speed is derived from the electrical frequency using a zero crossing detection algorithm developed in MATLAB.

Due to the co-axial shaft design of the generator the PMG does not provide true speed data for the generator but indicates speed part way along the generator shaft, this discrepancy is assumed to be negligible here. It is assumed that the coupling between the PMG and the main unit of the generator is rigid.

3.4.2.1 Identification of frequencies

Figure 3.27 shows the electrical generator shaft speed during an electrical load step from 20% to 40% rated load. This electrical load is applied simultaneously to both generators and so the response of both generators is very similar.
Figure 3.27: Engine test data - electrical generator 1 shaft speed during electrical load step (20% to 40%)

The generator angular velocity decreases after the electrical loading has been increased, at time $t=1$ s, but recovers after approximately 10 seconds. This dip in angular velocity is a result of the power being extracted from the mechanical system to feed the electrical load, the recovery occurs as fuel burn is increased. After the electrical load step speed oscillations occur, which eventually decay away.

Note: the gas turbine Full Authority Digital Engine Control (FADEC) controls numerous elements including fuel flow to the combustors based on many environmental inputs and so spool speed should not be expected hold a constant value or return to it once electrical loading has been varied.

Significant noise can also be seen on the signal, this is a result of the post-processing routines and the mechanics of the PMG which produce disturbance in the measured mechanical shaft frequency.

Angular acceleration of the electrical generator is directly proportional to the torque imposed on the generator inertia, and so oscillations in angular velocity can be considered indicative of torque oscillations within the drivetrain system. Frequency
analysis of angular velocity identifies the frequency of angular velocity oscillations and hence the frequency of torque oscillations.

Figure 3.28 shows the frequency content in the angular velocity for the electrical load step shown in Figure 3.27. Frequency content is derived using a post processing function to provide the power spectrum, traces are scaled individually to give detail of the most significant harmonics within the frequency range.

![Figure 3.28: Engine test data - electrical generator 1 shaft speed frequency content before (left) and after (right) electrical load step (20% to 40%)](image)

The frequency spectrum windows are: t=0 s to t=1 s (before) and t=1 s to t=3 s (after). The signal noise can be seen both before and after the load step at approximately 123 Hz which corresponds to the shaft speed. Frequencies at 25.8 Hz and 35.5 Hz, which are not present before the load step, can be detected after the step and are assumed to be the result of drivetrain oscillations.

Note: shaft speed variation after the electrical load step also results in DC content on the frequency spectrum and is the result of the fuel control loop which is slow compared to mechanical resonances.

### 3.4.2.2 Categorisation of frequencies

Frequency analysis is carried out on the difference between the two generators shaft speed, for the same electrical load step, and shown in Figure 3.29.
The frequency spectrum window is t=1 s to t=2 s. Again signal noise can be seen both before and after the load step at approximately 123 Hz. Of the two drivetrain modes visible in Figure 3.28 the 2nd at 35.5 Hz can be seen, while the 1st mode is not detectable. This indicates that the 1st mode oscillation acts with the two generators in-phase, while the 2nd mode is an out-of-phase oscillation between the generators.

### 3.4.3 Drivetrain model behaviour

A drivetrain model is produced using the modelling techniques selected in Section 3, component parameters are taken from data provided by the manufacturer. A screen shot of this model is shown in Appendix 3.C.

The model is configured to provide observation of angular position and torque levels on all components within the drivetrain, torque loading can be simulated on any of the auxiliary loads and from the spool offtake point. Of particular interest are torque levels throughout the mechanical network, which relate to component stress, potentially in response to an electrical generator load. Analysis is carried out of both the frequency domain and time domain response of the model.
3.4.3.1 Frequency response

The frequency response of the drivetrain, with respect to an excitation torque at electrical generator 1, is used to identify mechanical resonant modes. Peak torque (under steady conditions) is recorded for each element in response to an excitation frequency. Due to the symmetrical nature of the drivetrain the system response is very similar for excitation from either generator.

For the collection of a frequency response data the model is considered in a speed stiff mode. The dominant nature of the IP spool inertia means that it may be considered a mechanical ground point for the network, and so the spool offtake point is actually connected to ground via a high torsional stiffness. Under these conditions the drivetrain vibrations appear as angular position fluctuations around a zero net speed.

It has been shown with the continuous driveshaft model that any mechanical component has an infinite number of resonant frequencies. Hence when combined the mechanical drivetrain mode also has a high number of resonance modes. However, inherent damping within the system will also act to suppress resonances, particularly at higher frequencies. A frequency sweep found no significant resonances above 140 Hz and so the frequency response discussed here is limited to a range of 1 Hz to 140 Hz. A sinusoidal torque is applied at generator 1, the amplitude is kept constant but the frequency varied in finite steps to create a frequency sweep, the amplitude of response for each of the drivetrain elements is recorded at each frequency.

The frequency response of components throughout the drivetrain is shown in Figure 3.30.
The frequency sweep (with resolution of 0.1 Hz) presents resonances at 26.6 Hz, 37.2 Hz, and 87.2 Hz. The amplitude of response can be used to compare the resonances at each element, but should not be considered when comparing between elements as gearing leads to non-consistent excitation amplitude throughout the system.

A 1st frequency at 26.6 Hz is present throughout the mechanical system appearing at each element in the network. The 2nd frequency at 37.2 Hz can be observed only in the elements directly between the two electrical generators. A 3rd response, appearing to
derive from the fuel pump and at 87.2 Hz, can be seen in components linking the fuel pump and spool offtake point.

Resonant frequencies observed throughout the drivetrain are shown in Table 3.9.

<table>
<thead>
<tr>
<th>Component</th>
<th>1st Mode</th>
<th>2nd Mode</th>
<th>3rd Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spool offtake</td>
<td>26.6</td>
<td>-</td>
<td>87.2</td>
</tr>
<tr>
<td>RDS</td>
<td>26.6</td>
<td>-</td>
<td>87.2</td>
</tr>
<tr>
<td>ADS</td>
<td>26.6</td>
<td>-</td>
<td>87.2</td>
</tr>
<tr>
<td>TGB</td>
<td>26.6</td>
<td>-</td>
<td>87.2</td>
</tr>
<tr>
<td>Generator 1</td>
<td>26.6</td>
<td>37.2</td>
<td>87.2</td>
</tr>
<tr>
<td>Oil Pump</td>
<td>26.6</td>
<td>-</td>
<td>87.2</td>
</tr>
<tr>
<td>Generator 2</td>
<td>26.6</td>
<td>37.2</td>
<td>87.2</td>
</tr>
<tr>
<td>Fuel Pump</td>
<td>26.6</td>
<td>-</td>
<td>87.2</td>
</tr>
<tr>
<td>Hydraulic Pump</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 3.31 shows the time domain torque response for components throughout the drivetrain, when excited by a sinusoidal torque from generator 1, at each of the three resonant modes. The sinusoidal amplitude is identical at each frequency.

The 1st mode produces a response which is in phase for all drivetrain components. 2nd mode excitation produces high level responses from both of the electrical generators with these acting out of phase, all other components have a low level response. The 3rd mode gives a response which is at a lower level than other modes and indicates the two generators acting out-of-phase with the rest of the drivetrain, causing a higher response.
from the fuel pump in particular. The hydraulic pump has a negligible response at all frequencies due to its low inertia.

Figure 3.31 shows that the 1st and 2nd modes are more significant than the 3rd due to their high amplitude. The drivetrain is oscillating collectively in the 1st mode, with oscillations occurring only between the generators in the 2nd mode, at the 3rd mode the two generators act together and out-of-phase with the rest of the drivetrain / against the fuel pump.

3.4.3.2 Time domain response

The drivetrain model is here considered in true variable speed operation, the spool offtake point is coupled to a realistic IP spool inertia with torque control applied to replicate the mechanical behaviour of the IP spool seen in the engine test data. Spool control is not achieved using a full thermodynamic matching model but instead a PI control loop which is manually tuned to reproduce the response of the fuel control loop, this is representative of the true system and does not interfere with drivetrain behaviour due to the slow speed of the response. The model is run at 'low idle' speed, while typical continuous-torque loads are simulated on each of the auxiliary loads, as shown in Table 3.10. The load at electrical generator 1 is stepped from 20% to 40% load to provide a torque step on to the mechanical network. Torque is monitored on selected drivetrain components, as well as shaft speed on both of the electrical generators.

<table>
<thead>
<tr>
<th>Table 3.10: 3 spool GT Auxiliary loads nominal torque load</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Auxiliary Load</strong></td>
</tr>
<tr>
<td>----------------------</td>
</tr>
<tr>
<td>Oil Pump</td>
</tr>
<tr>
<td>Fuel Pump</td>
</tr>
<tr>
<td>Hydraulic Pump</td>
</tr>
<tr>
<td>Electrical generator</td>
</tr>
</tbody>
</table>

Torque Measurement

Electrical loading produces an electro-magnetic torque onto the generator shaft. This shaft torque is transferred into the drivetrain producing a step load. A torque is applied to the generator driveshaft to simulate this electrical loading. Figure 3.32 shows the applied electro-magnetic torque and the resultant torque on the RDS.
As electrical loading is increased at t=1 s, electro-magnetic torque within the generator increases accordingly. RDS torque increases to meet the power load, and the rapid change in torque induces oscillations within the drivetrain. These oscillations occur at 26.6 Hz. Due to the low levels of damping within the system these oscillations take approximately 10 seconds to decay. High levels of torque, and sustained a period of oscillation such as this would result in the fatigue and reduced lifespan of drivetrain components [118].

Angular Velocity

Measurements of shaft angular velocity are considered for both of the electrical generators so that they can be directly compared with actual engine test data.

Figure 3.33 shows time and frequency domain data for generator 1 shaft speed when the electrical load is stepped from 20% to 40% at t=1 s.
Generator shaft speed is reduced as mechanical potential energy is extracted as electrical power. Speed recovers as the control scheme counteracts the dip. No oscillations are present before the load step, after the load step frequencies at 27.2 Hz and 37.2 Hz can be detected within the generator shaft speed (an FFT resolution of 0.1 Hz is achieved). These oscillations decay almost entirely within 6.5 seconds.

The shaft speeds of the two electrical generators are compared during the same electrical load step. Figure 3.34 shows the frequency content of the difference in shaft speed between the two generators after the load step, with a resolution of 0.1 Hz.
The frequency at 37.2 Hz is visible between the two generators, oscillations at 27.2 Hz are not detected. It is concluded that the 1st mode oscillation occurs with the generators in-phase while the 2nd mode oscillation occurs with the generators out of phase as the 1st mode cannot be seen in the difference between shaft speeds.

The difference in detected resonant modes for the frequency sweep and FFT analysis is due to the methods of identifying frequency. Resolution of the frequency sweep is limited by the number of frequency values tested. Frequencies identified by FFT analysis should be considered more accurate. Furthermore the load step response is taken with a true variable speed model, whereas the frequency sweep is undertaken using a speed stiff configuration (which assumes an infinite spool inertia) so producing a different 1st and 3rd mode frequencies. This highlights the limitations of a speed stiff assumption, although both results are sufficiently similar in this case, and figures from the frequency sweep method are used.

3.4.3 Model validation

Analysis of data from engine test run has shown resonances at 25.8 Hz and 35.5 Hz. It has also been demonstrated that the 1st mode oscillation occurs with the generators in-phase while the 2nd mode occurs with the generators out-of-phase.
model exhibits resonances at 26.6 Hz, which can be observed throughout the transmission, AGB, and the electrical generators, and 37.2 Hz which is only apparent in components directly linking the two electrical generators. The model can be considered validated against real engine data by the similarity of these two resonance modes. A third frequency, at 87.2 Hz, is observable between the spool and fuel pump from where it appears to originate. This resonance is at much lower amplitude to the 1st and 2nd modes and is not observable in the real engine test data. Frequencies apparent in the real and simulated drivetrains are compared in Table 3.11.

<table>
<thead>
<tr>
<th></th>
<th>1st Mode (Hz)</th>
<th>2nd Mode (Hz)</th>
<th>3rd Mode (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine test data</td>
<td>25.8</td>
<td>35.5</td>
<td>-</td>
</tr>
<tr>
<td>Full definition drivetrain model (sweep)</td>
<td>26.6</td>
<td>37.2</td>
<td>87.2</td>
</tr>
<tr>
<td>Full definition drivetrain model (impulse, FFT)</td>
<td>27.2</td>
<td>37.2</td>
<td>87.7</td>
</tr>
</tbody>
</table>

The small differences in system resonances between modelled and real systems can be attributed to uncertainties in drivetrain parameters of stiffness and inertia. Damping may limit the visibility of the 3rd mode from the engine test data. Drivetrain parameters vary slightly between development engine numbers and hence may differ slightly from the parameters provided for modelling. Lumped parameter analysis has demonstrated the simplicity of the resonant modes present, in particular the 2nd mode. It is therefore possible to suggest the changes to model parameters which will give resonance modes more closely matching the real system; reducing the ratio of generator shaft stiffness to generator inertia \( \left( \frac{k}{J} \right) \) by 10% in both generators for example.

### 3.5 Analysis of drivetrain behaviour

A full-definition GT drivetrain model has been produced and validated against the available real gas turbine test data. While being accurate, the behaviour of the system is produced by replicating the existing system rather than being derived from knowledge of the principles of torsional behaviour. This section outlines work carried out to derive an understanding of the torsional behaviour of the mechanical system in question, the key influences on this behaviour, leading to mathematical derivation of the system response.
3.5.1 Identification of key influences

High frequency vibrations introduce low energy content into the drivetrain as they are more rapidly mitigated by cyclic damping and so have a lesser impact on drivetrain behaviour. Therefore lower frequencies have more significance on the resonances displayed by the complex drivetrain systems. Lower frequencies are generated by the combination of low torsional stiffness and high inertia, as shown in (3.13).

3.5.2 Resonance of drivetrain components

The natural resonance of each drivetrain component is calculated using (3.13). Vibration at each of the accessory loads is a result of the load inertia and the stiffness of it connecting driveshaft, resonances are therefore calculated for the combined shaft and inertia of the accessory load. Figure 3.35 shows the natural frequencies of individual elements within the drivetrain.

![Drivetrain component natural resonances](image)

Figure 3.35: Drivetrain component natural resonances

The majority of drivetrain components have natural resonance above 200 Hz, the one exception to this is the electrical generators which due to their high inertia and low stiffness driveshaft have a low natural frequency.

3.5.3 Gearbox referral

The drivetrain however consists of multiple interconnected elements. Sub-systems within drivetrains are often interlinked via gearboxes; while providing torque
transmission these also alter each elements drive speed and hence its apparent, or referred, torsional stiffness and inertia. When considering the frequency of torsional response in complex drivetrains referred parameters must be considered. While the selected referred nominal speed is unimportant is makes sense to take the speed at the root / ground point of the network, in this case the IP spool.

The stiffness and inertia of an element are referred through a gearbox to appear higher, if speed is stepped up, or lower if speed is stepped down [123]. This referral is demonstrated in Figure 3.36.

Given that on their own elements with either a high inertia or low torsional stiffness have the lowest resonant frequencies it is the influence of elements with the highest referred inertia and elements with the lowest referred stiffness which have the greatest influence within multi-element drivetrains.

\[
k_{2ref} = \left(\frac{n_1}{n_2}\right)^2 k_2, \quad J_{2ref} = \left(\frac{n_1}{n_2}\right)^2 J_2
\]
3.5.4 Referred parameters

The inertia of the IP spool is significantly larger than that of any other component within the network. Given the large inertia in comparison to all other mechanical systems it is a valid assumption that oscillations within the drivetrain will excite negligible oscillations within the IP spool having little influence on its behaviour. The IP coupling is therefore effectively a ground point for the mechanical network, all components are referred to this point and with speed to 100% N2.

Figure 3.37 shows the speed of the various components of the drivetrain in comparison to the 100% N2 spool speed.

The high drive speed of the RDS and its connected gears, gear B and gear C can be seen, as well as the low speed hydraulic pump, and gear 1 through which it connects to the AGB.

Drivetrain speed data is combined with component properties to provide both real and referred torsional properties for each of the components within the mechanical drivetrain.
Figure 3.38 shows the true inertias of the various drivetrain components and their referred inertias (referred to N2 and normalised to the lowest inertia in the drivetrain, the RDS).

![Drivetrain Component Inertias and Referred Inertias](image-url)

By far the dominating inertias within the drivetrain are those of the electrical generators, with a relatively high drive speed, at 1.8 times N2, their referred inertia is dramatically higher than all other components. Although the RDS has a very high speed, its low inertia means that its referred inertia is still not sizable. Gears B and C which couple the RDS do have higher inertias and so with their high speed have a considerable referred inertia. The ADS operates at one of the higher speeds within the geartrain but has a relatively low inertia, its driving gears, in particular Gear D and to a lesser extent Gear E, have significant referred inertia. The majority of other gears and auxiliary loads have negligible inertias (referred or otherwise) other than the fuel pump which is sizeable in nature and so has a noticeable inertia despite its low speed.

Figure 3.39 shows the stiffness of all the components throughout the drivetrain and their referred stiffness (normalised to the highest stiffness in the drivetrain, Gear A).
The driveshafts coupling the electrical generators and fuel pump to the AGB have the lowest stiffness. It is therefore the driveshafts for the electrical generators and fuel pump as well as the RDS, ADS and TGB which have the greatest influence on drivetrain behaviour. As a result of the high speeds and design for power transmission, the gears have the highest referred stiffness. Most of the transmission and auxiliary load driveshafts have a similar referred stiffness, this is lower than the gears as their length and low cross section introduces significant torsional compliance.

3.5.5 Derivation of drivetrain resonances

Analysis of the torsional properties of drivetrian components has highlighted subsystems likely to have the greatest influence on drivetrain behaviour. A complex torsional network can be simplified by neglecting elements with the least influence on drivetrain behaviour. Hence a mathematical frequency response can be derived by hand, should sufficient simplification be possible. Such analysis is carried out within this section.

Real system data and simulated, full definition, models have highlighted two main resonant modes. Frequency analysis of the drivetrain is carried out for the selection of mechanical components which lead to analysis of both 1st and 2nd mode resonance.
3.5.5.1 Transmission and AGB oscillations

The accessory load forms the bulk of the inertia within the drivetrain, with the transmission providing the substantial compliance. Hence a fundamental resonance will occur with the accessory load oscillating in unison with respect to the spool offtake point.

As previously shown, the inertia of the electrical generators dominate the mechanical accessory load and so the drivetrain as a whole. Inertias of auxiliary loads are relatively low and so may be neglected to form a lumped parameter accessory load model as shown in Figure 3.40. The gear inertias are lumped, and the generator driveshafts and inertias are combined to form a single stiffness and inertia.

![Figure 3.40: Accessory load lumped parameter model derivation diagram](image)
Similarly the transmission represents the majority of the compliance within the drivetrain with little inertia. Therefore the referred stiffness of the RDS, ADS and TGB can be combined, as shown in Figure 3.41, while neglecting the inertia of all these components.

The lumped parameter transmission and accessory load models are combined to form the fully lumped drivetrain model shown in Figure 3.42. The system is further simplified by neglecting the AGB lumped inertia which is in fact several orders of magnitude lower than the combined generator inertias.
Analysis of this system, following (3.13), indicates a natural resonance of 27.90 Hz.

### 3.5.5.2 Inter-generator oscillations

The two electrical generators and their shafts are identical, and mounted equidistant from the transmission (TGB) coupling. A 2nd mode of resonance will therefore exist between the two generators independent of the drivetrain oscillations already considered.

The dominance of the generator inertias compared to other auxiliary systems and the AGB means that all other accessory loads may be neglected. Likewise the gearing coupling the two generators has minimal inertia and high stiffness compared to the generators and their driveshafts so may also be neglected. This approach to model simplification is shown in Figure 3.43 and results in a free-free torsional system with two inertias separated by a torsional compliance.
Figure 3.43: Electrical generator lumped parameter model derivation diagram

The resonance frequency of the system, shown as a lateral system in Figure 3.43, can be found using equation (3.14), which gives a natural frequency of 37.22 Hz. Free-free systems of this nature oscillate in anti-phase [124]. This oscillation occurs with the two generators acting in anti-phase and is not affected by the transmission.

3.5.5.3 Analysis of parameter lumping technique

Resonances within the mechanical system have been derived from real engine data (Figure 3.28), a full definition drivetrain model (Figure 3.30), and mathematically using a parameter lumping approach (Figure 3.42 and Figure 3.43). The findings of each of these strategies are compared in Table 3.12.
Table 3.12: Drivetrain modelled lumped analysis resonant mode comparison

<table>
<thead>
<tr>
<th></th>
<th>1st Mode (Hz)</th>
<th>2nd Mode (Hz)</th>
<th>3rd Mode (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine test data</td>
<td>25.8</td>
<td>35.5</td>
<td>-</td>
</tr>
<tr>
<td>Full definition drivetrain model</td>
<td>26.6</td>
<td>37.2</td>
<td>87.2</td>
</tr>
<tr>
<td>Parameter lumping analysis</td>
<td>27.90</td>
<td>37.22</td>
<td>-</td>
</tr>
</tbody>
</table>

The parameter lumping derivation has indicated two resonant frequencies which are both within 2 Hz of those derived from the full definition drivetrain model and real engine data. This indicates that the parameter lumping method used is valid and hence that within the wider drivetrain the assumptions made during derivation can be considered accurate. These are that other inertias are negligible compared to the electrical generators, and that only the compliance of the transmission and generator shafts need be considered.

A third resonance, at 87.2 Hz, is seen only within the full definition drivetrain model. It is likely that this frequency is too low in amplitude to be detected with the real engine data, and that the lumping simplification approach removes it from the lumped parameter equations.

3.5.5.4 Summary

Parameter lumping has also been able to confirm the phase of oscillations on the electrical generators which experience both 1st and 2nd mode resonances. The 1st mode resonance results in the generators acting in-phase, the 2nd mode resonance leads to anti-phase oscillation between the two generators. This lumping approach is less practical for analysis of resonances occurring from other, low inertia, auxiliary loads as other inertias cannot be neglected so confidently.

3.6 Drivetrain lumped parameter model

Analysis of drivetrain behaviour has indicated two main mechanical resonances and identified key sub-systems which contribute to this behaviour. With this knowledge a simplified, or lumped parameter, drivetrain model is produced which exhibits the same key behaviours as the full GT drivetrain but with a reduced order. This lumped parameter model is validated against the full definition drivetrain model.
3.6.1 Key Assumptions

For the lumped parameter drivetrain model the following assumptions are made. These assumptions are based on research carried out on a 3 spool GT but can be expected to hold true for other high bypass gas turbine engines.

- The IP spool inertia, at 60 times the magnitude, is larger than any other system within the drivetrain. Hence, for the observation of vibration, the system can be considered speed stiff with the IP offtake an effective grounding point.
- Gears are stiffer when referred (≈2x) than other components, their inertia is also suitably low to be neglected, and so their parameters can be ignored.
- The electrical generators have a significantly larger inertia than all other auxiliary loads (≈6x). Other loads can be neglected as the generators dominate.
- Gear meshing frequencies are not present, gearing is assumed as an ideal ratio.

3.6.2 Model

The lumped parameter model takes a similar form to the full drivetrain with a transmission connecting the spool offtake point to the AGB which drives two electrical generators. However, this system is greatly simplified. A single high compliance driveshaft replaces the ADS, RDS and TGB, its torsional stiffness is combined from the properties of these three shafts. Only the electrical generators are considered so reducing the gearbox to three idealised gears.

Component stiffness and inertia are determined using the methods described previously. The gearbox ratio is set as 1/1, and so the electrical generators and their shafts take the N2 referred properties from the full model. Similarly transmission driveshaft properties are the combined referred parameters of the ADS, RDS and TGB.

The lumped parameter model diagram is shown in Figure 3.44, as per the full definition model, a frequency sweep is used to determine the models response to a sinusoidal torque disturbance at generator 1, this frequency response up 140 Hz is shown.
The lumped parameter model exhibits two modes of resonance. The 1st mode appears throughout the drivetrain at 26.6 Hz, a 2nd mode appears only between the two electrical generators and at 37.2 Hz. The 3rd mode, observable in the full definition drivetrain model, is not present due to the simplified architecture.

<table>
<thead>
<tr>
<th>Table 3.13: Drivetrain lumped parameter model resonant mode comparison</th>
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</thead>
<tbody>
<tr>
<td>Mode (Hz)</td>
</tr>
<tr>
<td>Full definition drivetrain model</td>
</tr>
<tr>
<td>1st Mode</td>
</tr>
<tr>
<td>26.6</td>
</tr>
<tr>
<td>2nd Mode</td>
</tr>
<tr>
<td>37.2</td>
</tr>
<tr>
<td>3rd Mode</td>
</tr>
<tr>
<td>87.2</td>
</tr>
<tr>
<td>Lumped parameter model</td>
</tr>
<tr>
<td>1st Mode</td>
</tr>
<tr>
<td>26.6</td>
</tr>
<tr>
<td>2nd Mode</td>
</tr>
<tr>
<td>37.2</td>
</tr>
<tr>
<td>3rd Mode</td>
</tr>
<tr>
<td>-</td>
</tr>
</tbody>
</table>

The lumped parameter model exhibits identical 1st and 2nd mode resonances to the full definition model, and achieves this by modelling only 5 elements compared to the 29 found in the real drivetrain. Assumptions made and methods used for drivetrain analysis appear to be justified.
The methods used to generate this lumped parameter model can readily be applied to other similar engine architectures. Analysis of referred properties identifies components with high compliance or inertia, which can then be combined to produce a reduced order model. Elements with low referred inertia and high referred stiffness can be neglected as typically the resonance modes of a complex system are dictated by a few key elements.

GT spool inertia is typically much greater than drivetrain inertia and so the drivetrain can be considered as a grounded network with low stiffness transmission connecting a high inertia accessory load. The makeup of the accessory load will vary from engine to engine, the trend of increasing airframe electrification means that electrical generators for the most part form the dominant inertia.

### 3.7 Summary and conclusions

The 3 spool GT drivetrain is comprised of three main subsystems, driveshafts, auxiliary loads and gearboxes. Modelling strategies have been identified and compared for each of these subsystems. Optimal modelling strategies are identified and combined to form a full definition drivetrain model, in Simulink, suitable for the investigation of electromechanical interaction. Mechanical properties for the drivetrain model are provided by the manufacturer and the model is validated, in both the frequency and time domains, against real data from an engine test run. The full definition model predicts a 1st mode resonance between the IP spool and accessory load inertia leading to an in-phase oscillation in the generators which is visible throughout the drivetrain, a 2nd mode resonance occurs solely between the two generators creating an anti-phase oscillation, and a 3rd resonance which stems from the fuel pump, with an amplitude which is sufficiently low to have a negligible impact on the wider drivetrain. The full definition drivetrain model correctly predicts modes of resonance to within 1.7 Hz and is used to identify the phase action of each mode.

Analysis of drivetrain component properties is carried out to identify elements with the greatest influence on the behaviour of the system. Torsional compliance and inertia within the drivetrain comes mainly from the longest driveshafts (ADS, RDS, TGB) and the highest inertias (electrical generators), respectively. Furthermore a speed-stiff approximation is appropriate due to the relatively large inertia of the IP spool. Other
systems may therefore be neglected and key parameters lumped to produce simplified models which are mathematically solved to predict each resonance model. This lumping analysis predicts resonance modes to within 1.3 Hz of the validated full definition model. These methods are then used to produce a lumped parameter model which accurately replicates the 1st and 2nd modes of the drivetrain but contains only 5 elements. The lumped parameter model forms the design basis for the mechanical system of the electro-mechanical test rig designed in Chapter 5.

The resonance methods predicted by each of the modelling strategies are compared to the real resonance modes in Table 3.14.

<table>
<thead>
<tr>
<th></th>
<th>1st Mode (Hz)</th>
<th>2nd Mode (Hz)</th>
<th>3rd Mode (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine test data</td>
<td>25.8</td>
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<tr>
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<td>27.2</td>
<td>37.2</td>
<td>87.7</td>
</tr>
<tr>
<td>Lumped Analysis</td>
<td>27.90</td>
<td>37.22</td>
<td>-</td>
</tr>
<tr>
<td>Lumped parameter model (sweep)</td>
<td>26.6</td>
<td>37.2</td>
<td>-</td>
</tr>
</tbody>
</table>

The lumped parameter model, developed using modelling strategies which can be applied to any engine, replaces the full drivetrain behaviour but considers only 5 elements compared to the 29 elements in the real drivetrain. This simplification reduces model solving time approximately 10 fold, which is necessary as the mechanical model is to be integrated into a wider electro-mechanical model with the added complexities of a generator model and electrical systems model.

Validation of the lumped parameter model (by frequency) validates the modelling and analysis methods used to predict drivetrain resonances and so demonstrates that mechanical modes can be predicted for any engine drivetrain. Knowing the key elements which dictate mechanical resonance frequencies allows an electro-mechanical system to be designed holistically, taking into account resonances within each subsystem. A generator control scheme can therefore be designed which does not excite these mechanical resonances, reducing interaction and extending system life. Generator modelling and the design of such a control scheme are detailed in Chapter 4.
Chapter 4  Modelling and Control of a Doubly-Fed Induction Machine

This chapter covers the novel use of a DFIG as an aero generator. A mathematical model of the generator is introduced, and implemented in Simulink. The model is used to identify the power flow and converter rating across the speed range of an aero application. A generator control scheme is also developed which is later implemented onto the real machine using lab hardware.

4.1 Machine Modelling and Characterisation

A mathematical model of the power conversion carried out by a DFIG, based on examples in literature, is developed. Machine characterisation is carried out and included within the model to produce a sufficiently accurate simulation of the DFIG which can be used to analyse its performance as an aero generator and develop a suitable control scheme.

Research is carried out using a lab machine rated at 14.4 kW (50 Hz) which is assumed to be representative of a 250 kVA (360-800 Hz) aero generator, validation of this assumption offers potential future work. A similar approach is considered viable in [84] where 60 Hz hardware rig is discussed to replicate a 400 Hz simulated aero system.

4.1.1 Derivation of DFIG model

A mathematical model for the DFIG is introduced based on the dynamic, 2-axis equivalent circuit model of the induction machine. This is designed so that it may be implemented on the differential-equation solving software package, Simulink (version 7.3) by MathWorks.

The terms used for machine modelling are given below:

\[ V_s : \text{Stator terminal voltage (V)} \]
\[ V_r : \text{Rotor terminal voltage (V)} \]
\[ I_m : \text{Magnetising current (A)} \]
\[ R_s : \text{Per-phase resistance of stator windings (Ω)} \]
\( X_s \): Per-phase reactance of stator windings (\( \Omega \))

\( R'_r \): Per-phase resistance of rotor windings (stator referred) (\( \Omega \))

\( X'_r \): Per-phase reactance of rotor windings (stator referred) (\( \Omega \))

\( R_m \): Per-phase equivalent magnetising resistance (\( \Omega \))

\( X_m \): Per-phase equivalent magnetising reactance (\( \Omega \))

pp: pole pairs
s: Slip, from (4.1) and (4.2).

\[
\omega_s = \frac{2\pi f}{pp} \tag{4.1}
\]

Where:

\( f \) = Electrical frequency (Hz), \( \omega_s \) = Mechanical synchronous speed (rad.s\(^{-1}\))

\[
s = \frac{\omega_r - \omega_s}{\omega_s} \tag{4.2}
\]

Where:

\( \omega_r \) = Rotor speed (rad.s\(^{-1}\))

4.1.1.1 Clarke transform

Each of the three electrical phases within the DFIG create a field which produces interaction with all other phases in both the stator and rotor. Although it is assumed the machine has well-balanced windings, the computation of a full three phase model is complex as a result of the number of interactions being considered.

The full, three phase, model is simplified by utilising the Clarke transform [132], fully detailed in Appendix 4.A. This converts the three phase (abc) system into a two phase (\( \alpha\beta \)) system of equivalent power (power invariant transform). The original three phase system had 120° separation between phases resulting in interaction between them, the resulting two phases are decoupled due to their 90° separation. The zero sequence term (\( \gamma \)) is not considered as the machine in question has an isolated star-connection and is well balanced, so it remains zero.
The Clarke transformed model uses the same equivalent circuit parameters as the steady-state model, but greatly simplifies the complexity of model calculation. A $dq$ DFIG machine model is developed.

### 4.1.1.2 Park transform

Park's reference frame transform [132] [133], detailed in Appendix 4.B is used to provide a reference frame conversion on the two axis machine model. As with the Clark transform power remains consistent between reference frames. This reference frame conversion from a two axis ($\alpha\beta$) model, rotating at arbitrary speed $\omega$, to a two axis ($dq$) model rotating at speed $\omega_0$ is shown in Figure 4.2.

Where the angle, $\theta$, is determined by (4.3).
\[ \theta = \int (\omega - \omega_o) \, dt \]  

(4.3)

Park’s transform provides reference frame conversion allowing a desired frequency of rotation, \( \omega_o \), to be imposed. This frequency can be chosen arbitrarily but several preferred reference frames exist which are normally selected to simplify control schemes. Some common reference frames are given in Table 4.1.

<table>
<thead>
<tr>
<th>Reference frame</th>
<th>Reference frame frequency (( \omega_o ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arbitrary</td>
<td>( \omega ) (arbitrary)</td>
</tr>
<tr>
<td>Stationary</td>
<td>0</td>
</tr>
<tr>
<td>Rotor</td>
<td>( \omega_r )</td>
</tr>
<tr>
<td>Synchronous</td>
<td>( \omega_s )</td>
</tr>
</tbody>
</table>

### 4.1.1.3 Implementation of control using transformations

Control of oscillatory signals requires a high bandwidth and so reference frame transformations are applied to the dq machine model. Selection of the synchronous reference frame is used to produce DC signals for the control scheme to be built around.

The reference frame conversion allows control schemes to be designed to operate on a simplified two phase DC systems, an inverse conversion is then used to return the control outputs to the original reference frame in three phase form.

Figure 4.3 shows how a reference frame conversion, can be using in a generic control system which incorporates a power electronic converter.
Real (abc) signals are monitored and transformed to dq signals before being fed into the controller, the controller outputs dq signals which are transformed into real (abc) references for the converter.

**4.1.1.4 Derivation of mathematical model**

The DFIG is modelled as given in [135], this dynamic machine model is shown in Figure 4.4.

Where it is assumed that:

- Saturation is neglected and therefore a linear magnetic circuit is assumed
- Line harmonics greater than a multiple of 2 are ignored
- The rotor and stator have sinusoidally distributed windings

A mathematical model of the DFIG, in a stationary reference frame, is provided by equations (4.4) to (4.13).

\[
V_{sd} = i_{sd} R_s + \frac{d\psi_{sd}}{dt} \quad (4.4)
\]
Dynamic model

The model is implemented in integral form for numerical stability. Currents are chosen as the state variables, since rotor and stator currents are directly measurable. Rearranging equations (4.4) to (4.13) produces (4.14), (4.15), (4.16), and (4.17), again in a stationary reference frame, which are implemented in Simulink.

\[
\begin{align*}
    i_{rd} &= \frac{L_y}{(L_y - L_m L_m)} \int \left[ \frac{V_{rd} - L_m}{L_y} V_{rd} - i_{rd} R_t - \omega_t L_m i_{sq} - \omega_t L_l i_{eq} + i_{dd} \frac{L_m}{L_y} R_t \right] dt \\
    i_{eq} &= \frac{L_y}{(L_y - L_m L_m)} \int \left[ \frac{V_{eq} - L_m}{L_y} V_{eq} - i_{eq} R_t + \omega_t L_m i_{rd} + \omega_t L_l i_{rd} + i_{dq} \frac{L_m}{L_y} R_t \right] dt \\
    i_{sd} &= \frac{L_y}{(L_s - L_m L_m)} \int \left[ \frac{V_{sd} - L_m}{L_s} V_{sd} + i_{rd} R_t - i_{rd} R_t + \omega_t L_m i_{sq} + \omega_t L_m i_{eq} \right] dt \\
    i_{sq} &= \frac{L_y}{(L_s - L_m L_m)} \int \left[ \frac{V_{sq} - L_m}{L_s} V_{sq} + i_{eq} R_t - i_{eq} R_t - \omega_t L_m i_{sd} - \omega_t L_m i_{rd} \right] dt
\end{align*}
\]
Steady-state model

For completion a steady-state model is derived from the dynamic machine model, this is detailed in Appendix 4.C.

4.1.2 Simulation of DFIG model

The machine equations given in (4.14), (4.15), (4.16), and (4.17) provide a mathematical model of the dynamic electro-magnetic behaviour of the DFIG. These equations are implemented in Simulink to produce a simulation of the electro-magnetic action of the machine core. Further adaption, by reference frame conversion, is required to reproduce the true behaviour of the lab machine.

4.1.2.1 Reference frame selection

Slip-rings on the DFIG provide a mechanical means of reference frame conversion on the rotor side. The derived machine core model assumes both the rotor and stator are monitored in the stationary reference frame where in fact the slip-rings provide a connection in a reference frame derived by the mechanical shaft speed. Therefore in order to be truly representative of the real machine a reference frame conversion, the equivalent of that carried out by the slip-rings, must also be modelled. This reference frame conversion is carried out using the Park transform, as given in Appendix 4.B.

4.1.3 Implementation in modelling environment

The machine core differential equations are implemented in the Simulink modelling environment. Clarke and Park's transforms are also implemented to create a three phase model in the correct reference frames. The combination of machine core model and transforms is shown in Figure 4.5.
The dynamic machine model equations produces a dq machine core model in the stationary reference frame. Reference frame conversions are applied to convert rotor elements into a reference frame determined by rotor mechanical shaft speed, stator elements are maintained in the stationary reference frame. Clarke transforms are used to convert the two phase model into a three phase model with real terminal quantities.

The Simulink implementation of the machine core equations, and the combination of this machine core model and transforms to produce a three phase DFIG model are shown in Appendix 4.D.

4.1.4 Machine characterisation

To ensure the behaviour of the model matches that of the real machine as closely as possible, parameters required within the model must be determined from the real machine. A number of characterisation methods are used to determine the electrical and mechanical parameters from the laboratory machine.

4.1.4.1 Electrical parameters

The steady-state machine model, given in [88], is shown in Figure 4.6.
Unlike the dynamic machine model described previously a term for magnetising resistance is included in the steady-state model.

With the rotor short circuited, the equivalent circuit is rearranged so that the magnetising terms are directly across the stator voltage input. Mechanical machine parameters of torque, $T$, and rotor speed, $\omega_r$, are linked to the electrical equivalent circuit by the rotor slip term, $s$, and by equations (4.18) and (4.19).

\[
|z| = \sqrt{(R_s + R_r / s)^2 + (X_s + X_r)^2} \quad (4.18)
\]

\[
T = \frac{3 V_s^2 R_r}{|z|^2 s \omega_r} \quad (4.19)
\]

Characterisation of induction machines is described in [136]. Access to the rotor coils, through the slip-rings, on the DFIG allows these tests to be expanded by permitting the DC resistance tests and locked rotor / no-load tests to be carried out from the rotor side as well as the stator side. Further strategies have also been suggested for the special case of characterising DFIGs, these are compared in [137] and [138]. Little difference is found between them, and so the IEEE Standard 112 tests [139] (method 1), which gives an iterative process, are used alongside the traditional induction machine tests to characterise the DFIG. Three main tests are carried out:

- DC resistance tests
- Locked rotor / no-load tests
- IEEE Standard 112 tests (method 1)
Results from these tests are compared to determine the machine equivalent parameters. The machine electrical characterisation is fully documented in Appendix 4.E, consideration is also given to the range of parameters for example due to thermal conditions. Table 4.2 gives a summary of the DFIG electrical parameters, and where appropriate their variation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>0.297 Ω</td>
<td>0.297 Ω to 0.391 Ω</td>
</tr>
<tr>
<td>$R'_2$</td>
<td>0.449 Ω</td>
<td>0.449 Ω to 0.591 Ω</td>
</tr>
<tr>
<td>$X_1$</td>
<td>1.13 Ω</td>
<td>1.13 Ω</td>
</tr>
<tr>
<td>$X'_2$</td>
<td>1.25 Ω</td>
<td>1.25 Ω</td>
</tr>
<tr>
<td>$R_m$</td>
<td>250 Ω</td>
<td>250 Ω</td>
</tr>
<tr>
<td>$X_m$</td>
<td>25.8 Ω</td>
<td>17.6 Ω to 27.1 Ω</td>
</tr>
<tr>
<td>$pp$</td>
<td>3</td>
<td>-</td>
</tr>
</tbody>
</table>

Comparison of Equivalent Circuit Model with Experimental Data
The torque versus speed performance of the real DFIG is compared to that predicted by the steady-state equivalent circuit machine model, using the electrical parameters identified by the processes described previously. Torque generation across the machine (motor) speed range is shown in Figure 4.7, the rotor is short circuited and stator supplied through a variac.
Predicted torque data compares well with the experimental data, both having peak torque at the same speed. Higher torque levels could not be observed due to limitations in the connected torque sensor.

The correlation between the real machine and equivalent circuit model demonstrate a good level of accuracy in the derivation of equivalent circuit parameters. However it is the transient response of the eventual model which is of particular interest, and it should be remembered that the DFIG has been characterised in a steady-state.

**4.1.4.2 Mechanical parameters**

Work undertaken in Chapter 3 has demonstrated that mechanically the behaviour of the machine is dictated by its inertia and the damping, the influence of shaft stiffness can be considered negligible due to the relatively high cross section. Simplistically, the inertia of the machine opposes its angular acceleration, and so inertia can be determined by the acceleration of the machine under constant torque. Damping, predominantly from bearing friction, but also windage, opposes the rotation of the machine and so can be determined by the rate of decay in shaft angular velocity under zero torque conditions.

Mechanical parameters for the DFIG are determined using two tests:
- Falling mass test
- Spin-down test

These tests are detailed in Appendix 4.F. The characterised mechanical properties of the DFIG are given in Table 4.3.

Table 4.3: DFIG mechanical properties

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Inertia</td>
<td>0.411 kg.m²</td>
</tr>
<tr>
<td>Damping</td>
<td>0.015 Nm.s.rad⁻¹</td>
</tr>
</tbody>
</table>

4.2 Analysis of DFIG performance

The DFIG model is used to further understand the behaviour of the machine in generator mode using time domain simulation. For the development of a reliable control scheme it is necessary to understand the output response and power flow throughout the DFIG at differing shaft speeds. Stator voltage and frequency are to be regulated, from the rotor side, by a control scheme across the full speed range of the DFIG and so the relationship between stator and rotor voltages and frequencies must be identified for the full speed range. As has been described the rating of the rotor side converter is dependent on the designed slip range. Power flow throughout the DFIG across the full speed range (±40%) is identified so that the converter can be rated correctly.

4.2.1 Stator frequency response

Generated stator electrical frequency is the sum of applied rotor frequency and rotor shaft speed, as shown in (4.20), [90].

\[ f_s = \frac{\omega_m \cdot Pp}{2\pi} + f_r \]  
(4.20)

Where:

- \( f_s \) = Stator electrical frequency (Hz),  
- \( \omega_m \) = Shaft speed, mechanical (rad.s⁻¹),  
- \( f_r \) = Rotor frequency (Hz)
The machine model is supplied with a constant rotor voltage and frequency (10 Hz), the stator side load is a fixed resistance. The rotor and stator electrical frequencies are determined via post-processing routines from the sinusoidal waveforms and are shown against shaft speed in Figure 4.8.

Stator current frequency is seen to be the sum of shaft speed (electrical) and rotor current frequency, where rotor current is in antiphase to shaft speed, and therefore shown as a negative frequency. The simulation is seen to match the relationship (4.20).

### 4.2.2 Stator voltage response

A constant voltage ($100 \, V_{\text{peak}}$) and frequency (10 Hz) is applied to the rotor using the DFIG dynamic machine model. Stator voltage (peak) is recorded against speed and shown in Figure 4.9.
For a constant applied rotor voltage stator voltage varies approximately linearly with shaft speed. The exception to this occurs at the highest speeds where the rise in stator voltage is less so with the rise of speed. This matches the response described by the steady-state machine model. At higher speeds, increased currents will cause increased losses resulting in the slight non-linearity of prediction at high speeds.

4.2.3 Power flow

As the DFIG is electrically doubly-fed, and including the mechanical rotor connection, there are three routes of power flow within the machine. Control is to be introduced via the rotor side converter. For the rating of this device, the relationship between mechanical input power and electrical output power, from the stator and rotor, must be understood across the designed mechanical speed range.

DFIGs are usually considered for speed ranges of synchronous ±30% [90] [93], however for the test rig a range of ±40% is considered to ensure that a robust control scheme is developed.
4.2.3.1 Simulation

The DFIG being used in the test system is thermally rated at 20 A_{\text{rms}} on both the rotor and stator sides. With a network voltage at 240 V_{\text{rms}} it can drive a continual rated output of 14.4 kW.

The DFIG model is used to identify power flow (mechanical input power, stator output power and rotor power) for a variable drivespeed system delivering stator voltage regulation. The model is run over a ±40% speed range and rotor current and frequency are adjusted at each speed to maintain a stator voltage of 240 V_{\text{rms}} at 50 Hz, an electrical load is applied to the stator. Several loading levels are simulated for each speed and the operating point selected which provides a 14.4 kW electrical output from the machine, where electrical output is the sum of the stator power and rotor power.

Mechanical input power, stator output power, and rotor power are shown against shaft speed in Figure 4.10. Data is plotted so that power input to the generator is negative (mechanical shaft power) and power output from the generator is positive (stator electrical power). A maximum electrical output of 6.6 kW is selected so that rotor and stator currents stay below their 20 A rating.

![Figure 4.10: DFIG power flow against shaft speed for a constant electrical frequency and voltage](image)
Electrical output power is the sum of the stator and rotor powers. It can be seen that in delivering constant stator frequency, power is extracted from the rotor windings during supersynchronous operation and power is injected into the rotor windings during subsynchronous operation, therefore zero power would be expected from the rotor at synchronous speed. However this point occurs at 1,050 rpm rather than 1,000 rpm as a result of additional rotor power required to compensate for resistive losses, and is expected to be further still from 1,000 rpm in the real system which also includes mechanical losses. The injected rotor power must be met by additional generation from the stator side. Mechanical input power does not remain constant across the speed range for a constant output electrical power. Machine efficiency is lower at low speeds, and drops particularly below 800 rpm.

The DFIG system here is designed for standalone network regulation rather than maximum energy transfer, however this data shows that maximum power is available from the DFIG in supersynchronous conditions as the stator can be operated at full rated load while slip energy can also be recovered through the rotor.

Converter rating
The rotor converter rating is dictated primarily by speed range extension in the subsynchronous speed range. A rating of 3.5 kW (53% full load rating) is required to give a speed range of ±30%, this is greater than the 33% rating suggested by [93] for this speed range. Aero generators are required to provided full electrical power across their whole speed range and so the converter must be rated accordingly. Wind generation systems, however, are able to deliver less power at low speeds because of a reduction in shaft power available from the blades. This makes DFIGs especially well suited to wind generation systems as the rotor side converter does not need to be rated to achieve full output power at low speeds.

For a speed range of ±40% a converter rating approximately equal to full machine rating is required, this clearly negates the benefits of a DFIG system making the range too large for consideration on a practical system. Given that rotor side inverter rating requirements rise at a faster rate for subsynchronous speed extension than for supersynchronous speed extension, a non-symmetrical speed range would offer the
greatest benefits for the lowest converter rating. A converter rated at only 26% of system power is required to provide a speed range of -15% and +40%.

The greatest benefits can be achieved from the DFIG system if it is normally operated in a supersynchronous condition, where it is potentially more efficient, with a greater speed range extension above synchronous speed than below it.

Efficiency

Machine efficiency is defined as the ratio of useful electrical power output to the mechanical input shaft power and is calculated from the same data used to produce Figure 4.10, this is shown for a regulated stator voltage in Figure 4.11.

![Figure 4.11: DFIG efficiency against shaft speed for a constant electrical frequency and voltage](image)

The drop in efficiency at subsynchronous speeds occurs as a result of the power circulation from stator back to rotor adding to the losses. It should be remembered that the machine model neglects core loss which increases with frequency (stator) and slip (rotor) and therefore system level efficiency may be more constant and independent of speed. Also inverter efficiency, and possibly bearing damping (although this will be relatively low), must be included to determine the true system efficiency.
4.2.3.2 Mathematical derivation of rotor power

Rotor power flow is mathematically derived to provide validation for the simulated findings shown in Figure 4.10 and to identify converter rating requirements for the test rig.

Real power terms are derived from the steady-state DFIG model for both the stator and rotor sides.

\[
\text{Re}\left[\vec{V}_s \vec{I}_s^*\right] = |I_s| R_s + \omega_s L_m \text{Im}\left[\vec{I}_r \vec{I}_s^*\right]
\]

\[
P_{\text{stator}} = P_{\text{stator-couloss}} + P_{\text{gap}}
\]

\[
\text{Re}\left[\vec{V}_r \vec{I}_r^*\right] = |I_r| R_r - \omega_s L_m \text{Im}\left[\vec{I}_r \vec{I}_s^*\right] + \omega_s L_m \text{Im}\left[\vec{I}_r \vec{I}_s^*\right]
\]

\[
P_{\text{rotor}} = P_{\text{rotor-couloss}} - P_{\text{gap}} + P_{\text{mechanical}}
\]

Where machine torque is defined by (4.25).

\[
T_{em} = pp.L_m \text{Im}\left[\vec{I}_r \vec{I}_s^*\right]
\]

Hence stator power, rotor power, gap power and mechanical power are derived.

\[
P_{\text{stator}} = \text{Re}\left[\vec{V}_s \vec{I}_s^*\right]
\]

\[
P_{\text{rotor}} = \text{Re}\left[\vec{V}_r \vec{I}_r^*\right]
\]

\[
P_{\text{gap}} = \omega_s L_m \text{Im}\left[\vec{I}_r \vec{I}_s^*\right]
\]

\[
P_{\text{mechanical}} = \omega_s L_m \text{Im}\left[\vec{I}_r \vec{I}_s^*\right] = P_{\text{gap}} \frac{\omega_s}{\omega_s}
\]

Through rearrangement of (4.24), and neglecting copper loss in both the stator and rotor, a term for rotor power dependant on rotor speed is produced.
\[ P_{\text{rotor}} \approx P_{\text{gap}} \left( \frac{\omega_s - \omega_r}{\omega_s} \right) \]  \hspace{1cm} (4.30)

A more complete derivation is given in Appendix 4.G.

Equation (4.30) demonstrates that, when operating as a generator, during subsynchronous operation power is injected into the rotor and during supersynchronous operation power may be extracted from the stator as shown in Figure 4.12.

A Doubly-Fed machine is considered as a motor in [135] and as a generator in [91] and [140], the same relationship in power flow is described.

4.3 DFIG as an aero Generator

The potential benefits of a DFIG aero generator are highlighted, as well as detailed consideration of generation operating modes and integration with the engine.

4.3.1 Benefits of DFIG in aero applications

High bandwidth frequency and voltage regulation

DFIGs offer a benefit to an aero system of providing a regulated electrical voltage and frequency from a variable-speed mechanical drive without the need for complex mechanical regulation or a fully-rated electrical converter. Furthermore, the direct power connection to the rotor allows the rotor field to be controlled at a much higher bandwidth than for other generator types at a similar rating (assuming a machine with slip-rings). For example, a conventional aero synchronous generator uses rotating power electronics to feed the rotor windings and so the control must contend with the time constant of the exciter stage when influencing the rotor field. The increased dynamic
control of the rotor field makes the DFIG well suited to respond to high bandwidth torque fluctuations [91] as seen with electro-mechanical interaction.

Aircraft Systems Level Considerations
The benefits of introducing a DFIG generator system to an aircraft come at a cost; the DFIG has greater size and mass than an equally rated synchronous generator and requires additional, more complex, control. However these drawbacks are counteracted by the weight-saving benefits which result from a potential constant frequency electrical distribution system. VF distribution simplifies the generation system but requires many of the electrical systems on the airframe (such as constant-speed drives) to have individual power electronics to maintain their operation. The necessity for power electronics on multiple electrical loads adds weight throughout the airframe. For example electrically-driven fuel pumps throughout the airframe require 5 kg of power electronics to function on a VF supply and there maybe four or more of them on an aircraft. This weight is reduced if a DFIG system is introduced, providing a CF network. Rather than creating a full constant frequency system, a VF network can be created but with a limited frequency range. Weight saving on each of the loads will be lower, but transformer systems are sized for minimum frequencies so these can be reduced in size.

Increased speed range / reduced frequency range
There are three key modes of operation within an civil aircraft's mission profile:

- Climb, which includes takeoff, requires high (possibly maximum) thrust from the GTs
- Cruise, where the aircraft is operating for the majority of time, requires low levels of thrust at maximum efficiency
- Descent, requiring minimum levels of thrust (and noise) as the aircraft descends and lands

It can reasonably be assumed that the spool speed is proportional to thrust delivered by the GT, and therefore the generator drive speed varies according to thrust. The electrical frequency range of the airframe distribution is defined at 360-800 Hz by specification. Drivetrain systems are geared so that at top end spool speeds (climb) the electrical
frequency produced by a synchronous generator is within the electrical frequency limit, and this creates the situation where sufficient spool speed is always required so as not to drop below the minimum specified electrical frequency. As a result of this, during descent when little or no thrust is required for flight the GT may actually be producing significant thrust so as to provide sufficient spool speeds. This unnecessarily increases both fuel burn and noise during a substantial portion of the aircraft's mission profile. The spool mechanical speed range is constrained by the specified electrical frequency range.

The DFIG system is able to decouple electrical frequency from mechanical drive speed and so can be utilised in this circumstance to extend the mechanical speed range while still allowing the electrical frequency to meet the specification. The benefit of a DFIG over a synchronous generator with fully rated converter is that the converter need only be rated for slip power. On a VF system a DFIG can be used to increase the mechanical speed range, reducing unnecessary fuel burn during descent, while still providing an electrical network that is within the specified frequency range.

4.3.2 DFIG operation in an aero system

Two significant modes of operation can be considered which provide either a constant frequency or a frequency limited electrical network as shown in Figure 4.13.

![Figure 4.13: DFIG aero generator operation modes](image)

The constant frequency mode requires a rotor-side converter which is rated for the whole mechanical speed range. Whereas the frequency limiting mode can require a lower rated converter if electrical frequency limiting is only required around the upper
and/or lower limits of mechanical speed. The nominal operating point of the DFIG is dependent on the chosen machine synchronous speed, this in turn dictates the converter rating and whether a bidirectional or unidirectional converter may be used. Figure 4.13 shows a frequency limited system normally operating with VF proportional to shaft speed but CF above and below threshold speeds, the frequency / speed gradient does not need to be constant although it is in this example.

**Speed and frequency matching**

The GT operates for the majority of the mission profile at cruise speed, using maximum and minimum speeds for takeoff and descent (and emergency conditions). Figure 4.14 shows potential implementations of the constant frequency and frequency limiting modes in comparison to the IP spool speed of a three spool GT. For the constant frequency mode is shown with the DFIG synchronous speed set at cruise speed, there is somewhat of a mismatch between the IP spool speed range and that usually considered for DFIG constant frequency operation.

![Figure 4.14: Comparison of IP spool speed range and DFIG speed range](image)

The IP spool operates over a speed range of more than 2:1, which is wider than that normally considered for the DFIG at ±30% (DFIG ranges of 2:1 are considered [91], however it is not normal). As this range is increased the power rating of the rotor side converter must also be increased, eventually negating the benefits of the DFIG system. The frequency limited mode is again set with synchronous speed at cruise speed and
operates with a converter rated for ±30% operation, it covers the full mechanical speed range of the IP spool but does produce a variable frequency operation.

Two spool GTs
On a two spool GT such as the GEnx mechanical offtake, which supplies the generators, power is taken from the HP spool and has a speed range of closer 1.6:1 (or ±24% around a mid point). The narrower speed range makes HP offtake more suitable for DFIG generation as it is well within the speed range normally considered for constant frequency generation with room for mechanical speed range extension. It should be noted, however, that a two spool configuration offers lower efficiency than a three spool system especially during ascent and descent, and therefore it may not be feasibly to obtain performance benefits by increasing mechanical speed range.

Open rotor Gas Turbines
There has been a recent rise in interest in open rotor GTs as they offer a reduced fuel burn when compared to conventions ducted fan GTs [20] [141] [142]. Open rotor GTs have a narrower spool speed range (approximately 1.8:1) due to their ability to vary thrust by blade pitch control. This range fits within the considered DFIG speed range of ±30% and so would be well suited to DFIG constant frequency architecture.

Aero DFIG Operation Modes
Two generating modes have been presented for the DFIG system; many permutations exist for both of them to achieve optimum benefits for airframe design. The selection of DFIG synchronous speed dictates the nominal operating speed of the DFIG (affecting machine efficiency) and also the rating of the rotor-side converter. The range of frequency control required (from constant frequency to lightly limited frequency) dictates converter design in terms of both power rating and directional requirements.

4.3.3 Summary
Aero generator systems are required to supply the airframe electrical network using variable speed mechanical power from the Gas Turbine. A constant frequency electrical network is beneficial, because it reduces the weight of power electronic systems required for certain electrical loads. While a DFIG system does not offer direct weight
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savings, it is able to decouple mechanical drive speed and electrical frequency, without the need for mechanical speed regulation or fully-rated power electronics. This can be used to provide CF distribution or a frequency limited distribution on a variable speed system. Alternatively a DFIG can be used with a VF network to increase the mechanical speed range, reducing aircraft fuel burn at key operating points.

The design requirements are more complex for an aero DFIG than that of a wind turbine system (standalone wind turbines to exists but are not common) as it does not provide power to a rigid distribution network. Electrical distribution on a aircraft is provided by a single generator per bus and so the DFIG must be designed and controlled to operate as a standalone generator, this is done based on existing literature.

4.4 DFIG Control

A generator control scheme is necessary to meet the operational requirements of an aero DFIG. The control structure is selected based on existing strategies documented in literature and is tuned to meet the generator specifications. The designed controller is implemented on the modelled DFIG system and later deployed using real hardware with the laboratory DFIG.

4.4.1 Controller specification

The DFIG is to be used for stand alone, variable speed, operation. A control scheme is therefore required to deliver a constant voltage ($240 \, V_{LN}$) at a constant frequency (50 Hz) from the stator windings across a full mechanical speed range and independent of electrical loading. Controller bandwidth must be sufficient to maintain a well regulated voltage and electrical frequency during mechanical speed transients and beyond this so that it may be used to in the mitigation of electro-mechanical interaction. Given the frequency regulation at 50 Hz, any unbalance could lead to power oscillations at 100 Hz so a controller bandwidth of 1,000 Hz is required.

4.4.2 Control scheme selection

Stator-flux orientated FOC is selected using the general schemes described in [113] [111] [89], for simplicity indirect FOC is used, although a in service implementation of
this system could use sensor-less position estimation, as described in [108], so as to avoid the necessity of a position encoder.

### 4.4.3 Control scheme derivation

A controller is designed to regulate stator voltage magnitude and frequency using the rotor current. It is shown later that voltage magnitude is controlled by direct axis rotor current and voltage frequency maintained by quadrature axis rotor current. Control schemes are designed which are then tuned to achieve optimum performance.

#### 4.4.3.1 Field orientation / voltage frequency control

FOC maintains a constant, or at most a slowly varying, flux magnitude and angle. This is well suited to the generator application as a constant voltage and frequency are required from the stator side and changes in drive speed are relatively slow due to the dynamics of the GT (drivetrain vibrations have the potential to disrupt this, however). Stator flux orientation is selected so that the stator flux, and so stator current, is held at the defined synchronous frequency. This stator flux orientation is demonstrated in Figure 4.15.

![Figure 4.15: Stator flux orientation](image)

Where the angle, $\theta_s$, is determined by (4.31).

$$\theta_s = \int \omega_s \, dt$$  \hspace{1cm} (4.31)
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The machine stator flux terms are given in equations (4.8) and (4.9). To achieve the stator flux orientation shown in Figure 4.15 the stator flux quadrature axis component must remain zero therefore the direct axis value is equal to the total flux magnitude, the direct and quadrature components of stator flux are described by (4.32) and (4.33).

\[
\psi_{sd} = L_s i_{sd} + L_m i_{rd} = |\psi_s| \tag{4.32}
\]

\[
\psi_{sq} = L_s i_{sq} + L_m i_{rq} = 0 \tag{4.33}
\]

Where stator flux magnitude, \(|\psi_s|\), is approximately constant.

Hence the stator flux orientation is achieved by maintaining the rotor quadrature current depending on the quadrature stator current [113] [143] using the relationship given in (4.34).

\[
i_{rq} = -\frac{L_s}{L_m} i_{sq} \tag{4.34}
\]

Controller implementation

A simple, single stage, PI controller is used to maintain the required current, this provides the voltage set point for the inverter which drives a current in the DFIG. For the purpose of controller design, an ideal (gain \(\approx 1\)) Voltage Source Inverter (VSI) is assumed. The quadrature rotor current control system is shown in Figure 4.16.

![Flux orientation controller schematic](image)

**Figure 4.16: Flux orientation controller schematic**

**4.4.3.2 Voltage magnitude Control**

Stator voltage regulation is required to be accurate for the full load and mechanical speed ranges. As a lab demonstrator the control scheme is designed to deliver 240 V_{LN}
in line with UK mains voltages. The rotor currents are controlled to influence the stator currents and therefore voltages.

Stator voltage magnitude is defined as shown in (4.35), [91] [113].

\[
|v_s| = \sqrt{v_{sq}^2 + v_{sd}^2} \tag{4.35}
\]

The two contributing terms, \(v_{sq}\) and \(v_{sd}\), must be controlled to regulate stator voltage magnitude. These two terms are defined by equations (4.36) and (4.37), given in a synchronous reference frame.

\[
v_{sd} = R_{sd}i_{sd} + \frac{d\psi_{sd}}{dt} - \omega_s\psi_{sq} \tag{4.36}
\]

\[
v_{sq} = R_{sq}i_{sq} + \frac{d\psi_{sq}}{dt} + \omega_s\psi_{sd} \tag{4.37}
\]

Given that \(\psi_{sq} = 0\), and hence \(\frac{d\psi_{sq}}{dt} = 0\), as a result of the flux orientation described previously, and that \(\frac{d\psi_{sd}}{dt}\) is small, the equations are reduced and given in (4.38) and (4.39).

\[
v_{sd} = R_{sd}i_{sd} \tag{4.38}
\]

\[
v_{sq} = R_{sq}i_{sq} + \omega_s\psi_{sd} \tag{4.39}
\]

Combining with the stator flux equation, (4.8), gives equations (4.40) and (4.41)

\[
v_{sd} = R_{sd}i_{sd} \tag{4.40}
\]

\[
v_{sq} = R_{sq}i_{sq} + \omega_sL_{sd}i_{sd} + \omega_sL_{md}i_{md} \tag{4.41}
\]
It can be seen that the values of $v_{sq}$ and $v_{sd}$, and hence $|v_s|$, are dependent on $i_{sd}$, $i_{sq}$ and $i_{rd}$. As $i_{sq}$ and $i_{rd}$ are load dependant they are considered as disturbances and removed at this stage which allows the equations to be simplified

$$v_{sd} \approx 0 \quad (4.42)$$

$$v_{sq} \approx \omega_s L_m i_{rd} \quad (4.43)$$

Equations (4.42) and (4.43) are combined with (4.35) to produce (4.44), [101], which shows that stator voltage can be controlled by regulation of direct axis rotor current.

$$|v_s| \approx \omega_s L_m i_{rd} \quad (4.44)$$

Given the disturbances included within this relationship it is necessary to have a two stage control system for voltage regulation to improve disturbance rejection and stability. A high speed inner control loop is used to precisely control the direct axis rotor current, with an outer, slower speed, control loop used to regulate the voltage. This method is also preferred in [111].

**Controller implementation**

A two stage control system is used to maintain the required stator voltage, this provides the voltage set point for the inverter which drives a current in the DFIG. For the purpose of controller design an ideal (gain = 1) VSI is assumed. The PI controller for this current control system is tuned to give a high bandwidth tracking for the reference value. The reference current is provided by a PI controller which is fed the voltage error. This outer voltage control loop is designed to have a lower bandwidth than the inner current controller to enable stability.

The direct rotor current control system, including inner loop current control and outer loop voltage control, is shown in Figure 4.17.
4.4.3.3 Control Loop Tuning of Field orientation and Voltage control systems

Both the flux orientation control and voltage control systems described require precise current control. In order to tune the current controllers a transfer function of the complete system is defined.

It is again assumed that the inverter behaves ideally, being able to hold to the voltage exactly at the machine voltage at the reference. This approximation is acceptable assuming the inverter switching frequency and power rating are sufficient. The inverter gain is therefore assumed to be 1. However a saturation limit is included within the simulation to represent the maximum voltage which may be applied to DFIG windings. Inverter arrangements and control scheme are not considered at this stage but will be designed for the laboratory test rig.

The response of the DFIG winding current, in respect to a change voltage, must be defined (for both direct and quadrature axes). This provides the transfer function for the DFIG in the current control system.

Direct Rotor Current Transfer Function

The direct axis current control system is shown in Figure 4.18.

The direct component of rotor voltage is given by equation (4.45) (in stator flux orientation).
\[ v_{rd} = R_i i_{rd} + \frac{L_m}{L_s} \frac{d\psi_{ad}}{dt} + \left( L_d - \frac{L_m^2}{L_s} \right) \frac{di_{rd}}{dt} - (\omega_s - \omega_r) \left( L_d - \frac{L_m^2}{L_s} \right) i_{eq} \] (4.45)

Considering the impact of \( i_{rd} \) on \( v_{rd} \), assuming a constant stator flux and neglecting the cross coupling term (the impact of \( i_{eq} \) on \( v_{rd} \)) the relationship is simplified.

\[ v_{rd} = R_i i_{rd} + \left( L_d - \frac{L_m^2}{L_s} \right) \frac{di_{rd}}{dt} \] (4.46)

(4.46) provides a linear relationship between \( i_{rd} \) and \( v_{rd} \) which simplifies the control scheme design. The decoupling terms are reintroduced later to the output of the controller.

Performing a Laplace transform on this relationship produces an s-domain transfer function which can then be included in the controller system as shown in Figure 4.19.

\[ \begin{align*}
\dot{i}_{rd} & = k_p + k_i \frac{i_{rd}}{s} \\
 v_{rd} & = R_i i_{rd} + s \left( L_d - \frac{L_m^2}{L_s} \right) i_{rd} \\
\end{align*} \]

\[ v_{rd}' = R_i i_{rd} + s \left( L_d - \frac{L_m^2}{L_s} \right) i_{rd} \]

Figure 4.19: Direct axis current controller with DFIG transfer function schematic

By combining the controller and machines transfer functions with the feedback (as shown in Figure 4.19) the transfer function of the whole system can be defined as given in (4.47).

\[ \begin{align*}
\frac{i_{rd}}{i_{rd}'} & = \frac{L_s \left( k_p s + k_i \right)}{L_s L_d - L_m^2} \\
\frac{i_{rd}'}{s^2} & = \frac{L_s \left( k_p s + k_i \right)}{L_s L_d - L_m^2} + \frac{L_s k_i}{L_s L_d - L_m^2} \\
\end{align*} \] (4.47)
Controller Tuning

To simplify controller design, the system is compared to the standard second-order transfer function response given in [144] and [145], (4.48). This is achieved by ignoring the system zero, the assumption is valid given that the zero is sufficiently far away from the system poles on a pole-zero plot.

\[ Y(s) = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \quad (4.48) \]

Where:

\[ \omega_n = \text{Natural frequency (rad.s}^{-1}), \quad \zeta = \text{Damping ratio} \]

By equating the system transfer function with the standard second-order transfer function, the damping ratio and bandwidth can be described by (4.49) and (4.50).

\[ \frac{L_s (R_s + k_p)}{L_s L_r - L_m^2} = 2\zeta \omega_n \quad (4.49) \]

\[ \frac{L_s k_i}{L_s L_r - L_m^2} = \omega_n^2 \quad (4.50) \]

The current controller is required to have the fastest settling time possible and to have a bandwidth at least an order of magnitude higher than mechanical resonances present within the wider system. Critical damping (\( \zeta = 0.7 \)) is selected as this offers the best trade off between settling time and overshoot for a second order system. The possibility of electrical unbalance introduces the possibility of torque oscillations at the second harmonic with respect to electrical frequency. Taking the electrical frequency as 50 Hz the bandwidth of the system should be designed for 1,000 Hz.

Designing the current controller with these parameters leads to the following proportional and integral controller gains:

\[ k_i = 292480 \]

\[ k_p = 65 \]
This high value of integral gain leads to saturation and instability. It is proven to be unfeasible to achieve the high bandwidth required with critical damping whilst still maintaining a stable system. Therefore it is opted to reduce the integral gain to the point of stability (by a factor of 10).

\[ k_i = 29248 \]
\[ k_p = 65 \]

For comparison, the step response of the implemented control scheme and controller models is shown in Figure 4.20. Real system is the current control system implemented with the DFIG model, full model is the controller transfer function including the system zero (4.47), and second order model is the second order approximation of the controller transfer function given by (4.49) and (4.50).

![Figure 4.20: Tuned direct axis current controller, step response](image)

Analysis of the real system step response shows a damping ratio of 2 and a bandwidth of 492 Hz. Although this bandwidth is not as high as initially desired it is high enough to avoid interaction with mechanical frequencies. The damping ratio is greater than the desired 0.7, high bandwidth is taken in preference over optimal damping ratio (and so settling time).
Quadrature Rotor Current Transfer Function

Generated frequency is determined by field frequency, as a synchronous reference frame is used the reference angle is used to provide frequency control. The quadrature current controller is tuned in an identical fashion to the direct current controller tuning detailed previously. The quadrature component of rotor voltage is given by equation (4.51) (in stator flux orientation).

\[ v_{rq} = R_r i_{rq} + \left( \frac{L_r}{L_s} \right) \frac{di_{rq}}{dt} + (\omega_s - \omega_r) \frac{L_{sm}}{L_s} \psi_{sd} + (\omega_s - \omega_r) \left( \frac{L_r}{L_s} \right)^2 i_{sd} \]  

(4.51)

Considering the impact of \( i_{rq} \) on \( v_{rq} \), and neglecting the cross coupling term (impact of \( i_{rd} \) on \( v_{rq} \)) and the stator flux orientation term the relationship is simplified.

\[ v_{rq} = R_r i_{rq} + \left( \frac{L_r}{L_s} \right)^2 \frac{di_{rq}}{dt} \]  

(4.52)

The dynamics of this relationship are identical to that for the direct current system, with a difference in coupling terms. Therefore the controller design is as detailed in the direct current tuning section. Identical controller parameters and values are used.

4.4.4 Control scheme implementation

The current controllers are combined with the outer control loops for voltage and frequency regulation in a form which can be implemented in Simulink.

4.4.4.1 Combined rotor current controller

Current Decoupling equations

Cross coupling terms were removed to simplify controller design, they are now reintroduced into the controller system to contribute to accurate performance. This is most necessary at extreme operational ranges such as high and low generator shaft speeds.

The following decoupling terms must be reintroduced:
\begin{align*}
v_{rd} & := -(\omega_s - \omega_r) \left( L_r - \frac{L_m^2}{L_s} \right) i_{rq} \\
v_{rq} & := + (\omega_s - \omega_r) \left( L_r - \frac{L_m^2}{L_s} \right) i_{rd} \\
\end{align*}

The following feed forward term is reintroduced to help eliminate tracking errors:

\begin{equation}
  v_{rq} := + (\omega_s - \omega_r) \frac{L_m}{L_{stat}} \psi_{rd}
\end{equation}

The tuned current controller systems for both the direct and quadrature axis are combined to produce the overall control scheme shown in Figure 4.21.
Stator flux is approximated, as given in [107] and [146], using (4.53).

\[
\psi_{sd} \approx \frac{v_{sq}}{\omega_r}
\]  

(4.53)

### 4.4.4.2 Voltage magnitude Controller

The voltage controller forms the outer loop of a two stage controller acting on the direct axis rotor current.

Voltage magnitude relationship to direct rotor current has been derived previously and is given in (4.44). Steady-state currents within the machine are assumed. This approximation is valid for a current control loop with a significantly higher bandwidth than the outer, voltage, controller.
Voltage decoupling equations
The following decoupling terms are not reintroduced, and this may contribute to voltage regulation instability:

\[ v_{sd} = R_s i_{sd} \]
\[ v_{sq} = R_s i_{sq} + \omega_L L_s i_{sd} \]

A similar strategy to controller design can be followed to that used when designing the current control system. However this requires a derivation of a transfer function for voltage magnitude with respect to direct axis rotor current. This is complex to derive, given that a high bandwidth is not required the voltage control system is tuned manually according to Ziegler-Nichols PI tuning rules leading to the following parameters for the voltage regulation controller:

\[ k_i = 10 \]
\[ k_p = 0.1 \]

4.4.4.3 FOC model implementation
The DFIG FOC scheme with combined current and voltage control is implemented in Simulink as shown in Figure 4.22.
Figure 4.22: FOC scheme implementation in Simulink (screen shot)

4.4.4.4 Controller Response

With the FOC scheme implemented on the machine model simulation is used to assess stator voltage regulation during load variation and speed variation.

Figure 4.23 shows the stator voltage (rms) and frequency during an electrical load step from 0.1 kW to 14.4 kW at both maximum and minimum drive speeds. Figure 4.24 shows the stator voltage (rms) and frequency during a drive speed step from 600 rpm to 1,400 rpm for loads of both 0.1 kW and 14.4 kW. Due to mechanical inertia a speed step of this kind will never be realised in a practical system, however it is used as an extreme case to appraise the control system. Although the DFIG is described as having a rating of 14.4 kW, simulation has shown that a maximum power of 6.6 kW is achievable for constant frequency operation when rotor power is supplied from the stator. As the rotor and stator are not interconnected in the practical realisation of the DFIG system the full rating of 14.4 kW is considered here to ensure thorough verification of the controller.
The voltage and frequency are seen to be held at 240 V\textsubscript{rms} and 50 Hz well with both recovering rapidly after electrical load steps and speed steps. It is noted that the worst case fluctuations occur with light electrical loading during the speed step and at high speeds with the electrical load step.

### 4.5 Integration within Electro-Mechanical Model

The DFIG model is integrated with the lump parameter drivetrain model, described in Chapter 3, by coupling the DFIG model drive speed to the speed of the DFIG inertia in
the mechanical model and by applying the DFIG model electro-magnetic torque on the same inertia.

4.5.1 Electrically induced drivetrain vibration

The lumped parameter model has already been shown to have 1st and 2nd mode resonances at 26.6 Hz and 37.2 Hz respectively. The combined electro-mechanical model is now tested to ensure that electrical loading on the generator is capable of exciting these resonances as has been observed in the real GT data.

Figure 4.25 shows the response of the combined electro-mechanical model when stator power is stepped from 0.1 kW to 14.4 kW at time t=1 s for a drive speed of 700 rpm.

As the increased electrical load is applied speed drops as energy is extracted. The basic GT emulating controller increases torque to return the drivetrain to its original speed.
after approximately 8 seconds. A real FADEC may not return the spool speed to an identical level. The high bandwidth of the DFIG controller ensures that the electrical power step is transferred to the mechanical system as a fast torque step which triggers oscillations which can be seen in both the DFIG shaft torque and DFIG speed.

Figure 4.26 shows the frequency content of the DFIG speed for the second after the electrical load step.

Note: frequency content is derived using a post processing function to provide the power spectrum, where appropriate traces are scaled to give detail of the most significant harmonics within the frequency range. Therefore, individual frequency components can be compared in each plot but not across plots.

A large low frequency component is visible due to the drive speed. Resonances are noted peaking at 27.2 Hz and 37.2 Hz.

Figure 4.27 shows the frequency content of the DFIG shaft and the transmission shaft for the second after the electrical load step.
Frequencies peaking at 27 Hz and 37 Hz can be seen in the DFIG shaft and a single frequency at 27 Hz can be seen in the transmission shaft. The 1st mode resonances are marginally higher that those identified in the isolated drivetrain model but are close enough for the difference to be put down to derivation methodology.

4.5.2 Electrical regulation during drivetrain vibration

Drivetrain vibrations lead to generator shaft speed oscillations which can potentially result in instability in the electrical network.

Figure 4.28 shows the stator voltage and frequency regulation before and during an electrical load step from 0.1 kW to 14.4 kW at time t=1 s for a drive speed of 700 rpm.
Stator voltage and frequency are tightly regulated with only a momentary fluctuation in voltage and frequency as a result of the load step. Oscillations in stator frequency can be seen at a very low level for approximately 2 seconds after the step, these occur at the mechanical resonances of 27 Hz and 37 Hz. Despite the drive speed oscillations the control scheme is robust enough to ensure tight voltage and frequency regulation during the electrical load step.

4.5.3 Pulsed electrical loading
To demonstrate the effect of non-continuous electrical loading, possibly from a radar type system, on the electro-mechanical system a pulsed electrical load is applied to the DFIG.
With the DFIG at 700 rpm a base load of approximately 0.1 pu is applied, at time t=30 s a further load of 0.2 pu is activated which pulsates at a frequency of approximately 5 Hz. The electrical loading and DFIG shaft torque are shown in Figure 4.29.

![Figure 4.29: Combined electro-mechanical model pulsed electrical load, (a), and mechanical response, (b)](image)

After pulsed load activation the DFIG shaft torque is seen to grow in amplitude for approximately 6 seconds, and rises to in excess of 200% rating (where rating corresponds to the 1.0 pu electrical load torque). Electrical load frequency is set at one fifth the 1st torsional resonant frequency of the mechanical system, and so the relatively low level electrical load results high amplitude torque. The torque amplitude, and reversal, is likely to introduce considerable fatigue to mechanical components.

These results show that, when the DFIG model is combined with a drivetrain model, simulated electrical loading mechanical resonances, successfully replicating the electromechanical interaction observed in real GT systems. The DFIG control scheme is able to reject mechanical disturbances to produces a well regulated and stable electrical network. The pulsed electrical loading demonstrates that electrical loading, apparently well within rating, is able to create damaging torque levels in the drivetrain if frequencies (or harmonics of) are close to mechanical resonances.
4.6 Summary and conclusions

A modelling strategy has been identified for a Doubly-Fed machine and a 14.4 kW lab machine characterised though a range of electrical and mechanical tests. This machine model is implemented in Simulink and has received some validation against the lab machine. This model has been used to understand the performance and behaviour of a DFIG and identify the rotor side converter rating (8.7 kW) necessary to maintain a constant electrical frequency for a speed range of ±30% around synchronous, and consideration is also extended to ±40%. Several modes of operation are identified for an aero DFIG, including constant frequency generation, these are compared with the mechanical speed range of a typical 3-spool GT and the benefits of such a system are considered at an aircraft system level. A FOC scheme is implemented, based on existing literature, and is tuned to achieve high bandwidth voltage and frequency regulation for the DFIG in a standalone condition and over a drive speed of ±40%. The DFIG model is integrated with a drivetrain model and used to simulate electro-mechanical interaction whereby electrical loading triggers drivetrain vibration.
Chapter 5  Design and Construction of an Electro-Mechanical Test Platform

This chapter details the design and construction of an electro-mechanical test platform to be used in the validation of simulated drivetrain and DFIG and to further the understanding of electro-mechanical interaction. The test platform is designed to replicate the behaviour of an aero electro-mechanical network, this architecture typically consists of a fan case mounted accessory gearbox, driven from a variable speed GT spool, delivering power to an accessory load which includes two identical generators. The electrical generators are controlled to provide stand alone voltage regulation, independent of load variation during the aircraft mission.

The mechanical design of the test rig is based on the reduced order mechanical model proposed in Chapter 3, Electro-mechanical conversion is carried out by a DFIG, and control scheme, described and modelled in Chapter 4.

The processes of design and construction for the test platform can be considered in three steps. Initially the functional design of the test platform is specified, primarily this details a system which behaves in a similar manner to an aero GT. Physical properties are then detailed for each component and designs drawn up so as to realise the desired functional design. Finally, consideration is also given to ensure that the physical designs are achievable with the available manufacturing capabilities and that assembly is practical, this step is carried out partly in parallel with the physical design of components.

5.1 Test platform overview description

The test platform consists of a mechanical rig, a controlled DFIG, and electrical loading from a resistive load bank. The mechanical rig represents a GT drivetrain with a DC drive and flywheel providing spool emulation and a gearbox driving the DFIG, a generator flywheel which is identically sized to the DFIG, and a fuel pump flywheel. An overview of the test platform is shown in Figure 5.1.
The DC machine prime mover is controlled to replicate the variable speed operation of a GT spool. FOC is implemented onto the DFIG via a rotor side converter. A data acquisition system records mechanical and electrical conditions throughout the rig.

### 5.1.1 Mechanical rig

The mechanical test rig appears as a close representation of a 3-spool GT drivetrain. GT spool mass and behaviour is provided by a prime mover (DC drive machine with commercial controller) with additional inertia from a flywheel. The compliance of the transmission system is achieved by a driveshaft which provides power to a gearbox, stepping up the drive speed to two, identically size and coupled, electrical generators. A third load, representing a fuel pump, is also driven from the gearbox. The drivetrain model on which the mechanical rig is based is shown in Figure 5.2, this is identical to the reduced order drivetrain model shown in Figure 3.44 but with additional idler gears which are required to provide spacing between the two generators and also increase its representation of a real AGB.
Figure 5.2: Mechanical rig schematic

5.1.2 Electrical generator

A DFIG is controlled to produce a regulated voltage and frequency independent of both electrical loading and mechanical drivespeed. The control scheme is designed and developed for the DFIG in question in Chapter 4.

The scheme controls current in the DFIG rotor winding to achieve stator voltage generation. Control is implemented in Simulink with dSPACE used to produce PWM inverter switching signals to drive a commercial VSI fed from a rectified DC link. The converter is unidirectional so a brake resistor is used during supersynchronous conditions.

The control scheme regulates stator side voltage and frequency for standalone operation with power being provided by a mains connection. On a real aero implementation a bi-
directional converter would be used to link rotor side power to stator side, however this additional complexity isn't required to assess the proposed DFIG control scheme and investigate electro-mechanical interaction.

5.1.3 Electrical loading
Electrical loading is provided by a resistive load bank. Non-unity power factor loads are beyond the scope of this research.

5.1.4 Control and Data Acquisition
As well as controlling the DFIG rotor side converter, the same Simulink and dSPACE system also controls the prime mover through a commercial motor drive unit. Data is recorded from position encoders, torque sensors, voltage sensors and current sensors throughout the test rig using a PC LabVIEW DAQ system, analysis is carried out using post-processing routines in MATLAB. Where necessary sensor signals are shared between the control system and DAQ system.

5.2 System Functional Design
It is desirable for the test platform to exhibit the same behaviour seen in an aero GT, however the lab environment and available equipment place some limitations on this. An existing DFIG is used, which is not optimised for low weight so it has a higher inertia than a typical aero generator, despite its lower power rating. It is also undesirable, from a safety point of view, to replicate the high mechanical speeds seen in an GT drivetrain. For these reasons the platform is scaled with respect to frequency, exhibiting resonances at similar frequencies to a typical 3-spool GT drivetrain, with mechanical parameters being designed using the DFIGs properties as an initial reference.

5.2.1 DFIG system
The control scheme ensures DFIG stator side electrical frequency is regulated at 50 Hz, in keeping with UK mains frequency. The 3 pole pair machine construction puts the mechanical synchronous speed at 1000 rpm. DFIG voltage and frequency regulation is controlled over a drive speed ±40% which requires a converter rating of 1.8 kW in supersynchronous operation and 5.4 kW in subsynchronous operation kW (as
demonstrated in Chapter 4), to give ample power capabilities a 14.0 kW rotor side converter is used.

5.2.2 Mechanical rig

Simulation shows that transient peak torque from the DFIG is 240 Nm and so the generator shaft and couplings must achieve this rating. With a generator drive speed ratio of 1.5 to the gearbox input the transmission torque rating is 390 Nm (simulated, transient). This also gives a prime mover drive speed range of 400 rpm to 933 rpm. The DFIG inertia has been determined as 0.411 kg.m$^2$, or 0.925 kg.m$^2$ when referred through the gearbox. This is larger than that of a real aero generator so it is impractical to achieve the same scaling seen in a real GT system, where spool inertia is approximately 60 times greater than referred generator inertia. The combined inertias of the GT flywheel and prime mover should be an order of magnitude greater than the DFIG so that it may be considered a ground point, ensuring the fixed-free torsional representation is accurate. The flywheel designs will be considered in later and a GT flywheel of 7 kg.m$^2$ is selected which is in part limited by manufacturing capabilities and safety considerations.

5.2.3 Torsional modes

The test rig is designed to exhibit the two modes of torsional resonance discussed in Chapter 3. The 1st mode occurs with the two generators acting in-phase and the second with the generators acting out-of-phase. Effects introduced by the additional, fuel pump load are neglected for the design of resonance modes, however its impact on said modes will later be considered. It is desirable for the resonance modes to be within the same range as a typical 3-spool GT system, and have a similar frequency relationship with respect to each other and also to have modes which are easily detectable and separate from other frequencies appearing throughout the rig.

Higher frequencies can be achieved with physically smaller mechanical systems, however, resonant frequencies must appear very separately to the other frequencies existing in the rig, for example the fundamental electrical frequency at 50 Hz, so that they can be readily isolated.
Resonant modes are selected at 20 Hz and 30 Hz for the 1st and 2nd modes respectively. These frequencies are not harmonics of each other and are separate from other frequencies expected in the rig.

5.2.4 Sensors

Data from both the mechanical and electrical domains is required for analysis. Angular position encoders provide data which can be used to derive speed (and speed fluctuations), providing mechanical vibration information, in the same way position information is used for analysis of the GT drivetrain oscillations (see Chapter 3). Unlike the real GT system, torque sensors can be mounted throughout the drivetrain to increase the observation of mechanical oscillations. Similarly electrical data is currently not available from real GT system, the test platform gives access to DFIG rotor and stator side voltages and currents.

Sensor signals are shared between both the control system and DAQ system. Sensor rating and bandwidth are considered for the four sensor types to ensure that accurate data collected from the test is both accurate and useful, full sensor detail is provided later.

5.2.5 Functional design specification summary

Test platform functional specifications are summarised in Table 5.1.

<table>
<thead>
<tr>
<th>Property</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>DFIG mechanical drive speed</td>
<td>1000 rpm±40%</td>
</tr>
<tr>
<td>DFIG rotor side converter rating</td>
<td>14.0 kW</td>
</tr>
<tr>
<td>Electrical generation</td>
<td>240 V rms, 50 Hz</td>
</tr>
<tr>
<td>Electrical load rating</td>
<td>14.4 kW</td>
</tr>
<tr>
<td>Electrical thermal current rating</td>
<td>20 A rms</td>
</tr>
<tr>
<td>DFIG torque rating</td>
<td>240 Nm</td>
</tr>
<tr>
<td>Transmission torque rating</td>
<td>390 Nm</td>
</tr>
<tr>
<td>DFIG drive speed ratio</td>
<td>1.5</td>
</tr>
<tr>
<td>Fuel pump drive speed ratio</td>
<td>0.9 and 2.0</td>
</tr>
<tr>
<td>Prime mover drive speed</td>
<td>400 rpm to 933 rpm</td>
</tr>
<tr>
<td>Representative GT spool inertia</td>
<td>7 kg.m²</td>
</tr>
</tbody>
</table>
5.3 Component physical design

Individual components of the test rig are designed so that they combine to meet the test platform functional design specifications.

5.3.1 Gearbox

The gearbox is designed to have a similar parallel axis structure to the AGB. On the typical 3-spool GT system the drive speed ratio of the electrical generators is 1.9 from the input bevel, the drive speed ratio for the fuel pump is 0.8 from the input bevel.

5.3.1.1 Specification

A drive speed ratio of 1.5 is selected for the test platform electrical generators. The drive speed ratio of the test rig fuel pump is also based on that of the real system but is variable (by way of exchanging gears) from 0.9 to 2.0.

The gearbox is designed based on consideration of load rating, physical sizing, meshing frequency and backlash. It is required to be capable of transmitting 15 kW (electrical rating plus safety factor) at a DFIG drivespeed between 500 rpm and 1,500 rpm. Simulation shows peak transient torques of 395 Nm and 240 Nm for the transmission input and DFIG output respectively. Gear diameters must allow room for the DFIG and flywheels alongside each other. The gearbox must have minimal influence on designed torsional resonances by ensuring its total referred inertia is an order of magnitude less than that of the DFIG (at 0.411 kg.m²). Gear mesh frequency should be an order of magnitude higher than designed 1st and 2nd mode torsional resonances (at 20 Hz and 30 Hz). As a non-linearity, backlash must be minimised.

Gearbox design tradeoffs

Matching the load rating is essential as a design start point, and is used to dictate the gear teeth strength, and hence teeth size, required. Larger gear teeth, however, reduce gear mesh frequency (compared to an identical circumference gear) and increase backlash which are both undesirable. The physical spacing between gear centre points must be large enough to ensure the DFIG and flywheel fit side by side, bigger gears increase this spacing and provide a greater number of teeth giving a higher gear mesh
frequency, but this does have the drawback of increasing gearbox inertia. All these considerations must be balanced and prioritised in the design of the test rig gearbox.

### 5.3.1.2 Design
A full range of gearbox configurations and ratios are assessed against the design requirements. A gear train consisting of 7 parallel axis gears is selected, idler gears are included to ensure spacing between input and output shafts (D2 and D1) while reducing gear diameter, and hence gearbox inertia. This gearbox design is shown in Figure 5.3.

Power is input to the gearbox through gear 5, where idler gears 6 and 4 exist to provide a 550 mm spacing, D1. The fuel pump is driven either directly by gear 1b or by gear 1 and gear 2 this provides the two different configurations to achieve drive speed ratios of 0.9 and 2.0. Spacing between the fuel pump output shaft and input shaft is 410 mm in both gear configurations.

**Parallel axis gear teeth configurations**
There are several common teeth configurations for parallel axis gears, these are shown in Figure 5.4.
Herring bone and double helical gears have the advantage of producing zero net axial thrust, however assembly is more challenging as they must be slotted together radially. Staggered double helical gears pose the same assembly challenges but they produce lower levels of backlash due to the gear tooth offset. Helical gears have been chosen as they can transmit higher loads and have a quieter, smoother action in comparison to similar straight cut spur gears, although this does come at a cost in terms of a slight efficiency reduction [129]. The low levels of axial thrust produced by the helical cut are tolerated as construction is simplified.

The angle of the helical cut is referred to as the pressure angle. Pressure angles are commonly in the region of 20°, however a lower pressure angle increases contact and so reduces vibration [129]. For this reason a low pressure angle of 17.75° is used for the rig gearbox.

The selection of helical cut gears allows a smaller gear tooth size to be used for the same power transmission and increases the teeth number for the same gear diameter so increasing mesh frequency. The input gear has 48 teeth providing a mesh frequency of between 320 Hz and 747 Hz across the rig speed range. Gear teeth numbers are shown in Table 5.2.

![Figure 5.4: Parallel axis gears, teeth configurations - top view (data from [129])](image)

<table>
<thead>
<tr>
<th>Gear</th>
<th>1</th>
<th>1b</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teeth number</td>
<td>24</td>
<td>53</td>
<td>24</td>
<td>32</td>
<td>48</td>
<td>48</td>
<td>48</td>
<td>32</td>
</tr>
</tbody>
</table>
Gearbox inertia

Gear manufacturers data is used to estimate total referred (to input side) gearbox inertia which is found to be 0.052 kg.m$^2$; suitably lower than referred DFIG inertia, 0.929 kg.m$^2$.

5.3.1.3 Manufacture

Standard 3 MOD helical gears, to the specifications detailed above, are supplied by HPC Gears. The gears are manufactured from BS 970: 214M15 steel and case hardened to ensure that minimal gear mass is required for sufficient power. The parallel axis gearbox arrangement, along with gear teeth numbers, is shown in Figure 5.5.

![Figure 5.5: Test rig gearbox arrangement side view - CAD image](image)

The configuration allows the fuel flywheel to be driven at either 2.0 input speed through the 24 teeth gears or 0.9 input speed through the single 53 tooth gear. This reconfiguration can be achieved without altering the alignment of the fuel pump flywheel.

Gear alignment

Careful gear alignment is necessary to ensure that the required radial distance between gears is achieved. If the gear separation is too large the gear teeth will not fully engage, which will cause increased vibration and wear to the gears [120]. Should the separation be too low the gear teeth will jam increasing system damping and potentially leading to gear tooth failures [120]. High tolerances are therefore required for precision manufacture. Gear alignment calculations are given in more detail in Appendix 5.A.
The gearbox side plates are milled out by a digital milling machine with a repeatability of ±0.0001 mm. The clearance between each set must therefore be set at twice this distance to ensure that at maximum position inaccuracy the clearance remains non-negative.

**Encasement**

The parallel axis gear train is mounted in a casing of 20 mm thick aluminium plate to minimise lateral vibration. Press fitted bearings are used to support gear shafts with power transmission provided with keyways.

### 5.3.2 Flywheels

The test platform requires two flywheels, one at 7 kg.m² representing the GT spool inertia, and the other at 0.411 kg.m² matching the DFIG inertia and acting as a second electrical generator.

#### 5.3.2.1 Design

A common design is used for both flywheels, the GT spool flywheel design is shown in Figure 5.6.
The flywheels are assembled from two components, a hub and a disk. The hub provides torque transmission through keyed shafts and supports the mass and rotary forces of the flywheel. One or more disks are mounted on the hub to provide the correct levels of inertia. The disk to hub fit is sufficient to ensure that torque is only transmitted through the securing bolts which are threaded on the hub side.

Inertia calculation
Flywheel disc dimensions are designed to meet the inertia requirements using equations (5.1) and (5.2).

\[
J = \frac{1}{2} M r^2 \tag{5.1}
\]

\[
M = L \rho \pi r^2 \tag{5.2}
\]

Where:

\[ r = \text{radius (m)}, \ L = \text{length (m)} \]

Steel is used for both the flywheel disc and hub, with \( \rho = 7850 \text{ kg.m}^3 \) [118].
Relationships (5.1) and (5.2) show that the highest inertia, with lowest material mass, is achieved by using the largest possible radius, while sufficient length is required to ensure disk strength. The radius itself is limited by machining capabilities and physical spacing within the test rig. A range of flywheel dimensions were considered before selecting those given in Table 5.3. Flywheel hub inertia is neglected and the disk is assumed to be complete as the central hole is filled by the hub and the protruding shafts have a very low diameter.

<table>
<thead>
<tr>
<th>Flywheel</th>
<th>Inertia (kg.m²)</th>
<th>Mass (kg)</th>
<th>Radius (m)</th>
<th>Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GT flywheel</td>
<td>7.000</td>
<td>117.6</td>
<td>0.345</td>
<td>0.040</td>
</tr>
<tr>
<td>Generator flywheel</td>
<td>0.411</td>
<td>42.6</td>
<td>0.160</td>
<td>0.051</td>
</tr>
</tbody>
</table>

The generator flywheel disc is manufactured as three thinner discs of radius 160 mm and diameter of 22.5 mm. This produced a total inertia which is slightly higher than intended but which was later machined down to match the inertia of the DFIG based on a hammer resonance test. After testing the generator flywheel mass was reduced to 32.1 kg to achieve the correct inertia.

5.3.2.2 Safety
Kinetic energy, material stress, and vibration are considered to ensure the flywheel designs are within safe bounds, details of these calculations are given in Appendix 5.B. Resonances in both the GT flywheel and generator flywheel are summarised in Table 5.4.

<table>
<thead>
<tr>
<th>Flywheel</th>
<th>Torsional vibration frequency (Hz)</th>
<th>Lateral vibration frequency (Hz)</th>
<th>Shaft whirl half critical speed (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GT flywheel</td>
<td>37.5</td>
<td>230.3</td>
<td>115.2</td>
</tr>
<tr>
<td>Generator flywheel</td>
<td>95.7</td>
<td>467.1</td>
<td>233.6</td>
</tr>
</tbody>
</table>

5.3.3 Driveshafts
The rig uses three main driveshafts to couple the prime mover, flywheels, gearbox and load machine. The transmission shaft connects the GT flywheel to the gearbox, while
two identical generator shafts connect the gearbox to the DFIG and generator flywheel. Driveshaft compliance combines with system inertias to dictate torsional system modes, and must therefore be designed to achieve the required system modes. The driveshafts must also be rated for the peak torque loading, with a sufficient safety factor to survive cyclic fatigue failure due to load oscillation.

5.3.3.1 Design
Test rig components are designed using a reverse process of that outlined in Chapter 3 for the identification of resonant frequencies from torsional properties.

2nd mode
The 2nd mode is modelled with the two generators coupled as a free-free system as shown in Figure 5.7.

![Figure 5.7: 2nd torsional mode frequency analysis schematic](image)

The inertia of the DFIG has been determined (0.411 kg.m²) and the generator flywheel inertia is defined as being identical. Generator driveshaft stiffness of 14,603 N.m.rad⁻¹ is calculated to achieve a natural resonance of 30 Hz.

1st mode
The 1st mode is modelled by lumping the inertia of the DFIG and generator flywheel and combining the stiffnesses of the transmission shaft and generator driveshafts as shown in Figure 5.8.
As the generators are driven through a gearbox, the drive speed ratio of 1.5 must be taken into account by referring both the summed inertias of the DFIG and generator flywheel and the parallel combination of two generator driveshafts. The inertia of the DFIG and generator flywheel are known (0.411 kg.m²) and the stiffness of the generator shaft has been determined previously (14,603.1 N.m.rad⁻¹), transmission shaft stiffness stiffness of 52,571 N.m.rad⁻¹ is calculated to achieve a 1st mode resonance of 20 Hz.

The calculated driveshaft stiffness are summarised in Table 5.5.

<table>
<thead>
<tr>
<th>Driveshaft</th>
<th>Stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission shaft</td>
<td>52,571 N.m.rad⁻¹</td>
</tr>
<tr>
<td>Generator shaft</td>
<td>14,603 N.m.rad⁻¹</td>
</tr>
</tbody>
</table>

5.3.3.2 Selection
Torque sensing action is incorporated within the shafts so that measurements can be made without needing to include additional components within the rig. Contactless torque sensors are provided by NCTEngineering GmbH (NCTE), best matching products are selected and the shafts customised to best achieve the stiffness requirements.

Torque rating
Rig simulation shows peak torque requirements for the transmission shaft and generator driveshafts of 395 Nm and 240 Nm respectively. The NCTE Q4-0500 and Q4-0200
sensors meet the criteria for these shafts. This means that the transmission shaft has a
diameter of 35 mm and generator shaft a diameter of 19 mm.

Torsional stiffness
Shaft torsional stiffness is defined by the shaft properties according to equation (5.3).

\[ k.L = G.J_p \] (5.3)

The shear modulus steel as taken as 77.5 GPa, [118], and so shaft lengths are
customised to achieve the stiffness as detailed in Table 5.6.

<table>
<thead>
<tr>
<th>Shaft</th>
<th>Peak torque (Nm)</th>
<th>Sensor</th>
<th>Diameter (mm)</th>
<th>Length (mm)</th>
<th>Stiffness (Nm.rad-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission shaft</td>
<td>395</td>
<td>Q4-0500</td>
<td>35</td>
<td>280.0</td>
<td>40,777.0</td>
</tr>
<tr>
<td>Generator shaft</td>
<td>240</td>
<td>Q4-0200</td>
<td>19</td>
<td>167.2</td>
<td>5,930.3</td>
</tr>
</tbody>
</table>

These shaft stiffness are lower than the intended levels which appears problematic,
however, as is shown later, when mounted with couplings the effective shaft stiffness
more closely resemble the intended value. Furthermore the shaft stiffness model used
has assumed a constant shaft diameter, in fact due to manufacturing and assembly
processes the shafts have a varying diameter, which can be seen in the Transmission
shaft engineering drawing is shown in Figure 5.9.

![Figure 5.9: Transmission shaft (Q4-0500) - engineering drawing](image)
5.3.4 Couplings

Couplings act as an interface between torsional components, transmitting power between them. The ideal coupling presents negligible inertia and infinite stiffness while transmitting torque in a constant linear manner (without backlash).

Couplings are required throughout the rig at the positions (A - E) shown in Figure 5.10.

![Figure 5.10: Coupling positions on test rig schematic (top view)](image)

Torque transmission

Keyways are commonplace as a means of torque transmission with couplings, and are used in this instance, however the desire to remove backlash means that clamping hub couplings are used (as well as keyways) which are fully rated for torque transmission.

5.3.4.1 Specification

Coupling inertia and torsional stiffness are typically related. For the test rig high and predictable stiffness are required for minimal possible inertia, a high stiffness is more critical for rig design, and so this is considered the priority. Couplings must be capable of continually operating at peak speeds up to 1,400 rpm and rated torques.
5.3.4.2 Selection

The rig is designed to exhibit precise torsional behaviour, the stiffness of each coupling will influence this. Coupling stiffness must be carefully chosen so that when two couplings are paired with each shaft the overall system stiffness remains close to the intended value. This is shown in Figure 5.11.

![Diagram of shaft and couplings combined torsional stiffness](image)

\[
k_{\text{system}} = \frac{k_A \cdot k_B \cdot k_C}{k_B \cdot k_C + k_A \cdot k_C + k_A \cdot k_B}
\]

Figure 5.11: Shaft and couplings combined torsional stiffness

Coupling fit length reduces the apparent length of each driveshaft hence increasing its stiffness. The stiffness of the two couplings and, now reduced length, driveshaft is considered in series in order to determine the system stiffness. The active stiffness of the driveshaft is higher than the original driveshaft stiffness as its length is reduced by the coupling fit length at either end. Couplings are individually specified for each area of the test rig as shown in Table 5.7.

<table>
<thead>
<tr>
<th>Coupling</th>
<th>Inertia (kg.m²)</th>
<th>Stiffness (Nm.rad⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8.5x10⁻³</td>
<td>450E3</td>
</tr>
<tr>
<td>B</td>
<td>8.5x10⁻³</td>
<td>450E3</td>
</tr>
<tr>
<td>C</td>
<td>4.5x10⁻³</td>
<td>191E3</td>
</tr>
<tr>
<td>D</td>
<td>7.5x10⁻³</td>
<td>157E3</td>
</tr>
<tr>
<td>E</td>
<td>7.5x10⁻³</td>
<td>157E3</td>
</tr>
</tbody>
</table>

Bellows couplings from R+W Coupling Technology are used as they quote high levels of torsional stiffness. The BKH product range is selected as the clamped hub fitting
simplifies assembly, these are supplied couplings with customised fits for use on the test rig. A CAD image of a coupling and shafts is shown in Figure 5.12.

![Figure 5.12: Torsional coupling and shaft keyway - CAD image](image)

### 5.3.5 Fuel pump load

The purpose of the test rigs 3rd auxiliary load, with similarities to a fuel pump, is to enhance the capabilities of the test platform to include the consideration of auxiliary loads beyond just the electrical generators. It has been demonstrated, in Chapter 3, that mechanical resonance behaviour is dominated by the electrical generators because of their larger inertias. The presence of another significant auxiliary load adds an additional mode and alters the 1st mode.

For a typical 3-spool GT accessory load the fuel pump is the next significant inertia, after the electrical generators. It has a drive speed ratio of 0.9 giving a referred inertia of $1.818 \times 10^{-2} \text{kg.m}^2$, this is approximately 1/3 the referred inertia of the generators which indicates why it has the lesser impact on the drivetrain response. However, as referred inertia has a squared relationship to drive speed ratio a small inertia may develop a significant impact on system behaviour if driven at a slightly higher speed. The test platform fuel pump can be geared to run at a drive speed ratio of either 0.9 or 2.0, and is designed so that the inertia is low enough not to interfere with system behaviour when driven at low speed but high enough that it will alter system behaviour at higher speeds.

The 3rd gearbox output allows the impact of additional loads to be ascertained on an aircraft electrical system. Hydraulic and fuel pumps introduce torque oscillations at a
harmonic of rotation speed and so potentially form a further source of excitation for mechanical resonances. However this is beyond the scope of this research.

5.3.5.1 Fuel pump inertia and shaft stiffness

Simulation is used to demonstrate that the increasing referred inertia of a fuel pump reduces the 1st mode resonance while having no effect on the 2nd mode resonance. A fuel pump inertia of 0.15 kg.m² on the test platform has a low impact on the 1st mode when driven at a drive speed ratio of 0.9 but this frequency is reduced by 2% when drive speed ratio is increased to 2.0. A high stiffness connection is used between the fuel pump and gearbox to reduce local vibrations.

Table 5.8 gives the resonances, detected at various points throughout the drivetrain, triggered by an electrical load step at 1,400 rpm.

<table>
<thead>
<tr>
<th>Drive speed</th>
<th>Transmission frequencies (Hz)</th>
<th>DFIG frequencies (Hz)</th>
<th>Fuel pump frequencies (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>21.5, 99.3</td>
<td>21.5, 30.0, 100.4</td>
<td>21.5, 99.7</td>
</tr>
<tr>
<td>2.0</td>
<td>21.0, 64.6</td>
<td>21.0, 30.0, 64.9</td>
<td>21.0, 64.8</td>
</tr>
</tbody>
</table>

The 2nd mode, at 30 Hz, is seen between the DFIG and generator flywheel, this is unaffected by the fuel pump. The 1st mode, at 21.5 Hz or 21.0 Hz is reduced when fuel pump drive speed is increased. The presence of the fuel pump inertia increases the frequency of the 1st mode compared to the system operating without it, and also creates an additional, 3rd, mode. The 3rd mode frequency decreases with drive speed because referred inertia is increased.

5.3.6 Control

Control is derived in Simulink and implemented on a dSPACE interface unit. Signals are read in from torque sensors, position encoders, voltage sensors and current sensors while actuation is achieved through a commercial DC drive and a VSI. This control architecture is shown over a CAD image of the test platform in Figure 5.13.
The general control structure is a three-tier scheme. At the lowest level, a DC drive is controlled to replicate the mechanical behaviour of a GT IP spool, this control is based on the mechanical response times of a real GT but is deliberately an open loop control system so as not to interfere with interaction which occurs. Also at the lowest level is a DFIG controller which takes input from a position encoder, voltage measurements and current measurements to produce PWM switching signals to the rotor side VSI, hence regulating stator side voltage and frequency. Operating above this, an aircraft mission control scheme is envisaged to replicate inflight conditions, directing the GT spool speed controller and electrical loading, presently only speed control is implemented with the full mission profile control to be developed later. A safety system prevents over speed from the mechanical systems and offers cut-off in the case of over voltage and over current on the electrical side.

5.3.6.1 GT spool mechanical emulation

The GT flywheel is driven by a DC machine supplied from a commercial thyristor control unit which accepts a speed reference signal from the dSPACE unit. A real GT is controlled by the FADEC which controls fuel flow to the combustors based on a thrust demand and a number of other conditions. It is reasonably assumed that thrust can be directly related to spool speed and hence spool speed will be maintained at set levels during each section of a mission (takeoff/climb, cruise, descent/land). The test platform is designed, crucially, not to be a speed stiff system and hence the GT flywheel must replicate the speed fluctuations found in a typical 3-spool GT during electrical load step. This is shown in Figure 5.14.
5.3.6.2 DFIG control

The DFIG control scheme, devised in Chapter 4, is implemented using the dSPACE controller and a commercial VSI. The control scheme regulates stator phase voltages to 240 V\textsubscript{rms} at 50 Hz and takes input from the DFIG position encoder and voltage and current sensors on both the rotor and stator side. Screen shots of the test platform controller, built in Simulink, are shown in Appendix 5.C.

Passive (RC) filtering is used on the voltage and current measurements fed into the controller, and care is taken to ensure that the phase changes these induce do not unduly influence the desired controller response. A cut off frequency of 770 Hz is achieved, giving a phase lag of -3.68° at 50 Hz.
5.3.7 Data acquisition

Data Acquisition (DAQ) is carried out by computer connected ADC LabVIEW system with the raw digitised signals being recorded as Comma Separated Variable (CSV) files which are then transferred into MATLAB for post-processing and analysis. A strategy of over sampling is used so that anti-aliasing filters are not required, the sample rate of the DAQ system is 100 kHz per channel. Raw data is converted into the real signals with the correct units by a post processing routine in MATLAB. Data is acquired from the sensors throughout the rig which are shown in Figure 5.13.

5.3.8 Sensor signals and post processing

Table 5.9 lists the sensors from which data is collected from the rig as well as input signal range and the eventual post-processed output signal. Sensor labels are given with reference to Figure 5.13.

<table>
<thead>
<tr>
<th>Label</th>
<th>Sensor</th>
<th>Input Data</th>
<th>Output Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>Torque (transmission driveshaft)</td>
<td>0 V → 5 V</td>
<td>Torque (Nm) ±650 Nm</td>
</tr>
<tr>
<td>T2</td>
<td>Torque (DFIG driveshaft)</td>
<td>0 V → 5 V</td>
<td>Torque (Nm) ±263 Nm</td>
</tr>
<tr>
<td>T3</td>
<td>Torque (generator flywheel driveshaft)</td>
<td>0 V → 5 V</td>
<td>Torque (Nm) ±263 Nm</td>
</tr>
<tr>
<td>T4</td>
<td>Torque (Fuel pump flywheel driveshaft)</td>
<td>0 V → 5 V</td>
<td>Torque (Nm) ±650 Nm</td>
</tr>
<tr>
<td>V_s</td>
<td>Voltage (DFIG stator)</td>
<td>-7 V → 7 V</td>
<td>Voltage (V) ±700 V</td>
</tr>
<tr>
<td>V_r</td>
<td>Voltage (DFIG rotor)</td>
<td>-7 V → 7 V</td>
<td>Voltage (V) ±700 V</td>
</tr>
<tr>
<td>i_s</td>
<td>Current (DFIG stator)</td>
<td>-10 V → 10 V</td>
<td>Current (A) ±35 A</td>
</tr>
<tr>
<td>i_r</td>
<td>Current (DFIG rotor)</td>
<td>-10 V → 10 V</td>
<td>Current (A) ±35 A</td>
</tr>
<tr>
<td>ω2</td>
<td>Position (DFIG)</td>
<td>0 V → 10 V</td>
<td>Speed (rpm) 1/1024 °</td>
</tr>
<tr>
<td>ω3</td>
<td>Position (generator flywheel)</td>
<td>0 V → 10 V</td>
<td>Speed (rpm) 1/1024 °</td>
</tr>
<tr>
<td>ω4</td>
<td>Position (fuel pump flywheel)</td>
<td>0 V → 10 V</td>
<td>Speed (rpm) 1/1024 °</td>
</tr>
<tr>
<td>ω1</td>
<td>Position (prime mover)</td>
<td>0 V → 10 V</td>
<td>Speed (rpm) 1/1024 °</td>
</tr>
</tbody>
</table>
5.3.8.1 Specification, calibration and post processing

Raw voltage signals are recorded from the various sensors throughout the rig, the range and bandwidth of which are careful considered to ensure sufficient data fidelity is available. Post-processing is then used to output signals which are calibrated and in the correct units.

Torque sensors

Torque sensing ranges of up to 390 Nm are required to monitor peak torques throughout the rig. The bandwidth of the sensors must ensure accurate replication of the highest frequency torques throughout the rig, which is the gearmesh frequency, giving a bandwidth requirement of 5 kHz. The torque sensors must be able to mechanically operate at rig speeds of at least 1,400 rpm.

The commercial torque sensors are pre calibrated delivering a voltage signal which is proportional to torque. Parameters from the calibration certificate are then used to produce the torque signal from the acquired voltage data.

Angular position

Incremental encoders are sufficient as it is necessary to identify fluctuations in speed not absolute position. These encoders are required to have a resolution to identify torsional vibrations across the full speed range of the encoders (400 rpm to 1,400 rpm). Simulation is used to determine that frequencies up to the electrical frequency 2nd harmonic of 100 Hz can be detected with a 1,024 pulse position encoder.

Angular position measurements are taken by a series of 1,024 pulse encoders. These provide a square wave voltage output with a frequency which is proportional to shaft speed. Processing routines are used to convert frequency to a shaft speed signal in rpm by a method of zero crossing detection.

Current

The inductive nature of both the stator and rotor mean that current fluctuations will not have a high bandwidth. Therefore current sensors with a bandwidth an order of magnitude greater than 50 Hz are sufficient. DFIG thermal rating means that peak
currents will not be above $28 \text{ A}_{\text{peak}}$. Passive filtering is used for control signals, but not on the DAQ system.

Current measurement is made by commercial current transducer and in-house manufactured power and amplifier unit. The unit returns a voltage output which is proportional to current and calibration data is used by processing routine to produce the current signal from the acquired voltage data.

**Voltage**

High frequency voltage transients are present due to the 2.4 kHz converter switching frequency, however a low pass filter (passive RC) with 700 Hz cut-off is used for the control system. The maximum voltage appearing on the rig is $600 \text{ V}_{\text{peak}}$ at the DC link and so isolation must be ensured.

Commercial isolated differential probes are used to measure and attenuate voltages, which are then converted by processing routine to produce real signals.

### 5.4 Interfacing and Assembly

The design of individual components making up the test rig have been detailed in the previous sections, with many of these designs influenced by limitations placed on manufacture and safe operation. Further to their individual design, the assembly of multiple mechanical systems requires additional consideration to ensure precise interfacing and assembly.

#### 5.4.1 Tolerance and fit

Preferred mechanical fits are provided in [148] and [149]. From these, H7/h6 fits are used between the couplings and shafts to provide an accurate fit with ease of assembly. H7/g6 are used on the flywheel disk / hub interface to ensure a positive location.

#### 5.4.2 Key fitting

Slip between the couplings and driveshafts must be prevented. It is traditionally common practice to introduce a key (with keyway in both the coupling and shaft) in a torsional coupling to transmit the torque without slip. The selected couplings are,
however, capable of transmitting rated torques without keyways [150]. Furthermore the introduction of a keyway reduces the area of the shaft circumference in which the coupling hold operates. It can be rationalised that should a keypiece come into action during loading, then slip has already occurred, however, given the high torque reversals expected throughout the rig a traditional keypiece interface is used. High tolerance, H9 and P9, fits are used for the keyways to ensure that no backlash is possible within the coupling.

5.4.2.1 Torque requirements

The keyways are designed so that they are capable of transmitting the levels of torque required without shearing. DIN standard keyway dimensions are used for the shaft diameters in question [120].

The maximum transmissible torque for each interface is calculated from the shear force exerted on each keypiece in regard to its yield stress, where the yield stress of steel is taken as 280 GPa [118]. Force exerted on the keypiece is derived from the torque loading and shaft radius giving (5.4), [118].

\[
\tau = \frac{F}{wL} \quad (5.4)
\]

Where:

\[ \tau = \text{shear stress (N.m}^2 \text{ or Pa)}, \]  \[ F = \text{Shear force (Nm)}, \]  \[ W = \text{Width (m)}, \]  \[ L = \text{Length (m)} \]

The keyways have a maximum rated torque holding capability of 1,617 Nm and 583 Nm for the 35 mm and 19 mm shafts respectively. Given the loading requirements of 390 Nm for the 35 mm shaft and 240 Nm for the 19 mm shaft a safety factor of 4 and 2 is achieved respectively.

5.4.3 Alignment and Balancing

Alignment and balancing is certified by an external company, Pruftechnik. This is necessary to minimise non-torsional vibration and also to reduce mechanical wear and conform to ensure that the flywheel stress calculations carried out are valid.
5.4.3.1 Alignment
Accurate alignment minimises loading on bearings and couplings throughout the rig. High levels of misalignment are likely to add additional resonances to the system which is undesirable. Alignment was carried out during construction using the reverse dial gauge method. These alignments were checked by the external company.

5.4.3.2 Balancing
Out of balanced results in torsional and lateral vibration in the system, the frequency of which is directly related to rig drive speed. Balancing on the GT flywheel reduced imbalance to 8% of initial levels by the addition of mass on the hub. The full balancing report is given in Appendix 5.D.

5.5 Rig overview and evaluation
Engineering drawings for mechanical components throughout the test rig are given in Appendix 5.E. The completed test platform is shown in Figure 5.15.

![Figure 5.15: Assembled test platform - CAD image](image)

The addition of safety enclosures can be seen on the test platform photograph, Figure 5.16.
Before being commissioned as an electro-mechanical test platform a number of tests were carried out to appraise the system in comparison to the functional specifications, and also to accurately determine its true behaviour.

5.5.1 Identification of true torsional properties

Real torsional resonances are measured by way of a hammer test. Calculations are carried out, taking into account the actual construction of the rig, to identify true stiffness and inertia where possible.

5.5.1.1 Hammer test

This test identifies the DFIG and generator shaft natural resonance which allows the generator flywheel mass to be altered to achieve an identical natural resonance. When these two resonances are identical, and they are coupled through the gearbox, this forms the 2nd mode resonance of the rig system.

The gearbox is locked off by inserting a wedge at the meshing point between gears 6 and 7. This isolates the fixed-free torsional system shown in Figure 5.17.
A hammer is used to apply an impulse torque to the system, at the DFIG shaft, exciting the natural resonance of the system, this resonance decays over time due to inherent system damping. Torque fluctuations induced by the natural vibrations are recorded and analysed to identify the frequency of natural resonance. The rate of decay of the oscillation is not important here, although it could be used to determine damping levels.

Figure 5.18 shows the DFIG generator driveshaft torque as a result of an applied impulse torque.

The natural resonance of the DFIG and generator driveshaft system is identified as 22.7 Hz. A high level of damping is present as the oscillations decay rapidly.

A hammer test was tried with the GT flywheel locked, and the gearbox not locked, in order to observed both the 1st and 2nd mode oscillations. The results were inconclusive because high levels of gear chatter due to backlash. When driven with a finite base load during rig operation, less backlash is expected.
Generator flywheel inertia tuning

This test aims to match the resonance of the generator flywheel system with that of the DFIG system so that the test rig behaves as the intended two mode system. The hammer tests were repeated on the generator flywheel system. Gears 1, 1b and 2 are removed from the gearbox and the gearbox was locked at the interface between gears 3 and 4. An identical fixed-free torsional system is created this time with the inertia of the generator flywheel and stiffness of the generator flywheel driveshaft. The generator driveshafts, and couplings, used on the DFIG and generator flywheel are identical and so stiffness is assumed to be the same. Generator flywheel mass is reduced to increased natural mode using a process of trial and improvement. A final natural system resonance of 22.2 Hz is achieved, which is sufficiently close to that of the DFIG system.

The newly sized generator flywheel has a calculated inertia of 0.359 kg.m\(^2\), which is lower than the predicted DFIG inertia of 0.411 kg.m\(^2\), perhaps indicating that true DFIG inertia is lower than originally predicted.

**5.5.1.2 Derivation of true torsional stiffness**

During the design process system compliance was assumed to be confined simply to the driveshafts, as all other components were considerably stiffer. Furthermore driveshaft stiffness was assumed based on a consistent driveshaft geometry. In reality torsional compliance throughout the system should be considered and the non-consistent geometry of the driveshafts must be taken in to account to identify the true system stiffnesses.

The chain of components creating the stiffness which links each inertia to the gearbox is investigated at a higher fidelity. Only the active stiffness of shafts is considered. Figure 5.19 and Figure 5.20 show the stiffness chain for DFIG and generator flywheel respectively.
Similarly, the stiffness chain coupling the GT flywheel to the gearbox is shown in Figure 5.21.
The stiffness chain coupling the DFIG inertia to the gearbox is shown as a torsional system in Figure 5.22.

As the gear diameter is much larger than other components within the chain its stiffness is assumed to be infinite. The stiffness of each coupling is assumed to match the value quoted in the associated data sheets. DFIG and flywheel shaft stiffness is estimated from known cross section and assumed material properties. Shaft cross section reduction caused by keyways is not considered, all shafts are assumed to be solid.

The derived component stiffnesses are shown in Table 5.10.

<table>
<thead>
<tr>
<th>Component</th>
<th>Gear 7 shaft</th>
<th>Coupling C</th>
<th>Generator Driveshaft</th>
<th>Coupling D</th>
<th>DFIG shaft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness (Nm.rad(^{-1}))</td>
<td>95,874</td>
<td>191,000</td>
<td>13,736</td>
<td>157,000</td>
<td>587,214</td>
</tr>
</tbody>
</table>
Adding each component in series allows the overall active stiffness values for each of the systems to be identified, as shown in Table 5.11.

### Table 5.11: Derived system stiffness

<table>
<thead>
<tr>
<th>System</th>
<th>Stiffness (Nm.rad(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>DFIG</td>
<td>10,358.6</td>
</tr>
<tr>
<td>Generator flywheel</td>
<td>10,395.7</td>
</tr>
<tr>
<td>Transmission</td>
<td>30,144.3</td>
</tr>
</tbody>
</table>

There is a slight discrepancy between the DFIG and generator flywheel system stiffness, this is a result of the marginal difference in axial positioning of couplings in the two systems. Both stiffness are lower than the designed values, which will inevitably lead to lower torsional resonances. The higher than anticipated compliance on the gear shafts is a major contributor to this.

#### 5.5.1.3 Summary of torsional modes

Test rig resonances have been identified by practical test (hammer test) and high fidelity derivation, the identified resonances are shown in Table 5.12. The 2nd mode is formed by the combined, and identical, oscillations of the DFIG and generator flywheel. Their individual resonances are listed here for comparison.

### Table 5.12: Test rig torsional resonances

<table>
<thead>
<tr>
<th></th>
<th>1st mode (Hz)</th>
<th>2nd mode (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DFIG</td>
<td>generator flywheel</td>
</tr>
<tr>
<td>Specified</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>Hammer test</td>
<td>-</td>
<td>22.7</td>
</tr>
<tr>
<td>Calculation</td>
<td>18.7</td>
<td>25.3</td>
</tr>
</tbody>
</table>

1 Hz
The hammer test is only able to identify the 2nd mode frequencies corresponding to the DFIG and generator flywheel, the 1st mode cannot be seen because of the non-linearity in the unloaded gearbox. 2nd mode resonances derived by calculation compare well with the true frequencies identified by the hammer test. Calculated resonances are derived using the best estimate of inertia for the DFIG and generator flywheel as well as the associated stiffness, inaccuracies in both of these values limit the overall accuracy of resonant frequency.

The hammer test and calculated resonances appear slightly lower than the specified frequencies due mainly to the lower than intended gear shaft stiffness. The hammer test resonances are the true system values and appear lower than the calculation most probably as it is unlikely that the couplings truly achieve their quoted stiffness levels.

When the rig is in full operation the 2nd mode should be expected to be marginally lower as the compliance of the full gearbox must also be included. The inertia of the DFIG and generator flywheel appear lower than expected, and with further compliance in the transmission couplings it can reasonably be estimated that the 1st mode will appear in the region of 15 Hz.

5.5.2 System wide frequency analysis
Analysis during the design process has highlighted the high number of resonances within the test platform. As well as the designed torsional resonances, lateral and torsional resonances have also been derived for each of the flywheels, and also critical drive speeds which must be avoided.

The test platform receives a wide range of excitation frequencies from once per revolution drive frequencies to gear meshing frequencies as well as potential electrical excitation from voltage regulation and power electronic systems.

The test platform frequency spectrum is shown in Figure 5.23.
Chapter 5 - Test Platform

Figure 5.23: Test platform resonance and excitation frequencies
The wide number of resonances within the mechanics of the test platform are clearly shown as well as the proximity of many of the possible excitations. Gear meshing frequencies and electrical loading may offer excitation to the generator flywheel lateral and torsional modes, although these are both considered to be low amplitude. Elsewhere resonances are generally well separated from possible excitation. The low 1st and 2nd torsional modes now fall within the mechanical drive speed range and so care must be taken to separate torsional oscillations from drive frequency during analysis. It does however offer a potential means of resonance identification on the practical system by varying drive speed to produce a frequency sweep.

5.6 Test platform design and construction summary

A test platform has been designed, constructed and commissioned for the analysis of electro-mechanical interaction. The system has been specified to replicated the behaviour of a 3-spool GT and aero generator system, presenting two torsional modes of torsional resonance. The design and manufacture of flywheels, gearbox and driveshafts has been achieved to match these specifications, and a thorough safety analysis has been carried out. Control and data acquisition systems have been applied to the test rig to produce a test platform able to emulate the behaviour of an aero GT generator system. Analysis has been undertaken of the test platform to identify actual resonances and confirm it meets the specifications.
Chapter 6  Results

This chapter presents results obtained from the electro-mechanical test platform. The results cover three areas, the practical performance of the DFIG and control scheme implementation, the behaviour of the mechanical test rig, and the occurrence of electro-mechanical interaction between drivetrain and DFIG.

6.1 DFIG performance

The DFIG simulation in Chapter 4 neglects converter switching, sampling delays, and sensing bandwidths, and hence the response of the practical system will differ. Both the inner current control loop, and outer voltage and frequency control loop response are analysed for load and speed variation. The power flow through the rotor, stator and shaft are compared to simulated results and system efficiency estimated.

6.1.1 Current control

High speed current control response is necessary to enable the DFIG to offer voltage and frequency regulation. The DFIG control characteristics are known to vary with shaft speed and so current controller response is assessed at synchronous speed as well as minimum and maximum speeds.

1 A load steps in reference current are applied separately to both the d and q axis current controllers at several different drive speeds in Figure 6.1.

![Figure 6.1: DFIG current controller 1 A step](image-url)
No post-processing is applied to the signals and so high frequency noise can be seen. The response is seen to be fast, settling in approximately 0.01 seconds, however the high overshoot (approximately 20%) shows a system which is less than critically damped. Damping is sacrificed to ensure a fast controller response and so the overshoot is tolerated. No difference can be seen in the controller response at different drive speeds.

Extreme electrical load steps on the DFIG require a higher amplitude step from the current controllers and so a 10 A step is carried out in Figure 6.2.

Again, no filtering is carried out on the measurements and so noise can be seen on the signals, however this is at a lower amplitude relative to the signal amplitude. The controller response is slower than for the 1 A step, settling in approximately 0.03 s. Overshoot is larger for the 10 A step than for the 1 A step at approximately 40% compared to 20%. For the larger step the current rise rate is limited by the converter and so the higher overshoot is likely to be a result of the simple controller anti-windup strategy applied. A marginally lower overshoot can be seen at the minimum drive speed of 600 rpm, at these speeds back emf from the DFIG is lower and so the converter is working with a greater voltage gradient and so able to achieve a marginally higher current rise rate (meaning the gain of the inverter is closer to unity) which leads to the reduced overshoot.
Simulation of the current controller, in Chapter 4, uses higher gains, these are shown alongside the gains implemented for the practical system in Table 6.1.

<table>
<thead>
<tr>
<th>Table 6.1: Current controller gains, modelled system and practical system</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Model</td>
</tr>
<tr>
<td>Practical system</td>
</tr>
</tbody>
</table>

Integral gain had to be reduced as a result of the unmodelled system dynamics. Proportional gain was increased by trial and error to give acceptable damping. The bandwidth is still lower than the modelled system which settles in approximately 0.005 seconds (half the time of the practically realised system).

Analysis of equations (4.49) and (4.50) identifies the relationships between controller gains and bandwidth / damping ratio, these are shown in (6.1) and (6.2)

\[
\zeta \propto \frac{k_p}{\omega_n} \quad \text{(6.1)}
\]

\[
\omega_n \propto \sqrt{k_i} \quad \text{(6.2)}
\]

Damping ratio is dependent on both proportional gain and (inversely) bandwidth, while bandwidth is only dependant on integral gain (with a square root relationship).

The observed response of the full real system model (Figure 4.20) and practical (Figure 6.1) systems to a 1 A step are compared in Table 6.2.

<table>
<thead>
<tr>
<th>Table 6.2: Current controller response, modelled system and practical system</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Model</td>
</tr>
<tr>
<td>Practical system</td>
</tr>
</tbody>
</table>

Integral gain for the modelled system is approximately 4 times greater than that on the practical system, and as a result the modelled system is approximately twice as fast in response, conforming to the relationship given in (6.2). The reduced bandwidth of the practical system and higher proportional gain give increased damping compared to the modelled system, (6.1), although overshoot remains higher despite this increased gain.
The higher damping may assist with disturbance rejection, allowing stable current control to be achieved.

### 6.1.2 Stator voltage and frequency regulation

Stator voltage and frequency regulation are carried out by control of the d and q axis rotor currents respectively. Stability is required over the full speed range of the DFIG, 1,000 rpm ±40%.

The test platform is driven through its full speed range with a low and near constant acceleration, the DFIG stator is left open circuit. The results are shown in Figure 6.3.

![Figure 6.3: DFIG stator voltage regulation](image)

A low pass filter is used, with cut-off at 1,000 Hz, to remove signal noise from the stator voltage measurement. Frequency is derived using a zero crossing detection algorithm on the filtered stator voltage.
Stator voltage is held at 240 V\textsubscript{rms} for all speeds, a slight apparent increase in amplitude at speeds away from synchronicity is a result of additional signal noise caused by the higher power levels being transferred through the rotor. Rotor current maintains a near constant amplitude but frequency is accurately controlled to maintain stator frequency at 50 Hz. Stator frequency regulation is within ±0.5% with some of this variation attributed to measurement error, as the method of zero crossing detection is limited in its accuracy by the original sample rate, and the frequency of the measured signal.

Voltage and frequency regulation is assessed during electrical loading. The test platform is allowed to settle with a DFIG shaft speed of 1,100 rpm but no load, at time t=1 s a 2.7 kW load is applied, the response is shown in Figure 6.4.

Figure 6.4: DFIG regulation, load step at 1,100 rpm

As before, stator voltage measurement is filtered to remove noise above 1,000 Hz, and a zero crossing detection algorithm used on this signal to determine the frequency of the voltage.
Stator voltage shows only a momentary dip which recovers in a fraction of a cycle due to the high current controller bandwidth. Stator frequency drops by approximately 10% and recovers in a couple of cycles. Although the electrical loading causes a change in machine electro-magnetic torque which slows the DFIG, this change in speed is at a low rate due to the machine inertia, and so only slow variation in rotor frequency is required to maintain stator frequency at 50 Hz. The stator voltage measurement is taken from the A phase and slight differences should be expected between phases according to the point-on-wave during the load step. Stator frequency is also derived from a single phase only and so has a low resolution (which is limited by the fundamental frequency), a more favourable / representative result could be achieved by considering the average frequency across all 3 phases.

As with the current controller, voltage controller gains are reduced to ensure stability, these are shown in Table 6.3.

<table>
<thead>
<tr>
<th></th>
<th>Proportional gain</th>
<th>Integral gain</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model</strong></td>
<td>0.1</td>
<td>10</td>
</tr>
<tr>
<td><strong>Practical system</strong></td>
<td>0.001</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Proportional gain was reduced by a factor of 100 to achieve stability, this is partly necessary as a cumulative effect of the reduced current controller bandwidth. The DFIG response was found to be particularly unstable at higher electrical loads, at which point disturbances from the sensors, perhaps mechanically induced, are at sufficient levels to cause instability. The use of active filters on control signals may offer increased stability, although this investigation is beyond the scope of this research.

The implemented control scheme provides good current following as well as frequency and voltage regulation at no load. The basic control scheme is effective but could be improved for high load rating by further work.

### 6.1.3 Power flow and efficiency

Power into and out of the DFIG is measured at a number of points over the full speed range of the DFIG, this is shown in Figure 6.5 for a stator load of 2.3 kW. Mechanical power calculated from the product of the DFIG speed and shaft torque measurements.
Stator power and rotor power are calculated as the sum of the products of d and q axis current and voltage, where rotor direct voltage is estimated from DC link voltage and modulation index, using values from the control scheme.

The rotor changes from drawing power, at subsynchronous speeds, to supplying power, at supersynchronous speeds. Where as the model predicts a linear variation of power crossing the origin at 1,050 rpm the practical system crosses at more like 1,150 rpm and shows an upward curve with greater than predicted power extracted from the rotor at supersynchronous speeds. The indication is that machine is working harder than predicted under loading and hence the zero power speed appears marginally higher. Mechanical power is consistently higher on the practical system than the model but increases linearly with shaft speed. Higher levels of input shaft power are seen from the practical system compared to the model, for the same output electrical power, because of core losses and mechanical losses which aren’t modelled.

The data presented in Figure 6.5 is used to derive DFIG efficiency. Efficiency is considered to be the ratio of useful output power (stator power) to input power (mechanical power + rotor power), efficiency across the full speed range is shown in Figure 6.6.
Efficiency is significantly lower for the practical system than for the model, again, as the model excludes core losses and mechanical losses. Efficiency is lowest near synchronous speeds unlike the model. Copper losses are greater for higher levels of slip and so efficiency would be expected to be lowest at maximum and minimum drive speeds. However, the model also excludes the converter which has low efficiency around synchronicity as it must supply magnetising current even though little real power is required.

6.1.4 DFIG performance summary

Current control is successfully implemented using a converter and DFIG. This response is both fast and stable across the full drive speed range of the DFIG and at high current levels. The high speed current controllers are used to implement stator voltage and frequency regulation which is accurate and reliable across the full speed range of the test platform. DFIG power flow follows a similar trend to the modelled system although losses which are not modelled are apparent in the practical system. Measurements are used to approximate machine efficiency which is significantly lower than the modelled system, although this comparison is somewhat unrepresentative as the measurements are taken from the control scheme and so somewhat distant from the actual power flow.
The practical results have confirmed the accuracy of the machine modelling and characterisation carried out in Chapter 4. The generator control scheme, assembled and tuned in Chapter 4, provides accurate voltage and frequency control over the full mechanical speed range of the practical system. However, significant de-tuning of the voltage control loop was required to reject measurement noise, and the resultant controller was unable to achieve load steps at very high load levels.

6.2 Mechanical rig behaviour

The test rig is designed to have two torsional modes, however due to the system complexity the rig is expected to contain a wide range of further resonant frequencies. These are identified by carrying out frequency sweeps considering both torsional and lateral vibration. As well has static resonances the test rig will also produce a wide range of harmonics during operation, these too are identified. These tests validate the mechanical systems analysis and reduced order model extraction in Chapter 3, as well as the test rig design undertaken in Chapter 5.

6.2.1 Torsional vibration

Torsional resonances are identified by applying a sinusoidal excitation torque to the drivetrain and monitoring the response. The excitation torque is created by applying a sinusoidal disturbance (amplitude = 5 A) to the q axis rotor current control loop on the DFIG. A constant electrical load and drive speed are applied to the test platform so that the gearbox remains under continual load, with negligible torque reversal, so minimising the backlash effects. This procedure relies on the fast response of the current controller to produced shaft torque and so the test has a limited frequency range.

The torsional response of the drivetrain, under operational conditions, is identified by applying the excitation disturbance and monitoring the amplitude of that frequency component throughout the rig. This ensures that excitation from frequencies other than the applied torque (such as drive frequency) are not overly considered. Excitation is carried out at a finite number of frequencies to form a frequency sweep with a 1 Hz resolution. The range of 6 Hz to 25 Hz is selected as modelling shows torsional modes to appear in this range. The test is conducted at more than one drive speed to create a separation between derived torsional response and drive speed excitation and drive
speeds are selected to avoid predicted resonances. Only a limited range of drive speeds are therefore possible, DFIG shaft speeds of 930 rpm and 1,000 rpm are selected. A 30 Ω load is applied to the DFIG stator phases which, along with the disturbance, gives an approximate equivalent electrical load varying between 2.1 kW and 2.5 kW. Data from the generator flywheel torque sensor is shown in Figure 6.7.

Note: frequency analysis is carried out using a post processing function deriving the power spectrum; this function is used consistently for all frequency analysis. In some cases traces are scaled to show details of the most significant harmonics within the frequency range, and individual frequency components can be compared in each plot but not across plots.

![Figure 6.7: Generator flywheel torque sensor frequency sweep at 930 rpm (a) and 1000 rpm (b)](image)

Frequencies corresponding to drive speeds at 10 Hz and 15 Hz and at 11 Hz and 17 Hz can be seen for the GT flywheel and DFIG at 930 rpm and 1,000 rpm (DFIG speed) respectively. A consistent and strong resonance at 13 Hz is clear at both drive speeds, additional resonances can also be seen between 7 Hz and 8 Hz and between 21 Hz and 22 Hz.

Data from the DFIG torque sensor is shown in Figure 6.8.
The log scale used in this instance to better show lower amplitude peaks. A strong response is again seen at 13 Hz and drive speeds are partially visible, responses at 8 Hz and 22 Hz are again visible for the 930 rpm test but not at 1,000 rpm. Other frequencies around 18 Hz and 19 Hz are visible but not at significant amplitude.

Data from the transmission driveshaft is not shown as disturbances are at such low levels compared to the shaft torque range that the signals are in the noise band. Signals can be better interpreted from the generator flywheel as this is an unloaded system (net torque is zero) and hence oscillations have a greater response here.

**6.2.1.1 Summary of identified torsional modes**

The frequency sweeps produces a consistently high response at 13 Hz and a lesser response around 22 Hz in the majority of traces. Drive speeds are also nearly always visible which provides a good reassurance that the test methodology is working correctly. An additional mode around 8 Hz can also be seen.

Given that the 1st and 2nd torsional modes have previously been estimated at 15 Hz and between 25 Hz and 27 Hz in Chapter 5, it is reasoned that the frequency sweeps have identified torsional resonant modes at 13 Hz and 22 Hz within the operating drivetrain. These resonances are lower than intended in both cases, perhaps due to the additional compliance associated with the gearbox.
6.2.2 Lateral vibration

Lateral vibration is measured by the addition of an accelerometer mounted temporarily fixed to the bedplate, measurements are taken from the vertical axis. Excitation is created by the natural imbalance in the rotating components and so a frequency sweep is provided by driving the test platform over its full speed range with a low constant acceleration.

The frequency component of the accelerometer data for the full frequency sweep is shown in Figure 6.9.

![Figure 6.9: Bedplate lateral vibration, speed sweep](image)

While not particularly conclusive it does show the high levels of resonance in the test rig with the highest levels at approximately 7 Hz. Excitation frequencies are 6.7 Hz to 15.6 Hz and 10.0 Hz to 23.3 Hz from the gearbox input and output respectively and so between 10.0 Hz and 15.6 Hz, where these two overlap, the total excitation experienced will be higher than average leading to higher levels on the trace.

6.2.3 Drive frequencies

The operation of the test rig creates significant frequencies which are inherent to the system and as such essentially constant excitation frequencies.
Torque throughout the test rig is shown in Figure 6.10 in response to an electrical load step (at \( t=1 \) s) on the DFIG.

![Torque throughout the test rig](image)

Figure 6.10: Torque due to load step (1,100 rpm)

A low pass filter is applied to all traces with a cut off frequency at 50 Hz. The electrical load step causes a step in mean DFIG shaft torque and a momentary oscillation, this torque step passes through the gearbox and is felt by the generator flywheel which has a net torque of zero but shows this oscillation clearly. Due to its low mechanical loading (bearing damping only) torque reversals lead to backlash in the gearbox which can be seen in non linearity in the torque oscillations. A regular negative torque can be seen on the generator flywheel shaft both before and after the load step. The variation in torque appears on the transmission shaft at a low level which cannot be distinguished on the torque sensor.

The response of each driveshaft is considered in more detail below.
6.2.3.1 Generator flywheel shaft

Generator shaft torque from Figure 6.10 is shown for a shorter time period in Figure 6.11.

![Generator flywheel torque (1,100 rpm)](image)

The torque signal is filtered above 3,000 Hz to remove noise from the signal. Two significant frequencies can be seen, a regular low frequency ‘beating’ at approximately 6 Hz and a higher frequency around 50 Hz which appears to decay after the lower frequency impulse. To identify the source of these frequencies, generator flywheel torque is measured for a range of drive speeds with the frequency content of these shown in Figure 6.12.
Note: indicated drive speeds are nominal, as the speed controller is open loop, and should be used for guidance only.

The drive frequencies can be clearly identified and also the sub harmonic beating which varies with drive speed. Frequencies are indicated which correspond to DFIG drive speeds, some associated GT flywheel drive frequencies can also be seen. Due to its variation with drive speed the beating is assumed to be as a result of sticking points in the gearbox rotation as a result of the imperfection in the gear alignments. A strong oscillation can be seen at 50 Hz.

**6.2.3.2 DFIG shaft torque**

Frequency content from the DFIG shaft torque sensor, for a range of drive speeds, is shown in Figure 6.13, frequencies corresponding to fundamentals of shaft speed have been marked.
The low frequency component can be associated with the non-zero net torque. Drive speeds can be identified but no other frequencies are clear.

### 6.2.3.3 Transmission shaft torque

The frequency components of the transmission shaft, for a range of drive speeds, are shown in Figure 6.14.
The low frequency component is much lower for the transmission shaft than for the DFIG shaft as a result of the relatively low levels of torque. It is hard to identify drive speeds, however oscillations at 50 Hz are clear.

### 6.2.3.4 Summary of drive frequencies

Regrettably the transmission shaft torque sensor is operating at the lowest end of its range for the power levels achieved in the test platform making it hard to determine signals. The generator flywheel provides the most visible means of identifying frequencies within the test rig as it oscillates with little loading. Once per revolution drive frequencies can be detected on both sides of the gearbox, this imbalance is not modelled. Gearbox backlash can clearly be seen as well as sub harmonic beating as a result of manufacturing imperfections. A 50 Hz oscillation can be seen although this is not visible within the DFIG driveshaft.

Running the test platform without activating the DFIG rotor side converter this 50 Hz component is no longer present. It can therefore be concluded that the electrical line frequency is introduced to the drivetrain through the DFIG, and this could be through any number of routes, from the DC drive, inverter, or signal pickup. However it does not provide proof that a resonance does not exist within the drivetrain at around the same frequency and this assumption is somewhat speculative, however, without further modelling and analysis which is beyond the scope of this research.

### 6.2.4 Summary of mechanical rig behaviour

Analysis of torsional resonances within the drivetrain identifies modes at 13 Hz and 22 Hz which are believed to be the designed 1st and 2nd modes. A mode around 8 Hz is also identified which corresponds closely with that identified by the lateral vibration frequency sweep and is believed to be a fundamental resonance within the test rig such as structural vibration or bedplate vibration. A high number of resonances are identified throughout the test rig as each component within the test platform provide an additional mode, creating a system which is very frequency rich. Excitation is also present from a number of sources, especially the gearbox, which although manufactured to a high precision produces backlash and drive beating at noticeable levels.
Despite the significant harmonic pollution from gearbox and a large number and range of frequencies existing within the rig the two designed torsional modes can be identified, validating the reduced-order modelling and test rig design carried out in Chapter 3 and Chapter 5 respectively. However unexplained frequencies have been identified at 8 Hz and 50 Hz which could be characterised with further work.

6.3 Electro-mechanical interaction

The purpose of the test platform is to further investigate electro-mechanical interaction. This section details the response of the mechanical rig to electrical loading on the DFIG. The affect of electrical loading is first considered at a high level, looking at the speed and torque response throughout the rig, and then at a lower level, identifying short duration torsional oscillations which are triggered.

6.3.1 Mechanical system response

The behaviour of the mechanical test rig is now investigated in response to a variation in electrical loading on the DFIG. Figure 6.15 shows the mechanical rig response when a 30 \(\Omega\) (per phase) load is applied to the open circuit terminals of the DFIG stator at time \(t=1\) s.

![Figure 6.15: Electro-mechanical system load step (650 rpm)](image-url)
The DFIG shaft torque and speed results have been filtered with a low-pass cut-off at 600 Hz and 500 Hz respectively. The electrical load step on the DFIG causes a change in electro-magnetic torque from the machine (due to the fast current controller response this is a high rate torque step) which is applied to the DFIG shaft, a short duration oscillation can be seen at this torque step which lasts for approximately 0.5 seconds. The DFIG torque step change causes the test rig to slow to a minimum after approximately 2.5 seconds, before increasing in speed again as the drive power is increased before settling to a slightly lower level after approximately 7 seconds. This replicates the true variable speed response seen by a real GT in Chapter 3, and validates the rig prime mover control design of Chapter 5.

6.3.2 Electrical load step

The short duration oscillation appearing on the DFIG shaft after the electrical load step are investigated in more detail, initially over a wide frequency range.

6.3.2.1 Wide frequency range analysis

DFIG torque sensor frequency content is analysed before and after an identical electrical load step at a range of DFIG drive speeds, this data is shown in Figure 6.16.

Note: indicated drive speeds are nominal.

![Figure 6.16: DFIG torque sensor frequency content for 0 s to 1 s, (a), and 1 s to 2 s, (b), electrical load step](image-url)
A frequency at 12,000 Hz is identifiable at all DFIG drive speeds, and is a function of the transducer electronics, no other significant frequencies are observable above 1,000 Hz. Gear meshing frequencies are clearly identifiable corresponding to 48 times the gearbox input drive speed. At approximately 140 Hz and below 40 Hz frequencies can be seen which are common to all drive speeds, these are assumed to be system resonances, although fundamental drive frequencies (for both gearbox input and output) will also be visible between 6.7 Hz and 23.3 Hz. The oscillation at 140 Hz is at a much lower amplitude to the lower torsional resonances being considered, and does not correspond to a predicted resonance of the system. Gear meshing is lower amplitude after the step than before as drive speed changes, increasing the spread.

6.3.2.2 Narrow frequency range analysis

Electrical load steps (of the type shown in Figure 6.15) are applied and the frequency content of the torque in the second before the step, directly after the step, and after the step has decayed are identified. This method allows inspection of the transient response in comparison to the original steady-state and post-step steady-state.

The DFIG shaft torque for an electrical load step at 650 rpm is shown in Figure 6.17 and Figure 6.18.

![Figure 6.17: Electrical load step DFIG torque at 650 rpm, time domain](image-url)
Frequencies at 7 Hz and 11 Hz represent drivespeeds at the input and output sides of the gearbox respectively and are present before and after the load step. Note the reduction in drivespeed to 10 Hz after the load step (this variation is too small to show on the input side drivespeed). A frequency at 13 Hz is present after the load step but not before, this has the greatest amplitude in the second after the load step and then reduces. This response suggests a torsional oscillation triggered by the load step which is attenuated by the system damping.

The DFIG shaft torque for an electrical load step at 1,400 rpm is shown in Figure 6.19 and Figure 6.20.
Figure 6.19: Electrical load step DFIG torque at 1400 rpm, time domain

Figure 6.20: Electrical load step DFIG torque at 1400 rpm, frequency domain

Frequencies of 16 Hz and 24 Hz can be seen both before and after the load step and correspond to drive speed, again a reduction in drive speed can be seen after the load step. An oscillation at 13 Hz can be seen only for the period directly after the load step. The frequency at 10 Hz appears only before the step and at a low amplitude.
6.3.3 Summary of electro-mechanical interaction

The DFIG controller is robust enough not to respond to torsional resonances so interaction is only in one direction, with the electrical loading triggering mechanical vibration. Electrical load variation is shown to trigger an oscillation which has already been identified as the 1st mode of the drivetrain. Damping levels are seen to be high by the rapid decay of this oscillation. The drivetrain 2nd mode is not detected, with the methods used, when an electrical load step is applied.

6.4 Summary of results

Both the DFIG control scheme and drivetrain model have successfully been realised as a practical system, validating the modelling work carried out in Chapter 5.

The DFIG controller has been shown to be robust to speed and load variation and as a result electro-mechanical interaction has been unidirectional with electrical loading resulting in drivetrain oscillation, but these oscillations do not lead to instability in the electrical network. The exception to this is during high electrical loads where measurement disturbance is sufficient to cause controller instability, these disturbances may well be the result of electro-mechanical interaction but do not appear at known torsional resonances. Current control loop gains have been lowered, reducing bandwidth, and voltage control loop gains have been reduced to improved disturbance rejection.

Once per revolution drive speeds, representing both the input and output of the gearbox, can be seen throughout the rig, as can gear meshing frequencies and low frequency gearbox beating. These effects are not modelled. Suspected structural vibration at 8 Hz and generation frequency at 50 Hz can also be seen.

Electro-mechanical interaction has been demonstrated in the form of transient torsional vibrations triggered by electrical load steps. This validates the mechanical modelling and analysis work carried out in Chapter 3 and the test platform design carried out in Chapter 5, however these oscillations are well damped compared to those in a real GT system. Efforts to reproduce the simulated impact of pulsed electrical loads (in Chapter 4) were unsuccessful because of the high damping levels.
Testing of the test platform has confirmed a system which is frequency-rich. As well as the designed 1st and 2nd mode resonances, which have been successfully identified, numerous other frequencies exist, those categorised are given in Table 6.4.

**Table 6.4: Summary of identified test platform frequencies.**

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5 - 6.3</td>
<td>Gearbox sub harmonic beating</td>
</tr>
<tr>
<td>8</td>
<td>Structural / bedplate vibration</td>
</tr>
<tr>
<td>13</td>
<td>1st mode</td>
</tr>
<tr>
<td>22</td>
<td>2nd mode</td>
</tr>
<tr>
<td>50</td>
<td>Electrical line frequency</td>
</tr>
<tr>
<td>140</td>
<td>Low amplitude vibration (source unknown)</td>
</tr>
<tr>
<td>320 - 747</td>
<td>Gear meshing frequency</td>
</tr>
</tbody>
</table>

Sensing and measurement for the most part has been successful in recording the necessary signals at sufficient resolution.

**6.4.1 Comparison to real GT system**

Torsional oscillations have been more challenging to detect on the test rig than on the real GT system. The construction of the electro-mechanical test platform includes proportionally higher levels of damping and so these oscillations are present for a much shorter period. While the gearbox was manufactured to a high level of precision this is still somewhat short of aero tolerances leading to higher levels of backlash and the sub harmonic beating effect. Gear teeth size is large because of limitations with gear material strength which further adds to backlash levels. The dimension of laboratory systems required additional gearbox spacing to be created by idler gears which added additional meshing stages within the drivetrain. Drive speed is significantly lower in the laboratory system than the real GT system and so drive frequencies and gear meshing frequencies are closer to the resonant frequencies adding disturbance.
Chapter 7  Conclusions
This chapter presents a summary of the research undertaken, and significant findings. Areas for further study are also discussed.

7.1 Review of presented work
This section summarises work undertaken and presented in this thesis.

7.1.1 Literature review
A thorough literature review has been carried out covering the areas of gas turbine operation, aircraft secondary power systems, electro-mechanical interaction, and the doubly-fed induction machine.

The electrification of aircraft secondary power systems is identified as a strategy to reduce aircraft emissions / operating costs. The so called ‘More-Electric Aircraft’ requires greater levels of electrical power for the airframe, and this power is delivered by a flight critical, yet fragile, gas turbine-generator system. High power rating and functional design restrictions make this system prone to electro-mechanical interaction. Understanding and mitigating electro-mechanical interaction is a key to barrier to the adoption of the MEA strategy. Due to the high rating of aircraft electrical power networks constant frequency AC is the preferred distribution standard. However, generating this from a variable speed gas turbine requires additional, undesirable, components. The DFIG is able to decouple mechanical drive speed and electrical generation to produce a constant frequency electrical network with reduced additional systems.

7.1.2 Drivetrain analysis
Modelling strategies for each of the significant drivetrain components have been compared, and the optimal choices integrated to form a full definition model of a gas turbine drivetrain. This has been validated against real engine test data. The behaviour of this drivetrain has been investigated and key resonances determined. Components which are most significant in forming these behaviours have been identified, which has facilitated the development of a reduced order drivetrain model which is able to exhibit the same behaviour as the full system.
Chapter 7 - Conclusions

7.1.3 DFIG evaluation
A laboratory DFIG has been characterised and modelled to produce a dynamic machine model based on standard theory. Power flow through the machine is evaluated for constant frequency operation. Further analysis is carried out of the machines performance as an aero generator and different generation modes (constant frequency and frequency limited) are considered. A control scheme is implemented to simulate voltage and frequency regulation for varying drive speeds and electrical loads. The controlled DFIG model is incorporated with the reduced order drivetrain model to produce a high fidelity electro-mechanical model.

7.1.4 Development of electro-mechanical test platform
A test platform to replicate the behaviour of a large gas turbine-generator system is designed. Components are manufactured and assembled, and the generator control scheme, designed earlier, is implemented using electronic hardware. Safety analysis, balancing and alignment is undertaken before the test platform is commissioned.

7.1.5 Experimental Results
The practical implementation of a DFIG, controlled for constant frequency, is compared to the simulated system. The control method is proven for small load steps, however the voltage regulation is unsuitable for higher load steps. A good match between the trend in power flow is identified, however losses are consistently higher in the real system as machine losses and inverter dynamics are not modelled.

Resonant modes are identified within the mechanical rig. The 1st and 2nd, designed, torsional modes are seen although they appear at slightly lower frequencies than originally intended. It is suspected that components (predominantly the couplings) within the drivetrain are less stiff than intended hence the modes appearing at lower frequencies. Numerous other frequencies are seen which are both resonant in the mechanical system and induced by the platforms operation. These are identified as structural vibration and an electrical frequency, however their further classification (perhaps through comparison of all sensors) is beyond the scope of this research.
Electro-mechanical interaction in the test platform is considered, however the DFIG controller is sufficiently robust to maintain electrical network stability during mechanical disturbance for small loads steps and so interaction is only seen as a mechanical disturbance triggered by electrical loading. Higher load levels do however lead to controller instability, which are suspected to be a result of electro-mechanical interaction, but this has not been confirmed. The test platform achieves representative variable speed operation, with speed dipping as electrical load is increased. A step change in electrical load causes mechanical vibration at the 1st torsional mode, but this is only apparent for a short period, the 2nd torsional mode is not clearly seen. The test rig includes high levels of damping which limits the presence of the 1st mode and makes 2nd mode resonance undetectable.

Care has been taken to produce a high tolerance gearbox, however its damping and non-linearity have a significant influence on the test platform behaviour. Aero grade gearboxes are manufactured to a higher tolerance still and incorporate lower levels of damping, they are also operated a much greater speed, and this reduces the impact of the affects such as backlash on the performance of a real gas turbine-generator system. The test platform is extremely frequency-rich producing a range of resonances beyond that of a real aero system. This has highlighted the care needed in gas turbine drivetrain design if it is hoped to avoid electro-mechanical interaction. The electro-mechanical test platform has been successful in validating both the drivetrain and DFIG modelling work and offers plenty of scope for further work.

7.2 Significant findings

The drivetrain modelling strategy, which is used to form a high fidelity model, is validated against real GT test data. A reduced order model is produced and the methods for its extraction are validated against a practical test rig.

The FOC scheme has been applied to a DFIG and shown to provide voltage and frequency regulation over a speed range of ±40% (around synchronous speed) for standalone operation. This control scheme has been validated by practical work. Rotor-side converter rating is determined by the speed range and restriction on the frequency of electrical generation, a smaller speed range or a wider electrical frequency range
reduces converter rating. Converter rating is determined by speed extension above synchronous speeds (supersynchronous operation) rather than subsynchronous operation. Doubly-fed machines are well suited to aircraft applications due to their ability to decouple electrical and mechanical frequencies, however GT spool speed range requires a near full rating converter for a constant frequency system. A reduced frequency range system will offer benefits for an airframe while having a lower converter rating.

Electro-mechanical interaction has been highlighted as an issue for aero systems, due to their drivetrain design and high generator loading, further more these components are safety critical. The aero gas turbine-generator architecture is relatively unique having a variable speed prime mover, drivetrain, and a standalone controlled generator. However elements of the research are applicable to other areas such as drivetrain vibration in HEVs, drivetrain fatigue in renewable energy systems, and electrical power control in marine systems.

### 7.3 Contribution

Electro-mechanical interaction in aero gas turbine-generator systems is considered at a high fidelity. Simulated electro-mechanical systems have incorporated both accurate drivetrain models and machine models. The DFIG is considered in detail as an aero generator and practical validation carried out. An electro-mechanical test platform is developed which is highly representative of a large diameter gas turbine-generator system, results from this have been used to validate drivetrain analysis and electro-mechanical modelling.

### 7.4 Areas for further work

**Generator bandwidth**

The affect of frequencies from electrical loading have been identified in the mechanical rig. The impact of this on electro-mechanical interaction could better understand if the range of frequencies able to pass through the generator (from electrical load to mechanical drivetrain) could be parameterised.
Chapter 7 - Conclusions

Alternative drivetrain configurations
The use of the third, fuel pump, load on the test platform has been modelled as a method of altering the 1st, but not 2nd, torsional resonance, this can be validated using existing hardware. The addition of a hydraulic pump, and system, will add damping as well has further excitation frequencies and can be investigated using existing hardware.

Alternative generator configurations
Generation has been considered for two equally sized machines, electro-mechanical interaction will be altered if these are sized or geared unequally. This can be achieved through the use of non-matching machine types.

Generator control
A simplistic FOC scheme has been implemented to provide voltage and frequency regulation. Altering these controller gains will affect the coupling between the mechanical and electrical networks potential reducing interaction but also reducing electrical network regulation. More complex control schemes may be used to reduce electro-mechanical interaction, for example notch filtering at mechanical resonant frequencies could be applied to control signals in a similar fashion to that described in industrial automation literature. Disturbances have been added to the current controller on the test platform, creating a frequency sweep to identify torsional modes, an inverse approach could be implemented to provide disturbance rejection.

Characterisation of system resonances
The test platform is very frequency rich, and not all frequencies have been fully characterised. In particular frequencies appearing from the gearbox and electrical systems are of interest and further modelling / investigation is required to fully understand these.

Real electrical loading
Only simple resistive loads have been used on the test platform. Non-resistive loads will pose an additional challenge for the generator controller.
Alternative generation schemes
Constant frequency generation has been implemented for the full mechanical speed range of the test platform, although a frequency limited generation scheme has been suggested which required a lower converter rating. Such a control scheme can be tested using the existing test platform.

Converter interfacing to power network
The test platform uses power from the laboratory network to supply the rotor side converter. True standalone operation, as seen in an aircraft will require the rotor and stator power to be connected, potentially with the addition of a PM machine stage.

Power and frequency scaling
It has been assumed for this research that the test platform at 14.4 kW and 50 Hz provides an accurate representation of a full scale aircraft system in the region of 250 kW and 400 Hz. Further work should be carried out to validate this assumption.

Slip-rings
As mentioned the use of slip-rings is not appropriate for aero systems and so further research would be to compare the performance of a brushless doubly-fed machine within the same electro-mechanical scenario.

Machine characterisation
Parameters determined through the various characterisation tests (both electrical and mechanical) have been used extensively for modelling and test platform design. Further work would allow these parameters to be determined to a greater accuracy and analyse the impact of their variation on system performance.
A. Appendix

Appendix 3.A - State Space Model MATLAB Function

```
function [wn1] = ssNinertia(Ninertia)

% Driveshaft properties
J = 1.0190;
k = 16723;
c = 3.2630;
D = 0.4000;

% Define lumped properties
n = Ninertia; % number of inertia lumps
Jef = J/n;
kef = k*(n-1);
cef = c/(n-1);

% Define SS tf
I = eye(n);
Z = zeros(n);

% Create tridiagonal matrix
Q = sparse(1:n,1:n,-2*ones(1,n),n,n);
E = sparse(2:n,1:n-1,ones(1,n-1),n,n);
S = E+Q+E';
Y = full(S);

% Complete definition of Y matrix
Y(1,1) = -1;
Y(n,n) = -1;
Ass = [(cef/Jef)*Y, (kef/Jef)*Y;I,Z];
Ass(1,1) = Ass(1,1) - (D/Jef);
Bss = zeros(2*n);
Bss(1,1) = 1;
Css = eye(2*n);
Dss = zeros(2*n);
[num,den] = ss2tf(Ass,Bss,Css,Dss,1);
num = num(1,:);

sys = tf(num,den);
bode(sys);
grid on;

[mag,phase,w] = bode(sys);

% convert to pole zero format
[n,d] = eqtflength(num,den);
[z,p,kg] = tf2zp(n,d);

% calculate mode frequencies
wn = abs(p);
damp = real(p)/wn;
wn1 = wn(((2*Ninertia)-2),1);
end
```
Appendix 3.B - Receptance Derivation Example

Receptance derivation example
A single lumped inertia driveshaft, as modelled in Section 3.1 is considered, and a receptance matrix produced. Torques, $T_1$ and $T_2$, are applied at opposite ends of the shaft achieving the responses, $\theta_1$ and $\theta_2$, as shown in Figure A.1.

![Figure A.1: Inertia receptance diagram](image)

When a torque, $T_1$, is applied at coordinate 1, a response occurs at coordinate 1 as described in (A.1). $T_2$ is assumed to be zero.

$$T_i = J\ddot{\theta}_i + k\dot{\theta}_i + c\theta_i$$  \hspace{1cm} (A.1)

If the applied torque, $T_1$, is sinusoidal, then the response, $\theta_1$, in a linear system can also be assumed to be sinusoidal. This leads to the frequency domain equation shown in (A.2), and the direct receptance shown in (A.3).

$$T_1 = \Theta_1(J\omega^2 + c\omega + k)$$  \hspace{1cm} (A.2)

$$\alpha_{11} = \frac{\Theta_1}{T_1} = \frac{1}{J\omega^2 + c\omega + k}$$  \hspace{1cm} (A.3)

Where:

- $T_1$ = amplitude of excitation (Nm), $\Theta_1$ = amplitude of response (rad),
- $\omega$ = frequency of excitation (rad.s$^{-1}$)
It is understandable in this case that both the direct receptances are the same, and since there is no flexibility within the inertia, both the cross receptances are equal and identical to the direct receptances. Therefore the inertia may be described fully by the following receptance matrix:

\[
\alpha = \begin{bmatrix}
\frac{1}{J\omega^2 + c\omega + k} & \frac{1}{J\omega^2 + c\omega + k} \\
\frac{1}{J\omega^2 + c\omega + k} & \frac{1}{J\omega^2 + c\omega + k}
\end{bmatrix}
\]  

(A.4)

Combining receptance subsystems
Receptance modelling allows complex systems to be described in terms of many simplified subsystems. Each component is considered individually to develop a receptance description in terms of its coordinate which are then combined to form multi degree of freedom systems.

Two systems, A and B, have the receptances \(\alpha\) and \(\beta\) respectively. These two subsystems are combined to produce the system C with receptance \(\gamma\) which represents the combined system. This is shown in Figure A.2.
For Figure A.2:

System A:

\[
\theta_1 = \alpha_1 T_1 + \alpha_{12} T_{2A} \quad (A.5)
\]

\[
\theta_{2A} = \alpha_{12} T_1 + \alpha_{22} T_{2A} \quad (A.6)
\]

System B:

\[
\theta_{2B} = \beta_{22} T_{2B} + \beta_{23} T_3 \quad (A.7)
\]

\[
\theta_3 = \beta_{23} T_{2B} + \beta_{33} T_3 \quad (A.8)
\]

At the connecting coordinate, 2, the angular displacements are identical, and the net torque is the sum of the torques applied by each subsystem.

\[
\theta_{2A} = \theta_{2B} = \theta_2 \quad (A.9)
\]

\[
T_{2A} + T_{2B} = T_2 \quad (A.10)
\]
Hence, through rearrangement:

\[ \frac{1}{\alpha_{22}} = \frac{1}{\beta_{22}} + \frac{1}{\gamma_{22}} \]  

(A.11)

It is shown in [151] and [131] that for the combined system, C, the receptances are shown in (A.12).

\[
\gamma = \begin{bmatrix}
\alpha_{11} - \frac{\alpha_{12}^2}{\alpha_{22} + \beta_{22}} & \alpha_{12} - \frac{\alpha_{12}\alpha_{22}}{\alpha_{22} + \beta_{22}} & \frac{\alpha_{12}\beta_{23}}{\alpha_{22} + \beta_{22}} \\
\alpha_{12} - \frac{\alpha_{12}\alpha_{22}}{\alpha_{22} + \beta_{22}} & \frac{\alpha_{22}\beta_{22}}{\alpha_{22} + \beta_{22}} & \beta_{23} - \frac{\beta_{22}\beta_{23}}{\alpha_{22} + \beta_{22}} \\
\frac{\alpha_{12}\beta_{23}}{\alpha_{22} + \beta_{22}} & \beta_{23} - \frac{\beta_{22}\beta_{23}}{\alpha_{22} + \beta_{22}} & \beta_{33} - \frac{\beta_{23}^2}{\alpha_{22} + \beta_{22}}
\end{bmatrix}
\]  

(A.12)

The complexity of the receptances for the combined system increases rapidly if full observation is required for each coordinate. Despite the apparent complexity of a system described by combined receptances, their mathematical structure means they can be solved quickly by a computer.

The process of combining various system arrangements is detailed in [131]. When combined with a derived receptance model of each individual subsystem, and details of combining a receptance network, an accurate mechanical model can be coded and solved with the aid of a computer.
Appendix 3.C - Simulink / SimDriveline Full drivetrain model

Figure A.3 shows the full drivetrain model as implemented in Simulink.

Figure A.3: Screen shot of full drivetrain model in Simulink
Appendix 4.A - Clarke Transform (abc -> αβ)

\[
\begin{bmatrix}
    i_a \\
    i_b \\
    i_c \\
\end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix}
    1 & -\frac{1}{2} & -\frac{1}{2} \\
    0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\
    \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\end{bmatrix} \begin{bmatrix}
    i_a \\
    i_b \\
    i_c \\
\end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix}
    1 & 0 & \frac{1}{\sqrt{2}} \\
    -\frac{1}{2} & \frac{\sqrt{3}}{2} & 2 \\
    -\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{2} \\
\end{bmatrix} \begin{bmatrix}
    i_a \\
    i_b \\
    i_c \\
\end{bmatrix}
\]

A scale factor of \( \sqrt{\frac{2}{3}} \) is chosen to ensure 'invariance of power' in the conversion, input power to the two equivalent model is identical to that of the real three winding system.

Figure A.4 is a vector diagram of the Clarke transform.

![Figure A.4: Clarke transform vector diagram](image-url)
Figure A.5 shows Clarke transform on a three winding system to produce a two winding representation with identical rotation frequency.
Appendix 4.B - Park Transform ($a\beta \rightarrow dq$)

\[
\begin{bmatrix}
i_d \\
i_q \\
i_r
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}\begin{bmatrix}
i_d \\
i_\beta \\
i_r
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_q \\
i_r
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & \sin \theta & 0 \\
-\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}\begin{bmatrix}
i_\beta \\
i_q \\
i_r
\end{bmatrix}
\]

Angle, $\theta$, is given by (A.13).

\[
\theta = \int (\omega - \omega_o) \, dt
\]  
(A.13)

Where:

$\omega$ = rotating electrical frequency of system (rad.s$^{-1}$),

$\omega_o$ = rotating frequency of transformed reference frame (rad.s$^{-1}$).

Park transform enables reference frame transformation by setting a desired frequency of rotation, $\omega_o$. Electrical frequency, $\omega$, is set by the supply frequency and machine design as shown in (A.14).

\[
\omega = \frac{2\pi f}{pp}
\]  
(A.14)

Where:

$f$ = machine supply frequency (Hz),  $pp$ = number of machine pole pairs

Figure A.6 shows the Park transform altering the rotational frequency of a two winding system.
Figure A.6: Park transform winding diagram
Appendix 4.C - DFIG steady-state model derivation

Combining equations (4.4) and (4.5) with (4.6) and (4.7), provides equations (A.15) and (A.16).

\[
\begin{align*}
V_s &= i_s R_s + \frac{d\psi_s}{dt} \quad (A.15) \\
V_r &= i_r R_r + \frac{d\psi_r}{dt} - j\omega \psi_r \quad (A.16)
\end{align*}
\]

Where:

\[
\begin{align*}
V_s &= \tilde{V}_s e^{j\alpha} = |V_s| e^{j\phi} e^{j\alpha} = V_{sd} + jV_{sq} \\
i_s &= \tilde{i}_s e^{j\alpha} = |I_s| e^{j\phi} e^{j\alpha} = I_{sd} + jI_{sq} \\
\psi_s &= \tilde{\psi}_s e^{j\alpha} = |\psi_s| e^{j\phi} e^{j\alpha} = \psi_{sd} + j\psi_{sq}
\end{align*}
\]

In steady-state, \(\tilde{V}, \tilde{i}\), and \(\tilde{\psi}\) are constant complex values. Replacing vectors in (A.15) with complex notations gives (A.20), which is integrated leaving (A.21).

\[
\begin{align*}
\tilde{V}_s e^{j\alpha} &= \tilde{I}_s e^{j\alpha} R_s + \frac{d\tilde{\psi}_s e^{j\alpha}}{dt} \quad (A.20) \\
\tilde{V}_s &= \tilde{I}_s R_s + j\omega \tilde{\psi}_s \quad (A.21)
\end{align*}
\]

(4.8) and (4.9) are incorporated with (A.21) to give (A.22) which describes the stator side steady-state equivalent circuit which is shown in Figure A.7.

\[
\tilde{V}_s = \tilde{I}_s R_s + j\omega \left( L_s \tilde{i}_s + L_m \tilde{i}_r \right) \quad (A.22)
\]
Replacing vectors in (A.16) with complex notations gives (A.23) which is integrated leaving (A.24).

\[
\tilde{V}_r e^{j\omega t} = \tilde{I}_r e^{j\omega t} R_r + \frac{d\tilde{\psi}_r e^{j\omega t}}{dt} - j\omega \tilde{\psi}_r, e^{j\omega t}
\]  
\[
(A.23)
\]

\[
\tilde{V}_r e^{j\omega t} = \tilde{I}_r e^{j\omega t} R_r + (j\omega \tilde{\psi}_r - j\omega \tilde{\psi}_r) e^{j\omega t}
\]  
\[
(A.24)
\]

(A.24) is simplified to give (A.26).

\[
\tilde{V}_r = \tilde{I}_r R_r + (j\omega \tilde{\psi}_r - j\omega \tilde{\psi}_r)
\]  
\[
(A.25)
\]

\[
\tilde{V}_r = \tilde{I}_r R_r + j(\omega - \omega_r) \tilde{\psi}_r
\]  
\[
(A.26)
\]

The rotor slip function (A.27) is introduced.

\[
\frac{\omega}{\omega - \omega_r} = \frac{1}{s}
\]  
\[
(A.27)
\]

Where:

\(\omega\) = Synchronous frequency (rad.s\(^{-1}\)), \(\omega_r\) = Rotor speed (electrical) (rad.s\(^{-1}\)), \(s\) = slip

(A.27) is incorporated with (A.26) to give (A.28) which is combined with (4.10) and (4.11) to give (A.29) describing the rotor side steady-state equivalent circuit shown in Figure A.8.

\[
\frac{\tilde{V}_r}{s} = \frac{\tilde{I}_r R_r}{s} + j\omega \tilde{\psi}_r
\]  
\[
(A.28)
\]
\[
\frac{\vec{V}_r}{s} = \bar{i}_r \frac{R_r}{s} + j\omega \left( L_r \bar{i}_r + L_m \bar{i}_s \right)
\]  

(A.29)

**Figure A.8:** DFIG rotor side steady-state equivalent circuit

Stator side, and rotor side equivalent circuits are combined to produce the full DFIG steady-state equivalent circuit shown in Figure A.9.

**Figure A.9:** DFIG steady-state equivalent circuit model (dq)
Appendix 4.D - DFIG model Simulink implementation

Figure A.10: Machine core model in Simulink (screen shot)
Figure A.11: Full DFIG model in Simulink (screen shot)
Appendix 4.E - DFIG electrical characterisation

DC resistance test

This test identifies the phase resistance of both the stator and rotor circuits, the current is measured and used to determine the resistance of each phase. As no neutral point is available DC voltage is applied between two phases, and the actual resistance assumed to be half of the measured value. Ideally this test would be carried out a working current and temperature however low current, ambient temperature tests are considered sufficiently accurate.

The winding resistances and derived single phase resistances are shown in Table A.1

<table>
<thead>
<tr>
<th>Table A.1: DFIG rotor and stator resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator resistance (Ω)</td>
</tr>
<tr>
<td>-----------------------</td>
</tr>
<tr>
<td>R-Y</td>
</tr>
<tr>
<td>R-B</td>
</tr>
<tr>
<td>Y-B</td>
</tr>
<tr>
<td>Phase mean</td>
</tr>
</tbody>
</table>

The resistance of individual phases are identical within the tolerances of the experiment and so it can be assumed phase windings are well balanced.

Testing on the rotor side excludes the slip-ring resistance as they renowned for being highly non-ohmic with a resistance that also varies with rotor speed. A comparison of mean rotor winding resistance measurements with (0.525 Ω) and without (0.449 Ω) slip-rings included indicates a stationary slip-ring resistance of 0.076 Ω. However it would be inaccurate to assume a slip-ring resistance of this value as it is know that slip-ring resistance varies substantially with rotor speed and supply voltage. Their resistance is sufficiently low and non-consistent and so neglected from the electrical machine model.

No-load test

The machine is driven at synchronous speed by a coupled motor ensuring minimal slip. This causes rotor resistance to become sufficiently high that the equivalent circuit can be reduced and is shown in Figure A.12.
A 50 Hz supply is applied to the stator side, measurements are taken of the supply voltage, current and real power from the supply side. The test is performed at a range of voltages from zero to the rated 240 V to replicate the range of operating conditions. The ability, provided by the slip-rings, to leave the rotor side open circuit further ensures that the no-load reduced equivalent circuit is true. Stator reactance ($X_s$) is neglected, however the derived value of R1 is used. $X_m$ and $R_m$ can therefore be derived for each phase, and an average taken. Connection to the rotor side allows this test to be repeated with the supply connected to the rotor side and the stator side left open circuit.

The stator side no-load test results are shown in Figure A.13. The rotor side no-load test results are shown in Figure A.14.
$x_m$ can be seen to increase with supply voltage initially but to reduce above 160V due to saturation. Similarly the general trend of $R_m$ is seen to increase with supply voltage. There is also a considerable variation in the derived value of $R_m$ between phases. In some cases it was not possible to get results for each phase because of the marginal unbalance between phase windings. These erroneous results have been removed.

Locked-rotor test

The locked rotor test involves short-circuiting one of the windings while supplying power to the other, with the rotor mechanically locked. It is assumed that $R_m$ and $X_m$ are significantly larger than $R_s$ and $X_s$ and $R_r'$ and $X_r'$ respectively allowing the equivalent circuit to be simplified, by the removal of $R_m$ and $X_m$, as given in Figure A.15.

The ratio of $R_s$, $R_r'$, and $X_s$, $X_r'$ is unknown and is assumed to be 1:1 in both cases. Values of $R_s$, $R_r'$, $X_s$, and $X_r'$ are derived using the measure values of real and apparent power.
Again measurements the tests are carried out from both the rotor and stator sides, and over a range of currents up to the rated 20 A. The results are shown in Figure A.16 and Figure A.17.

![Figure A.16: Locked rotor test - rotor short circuited](image)

![Figure A.17: Locked rotor test - stator short circuited](image)

Resistances and reactances are both approximately constant with respect to current. As with previous tests the rotor side tests derive a higher value of both resistance and reactance.

The results of the no-load and locked rotor tests are summarised in Table A.2.
Appendix

Table A.2: No-load and locked rotor test results

<table>
<thead>
<tr>
<th></th>
<th>Stator powered</th>
<th>Rotor powered</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_s$ and $R'_s$</td>
<td>0.421</td>
<td>0.467</td>
</tr>
<tr>
<td>$X_s$ and $X'_s$</td>
<td>1.100</td>
<td>1.218</td>
</tr>
<tr>
<td>$R_m$</td>
<td>250</td>
<td>250</td>
</tr>
<tr>
<td>$X_m$</td>
<td>21.095</td>
<td>24.346</td>
</tr>
<tr>
<td>Resistance, stator:rotor</td>
<td>1:1.110</td>
<td>1:1.108</td>
</tr>
<tr>
<td>Reactance, stator:rotor</td>
<td>1:1.108</td>
<td>1:1.108</td>
</tr>
</tbody>
</table>

IEEE 112 Standard

This standard outlines a thorough testing procedure for induction machines. This utilises initial assumptions based on machine structure and an iterative mathematical process is then used to identify machine parameters. Apparent power and real power measurements are taken for a range of voltages under both locked rotor and no load testing conditions. Four methods are offered and the choice is dependant on practicalities. Here Method 1 is used with 5 iterations.

The IEEE 112 Std test does not offer a method of determining magnitising resistance, as it is common to omit this term in machine models.

Stator side (rotor short circuited) results are shown in Figure A.18, and rotor side (stator short circuited) results in Figure A.19.

Figure A.18: IEEE 112 Std test - stator side
As per the standard machine tests, $X_s$ and $X'_r$ are again constant with current, while $X_m$ shows the effects of saturation above 160 V. The selected value is taken between this peak value, at 160 V, and rated voltage, 240 V.

**Table A.3: IEEE 112 Std results**

<table>
<thead>
<tr>
<th></th>
<th>Stator powered</th>
<th>Rotor powered</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_s$</td>
<td>1.069</td>
<td>1.183</td>
</tr>
<tr>
<td>$X'_r$</td>
<td>1.185</td>
<td>1.312</td>
</tr>
<tr>
<td>$X_m$</td>
<td>24.174</td>
<td>27.500</td>
</tr>
</tbody>
</table>

Rotor side parameters are again higher than the stator side parameters, this is especially true in the case of the magnetising reactance. Winding reactances are both slightly higher for the rotor powered system. This is a direct result of the calculation method employed where $X'_r$ is simply taken as a scaled version of $X_s$ after the last iteration.

**Selection of parameters**

Winding resistances, $R_s$ and $R'_r$, are taken from the DC resistance tests. The sum of winding reactances, $X_s$ and $X'_r$, is determined by taking a mean from the IEEE 112 Std tests, this value is split according to the ratio determined from the locked rotor machine test to find the individual terms. Magnetising resistance is taken from the no-load tests, as it is not calculated using the IEEE 112 Std tests. Magnetising reactance is taken from the mean of the IEEE 112 Std test results.
While every effort is made to ensure the accuracy of the machine characterisation, and best methods are used, there are approximations and inaccuracies associated with their derivation. It has been shown that these values are not consistent, and in fact vary with operating conditions.

Parameter range

With the aim of appreciating the limited accuracy of the machine model the parameters are considered over different operating conditions.

Winding Resistance

Each characterisation test was undertaken at room temperature (≈20°C). It is therefore predictable that during machine operation, as the temperature rises, winding resistance will also change. The resistance of copper varies with temperature in the predictable manner described by (A.30).

\[
R = R_o \left(1 + \alpha \left(T - T_o \right) \right)
\]

(A.30)

where:

\(R\) = resistance (Ω), \(R_o\) = resistance at temperature \(T_o\) (Ω), \(T\) = temperature (°C),

\(T_o\) = Initial temperature (°C), \(\alpha\) = Temperature coefficient of resistance

The temperature coefficient of resistance for copper, \(\alpha = 0.00393\) [152]. A maximum winding temperature, limited by the winding insulation, is assumed to be 100°C (assumed based on THHN cable insulation which is thermally rated at 90° for continual operation). The predicted maximum winding resistances are derived based on the maximum operating temperature in relation to the minimum operating temperature. Results are shown in Table A.5.
Magnetising Reactance

Magnetising reactance, \( X_m \), varies with supply voltage as shown in Figure A.13 and Figure A.14.

The rotor open-circuited configuration ranges from 17.624 \( \Omega \) at 238.170 V to 24.062 \( \Omega \) at 118.760 V. The stator open-circuited configuration ranges from 20.610 \( \Omega \) at 238.390 V to 27.111 \( \Omega \) at 121.850 V. The maximum and minimum values for \( X_m \) are shown in Table A.5.

| \( R_s \) | 0.297 to 0.391 |
| \( R'_r \) | 0.449 to 0.591 |
| \( X_s \) | 1.13 |
| \( X'_r \) | 1.25 |
| \( R_m \) | 250 |
| \( X_m \) | 17.6 to 27.1 |
Appendix 4.F - DFIG mechanical characterisation

Falling mass test

This experiment is used to determine the machine's inertia. A hanging mass is used to create a constant torque on the machine shaft, and the rate of fall of this mass is directly proportional to the machine angular acceleration, which is related to rotor inertia. The experimental arrangement shown in Figure A.20.

A cable is wound round the machine shaft and over a pulley mounted higher than the machine. A known mass, \( m \), is attached to the end of the cable so that under gravitational acceleration, \( g \), it applies tension in the cable, producing torque, \( T \), and causing an angular rotation, \( \omega \), of the machine inertia, \( J \).

The machine shaft is held stationary with the mass at position \( x_0 \). The shaft is then released, allowing it to accelerate as the mass falls. The time taken from release at position \( x_0 \) to crossing point \( x_1 \) is recorded.

Machine damping (bearing and windage) are neglected, the machine slip-rings are detached to remove their friction from the system. Pulley mass and friction as well as cable mass are neglected.

Machine inertia is derived and given in (A.31).
A number of tests are carried out with differing mass and fall distance, the calculated inertias are shaft against mass and fall distance in Figure A.21.

![Diagram showing inertia against mass and fall distance]

Figure A.21: Falling mass inertia calculation results

Although the calculated inertia is similar for most values of mass used, it can be seen that it is substantially higher than this trend when the lowest mass is used (1 kg). It is probable that due to the lower accelerating force, damping and losses (machine bearing and pulley) have a relatively higher impact on the calculation, increasing the derived inertia.

The inertia calculation against fall distance shows two distinct groups, at the fall heights just above 2 m and approximately 0.7 m. Experiments with the longer fall distance produce a slightly higher value of inertia. Bearing damping is often considered to be proportional to speed. The higher drop will allow the mass to accelerate to a higher speed, therefore overall system damping will be greater, and the predicted inertia higher.

Inertia values derived at the lowest inertia are neglected as their results do not match the other trends. Values calculated from both heights are included. The mean of these values produces an estimated machine inertia of 0.411 kg.m².
Spin-down test

A similar method is described in [135]. The rotor side of the DFIG is short-circuited, and the stator is connected to a 50 Hz supply, this drives the machine up near synchronous speed at 1,000 rpm. The electrical supply is then cut, removing the drive torque, and causing the rotor to continue rotating as a result of its inertia. Speed is recorded against time as the rotor speed slows due to the effects of damping (including bearing and windage).

The mechanical response of the machine is modelled by (A.32).

\[ T = J \ddot{\theta} + D \dot{\theta} \]  
\[ \quad (A.32) \]

\( T \) = Driving torque (Nm), \( J \) = Rotor inertia (Nm.m\(^2\)), \( D \) = System damping (Nm.s.rad\(^{-1}\)), \( \theta \) = Shaft angular (rad)

Solving this differential equation, through Laplace transform, predicts an exponential decay in shaft speed when the motor supply is disconnected, as given in (A.33).

\[ \omega = \omega_o e^{-\frac{D}{J}} \]  
\[ \quad (A.33) \]

Figure A.22 shows shaft speed versus time for both the experimental data and mathematical model. Parameters (\( J = 0.411 \text{ kg.m}^2 \) and \( D = 0.015 \text{ Nm.s.rad}^{-1} \)) for the mathematical model are selected to best match the trend of the experimental data.
A reasonable match between experimental data and modelled data can be seen. The derived value of damping, $D = 0.015 \text{ Nm.s.rad}^{-1}$, is taken for the machine.

The mathematical model assumes damping is proportional to speed, although as discussed in Chapter 3 this is rarely true. The experimental data shows a more linear speed decay which suggests higher proportion of constant damping across the whole speed range.

**Summary of mechanical parameters**

The mechanical parameters of the machine, derived using the falling mass test and spin-down tests, are shown in Table A.6.

<table>
<thead>
<tr>
<th>Table A.6: DFIG mechanical properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inertia</td>
</tr>
<tr>
<td>Damping</td>
</tr>
</tbody>
</table>

It is acknowledged that both the damping and inertia terms are approximate due to inaccuracies associated with the testing methods available. They are however considered accurate enough for the production of a DFIG model.
Appendix 4.G - DFIG power flow derivation

The steady-state DFIG model, in the stationary reference frame, as described by (A.21) and (A.26), is taken.

\[ \tilde{V}_s = \tilde{I}_s R_s + j \omega_s \tilde{\psi}_s \]  \hspace{1cm} (A.34)

\[ \tilde{V}_r = \tilde{I}_r R_r + j (\omega_s - \omega_r) \tilde{\psi}_r \]  \hspace{1cm} (A.35)

Where:

\[ \tilde{\psi}_s = L_s \tilde{I}_s + L_m \tilde{I}_r \]  \hspace{1cm} (A.36)

\[ \tilde{\psi}_r = L_r \tilde{I}_r + L_m \tilde{I}_s \]  \hspace{1cm} (A.37)

The stator side equation, (A.34), is combined with (A.36) and multiplied by \( \tilde{I}_s^* \). The real components selected to derive real power.

\[ \text{Re}\left[ \tilde{V}_s \tilde{I}_s^* \right] = |I_s|^2 R_s + \omega_s L_m \text{Im}\left[ \tilde{I}_r \tilde{I}_s^* \right] \]  \hspace{1cm} (A.38)

\[ P_{\text{stator}} = P_{\text{stator loss}} + P_{\text{gap}} \]  \hspace{1cm} (A.39)

Similarly the rotor side equation, (A.35), is combined with (A.37) and multiplied by \( \tilde{I}_r^* \). The real components are selected to derive real power.

\[ \text{Re}\left[ \tilde{V}_r \tilde{I}_r^* \right] = |I_r|^2 R_r - \omega_s L_m \text{Im}\left[ \tilde{I}_r \tilde{I}_s^* \right] + \omega_r L_m \text{Im}\left[ \tilde{I}_s \tilde{I}_s^* \right] \]  \hspace{1cm} (A.40)

\[ P_{\text{rotor}} = P_{\text{rotor loss}} - P_{\text{gap}} + P_{\text{mechanical}} \]  \hspace{1cm} (A.41)

It can be seen that rotor power, stator power, mechanical power, and gap power is given by (A.42), (A.43), (A.44), (A.45).

\[ P_{\text{rotor}} = \text{Re}\left[ \tilde{V}_r \tilde{I}_r^* \right] \]  \hspace{1cm} (A.42)
\[
\begin{align*}
\mathcal{P}_{\text{stator}} & = \Re\left[ V_s^* I_s \right] \quad (A.43) \\
\mathcal{P}_{\text{mechanical}} & = \omega_r L_{m} \Im\left[ I_r^* I_s \right] \quad (A.44) \\
\mathcal{P}_{\text{gap}} & = \omega_r L_{m} \Im\left[ I_r^* I_s \right] \quad (A.45)
\end{align*}
\]

Mechanically torque is generated by the machine as given in \((A.46)\) which leads to the derivation of mechanical power given in \((A.44)\).

\[
\begin{align*}
T_{em} & = ppL_{m} \left( i_r \wedge i_s \right) \quad (A.46) \\
T_{em} & = ppL_{m} \Im\left[ I_r^* I_s \right] \quad (A.47)
\end{align*}
\]

It is noted that gap power is related to mechanical power as shown in \((A.48)\)

\[
\mathcal{P}_{\text{mechanical}} = \omega_r L_{m} \Im\left[ I_r^* I_s \right] = \mathcal{P}_{\text{gap}} \frac{\omega_r}{\omega_s} \quad (A.48)
\]

Figure A.23 shows the power flows in the DFIG as well as the power direction conventions as set out by the steady-state equivalent circuit model current directions.

![DFIG schematic with power flow directional convention](image)

Combining power equations \((A.39)\) and \((A.41)\) provides an equation for power within the DFIG, \((A.49)\)

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At this point rotor and stator copper losses are neglected.

The rotor power equation, (A.41), is combined with (A.48).

\[
\begin{align*}
P_{\text{rotor}} + P_{\text{stator}} &= P_{\text{rotorloss}} + P_{\text{statorloss}} + P_{\text{mechanical}} \\
(A.49)
\end{align*}
\]

Given that the machine is operating as a generator, and so \( P_{\text{gap}} \) is negative, (A.51) demonstrates that power must be injected into the rotor during subsynchronous operation, and that power can be extracted from the rotor during supersynchronous operation.
Appendix 5.A - Gear alignment calculations

Figure A.24 shows the gear alignments and dimension labels.

![Gear placement design](image)

Figure A.24: Gear placement design

The outer diameter (OD) of the gear indicates the physical space which the gear requires to operate in. Gear positions are defined by the PCD of the two meshing gears plus the clearance required between the two. This clearance allows space for lubrication to act. For the selected gear range (HPC, SH3) suggested gear dimensions are taken from the supplier catalogue, [153], and given in Table A.7.

<table>
<thead>
<tr>
<th>Gear</th>
<th>PCD (mm)</th>
<th>OD (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SH3-24</td>
<td>75.5988</td>
<td>81.5988</td>
</tr>
<tr>
<td>SH3-32</td>
<td>100.7984</td>
<td>106.7984</td>
</tr>
<tr>
<td>SH3-53</td>
<td>166.9474</td>
<td>172.9474</td>
</tr>
</tbody>
</table>

The centre displacement is calculated as the sum of the half PCD of each gear plus a designed clearance, as given in

\[ \text{Displacement} = \frac{PCD_A}{2} + \frac{PCD_B}{2} + \text{clearance} \quad (A.52) \]

Where:
Appendix

Displacement = displacement between gear centres (mm), clearance = designed clearance between gears (mm), \( PCD_A \) = PCD of gear A, \( PCD_B \) = PCD of gear B

HPC helical gears are designed to have a clearance of between 0.00 mm and 0.05 mm, [153]. As the gearbox mechanism is designed for a limited lifespan and low duty cycle running only minimal lubricating is required, while at the same time it is desirable also to reduce backlash. Clearance is therefore designed to the minimal size at 0.00 mm.
Appendix 5.B - Flywheel safety calculations

Energy

The energy stored in a flywheel is given by (A.53).

\[ E = \frac{1}{2} J \omega^2 \]  

(A.53)

Where:

- \( E \) = Kinetic energy (J),
- \( \omega \) = rotational speed of flywheel (rad.s\(^{-1}\))

The maximum energy stored in the GT flywheel and generator flywheel is 33.4 kJ and 4.4 kJ respectively. Despite the lower maximum speed, significantly more energy is stored in the GT flywheel than generator flywheel so as the worst case the GT flywheel will be considered here for safety analysis. Similar analysis is also carried out on the generator flywheel.

When spinning at maximum speed the high levels of stored energy and a circumferential speed of 34.2 m.s\(^{-1}\) (77 mph) make it essential to ensure that the flywheel and mounting will not fail. Care is taken to ensure that bearings and supports are rated (with a suitable safety factor) to take the load weight of the flywheel, however failure on such a rotating system is most likely to be induced in rotary components by vibration.

Vibration

Fundamentally it is desirable to design any system so that excitation frequencies, induced during operation, are kept well below that of any resonant frequencies. When this is not possible, the system can be designed to operate above resonant frequencies, in this instance elements must be accelerated through the critical speed quickly before resonant modes have time to build amplitude. This method is used in power station systems which exhibit low resonant frequencies and are driven at a higher speed. If drive power is insufficient to accelerate the system rapidly through the critical speed, however, the system may stall in resonance, this is known as the Sommerfield effect [120]. The resonant modes of the flywheel and hub design must be identified to ensure that the test rig can be operated safely at the required rotational frequencies.
The flywheel is essentially a large mass supported via a sprung coupling at either side. Hence, there are three types of vibration which much must be considered, torsional, lateral and shaft whirl.

**Torsional vibration**

The torsional natural frequency of the GT flywheel and supporting shaft is derived as shown in Figure A.25.

The two stiffness elements are considered in parallel as a fixed-fixed torsional model is assumed. From the known material properties of the supporting shafts torsional stiffness is calculated as 389,432 Nm.rad\(^{-1}\). With inertia of 7.0 kg.m\(^2\), the torsional natural frequency of the flywheel system is derived as 37.5 Hz. This is close to the designed 2nd mode torsional resonance of the rig, however this mode exists between the DFIG and generator flywheel only, so is not expected to excite resonance within the GT spool flywheel structure. This frequency is also above the flywheel drive speed 2nd harmonic preventing excitation from this source.

**Lateral vibration**

Lateral vibration is an externally excited vibration which results in a lateral displacement of the system. The frequency of natural lateral vibration is derived from the lateral stiffness of the beam and mass of the flywheel. Free lateral vibration is shown in for the GT flywheel Figure A.26.
It is assumed that the flywheel has a point mass, and that the support bearings are placed equidistant either side of it. The distance to the centre of the bearings is used (they are considered short bearings).

Note: the shaft length used to derive torsional stiffness is greater than that used to derive lateral stiffness. This is because the bearings support the lateral load close to the flywheel, while the torsional effect extends beyond the bearing to the coupling.

The lateral stiffness is given by (A.54), from [118].

\[ k_{lat} = \frac{48.\pi \cdot E \cdot I_A}{L^2} \]

(A.54)

Where:

- \( k_{lat} \) = Lateral stiffness (N/m),
- \( L \) = Length between supports (m),
- \( E \) = Young's modulus (N/m²),
- \( I_A \) = Second moment of area (m⁴)

The Young's modulus (or modulus of elasticity) for steel is taken as 200 GPa, [118].

The second moment of area for the shaft in this action is given in (A.55), [118].

\[ I_A = \frac{\pi \cdot d^4}{64} \]

(A.55)

Where:

- \( d \) = shaft diameter (m)

The lateral stiffness of the flywheel system is calculated as \( 241 \times 10^6 \text{N.m}^{-1} \) and the flywheel mass is approximately 115 kg giving a natural frequency of 230.3 Hz. This
frequency is well separated from other resonant frequencies and excitation frequencies throughout the rig and so unlikely to be excited.

Shaft whirl

It is impossible to perfectly balance a rotating mechanical system. Hence the true centre of mass will be eccentric from the axis of rotation. When the flywheel is rotated this eccentricity causes centrifugal force which deflects the shaft in the direction of the eccentricity, further increasing the eccentricity and centrifugal force. It is the lateral elastic stiffness of the shaft which opposes the forces due to the induced eccentricity.

Shaft whirl is a self excited vibration caused by the rotating shaft and inevitable eccentricity where centrifugal force due to imbalance induces lateral vibration.

This process is shown schematically in Figure A.27.

Equating centrifugal force and lateral stiffness produces equation (A.56), [147].

\[
\delta = \frac{\omega^2 e}{k_{lat} \frac{m - \omega^2}{m}} \tag{A.56}
\]

Where:
$m$ = flywheel mass (kg), $\delta$ = deflection due to rotation (m), $e$ = natural eccentricity between axis of rotation and centre of mass (m), $\omega$ = angular rotation (rad.s$^{-1}$),

$$k_{lat} = \text{shaft lateral stiffness (N.m}^{-1}\text{)}$$

It can be seen that as the value of $\frac{k_{lat}}{m}$ approaches $\omega^2$, the deflection becomes maximal. The shaft speed at which this occurs is known as the critical speed, in the case of the GT spool flywheel this is 230.3 revolutions per second, or just over 13,800 rpm.

The rotation of the system around this critical speed is likely to exert excess stress within the support structure and must be avoided. Operating speed should be kept below half of the critical speed [118], hence the failure speed of the flywheel is considered to be 6,900 rpm. The maximum speed of the GT flywheel is 933 rpm, therefore a safety factor of 7 is achieved.

Material failure
During operation centripetal forces act on the flywheel, affectively pulling it apart. Should centripetal forces exceed the ultimate strength of the flywheel it could be considered a dangerous and catastrophic failure. The flywheel is designed so that the material and structure provide sufficient strength to ensure that tangential and radial stresses do not overcome the yield strength of the material (steel). Although cyclic loading will reduce the yield strength in the long term, for the system being considered (low operational time and low level speed fluctuations) safe operation can be achieved by utilising a substantial safety factor against original material sheer strength.

Stresses on flywheel
While rotating, elements of material throughout the flywheel experience radial and tangential stress, this is shown in Figure A.28.
Tangential and radial stresses are constant around the disk at any given radius, but vary throughout the radius of the flywheel as given in (A.57) and (A.58), [120].

\[
\sigma_t = \rho \omega^2 \left( \frac{3 + \nu}{8} \right) \left( r_t^2 + r_o^2 + \frac{r_t^2 r_o^2}{r^2} - \frac{1 + 3\nu}{3 + \nu} r^2 \right),
\]

(A.57)

\[
\sigma_r = \rho \omega^2 \left( \frac{3 + \nu}{8} \right) \left( r_t^2 + r_o^2 - \frac{r_t^2 r_o^2}{r^2} - r^2 \right),
\]

(A.58)

Where:

\( \sigma_t, \sigma_r \) = Tangential / radial stress (Pa), \( \nu \) = Poisson’s ratio, \( \rho \) = Material mass density (kg.m\(^{-3}\)), \( \omega \) = Angular velocity (rad.s\(^{-1}\))

For the GT flywheel in these stresses are shown in Figure A.29.
Radial stress peaks at approximately one third radius. Tangential force is highest at the inner radius of the flywheel and drops linearly around one third radius and thereafter. The highest stress in the flywheel occurs tangentially at the inner radius, it is this stress which must be considered for safe, reliable, flywheel wheel design. The 6 bolt holes for securing the disk to the hub are not considered, their presence would increase local stress concentration [120], however due to their low diameter compared to material area their impact is too low to consider.

A peak tangential stress of 7.5 MPa (occurring at the inner radius) is substantially less than the typical ultimate tensile strength of steel at 500 MPa [118], hence the design achieves a safety factor of 66.

These calculations do not take into account the stresses on the separate flywheel hub during rotation. It is assumed that the flywheel and hub is perfectly balanced (although they are not be perfectly balanced by manufacture, balancing is undertaken by an external company) and sufficiently well supported, therefore only balanced centrifugal force is considered. As well has having a much lower radius, the hub is effectively much thicker, so stress is far lower than for the disc itself (tangential stress on the hub = 0.034 MPa). Variations in hub cross-sectional geometry along its length lead to
localised stress concentrations [120], this are minimised by the use of large radii [120]. Engineering drawings for the GT flywheel hub are shown in Appendix 5.E.

Peak tangential and radial stresses for both the GT flywheel and generator flywheel are given in Table A.8.

<table>
<thead>
<tr>
<th>Flywheel</th>
<th>Peak tangential stress</th>
<th>Peak radial stress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MPa</td>
<td>% ultimate</td>
</tr>
<tr>
<td>GT flywheel</td>
<td>7.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Generator flywheel</td>
<td>1.4</td>
<td>0.3</td>
</tr>
</tbody>
</table>

While the GT flywheel has higher peak tangential and radial stresses than the generator flywheel, both are well below their ultimate ratings.
Appendix 5.C - Test platform controller

Figure A.30: Test platform control, top view - screen shot

Figure A.31: Test platform control, inverter control (screenshot)
Figure A.32: Test platform control, inverter interface (screenshot)
Appendix 5D - Pruftechnik Balance Results

27th June 2011

Priority Level: Satisfactory

Test System

Findings:

The initial set of readings found the motor drive end bearing to be the highest with regards to an imbalance with a value of 0.30 mm/s RMS at 960rpm.

After the balance had been carried out the amplitude had been reduced to 0.10 mm/s RMS at 960rpm however the overall vibration is still trending at 1.5 mm/s RMS.

As you can see in the trend below there was a slight increase in trend on the 3rd reading this is due to the unit being on load and is normal.

The balance was achieved by adding mass to the bolts on the face of the fly wheel.

Please see below graphs for balance confirmation.
Appendix 5.E - Test rig engineering drawings

A collection of engineering drawings for mechanical components of the test rig.

Figure A.33: Gearbox - engineering drawing

Figure A.34: GT flywheel hub - engineering drawings
Figure A.35: Generator flywheel hub - engineering drawing

Figure A.36: GT flywheel and mount - engineering drawing
Figure A.37: Generator flywheel and mount - engineering drawing

Figure A.38: Full test rig - engineering drawing
References


[101] F. Khatounian, E. Monmasson, F. Berthereau, E. Delaleau, and J. P. Louis, "Control of a doubly fed induction generator for aircraft application," in
References


[105] W. Leonhard, "Field orientated control of a variable speed alternator connected to the constant frequency line,“ in IEEE conference on Control of power systems Houston, 1979, pp. 149-153.


