CONTROL SYSTEMS FOR SWITCHED RELUCTANCE
AND PERMANENT MAGNET MACHINES IN
ADVANCED VEHICULAR ELECTRIC NETWORKS

A thesis submitted to the University of Manchester
for the degree of Doctor of Philosophy
in the Faculty of Engineering and Physical Sciences

2012

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N Simulation Results: IPM Motor

O FFT Analysis: SPM Motor

P FFT Analysis: IPM Motor

Q Simulation Results: SPM Generator

R Simulation Results: IPM Generator

S FFT Analysis: SPM Generator

T FFT Analysis: IPM Generator

Word Count: 77987
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Abstract

This thesis presents the design and analysis of specialised control systems for switched reluctance (SR) and permanent magnet (PM) machines in vehicular electric applications. Control systems for operation in motoring and power generation are considered for both the types of machines. The SR machine operation considered in this thesis is mainly focused towards the application of aero-engine starter/generators. The control designs for PM machines are formulated considering general fault-tolerant and isolated multiphase PM machines which can be applied in the majority of safety-critical vehicular power and propulsion applications.

The SR motoring mode presented in this thesis considers the control design for operation from zero speed to a high speed range, while SR generation mode is confined to the high speed range, such as for the requirements of aero-engine starter/generator operation. This thesis investigates applied control methods for both single-pulse and chopping modes of operation. Classical excitation control versus peak current control, and the introduction of a zero-voltage interval are compared for SR motor operation. Optimized excitation control versus two classical forms of excitation control are developed and compared for SR generator operation. Studies include simulation of a 12/8 250kW machine and experimental work on a 6/4 300W machine.

The PM motoring and power generation considered in this thesis focuses on a special class of PM machines and drives which are specifically designed for fault-tolerant operation. Optimized control strategies for the operation of PM machines with the parallel H-bridge per-phase converter architecture are investigated. Mathematical modelling of the machine and drive with a consideration of harmonics is presented. The developed control methods are then evaluated by means of finite-element model based simulations of a 125kW five phase surface PM rotor machine and an interior PM rotor machine.
Declaration

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Dedication

This thesis is dedicated to my father, Professor Noel Fernando and my mother, Mrs. Nalini Fernando for their unconditional love and their constant support. This thesis is also dedicated to my sister, Ayanthi and her family for their encouragement throughout.
I owe my deepest gratitude to my supervisor Dr. Mike Barnes for his guidance, intellectual advice and insight which appreciably assisted the research to progress on course. His admirable efficiency, dedication and punctuality deserve respect and have inspired me to advance further in engineering and research.

I would like to acknowledge the valuable advice and suggestions given by Dr. Ognjen Marjanovic in relation to control systems design, simulation and analysis. I would also like to acknowledge the constructive criticisms and comments given by Dr. Judith Apsley on the mathematical modelling of permanent magnet machines, which form the basis for a part of the thesis. Finally, I would like to thank Dr. Prabath Binduhewa for the time spent with general discussions in the initial part of the research which undoubtedly formulated the theme of the overall thesis. I would also like to acknowledge the support given by all the members of the power conversion group of the University of Manchester during the course of this research.

I would also like to thank the examiners, Professor Sandy Smith and Professor Barrie Mecrow for their valuable feedback and discussion for the revision of this thesis.

I would like to express my gratitude to the funding bodies of the Overseas Research Students Awards Scheme (ORSAS) and the University of Manchester, School of Electrical and Electronic Engineering Bursaries, without which this research would not have been possible.
Chapter 1

Introduction

The transportation industry is at present moving towards the use of sustainable energy technologies. This is mainly due to the issues associated with greenhouse gas (GHG) emissions with the transportation sector and its effects on climate change. The Kyoto Protocol established in 1997 is the main environment treaty dealing with the reduction of GHG emissions. Under the Kyoto Protocol, industrialized countries have agreed to reduce their GHG emissions by 5.2%. The main sectors that dominate GHG emissions are transportation, electricity, heating, fuel combustion, land use change, agriculture, industry and industrial processes.

Many initiatives have been brought forward with the intent of mitigating GHG emissions. Such attempts in the sectors of power, energy, and transportation are based on the green power concept where environmentally friendly power generation for electricity and transportation is emphasised. More-electric (ME) systems for aircraft and marine applications, and different electrical vehicle (EV) technologies have gained much attention in the recent past mainly for their predicted fuel economy and reduced carbon emissions. In the case of EVs, the GHG emissions are shifted to centralized power stations from the road transportation sector. This provides the advantage of the capability to focus on cleaner power plants to achieve clean energy goals of the future.

Electrical machines and drives are a primary component of ME and EV technologies. Unlike industrial systems, the machines and drives in ME and EV systems operate under variable speed and load conditions. In addition, starter-generator operation, in the case of aero-engine applications, and motoring/regenerative braking, in the case EV applications, may require the machine and drive to
operate in multiple quadrants. Hence, guaranteed high-efficiency can only be obtained at the cost of more complicated control systems. In addition, fault-tolerant operation of machines and drives is essential for safety critical applications such as in aero-engine or ship propulsion applications. Recently, fault-tolerant machine design techniques have also been proposed for EV applications [8–12]. The introduction of fault-tolerant machines in ME and EV systems brings about an added complexity in the design of drives and their control.

This thesis provides an investigation of the formulation of control for two of the main candidate fault-tolerant machines for EV and ME applications, i.e., the switched reluctance (SR) machine and the permanent magnet (PM) machine. Motoring mode operation and power generation mode operation are separately considered for both the types of machines. The potential areas of application of these machines are identified in aero-engine electric power generation and engine starter operation, electric ship propulsion, and hybrid electric vehicle propulsion. The content of this thesis is mainly inclined towards the application in aero-engine embedded power generation. However, the applicability of the techniques in other areas is considered in the literature review.

1.1 Objectives of the thesis

This thesis investigates more-electric starter-generator concepts and is formulated on four objectives. The first objective is to develop a SR motor control algorithm that caters for aero-engine starter requirements. Hence, an excitation control strategy that provides wide speed-range operation capability with characteristics such as minimal discontinuity at mode transition and enhanced overload capability of the SR machine is sought.

The second objective is to develop a SR generator control algorithm that caters for aero-engine embedded generator requirements. These include the capability to provide certain degree of optimal generator performance for operation above a certain speed. Hence, SR generator performance under different optimal conditions is analysed and compared with classical fixed turn-on or fixed turn-off angle based single pulse mode excitation control methods.

The third objective is to develop a control strategy for fault-tolerant operation of multiphase PM machines in motoring mode. The fourth objective is to develop the same for PM machines in generator mode of operation. The third and fourth
objectives cater for applications in general EV, ship propulsion in addition to aero-
engine starter generator applications. Section 1.4 presents an in-depth overview of
the contributions of this thesis in relation to the achievement of these objectives.

1.2 Research method

The overall objective of this thesis is to develop, analyse and validate spe-
cialised control strategies for SR and PM machines. In this investigation, the
control strategies developed for SR motoring and generating modes are focused
on a 250kW SR machine. The control strategies are validated by means of im-
plementation on a pre-existing 300W test-bed scaled-down SR machine. The PM
motoring and generator modes of operation are analysed for a 125kW surface
PM machine and an interior PM machine. The 250kW SR and 125kW PM ma-
chines studied in this thesis are based on preliminary designs and finite element
(FE) modelling performed as part of the research. The simulations performed on
these machines are based on electromagnetic characteristics extracted from the
FE modelling.

Time domain and frequency domain characteristics are analysed for the eval-
uation of the different control algorithm performances. Command response, i.e.,
speed command response in SR and PM motoring mode, and DC-link voltage re-
sponse in SR and PM generator mode is considered in a time domain evaluation.
In addition, efficiency, RMS current magnitude, real power and reactive power
magnitudes are also considered in the analysis. Analyses based on fast Fourier
transform (FFT) results are performed for the evaluation of frequency domain
performance of the different control algorithms. These mainly include FFTs of
flux-linkage, torque ripple and DC-link current ripple.

1.3 Experimental work

The experimental work performed in relation with this thesis is mainly for the
validation of the control algorithms for the SR motoring and generating modes
of operation. An asymmetric half-bridge converter was constructed with discrete
MOSFET devices for the SR machine operation. The control algorithms are im-
plemented on a dSpace 1103 rapid control prototyping system and the switching
/control signals are issues via the dSpace 1103 controller board. The SR machine
is loaded by means of a UniDrive PM load machine in motoring mode experiments of the SR machine. The SR machine is driven by this UniDrive PM motor in generation mode. The DC-link loading is performed by a voltage chopped resistive load in SR generator mode experiments. Further details are presented in chapters four and five. Figure 1.1 presents a photograph of the test-rig and the experimental setup.

Figure 1.1: Photograph of the test-rig with the 300W SR machine coupled with the PM motor drive, asymmetric half bridge converter, DC-link and DC-link chopped resistive load

1.4 Thesis organisation and contributions

The second chapter of this thesis presents the background of the more-electric aircraft (MEA) concept. In addition, a short outline of the state-of-development within electric ship propulsion and hybrid electric vehicles, and the possibility to extend the use of fault-tolerant machines and drives in such areas is also presented in the second chapter.

The control designs presented in this thesis for the SR machine operation is focused on a preliminary design of a 250kW 12/8 SR machine outlined in the third chapter. The control designs presented in this thesis for PM machines are focused on the preliminary designs of two 125kW five-phase PM machines, viz., a surface PM machine and an interior PM machine which are outlined in the third chapter. The third chapter also presents the FE modelling of these
machines, their electromagnetic characteristics and electromagnetic comparison of the 250kW 12/8 SR machine with the scaled down test-bed 300W 6/4 SR machine. The validation of the simulation technique is also presented in the third chapter.

The fourth and the fifth chapters investigate the operation of the SR machine in motoring and power generation respectively. The SR motoring operation in two modes commonly known as current chopping control and single pulse mode of operation are combined by means of a classical speed scheduled control strategy. The issues associated with this form of switched control strategy are addressed by considering a current-peak control strategy which provides seamless transition between the two modes. The advantages and disadvantages are discussed in chapter four. The possibility to enhance certain performance characteristics by means of incorporating a zero-volts-loop (ZVL) period within each stoke is examined. The speed and excitation control strategies are experimentally validated with the 300W 6/4 test-bed SR machine. The application of the developed techniques in a high power environment is evaluated by means of the 250kW 12/8 SR machine FE model based simulation.

The fifth chapter investigates SR generator operation in single pulse mode. The optimized variation of excitation parameters and the possibility to implement optimal excitation controllers by means of a general curve fitted function is investigated. The excitation control of SR generators under curve-fitted optimal conditions and with two classical techniques is implemented. The DC-link voltage regulator operation together with the excitation control strategies are experimentally validated with the 300W test-bed 6/4 SR machine. The application of the control strategies in a high power environment is evaluated by means of the 250kW 12/8 SR machine FE model based simulation.

The sixth chapter concerns mathematical modelling of fault-tolerant PM machines for representation of multiple harmonics and a high number of phases. This chapter serves as the foundation for the optimization and control designs presented in the following chapters seven and eight. The mathematical modelling of PM machines considers a modular approach for the representation of the machine phase dynamics and the voltage imposed by the per-phase H-bridge converter. The converter control is considered in the general format of harmonic injection pulse width modulation (HIPWM). While the PM machine back-EMF
is modelled with an arbitrary number of harmonics, the inductance profile representation is limited to a second order variation with minimal loss of generality. The mathematical model derives the dynamic interplay between harmonic components of currents and voltages. The model is confirmed by comparison with FE model based dynamic simulations of the two machines developed in chapter three.

The seventh and the eighth chapters investigate the operation of the PM machines in motoring and power generation modes respectively. The PM machine motoring mode operation considers the control implementation for three different optimization criteria, viz., torque ripple minimization, reactive power transfer minimization, and a multi-objective optimization. The optimal control signals for motoring are calculated by means of extending the mathematical model to represent the torque ripple waveform and reactive power transfer. The optimization is performed for normal operation and three open-circuit fault conditions. The speed control is implemented and simulated for normal operation and the three open-circuit fault conditions, and different optimization criteria.

The PM machine generator mode operation presented in chapter eight considers the control implementation for three different optimization criteria, viz., DC-link current ripple minimization, reactive power transfer minimization and a multi-objective optimization. The optimal control signals for generating mode are calculated by means of extending the mathematical model to represent the DC-link current ripple waveform and reactive power transfer. The optimization is performed for normal operation and three open-circuit fault conditions. The DC-link voltage regulation is implemented and simulated for normal operation and the three open-circuit fault conditions, and different optimization criteria. The possibility to analytically obtain the control signals for the minimized reactive power transfer operation for a simplified case of a surface PM-machine is also investigated.

The ninth chapter concludes this thesis.
Chapter 2

Literature Review

This chapter presents a literature review of the application of electrical machines and drives in vehicular electric systems and is mainly focused on the more-electric aircraft (MEA). The evolution of the more-electric (ME) concept from ME actuators, the implications on aero-engine power off-take and the improvement of aero-engine performance is explained. The present state-of-application of the MEA concept in the aircraft manufacturing industry is reviewed. The need for higher electrical power generation in MEA systems is explained and aero-engine embedded electric power generation topologies are outlined. The different generator options available for such aero-engine embedded power generation applications are discussed.

The latter part of this chapter presents an outline of the state-of-development within electric ship propulsion and hybrid electric vehicles. The application of machines and drives, specifically the application of permanent magnet and switched reluctance machines in vehicular electric systems is also briefly discussed.

2.1 More-electric aircraft systems

The aviation sector has a direct link with climate change due to GHG emissions at higher altitudes. The GHG emission by the aviation sector is 1-2% of the total GHG emissions and the overall effect of this on climate change is still being debated. The notion that conceivably GHG emission at higher altitudes plays a more significant role in climate change than their magnitude would predict is being scrutinized by environmentalists.

GHG emission reduction in the aviation sector is one of the challenges faced
by aircraft manufacturers. The British government has invested a significant amount of resources in low emissions aircraft technology. Some of the projects funded recently are:

1. 340 Million pounds for the development and introduction of eco-efficient passenger aircraft with the Airbus A350 XWB programme [13].
2. 40 Million pounds for development of the Rolls-Royce’s new open rotor engine with a predicted 30 per cent reduction in carbon emissions per aircraft [14].

The MEA concept had been proposed for enhanced aircraft efficiency, fuel economy and reduced carbon emissions. The basis of the ME concept is to replace the aircraft hydraulic/pneumatic power transfer systems and loads with electrical systems leading to a reduction in weight, higher reliability, enhanced service life and a low level of maintenance.

The present research undertaken in MEA systems can be broadly categorized into the areas of:

1. More-electric actuation techniques.
2. More-electric aircraft services (de-icing, environmental control etc.)
3. Power generation techniques.
4. Power distribution and management.
5. Power converters for rectification and inversion.
6. Power system reliability (fault-tolerance, protection etc)

It has been reported that a typical civil aircraft jet engine of the size of Rolls Royce IAE Model V2500 (e.g. A320 engine) produces about 40MW of thrust [15]. In addition to that, approximately 200kW of power is extracted by electrical generators, 1.2MW of power by the bleed-air system, 240kW of power by the hydraulic pump, and 100kW by fuel pumps and oil pumps within the jet engine. Therefore a total of 1.7MW of power maybe extracted from the jet engine for aircraft non-thrust applications [15].

Recent developments in MEA technology have motivated the gradual introduction of electrical systems in to aircraft designs. Partial replacements of hydraulic and pneumatic systems are seen in new designs of aircraft such as the Boeing 787 and the Airbus A380. The shift of dependency from hydraulic/pneumatic to electricity has resulted in the need for higher electrical power generation within the ME aircraft. This is predicted to be in the range of 1 to 2 MW.
CHAPTER 2. LITERATURE REVIEW

2.1.1 The more-electric aircraft and electrical loads

The MEA concept is founded on the replacement of hydraulic and pneumatic systems with electrical systems. To date the ME concept is commercially adopted by many aircraft manufacturers in their new designs, namely in the Boeing 787 Dreamliner, Airbus A380 as well as in the A400M military transport aircraft, Airbus A321 and Bombardier Global 5000.

In civil aviation, the Boeing 787 Dreamliner has employed ME concepts resulting in the elimination of the bleed-air system. The environmental control system (ECS) is supplemented with electrically driven air compressors, the technology which promises the most fuel savings for the Boeing 787 Dreamliner [16]. The de-icing system which bleeds hot air from the engine in conventional aircraft is replaced with a heater-mat construction. The heater-mat is effectively a spray-on metal material that embeds a conductive carbon fibre composite structure, and can be heated with the passage of a current.

The aircraft manufacturer Airbus has adopted the MEA concept, mainly in the designs of A321, A400M and A380. The Airbus A380 has employed backup electro-hydrostatic actuators for the primary hydraulic systems [17], and the Airbus A321, Airbus A400M has used MEA concepts for aileron control and flap actuation respectively. The Rolls-Royce Trent-900 turbofan jet engine of the Airbus A380 uses electrically actuated thrust reversers [18], and is based on variable speed constant frequency (VSCF) electrical power generation by elimination of the hydraulic constant speed drive (CSD) gearbox.

The aircraft manufacturer Bombardier Aerospace has advanced towards MEA technology by equipping the Bombardier Global 5000 test aircraft with an electrically actuated braking system (Ebrake) [16]. It is expected that Bombardier may adopt the Ebrake concept in future aircraft designs.

The ideal all-electric aircraft would employ electrically actuated systems replacing hydraulic and pneumatic actuators [19]. Extensive research has been undertaken in the area of replacing hydraulic and pneumatic systems with reliable electrical systems. These include a consideration of replacing primary flight control surfaces (FCSs), landing gear and brake actuation of future aircraft, however these have yet to be commercially adopted in the civil aviation sector.
CHAPTER 2. LITERATURE REVIEW

Flight control surfaces actuation in a more-electric aircraft

The actuation of FCSs via electrical means have been extensively researched due to its association with aircraft stability and control [20]. FCSs of a civil aircraft are shown in Figure 2.1, and include the aileron, rudder, elevator, slats, flaps and the spoiler.

Electro-hydrostatic aircraft actuators (EHAs) Figure 2.2 (a), and electromechanical actuators (EMAs) Figure 2.2 (b), are the main candidates for the application in actuation of the FCS. EHAs involve driving of an electric pump coupled to a hydraulic ram. The position of the hydraulic ram is controlled by the hydraulic flow pressure via the speed of the electric pump motor. On the other
CHAPTER 2. LITERATURE REVIEW

hand an EMA involves direct mechanical linkage of the actuator via a gearbox or ball-screw. There are many advantages of using EHAs and/or EMAs instead of centrally driven hydraulic actuators. These are:

1. Better energy efficiency.
2. Lower weight.
3. Reduced maintenance costs.
4. Increase of safety due to elimination of hydraulic liquid flow.

At present the EHA option is favoured to drive primary control surfaces over EMAs [21, 22]. The main issue with EMAs is found to be the jam-susceptibility of screw rotary-linear motion conversion. In contrast EHAs can be operated in passive mode [22] which makes the ram movable under external force. The EHA passivity is enabled by the mode selector valve (MSV) which connects the actuator chambers. Thus the jam-susceptibility of an EHA is very low compared with an EMA. However, EHAs require higher maintenance due to the utilization of an intermediate hydraulic medium within the actuator. Actuation of the aileron of a civil aircraft with EHAs has been demonstrated in [20]. The authors of [20] propose a matrix converter fed permanent magnet motor for the driving of the EHA pump. The control system for the EHA consists of three loops. These are mainly for motor current control, motor speed control and for EHA ram position, which is also the commanded aileron surface position. The matrix converter topology presents reduction in converter size, elimination of electrolytic capacitors and a potential saving in weight [20].

The authors of [23, 24] present the developments of the AWIATOR project (Aircraft Wing with Advanced Technology Operation, 2002-2006). The AWIATOR project develops a distributed flight control surface actuation technique and is concentrated on actuation of secondary control surfaces, i.e., mainly the slats and flaps.

Actuation of aircraft spoiler surfaces with EMAs is demonstrated in [25]. The authors of [25] propose a switched reluctance machine (SRM) for the EMA. The SRM is selected due to its fault-tolerance, reliability and drive converter simplicity of the machine. The inverter of the SRM can be powered from a DC bus in contrast with the matrix converter [20], which is mainly for the use with an AC supply.

The authors of [22] present a hybrid configuration of EHAs and EMAs operating on a single control surface. The hybrid configuration and control system
presented in [22] enables the use of the EHA as the main actuator and the EMA operates as backup in the event of an EHA failure. The control system presented in [22] implements position control of both actuators in the hybrid arrangement. In addition to position control, actuator load control is also implemented such that the EHA actuator bears the majority of the load during normal operation.

As explained earlier, jam-susceptibility is the main issue related to the application of EMA in aircraft FCSs. Thus actuator failure detection and isolation (FDI) has become a major sub-area of interest within EMA research as well as EHA research. Several authors have proposed different methods for FDI and control [26–28]. These are largely model based methods that employ observers to detect any lock-in-place fault of the actuator, and produce an FDI alarm. Knowledge of an actuator fault will enable fast operation of backup actuation schemes and avoid any catastrophic failures.

The more-electric environment control system

The conventional aircraft cabin environmental control system is supported by a bleed-air system. The bleed system is based on extracting air from the core of the jet engine. The point of bleed-air extraction varies depending on the engine type. The extracted bleed-air is then passed through a heat exchanger (also referred to as the pre-cooler), which outputs air at the correct temperature and pressure required for the pneumatic systems. The bleed-air required for the ECS is then cooled via the air cycle machine (ACM) before entering the ECS mixing stage.

The ME environment control system replaces the bleed-air system with a bleedless aircraft design. This also involves the design of bleedless jet engines such as the Rolls-Royce Trent-1000 jet engine. The bleedless technology foresees potential advantages in fuel economy, service life, maintenance and higher passenger comfort. Early generations of jet engines with a lower bypass ratio such as 2:1 did not see the benefit of the elimination of engine bleed. In contrast, new generation of jet engines are designed with high bypass ratios as 11:1 (the Trent-1000 has a bypass ratio of 11:1). Extracting bleed-air from the core of an engine with high bypass ratio has a significant effect on performance and the elimination of the bleed will lead to increased core efficiency and fuel savings.

The bleedless design of the ECS will require electrically driven compressors serving the air conditioning and pressurization systems. The bleedless aircraft
design will require additional electrical power generation capability to support the ECS. In addition to the ECS, the ME aircraft will demand a power system with higher power quality and reliability.

### 2.1.2 The more-electric aircraft electrical power demand

![Figure 2.3: Electrical load power demand trend on civil aircrafts [2]](image)

So far, it has been reported that certain aircraft manufacturers have embarked on replacing the aircraft bleed-air system with electrical systems. i.e., the required pneumatics that was conventionally produced by the engine bleed-air is being produced by electric compressors [29]. FCSs are yet to be replaced with electric actuation systems in commercial aircraft.

With increased reliance on electric power, the need for higher power generation capability with higher reliability in MEA systems has become obvious. Figure 2.3 shows the trend in which the aircraft electrical power demand has risen, and is expected to rise in future. The main engine manufacturers Rolls Royce and General Electric are further developing their own versions of jet engines for the powering of future MEA, most recently the Trent-1000 from Rolls Royce and the GEnx engine from General Electric respectively [29,30]. The Rolls Royce Trent-1000 is found to consume 15% less fuel than earlier generations of turbofan jet engines [31].
2.1.3 The more-electric jet engine

With the increase of aircraft electrical power demand, the power generation capability of jet engines has to be improved. This task is at present being widely researched and the engine is referred to as the More-electric / All-electric jet engine. In addition to increased power generation capability the All-electric jet engine may also feature internal electro-mechanical replacements for hydraulic services within the engine. This has already been attempted in the Rolls-Royce Trent-1000 and Trent-1700 jet engines developed for the Boeing 787 Dreamliner and the Airbus A350 respectively. The Trent-1000 and Trent-1700 three spool jet engines replaces the conventional mechanical drives for starting with an integrated starter-generator system [30]. The competing generation of jet engines by General Electric, i.e., GenX jet engines are based on the conventional two spool architecture and the mechanical drives for tapping into the high-pressure compressor spool for electrical power off-take. In contrast, the Trent-1000 and Trent-1700 engine electrical power extraction is undertaken from the intermediate pressure spool while future developments in ME jet engines may attempt the electrical power extraction from multiple spools.

Figure 2.4 shows a basic twin-spool ME aero-engine block diagram with electrical power off-take from both spools. Future three spool electrical power off-take architectures may be an extension of the two spool version shown in Figure 2.4. Considering the twin spool architecture, the HP spool machine operates as a
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The power extraction limit of the HP spool is low due to jet engine stability restrictions [30]. Therefore the LP spool machine may be of higher rated power compared with the rated power of the HP spool machine.

2.1.4 Basic operation of a two spool turbo-fan jet engine

Figure 2.5: Simplified block diagram representation of a two spool turbofan engine

Figure 2.5 shows the basic block diagram of a two spool turbofan jet engine. The turbofan jet engine effectively divides the compression system into two, i.e. the low pressure compressor (LPC) and a high pressure compressor (HPC). The LP compressor and the LP turbine are mounted on one shaft forming the LP spool. Similarly the HP compressor and the HP turbine are mounted on another shaft forming the HP spool. Each compressor unit is mounted on a coaxial shaft. The turbofan jet engine utilizes the turbojet engine principle but with an added effect of air bypassing the core. This enables production of more thrust at the expense of no significant additional fuel consumption. The air passing through the core enters the HP compressor and combustor. Typically the low pressure turbine (LPT) drives the LPC and the core is formed within the HP shaft [32]. The high pressure air flow from the high HPC is combusted and expanded through the high pressure turbine (HPT). The HP expansion process drives the HP spool. The airflow further expands through the LP turbine driving the LP compressor and thus the input fan.
2.1.5 Electrical power off-take and jet engine control

Figure 2.6: Simplified representation of the jet engine with system inputs, outputs and disturbances

More-electric aircraft technology foresees an increase in the utilization of electrical power within the aircraft. The power off-take from the jet engine may increase up to 800kVA [2] in the future. Therefore the interaction between the jet engine dynamics and the electrical system is inevitable. Thus a future jet engine control designer will have to thoroughly consider the impact of electrical power off-take in their design [33]. To-date no research has been published in the area of jet engine control or modifications to existing controllers for the application in ME jet engines.

The main control aspect of a turbofan jet engine is to regulate the output thrust, spool speeds and temperature of the engine. Figure 2.6 shows a simplified representation of the jet engine with system inputs, outputs and disturbances. The key inputs to the engine include the fuel flow rate, inlet guide vane position and exogenous inputs of Mach number and altitude [34], air speed, temperature and pressure. In a MEA jet engine an additional exogenous disturbance of electrical power off-take from the spools has to be considered. The possibility exists to utilize the information of electrical power off-take in the control algorithm of the jet engine [33]. This will effectively be in the format of decoupling the jet engine dynamics and the generator and electrical system dynamics from each other such that the thrust is not affected by the electrical system power off-take.

Figure 2.7 shows a simplified block diagram of a jet engine high pressure spool isochronous control system expressed in continuous-time [3]. Refer to table
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Table 2.1: Jet engine isochronous control system terminology

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_g$</td>
<td>Isochronous governor transfer function</td>
</tr>
<tr>
<td>$K_s$</td>
<td>Fuel metering valve servo loop gain</td>
</tr>
<tr>
<td>$NE$</td>
<td>Nonlinear element</td>
</tr>
<tr>
<td>$K_m$</td>
<td>Fuel metering valve velocity constant</td>
</tr>
<tr>
<td>$G\Delta p$</td>
<td>Regulation valve transfer function</td>
</tr>
<tr>
<td>$G_e$</td>
<td>Engine transfer function</td>
</tr>
<tr>
<td>$N_{H,\text{ref}}$</td>
<td>Commanded high pressure shaft speed</td>
</tr>
<tr>
<td>$N_{H,\text{act}}$</td>
<td>Actual high pressure shaft speed</td>
</tr>
<tr>
<td>$X_{H,\text{ref}}$</td>
<td>Fuel metering valve position reference</td>
</tr>
<tr>
<td>$X_{H,\text{act}}$</td>
<td>Fuel metering valve actual position</td>
</tr>
<tr>
<td>$WF$</td>
<td>Fuel flow</td>
</tr>
</tbody>
</table>

Figure 2.7: High pressure spool isochronous control system expressed in continuous-time [3]

2.1 for terminology. The engine control task is conventionally performed by the FADEC (full authority digital electronic control), and is well established. PI control methods can be found at the core of the FADEC and may also include limit logic schemes for constraining the commanded fuel flow rate, rate of change of spool speed and acceleration [35]. The turbofan jet engine is a nonlinear system. The method of gain selection in many of the PI controllers in the FADEC is based on experimental techniques and a systematic way of calculating parameters is still unavailable [36]. Although different control techniques are being developed based on advanced control theory, the utilization of these techniques is impeded due to the time and cost involved in developing and certifying a new industrial FADEC-like controller. Therefore a number of researchers base their new developments on a retrofit control architecture which is added onto the existing FADEC controller without significant modification [3, 35, 37]. Future developments in a FADEC
for ME jet engine would most likely follow a similar approach. While the basic FADEC functions are unaltered an electric power off-take signal may be utilized to make the jet engine control decoupled from the electrical generator dynamics. Such alterations or improvements to the FADEC are seen in literature. Research at NASA [35] develops a technique to adjust the FADEC operation with engine degradation so that degraded throttle-to-thrust relationship is taken into account in the FADEC in the long term operation. Authors of [3] propose a modification to the FADEC with an additional feedback loop of compressor discharge pressure and presents improvements with such an approach.

2.1.6 Generators used in conventional jet engines

![Diagram of constant speed drive with synchronous generator in conventional aircraft power generation](image)

Figure 2.8: Constant speed drive with synchronous generator in conventional aircraft power generation [4,5]

In conventional aircraft a synchronous generator coupled to a constant speed drive (CSD) operates as the main source of electrical energy at 400Hz [4,5]. The CSD operates at constant speed while the prime-mover shaft speed varies with engine throttle. The CSD operation requires additional mechanical components such as an epicyclical differential gear box and hydraulics (see Figure 2.8). The CSD is also known as a constant velocity gearbox (CVG).

The generator scheme employed in conventional aircraft electrical systems (see Figure 2.9) is a three stage permanent magnet (PM) excited wound field synchronous machine [2]. The generator terminal voltage is controlled by the excitation stage chopper. The generated power is directly distributed at 400Hz.
Elimination of the CSD and direct coupling of the generator to the turbofan jet engine spool shaft (direct-drive) has been considered as an option in the past [38]. Power generation at variable frequency (VF) is also known as variable speed variable frequency (VSVF) generation. Frequency insensitive loads such as de-icing and galley loads can be supplied with VF power. However, a large portion of aircraft loads, such as avionics, navigation equipment and flight control instruments require power at constant frequency or DC. Electrical motors and transformers designed to operate with a wide frequency range are known to have a weight penalty in their design [38]. Therefore constant frequency power generation is preferred for electrical generating systems for aircraft.

Variable frequency to fixed frequency power conversion is a well established technology and is also referred to as variable speed constant frequency (VSCF)
CHAPTER 2. LITERATURE REVIEW

systems [4, 5, 39]. Figure 2.10 shows the block diagram of a typical VSCF system. VSCF systems have created the possibility to eliminate the CSD and have direct coupling (direct-drive) of the generator to the turbofan jet engine spool shafts. This yields the advantages of reduced weight with the elimination of the CSD and the reduction of maintenance. With the elimination of the hydraulic and pneumatic CSD system, the reliability of the electrical generation system is improved [40]. VSCF operation enables the generator to be used in motoring mode for starting of the turbofan jet engine [4], typically referred to as a starter/generator system.

2.1.7 Generator options for future more-electric jet engines

Three types of generator systems have been extensively researched for the application in ME jet engines, viz.,

1. Permanent Magnet Synchronous Motor Drives [41–44].
2. Switched Reluctance Motor Drives [45–47].
3. Induction Motor Drives [48–50].

The permanent magnet synchronous generator option

The permanent magnet (PM) generator is strong a candidate for the ME jet engine due to its high power-to-weight ratio [42], brushless operation, efficiency, reliability and ruggedness. Furthermore it can be manufactured to be suited for the extreme mechanical environment within the turbofan jet engine. More-electric systems that employ PM machines opt for higher number of phases, as this gives partial redundancy. i.e., the occurrence of a fault in a phase or converter will be confined to that phase. Typically higher phase reactance and lower mutual coupling is sought in fault-tolerant PM generators [42]. Research undertaken with Rolls-Royce [41–44] so far has been directed towards the use of PM generators with five phases.

Two constructions of PM generators have been considered in application to ME systems, viz., interior permanent magnet (IPM) construction [51] and surface mounted permanent magnet (SPM) construction [44]. Both constructions feature their own merits and demerits. IPM construction is found to be rugged and robust [51] while the SPM construction is simple, cost effective [52], and the
mutual coupling between phases can be minimized [53]. These features of the SPM have drawn special attention and have been widely researched [53, 54].

SPM construction is prone to damage at high speeds or during fabrication and thus special attention is needed to ensure proper magnet retention. Different sleeve structures are available for magnet containment. These are typically constructed using nickel based alloys or composite carbon-fibre material. The PM generator is required to operate in extreme thermal environments within the turbofan jet engine (temperatures in excess of 350°C). Therefore magnetic materials with high thermal tolerances such as Samarium Cobalt (SmCo) based compounds [2, 52, 54] are employed.

The switched reluctance generator option

The switch reluctance (SR) machine option for the application in aircraft electrical power generation offers competitive advantages over PM generators. In general the SR machine is considered as the most suited for harsh environments [55, 56]. The mechanical ruggedness and the inherent fault-tolerant operating conditions are well suited for the application in ME turbofan jet engines. The SR machine has a high degree of independence between the motor phases and between inverter phases [45, 56]. Some researchers have claimed that the power electronic converters required to achieve the same level of fault-tolerance for the standard SR machine is simpler and more reliable than those required for the PM machine and the induction machine [57]. The SR machine operation as a generator is still being researched. Much of the research has demonstrated the superior fault-tolerant performance of the SRM under certain conditions that conventional generator technology has failed to achieve [45, 57].

Closed-loop control of SRM is known to be difficult due to the nonlinear magnetic properties of the machine. Thus different control schemes such as tolerance band control, PI control [46], linearising control based on a SRM inverse model [45] and fuzzy logic control [47] have been developed. Suitable SRM control schemes for the application in ME jet engine power generation have yet to be developed.

The induction generator option

The induction machine (IM) has been considered as an option for the application in aircraft electrical power generation [48–50]. Characteristics such as
rugged construction and the absence of magnetic material make the IM an attractive option. However, the PM and SR machines offer better performance compared with the induction generator. Present research done with induction generator construction as well as control with application to ME aero-engine embedded power generation systems is limited. The complexity of control of an IM is higher than that of both the PM and SR machines [57]. Thus much research attention is needed in the areas of IM thermal robustness, fault-tolerant operation, converter reliability and sensor-less operation to bring the IM to an equal standing with the prospects offered by the PM and SR machines.

2.1.8 More-electric aircraft power quality

With the increased electrical power utilization, the ME concept heavily relies on the power generation, distribution and protection system. Thus it is essential to guarantee high reliability and high power quality to ensure optimum fuel economy, service life and safe operation of avionics. At present the electrical power quality standards on-board an aircraft are specified by [58]:

1. RTCA DO-160C  US civil specification [59]
2. ADB-0100  Airbus specification [60]
3. MIL-STD-740E  US military specification [61]

Although the primary focus of this thesis is on the application of fault-tolerant PM and SR machines in aero-engine embedded power generation, the fault-tolerant drives technology has the potential for application in other vehicular electric systems. The next section briefly reviews the state-of-development of the application of machines and drives in electric ship propulsion and in the area of land transportation.


2.2 Machines and drives in electric ship propulsion

The use of electric motors and generators in ship power and propulsion has been of interest since the early 1900s [62]. However, the revival of interest in this area has been brought about by the availability of power electronics and the possibility of rapid control of electrical machines. Electric ship propulsion is seen in two distinct architectures [62], viz.,

1. Full-electric propulsion [63]
2. Hybrid electric propulsion. [64]

Figures 2.11 (a) and (b) represent the basic structure of these two configurations.

In full-electric ship propulsion, prime movers coupled with generators provide the required power for electric propulsion as well as ship service loads. The propeller is fully powered by an electric motor. In the hybrid electric propulsion systems, electric motors and propulsion prime movers are coupled via a mechanical gear-boxes and the load is shared between these two systems. The prime movers in an electric ship may consist of a combination of gas-turbine engines and diesel engines. The performances of these engines are a function of speed and load. The optimum performance is typically achieved at a specific operating speed and load [64,65]. The electric ship concept provides the capability to operate the engines at the most optimum point, irrespective of the speed of the ship. In addition, the electric ship propulsion systems provide the capability to operate in regenerative braking mode, which returns power to the ship electric system. Under low-speed manoeuvres, the number of engines in operation can be reduced, e.g. the authors of [64] investigate different operating profiles for multiple gas-turbine system for enhanced efficiency by operating a reduced number of engines at low speed conditions. In addition to such advantages, the electric ship concept also provides more flexibility to design the vessel with more stability by positioning the non-propulsion engines appropriately.

The possibility to enhance the system reliability of both full-electric and hybrid-electric ship propulsion systems have been identified by many researchers [66,67]. While the application of PM machines, synchronous machines, and induction machines in such high power propulsion applications have taken the
Figure 2.11: Simplified block diagrams of a (a) full-electric ship propulsion system (b) hybrid electric ship propulsion system
lead [62, 65], the possibility to use high power SR machines have also been proposed [68]. In some recent investigations, the authors of [63] present the design of a 24.5kW high torque-to-volume ratio surface PM machine for ship propulsion, while the authors of [65] present the analysis on a 750kW interior PM motor for ship propulsion. The authors of [66] analyse a surface PM machine of five phases, however the fault-tolerant capability is not considered.

The researchers of [65] identify specific characteristics of the interior PM machine for ship propulsion applications compared with the surface PM machine, such as rugged construction, better protection against demagnetization, enhanced field-weakening capability and the lower probability of damaging the magnets during installation. Both the authors of [65] and [67] identify the capability to utilize the reluctance torque component present in interior PM machines for torque production, while the authors of [67] further recognize additional degrees of freedom for torque enhancement by current harmonic injection.

Although, the high reliability design of ship propulsion systems with fault-tolerant machines have been considered in the past, the standard control design technique has not yet been identified. Although general control methods for surface PM machines are available, these techniques do not provide any path for optimized operation techniques with consideration on faults. The mathematical modelling and control design presented in this thesis on PM machines and SR machines can thus be equally applied in the area of ship propulsion.

2.3 Electrical machines and drives in land transportation

The electric vehicle (EV) technology can be considered as the most vibrant area of application and development in the electric transportation research. The interest in EVs has arisen due to the need for mitigation of the high GHG emission contributed by road transportation sector (79.5% [69]) compared with 13% from air transportation and 7% from sea transportation. Major automotive companies have developed EV/HEV vehicles, e.g., the Toyota RAV, Toyota Prius, Nissan Altra, Mazda Bongo, Honda EV plus, Honda Insight, Fiat Seocemto Electra and Fiat Panda Electra [70, 71], to name a few. The Toyota Prius and the Honda Insight are in the popular hybrid electric vehicle (HEV) category.

The basic design of the HEVs can be divided into two categories, viz. series
HEV technology and the parallel HEV technology. An analogy can be drawn from the concepts of full-electric ship propulsion and the hybrid electric ship propulsion configurations. In series HEVs, a generator is driven by the engine and the power is electrically transmitted to storage and to the propulsion motor. In contrast, in parallel HEVs, the engine shaft and the motor are mechanically coupled. Each configuration has advantages in terms of city driving cycles and highway driving cycles [71]. These two topologies of HEVs are shown in figures.
2.12 (a) and (b).

In addition, new developments such as crankshaft mounted machines [71], in-wheel topologies [72], and other methods have been proposed in alternative configurations. Each of these HEV and EV methods feature a propulsion drive with motoring capability. To-date the induction motor, PM brushless motor, and DC machines have been commercially applied in different EV systems. In contrast the application of the SR machine for EV applications is at an early stage of being researched [73, 74]. The application of fault-tolerant machines and enhancement of EV reliability has been considered by many researchers and remains to be fully investigated. Similar to the area of electric ship propulsion, the control design presented in this thesis on PM machines and SR machines can be extended for the application in the areas of EV and HEV propulsion.

2.4 Conclusion

In this chapter, the need for electric systems in transportation was explained. The potential application domains of fault-tolerant machines in electric transportation were also reviewed.

Vehicular electric networks such as MEA systems, ship propulsion systems, and EV/HEV systems consist of electrical machines. The reliability of these machines and drives can be increased by the application of fault-tolerant concepts, which provide the capability to “limp-back-home” upon the occurrence of a fault. MEA systems require high reliability due to the nature of the application. As a result, the application of fault-tolerant machines and drives in this area is of higher importance than other areas, and has been reviewed in this chapter in greater detail. The thesis is inclined towards the MEA aero-engine starter/generator application. However, the review conducted in this chapter demonstrates that the general techniques developed can also be extended for similar areas of interest such as ship propulsion and EV/HEV propulsion.

Chapter three present the preliminary design of a 250kW SR machine and two five-phase 125kW PM machines, which are used in the thesis for the purpose of simulation and validation of the proposed control methods.
Chapter 3

Design and Simulation of Switched Reluctance and Permanent Magnet Machines

In this chapter, the design of a SR machine for the application in high pressure (HP) spool starter generator applications and two designs of PM machines for low pressure (LP) spool power generator are outlined. Two rotor constructions, viz., a surface permanent magnet (SPM) rotor and an interior permanent magnet (IPM) rotor construction are presented for the PM machine design. Finite element models are developed for the three machines and the flux-linkage, inductance, and torque characteristics are obtained. The basis of this chapter is to obtain the electrical characteristics of typical SR and PM machines that would be employed in aero-engine embedded starter/generator applications. It should be noted that the basic designs presented have only been iteratively optimized to a limited degree and leave scope for further improvement. The characteristics of the SR and PM machines obtained are utilized in the rest of the thesis for analysis, control system design and for the formulation of high-fidelity dynamic simulations of the machines.

The latter part of this chapter describes the simulation technique adopted throughout this thesis. The utilization of the FE data for the electrical machine simulation is described. The implementation of the switching model for the PM machine fault-tolerant converter and the SR machine asymmetric half bridge converter is described. The test-bed SR machine and converter are presented. The simulation technique is validated via comparison with experimental data for
the SR machine and converter.

3.1 Switched Reluctance machine design

The standard techniques [75–78] can be applied for the design of SR machines for aircraft starter/generator applications. However, certain design parameters are constrained due to the high speed, high temperature operation and space constraints within the engine, e.g., bore diameter, stack length, and stator and rotor pole numbers [76,79]. The appendix A outlines the basic design process for a SR starter/generator for the operation in a HP spool of a twin-spool jet engine with a nominal HP turbine speed above 15000 rpm. The values for rated power and the torque requirement for starter operation are adopted from the machine design presented in [79], and are 250kW and 177Nm constant torque production capability from zero speed to the rated speed.

The calculations of appendix A arrives at a machine with the dimensions given in table 3.1. The cross section of the SR machine and the corresponding dimensions are shown in figure 3.1. The number of turns per-pole-pair is calculated as
12 turns such that the approximate back-EMF at the design base-speed is approximately equal to the required DC-link voltage of 540V. The slot average current density is obtained as 5.14A/mm². Simulations performed in the later part of this thesis show that the motoring RMS current value at full load may reach 260A (RMS). This would result in current densities up to 34.7A/mm² and is considered normal in engine starter motors due to the short duty operation. Furthermore the oil-cooling available within the aero-engine [79] provides the capability to operate the SR machine with such high current densities and associated thermal effects.

### Table 3.1: Physical parameters of the 250kW SR machine

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft diameter ($D_{sh}$)</td>
<td>144.2 mm</td>
</tr>
<tr>
<td>Internal rotor diameter</td>
<td>206.6 mm</td>
</tr>
<tr>
<td>Rotor outer diameter ($D_i - g$)</td>
<td>238 mm</td>
</tr>
<tr>
<td>Air-gap</td>
<td>1 mm</td>
</tr>
<tr>
<td>Internal stator diameter ($D_i$)</td>
<td>240 mm</td>
</tr>
<tr>
<td>Outer diameter ($D_o$)</td>
<td>302.8 mm</td>
</tr>
<tr>
<td>Stator/ rotor stack length</td>
<td>120 mm</td>
</tr>
<tr>
<td>Stator pole angle</td>
<td>15 deg</td>
</tr>
<tr>
<td>Rotor pole angle</td>
<td>22.5 deg</td>
</tr>
<tr>
<td>Number of turns per pole-pair</td>
<td>12</td>
</tr>
<tr>
<td>Number of parallel paths per-phase</td>
<td>2</td>
</tr>
<tr>
<td>Net slot-fill factor</td>
<td>0.5</td>
</tr>
</tbody>
</table>

A cross sectional flux-density plot of the machine designed in appendix A is shown in figure 3.2. The corresponding parameter values are summarized in table 3.1. Figures (a) and (b) of 3.3 show the resultant flux-linkage characteristics and the torque characteristics obtained from finite element modelling. The ripple effects in figure 3.3 (b) torque waveform maybe due to the finite element model mesh discretization and are not characteristics of the machine itself.

#### 3.1.1 Comparison of electromagnetic characteristics of the 250kW SR machine with a 300W test-bed SR machine.

The SR machine control design and optimization performed in this thesis is focused on the 250kW SR machine design presented above. The developed control methods are experimentally validated via a 300W test-bed SR machine. The cross
Figure 3.2: Cross sectional flux-density plot for the 250kW SR machine, phase-A excited with 200A.

Figure 3.3: (a) flux-linkage characteristics (b) torque vs rotor position characteristics for the 250kW SR machine.

section of the test-bed SR machine is shown in figure 3.5. The corresponding parameter values are given in table 3.2.

For comparison of the experimental variables to the simulated large machine, the electromagnetic characteristics require normalization. The normalized flux-linkage characteristics and the torque production at different per-unit current values for both the 250kW machine and the test-bed 300W machine are shown in figures 3.4 (a) to (d). In general, the base quantities for the normalization are selected as follows. The base real power $P_{\text{base}}$ is selected as,

$$ P_{\text{base}} = P_{\text{rated}} $$

(3.1)
Figure 3.4: (a) and (c) Normalized characteristics for the 250kW SR machine, (b) and (d) Normalized characteristics for the 300W test-bed SR machine

Figure 3.5: Cross section of the 300W test-bed SR machine
### Table 3.2: Physical parameters of the test-bed SRM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft diameter</td>
<td>10.1 ( \pm ) 0.01 mm</td>
</tr>
<tr>
<td>Internal rotor diameter</td>
<td>34.6 ( \pm ) 0.05 mm</td>
</tr>
<tr>
<td>Rotor outer diameter ( D_i - g )</td>
<td>51.22 ( \pm ) 0.01 mm</td>
</tr>
<tr>
<td>Air-gap</td>
<td>0.29 ( \pm ) 0.02 mm</td>
</tr>
<tr>
<td>Internal stator diameter ( D_i )</td>
<td>84.7 ( \pm ) 0.1 mm</td>
</tr>
<tr>
<td>Outer diameter</td>
<td>100.7 ( \pm ) 0.05mm</td>
</tr>
<tr>
<td>Stator/ rotor stack length</td>
<td>60.1 to 59.6 ( \pm ) 0.05mm</td>
</tr>
<tr>
<td>Stator pole angle</td>
<td>30 deg</td>
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<tr>
<td>Rotor pole angle</td>
<td>45 deg</td>
</tr>
<tr>
<td>Number of turns</td>
<td>52</td>
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<tr>
<td>Wire diameter</td>
<td>1.9mm</td>
</tr>
<tr>
<td>Net slot-fill factor</td>
<td>0.387</td>
</tr>
<tr>
<td>Coil type</td>
<td>Pushed through coils</td>
</tr>
<tr>
<td>Coil side shape</td>
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<tr>
<td>Steel type</td>
<td>Transil 300-35</td>
</tr>
<tr>
<td>Frame structure</td>
<td>Smooth round frame</td>
</tr>
</tbody>
</table>

Assuming a full angle current conduction at a nominal DC link voltage of \( v_{dc,nom} \), the base current can be defined as,

\[
I_{\text{base}} = \frac{P_{\text{rated}}}{v_{dc,nom}} \tag{3.2}
\]

\( I_{\text{base}} \) represents the magnitude of the idealized rectangular current in one phase.

The base-speed \( \omega_{\text{base}} \) is typically considered as the speed at which the generated back-EMF magnitude is equal to the DC link voltage. Thus, a base flux-linkage value can be obtained by considering the relationship between the back-EMF \( E_b \) and the flux-linkage \( \psi \) associated with one phase of a SR machine, i.e.,

\[
E_b = \omega \frac{\partial \psi}{\partial \theta} \tag{3.3}
\]

At base-speed and \( E_b \approx v_{dc,nom} \), the corresponding base partial differential is given by,

\[
\left( \frac{\partial \psi}{\partial \theta} \right)_{\text{base}} = \frac{v_{dc,nom}}{\omega_{\text{base}}} \tag{3.4}
\]
For a full angle conduction of $\beta_s$ at a constant current value below the knee point of the flux-linkage characteristics, (3.4) can be approximated to,

$$\frac{\psi_{u,\text{base}} - \psi_{u,\text{base}}}{\beta_s} = \frac{v_{\text{dc,nom}}}{\omega_{\text{base}}}$$ \hspace{1cm} (3.5)

Then the base $\psi$ value for normalization of the flux-linkage characteristics is obtained as,

$$\psi_{\text{base}} = \beta_s \frac{v_{\text{dc,nom}}}{\omega_{\text{base}}}$$ \hspace{1cm} (3.6)

The base torque is calculated as,

$$T_{\text{base}} = \frac{P_{\text{rated}}}{\omega_{\text{base}}}$$ \hspace{1cm} (3.7)

For the 250kW, 15000rpm machine with $\beta_s = \left(\frac{15}{180}\pi\right)\text{rad}$, $v_{\text{dc,nom}} = 540\text{V}$, and $\omega_{\text{base}} = 1570.8\text{ rad/s}$, the base values are obtained as, $I_{\text{base}} = 462.9\text{A}$, $\psi_{\text{base}} = 0.09\text{Vs}$, and $T_{\text{base}} = 159.15\text{Nm}$.

For the 300W, 2000rpm machine with $\beta_s = \left(\frac{30}{180}\pi\right)\text{rad}$, $v_{\text{dc,nom}} = 27\text{V}$, and $\omega_{\text{base}} = 209.4\text{ rad/s}$, the base values are obtained as, $I_{\text{base}} = 11.1\text{A}$, $\psi_{\text{base}} = 0.0675\text{Vs}$, and $T_{\text{base}} = 1.43\text{Nm}$.

### 3.2 Permanent Magnet machine design

PM machines have been considered for the application in aircraft embedded generator operation especially in the low pressure spool [42, 80, 81]. The aero engine electrical power generator may be interfaced with the spool shaft via the direct drive option or the geared drive option. Such techniques have been compared in [42] and the direct drive option can be identified as the ultimate choice for high reliable operation. Other fault-tolerant requirements in PM machine design are presented in [53] such as the need for high inductance for implicit fault current limiting capability, low mutual inductance for magnetic isolation between phases and high number of phases for partial redundancy.

In this section two preliminary versions of PM machines are considered for the application in aircraft LP spool power generation, viz., a SPM machine and an IPM machine of equal size. The SPM machine is reputed for low back-EMF
harmonic content. In contrast, the IPM machine is known to have superior field-weakening capability and the disadvantage of having a high back-EMF harmonic content.

The calculations of appendix B arrive at two PM machines with the dimensions given in table 3.3. Fault-tolerant operation requires multiphase design for increased partial redundancy. It is known that out of three, four, five and six phase machines, the five phase machine exhibits the lowest torque ripple [82]. Thus a five phase machine design is considered in the appendix B. Due to the requirement of physical isolation of phases, conductor placement according to one-phase per-slot concentrated winding arrangement is adopted with a 20 slot stator structure. The rotor is designed with 12 poles. The machine sizing and the number of turns are calculated considering a peak back-EMF of 540V at a rated speed of 3000rpm and a specific electrical loading of $A_{s,\text{rms}} = 12500\text{Am}^{-1}$. The number of turn per-phase is calculated as 24. The slot dimensions are calculated for a current density of 6Amm$^{-2}$. The cross sections of the PM machines and the corresponding dimensions associated with table 3.3 are shown in figures 3.6 and 3.7. The electromagnetic characteristics of these PM machines are obtained via FE modelling. These are presented in the following section.

Figure 3.6: Cross section of the surface PM machine
Table 3.3: Physical parameters of the SPM and IPM machine designs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator dimensions</td>
<td></td>
</tr>
<tr>
<td>Bore diameter $D_i$</td>
<td>420mm</td>
</tr>
<tr>
<td>Stator/ rotor stack length</td>
<td>250mm</td>
</tr>
<tr>
<td>Air-gap height</td>
<td>1mm</td>
</tr>
<tr>
<td>Stator pole height $h_{ps}$</td>
<td>35.8mm</td>
</tr>
<tr>
<td>Stator pole-edge height $h_{p,edge}$</td>
<td>22mm</td>
</tr>
<tr>
<td>Stator back iron height $h_{bs}$</td>
<td>55mm</td>
</tr>
<tr>
<td>Stator pole width $w_{ps}$</td>
<td>44mm</td>
</tr>
<tr>
<td>Stator slot-opening $w_{os}$</td>
<td>1mm</td>
</tr>
<tr>
<td>SPM rotor dimensions</td>
<td></td>
</tr>
<tr>
<td>PM thickness $h_{pm}$</td>
<td>5.1mm</td>
</tr>
<tr>
<td>Rotor back iron height $h_{br}$</td>
<td>61.1mm</td>
</tr>
<tr>
<td>IPM rotor dimensions</td>
<td></td>
</tr>
<tr>
<td>PM thickness $h_{pm}$</td>
<td>20mm</td>
</tr>
<tr>
<td>Rotor back iron height $h_{br}$</td>
<td>61.1mm</td>
</tr>
<tr>
<td>PM width $w_{m}$</td>
<td>92.6mm</td>
</tr>
<tr>
<td>Bridge width $w_{bdg}$</td>
<td>4mm</td>
</tr>
<tr>
<td>Flux barrier width $d_{bdg}$</td>
<td>5.2mm</td>
</tr>
<tr>
<td>Flux barrier height $h_{1}$</td>
<td>10.1mm</td>
</tr>
<tr>
<td>Flux barrier height $h_{2}$</td>
<td>22.1mm</td>
</tr>
</tbody>
</table>

Figure 3.7: Cross section of the interior PM machine
3.2.1 Finite element modelling of permanent magnet machines

Figure 3.8: FE mesh and flux lines of the SPM machine. (Aligned position of the first phase with balanced full load currents of -100A, -30.9A, 80.9A, 80.9A, -30.9A in five phases)

Figure 3.9: Flux density plot of the SPM machine. (Aligned position of the first phase with balanced full load currents of -100A, -30.9A, 80.9A, 80.9A, -30.9A in five phases)
Figure 3.10: FE mesh and flux lines of the IPM machine. (Aligned position of the first phase with balanced full load currents of -100A, -30.9A, 80.9A, 80.9A, -30.9A in five phases)

Figure 3.11: Flux density plot of the IPM machine. (Aligned position of the first phase with balanced full load currents of -100A, -30.9A, 80.9A, 80.9A, -30.9A in five phases)

Finite element models of the two PM machines are developed. Figures 3.8 and 3.10 show the FE mesh and flux lines and Figures 3.9 and 3.11 show the flux density plots of the SPM machine and the IPM machine respectively. These plots
represent the state of the machines when the first phase is aligned with the stator pole and the current of the first phase is at full-load peak value. The currents of the other four phases are balanced.

The FEMM 4.2 (Finite Element Method Magnetics) software with Lua scripting is used to iteratively simulate the two machines for different rotor positions and currents beyond 200% full-load peak value. The flux-linkage characteristics and torque production information of the two machines are extracted.
Figure 3.12: Flux-linkage and inductance vs rotor position and current characteristics for: (a), (c), (e) The Surface PM machine and (b), (d), (f) The Interior PM machine.
Figure 3.13: Mutual flux-linkage vs rotor position and current characteristics between the surface PM machine phases (a) 1 & 2 (c) 1 & 3 (e) 1 & 4 (g) 1 & 5 and between the interior PM machine phases (b) 1 & 2 (d) 1 & 3 (f) 1 & 4 (h) 1 & 5
Figure 3.14: Mutual inductance vs rotor position and current characteristics between the surface PM machine phases (a) 1 & 2 (c) 1 & 3 (e) 1 & 4 (g) 1 & 5 and between the interior PM machine phases (b) 1 & 2 (d) 1 & 3 (f) 1 & 4 (h) 1 & 5
Figure 3.15: Torque per-phase vs rotor position for
The Surface PM machine (a) positive currents (c) negative currents and
The Interior PM machine (b) positive currents (d) negative currents

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FEMM generated data for ±0A, ±25A ±50A and ±100A

---
CHAPTER 3. DESIGN AND SIMULATION OF SR & PM MACHINES

3.3 Finite element model based high fidelity dynamic simulation of PM and SR machines

The general technique adopted by commercial software (e.g. Simulink simpowersystems / PLECS) for the simulation of electrical machines is based on the dynamic equation,

\[ v_n = R_n i_n + \frac{d\psi_n(\theta_e + \gamma_n, i_n)}{dt} \]  \hspace{1cm} (3.8)

where \( v_n, i_n, \) and \( \psi_n(\theta_e + \gamma_n, i_n) \) are the per-phase instantaneous terminal voltage, current, and flux-linkage. \( \theta_e \) is the electrical angle and \( \gamma_n = \frac{2\pi}{N} (n - 1), \) \( n = 1, 2, ..., N \) represent the phase shift of the \( n^{th} \) phase of the machine with respect to the electrical angle \( \theta_e \) for \( N \) number of phases. Machine models in commercial software are also found in the format of direct-quadrature (d-q) representation and are mainly for three phase machines.

In the case of SR machines, the modelling of the flux-linkage characteristics is complicated. Simulation techniques seen in the literature favour the option of curve fitting of magnetization data and evaluation of the partial differential coefficients for the implementation of the full SR machine model [86–88]. A cubic-spline modelling technique is used in the popular industrial SR machine simulation program, PC-SRD [86, 89]. Each \( \psi - i - \theta \) curve is approximated to a linear segment and a parabolic segment. The flux-linkage values for intermediate rotor positions are divided into three regions and interpolated by a linear relationship and a Frohlich-like relationship [86].

Alternative techniques developed for the approximation of SR machine magnetization characteristics such as the bi-cubic spline approximation presented in [90, 91], the geometry-based analytical model presented in [92] and the artificial intelligence based methods presented in [93] are some notable examples.

The simulation algorithm reported in [94] concentrates on a look-up table based implementation of flux-linkage and torque models for the SR machine. Intermediate values within each of the look-up tables are obtained by cubic spline interpolation. This technique is adopted in this thesis for the implementation of the SR machine dynamic simulation as well as the PM machine dynamic simulation.

The FE model of a SR machine and two PM machines were presented earlier.
Given the \((i, \theta)\) pair, the FE data provides the corresponding flux-linkage value \(\psi\) of a certain phase. This data is numerically manipulated to provide the inverse function of flux-linkage, i.e., for a given \((\psi, \theta)\) pair, the current value \(i\) can be reproduced. Consequently, the \((i, \theta)\) pair can be used to reproduce the torque at a given \(i\) and \(\theta\). Since the machines considered are of fault-tolerant in design, the mutual inductances should be negligible and are not considered in the simulation. The flux-linkage associated with a phase is calculated by the integration,

\[
\psi_n = \int (v_n - R_n i_n) \, dt \quad (3.9)
\]

The block diagram representation of the per-phase simulation is shown in figure 3.16.

The value of \(v_n\) is produced by a switching model of a voltage source converter. In the case of the SR machine, this is an asymmetric half bridge converter and for the PM machines this is a parallel, H-bridge converter structure.

### 3.3.1 Asymmetric half bridge converter switching model

One phase of an asymmetric half bridge converter used with the SR machine is shown in figure 3.17. This converter consists of two semiconductor switches and two diodes per-phase. The relationship between the applied voltage \(v_n\) of a SR machine phase and the switching signals issued to the high-side and low-side switches \((H\ and\ L)\) is represented in table 3.4. Here \(R_{sw}\) and \(R_d\) represent the on-state resistance of each semiconductor switch and the conduction resistance...
of each free-wheeling diode. Typically, negative current does not occur and is represented here for completeness. Turn-on and turn-off effects and the dynamics associated with the parasitic capacitances and inductances of the switches are neglected in this converter simulation model.

### 3.3.2 H bridge converter switching model

One phase of a H bridge converter used with the PM machines is shown in figure 3.18. This converter consists of four semiconductor switches with anti-parallel diodes per-switch. The relationship between the applied voltage $v_n$ of a PM machine phase and the switching signals issued to the converter switches is given in Table 3.4.

![Asymmetric half bridge converter of one phase of the SR machine](image)

Table 3.4: Relationship between the applied voltage $v_n$ of an asymmetric half bridge converter phase and the switching signals $H$ and $L$

<table>
<thead>
<tr>
<th>$H$</th>
<th>$L$</th>
<th>$i_n$ state</th>
<th>$v_n$</th>
<th>$i_{dc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>$i_n &lt; 0$</td>
<td>$v_{dc} - 2i_n R_d$</td>
<td>$i_n$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>$i_n = 0$</td>
<td>$v_{dc}$</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>$i_n &gt; 0$</td>
<td>$v_{dc} - 2i_n R_{sw}$</td>
<td>$i_n$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>$i_n &lt; 0$</td>
<td>$v_{dc} - 2i_n R_d$</td>
<td>$i_n$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>$i_n = 0$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>$i_n &gt; 0$</td>
<td>$-i_n R_d - i_n R_{sw}$</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>$i_n &lt; 0$</td>
<td>$v_{dc} - 2i_n R_d$</td>
<td>$i_n$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>$i_n = 0$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>$i_n &gt; 0$</td>
<td>$-i_n R_d - i_n R_{sw}$</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>$i_n &lt; 0$</td>
<td>$v_{dc} - 2i_n R_d$</td>
<td>$i_n$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>$i_n = 0$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>$i_n &gt; 0$</td>
<td>$-v_{dc} - 2i_n R_d$</td>
<td>$-i_n$</td>
</tr>
</tbody>
</table>
and \( B \) are represented in the table 3.5. Here \( R_{sw} \) and \( R_d \) represent the on-state resistance of each semiconductor switch and the conduction resistance of each free-wheeling diode. Similar to the converter in Section 3.3.1, the turn-on and turn-off effects and the dynamics associated with the parasitic capacitances and inductances of the switches are neglected in this converter simulation model.

Table 3.5: Relationship between the applied voltage \( v_n \) of a PM phase and the switching signals \( A \) and \( B \)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>( i_n ) state</th>
<th>( v_n )</th>
<th>( i_{dc} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>( i_n &lt; 0 )</td>
<td>(-v_{dc} - 2i_n R_{sw})</td>
<td>(-i_n)</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>( i_n = 0 )</td>
<td>(-v_{dc})</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>( i_n &gt; 0 )</td>
<td>(-v_{dc} - 2i_n R_d)</td>
<td>(-i_n)</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>( i_n &lt; 0 )</td>
<td>( v_{dc} - 2i_n R_d)</td>
<td>( i_n)</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>( i_n = 0 )</td>
<td>( v_{dc})</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>( i_n &gt; 0 )</td>
<td>( v_{dc} - 2i_n R_{sw})</td>
<td>( i_n)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>( i_n &lt; 0 )</td>
<td>( v_{dc} - 2i_n R_d)</td>
<td>( i_n)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>( i_n = 0 )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>( i_n &gt; 0 )</td>
<td>(-v_{dc} - 2i_n R_d)</td>
<td>(-i_n)</td>
</tr>
</tbody>
</table>

### 3.3.3 Diode bridge rectifier model

The asymmetric half bridge converter used with the experimental SR machine is sourced via a single phase diode bridge rectifier. In order to replicate the DC-link dynamics in motoring mode and the associated DC-link voltage ripple, a diode bridge rectifier model is developed for the simulation of SR motor drive. An AC source with AC-side inductance is considered and is shown in figure 3.19.
Section 3.4 presents experimental validation of the simulation of a testbed SR machine operation in motoring mode with the asymmetric half bridge converter interfaced with the a diode bridge rectifier.

Given the AC source voltage $v_{ac}$, AC-side inductance $L_{ac}$, and resistance $R_{ac}$, the diode bridge rectifier terminal voltage $v_{rt}$ is related to the DC-link voltage $v_{dc}$ and AC-side current according to the logic relationship given in table 3.6.

### Table 3.6: Relationship between the diode bridge rectifier terminal voltage $v_{rt}$, the DC-link voltage $v_{dc}$, and the rectifier current $i_{s}$

<table>
<thead>
<tr>
<th>$i_{ac}$ state</th>
<th>$v_{rt}$</th>
<th>$i_{s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_{ac} &lt; 0$</td>
<td>$-v_{dc} + 2i_{ac}R_{rd}$</td>
<td>$-i_{ac}$</td>
</tr>
<tr>
<td>$i_{ac} = 0$</td>
<td>$v_{dc}$</td>
<td>$i_{ac}$</td>
</tr>
<tr>
<td>$v_{ac} &gt; v_{dc}$</td>
<td>$-v_{dc}$</td>
<td>$i_{ac}$</td>
</tr>
<tr>
<td>$v_{ac} &lt; -v_{dc}$</td>
<td>$v_{ac}$</td>
<td>$i_{ac}$</td>
</tr>
<tr>
<td>$-v_{dc} \leq v_{ac} \leq v_{dc}$</td>
<td>$v_{dc} + 2i_{ac}R_{rd}$</td>
<td>$i_{ac}$</td>
</tr>
</tbody>
</table>

where

$$v_{ac} = \hat{V}_{ac} \sin (2\pi ft + \varphi)$$  \hspace{1cm} (3.10)

where $\hat{V}_{ac}$, $f$, and $\varphi$ represent the AC-side voltage peak, frequency, and reference phase shift. The AC-side current is then calculated by the integration of the dynamic equation,

$$L_{ac} \frac{di_{ac}}{dt} + R_{ac}i_{ac} = v_{s} - v_{rt}$$  \hspace{1cm} (3.11)
\[ i_{ac} = \frac{1}{L_{ac}} \int (v_{ac} - v_{rt} - R_{ac}i_{ac}) \, dt \]  

(3.12)

The DC-side voltage is then calculated by the integration of the dynamic equation,

\[ C \frac{dv_{dc}}{dt} = i_s - i_{dc} \]  

(3.13)

### 3.4 Experimental validation of the FE model based simulation technique

The FE model based simulation technique and the converter switching model is validated considering a 300W test-bed SR machine. The SR machine FE model and the corresponding machine parameters were presented earlier in table 3.2. The converter parameters are given below:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_{ac} )</td>
<td>1.75mH</td>
</tr>
<tr>
<td>( R_{ac} )</td>
<td>0.1Ω</td>
</tr>
<tr>
<td>( v_{ac} )</td>
<td>37 ( \sin (2\pi ft) ) V</td>
</tr>
<tr>
<td>( R_{sw} )</td>
<td>0.01Ω</td>
</tr>
<tr>
<td>( R_d )</td>
<td>0.01Ω</td>
</tr>
<tr>
<td>( R_{rd} )</td>
<td>0.01Ω</td>
</tr>
<tr>
<td>( C )</td>
<td>10mF</td>
</tr>
</tbody>
</table>

A lookup table based SRM magnetization characteristics is implemented as explained in section 3.3. The motor is operated at different speeds and load torque levels. The corresponding phase current waveforms, DC-link voltage waveforms and phase voltages were recorded. The different parameters considered in the series of tests are given in table 3.8 below.

The validation is performed in two steps. In the first step, the experimental phase voltage waveforms are fed to the FE flux-linkage model based simulation. The resultant current waveforms are compared with the actual experimental current waveforms. In the second step, the asymmetric half bridge converter model presented in section 3.3.1 and the diode bridge rectifier model presented in section 3.3.3 are combined with the FE flux-linkage model to generate a more complete
Table 3.8: Different excitation parameters, speed and load conditions considered in the experiment. (turn-on angle specified with reference to unaligned position -12.5° rotor position)

<table>
<thead>
<tr>
<th>Condition</th>
<th>Speed [rpm]</th>
<th>turn-on angle [deg]</th>
<th>Dwell angle [deg]</th>
<th>Average load torque [Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500</td>
<td>8.5</td>
<td>2.40</td>
<td>0.11065</td>
</tr>
<tr>
<td>2</td>
<td>500</td>
<td>8.5</td>
<td>6.03</td>
<td>0.57258</td>
</tr>
<tr>
<td>3</td>
<td>500</td>
<td>9.5</td>
<td>11.02</td>
<td>1.41350</td>
</tr>
<tr>
<td>4</td>
<td>1000</td>
<td>5</td>
<td>3.96</td>
<td>0.14941</td>
</tr>
<tr>
<td>5</td>
<td>1000</td>
<td>5</td>
<td>8.61</td>
<td>0.61526</td>
</tr>
<tr>
<td>6</td>
<td>1000</td>
<td>0</td>
<td>16.34</td>
<td>1.66450</td>
</tr>
<tr>
<td>7</td>
<td>1100</td>
<td>6.5</td>
<td>21.00</td>
<td>1.51640</td>
</tr>
<tr>
<td>8</td>
<td>1500</td>
<td>5.5</td>
<td>15.90</td>
<td>0.67805</td>
</tr>
<tr>
<td>9</td>
<td>1500</td>
<td>14.5</td>
<td>12.84</td>
<td>0.21280</td>
</tr>
<tr>
<td>10</td>
<td>1963</td>
<td>-1.5</td>
<td>26.60</td>
<td>1.24250</td>
</tr>
<tr>
<td>11</td>
<td>2380</td>
<td>-4.5</td>
<td>36.00</td>
<td>1.25100</td>
</tr>
<tr>
<td>12</td>
<td>2500</td>
<td>-4.5</td>
<td>18.54</td>
<td>0.71594</td>
</tr>
<tr>
<td>13</td>
<td>2500</td>
<td>4.5</td>
<td>12.70</td>
<td>0.22738</td>
</tr>
<tr>
<td>14</td>
<td>3000</td>
<td>5.5</td>
<td>30.75</td>
<td>0.19398</td>
</tr>
<tr>
<td>15</td>
<td>3000</td>
<td>-4.5</td>
<td>21.00</td>
<td>0.55445</td>
</tr>
<tr>
<td>16</td>
<td>3000</td>
<td>-6.5</td>
<td>29.14</td>
<td>1.11220</td>
</tr>
</tbody>
</table>

In general, the simulation results of both these steps exhibit a satisfactory match with the experimental results. The results of the first step, i.e. appendix C, exhibit a higher degree of accuracy in contrast with the second step (appendix D). Since the FE model based flux-linkage characteristics are only considered in the first step, the excellent matching of waveforms confirm the accuracy of the FE model. Thus the inaccuracies in the second step can be mainly attributed to the discrepancy of the DC-link voltage.

The experiments consider single pulse mode of operation as well as chopping operation. Conditions, 2, 3, 6, and 7 of table 3.8 occur near 500rpm and 1000rpm and chopping operation can be seen in the associated waveforms. All the other waveforms operate in single pulse mode. The attempt to match the number of switching operations with the current hysteresis control is also shown.
to be successful and is dependent on matching the experimentally implemented hysteresis band. Thereby, high degree of simulation accuracy is obtained in both the chopping operation and the single pulse mode of operation.

A high degree of accuracy can be seen in the second step, especially under low load torque conditions. Figure 3.20 present the operation of the SR machine and converter under low load conditions 1, 4, 9, 13 and 14 of table 3.8. Figure 3.21 present the operation of the SR machine and converter under high load conditions 3, 6, 11 and 16 of table 3.8. The simulated current waveforms exhibit a very high resemblance with the experimental waveforms in these figures. It should also be noted that figure 3.21 waveforms are of single pulse mode of operation.

DC-link voltage variation and resultant deviation of current waveforms can also be seen in certain instances. For example, the SR machine operation in chopping mode shown in figures 3.21 (a1) to (d1) exhibit a slight variation in the DC-link voltage. This is a condition at which a higher load torque is imposed with a chopped pulse of 40A. As a result, the DC-link current ripple is higher and hence the simulated DC-link voltage ripple is higher. This leads to a higher discrepancy between the experimental and simulated waveforms of the DC-link voltage. Consequently leads to a discrepancy in the current waveform. A similar observation can also be made with the figures 3.21 (a2) to (d2).

The degree of resemblance between the simulation and experimental waveforms do not vary considerably with speed. However, the time taken for the current to extinguish following the turn-off occurs at a shorter time than the time predicted by the simulation. This can be attributed to the unmodelled behaviour of the free-wheeling diodes. The diode model considered in the simulation does not consist of a forward voltage, as a result the additional voltage components across the diode are not simulated. The SR machine model in this simulation is considered to be a fault-tolerant design and the mutual inductances are assumed to be negligible. However, a voltage transient can be observed in the experimental voltage waveforms. For current pulses of higher conduction angle, such as in conditions 11 and 16 of table 3.8, i.e. in figures 3.21 (a3) to (d3) and (a4) to (d4) the effect of the mutually induced voltages on the current pulses are higher in contrast with the other waveforms. However, compared with the simulation, such impacts of mutual effects have not made a significant difference in the current waveforms. The mutual voltage is considerably small and thus the negligence of mutual effects can be justified.
Figure 3.20: Comparison of experimental and simulation waveforms for the phase current vs rotor position and flux-linkage vs current trajectory for the low load conditions 1, 4, 9, 13 and 14 of table 3.8. Subplots: (a), (b) condition 1, (c), (d) condition 4, (e), (f) condition 9, (g), (h) condition 13 and (i), (j) condition 14.
Figure 3.21: Comparison of experimental and simulation waveforms for the phase voltage, phase current, DC-link voltage and flux-linkage vs current trajectory for the high load conditions 3, 6, 11 and 16 of table 3.8. (a1) to (d1) condition 3, (a2) to (d2) condition 6, (a3) to (d3) condition 11 and (a4) to (d4) condition 16.
3.5 Conclusion

In this chapter, a 250kW SR machine design was outlined. The electromagnetic characteristics of this SR machine were presented with the aid of FE modelling. The comparison of the 250kW SR machine with a test-bed 300W SR machine was performed.

Two designs of 125kW PM machines were outlined. A machine with a surface PM rotor construction and another machine with an interior PM rotor construction were presented. The electromagnetic characteristics of these two PM machines were obtained with the aid of FE modelling.

The FE model based simulation technique used in this thesis was presented. This modelling technique was validated with experimental results obtained from the test-bed 300W SR machine and is found to be accurate. The simulation waveforms match the experimental waveforms under single pulse mode of operation as well as chopping-mode of operation. The number of switching instances in chopping-mode is also accurately replicated by the simulation. The DC-link voltage is found to be a key factor in determining the accuracy between the experimental and simulation results. The switching simulation technique replicates the phase voltage and flux-linkage to a considerable accuracy which is also dependent on the DC-link voltage accuracy. It can be seen that the FE modelling and simulation technique performs satisfactorily even under high load conditions and thus can be used for the development and evaluation of different performance characteristics associated with fault-tolerant electrical machines and drives presented in the following chapters.
Chapter 4

Switched Reluctance Machine
Operation in Motoring Mode

This chapter investigates the operation of switched reluctance machines in motoring mode. The dynamic equations and electro-magnetic energy conversion of a typical SR machine is described. The conventional methods of excitation control for operation below base-speed and above base-speed are outlined. Due to the wide speed range operation requirement in many vehicle electric systems, e.g. aero-engine starter generators, the control of SR motors above base-speed and below base-speed is investigated. An excitation controller which switches from one structure to another at base-speed based on the conventional excitation method is presented. An improved turn-on angle controller which provides superior continuity in the transition from below base-speed to above base-speed operation is developed based on the current-peak control method. The current-peak controller validation via simulation and experimental results are presented. The possibility to incorporate a zero-volts loop interval within the stroke period and its potential benefits are investigated.

In each of these excitation control methods, the relative degree of input-output linearity of average torque production and commanded excitation parameters are presented. Commanded torque versus excitation parameter calculation based on fixed gain techniques and the capability to improve the input-output linearisation of average torque production by means of variable gain techniques is also considered. Speed control with these three excitation control methods are compared and evaluated by means of simulations and further validated by means of experimental results. The fault-tolerance and the capability to guarantee stability for
certain speed ranges under open-circuit fault conditions are presented.

4.1 Introduction

![Diagram of SRM winding connections to an asymmetric half bridge converter]

Figure 4.1: The SRM winding connections to an asymmetric half bridge converter

The SRM has gained much attention in the past few decades due to its simple construction, low manufacturing cost, low maintenance requirement, wide operational speed range, high torque density and high efficiency. The revival of interest in SRM drives is mainly due to the development of fast power electronic devices and microprocessor based advanced control methods that make rapid control of the SRM possible. The SRM is especially useful for high speed operation due to its rugged construction, brushless operation and the absence of permanent magnets. In addition to these characteristics the SR machine is capable of high torque production at high speed. Thus the SR machine is considered as a competitor to the PM machine and the cage induction machine for the application in many electric vehicle applications such as hybrid-electric vehicles [95], integrated starter/alternator systems [96,97], aircraft power generation applications, vehicular/industrial compressor systems [98–100], where operating conditions reach extreme levels of speed and temperature.

Although the control structure is simpler, the control design for SR machines is complicated in contrast to conventional machines. The direct model-based control design approach used in conventional machines is not predominantly used for SR machines, rather the control is formulated according to measured or simulated information [101]. This has lead to higher complexity in optimizing the SR motor performance.
A typical SRM consist of concentrated windings in each stator pole. No conductors exist on the rotor. Figure 4.1 shows the typical construction of 6/4 three phase SRM and winding connections. The electromagnetic energy conversion of SR machines and the corresponding mechanical and electrical dynamics in both motoring and generating modes of operation are explained in appendix E and is not repeated here. The next section presents brief review of the conventional methods of excitation control for SR motors.

### 4.2 Rotor position and SR motor electromagnetic characteristics

![Inductance variation with rotor position](image)

Figure 4.2: The SRM inductance variation with rotor position at constant current

Figure 4.2 shows the idealized linear inductance profile of a typical SRM, where $\beta_s$ and $\beta_r$ represent the stator pole angle and the rotor pole angle respectively. $\theta_u$ and $\theta_a$ represent the unaligned and aligned positions. $\theta_{rpp}$ represents the rotor pole pitch angle where $\theta_u + \theta_{rpp}$ corresponds with the consequent unaligned position in phase A. The same electromagnetic characteristics are repeated in phase B following a step-angle $\theta_{st}$,

$$\theta_{st} = \frac{2\pi (N_s - N_r)}{N_s N_r}$$  \hspace{1cm} (4.1)

Figure 4.3 shows the idealized linear inductance profile of a SRM phase and typical current waveforms at two different modes of operation. The rise of the inductance occurs at the initiation of the overlapping of the rotor and stator poles...
Figure 4.3: Current waveforms of a SRM (a) operating in current chopped mode (b) operating in single pulse mode

Figure 4.3: Current waveforms of a SRM (a) operating in current chopped mode (b) operating in single pulse mode

at $\theta = \theta_1$ till $\theta = \theta_1 + \beta_s$. Maximum inductance is achieved when the rotor pole and the stator pole are fully aligned and remains constant till $\theta = \theta_1 + \beta_r$. At $\theta = \theta_1 + \beta_r$, the rotor pole moves out of overlapping the stator pole and this results in reduction in the inductance until $\theta = \theta_1 + \beta_r + \beta_s$. Beyond $\theta = \theta_1 + \beta_r + \beta_s$ the minimum inductance prevails until $\theta = \theta_1 + \theta_u + \theta_{rpp}$. The region in which the inductance rises ($\frac{dL}{d\theta} > 0$) is referred to as the torque production region. In order to produce the required average torque a certain level of current must be maintained in the torque region.

The SRM excitation is achieved by three parameters, i.e., turn-on angle $\theta_{on}$, turn-off angle $\theta_{off}$ and the current reference $i_{ref}$. Parameters $\theta_{on}$ and $\theta_{off}$ define the period in which a positive voltage pulse is applied. The current is limited to $i_{ref}$ by means of hysteresis control or voltage PWM. The strategy used to maintain this current level is typically found to be done in one of two modes.
The first mode, as shown in figure 4.3 (a), is known as the chopping mode where the current is kept at a desired level by means of voltage chopping. This is suitable as long as the peak of the generated back-EMF is smaller than the supply voltage. In the simplest of methods for excitation control algorithms for operation below base-speed, the dwell angle $\theta_{dwell} = \theta_{off} - \theta_{on}$ is fixed, typically less than $\beta_s$ allowing the defluxing to occur within an angle $\theta_{deflux}$ such that the current tail does not continue to negative torque region. The current reference $i_{ref}$ is adjusted according to the torque demand from the speed control loop [102,103].

However at high speeds, the peak back-EMF will exceed the supply voltage and thus any chopping action will severely reduce the torque production. The current waveform for the single pulse mode of operation is shown in figure 4.3 (b). Thus a voltage pulse is imposed on the phase at each stroke of the SRM and this mode is referred to as the single pulse mode of operation. Torque control in this mode is achieved by regulating the time period the voltage pulse is applied by means of modifying the dwell angle $\theta_{dwell}$ via turn-on angle $\theta_{on}$, turn-off angle $\theta_{off}$ values. Typically the pulse is initiated in advance to the rising edge of the inductance, and this angle is referred to as the advance angle $\theta_{adv}$. The advancing of the turn-on angle enables the build up of current and flux to the required level to produce the demanded torque.

4.3 Literature review of SR motor control strategies

![Figure 4.4: Regions of interest in the torque-speed operating area of a SRM](image)
Figure 4.4 shows regions identified in [104] for low speed current chopped operation (I), medium speed current chopped operation (II), high speed single pulse operation (III) and high speed low load operation (IV). Different criteria are optimized in each region. In low speed operation, lower torque ripple is prioritized while in high speed single-pulse region, the efficiency is prioritized. This is due to the minimal effect of torque ripple on speed ripple in high speed operation. It follows that SR drive control objectives are application oriented in nature depending on the region of operation of priority. Applications that do not require high-dynamic speed/torque control and prefer high energy efficiency may consider optimization of efficiency at the cost of higher torque ripple. On the other hand, certain applications that require high-dynamic performance, and low speed/torque ripple may consider a trade-off of efficiency for reduced torque ripple. For example, in an application of SR motors for aircraft starter operation, the SR motor may transverse through the low speed region into the high speed region. While optimized speed control is not essential in the low speed region, the high speed region may be prioritized for control design and optimization. As a result, torque ripple minimization may be considered as a secondary objective than system efficiency. Another example is found in EV application where high dynamic performance is not essential in general electric vehicle traction control systems [101]. However, certain vehicle dynamic operations such as anti-slip control may require a certain degree of performance from the drive.

Early work considered the control of SRM in the low speed region. The turn-on angle or the turn-off angle or both are fixed while the voltage applied across the phase winding is controlled [102]. The SR drives presented in [105–107] consider fixing the turn-on and turn-off angles at specific values to achieve certain operational characteristics. Many researchers have developed techniques to operate the SRM optimally by dynamic variation of the turn-on and/or turn-off angles. The authors of [108] consider dynamic adjustment of the advance angle to increase the torque-per-ampere. Different techniques for the selection of excitation for torque ripple minimization [87, 109, 110] have also been investigated. In addition to minimization of torque ripple, a secondary objective may also be imposed in the selection of excitation parameters, such as maximization of efficiency [111–113], or minimization of the phase voltage requirement. Intelligent control methods have also been used by certain researchers to dynamically specify the turn-on and turn-off angles [114–117]. Certain techniques are suitable
for steady loads and will not perform with the required optimality for rapidly changing loads. Alternative control methods such as flux-linkage control [118], voltage vector control [119], direct torque control [120], discrete energy pulse control [121] and torque sharing function based control methods can also be found in literature.

One interesting feature identified in [104] is the fact that the main cause of reduction in the overall efficiency of the drive system is lowering of power factor. Here, the notion of power factor refers to the ratio between the utilized real power and the total power transferred between the DC-link and the converter [122]. It has been shown that the effect on efficiency due to low power factor in chopping operation at medium speeds is more prominent than the effect caused by the high current spike in single pulse operation at the same speed. The same fact is validated by the authors of [123], where they identify that the optimization for efficiency naturally leads to single pulse mode of operation. Thus drive systems that prioritize efficiency rather than the minimization of torque ripple at high speeds may opt for a control strategy that operates with an acceptable magnitude of current spike in single pulse mode of operation rather than operating in chopping mode.

SR motor excitation control by means of soft-chopping has been extensively investigated in the past as a means of increasing efficiency. Soft-chopping involves application of the positive voltage $+v_{dc}$ and zero volts, while hard-chopping involves application of the positive voltage $+v_{dc}$ and negative voltage $-v_{dc}$. Many researchers opt for the utilization of soft-chopping control in motoring mode and hard-chopping for braking operation in order maintain current controllability [110, 124]. Soft-chopping is preferred in the motoring mode due to factors such as lower current ripple, torque ripple, lower acoustic noise and chopping frequency [125]. The authors of [126] investigate torque ripple reduction of SR motors and the relationship with the current waveforms. Results for both hard-chopping and soft-chopping operation are presented and it is found that the torque ripple caused by the switching operation can be further mitigated by the use of soft-chopping. The authors of [127] claim that the acoustic noise is reduced with soft switching due to the smooth change is flux-linkage. Out of the 6% improvement in efficiency in [127], a 5% is achieved by application of soft-chopping. Furthermore a reduction of 23% in core losses is achieved with soft-chopping. The authors of [128] present a new switching scheme which includes a third state.
within the hysteresis band. Hard turn-on or turn-off is imposed outside the hysteresis band while zero volts loop operation is imposed within the hysteresis band. The authors claim that this switching technique achieves superior current tracking capability, efficiency and low EMI compared with the standard techniques.

The following section attempts to combine below base-speed current-chopped operation and above base-speed single pulse mode operation into one controller with variable structures for the application in wide speed range SR motor drive.

4.4 Design of conventional excitation control for SR motors

![Block diagram of the excitation control strategy for (a) current chopped SR motor operation below base-speed and (b) dwell angle control in SR motor operation above base-speed](image)

Current chopping is the most commonly used SR motor excitation method below base-speed. Typically, this technique considers a fixed turn-off angle at \( \theta_{off} = \theta_1 + \theta_{r,max} \) (see figure 4.2 for the angles). Due to the low speed of operation, the current tail during the defluxing period does not continue into the negative torque region. The turn-on angle is advanced as a function of speed \( \omega \), DC-link voltage \( v_{dc} \) and the commanded reference current \( i_{ref} \). The block diagram representation of this strategy is shown in figure 4.5 (a). The required reference current is calculated as a function of the control signal \( u^* \) which is representative of the demanded torque:

\[
i_{ref} = k_{u1} u^*
\]  

(4.2)
The parameter $k_u$ represents a simple linear conversion factor of torque command $u^*$ to the reference current command $i_{ref}$.

Above base-speed, the operation of the SR motor is typically in single pulse mode where chopping is not performed. The reference current value is set at $i_{max}$, the maximum allowable current. The turn-on angle is advanced linearly based on speed. The advancing factor $k_{adv}$ is determined such that the current reaches nearly the maximum value for all the speeds above base-speed. The block diagram representation of this strategy is shown in figure 4.5 (b). Torque control is implemented by means of regulating the dwell period. The required dwell angle is calculated by the demanded torque signal $u^*$ as:

$$
\theta_{dwell} = k_u u^*
$$

The parameter $k_u$ represents a simple linear conversion of torque command to the dwell angle command.

These two excitation control methods presented above can be combined by switching the excitation control structure from chopping control to single pulse mode control based on the operational speed. To avoid rapid switching between the two controllers around base-speed, the transition can be performed based on a speed-based hysteresis band $\delta_h$. The switching between the two excitation control strategies in the first quadrant positive torque production (i.e positive torque production $u^* > 0$) can be represented as,

$$
\theta_{on} = \begin{cases} 
\theta_1 - \frac{k_u i_{ref}}{v_{dc}} & \text{if } \omega \leq \omega_{base} - \frac{\delta_h}{2} \\
\theta_1 - k_{adv} \omega & \text{if } \omega \geq \omega_{base} + \frac{\delta_h}{2}
\end{cases}
$$

$$
\theta_{off} = \begin{cases} 
\theta_1 + \theta_{r,max} & \text{if } \omega \leq \omega_{base} - \frac{\delta_h}{2} \\
\theta_{on} + k_u u^* & \text{if } \omega \geq \omega_{base} + \frac{\delta_h}{2}
\end{cases}
$$

$$
\theta_{ref} = \begin{cases} 
k_u u^* & \text{if } \omega \leq \omega_{base} - \frac{\delta_h}{2} \\
i_{max} & \text{if } \omega \geq \omega_{base} + \frac{\delta_h}{2}
\end{cases}
$$

Negative torque production ($u^* < 0$) in the fourth quadrant is provided by means of fixed turn-on and turn-off angle based braking.
4.4.1 Investigation of torque production characteristics with the conventional excitation control strategy

The 250kW SR machine design presented in chapter three is analysed here with the conventional excitation control strategy. The FE model based simulation technique presented in chapter three is configured for the 250kW SR motor and simulated at different speeds. Similar analysis is also performed on the 300W test-bed SR machine for comparison. The machine parameters associated with the excitation controller for both the machines are given in table 4.1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>250kW SR motor</th>
<th>300W test-bed SR motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>kVA rating</td>
<td>250kW</td>
<td>300W</td>
</tr>
<tr>
<td>Base-speed $\omega_{\text{base}}$</td>
<td>18000rpm</td>
<td>1800rpm</td>
</tr>
<tr>
<td>DC-link voltage $v_{\text{dc}}$</td>
<td>540V</td>
<td>24V</td>
</tr>
<tr>
<td>Unaligned inductance $L_u$</td>
<td>0.0363mH</td>
<td>0.6073mH</td>
</tr>
<tr>
<td>maximum current $i_{\text{max}}$</td>
<td>500A</td>
<td>40A</td>
</tr>
<tr>
<td>maximum speed $\omega_{\text{max}}$</td>
<td>30000rpm</td>
<td>3000rpm</td>
</tr>
<tr>
<td>$T_{\text{max}}</td>
<td><em>{\omega=\omega</em>{\text{rated}}}$</td>
<td>160Nm</td>
</tr>
<tr>
<td>$T_{\text{max}}</td>
<td><em>{\omega=\omega</em>{\text{max}}}$</td>
<td>80Nm</td>
</tr>
<tr>
<td>$\theta_{r,max}$</td>
<td>12.5deg</td>
<td>25deg</td>
</tr>
<tr>
<td>$k_{\text{adv}}$</td>
<td>0.0024deg/(rad/s)</td>
<td>0.0644deg/(rad/s)</td>
</tr>
<tr>
<td>$k_{u1}$</td>
<td>3.12A/(Nm)</td>
<td>17.78A/(Nm)</td>
</tr>
<tr>
<td>$k_{u2}$</td>
<td>0.17deg/(Nm)</td>
<td>18.60deg/(Nm)</td>
</tr>
<tr>
<td>$\theta_{\text{on,brake}}$</td>
<td>26.25deg</td>
<td>52.5deg</td>
</tr>
<tr>
<td>$\theta_{\text{off,brake}}$</td>
<td>41.25deg</td>
<td>82.5deg</td>
</tr>
</tbody>
</table>

Figures 4.6 (a) and (b) show the envelope of maximum torque-speed for both
the SR machines with the selected maximum turn-off angle values. It can be seen that constant torque production can be guaranteed in the majority of the current chopped region. This is shown by the dashed line in figure 4.6 (a), and is approximately 160Nm. At base-speed, the transition from current chopping to single pulse mode induces a discontinuity in the envelope in both the graphs.
4.4.2 Approximate input-output linearization with fixed gains $k_{u1}$ and $k_{u2}$ and open-loop characteristics

The variable $u^*$ represents the average torque command. This command is appropriately translated to an $i_{ref}$ command below base-speed or to a $\theta_{dwell}$ command above base-speed such that the relationship between $u^*$ and the actual average torque production $T_{avg}$ exhibits a certain degree of linearity. Typically this is based on fixed gains $k_{u1}$ and $k_{u2}$ of (4.5) and (4.6) which can be calculated by:

$$k_{u1} = \frac{i_{\text{max}}}{T_{\text{max}}|_{\omega=\omega_{\text{rated}}}}$$
$$k_{u2} = \frac{\theta_{r,\text{max}} + k_{\text{adv}}\omega_{\text{max}}}{T_{\text{max}}|_{\omega=\omega_{\text{max}}}}$$

The calculated values of $k_{u1}$ and $k_{u2}$ for the 250kW SR motor and the 300W test-bed SR motor are given in table 4.1.

Figure 4.8: Command $u^*$ vs output average torque $T_{avg}$ characteristics for (a) the 250kW 12/8 SR motor and (b) test-bed 300W 6/4 SR motor with conventional excitation control.

It can be seen from figures 4.8 (a) and (b) that for both the motors the open-loop $u^*$ vs $T_{avg}$ characteristics are considerably linear for low speeds (below base-speed). Certain degree nonlinearity is observed above base-speed conditions.
4.5 Enhanced excitation control with current-peak feedback

The conventional excitation control strategy switches from one control structure to another at base-speed. This leads to a discontinuity in torque production and the current reference transits from a low value to the maximum value. As a result, a sudden jerk may be observed in the shaft at base-speed and the power electronic switches may be subjected to high stresses due to the sudden increase in current.

This section presents an enhanced excitation control method which eliminates the drawbacks seen in this conventional excitation control method. In the proposed method, the current-peak value is controlled by current-peak feedback. This enables the use of the current-peak value as a manipulated variable for torque control. Hence, the transition at base-speed from current-chopping to single pulse mode does not produce additional power electronic stresses or a discontinuous torque production. A block diagram representation of this inner current-peak controller is shown in figure 4.9. Variable $i_{k+1}^*$ represents the current-peak command and $i_k$ is the sampled current-peak value. $k_{p,c}$ and $k_{i,c}$ are variable gains in the form of a PI controller which outputs an advance angle value. The detailed explanation on the formulation of the controller and gain selection of this current-peak controller is given in the appendix F. The validation of the current-peak controller with simulations for the 250kW SR motor and the 300W test-bed SR motor are also presented in the appendix F. The following section presents the experimental validation of the current-peak controller.
4.5.1 Experimental validation of the current-peak controller

The current-peak controller is implemented for the test-bed 300W 6/4 SR motor. A step current-peak command of 7A, 14A and 21A is given sequentially as shown in figure 4.10 (a) for the speeds shown in figure 4.10 (b). The variation of advance angle of the experimental system with the current-peak controller and without the current-peak controller are shown in figures 4.10 (c) and (d). Note that (a) and (b) are plotted with time and (c) and (d) with rotor position.

Figure 4.10: Experimental results of the current-peak controller validation at different speeds for the 300W 6/4 SRM. (a) current-peak command, (b) operating speed, (c) advance angle variation with current-peak control and (d) advance angle variation without current-peak control
Figure 4.11: Variation of phase current (a) 2000rpm with the current-peak controller, (b) 2800rpm with the current-peak controller, (c) 2000rpm without the current-peak controller (d) 2800rpm without the current-peak controller.
Figure 4.12: Variation of phase current (a) 400rpm with the current-peak controller, (b) 1200rpm with the current-peak controller, (c) 400rpm without the current-peak controller, (d) 1200rpm without the current-peak controller.
Experimental results for the test–bed 300W SR motor. Phase current with control (a,c,e,f) and without control (b,d,f,h) for a 21A current-peak command.

Figure 4.13: Experimental phase current waveforms extracted from Figures 4.12 and 4.11 at a specific operating point of a 21A current-peak command. (a), (c), (e), and (g) with current-peak control, and (b), (d), (f) and (h) without current-peak control.
Figures 4.12 and 4.11 present the phase current waveforms obtained from the experimental validation. Graphs (a) and (b) of figure 4.12 and (a) and (b) of figure 4.11 show the variation of phase current at 400rpm, 1200rpm, 2000rpm and 2800rpm with the current-peak controller. Graphs (c) and (d) of the same figures show the variation of phase current without the current-peak controller at the same set of speeds.

Figure 4.13 presents an enlarged versions of Figures 4.12 and 4.11 at selected operating points where a 21A current-peak command is issued. Figure 4.13 (a), (c), (e), and (g) are for the operation with current-peak control, and (b), (d), (f) and (h) for the operation without current-peak control.

In general, the experimental results corroborate the simulations. The current-peak controller successfully minimizes the error between the commanded current-peak and the value of current sampled at a rotor position of $\theta = \theta_1$. This can be seen in detail from figure 4.13. The action of the current-peak controller is more pronounced at speeds near and above the base-speed and thus more torque production capability can be expected from the SR motor from these operating regions. The advance angle variation shown in figure 4.10 (c) validates the rapid convergence of the advance angle for a step current-peak command and the operational stability of the multi-rate controller which is executed at a variable rate. In contrast, it can be seen that at high speeds the advance angle achieves a higher magnitude with the current-peak controller than without the control as shown in figures 4.10 (c) and (d). The higher advance angle counteracts the effects of high back-EMF and other disturbances. The current-peak controller can also be employed without accurate knowledge of machine parameters such as unaligned inductance. The controller can be designed for convergence with approximate inductance values thus enabling the control of the advance-angle of SR motors without detailed analysis of the inductance profile. The following section analyses the torque production capability and extends to speed control design. The possibility utilize the current-peak command as a manipulated variable is investigated.
4.5.2 Torque production characteristics with the current-peak controller

The 250kW SR motor and the test-bed 300W SR motor are simulated at different speeds and different turn-off angles. The maximum torque production capability variation with the turn-off angle is shown in figures 4.14 (a) and (b) for the two machines. Comparison with figures 4.6 (a) and (b) show that the current-peak feedback based turn-on angle correction technique presents an increased maximum torque production capability especially at speeds in the neighbourhood of the base-speed. The maximum torque production occurs at turn-off angles in the range of 12.5$^\circ$ to 15$^\circ$ for the 12/8 SR machine and at 25$^\circ$ to 30$^\circ$ in the 6/4 SR machine, similar to the conventional turn-on angle control strategy.

Figures 4.15 (a) and (b) show the variation of the maximum torque production with speed for a turn-off angle of 12.5$^\circ$ for the 250kW 12/8 SR motor and 25$^\circ$ for the 300W 6/4 SR motor. The dashed blue line shows the maximum torque production with the conventional turn-on angle control strategy, while the darker line shows the maximum torque production envelope with the current-peak control. It can be clearly seen that the adjustment of the turn-on angle to satisfy
the current-peak demand enhances the maximum torque production at all the speeds. The advantage of turn-on angle correction is more pronounced in the neighbourhood of the base-speed, where more than a 30% increase in the torque production is observed. Furthermore, the current-peak based turn-on angle control does not impose such a strong discontinuity in the torque production at the transition at base-speed. Thus smooth speed variation can be expected at the base-speed. Figure 4.16 (a) and (b) show the variation of torque production with current-peak demand at different speeds for the two SR machines considered. The torque production varies in a quasi-linear manner with the current-peak value. Thus, current-peak value can be considered as a good manipulated variable for torque control in both chopping operation as well as single pulse mode control, thereby avoiding the need to switch control variables from a current reference to dwell angle command at the transition near base-speed. Such a torque control method directly links the current-peak value with manipulation of the turn-on angle for torque control. Traditionally, turn-on angle based torque control is typically avoided in high speed SR motor applications due to the high sensitively of the turn-on angle with the torque production, i.e., a large variation of torque production is seen with only small variations of the turn-on angle and hence is not
Figure 4.16: Torque vs current-peak variation with current-peak control for (a) the 250kW 12/8 SR motor and (b) test-bed 300W 6/4 SR motor. $\theta_{off}$ fixed at the maximum value of $\theta_{r,max}$.

considered as a suitable manipulated variable. However, the effect of the high sensitivity of the turn-on angle is suppressed by the current-peak feedback controller action. Hence, the current-peak command can be considered as a manipulated variable for the outer torque control loop. Different methods to translate the torque command $u^*$ to the output average torque production are investigated in the following section.

4.5.3 Approximate input-output linearization with a variable gain $k_u$

Closer examination of the characteristics of figures 4.16 (a) and (b) reveal that the change of the curve with speed is minimal in operation below base-speed. This is due to the minimal variation of maximum torque production capability in this speed domain. However, the maximum torque production capability varies approximately in a constant power characteristic at speeds above base-speed. Hence, by incorporation of this speed dependent relationship in the input-output linearization by means of a variable gain $k_u$, in $i_k^* = k_u u^*$, a higher degree of
Figure 4.17: Command $u^*$ vs output average torque $T_{avg}$ characteristics for (a) the 250kW 12/8 SR motor and (b) test-bed 300W 6/4 SR motor with current-peak control and variable $u^*/i^*$ gain.

Linearity can be expected from the $u^*$ vs $T_{avg}$ characteristics in the first quadrant of positive torque production.

The maximum average shaft power at base-speed is given by:

$$P_{\max|\omega=\omega_{\text{base}}} = (T_{\max|\omega=\omega_{\text{base}}}) \omega_{\text{base}}$$  \hspace{1cm} (4.9)

It follows from (4.9) that maximum torque production capability above base-speed can related to torque production at base-speed by:

$$(T_{\max|\omega=\omega_{\text{base}}}) \omega_{\text{base}} = (T_{\max|\omega>\omega_{\text{base}}}) \omega$$  \hspace{1cm} (4.10)

which is a constant power region of the motor torque-speed characteristics. The maximum torque production at a speed $\omega > \omega_{\text{base}}$ can be approximated as:

$$T_{\max|\omega>\omega_{\text{base}}} = \frac{(T_{\max|\omega=\omega_{\text{base}}}) \omega_{\text{base}}}{\omega}$$  \hspace{1cm} (4.11)
The speed dependent gain $k_u$ can be formulated as:

$$k_u = \begin{cases} \frac{i_{\text{max}}}{T_{\text{max}} |\omega - \omega_{\text{base}}} \omega \text{ if } \omega \leq \omega_{\text{base}} \\ \frac{i_{\text{max}}}{T_{\text{max}} |\omega - \omega_{\text{base}}} \omega \text{ if } \omega > \omega_{\text{base}} \end{cases}$$

(4.12)

The excitation controller with fixed turn-off angle and variable $k_u$ is simulated for both the motors. Figures 4.17 (a) and (b) present the $u^*$ vs $T_{\text{avg}}$ characteristics for the 250kW motor and the test-bed 300W motor. It can be clearly seen that these $u^*$ vs $T_{\text{avg}}$ characteristics are of higher degree of linearity compared with the conventional characteristics of figure 4.8. Hence, the torque command to current-peak demand translation (4.12) provides the capability to maintain a consistent command response under variable speed conditions. Furthermore, torque control applications such as in electric vehicle propulsion can be implemented without any torque feedback with considerable accuracy for operation below and above base-speed.

### 4.6 Incorporation of an extended zero-voltage loop interval for the advancement of operational characteristics

![Diagram of zero-voltage loop](image)

Figure 4.18: (a) Positive voltage turn-on (b) Zero voltage loop (c) hard turn-off with negative voltage

The zero volt loop (ZVL) refers to the application of zero-voltage across the
phase winding of the SR motor, i.e., short circuiting the phase windings through the commutation switches and diodes during part of the stroke period. The application of a ZVL has been used in various commutation methods in current control of SR motors. A ZVL is typically used instead of application of a negative voltage in soft chopping operation to drive the phase current below the reference current. A ZVL has been presented in [102] to keep phase flux-linkage at a constant value through part of the stroke period. A converter topology is developed in the patent [129] for a bifilar wound SR motor and utilizes ZVL based operation for increased efficiency, increasing the area of flux-linkage vs current trajectory for a given peak current. The authors of [130] present a uni-polar circuit that utilizes ZVL in their current control strategy. The authors of [131] utilize a ZVL for noise cancellation of SR motors. Both [129] and [131] utilizes a commutation technique with ZVL, where in the first stage involves building-up the current, in the second stage the current is chopped and in the third stage ZVL is applied for a certain period after which a negative voltage is applied. The authors of [131] utilize the ZVL period for noise cancellation, while the emphasis in [129] is on the circuit topology. During the ZVL period of the phase, current circulates within the phase winding producing torque and does not return to the DC-link. The authors of [104] claim that this improves the energy utilization, while the reduction in the number of switching instances reduces the power factor and increases overall efficiency further.

Figure 4.18 shows the different commutation stages of a typical asymmetric half bridge converter used in this research for the SR motor control. Figure 4.18 (a) shows the application of full voltage where the current is built-up, figure 4.18 (b) shows the application of zero volts by keeping the lower switch turned-on while the upper switch is turned-off. Figure 4.18 (c) shows the hard turn-off of the phase current by application of the negative voltage.

The following section considers the incorporation of a ZVL interval within each stroke in order to examine its potential to improve efficiency. Figure 4.19 shows the single pulse mode current waveforms, flux-linkage waveforms and the corresponding flux-linkage/current loops for a stroke with and without the ZVL period. The different commutation stages during ZVL operation correspond to figures 4.18 (a) to (c) where the application of positive voltage, zero-voltage followed by hard turn-off is shown sequentially.

During conventional chopping operation, hard turn-off can be performed prior
Figure 4.19: (a) Current waveform (b) flux-linkage waveform and (c) flux-linkage / current loop for a SR motor phase commutation with the a ZVL period during single pulse operation
to the aligned position to mitigate the flux-linkage peak and improve efficiency. However, during such conditions the maximum energy conversion maybe limited. In contrast, hard turn-off is performed during single pulse operation based on the dwell angle command to deliver the required average torque demand. Application of the ZVL period increases the energy conversion loop for the same flux-linkage peak. This also lowers iron losses for the same average torque production.

In this investigation, the ZVL interval is varied in linear manner to the current-peak command $i_k^*$ and dwell angle $\theta_{dwell}$ as:

$$\theta_z = \theta_{dwell} \left( \frac{i_{max} - i_k^*}{i_{max}} \right)$$

(4.13)

where $\theta_{dwell} = \theta_{off} - \theta_{on}$, $\theta_{off} = \theta_{r,max}$ and $\theta_{on}$ is determined by the current-peak controller.

![Command $u^*$ vs output average torque $T_{avg}$ characteristics for the 250kW SR motor](image1)

![Command $u^*$ vs output average torque $T_{avg}$ characteristics for the 300W test-bed SR motor](image2)

Figure 4.20: Command $u^*$ vs output average torque $T_{avg}$ characteristics for (a) the 250kW 12/8 SR motor and (b) test-bed 300W 6/4 SR motor with current-peak control, variable ZVL and variable $u^*/i_k^*$ gain.

The variable gain input-output linearization presented in section 4.5.3 is also used here for the translation of the average torque demand to a current-peak command. The 250kW SR motor and the test-bed 300W SR motor are simulated at different speeds. Figures 4.20 (a) and (b) present the $u^*$ vs $T_{avg}$ characteristics
for the 250kW motor and the test-bed 300W. The $u^*$ vs $T_{avg}$ characteristics present a lower grade of linearity compared with the version without ZVL in section 4.5.3. $T_{avg}$ monotonically increases with $u^*$ command and hence the design of an outer speed control loop can be considered with $u^*$.

4.7 Speed control design of SR motors and fault-tolerant operation

This section considers the design of speed control for SR motors. The excitation control methods presented earlier are simulated and tested in experiments for the outer speed loop operation. The translation of the average torque command $u^*$ to the excitation parameters is ensured by the excitation controller. Assuming that the outer speed control loop is of lower bandwidth, the relationship between the average torque command $u^*$ and the actual average torque production $T_{avg}$ can be approximated to:

$$T_{avg} = xu^* + h_u (\omega, i_k) \quad (4.14)$$

where $x$ represents the healthy fraction of the machine. For a 6/4 SR machine $x \in \left\{1, \frac{2}{3}, \frac{1}{3}\right\}$ and for a 12/8 SR machine with two parallel paths per-phase the possible fractions for $x$ are $x \in \left\{1, \frac{5}{6}, \ldots, \frac{1}{6}\right\}$. The function $h_u (\omega, i_k)$ incorporates the deviation of the average torque command with the actual torque production. The instantaneous torque production can be represented as:

$$T_m = T_{avg} + h_i (\omega, \theta, i_k) \quad (4.15)$$

where $h_i$ incorporates the torque ripple components. Substitution of $T_{avg}$ of (4.14) in (4.15) yields:

$$T_m = xu^* + h \quad (4.16)$$

where $h$ represent the high frequency ripple and the modelling inaccuracies.

$$h = h_i (\omega, \theta, i_k) + h_u (\omega, i_k) \quad (4.17)$$
The shaft dynamics are given by:

$$J_m \frac{d\omega}{dt} + B_m \omega = T_m - T_{load}$$

Substitution of (4.16) yields:

$$J_m \frac{d\omega}{dt} + B_m \omega = xu^* + h - T_{load} \quad (4.18)$$

The speed control can be formulated as a proportional/integral law:

$$u^* = k_{p,s} (\omega^* - \omega) + k_{i,s} \int (\omega^* - \omega) \, dt \quad (4.19)$$

Substitution of (4.19) in (4.18) yields:

$$J_m \frac{d\omega}{dt} + B_m \omega = xk_{p,s} (\omega^* - \omega) + xk_{i,s} \int (\omega^* - \omega) \, dt + w_s \quad (4.20)$$

where $w_s$ is disturbance to the system (4.20). Assuming $w_s$ is an exogenous disturbance, the closed-loop transfer function for the command input is given by:

$$G_{cl}(s) = \frac{\omega(s)}{\omega^*(s)} = \frac{xk_{p,s}J_m}{s^2 + (B_m + xk_{p,s})J_m} + \frac{xk_{i,s}J_m}{s^2 + (B_m + xk_{p,s})J_m} \quad (4.21)$$

The disturbance transfer function is given by:

$$G_d(s) = \frac{\omega(s)}{w_s(s)} = \frac{\frac{1}{J_m} s}{s^2 + \left(\frac{B_m + xk_{p,s}}{J_m}\right) s + \frac{xk_{i,s}}{J_m}} \quad (4.22)$$

The closed-loop transfer function can approximately cast into the standard second order format:

$$G_{std}(s) = \frac{\Omega_n^2}{s^2 + 2\varsigma \Omega_n s + \Omega_n^2} \quad (4.23)$$

where $\varsigma$ is the damping factor and $\Omega_n$ is the undamped natural frequency. Considering a healthy machine with $x = 1$ and for $\varsigma = \varsigma_s$ and $\Omega_n = \Omega_{s,n}$ the proportional
and integral gains of the controller (4.19) for the speed system (4.20) can be approximated to:

\[ k_{p,s} = 2s_0 \Omega_{s,n} J_m - B_m \]  

(4.24)

\[ k_{i,s} = J_m \Omega_{s,n}^2 \]  

(4.25)

During chopping operation, the torque production is nearly continuous and thus system bandwidth is limited by the switching frequency and the digital controller step-time. However, the torque production develops into a pulsating waveform near and above base-speed, during which instantaneous torque control or current control may not be achievable. Hence, the bandwidth of the speed system (4.20) must be designed with consideration on such limitations. Based on the Nyquist theorem, the system maximum bandwidth requires to be limited by:

\[ \Omega_{s,n,\text{max}} < \begin{cases} 
\pi \frac{1}{T_s} & \text{if } \omega < \omega_{\text{base}} \\
\frac{N_p N_r \omega_{\text{base}}}{2} & \text{if } \omega \geq \omega_{\text{base}}
\end{cases} \]  

(4.26)

where the above base-speed bandwidth is limited by the minimum stroke frequency of the SR motor \( \frac{N_p N_r \omega_{\text{base}}}{2} \) due to operation in single pulse mode. For higher phase margin and gain margin, the system bandwidth maybe selected as

\[ \Omega_{s,n} = \gamma \min (\Omega_{s,n,\text{max}}) \]  

(4.27)

where \( 0 < \gamma < 1 \) is a design parameter.

**Consideration for fault-tolerant operation**

For a healthy SR motor, the low speed torque production is nearly continuous with minimal torque ripple at the turn-off and turn-on points of commutation. However, under faulted conditions the torque production is approximately a pulsating square waveform. For the 6/4 and 12/8 SR machines considered in this chapter, loss of one phase or two phases result in a torque pulsation of fundamental frequency \( N_r \omega \) rad/s. Hence, the bandwidth limitation condition can be rewritten for faulted operation as:

\[ \Omega_{s,n,\text{max}} < \frac{N_r}{2} \omega \]  

for all \( \omega \)  

(4.28)
For a given system bandwidth $\Omega_{s,n}$, the minimum speed which satisfies the Nyquist condition is given by:

$$\omega_{nyq,\text{min}} = \frac{2\Omega_{s,n}}{N_r}$$

(4.29)

Substitution of (4.27) in (4.29) yields $\gamma$ as:

$$\gamma = \frac{N_r}{2 \min (\Omega_{s,n,\text{max}})} \omega_{nyq,\text{min}}$$

(4.30)

where stable fault-tolerant operation can be guaranteed for all speeds $\omega > \omega_{nyq,\text{min}}$.

For a maximum torque ripple / disturbance magnitude of $\hat{w}_s$, the magnitude of speed ripple is given by (4.22) as:

$$\hat{\omega}_{\text{rip}} = |G_d (j\omega_e)| \hat{w}_s$$

(4.31)

$\omega_e$ represents the ripple frequency given by $\omega_e = N_r \omega$ under faulted conditions. For stability, the speed must be maintained above the minimum speed $\omega_{nyq,\text{min}}$ and this can be represented by the inequality:

$$\omega - \hat{\omega}_{\text{rip}} > \omega_{nyq,\text{min}}$$

(4.32)

Substitution of (4.31) in (4.32) and rearrangement yield the inequality:

$$|G_d (j\omega_e)| < \frac{\omega - \omega_{nyq,\text{min}}}{\hat{w}_s}$$

(4.33)

where the rotor speed and the faulted operation torque ripple frequency are related by $\omega_e = N_r \omega$.

Inequality (4.33) is satisfied at frequencies above the intersection of the bode magnitude curve $Y' = |G_d (j\omega_e)|$ and the line given by:

$$Y = \frac{1}{N_r \hat{w}_s} - \frac{\omega_{nyq,\text{min}}}{\hat{w}_s}$$

(4.34)

where $\omega_{nyq,\text{min}}$ is given by (4.30) and is a function of the design parameter $\Omega_{s,n}$ imposed via $k_{p,s}$ and $k_{i,s}$ of equations (4.24) and (4.25).

Given the frequency at the intersection as $\omega_{e,0}$, stability can then be guaranteed at all speeds beyond $\omega_{e,0}$. 
4.7.1 Case study: Speed control design for the 250kW 12/8 SR motor

The system parameters considered for the 250kW motor drive are given in table 4.2.

Table 4.2: 250kW SR motor mechanical parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft inertia $J_m$</td>
<td>0.025 kgm$^2$</td>
</tr>
<tr>
<td>Frictional coefficient $B_m$</td>
<td>0.0050 Nms</td>
</tr>
</tbody>
</table>

The digital controller step time is $T_s = 40\mu s$ and the base-speed is 18000rpm ($\omega_{base} = 1885$ rad/s). Hence, the maximum bandwidth limit can be defined from (4.26) as:

$$\Omega_{s,n,max} < \begin{cases} 
7854 \text{rad/s} & \text{if } \omega < \omega_{base} \\
22619 \text{rad/s} & \text{if } \omega \geq \omega_{base} 
\end{cases}$$

Figure 4.21: Bode magnitude plot of the disturbance transfer function and the system disturbance boundary line for the 250kW motor drive

In this design $\Omega_{s,n} = 200 \text{rad/s}$ is selected such that $\gamma = 0.0255$ and $\Omega_{s,n} < \Omega_{s,n,max}$ is satisfied. For a damping factor of $\zeta_s = 0.707$, the PI controller parameters are calculated from (4.24) and (4.25) as $k_{p,s} = 7.0650$ and $k_{i,s} = 1000$. 

Faulted operational stability can be guaranteed to a lower boundary of \( \omega = \omega_{nyq, \text{min}} \), where \( \omega_{nyq, \text{min}} = 50 \text{rad/s} \) or 477.46rpm. The disturbance boundary line (4.34) is plotted for a disturbance magnitude of \( \hat{w}_s = 4T_{\text{rated}} \) and \( \hat{w}_s = 8T_{\text{rated}} \) and is shown in figure 4.21 by the two red-lines. The intersections occur at \( w_e = 581 \text{rad/s} \) (speed of 5548.1rpm) at \( \hat{w}_s = 4T_{\text{rated}} \) and \( w_e = 692 \text{rad/s} \) (speed of 6608rpm) at \( \hat{w}_s = 8T_{\text{rated}} \) respectively. Therefore stability can be guaranteed for speeds above 6608rpm for all disturbance ripple below \( 8T_{\text{rated}} \).

### 4.7.2 Case study: Speed control design for the test-bed 300W experimental 6/4 SR motor

The test-bed consists of the SRM coupled to a PM motor acting as a load machine. The complete system parameters are given in table 4.3.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft inertia ( J_m )</td>
<td>0.0025 \text{kgm}^2</td>
</tr>
<tr>
<td>Frictional coefficient ( B_m )</td>
<td>0.0025 \text{Nms}</td>
</tr>
</tbody>
</table>

![Figure 4.22: Bode magnitude plot of the disturbance transfer function and the system disturbance boundary line for the 300W motor drive](image)

The digital controller step time is \( T_s = 40 \mu s \) and the base-speed is 1800rpm (\( \omega_{\text{base}} = 188.5 \text{ rad/s} \)). Hence, the maximum bandwidth limit can be defined from
CHAPTER 4. SR MACHINE OPERATION IN MOToring MODE

(4.26) as:
\[
\Omega_{s,n,\text{max}} < \begin{cases} 
7854 \text{rad/s} & \text{if } \omega < \omega_{\text{base}} \\
1131 \text{rad/s} & \text{if } \omega \geq \omega_{\text{base}}
\end{cases}
\]  \quad (4.36)

In this design \( \Omega_{s,n} = 25 \text{rad/s} \) is selected such that \( \gamma = 0.0221 \), and \( \Omega_{s,n} < \Omega_{s,n,\text{max}} \) is satisfied. For a damping factor of \( \zeta_s = 0.707 \), the PI controller parameters are calculated from (4.24) and (4.25) as \( k_{p,s} = 0.0859 \) and \( k_{i,s} = 1.5625 \).

Faulted operational stability can be guaranteed to a lower boundary of \( \omega = \omega_{\text{nyq},\text{min}} \), where \( \omega_{\text{nyq},\text{min}} = 12.5 \text{rad/s} \) or 119.36rpm. The disturbance boundary line (4.34) is plotted for a disturbance magnitude of \( \hat{w}_s = 4T_{\text{rated}} \) and \( \hat{w}_s = 8T_{\text{rated}} \) and is shown in figure 4.22. The intersections occur at \( w_e = 140 \text{rad/s} \) (speed of 334rpm) at \( \hat{w}_s = 4T_{\text{rated}} \) and \( w_e = 183 \text{rad/s} \) (speed of 437rpm) at \( \hat{w}_s = 8T_{\text{rated}} \) respectively. Therefore stability can be guaranteed for speeds above 437rpm for all disturbance ripple below \( 8T_{\text{rated}} \).

4.8 Experimental validation and simulation results

Simulation result for the speed control of the 250kW SR motor and the 300W test-bed SR motor in addition to experimental results for the 300W test-bed SR motor are presented in this section. Experimental validation for the speed control and the excitation control are obtained by implementing these algorithms in the 300W test-bed SR motor drive. The three main excitation control methods presented earlier are considered for the simulation and experimentation with the outer speed loop. These are namely,

1. Conventional excitation control method with below base-speed chopping and above base-speed single pulse operation.
2. Current-peak feedback based turn-on angle control with fixed turn-off angle.
3. Current-peak feedback based turn-on angle control, variable ZVL and fixed turn-off angle.
Figure 4.23: (a) Speed command and (b) mechanical load torque used in the simulation of the 250kW SRM

Figure 4.24: Speed error with respect to the command in figure 4.23 for the 250kW SRM, based on simulation results

4.8.1 Simulation result for the 250kW 12/8 SR motor drive

The 250kW SR motor drive is simulated for the speed command profile and the load torque profile shown in figures 4.23 (a) and (b) respectively. The full simulation results for the operation of the 250kW SRM with the three excitation controllers are shown in the appendix G. Figure 4.25 shows an example of the simulation results for the operation of the SRM with the current-peak feedback turn-on angle controller. The comparison of power consumption and efficiency are discussed in section 4.8.3.

FFT analyses on the flux-linkage, torque production and DC-link current
waveform are performed on the 250kW SRM operation at different speeds and loads. These are also given in the appendix H. The frequency domain performance characteristics of the three different excitation controllers are discussed in section 4.8.3.

Figure 4.24 presents the speed error of the 250kW SRM with respect to the command and load given in figure 4.23. In general it can be concluded that the speed controller operates with excellent stability for all three excitation control methods. The conventional method has a higher error compared with the two other methods. This is mainly due to the higher deviation from linearity of the torque production characteristics, especially at speeds above base-speed. A speed error reaches 2000rpm at the instant of acceleration and switch-over from current chopping control to single pulse mode. In contrast the operation of the 250kW SRM with the current-peak feedback based excitation control achieved only a speed error of approximately 750rpm at this instant.

4.8.2 Experimental validation and simulation result for the 300W test-bed SR motor drive

The simulation and the test-bed experiment are both performed with the speed command profile shown in figure 4.26. At each speed, the drive is subjected to a three level step loading with an upper limit close to the maximum torque production capability. Oscilloscope waveforms showing the experimental results of the 300W SRM operation with the three different excitation control methods are given in figures 4.27 (a), (b) and (c). The motor phase current, DC-link voltage, load-machine command and the drive actual speed are shown in these figures. The full experimental and simulation results for the operation of the 300W SRM with the three excitation controllers are shown in the appendix G. Figure 4.28 shows the comparison of the experimental results and simulation results for the operation of the SRM with the current-peak feedback turn-on angle controller. The comparison of power consumption and efficiency are discussed in section 4.8.3.

FFT analyses are performed on the flux-linkage, torque production and experimentally obtained DC-link current waveforms of the 300W SRM operating at different speeds and loads. These are also given in the appendix H. The frequency domain performance characteristics of the three different excitation
Simulation results of the 250kW SRM speed control. Excitation control with current-peak feedback based turn-on angle control with fixed turn-off angle.

Figure 4.25: Simulation results of the 250kW SRM speed control. Excitation control with current-peak feedback based turn-on angle control with fixed turn-off angle. (a) speed command and actual speed, (b) phase current (c) advance angle (d) power consumption
controllers are also discussed in section 4.8.3.

Figures 4.29 (a) and (b) presents the speed error of the 300W SRM based on experimental results and simulations respectively. In general it can be concluded that the SRM performs with excellent response characteristics for with all three excitation control methods. The excitation controller with current-peak feedback and ZVL exhibits a higher speed ripple at low speeds, and this is shown in both experimental and simulation results. However, it should be noted that certain portion of the speed ripple shown in the experimental waveform is also due to the load-machine characteristics which is not modelled in the simulations.

4.8.3 Discussion

Power consumption and efficiency

Figures 4.30 and 4.31 present the power consumption and efficiency based on simulation results for the 250kW and 300W SR motors and experimental results for the 300W SR motor. The electro-mechanical energy conversion efficiency is considered here and hence the frictional losses are not considered. The experimental electro-mechanical energy conversion efficiency in motoring mode is calculated by:

\[
\eta = \frac{P_{out}}{P_{in}} = \frac{T_{em} \omega}{v_{dc}i_{dc}}
\]  

(4.37)
Figure 4.27: Oscilloscope waveforms for the 300W SRM operation with (a) with the conventional excitation control, (b) with current-peak feedback turn-on control and (c) current-peak feedback turn-on control and added ZVL interval.
Figure 4.28: Comparison of simulation and experimental results for the 300W SRM operation with PI speed control and current-peak feedback turn-on control combined. (a), (e) speed command and actual speed, (b), (f) phase current (c), (g) advance angle and (d), (h) power consumption.
where $T_{em}$ is the electromagnetic torque production. In the evaluation, the electromagnetic torque production is estimated by:

$$T_{em} = T_{meas} + B_{estim}\omega$$  \hspace{1cm} (4.38)

where $T_{meas}$ is the shaft torque measured via the transducer and $B_{estim}$ is an estimated frictional coefficient value.

A similar pattern of efficiency can be identified from the 250kW SR motor and the test-bed 300W SR motor. The experimental results on efficiency validate the simulation results to a certain degree with explainable deviations.

The simulation results for the 250kW SR motor and the 300W SR motor both predict very similar efficiencies during below base-speed chopping operation,
and a lower efficiency for the ZVL technique in certain instances below base-speed. However, the experimental results show that the ZVL technique achieves a higher efficiency of 3% to 5% in chopping operation compared with the other two excitation control methods. This can be attributed to the fact that the simulations do not reproduce the iron losses. The efficiencies achieved by the experimental result are slightly less than the predictions by the simulation results due to the fact that the iron losses are not taken into account by the simulations.

Both the simulation and experimental results show that for all above base-speed operating conditions (i.e., single pulse mode), the excitation control methods featuring the current-peak feedback achieves higher efficiency compared with the conventional excitation control method. Closer examination of the experimental results for efficiency obtained for the excitation control methods featuring the current-peak feedback and ZVL reveal that the excitation controller with ZVL achieves higher efficiency during above base-speed single-pulse mode operation. Similar to the below base-speed results, the simulations fail to predict this improvement in efficiency due to the ZVL. This can also be attributed to the fact that the simulations do not reproduce the iron losses. As explained in the following section with the aid of FFT results, the ZVL technique achieves lower iron losses compared with the other two excitation methods. Hence the experimental results for the ZVL technique achieve higher efficiency. This increase in efficiency
Figure 4.31: (a) Power consumption (b) Efficiency based on experimental results of the 300W SRM and (a) Power consumption (d) Efficiency based on simulation results of the 300W SRM due to the ZVL is also relevant for the 250kW machine and provides a prediction of 4% to 5% increase in efficiency. This would translate into 10kW to 12.5kW of energy saving under full load operation.

**Frequency domain analysis**

Frequency domain results based on the 250kW SR motor and the 300W SR motor simulations are presented in appendix H. Examples of the flux-linkage FFT spectrums for the 250kW SR motor and the 300W SR motor are shown in figures 4.32 (a) and (b) respectively. These represent motor operation above
base-speed. In general it can be seen that the conventional excitation method
achieves the lowest flux-linkage magnitude at the fundamental frequency and at
high speeds due to the short dwell period (period of application of full positive
voltage). The current-peak feedback excitation method achieves a higher flux-
linkage as the dwell period is at a higher value. The ZVL technique successfully
lowers the flux-linkage fundamental magnitude as well as the other high frequency
magnitudes. Based on this analysis it can be predicted that the conventional
excitation achieves a lower iron loss compared with the ZVL. However, the overall
copper losses incurred by the conventional method are considerably higher than
the difference in iron losses. Hence the conventional method achieves lower overall
efficiency than the ZVL technique as explained earlier.

An example of the torque production FFT analysis is shown in figures 4.33
(a) and (b) for the 250kW SR motor and the 300W SR motor respectively. The
low speed performance of the conventional method and current-peak feedback
technique are similar. However, the introduction of a ZVL increases the torque
ripple significantly under low speed operation. The conventional method transits
into single pulse mode of operation as the speed increase above base-speed and
hence produces a high torque ripple compared with the other two methods.

An example of the FFT analysis on the DC-link current is shown in figures 4.34
Conventional excitation control method
Current-peak feedback based turn-on angle control
Current-peak feedback based turn-on angle control, combined with variable ZVL

Figure 4.33: Sample FFT spectrum of electromagnetic torque waveform. (a) 250kW SRM operating at 28000rpm and at a load of 48Nm. (b) 300W SRM operating at 2800rpm and at a load of 0.4Nm.

Figure 4.34: Sample FFT spectrum of dc-current waveform. (a) 250kW SRM operating at 28000rpm and at a load of 48Nm. (b) 300W SRM operating at 2800rpm and at a load of 0.4Nm.

(a) and (b) for the 250kW SR motor and the 300W SR motor respectively. The frequency domain performance of the DC-link current follows a similar pattern to that of the torque waveform where a high DC-link current ripple is seen with
the ZVL technique at lower speeds and with the conventional method at higher speeds respectively.

4.9 Conclusion

SR machine operation in motoring mode has been investigated in this chapter. The conventional method of excitation control was presented. Two improved versions of excitation control were developed. Speed control for SR motor operation with these three excitation control methods was considered.

Speed control combined with excitation control was developed for a 250kW SR machine and 300W test-bed SR machine. The speed control and excitation control has been experimentally validated. The performances in terms of command response, stability, power consumption, efficiency, and frequency domain analysis were performed on the two machines based on simulation and experimental results. The variations of performance and the advantages of the different excitation control methods were discussed.

In conclusion, it can be seen that the current-peak feedback technique achieves a higher maximum torque production envelope compared with the conventional method. This provides an added advantage in motoring operation especially for applications that require short-term overload operation such as in aero-engine starter operation where high starting torque as well as short term high speed motoring is also required.

The improvement in efficiency with the ZVL technique is achieved by the mitigation of the flux-linkage peak while maintaining the energy conversion loop area to produce the required average torque. The flux densities within the machine are a function of flux-linkage and a reduction in the flux-linkage peak will also result in a reduction in the harmonic content of the magnetic flux densities distribution within the machine. This results in a reduction in iron losses during ZVL operation. The ZVL technique provides an improved efficiency, however at the cost of additional torque ripple and DC-link current ripple. An improvement of approximately 4% to 5% can be achieved with the ZVL technique at high speeds compared with the current-peak feedback excitation without ZVL. In comparison with the conventional method, the ZVL technique achieves approximately 20% to 30% improvement in efficiency. The ZVL technique reaches near 92% to 93% efficiency at high speed, full-load operation.
Chapter 5

Switched Reluctance Machine Operation in Generation Mode

5.1 Introduction

Electrical systems for vehicle power and propulsion have become a major area of interest due to their benefits in the transportation industry, e.g., high efficiency, high fuel economy, low maintenance and enhanced life time. The SR machine is an option for the more-electric aero-engine starter/generator [132–135], automotive starter/generator [96, 136], regenerative braking [137] and hybrid electric vehicle applications [138, 139]. The SR machine is an option in such applications due to its simple construction, low manufacturing cost, brushless operation, wide operational speed range, high torque density, high efficiency and the independence from rare earth permanent magnet metals [55].

However, the control of SR machines is complicated in both the motoring and power generation modes of operation. This is mainly due to the inherent nonlinear nature and the difficulty in adopting standard linear control methods for the excitation control of the machine. Furthermore, the optimization of SR machine control for high efficiency and performance requires more complex analysis compared with that of the PM machine. As a result, the adoption of the SR machine technology by the transportation industry has been hindered.

The majority of vehicle power and propulsion applications require high speed operation. During high speed operation, SR machines perform best in single pulse mode. The instantaneous output current of the system cannot be directly controlled and average current production during each stroke period of the SR
The objective of this chapter is to investigate SR generator excitation control strategies suitable for aero-engine power generation which offer enhanced efficiency and performance. Due to the high speed nature of the application, only single pulse mode of operation is investigated in this study. Two of the structurally simple, classical excitation control strategies, viz.,

1: fixed turn-on angle/variable turn-off angle based control and
2: fixed turn-off angle/variable turn-on angle based control

are designed in this chapter. These two excitation control strategies are compared with simulation and experimental results. In addition to these two classical excitation controllers, four optimal excitation controllers are also developed. These optimal excitation controllers are designed with piecewise continuous functions (PCFs) fitted to optimal data points extracted from simulated data. The optimal excitation angle controller which provides superior overall efficiency and performance is selected based on experimental data. This is then compared with the two classical excitation controllers in order to determine the near-optimal design and operational capability of the two classical forms. In each of these simulations and experiments, the excitation angle controller is implemented in combination with the voltage regulator. The simulation results are validated by the 300W test-bed SR generator.

The topology of the SR generator system considered in this study is shown in figure 5.1. Each phase of the SR machine is interfaced with an asymmetric half bridge converter. The operational dynamics of this system in generation mode is
briefly explained in section 5.2. Section 5.3 defines four optimizations of interest in the operation of a SR generator and discusses the variation of the excitation parameters for these based on simulations performed for the 250kW SR generator and the 300W SR generator. Section 5.4 presents the design of the two classical excitation controllers. Section 5.5 presents the PCF curve fitting of optimal data and the design of the optimal excitation controllers. Section 5.6 presents the design of the voltage regulator for the operation in combination with these excitation control methods. Section 5.7 presents simulation results and experimental validation of these excitation controllers, and discusses the performance and efficiency with the aid of time-domain and frequency-domain analyses.

5.2 SR generator operation

Two classes of current control methods are available for SR generators, viz., current chopping operation and single pulse mode of operation. Current chopping is successful with low speeds where the generator back-EMF is significantly lower than the DC-link voltage $v_{dc}$. However, when the magnitude of the back-EMF is in the same range as $v_{dc}$, chopping operation becomes difficult due to the insufficiency of DC-link voltage. In such situations, instantaneous current control
is typically relinquished and per-stroke average current control via single pulse mode of operation is established.

The SR generator excitation in single pulse mode of operation is controlled by manipulation of the turn-on (θ_{on}) and turn-off (θ_{off}) angles with respect to the rotor angle θ. Figure 5.2 shows the idealized inductance profile of a typical SR machine and the current waveforms for three cases of excitation pulses. Parameters β_s and β_r represent the stator pole angle and the rotor pole angle respectively. θ_u and θ_a represent the unaligned and aligned positions. θ_{rpp} represents the rotor pole pitch angle where θ_u + θ_{rpp} corresponds with the consequent unaligned position in phase A. The stroke angle θ_{st} represents the phase shift of the adjacent phase, at which a similar inductance profile is repeated.

Given the applied voltage on a certain phase v_{ph}, the instantaneous dynamics is governed by,

\[ v_{ph} = R i_{ph} + \frac{d\psi}{dt} \]  

Equation (5.1) can also be written as:

\[ v_{ph} = R i_{ph} + \left( \frac{\partial\psi}{\partial i_{ph}} \right) \frac{di_{ph}}{dt} + \left( \frac{\partial\psi}{\partial \theta} \right) \frac{d\theta}{dt} \]  

The two partial differentials \( L = \frac{\partial \psi}{\partial i_{ph}} \) and \( e = \frac{\partial \psi}{\partial \theta} \) are considered as the inductance and back-EMF terms which are functions of \( i_{ph} \) and \( \theta \).

For a typical SR machine, negative back-EMF (\( e < 0 \)) results during the region \( \frac{dL}{d\theta} < 0 \) as shown in figure 5.2 and is utilized for power generation. The SR machine current is built-up during the period from \( \theta_{on} \) to \( \theta_{off} \) while the switches are turned on. Following turn-off, energy is returned to the DC-link via the free-wheeling diodes until the phase current extinguishes at \( \theta_{ext} \). The selection of the excitation parameters \( \theta_{on} \) and \( \theta_{off} \) is a challenging task since these parameters are not unique for the same average DC current production. Techniques used for the calculation of these excitation parameters are briefly reviewed in the section 5.3. Figure 5.2 shows three cases where the same average DC current is produced. While current waveforms a and b are for the same speed with \( e < v_{dc} \) and different excitation times, waveform c represents a typical situation where \( e > v_{dc} \). In waveform a and b, the back-EMF \( e < v_{dc} \) and as a result the current decays following \( \theta > \theta_{off} \). In contrast, waveform c increases
for a certain duration beyond $\theta > \theta_{\text{off}}$ due to the back-EMF $e > v_{\text{dc}}$ condition.

Although, the three waveforms in figure 5.2 produce the same average DC current, the efficiencies and the power electronic converter stresses are at three different levels. Four parameters that can be used to characterize the SR generator and converter performance are the excitation penalty $\varepsilon$, root mean square (RMS) current level $I_{\text{rms}}$, peak phase current value $I_{pk}$, and peak flux-linkage $\psi_{pk}$. The excitation penalty is defined by [140],

$$
\varepsilon = \frac{I_{\text{in}}}{I_{\text{out}}} = \frac{1}{\theta_{\text{rpp}}} \frac{\theta_{\text{off}}}{\theta_{\text{on}}} \int_{\theta_{\text{on}}}^{\theta_{\text{off}}} i_{\text{ph}} d\theta
$$

The per-stroke per-phase RMS current, peak current and peak flux-linkage values are calculated by,

$$
I_{\text{rms}} = \sqrt{\left\{ \frac{1}{\theta_{\text{rpp}}} \int_{\theta_{\text{on}}}^{\theta_{\text{ext}}} i_{\text{ph}}^2 d\theta \right\}}
$$

$$
I_{pk} = \max_{\theta_{\text{u}} < \theta < \theta_{\text{u}} + \theta_{\text{rpp}}} \{ i_{\text{ph}} \}
$$

$$
\psi_{pk} = \max_{\theta_{\text{u}} < \theta < \theta_{\text{u}} + \theta_{\text{rpp}}} \left\{ \frac{\theta_{\text{off}}}{\theta_{\text{on}}} \int_{\theta_{\text{on}}}^{\theta_{\text{off}}} (v_{\text{ph}} - Ri_{\text{ph}}) d\theta \right\}
$$

The excitation penalty is a measure of the level of power exchange with the DC-link. Low $\varepsilon$ is preferred, as this will lead to lower power transfer through the converter for the same average DC-link current. The RMS current is a proportionate measurement of the level of copper losses within the generator and the converter. Obviously low RMS current is preferred. Peak phase current level is a measure of the power electronic converter stresses. Higher peak currents will lead to premature failure of the converter and a lower value of peak current is always preferred in continuous duty operation of the system. The peak flux-linkage is a measure of the level of core losses of the SR machine. The magnitude of flux-linkage and the frequency directly effect eddy current losses and hysteresis losses. Lower value of peak flux-linkage is preferred for high efficiency operation.
Consider the waveforms given in figure 5.2. Comparison of $a$ and $b$ show that current waveform $a$ yields high $\varepsilon$, $I_{\text{rms}}$, and $I_{\text{pk}}$ and is not desirable. Waveform $b$ achieves the lowest level of peak flux-linkage. The selection of excitation parameters can therefore be considered as an optimization problem that achieves a balance between these different factors.

5.3 Variation of excitation parameters for different objectives

The 250kW 12/8 SR machine and the test-bed 300W 6/4 experimental SR machine are simulated in generation mode. The FE model based switching simulation technique presented in chapter three is utilized here for the simulation of both the machines. The turn-on and turn-off angles are varied while the hysteresis current level is set at the maximum for single pulse mode operation. The simulations are performed for a range of turn-on and turn-off angle values from a minimum turn-on angle value of $\theta_a - \frac{\beta_r}{2}$ to a maximum turn-off angle value of $\theta_a + \beta_r$. For a given average DC current production, the values of $\varepsilon$, $I_{\text{rms}}$, $\psi_{\text{pk}}$ and $I_{\text{pk}}$ which satisfy the optimizations below are extracted from the simulated data. Appendix I present the optimal variation of excitation parameters of the 250kW SRG and the 300W SRG systems.

\[
\text{Optimization 1 : minimize } \{\varepsilon\} \\
\text{Optimization 2 : minimize } \{I_{\text{rms}}\} \\
\text{Optimization 3 : minimize } \{\psi_{\text{pk}}\} \\
\text{Optimization 4 : minimize } \{I_{\text{pk}}\}
\]

5.3.1 Observations on the optimal variations of SR generator variables

The following observations can be made based on the optimum variations of excitation parameters presented in appendix I.

1. The dwell angle rapidly approaches the maximum value during min $\{I_{\text{pk}}\}$ operation, for the majority of the corresponding operating regime, while the variation of $\theta_{\text{on}}$ and $\theta_{\text{off}}$ regulates the average DC current production. At low load conditions, the dwell angle approaches zero where zero DC current would be established.
2. The general trend of the variables for min $\{I_{\text{rms}}\}$ follows a similar trend to that of min $\{I_{\text{pk}}\}$ with slight variations. Obviously, this is due to the relationship between the phase peak current value and the RMS current value. Minimizing peak current will generally lower RMS current.

3. The operation with min $\{I_{\text{pk}}\}$ or min $\{I_{\text{rms}}\}$ increases the full-load peak flux-linkage value by about 0.2pu to 0.5pu of the minimum value at the lower two speeds considered in the simulations. This is also corroborated by the operating points 1, 3, 7 and 9 of table 5.1 and the corresponding simulation waveforms shown in the appendix I.

4. The peak-current value and the RMS current value varies in a nearly linear manner with average DC-current production for the 250kW generator, while a deviation from linearity can be seen for the 300W machine min $\{\psi_{\text{pk}}\}$ operation. The operation at the lower two speeds increases the full-load peak current value and RMS current value by about 0.8pu and 0.4pu respectively in comparison with the achievable minimum.

5. The min $\{\psi_{\text{pk}}\}$ operation lowers the dwell angle value while min $\{\varepsilon\}$ operation has a slightly higher dwell angle value under all operating conditions. In both the cases of min $\{\psi_{\text{pk}}\}$ and min $\{\varepsilon\}$ the dwell angle varies in quasi-linear manner with the average DC current production. Hence the manipulation of dwell angle for the control of average DC-current production is a suitable technique for the achievement of a secondary objective of min $\{\psi_{\text{pk}}\}$ or min $\{\varepsilon\}$ operation.

6. In contrast, it follows from observation (1) that the usage of the dwell angle as a sole manipulated variable is not suitable for min $\{I_{\text{pk}}\}$ or min $\{I_{\text{rms}}\}$ operation. Control of the average DC current production can be made by shifting a constant dwell period appropriately by variation of $\theta_{\text{on}}$ and $\theta_{\text{off}}$ values.

The next section considers few operating points of interest and outlines further characteristics of these optimizations.

5.3.2 Observations on optimal operating point waveforms

A few optimum operating points of interest are given in table 5.1 and the corresponding simulation waveforms are given in appendix I. The current waveforms and flux-linkage/current loops for the 250kW SR generator operating points are also shown in figures 5.3 (a) to (d). The main resulting observations are discussed below.
Figure 5.3: Operating waveforms during three different optimal excitation conditions: (a), (b) current waveforms and (c), (d) flux-linkage vs. current trajectories for the operating points 1 to 6 of the 250kW SR generator given in table 5.1. Plots (a) and (c) correspond with the 20000rpm operating points and (b) and (d) with the 50000rpm operating points.

1. The operating points 1 and 7 present near full load and close to base-speed operation of the 250kW SR generator and the 300W SR generator with the minimized RMS current condition. Operating points 3 and 9 are for a similar loading and speed with the minimized peak flux-linkage condition. It can be seen that the minimization of RMS current leads to a peak flux-linkage value of approximately 144% for the 250kW SRG and 122% for the 300W SRG compared with the minimum achievable peak flux-linkage value obtained by the operating points 3 and 9. This can also be seen in figure 5.3 (c).

2. In contrast, at high speeds the relative difference from the achievable
Table 5.1: Sample of optimized SR generator operating points

<table>
<thead>
<tr>
<th>Optimization</th>
<th>Speed</th>
<th>( \theta_{on} )</th>
<th>( \theta_{off} )</th>
<th>( i_{dc,avg} )</th>
<th>( \psi_{pk} )</th>
<th>( \varepsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( \min { I_{rms} } )</td>
<td>20000rpm</td>
<td>-6.4°</td>
<td>8.6°</td>
<td>465.08A</td>
<td>0.0675Vs</td>
<td>0.34</td>
</tr>
<tr>
<td>2 ( \min { I_{rms} } )</td>
<td>50000rpm</td>
<td>-4.4°</td>
<td>10.6°</td>
<td>245.33A</td>
<td>0.0270Vs</td>
<td>0.31</td>
</tr>
<tr>
<td>3 ( \min { \psi_{pk} } )</td>
<td>20000rpm</td>
<td>8°</td>
<td>19.0°</td>
<td>463.72A</td>
<td>0.0470Vs</td>
<td>0.62</td>
</tr>
<tr>
<td>4 ( \min { \psi_{pk} } )</td>
<td>50000rpm</td>
<td>5.0°</td>
<td>16.5°</td>
<td>227.90A</td>
<td>0.0211Vs</td>
<td>0.33</td>
</tr>
<tr>
<td>5 ( \min { \varepsilon } )</td>
<td>20000rpm</td>
<td>1.5°</td>
<td>13°</td>
<td>472.50A</td>
<td>0.0518Vs</td>
<td>0.31</td>
</tr>
<tr>
<td>6 ( \min { \varepsilon } )</td>
<td>50000rpm</td>
<td>-0.5°</td>
<td>13.0°</td>
<td>227.94A</td>
<td>0.0243Vs</td>
<td>0.29</td>
</tr>
</tbody>
</table>

300W 6/4 SR generator operating points

<table>
<thead>
<tr>
<th>Optimization</th>
<th>Speed</th>
<th>( \theta_{on} )</th>
<th>( \theta_{off} )</th>
<th>( i_{dc,avg} )</th>
<th>( \psi_{pk} )</th>
<th>( \varepsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 ( \min { I_{rms} } )</td>
<td>2000rpm</td>
<td>-10°</td>
<td>20°</td>
<td>12.08A</td>
<td>0.0582Vs</td>
<td>0.40</td>
</tr>
<tr>
<td>8 ( \min { I_{rms} } )</td>
<td>5000rpm</td>
<td>-7.25°</td>
<td>22.75°</td>
<td>6.21A</td>
<td>0.0237Vs</td>
<td>0.33</td>
</tr>
<tr>
<td>9 ( \min { \psi_{pk} } )</td>
<td>2000rpm</td>
<td>5°</td>
<td>29.8°</td>
<td>12.04A</td>
<td>0.0477Vs</td>
<td>0.41</td>
</tr>
<tr>
<td>10 ( \min { \psi_{pk} } )</td>
<td>5000rpm</td>
<td>10°</td>
<td>34°</td>
<td>6.23A</td>
<td>0.0189Vs</td>
<td>0.36</td>
</tr>
<tr>
<td>11 ( \min { \varepsilon } )</td>
<td>2000rpm</td>
<td>-5°</td>
<td>23°</td>
<td>12.16A</td>
<td>0.0542Vs</td>
<td>0.38</td>
</tr>
<tr>
<td>12 ( \min { \varepsilon } )</td>
<td>5000rpm</td>
<td>0°</td>
<td>27°</td>
<td>6.28A</td>
<td>0.0213Vs</td>
<td>0.31</td>
</tr>
</tbody>
</table>

minimum tends to be in the range of 0.2pu to 0.25pu. Operating points 2, 4, 8 and 10 present the 250kW SR generator and the 300W SR generator operation near 50% full-load and at extended speeds of 50000rpm and 5000rpm respectively. Operating points 2 and 8 correspond with the operation with minimized RMS current condition and operating points 4 and 10 with the minimized peak flux-linkage condition. In this case, the minimization of RMS current leads to a peak flux-linkage value of approximately 128% for the 250kW SRG and 125% for the 300W SRG compared with the minimum achievable peak flux-linkage value at operating points 4 and 10. This can also be seen in figure 5.3 (d). Iron losses are a function of frequency as well as the flux-linkage harmonic content. Lowering of fundamental flux-linkage magnitude at high-speed will result in significant improvement in iron losses of the SR generator performance.

3. Operating points 5 and 11 present the operation of the 250kW SR generator and the 300W SR generator near full-load and at 20000rpm and 2000rpm respectively. In this case, it can be seen that the minimization of excitation penalty leads to a peak flux-linkage value of approximately 110% for the 250kW SRG and 113% for the 300W SRG compared with the minimum achievable peak flux-linkage value obtained by the operating points 3 and 9. Operating point 3 waveforms are shown in figures 5.3 (a) and (c).
4. In contrast, the peak flux-linkage at higher speeds with \( \min \{ \varepsilon \} \) follows a similar pattern. Operating points 6 and 12 present the operation of the 250kW SR generator and the 300W SR generator near 50\% full-load and at extended speeds of 50000rpm and 5000rpm respectively. In this case, it can be seen that the minimization of excitation penalty leads to a peak flux-linkage value of approximately 115\% for the 250kW SRG and 112\% for the 300W SRG compared with the minimum achievable peak flux-linkage value obtained by the operating points 5 and 11. Operating point 5 waveforms are shown in figures 5.3 (b) and (d).

These observations are further clarified by the simulation and experimental results presented in the latter part of this chapter. Two of the classical forms of excitation control are presented in the following section.

\section*{5.4 Excitation control in single pulse mode of SR generator operation}

Traditional SR generator excitation control in single pulse mode operation is based on fixing one of the two excitation parameters, turn-on angle \([141]\) or the turn off angle \([142]\) at a specific value. The remaining variable is then manipulated according to the required DC-link power output \([142]\). The authors of \([142]\) fix the turn-off angle and manipulate the turn-on angle via a fuzzy logic strategy for constant power and variable speed operation. Efficiencies as high as 90.9\% are reported with 5kW output power. The technique adopted in \([143]\) and \([132]\) achieves voltage control with fixed turn-on and turn-off angles and chopping operation even near base-speed.

The authors of \([136, 144, 145]\) present an optimal excitation strategy for SR generators which requires finite element modelling of the machine and simulation of the system for all possible excitation parameters to gather the optimal combinations of turn-on and turn-off angles. The controller then obtains this information from either a look-up table \([144]\) or via a curve-fit as a function of speed and load \([146]\). Different criteria, such as minimization of RMS current, torque ripple, DC-link voltage ripple and copper losses may be utilized in the calculation of the optimal excitation parameters. Successful operation of a SR generator with the curve fitted optimal excitation function is demonstrated in \([146]\) at variable speeds ranging from 2000rpm to 6000rpm and output power
levels of 1kW to 1.2kW. Generator efficiencies in the range of 90.7% to 88% are achieved in [144] for variable speeds and load conditions ranging from 1000rpm to 5000rpm and 3kW to 6kW.

5.4.1 Fixed turn-on angle based excitation control design

In this form of excitation control the turn-on angle is fixed at an optimal value. The turn-off angle is then manipulated in-order to control the average DC current production. A smaller turn-on angle (higher level of advancing) will lead to high excitation penalty and high values of peak flux-linkage. However, a lower level of advancing will lead to a reduction in the maximum average DC-current production capability. In this case, the fixed turn-on angle values for the 250kW SRG and the 300W test-bed are selected as $\theta_{on} = 1^\circ$ and $\theta_{on} = 1.5^\circ$ respectively. These turn-on angles are selected such that the current peak values are kept within the maximum allowable limits for the full operating speed region. This achieves a maximum average DC current production of 180% and 130% of rated current at base-speed for the two machines.

![DC current vs dwell angle characteristics for the 250kW SRG](image)

![DC current vs dwell angle characteristics for the 300W SRG](image)

Figure 5.4: Average DC current production variation with dwell angle for (a) the 250kW 12/8 SR generator and (b) test-bed 300W 6/4 SR generator.

Figures 5.4 (a) and (b) present the average DC current production variation with dwell angle. The dots shown in figures 5.4 (a) and (b) show the simulated
average DC current production and the line corresponds to the curve-fit explained in the following section. The maximum dwell angle for the 250kW SRG and the 300W SRG is approximately at $\theta_{dwell, max} = 16.5^0$, $\theta_{dwell, max} = 32.5^0$ respectively. Figures 5.5 (a) and (b) show the variation of the maximum DC-current generation capability with speed for a fixed maximum dwell angle value.

At low speeds, high excitation current is required to generate positive average DC current. Limitation of the maximum phase current also limits the excitation at such speeds. Hence, a sharp drop in the average DC current production capability is noted in figures 5.5 (a) and (b).

Approximate input-output linearization for the fixed turn-on angle based excitation controller

Let the average DC-link current production be given by a function of speed $\omega$ and dwell angle $\theta_{dwell}$ as:

$$i_{dc,avg} = f(\omega, \theta_{dwell}) \tag{5.7}$$

The DC-link voltage regulator design is typically performed assuming the availability of the inverse function of the average DC-link characteristics, i.e. $f^{-1}(\omega, \theta_{dwell})$. This provides the capability to form a linear relationship between...
the commanded average DC-current and the actual average DC-current.

In this fixed turn-on angle based excitation controller, the SR generator operation above the base-speed is considered. The function $f(\omega, \theta_{dwell})$ of (5.7) has an inverse relationship to speed. Hence, this can be represented as:

$$f(\omega, \theta_{dwell}) = \frac{1}{\omega} g(\theta_{dwell}) \quad \text{for all } \omega > \omega_{\text{base}}$$

(5.8)

Here $g(\theta_{dwell})$ is a function of $\theta_{dwell}$. The simplest form for $g(\theta_{dwell})$ which provides a satisfactory fit for (5.7) is given by the relationship:

$$g(\theta_{dwell}) = k_{dc} \theta_{dwell}^\alpha$$

(5.9)

Substitution of (5.9) and (5.8) in (5.7) yields:

$$i_{dc,\text{avg}} = k_{dc} \frac{1}{\omega} \theta_{dwell}^\alpha$$

(5.10)

Figures 5.4 (a) and (b) show the simulated average DC current production with dots and curve-fit based on (5.10). Figures 5.4 (a) and (b) confirm satisfactory approximation of the average DC current production by $f(\omega, \theta_{dwell})$ of (5.7).

Given the average DC-current demand $u^*$, the dwell angle command can be found by (5.10) as:

$$\theta_{dwell} = \left( \frac{1}{k_{dc} \omega |u^*|} \right)^{1/\alpha}$$

(5.11)

The input-output linearization (5.11) is simulated for the 250kW SR generator and the 300W SR generator two quadrant operation with the parameters given in table 5.2. The first quadrant generation mode and the fourth quadrant motoring mode turn-on angles are defined by:

$$\theta_{on} = \begin{cases} \theta_2 + \theta_{on,fixed} & \text{if } u^* \geq 0 \\ \theta_1 - k_{adv} \omega & \text{if } u^* < 0 \end{cases}$$

(5.12)

where $\theta_1$ and $\theta_2$ rotor positions are shown in figure 5.2.

Figures 5.6 (a) and (b) show the variation of the actual average DC-current production $i_{dc,\text{avg}}$ with average DC-current command $u^*$ for these two machines. Compared with the dwell angle based average current variation of figures 5.4
Figure 5.6: Average DC current production variation with average current command $u^*$ with fixed turn-on angle excitation control (a) the 250kW 12/8 SR generator and (b) test-bed 300W 6/4 SR generator.

(a) and (b), the nonlinear transformation (5.11) has improved the input-output linearity in figures 5.6 (a) and (b). Considerable linearity is achieved in the regions from no-load to full-load operation at all the speeds considered in the simulation. These linear regions are highlighted by the darker lines in figures 5.6 (a) and (b). Hence, linear DC-link voltage regulator formulation can be considered and consistent voltage quality can be expected within these ranges of speeds with this excitation control method.

Table 5.2: Input-output linearization parameters for the fixed-turn-on angle based excitation controller

<table>
<thead>
<tr>
<th>Symbol</th>
<th>250kW SR generator</th>
<th>300W test-bed SR generator</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{dc}$</td>
<td>400 A[rpm][deg]$^{-\alpha}$</td>
<td>0.0561 A[rpm][deg]$^{-\alpha}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$k_{adv}$</td>
<td>0.0015 deg/[rad/s]</td>
<td>0.0435 deg/[rad/s]</td>
</tr>
<tr>
<td>$\theta_{on,fixed}$</td>
<td>1 deg</td>
<td>1.5 deg</td>
</tr>
</tbody>
</table>
CHAPTER 5. SR MACHINE OPERATION IN GENERATION MODE

5.4.2 Fixed turn-off angle based excitation control design

In this form of excitation control, the turn-off angle is fixed at an optimal value. The fixed turn-off angle value determines the operational efficiency, and
other performance criteria such as excitation penalty, peak flux-linkage, RMS current, peak current and torque ripple. The optimized variations presented in appendix I reveal that the optimum turn-off angles occur in the neighbourhood of approximately \( \frac{3}{4} \) of the decreasing inductance region. Hence, the fixed turn-off angle values are selected at \( \theta_{off} = 15^0 \) and \( \theta_{off} = 30^0 \) for the 250kW SR generator and the 300W SR generator respectively (\( \theta = 0 \) position represents the aligned position). The turn-on angle is then manipulated in order to control the average DC current production. A clear advantage of a fixed turn-off angle based excitation control strategy is the capability to over-load the machine. Figures 5.7 (a) and (b) present the average DC current production capability for the 250kW SR generator and the 300W test-bed SR generator.

Figures 5.8 (a) and (b) present the maximum current-speed envelopes for the 250kW SR generator and the 300W test-bed SR generator respectively. The blue line in each figure represent the maximum with the fixed turn-on angle based strategy. The black lines represent the maximum with the fixed turn-off angle and discontinuous conduction. The red lines represent the maximum with the fixed turn-off angle and continuous conduction. Given the thermal and mechanical environment to operate at such extreme levels, the continuous conduction mode provides the capability to overload the machine up to high levels.

In contrast with an optimal fixed-turn-on angle based excitation strategy, an optimal fixed turn-off strategy achieves a higher average DC current production capability under discontinuous conduction. This strategy also has the capability to drive the machine into continuous conduction mode, which cannot be performed effectively with a fixed-turn-on angle based strategy. Unlike the fixed turn-on angle based excitation control, the system does not enter into an unstable region prior to continuous conduction. Hence, fixed turn-off angle based excitation control can be considered as a strategy which can yield superior stability even as the dwell angle is increased beyond the nominal maximum. In the excitation control strategy considered here, the dwell angle is approximately related to the average DC current demand by:

\[
\theta_{dwell} = \frac{\theta_{dwell,\text{max}}}{k_{dc}} u
\]  

The nominal maximum dwell angle is selected as \( \theta_{dwell,\text{max}} = \beta_s \) and \( k_{dc} \) is selected
as $k_{dc} = i_{dc,rated}$. The turn-on angle is determined by:

$$\theta_{on} = \theta_{off} - \theta_{dwell} \tag{5.14}$$

The performance of the fixed turn-off angle based excitation control is analysed and compared with the fixed turn-on based control by means of experimental and simulation results in the latter part of this chapter.

### 5.5 Curve fitted optimal variation based excitation control

The four optimal variations of excitation parameters for the 250kW SR generator and the 300W SR generator were presented in section 5.3. This section presents the piecewise continuous function (PCF) based curve fitting of the optimal data points of the optimized variations presented in appendix I. A general PCF format applicable to the four optimizations is proposed. The input-output characteristics obtained with the use of the PCF based excitation control for the different optimizations are also investigated.

The proposed function for the approximation of the dwell angle variation is of the format given by:

$$\theta_{dwell} = \theta_{d,max} \left(1 - e^{-\lambda_1 i_{dc,pu} \hat{\omega}}\right) \tag{5.15}$$

where $i_{dc,pu} = \frac{i_{dc,avg}}{i_{dc,rated}}$ represent the per-unit average DC-current production and $\hat{\omega} = \frac{\omega}{\omega_{base}}$ represents speed as a ratio to base-speed.

It was observed in the optimized variations presented in appendix I that in general the turn-on angle declines to a minimum point at low-load conditions, after which the turn-on angle may increase again for higher loads. The low-load variation of turn-on angle can be approximated to the exponential function:

$$\theta_{on} = \theta_{on,min} + (\theta_{on,max} - \theta_{on,min}) e^{-\lambda_2 i_{dc,pu} \hat{\omega}} \tag{5.16}$$

Consider a turn-on angle value $\theta_{on,0}$ close to the minimum point:

$$\theta_{on,0} = \theta_{on,min} + \kappa (\theta_{on,max} - \theta_{on,min}) \quad 0 < \kappa \ll 1 \tag{5.17}$$
The corresponding per-unit DC-current value associated with $\theta_{on} = \theta_{on,0}$ is given by:

$$i_{dc,pu,0} = -\frac{1}{\lambda_2 \omega} \log (\kappa)$$  \hspace{1cm} (5.18)

For small $\kappa$, $0 < \kappa \ll 1$, the approximation $\theta_{on,0} \approx \theta_{on,\text{min}}$ is valid. Hence the parameter $\kappa$ is appropriately selected to define the bifurcation point of the PCF for turn-on angle variation:

$$\theta_{on} = \begin{cases} 
\theta_{on,\text{min}} + (\theta_{on,\text{max}} - \theta_{on,\text{min}}) e^{-\lambda_2 i_{dc,pu} \omega} & \text{if } i_{dc,pu} \leq i_{dc,pu,0} \\
\theta_{on,\text{min}} + \kappa (\theta_{on,\text{max}} - \theta_{on,\text{min}}) + \gamma \left( \frac{i_{dc,pu} - i_{dc,pu,0}}{i_{dc,pu,0}} \right) & \text{if } i_{dc,pu} > i_{dc,pu,0}
\end{cases} \hspace{1cm} (5.19)$$

Beyond $i_{dc,pu} > i_{dc,pu,0}$, $\theta_{on}$ is varied in a linear manner. The gradient of this linear variation is specified by the parameter $\gamma$.

The turn-off angle follows from the relationship:

$$\theta_{off} = \theta_{on} + \theta_{dwell}$$  \hspace{1cm} (5.20)

### 5.5.1 Curve fitted optimal variation of excitation angles case study: 250kW SR generator and 300W test-bed SR generator optimum excitation angle control

The 250kW SR generator and the 300W SR generator optimal variations are curve fitted to the PCFs defined earlier. The corresponding parameters associated with the curve-fit is given in table 5.3. Figures 5.9 and 5.10 present the optimal data points and the corresponding fitted curve. The subplots (a-4) to (d-4) in each of the figures 5.9 and 5.10 present the average DC current production characteristics with the command $u^*$. 
Figure 5.9: PCF fitted variation of the optimized excitation data points for the 250kW SR generator.

Figure 5.10: PCF fitted variation of the optimized excitation data points for the 300W SR generator.
Table 5.3: Curve-fit parameter values for the four different optimizations of the 250kW SR generator and the 300W test-bed SR generator

<table>
<thead>
<tr>
<th>Variable</th>
<th>min ${\varepsilon}$</th>
<th>min ${I_{\text{rms}}}$</th>
<th>min ${\psi_{\text{pk}}}$</th>
<th>min ${I_{\text{pk}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>250kW SR generator optimal curve-fit parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_{d,\text{max}}$ [deg]</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>1.6</td>
<td>2.9</td>
<td>1.5</td>
<td>5.4</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>1</td>
<td>3.3</td>
<td>0.4</td>
<td>3.6</td>
</tr>
<tr>
<td>$\theta_{\text{on, max}}$ [deg]</td>
<td>15</td>
<td>12.5</td>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>$\theta_{\text{on, min}}$ [deg]</td>
<td>-5</td>
<td>-6</td>
<td>-2</td>
<td>-10</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.15</td>
<td>0.01</td>
<td>0.01</td>
<td>0.1</td>
</tr>
<tr>
<td>$\gamma$ [deg]</td>
<td>12</td>
<td>10</td>
<td>20</td>
<td>3.5</td>
</tr>
<tr>
<td>$\omega_{\text{base}}$ [rpm]</td>
<td>18000</td>
<td>18000</td>
<td>18000</td>
<td>18000</td>
</tr>
<tr>
<td>300W test-bed SR generator optimal curve-fit parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_{d,\text{max}}$ [deg]</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>2</td>
<td>3.6</td>
<td>1.6</td>
<td>5</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>2</td>
<td>3.6</td>
<td>0.9</td>
<td>3.5</td>
</tr>
<tr>
<td>$\theta_{\text{on, max}}$ [deg]</td>
<td>35</td>
<td>25</td>
<td>28</td>
<td>10</td>
</tr>
<tr>
<td>$\theta_{\text{on, min}}$ [deg]</td>
<td>-6</td>
<td>-10</td>
<td>0</td>
<td>-15</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.05</td>
<td>0.02</td>
<td>0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>$\gamma$ [deg]</td>
<td>15</td>
<td>10</td>
<td>20</td>
<td>11</td>
</tr>
<tr>
<td>$\omega_{\text{base}}$ [rpm]</td>
<td>1800</td>
<td>1800</td>
<td>1800</td>
<td>1800</td>
</tr>
</tbody>
</table>
CHAPTER 5. SR MACHINE OPERATION IN GENERATION MODE

5.6 Voltage regulator design for SR generators and fault-tolerant operation

The DC-link voltage regulator design follows a similar argument to that of SR motor speed control design presented in chapter four. However, the SR generator operation in this chapter does not consider the operation in chopping mode due to the high-speed region of interest. Hence the bandwidth limitation of the voltage regulator is directly related to the operational speed. The excitation control methods presented earlier are simulated and experimentally validated for the outer voltage regulation loop operation. The translation of the average DC current command $u^*$ to the excitation parameters is ensured by the excitation controller. Assuming that the outer voltage control loop is of lower bandwidth, the relationship between the average current command $u^*$ and the actual average current production $i_{dc,avg}$ can be approximated to:

$$i_{dc,avg} = xu^* + h_a (\omega, i_k)$$  \hspace{1cm} (5.21)

where $x$ represents the healthy fraction of the machine. The function $h_a (\omega, i_k)$ incorporates the deviation of the average DC current command with the actual average DC current production. The instantaneous current production can be represented as:

$$i_{dc} = i_{dc,avg} + h_i (\omega, \theta, i_k)$$ \hspace{1cm} (5.22)

where $h_i$ incorporates the current ripple components. Substitution of $i_{dc,avg}$ of (5.21) in (5.22) yields:

$$i_{dc,avg} = xu^* + h$$ \hspace{1cm} (5.23)

where $h$ represent the high frequency ripple and the modelling inaccuracies.

$$h = h_i (\omega, \theta, i_k) + h_a (\omega, i_k)$$ \hspace{1cm} (5.24)

The DC-link dynamics are given by:

$$C_{dc} \frac{dv_{dc}}{dt} = i_{dc} - i_{load}$$
Substitution of (5.23) yields:

\[ C_{dc} \frac{dv_{dc}}{dt} = xu^* + h - i_{load} \tag{5.25} \]

The speed control can be formulated as a proportional/integral law:

\[ u^* = k_{p,dc} (v^*_{dc} - v_{dc}) + k_{i,dc} \int (v^*_{dc} - v_{dc}) \, dt \tag{5.26} \]

Substitution of (5.26) in (5.25) yields:

\[ C_{dc} \frac{dv_{dc}}{dt} = xk_{p,dc} (v^*_{dc} - v_{dc}) + xk_{i,dc} \int (v^*_{dc} - v_{dc}) \, dt + w_s \tag{5.27} \]

where \( w_s \) is disturbance to the system (5.27). Assuming \( w_s \) is an exogenous disturbance, the closed-loop transfer function for the command input is given by:

\[ G_{cl}(s) = \frac{v_{dc}(s)}{v^*_{dc}(s)} = \frac{\frac{zk_{p,dc}s + zk_{i,dc}}{C_{dc}}}{s^2 + \left(\frac{zk_{p,dc}}{C_{dc}}\right)s + \frac{zk_{i,dc}}{C_{dc}}} \tag{5.28} \]

The disturbance transfer function is given by:

\[ G_d(s) = \frac{v_{dc}(s)}{w_s(s)} = \frac{1}{s^2 + \left(\frac{zk_{p,dc}}{C_{dc}}\right)s + \frac{zk_{i,dc}}{C_{dc}}} \tag{5.29} \]

The closed-loop transfer function can be approximately cast into the standard second order format:

\[ G_{std}(s) = \frac{\Omega_n^2}{s^2 + 2\varsigma\Omega_n s + \Omega_n^2} \tag{5.30} \]

where \( \varsigma \) is the damping factor and \( \Omega_n \) is the undamped natural frequency. Considering a healthy machine with \( x = 1 \) and for \( \varsigma = \varsigma_s \) and \( \Omega_n = \Omega_{s,n} \) the proportional and integral gains of the controller (5.26) for the voltage regulation can be approximated to:

\[ k_{p,dc} = 2s_{dc}\Omega_{dc,n}C_{dc} \tag{5.31} \]

\[ k_{i,dc} = C_{dc}\Omega_{dc,n}^2 \tag{5.32} \]
As mentioned earlier, the single pulse mode SR generator operation produces a pulsating current waveform. This current pulse is a function of the system control variables. Hence, the bandwidth of the DC current / voltage system (5.27) must be designed with consideration on such limitations. Based on the Nyquist theorem, the system maximum bandwidth must be limited by:

$$\Omega_{dc,n,max} < \frac{N_p N_r \omega_{base}}{2} \text{ if } \omega \geq \omega_{base}$$  \hspace{1cm} (5.33)

where \(\frac{N_p N_r \omega_{base}}{2}\) is the minimum stroke frequency of the SR motor above base-speed. For higher phase margin and gain margin, the system bandwidth maybe selected as

$$\Omega_{dc,n} = \gamma \Omega_{dc,n,max}$$ \hspace{1cm} (5.34)

where \(0 < \gamma < 1\) is a design parameter.

**Consideration for fault-tolerant operation**

Similar to the SR motoring mode, loss of one or two phases will result in a DC-link current generated with a fundamental frequency \(N_r \omega\) rad/s. Hence, the bandwidth limitation condition can be rewritten for faulted operation as:

$$\Omega_{dc,n,max} < \frac{N_r \omega}{2} \text{ for all } \omega$$  \hspace{1cm} (5.35)

It is assumed that the prime mover is of high inertia and controlled ideally to maintain a fixed speed under SR generator loading. Hence, the effects of speed dynamics on the generator voltage stability are not analysed.
5.6.1 Case study: Voltage regulator design for the 250kW 12/8 SR generator

The 250kW SR generator system considered here consists of a DC-link capacitor of $C_{dc} = 55\text{mF}$. The maximum bandwidth limit is $\Omega_{dc,n,\text{max}} = 22619\ \text{rad/s}$ for operation above base-speed 18000rpm.

Similar to the motoring mode speed control design, $\Omega_{dc,n} = 200\text{rad/s}$ is selected such that $\gamma = 0.0255$, and $\Omega_{dc,n} < \Omega_{dc,n,\text{max}}$ is satisfied. For a damping factor of $\zeta_s = 0.707$, the PI controller parameters are calculated from (5.31) and (5.32) as $k_{p,dc} = 15.4$ and $k_{i,dc} = 2200$.

The bandwidth limitation for faulted operation can be calculated as $\Omega_{dc,n,\text{max}} = 7539.8\ \text{rad/s}$. $\Omega_{dc,n} \ll \Omega_{dc,n,\text{max}}$, and hence stability under phase open-circuit faults can be maintained.

5.6.2 Case study: Voltage regulator design for the test-bed 300W 6/4 experimental SR generator

The 300W SR generator system considered here consists of a DC-link capacitor of $C_{dc} = 12.1\text{mF}$. The maximum bandwidth limit is $\Omega_{dc,n,\text{max}} = 1131\ \text{rad/s}$ for operation above base-speed 1800rpm.

$\Omega_{dc,n} = 50\text{rad/s}$ is selected such that $\gamma = 0.0442$, and $\Omega_{dc,n} < \Omega_{dc,n,\text{max}}$ is satisfied. For a damping factor of $\zeta_s = 0.707$, the PI controller parameters are calculated from (5.31) and (5.32) as $k_{p,dc} = 0.8470$ and $k_{i,dc} = 30.25$.

The bandwidth limitation for faulted operation can be calculated as $\Omega_{dc,n,\text{max}} = 377\ \text{rad/s}$. $\Omega_{dc,n} \ll \Omega_{dc,n,\text{max}}$, and hence stability under phase open-circuit faults can be maintained.
5.7 Experimental validation and simulation results

This section presents the experimental validation and simulation results for the different excitation controllers operating with the voltage regulator. In summary, the six excitation controllers developed earlier are:

1. PCF fitted min $\{\varepsilon\}$ optimization based excitation controller operation with voltage regulation.
2. PCF fitted min $\{I_{\text{rms}}\}$ optimization based excitation controller operation with voltage regulation.
3. PCF fitted min $\{\psi_{pk}\}$ optimization based excitation controller operation with voltage regulation.
4. PCF fitted min $\{I_{pk}\}$ optimization based excitation controller operation with voltage regulation.
5. Fixed turn-on angle based excitation controller operation with voltage regulation.
6. Fixed turn-off angle based excitation controller operation with voltage regulation.

![Figure 5.11](image)

Figure 5.11: (a) Speed and (b) electrical loading on the DC-link considered in the simulations of the 250kW SRG

The simulation of the 250kW SR generator system considers a generator starting speed of 20000rpm. The speed is then sequentially increased to 35000rpm and 50000rpm. The DC-link is loaded in steps to maximums of 100%, 83.3%, and 66.7% of full load DC current at each speed as shown in figure 5.11. The DC-link voltage is regulated at a fixed value of 540V in this 250kW system.

Experimental validation of the voltage regulation and excitation controller
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155

Asymmetric half bridge converter
dcv
sv
Chopped resistive load
R
source of initial excitation

Figure 5.12: Block diagram of the experimental setup with the DC-link capacitor, chopped resistive load, asymmetric half bridge converter, and the initial excitation source from a diode-bridge rectifier

Figure 5.13: (a) Speed and (b) electrical loading (blue-line) on the DC-link used in the experiments and simulations of the 300W test-bed SRG. (b) Duty (green-line) issued to the chopper with the resistive load on the DC-link of the experimental setup.

operation is obtained by implementing the control algorithms in the 300W test-bed SR generator drive. The SR generator is driven by a PM machine (prime mover). The DC-link is loaded with a chopped 22A, 1.1Ω resistive load as shown in figure 5.12. The chopping duty is issued externally is relation to the required DC-link loading. The DC-link voltage is regulated at a fixed value of 24V. The PM machine speed is varied from a starting speed of 2000rpm and increased sequentially to 3500rpm and 5000rpm respectively. Figure 5.13 shows the speed command and the DC-link loading command issued for the 300W test-bed SR generator drive system. The simulation of the 300W SR generator uses the same speed profile and the DC-link loading shown in figure 5.13. The initial excitation for the SR generator system is provided by the capacitor charged to a low voltage between 9 – 13V. This initial charge is provided by a rectified AC voltage source.
Figure 5.14: Voltage response based on simulation results of (a)-(f) 250kW SRG system and (g)-(l) 300W SRG system for the speed and load profile given in figures 5.11 and 5.13, and for the six different excitation control strategies.
Figure 5.15: Oscilloscope waveforms for the test-bed 300W SRG operation for the speed and load profiles given in figure 5.13, and PCF fitted optimal excitation controllers (a) $\min \{\varepsilon\}$, (b) $\min \{I_{\text{rms}}\}$, (c) $\min \{\psi_{pk}\}$, and (d) $\min \{I_{pk}\}$. 
Figure 5.16: Oscilloscope waveforms for the test-bed 300W SRG operation for the speed and load profiles given in figure 5.13, and (a) fixed turn-on angle based excitation control and (b) fixed turn-off angle based excitation control.
Figure 5.17: Experimental and simulated phase current waveforms of the SRG operation at one operating point of 10A DC-link loading and speed of 3500rpm. Current waveforms are shown for (a) minimized excitation penalty, (b) minimized RMS current, (c) minimized peak flux-linkage and (d) minimized peak current.

Figure 5.18: Simulated flux-linkage/current loops at one operating point of the SRG system at 10A DC-link loading and speed of 3500rpm. Flux-linkage/current loops are shown for minimized excitation penalty, minimized RMS current, minimized peak flux-linkage and minimized peak current.
5.7.1 Discussion

The full simulation results for the 250kW SRG and the full simulation and experimental results for the 300W SRG are given in appendix J. Figure 5.14 summarizes the system voltage waveforms for the 250kW SRG and the 300W SRG simulations. Oscilloscope waveforms presented in figures 5.15 (a) to (d) show the experimental results of the 300W SRG operation with the four different PCF fitted optimal excitation angle controllers. Oscilloscope waveforms presented in figure 5.16 (a) and (b) show the experimental results of the 300W SRG operation with the fixed turn-on angle based excitation control and fixed turn-off angle based excitation control respectively. The figures 5.17 and 5.18 present a comparison of the four optimized excitation angle controllers at one operating point of 10A DC-link loading and speed of 3500rpm.

In general, it can be concluded that the experimental results corroborate the simulation results. Examination of the full results presented in appendix J reveal that the turn-on and turn-off angles of the experimental results follow a similar pattern to that produced by simulations. In addition, the current waveform obtained by experiments follow a similar pattern to that of simulation results in majority of the operating conditions. Waveforms in figures 5.17 (a) to (d) show one such example. Hence, the characteristics anticipated by excitation angle control design and simulations are confirmed by the experimental results. The experimental validation is further discussed with the aid of power consumption, efficiency and simulated FFT relationships in this section.

The voltage regulator operation combined with each of the six excitation angle control strategies perform with excellent stability within the considered speed and load regime for both the 250kW SRG and the 300W SRG. The experimental results validate the stable operation of the control strategies. Comparison of the phase current waveforms of the PCF fitted excitation angle controller operation in figures 5.15 (a) to (d) reveal that the minimized flux-linkage operation leads to higher peak currents. This is also confirmed by the flux-linkage/current loops shown in figure 5.18. Such high peak currents also result in a higher DC-link current ripple leading to a higher level of DC-link voltage ripple especially in the 300W SRG, 24V system. Minimization of peak flux-linkage leads to a lowering of the dwell angle, causing a higher peak-current requirement in order to satisfy the DC load current demand. The phase current waveforms of PCF fitted $\min \{I_{\text{rms}}\}$ operation and $\min \{I_{\text{pk}}\}$ operation are very similar in terms of magnitude. This
fact is also reflected by the operating point current waveforms shown in figure 5.17 (b) and (d). Hence, similar copper losses can be expected and this is found to be true in the following section.

**Calculation of efficiency**

The electro-mechanical energy conversion efficiency is considered here and hence the frictional losses are not considered. The electro-mechanical energy conversion efficiency in power generation mode is calculated by:

\[ \eta = \frac{P_{out}}{P_{in}} = \frac{v_{dc}i_{dc}}{T_{em}\omega} \]  

where \( T_{em} \) is the electromagnetic torque production. In the experimental evaluation, the electromagnetic torque production is estimated by:

\[ T_{em} = T_{meas} - B_{estim}\omega \]

where \( T_{meas} \) is the shaft torque measured via the transducer and \( B_{estim} \) is an estimated frictional coefficient value.

**Power consumption and efficiency: Comparison of the PCF fitted excitation angle controllers**

Figures 5.19 (a) and (b) present the shaft power input and efficiency based on simulation results for the 250kW SR generator operation with the PCF fitted excitation angle controllers. Figures 5.20 (a) to (d) present the shaft power input and efficiency for the 300W test-bed SR generator based on simulation results and experimental results respectively, and are based on operation with the PCF fitted excitation angle controllers. Simulation results for the 250kW SR generator follow a similar pattern to that of the 300W SR generator. However, the efficiency obtained by experimental results present certain variations from the efficiency results of simulations.

As explained in chapter four, the simulation incorporates only the copper losses. Hence, the efficiencies predicted by simulations are higher than those obtained by experimental results. As expected, the efficiency predicted by the simulation results for the PCF fitted \( \min \{I_{rms}\} \) optimization based excitation controller (green line) operation achieves the highest efficiency in both figures 5.19
Figure 5.19: (a) Power consumption and (b) Efficiency based on simulation of the 250kW SRG system operation with the four PCF fitted optimal excitation angle control strategies.

The efficiency achieved by the PCF fitted min $\{I_{pk}\}$ optimization based excitation controller (black line) is very close to the min $\{I_{rms}\}$ operation for both the 250kW SRG and the 300W SRG. For practical conditions, the difference between the copper loss reduction achieved by min $\{I_{pk}\}$ and min $\{I_{rms}\}$ can be considered negligible. However, the experimental results on the 300W SRG reveal that the min $\{I_{pk}\}$ operation leads to higher overall losses under low load conditions. The difference between the overall efficiency achieved by min $\{I_{pk}\}$ and min $\{I_{rms}\}$ is in the range of 13%−17% in the neighborhood of 25% full load, 6%−10% in the neighborhood of 50% full load and 0%−3% in the neighborhood of 75% full load. For the 250kW SR machine this would translate into a difference of 5kW to 12.5kW of overall losses between the two PCF fitted excitation angle controllers.

The simulation results for the 250kW SRG and the 300W SRG predict a significantly lower efficiency for the system operation with the PCF fitted min $\{\psi_{pk}\}$ excitation angle controller (blue line) compared with the operation with the other three PCF fitted excitation angle controllers. This is mainly due to the increase of simulated copper losses due to the minimization of peak flux-linkage. However, the experimental results on the 300W SRG reveal that the min $\{\psi_{pk}\}$ operation leads to considerably higher overall efficiency especially at higher speeds, compared with the min $\{I_{rms}\}$ and min $\{I_{pk}\}$ operations. This can be mainly
attributed to the fact that $\min \{ \psi_{pk} \}$ leads to a lowering of iron losses leading to a significant overall improvement in efficiency. A further discussion on this is presented in a latter section with the aid of FFT results based on the simulations.

Superior overall efficiency is obtained by the PCF fitted $\min \{ \varepsilon \}$ optimization based excitation angle controller operation at all speeds. The experimental results predict the capability to maintain the efficiency in a narrow range of 79% to 83%, whereas the variation of efficiencies in the other three PCF fitted excitation angle controllers are of wider range. Furthermore, the $\min \{ \varepsilon \}$ operation successfully provides a superior trade-off of copper losses for the improvement
in the overall efficiency at all speeds compared with the other three PCF fitted excitation angle controllers. Hence the next section compares the fixed turn-on angle based excitation control and fixed turn-off angle based excitation control with the min $\{\varepsilon\}$ operation.

**Power consumption and efficiency:** Comparison of the fixed turn-on angle based and fixed turn-off angle based excitation angle controllers

![Simulation results](image)

Figure 5.21: (a) Power consumption and (b) Efficiency based on simulation of the 250kW SRG system operation with minimized excitation penalty, fixed turn-on angle based and fixed turn-off angle based excitation angle controllers.

Figures 5.21 (a) and (b) present the shaft power input and efficiency based on simulation results for the 250kW SR generator operation with the fixed turn-on angle based excitation controller, fixed turn-off angle based excitation controller and PCF fitted min $\{\varepsilon\}$ optimization based excitation angle controller. Figures 5.22 (a) to (d) present the shaft power input and efficiency for the 300W test-bed SR generator based on simulation results and experimental results respectively for the same three excitation controllers.

Simulation results for the 250kW SR generator and the 300W SR generator predict higher efficiencies for the fixed turn-on angle based excitation controller in figures 5.21 (b) and 5.22 (d) whilst the fixed turn-off angle based excitation controller achieves lower efficiency in simulations due to higher copper losses. This difference is approximately in the range of 1% to 5% for both the machines with a reduction at high speeds. However, the experimental results predict a
Figure 5.22: (a) Power consumption (b) Efficiency based on experimental results of the 300W SRG system and (c) Power consumption (d) Efficiency based on simulation results of the 300W SRG system operation with minimized excitation penalty, fixed turn-on angle based and fixed turn-off angle based excitation angle controllers.
higher overall efficiency for the operation with the fixed turn-off angle based excitation angle controller compared with the fixed turn-on angle based excitation angle controller. This can be attributed to reduction in iron losses. At low loads, the overall efficiency of the fixed turn-on angle based excitation angle controller drops by 6% to 10%, and has similarities with the earlier presented PCF fitted min $\{I_{rms}\}$ excitation controller operation. In contrast, the fixed turn-off angle based excitation controller achieves efficiencies close to the PCF fitted min $\{\epsilon\}$ optimization based excitation angle controller operation especially at high speeds. Furthermore, the fixed turn-off angle based excitation controller presents enhanced capability to trade-off copper losses for the improvement in the overall efficiency compared with the fixed turn-on angle based excitation controller. The next section further discusses this with the aid of FFT results based on simulations of the two SR generators.

**Frequency domain analysis**

Frequency domain results based on simulations of the 250kW SRG and the 300W SRG are presented in appendix K. The FFT results based on the simulations on the four PCF fitted optimal excitation angle controllers are discussed in the following section. The comparison of the FFT results based on the fixed turn-on angle based excitation controller and the fixed turn-off angle based excitation controller are presented in the latter part of the discussion.

**Comparison of the frequency domain performance of the four PCF fitted optimal excitation angle controllers**

A sample comparison of the FFT spectrums of flux-linkage, electromagnetic torque production and DC-side current production are shown in figures 5.23, 5.24 and 5.25 respectively. These are for the two SR generators, considering operation with the four PCF fitted optimal excitation angle controllers. The examples provided in these figures mirror similar characteristics to that of the full FFT results of the PCF fitted optimal excitation angle controllers presented in appendix K.

As anticipated by the optimization, the PCF fitted min $\{\psi_{pk}\}$ operation achieves the minimum fundamental flux-linkage magnitude. The PCF fitted min $\{I_{rms}\}$ and min $\{I_{pk}\}$ operations achieve higher flux-linkage magnitude, while the min $\{\epsilon\}$ achieves an intermediate level of flux-linkage magnitude. It was explained in the
earlier discussion on experimental results, that the PCF fitted \( \min \{ I_{rms} \} \) and \( \min \{ I_{pk} \} \) are of lower efficiency compared with the other two PCF fitted excitation angle controllers. Observations on the frequency domain results reveal the relationship between the efficiency and flux-linkage magnitude. That is, the sole minimization of flux-linkage achieves high copper losses and hence does not achieve high overall efficiency at all speeds. In contrast, sole minimization of RMS current achieves low copper losses at the cost of high iron losses. This confirms the argument that the PCF fitted \( \min \{ \varepsilon \} \) operation successfully trades-off the flux-linkage magnitude for a slightly higher copper loss yielding a lower overall power loss at all speeds of operation. Hence, a superior overall efficiency of the PCF fitted \( \min \{ \varepsilon \} \) controller is observed at all speeds.

The frequency domain performance of the DC-link current follows a similar pattern to that of torque production. A higher DC-side current fundamental will also be reflected as a high torque fundamental. PCF fitted \( \min \{ I_{pk} \} \) operation achieves the minimal torque and DC current fundamental magnitude. PCF fitted \( \min \{ I_{rms} \} \) follows a close relationship to the PCF fitted \( \min \{ I_{pk} \} \) frequency domain performance. The PCF fitted \( \min \{ \psi_{pk} \} \) operation yields the highest fundamental magnitude for torque and DC current. This is caused by the lowering of the conduction period which results in a higher phase current in order to satisfy

Figure 5.23: Sample FFT spectrum of flux-linkage: (a) 250kW SRG operating at 37.5krpm and at an electrical load of 288.75A, and (b) 300W SRG operating at 3750rpm and at an electrical load of 7.5A.
CHAPTER 5. SR MACHINE OPERATION IN GENERATION MODE

Figure 5.24: Sample FFT spectrum of electromagnetic torque waveform: (a) 250kW SRG operating at 37.5krpm and at an electrical load of 288.75A, and (b) 300W SRG operating at 3750rpm and at an electrical load of 7.5A.

Figure 5.25: Sample FFT spectrum of dc current waveform: (a) 250kW SRG operating at 37.5krpm and at an electrical load of 288.75A, and (b) 300W SRG operating at 3750rpm and at an electrical load of 7.5A.

the average load demand. PCF fitted $\min \{I_{rms}\}$ operation achieves an intermediate level of fundamental torque magnitude and DC current magnitude. Hence minimization of the excitation penalty can be considered as a multi-objective solution to the problems of minimization of overall power loss, torque ripple and DC current ripple.
Comparison of the frequency domain performance of the fixed turn-on angle based excitation controller and fixed turn-off angle based controller.

![Figure 5.26: Sample FFT spectrum of flux-linkage: (a) 250kW SRG operating at 37.5krpm and at an electrical load of 288.75A, and (b) 300W SRG operating at 3750rpm and at an electrical load of 7.5A.](image)

Figures 5.26 (a) to (d) present sample comparison of the FFT spectrums of flux-linkage for the 250kW SR generator and the 300W SR generator. Figures 5.26 (a) and (c) represent sample operation at a low-speed/high-load condition and figures 5.26 (b) and (d) represent operation at a high-speed/medium-load condition respectively. Both the generators follow a similar pattern. During low-speed/high-load operation, the relative differences between the flux-linkage magnitudes are small. However, at high-speed/medium-load operation, the fixed turn-on angle based excitation controller yields a higher flux-linkage magnitude.
relative to the other two excitation controllers. As a consequence, during low-load operation, the fixed turn-on angle based excitation controller exhibits poor performance, and this is also validated by the experimental results on efficiency presented in figure 5.22 (b).

In contrast, the fixed turn-off angle based controller achieves a flux-linkage magnitude close to the min \( \{ \varepsilon \} \) optimization. Especially at low speeds, the fixed turn-off angle based controller achieves a lower magnitude of flux-linkage than that with the min \( \{ \varepsilon \} \) optimization. This can be observed in the low-speed FFT shown in figure 5.26 (a) and (c). This produces high RMS currents at low speeds, which result in a slight reduction in efficiency during low-speed/high-load conditions. However, during high-speed operation, the fixed turn-off angle based excitation controller exhibits superior performance par with the min \( \{ \varepsilon \} \) optimization.

The frequency domain results of DC-link current and the torque production of these three excitation controllers are also given in appendix K. Similarly to the earlier case, it can be seen that the DC-link current frequency domain performance follows a similar pattern to that of torque production. The fixed turn-on angle based excitation controller achieves a lower level of fundamental torque ripple magnitude and DC current ripple magnitude at the cost of higher flux-linkage magnitude. In contrast, the fixed turn-off angle based controller achieves a higher level of fundamental torque ripple magnitude. In conclusion, it can be deduced that the fixed turn-off angle based excitation controller provides a better balance of efficiency and frequency domain performance compared with the fixed turn-on angle based excitation controller.

5.8 Conclusion

In this chapter, SR machine operation in generation mode has been investigated. The objective of evaluating structurally simple excitation control methods which offer enhanced efficiency and performance in high speed operation was achieved considering two of the classical excitation control strategies, viz.,

1: fixed turn-on angle/variable turn-off angle based control and
2: fixed turn-off angle/variable turn-on angle based control

The performance of these two excitation control methods was compared with optimized excitation controller performance. For this, SR generator operation
under four different optimizations that minimize excitation penalty, RMS current, peak flux-linkage value and peak current value were investigated. The variations of excitation parameters for these four optimizations were presented. The relative differences of each of these optimizations have been explained with the aid of simulations.

For the implementation of the optimal excitation controllers, a piecewise continuous function (PCF) format for the general variation of the optimal excitation parameters was proposed. The optimal excitation parameter variations were fitted to this general PCF format considering the optimal data points obtained from simulations for the 250kW SR generator and the 300W SR generator.

The voltage regulator was designed for the operation of the system in combination with these excitation controllers. Each of these controllers were simulated for the 250kW SR generator and the 300W SR generator, and experimentally validated with the test-bed 300W SR generator system.

Table 5.4: Summary of different performance trade-off characteristics at two extreme load levels derived from the 300W test-bed SRG experimental and simulation results.

<table>
<thead>
<tr>
<th>Load level</th>
<th>$\eta$ [%]</th>
<th>$\psi_{pk}$ [%]</th>
<th>$i_{dc}$ FFT [%]</th>
<th>$T_{em}$ FFT [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>High</td>
<td>Low</td>
<td>High</td>
<td></td>
</tr>
<tr>
<td>min ${\varepsilon}$</td>
<td>84.9</td>
<td>79.6</td>
<td>23.6</td>
<td>69.3</td>
</tr>
<tr>
<td>min ${I_{rms}}$</td>
<td>70.8</td>
<td>80.1</td>
<td>66.0</td>
<td>100.0</td>
</tr>
<tr>
<td>min ${\psi_{pk}}$</td>
<td>73.7</td>
<td>81.3</td>
<td>13.2</td>
<td>58.7</td>
</tr>
<tr>
<td>min ${I_{pk}}$</td>
<td>60.9</td>
<td>79.9</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>Fixed $\theta_m$</td>
<td>70.9</td>
<td>80.7</td>
<td>47.2</td>
<td>65.3</td>
</tr>
<tr>
<td>Fixed $\theta_{off}$</td>
<td>78.8</td>
<td>80.2</td>
<td>19.8</td>
<td>53.3</td>
</tr>
</tbody>
</table>

Time-domain performances of these controllers are evaluated by means of voltage response and efficiency. The table 5.4 present a summary of different performance trade-off characteristics at two extreme load levels derived from the 300W test-bed SRG experimental and simulation results. The normalized percentage values of the peak flux-linkage, DC-link current ripple fundamental FFT-power value and the torque ripple fundamental FFT-power value are shown in table 5.4. A value of 100% represents the maximum value in each of these columns. It has been shown that the minimization of peak current yields the highest flux-linkage while minimization of the peak flux-linkage yields the highest fundamental DC current ripple and torque ripple values. Minimization of the RMS current, peak current or the peak flux-linkage results in the efficiency variation within a
wider range than that of excitation penalty. Experimental results also show that
the sole minimization of either the RMS current magnitude, peak flux-linkage
magnitude or the peak current-peak magnitude does not provide good overall
efficiency. However, the minimization of excitation penalty is found to be an
excellent multi-objective solution, which provide superior efficiency for a range of
speed and load conditions while balancing the torque ripple and DC-link current
ripple magnitudes.

Minimization of the peak current value reduces the DC-link current ripple
fundamental power by 85.3% and the torque ripple fundamental power by 89.2%
compared with that achieved with the minimized peak flux-linkage operation at
high loads. However, the efficiency is at 79.9% where a drop of 1.4% is observed.
At low loads, the drop in efficiency is 12.8%. However, the minimized excitation
penalty operation achieves higher efficiency at 84.9% with a DC-link current ripple
fundamental power of 88.9% and torque ripple fundamental power at 54.5%.
Similar observations can be made for the other optimization conditions.

It shows that the fixed turn-on angle based controller exhibits poor perfor-
mance during low-load operation. However, the fixed turn-off angle based exci-
tation controller performs with a better balance between the losses and is com-
parable to the minimized excitation penalty based controller in certain regions of
operation.

In conclusion, it can be deduced that the controller which offers the simplest
structure and the balance between efficiency, torque ripple and DC-link current
ripple in high speed power generation is the fixed turn-off angle based excitation
controller. This excitation control strategy achieves efficiencies within the narrow
range of 79% to 85% for the majority of the operating conditions. Furthermore,
the fixed turn-off angle excitation controller provides the capability to overload
the machine and even extend operation to the continuous conduction mode.
Chapter 6

Modelling of Fault-Tolerant Permanent Magnet Machines and Active Rectifier Systems

This chapter develops a generalized model of a fault-tolerant permanent magnet (PM) machine interfaced with an active rectifier system. Active rectifier operation with PWM switching is considered. Special attention is given to the task of modelling an arbitrary number of phases in operation. In addition the model is derived in the format where an arbitrary number of current, voltage and PM flux-linkage harmonics can be considered. This model is validated by means of the high fidelity dynamic simulation technique presented in chapter three and the FE models developed therein.

6.1 Introduction

Fault-tolerant PM machines ideally possess the characteristics of a high number of phases for enhanced partial redundancy, high inductance for fault current limiting capability and negligible mutual coupling. Significant effort has been devoted in the past to the design of fault-tolerant machines with such characteristics [42,53,81,147]. However, the fault-tolerant nature of the machine alone does not assure the sustained operation of the system under faulted condition. Both the power conversion technology and the converter control play a major role in achieving satisfactory continued operation.

Different fault-tolerant converter structures have been proposed in the past.
Complete electrical isolation is obtained by use of a parallel converter topology where each phase consists of a full bridge module [53,148]. The topology of such a converter is shown in figure 6.1, and has been considered as a candidate for fault-tolerant power generation and rectification in [41, 43, 53, 134, 135]. PM machine and rectifier models that can be easily adapted to represent a system under partial operation have not yet been extensively studied, and will therefore be of high importance in future fault-tolerant control systems design for PM machines. In particular, specialised control strategies for the parallel H-bridge converter combined with multiphase PM machines has not been thoroughly considered in the past. One objective of this thesis is to develop novel specialised control methods for fault-tolerant PM machines operating with such a converter topology. For this purpose, a mathematic model is required that represents PM machine electrical and mechanical dynamics in the format where arbitrary number of phases can be considered. In addition to control system design, such models are essential for stability assessment and fault detection and isolation (FDI) tasks. The following section develops a mathematical representation of PM machine electrical dynamics for a general case where non-sinusoidal phase voltages are imposed via harmonic injection pulse width modulation (HIPWM). The insight into controllability of machine harmonics will facilitate the control design objectives such as DC-link voltage ripple reduction and torque ripple reduction [149,150]. Dynamic interaction of harmonics, mean rectifier DC-side current and the mean torque production are mathematically modelled.
CHAPTER 6. MODELLING OF PM MACHINES

Typical control systems for PM machine drives are designed with the assumption of a balanced machine. Under balanced operation, the DC-link current, DC-link voltage and electromagnetic torque production can be driven to a smooth, minimal ripple level by supplying the machine phases with symmetrical voltages. However, under faulted conditions, the same level of DC-link voltage ripple and electromagnetic torque ripple cannot be attained with the same phase currents as in the balanced case. Special optimization strategies are needed to calculate the required currents and voltages of each phase in operation to achieve a minimal ripple condition, together with other constraints such as minimized reactive power consumption and copper losses. Chapters seven and eight utilize the model developed in this chapter to formulate optimal control strategies for motoring and generating, and also considers the minimization of reactive power transfer, torque ripple and DC-link current ripple under faulted operating conditions.

The approach adopted in this chapter is to model such a fault-tolerant PM machine and active rectifier systems by utilizing the inherent modularity of the circuit structure. Each phase of the PM machine and the H-bridge are identical, and thus based on symmetry, a per-phase model can be used to describe the system adequately. Single phase AC power source models are reported in literature, and have been the subject of considerable research. However, the majority of these models are confined to the modelling of the effect of the fundamental frequency [151, 152] and are for a fixed inductance values. Therefore the effect of inductance variation, as seen in PM machines with magnetic saliency cannot be analysed with such models. The authors of [134, 152] utilize steady state power transfer models for control design, while the analysis in [153] on a single phase PWM rectifier is performed based on a small signal approximated model. In contrast, the model developed in this chapter considered multiple harmonics and a second order variation of inductance, and therefore can be applied for the modelling of a wide class of fault-tolerant PM machines.

6.2 System instantaneous dynamic model

Figure 6.1 shows the PM generator and parallel active rectifier interconnection for fault-tolerant power generation. For a general \(n\)th phase, the terminal voltage \(v_n\) and phase current \(i_n\) are depicted in this figure.

The continuous time dynamic model of the \(n\)th phase of a PM generator can
be written as:

$$v_n = R_n i_n + \frac{d\psi_n (\theta_e + \gamma_n, i_n)}{dt}$$  \hspace{1cm} (6.1)

$$\psi_n (\theta_e + \gamma_n, i_n) = l_{nn} i_n + \sum_{j=1 \atop j \neq n}^{N} l_{nj} i_j + \psi_{pm,n}$$  \hspace{1cm} (6.2)

\(\gamma_n\) represent the phase shift of the \(n^{th}\) phase of the machine with respect to the electrical angle \(\theta_e\). The phase shift \(\gamma_n = \frac{2\pi}{N} (n - 1)\), \(n = 1, 2, \ldots, N\), \(N\) is the number of phases of the machine.

Since the emphasis of this chapter is based on a power generation perspective, currents generated out of the PM machine are taken as positive. Replacing \(i_n \rightarrow -i_n\) equations (6.1) and (6.2) are rewritten as:

$$v_n = -R_n i_n + \frac{d\psi_n (\theta_e + \gamma_n, i_n)}{dt}$$  \hspace{1cm} (6.3)

$$\psi_n (\theta_e + \gamma_n, i_n) = -l_{nn} i_n - \sum_{j=1 \atop j \neq n}^{N} l_{nj} i_j + \psi_{pm,n}$$  \hspace{1cm} (6.4)

The flux-linkage \(\psi_n\) incorporates the effects of both the stator flux and the rotor PM flux. Self inductances and mutual inductances are considered as time varying and are represented by \(l_{nn}\) and \(l_{nj}\). Here subscripts \(n\) and \(j\) represents a phase number. The flux-linkage of the \(n^{th}\) phase established by the PM flux \(\psi_{pm,n}\) is considered to have multiple harmonics. This is represented by the complex exponential Fourier series:

$$\psi_{pm,n} = \sum_{k=1}^{K_1} \left\{ \tilde{\psi}_{n,k} e^{-jk(\theta_e + \gamma_n)} + \tilde{\psi}_{n,k}^* e^{jk(\theta_e + \gamma_n)} \right\}$$  \hspace{1cm} (6.5)

where \(\tilde{\psi}_{n,k} = \psi_{\alpha,n,k} + j\psi_{\beta,n,k}\) represents a magnitude vector of the flux-linkage due to the PM flux with

$$\psi_{\alpha,n,k} = \psi_{pk,n,k} \cos k\nu_{n,k} \hspace{1cm} \text{and} \hspace{1cm} \psi_{\beta,n,k} = \psi_{pk,n,k} \sin k\nu_{n,k}$$  \hspace{1cm} (6.6)

The parameter \(\nu_{n,k}\) represents the phase shift of the PM flux-linkage with respect to an arbitrary reference angle. Typical PM machines are symmetrical and will see identical PM flux-linkage magnitudes, i.e., \(\tilde{\psi}_{x,k} = \tilde{\psi}_{y,k} = \tilde{\psi}_{k}\) for
all \( x, y = 1, 2, \ldots, N \). Therefore the subscript of phase number can be omitted. With the notation \( \vec{\psi}_{-k} = \vec{\psi}_k^* \) and taking \( \vec{\psi}_0 = 0 \), equation (6.5) can be rewritten as a two sided complex exponential Fourier series:

\[
\psi_{\text{pm}, n} = \frac{1}{2} \sum_{k=-K_1}^{K_1} \left\{ \vec{\psi}_k e^{-jk(\theta_e + \gamma_n)} \right\} \tag{6.7}
\]

The fault-tolerant machine design requirement of magnetic isolation will permit the assumption that the mutual inductances are negligible. Thus in this chapter we assume:

\[
\sum_{j=1, j \neq n}^{N} l_{nj} i_j = 0 \tag{6.8}
\]

The self inductance term of a majority of the PM machines can be described by a second harmonic variation \([84, 85]\), i.e.:

\[
l_{nn} = L_{nn, l} + L_{nn, a0} + L_{nn, a2} \cos 2(\theta_e + \gamma_n) \tag{6.9}
\]

where the total self inductance is the addition of inductances arising due to the leakage flux, \( L_{nn, l} \) and the air-gap flux, \( L_{nn, a0}, L_{nn, a2} \). As explained earlier, it should be noted here that the self inductance of a surface PM machine can be written as a constant term \([83]\), i.e., assuming \( L_{nn, a2} = 0 \).

\[
l_{nn} = L_{n,0} = L_{nn, l} + L_{nn, a0} \tag{6.10}
\]

Without loss of generality, the notation \( \theta = \theta_e + \gamma_n \) is introduced for the \( n \)th phase considered in this analysis. Then (6.9) can also be written as:

\[
l_{nn} = L_{n,0} + L_{n,2} \cos 2\theta \quad \text{or} \quad l_{nn} = L_{nn,0} + \frac{L_{n,2}}{2} (e^{j2\theta} + e^{-j2\theta}) \tag{6.11}
\]

where \( L_{n,0} = L_{nn, l} + L_{nn, a0} \) and \( L_{n,2} = L_{nn, a2} \).

The rectifier/inverter representation is based on the switch averaged lossless PWM rectifier model described by the rectifier instantaneous AC-side terminal voltage \( v_n \), phase current \( i_n \), the DC-link voltage \( v_{dc} \) and the rectified current \( i_{\text{rect}, n} \). The relationship between the input power \( p_{\text{in}}(t) = v_n i_n \) and the output power \( p_{\text{out}}(t) = v_{dc} i_{\text{rect}, n} \) can be written by assuming a lossless converter stage
power balance.

\[ v_n i_n = v_{dc} i_{\text{rect},n} \]  \hspace{1cm} (6.12)

The rectifier AC-side terminal voltage of the \( n^{\text{th}} \) phase can be written in terms of the general PWM switching function \( d_n(t) \) as:

\[ v_n = v_{dc} d_n(t) \]  \hspace{1cm} (6.13)

Substitution of (6.13) in (6.12) and cancellation of \( v_{dc} \) yields the rectified switch-averaged dc current as:

\[ i_{\text{rect},n} = d_n(t) i_n \]  \hspace{1cm} (6.14)

Then the DC-link voltage dynamics can be written as:

\[ C \frac{dv_{dc}}{dt} = \left\{ \sum_{n \in S} i_{\text{rect},j} \right\} - i_{\text{load}} \]  \hspace{1cm} (6.15)

where \( S \) is the set of operational phases of the considered fault-tolerant PM generator and \( i_{\text{load}} \) represent the load current disturbances.

### 6.3 Per-phase combined mathematical model of a PMG and PWM rectifier

Often average dynamics are required for the design of stabilizing controllers for DC-link voltage control in fault-tolerant rectifier systems or torque control in fault-tolerant drive systems. This section focuses on derivation of mean value rectifier DC current and mean value electromagnetic torque production. Expressions for DC-link ripple current, electromagnetic torque ripple and mean real and reactive power are derived in terms of the harmonic injection pulse width modulation (HIPWM) input.
6.3.1 Rectifier mean-value model

Let the HIPWM switching signal issued to the \( n \)th phase be of the generalized form:

\[
d_n(t) = \sum_{k=1}^{K} \{u_{\beta,n,k} \sin k\theta + u_{\alpha,n,k} \cos k\theta\}
\] (6.16)

The input \( (u_{\beta,n,k}, u_{\alpha,n,k}) \) represents the \( k \)th harmonic Fourier coefficients of the PWM switching function of the \( n \)th phase. Let the \( k \)th input be represented as a vector in the complex rotating reference frame with basis \( e^{jk\theta} \), \( k \theta \in \mathbb{R} \):

\[
\vec{u}_{n,k} = u_{\alpha,n,k} + ju_{\beta,n,k}
\] (6.17)

It can be easily shown that the basis function \( e^{jk\theta} \) at different \( k \)th harmonics are mutually orthogonal [154]. Then the input vector \( (\vec{u}_{n,1} \quad \vec{u}_{n,2} \quad \ldots \quad \vec{u}_{n,K}) \) forms a multidimensional space with mutually orthogonal planes with vector space basis \( (e^{j\theta} \quad e^{j2\theta} \quad \ldots \quad e^{jK\theta}) \). Value \( K \) represents the maximum harmonic order that we may require to consider. Then the switching function (6.16) can also be written in terms of the complex exponential function as:

\[
d_n(t) = \frac{1}{2} \sum_{k=1}^{K} \{\vec{u}_{n,k} e^{-jk\theta} + \vec{u}_{n,k}^* e^{jk\theta}\}
\] (6.18)

where \( \vec{u}_{n,k} \) is defines as,

\[
\vec{u}_{n,k} = u_{\alpha,n,k} + ju_{\beta,n,k}, \quad \vec{u}_{n,0} = 0 \quad \text{and} \quad \vec{u}_{n,-k} = \vec{u}_{n,k}^*
\] (6.19)

The switching function (6.18) can also be written as a two sided complex series,

\[
d_n(t) = \frac{1}{2} \sum_{k=-K}^{K} \{\vec{u}_{n,k} e^{-jk\theta}\}
\] (6.20)

Then the switch average terminal voltage of the \( n \)th phase can be written as a two sided complex exponential Fourier series:

\[
v_n = \frac{1}{2} v_{dc} \sum_{k=-K}^{K} \{\vec{u}_{n,k} e^{-jk\theta}\}
\] (6.21)
Let the current of the $n^{th}$ phase, $i_n$ be written as a current waveform of the form:

$$i_n = \sum_{k=1}^{K} \{i_{pk,n,k} \sin k(\theta - \varepsilon_k)\} \quad (6.22)$$

where $i_{pk,n,k}$ represents the magnitude of the $k^{th}$ current harmonic. Angle $\varepsilon_k$ represents the phase difference of the $k^{th}$ harmonic with respect to the reference angle $\theta$. Introducing the notation $i_{pk,n,k} = -i_{pk,n,-k}$ (6.22) can be written as a complex exponential two sided series,

$$i_n = -\frac{1}{2j} \sum_{k=-K}^{K} \{i_{pk,n,k}e^{-jk(\theta - \varepsilon_n,k)}\} \quad (6.23)$$
Further simplification yields,

\[ i_n = \frac{1}{2} \sum_{k=-K}^{K} \left\{ (ji_{pk,n,k}e^{jk\varepsilon_n,k}) e^{-jk\theta} \right\} \]  \hspace{1cm} (6.24)

Then (6.24) can be alternatively written in terms of the complex exponential function as:

\[ i_n = \frac{1}{2} \sum_{k=-K}^{K} \left\{ \vec{i}_{n,k} e^{-jk\theta} \right\} \]  \hspace{1cm} (6.25)

Each frequency component of (6.25), \( \vec{i}_{n,k} = ji_{pk,n,k}e^{jk\varepsilon_n,k} \) can be written as a current vector in the complex rotating reference frame with basis \( e^{jk\theta} \):

\[ \vec{i}_{n,k} = -i_{\alpha,n,k} + ji_{\beta,n,k} \]  \hspace{1cm} (6.26)

where the current component in phase with the generator back-EMF and in anti-quadrature phase with the back-EMF can be written as:

\[ i_{\beta,n,k} = i_{pk,n,k} \cos k\varepsilon_n,k \]  \hspace{1cm} (6.27)

\[ i_{\alpha,n,k} = i_{pk,n,k} \sin k\varepsilon_n,k \]  \hspace{1cm} (6.28)

The vector diagram representation of \( \vec{\psi}_{n,k}, \vec{u}_{n,k} \) and \( \vec{i}_{n,k} \) in the complex plane is shown in figure 6.2. The instantaneous value of the rectified current can be derived by substitution of (6.20) and (6.25) in (6.14) as:

\[ i_{\text{rect},n} = \frac{1}{4} \sum_{l=-K}^{K} \sum_{k=-K}^{K} \left\{ \vec{u}_{n,l} \vec{i}_{n,k} e^{-j(k+l)\theta} \right\} \]  \hspace{1cm} (6.29)

The mean-value of the rectified current can be derived by considering the terms where indices \( k = l \) of (6.29), which yield unity for all terms with \( e^{j(k-l)\theta} \). Then the mean component of the per-phase dc current \( i_{\text{rect},n} \) can be written as:

\[ i_{\text{dc},n} = \frac{1}{4} \sum_{k=-K}^{K} \left\{ \vec{u}_{n,k} \vec{i}_{n,k}^{*} \right\} \]  \hspace{1cm} (6.30)
CHAPTER 6. MODELLING OF PM MACHINES

The DC-link mean voltage dynamics can be written as:

\[
C \frac{dv_{dc,\text{mean}}}{dt} = \left\{ \sum_{n \in S} i_{dc,n} \right\} - i_{\text{load}} \tag{6.31}
\]

where \( S \) is the set of operational phases of the considered fault-tolerant PMG.

6.4 Generalized Per-Phase Electrical Model of a Fault-Tolerant PM Machine

The interaction between the HIPWM control variables \( \vec{u}_{n,k}, k > 0 \) and the current harmonics \( \vec{i}_{n,k}, k > 0 \) in one phase, can be derived from the equations (6.3) to (6.29). The detailed derivation is given in appendix L. In summary, the general per-phase \( k^{th} \) harmonic dynamics can be written as:

\[
\frac{L_n}{2} \frac{d\vec{i}_{\alpha,n,k+2}}{dt} + L_{n,0} \frac{d\vec{i}_{\alpha,n,k}}{dt} + \frac{L_n}{2} \frac{d\vec{i}_{\alpha,n,k-2}}{dt} - jk\omega \frac{L_n}{2} \vec{i}_{\alpha,n,k+2} \\
- jk\omega \frac{L_n}{2} \vec{i}_{n,k-2} - jk\omega L_{n,0} \vec{i}_{n,k} + R_s \vec{i}_{n,k} = -v_{dc} \vec{u}_{n,k} - jk\omega \vec{\psi}_k \tag{6.32}
\]

Substitution of (6.19) and (6.26) in (6.32) and by separation of the real and imaginary parts, a new dynamic model representing the mutually orthogonal harmonic dynamics of the PMG is obtained as:

\[
- \frac{L_n}{2} \frac{di_{\alpha,n,k+2}}{dt} - L_{n,0} \frac{di_{\alpha,n,k}}{dt} - \frac{L_n}{2} \frac{di_{\alpha,n,k-2}}{dt} + k\omega \frac{L_n}{2} i_{\beta,n,k+2} \\
+ k\omega \frac{L_n}{2} i_{\beta,n,k-2} + k\omega L_{n,0} i_{\beta,n,k} - R_s i_{\alpha,n,k} = -v_{dc} u_{\alpha,n,k} + \omega k\psi_{\beta,n,k} \tag{6.33}
\]

\[
\frac{L_n}{2} \frac{di_{\beta,n,k+2}}{dt} + L_{n,0} \frac{di_{\beta,n,k}}{dt} + \frac{L_n}{2} \frac{di_{\beta,n,k-2}}{dt} + k\omega \frac{L_n}{2} i_{\alpha,n,k+2} \\
+ k\omega \frac{L_n}{2} i_{\alpha,n,k-2} + k\omega L_{n,0} i_{\alpha,n,k} + R_s i_{\beta,n,k} = -v_{dc} u_{\beta,n,k} - \omega k\psi_{\alpha,n,k} \tag{6.34}
\]
6.5 Representation of mean electromagnetic torque production per-phase

The electromagnetic torque produced by the $n^{\text{th}}$ phase of a PMG can be calculated from the partial derivative of the magnetic co-energy. Assuming no saturation effects are present, the co-energy can be written as:

$$W_n = \int_{i=0}^{i_n} \psi_n(\theta, i) \, di$$  \hspace{1cm} (6.35)

Note that the co-energy calculated here conforms to the convention adopted in this chapter, i.e., energy delivered into the machine is negative, while energy delivered out of the machine is positive. Substitution of (6.4) in (6.35) yields,

$$W_n = \int_{i=0}^{i_n} \left\{ -l_{nn}i_n + \frac{1}{2} \sum_{k=-K}^{K} \left\{ \bar{\psi}_k e^{-jk\theta} \right\} \right\} \, di$$  \hspace{1cm} (6.36)

Evaluation of the integral operation yields,

$$W_n = \left[ -\frac{1}{2}l_{nn}i_n^2 + \frac{1}{2}i_n \sum_{k=-K}^{K} \left\{ \bar{\psi}_k e^{-jk\theta} \right\} \right]_{i_n=0}$$  \hspace{1cm} (6.37)

$$W_n = -\frac{1}{2}l_{nn}i_n^2 + \frac{1}{2}i_n \sum_{k=-K}^{K} \left\{ \bar{\psi}_k e^{-jk\theta} \right\}$$  \hspace{1cm} (6.38)

Then torque produced by the $n^{\text{th}}$ phase is then given by:

$$T_n = k_{pp} \frac{\partial W_n}{\partial \theta_e}$$  \hspace{1cm} (6.39)

where $k_{pp}$ is the number of pole-pairs. The torque calculated by this expression here represents the shaft loading via the electromagnetic torque. A positive value represents positive loading and electrical power generation and vice versa.

Substitution of (6.38) in (6.39) yields the torque per-phase in the form:

$$T_n = T_{pm,n} + T_{rel,n}$$  \hspace{1cm} (6.40)
where $T_{pm,n}$ and $T_{rel,n}$ represent the PM torque and the reluctance torque components of the $n^{th}$ phase, i.e.,

\[
T_{pm,n} = \frac{k_{pp} i_n}{2} \sum_{k=-K_1}^{K_1} \frac{\partial}{\partial \theta} \left\{ \vec{\psi}_k e^{-jk(\theta + \gamma_n)} \right\} \tag{6.41}
\]

\[
T_{rel} = -\frac{k_{pp} i_n^2}{2} \frac{\partial l_{nn}}{\partial \theta} \tag{6.42}
\]

It is worth noting here that the reluctance torque expression inherits a negative sign, as the flux-linkage produced by the inductance variation is negative for a positive current.

### 6.5.1 Mean permanent magnet torque per-phase

Equation (6.41) can be expanded by substitution of (6.25),

\[
T_{pm,n}(\theta) = \frac{k_{pp} i_n}{4} \left[ \sum_{k=-K_1}^{K_1} \frac{\partial}{\partial \theta} \left\{ \vec{\psi}_k e^{-jk\theta} \right\} \right] \left[ \sum_{k=-K}^{K} \left\{ \vec{i}_{n,k} e^{-jk\theta} \right\} \right] \tag{6.43}
\]

Performing the partial differentiation operation yields:

\[
T_{pm,n}(\theta) = -\frac{j k_{pp} i_n}{4} \left[ \sum_{k=-K_1}^{K_1} \left\{ k \vec{\psi}_k e^{-jk\theta} \right\} \right] \left[ \sum_{k=-K}^{K} \left\{ \vec{i}_{n,k} e^{-jk\theta} \right\} \right] \tag{6.44}
\]

or alternatively written as,

\[
T_{pm,n}(\theta) = k_{pp} \text{Im} \left[ \sum_{k=0}^{K_1} \left\{ k \vec{\psi}_k e^{-jk\theta} \right\} \right] \text{Re} \left[ \sum_{k=0}^{K} \left\{ \vec{i}_{n,k} e^{-jk\theta} \right\} \right]
\]

Equation (6.44) can be written as a double summation:

\[
T_{pm,n}(\theta) = -\frac{j k_{pp} i_n}{4} \sum_{l=-K_1}^{K_1} \sum_{k=-K}^{K} \left\{ l \vec{\psi}_l \vec{i}_{n,k} e^{-j(k+l)\theta} \right\} \tag{6.45}
\]

The mean PM torque produced by the $n^{th}$ phase $T_{pm,avg,n}$, can be obtained by considering the terms consisting of $e^{j(k-l)\theta} = 1$ and $e^{-j(k-l)\theta} = 1$ when $k - l = 0$.
of the series (6.45). This can be written by assuming \( K \leq K_1 \) as:

\[
T_{\text{pm,avg},n} = \frac{jk_{\text{pp}}}{4} \sum_{k=-K_1}^{K_1} \left( k\tilde{\psi}_k \tilde{i}_{n,k} \right)
\]  

(6.46)

### 6.5.2 Mean reluctance torque per-phase

Equation (6.42) can be expanded by substitution of (6.25) and, (6.11) in exponential form,

\[
T_{\text{rel},n} (\theta) = -\frac{k_{\text{pp}}}{8} \left[ \sum_{k=-K}^{K} \{ \tilde{i}_{n,k} e^{-jk\theta} \} \right]^2 \frac{L_{n,2}}{2} \frac{\partial (e^{j2\theta} + e^{-j2\theta})}{\partial \theta}
\]  

(6.47)

Performing the partial differentiation operation yields,

\[
T_{\text{rel},n} (\theta) = -\frac{jk_{\text{pp}}L_{n,2}}{8} \left[ \sum_{k=-K_1}^{K_1} \{ i_k e^{-jk\theta} \} \right]^2 \left( e^{j2\theta} - e^{-j2\theta} \right)
\]

which can be alternatively written as,

\[
T_{\text{rel},n} (\theta) = k_{\text{pp}} L_{n,2} \left[ \text{Re} \left\{ \sum_{k=-K_1}^{K_1} i_k e^{-jk\theta} \right\} \right]^2 \text{Im} (e^{j2\theta})
\]

Simplification of the squared term within brackets of equation (6.47) yields:

\[
T_{\text{rel},n} (\theta) = -\frac{jk_{\text{pp}}L_{n,2}}{8} \left[ \sum_{l=-K_1}^{K_1} \sum_{k=-K}^{K} \left\{ \tilde{i}_{n,l} \tilde{i}_{n,k} e^{-j(k+l)\theta} \right\} \right] \left( e^{j2\theta} - e^{-j2\theta} \right)
\]  

(6.48)

Absorption of the the exponential terms into summation operation yields:

\[
T_{\text{rel},n} (\theta) = -\frac{jk_{\text{pp}}L_{n,2}}{8} \sum_{l=-K_1}^{K_1} \sum_{k=-K}^{K} \left( \tilde{i}_{n,l} \tilde{i}_{n,k} e^{-j(k+l)\theta} - \tilde{i}_{n,l} \tilde{i}_{n,k} e^{-j(k+l+2)\theta} \right)
\]  

(6.49)

The mean reluctance torque produced by the \( n^{\text{th}} \) phase \( T_{\text{rel,avg},n} \), can be obtained by considering the terms consisting of \( e^{j(k+l-2)\theta} = 1 \) and \( e^{-j(k+l+2)\theta} = 1 \) when \( k+l-2 = 0 \) and \( k+l+2 = 0 \) of the series (6.49). Assuming \( K \leq K_1 \), this
can be written as:

\[
T_{\text{avg,rel},n}(\theta) = -\frac{j k_{pp} L_{n,2}}{8} K_1 \sum_{k=-K_1}^{K_1} \left( \vec{i}_{n,k} \vec{i}^*(n,2-k) - \vec{i}_{n,k} \vec{i}^*(n,-2-k) \right)
\]
or,

\[
T_{\text{avg,rel},n}(\theta) = -\frac{j k_{pp} L_{n,2}}{8} K_1 \sum_{k=-K_1}^{K_1} \left( \vec{i}_{n,k} \vec{i}^*(n,k-2) - \vec{i}_{n,k} \vec{i}^*(n,k+2) \right) \tag{6.50}
\]

### 6.5.3 Total torque per-phase and torque ripple

The total average torque per-phase is given by:

\[
T_{\text{avg},n} = \frac{j k_{pp}}{4} \sum_{k=-K_1}^{K_1} \left( k \vec{\psi}^*_k \vec{i}_{n,k} \right) - \frac{j k_{pp} L_{n,2}}{8} \sum_{k=-K_1}^{K_1} \left( \vec{i}_{n,k} \vec{i}^*(n,k-2) - \vec{i}_{n,k} \vec{i}^*(n,k+2) \right) \tag{6.51}
\]

\[
T_{\text{avg},n} = \frac{j k_{pp}}{8} \sum_{k=-K_1}^{K_1} \left\{ \vec{i}_{n,k} \left( 2k \vec{\psi}^*_k - L_{n,2} \vec{i}^*(n,k-2) + L_{n,2} \vec{i}^*(n,k+2) \right) \right\} \tag{6.52}
\]

The torque ripple produced by the \(n^{th}\) phase is given by,

\[
T_{\text{rip},n} = T_{\text{pm},n} + T_{\text{rel},n} - T_{\text{avg},n} \tag{6.53}
\]

The total average torque production is given by,

\[
T_{\text{tot,avg}} = \sum_{n \in S} T_{\text{avg},n} \tag{6.54}
\]

### 6.6 Steady state DC-link current and voltage ripple

The current ripple produced by the \(n^{th}\) phase is given by,

\[
i_{\text{rip},n} = i_{\text{rect},n} - i_{\text{dc},n} \tag{6.55}
\]
Then the total DC-link current ripple produced by all operation phases can be derived by substitution of (6.29) and (6.30) in (6.55) for all $n \in S$ as,

$$i_{\text{rip}} = \sum_{n \in S} \left\{ \frac{1}{4} \sum_{l=-K}^{K} \sum_{k=-K}^{K} \left( \vec{u}_{n,l} \vec{i}_{n,k} e^{-j(k+l)\left(\theta_e + \gamma_n\right)} \right) - \frac{1}{4} \sum_{k=-K}^{K} \left( \vec{u}_{n,k} \vec{i}_{n,k}^* \right) \right\} \quad (6.56)$$

Further simplification yields:

$$i_{\text{rip}} = \frac{1}{4} \sum_{n \in S} \left\{ \sum_{l=-K}^{K} \left( \sum_{k=-K}^{K} \vec{u}_{n,l} \vec{i}_{n,k} e^{-j(k+l)\left(\theta_e + \gamma_n\right)} \right) - \left( \vec{u}_{n,l} \vec{i}_{n,l} \right) \right\} \quad (6.57)$$

Equation (6.57) can be written in compact form:

$$i_{\text{rip}} = \frac{1}{4} \sum_{n \in S} \left\{ \sum_{l=-K}^{K} \sum_{k=-K}^{K} \left( \vec{u}_{n,l} \vec{i}_{n,k} e^{-j(k+l)\left(\theta_e + \gamma_n\right)} \right) \right\} \quad (6.58)$$

Assuming the effect of the DC-link voltage ripple on the load current is negligible, we may calculate the DC-link voltage via (6.15). Under steady state conditions we may assume that the DC-link load current is sourced by the total mean rectified current, i.e., $i_{\text{load}} = \sum_{n \in S} i_{\text{dc},n}$. Then the DC-link voltage ripple can be calculated as,

$$v_{\text{rip}} = \frac{1}{C} \int_{t=0}^{t} i_{\text{rip}} \, dt \quad (6.59)$$

where $C$ is the total DC-link capacitance. Substitution of (6.58) in (6.59) yields:

$$v_{\text{rip}} = \frac{1}{C} \int_{t=0}^{t} \sum_{n \in S} \left\{ \frac{1}{4} \sum_{l=-K}^{K} \sum_{k=-K}^{K} \left( \vec{u}_{n,l} \vec{i}_{n,k} e^{-j(k+l)\left(\theta_e + \gamma_n\right)} \right) \right\} \, dt \quad (6.60)$$

Specification of the integral limits in terms of the electrical angle yields:

$$v_{\text{rip}} = \frac{1}{4\omega C} \sum_{n \in S} \left\{ \int_{\theta_e=0}^{\theta_e} \left\{ \sum_{l=-K}^{K} \sum_{k=-K}^{K} \left( \vec{u}_{n,l} \vec{i}_{n,k} e^{-j(k+l)\left(\theta_e + \gamma_n\right)} \right) \right\} \, d(\theta_e + \gamma_n) \right\} \quad (6.61)$$
Evaluation of the integral operation yields:

\[ v_{\text{rip}} (\theta_e) = \frac{1}{4\omega C} \sum_{n \in S} \left\{ \sum_{l=-K}^{K} \sum_{k=-K}^{K} \left( \frac{\vec{u}_{n,l} \vec{\imath}_{n,k}}{-j(k+l)} e^{-j(k+l)(\theta_e+\gamma_n)} \right) \right\} \] (6.62)

Further simplification yields:

\[ v_{\text{rip}} (\theta) = \frac{j}{4\omega C} \sum_{n \in S} \left\{ \sum_{l=-K}^{K} \sum_{k=-K}^{K} \left( \frac{1}{(k+l)} \vec{u}_{n,l} \vec{\imath}_{n,k} \left( e^{-j(k+l)(\theta_e+\gamma_n)} - e^{-j(k+l)\gamma_n} \right) \right) \right\} \] (6.63)

### 6.7 Real power and reactive power

The generated real and reactive power associated with the \( k^{\text{th}} \) harmonic of the \( n^{\text{th}} \) phase, \( P_{n,k} \) and \( Q_{n,k} \) can be calculated by considering the component of \( \vec{\imath}_{n,k} \) in the direction of \( \vec{u}_{n,k} \) and perpendicular to \( \vec{u}_{n,k} \). This can be found by the dot products \( \langle \vec{\imath}_{n,k}, \vec{u}_{n,k} \rangle \) and \( \langle \vec{\imath}_{n,k}, j\vec{u}_{n,k} \rangle \). Evaluation of the dot product yields,

\[ P_{n,k} = \frac{v_{dc}}{2} \text{Re} \left\{ \vec{u}_{n,k} \vec{\imath}_{n,k}^* \right\} \] (6.64)

\[ Q_{n,k} = \frac{v_{dc}}{2} \text{Im} \left\{ \vec{u}_{n,k} \vec{\imath}_{n,k}^* \right\} \] (6.65)

Summation for multiple harmonics yields the total real power \( P_n \) associated with the \( n^{\text{th}} \) phase,

\[ P_n = \frac{v_{dc}}{2} \text{Re} \left\{ \sum_{k=0}^{K} \vec{u}_{n,k} \vec{\imath}_{n,k}^* \right\} \] (6.66)

The total real power and the notional reactive power associated with each \( k^{\text{th}} \) harmonic component delivered by all the operational phases can be found by summations,

\[ P_{\text{tot}} = \sum_{n \in S} P_n \quad \text{and} \quad Q_{\text{tot},k} = \sum_{n \in S} Q_{n,k} \] (6.67)

In general, reactive power represents an oscillatory power interchange between the DC-link and the machine. Individual harmonic components possess different
magnitude and phase. Thus, the reactive power harmonics are not summed, and are treated as separate output variables.

6.8 Model approximations

The generic model presented by equations (6.33) to (6.34) can be used to formulate different versions of simpler PMG models. Three versions of model variations are considered in this section. The first model contains the electrical dynamics of the fundamental and the third harmonic. The second model neglects the inductance variation and the third harmonic of the back-EMF. However, the third harmonic voltage injection is incorporated in the second model. The third model contains only the fundamental dynamics and is presented here to establish the significance of modelling additional harmonics. The following section describes these two versions of the models in more detail. The two models are simulated simultaneously for the SPM machine and the IPM machine developed in chapter three, and compared with the FE model based high-fidelity switching simulation technique.

6.8.1 PMG machine model Version 1 - Fundamental and third harmonic model

It was shown in chapter three that the back-EMF contains a significant third harmonic in both the SPM and the IPM machines designed therein. The model approximation performed here is based on these two machines, and therefore it is essential to incorporate the effect of the third harmonic. This can be achieved by expanding the model (6.33) to (6.34) for $k = 3$ in addition to the fundamental $k = 1$. The input PWM switching function is also represented with a vector with a third harmonic component $\vec{u}_3 = u_{a,3} + j u_{\beta,3}$ such that controllability over the rectified third harmonic current is also achieved. The resulting model of the $n^{th}$ phase electrical dynamics of the fundamental and the third harmonic are then given by:
\[
\frac{L_{n,2}}{2} \frac{di_{\alpha,n,3}}{dt} + \left( L_{n,0} + \frac{L_{n,2}}{2} \right) \frac{di_{\alpha,n,1}}{dt} = \omega \left( \frac{L_{n,2}}{2} i_{\beta,n,3} + \left( L_{n,0} - \frac{L_{n,2}}{2} \right) i_{\beta,n,1} \right) \\
- R_s i_{\alpha,n,1} + v_{dc} u_{\alpha,n,1} - \omega \psi_{\beta,n,1} \quad (6.68)
\]
\[
\frac{L_{n,2}}{2} \frac{di_{\beta,n,3}}{dt} + \left( L_{n,0} - \frac{L_{n,2}}{2} \right) \frac{di_{\beta,n,1}}{dt} = - \omega \left( \frac{L_{n,2}}{2} i_{\alpha,n,3} + \left( L_{n,0} + \frac{L_{n,2}}{2} \right) i_{\alpha,n,1} \right) \\
- R_s i_{\beta,n,1} - v_{dc} u_{\beta,n,1} - \omega \psi_{\alpha,n,1} \quad (6.69)
\]
\[
\frac{L_{n,2}}{2} \frac{di_{\alpha,n,1}}{dt} + L_{n,0} \frac{di_{\alpha,n,3}}{dt} = 3 \omega \left( \frac{L_{n,2}}{2} i_{\beta,n,1} + L_{n,0} i_{\beta,n,3} \right) \\
- R_s i_{\alpha,n,3} + v_{dc} u_{\alpha,n,3} - 3 \omega \psi_{\beta,n,3} \quad (6.70)
\]
\[
\frac{L_{n,2}}{2} \frac{di_{\beta,n,1}}{dt} + L_{n,0} \frac{di_{\beta,n,3}}{dt} = - 3 \omega \left( \frac{L_{n,2}}{2} i_{\alpha,n,1} + L_{n,0} i_{\alpha,n,3} \right) \\
- R_s i_{\beta,n,3} - v_{dc} u_{\beta,n,3} - 3 \omega \psi_{\alpha,n,3} \quad (6.71)
\]

Equations (6.68) to (6.71) can be written as:

\[
E \dot{x} = Fx + Gu + Gw \quad (6.72)
\]

where \( x = [i_{\alpha,n,1}, i_{\beta,n,1}, i_{\alpha,n,3}, i_{\beta,n,3}] \) represents the state vector. 
\( u = [u_{\alpha,n,1}, u_{\beta,n,1}, u_{\alpha,n,3}, u_{\beta,n,3}] \) and \( w = [\psi_{\beta,n,1}, \psi_{\alpha,n,1}, \psi_{\beta,n,3}, \psi_{\alpha,n,3}] \) represent the input vectors respectively.

A block diagram representation of this model combined with the DC-link voltage dynamics (6.31) is shown in figure 6.3.
Figure 6.3: PMG per-phase dynamic model extended to represent $k = 1$ and $k = 3$ dynamics, i.e., considering 1st to 3rd harmonics only (2nd harmonics negligible)
The matrices $E$, $F$, $G_u$ and $G_w$ are given by:

$$E = \begin{bmatrix}
(L_{n,0} + \frac{L_{n,2}}{2}) & 0 & \frac{L_{n,2}}{2} & 0 \\
0 & (L_{n,0} - \frac{L_{n,2}}{2}) & 0 & \frac{L_{n,2}}{2} \\
\frac{L_{n,2}}{2} & 0 & L_{n,0} & 0 \\
0 & \frac{L_{n,2}}{2} & 0 & L_{n,0}
\end{bmatrix}$$

(6.73)

$$F = \begin{bmatrix}
-\omega (L_{n,0} + \frac{L_{n,2}}{2}) & -R_s & -\omega \frac{L_{n,2}}{2} & 0 \\
-\omega (L_{n,0} + \frac{L_{n,2}}{2}) & -R_s & -\omega \frac{L_{n,2}}{2} & 0 \\
-3\omega \frac{L_{n,2}}{2} & 3\omega \frac{L_{n,2}}{2} & -R_s & 3\omega L_{n,0} \\
0 & 0 & -3\omega L_{n,0} & -R_s
\end{bmatrix}$$

(6.74)

$$G_u = \text{diag}(v_{dc}, -v_{dc}, v_{dc}, -v_{dc})$$

(6.75)

and

$$G_w = \text{diag}(-\omega, -\omega, -3\omega, -3\omega)$$

(6.76)

Note that $E$ is invertible for all $L_{n,0} > \left| \frac{L_{n,2}}{2} \right|$. Then the model (6.72) can be be written in canonical state-space form:

$$\dot{x} = Ax + Bu + Bw$$

(6.77)

where $A = E^{-1}F$, $B_u = E^{-1}G_u$ and $B_w = E^{-1}G_w$.

The mean DC current produced by the $n$th phase is given by:

$$i_{dc,n} = \frac{1}{4} \sum_{k=-3}^{3} \{ \bar{u}_{n,k} \bar{i}_{n,k} \}$$

(6.78)

$$i_{dc,n} = \frac{1}{2} (u_{\beta,n,1}i_{\beta,n,1} + u_{\beta,n,3}i_{\beta,n,3} - u_{\alpha,n,1}i_{\alpha,n,1} - u_{\alpha,n,3}i_{\alpha,n,3})$$

(6.79)

The total mean DC current generated by all functional phases $n \in S$ is given by:

$$i_{dc,tot} = \frac{1}{2} \sum_{n \in S} (u_{\beta,n,1}i_{\beta,n,1} + u_{\beta,n,3}i_{\beta,n,3} - u_{\alpha,n,1}i_{\alpha,n,1} - u_{\alpha,n,3}i_{\alpha,n,3})$$

(6.80)

The mean torque produced by the $n$th phase is given by:

$$T_{avg,n} = \frac{jK_{ip}}{8} \sum_{k=-3}^{3} \{ i_{n,k} (2K_{ip} \psi_{n,k} - L_{n,2}i_{n,k-2} + L_{n,2}i_{n,k+2}) \}$$

(6.81)
Expanding the summation term yields:

\[ T_{\text{avg},n} = \frac{j k_{\text{pp}}}{8} \left\{ 2(3\psi_{n,3}^* i_{n,3}^* - 3\psi_{n,3} i_{n,3} - \psi_{n,1}^* i_{n,1} + \psi_{n,1} i_{n,1}^*) \\
+ (2L_{n,2} i_{n,1}^* i_{n,3}^* - 2L_{n,2} i_{n,3} i_{n,1}^* + L_{n,2} i_{n,1}^* i_{n,1} - L_{n,2} i_{n,1} i_{n,1}^*) \right\} \] (6.82)

Substitution of (6.6) and (6.26) in (6.82) yields the mean torque production by the \( n \)th phase of the machine,

\[ T_{\text{avg},n} = -\frac{k_{\text{pp}}}{2} \left\{ (\psi_{\beta,n,1}^* i_{\alpha,n,1} + \psi_{\alpha,n,1} i_{\beta,n,1}) \\
+ 3 (\psi_{\beta,n,3}^* i_{\alpha,n,3} + \psi_{\alpha,n,3} i_{\beta,n,3}) \\
+ L_{n,2} (i_{\alpha,n,1} i_{\beta,n,1} + i_{\alpha,n,1} i_{\beta,n,3} - i_{\beta,n,1} i_{\alpha,n,3}) \right\} \] (6.83)

The real power delivered by the \( n \)th phase can be written as:

\[ P_n = \frac{v_{\text{dc}}}{2} (u_{\beta,n,1} i_{\beta,n,1} - u_{\alpha,n,1} i_{\alpha,n,1} + u_{\beta,n,3} i_{\beta,n,3} - u_{\alpha,n,3} i_{\alpha,n,3}) \] (6.84)

The \( n \)th phase reactive power associated with the fundamental and the notional reactive power associated with the 3rd harmonic are given by:

\[ Q_{n,1} = \frac{v_{\text{dc}}}{2} (u_{\beta,n,1} i_{\alpha,n,1} + u_{\alpha,n,1} i_{\beta,n,1}) \]
\[ Q_{n,3} = \frac{v_{\text{dc}}}{2} (u_{\beta,n,3} i_{\alpha,n,3} + u_{\alpha,n,3} i_{\beta,n,3}) \] (6.85)

### 6.8.2 PMG machine model Version 2 - neglected inductance variation and neglected back-EMF third harmonic

This model neglects the inductance variation and the third harmonic of the back-EMF. This is achieved by setting \( L_{n,2} = 0 \) and \( \vec{\psi}_{n,3} = 0 \) in model version 1. The simulation of this model in section 6.9 shows the extent of accuracy achieved by model version 1 due to the modelled inductance variation and the back-EMF third harmonic. In certain situations, where the fundamental magnitude of the back-EMF and the average inductance is only available from machine data, this model can be used to formulate the electrical dynamics of the system with a certain degree of accuracy. Output equations (6.79) to (6.85) remain the same for this model.
6.8.3 PMG machine model Version 3 - Fundamental model

This model considers only the fundamental dynamics. The effects of the third harmonic are completely neglected. This model can be derived by expanding the model (6.33) to (6.34) for the fundamental $k = 1$ only. The inductance variation and the third harmonic of the back-EMF as well as the third harmonic input $\vec{u}_{3,n}$ is considered zero. Certain SPM machines are designed with constant inductance and pure sinusoidal back-EMF. In such cases, this model can be used to describe electrical dynamics with a high degree of accuracy. Output equations (6.79) to (6.85) remain the same for this model.

6.9 Model validation

Figure 6.4: HIPWM input signals used in the simulation of the surface PM and interior PM machine models.

This section validates the mathematical model versions 1, 2 and 3 described in the preceding section. The two PM machines designed in chapter three are considered in this simulation. The FE model based switching simulation outputs are compared with the outputs of these three mathematical model versions.

As explained earlier, the mathematical model of a fault-tolerant machine can be considered in a modular per-phase representation. Likewise, the numerical validation of the model can also be performed considering the simulation of one phase of the machine. The analysis of one phase can then be extended to a machine with all phases in operation or with number of phases open-circuited.

In this simulation, the different models are issued with the same control signals shown in figure 6.4. A fixed speed of 2500 rpm and a fixed voltage of 540V is considered for this simulation. The machine parameters have been presented in chapter three and are not repeated here.
6.9.1 Simulation results for the SPM machine

Figure 6.5: Comparison of different output variables of the model versions 1, 2 and 3 with the FE model based simulation for the surface PM machine.
Figure 6.6: Comparison of different output variables of the model versions 1, 2 and 3 as error% with the FE model based simulation for the surface PM machine.
Figure 6.7: Instantaneous AC-side current, DC-side current and torque production waveforms at different operating points associated with the simulations of figures 6.5 and 6.6. Comparison of the model version-1 with the FEM based simulation.
Figure 6.8: Instantaneous AC-side current, DC-side current and torque production waveforms at different operating points associated with the simulations of figures 6.5 and 6.6. Comparison of the model version 2 with the FEM based simulation.
Figure 6.9: Instantaneous AC-side current, DC-side current and torque production waveforms at different operating points associated with the simulations of figures 6.5 and 6.6. Comparison of the model version-3 with the FEM based simulation.
6.9.2 Simulation results for the IPM machine

Figure 6.10: Comparison of different output variables of the model versions 1, 2 and 3 with the FE model based simulation for the interior PM machine.

- **IPM machine Model version-1** (fundamental and third harmonic model) simulation output
- **IPM machine Model version-2** ith forced \( L_{a_2} = 0 \) and \( \psi_{a_3} = \psi_{r_3} = 0 \) simulation output
- **IPM machine Model version-3** with forced \( L_{a_2} = 0, \psi_{a_3} = \psi_{r_3} = 0 \) and \( u_{a_3} = u_{r_3} = 0 \) simulation output
- **FE model based simulation output with switching converter model**
Figure 6.11: Comparison of different output variables of the model versions 1, 2 and 3 as error% with the FE model based simulation for the interior PM machine.
Figure 6.12: Instantaneous AC-side current, DC-side current and torque production waveforms at different operating points associated with the simulations of figures 6.10 and 6.11. Comparison of the model version-1 with the FEM based simulation.
Figure 6.13: Instantaneous AC-side current, DC-side current and torque production waveforms at different operating points associated with the simulations of figures 6.10 and 6.11. Comparison of the model version-2 with the FEM based simulation.
Figure 6.14: Instantaneous AC-side current, DC-side current and torque production waveforms at different operating points associated with the simulations of figures 6.10 and 6.11. Comparison of the model version-3 with the FEM based simulation.
6.10 Discussion

Figures 6.5 and 6.6 present the comparison of the surface PM machine output variables and figures 6.10 and 6.11 present the comparison of the interior PM machine output variables respectively. In these simulations, the output values of the three mathematical models are compared with the output values of the FE model based simulation.

In general, it can be seen from plots (a) and (b) of figures 6.5, 6.6, 6.10 and 6.11 that the fundamental real power and the fundamental current magnitudes are of high accuracy for all the three mathematical model versions and the error is within the range of 0% – 5%. However, for the interior PM machine model version with neglected inductance variation $L_{n,2}$ leads to an inaccuracy in the range of 0% – 5% error, while for the model with the inductance variation achieves an accuracy within 0% – 1% error of the same machine. For the surface PM machine, the effect of neglected inductance variation, third harmonic back-EMF component or the third harmonic PWM has no effect on the fundamental real or the reactive power accuracies.

In contrast, it can be seen from plot (e) of figures 6.10 and 6.11 that the fundamental reactive power transfer of the interior PM machine yields an error in the range of 50% to 75% for the models where the inductance variation is neglected. The first model of the same machine with the inductance variation achieves an accuracy for the fundamental reactive power transfer within the range of 0% – 8% error. A similar observation can be made for the reactive power value associated with the third harmonic.

As expected, the modelling of the inductance variation and the third harmonic has a significant effect on the mathematical modelling accuracy of the interior PM machine. The real power associated with the third harmonic can be mathematically modeled to an accuracy of 0% – 25% error for both the machines. However, the model with neglected inductance variation or third harmonic back-EMF yields real power and third harmonic current magnitude errors in the range of 90% – 110%. The effective difference between neglecting the control inputs is insignificant in this simulation due to the high inductances of the machines considered. The third harmonic real power component follows a similar pattern.

The model version 1 achieves an accuracy of 0% – 3% error for the DC current for both the surface and interior PM machines. The models with neglected inductance variation $L_{n,2}$ and third harmonic back-EMF yields an error in the range
of 0% – 8%. A similar observation can also be made for the torque production shown in plot (f) of figures 6.5, 6.6, 6.10 and 6.11.

Figures 6.7, 6.8 and 6.9 present the comparison of the three mathematical models with the FE model based simulation instantaneous waveforms for the surface PM machine. Figures 6.12, 6.13 and 6.14 present the same for the interior PM machine. The instantaneous waveforms corroborate the observations made with the averaged results. Model version-1 generates the most accurate waveform for both phase current and torque production compared with the FE model based simulation. Model version-2 neglect certain features while the model version-3 particularly replicates the fundamental variations of the associated variables.

6.11 Conclusion

In this chapter, the modelling of fault-tolerant PM machines consisting of an arbitrary number of phases has been presented. The modelling was performed considering PM machines interfaced with the parallel H-bridge topology, i.e., where each phase consist of an H-bridge module. Per-phase mathematical representation capable of modelling partially functional systems, such as machines with phase open-circuit conditions was developed. The modelling also considers non-sinusoidal flux-linkage distribution in terms of back-EMF harmonics and second order inductance variations due to rotor saliency. The input to the mathematical model is also specified in a general format of a HIPWM voltage source and hence controllability of multiple harmonic currents was established.

Three versions of simplifications of the mathematical model were defined with neglected harmonic components, neglected inductance variation and neglected third harmonic voltage input. These three model versions are compared with simulation outputs of a high-fidelity FE model based simulation. The importance of modelling the additional harmonics was identified. Based on the simulations performed, the fundamental current and real power error can be reduced by 4% to 5%, and the fundamental reactive power by 75% by incorporation of the third harmonic and the inductance variation respectively, for an interior PM machine which possess magnetic saliency and high back-EMF harmonic content. Hence, in conclusion it can be deduced that the developed mathematical model is of importance for the modelling of PM machines with magnetic saliency and non-sinusoidal flux-linkage distribution.
Chapter 7

Optimized Operation of Fault-Tolerant Permanent Magnet Machines in Motoring

This chapter investigates the optimized operation of fault-tolerant PM machines in motoring. The generalized harmonic model for fault-tolerant PM machines developed in chapter six is used here in the optimization process. A generic expression for torque ripple under faulted conditions is derived and the harmonic components of the torque ripple waveforms are quantified. The optimization problems of torque ripple minimization, reactive power minimization and copper loss minimization are defined. The possibility to improve performance via harmonic injection is considered. A multi-objective optimization is formulated, and the solution of these different optimization problems via numerical techniques is investigated.

This model based optimization technique is evaluated for the concentrated wound PM machine designs presented in the chapter three. A speed control technique for the operation of the PM machines with optimized performance under faulted conditions is presented. The performances of the two PM machines under different open-circuit fault conditions is evaluated by means of the high-fidelity finite element model based simulation technique presented in chapter three. The continuous time domain performance, and the frequency domain torque ripple and DC-link current ripple performances are evaluated and discussed.
CHAPTER 7. OPTIMIZED OPERATION OF PM MOTORS

7.1 Introduction

Permanent magnet machine technology is a major area of research in vehicular power and propulsion applications such as marine propulsion [155], aero-engine starter generators [134,135] and hybrid electric vehicle propulsion [11]. PM machines with fault-tolerant characteristics have presently gained much attention in applications that demand high reliability and high power density. Different constructions of fault-tolerant PM machines for such applications have been developed in the past [82,156–158] as explained in chapter three. These designs seek characteristics such as potentially near complete electrical, magnetic, physical and thermal isolation between each phase of the machine. The power electronic circuitry may also be designed to achieve high overall fault-tolerance. The parallel H-bridge topology considered in this study was explained in chapter six. This topology allows each phase of the machine to be operated independently under faulted conditions.

General techniques to control partially functional PM machines have not been developed to date. The main type of candidate PM machines for vehicular applications feature magneto-motive force (MMF) harmonics in the air gap [159]. This is due to the non-sinusoidal PM flux-linkage and significant inductance saliency due to flux path variation with rotor position. These effects induce additional harmonics in the total electromagnetic torque. Much of the research on the control of partially functional PM motors is limited to machines with sinusoidal back-EMF or with non-salient inductance characteristics. The authors of [160] and [161] consider the third harmonic currents and back-EMFs in their fault-tolerant control strategies, and the inductance saliency is assumed negligible. The authors of [11] consider the effect of inductance saliency via a direct-quadrate (dq) reference frame based controller and confine to a PM motor with sinusoidal back-EMF. The majority of research attempts the minimization of torque ripple and copper losses [161], while minimization of inverter reactive power has not received much attention. It is well known that high reactive power transfer at the inverter AC-side causes high oscillatory currents in the DC-link [135]. This would effectively increase inverter power losses and therefore it is essential to minimize the AC-side reactive power transfer. Above base-speed operation may naturally demand high reactive power for field weakening operation. This can be somewhat mitigated by application of switching strategies that provide high DC-link voltage utilization, e.g., space vector modulation, six-step mode of operation and third harmonic.
injection pulse width modulation (THIPWM) [162, 163]. While the model developed in chapter six provides the capability to consider a multiple harmonic injection strategy, this study is confined to the calculation of the optimum third harmonic injection for superior DC-link voltage utilization and the minimization of reactive power transfer.

The rationale of this chapter is to develop a control strategy that minimizes torque ripple, copper losses and inverter reactive power transfer under normal and faulted conditions. Multiple back-EMF harmonics and inductance saliency are considered, thereby providing applicability to a majority of fault-tolerant PM machines. The section 7.2 outlines the general harmonic model of a PM machine developed in chapter six configured for motoring mode. Section 7.3 presents the optimization of the THIPWM input for a surface permanent magnet (SPM) motor and an interior permanent magnet (IPM) motor. Section 7.4 develops the speed controller of the fault-tolerant drive. Section 7.6 evaluates the controller operation for both the machines under fault conditions via FEM model based dynamic simulations.

7.2 Permanent magnet motor harmonic model

7.2.1 Electrical system dynamics

The fault-tolerant PM machines and the application of the parallel H-bridge system was explained in chapter six. Due to the negligible mutual coupling, a per-phase continuous-time model of the $n^{th}$ phase of a PM machine was used in
The derivations presented in chapter six consider the currents out of the machine as positive in a power generation perspective. By replacing $i_n \rightarrow -i_n$ in the dynamic model (7.1) and (7.2), the corresponding output variables can be adjusted to confirm to a motoring mode convention where currents into the machine are considered positive. This results in,

$$\begin{align*}
v_n &= R_n i_n + \frac{d\psi_n}{dt} \quad (7.3) \\
\psi_n &= l_{nn} i_n + \psi_{pm,n} \quad (7.4)
\end{align*}$$

The same derivation of the harmonic modelling holds and yields a two sided geometric series of the format of

$$\sum_{k=-K-2}^{K+2} \hat{a}_{n,k} e^{-jk\theta} = 0 \quad (7.5)$$

where the coefficient $\hat{a}_{n,k}$ can be written as:

$$\begin{align*}
\hat{a}_{n,k} &= -L_{n,0} \left( -jk\omega \vec{i}_{n,k} + \frac{d\vec{i}_{n,k}}{dt} \right) - \frac{L_{n,2}}{2} \left( \frac{d\vec{i}_{n,k-2}}{dt} + \frac{d\vec{i}_{n,k+2}}{dt} \right) \\
&\quad + jk\omega \frac{L_{n,2}}{2} \left( \vec{i}_{n,k-2} + \vec{i}_{n,k+2} \right) - R_s \vec{i}_{n,k} + v_{dc} \vec{u}_{n,k} + j\omega k\vec{\psi}_k
\end{align*} \quad (7.6)$$

Assuming $\frac{d\omega}{d\theta} = \frac{1}{\omega} \frac{d\omega}{dt} \approx 0$ we note that $\frac{d\hat{a}_{n,k}}{d\theta} \approx 0$, and also $\frac{d^r \hat{a}_{n,k}}{d\theta^r} \approx 0$ for any $r > 0$. Then we can easily prove that this is true for all $\theta \in \mathbb{R}$ if and only if $a_{n,k}$ equate to zero. $a_{n,k} = 0$ represents the dynamic interaction of the $k^{th}$ harmonic with the $k-1$ and $k+1$ harmonic currents. Steady state relationship between these harmonic components can be obtained by setting the derivative terms to zero, i.e.

$$L_{n,0}jk\omega \vec{i}_{n,k} + jk\omega \frac{L_{n,2}}{2} \left( \vec{i}_{n,k-2} + \vec{i}_{n,k+2} \right) - R_s \vec{i}_{n,k} + v_{dc} \vec{u}_{n,k} + j\omega k\vec{\psi}_k = 0 \quad (7.7)$$
CHAPTER 7. OPTIMIZED OPERATION OF PM MOTORS

The $n^{th}$-phase real power transfer ($P_{n,k}$) and the reactive power transfer ($Q_{n,k}$) harmonic components at the inverter AC-side can be found by the dot products $\langle \vec{i}_{n,k}, \vec{u}_{n,k} \rangle$ and $\langle \vec{i}_{n,k}, j\vec{u}_{n,k} \rangle$. Evaluation of the dot product yields,

$$P_{n,k} = \frac{v_{dc}}{2} \text{Re}\left\{\vec{u}_{n,k}^{*} \vec{i}_{n,k}\right\} \quad (7.8)$$

$$Q_{n,k} = \frac{v_{dc}}{2} \text{Im}\left\{\vec{u}_{n,k}^{*} \vec{i}_{n,k}\right\} \quad (7.9)$$

In contrast with generation mode, power delivered into the machine is positive.

### 7.2.2 Electromagnetic torque production

The total electromagnetic torque per-phase can be written in the form $T_{e,n} = T_{pm,n} + T_{rel,n}$, where $T_{pm,n}$ and $T_{rel,n}$ represent the PM torque and the reluctance torque components of the $n^{th}$ phase. From principle of co-energy, these instantaneous torque components at an electrical angle of $\theta = (\theta_e + \gamma_n)$ can be written as,

$$T_{pm,n} (\theta) = \frac{jk_{pp}}{4} \sum_{l=-K_1}^{K_1} \sum_{k=-K}^{K} \left\{ l \vec{\psi}_{l} \vec{i}_{n,k} e^{-j(k+l)\theta} \right\} \quad (7.10)$$

$$T_{rel,n} (\theta) = -\frac{jk_{pp} L_{n,2}}{8} \sum_{l=-K_1}^{K_1} \sum_{k=-K}^{K} \left( \vec{i}_{n,l} \vec{i}_{n,k} e^{-j(k+l-2)\theta} - \vec{i}_{n,l} \vec{i}_{n,k} e^{-j(k+l+2)\theta} \right) \quad (7.11)$$

Equations (7.10) and (7.11) correspond with the electromagnetic torque equations derived in chapter six. However, it should be noted that the expression for the torque production due to the permanent magnet yields a positive sign as the convention adopted in this chapter is such that currents into the machine are taken as positive.

The average torque produced by the $n^{th}$ phase ($T_{avg,n}$), can be obtained by considering the terms consisting of $e^{j(k+l-2)\theta} = 1$, $e^{-j(k+l+2)\theta} = 1$ and $e^{j(k+l)\theta} = 1$ when $k + l - 2 = 0$, $k + l + 2 = 0$ and $k + l = 0$ of the series (7.10) and (7.11). Assuming $K \leq K_1$, this can be written as:

$$T_{avg,n} = \frac{-jk_{pp}}{8} \sum_{k=-K_1}^{K_1} \left\{ i_k \left( -2k\psi_{\alpha,n,k} + L_{nn,2} i_k^{*} (n,k-2) - L_{nn,2} i_k^{*} (n,k+2) \right) \right\} \quad (7.12)$$
The \( r \)th order permanent magnet torque ripple component \( T_{pm,n,r}(\theta) \) for \((r = 2, 4, \ldots, 2K)\) produced by the \( n \)th phase can be written in general by summing the terms of the series (7.10) when \((k + l) = r, (k + l) = -r \) as,

\[
T_{pm,n,r}(\theta) = \frac{j k_{pp}}{4} \sum_{k=-2K}^{2K} \left\{ (r - k) \psi_{r-k} i_k e^{-j r \theta} - (k + r) \psi_{k+r}^* i_k e^{j r \theta} \right\}
\]

(7.13) can be simplified as,

\[
T_{pm,n,r}(\theta) = -\frac{k_{pp}}{2} \text{Im} \left\{ \sum_{k=-2K}^{2K} (r - k) \psi_{r-k} i_k e^{-j r \theta} \right\}
\]

The \( r \)th order reluctance torque ripple component \( T_{rel,n,r}(\theta) \) for \((r = 2, 4, \ldots, 2K)\) produced by the \( n \)th phase can be written in general by summing the terms of the series (7.11) when \((k + l - 2) = r, (k + l - 2) = -r, (k + l + 2) = r, (k + l + 2) = -r \).

This yields,

\[
T_{rel,n,r}(\theta) = -\frac{j k_{pp} L_{nn,2}}{8} \sum_{k=-2K-2}^{2K+2} \left\{ \left( i_{n,k} i_{n,r-k+2} - i_{n,k} i_{n,r-k-2} \right) e^{-j r \theta} \right.
\]

\[
+ \left( i_{n,k} i_{n,-r-k+2} - i_{n,k} i_{n,-r-k-2} \right) e^{j r \theta} \right\}
\]

(7.15) can be simplified as,

\[
T_{rel,n,r}(\theta) = \frac{k_{pp} L_{nn,2}}{4} \text{Im} \left\{ \sum_{k=-2K-2}^{2K+2} \left( i_{n,k} i_{n,r-k+2} - i_{n,k} i_{n,r-k-2} \right) e^{-j r \theta} \right\}
\]

(7.16)

Summation of (7.14) and (7.16) yields the total \( r \)th order torque ripple component of the \( n \)th phase as,

\[
T_{n,r}(\theta) = \frac{k_{pp}}{4} \text{Im} \left\{ e^{-j r \theta} \sum_{k=-2K-2}^{2K+2} \left( -2 (r - k) \psi_{n,n,(r-k)} i_{n,k} \right.ight.
\]

\[
+ L_{nn,2} i_{n,(r+2-k)} i_k - L_{nn,2} i_{n,(r-2-k)} i_n \left) \right\}
\]

(7.17)

### 7.2.3 Electromagnetic torque ripple model

For an \( N \)-phase machine with \( N_f \) functional phases, the total average torque production is the summation of the average torque components contributed by
each phase. It follows from (7.17) that for a balanced machine, the summation of all \( r^{th} \) order torque ripple components is zero if \( \sum_{n=1}^{N} e^{r \gamma n} = 0 \) is true. However, higher torque ripple will be prevalent if the machine is partially operational, i.e., with a reduced number of functional phases.

Let the functional state of any \( n^{th} \) phase be represented by variable \( x_n \), where

\[
x_n = \begin{cases} 
1 & \text{if } n^{th} \text{ phase is functional} \\
0 & \text{if } n^{th} \text{ phase is open circuit}
\end{cases}
\]  

(7.18)

By extension of (7.17) to \( n = 1, 2, \ldots, N \), total \( r^{th} \) order torque ripple component can be represented as,

\[
T_r(\theta) = k_{pp} \frac{4}{4} \text{Im} \left\{ e^{-jr\theta} \sum_{n=1}^{N} x_n \hat{T}_{n,r} \right\}
\]  

(7.19)

where

\[
\hat{T}_{n,r} = k_{pp} e^{-jr\gamma n} \sum_{k=-2K-2}^{2K+2} (-2(r-k)\psi_{r-k}i_k + L_{n,2} (i_{n,k}i_{n,r-k+2} - i_{n,k}i_{n,r-k-2}))
\]  

(7.20)

Therefore, the magnitude and phase of the total \( r^{th} \) order torque ripple waveform produced by the motor can be characterized by the vector,

\[
\hat{T}_{\text{char},r} = \sum_{n=1}^{N} x_n \hat{T}_{n,r}
\]  

(7.21)

### 7.3 Optimization for fault-tolerant control

As explained in section 7.1, smooth electromagnetic torque production, reduced copper losses and low reactive power transfer are preferred in the motor drive system under both normal and faulted operation conditions. Low reactive power transfer can be achieved by maintaining (7.9) at zero during normal operation, while commanding a minimized magnitude of reactive power under field weakening and field strengthening modes of operation. Similarly the copper losses can be reduced by maintaining the magnitude of \( i_{n,k} \), (for all \( k \) and \( n \)) at the lowest possible value under normal operating conditions. However, any
requirement for torque ripple minimization under faulted operation dictates the optimal variation of the control signals.

This chapter investigates the optimization of PM motor operation with different objectives under normal operation and under different open-circuit fault conditions. The mathematical modelling and the electromagnetic torque ripple model presented earlier is utilized in the optimization presented in the following sections.

The generalized optimization is then applied to the fault-tolerant control design of the five phase SPM and IPM machines developed in chapter three. Normal operation and three cases of open-circuit faulted operation are considered for both the machines. The SPM and IPM machine operation under normal and faulted conditions is then simulated with high-fidelity FE model based switching simulations.

7.3.1 Operation with minimized torque ripple

The objective of torque ripple minimization for a given torque demand can be formulated as,

\[
\min_{\{u_{n,k}\}} \sum_{r=2}^{2K+2} \lambda_r \left( \frac{\text{Re} \left( \hat{T}_{\text{char},r} \right)}{T_{\text{rated}}} \right)^2 + \lambda_r \left( \frac{\text{Im} \left( \hat{T}_{\text{char},r} \right)}{T_{\text{rated}}} \right)^2
\]

Subject to

\[
\Gamma^* T_{\text{rated}} - \sum_{n=1}^{N_f} T_{\text{avg},n} = 0 \quad \text{for all } r
\]

and \( d_n(t) \leq 1 \) for all \( n \) and \( t \)

Here, \( T_{\text{rated}} \) is the rated torque of the machine and \( \Gamma^* \) represents the per-unit torque demanded by an outer-loop. The factor \( \lambda_r \) provides prioritization of certain torque ripple components. Higher \( \lambda_r \) value will impose a higher weight on the minimization of specific torque ripple components.

This form of optimization is solely for the purpose of torque ripple minimization and does not consider copper-loss minimization or reactive power minimization. The quality of torque production is given high priority and it is assumed that the DC-link capacitance is sufficiently high in order to assume the effect of
DC-link current ripple on the DC-link voltage negligible.

### 7.3.2 Operation with minimized reactive power transfer

The objective of reactive power transfer minimization for a given torque demand can be formulated as,

$$\min \left\{ x_n \left( \frac{Q_{n,1}}{S_{\text{base}}} \right)^2 \right\} \quad (7.25)$$

Subject to:

$$\Gamma^* T_{\text{rated}} - \sum_{n=1}^{N_f} T_{\text{avg},n} = 0 \text{ for all } r \quad (7.26)$$

and $$d_n(t) \leq 1 \text{ for all } n \text{ and } t \quad (7.27)$$

Here, $$S_{\text{base}}$$ and $$I_{\text{base}}$$ represents the kVA rating and the base current values of the considered PM motor. Variable $$Q_{n,1}$$ is the fundamental reactive power transfer from the $$n^{th}$$ phase. It is worth noting that only the fundamental reactive power transfer is consider here, as this is the major contributor to higher current interchange with the DC-link. The effects of $$Q_{n,r}, r > 1$$ are negligible and are of notional expressions of harmonic reactive power components. The parameter $$\kappa_k$$ enables a trade-off between $$Q_{n,1}$$ and the current magnitude.

This form of optimization prioritizes the electrical system efficiency while torque ripple minimization is not attempted.

### 7.3.3 Multi-objective optimization

Many motor and drives design applications encounter the requirement to maintain torque ripple below a certain boundary. This can be incorporated as an additional objective into the minimization of reactive power and copper loss problem, which leads to a multi-objective optimization of the form of,

$$\min \left\{ x_n \left( \frac{Q_{n,1}}{S_{\text{base}}} \right)^2 + \sum_{k=1}^{K} \kappa_k^2 \left( \frac{i_{n,k}^k i_{n,k}^*}{I_{\text{base}}^2} \right) \right\} \quad (7.28)$$
Subject to:

\[
\Gamma^* T_{\text{rated}} - \sum_{n=1}^{N_f} T_{\text{avg},n} = 0 \quad (7.29)
\]

\[
\left( \frac{\text{Re}(\hat{T}_{\text{char},r})}{T_{\text{rated}}} \right)^2 + \left( \frac{\text{Im}(\hat{T}_{\text{char},r})}{T_{\text{rated}}} \right)^2 < \chi_r^2 \quad \text{for all } r \quad (7.30)
\]

and \(d_n(t) \leq 1\) for all \(n\) and \(t\) \( (7.31)\)

Here, \(S_{\text{base}}\) and \(I_{\text{base}}\) represent the kVA rating and the base current values of the considered PM motor. Variable \(Q_{n,1}\) is the fundamental reactive power transfer from the \(n^{th}\) phase. It is worth noting that only the fundamental reactive power transfer is considered here as this is the major contributor to large current interchange with the DC-link. The effects of \(Q_{n,r}, r > 1\) are negligible and are notional expressions of harmonic reactive power components.

The parameter \(\kappa_k\) enables a trade-off between \(Q_{n,1}\) and the current magnitude.

### 7.3.4 Solution of the optimization problems

Standard optimization algorithms such as sequential quadratic programming (SQP) [164] can be readily applied for the solution of these optimization problems utilizing the model equations (7.7), (7.9), (7.12) and (7.17). The reader is referred to [164] for further details and the detailed technique of the solution is not considered here. Section 7.5 presents examples of the application of SQP based optimization for the solution and design of the control strategy for the two PM machines considered in this study.
The speed control of fault-tolerant PM machines can be formulated in a cascaded loop feedback / feedforward structure. The speed loop is implemented as a proportional integral (PI) controller, where the average torque demand is calculated by,

\[ T^* = k_{p,s} (\omega_r^* - \omega_r) + k_{i,s} \int (\omega_r^* - \omega_r) dt \]  

(7.32)

The optimal phase voltage harmonic components are obtained by a multidimensional lookup table that is generated by the optimization explained in section 7.3. The expected current waveform harmonic components can be calculated online by solution of equation (7.7). The PWM switching is then calculated by a feedback/feedforward current control law,

\[ d(t) = d(t)' + k_{p,c} (i_n^* - i_n) + k_{i,c} \int (i_n^* - i_n) dt \]  

(7.33)

where \(d(t)\) is the PWM switching waveform generated by the model (7.7). Under ideal conditions where the model exactly matches the PM motor, \(d(t) = d(t)'\) will result. However, effects of saturation, thermal variation of parameters and other unmodelled dynamics may result in a discrepancy between the model (7.7) and the actual PM motor dynamics. The variation of the phase current from the required waveform can be alleviated by incorporation of PI current control in the form of (7.7). The block diagram of figure 7.2 shows the interconnection of a PM
motor phase with a H bridge module and the controller of that phase.

### 7.4.1 Current controller gain selection

The current controller gains are selected considering the dynamics,

\[ v_n = R_n i_n + L_{n,0} \frac{di_n}{dt} + e_b(t) \]  

(7.34)

where \( e_b(t) \) incorporates the back-EMF produced by the permanent magnet flux and the reluctance component. The switch-average voltage generated by the PWM inverter is given by,

\[ v_n = v_{dc}d(t) \]  

(7.35)

Therefore, by substitution of (7.33) and (7.35) in (7.34) yields:

\[ R_n i_n + L_{n,0} \frac{di_n}{dt} + e_b(t) = v_{dc}d'(t) + v_{dc}k_{p,c}(i^*_n - i_n) + v_{dc}k_{p,c} \int (i^*_n - i_n) dt \]  

(7.36)

It follows from the model developed in chapter six that under steady state conditions, i.e, when \( \frac{di_n}{dt} = 0 \), the voltage demanded by the mathematical model satisfy,

\[ v_{dc}d'(t) = R_n i_n + e_b(t) \]  

(7.37)

Substitution of (7.37) in (7.36) and simplification yields:

\[ L_{n,0} \frac{di_n}{dt} = v_{dc}k_{p,c}(i^*_n - i_n) + v_{dc}k_{p,c} \int (i^*_n - i_n) dt \]  

(7.38)

Assuming constant DC-link voltage, the resultant transfer function for the current controller is then given by:

\[ \frac{I_n(s)}{I_n^*(s)} = \frac{v_{dc}k_{p,c}s + v_{dc}k_{p,c}}{L_{n,0}s^2 + v_{dc}k_{p,c}s + v_{dc}k_{p,c}} \]  

(7.39)
Casting (7.39) into the standard second order format (and assuming the coefficient of the \( s \) term in the numerator is small) yields,

\[
\frac{I_n(s)}{I^*_n(s)} = \frac{\frac{v_{dc}k_{p,c}}{L_{n,0}}s + \frac{v_{dc}k_{p,c}}{L_{n,0}}}{s^2 + \frac{v_{dc}k_{p,c}}{L_{n,0}}s + \frac{v_{dc}k_{p,c}}{L_{n,0}}}
\]  

(7.40)

where the proportional and integral gains can be written in terms of damping factor \( \varsigma_c \) and un-damped natural frequency \( \omega_{n,c} \):

\[
k_{p,c} = \frac{2\varsigma_c\omega_{n,c}L_{n,0}}{v_{dc}}
\]  

(7.41)

\[
k_{p,c} = \frac{L_{n,0}\omega^2_{n,c}}{v_{dc}}
\]  

(7.42)

7.4.2 Speed controller gain selection

The instantaneous torque production of the machine can be rewritten in the form of:

\[
T_e(\theta) = T_{\text{avg,tot}} + T_{\text{rip}}(\theta)
\]  

(7.43)

where \( T_{\text{avg,tot}} \) and \( T_{\text{rip}}(\theta) \) are given by summation of (7.12) for all the phases in operation, and (7.12) for all the ripple components considered. Assuming that the commanded average torque is produced without lag i.e, \( T^* = T_{\text{avg,tot}} \) the shaft dynamics can be written as,

\[
T^* + T_{\text{rip}}(\theta) - T_{\text{load}} = J_m \frac{d\omega_r}{dt} + B_m \omega_r
\]  

(7.44)

Substitution of (7.32) in (7.44) yields:

\[
J_m \frac{d\omega_r}{dt} = -B_m \omega_r + k_{p,s} (\omega^*_r - \omega_r) + k_{i,s} \int (\omega^*_r - \omega_r) dt + w(t)
\]  

(7.45)

where \( \omega^*_r \) is the speed command, \( w(t) \) represents the torque ripple and load torque as a disturbance. The corresponding input-output transfer function is given by,

\[
\frac{\omega_r(s)}{\omega^*_r(s)} = \frac{k_{p,s}s + k_{i,s}}{J_ms^2 + (B_m + k_{p,s})s + k_{i,s}}
\]  

(7.46)
CHAPTER 7. OPTIMIZED OPERATION OF PM MOTORS

Casting (7.46) into the standard second order format (again assuming the coefficient of the $s$ term in the numerator is small) yields,

$$
\frac{\omega_r(s)}{\omega^*_r(s)} = \frac{\frac{k_{p,s}}{J_m}s + \frac{k_{i,s}}{J_m}}{s^2 + \frac{(B_m + k_{p,s})}{J_m}s + \frac{k_{i,s}}{J_m}} \tag{7.47}
$$

where the proportional and integral gains can be written in terms of damping factor $\varsigma_s$ and un-damped natural frequency $\omega_{n,s}$:

$$
k_{p,s} = 2\varsigma_s \omega_{n,s}J_m - B_m \tag{7.48}
$$

$$
k_{i,s} = J_m \omega_{n,s}^2 \tag{7.49}
$$

Transfer function (7.47) can be used to calculate the proportional and integral gains for the required speed command response. Although the poles of the above transfer function may be placed on the stable left half s-plane, this does not guarantee stability due to the nonlinear relationship between the operating point and the torque ripple component $T_{rip,r}(\theta)$. However, stable operation can be obtained by placing the poles at undamped natural frequencies below the minimum torque ripple frequency. Thereby maintaining the speed ripple caused by the torque ripple within bounded limits. This can also be analysed by means of the sensitivity transfer function:

$$
\frac{\omega_r(s)}{w(s)} = \left( \frac{s}{J_m s^2 + (B_m + k_{p,s}) s + k_{i,s}} \right) \tag{7.50}
$$

Stable operation can be expected by designing the controller sensitivity to satisfy $\left| \frac{\omega_r(s)}{w(s)} \right| f_r \ll 1$ at all ripple frequencies $f_r$. 
CHAPTER 7. OPTIMIZED OPERATION OF PM MOTORS

7.5 Case study

7.5.1 Optimization

The case study presented here considers the surface PM machine and the interior PM machine designs presented in Chapter three. The machine parameters from chapter three are given in table 7.1 below for both the machines.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>SPM Motor Parameters</th>
<th>IPM Motor Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>kVA rating</td>
<td>125kVA</td>
<td>125kVA</td>
</tr>
<tr>
<td>kVA rating per-phase</td>
<td>25kVA</td>
<td>25kVA</td>
</tr>
<tr>
<td>Base phase current</td>
<td>66.75A</td>
<td>66.75A</td>
</tr>
<tr>
<td>Rated torque</td>
<td>397.88Nm</td>
<td>397.88Nm</td>
</tr>
<tr>
<td>PM flux</td>
<td>−0.3249Vs</td>
<td>−0.2810Vs</td>
</tr>
<tr>
<td>Maximum speed</td>
<td>3000rpm</td>
<td>3000rpm</td>
</tr>
<tr>
<td>$R_s$</td>
<td>0.01Ω</td>
<td>0.01Ω</td>
</tr>
<tr>
<td>$L_{nn,0}$</td>
<td>1.3976mH</td>
<td>2.7234mH</td>
</tr>
<tr>
<td>$L_{nn,2}$</td>
<td>0mH</td>
<td>−0.5561mH</td>
</tr>
<tr>
<td>pole pair number</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>total number of phases</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Inertia $J_m$</td>
<td>0.05kgm²</td>
<td>0.05kgm²</td>
</tr>
<tr>
<td>Frictional coefficient</td>
<td>0.01Ns</td>
<td>0.01Ns</td>
</tr>
</tbody>
</table>

The three forms of optimizations explained earlier are performed for the two machines, i.e. (a) Minimized torque ripple, (b) Minimized reactive power and (c) Multi-objective optimization, for the following fault conditions:

i. All phases in operation.
ii. Phase a open-circuited.
iii. Phases a and b open-circuited.
iv. Phases a and c open-circuited.

The parameters associated with these three optimization are selected as, $\lambda_r = 1$ for $r = 2, 4, 6, 8$, $\kappa_k^2 = 0.1$ for $k = 1, 3$, and $\chi_k^2 = 0.00001$ for $r = 2, 4, 6, 8$. The optimization is performed considering the fundamental and the third harmonic and for a speed range 750rpm to a maximum speed 3750rpm. An average torque production from -1pu to +1pu is considered. The variation of $\vec{u}_{n,1}$ and $\vec{u}_{n,3}$ for all the five phases, i.e., $n = 1, 2, ..., 5$ are obtained for the four different fault conditions and the three different optimization criteria.
Figure 7.3: Sample variation of HIPWM input signals for minimized torque ripple operation at 2250 rpm. (a) - (d) healthy machine, (e) - (h) phase a faulted, (i) - (l) phases a and b faulted, and (m) - (p) phases a and c faulted.

Figure 7.3 shows a sample variation of HIPWM input signals for minimized torque ripple operation of the interior PM motor at 2250 rpm, for the four different
fault conditions. Similar variations are obtained for the surface PM machine, and at different speeds and optimization criteria. These are not discussed here.

### 7.5.2 Control design

For the machine parameters given in table 7.1, the current controller proportional and integral gains are calculated using (7.41) and (7.42) for the damping factor and un-damped natural frequency shown in table 7.2. The resultant gains are also shown in table 7.2.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>SPM Motor Parameters</th>
<th>IPM Motor Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{dc}$</td>
<td>nominal DC-link voltage 540V</td>
<td>540V</td>
</tr>
<tr>
<td>$\omega_{n,c}$</td>
<td>6283.2 rad/s</td>
<td>6283.2 rad/s</td>
</tr>
<tr>
<td>$\varsigma_c$</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>$k_{p,c}$</td>
<td>0.0228</td>
<td>0.0444</td>
</tr>
<tr>
<td>$k_{i,c}$</td>
<td>102.1760</td>
<td>199.1028</td>
</tr>
</tbody>
</table>

Similarly the speed controller proportional and integral gains are calculated using (7.48) and (7.49) and the corresponding parameters are shown in table 7.3.

![Bode Diagram](image)

Figure 7.4: Bode plot of sensitivity function of the speed loop
Table 7.3: Current controller parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>SPM Motor and IPM Motor Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_n,s$</td>
<td>125.6637 rad/s</td>
</tr>
<tr>
<td>$\varsigma_s$</td>
<td>1</td>
</tr>
<tr>
<td>$k_{p,s}$</td>
<td>12.5564</td>
</tr>
<tr>
<td>$k_{i,s}$</td>
<td>789.5684</td>
</tr>
</tbody>
</table>

Figure 7.4 shows the bode plot of the speed loop sensitivity function. For the operating speed ranges from 750 rpm to 3750 rpm, the lowest theoretical ripple occurs at 150Hz or 942.47 rad/s, i.e., the second order harmonic. The sensitivity function at this frequency is approximately 0.08 ($\ll 1$). The sensitivity of higher order harmonics is even lower. Thus stable operation can be expected.

### 7.6 Simulation results

Simulations were performed for the surface PM motor and the interior PM motor for the different optimization criteria and fault conditions specified earlier. In addition to this, another set of simulations where performed for the faulted machine operation with the control signals derived for a healthy machine, thereby providing a comparison of the effectiveness of the optimization. The full simulation results for the surface PM motor and the interior PM motor are attached in appendices M and N. Tables M.1 and N.1 outline the set of simulations performed and the corresponding figures in the appendices. Frequency domain characteristics were obtained by fast Fourier transform (FFT) of the torque ripple and DC-link current waveforms generated by the simulations. The FFT results for the surface PM motor and the interior PM motor are attached in the appendices O and P respectively.

The frequency components of interest associated with the torque ripple at five different speeds is shown in table 7.4. It should be noted that the 20 slot, 6 pole-pair machines considered here also generate a tenth order torque harmonic. Since this optimization considers only the electromagnetic torque production, the cogging torque component is subtracted from the results and thereby the FFT depicts only the effect of electromagnetic torque production. The optimization only considers the fundamental and the third harmonic currents and the controllability of the tenth harmonic torque component is not considered in this study, as
this would require a further extension of the mathematical model to the fifth harmonic. In certain instances, such as the one shown in figure 7.10 (a) a significant tenth order torque ripple component is seen. Figure 7.10 shows a the frequency domain analysis performed on the IPM motor torque production and the DC-link current ripple under different fault conditions and different optimizations and operation at 2250 rpm and 140 Nm loading.

Table 7.4: Important frequency components at the speeds considered in the simulation

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>Speeds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>750rpm</td>
</tr>
<tr>
<td>2</td>
<td>150Hz</td>
</tr>
<tr>
<td>4</td>
<td>300Hz</td>
</tr>
<tr>
<td>6</td>
<td>450Hz</td>
</tr>
<tr>
<td>8</td>
<td>600Hz</td>
</tr>
<tr>
<td>10</td>
<td>750Hz</td>
</tr>
</tbody>
</table>

7.6.1 Healthy machine operation

The three sets of simulations for the surface PM machine operation under the three different optimization criteria and the same for the interior PM machine are shown in appendices M and N in figures M.1, M.8, and M.12, and figures N.1, N.8 and N.12 respectively. A sample result is shown in figure 7.5 for the operation of a healthy interior PM machine with minimized torque ripple optimization.

It can be seen from the set of results for healthy operation, the real power and reactive power are all balanced in all the phases, i.e. for example in figures 7.5 (f) and (g) the curves for the individual phase quantities overlap. In general the torque ripple is minimal during balanced machine operation and this is also confirmed by the FFT analysis in appendix P. A typical example of the torque ripple FFT under such conditions is shown in figure 7.10 (a).
Figure 7.5: Variables for the interior PM machine operation with minimized torque ripple optimization with all phases healthy. (a) speed, (b) torque, (c) DC-side current, (d) total real power, (e) total reactive power, (f) phase real power and (g) phase reactive power.
Figure 7.6: Comparison of reactive power transfer for the surface PM machine operation under different optimization criteria and four different fault conditions, (a) all phases healthy, (b) phase a faulted, (c) phases a and b faulted and (d) phases a and c faulted.

Figures 7.6 (a) and 7.7 (a) shows the comparison of reactive power transfer for the surface PM machine and the interior PM machine operation under different optimization criteria under healthy conditions. For the surface PM motor, the reactive power is zero throughout 1500 rpm and 2250 rpm and is close to zero for the interior PM motor for the operation with minimized reactive power transfer (blue line in figures 7.6 (a) and 7.7 (a)). A transient demand of reactive power is delivered in 750 rpm operation. In contrast a high steady state negative reactive power is delivered to the machine for field weakening operation when the speed exceeds 3000 rpm. This is also observed in figure 7.5 (e). The magnitude of reactive power delivered during 750 to 3000 rpm with minimized reactive power
CHAPTER 7. OPTIMIZED OPERATION OF PM MOTORS

Reactive power of interior PM machine operation under different fault and optimization criteria

Figure 7.7: Comparison of reactive power transfer for the interior PM machine operation under different optimization criteria and four different fault conditions, (a) all phases healthy, (b) phase a faulted, (c) phases a and b faulted and (d) phases a and c faulted.

operation is always lower than that of other optimization criteria.

7.6.2 One phase faulted operation

Figure 7.8 shows the operation of a one phase faulted interior PM machine with control signals optimized for minimized torque ripple. Similar results for operation with the other optimizations for the surface PM machine and the interior PM machine are attached in appendices M and N in figures M.2, M.5, M.9 and M.13, and figures N.2, N.5, N.9 and N.13 respectively.
Results of the interior PM motor operation with four healthy phases (phase a open-circuited)
Control signal optimized for minimized torque ripple for the four healthy phases

Figure 7.8: Variables for the interior PM machine operation with minimized torque ripple optimization with phase a open-circuited. (a) speed, (b) torque, (c) DC-side current, (d) total real power, (e) total reactive power, (f) phase real power and (g) phase reactive power.
It can be seen from figure 7.8 that the operation is unbalanced. The real power and reactive power consumed by each phase are different from one another. This unbalanced loading of the remaining four phases occur in such a manner that it produces a torque which attempts to minimize the major second, fourth, sixth and the eighth order torque ripple components arising due to the unavailability of one phase. The multi-objective optimization achieves similar range of torque ripple reduction, and a lower reactive power consumption to that under operation with torque ripple minimization shown in figures 7.6 (b) and 7.7 (b). The multi-objective optimization thus offers the capability to trade-off reactive power transfer for torque ripple reduction under this faulted condition.

According to the sample results shown in figure 7.10 (b), the lines with the red stems representing the minimized torque ripple, are at a lower value compared with the other stem lines at the major frequency components of 450Hz, 900Hz, 1350Hz, 1800Hz and 2250Hz. A higher torque ripple is observed in the blue stem lines, i.e. during operation with minimized reactive power transfer and when the machine is operated without consideration for the fault condition (green stem lines). Torque ripple reduction is achieved for majority of the cases of the surface PM motor and the interior PM motor. However, certain instances can be seen at high speed and high load conditions where the torque ripple reduction is not achieved in the interior PM machine, while torque ripple reduction is successfully achieved in the surface PM machine. This phenomenon seen particularly in the interior PM operation, can be attributed to the high level of saturation of the machine. At higher loads and speeds the interior PM machine saturates and as a result the mathematical model and the optimization become inaccurate. As a result the machine torque ripple reduction control does not reduce the ripple as expected. In contrast, the surface PM machine saturation occurs at comparatively higher speeds and higher load conditions, and as a result the torque ripple reduction is successful for majority of the cases.
Figure 7.9: Variables for the interior PM machine operation with minimized torque ripple optimization with phases a and b open-circuited. (a) speed, (b) torque, (c) DC-side current, (d) total real power, (e) total reactive power, (f) phase real power and (g) phase reactive power.
7.6.3 Two phase faulted operation

The operation of the surface PM machine and the interior PM machine with two adjacent phase faults and two non-adjacent phased faults is considered in this study. The simulation results for the three different optimizations and without optimization are given in the appendices M and M in figures M.3, M.4, M.6, M.7, M.10, M.11, M.14 and M.15, and N.3, N.4, N.6, N.7, N.10, N.11, N.14 and N.15 respectively.

A sample result is given in figure 7.9 for the operation of the interior PM motor with two adjacent phases faulted. Similar to the case with one faulted phase operation with minimized torque ripple, here the real and reactive power levels of the three healthy phases are different, and work towards minimizing torque ripple. The torque ripple minimization is successfully achieved for the speed of 2250 rpm and load of 140Nm shown in figures 7.10 (c) and (d). The highest torque ripple is achieved during the operation with minimized reactive power transfer. Analysis of the FFTs at the other operating points confirm excellent torque ripple minimization during two phase faulted operation for both the interior PM and surface PM machines.

Figures 7.6 (c) and (d), and figures 7.7 (c) and (d) compare the reactive power transfer for the surface PM and the interior PM motors. This show that the minimized reactive power transfer operation yields lower reactive power values than that of operation without consideration of the fault. During higher loads, reactive power consumed by the system with control assuming a healthy machine, i.e., the green curves of figures 7.6 (c), (d) and 7.7 (c), (d) seem to achieve a lower value than the torque ripple minimization and the multi-objective optimization (red and black lines). However, careful consideration will reveal that at higher loads, the non-optimized control signal saturates and results in an inability to produce the required torque for acceleration, resulting in reduction of speed. This can also be seen in the surface PM machine operation. The optimized operation takes into account the fault and the reduction of torque, and as a result the commanded torque is delivered by the system.
Minimized torque ripple operation, considering the corresponding fault condition
Minimized torque ripple operation, assuming a healthy machine
Minimized reactive power operation, considering the corresponding fault condition
Multiobjective optimized operation, considering the corresponding fault condition

Figure 7.10: Example FFT analysis: interior PM motor operation at 2250rpm and at 140Nm for four different fault conditions. (a), (b), (c), (d) torque ripple FFT and (e), (f), (g), (h) DC-link current ripple FFT.
7.6.4 Observations on DC-link current ripple

In general the FFT analysis of the current ripple for the surface PM machine weakly follows the same trend as the torque ripple. That is, in many instances where the machine is operated with the control associated with reactive power minimization, a high second order torque ripple component is present and the DC-link voltage ripple also contains a ripple component of the same frequency. In contrast, when the motor is operated with torque ripple minimization or the multi-objective optimization, the same torque ripple component is reduced in the DC-link. However, for the interior PM machine this trend does not occur.

On the contrary, the figures 7.8 (g) and (h) show that the reductions in torque ripple generate a higher DC-link voltage ripple. In addition, lower order harmonic components are also seen in the DC-link voltage ripple. These can be attributed to the current controller, which attempts to track the current waveform generated by the mathematical model. As the mathematical model does not replicate saturation effects, the current controller imposes an additional voltage on the phase to force the current to track the required current waveform, thus generating lower order harmonic component in the DC-link current. This phenomena is particularly observed in conditions when reactive power in not minimized, i.e., the current magnitude is high, such as the instance shown in figures 7.8 (g) and (h).

7.7 Conclusion

In this chapter, the control of PM machines with different optimization criteria has been presented. The key optimizations considered are namely, PM machine operation with minimized torque ripple, minimized reactive power transfer and a multi-objective optimization. In addition, the control of PM machines under different open-circuit faulted conditions was also investigated while these different optimization criteria were also considered simultaneously with speed control.

The general mathematical representation of a multiphase fault-tolerant PM machine developed in chapter six was utilized for the task of obtaining optimized control for the PM machines considered in this chapter. This was performed by considering mathematical representations for the reactive power transfer, RMS current and by characterization of the torque ripple components such that the objective functions of different optimizations are easily represented mathematically.
The optimization problems were solved with standard numerical techniques.

The operation of a surface PM machine and an interior PM machine were investigated for three different optimizations, i.e. minimized torque ripple, minimized reactive power transfer and for a multi-objective optimization. The different operational characteristics of the PM machines with each of these optimizations were presented by analysis of simulation results. The operation with torque ripple minimization is found to consume high RMS current and reactive power while the operation with minimized reactive power transfer tend to produce high torque ripple components. In contrast, operation with the multi-objective optimization is found to provide the capability to trade-off different performance characteristics and balance the quality of torque production and the magnitudes of reactive power transfer and RMS current. Hence in conclusion it can be deduced that the multi-objective optimization provides more flexibility for control design and generality for the application in obtaining required performance characteristics for vehicular electric systems.
Chapter 8

Optimized Operation of Fault-Tolerant Permanent Magnet Machines in Power Generation

This chapter presents the optimized operation of fault-tolerant PM machines in power generation and serves as a complement to chapter seven. The generic expression for the DC-side rectifier current derived in chapter six is extended for the characterization of harmonic components of the DC-link current ripple. The optimization problems of DC-link current ripple minimization, reactive power minimization and copper loss minimization are defined considering operation under different open-circuit fault conditions. These optimization problems are formulated in a similar manner to the format of the optimizations presented in chapter seven. A multi-objective optimization is considered for the trade-off of DC-link current ripple and reactive power transfer under certain operating conditions.

The mathematical model of chapter six is utilized for the solutions of the optimization problems. The two PM machines presented in chapter three are considered as case studies. The optimized control signals are calculated for these two machines and the DC-link voltage regulator is formulated in the format of a PI controller. The performance of the DC-link voltage regulator is evaluated with simulations for different operating speeds, loads and different open-circuit fault conditions with the three different optimization criteria. The high-fidelity FE model based simulation technique presented in chapter three was reconfigured.
in power generation mode for the simulations presented in this chapter. The continuous time domain performances of the PM generator and AC-DC active rectification, and the frequency domain DC-link current ripple performances are evaluated and discussed.

8.1 Introduction

Fault-tolerant PM generator technology is presently being researched for application in environments that require highly reliable operation. One such area is the application of PM machines in aero-engine embedded power generation for future more-electric aircraft (MEA). The PM generator is a strong candidate due to its high power-to-weight ratio [42, 54], brushless operation, efficiency, reliability and ruggedness. More-electric systems that employ PM generators opt for a high number of phases, high phase reactance and low mutual coupling [42,53,54]. High phase reactance in the design of fault-tolerant PM generators will result in a low fault-level at the generator terminals. Such machines, designed with partial redundancy, allow faulted operational capability. Hence, the occurrence of a fault in a phase or converter will be confined to that phase and will not severely compromise the overall operation.

Chapter two introduce the need for the MEA, the increase in electrical power demand within the MEA, and the present MEA power generation architecture under consideration. In addition to MEA applications, the fault-tolerant PM machines for power generation can also be extended to applications in electric vehicles, ship propulsion, wind power generation and high-reliable gas-turbine power generation especially for nuclear reactors.

The rationale introduced in chapter seven for fault-tolerant PM motoring and control also holds for PM generation presented in this chapter. The fault-tolerant converter topology for PM machines adopted in this thesis, i.e. the parallel H-bridge topology which is shown in figure 8.1 is also considered in this chapter for power generator mode of PM machine operation. To date, the proposed techniques for the control of power generation with multiphase fault-tolerant PM machines have been confined to the basic surface PM machine [135, 165], and the available advanced forms of control for PM machines are only applicable for three phase machines (e.g., dq control of PM machines). A generalized technique applicable to machines with arbitrary number of phases is sought in this thesis,
and the mathematical model developed in chapter six serves as the foundation for this.

More advanced forms of PM machines offer advantages in the ease of construction of the machine. One example is the interior PM machine which is also considered in this thesis. Other forms of PM machine rotor topologies are available with different characteristics, and a review of these is out of the scope of this chapter. However, it is worth noting that the interior PM machine possesses attractive features such as higher mechanical robustness, better protection against demagnetization, higher field weakening capability and higher mechanical protection of the rotor PMs [65]. The existing fault-tolerant control methods developed for surface PM machines, which consider a sinusoidal back-EMF and non-salient flux-linkage characteristics does not utilize the full potential of more advanced machine designs. For example the interior PM machine contains additional harmonics in the back-EMF and single-saliency inductance characteristics, i.e., variation of inductance with rotor position. Such characteristics can be utilized for additional degrees of freedom in fault-tolerant control, enhanced DC-link voltage utilization with harmonic injection and enhanced DC-link current ripple minimization during faulted operation. Open-circuit faults are the main form of faults encountered and control presented in this chapter considers PM machine operation under such conditions. The generic mathematical model of PM machines developed in chapter six presents the interplay between different current harmonics and phase flux-linkage harmonics. The mathematical model considers a terminal voltage controlled in the form of harmonic injection pulse width modulation (HIPWM) which provides the control over converter AC-side voltage
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8.2 Permanent magnet generator harmonic model

8.2.1 Electrical system dynamics

The electrical system dynamics of a generic fault-tolerant PM generator was presented in chapter six and is thus not repeated here. In summary, the per-phase harmonic representation of electrical system dynamics of a PM machine was obtained in chapter six as,

\[
L_{n,0} \left( -jk\omega \vec{i}_{n,k} + \frac{d\vec{i}_{n,k}}{dt} \right) + \frac{L_{n,2}}{2} \left( \frac{d\vec{i}_{n,k-2}}{dt} + \frac{d\vec{i}_{n,k+2}}{dt} \right) - jk\omega \frac{L_{n,2}}{2} \left( \vec{i}_{n,k-2} + \vec{i}_{n,k+2} \right) + R_s \vec{i}_{n,k} + v_{dc} \vec{u}_{n,k} + j\omega k \vec{\psi}_k = 0 \quad (8.1)
\]

In contrast with chapter seven (motoring mode), the currents out of the machine are considered as positive. Hence the expression for real and reactive power (8.2), (8.3), represents power delivered out of the system as positive. i.e.,

\[
P_{n,k} = \frac{v_{dc}}{2} \text{Re} \left\{ \vec{u}_{n,k} \vec{i}_{n,k}^* \right\} \quad (8.2)
\]
\[
Q_{n,k} = \frac{v_{dc}}{2} \text{Im} \left\{ \vec{u}_{n,k} \vec{i}_{n,k}^* \right\} \quad (8.3)
\]

where \( P_{n,k}, Q_{n,k} \) represents the \( n \)th-phase real power transfer and the reactive power transfer associated with the \( k \)th harmonic component at the inverter AC-side.

8.2.2 DC-link current ripple model

The DC-link current generated by a healthy phase was derived in chapter six as,

\[
i_{\text{rect},n} = \frac{1}{4} \sum_{l=-K_1}^{K_1} \sum_{k=-K}^{K} \vec{u}_{n,l} \vec{i}_{n,k} e^{-j(k+l)\theta} \quad (8.4)
\]

The average DC-link current produced by one phase was derived in chapter six by considering the terms consisting of \( e^{-j(k+l)\theta} = 1 \) when \( k + l = 0 \). Assuming
\( K \leq K_1 \) this can be written as,

\[
i_{dc,n} = \frac{1}{4} \sum_{k=-K}^{K} \vec{u}_{n,k}^* \vec{t}_{n,k}^* \tag{8.5}\]

The \( r^{th} \) order DC-link current ripple component produced by the \( n^{th} \) phase can be written in general form by summing the terms of the series (8.4) when \( k + l = r \) and \( k + l = -r \). This results in,

\[
i_{dc,n,r}(\theta) = \frac{1}{4} \sum_{k=-2K}^{2K} \{ \vec{u}_{n,r-k}^* \vec{t}_{n,k} e^{-jr\theta} + \vec{u}_{n,r+k}^* \vec{t}_{n,k} e^{jr\theta} \} \tag{8.6}\]

Equation (8.6) can be simplified as,

\[
i_{dc,n,r}(\theta) = \frac{1}{2} \text{Re} \sum_{k=-2K}^{2K} \{ \vec{u}_{n,r-k}^* \vec{t}_{n,k} e^{-jr\theta} \} \tag{8.7}\]

Analysis of the DC-link current ripple follows a similar argument to that of the torque ripple presented in chapter seven. For an \( N \)-phase machine with \( N_f \) functional phases and \( (N-N_f) \) number of open-circuited phases, the total average DC current production due to the operation of the active rectifiers is given by the summation of individual DC current components contributed by each of the \( N_f \) phases. It follows from (8.7) that for a balanced machine, the summation of all \( r^{th} \) order current ripple components is zero if \( \sum_{n=1}^{N} e^{jr\gamma_n} = 0 \) is true. However, higher current ripple will be prevalent if the machine is partially operational, i.e., with a reduced number of functional phases.

Let the functional state of any \( n^{th} \) phase be represented by variable \( x_n \), where

\[
x_n = \begin{cases} 1 & \text{if } n^{th} \text{ phase is functional} \\ 0 & \text{if } n^{th} \text{ phase is open circuit} \end{cases} \tag{8.8}\]

By extension of (8.7) to \( n = 1, 2, \ldots, N \), the total \( r^{th} \) order current ripple component can be represented as,

\[
i_{dc,r}(\theta) = \frac{1}{2} \text{Re} \left\{ e^{-jr\theta} \sum_{n=1}^{N} x_n i_{dc,n,r} \right\} \tag{8.9}\]
where

\[ \hat{i}_{dc,n,r} = e^{-jr_\gamma n} \sum_{k=-2K}^{2K} \{ \vec{u}_{n,r-k} \vec{t}_{n,k} \} \]  

(8.10)

The magnitude and phase of the total \( r^{th} \) order DC-link current ripple waveform produced by the generator and active rectifier can be characterized by the vector:

\[ \hat{i}_{dc,\text{char},r} = \sum_{n=1}^{N} x_n \hat{i}_{dc,n,r} \]  

(8.11)

### 8.3 Optimization for fault-tolerant control

Equations (8.3), (8.5) and (8.11) together with the model (8.1) can be utilized for the calculation of different voltage and current control signals yielding different optimization criteria. The output variable in power generation mode is the DC-link voltage. In analogous form to the torque production considered in motoring mode, the enhancement of DC-link voltage quality is given higher priority in this chapter instead of torque production as in motoring mode. It is assumed that the inertia of the prime mover is sufficiently high in order to assume the effect of torque ripple on the prime mover speed is negligible. The DC-link ripple is directly influenced by the load current and the rectified current waveforms. As the load dynamics are not known, we may consider the load current as an exogenous disturbance and attempt to minimize the rectifier current harmonics under faulted conditions. Thus one form of optimization considered in generation mode is the DC-link ripple minimization.

In certain instances, the DC-link maybe sufficiently stiff due to the availability of large energy storage capability or the DC-link voltage ripple maybe inconsequential. In such instances, the priority maybe shifted to minimization of copper losses and power electronic losses of the converter. This can be attempted by the minimization of the reactive power transfer and the magnitude of the AC-side current magnitude.

As an intermediary solution, the optimization problems of DC-link current ripple minimization and the minimization of reactive power transfer can be formulated as a multi-objective optimization, and is also presented in this chapter.

These three forms are optimizations are described in general format below.
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This is then applied to the fault-tolerant control design of the five phase surface PM generator and the interior PM generator developed in chapter three. The optimization is performed on the operation of the machines with different open-circuit fault conditions. The DC-link voltage regulator is designed for both the systems and the operation of each PM generator and active rectifier is evaluated with high-fidelity FE model based switching simulations.

8.3.1 Operation with minimized DC-link current ripple

The objective of DC-link current ripple minimization for a given average DC current demand can be formulated as,

\[
\min \{u_{n,k}\} = \min \left[ \sum_{r=2}^{2K+2} \left( \lambda_r \left( \frac{\text{Re} \left( \frac{i_{\text{dc, char}, r}}{i_{\text{dc, rated}}} \right)}{\text{i}_{\text{dc, rated}}} \right)^2 + \lambda_r \left( \frac{\text{Im} \left( \frac{i_{\text{dc, char}, r}}{i_{\text{dc, rated}}} \right)}{\text{i}_{\text{dc, rated}}} \right)^2 \right) \right] (8.12)
\]

Subject to

\[
i_{\text{avg, tot}}^* - \sum_{n=1}^{N_f} i_{\text{dc, n}} = 0 \text{ for all } r (8.13)
\]

and \( d_n(t) \leq 1 \text{ for all } n \text{ and } t \) (8.14)

Here, \( i_{\text{dc, rated}} \) is the rated DC-link current production for the full machine and active rectifier system. The variable \( i_{\text{avg, tot}}^* \) represents the average DC current demanded by an outer-loop. The factor \( \lambda_r \) provides prioritization of certain current ripple components. Higher \( \lambda_r \) value will impose a higher weight on the minimization of specific current ripple components.

This optimization does not consider the magnitude of the current or reactive power transfer, and is solely for the purpose of DC-link current ripple minimization.
8.3.2 Operation with minimized reactive power transfer

The objective of reactive power transfer minimization for a given torque demand can be formulated as,

\[
\text{minimize } \left\{ u_{n,k} \right\} \left[ N \sum_{n=1}^{N} x_{n} \left\{ \left( \frac{Q_{n,1}}{S_{\text{base}}} \right)^{2} \right\} \right]
\]

(8.15)

Subject to:

\[
i_{\text{avg,tot}}^{*} - \sum_{n=1}^{N_{f}} i_{\text{dc,n}} = 0 \text{ for all } r
\]

(8.16)

and \( d_{n}(t) \leq 1 \) for all \( n \) and \( t \)

(8.17)

Here, \( S_{\text{base}} \) and \( I_{\text{base}} \) represents the kVA rating and the base current values of the considered PM generator. Variable \( Q_{n,1} \) is the fundamental reactive power transfer from the \( n^{th} \) phase. Similar to the reactive power minimization problem considered in the earlier chapter in motoring mode, only the fundamental reactive power transfer is consider here as this is the major contributor to higher current interchange with the DC-link. The effects of \( Q_{n,r}, r > 1 \) are generally negligible and are of notional expressions of harmonic reactive power components. This form of optimization prioritizes the power electronic system efficiency, while DC-link voltage ripple minimization is not attempted.

8.3.3 Multi-objective optimization

This form of optimization provides the capability to trade-off the objective of DC-link current ripple minimization with the objective of minimization of reactive power transfer and copper losses. Certain vehicle electric network power quality standards dictate the acceptable level of DC-link voltage ripple under operating conditions, e.g., RTCA DO-160C US civil specification [59], ADB-0100 Airbus specification [60] and MIL-STD-740E US military specification [61] to name a few. These do not dictate any power quality requirement during generator faulted operation, probably due to the fact that faulted operation has not been considered a possibility earlier. However, the multi-objective optimization format provides the capability to maintain the DC-link ripple within certain limits while minimizing the reactive power transfer and copper losses under faulted generator
operation. The optimization below is also formulated for open-circuited fault conditions only.

\[
\text{minimize} \left\{ u_{n,k} \right\} \left[ \sum_{n=1}^{N} x_n \left\{ \left( \frac{Q_{n,1}}{S_{\text{base}}} \right)^2 + \sum_{k=1}^{K} \kappa_k^2 \left( \frac{i_{n,k} i_{n,k}^*}{I_{\text{base}}^2} \right) \right\} \right]
\]  

(8.18)

Subject to:

\[
i_{\text{avg,tot}}^* - \sum_{n=1}^{N_f} i_{\text{dc,n}} = 0
\]  

(8.19)

\[
\left( \frac{\text{Re}\left( \frac{i_{\text{dc, char,r}}}{i_{\text{dc, rated}}} \right)}{i_{\text{dc, rated}}} \right)^2 + \left( \frac{\text{Im}\left( \frac{i_{\text{dc, char,r}}}{i_{\text{dc, rated}}} \right)}{i_{\text{dc, rated}}} \right)^2 < \chi_r^2 \text{ for all } r
\]  

(8.20)

and \( d_n(t) \leq 1 \) for all \( n \) and \( t \)  

(8.21)

Here, \( S_{\text{base}} \) and \( I_{\text{base}} \) represents the kVA rating and the base current values of the considered PM generator. Variable \( Q_{n,1} \) is the fundamental reactive power transfer from the \( n^{th} \) phase. As in the earlier cases, only the fundamental reactive power transfer is considered here as this is the major contributor to large current interchange with the DC-link. The parameter \( \kappa_k \) enables a trade-off between \( Q_{n,1} \) and the current magnitude.

### 8.3.4 Solution of the optimization problems

In analogous form to chapter seven, the optimization problems presented above can be solved with numerical algorithms such as sequential quadratic programming (SQP) [164]. In this chapter, SQP based algorithm with the output equations (8.3), (8.5) and (8.11), and model (8.1) are used in the solution and calculation of the required phase current and voltages for a given average current demand. A simplified version of the basic optimization strategy can be analytically evaluated for a PM machine with neglected third harmonic back-EMF component and neglected magnetic saliency.
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E.g. Analytical solution of the reactive power transfer minimization problem for a simplified PM machine model

For a certain class of PM machines, the back-EMF can be approximated to a fundamental sinusoidal variation and the inductance can be considered as constant. One such example is the surface PM machine. Under such approximations the optimization problem can be simplified to finding the minimum point of a nonlinear function, which can be easily solved via standard analytical techniques. As part of this research, one such example is published in [135] where the standard power transfer equations with rectifier phase angle and modulation index control strategy is developed. The technique presented below is an alternative improved version compared to that presented in [135]. The use of the PM machine model developed in chapter six has several advantages in control design, which will be outlined below.

The simplified representation of PM machine and PWM active rectifier dynamics can be obtained by setting $K = K_1 = 1$ in the model presented in chapter six, i.e., confining the model representation to the fundamental current component and the fundamental flux-linkage component.

Then the DC-link average current (8.5) can be simplified to:

$$i_{dc,n} = \frac{1}{4} \left( u_{n,1} i_{n,1}^* + u_{n,1}^* i_{n,1} \right) \quad (8.22)$$

Substitution of $\vec{u}_1 = u_{\alpha,1} + ju_{\beta,1}$ and $\vec{i}_1 = -i_{\alpha,1} + ji_{\beta,1}$ in (8.22) and simplification yields,

$$i_{dc,n} = \frac{1}{2} \left\{ -u_{\alpha,1} i_{\alpha,1} + u_{\beta,1} i_{\beta,1} \right\} \quad (8.23)$$

The reactive power transfer is given by:

$$Q_{n,1} = -\frac{1}{2} v_{dc} (u_{\beta,1} i_{\alpha,1} + u_{\alpha,1} i_{\beta,1})$$

(8.24)

The dynamic equations (1.59) and (1.60) of chapter six can also be simplified to,
\[-L_{n,0} \frac{di_{\alpha,n,1}}{dt} + \omega L_{n,0} i_{\beta,n,1} - R_s i_{\alpha,n,1} = -v_{dc}u_{\alpha,n,1} + \omega \psi_{\beta,1} \quad (8.25)\]
\[L_{n,0} \frac{di_{\beta,n,1}}{dt} + \omega L_{n,0} i_{\alpha,n,1} + R_s i_{\beta,n,1} = -v_{dc}u_{\beta,n,1} - \omega \psi_{\alpha,1} \quad (8.26)\]

The resistance terms are small, and can be neglected for simplicity. Then the current and voltage relationship under steady state operation can be written by neglecting the derivative terms and the resistance terms as:
\[\omega L_{n,0} i_{\beta,1} + v_{dc}u_{\alpha,1} - \omega \psi_{\beta,1} = 0 \quad (8.27)\]
\[\omega L_{n,0} i_{\alpha,1} - v_{dc}u_{\beta,1} - \omega \psi_{\alpha,1} = 0 \quad (8.28)\]

Substitution of \(i_{\alpha,1}\) and \(i_{\beta,1}\) from (8.27) and (8.28) in the average DC-link current production equation (8.23) yields:
\[i_{dc,n} = \frac{1}{2\omega L_{n,0}} \left\{ u_{\alpha,1} \left(v_{dc}u_{\beta,1} + \omega \psi_{\alpha,1}\right) - u_{\beta,1}v_{dc}u_{\alpha,1} \right\} \quad (8.29)\]
which reduces to:
\[i_{dc,n} = \frac{1}{2L_{n,0}} \psi_{\alpha,1} u_{\alpha,1} \quad (8.30)\]

Substitution of \(i_{\alpha,1}\) and \(i_{\beta,1}\) from (8.27) and (8.28) in the reactive power transfer equation (8.24) yields:
\[Q_{n,1} = -\frac{v_{dc}}{2} \left( \frac{1}{\omega L_{n,0}} \right) \left( -v_{dc}u_{\beta,1}^2 - \omega \psi_{\alpha,1} u_{\beta,1} - v_{dc}u_{\alpha,1}^2 + \omega \psi_{\beta,1} u_{\alpha,1} \right) \quad (8.31)\]

Equations (8.27) and (8.28) represents a steady state simplified per-phase PM machine electrical model with system outputs (8.30) and (8.32). The control problem of DC-link voltage regulation involves calculation of the control signals \(u_{\alpha,1}\) and \(u_{\beta,1}\). This dual-input system achieved by the application of the model presented in chapter six has a average DC-link current production equation with only one control input \(u_{\alpha,1}\), and provides ease of control design. The power transfer equation based model utilized in [135] deals with a nonlinear DC-link current production equation as a function of the modulation index and the phase angle. Thus the feedback control design is complicated in [135] in contrast with
(8.30) where feedback control of DC-link voltage can be implemented considering a linear parameter varying system (LPV).

PM generators do not require reactive current for magnetization due to the PM excitation. Thus \( u_{\alpha,1} \) and \( u_{\beta,1} \) should to be selected such that reactive power transfer is minimal for any demanded DC-link current. Ideally zero reactive power transfer is preferred so that the overall power electronic and copper losses are minimized. However due to the high inductance of the fault-tolerant PM generator phases, a non-zero level of reactive power transfer has to be established to avoid voltage collapse at high DC-link current demands. Furthermore zero reactive power transfer cannot be guaranteed due to the variable speed nature of the system, especially at higher speeds (above 100% nominal speed).

The solution to the minimized reactive power transfer problem provides the variation of \( u_{\beta,1} \) with \( u_{\alpha,1} \). The minimization of overall reactive power transfer is also the minimization of reactive power transfers from individual phases. Thus the optimization (8.15) - (8.17) can be simplified to:

\[
\text{minimize} \ [q(u_{\alpha,1}, u_{\beta,1})] \quad (8.32)
\]

Subject to:

\[
p(u_{\alpha,1}) = i_{\text{avg,tot}}^* - N_f \frac{1}{2L_n,0} \psi_{\alpha,1} u_{\alpha,1} = 0 \quad (8.33)
\]

and \( d_n(t) \leq 1 \) for all \( n \) and \( t \) (8.34)

where \( p(u_{\alpha,1}) \) is the constraint surface and \( q(u_{\alpha,1}, u_{\beta,1}) \) is a objective function related to the magnitude of the reactive power transfer given by (8.31),

\[
q(u_{\alpha,1}, u_{\beta,1}) = Q_{n,1}^2 \quad (8.35)
\]

The widely used Lagrange’s multiplier theorem for two variables [166] is used in [135] for the calculation of the optimum control input of modulation index \( M \) as a function of phase angle \( \delta \). However, in this case the constraint surface is only a function of \( u_{\alpha,1} \). Hence, the minimization problem is to find the minimum trajectory of function \( q(u_{\alpha,1}, u_{\beta,1}) \) over all \( u_{\alpha,1}, u_{\beta,1} \). This is defined by the function \( \frac{\partial q(u_{\alpha,1}, u_{\beta,1})}{\partial u_{\beta,1}} = 0 \), given by:
\[
\frac{\partial q(u_{\alpha,1}, u_{\beta,1})}{\partial u_{\beta,1}} = 2 \left( \frac{v_{dc}}{2\omega L_{n,0}} \right)^2 \left( v_{dc} u_{\beta,1}^2 + \omega \psi_{\alpha,1} u_{\beta,1} + v_{dc} u_{\alpha,1}^2 \right) \left( 2 v_{dc} u_{\beta,1} + \omega \psi_{\alpha,1} \right)
\]

(8.36)

This equation is zero if and only if,

\[
\left( v_{dc} u_{\beta,1}^2 + \omega \psi_{\alpha,1} u_{\beta,1} + v_{dc} u_{\alpha,1}^2 \right) = 0
\]

(8.37)

or

\[
(2 v_{dc} u_{\beta,1} + \omega \psi_{\alpha,1}) = 0
\]

(8.38)

From (8.31) it can be seen that the condition (8.37) drives the reactive power to zero. This can be achieved by setting \( u_{\beta,1} \) as a function of \( u_{\alpha,1} \) as:

\[
u_{\beta,1} = -\frac{\omega \psi_{\alpha,1}}{2 v_{dc}} \pm \sqrt{\left(\frac{\omega \psi_{\alpha,1}}{2 v_{dc}}\right)^2 - 4 v_{dc}^2 u_{\alpha,1}^2}
\]

(8.39)

Solution (8.39) is valid only if:

\[(\omega \psi_{\alpha,1})^2 - 4 v_{dc}^2 u_{\alpha,1}^2 > 0\]

This can be guaranteed in the region:

\[
\frac{\omega \psi_{\alpha,1}}{2 v_{dc}} < u_{\alpha,1} < -\frac{\omega \psi_{\alpha,1}}{2 v_{dc}}
\]

(8.40)

Beyond the region specified by (8.40) a non-zero minimum reactive power transfer can be established by maintaining the condition (8.38) by fixing \( u_{\beta,1} \) at a value of:

\[
u_{\beta,1} = -\frac{\omega \psi_{\alpha,1}}{2 v_{dc}}
\]

(8.41)

At high speeds the back-EMF exceeds the DC-link voltage and the condition \( d_n(t) \leq 1 \) or alternatively \( u_{\beta,1}^2 + u_{\alpha,1}^2 \leq 1 \) cannot be satisfied. In such conditions \( u_{\beta,1} \) can be limited at the maximum value indirectly bounded by the \( u_{\alpha,1} \)
command, which is given by:

\[ u_{\beta,1} = \sqrt{1 - u_{\alpha,1}^2} \]  

(8.42)

Figure 8.2: Block diagram of obtaining control signals via (a) the analytical technique (b) look-up table based off-line calculated values

Figure 8.2 (a) represents the block diagram of the optimization control signal calculation. Alternatively, the variation of \( u_{\beta,1} \) and \( u_{\beta,1} \) for a given average DC-link current demand can be calculated off-line and then implemented as a look-up table and is shown in figure 8.2 (b). The control systems simulated in this chapter considers higher order harmonics and obtains the solutions of the optimizations in-terms of standard SQP algorithms and is performed off-line. The following subsection presents the design of the overall outer control structure of the PM generator and active rectifier system for DC-link voltage regulation.
8.4 DC-link voltage regulation of fault-tolerant PM generator and parallel H-bridge active rectifier systems

The DC-link voltage regulation problem of a multiphase fault-tolerant PM generator interfaced with a parallel H bridge system can be formulated in a cascaded loop feedback / feedforward structure. The voltage regulator is implemented as a proportional integral (PI) controller, where the average DC-link current demand is calculated by,

\[ i_{\text{avg,tot}}^* = k_{p,dc} (v_{dc}^* - v_{dc}) + k_{i,dc} \int (v_{dc}^* - v_{dc}) \, dt \] (8.43)

The optimal phase voltage harmonic components are obtained by a multi-dimensional lookup table that is generated by the optimization explained in section 8.3. The expected current waveform harmonic components can be calculated online by solution of machine phase steady state electrical equations derived from (8.1). The PWM switching is then calculated by a feedback/feedforward current control law,

\[ d(t) = d(t)' - k_{p,c} (i_n^* - i_n) - k_{i,c} \int (i_n^* - i_n) \, dt \] (8.44)

Figure 8.3: Permanent magnet generator and parallel H-bridge module active rectifier topology with controller
where \(d(t)\) is the PWM switching waveform generated by HIPWM switching waveform generator. Under ideal conditions where the model exactly matches the PM motor, \(d(t) = d(t)\) will result. However, effects of saturation, thermal variation of parameters and other unmodelled dynamics may result in a discrepancy between the model (8.1) and the actual PM motor dynamics. The variation of the phase current from the required waveform can be alleviated by incorporation of the PI current control in the form of (8.44). The block diagram of figure 8.3 shows the interconnection of a PM generator phase with an Hbridge module and the controller of that phase.

8.4.1 Current controller gain selection

The current controller gain selection follows a similar argument to that presented in chapter seven in motoring mode. Since the currents out of the machine are considered as positive, the corresponding dynamic equation for generation mode is given by:

\[
v_n = -R_n i_n - L_{n,0} \frac{di_n}{dt} + e_b(t)
\]  

(8.45)

where \(e_b(t)\) incorporates the back-EMF produced by the permanent magnet flux and the reluctance component. The switch-average voltage generated by the PWM inverter is given by,

\[
v_n = v_{dc} d(t)
\]  

(8.46)

Therefore, by substitution of (8.44) and (8.46) in (8.45) yields:

\[
R_n i_n + L_{n,0} \frac{di_n}{dt} - e_b(t) = -v_{dc} d'(t) + v_{dc} k_{p,c} (i_n^* - i_n) + v_{dc} k_{p,c} \int (i_n^* - i_n) dt
\]  

(8.47)

It follows from the model developed in chapter six that under steady state conditions, i.e., when \(\frac{di_n}{dt} = 0\), the voltage demanded by the mathematical model by \(v_{dc} d'(t)\) satisfy,

\[
v_{dc} d'(t) = R_n i_n + e_b(t)
\]  

(8.48)
Substitution of (8.48) in (8.47) and simplification yields:

\[ L_{n,0} \frac{di_n}{dt} = v_{dc}k_{p,c} (i_n^* - i_n) + v_{dc}k_{p,c} \int (i_n^* - i_n) dt \]  \hspace{1cm} (8.49)

Assuming constant DC-link voltage, the resultant transfer function for the current controller is then given by:

\[ \frac{I_n(s)}{I_n^*(s)} = \frac{v_{dc}k_{p,c}s + v_{dc}k_{p,c}}{L_{n,0}s^2 + v_{dc}k_{p,c}s + v_{dc}k_{p,c}} \]  \hspace{1cm} (8.50)

Casting (8.50) into the standard second order format yields,

\[ \frac{I_n(s)}{I_n^*(s)} \approx \frac{v_{dc}k_{p,c}}{L_{n,0}s^2 + \frac{v_{dc}k_{p,c}}{L_{n,0}}s + \frac{v_{dc}k_{p,c}}{L_{n,0}}} \]  \hspace{1cm} (8.51)

where the proportional and integral gains can be written in terms of damping factor \( \varsigma_c \) and un-damped natural frequency \( \omega_{n,c} \):

\[ k_{p,c} = \frac{2\varsigma_c\omega_{n,c}L_{n,0}}{v_{dc}} \]  \hspace{1cm} (8.52)

\[ k_{i,c} = \frac{L_{n,0}\omega_{n,c}^2}{v_{dc}} \]  \hspace{1cm} (8.53)

### 8.4.2 Voltage regulator gain selection

The voltage regulator gain selection in PM generator mode of operation follows a similar argument to that of the speed controller gain selection in PM motoring mode operation of chapter seven. The instantaneous DC current generation from the active rectifier can be rewritten in the form of:

\[ i_{dc}(\theta) = i_{avg, tot} + i_{rip}(\theta) \]  \hspace{1cm} (8.54)

where \( i_{avg, tot} \) and \( i_{rip}(\theta) \) are given by summation of (8.7) for all the phases in operation, and (8.11) for all the ripple components considered. Assuming that the commanded average DC current is produced without lag i.e, \( i_{avg, tot}^* = i_{avg, tot} \) the DC-link voltage dynamics can be written as,

\[ i_{avg, tot}^* + i_{rip}(\theta) - i_{load} = C_{dc} \frac{dv_{dc}}{dt} \]  \hspace{1cm} (8.55)
CHAPTER 8. OPTIMIZED OPERATION OF PM GENERATORS

Substitution of (8.43) in (8.55) yields:

\[ C_{dc} \frac{dv_{dc}}{dt} = k_{p,dc} (v_{dc}^* - v_{dc}) + k_{i,dc} \int (v_{dc}^* - v_{dc}) dt + w(t) \]  

(8.56)

where \( v_{dc}^* \) is the voltage command, \( w(t) \) represents the rectified current and load current as a disturbance. The corresponding input-output transfer function is given by,

\[ \frac{V_{dc}(s)}{v_{dc}^*(s)} = \frac{k_{p,dc}s + k_{i,dc}}{C_{dc}s^2 + k_{p,dc}s + k_{i,dc}} \]  

(8.57)

Casting (8.57) into the standard second order format yields,

\[ \frac{v_{dc}(s)}{v_{dc}^*(s)} \approx \frac{k_{i,dc}}{s^2 + \frac{k_{p,dc}}{C_{dc}} s + \frac{k_{i,dc}}{C_{dc}}} \]  

(8.58)

where the proportional and integral gains can be written in terms of damping factor \( \varsigma_{dc} \) and un-damped natural frequency \( \omega_{n,dc} \):

\[ k_{p,dc} = 2\varsigma_{dc}\omega_{n,dc}C_{dc} \]  

(8.59)

\[ k_{i,dc} = C_{dc}\omega_{n,dc}^2 \]  

(8.60)

Transfer function (8.58) can be used to calculate the proportional and integral gains for the required voltage regulator performance. Although, the poles of the above transfer function maybe placed on the stable left half s-plane, this does not guarantee stability due to the nonlinear relationship between the operating point and the current ripple component \( i_{rip}(\theta) \). However, stable operation can be obtained by placing the poles at undamped natural frequencies below the minimum DC-link current ripple frequency. Thereby maintaining the voltage ripple caused by the current ripple low such that the nonlinearity of \( w(t) \) is not augmented. This can also be analysed by means of the sensitivity transfer function:

\[ \frac{V_{dc}(s)}{w(s)} = \left( \frac{s}{C_{dc}s^2 + k_{p,s}s + k_{i,s}} \right) \]  

(8.61)

Stable operation can be expected by designing the controller sensitivity to satisfy

\[ \left| \frac{V_{dc}(s)}{w(s)} \right| \ll \gamma_{dc} \]  

at all ripple frequencies \( f_r \). \( \gamma_{dc} \) is a design parameter.
CHAPTER 8. OPTIMIZED OPERATION OF PM GENERATORS

8.5 Case study

8.5.1 Optimization

The case study presented in this chapter considers generator mode of operation of the surface PM machine and the interior PM machine presented in chapter three. The generator and the DC-link parameters from chapter three are given in Table 8.1 below for both the machines.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>SPM Gen Parameters</th>
<th>IPM Gen Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>kVA rating</td>
<td>125kVA</td>
<td>125kVA</td>
</tr>
<tr>
<td>kVA rating per-phase</td>
<td>25kVA</td>
<td>25kVA</td>
</tr>
<tr>
<td>Base phase current</td>
<td>66.75A</td>
<td>66.75A</td>
</tr>
<tr>
<td>Rated torque</td>
<td>397.88Nm</td>
<td>397.88Nm</td>
</tr>
<tr>
<td>PM flux</td>
<td>-0.3249Vs</td>
<td>-0.2810Vs</td>
</tr>
<tr>
<td>Maximum speed</td>
<td>3000rpm</td>
<td>3000rpm</td>
</tr>
<tr>
<td>( R_s )</td>
<td>0.01Ω</td>
<td>0.01Ω</td>
</tr>
<tr>
<td>( L_{m,n,0} )</td>
<td>1.3976mH</td>
<td>2.7234mH</td>
</tr>
<tr>
<td>( L_{m,n,2} )</td>
<td>0mH</td>
<td>-0.5561mH</td>
</tr>
<tr>
<td>pole pair number</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>total number of phases</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>DC-link capacitance</td>
<td>10mF</td>
<td>10mF</td>
</tr>
</tbody>
</table>

The three forms of optimizations explained earlier are performed for the two machines, i.e., (a) Minimized DC-link current ripple, (b) Minimized reactive power and (c) Multi-objective optimization, for the following fault conditions:

i. All phases in operation.

ii. Phase a open-circuited.

iii. Phases a and b open-circuited.

iv. Phases a and c open-circuited.

The parameters associated with these three optimization are selected as, \( \lambda_r = 1 \) for \( r = 2, 4, 6 \), \( \kappa_k = 1 \) for \( k = 1, 3 \), and \( \chi_k^2 = 0.01 \) for \( r = 2 \) and \( \chi_k^2 = 0.001 \) for \( r = 4, 6 \). The optimization is performed considering the fundamental and the third harmonic for a fixed DC-link voltage of 540V and speed range of 750rpm to a maximum speed of 3750rpm. An average DC-link current production from -1pu to +1pu is consider. The variation of \( \vec{u}_{n,1} \) and \( \vec{u}_{n,3} \) for all the five phases, i.e.,
Figure 8.4: Sample variation of control signals obtained via SQP algorithms for minimized DC-link current ripple operation at 2250 rpm. (a) - (d) healthy machine, (e) - (h) phase a faulted, (i) - (l) phases a and b faulted, and (m) - (p) phases a and c faulted.

$n = 1, 2, ..., 5$ are obtained for the four different fault conditions and the three different optimization criteria.
Figure 8.4 shows a sample variation of control signals for minimized DC-link current ripple operation of the interior PM motor at 2250 rpm, for the four different fault conditions. Similar variations are obtained for the surface PM machine, and at different speeds and optimization criteria. These optimal variations are obtained considering the objective functions defined earlier and using standard SQP algorithms for the solution of these optimization problems. Details of the solution of the optimization problems are not discussed here.

### 8.5.2 Control design

For the machine parameters given in table 8.1, the current controller proportional and integral gains are calculated using (8.52) and (8.53) for the damping factor and un-damped natural frequency shown in table 8.2. The resultant gains are also shown in table 8.2.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>SPM Motor Parameters</th>
<th>IPM Motor Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v_{dc}) nominal DC-link voltage</td>
<td>540V</td>
<td>540V</td>
</tr>
<tr>
<td>(\omega_{n,c})</td>
<td>2513.3rad/s</td>
<td>2513.3rad/s</td>
</tr>
<tr>
<td>(\varsigma_c)</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>(k_{p,c})</td>
<td>0.0091</td>
<td>0.0177</td>
</tr>
<tr>
<td>(k_{i,c})</td>
<td>16.3482</td>
<td>31.8565</td>
</tr>
</tbody>
</table>

Similarly the DC-link voltage regulator proportional and integral gains are calculated using (8.59) and (8.60) and the corresponding parameters are shown in table 8.3.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>SPM Motor and IPM Motor Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega_{n,s})</td>
<td>125.6637rad/s</td>
</tr>
<tr>
<td>(\varsigma_s)</td>
<td>0.7</td>
</tr>
<tr>
<td>(k_{p,dc})</td>
<td>1.7593</td>
</tr>
<tr>
<td>(k_{i,dc})</td>
<td>157.9137</td>
</tr>
</tbody>
</table>

Figure 8.5 shows the bode plot of the voltage loop sensitivity function. For the operating speed ranges from 750 rpm to 3750 rpm, the lowest theoretical ripple occurs at 150Hz or 942.47rad/s i.e., the second order harmonic. The magnitude of the sensitivity function above this frequency is approximately smaller than 0.11.
(i.e., $\gamma_{dc} = 0.11$). This implies that for a 1A DC-link current ripple component above 150Hz, the effective voltage ripple is below 0.11V. This is a considerably low value and hence operation with stable DC-link voltage can be expected.

8.6 Simulation results

The surface PM generator and interior PM generator are separately simulated by means of the FE model based technique, and with the parallel H-bridge converter switching model implemented in Simulink. The controller for DC-link voltage regulation is implemented as explained earlier and the optimum control inputs are stored in a multi-dimensional look-up table which is accessed dynamically in the simulation. The speed of the prime-mover driving the generator is varied from a minimum speed of 750rpm to a maximum of 3750rpm in five steps, and is shown in figure 8.6. At each speed the DC-link is loaded with a three level step load of 12%, 36%, and 60% of the full system rated DC current production capability.

The four different operating conditions with three different optimized control signals mentioned earlier, i.e., (a) Minimized DC-link current ripple, (b) Minimized reactive power and (c) Multi-objective optimization, are simulated for the
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Figure 8.6: Prime-mover speed variation used in the simulation of the PM generator system

following fault conditions:

i. All phases in operation.
ii. Phase a open-circuited.
iii. Phases a and b open-circuited.
iv. Phases a and c open-circuited.

For the comparison of the effectiveness of the optimization, a separate simulation is performed by considering the open-circuit fault conditions (ii), (iii) and (iv) with the control signals of a healthy machine. The full simulation results are presented in appendices Q and R.

Similar to the analysis and observations performed on the torque waveform in motoring mode, the frequency domain performance is evaluated by observations of the FFT of the DC-link current waveform in each of the speed and load conditions of the simulation. The corresponding FFT results for the surface PM machine and the interior PM machine are presented in appendices S and T.

As mentioned earlier, the torque ripple is analogous to the DC-link current ripple. The modelling and optimization considered in this analysis only deals with the fundamental and third harmonic currents, which effects the 2\textsuperscript{nd}, 4\textsuperscript{th}, 6\textsuperscript{th} and the 8\textsuperscript{th} harmonic components of the DC-link current waveform. The frequency components of interest in the DC-link current waveform at the different speeds of the simulation are shown in table 8.4.

Figures 8.8 (a) to (d) and figures 8.9 (a) to (d) present the reactive power transfers of the surface PM generator and the interior PM generator systems under different operating conditions. The lines shown in four different colours provide a comparison of reactive power levels under different optimization criteria. In general it can be seen that the blue-line representing the minimized
Table 8.4: Important frequency components of the DC-link current waveform at the speeds considered in the simulation

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>750rpm</th>
<th>1500rpm</th>
<th>2250rpm</th>
<th>3000rpm</th>
<th>3750rpm</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>150Hz</td>
<td>300Hz</td>
<td>450Hz</td>
<td>600Hz</td>
<td>750Hz</td>
</tr>
<tr>
<td>4</td>
<td>300Hz</td>
<td>600Hz</td>
<td>900Hz</td>
<td>1200Hz</td>
<td>1500Hz</td>
</tr>
<tr>
<td>6</td>
<td>450Hz</td>
<td>900Hz</td>
<td>1350Hz</td>
<td>1800Hz</td>
<td>2250Hz</td>
</tr>
<tr>
<td>8</td>
<td>600Hz</td>
<td>1200Hz</td>
<td>1800Hz</td>
<td>2400Hz</td>
<td>3000Hz</td>
</tr>
</tbody>
</table>

reactive power operation is lower than the other lines in majority of the operating conditions except certain extreme conditions such as low-speed and high-load conditions. Thus in general it can be deduced that the reactive power minimization operation is successful.
CHAPTER 8. OPTIMIZED OPERATION OF PM GENERATORS

Figure 8.7: Results of the interior PM generator operation with all phases healthy. Control signal optimized for minimization of reactive power transfer.
8.6.1 Healthy machine operation

Figures 8.7 (a) to (g) show the variation of different variables of the interior PM generator and drive system for operation with all phases healthy. The control signal in the case shown in figure 8.7 is optimized for minimized reactive power transfer. The operation with other control signals for the surface PM generator as well as the interior PM generator are given in the appendices Q and R respectively. Under healthy conditions, the PM generator operation is balanced. Hence, the per-phase real power and reactive power in all the phases are balanced. Furthermore, the total DC-link current ripple is minimal as the current components contributed by individual phases are symmetrical. However, under high load conditions the machine is driven into saturation which results in additional

Figure 8.8: Comparison of reactive power transfer for the surface PM generator operation under different optimization criteria and four different fault conditions, (a) all phases healthy, (b) phase a faulted, (c) phases a and b faulted, and (d) phases a and c faulted.
CHAPTER 8. OPTIMIZED OPERATION OF PM GENERATORS

Figure 8.9: Comparison of reactive power transfer for the interior PM generator operation under different optimization criteria and four different fault conditions, (a) all phases healthy, (b) phase a faulted, (c) phases a and b faulted, and (d) phases a and c faulted.

unmodeled DC-link current harmonic components which are not minimized in the optimization process.

The figure 8.8 (a) and the figure 8.9 (a) present the comparison of the reactive power transfer of the surface PM generator system and the interior PM generator system under the three different optimization criteria, and are denoted in three different colours in these figures. It can be seen that in both the cases, the blue line representing the operation with minimized reactive power transfer is of lower magnitude than those with other optimizations. However, certain extreme conditions such as low speed and high load situations, the minimization of reactive power is less effective. During low speed operation a negative level of reactive power is required for the strengthening of the magnetic field. The interior PM generator requires more reactive power for the strengthening of the
magnetic field in low speed operation compared with the surface PM generator, which is obviously due to the weaker air-gap flux of the interior PM machines. Both machines succeed in maintaining the reactive power level very close to zero during 2250rpm and 3000rpm operation. During above rated-speed operation, i.e. at 3750rpm, the surface PM machine demands more reactive power for field weakening compared with the interior PM machine which maintains the reactive power level at a very low value. This can be again attributed to the relative difference of the air-gap fluxes of the two machines. In addition, the optimization for the interior PM machine automatically utilizes the third harmonic component for the improvement of the DC-link voltage utilization during high-speed operation, which would result in lowering of the reactive power requirement of the interior PM generator system.

8.6.2 One-phase open-circuit faulted operation

The operation of the surface PM generator and the interior PM generator under a one-phase open-circuit condition was simulated with the different optimization criteria. Figures 8.10 (a) to (g) show the variation of different variables of the interior PM generator and drive system under one-phase open-circuit fault condition and with control signals optimized for minimized DC-link current ripple. The operation with other control signals for the surface PM generator as well as the interior PM generator are given in the appendices Q and R respectively. In general, under all the different optimization criteria simulated with one-phase fault, stable DC-link voltage with satisfactory disturbance response characteristics is seen similar to the case shown in figure 8.10 (a).

Figure 8.8 (b) and 8.9 (b) present the comparison of the reactive power transfer of the surface PM generator system and the interior PM generator system under a one-phase open-circuit fault condition and operation with the three different optimization criteria. As explained earlier, the blue line representing the minimization of reactive power transfer is successful as the magnitude is lower than those operating with the other optimization criteria in majority of the speed and load conditions. Moreover, a similar observation of the magnitude of reactive power during healthy operation can be seen in this case, where the minimization is less effective during low-speed high-load conditions. Loss of one phase also leads to an increase in the magnitude of the total reactive power demand.
Results of the interior PM generator operation with four healthy phases (phase-a open circuited)
Control signal optimized for minimized DC link current ripple

Figure 8.10: Results of the interior PM generator operation with a one phase fault (phase a open-circuited). Control signal optimized for minimization of DC-link current ripple.
It can be seen from figures 8.10 (f) and (g) that the real power and reactive power of different phases are not identical. This occurs as a result of the optimization which attempts to minimize the DC-link current ripple produced
due to the loss of one phase. The load sharing between phases are unequal and this may lead to unequal heating under continuous duty operation under faulted condition. Similar observations can be made with the surface PM generator operation, which is presented in the appendix Q. The per-phase real power and reactive power transfer for the interior PM generator and drive system under one-phase open-circuit fault condition and with multi-objective optimized control signals is shown in figures 8.11 (a) and (b). The operation of systems under the multi-objective optimization shown in figure 8.11 follows a similar pattern,

Figure 8.13: FFT results on the DC-link current waveform of the interior PM generator system operating at 60% of the rated load and at different speeds. (a) at 1500rpm, (b) at 2250rpm, (c) at 3000rpm and (d) at 3750rpm respectively.
however with a higher level of load sharing. In contrast, the operation of the PM generators with the reactive power minimization as shown in figure 8.12 achieves equal sharing of load between the operational phases.

Frequency domain FFT analysis on the DC-link current waveform under one-phase fault condition was performed. Figures 8.13 (a) to (d) present a sample of the results associated with the interior PM generator system operation with one-phase fault at a 60% of the rated loading. Figures 8.13 (a) to (d) correspond to the FFT plots at speeds of 1500rpm, 2250rpm, 3000rpm and 3750rpm respectively. Further results at different loading levels for the surface PM generator system and the interior PM generator system are presented in appendices S and T. In each figure, the red-stem-line represents the FFT under operation with the DC-link current ripple minimization control. In general, a clear advantage in the DC-link current ripple minimization operation is evident. The major second order DC-link current ripple components at 300Hz, 450Hz, 600Hz and 750Hz corresponding to the four different speeds presented in figures 8.13 (a) to (d) testifies to this claim. The red-stem-lines of these four figures are significantly lower than those with the other optimizations. The system operation with minimized reactive power (blue-stem-line) and the operation without consideration for the fault condition (green-stem-line) has the highest DC-link current ripple magnitude in all the cases presented in figure 8.13. The multi-objective optimization (black-stem-line) has an intermediate torque ripple magnitude compared with the other components. Hence, the capability to trade-off reactive power with torque ripple with the multi-objective optimization is confirmed by this simulation. However, close examination of the FFT results may reveal that the red-stem-line has a higher magnitude at certain frequencies compared with the other stem-lines. These typically occur at frequency components not considered in the optimization process. These mainly occur due to the effect of saturation and other mathematically unanalysed dynamics which are replicated by the FE model based simulation. The current controller imposes a voltage component in addition to the feedforward command in order to track the reference current waveform. This will cause additional DC-link current harmonics. These occur at very few instances and therefore it can be deduced that the proposed technique is suitable for the DC-link current ripple minimization under one-phase fault conditions.
Results of the interior PM generator operation with three healthy phases (phases a and b open-circuited)  
Control signal optimized for minimized DC-link current ripple

Figure 8.14: Results of the interior PM generator operation with a two consecutive phase fault (phase a and b open-circuited). Control signal optimized for minimization of DC-link current ripple.
Figure 8.15: Results of the interior PM generator operation with a two non-consecutive phase fault (phase a and c open-circuited). Control signal multi-objective optimized.
8.6.3 Two-phase open-circuit faulted operation

The simulations presented in appendices Q and R consider a consecutive phase faulted condition and a non-consecutive phase faulted condition. Operation of the surface PM generator and the interior PM generator are simulated with different optimization criteria. In all these cases stable DC-link voltage is verified by the simulations. The disturbance response characteristics under step loading conditions are seen to be consistent for majority of the operating conditions and show satisfactory performance.

It can be seen from figures 8.8 and 8.9, (c) and (d) that the blue-line representing the operation with minimized reactive power is lower than the reactive power levels of the other optimizations. Hence it can be deduced that the minimized reactive power operation is successful for two-phase faulted operation.

Figures 8.14 (a) to (g) and figures 8.15 (a) to (g) present two samples of simulations viz., the variation of variables for the interior PM generator and drive system for operation with consecutive phases faulted condition and non-consecutive phases faulted condition respectively. Figure 8.14 presents minimized DC-link current ripple operation and figure 8.15 presents the operation with the multi-objective optimization. It can be seen that in both the figures 8.14 and 8.15, (f)
and (g) the real power and reactive power sharing between the three operational phases are unequal. This is due to the attempt made by the optimization to minimize the DC-link current ripple components.

In contrast, real power and reactive power sharing during reactive power minimized operation as shown in figure 8.16 (a) and (b) is equal and overlaps. The magnitude of the total negative reactive power consumption for field strengthening operation during two phase open-circuit condition is higher than the reactive

![Figure 8.17: FFT results on the DC-link current waveform of the interior PM generator system operating at 36% of the rated load and at different speeds. (a) at 1500rpm (b) at 2250rpm (c) at 3000rpm and (d) at 3750rpm respectively.](image-url)
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power consumption during one-phase faulted operation or healthy machine operation. In contrast the reactive power requirement for field weakening is reduced during the two-phase fault conditions. Similar observations can be made for the surface PM generator operation also.

Frequency domain FFT analysis on the DC-link current waveforms under two-phase fault conditions was performed. Figures 8.17 (a) to (b) present a sample of the results associated with the interior PM generator system operation with two consecutive phase faults at a 36% of the rated loading. Further results at different loading levels for the surface PM generator system and the interior PM generator system are presented in appendices S and T. Similar to the observations made in the earlier section for one-phase fault conditions, a clear advantage in the DC-link current ripple minimization operation is evident in figures 8.17 (a) to (b) frequency components represented by the red-stem-lines. The DC-link current ripples at the major frequencies are reduced. At higher loading, e.g. 60% rated loading under a two phase fault represents 100% loading of individual phases with the per-phase rated value. Under such loading conditions the machine operates in the saturated region which results in a higher discrepancy from the single-saliency mathematical model. Hence, the torque ripple minimization is less effective during very high loads. However, the major frequency components are reduced during majority of the operating cases simulated and thus the torque ripple minimization for the two-phase faulted conditions is successful.

8.7 Conclusion

In this chapter, a general control strategy for multiphase fault-tolerant PM generators was developed. The PM machine operation with different open-circuit fault conditions has been considered, and the mathematical modelling from chapter six has been utilized for the characterization of the rectifier DC-side current waveform and the corresponding harmonic components. PM generator control was formulated with different optimization characteristics and the possibility to operate with minimized DC-link current ripple or minimized reactive power transfer has been investigated. The possibility to trade-off DC-link current ripple with reactive power transfer was investigated by means of a multi-objective optimization. The control designs were presented for a parallel H-bridge active rectifier system interfaced with a fault-tolerant surface PM generator and an interior PM
generator. The systems were simulated with the FE model based technique presented in chapter three. The effectiveness of the reactive power minimization, DC-link current ripple minimization and the multi-objective optimization were discussed, and shown to be successful under the majority of the operating conditions. It can be deduced that the multi-objective optimization provides more flexibility for control design and generality for the application in obtaining required performance characteristics for vehicular electric systems.
Chapter 9

Conclusions and Future Work

9.1 Thesis conclusions

In this thesis, different strategies for the control of SR and PM machines have been investigated particularly with application to vehicular electric systems. The four objectives of this thesis outlined in the introduction were,

1. Development of a SR motor control strategy for aero-engine starter requirements.
3. Development of fault-tolerant PM motor control strategies and,
4. Development of fault-tolerant PM generator control strategies that cater for a broader class of vehicular electric applications.

The need for electric systems in transportation was explained in chapter two. In addition the potential application domains of fault-tolerant machines in vehicular electric systems have been reviewed. The progression of the concept of the MEA and ME systems was explained and the state-of-development within electric ship propulsion and hybrid electric vehicles was examined. The applicability of fault-tolerant machines to such areas was outlined.

The objectives of this thesis were achieved by focusing on a 250kW SR machine design and two 125kW PM machine designs presented in chapter three. The control designs for SR motoring and power generation were experimentally
validated by a 300W test-bed SR motor / generator system. Analyses on the control designs presented in this thesis for the SR and PM machines were performed with the aid of high-fidelity FE-model based simulations. These simulation models were formulated with FE-model based information and hence replicate major nonlinearities such as magnetic saturation and fringing effects.

This thesis realizes the first objective by proposing a current-peak control strategy for SR motoring for the application to aero-engine starter operation. The aero-engine starter requirement of operation from below base-speed to above base-speed is thus satisfied by this excitation control strategy. This thesis proceeds further to evaluate the efficiency and performance of the current-peak excitation control strategy supplemented with a ZVL switching approach. These two excitation control strategies are compared with the classical form of excitation control. The three controllers combined with a speed controller were simulated for the 250kW SR motor and experimentally validated with the 300W test-bed system. It is shown that the current-peak control strategy achieves a higher torque production envelope compared with the classical method and avoids a large discontinuity in the transition from below base-speed to above base-speed operation. The ZVL based excitation control strategy achieves an improvement in efficiency in the range of 20% to 30% compared with the conventional excitation control strategy. In contrast, the current-peak feedback strategy achieves an improvement in efficiency in the range of 16% to 25% compared with the conventional excitation control strategy. The conventional strategy also suffers from high torque ripple fundamental and DC-link current ripple fundamental FFT-power and is more than 10 times that with the ZVL method. However, the current-peak feedback excitation strategy achieves the lowest torque ripple fundamental and DC-link current ripple fundamental FFT-power, which is about 30-80% of the ZVL method. The different frequency domain performance characteristics are also discussed and it is shown that incorporation of the ZVL reduces the quality of torque production due to high ripple, and also increases the current ripple of the DC-link. The inferior frequency domain performance of the ZVL technique is not seen as a challenge to the application of this technique for aero-engine starter operation due to the high inertia involved and the short duty of an aero-engine start operation.

The second objective of SR generator control design for aero-engine embedded power generation is achieved by evaluation of different excitation control
strategies. Two classical forms of excitation control and four different optimized excitation controllers are evaluated. In this part, only single pulse mode of SR generator operation is considered due to the high speed range requirement of direct-drive aero-engine power generators. Excitation control with minimized excitation penalty, minimized RMS current, minimized peak flux-linkage and minimized peak current were investigated. The optimal variation of excitation parameters was extracted from iterative simulations performed using high-fidelity FE model based SR generator representations for the 250kW system and the 300W test-bed system. The optimal variations were then fitted to a PCF (piece-wise continuous function) format and implemented as excitation controllers. The performance of the different optimal excitation control strategies combined with a voltage regulator was evaluated by means of simulations and validated with 300W test-bed experimental results.

In conclusion it was shown that excitation control with minimized excitation penalty outperforms the other optimizations of minimized RMS current, minimized peak flux-linkage or minimized peak current. Minimization of RMS current or peak current value leads to an increase in peak flux-linkage while minimization of peak flux-linkage leads to an increase in the RMS current value. Furthermore, minimization of the peak flux-linkage also leads to higher torque ripple and DC-link current ripple. In contrast, the minimization of excitation penalty is shown to perform with a superior balance between efficiency and the trade-off of torque ripple and peak flux-linkage. It has been shown that at high loads the minimization of the peak current value reduces the DC-link current ripple fundamental and the torque ripple fundamental FFT-power in the range of 90% at the expense of about 1.4% reduction in efficiency. In contrast, the drop in efficiency is in the range of 10% to 20% during low load conditions. It has also been shown that at high loads the minimization of excitation penalty achieves efficiencies in the range of 79% to 86% at the expense of higher DC-link current ripple fundamental and the torque ripple fundamental FFT-power in the ranges of 80% to 90% and 50% to 60% respectively. In conclusion, it is shown that the minimization of excitation penalty operation performs with superior trade-off characteristics. The performances of the two simple excitation control strategies are compared with the performance of a minimized excitation penalty controller. In comparison, it was shown that the excitation control based on a fixed turn-on angle / variable
CHAPTER 9. CONCLUSIONS AND FUTURE WORK

turn-off angle exhibits poor performance during low loads while the fixed turn-off / variable turn-on angle based excitation controller performs with superior overall efficiencies close to the minimized excitation penalty controller. Hence it was deduced that this controller offers the simplest structure, while also maintaining a balance between different frequency domain performance characteristics such as torque ripple / DC-link current ripple and peak flux-linkage. This thesis claims that the classical fixed turn-off angle / variable turn-on angle based excitation control strategy can be designed with characteristics close to a minimized excitation penalty operation.

The third and fourth objectives, i.e. the development of fault-tolerant PM motor and generator control strategies are achieved mainly by application of the modelling performed in chapter six. Part of this thesis presents the development of a mathematical model for the representation of PM machines fed by the parallel H-bridge per-phase converter topology. The model developed in this thesis is formulated in general harmonic format which may also be interpreted with multi-phase orthogonal components. The mathematical model was utilized to represent healthy and open-circuit faulted PM machines with non-sinusoidal flux-linkage distribution. In addition, equations for per-phase real power, reactive power, average DC current production and torque production were developed. Frequency domain characterization of harmonic components of the torque ripple and DC-link current ripple was performed. These model equations were then utilized for the formulation of different optimized control strategies with a HIPWM (harmonic injection pulse width modulation) input for motoring and generating under different faulted conditions. The PM machine motoring with speed control and generating with DC-link voltage regulation was simulated for the surface PM machine system and the interior PM machine system presented earlier in the thesis. Performance characteristics with different optimizations such as minimized reactive power transfer, minimized torque ripple in motoring mode and minimized DC-link current ripple in generating mode were analysed. Based on the analysis performed, the multi-objective optimization is found to offer the capability to trade-off different parameters successfully. For electric vehicular power and propulsion applications, a controller based on the multi-objective optimization is proposed due to its flexibility to conform to given specifications of torque ripple / DC-link current ripple limits and field-weakening reactive power trade-off criteria.

In summary the following contributions are made by this thesis:
1. Development of a current-peak regulation based excitation control strategy for SR motors with application to aero-engine starter operation.

2. Enhancement of efficiency by the application of ZVL to the current-peak control strategy.

3. Voltage regulator design and the investigation of different optimal excitation control performances for SR generator operation.

4. Application of fixed turn-on / variable turn-off and fixed turn-off / variable turn-on angle based excitation control and comparison with optimized performance characteristics.

5. Development of a general mathematical model for the representation of healthy or faulty multiphase fault-tolerant PM machines considering the interface with the H-bridge per-phase converter architecture.

6. Development of multiphase PM motor operational and control strategies for normal and faulted conditions with consideration of the formulation of control signals for different optimization criteria.

7. Development of multiphase PM generator operational and control strategies for normal and faulted conditions with consideration of the formulation of control signals for different optimization criteria.

In addition to these contributions, the load sharing between multiple DC sources was also investigated as a part of the project. Although, this is not presented in this thesis, the outcomes of this study have been published in the first paper associated with the project given in section 9.2. This paper investigates two different load sharing control strategies for DC distribution systems such as that proposed for MEA. Droop based load sharing and master slave load sharing have been analysed in this publication.
9.2 Publications associated with the thesis


9.3 Recommendations for further work

1. The power generation with SR machines considered in this thesis is mainly in discontinuous conduction mode. However, certain advantages may be present in designing SR machines for continuous conduction mode of operation. Further scope exists in the area of machine design and development of optimal control strategies for the operation of SR machines in continuous condition mode.

2. The PM machine modelling of this thesis has been utilized for the design of control with HIPWM. The possibility exists to consider different forms of switching strategies as an alternative to HIPWM. For example, certain fault-tolerant PM machines / BLDC machines may opt for a quasi-square wave voltage waveform. Control design can be performed by modification of the model input variables to represent such a voltage waveform.
3. The investigation of the PM machine operation has been performed considering only open-circuit faulted conditions. However, the modelling can be also applied for the control design for short-circuit faulted operation and may follow a similar argument to that presented in this thesis.

4. The modelling performed on PM machines focus on the parallel H-bridge per-phase converter topology. This modelling may also be extended for other fault-tolerant converter topologies that exist or arise in the future.

5. The optimized control signals for the PM machine operation are calculated using standard SQP algorithms. This must be performed off-line at the stage of experimental implementation. However, further scope exists to develop strategies and simplified algorithms which compute the optimal control signals on-line.
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Appendix A

Design of a 12/8, 250kW SR Machine

This appendix presents the design calculations associated with the SR machine outlined in chapter three. This design considers an SR machine of 250kW rated power with 12/8 construction. The base-speed is selected at 15000rpm for the operation in single pulse mode of power generation in the range of 20000rpm to 60000rpm. The resulting FE modelling is presented in chapter three.

![Magnetization curve of Hiperco 50HS](image)

Figure A.1: Magnetization curve of Hiperco 50HS

Soft magnetic lamination materials such as cobalt iron are preferred for the design of machines that operate in high temperature environments [76]. Furthermore cobalt-iron materials offer superior mechanical and magnetic properties
compared with silicon-iron. Therefore in this study Hiperco-50HS iron-cobalt-vanadium soft magnetic alloy with high magnetic saturation characteristics is considered for the design of the SR machine. The corresponding magnetization curve of Hiperco 50HS is shown in figure A.1.

\section*{A.1 SR Machine sizing}

The design power equation for a SR machine is given by \cite{75}:

\[ P_o = \eta k_d k_2 \left( \frac{\pi}{4} \right) B_g A_{sp} D_i^2 L \omega_m \]  

(A.1)

where \( \eta \) is the efficiency, \( B_g \) is the air gap magnetic flux density at the aligned position, and \( A_{sp} \) is the specific electrical loading. The duty factor \( k_d \) and the inductance factor \( k_2 \) are given by \cite{75}:

\[ k_d = \frac{\theta_i N_s N_r}{4\pi} \quad \text{and} \quad k_2 = 1 - \frac{1}{\sigma \lambda_u} \]  

(A.2)

\( \theta_i \) is the nominal conduction angle, \( N_s \) and \( N_r \) are the number of stator and rotor poles. \( \sigma = L^{s}_{u}/L^{u}_{a} \) and \( \lambda_u = L^{u}_{a}/L_{a} \) are the ratios of aligned saturated inductance to aligned unsaturated inductance and aligned unsaturated inductance to unaligned inductance.

With assumed values for \( \eta \), \( k_d \), \( k_2 \) and design air gap flux density \( B_g \) and rated speed \( \omega_m \) and power \( P_o \), the product \( (D_i^2 L) \) can be found by,

\[ D_i^2 L = \frac{P_o}{\eta k_d k_2 \left( \frac{\pi}{4} \right) B_g A_{sp} \omega_m} \]  

(A.3)

\section*{A.2 Stator and rotor design for a 250kW SR Machine.}

The initial design assumes an efficiency of \( \eta = 0.95 \) and duty factor \( k_d = 1 \). The constant \( k_2 \) is typically in the range of \( 0.65 < k_2 < 0.75 \) \cite{75}, and it is assumed that \( k_2 = 0.7 \). For an aligned position operating air gap flux density of \( B_g = 1.75T \) and specific electrical loading of \( A_{s, rms} = 25000\text{Am}^{-1} \), the product
$D_i^2 L$ can be approximated to

$$D_i^2 L = \frac{250 \times 10^3}{0.95 \times 1 \times 0.7 \times \left(\frac{\pi}{4}\right) \times 1.75 \times 25000 \times 1570.8} = 0.007 m^3 \quad (A.4)$$

Aero-engine HP spool space constraints dictate the maximum length of the generator [76]. For a length $L = 0.12 m$, the resultant stator internal diameter can be approximated to $D_{si} = 0.24 m$. A 12/8 SR machine with three phases, i.e., $q = 3$ is chosen here instead of the basic 6/4 machine, as a high number of poles achieve a reduction in stator and rotor back-iron [76]. Furthermore the radial forces around a 12/8 machine are more evenly distributed compared with a 6/4 SR machine. A cross section of this machine and the physical parameters are shown in figure A.2.
APPENDIX A. DESIGN OF A 12/8, 250KW SR MACHINE

The rotor and stator pole-arcs, $\beta_r, \beta_s$ are selected as $\beta_s = 15^0$ and $\beta_r = 22.5^0$. These values satisfy the conditions stipulated in [77], i.e., $\min(\beta_r, \beta_s) \geq \frac{360}{qN_r}$ and $N_r + N_s \leq \frac{360}{N_r}$ and achieves the largest pole area. The rotor tooth height $h_{pr}$ is selected according to [167], where minimized unaligned inductance is sought. This is achieved by selecting the ratio between the circumferential distance between the edges of the rotor and state teeth to rotor pole height to be in the range of 1.5 to 2.

The circumferential distance between the edges of the rotor and state teeth for the selected pole-arcs can be calculated as

$$l_{rs,\text{edge}} = \frac{(22.5 - 15)}{2} \times \frac{\pi}{180} \times \frac{0.24}{2} = 0.0079m \tag{A.5}$$

Selecting the ratio $\frac{h_{rs,\text{edge}}}{h_{pr}} = 2$, the height of the rotor teeth can be calculated as:

$$h_{pr} = 2 \times 0.0079 = 0.0157m \tag{A.6}$$

The stator and rotor pole widths, $w_{ps}$ and $w_{pr}$ are given by [168]:

$$w_{ps} = 0.24 \times \sin \left( \frac{15}{2} \times \frac{\pi}{180} \right) = 0.0313m \tag{A.7}$$

$$w_{pr} = 0.24 \times \sin \left( \frac{22.5}{2} \times \frac{\pi}{180} \right) = 0.0468m \tag{A.8}$$

The stator and rotor back iron widths $w_{bs}$, $w_{br}$ are taken as $\frac{2}{3}$ of the total pole widths to accommodate the flux of more than one phase [76].

$$w_{bs} = \frac{2}{3} w_{ps} = 0.0209m \tag{A.9}$$

$$w_{br} = \frac{2}{3} w_{pr} = 0.0312m \tag{A.10}$$

The for an air gap of $g = 0.001m$, the diameter of the nonmagnetic portion/shaft of the rotor can be calculated as

$$D_{sh} = D_i - 2(h_{pr} + g + w_{br}) = 0.1442m \tag{A.11}$$

Keeping equal ratios of $h_r/b_{pr}$ and $h_s/b_{ps}$ [78], the stator pole height can be
calculated as:

\[ h_{ps} = \frac{h_{pr}}{w_{pr}} w_{ps} = 0.0105 \] (A.12)

The SR machine outer diameter is then given by,

\[ D_o = D_i + 2 (h_{ps} + w_{bs}) = 0.3028m \] (A.13)

For the given stator pole widths and stack length, the nominal flux per-pole can be expressed as [78]:

\[ \phi_{p,nom} = B_{sat} w_{sp} L (1 + k_l) \] (A.14)

Where \( k_l \) is the leakage constant. For a nominal flux density of \( B_{nom} = 1.75T \) and \( k_l = 0.2 \), the flux per-pole can be calculated as:

\[ \phi_{p,nom} = 1.75 \times 0.0313 \times 0.1 \times (1 + 0.2) = 0.0079Vs \] (A.15)

For a full angle conduction of \( \beta_s \), with given \( N_1 \) number of series turns per-phase, the generated nominal back-EMF magnitude \( E_b \) can be calculated as,

\[ E_b = \frac{N_1 \phi_{p,nom}}{\beta_s / \omega_{m,rated}} \] (A.16)

A back-EMF equal to the DC-link voltage is preferred at the rated speed, as this will lead to a relative increase in energy conversion [78]. Thus the required number of series turns per-phase can be calculated as,

\[ N_1 = \frac{E_b \beta_s}{\omega_m \phi_{p,nom}} \] (A.17)

For \( E_b = V_{dc} = 540V \), \( \beta_s = (\frac{15}{180}\pi) \text{rad} \), \( \phi_{p,nom} = 0.0079Vs \) and rated speed \( \omega_m = 1570.8 \text{rad/s} \),

\[ N_1 = \frac{540 \times \frac{15}{180}\pi}{1570.8 \times 0.0079} = 11.4007 \] (A.18)

Selection of the nearest integer yields \( N_1 = 12 \) series turns per-phase, i.e., 6 turns per pole. The window area available for a coil is given by:
Substitution of the calculated values yields \( A_w = 180.0116 \text{ mm}^2 \). The average current in one coil is given by

\[
I_{avg} = \frac{1}{a} \times \frac{P_0}{3V_{dc}} = 10.2874 \text{A} \tag{A.20}
\]

where \( a = 2 \) is the number parallel paths per-phase. Then for a typical filling factor of \( k_{fill} = 0.5 \) the slot average current density can be expressed as

\[
J_{avg} = \frac{N_1 I_{avg}}{A_w k_{fill}} \tag{A.21}
\]

Substitution of the above calculated values yields,

\[
J_{avg} = 10.2874 \text{A/mm}^2 \tag{A.22}
\]

Chapter three presents the FE modelling and the resulting electromagnetic characteristics of the machine.
Appendix B

Design of 125kW PM Machines with SPM Rotor and IPM Rotor Constructions

This appendix presents the design calculations associated with the PM machines outlined in chapter three. These designs consider PM machines of 125kW rated power and rated speed of 3000rpm. The resulting FE modelling is presented in chapter three.

Soft magnetic lamination materials such as cobalt iron are preferred for the design of machines that operate in high temperature environments [76]. Therefore the Hiperco-50HS iron-cobalt-vanadium soft magnetic alloy is considered here as the lamination material of the PM stator and rotor.

<table>
<thead>
<tr>
<th>Table B.1: Characteristics of SmCo magnetic material [1]</th>
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<tbody>
<tr>
<td>Magnet remnant flux density $B_r$ [T]</td>
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<tr>
<td>Intrinsic coercivity $H_c$ [MA/m]</td>
</tr>
<tr>
<td>Curie temperature [$^\circ$C]</td>
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<tr>
<td>Energy density [kJm$^{-3}$]</td>
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<tr>
<td>Temperature coefficient of $B_r$ [%/$^\circ$C]</td>
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<tr>
<td>Temperature coefficient of $H_c$ [%/$^\circ$C]</td>
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</table>

Typically PM material such as Neodymium Iron Boron (NdFeB), Samarium Cobalt (SmCo), Aluminum Nickel Cobalt (AlNiCo) alloys and bonded versions of these are used in PM machines. Aero-engine applications require magnetic materials that are capable of operation in high temperature environments. The
candidate PM material for this application is SmCo [2], due to its high tolerance of heat and capability to operate in extreme environments with temperatures as high as 350°C to 400°C. Table B.1 gives the characteristics of SmCo magnets used in the design of the PM rotor in the subsequent sections.

B.1 PM machine sizing

Standard machine design equations are utilized for the sizing of the machine. The power equation for a PM machine is given by [169]:

\[ P_{\text{rated}} = \eta q E_{\text{ph}} I_{\text{ph}} \]  \hspace{1cm} (B.1)

where \( P_{\text{rated}} \) is the rated power output and \( \eta, q, I_{\text{ph}} \) are the efficiency, number of phases and the RMS phase current respectively. However, due to the requirement of fault-tolerance, the machine is designed for an over-rated value such that failure of one phase does not limit the power delivered to the system [53]. This design power value can be calculated by:

\[ P_0 = \left( \frac{q}{q - 1} \right) P_{\text{rated}} \]  \hspace{1cm} (B.2)

The RMS phase back-EMF can be represented as [170,171]

\[ E_{\text{ph}} = \frac{1}{\sqrt{2}} N_1 K_w p \omega_m \phi \]  \hspace{1cm} (B.3)

where \( \phi, N_1, K_w \) and \( p \) are respectively the magnetic excitation flux, the number of series turns per-phase, the winding factor, and the number of pole pairs. The fundamental component of the magnetic flux without the armature reaction is given by [171]:

\[ \phi_1 = \frac{2}{\pi} B_{g1} L \tau_r \]  \hspace{1cm} (B.4)

where \( \tau_r = \frac{\pi D_i}{2p} \) is the rotor pole-pitch with \( D_i \) stator bore diameter. Substitution of (B.3) and (B.4) in (B.1) yields:

\[ E_{\text{ph}} = \frac{1}{\sqrt{2}} N_1 K_w B_g L D_i \omega_m \]  \hspace{1cm} (B.5)
where $\omega_m$, $L$ and $B_g$ are respectively the rated speed, stack length and air gap magnetic flux density. The specific electrical loading of the machine is given by [171]:

$$A_{s,rms} = \frac{2N_1 I_{ph} q}{\pi D_i} \quad (B.6)$$

Substitution of $N_1$ of (B.6) in (B.5) yields:

$$E_{ph} = \frac{1}{\sqrt{2}} \left( \frac{\pi D_i A_{s,rms}}{2 I_{ph} q} \right) K_w B_g LD_i \omega_m \quad (B.7)$$

The product $D_i^2 L$ can be equated to:

$$D_i^2 L = \frac{2 \sqrt{2} E_{ph} I_{ph} q}{\pi A_{s,rms} K_w B_g \omega_m} \quad (B.8)$$

This study considers the design of a PM machine rated for 100kW capability under one phase fault and with a rated speed of 3000rpm for the operation in a large civil aircraft engine LP spool. To accommodate fault-tolerant operation, the machine is designed for an over-rated value of 125kW. The electrical power of each phase is rectified to 540V. The generator initial design back-EMF is thus selected as

$$E_{ph} = \frac{540}{\sqrt{2}} = 381.84V \quad (B.9)$$

Fault-tolerant operation requires multiphase design for increased partial redundancy. It is known that out of three, four, five and six phase machines, the five phase machine exhibits the lowest torque ripple [82]. Thus a five phase machine design is considered here.

Permanent magnet machines with concentrated windings exhibit certain characteristics that are favorable for fault-tolerant machine operation. For example, such machines exhibit higher inductance compared with machines with distributed windings [85]. Furthermore the effect of end windings of machines with concentrated windings is smaller than that of distributed wound machines [172]. In addition these windings offer a simpler manufacturing process. Thus the machine design considered here adopts the concentrated winding technique. The need for minimal phase-to-phase mutual coupling in fault-tolerant machines dictates the selection of coil placement. By placing the conductors of one phase in
one slot, the mutual inductance is reduced [53]. By selection of the number of stator poles \( N_s = 20 \), two opposite poles are available for one phase of a concentrated winding.

The selection of the number of rotor poles governs the electrical frequency. Higher number of poles reduces the cogging torque [171] while the high electrical frequency leads to increased iron losses. The feasible maximum number of rotor poles is given by \( 2p = N_s - 2 \). To keep the electrical frequency at an acceptable level, \( p = 6 \) is chosen.

Assuming a worst-case efficiency of \( \eta = 0.95 \), the rated RMS phase current can be estimated as:

\[
I_{ph} = \frac{125 \times 10^3}{0.95 \times 5 \times 381.84} = 68.92 \text{A} \quad (B.10)
\]

Assuming a typical winding factor of \( K_w = 0.866 \) [173], design air gap magnetic flux density of \( B_g = 0.8 \text{T} \), and a specific electrical loading of \( A_{s,\text{rms}} = 12500 \text{Am}^{-1} \), the initial value for the product \( D^2_i L \) can be calculated as:

\[
D^2_i L = \frac{2\sqrt{2} \times 381.84 \times 68.92 \times 5}{\pi \times 12500 \times 0.866 \times 0.8 \times 314.16} = 0.0435 \text{m}^3 \quad (B.11)
\]

Aero-engine space constraints dictate the maximum length of the generator. For a length \( L = 0.25 \text{m} \), the resultant stator internal diameter can be approximated to \( D_{si} = 0.42 \text{m} \). The rotor tip speed is given by:

\[
V_{\text{tip}} = \frac{1}{2} D_i \omega_m \quad (B.12)
\]

At \( \omega_m = 314.16 \text{rad}\text{s}^{-1} \), the rotor tip speed is calculated as \( V_{\text{tip}} = 65.97 \text{ms}^{-1} \). This speed is well below the typical acceptable upper limit of 200ms\(^{-1}\). The air gap shear stress value is given by:

\[
\tau_{sh} = \frac{P_0}{\pi D_i L V_{\text{tip}}} \quad (B.13)
\]

For the machine considered here, the shear stress value is obtained as 5.7kPa or 0.83psi. This value is within the acceptable limit (< 5psi) for air-cooled PM machines [174,175].
B.2 PM machine stator design

Stator design involves calculation of the winding parameters, pole and back-iron dimensions of the stator. Equation (B.6) can be used to express the number of series turns per-phase as:

\[ N_1 = \frac{A_{s,rms} \pi D_i}{2 I_{ph} q} \]  

(B.14)

The stator pole width \( w_{ps} \) is calculated such that the pole flux density is rated at a specific value. Given the air gap flux density, the stator pole flux density can be expressed as:

\[ B_{ps} = \frac{\tau_s}{w_{ps}} B_g \]  

(B.15)

where \( \tau_s \) is the stator slot pitch, given by \( \tau_s = \frac{\pi D_i}{N_{slot}} \) with \( N_{slot} \) number of slots.

Similarly the stator back-iron width is designed such that the flux density of the stator back-iron is rated at a specific value. Half of the stator pole flux passes through the stator back iron. Then the stator back-iron height for a rated flux density of \( B_{bs} \) is given by [78,176]:

\[ h_{bs} = \frac{\phi_p}{2LB_{bs}} \]  

(B.16)

where the PM flux per pole pitch \( \phi_p \) is given by:

\[ \phi_p = B_g \tau_r L \]  

(B.17)

\( \tau_r \) is the rotor pole-pitch. Typically \( h_{bs} \) is selected such that,

\[ h_{bs} \geq \frac{B_g \tau_r}{2B_{bs}} \]  

(B.18)

is satisfied. The slot height is calculated in order to satisfy the current density criteria of \( J_{rms} < J_{max} \), where \( J_{max} \) is the maximum allowable current density. Then for a filling factor of \( k_{fill} \) the slot current density can be expressed as:

\[ J_{rms} = \frac{N_1 I_{rms}}{2 A_w k_{fill}} \]  

(B.19)
where $A_w$, the window area available for a coil [6] is related to the stator pole height $h_{ps}$ by,

$$A_w = \frac{1}{2N_s} \left[ \frac{\pi}{4} ((D_i + 2h_{ps})^2 - D_i^2) - N_s w_{ps} h_{ps} \right]$$  \hspace{1cm} (B.20)

For a given value of $A_w$, the solution to (B.20) is given by:

$$h_{ps} = \frac{-(\pi D_i - N_s w_{ps}) + \sqrt{((\pi D_i - N_s w_{ps})^2 + 8\pi N_s A_w) / 2\pi}}{2\pi}$$ \hspace{1cm} (B.21)

For the calculated stator bore diameter of $D_i = 0.42\text{m}$, $N_1$ can be found by (B.14)

$$N_1 = \frac{12500 \times \pi \times 0.42}{2 \times 68.92 \times 5} = 23.93$$ \hspace{1cm} (B.22)

Rounding $N_1$ to the nearest integer yields $N_1 = 24$ turns per-phase or 12 turns per pole in series.

Substitution of $N_{slot} = 20$, $D_i = 0.42\text{m}$, $B_g = 0.8\text{T}$ and for a maximum of $B_{ps} = 1.2\text{T}$ flux density in the stator pole, the pole width can be calculated by (B.15) as:

$$w_{ps} = \frac{\pi \times 0.42 \times 0.8}{20 \times 1.2} = 0.044\text{m}$$ \hspace{1cm} (B.23)

The PM flux per pole pitch can be calculated by (B.17) as,

$$\phi_p = 0.8 \times 0.11 \times 0.25 = 0.022\text{Wb}$$ \hspace{1cm} (B.24)

Substitution of $B_g = 0.8\text{T}$ and for a maximum of $B_{bs} = 0.8\text{T}$ the minimum value of (B.18) is selected as the stator back-iron height.

$$h_{bs} = \frac{0.8 \times 0.11}{2 \times 0.8} = 0.055\text{m}$$ \hspace{1cm} (B.25)

For a typical filling factor of $k_{fill} = 0.6$, $J_{rms} = J_{max} = 6\text{Amm}^{-2}$ and for a RMS current value $I_{rms}$ of 150% of the rated value considering field-weakening power-factor, the required slot area can be calculated by (B.19) as:

$$A_w = \frac{24 \times 1.5 \times 68.92}{2 \times 6 \times 0.6} = 344.6\text{mm}^2$$ \hspace{1cm} (B.26)
With an allowance of 20% area for the slot wedge and an allowance of 0.005m for the slot insulation, the stator pole height can be calculated by (B.21) for $A_w = 1.2 \times 344.6\text{mm}^2$ as

$$h_{ps} = 0.0358\text{m} \quad (B.27)$$

Approximately half of the pole flux may pass through the pole edge at the rated pole flux. In order to accommodate the stator pole fluxes and avoid pole edge saturation, the pole edge height $h_{p,edge}$ (see figure B.2 and B.2 for the dimension) is selected as:

$$h_{p,edge} = \frac{w_{ps}}{2} = 0.022\text{m} \quad (B.28)$$

In order to reduce the cogging torque to a minimum, the stator slot opening $w_{os}$ is selected at approximately equal to the air gap width, $w_{os} \approx g [78]$.  

### B.3 PM machine rotor design

Rotor design for a PM machine involves the calculation of the permanent magnet width and the rotor back-iron height. Two rotor constructions are considered here, viz., the SPM construction and IPM construction.

#### B.3.1 SPM rotor design

Figure B.1 shows the magnetic equivalent circuit of a surface PM machine. Figure B.2 shows the corresponding physical dimensions of the machine cross section. $\phi_r$, $\phi_m$, and $\phi_g$, represents the PM magnet remnant flux, magnet flux and the air gap flux respectively. Reluctances $R_{mo}$, $R_l$ and $R_g$ represents the magnet internal reluctance, leakage reluctance and the air gap reluctance respectively.

The air gap flux is related to the remnant flux by:

$$\phi_g R_g = R_m (\phi_r - \phi_g) \quad (B.29)$$

The modified magnet internal permeance is given by [6]:

$$P_m = P_{mo} + 2P_l \quad (B.30)$$
Figure B.1: Magnetic equivalent circuit of a surface PM machine [6].

where $P_{m0} = \frac{1}{R_{m0}}$ and $P_l = \frac{1}{R_l}$. The air gap flux density $B_g$ can be related to magnet remnant flux density $B_r$ by:

$$B_g = \frac{C_\phi}{(1 + P_m R_g)} B_r \quad (B.31)$$

where $C_\phi$, $R_g$, and $P_m = \frac{1}{R_m}$ are the flux concentration factor, air gap reluctance and the modified magnet internal permeance respectively. The flux concentration factor is given in terms of $A_m$, $A_g$, i.e., the cross sectional areas per-pole of the magnet and air gap respectively.

$$C_\phi = \frac{A_m}{A_g} \quad (B.32)$$

$A_m$ and $A_g$ can be estimated by [6]:

$$A_m \approx \beta_r \times \left( \frac{D}{2} - g - \frac{h_{pm}}{2} \right) \times L \quad (B.33)$$

$$A_g \approx \left[ \beta_r \times \left( \frac{D}{2} - g \right) + 2g \right] \times (L + 2g) \quad (B.34)$$

$P_l$ is typically a fraction of $P_{m0}$. Thus $P_m$ can be written as $P_m = k_{pr} P_{m0}$. 
Figure B.2: Cross section of the surface PM machine

\( k_{pr} > 1 \) [6]. The magnet internal permeance \( P_{m0} \) is given by

\[
P_{m0} = \frac{\mu_0 \mu_{rec} A_m}{h_{pm}}
\]

where \( \mu_{rec} \) is the relative recoil permeance and \( \mu_0 \) is the permeability of free space. The air gap reluctance can be calculated by

\[
R_g = \frac{K_c g}{\mu_0 A_g}
\]

where \( g \) the air gap length, \( K_c \) is the carter coefficient which takes slot width and
length effects into account. The carter coefficient can be estimated as [176]

\[ K_c = \left(1 - \frac{w_s}{\tau_s} + \frac{4g}{\pi \tau_s} \ln \left(1 + \frac{\pi w_s}{4g}\right) \right)^{-1} \] (B.37)

where \( w_s = \tau_s - w_{ps} \). Substitution of (B.35), (B.36) and (B.37) in (B.31) yields:

\[ B_g = \frac{C_\phi}{\left(1 + \left(\frac{\mu_0 \mu_{rec} A_m}{h_{pm}}\right)\left(\frac{K_c g}{\mu_0 A_g}\right)\right) B_r} \] (B.38)

For a given magnet remnant flux density \( B_r \) and design air gap flux density \( B_g \), the required surface PM height can be calculated as:

\[ h_{pm} = \frac{\mu_{rec} C_\phi K_c g}{C_\phi B_r B_g - 1} \] (B.39)

The optimal ratio of pole-arc to pole pitch \( \alpha_p = \frac{\beta_r}{\tau_r} \) that minimizes the cogging torque is given by [177]

\[ \alpha_p = \frac{N_\alpha - k_1}{N_\alpha} + k_2 \] (B.40)

where \( k_1 = 1, 2, ..., N_\alpha - 1, 0.01 \leq k_2 \leq 0.03 \) and \( N_\alpha = \frac{LCM(N_{slot}, 2p)}{2p} \).

For the machine considered here, \( N_\alpha = 5, k_1 = 1 \) and \( k_2 = 0.03 \) yields \( \alpha_p = 0.83 \). Substitution of \( w_{ps} = 0.044m, \tau_s = 0.066m, g = 0.001m \) in (B.37) the carter coefficient is obtained as \( K_c = 1.36 \). For SmCo permanent magnets with \( B_r = 1.1T, \) recoil permeability \( \mu_{rec} = 1.05 \) and a design air gap flux density \( B_g = 0.8T \), the PM height can be iteratively estimated via equations (B.32), (B.33), (B.34) and (B.39) yielding \( C_\phi = 0.9624 \) with \( A_m = 0.0224m^2 \) and \( A_g = 0.0233m^2 \). PM height is obtained as \( h_{pm} = 0.0051m \).

The rotor back iron height \( h_{br} \) is selected such that the flux density is kept below a rated value of \( B_{br} \) by \( h_{br} \geq \frac{B_s \tau_{pm}}{2B_{br}} \). Selection of \( B_{br} = 0.8T \) and a 10% allowance for leakage, the rotor back iron height is selected as \( h_{br} = \frac{h_{bs}}{0.9} \), resulting in \( h_{br} = 0.0611m \).

Figure B.2 shows the cross section and dimensions of the machine with the surface PM rotor construction. Finite Element Modelling (FEM) of this machine is presented in chapter three.
Figure B.3: Magnetic equivalent circuit of an interior PM machine [7].

Figure B.3 shows the magnetic equivalent circuit of an interior PM machine with rotor topology as shown in figure B.4. Parameters $R_{mo}$, $R_{ml}$, $R_{mm}$ and $R_g$ represent the reluctances of the magnet, flux leakage through the air under the bridge, flux leakage through the bridge and the reluctance of the air gap respectively. The stator and rotor back iron is assumed to be infinitely permeable. These reluctances are given by [7]:

\[
R_{mo} = \frac{1}{P_{m0}} = \frac{h_{pm}}{\mu_0 \mu_{rec} A_m} \tag{B.41}
\]

\[
R_{ml} = \frac{4d}{\mu_0 L (h_1 + h_2)} \tag{B.42}
\]

\[
R_g = \frac{K_c g}{\mu_0 A_g} \tag{B.43}
\]

where $h_1$ and $h_2$ are shown in figure B.4. For a average flux density of $B_{bdg}$ through the bridge, $R_{mm}$ can be found by [178]:

\[
R_{mm} = \frac{1}{4} \left( \frac{A_{bdg}}{A_{bdg}} \left( \frac{B_{bdg}}{B_{bdg}} \right) - 1 \right) \tag{B.44}
\]

where the cross sectional areas of the bridge, per pole of the magnet and air gap
are given by $A_{bdg} = w_{bdg}L$, $A_m = w_mL$, and $A_g \approx [\beta_r \times (\frac{D}{2} - g) + 2g] \times (L + 2g)$. Parameters $w_{bdg}$ and $w_m$ correspond to the width of the bridge and the width per pole of the magnet as shown in figure B.4.

According to the magnetic equivalent circuit B.3, the air gap flux is related to the remnant flux by:

$$4\phi_g R_g = R_{eq} (\phi_r - \phi_g) \quad (B.45)$$

where $R_{eq} = \frac{1}{\frac{1}{R_{mo}} + \frac{1}{R_{ml}} + \frac{1}{R_{mm}}}$. Rearranging (B.45), the air gap flux-density $B_g$ can be related to $B_r$ via the
concentration factor $C_\phi = \frac{A_m}{A_g}$ by:

$$B_g = \left( \frac{C_\phi}{1 + \frac{4R_g}{R_{eq}}} \right) B_r$$

(B.46)

Substitution of $R_{eq}$ and (B.41) in (B.46) yields an expression for $h_{pm}$ as:

$$h_{pm} = \frac{\mu_0 \mu_{rec} A_m R_g}{C_\phi B_g - \left( 1 + \frac{2R_g}{R_{ml}} + \frac{4R_g}{R_{mm}} \right)}$$

(B.47)

Iterative solution of (B.41), (B.42), (B.43), (B.44) and (B.47) yields $h_{pm} = 0.02m$ for $C_\phi = 1.3574$, $B_{bg} = 0.43T$, $w_m = 0.0926m$, $d = 0.0052m$, $h_1 = 0.0101m$, and $h_2 = 0.0221m$. 
Appendix C

Validation of the FE Model Based Flux-linkage Characteristics Replication

This appendix presents the validation of the FE model generated flux-linkage characteristics obtained for the test-bed 300W SR machine. This is done by feeding the experimental phase voltage waveforms to a time-stepped simulation model incorporating an inverse flux-linkage look-up table of the SR machine. The experimentally obtained current waveforms are compared with the current waveforms output by the inverse flux-linkage look-up table. This comparison is performed at different speed and load conditions. Table C.1 gives the different parameters at which this simulation and experiments are performed. The results are discussed in chapter three.
### APPENDIX C. VALIDATION OF $\psi$-i REPLICATION

Table C.1: Different excitation parameters, speed and load conditions considered in the experiment. (turn-on angle specified with reference to unaligned position -12.5° rotor position)

<table>
<thead>
<tr>
<th>Condition</th>
<th>Speed [rpm]</th>
<th>turn-on angle [deg]</th>
<th>Dwell angle [deg]</th>
<th>Average load torque [Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500</td>
<td>8.5</td>
<td>2.40</td>
<td>0.11065</td>
</tr>
<tr>
<td>2</td>
<td>500</td>
<td>8.5</td>
<td>6.03</td>
<td>0.57258</td>
</tr>
<tr>
<td>3</td>
<td>500</td>
<td>9.5</td>
<td>11.02</td>
<td>1.41350</td>
</tr>
<tr>
<td>4</td>
<td>1000</td>
<td>5</td>
<td>3.96</td>
<td>0.14941</td>
</tr>
<tr>
<td>5</td>
<td>1000</td>
<td>5</td>
<td>8.61</td>
<td>0.61526</td>
</tr>
<tr>
<td>6</td>
<td>1000</td>
<td>0</td>
<td>16.34</td>
<td>1.66450</td>
</tr>
<tr>
<td>7</td>
<td>1100</td>
<td>6.5</td>
<td>21.00</td>
<td>1.51640</td>
</tr>
<tr>
<td>8</td>
<td>1500</td>
<td>5.5</td>
<td>15.90</td>
<td>0.67805</td>
</tr>
<tr>
<td>9</td>
<td>1500</td>
<td>14.5</td>
<td>12.84</td>
<td>0.21280</td>
</tr>
<tr>
<td>10</td>
<td>1963</td>
<td>-1.5</td>
<td>26.60</td>
<td>1.24250</td>
</tr>
<tr>
<td>11</td>
<td>2380</td>
<td>-4.5</td>
<td>36.00</td>
<td>1.25100</td>
</tr>
<tr>
<td>12</td>
<td>2500</td>
<td>-4.5</td>
<td>18.54</td>
<td>0.71594</td>
</tr>
<tr>
<td>13</td>
<td>2500</td>
<td>4.5</td>
<td>12.70</td>
<td>0.22738</td>
</tr>
<tr>
<td>14</td>
<td>3000</td>
<td>5.5</td>
<td>30.75</td>
<td>0.19398</td>
</tr>
<tr>
<td>15</td>
<td>3000</td>
<td>-4.5</td>
<td>21.00</td>
<td>0.55445</td>
</tr>
<tr>
<td>16</td>
<td>3000</td>
<td>-6.5</td>
<td>29.14</td>
<td>1.11220</td>
</tr>
</tbody>
</table>
Figure C.1: (a), (c), (e), (g) Comparison between current waveforms, at different speed, load, turn-on and turn-off angles. (b), (d), (f), (g) Corresponding $\psi-i$ trajectory comparison.
APPENDIX C. VALIDATION OF $\psi$-$i$ REPLICATION

Figure C.2: (a), (c), (e), (g) Comparison between current waveforms, at different speed, load, turn-on and turn-off angles. (b), (d), (f), (h) Corresponding $\psi$-$i$ trajectory comparison.
Figure C.3: (a), (c), (e), (g) Comparison between current waveforms, at different speed, load, turn-off and turn-off angles. (b), (d), (f), (g) Corresponding $\psi$-$i$ trajectory comparison.
Figure C.4: (a), (c), (e), (g) Comparison between current waveforms, at different speed, load, turn-of and turn-off angles. (b), (d), (f), (g) Corresponding $\psi$-$i$ trajectory comparison.
Appendix D

Validation of the FE Model Based Switching Simulation of the 300W SR Machine

This appendix presents the validation of the switching simulation of the asymmetric half bridge converter combined with the time-stepped SR machine inverse flux-linkage model. This comparison is performed with the same results of appendix C, i.e. at the same speed and load conditions. The results are discussed in chapter three.
Figure D.1: Comparison between FE model based switching simulation and experimental waveforms at a speed of 500 rpm and load torque of 0.11065 Nm
Figure D.2: Comparison between FE model based switching simulation and experimental waveforms at a speed of 500 rpm and load torque of 0.57258 Nm
Figure D.3: Comparison between FE model based switching simulation and experimental waveforms at a speed of 500 rpm and load torque of 1.4135 Nm
Figure D.4: Comparison between FE model based switching simulation and experimental waveforms at a speed of 1000 rpm and load torque of 0.14941 Nm
Figure D.5: Comparison between FE model based switching simulation and experimental waveforms at a speed of 1000 rpm and load torque of 0.61526 Nm.
APPENDIX D. VALIDATION OF THE SWITCHING SIMULATION

Figure D.6: Comparison between FE model based switching simulation and experimental waveforms at a speed of 1000 rpm and load torque of 1.6645 Nm
Figure D.7: Comparison between FE model based switching simulation and experimental waveforms at a speed of 1100 rpm and load torque of 1.5164 Nm
Figure D.8: Comparison between FE model based switching simulation and experimental waveforms at a speed of 1500 rpm and load torque of 0.67805 Nm
Figure D.9: Comparison between FE model based switching simulation and experimental waveforms at a speed of 1500 rpm and load torque of 0.2128 Nm
Figure D.10: Comparison between FE model based switching simulation and experimental waveforms at a speed of 1963 rpm and load torque of 1.2425 Nm
Figure D.11: Comparison between FE model based switching simulation and experimental waveforms at a speed of 2380 rpm and load torque of 1.251 Nm
Figure D.12: Comparison between FE model based switching simulation and experimental waveforms at a speed of 2500 rpm and load torque of 0.71594 Nm
Figure D.13: Comparison between FE model based switching simulation and experimental waveforms at a speed of 2500 rpm and load torque of 0.22738 Nm
Figure D.14: Comparison between FE model based switching simulation and experimental waveforms at a speed of 3000 rpm and load torque of 0.19398 Nm
Figure D.15: Comparison between FE model based switching simulation and experimental waveforms at a speed of 3000 rpm and load torque of 0.55445 Nm
Figure D.16: Comparison between FE model based switching simulation and experimental waveforms at a speed of 3000 rpm and load torque of 1.1122 Nm
Appendix E

General electro-magnetic energy conversion of SR motors

E.1 Introduction

Figure E.1: Typical magnetization characteristic curves of a SRM

The SRM is a doubly salient machine i.e., the machine characteristics vary with rotor position and flux density. Figure E.1 shows the typical magnetization characteristics of a SRM for different rotor positions. The level of saturation is insignificant in the unaligned position and the characteristic curve is effectively linear. In contrast, the aligned position exhibits significant saturation of the core
with a high level of flux linkage at higher current levels due to the overlapping of the stator and rotor poles. As a result, the dynamic model of the machine is nonlinear and control design becomes complicated. The basic electromagnetic model is:

\[ v = R_i + \frac{d\psi}{dt} \]  

(E.1)

Equation (E.1) can also be written as:

\[ v = R_i + \left( \frac{\partial \psi}{\partial i} \right) \frac{di}{dt} + \left( \frac{\partial \psi}{\partial \theta} \right) \frac{d\theta}{dt} \]  

(E.2)

The two partial differentials can be considered as inductance and back-EMF:

\[ e_b = \omega \left( \frac{\partial \psi}{\partial \theta} \right) \]  

(E.3)

\[ L(\theta, i) = \left( \frac{\partial \psi}{\partial i} \right) \]  

(E.4)

This results in

\[ v = R_i + L(\theta, i) \frac{di}{dt} + e_b(\theta, i) \]  

(E.5)

SRM electromechanical energy conversion is best analysed by means of the principle of virtual work (co-energy) due to the magnetic nonlinearity. Typically the two variables that represent stored energy \( W_f \) and co-energy \( W_c \) are defined as:

\[ W_f = \int_{\psi=0}^{\psi} i(\psi, \theta) \, d\psi \]  

(E.6)

\[ W_c = \int_{i=0}^{i} \psi(i, \theta) \, di \]  

(E.7)

Figure E.1 shows the areas in the magnetization characteristic curves that correspond to \( W_c \) and \( W_f \) for a fixed rotor position. Then the electromagnetic
torque produced is given by:

\[ T_e(\theta, i) = \left( \frac{\partial W_e}{\partial \theta} \right) \]  

(E.8)

The mechanical shaft dynamic equation is given by:

\[ J \frac{d^2 \theta}{dt^2} + B \frac{d\theta}{dt} = T_{tot} - T_{load} \]  

(E.9)

where \( T_{load} \) is the load torque and \( T_{tot} = \sum_{n=1}^{N_p} \{ T_e(n, \theta, i_{ph,n}) \} \) is the total electro-magnetic torque with \( N_p \) number of phases, \( N_r \) number of rotor poles and \( N_s \) number of stator poles. \( J \) and \( B \) represent the motor shaft inertia and friction coefficient. Equations (E.1) to (E.9) provide the continuous-time dynamic model for the SRM machine in motoring mode.

It can be noted that knowledge of the magnetization characteristics is essential to analyse the operation of the SRM. Typically this information is derived via finite element modelling (FEM) techniques.

### E.2 Averaged torque control of SR motors

The majority of SR motor control methods based on current chopping and single pulse mode of operation are average torque control applications [101, 109, 110, 112, 179–181], where the focus is on the energy conversion of each stroke. While high-grade control of SR motors [87] may attempt dynamic variation of the current profile, average torque control emphasizes the application of a fixed command at each stroke period. While current chopping operation can be successfully implemented in the low and medium speeds of operation, high speed operation of SRMs naturally imposes an electrical limitation on rapid controllability of current and torque. This is due to the generation of a high level of phase back-EMF during the stroke period, resulting in insufficient inverter voltage to perform current chopping. Thus the option of single pulse switching and control of average torque is typically adopted. At high speeds, the time of a stroke period is reasonably small and also the dynamic performance of the averaged control methods can be made comparable to high-grade control methods used in low speed operation of SRMs.
E.3 Per-stroke electro-magnetic characteristics

![Energy conversion loop per-stroke of a SR motor](image)

Per-stroke system variables such as energy conversion ratio, average torque per-stroke, torque ripple, RMS phase current and peak flux linkage are often used for evaluation of the drive performance. The relationship between these measures and the system efficiency is presented in the following section. Figure E.2 shows the energy conversion loop for one stroke of one SR motor phase for a reference current of $i_{ref}$. The total mechanical energy delivered per-stroke is given by the grey area enclosed in figure E.2. This is calculated by,

$$W_{mech} = \oint \psi di$$  \hspace{1cm} (E.10)

The total electrical energy delivered into the phase winding is given by the area OABCO, which is given by the integration,

$$W_{elec} = \int_{\theta=\theta_{on}}^{\theta=\theta_{off}} id\psi$$  \hspace{1cm} (E.11)

The unutilized magnetic field energy is returned to the DC link at the end of the stroke. This energy is given by the area OABO, which can be calculated by the
integration,

\[
W_{mag} = \int_{\theta=\theta_{off}}^{\theta=\theta_{ext}} id\psi 
\]  \hspace{1cm} (E.12)

The total mechanical energy delivered per-stroke is related to (E.11) and (E.12) by [182],

\[
W_{mech} = W_{elec} - W_{mag} 
\]  \hspace{1cm} (E.13)

The energy conversion ratio in motoring mode can then be expressed as a ratio between the energy delivered into the phase winding and the energy converted into mechanical energy as,

\[
\varepsilon = \frac{W_{mech}}{W_{elec}} 
\]  \hspace{1cm} (E.14)

The average torque production by the \(n^{th}\) phase is given by [182],

\[
T_{avg,n} = \frac{N_r W_{mech}}{2\pi} 
\]  \hspace{1cm} (E.15)

The torque ripple magnitude is defined as,

\[
T_{rip} = T_{max} - T_{min} 
\]  \hspace{1cm} (E.16)

where \(T_{max}\) and \(T_{min}\) are the minimum and maximum points of the total instantaneous torque waveform. The torque ripple factor is defined as [110],

\[
k_{rip} = \frac{T_{rip}}{T_{avg}} 
\]  \hspace{1cm} (E.17)

Where \(T_{avg}\) is the total average torque.

The RMS phase current is defined as,

\[
\dot{i}_{rms} = \sqrt{\frac{N_r \omega}{2\pi} \int_{\theta=\theta_{off}}^{\theta=\theta_{ext}} i^2 \, dt} 
\]  \hspace{1cm} (E.18)

The peak flux linkage value \(\psi_{max}\) is the maximum point of the \(\psi\) waveform.
E.4 Connection with system losses and efficiency

The parameters defined in section E.3 are directly related to the system losses. The total loss within the motor is characterized by [111],

\[ P_{\text{loss}} = P_{\text{cu}} + P_{\text{hy}} + P_{\text{ed}} \]  \hspace{1cm} (E.19)

where \( P_{\text{cu}} \) is the copper loss, \( P_{\text{hy}} \) is the hysteresis loss, and \( P_{\text{ed}} \) is the eddy current losses. These can be expressed as [111],

\[ P_{\text{cu}} = N_p R_i r_{\text{rms}}^2 \]  \hspace{1cm} (E.20)
\[ P_{\text{hy}} = k_1 f \psi^k_2 \]  \hspace{1cm} (E.21)
\[ P_{\text{ed}} = k_3 f^2 \psi \]  \hspace{1cm} (E.22)

Here, \( k_1, k_2, \) and \( k_3 \) are empirical constants. \( f \) represents the frequency of the flux-linkage waveform. The losses within the asymmetric half bridge converter are also related to \( i_{\text{rms}}, \varepsilon \) and the number of switching occurrences within one stroke. While \( i_{\text{rms}} \) is a representation of the copper losses within the motor and the part of the conduction losses within the converter, \( \varepsilon \) represents the level of energy interchange through the converter. A higher \( \varepsilon \) is preferred as this would yield a lower amount of energy return to the DC link than a lower \( \varepsilon \), thus lowering the converter losses. The eddy current and hysteresis losses are related to the magnitude of the flux-linkage \( \psi_{\text{max}} \). The maximum value of the flux-linkage waveform is thus representative of the level of eddy-current and hysteresis losses and a low value of \( \psi_{\text{max}} \) is preferred.

The high turn-on and turn-off losses within IGBTs make the number of switching occurrences at high power an important factor related to converter losses. Excitation control methods that achieve a lower number of switching instances are preferred and techniques such as soft-chopping have been proposed in the literature to mitigate the effect of converter switching losses.
Appendix F

Detailed description of the enhanced excitation controller formulation with the current-peak feedback technique

F.1 Description of technique

The basis of the turn-on angle calculation for the conventional excitation control strategy, is to achieve the first peak of the phase current at the starting angle of the rising inductance $\theta_1$. The advance angle calculation for operation below base-speed assumes that the effect of winding resistance, fringing effects and back-EMF are negligible. This assumption cannot be substantiated near base-speed and for operation above base-speed due to the high back-EMF of the SR motor. This leads to lowering of torque production capability of the SR motor in these regions, and can be clearly seen from figures 4.7 (a) and (b) where a significant drop in torque production can be seen near the base-speed. In this section, a turn-on angle calculation technique that automatically compensates these effects is developed. This provides the applicability of the excitation control to SR motors without accurate knowledge of the motor parameters, or even without measurement of the DC link voltage.

In order to rectify this issue, many alternative advance angle correction methods have been proposed in the past [109, 183, 184]. The technique proposed by [184] iteratively adjusts the turn-on angle by detecting the difference in the
rotor position of the actual first peak of the current and the required position. Upon the change of the $i_{ref}$ command or speed, the iteration is reset. The scheme in [184] requires measurement of the rotor position at the current-peak value. The technique proposed by [183] adopts a similar closed-loop advance angle calculation technique, where the position of the first current-peak is monitored and adjusted. In contrast, the technique proposed in this section considers sampling of the phase current value at the $\theta_1$ position and adjustment of the advance angle in the form of a discrete time dynamic controller. This technique is less computationally intensive compared with the methods proposed by [109,183,184] where detection of the current-peak and retrieval of the corresponding position value has to be accomplished every stroke.

![Figure F.1: Three current pulses at different speeds and turn-on angles in single pulse mode of operation](image)

It is well known that in single pulse mode of SR motor operation, the current-peak occurs at the $\theta_1$ position [109]. This is shown in figure F.1 for three different turn-on angle values for the 250kW SR motor simulation. However in below base-speed chopping operation, the rotor position at which the current-peak occurs is determined by the turn-on angle and is typically at $\theta_1$. The current-peak value occurs near the $i_{ref}$ command due to the hysteresis current controller operation.

Figures F.2 (a), (b) and (c) show three instances of chopping operation for the 250kW SR motor simulation. Figure F.2 (a) requires further advancing of the turn-on angle to set the first peak at $\theta_1$. This can be accomplished by a feedback of the difference of the current-peak command $i^*_k$ and the current value sampled
Figure F.2: Three current pulses at different speeds and turn-on angles in current chopped mode of operation

at position $\theta_1$, $i_k$. In the case of chopping control $i_{ref} = i_k$. Figure F.2 (b) shows perfect matching of the current-peak position with $\theta_1$, and the feedback would ideally be zero. Figure F.2 (c) shows an instance where the advance angle is excessive. However, the sampled current value at $\theta_1$ is equal to the $i_{ref}$ value and thus the feedback control of advance angle is ineffective. Excessive advancing of the turn-on angle can be treated by incorporation of dynamic attenuation of the turn-on angle, thus stabilizing the turn-on angle at a minimal error between the $i_{ref}$ and the current at $\theta_1$.

Figure F.3: Two current pulses at the same speed. Different current-peak commands.
In the conventional excitation control method explained earlier, the variable $i_{\text{ref}}$ represents the hysteresis current reference and is varied according to the torque requirement for operation below base-speed. In contrast, $i_{\text{ref}}$ is kept at a fixed maximum value in operation above base-speed. This discrete transition of $i_{\text{ref}}$ is implemented considering the maximum current-peak conduction condition, where the back-EMF exactly equals the DC-link voltage. In the proposed current-peak control strategy, the current-peak command $i_k^*$ is the same as the reference current value $i_{\text{ref}}$ for operation below base-speed. However, in the proposed method, as the motor transits above base-speed, the hysteresis current value is not increased to the maximum value as this would result in unnecessary peak currents. At high current-peak commands above base-speed operation, the feedback controller automatically adjusts the turn-on angle in-order to achieve the commanded current-peak at $\theta = \theta_1$, after which the current decays. However, lower current conduction at the same speed will produce a lower back-EMF resulting in an uncontrolled increase of current beyond the current-peak command $i_k^*$, with the hysteresis reference $i_{\text{ref}}$ at a maximum value. An example of this is shown in figure F.3 for the 250kW SR motor operation above its base-speed. The red current waveform is for a current-peak command of 100A, however the current progresses beyond 200A due to the higher hysteresis current reference. The black current waveform is for a 390A command which is successfully satisfied due to the high back-EMF. Thus the requirement of current chopping even above base-speed is noted. However, setting the $i_{\text{ref}}$ at $i_k^*$ will not be effective as this will lead to a conflict with the current-peak feedback control. Hence the hysteresis current reference is set slightly higher, i.e at:

$$i_{\text{ref}} = i_k^* + \Delta i_{\text{mgn}} \quad (F.1)$$

where $\Delta i_{\text{mgn}}$ represents the current margin to activate above base-speed low current chopping operation.
Figure F.4: Discrete-time index, sampling instant and state-variable relationship
F.2 Controller formulation

It was explained earlier in the chapter that the phase dynamics are given by the equation:

\[ v = Ri + L(\theta, i) \frac{di}{dt} + e_b(\theta, i) \]  \hspace{1cm} (F.2)

The phase current is the solution of (F.2):

\[ i(t) = i(t_0) + \int_{t=t_0}^{t} \frac{1}{L(\theta, i)} \{v - Ri - e_b(\theta, i)\} dt \]  \hspace{1cm} (F.3)

Taking \( t = t_0 \) at an angle of \( \theta = \theta_1 \) positions \( i(t_0) = 0 \). An approximate expression for the current at \( t = t_1 \) when \( \theta = \theta_1 \) can written by considering the interval \( \theta_1 - \theta_{adv} < \theta < \theta_1 \) as:

\[ i(t_1) = \frac{v_{dc}}{L_u}(t_1 - t_0) + \int_{t=t_0}^{t_1} \frac{1}{L(\theta, i)} \{-Ri - e_b(\theta, i)\} dt \]  \hspace{1cm} (F.4)

Assuming negligible speed change during the stroke, the time period \( (t_1 - t_0) \) can be approximated to:

\[ t_1 - t_0 = \frac{\theta_{adv}}{\omega} \]  \hspace{1cm} (F.5)

The advance angle command issued before \( \theta = \theta_1 \) angle results in a current \( i(t_1) \) ideally at an angle of \( \theta_1 \). The current value is sampled at an angle of \( \theta_1 \). Thus it is natural to consider a discrete time system for (F.4) with a priori state \( (k-1) \) in the interval \( \theta_1 - \theta_{rpp} \leq \theta < \theta_1 \) and a current state \( (k) \) in the interval \( \theta_1 \leq \theta < \theta_1 + \theta_{rpp} \). Figure F.4 shows the current pulse and the associated discrete-time state variables, indexes and the time-intervals. System (F.4) can be translated to a first order discrete-time format as:

\[ i_{k+1} = \frac{v_{dc}}{L_u \omega_k} \theta_{adv,k} + w_k \]  \hspace{1cm} (F.6)
where

\[ w_k = \int_{t=t_0}^{t_1} \frac{1}{L(\theta, i)} \{ -Ri - e_b(\theta, i) \} \, dt \quad (F.7) \]

is considered as a disturbance to the system (F.6). The current at \( \theta_1 \) is sampled and compared with the current-peak command \( i^*_k \). The error is given by:

\[ e_k = i^*_k - i_k \quad (F.8) \]

Then the advance angle controller can be formulated as:

\[ \vartheta_{k+1} = k_a \vartheta_k + \tau_k e_k \quad (F.9) \]

\[ \theta_{adv,k} = \frac{L_u \omega_k}{v_{dc}} i^*_k + 1 + k_p e_k + k_i \vartheta_k \quad (F.10) \]

Parameters \( k_p \) and \( k_i \) are proportional and integral gains where (F.9) acts as a discrete time integral action for \( k_a = 1 \). Parameter \( k_a \) is an attenuation constant of the integral action. For single pulse mode of operation \( k_a = 1 \) is selected. However, during current shopping operation \( k_a < 1 \) is selected such that excessive advancing of the turn-on angle is automatically reset. Variable \( \tau_k \) represents the time between two sampling instances.

Substitution of (F.10) in (F.6) yields:

\[ i_{k+1} = \left( \frac{v_{dc}}{L_u \omega_k} \right) \left( \frac{L_u \omega_k}{v_{dc}} i^*_k + 1 + k_p e_k + k_i \vartheta_k \right) + w_k \quad (F.11) \]

where the error dynamics in discrete-time is given by:

\[ e_{k+1} = - \left( \frac{v_{dc}}{L_u \omega_k} \right) k_p e_k - \left( \frac{v_{dc}}{L_u \omega_k} \right) k_i \vartheta_k - w_k \quad (F.12) \]

The error system can be written in state-space form as:

\[
\begin{bmatrix}
  e_{k+1} \\
  \vartheta_{k+1}
\end{bmatrix} =
\begin{bmatrix}
  -\left( \frac{v_{dc}}{L_u \omega_k} \right) k_p & -\left( \frac{v_{dc}}{L_u \omega_k} \right) k_i \\
  \tau_k & k_a
\end{bmatrix}
\begin{bmatrix}
  e_k \\
  \vartheta_k
\end{bmatrix} +
\begin{bmatrix}
  -1 \\
  0
\end{bmatrix} w_k \quad (F.13)
\]

The state space system (F.13) for \( k_a = 1 \) can be cast into the Euler discretized
standard second order format:

\[
\begin{bmatrix}
  x_{2,k+1} \\
  x_{1,k+1}
\end{bmatrix} = \begin{bmatrix}
  1 - 2\zeta\Omega_n\Delta t_k & -\Omega_n^2\Delta t_k \\
  \Delta t_k & 1
\end{bmatrix} \begin{bmatrix}
  x_{2,k} \\
  x_{1,k}
\end{bmatrix} + \begin{bmatrix}
  G\Omega_n^2\Delta t_k \\
  0
\end{bmatrix} u_k
\] (F.14)

where \( \tau_k = \Delta t_k \), \( x_{1,k} = \vartheta_k \), \( x_{2,k} = e_k \) and \( G\Omega_n^2\Delta t_k u_k = -w_k \). Parameters \( \Omega_n \) and \( \zeta \) represent the undamped natural frequency and the damping factor of the system. Controller gains \( k_p \) and \( k_i \) can be calculated from:

\[
k_p = \frac{1}{v_{dc}} \frac{(2\zeta\Omega_n\tau_k - 1) L_u \omega_k}{(2\zeta\Omega_n\tau_k - 1) L_u \omega_k} \] (F.15)
\[
k_i = \frac{\Omega_n^2\tau_k L_u \omega_k}{v_{dc}} \] (F.16)

Fault-tolerant control requires independent operation of the phases and the loss of one phase should not affect the operation of the other phases. Therefore, the advance angle controller is implemented for each phase, and the controller is updated based on the current feedback of the corresponding phase. Thus the controller is designed considering only one phase of the machine and multiplied for arbitrary number of phases. The controller update is performed in each stroke-per-phase which occurs within an angle of the rotor pole-pitch \( \theta_{rpp} \). Assuming negligible speed change within a stroke, the time difference between two sampling instances can be approximately related to speed by:

\[
\tau_k = \frac{\theta_{rpp}}{\omega_k} \] (F.17)

Substitution of (F.17) in (F.15) and (F.16) yields:

\[
k_p = \frac{1}{v_{dc}} L_u (2\zeta\Omega_n\theta_{rpp} - \omega_k) \] (F.18)
\[
k_i = \frac{\Omega_n^2\theta_{rpp} L_u}{v_{dc}} \] (F.19)

For a SR machine with \( N_r \) number of rotor poles, the current pulses occur at an electrical frequency of \( N_r \omega \), where \( \omega \) is the rotor speed. Based on the Nyquist theorem, the excitation angle controller bandwidth must not exceed half the electrical frequency in order to avoid instability. This can be achieved at all
speeds by specifying the undamped natural frequency of the system as:

$$\Omega_n = \lambda \omega_k, \quad 0 < \lambda < \frac{N_r}{2}$$

(F.20)

Substitution of (F.20) in (F.18) and (F.19) yields time-varying controller parameters:

$$k_p = \frac{1}{v_{dc}} L_u (2\lambda \theta_{rpp} - 1) \omega_k$$

(F.21)

$$k_i = \frac{\lambda^2 \theta_{rpp} L_u}{v_{dc} \omega_k^2}$$

(F.22)

and the advance angle attenuation in chopping operation is defined by:

$$k_a = \begin{cases} 
0 < \kappa_a < 1 & \text{if } \omega < \omega_{base} \\
1 & \text{if } \omega \geq \omega_{base}
\end{cases}$$

(F.23)

The discrete-time controller block diagram is shown in figure F.5.

### F.3 Current-peak controller verification with simulations

The excitation controller is designed to the 250kW 12/8 SR motor and the 6/4 test-bed SR motor. The corresponding parameters specific to this controller are given in table F.1. A series of simulations are performed for both the machines at different speed and load conditions. The operation of the current-peak controller is verified by the simulation results shown from figures F.6 to F.13 below.
Table F.1: SR motor current-peak controller parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>250kW SR motor</th>
<th>300W test-bed SR motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC-link voltage $v_{dc}$</td>
<td>540V</td>
<td>24V</td>
</tr>
<tr>
<td>Unaligned inductance $L_u$</td>
<td>0.0363mH</td>
<td>0.6073mH</td>
</tr>
<tr>
<td>Rotor pole-pitch $\theta_{rpp}$</td>
<td>90deg</td>
<td>45deg</td>
</tr>
<tr>
<td>System damping factor $\zeta$</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Bandwidth/speed ratio $\lambda$</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

In general it can be concluded that the excitation angle controller exhibits stable operation and achieves the objective of current-peak control. Close examination of the current pulses reveal that the peak value $i_{ref}$ is achieved at the rotor position $\theta_1$ in single pulse mode as well as in chopping mode of operation at steady state.
Simulation of the 250kW 12/8 SR machine with the current feedback turn-on angle controller 12500rpm.

(a) to (e) current waveform, (f) advance angle and (g) speed (blue) and current-peak command (green).
Figure F.7: Simulation of the 250kW 12/8 SR machine with the current feedback turn-on angle controller 25000rpm. (a) to (e) current waveform, (f) advance angle and (g) speed (blue) and current-peak command (green).
Figure F.8: Simulation of the 250kW 12/8 SR machine with the current feedback turn-on angle controller 37500rpm. (a) to (e) current waveform, (f) advance angle and (g) speed (blue) and current-peak command (green).
Figure F.9: Simulation of the 250kW 12/8 SR machine with the current feedback turn-on angle controller 50000rpm. (a) to (e) current waveform, (f) advance angle and (g) speed (blue) and current-peak command (green).
Figure F.10: Simulation of the 300W 6/4 test-bed SR machine with the current feedback turn-on angle controller 400rpm. (a) to (e) current waveform, (f) advance angle and (g) speed (blue) and current-peak command (green).
Simulation of the 300W 6/4 test-bed SR machine with the current feedback turn-on angle controller 1200rpm.

(a) to (e) current waveform, (f) advance angle and (g) speed (blue) and current-peak command (green).

Figure F.11: Simulation of the 300W 6/4 test-bed SR machine with the current feedback turn-on angle controller 1200rpm.
Figure F.12: Simulation of the 300W 6/4 test-bed SR machine with the current feedback turn-on angle controller 2000rpm. (a) to (e) current waveform, (f) advance angle and (g) speed (blue) and current-peak command (green).
Figure F.13: Simulation of the 300W 6/4 test-bed SR machine with the current feedback turn-on angle controller 2800rpm. (a) to (e) current waveform, (f) advance angle and (g) speed (blue) and current-peak command (green).
Appendix G

Experimental and Simulation Results for the Speed Control of SR MOTORS

The contents of this appendix is included in the PDF file: appendix_G.pdf.

This appendix presents the simulation results for the 250kW SR motor and a comparison of simulation and experimental results for the 300W SR motor.
Appendix H

Frequency Domain Results for the SR Motor Excitation Controllers

The contents of this appendix is included in the PDF file: appendix_H.pdf.

This appendix presents the frequency domain performance comparison of the different excitation controllers developed in chapter four. These results are based on the 250kW SR motor and the 300W SR motor simulations.
Appendix I

Simulation of Sample Optimal Operating Points of the 250kW and 300W SR Generators

The contents of this appendix is included in the PDF file: appendix_I.pdf.

This appendix presents the simulation waveforms associated with sample optimal operating points outlined in the table 5.1.
Appendix J

Simulation and Experimental Results for 250kW and 300W SR Generators

The contents of this appendix is included in the PDF file: appendix_J.pdf.

This appendix presents the simulation results for the 250kW SR generator and 300W SR generator in addition to experimental results for the 300W test-bed SR generator. The discussion on these simulation and experimental results is presented in Chapter five.
Appendix K

Frequency Domain Results for the SR Generator Excitation Controllers

The contents of this appendix is included in the PDF file: appendix_K.pdf.

This appendix presents frequency domain results based on the simulations of chapter five. The analysis is performed for the 250kW SR generator and the 300W SR generator at different speed and load conditions.
Appendix L

Generalized Per-Phase Electrical Model of a Fault Tolerant PM Machine

This appendix presents the derivation of a generalized per-phase electrical model of a fault tolerant PM machine. This appendix follows from equations defined in chapter six. The primary objective of this modelling is to facilitate the representation of partially functional fault tolerant PMG and active rectifier systems. The most suitable option is to consider the per-phase dynamics and application of superposition based on the number of operational phases. The per-phase PMG dynamic model can be written in terms of \((i_{\alpha,n,k}, i_{\beta,n,k})\) which can be directly used to calculate the mean rectified current via (6.30).

Substitution of (6.4) and (6.7) in (6.3) for \(l_{nj} = 0, j \neq n\) yields the general per-phase model of the \(n^{th}\) phase of the system,

\[ v_n = -R_n i_n - \frac{d}{dt} (l_{nn} i_n) + \frac{d\psi_{pm,n}}{dt} \]  \hspace{1cm} (L.1)

\[ \frac{d\psi_{pm,n}}{dt} = -\frac{j\omega}{2} \sum_{k=-K_1}^{K_1} \left\{ k\vec{\psi}_k e^{-jk\theta} \right\} \]  \hspace{1cm} (L.2)

Note that the subscript of phase number in \(\vec{\psi}_k\) is omitted due to identical PM
flux-linkage magnitudes in all phases. Substitution of (L.1) in (L.2) yields

\[
\frac{d}{dt}(l_{nn}i_n) = -R_n i_n - v_n - \frac{j \omega}{2} \sum_{k=-K_1}^{K_1} \{k \vec{\psi}_k e^{-jk\theta}\}
\]

(L.3)

Then by substitution of (6.13) and expansion of the derivative yields:

\[
l_{nn} \frac{di_n}{dt} + i_n \frac{dl_{nn}}{dt} = -R_n i_n - v_{dc} d_n(t) - \frac{j \omega}{2} \sum_{k=-K_1}^{K_1} \{k \vec{\psi}_k e^{-jk\theta}\}
\]

(L.4)

Consider a generalized case with second order inductance variation (6.11). Substitution of (6.11) in (L.4) yields:

\[
L_{n,0} \frac{di_n}{dt} + L_{n,2} \cos 2\theta \frac{di_n}{dt} = 2L_{n,2} \omega \sin 2\theta i_n - R_s i_n
\]

\[
- v_{dc} d_n(t) - \frac{j \omega}{2} \sum_{k=-K_1}^{K_1} \{k \vec{\psi}_k e^{-jk\theta}\}
\]

(L.5)

Introducing

\[
\cos(k\theta) = \frac{1}{2} (e^{jk\theta} + e^{-jk\theta})
\]

(L.6)

\[
\sin(k\theta) = -\frac{j}{2} (e^{jk\theta} - e^{-jk\theta})
\]

(L.7)

(L.5) can be written in terms of the complex exponential as:

\[
L_{n,0} \frac{di_n}{dt} + \frac{L_{n,2}}{2} (e^{j2\theta} + e^{-j2\theta}) \frac{di_n}{dt} = -jL_{n,2} \omega (e^{j2\theta} - e^{-j2\theta}) i_n - R_s i_n
\]

\[
- v_{dc} d_n(t) - \frac{j \omega}{2} \sum_{k=-K_1}^{K_1} \{k \vec{\psi}_k e^{-jk\theta}\}
\]

(L.8)
Substitution of (6.21) and (6.25) in (L.8) and differentiation yields:

$$\frac{1}{2} L_{n,0} \sum_{k=-K}^{K} \left(-j k \omega \tilde{i}_{n,k} e^{-jk\theta} + e^{-jk\theta} \frac{dt_{n,k}}{dt}\right)$$

$$+ \frac{1}{4} L_{n,2} \left(e^{j2\theta} + e^{-j2\theta}\right) \sum_{k=-K}^{K} \left(-j k \omega \tilde{i}_{n,k} e^{-jk\theta} + e^{-jk\theta} \frac{dt_{n,k}}{dt}\right) =$$

$$\frac{-j L_{n,2} \omega \left(e^{j2\theta} - \frac{1}{2} e^{-j2\theta}\right) \sum_{k=-K}^{K} \left(\tilde{i}_{n,k} e^{-jk\theta}\right) - \frac{1}{2} R_s \sum_{k=-K}^{K} \left(\tilde{i}_{n,k} e^{-jk\theta}\right)$$

$$- \frac{1}{2} \psi_{nc} \sum_{k=-K}^{K} \left(\tilde{u}_{n,k} e^{-jk\theta}\right) - \frac{1}{2} j \omega \sum_{k=-K_1}^{K_1} \left\{ k \tilde{\psi}_k e^{-jk\theta}\right\}$$

Multiplication of (L.9) by 2 and rearranging the terms yields:

$$L_{n,0} \sum_{k=-K}^{K} \left\{ \left(-j k \omega \tilde{i}_{n,k} + \frac{dt_{n,k}}{dt}\right) e^{-jk\theta}\right\} + \frac{1}{2} L_{n,2} e^{j2\theta} \sum_{k=-K}^{K} \left\{ \left(-j k \omega \tilde{i}_{n,k} + \frac{dt_{n,k}}{dt}\right) e^{-jk\theta}\right\}$$

$$+ \frac{1}{2} L_{n,2} e^{-j2\theta} \sum_{k=1}^{K} \left\{ \left(-j k \omega \tilde{i}_{n,k} + \frac{dt_{n,k}}{dt}\right) e^{-jk\theta}\right\} =$$

$$-j L_{n,2} \omega e^{j2\theta} \sum_{k=-K}^{K} \left(\tilde{i}_{n,k} e^{-jk\theta}\right) - R_s \sum_{k=-K}^{K} \left(\tilde{i}_{n,k} e^{-jk\theta}\right)$$

$$- \psi_{nc} \sum_{k=-K}^{K} \left(\tilde{u}_{n,k} e^{-jk\theta}\right) - j \omega \sum_{k=-K_1}^{K_1} \left\{ k \tilde{\psi}_k e^{-jk\theta}\right\}$$

Absorption of the exponential terms into the summation operations yields:

$$L_{n,0} \sum_{k=-K}^{K} \left\{ \left(-j k \omega \tilde{i}_{n,k} + \frac{dt_{n,k}}{dt}\right) e^{-jk\theta}\right\}$$

$$+ \frac{1}{2} L_{n,2} \sum_{k=-K}^{K} \left\{ \left(-j k \omega \tilde{i}_{n,k} + \frac{dt_{n,k}}{dt}\right) \left(e^{-j(k+2)\theta} + e^{-j(k-2)\theta}\right)\right\} =$$

$$-R_s \sum_{k=-K}^{K} \left(\tilde{i}_{n,k} e^{-jk\theta}\right) - j L_{n,2} \omega \sum_{k=-K}^{K} \left(\tilde{i}_{n,k} e^{-j(k-2)\theta} - \tilde{i}_{n,k} e^{-j(k+2)\theta}\right)$$

$$- \psi_{nc} \sum_{k=-K}^{K} \left(\tilde{u}_{n,k} e^{-jk\theta}\right) - j \omega \sum_{k=-K_1}^{K_1} \left\{ k \tilde{\psi}_k e^{-jk\theta}\right\}$$

Substitution of \( l = k+2 \) and \( l = k-2 \) in coefficients and powers of terms \( e^{-j(k+2)\theta} \).
and $e^{-j(k-2)\theta}$ respectively yields

$$
L_{n,0} \sum_{k=-K}^{K} \left\{ \left( -j k \omega \vec{i}_{n,k} + \frac{d\vec{i}_{n,k}}{dt} \right) e^{-j k \theta} \right\} 
$$

$$
+ \frac{1}{2} L_{n,2} \sum_{l=-K+2}^{K+2} \left\{ \left( -j \left( l - 2 \right) \omega \vec{i}_{n,l-2} + \frac{d\vec{i}_{n,l-2}}{dt} \right) e^{-jl \theta} \right\} 
$$

$$
+ \frac{1}{2} L_{n,2} \sum_{l=-K+2}^{K+2} \left\{ \left( -j \left( l + 2 \right) \omega \vec{i}_{n,l+2} + \frac{d\vec{i}_{n,l+2}}{dt} \right) e^{-jl \theta} \right\} = 
$$

$$
-R_s \sum_{k=-K}^{K} \left( \vec{i}_{n,k} e^{-jk \theta} \right) - j L_{n,2} \omega \sum_{l=-K+2}^{K+2} \left( \vec{i}_{n,l+2} e^{-jl \theta} \right) + j L_{n,2} \omega \sum_{l=-K+2}^{K+2} \left( \vec{i}_{n,l+2} e^{-jl \theta} \right) 
$$

$$
- v_{dc} \sum_{k=-K}^{-K} \left( \vec{u}_{n,k} e^{-jk \theta} \right) - j \omega \sum_{k=-K+1}^{K} \left\{ k \vec{\psi}_k e^{-jk \theta} \right\} 
$$

(L.12)

Assuming $\vec{u}_{n,k} = \vec{u}_{n,-k} = 0$ and $\vec{i}_{n,k} = \vec{i}_{n,-k} = 0$, for all $k > K$ and $\vec{\psi}_k = \vec{\psi}_{-k} = 0$ for all $k > K_1$ where $K_1 \leq K$, the equation (L.12) can be brought under a common summation operation as:

$$
\sum_{k=-K+2}^{K+2} \left\{ \left( -j k \omega \vec{i}_{n,k} + \frac{d\vec{i}_{n,k}}{dt} \right) e^{-j k \theta} \right\} + \frac{1}{2} L_{n,2} \left( -j \left( k - 2 \right) \omega \vec{i}_{n,k-2} + \frac{d\vec{i}_{n,k-2}}{dt} \right) 
$$

$$
+ \frac{1}{2} L_{n,2} \left( -j \left( k + 2 \right) \omega \vec{i}_{n,k+2} + \frac{d\vec{i}_{n,k+2}}{dt} \right) \right\} e^{-j k \theta} = 
$$

$$
+ \sum_{k=-K+2}^{K+2} \left\{ -R_s \vec{i}_{n,k} - j L_{n,2} \omega \vec{i}_{n,k+2} + j L_{n,2} \omega \vec{i}_{n,k-2} - v_{dc} \vec{u}_{n,k} - j \omega k \vec{\psi}_k \right\} e^{-j k \theta} 
$$

(L.13)

Further simplification yields:

$$
\sum_{k=-K+2}^{K+2} \left\{ \left( -j k \omega \vec{i}_{n,k} + \frac{d\vec{i}_{n,k}}{dt} \right) + \frac{1}{2} L_{n,2} \left( \frac{d\vec{i}_{n,k-2}}{dt} + \frac{d\vec{i}_{n,k+2}}{dt} \right) 
$$

$$
+ \frac{1}{2} L_{n,2} \left( -j k \omega \vec{i}_{n,k-2} - j k \omega \vec{i}_{n,k+2} + R_s \vec{i}_{n,k} + v_{dc} \vec{u}_{n,k} + j \omega k \vec{\psi}_k \right) \right\} e^{-j k \theta} = 0 
$$

(L.14)

Equation (L.14) is a two sided geometric series of the format of

$$
\sum_{k=-K+2}^{K+2} \vec{a}_{n,k} e^{-j k \theta} = 0 
$$

(L.15)
where the coefficient $\vec{a}_{n,k}$ can be written as:

$$\vec{a}_{n,k} = L_{n,0} \left( -jk\omega \vec{i}_{n,k} + \frac{d\vec{i}_{n,k}}{dt} \right) + \frac{L_{n,2}}{2} \left( \frac{d\vec{i}_{n,k-2}}{dt} + \frac{d\vec{i}_{n,k+2}}{dt} \right) - jk\omega L_{n,2} \left( \vec{i}_{n,k-2} + \vec{i}_{n,k+2} \right) + R_s \vec{i}_{n,k} + V_{dc} \vec{a}_{n,k} + j\omega k \vec{\psi}_k$$

(L.16)

It follows that the $r$th derivative of (L.16) is equal to zero. That is

$$\frac{\partial^r}{\partial \theta^r} \left\{ \sum_{k=K-2}^{K+2} \vec{a}_{n,k} e^{-jk\theta} \right\} = 0$$

(L.17)

$r = 1, 2, ...$. Expansion for $r = 1$ yields:

$$\frac{\partial}{\partial \theta} \left\{ \sum_{k=-K-2}^{K+2} \vec{a}_{n,k} e^{-jk\theta} \right\} = \sum_{k=-K-2}^{K+2} (-jk \vec{a}_{n,k} e^{-jk\theta}) + \sum_{k=-K-2}^{K+2} (e^{-jk\theta} \frac{\partial \vec{a}_{n,k}}{\partial \theta}) = 0$$

(L.18)

$\vec{a}_{n,k}$ is a function of $\omega$. Assuming $\frac{d\omega}{d\theta} = \frac{1}{\omega} \frac{d\omega}{dt} \approx 0$ we note that $\frac{\partial \vec{a}_{n,k}}{\partial \theta} \approx 0$. Then (L.19) can be reduced to:

$$\sum_{k=-K-2}^{K+2} (k \vec{a}_{n,k} e^{-jk\theta}) = 0$$

(L.19)

Similarly the $r$th derivative yields:

$$\sum_{k=-K-2}^{K+2} (k^r \vec{a}_{n,k} e^{-jk\theta}) = 0$$

(L.20)

Writing $2(K+2) + 1$ simultaneous equations with (L.20) from $r = 1, 2, ..., 2(K + 2) + 1$ we may formulate a matrix equation of the form:

$$H [E (\theta)] A = \bar{0}$$

(L.21)

where

$$A = [\vec{a}_{(n,-K-2)}, \vec{a}_{(n,-K-1)}, \ldots, \vec{a}_{n,-1}, 0, \vec{a}_{n,1}, \ldots, \vec{a}_{(n,K+1)}, \vec{a}_{(n,K+2)}]^T$$

(L.22)

$$E (\theta) = \text{diag} (e^{j(K+2)\theta}, e^{j(K+1)\theta}, \ldots, e^{j\theta}, 0, e^{-j\theta}, \ldots, e^{-j(K+1)\theta}, e^{-j(K+2)\theta})$$

(L.23)
and

\[ H = \begin{bmatrix}
(-K - 2) & (-K - 1) & \ldots & (K + 2) \\
(-K - 2)^2 & (-K - 1)^2 & \ldots & (K + 2)^2 \\
\vdots & \vdots & \ddots & \vdots \\
(-K - 2)^r & (-K - 1)^r & \ldots & (K + 2)^r \\
\vdots & \vdots & \ddots & \vdots \\
(-K - 2)^{2K+5} & (-K - 1)^{2K+5} & \ldots & (K + 2)^{2K+5}
\end{bmatrix} \] (L.24)

Matrix \( H \) is invertible and multiplication of (L.21) by \( H^{-1} \) yields

\[ [E(\theta)] A = \bar{0} \] (L.25)

The matrix \( E(\theta) \) is a function of \( \theta \) and \( E(\theta) \neq 0 \). Therefore \( A = \bar{0} \). This implies that the coefficients \( \vec{a}_{n,k} = 0 \) for all \( k \). Since \( \vec{a}_{n,k} \) and \( \vec{a}_{n,-k} = \vec{a}^*_{n,k} \) are complex conjugates, it is sufficient to consider only one set of coefficient to obtain the dynamic equations. The fundamental dynamics can be written by equating the coefficients of \( e^{-j\theta} \) to zero, i.e., \( \vec{a}_{n,1} = 0 \):

\[
\frac{L_{n,2}}{2} \frac{d\vec{i}_{n,3}}{dt} + L_{n,0} \frac{d\vec{i}_{n,1}}{dt} + \frac{L_{n,2}}{2} \frac{d\vec{i}_{n,-1}}{dt} - j\omega \frac{L_{n,2}}{2} \vec{i}_{n,3} \\
- j\omega L_{n,0} \vec{i}_{n,1} - j\omega \frac{L_{n,2}}{2} \vec{i}_{n,-1} + R_s \vec{i}_{n,1} = -v_{dc} \vec{u}_{n,1} - j\omega \vec{\psi}_{n,1}
\] (L.26)

The higher plane, \( k^{th} \) harmonic dynamics can be written by equating the coefficients of \( e^{-jk\theta} \) to zero, i.e., \( \vec{a}_{n,k} = 0 \):

\[
\frac{L_{n,2}}{2} \frac{d\vec{i}_{n,k+2}}{dt} + L_{n,0} \frac{d\vec{i}_{n,k}}{dt} + \frac{L_{n,2}}{2} \frac{d\vec{i}_{n,k-2}}{dt} - jk\omega \frac{L_{n,2}}{2} \vec{i}_{n,k+2} \\
- jk\omega L_{n,0} \vec{i}_{n,k-2} - jk\omega L_{n,0} \vec{i}_{n,k} + R_s \vec{i}_{n,k} = -v_{dc} \vec{u}_{n,k} - jk\omega \vec{\psi}_k
\] (L.27)

Second harmonic \((k = 2)\) dynamics are not coupled with the fundamental current vector \( \vec{i}_{n,1} \). Moreover it can be seen that \( \vec{i}_{n,1} \) does not appear in all the even harmonic equations. Even frequency components are negligible and are not of importance in this analysis. Therefore we may safely assume that for given \( \vec{u}_{n,k} = 0 \), the current vector \( \vec{i}_{n,k} \approx 0 \) for all even \( k \), \( k \geq 0 \).

Substitution of (6.19) and (6.26) in (L.27) and by separation of the real and imaginary parts, a new dynamic model representing the mutually orthogonal
harmonic dynamics of the PMG is obtained. i.e., for $k = 1$ the fundamental dynamics are given by:

$$
\begin{align*}
- \frac{L_{n,2}}{2} \frac{d i_{\alpha,n,3}}{dt} & - \left( L_{n,0} + \frac{L_{n,2}}{2} \right) \frac{d i_{\alpha,n,1}}{dt} + \omega \frac{L_{n,2}}{2} i_{\beta,n,3} \\
+ \omega \left( L_{n,0} i_{\beta,n,1} - \frac{L_{n,2}}{2} \right) i_{\beta,n,1} - R_s i_{\alpha,n,1} = -v_{dc} u_{\alpha,n,1} + \omega \psi_{\beta,1}
\end{align*}
$$

(L.28)

Higher plane $k^{th}$ harmonic dynamics are given by:

$$
\begin{align*}
- \frac{L_{n,2}}{2} \frac{d i_{\alpha,n,k+2}}{dt} & - L_{n,0} \frac{d i_{\alpha,n,k}}{dt} - \frac{L_{n,2}}{2} \frac{d i_{\alpha,n,k-2}}{dt} + k\omega \frac{L_{n,2}}{2} i_{\beta,n,k+2} \\
+ k\omega \frac{L_{n,2}}{2} i_{\beta,n,k-2} + k\omega L_{n,0} i_{\beta,n,k} - R_s i_{\alpha,n,k} = -v_{dc} u_{\alpha,n,k} + \omega k \psi_{\beta,n,k}
\end{align*}
$$

(L.30)
Appendix M

Simulation Results of the Surface PM Motor Fault-Tolerant Drive

The contents of this appendix is included in the PDF file: appendix_M.pdf.

This appendix presents the simulation results for the surface PM motor operation with different optimization criteria and with different fault conditions. The discussion of these simulation results are presented in chapter seven.
Appendix N

Simulation Results of the Interior PM Motor Fault-Tolerant Drive

The contents of this appendix is included in the PDF file: appendix_N.pdf.

This appendix presents the simulation results for the interior PM motor operation with different optimization criteria and with different fault conditions. The discussion of these simulation results are presented in chapter seven.
Appendix O

Frequency Domain Analysis of Simulation Results of the Surface PM Motor Operation

The contents of this appendix is included in the PDF file: appendix_O.pdf.

This appendix presents FFT analysis of the torque and DC-link current for the surface PM motor. The FFT analysis is based on the simulation results obtained earlier. A discussion on the torque ripple and DC-link current ripple performance under different speed and load conditions is presented in chapter seven.
Appendix P

Frequency Domain Analysis of Simulation Results of the Interior PM Motor Operation

The contents of this appendix is included in the PDF file: appendix_P.pdf.

This appendix presents FFT analysis of the torque and DC-link current for the interior PM motor. The FFT analysis is based on the simulation results obtained earlier. A discussion on the torque ripple and DC-link current ripple performance under different speed and load conditions is presented in chapter seven.
Appendix Q

Simulation Results of the Surface PM Generator Fault-Tolerant System

The contents of this appendix is included in the PDF file: appendix_Q.pdf.

This appendix presents the simulation results for the surface PM generator operation with different optimization criteria and with different fault conditions. The discussion of these simulation results are presented in chapter eight.
Appendix R

Simulation Results of the Interior PM Generator Fault-Tolerant System

The contents of this appendix is included in the PDF file: appendix_R.pdf.

This appendix presents the simulation results for the interior PM generator operation with different optimization criteria and with different fault conditions. The discussion of these simulation results are presented in chapter eight.
Appendix S

Frequency Domain Analysis of Simulation Results of the Surface PM Generator Operation

The contents of this appendix is included in the PDF file: appendix_S.pdf.

This appendix presents FFT analysis of the torque and DC-link current for the surface PM generator. The FFT analysis is based on the simulation results obtained earlier. A discussion on the torque ripple and DC-link current ripple performance under different speed and DC-link load conditions is presented in chapter eight.
Appendix T

Frequency Domain Analysis of Simulation Results of the Interior PM Generator Operation

The contents of this appendix is included in the PDF file: appendix_T.pdf.

This appendix presents FFT analysis of the torque and DC-link current for the interior PM generator. The FFT analysis is based on the simulation results obtained earlier. A discussion on the torque ripple and DC-link current ripple performance under different speed and DC-link load conditions is presented in chapter eight.