Advanced Ultrasonic Digital Imaging and Signal Processing for Applications in the Field of Non-Destructive Testing

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### Abbreviations

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<th>Description</th>
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<tr>
<td>CPU</td>
<td>Central Processing Unit</td>
</tr>
<tr>
<td>DDF</td>
<td>Dynamic Depth Focusing</td>
</tr>
<tr>
<td>EngD</td>
<td>Engineering Doctorate</td>
</tr>
<tr>
<td>EPSRC</td>
<td>Engineering and Physical Sciences Research Council</td>
</tr>
<tr>
<td>FIR</td>
<td>Finite Impulse Response</td>
</tr>
<tr>
<td>FMC</td>
<td>Full Matrix Capture</td>
</tr>
<tr>
<td>FPGA</td>
<td>Field Programmable Gate Array</td>
</tr>
<tr>
<td>FRD</td>
<td>Full Raw Data</td>
</tr>
<tr>
<td>GPU</td>
<td>Graphics Processing Unit</td>
</tr>
<tr>
<td>LORP</td>
<td>Lack of Root Penetration</td>
</tr>
<tr>
<td>NDT</td>
<td>Non-Destructive Testing</td>
</tr>
<tr>
<td>PGDip</td>
<td>Post-Graduate Diploma</td>
</tr>
<tr>
<td>POI</td>
<td>Point of Incidence</td>
</tr>
<tr>
<td>PSF</td>
<td>Point Spread Function</td>
</tr>
<tr>
<td>PZT</td>
<td>Lead Zirconate Titanate</td>
</tr>
<tr>
<td>RA</td>
<td>Receive Aperture</td>
</tr>
<tr>
<td>SDH</td>
<td>Side Drilled Hole</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>SPA</td>
<td>Sequential Phased Array</td>
</tr>
<tr>
<td>TA</td>
<td>Transmit Aperture</td>
</tr>
<tr>
<td>TFM</td>
<td>Total Focusing Method</td>
</tr>
<tr>
<td>VTFM</td>
<td>Vector Total Focusing Method</td>
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Abstract

Non-destructive testing (NDT) is the process of non-invasive material examination. Within this field, ultrasonic inspection is one method of examination used to detect flaws in structural and functional industrial components, to assess their structural integrity and fitness for service. Conventional NDT ultrasonic array techniques transmit on multiple elements in parallel, according to a focal law, which facilitates beam steering, focussing and scanning within the test component. Received signals are then ‘stacked’ to generate images of the test component.

With the advent of affordable high speed computing, novel data acquisition techniques based on sequential transmission are now able to be developed, which allow images to be generated using advanced signal processing and image reconstruction algorithms. One such data acquisition technique known as Full Matrix Capture (FMC), has received considerable research attention in recent years, largely because it allows fully focused images of test components to be generated.

This project provides an improved understanding of the FMC technique and associated signal processing algorithms. It achieves this through the development of novel inspection techniques and signal processing algorithms. Collectively algorithms developed within this work were termed Sequential Phased Array (SPA). Initially comparisons were made between conventional ultrasonic techniques and the SPA algorithms in terms of image quality and speed of inspection. A novel approach was then suggested to facilitate inspection through dual-layered media, separated by a refractive interface using the SPA algorithms. The use of sparse arrays to enhance the speed of inspection using the SPA algorithms was also investigated, and the trade-off between speed of inspection against image degradation was addressed. Finally a novel approach to calibration of a FMC based system was developed, in order to provide uniform image sensitivity during inspection.
Declaration

No portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

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Acknowledgements

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I am forever in debt to my parents for their permanent support and encouragement throughout my education; you have made this endeavour possible. The camaraderie from both family and friends has also been an essential ingredient and is gratefully acknowledged, as is the help and encouragement from my loving girlfriend.

For the project funding, thanks and recognition must also be apportioned to the Engineering and Physical Sciences Research Council (EPSRC) and TWI Ltd.

Finally, the acknowledgements would be incomplete without giving special thanks to Channa Nageswaran for all of his technical guidance, support and enlightening insights into the world of ultrasonics.
Chapter 1

Introduction

1.1 Motivation for research

Within industry there is a constant demand for reliable assessment of a component’s structural integrity. Unfortunately there is no single Non Destructive Testing (NDT) technique which is suitable for all purposes; therefore, it is often the case that multiple techniques will be used to inspect a single component. Inspection using ultrasonic arrays is one technique which is found to be effective at detecting and characterising flaws in many inspection scenarios and is the main focus of this thesis. Improved flaw detection and characterisation techniques will facilitate more reliable fitness for service assessments leading to improved safety levels and reduced costs for industry.

1.2 Ultrasonic testing

Phased array ultrasound is an inspection technique used in the field of NDT to determine the integrity of a component’s internal structure. A typical phased array transducer consists of a number of closely spaced piezoelectric elements. Elements are excited in parallel to allow a sound field to be generated in front of the array. The generated sound field is formed from the interference of individual sound fields originating from each element in the array. By controlling the excitation times of elements relative to each other, the sound field can be steered and focussed inside the test component.

The research presented in this thesis will largely focus on the development and analysis of the Full Matrix Capture (FMC) technique and the associated signal processing algorithms. FMC is a data acquisition technique which collects time domain signals for every transmit-receive element combination within an array transducer. The acquisition process proceeds by exciting a single element in the array and receiving on all elements. This process of transmit on one and receive on all is repeated until all elements have been excited. This approach differs from conventional phased array techniques as, due to the sequential transmission pattern, only the sound field from a single element is present in the test material at any moment in time. It is therefore necessary to simulate beam forming using signal processing algorithms in order to generate imagery of the test component.
In this thesis signal processing algorithms, collectively termed Sequential Phased Array (SPA), were developed to process data collected using the FMC acquisition technique.

1.3 Aims and objectives

The research literature on FMC suggests that the technique has the potential to be an effective ultrasonic imaging technique for defect detection, sizing and characterisation. In the majority, this is attributed to the techniques ability to generate fully focussed imagery and to acquire and store raw time-amplitude data. However, due to the large data sets associated with FMC, data acquisition and processing can be slow in comparison to existing ultrasonic imaging techniques.

The overall aim of the thesis is to research and develop methods to reduce the inspection time while maintaining a high imaging quality. The objectives of the research were as follows:

- To review the state of the art in ultrasonic imaging in relation to the FMC technique.
- To outline the fundamental differences between FMC and conventional phased array techniques.
- To develop time efficient algorithms to account for refractive boundary conditions in dual-layered media.
- To investigate sparse array transducers to minimise inspection time, while maintaining a high image quality.
- To develop calibration techniques to facilitate accurate flaw sizing and characterisation.
- To determine the industrial relevance of FMC.

It should be noted that throughout this thesis all experimentation was performed on isotropic and homogeneous materials using 1D linear arrays on assumed 2D surfaces.

1.4 Industrial context

The Engineering Doctorate (EngD) is a post-graduate, research based scheme established in the early 1990’s by the UK Engineering and Physical Sciences Research Council (EPSRC). It was developed to produce graduates at a level equivalent to the traditional PhD, but with a greater awareness of industry. Hence an EngD may be expected to spend around 75 percent of their time based in the sponsor company. In addition to the research aspect of study, graduates will have attended a number of taught
courses including a post-graduate diploma in business management (PGDip), personal and professional development training and technical modules relating to the EngD’s field of research. The expected timeframe for thesis submission and completion of all taught modules is 4 years.

The sponsor company for this EngD project was TWI Ltd. TWI is an independent research and technology organisation, founded in 1946. Headquarters are based in Cambridge, with regional facilities in South Wales, Yorkshire, Middlesbrough and several other offices spread worldwide. TWI has a broad range of expertise related to all aspects of engineering, materials and joining technologies. The TWI NDT Technology Centre located in Port Talbot, South Wales is part of the TWI NDT Group, and has expertise in a wide range of techniques including ultrasound, radiography, thermography and eddy current inspection amongst others.

1.5 Summary of contribution to knowledge

Whilst there has been a considerable amount of research to develop algorithms which account for refraction in dual and multi-layered media across a number of industries, many algorithms are based on solving Fermat’s principle using iterative methods (Long et al 2009; Calmon et al 1998). In the majority of cases the reason for this is due to the irregular geometry of the system. However, in the field of NDT there is a need to account for refraction through dual-layered media with a planar boundary. The most obvious example is the inspection of a weld body of a flat plate using an angled wedge. In this thesis an analytic solution to Snell’s law, which calculates the exit point of the longitudinal and shear wave modes between dual-layered media was developed. These algorithms account for the varying positions of elements in an array and allow the interface between dual-layered media to be located at any position or angle between the transducer and region of interest. With data collected using the FMC technique and curve-fitting algorithms, the boundary between dual-layered media was mapped. This allowed the wedge geometry to be determined without the need for an operator to manually input such parameters. These algorithms combined with algorithms developed to generate focussed imagery from the FMC data therefore allowed for automated focussing through dual-layered media. The only other literature found to suggest a similar approach, uses a numerical approach to solve Snell’s law for medical applications. Also, the geometry and required mathematics is simplified by the assumption that the transducer consists of a single element and that this element is located at the origin (Shin et al 2010).
On the topic of refractive boundaries, new algorithms were also devised as part of this thesis which allow the boundary in the active plane to be mapped and expressed as a polynomial using curve fitting algorithms. A novel algorithm based on solving Snell’s law using numerical methods was developed to predict the beam path of longitudinal and shear wave modes propagating through dual-layered media with an irregular boundary profile for a transducer containing multiple elements. In the NDT industry similar algorithms have been developed, but take an iterative approach to solve Fermat’s principle (Drinkwater and Bowler 2009; Long and Cawley 2007).

One potential disadvantage associated with the FMC technique is the slow data acquisition time. This is caused by the inherently large data sets in comparison with conventional phased array techniques. Much research has been published in the medical industry on sparse array design (Lockwood and Foster 1994; Gehlbach and Alvarez 1981; Lockwood et al (1996); Lockwood and Foster 1996). However, because human tissue consists largely of water, the literature concentrates on transducer designs based on the longitudinal wave mode, with the transducer face being in direct contact with the patient. Previously literature has also been published in the field of NDT (Moreau et al 2010); in this instance, ‘sparse’ often only refers to the transmit elements and only considers the longitudinal wave mode. The contribution to knowledge of this work was the development of a sparse array transducer using far field approximations, which permitted the spacing between adjacent elements to exceed half-wavelength, without the generation of grating lobes. All experiments were performed in the near field and the use of sparse transmit and receive apertures were investigated.

The final significant contribution to knowledge in this thesis was the development of an FMC calibration method to allow accurate defect sizing and characterisation. Calibration methods for conventional phased array inspection are already well established (British Standards Institution 2001). However, fundamental differences in the data capture process necessitate existing methods to be adapted for use with FMC. While it is feasible to calibrate using phased array techniques, then perform inspection using FMC, the calibration in this instance would require additional hardware to achieve parallel transmission. The suggested calibration method in this thesis does not require such hardware, therefore reducing the potential manufacture cost of a dedicated FMC pulser-receiver system.
1.6 Organisation of thesis

Chapter 1 provides an outline of the research covered by this thesis, as well as an overview of the EngD scheme and its relation to the more traditional PhD. There is also a description of the sponsor company and a discussion of the industrial benefit of this research to the organisation.

The physical principles of acoustics in relation to ultrasonic inspection are introduced in Chapter 2. A review of the literature up to the current state of the art, which covers the topics discussed in this thesis, are also presented in this chapter.

Chapter 3 describes the development and implementation of the FMC data acquisition technique and signal processing algorithms. A comparative study then investigates the main differences and similarities between the FMC technique and parallel transmission, phased array techniques.

Building on the fundamental FMC technique and associated signal processing algorithms, Chapter 4 investigates different mathematical methods which can be used to account for planar and irregular refractive boundaries in dual-layered media. Methods are compared on a variety of aspects including processing time, implementation and image quality. Included in this chapter is the development of polynomial curve-fitting algorithms to allow automated focussing through dual-layered media. The feasibility of integrating such algorithms into a commercial FMC system is also discussed.

Chapter 5 focuses on the design and implementation of sparse array transducers in order to minimise data acquisition and signal processing times during inspection using the FMC technique. Element configurations were designed using far-field approximations which potentially allow the relaxation of the half wavelength inter-element spacing criterion. Simulated results are presented which show how increased side lobes, caused as a result of the sparse array configuration, can be suppressed. The effects of grating lobes when using far-field approximations are also presented and discussed.

In Chapter 6 methods to calibration the FMC system to allow accurate defect sizing and characterisation are discussed and compared to existing parallel transmission ultrasonic array calibration procedures. Experimental results are also presented to validate the calibration procedure.

Chapter 7 draws together the conclusions from the research and discusses potential future work resulting from this thesis.
Chapter 2

A background into ultrasound

2.1 Introduction

In its generic sense, ultrasound is a pressure wave which propagates in a liquid, gas or solid at frequencies above the upper audible limit of human hearing (20 kHz). It has a wide range of applications in science and engineering, particularly in the fields of sonar, medical and non-destructive testing, where it is primarily used as a detection and characterisation tool. This chapter discusses in detail the fundamental principles of ultrasound which serve as a prerequisite to concepts developed later in this work. A review of the current literature, in relation to ideas and concepts developed later in this work, is also presented.

2.2 Wave theory

2.2.1 Introduction

Mechanical sound waves propagating through a material are composed of discrete particle disturbances. If this applied excitation is a single frequency then the movement of the particles as a function of time will be sinusoidal about their equilibrium position, provided that when under stress the tension or compression does not exceed the elastic limit. In other words the driving force, $F$, will increase proportionally with displacement, $r$, at a rate defined by the stiffness, $k$, according to Hooke’s law (Young and Freedman 2004) in Equation 2.1.

$$F_{Hooke} = -kr$$  \hspace{1cm} 2.1

2.2.2 The Wave Equation

Partial differential equations are used in mathematics to describe many of the phenomena which occur in nature. Some of the better known equations include Schrödinger’s equation, used to describe the quantum state of a system with changing time, and the heat equation which describes the distribution of heat in an area with respect to time. The motion of mechanical sound waves as a function of time and displacement from the equilibrium is governed by a second-order partial differential equation known as the wave equation. Assuming bulk waves propagating within the
elastic limit in an isotropic media, the wave equation in Equation 2.8 can be derived by equating Hooke’s law in Equation 2.1 with Newton’s second law (Young and Freedman 2004) in Equation 2.2.

\[ F_{\text{Newton}} = ma \]  

If we make the analogy between particle motion and a mass-spring system, illustrated in Figure 2.1, where \( \psi(r) \) is the displacement of a mass, \( m \), from its equilibrium position, \( r \), then the forces exerted on a particle at position, \( (r + h) \), are

\[
F_{\text{Newton}} = ma = m \frac{\partial^2}{\partial t^2} \psi(r + h, t) \quad 2.3
\]

\[
F_{\text{Hooke}} = k[\psi(r + 2h, t) - \psi(r + h, t)] + k[\psi(r, t) - \psi(r + h, t)] \quad 2.4
\]

Equating these two forces gives

\[
m \frac{\partial^2}{\partial t^2} \psi(r + h, t) = k[\psi(r + 2h, t) - \psi(r + h, t)] + k[\psi(r, t) - \psi(r + h, t)] \quad 2.5
\]

In the case of \( N \) particles spaced periodically over a length \( L \), we can say \( L = Nh \), the total mass is \( M = Nm \) and the total spring stiffness \( K = k/N \). Therefore Equation 2.5 can be written as

\[
\frac{\partial^2}{\partial t^2} \psi(r + h, t) = \frac{KL^2}{M} \frac{[\psi(r + 2h, t) - 2\psi(r + h, t) + \psi(r, t)]}{h^2} \quad 2.6
\]

In the limit that \( h \to 0 \)

\[
\frac{\partial^2 \psi}{\partial r^2} = \frac{KL^2}{M} \frac{\partial^2 \psi}{\partial t^2} \quad 2.7
\]

Since velocity, \( v = \sqrt{KL^2/M} \), then the wave equation becomes

\[
\frac{\partial^2 \psi}{\partial r^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \quad 2.8
\]

As with all second order differential equations there is a general solution, whose parameters are determined from the boundary conditions.

\[
\psi = Ae^{i(kr-\omega t)} \quad 2.9
\]

Inputting the test solution in Equation 2.9 which suggests a cyclic wave motion, we obtain the following
Therefore the wave equation becomes

\[ \frac{\partial^2 \psi}{\partial r^2} = -k^2 \psi \]  
\[ \frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi \]

Since \( \omega = 2\pi f \) and \( k = 2\pi/\lambda \) then we get the equation

\[ v = \frac{\omega}{k} \]

This is the fundamental equation of bulk wave propagation in isotropic media and describes the relationship between phase velocity, \( v \), frequency, \( f \), and wavelength, \( \lambda \).

![Diagram of particle displacement represented by a mass-spring system](image)

**Figure 2.1:** A diagram of particle displacement represented by a mass-spring system

### 2.2.3 Modes of propagation

Dependent on the way particles are excited and their state of matter, different modes of propagation can be produced. For a typical ultrasonic inspection the two most common bulk wave modes are the longitudinal and shear wave. The velocity with which these modes propagate in a solid will depend upon the material density, \( \rho \), and elastic constants, \( C \), according to Equation 2.14. The elastic constants of the material include the Young’s Modulus, \( E \), which defines the ratio between stress and strain in the elastic region where Hooke’s law is obeyed, and Poisson’s Ratio, \( \mu \), which represents the compressibility of an object under pressure.

\[ v = \sqrt{\frac{C}{\rho}} \]  

When the particles oscillation is in the direction of propagation, compression and rarefaction will occur and the longitudinal wave mode will be generated. In this instance the longitudinal wave velocity, \( v_L \), is defined by Equation 2.15 (Krautkramer and Krautkramer 1977). In a solid medium, particles may also oscillate at 90 degrees to the direction of propagation. In this case the shear wave mode will be generated, whose sound velocity \( v_S \) is defined by Equation 2.16 (Krautkramer and Krautkramer 1977).
Rearranging the general equation of acoustic velocity given by Equation 2.14, and dividing through by the velocity we can obtain another important equation which is fundamental to acoustics. The equation of characteristic impedance is given in Equation 2.17, and shows the relationship between impedance, $Z$, material density, $\rho$, and acoustic velocity, $v$. In the presence of material discontinuities, it is the change in acoustic impedance which gives rise to signal reflections which allow flaws to be identified using ultrasound (Krautkramer and Krautkramer 1977).

$$Z = \frac{C}{v} = \rho v$$  \hspace{1cm}  2.17

### 2.2.4 Reflection and transmission

When a sound field propagates with an incident angle normal to the interface a portion of its pressure amplitude will be reflected, $R$, while the remainder will be transmitted, $T$. The amount of reflected and transmitted energy will depend upon the relative acoustic impedance between the two materials as shown in Equation 2.18 and 2.19 respectively (Krautkramer and Krautkramer 1977). $Z_1$ and $Z_2$ represent the acoustic velocity of the first and second medium respectively.

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$  \hspace{1cm}  2.18

$$T = \frac{2Z_2}{Z_2 + Z_1}$$  \hspace{1cm}  2.19

While these two equations are valid for normal incident angles, it is often the case that ultrasonic inspection is performed at oblique angles. In this situation transmitted wave modes will undergo refraction and may also be mode converted (Section 2.2.4.3). Refraction is the change in direction of a propagating wave due to a change in its speed as illustrated in Figure 2.2. A direction change in the propagating wave occurs to conserve the periodic variation of the wave both sides of the boundary, and so ensures a plain wave front is maintained.
2.2.4.1 Fermat’s principle

In the field of acoustics Fermat’s principle can be used to determine properties relating to reflection and refraction of a sound field. It is often referred to as the ‘principle of least time’, and in some text books is described as the path between two points which yields the minimum time of flight. However this statement is incomplete. A more precise and general description of Fermat’s principle is

“A beam travelling in a region of variable refractive index follows a path such that the total path length is stationary” (Riley et al 2006)

The time taken for a sound field to propagate between two points separated by a refractive interface can be expressed in two dimensions as

\[
t(x_b) = \frac{\sqrt{(x_b - x_0)^2 + (z_b - z_0)^2}}{v_1} + \frac{\sqrt{(x_s - x_b)^2 + (z_s - z_b)^2}}{v_2} \quad 2.20
\]

Whereby \((x_0,z_0)\) is the position of the transducer, \((x_b,z_b)\) is the point of incidence, \((x_s,z_s)\) is the focal point in the test component, and \(v_1\) and \(v_2\) are the respective acoustic velocities of the two materials separated by a refractive interface (Figure 2.2). From Fermat’s principle we know that

\[
\frac{dt(x_b)}{dx_b} = 0 \quad 2.21
\]

Therefore, the point of incidence between dual layered media can be determined by satisfying Equation 2.21.

2.2.4.2 Snell’s law

The relationship between the angle of incidence, \(\theta_i\) and angle of refraction, \(\theta_r\) as sound propagates between two media can be described mathematically using Snell’s law given in Equation 2.28. It can be shown that Snell’s law is in fact derived from Fermat’s principle:

Referring to Figure 2.2, the time taken for a sound field to propagate between two points separated by a refractive interface is

\[
t(x) = \frac{\sqrt{x^2 + z_1^2}}{v_1} + \frac{\sqrt{(L - x)^2 + z_2^2}}{v_2} \quad 2.22
\]

From Fermat’s principle we know that the actual path taken will be stationary, so must satisfy
Taking the first derivative of Equation 2.22 yields

\[ \frac{dt(x)}{dx} = 0 \]

\[ \frac{dt(x)}{dx} = \frac{x}{v_1 \sqrt{x^2 + z_1^2}} + \frac{-(L - x)}{v_2 \sqrt{(L - x)^2 + z_2^2}} = 0 \]

From basic trigonometry we know that

\[ \frac{x}{\sqrt{x^2 + z_1^2}} = \sin(\theta_i) \]

\[ \frac{(L - x)}{\sqrt{(L - x)^2 + z_2^2}} = \sin(\theta_r) \]

Therefore Equation 2.24 becomes

\[ \frac{dt(x)}{dx} = \frac{\sin(\theta_i)}{v_1} - \frac{\sin(\theta_r)}{v_2} = 0 \]

This leads to Snell’s law (Young and Freedman 2004)

\[ \frac{\sin(\theta_i)}{v_1} = \frac{\sin(\theta_r)}{v_2} \]

**Figure 2.2:** A diagram of refraction at a planar interface between dual layered media

### 2.2.4.3 The critical angle and mode conversion

Let us consider the common inspection scenario of a liquid-solid interface (incident wave in liquid). Figure 2.3 illustrates the transmitted and reflected wave modes which can occur when an oblique longitudinal wave is incident at the interface (shear waves
can only exist in solids, while longitudinal waves can propagate in any state of matter). The reflected wave will remain longitudinal, while the transmitted energy will refract in the solid material as a longitudinal wave and mode converted shear wave. The reflected wave mode will have an angle of reflection equal about the normal to the angle of incidence, while the angles of the refracted wave modes can be described using Snell’s law in Equation 2.28.

Figure 2.3: A diagram of mode conversion at a liquid-solid interface

It can be shown using Snell’s law that at oblique angles of incidence the transmitted energy will be refracted as a shear and longitudinal wave up until the 1st critical angle. At angles close to the normal the longitudinal wave mode will be dominant. Then at some point approaching the 1st critical angle, the shear wave takes over as the dominant wave mode. At the first critical angle the energy of the longitudinal wave mode will fall to zero. Moving further away from the normal to between the first and second critical angles, only the shear wave mode will be transmitted. Then as the second critical angle is reached the energy of the shear wave mode will also fall to zero. Beyond the second critical angle neither the longitudinal nor shear wave modes exist, instead a surface wave is generated (which is theoretically the shear wave refracted at an angle of 90 degrees). Typical amplitude variations of the different wave modes with respect to incident angle are shown in Figure 2.4.
Snell’s law can be used to predict the 1st and 2nd critical angles, and so tell us when the energy of the shear and longitudinal wave modes fall to zero. In order to predict the energy of reflected and transmitted wave modes at all other oblique angles, we must determine the reflection and transmission coefficients. In the case of a liquid-solid planar interface, and ignoring the effects of absorption, the reflection coefficient and the transmission coefficients of the longitudinal and shear wave modes are given by Equations 2.29, 2.30 and 2.31 respectively (Krautkramer and Krautkramer 1977). Where $\rho_1$ and $\rho_2$ are the densities of the liquid and solid respectively, $v_L$ and $v_s$ are the longitudinal and shear velocities in solid, and $v$ is the longitudinal velocity in liquid. Finally, $\theta_L$ is the longitudinal wave angle in liquid, and $\theta_L$ and $\theta_S$ are the longitudinal and shear wave angles in solid respectively.

\[
R = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{2.29}
\]

\[
T_L = \frac{\rho_1 2Z_L \cos(2\theta_s)}{\rho_2 Z_2 + Z_1} \tag{2.30}
\]

\[
T_S = \frac{\rho_1 2Z_s \sin(2\theta_s)}{\rho_2 Z_2 + Z_1} \tag{2.31}
\]

Where

Figure 2.4: Relative amplitudes of transmitted wave modes vs incident angle for a typical water-steel interface (OlympusNDT 2010)
2.2.5 Attenuation

As a sound field propagates through a medium energy is lost as a function of distance due to attenuation. Attenuation is the combined effect of two causes, namely scattering and absorption (Krautkramer and Krautkramer 1977). Scattering results from sudden changes in impedance at grain boundaries and is highly dependent on the grain size. If the grain size is larger than the propagating sound field then reflections off the grain boundaries will occur. In cases where the wavelength of a propagating sound field is similar or smaller than the grain size, it may split into a variety of transmitted and reflected wave modes. At grain sizes which are considerably smaller than the wavelength of the insonifying sound field, the wave front will no longer split into different wave modes, but instead deviate from its original trajectory. To minimise the effects of scattering the frequency of the sound field must be reduced. However, this may have other unwanted effects such as loss in sensitivity to small flaws. The second cause of attenuation is absorption, which occurs when sound is converted into heat due to material vibration. To minimise the effects of absorption the frequency of the sound field can again be reduced, or a higher voltage may be applied across the transducer.

For inspection of a given material at a given temperature using a known frequency, an attenuation coefficient, \( \alpha \), can be determined. The loss in sound pressure due to attenuation can then be defined according to Equation 2.32, where \( p_0 \) is the initial pressure at the source and \( p \) is the pressure a distance, \( d \), from the source (OlympusNDT 2010).

\[
p = p_0 e^{-\alpha d}
\]

2.2.6 Beam spread

Similarly to attenuation, beam spread also causes a propagating sound field to lose energy as a function of distance. As a sound field propagates away from a source, its surface area will increase as a function of distance. If the total energy of the sound field remains constant with respect to distance, then the energy density will decrease with distance. This is because the total energy will be distributed over a greater area (Figure 2.5). This is a commonly known phenomenon, sometimes referred to as the inverse
power law. When a sound field is emitted from a source whose diameter is much smaller than the wavelength, the sound field will be highly divergent. It will also be a poor emitter of sound, and as such the peak amplitude of the propagating sound field will be low, particularly over large distances, which may reduce the sensitivity of the inspection (Krautkramer and Krautkramer 1977).

![Illustration of the inverse power law](image)

**Figure 2.5:** Illustration of the inverse power law

2.2.7 **Signal to noise ratio**

Signal to noise ratio (SNR) is used to measure signal intensity against the background noise according to Equation 2.33. In acoustics SNR is highly dependent upon the frequency of inspection. Higher inspection frequencies allow for tighter focal spot sizes to be generated, which therefore increases the energy density for a given point and so increases the SNR. However, a signal's ability to penetrate a material will decrease with frequency due to attenuation. At low frequencies where grain sizes are small relative to wavelength, scatter is negligible and absorption will be the dominant factor of attenuation. At high frequencies where grain size and wavelength are within an order of magnitude in relative size, then scatter becomes the dominant factor of attenuation which significantly degrades the SNR. However high frequencies are often favourable as they offer greater resolution. Therefore, at a given depth of inspection there will always be an optimum inspection frequency as illustrated by the graph in Figure 2.6.

\[
SNR = 20 \log_{10} \left( \frac{I_{signal}}{I_{noise}} \right) \quad 2.33
\]
2.3 Ultrasonic transducers

2.3.1 Introduction
In the field of ultrasonic NDT ultrasound is transmitted and received using a transducer. Conventional ultrasonic transducers typically consist of a single piezoelectric crystal, a backing material and a wear plate. The basic principle is that a potential difference is applied across a thin piezoelectric crystal for a short period of time, causing it to resonate. The wavelength emitted by a piezoelectric crystal can be calculated from its thickness, \( T \), using Equation 2.34.

\[
\lambda = 2T
\]

The resonant vibration of the crystal is transferred to the test material via a thin wear plate. The wear plate usually has a \( \lambda/4 \) thickness and protects the piezoelectric material from frictional effects when the transducer is in direct contact with the test specimen. The backing material is attached to the piezoelectric crystal and acts as the dampener which determines the ‘ring down’ time (i.e. time taken for the crystal to stop ringing after transmission).

2.3.2 The piezoelectric effect
The vast majority of available ultrasonic transducers rely on piezoelectric crystals to convert electrical pulses into mechanical vibration and vice versa. In an ultrasonic transducer two electrodes are attached to opposite faces of a piezoelectric crystal. When an electric field is applied between the electrodes, the polarised molecules of the crystal align themselves with the electric field. The effect of alignment will cause the crystal to
deform slightly. The effect is also reversible in as much as mechanical deformation of the crystal from an external source will induce an electrical current. This phenomenon is the piezoelectric effect. There are many types of naturally occurring and manmade materials which possess piezoelectric properties, each with its own set of advantages and disadvantages. Typical piezoelectric crystals used for pulse-echo inspections are manufactured from Lead Zirconate Titanate (PZT) as this material is a good transmitter and receiver of mechanical vibration.

2.3.3 Transducer configuration
There is a wide range of piezoelectric transducer configurations readily available on the market. These can be broadly categorised into contact and immersion transducers. Immersion transducers do not come into direct contact with test components, instead a liquid such as water is used as an intermediary medium to transfer ultrasound. These transducers are designed to be completely water resistant and often contain an impedance matched layer to maximise the amount of energy dissipated. To improve SNR and sensitivity, curved lenses can be attached at the end of the transducer to facilitate focussing.

Contact transducers, as the name suggests, operate in direct contact with the test component. To maximise the amount of energy transfer a thin layer (<\(\lambda/4\)) of liquid couplant is applied between the transducer and test component. Typically this will be a water or oil based substance. To increase their potential application, there are a number of different types of contact transducers including single crystal, dual crystal, delay line. Single crystal transducers transmit the longitudinal wave mode into a wedge material. Dependent on the wedge angle a longitudinal or shear wave may be transmitted into the test material or a surface wave generated along its surface. If the component is curved then a curved shoe can be attached to the end of the transducer to improve coupling. Dual crystal transducers operate using one element as the transmitter and the other as a receiver. This allows the two crystals to be made from different materials to optimise their transmit and receive capabilities and so improving SNR. These transducers are effective for inspection of near surface flaws since they have separate transmit and receive elements, so there is no associated ring down time. Similarly delay line transducers use a thick section of material which gives enough standoff between the crystal and test component to allow the crystal to stop ringing and start receiving before signals are reflected back from the test component. Angled beam transducers use a slanted wedge to transmit an oblique shear or longitudinal wave mode into a test
component. This is useful for applications such as welded joints, where the weld cap can prevent inspection from directly above the region of interest, therefore forcing inspection from a side angle. They are also used to obtain a more favourable incident angle on the most plausible planar defects, such as lack of sidewall fusion, found in the weld fusion face.

2.3.4 Ultrasonic radiation fields

A transducer radiating energy does so from a surface of finite area rather than a point source. Therefore, the ultrasonic sound field will be generated from displacements at different positions on the surface of the active element. As such, an interference pattern containing localised regions of high and low pressure will occur in front of the transducer. The near field is defined as the region between the transducer and the last on-axis sound pressure peak (OlympusNDT 2010). The sound pressure peak which occurs at the end of the near field is known as the ‘natural focus’ point. Focussed inspection cannot surpass this distance, however, using curved lenses the natural focus point (and therefore the end of the near field) can be moved closer to the transducer. For a flat circular transducer the end of the near field depends on the ratio between the square of the element diameter, D, and the wavelength as shown by Equation 2.35. Due to the interference pattern in the near field, flaw sizing and characterisation is not performed in this region (Krautkramer and Krautkramer 1977).

\[
\text{Near field length} = \frac{D^2}{4\lambda}
\]

The sound field that occurs after the end of the near field is known as the far field. Here the sound field decays steadily with distance and is more predictable, therefore flaw sizing and characterisation is performed in this region. An illustration of the near and far field generated from a piston source is shown in Figure 2.7.
2.3.5 Beam directivity

Beam directivity is the variation in pressure in the far field with respect to angle. For a sound field emitted from a transducer, beam directivity will depend upon the ratio between acoustic wavelength and the aperture (Krautkramer and Krautkramer 1977). A rectangular aperture whose width in the active plane is larger than its length in the passive plane will generate an asymmetric beam profile with greater beam directivity occurring in the active plane. For a rectangular aperture the beam half angle at -6 dB can be calculated using Equation 2.36 (OlympusNDT 2010), where $L$ represents the element length or width in the appropriate axis. Figure 2.8 illustrates how beam directivity changes with respect to the ratio between the acoustic wavelength and aperture for a piston source.

$$\theta_{-6dB} = \sin^{-1} \left( \frac{0.44\lambda}{L} \right)$$ 2.36
2.4 Ultrasonic arrays

2.4.1 Introduction
The basic principles behind ultrasonic arrays were first developed in the radar and sonar industries following World War II. The first publication which showed the technology applied to material inspection was by Somer (1968), who developed an array transducer for medical applications. In the majority of cases the medical industry has been at the forefront of ultrasonic array inspection technology due to high levels of funding. It is only in recent years with the advent of affordable high speed computers that this technology has become popular in the field of NDT.

The main advantage of arrays over single crystal transducers is that an array transducer can steer, focus and scan over a wide range of angles from a single position. This has the potential to reduce inspection time as it minimises the amount of mechanical scanning. It also allows for large regions to be inspected from a single location, which can be beneficial for inspection of components with limited surface access or complex geometries. The disadvantages of using phased arrays are that equipment can be expensive, so start up costs are high. Also software can be complicated to use and so requires skilled operators and a significant amount of staff training. Finally, because of
the increase in electronic components, phased array systems can be less reliable and more fragile than conventional ultrasonic systems.

2.4.2 Arrays

An ultrasonic array consists of a number of small elements arranged in close proximity of each other. A typical 1D linear array layout is shown in Figure 2.9, where \( g \) is the spacing between adjacent elements (the gap), \( p \) is the centre to centre distance between adjacent elements (pitch), \( w \) is the width, \( h \) is the height and \( a \) is the active aperture.

![Figure 2.9: Footprint of a 1D ultrasonic array transducer](image)

Each element in an array can be controlled independently by the array controller, and as such relative transmission and reception times can be set between elements. This is known as a delay law (or focal law) and facilitates beam steering, scanning and focussing. In order to focus a wave front at a given point using an array transducer, a delay law must be set where the outer most elements are fired first, followed by the inner elements as illustrated by Figure 2.10a. To steer a wave front to the left or right the delay law is set so that an initial excitation occurs at one end of the array, excitation of adjacent elements then continues in a linear fashion along the array as shown by Figure 2.10b.
There are three main types of scanning patterns commonly used in phased array ultrasound, these include linear scanning, sectorial scanning and Dynamic Depth Focussing (DDF). Linear scanning uses a fixed delay law which is multiplexed across the array using a small number of elements (sub-aperture). In other words, at every position along the array, the same delay law is applied to the sub-aperture (Figure 2.11a). Sectorial scanning applies multiple delay laws to sweep over a range of angles using a fixed group of elements. Delay laws are set to allow a constant depth of focus over the entire angular range (Figure 2.11b). DDF facilitates focussing at multiple depths per transmission. This is achieved using several receive delay laws per transmission (Figure 2.11c). This technique therefore increases the depth of field allowing for a uniform sensitivity over a larger area relative to other phased array focussing techniques. An overview of various scanning methods which incorporate different focussing and steering patterns is given by Cochran (2006) and OlympusNDT (2007).
Transducers discussed so far have been 1D linear arrays, these are the most commonly used transducers as they are versatile, cheap in comparison with other transducer configurations, and do not require complex delay law calculations. There are however a number of different transducer types readily available on the market. 1D annular transducers are often used for detection of small flaws, though they require complex delay laws and cannot be used for beam steering. 2D square and elliptical arrays have the advantage of being able to steer and focus in three dimensions, though they require complicated delay law calculations and a large number of transmission and receiver

**Figure 2.11:** Illustration of (a) linear scanning, (b) sectorial scanning and (c) dynamic depth focussing
channels. They can also be expensive compared to their 1D counterparts. Illustrations of the various transducer types are shown in Figure 2.12.

**Figure 2.12**: Illustration of different transducer configurations (Olympus NDT 2011)

Data acquired using ultrasonic arrays are generally displayed in three formats, namely A-scan, B-scan or C-scan presentation as shown in Figure 2.13. A-scans are the most basic analysis tool, but possibly require the most skill to interpret. Here data is presented in time-amplitude or distance-amplitude format (Figure 2.13b). A-scans can be stacked together using a number of methods to form a 2D cross-sectional image known as a B-scan (Figure 2.13c). If a transducer is scanned along the passive axis, then multiple B-scan images can be stacked to produce a C-scan which offers a plan view of the inspection (Figure 2.13d).
2.4.3 Transducer leakage

When deciding on the most appropriate transducer and delay law configuration to use for an inspection, a key consideration is the generation of grating lobes. Grating lobes are constructive interference patterns at angles away from the main lobe and occur when inter-element spacing is periodic and elements are small. During an inspection energy from grating lobes can be reflected back to the transducer, which potentially leads to false indications. The extent to which grating lobes occur depends upon the pitch and element width. If elements are wide, then their beam directivity is reduced and so steering capabilities will be limited. Therefore, any grating lobes at wide angles will be of low amplitude.

In order to totally avoid grating lobes the pitch between elements in an array must be less than $\lambda/2$. If beam steering angles are small then grating lobes can largely be avoided provided the pitch is smaller than $\lambda$. For pitch sizes which are greater than $\lambda$, then grating lobes may appear even without beam steering. The grating lobe angle, $\theta_g$, can be calculated from the angle of the main lobe, $\theta_s$, and the ratio of wavelength over pitch as shown by Equation 2.37, where $n$ takes integer values.
Side lobes are another type of energy leakage away from the main beam, caused by the highly divergent nature of sound. In general side lobes have low energy, however, if required they can be controlled using apodisation techniques, whereby a bias is applied to the voltage across certain elements to minimise leakage. This technique can also be used to control the width and length of the central lobe.

2.5 Advanced data acquisition and image reconstruction methods

2.5.1 Introduction

So far the fundamental concepts behind acoustics and ultrasonic inspection have been discussed. In the remainder of this chapter a review of the current literature, in relation to ideas and concepts developed later in this work are discussed. The aim is to offer the reader a comprehensive insight into the current state-of-the-art and existing technologies.

The array controller used to acquire data in this work was a Micropulse 5PA unit, developed by PeakNDT, Derby, UK. The system can operate in many of the standard NDT modes of inspection including sectorial, dynamic depth focussing and linear scanning, and has been used for both sight inspections and research purposes alike for many years. The system also facilitates the sequential transmission data acquisition technique, FMC, which is fundamental to this work. The name ‘Full Matrix Capture’ was coined at Bristol University by Holmes et al (2004), although as will be discussed later in this chapter, various organisations have developed similar acquisition processes under different names. Since the Micropulse MP5 was the system used in this work, we will refer to the sequential transmission data acquisition process as FMC.

2.5.2 Advanced pulse-echo data acquisition and reconstruction methods

FMC was first described by Holmes et al (2004) as a data acquisition technique which captures raw time-amplitude data for every transmitter receiver combination in an array. That is to say for a transducer containing $n$ elements, there will be $n^2$ received signals. The technique proceeds by transmitting on a single element within the array while receiving on all. This “transmit on one, receive on all” approach is then repeated until all elements in the array have fired once. Holmes also describes a data reconstruction
algorithm known as the Total Focusing Method (TFM). This algorithm initially simulates a discrete grid in front of the transducer, where every cell in the grid represents a pixel in the final image. Treating each pixel as a focal point, the round trip time of flight for every possible transmit receive combination is calculated. In other words, for every focal point there will be $n^2$ time of flight calculations. This time of flight information is then used to extract relevant amplitude information from the raw FMC data using a delay and sum beam forming approach. Since every pixel in the image acts as a focal point, then fully focused imagery in the region of interest can be obtained. If the assumption is made that all elements within an array are identical, there is some repetition within the data. The signal acquired when transmitting on an element $A$ and receiving on an element $B$, is equivalent to the signal when transmitting on element $B$ and receiving on element $A$. Therefore it is only necessary to acquire $n(n+1)/2$ A-scans.

An important consideration in terms of technique selection is SNR. Differences in SNR between FMC and parallel transmission techniques have been previously discussed by Wilcox et al (2006). Here it is expressed that in the case of coherent noise (e.g. mode conversion, grain scatter and wedge echoes), the performance of FMC and parallel transmission techniques will be identical providing the process is time invariant (i.e. transducer and test component remain stationary relative to each other during the acquisition process). In terms of incoherent noise (e.g. electronic and ambient), parallel transmission techniques will have an improved SNR of $\sqrt{n}$ over FMC. However with modern electronics incoherent noise is rarely a limiting factor.

Holmes et al (2005) compared FMC and its associated data processing algorithm (TFM) with existing parallel transmission techniques by quantifying their ability to image point like reflectors. Here it was shown that FMC was capable of generating fully focussed imagery everywhere within the field of view and could outperform all other conventional array techniques in terms of its focussing ability. Building on the TFM algorithm Wilcox et al (2007) goes on to describe the vector-Total Focusing Method (VTFM), an extension to the TFM used to aid in flaw characterisation. This algorithm was shown to calculate the orientation of artificial specular indicators, and also determined their degree of specularity. Extending on this work, Holmes et al (2008) also describes the VTFM. They showed how increasing the aperture of a transducer improved the accuracy with which defect orientation could be determined.
FMC and its associated TFM algorithm is one approach to material inspection which utilises some of the benefits of sequential transmission. There are however several other authors who use similar data acquisition and processing algorithms under a variety of different names. Full Raw Data (FRD) is a technique similar to the combination of FMC and TFM, developed by Lines (1982). The development of FRD allowed for fully focussed imagery to be generated without the need for multiple sweeps, and so reduced data acquisition times. Sampling Phased Array described by Bernus et al (2006) was developed at the Fraunhofer Institute for NDT applications. The fundamental acquisition and data reconstruction processes of this technique again operate in a similar manor to the combination of FMC and TFM. Kroning et al (2006) describes an extension to the fundamental Sampling Phased Array algorithm, which improves its ability to inspect anisotropic materials such as stainless steels and carbon fibre reinforced plastics. It achieves this by using an additional algorithm known as Reverse Phase Matching, which uses experimental data to account for variations in velocity as a function of direction.

Other algorithms which have been developed for material inspection include the Wavenumber algorithm and Inverse Wave-Field Extrapolation algorithm. Both of which operate on data acquired using the same sequential transmission approach as FMC, however, data reconstruction is performed in the frequency domain. Inverse Wave Field Extrapolation was originally developed for application in Seismology (Berkhout 1982), and has since been applied to NDT by Portzgen et al (2007). Similarly the Wavenumber algorithm also originated from the field of Seismology (Stolt 1978), and has only recently been applied to applications in NDT by authors including Chang and Chern (2000), Stepinski (2007) and Hunter et al (2008). In particular, Hunter et al (2008) makes a direct comparison between the time and frequency domain signal processing approaches of the TFM and Wavenumber algorithms. The Wavenumber algorithm was shown to be several orders of magnitude faster than the TFM. However, it also required a regularly sampled array, whereas the TFM could operate on any arbitrary element distribution, resulting in the Wavenumber algorithm being less flexible.

2.5.3 Commercialisation
The maximum frame rate of any pulse-echo ultrasonic inspection is physically limited by the round trip time-of-flight of sound and the number of transmissions required per image, as described by Equation 2.38. For a given depth of inspection the maximum
frame rate of sequential transmission techniques is therefore dependent on the number of elements in the array. The maximum frame rate of parallel transmission techniques however is dependent on the number of focal points. (With parallel transmission techniques every focal point requires a different delay law, which in turn requires its own transmission).

\[
frame rate_{\text{max}} = \frac{1}{\text{No of Transmissions} \times \text{time of flight}}
\]

For FMC factors relating to the array controller and computer processor tend to limit the frame rate long before this physical limit is approached. The Micropulse 5PA used in this work had a maximum data transfer rate via Ethernet of approximately 7 MB/s according to the manufacturer’s specification. For a typical FMC data set acquired using a 64 element transducer we could easily obtain a 4096 by 3000 matrix of data (4096 A-scans of length 3000 sample points). For an 8-bit amplitude resolution (1 byte per sample) this gives us a 12 MB file, yielding a maximum frame rate of approximately 0.6 Hz. To bring us up to the state of the art, the Micropulse FMC is the latest array controller to be released by PeakNDT. This instrument has a data transfer rate of 40 MB/s according to manufacturer’s specification, which for a 12 MB FMC data set would allow a maximum frame rate of 3.3 Hz.

Lines et al (2011) describes a setup which combined National Instruments hardware, including a NI 5752 digitizer and several NI FlexRIO Field Programmable Gate arrays (FPGAs), with other pulser boards and multi-channel preamplifiers. Data was captured and processed using the FRD technique, the algorithms of which had been highly parallelised for optimal processing performance with FPGAs. Using a 32 element array, frame rates of over 20 Hz were achieved.

An alternative method to achieve accelerated data processing via parallel processing was described by Romero-Laorden et al (2011). He demonstrated how the Graphical Processing Unit (GPU) could accelerate the processing speed of the TFM algorithm by over an order of magnitude compared with the Central Processing Unit (CPU). A general overview of the literature suggests that in the last decade many industries have started trialling and implementing GPU based systems for high performance processing. Of particular interest, GPU systems have already been shown to have performance benefits in various aspects of ultrasonics including data reconstruction, simulation and imaging (Dominguez and Ithurralde 2010; Elnokrashy et al 2009; Reichl et al 2008 and Romero et al 2009). It should be noted that while comparisons between software and
hardware configurations act as a useful guideline, the results are not absolute and depend highly on the implementation. Therefore, such results should only be used as a good guideline of what is possible.

There are two known manufacturers of ultrasonic NDT systems which offer a complete inspection package based on sequential transmission. They are Diagnostic Sonar and I-deal Technologies. Diagnostic Sonar offer a 64 channel system known as FlawInspecta. This system is based around National Instruments PXI hardware, and uses the FRD acquisition and data reconstruction technique, as mentioned previously. A ruggedized laptop is also included from which the hardware is controlled. The providers claim that real-time B-scan imaging can be obtained, with frame rates in excess of 100 Hz, though it is unclear whether this is in reference to sequential or parallel transmission techniques.

I-deal Solutions is a spin-off organisation from the Fraunhofer Institute and offers a range of products, including the A1550 IntroVisor, IDealsystem3D and IDealweld. All of these products utilise the Sampling Phased Array inspection technique, developed by the Fraunhofer Institute. The A1550 IntroVisor is a portable, 16 channel, self-contained array controller with built in function keys and a TFT monitor, specifically designed for site inspection and harsh environments. IDealsystem3D is a laptop controlled, two axes scanning system. It operates in contact mode and facilitates conventional phased array or Sampling Phased Array scanning techniques. IDealsystem3D: CFK-Line is similar in specification to the IDealsystem3D instrument, but operates in immersion mode and offers 3 axes of scanning. Similarly, IDealweld is a laptop operated system using the same technology as the IDealsystem3D instrument, but includes a pipe manipulator for inspection of circular welds on small pipes. Little information is provided about the hardware of these systems and their performance.

PeakNDT is a third organisation which offers several hardware systems with sequential transmission functionality which they call FMC. While FMC data can be acquired using the ArrayGen software (developed by PeakNDT to operate the MircoPulse array controller), there is no method of reconstructing the raw data into an image. For this reason the MicroPulse has an open interface which allows other organisations to develop their own software packages, specific to their needs.
2.5.4 Calibration

There are several types of calibration associated with ultrasonic testing. Equipment calibration refers to setting each channel of the ultrasonic instrument to a set voltage and is usually performed by the equipment manufacturer. Calibration of inspection sensitivity is used to compensate for variations in signal amplitude as a function of distance and angle, and allows a uniform signal response from anywhere within the field of view. The decision to calibrate an ultrasonic system prior to inspection depends on the required outcome. If only positional information is needed, then inspection can be conducted without calibration. If flaw sizing and characterisation is required then variations in acoustic energy density must be considered and compensated for. Other forms of calibration include setting time delays to compensate for the sound path in the wedge material, as well as determining the sound velocity of relevant wave modes in the wedge and test material.

For parallel transmission techniques, such as Phased Array, calibration methods are well established and are described in detail by various texts including Olympus (2007). Many modern array controller software also provide wizards which facilitate automated calibration, which take operators through step by step calibration tutorials. For sequential transmission techniques, however, there appears to be very little in the way of calibration procedures. The only literature found to explicitly discuss this topic was by Duxbury et al (2011), who demonstrates a calibration method to correct for sensitivity variations as a function of distance for the FMC acquisition technique. Though there was no mention of how amplitude variation as a function of angle (caused by beam directivity and the oblique angle transmission coefficient) might be accounted for. Holmes et al (2005) also alluded towards a method of calibration using two mathematical functions incorporated into the TFM algorithm. These functions accounted for sensitivity variations as a function of distance and directivity. However there was no suggestion of how the level of sensitivity compensation might be determined experimentally or otherwise.

2.6 Sparse arrays

2.6.1 Introduction

So far the literature has suggested some advantages of using sequential transmission over parallel transmission techniques, particularly in relation to focussing ability. One potential downside to sequential transmission however is the large data sets which need
to be acquired, stored and processed. For an array transducer containing $n$ elements, parallel transmission techniques such as phased array can transmit and receive on all elements in unison. This yields a total of $n$ A-scans which will need to be acquired and reconstructed. In comparison sequential transmission techniques will need to perform the same process on $n^2$ A-scans, making it a fundamentally slower process.

For applications where fast data acquisition rates are critical, such as high high-speed rail inspection, there may not be time to acquire a full data set using sequential transmission (Clark 2004). A common approach which has been adopted by some authors is to implement a sparse array setup (Lockwood and Foster 1994; Moreau et al 2009). As the name suggests, sparse arrays limit the number of transmit and/or receive elements in an array. A reduced data set is therefore obtained which can improve the speed of data acquisition and reconstruction. However, without careful design consideration sparse arrays can suffer from image degradation due to several reasons including, poor side lobe suppression, introduction of grating lobes and reduced energy density of the main lobe.

2.6.2 The effective aperture concept

Using a linear array transducer Moreau et al (2010) investigated the potential speed improvements when using a sparse transmit aperture (i.e. reduce the number of transmit elements while still receiving on all) for both the TFM and Wavenumber algorithm. Results showed that if some reduction in image quality was acceptable, significant gains in data acquisition and processing speeds could be achieved. Moreau et al (2009) then went on to show how far-field approximations and the Effective Aperture Concept could be used to minimise near field image degradation for a sparse transmit aperture.

The Effective Aperture Concept works on the basis that the two way radiation pattern at a given instance in time can be thought of as the locus of the round trip distance between the transmitter and receiver, as illustrated in Figure 2.14. Using far-field approximations the virtual source is defined as the point which will yield the same two way radiation pattern as the combined transmit-receive pair. It is also the mid-point distance between the transmitter and receiver, also shown in Figure 2.14. For a linear array transducer containing $n$ periodically spaced elements, this will yield a total of $2n-1$ virtual sources. There are more transmit-receive pairs than virtual sources as certain pairs will share the same virtual source. If there are $N$ transmit-receive pairs sharing the same virtual source, then the amplitude of the sound field emitted from that source will
be N times greater compared with that of a virtual source with only one associated transmit-receive pair. The effective aperture is simply the distribution of virtual sources and their associated weightings (Figure 2.15). It can be described mathematically as the convolution of the transmit aperture (TA) and receive aperture (RA) as shown by Equation 2.39. Taking the Fourier transform of the Effective Aperture, the radiation pattern of the sound field in the far-field can be predicted, also shown in Figure 2.15. This radiation pattern has many similarities to the frequency response of Finite Impulse Response (FIR) filters for equally spaced elements. The main benefit of applying the Effective Aperture Concept in the design of sparse arrays is that it can be used to predict many of the characteristics of the radiation pattern.

\[
E = TA \otimes RA
\]

Figure 2.14: Locus of the two way radiation pattern between transmitter and receiver, and virtual source
Figure 2.15: The Effective Aperture Concept and predicted radiation pattern obtained from the FFT of the effective aperture.

The Effective Aperture Concept was first proposed for ultrasound imaging by Gehlbach and Alvarez (1981) to aid in the design of a sparse array system for real-time medical imaging applications. However, much of the literature tends to cite Lockwood and Foster (1994) who describe the concept in greater depth. They also showed through simulation how the technique could be used to predict the main characteristics of the central lobe, side lobes and grating lobes. Lockwood et al (1996) and Lockwood and Foster (1996) then went on to prove the Effective Aperture Concept experimentally using linear arrays and 2D arrays respectively for medical applications. While Lockwood assumed elements in an array behaved as point sources, Wooh and Shi (1998) showed the influence of element size on the radiation pattern. They concluded that in fact element size had no effect on the main lobe width, and made only marginal differences to the side lobe amplitude at shallow beam steering angles. In the same work they also showed that element width had no effect on the positioning of grating lobes, although wider elements did help to suppress them.

The Effective Aperture Concept is now widely used for optimisation of sparse array transducer design in many fields including radar, medical ultrasound and NDT (Rattan et al 2008; Liu and Lin 2004; Moreau et al 2009).

2.7 Refraction through dual layered media

The fundamental principles of refraction have been previously covered in Section 2.2.4 of this chapter. Given the acoustic velocity of two media separated by a refractive interface, either the angle of incidence or refraction can be found using Snell’s law in Equation 2.28, provided the other angle is known. Typically, in ultrasonic inspection, the situation arises whereby the position of the transducer and the region of interest relative to the transducer are well defined parameters, but the point of incidence at the
interface is unknown. This leaves the problem that both the angle of incidence and the angle of refraction are undefined and so Snell’s law in the form presented in Equation 2.28 cannot be solved directly. Converting this equation into coordinate form and squaring both sides, Snell’s law takes the form given by Equation 2.40. A few simple steps of rearrangement and Snell’s law can be recognised as a quartic polynomial (Equation 2.42). Equation 2.42 was used by Shin et al (2010) for use in medical ultrasound, who calculated solutions to the quartic polynomial using numerical methods. However, the solutions he presented were for the simplified case of a single element transducer and target separated by a planar refractive interface, with the transducer located at the origin \((x_0 = z_0 = 0)\).

\[
\frac{\beta (x_b - x_0)^2}{(z_b - z_0)^2 + (x_b - x_0)^2} = \frac{(x_s - x_b)^2}{(z_s - z_b)^2 + (x_s - x_b)^2}
\]

\[\beta = \frac{v_2^2}{v_1^2}\]  \hspace{1cm} 2.40

\[p_4 x_b^4 + p_3 x_b^3 + p_2 x_b^2 + p_1 x_b + p_0 = 0\] \hspace{1cm} 2.42

An alternative approach of solving the quartic polynomial is to use the well known Ferrari’s method, which offers an exact analytical solution (Uspensky 1948). Amongst others, Beshers and Oppenheim (1981) applied Ferrari’s method to model the generation of acoustic harmonics caused by material dislocations. However, no literature was found which applied this method to material inspection.

In the NDT industry ultrasonic transducers are routinely coupled to test specimens via a bulk intermediary medium. Examples of this include contact mode and immersion mode testing. In immersion mode both the transducer and specimen are submersed in a liquid medium (usually deionised water) which acts as the couplant. In contact mode a transducer is attached to a plastic wedge, which allows sound to be introduced to the test component over a range of angles in accordance with Equation 2.28. Therefore the ability to account for refraction between materials of different acoustic velocity is fundamental to determining the sound path. In the literature specific to NDT it appears that algorithms developed to account for refraction at the interface between dual layered media (planar or non-planar) are focussed around solving Fermat’s principle using either iterative or numerical methods (Drinkwater and Bowler 2009; Long and Cawley 2007; Long et al 2009; Calmon et al 1998).
It should be mentioned that for contact mode inspection, of specimens with planar or fixed curvature boundaries, it is possible to couple a rigid transducer directly to the surface. This simplifies the delay law calculation as there is no refraction at the interface. However, only the longitudinal wave mode can be used for inspection, and the range of possible beam angles is limited. For components with unknown non-planar boundaries, flexible arrays have been developed which allow elements within the array to conform to arbitrary surface profiles. In this case reconstruction algorithms do not need to account for refraction; they do however require the location of elements relative to each other to been known. Rather than track the location of elements relative to each other, Hunter et al (2010) demonstrated how autofocus algorithms based on maximising image contrast could be used to accurately predict element position and produce focussed imagery using the TFM algorithm.

2.8 Conclusions

In this chapter topics have been discussed which underpin the research conducted in the later chapters of this work. A general background on the fundamental principles of acoustics was presented, followed by an overview of conventional and array ultrasound devices. Finally advanced ultrasonic data acquisition and reconstruction techniques were discussed.
Chapter 3

Full Matrix Capture and the Sequential Phased Array algorithm

3.1 Introduction

In Chapter 2 existing sequential and parallel transmission techniques used with ultrasonic arrays were described. Here parallel transmission techniques, already in use by industry, have been compared with the SPA algorithm. This algorithm utilises time-amplitude data acquired using the FMC acquisition process. Initially a detailed description of the SPA algorithm is given. Then qualitative and quantitative comparisons are made between the SPA algorithm and conventional Phased Array imaging techniques in terms of image quality and inspection speed. The acquisition system used in this work was a 128 channel MicroPulse 5PA array controller, manufactured by PeakNDT, Derby, UK. The system contained separate transmit and receive lines per channel and allowed the use of parallel and sequential transmission techniques. The data acquisition rate via Ethernet was 7 MB/s according to the manufacturer’s specification. Analogue signals were captured and discretised using an 8 bit amplitude resolution. A (0.5 - 10) MHz bandpass filter was applied to all signals output by the MicroPulse. This allowed a sample rate of 25 MHz to be implemented while still obeying the Nyquist criterion. Reconstruction of the raw data was performed in the MATLAB environment using a desktop PC containing two Quadcore 3 GHz CPUs. A diagram of the equipment setup is given in Figure 3.1. The transducer used throughout this thesis was an Olympus 5L64-A2, the key parameters of which are given in Table 3.1.
Table 3.1: Olympus 5L64-A2 transducer parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centre frequency (MHz)</td>
<td>5</td>
</tr>
<tr>
<td>-6dB Bandwidth (%)</td>
<td>80</td>
</tr>
<tr>
<td>Number of elements</td>
<td>64</td>
</tr>
<tr>
<td>Pitch (mm)</td>
<td>0.6</td>
</tr>
<tr>
<td>Element width (mm)</td>
<td>0.55</td>
</tr>
<tr>
<td>Element length (mm)</td>
<td>10</td>
</tr>
</tbody>
</table>

Figure 3.1: Diagram of equipment setup

3.2 Sequential Phased Array

The FMC data acquisition technique is described in Chapter 2. In this chapter and future chapters of work, data sets acquired using this technique will be referred to as the Full Matrix of Data. An algorithm known as Sequential Phased Array was developed, capable to generating full focussed cross-sectional imagery from the Full Matrix of Data. A flow diagram of the major steps involved with the SPA algorithm is shown in Figure 3.2. Initially an operator is required to input a number of setup parameters which define the transducer, the character of the signal applied across the transducer and the region of inspection. Once the operator initiates the inspection process, A-scan data
from every transmit-receive combination were acquired using the FMC technique and stored as the Full Matrix of Data. Information from the input parameters was also used by the focal law calculator to generate the focal law. Initially a discrete grid was simulated in front of the transducer, where every cell in the grid represented a pixel in the cross-sectional image. Treating each pixel as a focal point, the round trip time of flight for every possible transmit receive combination was calculated. Hence, for every focal point there were \( n^2 \) time of flight calculations. If the assumption is made that all elements within an array are identical, there is some repetition within the Full Matrix of Data, i.e. the signal acquired when transmitting on an element \( A \) and receiving on an element \( B \), will be equivalent to the signal when transmitting on element \( B \) and receiving on element \( A \). Therefore, it was only necessary to acquire \( n(n+1)/2 \) A-scans.

At the heart of the focal law calculator are Equations 3.1 and 3.2, which were used in the time of flight calculations. Here \( x_f \) and \( z_f \) represent pixel location, and \( x_T \) and \( x_R \) are the locations of the transmit and receive elements along the \( x \)-axis respectively. Element location in the \( z \)-axis was set to zero.

During the ‘data process’ phase, time of flight calculations were used to extract relevant amplitude information from the Full Matrix of Data. Since every pixel in the image acts as a focal point, fully focused imagery of the region-of-interest could be obtained. A mathematical description of the image intensity \( I(x_f, z_f) \) is given by Equation 3.3. Here \( F \) represents the Full Matrix of Data, and \( h \) is the well known Hilbert transform, used to convert signals from real into complex form. Finally a cross-sectional image could be rendered from the processed data which was used for analysis by an operator. If the algorithm was not interrupted then the acquisition process repeated and a new Full Matrix of Data was generated. The Full Matrix of Data was then processed as before using the focal law data and a cross-sectional image was rendered. In order to improve the appearance of the cross-sectional imagery, a lower amplitude threshold was applied, below which amplitudes were set to zero – typically this threshold ranged between \((20 – 40) \) dB. The process of acquire, process and image, looped until interrupted by the operator. It should be noted that in the scenario where the region of inspection remained constant relative to the transducer, the focal law was unchanged and did not need to be recalculated during the looping cycle.

\[
 r_T = \sqrt{(x_T - x_f)^2 + z_f^2} 
\]  

3.1
3.2.1 The Hilbert transform

In signal processing the Hilbert transform is a mathematical operator which acts on a real signal to return a complex signal. The benefit of this in the context of image reconstruction is it allows the signal magnitude to be determined as shown by Figure 3.3. Here it can be seen that if only the real signal is used to render an image then indications from a single reflector may show up as multiple responses due to the nature of the ultrasonic signal (typically a five cycle Gaussian). This can hinder data interpretation and in the worst case scenario lead to false flaw characterisation.
Complex signals can be represented mathematically by Euler’s Equation given in Equation 3.4. It is straightforward to show that this equation can be used to derive the well-known sine and cosine identities given in Equations 3.5 and 3.6. These identities show that sine and cosine waves consist of both negative and positive frequency components. From the Argand diagram in Figure 3.4 we can see that when positive and negative frequency components of a cosine signal are added together according to the identity in Equation 3.6, the imaginary component cancels, leaving a purely real signal. A similar occurrence happens in the case of a sine wave signal, although the process is slightly more complicated as a 90 degree phase shift is also required (Equation 3.5).

$$e^{j2\pi ft} = \cos(2\pi ft) + j \sin(2\pi ft) \quad 3.4$$

$$\sin(2\pi ft) = \frac{e^{j2\pi ft} - e^{-j2\pi ft}}{2j} \quad 3.5$$

**Figure 3.3:** Plot of (a) raw A-scan data, (b) complex A-scan data generated from the Hilbert transform of raw data, (c) cross-sectional image of a SDH generated from raw data, (d) cross-sectional image of a SDH generated from the modulus of complex data.
In order to determine the complex version of a signal \( \cos(2\pi ft) \), we must express it in the form \( e^{j2\pi ft} \). From the cosine identity we can see that to transform \( \cos(2\pi ft) \) into \( e^{j2\pi ft} \) we must get rid of the negative frequency term \( (e^{-j2\pi ft}/2) \) and double the remaining positive frequency term \( (e^{j2\pi ft}/2) \). This approach can be implemented in three steps: Firstly apply a Fourier transform on the original signal \( \cos(2\pi ft) \), then in the complex frequency domain apply the transformations given in Equation 3.7, where \( G \) represents the signal amplitude in the frequency domain. Finally the inverse Fourier transform should be performed to obtain the complex time domain signal \( e^{j2\pi ft} \). The signal magnitude, \( M \), can be found by calculating the modulus of the complex signal according to Equation 3.8.

\[
G = \begin{cases} 
2G(f) & \text{if } f > 0 \\
G(f) & \text{if } f = 0 \\
0 & \text{if } f < 0 
\end{cases} \quad 3.7
\]

\[
M = \sqrt{Re^2 + Im^2} \quad 3.8
\]

In terms of the SPA algorithm the Hilbert transform is applied to the Full Matrix of Data to return a complex signal. Next the focal law is applied which samples the Full Matrix of Data and determines pixel intensity in the cross-sectional image. Since the sampled data contains a real and imaginary component then the pixel intensity is complex. Finally the modulus of the pixel intensity is determined which returns an
image which is independent of phase and contains only the signal magnitude (Figure 3.3d). It is important that signals remain in raw full bandwidth form during image reconstruction as this allows incoherent noise contributions to be suppressed due to the inherent averaging in the SPA algorithm during the summation process (Equation 3.3).

3.3 Results and discussion: Comparison of techniques

3.3.1 Introduction
In this section the SPA algorithm was compared to existing parallel transmission ultrasonic array inspection techniques already used in industry. These included focussed and unfocussed linear scans and focussed sectorial scans. Specifically, techniques were compared in terms of their image quality and speed of inspection.

Inspections were carried out using three test samples, all of which were manufactured from low carbon steel. Test Block 1 contained a 1.2 mm diameter side drilled hole (SDH) at a depth of 31 mm (Figure 3.5a). Test Block 2 contained five 1.2 mm diameter SDHs positioned 15 mm apart along a horizontal plane at a depth of 50 mm relative to the transducer face (Figure 3.5b). Finally Test Block 3 contained 12 1.2 mm diameter SDHs, positioned 5 mm apart along a 60 degree plane relative to the horizontal (Figure 3.5c).

Figure 3.5: Illustration of (a) Test Block 1 (b) Test Block 2 and (c) Test Block 3
In order to quantitatively compare the focussing ability of different inspection techniques, a point spread function (PSF) was used. The PSF calculated the \(-6\)dB area, \(A_{-6\text{dB}}\), down from the peak response in the generated image according to Equation 3.9, where \(\lambda\) is the wavelength of the propagating wavefront and was used to normalisation the response. A \(-6\) dB amplitude drop was chosen as in industry this is typically the minimum separation required in order to resolve adjacent indicators as described by the British Standards Institution (2010).

\[
PSF = \frac{A_{-6\text{dB}}}{\lambda^2}
\]

3.3.2 Image Quality

Figure 3.6a-d show results from inspection of Test Block 1 using a focussed and unfocussed linear scan, a focussed sectorial scan and Sequential Phased Array. A focus depth of 31 mm was used for the focussed linear scan and sectorial scan. All inspections were performed with the centre of the transducer positioned directly above the 1.2 mm SDH. It is clear that Sequential Phased Array and the sectorial scanning method outperformed the focussed and unfocussed linear scanning approaches in terms of focussing ability. This was confirmed from the quantitative results presented in Table 3.2 which compares the focussing ability of techniques in terms of the PSF parameter. The table also shows that Sequential Phased Array outperformed the sectorial scan by over 25% in terms of its focussing ability. The superior focussing ability of Sequential Phased Array and the sectorial scanning method was related to the fact that focal spot size is a function of aperture size. Both Sequential Phased Array and the sectorial scanning method used the full 64 element aperture to insonify Test Block 1. However, linear scanning methods require a sub-aperture to scan across the full aperture, therefore, in this case only 16 elements were excited at any one time.
Figure 3.6: Inspection of Test Block 1 using (a) a focussed linear scan (b) an unfocussed linear scan, (c) a sectorial scan, and (d) Sequential Phased Array. All images employed a 20 dB threshold.

Table 3.2: Quantitative analysis of the focussing ability of various ultrasound inspection methods using a point spread function.

<table>
<thead>
<tr>
<th>Technique</th>
<th>Focussed linear scan</th>
<th>Unfocussed linear scan</th>
<th>Sectorial scan</th>
<th>Sequential Phased Array</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSF</td>
<td>1.10</td>
<td>1.30</td>
<td>0.42</td>
<td>0.30</td>
</tr>
</tbody>
</table>

While one of the main benefits of Sequential Phased Array is its ability to focus everywhere, delay laws used with parallel transmission are also capable of focussing over a range of positions. For example, with a sector scan a focal point can be set for every angle in the sweep. Typically commercial focal law calculators allow operators to focus at a constant depth, range or distance from the transducer. Inspection of Test Block 2 in Figure 3.7c illustrates the benefit of this. Here a delay law was set at a constant focal depth of 50 mm. Comparing the signal responses of the sector scan with
that of Sequential Phased Array in Figure 3.7d, it is clear that the two techniques have a similar focusing ability for this inspection scenario.

Figure 3.7a-b shows the results from inspection using a focused linear scan with a constant focal depth of 50 mm and an unfocused linear scan respectively. Here the inspection range in the x-axis was physically limited at -14 to +14 mm. This was due to the number of A-scans available to generate the B-scan being limited by the size of the transducer aperture, as discussed in Chapter 2. Comparison of these plots with those generated using a sector scan and the Sequential Phased Array algorithm illustrate how only linear scans suffer from a limited field of view in the lateral direction. While it is feasible to scan at angles away from the normal, the lateral range of inspection will remain constant.

![Figure 3.7](image)

**Figure 3.7:** Inspection of Test Block 2 using (a) a focused linear scan (b) an unfocused linear scan, (c) a sectorial scan, and (d) Sequential Phased Array. All images employed a 20 dB threshold.

Figure 3.8a-d shows the results from inspection of Test Block 3, where multiple SDHs were aligned at an angle of 60 degrees relative to the horizontal. A focal depth of 30
mm was chosen for the focussed linear and sectorial scan. This resulted in SDHs being located at various distances from the focal plane for these inspection techniques. It can be seen that using Sequential Phased Array and the focussed and unfocussed linear scanning methods, all SDHs in the test block could be identified and were correctly located. However Sequential Phased Array offered a much tighter focal spot size in comparison to both linear scanning methods. Using the sectorial scanning method, responses from all SDHs were visible but positional accuracy, amplitude response and focal spot size degraded rapidly as a function of distance from the focal plane (Figure 3.8c). The reason for this related to the size of the aperture. Since the sectorial scan utilised the full aperture it had a smaller depth of field in comparison to the linear scanning approaches (which used a 16 element sub-aperture). It is interesting to note that while the Sequential Phased Array algorithm also utilised the full aperture during image reconstruction, it did not suffer from the limited depth of field associated with the sector scan. The reason for this was that while the sector scan could only focus along a fixed plane, Sequential Phased Array had the ability to focus everywhere within the field of view.
3.3.3 Speed of inspection

So far techniques have been compared in terms of their image quality. In industry another consideration which can sometimes influence technique selection is speed. Examples where speed of inspection can be a key criterion include hazardous environments and production lines. There are many parameters which will affect the frame rate of the SPA technique; these can be broadly divided into three categories including round-trip propagation, data acquisition and image reconstruction. Round-trip propagation refers to the time of flight of ultrasound in a test material and ultimately governs the physical limit of inspection speed. Data acquisition is the time taken for signals received by the transducer to be transferred onto a computer storage device. If data is acquired for offline processing, it may be stored onto the computer hard disk. If however image reconstruction is required online, then such data is stored onto RAM for fast access speed. Image reconstruction refers to the time to convert raw signals into a 2D cross-sectional image.

3.3.3.1 Sequential Phased Array

In certain instances such as inspection in a hazardous environment, operators may only be required to acquire data. Processing and analysis can then be conducted elsewhere. The graph in Figure 3.9 shows the relationship between the file size of the Full Matrix of Data and acquisition time. It can be seen that there is a strong linear relationship between file size and the data acquisition time. Calculating the gradient of the graph the data acquisition rate was found to be 6.48 MB/s which matched closely with the manufacturers specification discussed in Section 3.1. Depending on the inspection scenario, the file size of the Full Matrix of Data may vary considerably due to a number of parameters including number of elements in an array, sample rate, amplitude resolution and A-scan length. In this chapter all Sequential Phased Array results presented were acquired using the half matrix acquisition mode (Section 3.2) in order to reduce file sizes and maximise frame rate.
The image reconstruction time of the Sequential Phased Array algorithm will depend on three main parameters, namely number of pixels in the image (image size), A-scan length and number of A-scans. The graphs in Figure 3.10 show how image reconstruction time varied with respect to each of these parameters. While one parameter was varied the others remained constant and took the following values: The number of A-scans was 2080 and each had a length of 769 sample points. Imagery generated from the raw A-scan data contained 17500 pixels. To put this into perspective, these were the values used to generate the image in Figure 3.8d, which is representative of what may be required in a typical industrial setup. The image reconstruction time for this figure was 0.88s.

The plots in Figure 3.10 show that the parameters which influenced image reconstruction time all followed a linear relationship with respect to time. Therefore the effect each parameter had on image reconstruction time can be determined from their relative gradients. From Table 3.3 it can be seen that the number of A-scans in the Full Matrix of Data has the greatest effect on the image reconstruction time. The number of pixels also has a significant effect whereas A-scan length appears to be negligible by comparison, for the range considered.

**Figure 3.9**: Plot of file size against data acquisition time
Figure 3.10: Plots of reconstruction time against (a) number of A-scans (b) number of pixels (c) A-scan length.

Table 3.3: Table showing the effect of parameters on image reconstruction time

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Gradient (milliseconds/increment)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of A-scans</td>
<td>425</td>
</tr>
<tr>
<td>Number of pixels</td>
<td>100</td>
</tr>
<tr>
<td>A-scan length</td>
<td>40</td>
</tr>
</tbody>
</table>

3.3.3.2 Comparison of techniques

As previously discussed, frame rate is determined from the sum of acquisition and image reconstruction time (acquisition time incorporates propagation time). Table 3.4 shows the acquisition time, image reconstruction time and frame rates which were achieved when generating the plots shown in Figure 3.8. Here, linear scans were generated from 49 A-scans, the sector scan from 161 A-scans and the Sequential Phased
Array algorithm operated on a Full Matrix of Data containing 2080 A-scans. For all techniques the length of each A-scan was 769 sample points, which yielded a total file size of 1.60 MB for Sequential Phased Array, 0.04 MB for the focussed and unfocussed linear scans and 0.12 MB for the sector scan.

**Table 3.4**: Comparison of inspection times between ultrasonic inspection methods

<table>
<thead>
<tr>
<th>Inspection method</th>
<th>Focussed linear scan</th>
<th>Unfocussed linear scan</th>
<th>Sector scan</th>
<th>Sequential Phased Array</th>
</tr>
</thead>
<tbody>
<tr>
<td>File size (MB)</td>
<td>0.04</td>
<td>0.04</td>
<td>0.12</td>
<td>1.60</td>
</tr>
<tr>
<td>Propagation time (ms)</td>
<td>0.42</td>
<td>0.42</td>
<td>1.4</td>
<td>0.54</td>
</tr>
<tr>
<td>Acquisition time (ms)</td>
<td>6</td>
<td>6</td>
<td>19</td>
<td>246</td>
</tr>
<tr>
<td>Reconstruction time (ms)</td>
<td>9</td>
<td>9</td>
<td>12</td>
<td>880</td>
</tr>
<tr>
<td>Total time (ms)</td>
<td>15</td>
<td>15</td>
<td>31</td>
<td>1146</td>
</tr>
<tr>
<td><strong>Frame rate (Hz)</strong></td>
<td><strong>66</strong></td>
<td><strong>66</strong></td>
<td><strong>32</strong></td>
<td><strong>0.9</strong></td>
</tr>
</tbody>
</table>

Comparing the number of A-scans acquired by each of these techniques it can be seen that Sequential Phased Array required data sets which are over an order of magnitude larger than any of the other techniques and hence had a much greater acquisition time. Equations 3.10-3.12 show the relationship between the number of elements in an array and the number of A-scans used for Sequential Phased Array (half matrix), a sector scan and a linear scan respectively, where \( n \) is the size of the array, \( m \) is the size of the sub-array, \( r \) is the angular range and \( i \) is the angular step size. It is clear that for any size of array (excluding the case of a single element) there will always be a larger data set associated with Sequential Phased Array in comparison to a linear scanning approach. For array sizes typical to industry (i.e. 32 elements or more) it is also the case that data sets associated with Sequential Phased Array will be considerably larger than those of sector scans. Since the data transfer rate is the same for all techniques, this means that data acquisition times will always be slower for Sequential Phased Array compared with parallel transmission approaches.
In terms of image reconstruction time, Table 3.4 shows that the linear scanning technique was fastest followed by the sector scan. These techniques require only a few milliseconds to generate imagery from the raw data. In comparison Sequential Phased Array took hundreds of milliseconds. There were two main reasons for this; firstly there was significantly less data for the parallel transmission techniques to process. The most significant reason however was that the Sequential Phased Array image reconstruction algorithm (Equation 3.3) was computationally more intensive in comparison to both linear and sector scanning techniques, which generate imagery by simply stacking A-scans (whereby each A-scan represents a column of pixels).

It has been shown that at present Sequential Phased Array is considerably slower than parallel transmission techniques, both in terms of data acquisition and image reconstruction. By comparison this results in a poor overall frame rate. However, with rapid developments over the past few years in computer technology, affordable hardware systems are now readily available on the market which have rapid acquisition and data processing speeds (National Instruments 2011). It is predicted that implementation of Sequential Phased Array onto such a system would offer a frame rate in excess of 20 Hz (Lines et al. 2011).

### 3.4 Conclusion

In this chapter an algorithm known as Sequential Phased Array has been developed. This algorithm is capable of generating fully focussed imagery from data captured using the Full Matrix Capture data acquisition process. It has been shown that Sequential Phased Array can outperform existing parallel transmission techniques in terms of focussing and defect positioning, and therefore offers an overall superior image quality. However, this is at the cost of inspection speed, for which parallel transmission techniques vastly outperform Sequential Phased Array at present.

\[
Ascan\ Number_{\text{Sequential}} = \frac{n(n + 1)}{2} \quad 3.10
\]

\[
Ascan\ Number_{\text{Sector}} = \frac{r}{l} + 1 \quad 3.11
\]

\[
Ascan\ Number_{\text{Linear}} = n - m + 1 \quad 3.12
\]
Chapter 4

Consideration of refractive boundary conditions

4.1 Introduction

In Chapter 3 the FMC data acquisition technique and SPA algorithm were discussed in the context of a transducer and test specimen in direct contact with each other. However, in the NDT industry it is common for a transducer and test specimen to be coupled via a bulk intermediary medium. In contact mode, a solid Perspex or Rexolite wedge is commonly used for this purpose. The appropriate wedge angle will depend on the characteristics of the component and the types of flaw which are likely to occur. In immersion mode both transducer and component are submerged in a tank filled with water, which acts as the couplant. A similar method of water couplant could be used without full immersion, using water filled inspection wheels or water spray with sledges. One advantage of a purely liquid intermediary medium is that test specimens with irregular surface profiles will still be fully coupled to the transducer.

In this chapter the development of algorithms to determine the path of a sound field propagating through a refractive interface between dual layered media are described. Algorithms were compared in terms of processing speed, imaging quality and their ability to locate discontinuities accurately. Curve-fitting algorithms to enable automatic focussing through dual layered media were also developed. Experimental results from planar and irregular geometry samples containing known artificial flaws were used to validate developed algorithms. Finally experimental results from inspection of a double ‘v’ butt welded plate are presented which demonstrate the industrial relevance of the algorithms.

4.2 Solutions to Snell’s law

In Chapter 2 it was shown how Snell’s law may be derived from Fermat’s principle and expressed as a quartic polynomial as shown in Equation 4.1 where \( x_b \) is the position of the interface along the x-axis.

\[
p_4 x_b^4 + p_3 x_b^3 + p_2 x_b^2 + p_1 x_b + p_0 = 0 \quad 4.1
\]
The interface between dual layered media can also be expressed as a polynomial, which takes the general form

\[ z_b(x_b) = m_n x_b^n + m_{n-1} x_b^{n-1} + \ldots + m_2 x_b^2 + m_1 x_b + m_0 \]  \hspace{1cm} 4.2

For the simplest case of the a planar interface parallel with the x-axis, Equation 4.2 can be expressed as

\[ z_b = m_0 \]  \hspace{1cm} 4.3

In this instance the coefficients of Equation 4.1 can be expressed as shown by Equation 4.4-4.8 (Weston et al 2012).

\[ p_4 = \beta - 1 \]  \hspace{1cm} 4.4

\[ p_3 = 2x_0 - 2\beta x_0 + 2x_s - 2\beta x_s \]  \hspace{1cm} 4.5

\[ p_2 = -x_0^2 + \beta x_0^2 - 4x_0x_s + 4\beta x_0 x_s - x_s^2 + \beta x_s^2 \]  \hspace{1cm} 4.6

\[ -(-z_0 + z_b)^2 + \beta(-z_b + z_s)^2 \]

\[ p_1 = 2x_0^2 x_s - 2\beta x_0^2 x_s + 2x_0 x_s^2 - 2\beta x_0 x_s^2 \]  \hspace{1cm} 4.7

\[ + 2x_s(-z_0 + z_b)^2 - 2\beta(-z_b + z_s)^2 \]

\[ p_0 = -x_0^2 x_s^2 + \beta x_0^2 x_s^2 - x_s^2(-z_0 + z_b)^2 + \beta x_0^2(-z_b + z_s)^2 \]  \hspace{1cm} 4.8

Where \((x_0, z_0)\) is the position of the transmitter or receiver, \((x_s, z_s)\) is the focal point and \(\beta\) is the ratio of the material velocities squared \((v_b^2/v_s^2)\). These coefficients assume a constant gradient, parallel with the x-axis; which therefore implies that the normal to the interface is a constant and parallel with the z-axis.
Figure 4.1: A diagram of refraction at a non-planar interface between dual layered media

In the general case of a non-planar refractive boundary described by Equation 4.2 and illustrated in Figure 4.1, the gradient can no longer be considered a constant, but is dependent on \( x_b \). Since the angle of incidence and angle of refraction are both parameters which are determined relative to the normal at a given point, then the gradient must be known for every point along the interface to determine the point of incidence. To calculate the gradient, it is necessary to differentiate Equation 4.2 which describes the interface. From the gradient of the tangent, seen in Figure 4.1, the coordinates \((x_{t1}, z_{t1}, x_{t2}, z_{t2})\) can then be calculated.

\[
\frac{dz_b}{dx_b} = m_n nx_b^{n-1} + m_{n-1} (n - 1)x_b^{n-2} + \cdots + 2m_2x_b + m_1 \quad 4.9
\]

Since the interface and its derivative described by Equations 4.2 and 4.9 respectively are dependent on \( x_b \), and Equation 4.9 contains terms involving division by the variable \( x_b \), Snell’s law in Equation 4.1 will take the form of a non-polynomial equation, whose coefficients are expressed by Equation 4.10-4.18 (Weston et al 2012).

\[
p_4 = \beta - 1 \quad 4.10
\]
\[
p_3 = 2x_0 + 2x_{t2} - 2\beta x_s - 2\beta x_{t1} \quad 4.11
\]
\[
p_2 = \beta x_s^2 + \beta x_{t1}^2 + 2\beta z_b^2 + \beta z_s^2 + \beta z_{t1}^2 - x_0^2 - x_{t2}^2 - z_0^2 \\
- 2z_b^2 - z_{t2}^2 - 4x_0x_{t2} + 2z_0z_b + 2z_bz_{t2} + 4\beta x_s x_{t1} - 2\beta z_b z_s - 2\beta z_b z_{t1} \quad 4.12
\]
\[ p_1 = 2x_0^2t_2 + 2x_0^2z_t + 2x_0z_t^2 - 4x_0z_0z_t^2 + 2x_0z_0^2 
- 2\beta x_0^2t_1 - 2\beta x_0z_0t_1 - 2\beta x_0z_0^2 
+ 4\beta x_0z_0z_t - 2\beta x_0z_t^2 - 2\beta x_0z_t^2 
+ 4\beta x_0z_0z_t - 2\beta x_0z_t^2 - 2\beta x_0z_t^2 
- 4x_0z_0z_t + 2x_0z_t^2 \quad 4.13 \]

\[ p_0 = \beta z_0^4 + 2z_0z_1^3 + 2z_1^3z_t - z_1^4 - x_0^2z_0^2 - x_0^2z_0^2 - x_0^2z_0^2 
- x_0^2z_0^2 - x_0^2z_0^2 - x_0^2z_0^2 
+ \beta x_0^2z_0^2 + \beta x_0^2z_0^2 + \beta x_0^2z_0^2 + \beta x_0^2z_0^2 
+ \beta x_0^2z_0^2 + \beta x_0^2z_0^2 + \beta x_0^2z_0^2 + \beta x_0^2z_0^2 
- 2\beta z_0z_0z_t - 2\beta z_0z_0z_t + 2x_0z_0z_0z_t + 2x_0z_0z_0z_t 
+ 2z_0z_0z_0z_t - 4z_0z_0z_0z_t + 2z_0z_0z_0z_t 
- 2\beta x_0z_0z_0z_t - 2\beta x_0z_0z_0z_t - 2\beta x_0z_0z_0z_t 
- 2\beta x_0z_0z_0z_t + 4\beta x_0z_0z_0z_t \quad 4.14 \]

Where

\[ x_{t1} = \frac{z_0 - z_0 + \frac{dz_0}{dx_0}x_0}{\left(\frac{dz_0}{dx_0} + \frac{dz_0}{dx_0}\right)^2 + 1} \quad 4.15 \]

\[ z_{t1} = -\frac{x_{t1}}{\left(\frac{dz_0}{dx_0}\right)} + z_0 + \frac{x_0}{\left(\frac{dz_0}{dx_0}\right)} \quad 4.16 \]

\[ x_{t2} = \frac{z_s - z_s + \frac{dz_s}{dx_s}x_s}{\left(\frac{dz_s}{dx_s} + \frac{dz_s}{dx_s}\right)^2 + 1} \quad 4.17 \]

\[ z_{t2} = -\frac{x_{t2}}{\left(\frac{dz_s}{dx_s}\right)} + z_s + \frac{x_s}{\left(\frac{dz_s}{dx_s}\right)} \quad 4.18 \]

### 4.3 Development of the SPA Dual-Media algorithm

Having derived mathematical expressions to account for refraction at the boundary between dual layered media, this section of the thesis describes different methods to solve these equations. Also discussed is how these equations were implemented to enhance the inspection capabilities of the FMC technique. Two main algorithms were
developed for this purpose, namely the SPA Dual-Media and SPA Autofocus algorithms.

The SPA Dual-Media algorithm was developed to facilitate focused inspection through dual-layered media separated by a refractive interface. A flow diagram which illustrates the major stages of the SPA Dual-Media algorithm can be found in Figure 4.2. The major steps of this algorithm are similar to that of the SPA algorithm described in Chapter 3. The only difference being the inclusion of the Point of Incidence Methods (POI Methods) as seen in Figure 4.2. In total there were three POI methods developed, these include the Iterative POI, Numerical POI and Analytical POI methods. As their names suggest, each algorithm has the capability of determining the point of incidence between dual layered media, but use different mathematical techniques to achieve this.
4.3.1 Iterative POI method: A mathematical approach to solve Fermat’s principle

From Fermat’s principle it is known that the total path length of a sound beam propagating between two points is stationary. Hence for a sound field propagating between two points separated by a refractive interface, the point of incidence can be determined by locating the position along the interface where the time of flight is at a localised minima or maxima.

The Iterative POI method was developed in the MATLAB environment and was capable of determining the point of incidence between dual layered media based on an iterative approach to solve Equation 2.20 with respect to position along the interface. The MATLAB function `findpeaks.m` was used to determine points along the interface.
corresponding to a local minima or maxima with respect to time of flight. Provided there was a known equation which described the interface between dual layered media, of the form shown in Equation 4.2, then the algorithm was valid for both planar and non-planar boundary scenarios. In order to check that the stationary points calculated using the POI Iterative method were correct, the constraint that solutions obeyed Snell’s law in Equation 2.28 was imposed. Since the algorithm was based on an iterative approach, the beam path error $\Delta x_s$ will largely depend on the iterative step size $\Delta x_b$. From Equation 2.40 it can be seen that the error in beam path when the sound field is in the secondary medium, will always be larger than the step size used. In other words the iterative step size $\Delta x_b$ must always be smaller than the allowable beam path error $\Delta x_s$, as illustrated by Figure 4.3.

**Figure 4.3:** An illustration showing how small errors in predicted beam path at the interface $\Delta x_b$, can cause significant errors in predicted beam path in the secondary medium $\Delta x_s$.

### 4.3.2 Analytical POI method: A Mathematical approach to solve Snell’s law

In the case of a planar boundary between dual layered media, Snell’s law takes the form of a quartic polynomial given by Equation 4.1. The Analytical POI method calculates the four roots to this equation using Ferrari’s method (Uspensky 1948), which proceeds as follows

\[
\alpha = -\frac{3p_3^2}{8p_4^4} + \frac{p_2}{p_4} \quad 4.19
\]

\[
\beta = \frac{p_3^3}{8p_4^3} - \frac{p_3p_2}{2p_4^2} + \frac{p_1}{p_4} \quad 4.20
\]
From these values the point of incidence can be found using the following equation

\[ x_b = -\frac{p_3}{4p_4} + \frac{\pm W \mp \sqrt{\left(3\alpha + 2V \pm \frac{2\beta}{W^2}\right)}}{2} \]  

Once the position of \( x_b \) is known, it is then straight forward to calculate \( z_b \) from the equation of the interface between dual layered media given by Equation 4.2. The unique solution to Snell’s law can be found through the constraint that the root is real and lies between the position of the transducer (\( x_0 \)) and the focal point (\( x_s \)). Alternatively, the constraint that the actual root corresponds to the minimum time of flight could be imposed. Both constraints will lead to a unique solution for Snell’s law, therefore the approach which requires the least computer processing and memory should be implemented. The main disadvantage of solving Snell’s law using an analytical approach is that it can only be implemented when the interface between dual layered media is planar. For a non-planar interface Equation 4.1 will alter its form and contain exponents of \( x_b \) which are negative and greater than 4th order, so cannot be solved analytically.

### 4.3.3 Numerical POI method: A Mathematical approach to solve Snell’s law

An alternative approach to solve Snell’s law is to implement an algorithm based on numerical methods. For a planar boundary the Numerical POI method was developed which uses the MATLAB function `roots.m` to solve Equation 4.1. This function
employs numerical methods based on computing the eigenvalues $\lambda$ of the companion matrix $M$ given in Equation 4.29, where $x$ is an eigenvector of the companion matrix (Williams M.P. 2010).

$$MX = \lambda x \quad 4.29$$

Similarly to the Analytical POI method, a unique solution to Snell’s law can be found through the constraint that the root is real and lies between the position of the transducer ($x_0$) and the focal point ($x_s$), or that the actual root is that which corresponds to the minimum time of flight.

![Figure 4.4: A plot of (a) time of flight against position along the interface and (b) beam path, for a sound field propagating between two points separated by a quadratic interface.](image)

Figure 4.4: A plot of (a) time of flight against position along the interface and (b) beam path, for a sound field propagating between two points separated by a quadratic interface.

For a non-planar interface Snell’s law in Equation 4.1 does not take the form of a polynomial, as seen through analysis of the coefficients of Snell’s law for a non-planar interface in Equations 4.10 – 4.18. Therefore the MATLAB function `roots.m` is no longer suitable. Instead the function `fsolve.m` was implemented as part of the Numerical POI method to determine the points of incidence. This function determines the roots of the equation based on a least squares approach.

In the case of a non-planar interface it is possible for a sound field to be incident at several localised minima or maxima as illustrated in Figure 4.4 and Figure 4.5. Figure 4.4a shows the time of flight for a sound field propagating through a quadratic interface between two points as a function of the point of incidence. The simulated beam path in Figure 4.4b shows the sound field to be incident at the point along the interface which corresponds to a local maximum time of flight.
Figure 4.5: A plot of (a) beam path and (b) stationary points (roots), for a sound field propagating between two points separated by a 5th order polynomial interface.

Table 4.1: Predicted roots of Snell’s law calculated using the MATLAB function `fsolve.m` to solve Equation 4.1 for a 5th order polynomial interface.

<table>
<thead>
<tr>
<th>Predicted root, $x_b$ (mm)</th>
<th>Incident angle ($^\circ$)</th>
<th>Refractive angle ($^\circ$) calculated by Equation 4.1</th>
<th>Refractive angle ($^\circ$) calculated by Equation 2.28</th>
<th>Valid root?</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4.36</td>
<td>16.23</td>
<td>-45.55</td>
<td>45.55</td>
<td>NO, negative refraction</td>
</tr>
<tr>
<td>-3.24</td>
<td>-21.80</td>
<td>-71.54</td>
<td>-71.54</td>
<td>YES</td>
</tr>
<tr>
<td>-0.47</td>
<td>-12.82</td>
<td>-34.52</td>
<td>-34.52</td>
<td>YES</td>
</tr>
<tr>
<td>0.15</td>
<td>4.22</td>
<td>-10.83</td>
<td>10.83</td>
<td>NO, negative refraction</td>
</tr>
<tr>
<td>3.54</td>
<td>15.75</td>
<td>43.89</td>
<td>43.89</td>
<td>YES</td>
</tr>
<tr>
<td>4.26</td>
<td>-10.25</td>
<td>27.02</td>
<td>-27.02</td>
<td>NO, negative refraction</td>
</tr>
<tr>
<td>7.81</td>
<td>72.82</td>
<td>-18.96</td>
<td>90.00 – 88.25i</td>
<td>NO, complex angle</td>
</tr>
</tbody>
</table>
The simulation in Figure 4.5a illustrates a scenario in which there are 3 valid paths for a sound field propagating through a non-planar interface between a fixed start and endpoint. Shown in Figure 4.5b are all the roots predicted by the function \texttt{fsolve.m}. Analysis of Table 4.1 shows that there are in fact seven predicted roots, however, only three are valid as one was complex and three predicted negative refraction. In order to establish the valid roots, the constraint that solutions obey Snell’s law in Equation 2.28 must be imposed. Using this restriction ensures that all solutions are real and stationary. This restriction also ensures that solutions which imply a negative refraction are ignored.

Negative refraction refers to a sound beam refracted with a negative angle of refraction \( \theta_{nr} \) (Figure 4.6); a phenomenon not physically possible in most materials (Iyer 2002). The prediction of negative refraction occurs in the algorithms developed due to the symmetry introduced to Snell’s law in Equation 2.40. Here both sides of the equation which defines Snell’s law have been squared, therefore it is no longer possible to differentiate between positive and negative angles of refraction. Equation 4.30 describes this explicitly.

\[
\frac{\sin^2(\theta_i)}{v_1^2} = \frac{\sin^2(\pm \theta_r)}{v_2^2}
\]  

4.30

Figure 4.6: An illustration of negative refraction

\[\theta_i = \theta_{nr}\]
4.3.4 Consideration of multiple solutions to the refractive boundary problem

For a sound field propagating through a non-planar interface, it is possible for multiple sound paths to exist between two points. This was illustrated by the simulation in Figure 4.5, where a sound field propagated along three different paths for a given start and end point. Signal contributions from all possible sound paths could be included in the SPA Dual-Media algorithm, however this would increase the required computer processing and may include signals which, while theoretically valid, may be at beam angles where only very weak signal contributions are obtainable (Wooh and Shi 1999). Introducing a criterion that permits only a single sound path between two points minimises the required computer processing. By also stipulating that the selected sound path is the path with greatest beam strength, ensures the SNR is as high as possible.

To determine the path of maximum beam strength several parameters must first be defined, including beam attenuation, beam angle relative to the transducer face $\theta_t$ and angle of incidence $\theta_i$, as illustrated in Figure 4.7. Factors which influence attenuation and beam spread are described in detail in sections 2.2.5 and 2.2.6 respectively. The angle at which sound is incident on a refractive interface will directly affect the amount of energy that is reflected or transmitted into the test specimen. For an incident beam normal to the boundary the ratio of energy transmitted and reflected can be described using the reflection and transmission coefficients in Equations 4.31 and 4.32 respectively. In the case where the incident beam is not normal with the interface, both the reflection and transmission coefficients become dependent on the angle of incidence (Section 2.2.4).

\[ R = \frac{Z_2 - Z_1}{Z_2 + Z_1} \]  \hspace{1cm} 4.31

\[ T = \frac{2Z_2}{Z_2 + Z_1} \]  \hspace{1cm} 4.32
Figure 4.7: A diagram illustrating beam angle relative to the transducer face $\theta_t$, and angle of incidence $\theta_i$.

4.3.5 Development of the SPA Autofocus algorithm

To calculate the beam path of a sound field propagating through dual layered media it is necessary to have knowledge of the transducer position relative to the interface between dual layered media, as well as the equation which defines the interface (Equation 4.2). In many cases however, the equation which defines the geometry of a component is unknown. This can be due to several reasons, such as manufacturers not wishing to release such information or the component changing shape over time due to continual wear or induced stresses.

In this work an autofocus algorithm known as the SPA Autofocus algorithm was developed, capable of mapping the geometry of irregular shaped components from data acquired using the FMC technique. It should be noted that this is the same data from which focussed imagery of the components sub-surface is later generated. This is useful for two reasons; firstly it minimises acquisition time as only a single set of data is required; secondly the same transducer can be used to map the geometry of a component and inspect its sub-surface, which saves equipment cost and space during inspection.

The SPA Autofocus algorithm uses many of the same processes as the SPA Dual Media algorithm. The main difference between them is that the SPA Autofocus algorithm uses the Full Matrix of Data in order to map the interface between dual layered media using a curve fitting process as shown in Figure 4.8. Once the interface has been defined the point of incidence can be determined and the algorithm continues in much the same manner as the SPA Dual Media algorithm in Figure 4.2.
The curve fitting process was implemented in the MATLAB environment, the major steps of which are shown by the process diagram in Figure 4.8. Curve fitting proceeds by generating a cross-sectional image of the components geometry from the Full Matrix of Data. Once a cross-sectional image of a component’s geometry had been generated (Figure 4.9a), a best-fit curve was used to express the geometry as a polynomial of the form shown in Equation 4.2 and illustrated by Figure 4.9b. A best-fit curve was fitted to the interface based on a trial and improvement method; starting with a first order polynomial. The function *polyfit.m* was used to estimate the polynomial coefficients which best fitted the data representing the interface between dual layered media. The function *polyval.m* was then used to determine the associated error with the estimated polynomial fit. If this error was above a user defined threshold, then the process repeated using a second order polynomial. The process of increasing the order of the polynomial was repeated until the error associated with the best-fit curve fell below the threshold, as illustrated by the simulated results in Figure 4.10.
Figure 4.8: Flow diagram of the SPA Autofocus algorithm
Figure 4.9: Images of (a) response from a planar interface and (b) line of best fit, predicted using the SPA Autofocus algorithm

Figure 4.10: Graphical representation of the curve-fitting algorithm fitting a (a) 1st order (b) 2nd order (c) 3rd order (d) 4th order polynomial (continuous line) to a simulated interface (dotted line). The error refers to the average error per point between the interface and the curve fit line.

In a real inspection scenario noise and near surface discontinuities would also be plotted in the cross-sectional image of the interface. Therefore a user defined amplitude
threshold was implemented which ignored signals below a given value. This ensured that the majority of signals included in the curve-fitting algorithm were indeed reflections from the interface. The amplitude threshold was based on the assumption that the reflected signal from the interface had greater amplitude than any other signal contribution in the generated cross-sectional imagery. While this assumption is not guaranteed it is likely to hold true as incoherent noise levels will be suppressed due to the inherent averaging of the SPA algorithms (Section 3.2). Also discontinuities positioned below the interface will be out of focus as the interface is not defined until after the curve fitting process and so focussing through it is not possible.

A useful feature of the SPA Autofocus algorithm was its ability to switch between the longitudinal and shear wave modes automatically during inspection, based on the angle of incidence. By knowing the ratio of velocity between dual layered media it was possible to determine the first critical angle, \( \theta_c \), as given by Equation 4.33.

\[
\theta_c = \sin^{-1} \left( \frac{v_1}{v_2} \right)
\]

The first critical angle is the angle of incidence above which the longitudinal wave mode will be totally reflected, and so no transmission will occur. If the angle of incidence was less than the first critical angle, then a focal law was generated based on the longitudinal wave velocity in the secondary medium. If the angle of incidence was greater than or equal to the first critical angle then the calculation was based on the shear wave velocity in the secondary medium.

**4.4 Results and discussion: Inspection through a planar refractive interface**

**4.4.1 Introduction**

As part of this chapter several algorithms have been developed which allow focussed inspection through dual layered media separated by a refractive interface. In this section the SPA Dual-Media and SPA Autofocus algorithms which facilitate focussed inspection through a planar interface are validated experimentally using test samples containing known artificial flaws. Finally, results from the inspection of a double ‘\( \text{v} \)’ butted welded plate containing real flaws are presented and discussed.

The acquisition system used was a Micropulse 5PA 128/128 array controller. Data were acquired using the FMC technique and processed using a desktop computer with two
quad core 3 GHz CPUs. The received data were sampled at 50 MHz and stored with an 8 bit dynamic range. The transducer used for inspection was an Olympus 5L64-A2, described in Table 3.1. This was mounted onto a 36 degree Perspex wedge which induced a shear wave mode into the test samples. A diagram of the setup is shown in Figure 4.11.

![Diagram of equipment setup](image)

**Figure 4.11:** A schematic diagram of the equipment setup

Before inspection of an industrial structure can take place, it is first necessary to validate the algorithms used in order to have confidence in the experimental results. The test blocks used in the validation process were specifically chosen to validate the inspection of a double ‘v’ butt welded plate, although did not contain a welded region themselves. Test blocks were manufactured from the same parent material (low carbon steel) and artificial flaws were located at depths which corresponded to the depth of the weld body (region of inspection) of the double ‘v’ butt welded plate. A total of three test samples were used for validation of the SPA Dual-Media and SPA Autofocus algorithm, all of which are illustrated in Figure 4.12. Test Block 4 was a 25 mm thick, low carbon steel block containing four 3 mm SDHs positioned 10 mm apart along a 60 degree plane. Using an orientation of 60 degrees meant that both skip and direct beam inspection could be performed while minimising interference between SDHs. A thickness of 25 mm was chosen for Test Block 4 as this was the thickness of the double ‘v’ butt welded plate. Therefore when performing a skipped beam inspection using the backwall, the beam path and beam path length would be similar to that in the double ‘v’ butt welded plate. Skipped beam inspection is a commonly used technique for plate and pipeline inspection as it allows the regions of the weld body that are inaccessible by direct beam inspection due to the weld cap to be interrogated. Test Block 5 was a low carbon steel block containing three 3 mm SDHs which were vertically aligned and spaced 10 mm apart. By aligning the holes in the vertical plane the positional accuracy of the algorithms could be checked visually, as any error in position would show up in
the generated imagery as misalignment or skew on the display. This is a common technique used onsite by NDT operators as it is fast and only requires visual interpretation rather than quantitative data analysis. Test Block 6 was a low carbon steel block containing a single 3 mm SDH at a depth of 14 mm. A test block containing a single flaw was chosen to ensure there could be no spurious indications imaged as a result of reflections from other discontinuities.

Figure 4.12: Schematic diagram of (a) Test Block 4 showing a skipped beam path (b) Test Block 5 showing a direct beam path (c) Test Block 6 showing a direct beam path.

It is impossible to place a definitive value on the acceptable accuracy of an inspection, as the allowable tolerance will ultimately be specified by the client and will vary greatly depending on the application. The test blocks used during validation all contained 3 mm SDHs. This type of artificial flaw was chosen because it appears in a number of European and British Standards calibration blocks (British Standards Institution 1986 and 2000) and is also omni-directional, so any incident sound field would always be reflected back to the transducer, irrespective of angle.
4.4.2 Validation of the SPA Dual-Media algorithm for a planar refractive interface

There were two main experiments performed to validate the SPA Dual-Media algorithm. In the first experiment the spatial response and positional accuracy of the SPA Dual-Media algorithm for each of the different POI methods were analysed using both qualitative and quantitative techniques. For inspection using a direct beam path the transducer was positioned with a 52 mm stand-off relative to the SDH which was at a depth of 15 mm in Test Block 4. For skipped beam inspection the same SDH in Test Block 4 was interrogated, but a stand-off of 57 mm was chosen to maintain optimum beam angle (as discussed in Section 4.3.4). In the second experiment a visual alignment check was performed using the 3 vertically aligned SDHs in Test Block 5. The transducer was positioned with a 61 mm stand-off relative to the SDHs which were located at a depth of 20, 30 and 40 mm below the surface.

To measure the spatial response the PSF function given in Equation 3.9 was used. The parameter Relative PSF, was also used for quantitative analysis and is simply a measure of the fractional difference in PSF of the numerical and iterative methods relative to the analytical method according to Equation 4.34, where $PSF_{Method}$ refers to the PSF of each method expressed in Table 4.2. The parameter $\Delta Peak\ Position$ is a measure of the distance between the maximum signal response generated using the numerical and iterative methods compared with that of the analytical method.

$$Relative\ PSF = \left| \frac{PSF_{Method} - PSF_{analytic}}{PSF_{analytic}} \right| \quad 4.34$$

4.4.2.1 Validation Test 1: Quantitative comparison of techniques

Figure 4.13 displays the spatial responses from a 3 mm SDH in Test Block 4, insonified using a direct beam path. All images were generated from the same raw data set to avoid any differences in signal response during the data acquisition process. Figure 4.13a shows the spatial response when using the Analytical POI method to determine the point of incidence at the interface. This algorithm was used as a benchmark, to which the numerical and iterative based algorithms were compared as it offers an exact solution to Snell’s law. As such there is no theoretical error associated with the calculation. In comparison, the numerical and iterative methods only offer an approximate solution, accurate to within a specified tolerance. Ultimately the interference pattern of sound waves relates to time of flight, which in turn relates to the
point of incidence determined by Snell’s law. Therefore consideration must be given to the specified tolerance when using the numerical and iterative based algorithms.

Figure 4.13b shows the spatial response when using the numerical methods algorithm to determine the point of incidence at the interface. The image is identical to that generated using the analytical algorithm. This was expected as the numerical methods algorithm determined the point of incidence to a point which was many orders of magnitude smaller than the resolution of the raw data and the resolution of the generated image. Figure 4.13c-h show the spatial response from the 3 mm SDH when using the Iterative POI method with a step size of 0.5, 1, 2, 3, 4 and 5 mm to determine the point of incidence at the interface respectively. Comparing the spatial responses generated by the iterative method with that of the analytical method, no visual differences could be seen when using a step size of 0.5 mm and 1 mm. For a step size of 2 mm and 3 mm only very slight differences in spatial response were visible. However with a step size of 4 mm there was considerable image degradation which included loss in resolution and slight misalignment of the indication. When the step size was further increased to 5 mm the response from the SDH was fragmented and could not be used to determine a single reflector.

Figure 4.13i-o show the -6 dB contour plots of the 3 mm SDH in Test Block 4. Each plot contains the response from the analytical method and an overlay of the response from either the numerical or iterative method. When using the iterative method a step size of 2 mm was sufficient to generate an image comparable to that of the analytical method. However, when using a step size of 3 mm or greater the positional accuracy of the technique was significantly affected, and for a step size of 4 mm or greater the accuracy of both the position and spatial response was degraded. Closer analysis also shows a bias in the positional error of the iterative method, which becomes more prominent with increasing step size. This effect can be explained through analysis of Figure 4.3 which illustrates how the magnitude of beam path error increases as a function of distance.

Quantitative analysis performed on each of the plots in Figure 4.13a-o can be found on the left hand side of Table 4.2. It can been seen that the numerical and iterative method up to and including a 0.5 mm step size have near identical sizing and positional accuracy in comparison to the analytical method. If a slight error in positional accuracy and spatial response can be tolerated, then the iterative method with a step size of up to 2 mm was also valid. From the Relative PSF calculations it can be seen that the PSF of
the numerical and iterative method up to a step size of 3 mm match that of the analytical method to within 10%.

**Table 4.2:** Quantitative analysis of the SPA Dual-Media algorithm in response to a 3 mm SDH in Test Block 4.

<table>
<thead>
<tr>
<th>Method</th>
<th>Direct beam path</th>
<th>Skipped beam path</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PSF</td>
<td>Relative PSF</td>
</tr>
<tr>
<td>Analytic</td>
<td>0.27</td>
<td>N/A</td>
</tr>
<tr>
<td>Numerical</td>
<td>0.27</td>
<td>0</td>
</tr>
<tr>
<td>Iterative</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.5 mm step)</td>
<td>0.25</td>
<td>0.07</td>
</tr>
<tr>
<td>Iterative</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1 mm step)</td>
<td>0.25</td>
<td>0.07</td>
</tr>
<tr>
<td>Iterative</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2 mm step)</td>
<td>0.24</td>
<td>0.10</td>
</tr>
<tr>
<td>Iterative</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3 mm step)</td>
<td>0.30</td>
<td>0.09</td>
</tr>
<tr>
<td>Iterative</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4 mm step)</td>
<td>0.37</td>
<td>0.36</td>
</tr>
<tr>
<td>Iterative</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5 mm step)</td>
<td>0.47</td>
<td>0.71</td>
</tr>
</tbody>
</table>
Figure 4.13: Direct beam path response from a SDH in Test Block 4 using the SPA Dual-Media algorithm with the (a) Analytic (b) Numerical (c-h) Iterative (0.5, 1, 2, 3, 4, 5) mm step size POI methods implemented. Also shown are the -6dB contour plots which compare the Analytic POI method with the (i) Numerical and (j-o) Iterative (0.5, 1, 2, 3, 4, 5) mm step size POI methods. All images employed a 20 dB threshold.
Figure 4.14 displays the spatial responses from a 3mm SDH in Test Block 4, insonified using a skipped beam path. Again, all images were generated from the same raw data set to avoid any differences in signal response during the data acquisition process.

Figure 4.14a-e shows the response from a 3 mm SDH in Test Block 4 when insonified using a skipped beam path. Images were generated using the analytical, numerical and iterative 0.5, 1 and 2 mm methods to determine the point of incidence respectively. It can be seen that the spatial response of the numerical and iterative 0.5, 1 and 2 mm methods match very closely to that of the analytical method. This is confirmed in Figure 4.14i-l which shows the similarity of the numerical and iterative 0.5, 1 and 2 mm methods with the analytical method in the form of -6dB contour plots. Analysis of the parameters on the right hand side of Table 4.2 shows that the numerical and analytical methods were identical in terms of their PSF and positioning. Also the iterative method up to a 1 mm step size only differed by a maximum of 6% in terms of PSF and 0.11 mm in terms of positioning relative to the analytical method. With a step size of 3 mm it can be seen from Figure 4.14f that the iterative method can still be used to identify and position the 3 mm SDH if a slight misplaced response can be accepted. This is confirmed by both Figure 4.14m, which shows the response from the analytical and iterative method using a -6 dB contour plot, and Table 4.2 which shows the relative PSF to be within 4% of the analytical method and the positional accuracy of the peak response to be within 0.29 mm. Further increasing the step size of the iterative method to 4 mm and 5 mm greatly degraded the image quality and it can be seen from Figure 4.14g-h that the response from the SDH is fragmented and can no longer be determined as a single reflector.

Comparison of the images generated using direct and skipped beam paths, as illustrated in Figure 4.13 and Figure 4.14 respectively, show a tighter focus is achieved when using the direct beam path. From Table 4.2 it can be seen that the PSF of the direct beam path is approximately half that of the skipped beam path regardless of the method used to determine the point of incidence. This result was expected, as the SDH imaged with the skipped beam path was effectively at a greater depth compared to the SDH imaged with the direct beam path due to the beam being skipped off the backwall. This had the effect of reducing the angular range with which the sound field could interrogate the SDH, which therefore increased the focal spot size. Further comparisons between the skipped and direct beam paths showed that the direct beam was able to image the SDH using a step size of 4 mm, whereas the skipped beam only managed a step size of 3 mm. The
two main reason for this was, firstly the increased beam path of the skipped beam which caused an increase in beam path error. This is illustrated in Figure 4.3 which shows how beam path error increases as a function of distance. Secondly skipping over a large area allows energy to be lost in the form of other wave modes being introduced after reflection off the backwall. The increased beam path of the skipped beam also meant that the effects of attenuation and beam spread were greater compared with the direct beam, although this will only have a significant effect if the signal strength approaches that of the noise level.
Figure 4.14: Skipped beam path response from a SDH in Test Block 4 using SPA Dual-Media algorithm with the (a) Analytic (b) Numerical (c-h) Iterative 0.5, 1, 2, 3, 4 and 5 mm step size POI methods implemented. Also shown are -6dB contour plots which compare the Analytic POI method with the (i) Numerical and (j-o) Iterative 0.5, 1, 2, 3, 4 and 5 mm step size POI methods. All images employed a 20 dB threshold.
4.4.2.2 Validation Test 2: Visual Interpretation

In this experiment a visual alignment check was performed using the 3 vertically aligned SDHs in Test Block 5. The transducer mounted on a 36 degree Perspex wedge was positioned with a 61 mm stand-off relative to the SDHs which were located at a depth of 20, 30 and 40 mm below the surface.

Figure 4.15 shows reconstructed images of the vertically aligned SDHs in Test Block 5, generated using the analytical, numerical and iterative 0.5, 1 and 2 mm step size methods respectively. The region imaged in each of the plots was equivalent to the area of the weld body in the double ‘v’ butt welded plate. Through qualitative visual comparisons of each plot it can be seen that the analytical, numerical and iterative 0.5 and 1 mm step size methods can be used to accurately position the SDHs which were spaced 10 mm apart throughout the region of interest. The iterative method implemented with a step size of 2 mm correctly positioned the SDH at a depth of 20 mm, however the SDHs at depths of 30 and 40 mm were fragmented and so did not suggest a response from a single reflector.
Figure 4.15: Response from the SDHs in Test Block 5 using the SPA Dual-Media algorithm, with the (a) Analytical (b) Numerical and (c-e) Iterative 0.5, 1 and 2 mm step size POI methods implemented. All images employed a 20 dB threshold.

Tests performed on Test Blocks 4 and 5 confirm that all three of the algorithms can be used to determine the point of incidence between dual layered media. However a step size of 1 mm or less should be used to ensure an accurate result for the SDHs examined when using the iterative method.
4.4.3 Comparison of processing speeds of the POI methods

POI methods have been analysed in terms of their processing speed, with the aim of determining the most appropriate algorithm with which to inspect the double ‘v’ butt welded plate. For the iterative method all tests were performed using a step size of 1 mm, as this was the coarsest increment which was validated, and therefore, involved the least amount of computer processing to calculate the point of incidence. Data acquisition speeds are not discussed here as the time taken to process a given data set was independent of the time taken to acquire it.

Figure 4.16: A plot displaying pixel number against time to determine the point of incidence between dual layered media, for the different POI methods.

The plot presented in Figure 4.16 shows the number of pixels in an image against the image reconstruction time. The range of pixels chosen was representative of the typical number of pixels which might be used in industrial applications. For example, using an image resolution of 0.25 mm, 5k pixels corresponds to an image size of approximately 18 x 18 mm, whereas using an image resolution of 1 mm, 20k pixels corresponds to an image size of approximately 140 x 140 mm.

Each image was generated from raw A-scan data acquired using the FMC technique. The raw data contained 4096 A-scans; one for every transmitter-receiver pair in the 64 element transducer. Each A-scan contained 6000 sample points and had an amplitude resolution of 8 bits, yielding a total file size of approximately 24 Mb. It can be seen that
for each method, the time take to generate an image was dependent on the number of pixels. This was expected as the number of times a given method was called within the algorithm depended directly on the number of pixels. It can also be seen that in all cases the analytical method outperformed the numerical and iterative methods. The reason for the efficiency of the analytical method was that in contrast to the numerical and iterative methods, it did not involve an iterative trial and improvement process. Therefore each calculation was only performed once, which minimised the required number of computer clock cycles. By far the slowest approach was the iterative method. This was partly because it required a larger number of cycles to calculate the point of incidence compared to the other techniques. It also related to the software platform on which the iterative process was implemented. Whereas the numerical method called the function \texttt{roots.m} which performed the trial and improvement process in C, the iterative method was entirely implemented in the MATLAB environment, which is fundamentally designed for matrix multiplication rather than loops.

For a planar interface between dual layered media it is often the case that the region of inspection remains constant relative to the transducer face (e.g. butt welded plate inspection). This being the case, the focal law calculator, which used one of the POI methods to determine the time of flight between the transducer and region of interest, only needed calculating once. The calculated values could then be stored in a library and referenced when required. This approach was known as the Reference Method. Figure 4.16 shows that using this approach, processing times were orders of magnitude quicker than any of the other methods. This was expected as the approach used predetermined values from a reference library and so did not involve any intensive mathematical calculations. It should be remembered however that this method is only valid for certain applications and requires either the iterative, numerical or analytical POI method to calculate the initial focal law, which it then references.

4.4.4 Validation of the SPA Autofocus algorithm for a planar refractive interface

In Section 4.4.2 the SPA Dual-Media algorithm, which allows for focussed inspection through dual layered media separated by a planar interface, was validated. To allow focussed inspection this algorithm requires the manual input of several initial parameters which define the interface as a mathematical equation. In the following text the SPA Autofocus algorithm, which allows focussed imagery to be generated through
dual layered media separated by an undefined refractive interface is validated. A
detailed description of the autofocus algorithm can be found in Section 4.3.5.

Since the autofocus algorithm works by generating a cross-sectional image of the
interface between dual layered media and curve fitting to that interface, then clearly the
resolution of the cross-sectional image will place a limit the accuracy with which the
curve fitting algorithm can work. The following results shows how varying the
resolution used to image the interface affects the performance of the curve fitting
algorithm for inspection using a 0 degree Perspex wedge, placed directly above the 3
mm SDH in Test Block 6. The image resolutions used to map the interface between
dual layered media in this test included 0.1, 0.2 and 0.5 mm. Table 4.3 shows that for an
image resolution of 0.1 and 0.2 mm the wedge angle was predicted to within 0.03
degrees and the perpendicular distance from the transducer face to the Perspex-steel
interface (wedge length) was accurate to within 0.1 mm. For an image resolution of 0.5
mm however, errors in wedge length and wedge angle were very large, suggesting the
interface could not be detected at this resolution.

**Table 4.3:** Quantitative analysis of the wedge parameters predicted by the SPA
autofocus algorithm

<table>
<thead>
<tr>
<th>Image resolution at interface</th>
<th>Error in wedge length (mm)</th>
<th>Error in wedge angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1 mm</td>
<td>0.10</td>
<td>0.003</td>
</tr>
<tr>
<td>0.2 mm</td>
<td>0.06</td>
<td>0.03</td>
</tr>
<tr>
<td>0.5 mm</td>
<td>5.6</td>
<td>4.3</td>
</tr>
</tbody>
</table>

Figure 4.17 illustrates the ability of the autofocussing algorithm to image the 3 mm
SDH in Test Block 6 for the range of curve fitting errors given in Table 4.3. It can be
seen from Figure 4.17b-c that when using an image resolution at the interface of 0.1 or
0.2 mm there is very little visual difference compared to Figure 4.17a, which shows the
1 mm SDH imaged using the actual wedge parameters. For an image resolution of 0.5
mm however there was considerable image degradation and the 3 mm SDH could not be determined.
Based on this result future experiments which require the use of the autofocus algorithm will use an image resolution of 0.2 mm to generate an image of the interface between dual layered media to avoid image degradation and incorrect positioning of flaws.

**Figure 4.17:** Responses from the 3 mm SDH in Test Block 6 generated using (a) the SPA Media-Dual algorithm and (b-d) the SPA Autofocus algorithm using an image resolution of 0.1, 0.2, 0.5 mm respectively to map the interface between dual-layered media. All images employed a 20 dB threshold.

Having established an acceptable resolution to use with the curve-fitting algorithm, inspection of Test Block 6 was then carried out using a range of 0, 20, 32 and 42 degree Perspex wedges which induced an angle of refraction of 0, 29, 48 and 70 degrees in low carbon steel test piece respectively. Wedge angles were chosen specifically to show the ability of the autofocus algorithm to inspect over a wide range of angles common to industry. The first critical angle at the Perspex-steel interface was calculated to be 23.5 degrees (Equation 4.33). Therefore for inspection with the 0 and 20 degree angle wedges, the longitudinal wave mode was used, while for the 32 and 42 degree angle wedges the shear wave mode was chosen. In this experiment images were generated from wedge parameters predicted by the SPA Autofocus algorithm. Images were then compared to those generated using the actual wedge parameters which were input into the SPA Dual-Media algorithm. The aim of this test was to determine the ability of the SPA Autofocus algorithm to generate accurate images of flaws in a test component.
Table 4.4 compares the predicted wedge parameters calculated using the SPA Autofocus algorithm, with the actual wedge parameters. In all cases the wedge angle predicted by the autofocus algorithm matched the actual wedge angle to within 0.3 degrees, and the wedge length was predicted to within 0.2 mm of the measured distance. Figure 4.18 illustrates the effects of these errors when imaging the 3 mm SDH in Test Block 6. Qualitative analysis showed that for all wedge angles used, there were no obvious visual differences in the spatial response. However, small deviations in position were visible, particularly when the 42 degree wedge was used. This was confirmed by quantitative analysis presented in Table 4.5.

**Table 4.4**: A comparison of actual wedge parameters against those predicted by the SPA Autofocus algorithm.

<table>
<thead>
<tr>
<th>Actual wedge parameters</th>
<th>Predicted wedge parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Induced wave mode</strong></td>
<td><strong>Wedge Angle (°)</strong></td>
</tr>
<tr>
<td>Longitudinal</td>
<td>0</td>
</tr>
<tr>
<td>Longitudinal</td>
<td>20</td>
</tr>
<tr>
<td>Shear</td>
<td>32</td>
</tr>
<tr>
<td>Shear</td>
<td>42</td>
</tr>
</tbody>
</table>
Table 4.5: Error in predicted position (measured as distance between peak responses) of the SDH in Test Block 6 using the SPA Autofocus algorithm relative to the SPA Dual-Media algorithm for a range of wedge angles.

<table>
<thead>
<tr>
<th>Wedge Angle (°)</th>
<th>Peak-to-Peak Positional error (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>20</td>
<td>0.1</td>
</tr>
<tr>
<td>32</td>
<td>0.1</td>
</tr>
<tr>
<td>42</td>
<td>1.2</td>
</tr>
</tbody>
</table>

It has been shown through experimental results and analysis that the SPA Autofocus algorithm can be used to inspect through an unknown planar refractive boundary over a wide range of incident angles which are common to industry. However, some known limitations of the current linear autofocus algorithm are that it is only implemented for two dimensional geometries using linear arrays, so does not account for geometric variations which occur in the passive plane of the transducer. Also, the algorithm can only curve-fit to regions of the interface where the incident sound field has been reflected directly back to the transducer. Therefore it is possible for some geometries to have ‘blind’ regions, which cannot be detected. A potential solution to this problem could be to use a larger aperture to increase the angular range of insonification. This could be implemented either by simply using a larger transducer or by synthesising a larger transducer using a scanning array (Doctor et al 1986).
Figure 4.18: Images of a 3 mm SDH at depth 14 mm, generated using a (a - b) 0 degree (c - d) 20 degree (e - f) 32 degree and (g - h) 42 degree Perspex wedge. Images on the left side were generated using the actual wedge parameters, while on the right images were generated using the wedge parameters predicted by the autofocus algorithm. All images employed a 20 dB threshold.
4.4.5 Comparison of processing times between the SPA Autofocus and SPA Dual-Media algorithms

Results from the tests performed using the autofocus algorithm showed that when curve fitting to the interface between dual layered media, an image resolution of 0.2 mm or smaller was sufficient to generate accurate imagery of the 3 mm SDH in test block block 6. Using an image resolution of 0.2 mm the autofocus algorithm was analysed in terms of its data processing speed and compared with the SPA Dual-Media algorithm. The fundamental difference between the SPA Autofocus and SPA Dual-Media algorithms was that the autofocus algorithm used curve fitting techniques to determine the parameters which defined the interface between dual layered media, whereas the SPA Dual-Media algorithm required parameters to be manually entered by the operator prior to inspection.

Since it has been shown that the analytical method of calculating the point of incidence is the fastest and most accurate of all the POI methods, this method was used to generate the focal law for both the SPA Dual-Media and SPA Autofocus algorithms. The test performed compared the processing time of the two algorithms when imaging the 3 mm SDH in Validation block 3 with a 0 degree wedge. Tests were repeated for a variety of image sizes which included 5k, 10k, 15k and 20k pixels, the results are presented in Table 4.6.

**Table 4.6:** Processing times for the SPA Autofocus and SPA Dual-Media algorithms with respect to image pixel number.

<table>
<thead>
<tr>
<th>Pixel number</th>
<th>Processing time (s) (SPA Autofocus)</th>
<th>Processing time (s) (SPA Dual-Media)</th>
<th>Difference in processing time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,000</td>
<td>38.9</td>
<td>31.2</td>
<td>7.7</td>
</tr>
<tr>
<td>10,000</td>
<td>63.5</td>
<td>56.3</td>
<td>7.2</td>
</tr>
<tr>
<td>15,000</td>
<td>87.8</td>
<td>79.3</td>
<td>8.4</td>
</tr>
<tr>
<td>20,000</td>
<td>91.0</td>
<td>83.6</td>
<td>7.4</td>
</tr>
</tbody>
</table>
Comparing the SPA Autofocus and SPA Dual Media algorithms in Table 4.6, it can be seen that regardless of the pixel number used, the difference in processing times between the algorithms remains approximately constant at between 7.2 - 8.4 s. This difference in processing time was a direct result of the inclusion of the curve-fitting process implemented in the autofocus algorithm. From the ratio of the SPA Dual Media and SPA Autofocus processing times it can be seen that curve fitting took between 8 – 20 % of the overall processing time of the autofocus algorithm. It is likely however, when curve-fitting to higher order polynomial surfaces, the overhead time of the curve fitting process will be more significant. This is due to the trial and improvement iterative process it uses, as shown by the process diagram in Figure 4.8.

4.4.6 Inspection of a double ‘v’ butt welded plate

In this section results from inspection of a low carbon steel plate containing a double ‘v’ butt weld with known flaws are presented and discussed. The types of flaws contained in the weld included lack of root penetration (LORP), porosity and a toe crack. The geometry of the plate as well as the size and locations of the flaws contained within it are shown Figure 4.19.

The transducer was mounted onto a 36 degree Perspex wedge which facilitated direct and skipped beam inspection of the weld region as shown by Figure 4.20. A fixed stand-off of 54 mm between the centre of the weld and the centre of the transducer was used for inspection. Inspection of the weld region using an angled wedge was necessary because of the proud undressed weld cap, which prevented inspection from the surface directly above the weld.
Figure 4.19: A diagram of a low carbon steel double ‘v’ butt welded plate, illustrating the type, size and location of flaws contained within the weld region.

Figure 4.20: Cross-sectional image of inspection scenario of double ‘v’ butt welded plate

Data were acquired using the FMC technique, and processing using the SPA Autofocus with the Analytical POI method used to account for refraction at the Perspex-steel interface. The analytical method was chosen based on previous validation tests which showed it out-performed the iterative and numerical methods in terms of image quality and processing speed. The SPA Autofocus algorithm was used to determine the wedge parameters, even though this information was provided by the wedge manufacturer. The reason being that frequent use over an extended period can cause wear which can alter
the original wedge dimensions. To maximise the speed of inspection the focal law was calculated once, then stored and referenced during the inspection.

During the inspection the transducer was moved in the direction of its passive axis, while its active axis remained orthogonal to the weld body. Unfortunately the developed algorithms did not facilitate the use of an encoder system and so the transducer was moved manually by the operator. While every effort was made to move the transducer at a constant velocity during the scanning process, this method was not as precise as encoded scanning systems, therefore dimensions in the passive axis (y-axis) were only approximate.

Figure 4.21: Images of the double ‘v’ butt welded plate showing (a) an unencoded, volume corrected C-scan of the weld region, and (b-d) cross-section imagery of lack of root penetration, porosity and (d) a toe crack respectively, imaged within the weld region (indicated by the dashed line). All images employed a 20 dB threshold.

Figure 4.21a shows a volume corrected, unencoded C-scan of the weld region using a 54 mm stand-off. Three dotted lines, parallel to the y-axis, have been overlaid on the image which represent the centre and edges of the weld body. Identified in the image
are the three flaws, which are clearly visible above the background noise level. Since the scan was unencoded, the positions and dimensions of the flaws in the y-axis are only approximate. To accurately determine flaw sizes in this axis an encoder would need to be incorporated into the scanning system. Other artefacts, present in the C-scan, were identified as geometric or as being located outside the weld region through B-scan interpretation and were ignored. Figure 4.21b-d show the B-scan images of the flaws LORP, porosity and toe crack respectively. For each image a weld overlay was generated for clarity. The dashed horizontal line represents the plate backwall. Any indications imaged above this line were in direct line of sight of the transducer. Below the horizontal line is a mirror image of the weld; any indications plotted in this region represented the beam skipping off the backwall before insonifying the weld region. Comparing B-scan images to the master drawing in Figure 4.19 showed that all flaws were accurately positioned in the weld in the plane perpendicular to the weld run (x-z plane). In the y-direction both the length and positioning of the flaws were inconsistent with master drawings, this was because scanning was unencoded.

4.5 Results and discussion: Inspection through a non-planar refractive interface

4.5.1 Introduction

In this section results are presented which validate the use of the SPA Autofocus algorithm for focussed inspection through an irregular boundary profile. Two methods were used to facilitate this inspection, the first was the Numerical POI method (Section 4.3.3), the second was the Iterative POI method (Section 4.3.1). An analytical approach could not be used as Snell’s law for a non-planar interface exceeded fourth order and no longer took the form of a polynomial equation. It was also not possible to use the SPA Dual-Media algorithm in this instance as the equation of the interface relative to the transducer (Equation 4.2) changed with transducer position. Therefore curve fitting was required to gain a mathematical expression of the interface for a given transducer location which required the SPA Autofocus algorithm.

Results from each POI method were compared in terms of frame rate and positional accuracy of SDHs. During the experimental setup the 64 element array transducer was placed on a Perspex wedge, which in turn was placed onto the curved geometry interface of Test Block 7 (Figure 4.22). The curvature of the wedge matched that of the test block, therefore creating an intimate contact. The wedge-test block interface obeyed
a quadratic relationship according to Equation 4.35. A known interface was used in order to determine the accuracy with which it could be mapped by the curve-fitting algorithm. This curved interface was chosen based on ray tracing which showed that a single wave mode (longitudinal) could be used by the majority of elements to insonify all five SDHs without blind zones. Had immersion mode been used for the experimental setup there would have been more flexibility in transducer positioning relative to the test block. However, it was decided that machining a Perspex wedge to fit the test block was more appropriate due to the reduced setup time, and the improved portability of this experimental setup.

\[ z = 0.01x^2 \]  

\[ 2.35 \]

**Figure 4.22:** Test Block 7 with fitted Perspex wedge

### 4.5.2 Validation of the SPA Autofocus algorithm for a non-planar interface

Figure 4.23a shows the response from the curved interface of Test Block 7. The plot also includes the predicted interface, calculated by the SPA Autofocus algorithm (dashed line). For clarity the same predicted interface is also shown in Figure 4.23b, but this time the reconstructed image response has been removed. The plot shows that all points used to predict the profile of the interface occurred in the region of $-10 < x < 10$. However Figure 4.22 shows that the actual width of the validation piece is much greater than this. The curve-fitting algorithm failed to detect the interface outside of the $-10 < x < 10$ region because incident beams in this region were not reflected directly back to the transducer. In order to detect a greater portion of the interface, a larger transducer could have been used, or an encoded scan from numerous transducer positions could have been performed. It should be noted that while only a limited surface region of Test
Block 7 was detected, this was enough to accurately predict the entire interface profile. This was probably due to the low order polynomial equation which defined the interface (quadratic interface), coupled with the fact that the curve fitting algorithm iterated through different order polynomials starting with 1st order and increasing by an order of one each time; therefore arriving at a quadratic equation on the second iteration. Had a more complex geometry interface been investigated then the fit would likely have been worse.

![Graph](image)

**Figure 4.23**: A plot of (a) the wedge-test block interface with a line of best fit (b) points illustrating the detected interface and line of best fit

Figure 4.24 shows inspection of Test Block 7 through the curved refractive interface using the Numerical POI method based on Snell’s law and the Iterative POI method based on solving Fermat’s principle. Strong correlation was found between images, with no noticeable differences found. In terms of flaw identification, both images clearly
show the three central SDHs. But the SDH at the far right of Test Block 7 was only just visible, and the SDH on the far left was not detected. This was due to many of the A-scans having low signals amplitudes due to the high beam angles, which in some cases approached and went beyond the first critical angle.

The imagery also shows that the three central SDHs were all imaged at the correct depth of 50 mm and with the correct spacing of 15 mm between holes. The response from the SDH on the far right of the image also had the correct inter-SDH spacing of 15 mm, but had a depth of 51 mm and so was misplaced vertically by 1 mm. The images also show that the SDHs responses were offset from the centre of the image by approximately 3 mm. This was likely to have been caused by the transducer position being offset from the middle SDH during the experimental setup rather than an error in the algorithms themselves. Also of interest in the images was the presence of long sweeping arcs of coherent noise. It was expected that these were a combined result of internal wedge reflections and mode conversion.

The images in Figure 4.24 showed the ability of the SPA Autofocus algorithm to focus automatically through a non-planar boundary. To further verify the result, the same inspection of Test Block 7 was repeated, but this time a planar interface was assumed. Figure 4.25 shows the results from inspection of Test Block 7 assuming a planar interface parallel to the transducer face (zero degrees) at a depth of 25 and 30 mm. The figure shows that the response from the SDHs was still visible, but there was a large amount of distortion and the response could not be used to size or characterise any of the SDHs. Nor could it be used to identify the number of flaws present. The assumption of a 25 mm and 30 mm planar interface was made as these approximately corresponded to the depth of the curved interface of Test Block 7 below the transducer (Figure 4.26).

![Figure 4.24](image)

*Figure 4.24: Cross-sectional image of SDHs in Test Block 7 generated using (a) the Numerical POI method based on Snell’s law and (b) the Iterative POI method based on...*
Fermat’s principle, to determine the points of incidence at the curved interface between dual layered media. Both images employed a 20 dB threshold.

Figure 4.25: Cross-sectional images of SDHs in Test Block 7, generated using the POI analytical method, but assuming a planar boundary parallel to the transducer face (zero degrees) at a depth of (a) 25 mm (b) 30 mm. Both images employed a 20 dB threshold.

Figure 4.26: Illustration of Test Block 7 with indications (dashed lines) showing a 25 and 30 mm depth from the transducer face.

It has been shown that the Numerical POI and Iterative POI methods generated almost identical images when inspecting the SDHs in Test Block 7. However the time taken to generate each of the images varied considerably, as shown in Table 4.7. The Iterative POI method based on Fermat’s principle took a total of $8.8 \times 10^3$ s to generate a single image. In comparison the Numerical POI method took $2.8 \times 10^5$ s and so was approximately two orders of magnitude slower. The reason for the slow processing time of the Numerical POI method was largely a result of the mathematically intensive operations required, some of which are shown in Equations 4.10-4.18. As well as the
Numerical POI method taking considerably longer to generate imagery, the method was also more convoluted to program and required many conditional statements in order to prevent it from outputting error messages. While it is difficult to quantify, it was felt that this method was more unstable in comparison to the Iterative POI method, which could be written in just a few lines of code and appeared highly stable.

**Table 4.7**: Table showing time to generate a single cross-sectional image using numerical methods to solve Snell’s law and an iterative method to solve Fermat’s principle

<table>
<thead>
<tr>
<th>POI method</th>
<th>Total time to generate image (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical POI method (Snell’s law)</td>
<td>$2.8 \times 10^7$</td>
</tr>
<tr>
<td>Iterative POI method (Fermat’s principle)</td>
<td>$8.8 \times 10^3$</td>
</tr>
</tbody>
</table>

### 4.6 Conclusions

It was shown that for a planar refractive interface the Analytical POI method used to solve Snell’s law (4th order polynomial) was fastest. This was in comparison with a numerical approach to solving Snell’s law, and an iterative approach to solving Fermat’s principle. For the case of a non-planar interface no analytical solution to Snell’s law exists. As such comparisons were only made between a numerical implementation to solve Snell’s law and an iterative implementation to solve Fermat’s principle. In this case Fermat’s principle was preferred, not only because it was fastest, but also it was far more stable and required significantly less code.

Curve-fitting algorithms developed within this chapter facilitated automated focussed inspection through dual layered media using signals reflected from the refractive boundary profile. Such signals were present within the Full Matrix of Data and so no additional firing was required to then generate focussed imagery beyond the refractive boundary. The curve-fitting algorithm allowed automated focussing through dual layered media; although this was limited to regions where incident beams were reflected directly back to the transducer from the refractive boundary.
Chapter 5

Sparse Arrays

5.1 Introduction
In Chapter 3 the SPA algorithm was compared with existing parallel transmission ultrasonic techniques. While the SPA approach offered several benefits in terms of image quality, the achievable frame rate was slow by comparison. This was shown to be a result of a combination of factors including data acquisition and data processing times. This chapter investigates the use of sparse arrays, designed using the Effective Aperture concept, which is based on far-field approximations (Section 2.6.2). The potential advantages of a sparse array configuration are: fewer transmissions result in a reduced physical limit of inspection (i.e time ultrasound is in specimen per scan). Also, smaller data sets will result in reduced acquisition and image reconstruction times yielding an overall improvement in the achievable image frame rate.

Two modes of sparse array inspections have been considered. The first utilised a sparse transmit with dense receive aperture. The second used a full sparse transmit and receive aperture. Results were compared to existing parallel transmission techniques and the original SPA algorithm, both of which make use of a dense array transducer configuration. For clarity the term ‘dense array’ will be used in reference to an array of elements where inter-element spacing is equal to or less than \(\lambda/2\). Also, data acquired using sparse arrays will be known as a Sparse Matrix of Data, in order to differentiate it from the Full Matrix of Data acquired using a dense array.

5.2 Sparse Sequential Phased Array
The Sparse Sequential Phased Array (Sparse-SPA) algorithm processes data acquired using the FMC technique in an equivalent manner to the SPA algorithm discussed in Chapter 3. The only difference being that it operates on a Sparse Matrix of Data, acquired using a sparse array which omits particular transmit-receive pairs during the acquisition process. In order to avoid grating lobes which may occur if the array is spatially under sampled (i.e. inter-element spacing exceeds \(\lambda/2\)), transmit-receive patterns were designed using the Effective Aperture Concept described in Section 2.6.2.
### 5.2.1 Inspection configurations

The sparse-SPA algorithm has several functionalities which include two data acquisition modes, namely Sparse Transmit and Sparse Transmit-Receive. Sparse Transmit mode transmits on a select number of elements while receiving on all elements, such that only the transmit aperture is sparse. During the setup an operator chooses the number of transmit elements \( n_{tx} \) from a list. The list comprises of all transmit combinations which yield a periodic element spacing. These combinations relate to the total number of elements in the array \( n \), according to Equation 5.1, where \( f \) denotes the factors of a number. For example, using a 64 element transducer the number of transmit elements which can be used include 2, 4, 8, 10 or 22. Sparse Transmit-Receive mode is similar in functionality, but facilitates transmission and reception on a select number of elements, such that both the transmit aperture and receive aperture may be sparse. Again transmit-receive element combinations were calculated using factorisation to utilise the full 64 elements of the array. For a 64 element array, available transmit receive configurations for both Sparse Transmit and Sparse Transmit-Receive modes are given in Table 5.1.

\[
n_{tx} = f [n - 1] + 1 \tag{5.1}
\]

For a typical linear ultrasonic array transducer, elements spacing is approximately \( \lambda/2 \). Therefore transmit and receive apertures used by the Sparse-SPA algorithm may be under sampled. However, since with pulse-echo inspection ultrasound takes a round trip sound path (i.e. received signals will propagate to and from the transducer), the two way radiation pattern must be considered. Using the Effective Aperture Concept, Lockwood and Foster (1994) which takes the convolution of the transmit and receive apertures, all sparse array configurations (Table 5.1) were designed so that their effective apertures were densely populated to avoid spatial aliasing. The graphs in Figure 5.1 illustrate the concept of the effective aperture using a 13 element array for clarity. Figure 5.1a shows a dense array with a pitch of \( \lambda/2 \), while Figure 5.1b,c show various under sampled transmit and receive apertures convolved to produce a dense effective aperture.
Figure 5.1: Effective aperture concept for different transmit-receive configurations including (a) 13 element aperture dense array (b) 2 transmit 13 receive sparse array (c) 4 transmit 13 receive sparse array
Table 5.1: Transmit and receive aperture configurations for the Sparse-SPA algorithm

<table>
<thead>
<tr>
<th>Inspection mode</th>
<th>Number of Transmit/receive elements</th>
<th>Number of A-scans</th>
<th>Element pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sparse Transmit</td>
<td>2/64</td>
<td>128</td>
<td>1, 64</td>
</tr>
<tr>
<td></td>
<td>4/64</td>
<td>256</td>
<td>1, 22, 43, 64</td>
</tr>
<tr>
<td></td>
<td>8/64</td>
<td>512</td>
<td>1, 10, 19 ... 46, 55, 64</td>
</tr>
<tr>
<td></td>
<td>10/64</td>
<td>640</td>
<td>1, 8, 15 ... 50, 57, 64</td>
</tr>
<tr>
<td></td>
<td>22/64</td>
<td>1408</td>
<td>1, 4, 7 ... 58, 61, 64</td>
</tr>
<tr>
<td>Sparse Transmit-Receive</td>
<td>6/22</td>
<td>132</td>
<td>1 – 3, 62 – 64</td>
</tr>
<tr>
<td></td>
<td>14/10</td>
<td>140</td>
<td>1 – 7, 58 – 64</td>
</tr>
<tr>
<td></td>
<td>18/8</td>
<td>144</td>
<td>1 – 9, 56 – 64</td>
</tr>
<tr>
<td></td>
<td>42/4</td>
<td>168</td>
<td>1 – 21, 44 – 64</td>
</tr>
</tbody>
</table>

5.2.2 Sidelobe suppression

The two-way radiation pattern of a given transmit and receive aperture configuration in the far-field can be determined from the Fourier transform of the effective aperture. Figure 5.2 shows the effective aperture and resulting two-way radiation pattern when transmitting and receiving on every element in a 64 element array. In the case of the dense array the effective aperture takes the form of a triangular window; this form of window and its Fourier transform are well known (Britton-Rorabaugh 1998). The resulting radiation pattern shows a main lobe with a sidelobe suppression of 27 dB. In
comparison the effective aperture of the sparse array configuration takes a similar form to a rectangular window, but with a central peak (Figure 5.1b). Taking the Fourier transform we find that the resulting radiation pattern has a narrow main lobe with sidelobe suppression of 14 dB. It is not uncommon in NDT to generate imagery using a 30 dB or 40 dB amplitude threshold, therefore sidelobe suppression can be an important consideration when designing a transducer and developing focal laws. (Amplitude threshold is defined as a cut-off point which sets to zero any signal below a given amplitude level relative to the peak response in an image). Conventional parallel transmission ultrasound techniques use a method known as apodisation to control sidelobe levels. This approach requires a voltage bias to be applied between elements in an array, and therefore is only applied to transmitted signals. This makes it inappropriate for FMC where energy transmitted from a single element will be received by many elements; each of which may require a different level of bias.

**Figure 5.2:** Plot of the (a) transmit aperture, (b) receive aperture, (c) Effective Aperture and (d) radiation pattern, for a 64 element dense array transducer.
To address the issue of sidelobe suppression, a function known as Gain Bias was developed and incorporated into the sparse-SPA algorithm. This function adjusted the gain applied to each A-scan in the Sparse Matrix of Data, based on the location of its associated virtual source (mid-point between the transmit and receive element) in the effective aperture. Initially the desired window shape of the effective aperture (e.g. triangular, Gaussian, Hanning etc.) is input. The Gain Bias function then calculates the required correction coefficients for each virtual source in the effective aperture using Equation 5.2. The virtual source associated with each A-scan is then determined from its transmit and receive pair and the relevant gain compensation is applied. An example of the Gain Bias function is given by Figure 5.3. Current windowing options include triangular, Gaussian or no window. The triangular window was included to allow the effective aperture of the sparse arrays to match that of a fully populated dense array. The benefit of the Gaussian window was that it offered greater sidelobe suppression than the triangular window, without significantly increasing the width of central lobe which would affect lateral resolution. While such window functions can be advantageous in terms of sidelobe suppression, they do have some disadvantages and so require careful design prior to implementation. In particular the system may become more sensitive to noise from channels with high corrective gain.

\[
\text{Correction coefficient} = \frac{\text{Desired window}}{\text{Actual window}} \tag{5.2}
\]
Figure 5.3: Plot of (a) the Effective Aperture from a sparse array transducer, (b) the desired Effective Aperture, (c) Effective Aperture compensation that (a) required to achieve (b), (d) amplitude of A-scans in the Sparse Matrix of Data, (e) Gain Bias applied to Sparse Matrix of Data to achieve desired window function in (b).

5.3 Results and Discussion

In this section results are presented which compare sparse array configurations with those of a densely populated array. Both Sparse Transmit and Sparse Transmit-Receive modes are compared to dense arrays in terms of frame rate and image quality. It is then shown experimentally how averaging and the Gain Bias function can be applied to improve the image quality, generated using a sparse array configuration.

Data were acquired using a low carbon steel test sample containing twelve 1.2 mm SDH’s which were spaced 5 mm apart along a 30 degree plane relative to the transducer face (Test Block 8), as shown in Figure 5.4. It should be noted that all images of Test Block 8 in this chapter were generated from a single Full Matrix of Data to ensure consistency in the results. Where sparse data sets were required, particular A-scans within the Full Matrix of Data were selected for use, while others were ignored.
Figure 5.4: Illustration of Test Block 8 containing multiple SDHs spaced 5 mm apart along a 30 degree plane relative to the probe face

5.3.1 Sparse transmit aperture

Figure 5.5 shows cross sectional imagery from inspection of Test Block 8 using various element configurations in the Sparse Transmit mode of the Sparse-SPA algorithm. It is clear that while all SDHs in all images could be identified, there were some significant differences in SNR levels between images. Figure 5.5a exhibited the poorest SNR levels, particularly in regions of close proximity to SDHs, where noise levels were as high as -12 dB. There were two main reasons for this; firstly the radiation patterns in Figure 5.6 showed that only a 14 dB sidelobe suppression was achieved for the element configuration used in Figure 5.5a. By comparison all other Sparse Transmit configurations achieved at least a -25 dB sidelobe suppression, which approached the imaging threshold level. Further analysis of Figure 5.6 showed that the reason for this particular poor sidelobe suppression was caused by the approximately rectangular shape of the effective aperture. In contrast all other Sparse Transmit element configurations exhibited an approximately triangular effective aperture. The second reason for the high noise levels in Figure 5.5a related to the fact that only two elements were used to transmit, and these were located at the ends of the array. Consequently SDHs located directly below the centre of the array were insonified by a sound field whose incident angle was as much as 30 degrees from the normal. This caused a significant drop in sound field intensity due to the large oblique angle of incident as discussed in Section 2.3.5.

For each of the element configurations in Figure 5.5b-f, a high SNR was achieved and all SDHs were clearly identifiable. Some low level noise (-20 dB) was still present when transmitting on 4 elements in regions of close proximity to SDHs (Figure 5.5b).
However when transmitting on 8 or more elements, virtually no noise appeared above the -30 dB image threshold (Figure 5.5c-f). The reasons for the improved SNR were partly the increase in the number of transmit elements, which resulted in an increase in the range of angles with which regions of interest were inspected. This lowered the maximum angle of incidence and increased the level of spatial averaging. It was also due to the approximately triangular effective aperture, which resulted in good sidelobe suppression (-25 dB).
Figure 5.5: Cross-sectional imagery from inspection of Test Block 8 using Sparse Transmit mode with configurations, (a) 2 transmit, 64 receive, (b) 4 transmit, 64 receive, (c) 8 transmit, 64 receive (d) 10 transmit, 64 receive, (e) 22 transmit, 64 receive. Image (f) was generated using a dense array. All images employed a 30 dB threshold.
Figure 5.6: Radiation patterns using Spare Transmit mode of the Sparse SPA algorithm with element configuration (a) 2 transmit 64 receive, (b) 4 transmit 64 receive, (c) 8 transmit 64 receive, (d) 10 transmit 64 receive, (e) 22 transmit 64 receive and (f) using a dense array.
5.3.2 Sparse transmit-receive aperture

Figure 5.7 shows cross sectional imagery from inspection of Test Block 8 using various element configurations in the Sparse Transmit-Receive mode of the Sparse-SPA algorithm. In each of the images all SDHs were identified, however SNR was poor by comparison to that of the dense array (Figure 5.5f). The radiation patterns generated for each of the Sparse Transmit-Receive element configurations (Figure 5.8) showed the low SNR was partially a result of poor sidelobe suppression. This was due to each element configuration having an approximately rectangular effective aperture. For all Sparse Transmit-Receive element configurations used, sidelobe suppression range between 14 dB and 17 dB with the greatest suppression being achieved when using the 42 transmit 4 receive element layout. This difference in sidelobe level was visible in the cross sectional imagery where Figure 5.7d does indeed show a higher SNR level compared to the other images in the Figure. It is interesting to note that this image was reconstructed from 168 A-scans, which was more than any other (range in number of A-scans used for reconstruction was 132 - 168). Therefore incoherent noise averaging, which occurs through summation of r.f. A-scan summation, was greatest for this image relative to others in the figure.

A second reason for the overall poor SNR of imagery in Figure 5.7 was that there were very few transmit elements in the centre of the array. For the sparse array setup used to generate Figure 5.7c.d there was also a limited number of receive elements in the centre of the array. In this case, while the virtual sources still formed a dense array the directivity of the elements meant poor sensitivity was achieved directly below the centre of the array.
Figure 5.7: Cross-sectional imagery from inspection of Test Block 8 using Sparse Transmit-Receive mode with configurations, (a) 6 transmit, 22 receive, (b) 14 transmit, 10 receive, (c) 18 transmit, 8 receive (d) 42 transmit, 4 receive. Image (e) was generated using a dense array. All images employed a 30 dB threshold.
Figure 5.8: Radiation patterns using Spare Transmit-Receive mode of the Sparse SPA algorithm with element configuration (a) 6 transmit 22 receive, (b) 14 transmit 10 receive, (c) 18 transmit 8 receive, (d) 42 transmit 4 receive, (e) using a dense array.
5.3.3 The Effective Aperture

It was shown that the SNR of images generated using sparse arrays can in some instances be considerably lower than that of a dense array element configuration. A discussion was also given which described how the cause of this low SNR could be partially attributed to poor sidelobe suppression. Figure 5.9 shows how using the Gain Bias function (Section 5.2.2) sidelobes may be suppressed, resulting in an improved SNR. Figure 5.9a,b depicts the original cross-sectional images generated using the 6 transmit 22 receive and 18 transmit 8 receive element configurations respectively of the Sparse-SPA algorithm in Sparse Transmit-Receive mode. Figure 5.9c,d and Figure 5.5e,f then demonstrate how a triangular window and Gaussian window Gain Bias function can be applied to improve the SNR. A comparison of the radiation patterns with and without the application of the Gain Bias function is given in Figure 5.10. For the element configurations shown, the most noticeable SNR improvement when using either the triangular or Gaussian window was in the region of z < 10 mm. Here we can see that the high noise levels at positions near the edges of the array at z < 10 mm have largely been suppressed below the -30 dB image threshold. This was due to the fact that both of the applied window functions applied a greater gain to elements in the centre of the array.

Of more importance is the significance of the Gain Bias function on the SNR in regions of close proximity to the SDHs. It is shown in Figure 5.9c,d,e,f how application of a triangular or Gaussian window greatly improved the SNR in this region, allowing individual SDHs to be better resolved. Comparing results between the triangular and Gaussian windows it was determined that there were very few features which could distinguish between images. This was largely because both window functions suppressed sidelobes to a level where they were reaching the imaging threshold level of -30 dB, below which they were truncated.

It has been shown that the Gain Bias function can be used to improve image quality by suppressing sidelobes. However comparison of Figure 5.9c,d,e,f with Figure 5.5f shows that even with the use of windowing, the noise level is still higher using the Sparse configuration relative to the dense array. This was due to sidelobe levels being only partly responsible for the low SNR levels in the imagery. The issue of poor image sensitivity in regions directly below the centre of the array due to element layout and directivity still remained.
Figure 5.9: Cross-sectional imagery from inspection of Test Block 8 using Sparse Transmit-Receive mode with configurations, 6 transmit 22 receive and 18 transmit 8 receive (a,b) without Gain Bias (c,d) with a triangular window Gain Bias, (e,f) with a Gaussian window Gain Bias respectively. All images employed a 30 dB threshold.
Figure 5.10: Radiation patterns using Spare Transmit-Receive mode of the Sparse SPA algorithm with element configurations of (a) 6 transmit 22 receive, (b) 18 transmit 8 receive, (c) dense array (64 transmit, 64 receive), (d) dense array with Gaussian window Gain Bias.

5.3.4 Averaging

In ultrasonic NDT averaging can be used to improve SNR through a process of acquisition and summation of multiple time invariant data sets. While coherent noise sources such as grain structure and mode conversions are unaffected by averaging, incoherent noise levels such as electronic noise may be suppressed. Within the SPA algorithm there is a certain amount of inherent averaging due to the A-scan summation process when reconstructing a cross-sectional image (Equation 5.3). The degree of averaging depends directly on the number of A-scans used to generate the cross-sectional image. For example, using the 2 transmit 64 receive element configuration of the Sparse-SPA algorithm yields a 128 A-scan data set, therefore generating 128 averages per image pixel. For a 64 element dense array configuration there would be 4096 acquired A-scans, resulting in 4096 averages per image pixel. Therefore to achieve the same level of averaging between these two configurations, the sparse configuration would require 32 data sets to be acquired and used during reconstruction (i.e. 128 x 32 = 4096).
Comparing all sparse array images generated using the Sparse Transmit and Sparse Transmit-Receive modes showed a strong correlation between SNR and the number of A-scans used during image reconstruction. Investigations were therefore carried out to see if averaging using multiple data sets could be used to enhance SNR levels. For a given transmit and receive element configuration $N$ time invariant data sets, $s_i$ were acquired and averaged according to Equation 5.3. This averaged data set $\bar{S}$ was then processed to generate a single cross-sectional image.

$$\bar{S} = \left( \sum_{i=1}^{N} s_i \right) / N \quad 5.3$$

Figure 5.11a shows a reconstructed image generated using a 2 transmit 64 receive sparse array element configuration. Comparing this with the same configuration scan, but with the addition of 32 averages (total of 4096 A-scans – equivalent to dense array) given in Figure 5.11b, showed significantly less noise in regions adjacent to SDHs. The result suggests that incoherent noise levels were therefore above the -30 dB image threshold when using only the inherent averaging ability of the SPA algorithm with a 128 A-scan data set. For completeness Figure 5.11c illustrates the effect of combining signal averaging with the triangular window Gain Bias function discussed Section 5.3.3. Using this approach additional reductions in noise levels compared to averaging alone were achieved. However significant noise was still visible above the -30 dB image threshold and image quality was still low in comparison to imagery generated using a dense array (Figure 5.11d).

The results demonstrate that for sparse arrays, low SNR is partially a result of incoherent noise levels, as well as element directivity and element layout as shown in Section 5.3.3.

While it has been shown that averaging can be used to improve SNR levels in sparse array designs, the approach is unlikely to be implemented for inspection purposes. The reason being that sparse array configurations are designed to increase frame rate by minimising data sets. However, averaging requires use of additional data sets per image and so reduces frame rate. Therefore an operator would be better off using a more densely populated array configuration which would increase angular range, rather than averaging. For example Figure 5.11b used a sparse array configuration with averaging and Figure 5.11d used a dense array. Both were generated using the same number of A-
scans (4096), but transmit elements in the dense array were spaced over a much greater area and so achieved a greater SNR.

![Images](image.png)

**Figure 5.11:** Plots of the 2 transmit 64 receive element configuration of the Sparse-SPA algorithm, using (a) no averaging (b) 32 data set averages (c) 32 data set averaging and triangular window Gain Bias function (d) dense array with no averaging. All images employed a 30 dB threshold.

### 5.3.5 Speed of Inspection

In Chapter 3 it was shown that the FMC acquisition technique combined with the SPA algorithm could achieve a frame rate of 0.9 Hz when operating using the half matrix function (2080 A-scans). In comparison, the linear and sector scans achieved frame rates of 66 and 32 Hz respectively. In this section of work it is shown how frame rate of the FMC technique combined with the SPA algorithm can be increased using a sparse array setup. The increase in frame rate is then contrasted against the quality of the generated imagery.

Table 5.2 shows the acquisition time, reconstruction time and frame rate when performing an inspection using the SPA-Sparse algorithm in Sparse Transmit and Sparse Transmit-Receive mode. For consistency with inspection speeds given in
Chapter 3, the length of each A-scan was again 769 sample points and data was digitised using an 8 bit amplitude resolution. There were also 17500 pixels per image. The table shows how reducing the number of acquired A-scans lowered both acquisition and reconstruction time, which consequently increased the achievable frame rate. The relationship between number of A-scans against acquisition and reconstruction time is shown graphically in Figure 5.12a. Here both acquisition and reconstruction time vary with an almost linear relationship as a function of data set size. From summation of the acquisition and reconstruction time the image frame rate can be calculated. A plot of A-scan number against frame rate is illustrated in Figure 5.12b. Here it is shown that for the sparse array element configurations incorporated into the Sparse-SPA algorithm frame rates of up to 5.5 Hz could be achieved. However, from the imagery generated in Figure 5.5 and Figure 5.7 it is clear that this increase in frame rate is at the cost of SNR. For inspection of SDHs in Test Block 8 analysis of Figure 5.5 and Figure 5.7 show that to achieve an image quality similar to that of the dense array, the fastest frame rate was 4.1 frames per second. This was achieved using the 4 transmit 64 receive element configuration in Sparse Transmit mode.

In this chapter sparse arrays have been used to significantly improve frame rate relative to a dense array configuration. However, comparison of these results with the sector scans and linear scans in Chapter 3 showed that the parallel transmission techniques still significantly outperformed the Sparse-SPA algorithm in terms of frame rate. It should be noted that frame rates described here refer to the frequency with which data can be acquired and reconstructed to form an image. However it is sometimes the case that data need only be acquired online, with reconstruction being conducted offline. In this instance frame rates could be significantly improved.

In this section of work acquisition time was calculated from the moment the first element in the array fired until the data was stored in memory. As such, this parameter inherently included round trip propagation time. The reason for this was that data storage and consecutive firings of elements in the array occurred in parallel. However, the round trip propagation time, \( T \), can be calculated theoretically as shown by Equation 5.4, where \( N \) is the number of transmissions per frame, \( d \) is the round trip material distance and \( v \) is the material velocity. This equation states that for a given region of interest in a given material, the round trip propagation time, which defines the physical limit of inspection speed, is purely dependent on the number of transmissions.
By comparing the number of transmissions for the various element configurations of the Sparse-SPA algorithm with those of the linear and sector scan in Chapter 3 (linear scan: 49 transmits, sector scan: 161 transmits), it can be seen that it is possible to use considerably fewer transmits with the sparse array setup. It therefore follows that FMC using a sparse array configuration has the potential to be faster than conventional phased array techniques, if the scan were limited by the round trip propagation time appose to the data transfer rate or processing time.

It should be noted that while it is feasible to incorporate a sparse array element configuration into a conventional parallel transmission scan there is less of a motivation. Frame rate could be increased using a sparse array linear scan by increasing the step size of the active aperture, however, this would directly affect lateral beam resolution. For a sector scan there would be no benefit to a sparse element configuration as the number of acquired A-scans and number of transmissions are both dependent on the angular range of inspection (Equation 3.11). Therefore, using fewer elements would have the effect of degrading image quality, with no benefit in terms of reduced data sets.
Table 5.2: Comparison of inspection speeds for different element configurations of the Sparse-SPA algorithm

<table>
<thead>
<tr>
<th>Inspection mode</th>
<th>Number of transmit/receive elements</th>
<th>Number of A-scans</th>
<th>Acquisition time (ms)</th>
<th>Reconstruction time (ms)</th>
<th>Frame rate (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sparse Transmit</td>
<td>2/64</td>
<td>128</td>
<td>15</td>
<td>168</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td>4/64</td>
<td>256</td>
<td>30</td>
<td>213</td>
<td>4.1</td>
</tr>
<tr>
<td></td>
<td>8/64</td>
<td>512</td>
<td>61</td>
<td>305</td>
<td>2.7</td>
</tr>
<tr>
<td></td>
<td>10/64</td>
<td>640</td>
<td>76</td>
<td>351</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>22/64</td>
<td>1408</td>
<td>166</td>
<td>632</td>
<td>1.25</td>
</tr>
<tr>
<td>Sparse Transmit-Receive</td>
<td>6/22</td>
<td>132</td>
<td>16</td>
<td>169</td>
<td>5.4</td>
</tr>
<tr>
<td></td>
<td>14/10</td>
<td>140</td>
<td>17</td>
<td>172</td>
<td>5.3</td>
</tr>
<tr>
<td></td>
<td>18/8</td>
<td>144</td>
<td>17</td>
<td>174</td>
<td>5.2</td>
</tr>
<tr>
<td></td>
<td>42/4</td>
<td>168</td>
<td>20</td>
<td>181</td>
<td>5.0</td>
</tr>
<tr>
<td>Dense array</td>
<td>64/64</td>
<td>4096</td>
<td>484</td>
<td>1625</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Figure 5.12: Graph of A-scan number against (a) Data acquisition and reconstruction times (b) frame rate using the Sparse-SPA algorithm.
5.4 Conclusion

An algorithm known as Sparse-SPA was developed in this chapter, capable of transmitting and receiving with a sparsely filled array of elements. It was shown how by reducing the number of transmit and receive elements the overall imaging frame rate could be improved. This was largely a result of smaller data sets which allowed for a reduced data acquisition and processing time. The effects of sparse arrays on image quality were also investigated and showed that certain sparse array configurations could achieve equivalent image quality to dense arrays (for a -30 dB image threshold). The quality of the sparse array imagery appeared to be dependent on the number of A-scans, the effective aperture, element layout and element directivity. It was shown how for certain sparse array imagery suffering from poor SNR, noise levels could be lowered by manipulating side lobe levels. This was achieved by applying the Gain Bias function between elements which altered the effective aperture so as to suppress side lobe levels and therefore increase SNR. However the SNR of these images after sidelobe suppression were still lower than that of images generated using a dense array. It was also shown how incoherent noise levels could be reduced via averaging. Though it was discussed that the approach is unlikely be used due to the decrease in speed performance, which was the primary motivation for implementing a sparse array element configuration.
Chapter 6

Calibration of the Full Matrix Capture inspection technique

6.1 Introduction

Calibration refers to setting an instrument’s precision and/or accuracy against a known set of values. In ultrasonic NDT, calibration is used to compensate for variations in sensitivity of the transmitter and receiver channels, and is crucial for reliable flaw sizing and characterisation. Calibration is also used to set absolute sensitivity levels to determine the magnitude of response. The decision to calibrate an ultrasonic system prior to inspection depends on the required outcome. If only positional information is required then inspection can be conducted without calibration. If flaw sizing and characterisation is required then variations in acoustic energy density must be considered and compensated for.

Methods and standards to calibrate array ultrasound equipment already exist for parallel transmission techniques such as Phased Array (British Standards Institution 2001). However, FMC acquires data using a sequential transmission pattern, for which there appears to be little documentation on calibration. It is feasible to use parallel transmission techniques to calibrate an FMC system. However, this would require the use of additional hardware to enable parallel transmission. For a dedicated FMC instrument this would result in additional weight and cost.

The aim of this chapter was to develop a complete calibration process which relied solely on the sequential transmission process of the FMC data acquisition technique. Initially development and implementation of calibration techniques in the MATLAB environment are described. Results from inspection of test blocks containing artificial flaws, designed to validate the calibration techniques and procedures are then discussed. Finally results from inspection of a double ‘v’ butt welded plate containing real flaw are presented which illustrate the industrial relevance of the developed techniques.

6.2 Full Matrix Capture calibration methods

In this chapter the acquisition system used was a Micropulse 5PA 128/128 array controller. Data were acquired using the FMC technique and processed using a desktop
computer with two quad core 3 GHz CPUs. The received data were sampled at 50 MHz and stored with an 8 bit dynamic range. The transducer used for inspection was an Olympus 5L64-A2 linear array probe with 64 elements, 0.6 mm pitch and a 5 MHz central frequency. A 36 degree Perspex wedge was also used for some of the calibration procedures. A diagram of the equipment setup is shown in Figure 3.1.

In the field of NDT there is a standard set of calibration blocks that exist for calibration of parallel transmission array ultrasound and single crystal inspection systems. The samples used for calibration in this chapter included the V1 and A5 calibration blocks as shown in Figure 6.1. Calibration blocks were manufactured from the same material as the double ‘v’ butt welded plate (low carbon steel) to make the calibration process as accurate as possible.

![Diagram of (a) V1 and (b) A5 calibration blocks, manufactured from low carbon steel](image)

**Figure 6.1:** Diagram of (a) V1 and (b) A5 calibration blocks, manufactured from low carbon steel

With array ultrasound inspections there are several aspects to calibration. Firstly the array controller must be calibrated to ensure it is operating correctly. This is usually conducted by the equipment manufacturer and so is not discussed in this thesis. The remaining aspects can be split into two parts, namely transducer and material calibration. Transducer calibration corrects for variations in sensitivity which relate to the elements in the array. Material calibration accounts for variation in sensitivity caused by the properties of the test component and coupling medium. The main benefit of a fully calibrated system in ultrasonic NDT is that it ensures a uniform signal sensitivity from anywhere in the field of view.
During calibration of parallel transmission systems, some of the techniques use a single element aperture linear scan and as such do not require parallel transmission. These techniques can therefore be implemented into an FMC system without the need to change the fundamental approach. It is still, however, important to validate these calibration techniques for FMC, as FMC and parallel transmission systems use different approaches to data acquisition, beam forming and imaging. They are therefore fundamentally different techniques, and so require independent proof that they can perform in the ways which are claimed. Other calibration techniques used by parallel transmission systems utilise averaged A-scans in order to obtain positional information. These approaches require parallel transmission and so different techniques and procedures must be developed for use with FMC.

In the sections that follow algorithms and procedures have been developed which enable the FMC technique to be fully calibrated. This was achieved without the need for parallel transmission.

6.2.1 Material calibration

In this section techniques have been developed which correct for variations in sensitivity which occur as a direct result of factors relating to the material properties of the test component and coupling medium. These include velocity measurements, attenuation compensation and wedge delay compensation.

6.2.1.1 Wedge velocity

In order to determine the beam path length and point of incidence at the interface between dual-layered media, it is necessary to know the velocity of sound in both media. Wedge velocity can be approximately determined from the material type. Perspex and Rexolite are two of the most commonly used materials and their acoustic velocities are well known. To calculate the acoustic wedge velocity experimentally, the width of the wedge was measured. The ultrasonic transducer was then coupled to one of its parallel side faces as illustrated in Figure 6.2.
The range on the instrument display should be set to twice the wedge width. Using a single element in the array transducer in pulse-echo mode the velocity was adjusted until the 2\textsuperscript{nd} backwall echo was at a distance corresponding to twice the width of the wedge on the instrument display. Figure 6.3 displays the 1\textsuperscript{st} and 2\textsuperscript{nd} backwall echoes from the wedge when wedge velocity was set to 2310 m/s. It can be seen that with this velocity the thickness of the wedge was accurately predicted at 40 mm. It should be noted that for thickness measurements the initial part of the backwall response should be used for calibration, rather than the peak amplitude, as the timing trigger on the equipment will start as soon as a voltage is applied across the piezoelectric crystal.

**Figure 6.3:** A graph to calibrate wedge velocity

### 6.2.1.2 Material velocity

Similarly to wedge velocity, the material velocity can be approximated from the material type. Alternatively the longitudinal wave velocity can be found experimentally
using the V1 calibration block, which has a known thickness of 25 mm. The ultrasonic transducer was initially coupled to one of the parallel faces of the V1 block, as shown in Figure 6.4, and the range on the instrument display was set to twice its thickness.

**Figure 6.4:** Illustration of an ultrasonic transducer coupled to the side face of the V1 calibration block

Using a single element in the array in pulse-echo mode, the velocity was adjusted until the 2nd backwall echo was at a distance corresponding to twice the thickness of the calibration block. Figure 6.5 shows the first and second backwall responses at 25 mm and 50 mm respectively when setting the velocity to 5810 m/s.

**Figure 6.5:** A graph to calibrate longitudinal velocity using the V1 calibration block

To determine the shear wave velocity a wedge with known dimensions was attached to the transducer. With the 100 mm radius of the V1 calibration block and using a single element in the array in pulse-echo mode, the transducer was moved back and forth until the maximum signal amplitude was found as shown by Figure 6.6. At this position the maximum intensity part of the sound field will exit the wedge at the notch on the V1 calibration block which represents the point of equidistance to anywhere 100 mm radius.
curvature. The path length in the calibration block will therefore be exactly 100 mm. The path length in the wedge will be the perpendicular distance from the transducer face to the point of equidistance, and can be measured or calculated using Equation 6.1 provided the wedge angle, $\theta_{\text{wedge}}$, and height, $h$, of the excited element are known.

$$Path \ length = h \cos(\theta_{\text{wedge}})$$  \hspace{1cm} 6.1

Figure 6.6: Illustration of the transducer coupled to the V1 calibration block via a 36° Perspex wedge.

The time travelled in the material is simply the total A-scan length minus the time travelled in the wedge. Therefore, the shear wave velocity can be determined by dividing the 100 mm material path length by the time travelled in the material. Figure 6.7 displays stacked A-scans generated as the transducer was moved back and forth on the V1 block. Also plotted is an envelope function, which plots the maximum signal amplitude from each of the stacked A-scans. It can be seen that a velocity of 3230 m/s correctly predicts the length of the 100 mm radius, whereas a velocity different of +/- 100 m/s incorrectly plots the 100 mm radius by 3 mm.
Figure 6.7: Graphs to calibrate shear wave velocity using the V1 calibration block. Distances calculated using a velocity of (a) 3230 m/s (b) 3130 m/s (c) 3330 m/s. Displayed are stacked A-scans from different positions on the V1 calibration block (blue), and an envelope of the peak amplitude of each stacked A-scan (red).

6.2.1.3 Wedge delays
Delays to account for variations in beam path in the Perspex wedge can be determined experimentally to enable the wedge geometry to be calculated. Once the geometry of the wedge is known the entry surface or “zero depth” can be set to the surface of the test sample, rather than at the transducer face, as shown by Figure 6.8. Also the delay laws, which account for refraction at the interface between dual-layered media, can then be calculated mathematically using Fermat’s principle or Snell’s law, as discussed in detail in Section 4.3.
Figure 6.8: Diagram illustrating the different planes that the “zero depth” can be taken from.

To calculate the wedge geometry the 100 mm radius on the V1 calibration block was used and a single element aperture linear scan was performed over the whole array. The transducer was moved back and forth until the maximum amplitude for each element was found and the corresponding A-scans were recorded. An example of the maximum signal amplitude for a single element in the array is shown in Figure 6.9. At maximum amplitude the beam path in the wedge will be perpendicular to the transducer face, and the beam path length in the V1 block will be exactly 100 mm. Therefore, by subtracting this beam path from the total length of each A-scan, the geometry of the wedge was determined.
Figure 6.9: An example of a graph used to determine wedge geometry. Displayed are stacked A-scans at different positions on the V1 calibration block (blue), and an envelope of the peak amplitude of each stacked A-scan (red).

6.2.1.4 Wedge attenuation and beam spread

For angled beam inspection, the path length travelled in the wedge will vary as a function of the position of the element in the array and the point of incidence at the interface. Since attenuation and beam spread are both dependent on path length, as described in Sections 2.2.5 and 2.2.6 respectively, then path length variations in the wedge will affect the overall sensitivity. This can be accounted for by adjusting the gain for each focal law prior to inspection to ensure a uniform signal response. The 100 mm radius on the V1 calibration block was used to calculate the gain compensation for each focal law. Transmitting and receiving on the first element in the array only, the transducer was moved back and forth until the maximum signal response was found. The signal amplitude was then recorded and the procedure repeated for several other elements spaced throughout the array. For a given element in the array, the point of maximum amplitude will correspond to a sound path length in the test material of 100 mm. Therefore any variations in maximum amplitude will be caused by path length differences in the wedge. Comparing time of flight in the wedge with signal amplitude, sensitivity variations resulting from wedge attenuation and beam spread were compensated for. Figure 6.10 compares the signal response of four elements spaced...
throughout the array and illustrates the effects of attenuation and beam spread for the Perspex wedge. The result shows that there was in fact very little difference in response caused as a result of changes in path length. This could have been because of the relatively small difference in path length (approximately 15 mm). Had a steeper angled wedge been used, where the difference in path length would be greater, then the effects of attenuation and beam spread may have been more prominent. It can also be seen from Figure 6.10 that some of the signals have two major peaks of similar amplitude. This is likely to be caused by geometric reflections in the V1 calibration block.

![Figure 6.10: A graph showing the effect of beam spread and attenuation in the Perspex wedge for element 1 (red), element 22 (blue), element 43 (green) and element 64 (black) in the ultrasonic array.](image)

### 6.2.1.5 Material attenuation and beam spread

To account for the effects of attenuation and beam spread in the test material, the A5 calibration block was used. By transmitting and receiving on a single element in the array, the transducer was moved back and forth until the maximum signal response for a SDH at a given depth was found. The procedure was then repeated for several SDHs at varying depths. By comparing the material path length against signal amplitude, material attenuation and beam spread were compensated for. The significance of beam spread and material attenuation is shown in Figure 6.11.
6.2.2 Transducer calibration

In this section two techniques have been developed to correct for sensitivity variations which occur as a direct result of factors relating to the transducer. These include an element linearity check and a method of accounting for changes in signal strength as a function of direction.

6.2.2.1 Element linearity assessment

In the field of ultrasonic NDT it is necessary for operators to perform an element linearity assessment periodically. This assessment is used to check that elements in the array are active and have a uniform sensitivity.

The assessment was carried out using the 25 mm thickness of the V1 calibration block. Initially the transducer was coupled to the side face of the V1 block (Figure 6.4). Each element was fired sequentially using a single element aperture linear scan. The amplitude from each element was recorded and compared to all other elements. There are no strict guidelines on the allowable amplitude difference between elements, nor on the acceptable number of inactive elements, as tolerances vary depending on the inspection scenario. Therefore, information regarding transducer linearity is usually provided in the written instruction which is unique to the inspection. Typically, all active elements should be within a 3 dB threshold of the mean value, with no more than 10% of elements being inactive. If these limits are not adhered to then the sensitivity

Figure 6.11: A graph showing the effect of beam spread and attenuation in the A5 calibration block
and steering ability of the transducer may be compromised. Figure 6.12 shows the linearity check performed on the 64 element transducer used in this chapter. The result confirms that all elements are active and fall with a 3 dB range of the mean.

Figure 6.12: A graph comparing the response from elements in an array (solid line). Also shown are the 3 dB amplitude thresholds from the mean (dashed line).

6.2.2.2 Beam divergence

The transducer used in this chapter had an element size in the active plane of approximately half the wavelength of the emitted sound field. Therefore as previously discussed in Section 2.2.4, signal amplitude will depend on the transmitted and received beam angle.

The dependence of signal amplitude on beam angle can be accounted for using the 1 mm SDH at a depth of 25 mm in the A5 calibration block. Initially values to account for material and wedge attenuation were applied to the system to ensure the only variable to effect signal strength was beam divergence. Next, FMC data was continuously acquired (using all elements in the array), processed and imaged while the transducer was moved back and forth. By processing the raw data, the position of the SDH can be to be tracked relative to the exit point for a given element (Figure 6.13). In conjunction with this A-scan information from the same given element was used to monitor the signal amplitude. Relating signal amplitude with position allowed the beam directivity to be determined.
Figure 6.13: Illustration of the transducer coupled to the A5 calibration block via a 36° Perspex wedge.

Figure 6.14a-b shows the relationship between directivity and angle of refraction in the test material. Figure 6.14a shows the maximum beam strength to be at an angle of refraction of approximately 48 degrees. However, this plot does not consider the effects of beam spread and attenuation, which will bias the result due to path length differences which will occur as the transducer is moved back and forth. Figure 6.14b shows the maximum beam strength to be at an angle of refraction of approximately 56 degrees when compensating for the effects of attenuation and beam spread. Comparing these two plots shows a difference in optimum beam angle of 8 degrees. This illustrates the importance of compensating for attenuation and beam spread in the test component prior to accounting for element directivity.

Figure 6.14: Plot of directivity (a) without attenuation compensation (b) with attenuation compensation. All plots were generated from a single element in the array transducer.
This approach calculates the beam divergence for each element in the array. This differs from parallel transmission techniques which calculate an average divergence by treating the elements in the array as a single large aperture as shown by Figure 6.15.

![Figure 6.15: Illustration of the difference in approach between Phased Array and FMC directivity calculations.](image)

### 6.2.3 Calibration of image sensitivity

In order to accurately size and characterise flaws during an ultrasonic array inspection it is useful to have a uniform image sensitivity. In this situation, for a flaw of given size and shape, a constant amplitude response may be achieved independent of position in the image. The SPA algorithm achieved a uniform image sensitivity by accounting for signal variations caused by beam attenuation, beam spread and beam divergence. The effect of each parameter on the signal was calculated using methods described in Section 6.2.1 and 6.2.2. For each parameter, a matrix of coefficients (known as a compensation matrix) was generated which represented the required amplitude compensation. Each matrix contained the same number of cells as pixels in the final image. Therefore a uniform image sensitivity could be achieved by superimposing each compensation matrix onto the original image generated using the SPA algorithm.

### 6.3 Results and discussion: Inspection using a calibrated FMC system

#### 6.3.1 Introduction

So far in this chapter algorithms and inspection techniques have been developed in order to achieve a calibrated FMC inspection technique. In this section techniques are validated using test components containing artificial flaws of known size and shape.
Validated algorithms are then used to characterise flaws in a double ‘v’ butt welded plate, the results of which are also presented and discussed here. A full description of the equipment setup used for inspection and analysis can be found in section 3.1. In this chapter the transducer was mounted onto a 36 degree Perspex wedge which induced the shear wave as the dominant mode into the test samples.

### 6.3.2 Validation of calibration techniques

Before the developed calibration techniques could be used to characterise real flaws in industrial components it was first necessary to validate the techniques against idealised flaws of known shape and size. Here test components containing known artificial defects were inspected using both calibrated and uncalibrated systems. Results were compared in order to assess the validity of the developed calibration techniques.

The test block used during the validation process was manufactured from low carbon steel and contained multiple 3 mm SDHs, vertically aligned and spaced 10 mm apart as shown by Figure 6.16. Their vertical alignment was suited to minimise interference and masking between adjacent holes when performing angled beam inspection.

![Test Block 9 containing multiple 3 mm SDHs with a vertical alignment](image)

**Figure 6.16:** Test Block 9 containing multiple 3 mm SDHs with a vertical alignment.

To test the validity of the calibration techniques an angled beam inspection was performed using a transducer mounted onto a 36 degree Perspex wedge, which generated a dominant shear wave mode in the test material. The vertically aligned SDHs in Test Block 9 were inspected using a standoff of 50 mm which corresponded to a typical standoff for inspection of the double ‘v’ butt welded plate. It also allowed for five SDHs to be located within the 40-70 degree angular window of inspection.

As well as visually analysing generated imagery of the test blocks, quantitative signal analysis was performed using the parameter Relative Amplitude. Here the peak
amplitude response from each SDH was normalised against the peak response of the whole image as shown by Equation 6.2.

\[
\text{Relative Amplitude} = \frac{\text{max. amplitude}_{SDH}}{\text{max. amplitude}_{image}} \tag{6.2}
\]

Figure 6.17a-b show cross-sectional images of five vertically aligned SDHs in Test Block 9, generated using an uncalibrated and calibrated inspection system respectively. The imaged SDHs ranged from 20 - 60 mm in depth, which, with a 50 mm stand-off, corresponded to an angular range of 40-70 degrees relative to the normal. This angular range is typical for the NDT industry (Olympus NDT 2010).

From the uncalibrated image in Figure 6.17a it is clear that the peak image response comes from one of the two SDHs located at angles of 59 and 68 degrees. The signal response then decays with increasing depth, with the weakest response coming from the SDH located at 40 degrees. Comparing signal responses using the Relative Amplitude tool as shown in Table 6.1, it can be seen that the peak response was from the SDH located at an angle of 59 degrees, with the SDH at 68 degrees being 0.3 dB weaker. The weakest response was from the SDH located at 40 degrees; here the signal was 8.4 dB below the peak response.

Visual analysis of the calibrated image in Figure 6.17b, showed there was strong amplitude correlation between the signal responses of all SDHs in the 40-70 degree angular window. This was confirmed by the results from the Relative Amplitude measurements in Table 6.1, which show a maximum variation in signal response between SDHs of 1.5 dB. The improved uniformity in amplitude distribution of the calibrated system is also confirmed by the standard deviation calculations, also in Table 6.1, which showed the calibrated system to have an average deviation of 0.7 dB. In comparison the standard deviation between SDHs in the uncalibrated image was 3.6 dB.

While it was clear that a calibrated system improved the uniformity of response, it should be noted that this was at the cost of a slight increase in noise in certain regions of the images. This was caused by high level gain used to create a uniform response. Such variations in noise can be seen when comparing the plots in Figure 6.17. To reduce amplification of noise during calibration it may be possible to apply frequency filters to the acquired data. Alternatively increasing the number of A-scans through averaging or larger arrays will reduce incoherent noise levels. Coherent noise levels can be reduced by increasing the angular range of inspection.
Figure 6.17: Cross-sectional images showing five vertically aligned SDHs in Test Block 9 using (a) an uncalibrated system (b) a calibrated system. All images employed a 20 dB threshold.

Table 6.1: Table displaying quantitative data from calibrated and uncalibrated inspection of Test Block 9

<table>
<thead>
<tr>
<th>Depth (mm)</th>
<th>Angle (°)</th>
<th>Relative Amplitude from SDHs (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Uncalibrated</td>
</tr>
<tr>
<td>20</td>
<td>68.2</td>
<td>-0.3</td>
</tr>
<tr>
<td>30</td>
<td>59.0</td>
<td>0</td>
</tr>
<tr>
<td>40</td>
<td>51.3</td>
<td>-2.7</td>
</tr>
<tr>
<td>50</td>
<td>45.0</td>
<td>-5.5</td>
</tr>
<tr>
<td>60</td>
<td>39.8</td>
<td>-8.4</td>
</tr>
<tr>
<td>Standard deviation</td>
<td></td>
<td>3.6</td>
</tr>
</tbody>
</table>

6.3.3 Inspection of a double ‘v’ butt welded plate

In this section a low carbon steel plate containing a double ‘v’ butt weld with known flaws was inspected as shown in Figure 4.19. The types of flaws contained in the weld included LORP, porosity and a toe crack. Results are presented and discussed which examine the inspection capability of the calibrated FMC system, which was validated in
section 6.3.2. The geometry of the plate as well as the size and locations of the flaws contained within it were shown in Figure 4.19. A full description of the ultrasonic equipment setup used can be found in Section 4.4.1.

Figure 6.18 illustrates the three known flaws in the double ‘v’ butt welded plate, generated using a calibrated and uncalibrated system. In both images a reference amplitude was set relative to the peak response of the 3 mm SDH in Test Block 9 at depth 30 mm and stand-off of 50 mm, this corresponded to the location of the weld body relative to the centre of the transducer. The amplitude range used for the inspection was between 6 and -25 dB, with 0 dB representing the same signal response as that of the reference amplitude. For each image a weld overlay was generated for clarity. The dashed horizontal line represented the plate backwall; any indications imaged above this line were in direct line of sight of the transducer, any indications plotted below this line represented the beam skipping off the backwall before insonifying the weld region.

From Table 6.2 it can be seen that the calibrated signal response of the toe crack in the double ‘v’ butt weld was -3.2 dB below the reference signal, in comparison the uncalibrated signal response had an amplitude of -11.2 dB. This difference in amplitude is clearly visible in the cross-sectional imagery shown in Figure 6.18a,d and demonstrates the importance of calibration. In this instance the difference in signal response is enough that an operator analysing both images may come to different conclusions about the severity of the flaw, leading to a different course of action being taken; for example the crack might be periodically monitored rather than cut out if the system was not properly calibrated prior to inspection. Since the toe crack was located at the furthest possible distance away from the transducer in the weld body, differences in signal amplitude were largely attributed to beam spread and attenuation.

Table 6.2 shows that for both the calibrated and uncalibrated systems, porosity was approximately 13 dB weaker than that of the reference amplitude, and as such was only just visible in the cross-sectional imagery in Figure 6.18b,e. The strong similarity in response was attributed to the fact that the porosity in the weld was located at a very similar position to the SDH in Test Block 9 at depth 20 mm (relative to the transducer), which was used to determine the reference amplitude. Therefore, little to no amplitude corrections were applied to the calibrated images in this region. Similar reasoning can also be used to explain the similarity in response between the calibrated and uncalibrated signals from the LORP indications shown in Figure 6.18c,f. In this case the
peak responses from both signals were approximately 7 dB lower than that of the reference amplitude.

Figure 6.18: Calibrated (left) and uncalibrated (right) cross-sectional images of (a,d) toe crack, (b,e) porosity, (c,f) LORP in a double ‘v’ butt welded plate. Amplitude range is -25 to 6 dB relative to the peak response SDH in Validation block 1.
Table 6.2: Table displaying quantitative data from calibrated and uncalibrated inspection of the double ‘v’ butt welded plate.

<table>
<thead>
<tr>
<th>Flaw type</th>
<th>Amplitude (dB)</th>
<th>Uncalibrated</th>
<th>Calibrated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toe crack</td>
<td>-11.2</td>
<td>-3.2</td>
<td></td>
</tr>
<tr>
<td>Porosity</td>
<td>-12.5</td>
<td>-12.8</td>
<td></td>
</tr>
<tr>
<td>LORP</td>
<td>-7.2</td>
<td>-6.6</td>
<td></td>
</tr>
</tbody>
</table>

6.4 Conclusion

The main factors which affected the sensitivity of FMC inspection were related to the test material and transducer itself. These included attenuation, beam spread, divergence and element linearity. Results presented in this chapter showed the FMC technique can be fully calibrated using purely sequential transmission patterns to yield a highly uniform image sensitivity (standard deviation of 0.7 dB) within the standard angular range of inspection. Experimental data from inspection of a double ‘v’ butt welded plate further illustrated the significance of calibration, with differences in sensitivity between calibrated and uncalibrated inspections of up to 8 dB. Validation of the FMC calibration technique was an important step forwards towards defect sizing and characterisation. It also moves the technique closer to an industrially viable and accepted alternative to the existing parallel transmission ultrasonic techniques. However, before full defect sizing and characterisation can be realised, encoders would need to be incorporated into the system to allow accurate sizing and positioning in the direction of mechanical scanning (passive axis).
Chapter 7

Conclusions and recommendations for future work

The research presented in this thesis is a result of an EPSRC Engineering Doctorate project between the University of Manchester, the University of Birmingham and TWI Ltd. The overall aim of the thesis was to realise the industrial relevance of the FMC acquisition technique and associated data processing algorithms through algorithm optimisation and technique development. To achieve this aim the FMC technique was compared to existing industrial techniques in terms of a number of assessment criteria. These were SNR, a point spread function and frame rate (Chapter 3). The use of sparse array element configurations were also investigated and compared to dense arrays using similar assessment criteria. The effective aperture concept was used to design sparse array element configurations, and image quality was enhanced using a windowing function termed the Gain Bias function (Chapter 5). Several techniques were also investigated which facilitated focussed inspection through dual layered media. Techniques were compared in terms of their positional accuracy and the time taken to determine the point of incidence (Chapter 4). Finally a calibration process was developed which allowed for a highly uniform image sensitivity to be achieved throughout the region of inspection (Chapter 6).

In summary the main conclusions of this work were:

- With parallel transmission techniques there is a trade-off between focal spot size and depth of field (which relates to the size of the active aperture). However, FMC and the associated SPA algorithm overcome this issue and allow a tight focal spot to be generated while still maintaining a fully focussed image.

- For the algorithms developed in this work using the MATLAB software package, parallel transmission techniques achieved frame rates which were over an order of magnitude faster than FMC and the associated SPA algorithm.

- A novel algorithm which solved Snell’s law using either an analytical or numerical method was developed to facilitate focussed inspection through a planar refractive interface. The analytical method offered an exact solution to determine the point of incidence at a planar refractive boundary. In comparison to the analytical method, existing iterative methods found only approximately solutions and took longer to process images of equivalent quality.
• A novel algorithm which solved Snell’s law using a numerical method was developed to facilitate focussed inspection through an arbitrary profile refractive interface. Generated images were equivalent in quality to existing iterative algorithms used to solve Fermat’s principle. However processing time was significantly slower.
• Curve fitting algorithms were implemented which facilitated automated inspection through arbitrary refractive interfaces. Though curve fitting was limited to regions where incident beams were reflected directly back to the transducer.
• Sparse array algorithms were developed to increase the achievable frame rate during inspection. A novel algorithm was developed which used far-field approximations and the effective aperture concept to allow transducer inter-element spacing to exceed half wavelength, while avoiding grating lobes and maintaining a high SNR. Existing algorithms in the field currently only allow element spacing on the transmit aperture to exceed half wavelength. The implementation presented in this work allow element spacing to exceed half wavelength on both transmit and receive apertures simultaneously.
• Sparse arrays were shown to greatly improve the speed of inspection in comparison to a dense array. The quality of sparse array images appeared to be dependent on the number of A-scans, the effective aperture, element layout and element directivity. It was shown how for certain sparse array images suffering from poor SNR, noise levels could be lowered by using window functions to manipulate side lobe levels.
• For the first time a fully functional method has been developed and implemented which allows full calibration of the FMC inspection technique. Algorithms were adapted and developed from existing parallel transmission methods and allow images to be generated with uniform sensitivity levels.

These achievements will now be described in more detail in the following section.

### 7.1 Main Conclusions

In chapter 3 quantitative comparisons were made between existing parallel transmission techniques and the FMC data acquisition technique and associated SPA algorithm in terms of imaging quality and inspection speed. It was found that while with focussed parallel transmission techniques the depth of field varied as a function of aperture size, the SPA algorithm overcame this barrier due to its ability to focus everywhere.
Therefore, the technique allowed for very large apertures to be used without suffering from a very shallow region of focus. Using unfocussed parallel transmission scans increased the depth of field to a similar level to the SPA algorithm, but this was at the cost of lateral resolution which was poor. Another advantage of the SPA algorithm was its superior focussing ability compared to existing techniques such as linear and sectorial scanning. This again related to the ability of the SPA algorithm to focus everywhere within the field of view, as well as it’s utilisation of all elements in the array during focussing.

The main disadvantage of the FMC acquisition process and SPA algorithm related to the time taken to acquire and process the data. These parameters were found to be up to an order of magnitude slower compared to existing industrial techniques, leading to a slower overall imaging frame rate. However it is expected that should a lower level programming language be used, or a parallel processing architecture be implemented then frame rates could be greatly increased.

In this work parallel transmission techniques achieved faster frame rates than FMC. However, in some instances it was shown that the physical limit of inspection (governed by the ultrasonic propagation time) was smaller for FMC in comparison to parallel transmission techniques. This suggests that if hardware and software improved to a level where the physical limit also became the limiting factor for inspection, then FMC and the associated SPA algorithm could outperform parallel transmission techniques in terms of frame rate.

In Chapter 4 novel algorithms were developed and implemented which minimised the time taken to determine the point of incidence between dual layered media. It was shown that for a planar refractive interface an analytical algorithm used to solve Snell’s law (4th order polynomial) was fastest for all image sizes tested. This was in comparison with a numerical approach to solving Snell’s law and an iterative approach to solving Fermat’s principle. For the case of a non-planar interface no analytical solution to Snell’s law exists. As such comparisons were only made between a numerical implementation to solve Snell’s law and an iterative implementation to solve Fermat’s principle. In this case Fermat’s principle was preferred, not only because it was faster, but also it was far more stable and required significantly less code.

Curve-fitting algorithms were also developed within Chapter 4. These facilitated automated focussed inspection through dual layered media using signals reflected from
the refractive boundary profile. Such signals were present within the full matrix of data and so no additional firing were required to then generate focussed imagery beyond the refractive boundary. The curve-fitting algorithm allowed automated focussing through dual layered media, although this was limited to regions where incident beams were reflected directly back to the transducer from the refractive boundary.

In Chapter 5 the use of sparse arrays were implemented in order to increase the frame rate of the FMC acquisition process and associated SPA algorithm. Investigations were performed which compared the increase in speed using sparse arrays against image quality. It was shown how reducing the number of transmit and receive elements could improve the overall imaging frame rate. This was largely a result of smaller data sets which allowed for a reduced data acquisition and processing time. It was also demonstrated that certain sparse array configurations could achieve equivalent image quality to dense arrays (for a -30 dB image threshold). The quality of the sparse array imagery appeared to be dependent on the number of A-scans, the effective aperture, element layout and element directivity. It was shown how for certain sparse array imagery suffering from poor SNR, noise levels could be lowered by manipulating side lobe levels. This was achieved by applying the Gain Bias function between elements which altered the effective aperture so as to suppress side lobe levels and therefore increase SNR. However, the SNR of these images after side lobe suppression were still lower than that of images generated using a dense array. It was also shown how incoherent noise levels could be reduced via averaging. However, the approach would unlikely be used in practice due to the decrease in speed performance, which was the primary motivation for implementing a sparse array element configuration.

In Chapter 6 a novel process was developed to facilitate calibration of the FMC inspection technique. Such processes were adapted from existing parallel transmission techniques. The main factors which affected the sensitivity of FMC inspection were related to the test material and transducer itself. These included attenuation, beam spread, divergence and element linearity. Results presented in this chapter showed the FMC technique can be fully calibrated using purely sequential transmission patterns to yield a uniform image sensitivity (standard deviation of 0.7 dB) within the standard angular range of inspection. Experimental data from inspection of a double ‘v’ butt welded plate further illustrated the significance of calibration, with differences in sensitivity between calibrated and uncalibrated inspections of up to 8 dB. Validation of the FMC calibration technique was an important step forwards towards defect sizing
and characterisation. It also moves the technique closer to an industrially viable and accepted alternative to the existing parallel transmission ultrasonic techniques. However, before full defect sizing and characterisation can be realised, encoders would need to be incorporated into the system to allow accurate sizing and positioning in the direction of mechanical scanning (passive axis).

7.2 Recommendations for future work

This project has improved the knowledge of FMC and its associated processing algorithms and has given insight to the potential industrial relevance. Direct comparisons have been made with existing industrial ultrasonic approaches. Techniques have also been developed which enhanced the knowledge and capability of FMC. These included sparse array designs, methods to account for inspection through dual layered media and a method of calibration. Time restrictions on the project, however, have forced this work to stop; the following is a description of considerations for future work.

It was identified within this work that one of the main downsides of FMC was the low frame rate in comparison to existing industrial techniques, which utilise a parallel transmission approach. This thesis has identified and developed novel ultrasonic techniques and algorithms to address this issue. Future work should aim to take these developed techniques and algorithms and transfer them onto a platform designed for high performance, i.e. intensive processing. Since the majority of algorithms developed are highly parallelisable then use of the GPU is one option for this. If portability is paramount then Field Programmable Gate Arrays (FPGAs) are another option which is readily affordable and available off the shelf.

Parallel transmission FMC is an area of research which appears to have had little investigation in the current NDT literature, though much research has been conducted in the medical and RADAR fields. Transmission on multiple elements in parallel using encoded signals (e.g. frequency modulation and Golay codes) allows for data to be superimposed during acquisition, then decomposed during data processing. This is possible due to the unique characteristics of each signal which are introduced by the encoding scheme during the transmission process. As such the approach offers the potential to reduce the propagation time (physical limit) and data transfer time.

It is often useful to be able to inspect large volumes of a component from a minimal number of transducer locations. This reduces mechanical movement during an inspection, decreases the risk of lift-off and allows inspection of components with
limited access. One way which the NDT industry has addressed this issue is to develop 2D matrix and annular arrays; capable of omnidirectional electronic beam steering and focussing. The problem with FMC is that 2D arrays often contain many more elements than linear arrays. Therefore, the issue of poor frame rate is even more apparent. Within the NDT industry limited research in this area has been undertaken in relation to FMC and there is much scope to develop novel focal laws and optimise data processing algorithms to achieve real time imaging. The use of sparse arrays may also become important to achieve this goal.
Publications


References


