Energy-Balancing-Based Control Design
for Power Systems
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Abstract

This thesis summarizes the MPhil dissertation on the title of energy-balancing-based control design for power systems, School of Electrical and Electronic Engineering, the University of Manchester.

This MPhil thesis reviews two years researches of Hamiltonian system and its applications and both adaptive and energy-balancing control designs for Hamiltonian system. The studies on Hamiltonian systems focused on the development of the Hamiltonian theory and building Hamiltonian model, especially power system. To obtain better control result of Hamiltonian system, adaptive control and energy-balancing-based control are considered. Combined those two methods with Hamiltonian control system, by using simulation, the better preforming result can be achieved.
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Chapter 1

Introduction

Hamiltonian systems are mechanical nonlinear control systems obtained from Euler-Lagrange equations in 1833 by Irish mathematician William Rowan Hamilton. Over a span of nearly two centuries, a large number of research works on the Hamiltonian systems have been done, and combined with other theories, more and newer attempts have been made to develop the system. For example, among other studies in [38] adaptive stabilization was used in a generalized Hamiltonian system with dissipation, [24] proposed the adaptive $H_{\infty}$ excitation control of multimachine power systems via the Hamiltonian function method, in [13] the decentralized PD control was associated with the Hamiltonian hybrid system, and so on.

In [23] – [28] and [33] – [34], Hamiltonian systems were applied in power systems, several nonlinear control strategies, such as energy-based $L_2$-disturbance attenuation control, adaptive $H_{\infty}$ excitation con-
control, and nonlinear decentralized saturated controller design and so on, were used in port-controlled Hamiltonian models for power systems.

In this present MPhil thesis, a power system is modelled by a port-controlled Hamiltonian system with dissipation (PCHD), as it applied in [23] – [28] and [33] – [34]. To determine its other properties, adaptive stabilization and energy-balancing-based control (EB-based control) are applied to control this Hamiltonian systems.

Adaptive control design is based on parameter estimation, and it is a control method that adapts itself to varying parameters. The designed adaptive control laws require an asymptotic trajectory tracking, which is bounded for all internal signals. Such adaptive control laws can be classified on the basis of control objective and the signal that drives parameter update law. Control objective divides the structure of controller, parameters of which are to be updated on-line. The parameter update law is driven by signals that calculate the error between estimated and true parameters or the error between desired and actual output. In present report, the adaptive control law is achieved by an error between tracking error and prediction error.[39]

An energy-balancing system is a kind of Euler-Lagrange system, in which the total closed-loop energy is the difference between open-loop
energy and supplied energy. In the energy-balancing-based control system, the energy-balancing control is designed for generalized canonical transformations of time-varying port controlled Hamiltonian systems, and the designed controller is based on the storage equation, which is a function of only the tracking error. At the same time, in Hamiltonian system, Hamiltonian function denotes the total energy of system, and satisfies the request of energy-balancing-based design.\cite{40}

In current thesis, a method called interconnection and damping assignment passivity-based control (IDA-PBC) is mentioned, and it is used to solve the tracking problem for port-controlled Hamiltonian systems, is referred to \cite{37}, \cite{40}, \cite{41}, \cite{44}. 
Chapter 2

Power System Modelling and Control Methods

To model and design power systems, the first step is to study the power systems. According to the mechanical and electrical characters of power systems, the state space equations can be obtained. Different control methods which are applied to control the state space equations are discussed.

This chapter can be divided into three parts. The first part analyzes the structure of power systems, the second builds the state space equations for power systems, and the third considers the appropriate control methods which are used to design power systems.
2.1 Power system analysis

A power system is defined as a network of sets of elements designed to convert nonelectrical energy to electrical energy effectively and continuously.

Frequently, a power system can be divided into the following five subsystems: \([4], [10]\)

Generation
Transmission
Subtransmission
Distribution (include Primary and Secondary Distribution)
Use

The generation part of the power system takes charge of converting other types of energy into electrical energy. An energy source produces mechanical torque or a rotating shaft which is transferred to the turbine. The turbine drives the generator. At the same time, the mechanical torque or rotating shaft can be turned into electrical energy. \([4], [10]\)

Transmission is used to transfer the produced electrical power from the generation system to subtransmission. The electrical power produced in the generator has lower voltage, and is transformed into a
2.1 Power system analysis

higher voltage by the transformer. This higher transmission voltage power is then moved to the subtransmission system by transmission lines. [4], [10]

Older, lower voltage networks can be replaced by new, high-capacity transmission networks, and the older transmission can be turned into subtransmission lines. [4], [10]

Electrical power is delivered from subtransmission lines to the substation where it can be decreased in the voltage. From the substation, individual circuits extend to the customer’s location. These circuits constitute the primary distribution system. The distribution system has two basic designs: radial, where power flows in only one direction in a given circuit from source to load, or loop, where the primary distribution system is a network, and the customer can receive power from more than one direction. Illustrations of these two designs are given in Figure 2.1 and Figure 2.2. [4], [10]

The use indicates the end-user, and, the end-user is usually called the load. [4], [10], [15]

There are two kinds of energy sources that can be used to produce electrical energy, and these thermal and non-thermal sources. Thermal
source includes coal, oil, natural gas, nuclear fission of uranium, solar and so on, while non-thermal sources include energies like hydro, wind, waves, and so on. \[15\]

All power systems have five subsystems, all of which influence the analysis of power system. For example, the consumed energy on the transmission lines, and the different energy transfers between different distributions of transmission lines, substations and loads.

The current MPhil thesis studies a simplified single machine with infinite bus power system which is shown in Figure\[2.3\]. Complicated conditions such as generator-network interactions will be considered in future research. \[15\]
2.2 State space model of power systems

Consider a single machine infinite bus power system consisting of \( n \) synchronous machines which is shown in Figure 2.3. According to some standard assumptions, the classical model with flux decay dynamics is used to describe interconnected generators. In this classical model with flux decay dynamics, the voltage behind the direct axis transient react-
2.2 State space model of power systems

The angle of the voltage synchronizes with the mechanical angle, and both the angle of the voltage and the mechanical angle are relative to the synchronously rotating reference frame, which is given in Figure 2.4. The mechanical equations of the power system can be obtained as follows: [4] – [15]

\[
\dot{\delta} = \omega - \omega_0 \tag{2.1}
\]

\[
\dot{\omega} = -\frac{D}{H}(\omega - \omega_0) - \frac{\omega_0}{H}(P_e - P_m) - w_1 \tag{2.2}
\]

where

\(w_1\) — is disturbance of the system,

\(\delta\) — is the power angle of the generator, in radians,
\[ \omega \]— is the relative speed of the generator, in rad/s,

\[ P_m \]— is the mechanical input power, in p.u., which is assumed to be constant,

\[ P_e \]— is the active electrical power delivered by the generator, in p.u.,

\[ \omega_0 \]— is the synchronous machine speed, in rad/s,

\[ D \]— is the per unit damping constant,

\[ H \]— is the per unit inertia constant, in seconds.

The electrical energy produced by the generator needs to be transferred to the rest of the power system. Each of phases of the open circuit diagram is predigested as in Figure 2.5.
From Figure 2.5, the equations of electrical power in the system can be expressed as follows [4] – [15]:

\[
V_a = E_a - rI_a - jX_s I_a \tag{2.3}
\]
\[
I_a = \frac{V_a}{X'_{ds}} \sin \theta_e \tag{2.4}
\]

Transforming Equation (2.3) into other expression [15], and equation for the open circuit voltage can be derived [4] – [15]:

\[
E_a = V_a + rI_a + jX_s I_a \tag{2.5}
\]

where

\( V_a \)—is the voltage of the rest of power system,
\( E_a \)—is the open circuit voltage,
\( r \)—is the resistance of the open circuit,
\( I_a \)—is the current of the open circuit,
\( jX_s \)—is the inductance of the open circuit,
\( x'_{ds} \)—is the mutual reactance between the excitation coin and the stator coil of the generator,
\( \theta_e \)—is the rotor angular position of the generator with respect to a stationary axis.
2.2 State space model of power systems

When the power system is in a synchronous, positive-sequence, and steady-state operation, assuming \( v_0 = i_0 = 0 \), the quadrature axis of the generator electrical dynamics can be devised as [15]:

\[
\dot{E}_q' = -\frac{1}{T_{d0}} E_q' + \frac{1}{T_{d0}} E_f + w_2
\]  

(2.6)

The electrical equations are [15]:

\[
E_q = x_{ds} V_s x'_q \frac{E_q'}{x_{ds}} - x_d' - x_d' V_s \cos \delta = x_{ad} I_f,
\]  

(2.7)

\[
E_f = k_c u_f,
\]  

(2.8)

\[
P_e = \frac{V_s}{x_{ds}} E_q' \sin \delta,
\]  

(2.9)

\[
I_q = \frac{V_s}{x_{ds}} \sin \delta,
\]  

(2.10)

\[
Q_e = \frac{V_s}{x_{ds}} E_q' \cos \delta - \frac{V_s^2}{x_{ds}}
\]  

(2.11)

\[
Q_e = \frac{V_s}{x_{ds}} E_q' \cos \delta - \frac{V_s^2}{x_{ds}}
\]  

(2.12)

where

\( w_2 \)—is disturbance of the system,

\( Q_e \)—is the reactive power, in p.u.,

\( E_q' \)—is the transient electromagnetic force in the quadrature axis of the
2.2 State space model of power systems

generator, in p.u.,

\( E_q \) — is the electromagnetic force in the quadrature axis of the generator, in p.u.,

\( E_f \) — is the equivalent electromagnetic force in the excitation coil, in p.u.,

\( T_{d0} \) — is the direct axis transient short circuit time constant, in seconds,

\( I_f \) — is the excitation current,

\( I_q \) — is the quadrature axis current,

\( k_c \) — is the gain of the excitation amplifier,

\( u_f \) — is the input of the SCR amplifier of the generator,

\( x_{ad} \) — is the mutual reactance between the excitation coil and the stator coil of the generator,

\( V_s \) — is the infinite bus voltage.

And

\[
\begin{align*}
x_{ds} &= x_T + x_d + 0.5x_L \\
x'_{ds} &= x_T + x'_d + 0.5x_L \\
x_s &= x_T + 0.5x_L
\end{align*}
\]  

(2.13)  

(2.14)  

(2.15)

where, \( x_L \) is the transmission line reactance, \( x_T \) is the transformer reactance, \( x_d \) is the direct axis reactance of the generator, and \( x'_d \) is the direct
2.3 Control methods

axis transient reactance of the generator. [15]

Considering above equations, the state space equations of the power system can be expressed as [16] – [28]:

\[
\begin{align*}
\dot{\theta} &= \omega - \omega_0 \\
\dot{\omega} &= \frac{\omega_0}{M} P_m - \frac{D}{M} (\omega - \omega_0) - \frac{\omega_0}{M} \frac{V_s}{x_{ds}'} E_{q'} \sin \theta \\
\dot{E}_{q'} &= -\frac{1}{T_{d0}} E_{q'} + \frac{1}{T_{d0}} \frac{x_d - x_{d}'}{x_{ds}} V_s \cos \theta + \frac{1}{T_{d0}} V_f
\end{align*}
\] (2.16)

The above state space equations of power system are the fundamant of control design in this thesis. Using the state space equations, obtain Hamiltonian expression of this power system. Discussing different control designs, apply to this Hamiltonian power system.

2.3 Control methods

2.3.1 Hamiltonian systems

William Rowan Hamilton first provided a reformulation of the Lagrangian dynamics in 1834. Hamiltonian dynamics provides a compact notation in which the concept of integrability is most naturally expressed and in which the perturbation theory can be efficiently carried out. The Hamiltonian formulation is also pivotal for the foundation of both statistical and quantum mechanics. [33], [34]
The Hamiltonian equation is obtained through the Euler-Lagrange equation, which represents the motion equation for a mechanical system:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}(q, \dot{q})\right) - \frac{\partial L}{\partial q}(q, \dot{q}) = \tau$$

(2.17)

where \( q = (q_1, ..., q_n)^2 \) are generalized configuration coordinates for the system. The Lagrangian variable \( L \) equals the difference between kinetic energy \( K \) and potential energy \( P \), Meanwhile, \( \tau \) is the vector of generalized forces acting on the system. [1] – [3]

According to the kinetic energy equation, in a mechanical system, the kinetic energy \( K \) can be represented as:

$$K(q, \dot{q}) = \frac{1}{2} \dot{q}^T M(q) \dot{q}$$

(2.18)

where the \( n \times n \) matrix \( M(q) \) is the mass matrix, and this matrix is symmetric and positive definite for all \( q \). [1] – [3]

The generalized momenta \( p \) is defined as:

$$p = M(q) \dot{q}$$

(2.19)
2.3.1 Hamiltonian systems

Transforming equation (2.17) into a $2n$ first-order equation, the following is obtained:

\[
\dot{q} = \frac{\partial H}{\partial p}(q, p) \quad (2.20)
\]

\[
\dot{p} = -\frac{\partial H}{\partial q}(q, p) + \tau \quad (2.21)
\]

where

\[
H(q, p) = \frac{1}{2}p^T M^{-1}(q)p + P(q) = \frac{1}{2}\dot{q}^T M(q)\dot{q} + P(q) \quad (2.22)
\]

is the total energy of the system. Equations (2.20) and (2.21) are called the Hamiltonian equations of motion, equation (2.22) is called the Hamiltonian function. [1] – [3]

The partial derivative of this Hamiltonian function (2.23) $\frac{d}{dt} H$ gives expression to the conservation of system energy, in that the increase in system energy is equal to the supplied work. [1] – [3]
2.3.1 Hamiltonian systems

\[
\frac{d}{dt}H = \frac{\partial T H}{\partial q}(q, p) \dot{q} + \frac{\partial T H}{\partial p}(q, p) \dot{p} \tag{2.23}
\]

\[
= \frac{\partial T H}{\partial p}(q, p) \tau \tag{2.24}
\]

\[
= \dot{q}^T \tau \tag{2.25}
\]

Considering Hamiltonian equations (2.20) and (2.21) to be fully actuated Euler-Lagrange equations in \( k \) configuration coordinates \( q = (q_1, q_2, \ldots, q_k) \), the following equations can be obtained [1] – [3]:

\[
\dot{q} = \frac{\partial H}{\partial p}(q, p) \tag{2.26}
\]

\[
\dot{p} = -\frac{\partial H}{\partial q}(q, p) + u \tag{2.27}
\]

\[
y = \frac{\partial H}{\partial p}(q, p)(= \dot{q}) \tag{2.28}
\]

where \( p = (p_1, p_2, \ldots, p_k) \), \( u = (u_1, u_2, \ldots, u_k) \), and \( y = (y_1, y_2, \ldots, y_k) \).

The trajectory of the above Hamiltonian equations is [1] – [3]:

\[
H(q(t_1), p(t_1)) = H(q(t_0), p(t_0)) + \int_{t_0}^{t_1} u^T(t)y(t)dt \tag{2.29}
\]

Equation (2.29) denotes the system energy, which means the increased energy of system \( H \) equals the work supplied to the system.
Rewriting Hamiltonian equations (2.26), (2.27) and (2.28) into matrix form, the following are derived:

\[
\begin{align*}
\dot{x} &= J(x) \frac{\partial H}{\partial x}(x) + g(x)u, \\
y &= g^T(x) \frac{\partial H}{\partial x}(x)
\end{align*}
\] (2.30) (2.31)

where \( J(x) = -J^T(x) \) is called structure matrix which is an \( n \times n \) skew-symmetric matrix, and \( x = (x_1, x_1, \ldots, x_n) \) are local coordinates for state space manifold. And equations (2.30) and (2.31) are called port-controlled Hamiltonian system. The Hamiltonian function \( H \) also satisfies the conservation of energy as equation (2.32), which shows that the internal energy equals the externally supplied power. [1] – [3]

\[
\frac{dH}{dt}(x(t)) = u^T(t)y(t)
\] (2.32)

This port-controlled Hamiltonian (PCH) system has energy-dissipation, when some of the ports in the system are terminated by state resistive elements. The port structure \( g(x) \) and the control input \( u \) can be represented as [1] – [3]:

\[
\begin{bmatrix}
g(x) & g_R(x)
\end{bmatrix}
\begin{bmatrix}
u \\
u_R
\end{bmatrix} = g(x)u + g_R(x)u_R
\] (2.33)
Correspondingly, the output equation (2.31) is extended as follows [1] – [3]:

\[
\begin{bmatrix}
y \\
y_R
\end{bmatrix} = \begin{bmatrix}
g^T(x) \frac{\partial H}{\partial x}(x) \\
g^T_R(x) \frac{\partial H}{\partial x}(x)
\end{bmatrix}
\] (2.34)

Considering that the resistive elements which terminate the ports of the port-controlled Hamiltonian system are linear resistive elements.

\[u_R = -S(y_R)\] (2.35)

where \(S = S^T \geq 0\) is a positive semi-definite symmetric matrix. [1] – [3]

Substituting equation (2.35) into equation (2.33):

\[
\dot{x} = [J(x) - R(x)] \frac{\partial H}{\partial x}(x) + g(x)u \\
y = g^T(x) \frac{\partial H}{\partial x}(x)
\] (2.36) (2.37)

where \(R(x) = g_R(x)S g^T_R(x)\) is a positive semi-definite symmetric matrix. Equations (2.36) and (2.37) are called port-controlled Hamiltonian system with dissipation (PCHD), and \([J(x) - R(x)]\) is a structure matrix
of the system, where $J(x)$ is the internal interconnection structure, and $R(x)$ is the additional resistive structure. [1] – [3]

The increase of system energy equals the difference between the work supplied to system and the dissipated energy of system: [1] – [3]

$$\frac{dH}{dt}(x(t)) = u^T(t)y(t) - \frac{\partial^T H}{\partial x}(x(t))R(x(t))\frac{\partial H}{\partial x}(x(t))^T \leq 0$$

(2.38)

In recent years, Hamiltonian systems have undergone tremendous development. Daizhan Chen and his group have focused on Hamiltonian system research for several years, particularly its applications in power systems.

In the later stages of Hamiltonian system development, many different nonlinear control strategies associated with port controlled Hamiltonian systems with dissipation were discussed, such as adaptive stabilization of generalized Hamiltonian systems [38], energy-based $L_2$-disturbance attenuation control [23], and adaptive $H_{\infty}$ excitation control [24].

Others’ research works, such as those discussing control by interconnection of mixed port Hamiltonian systems were introduced in [2].
2.3.2 Adaptive stabilization for Hamiltonian systems

The regulation problem for mixed finite and infinite dimensional port
controlled Hamiltonian systems has also been discussed. This kind of
system is used on systems with distributed parameters [2].

2.3.2 Adaptive stabilization for Hamiltonian systems

As stated in previous section, Hamiltonian systems are devised from
Euler-Lagrange (EL) systems. In this section, adaptive stabilization of
generalized Hamiltonian system with dissipation is considered.

In actual use, generalized Hamiltonian system with dissipation is
zero-state detectable, and it has a linearly parameterized Hamiltonian
function with known and unknown constants. Divided Hamiltonian func-
tion into two parts with known and unknown constants. This Hamilto-
nian system can be expressed in the following form:

\[
\dot{x} = (J - R)(\frac{\partial L_0}{\partial x} + \sum_{i=1}^{m} \theta_i \frac{\partial L_i}{\partial x}) + g(x)u \tag{2.39}
\]

Its energy function can be expressed as:

\[
H(x) = L_0(x) + \sum_{i=1}^{m} \theta_i L_i(x) \tag{2.40}
\]

where \(u\) is the control law, \(\theta = (\theta_1, \theta_2, \ldots, \theta_m)^T\) is the unknown constant
vector, and \(L_0(x)\) and \(L_i(x)\) are the known differentiable functions. [38],

[2]: Reference to a specific work.
[38]: Reference to another work.
Taking

\[ V(x, \hat{\theta}) = L_0(x) + \sum_{i=1}^{m} \theta_i L_i(x) + \frac{1}{2} \dot{\hat{\theta}}^T \Gamma^{-1} \dot{\hat{\theta}} \]  \hfill (2.41)

as a Lyapunov function, where \( \tilde{\theta} = \hat{\theta} - \theta \), we can obtain:

\[
\dot{V} = -\left( \frac{\partial L_0}{\partial x} + \sum_{i=1}^{m} \theta_i \frac{\partial L_i}{\partial x} \right)^T \Gamma^{-1} \Gamma^{-1} \left( \frac{\partial L_0}{\partial x} + \sum_{i=1}^{m} \theta_i \frac{\partial L_i}{\partial x} \right) \\
- \left( \frac{\partial L_0}{\partial x} + \sum_{i=1}^{m} \hat{\theta}_i \frac{\partial L_i}{\partial x} \right)^T \Gamma^{-1} \left( \frac{\partial L_0}{\partial x} + \sum_{i=1}^{m} \hat{\theta}_i \frac{\partial L_i}{\partial x} \right)
\]

\[
\leq 0 \hfill (2.42)
\]

This means that the closed-loop Hamiltonian system is bounded, and \( \Gamma \) and \( P \) are two appropriate dimension positive definite matrices. The system is asymptotically stable when

\[
A = \{(x, \hat{\theta}) : R \left( \frac{\partial L_0}{\partial x} + \sum_{i=1}^{m} \theta_i \frac{\partial L_i}{\partial x} \right) = 0, \quad g^T \left( \frac{\partial L_0}{\partial x} + \sum_{i=1}^{m} \hat{\theta}_i \frac{\partial L_i}{\partial x} \right) = 0 \} \hfill (2.43)
\]

Then the adaptive control law can be designed as: [38], [39]
This method uses an adaptive controller to design a class of generalized Hamiltonian systems with dissipation. These Hamiltonian systems usually have linearly parameterized Hamiltonian equations with both known and unknown constants.

2.3.3 EB-based control for Hamiltonian systems

Energy-balancing control requires determination of a certain function, that is rendered by the difference between control object and designed controller.\[37\]

An energy-balancing system is satisfied by a kind of Euler-Lagrange system, which the total closed-loop energy is the difference between the open-loop energy and the supplied energy. The Hamiltonian function denotes the total energy of the system, and satisfies the energy balancing equation. This method is based on the matching condition that is suitable for shaping potential energy only.\[40\]
2.3.3 EB-based control for Hamiltonian systems

The passivity-based control (PBC) is a kind of energy-balancing control, it is a well-established technique that is very effective designing controller for systems that are also described by Euler-Lagrange equations of motion. To solve the regulation problem of mechanical systems, using passivity-based control design, only potential energy must be stabilized by "shaping". Passivity-based control also keeps system in the Euler-Lagrange form, to obtain a closed-loop energy function. This energy function equals the difference between the energy in the system and the energy supplied by the controller. Hence, stabilization of the system can be explained in terms of energy-balancing, which means that in a passivity-based controlled system, when the system runs toward equilibrium point, dissipation of the system becomes zero.\[43], [37]

However, in some cases, shaping of total energy is required, and modification of the kinetic energy is necessary. Closed-loop system no longer satisfies the Euler-Lagrange structure, and energy function no longer represents the total energy. Interconnection and damping assignment passivity-based control (IDA-PBC) is concerned. IDA-PBC is satisfied by energy-balancing, and it may be used to solve the problem of stabilization of under-actuated mechanical systems.

In general, IDA-PBC is used to design Hamiltonian system, which aims to find a state-feedback control $u = \beta(x)$, so that the closed-loop
dynamics system becomes port-controlled Hamiltonian system with dissipation, such as in the following formula: \cite{43, 41}

\[
\dot{x} = \left[J_d(x) - R_d(x)\right] \frac{\partial H_d(x)}{\partial x} \tag{2.45}
\]

where the new energy equation $H_d(x)$ has a strict local minimum at the desired equilibrium $x^*_d$, $J_d(x) = -J_d^T(x)$ is the desired interconnection matrix and $R_d(x) = R_d^T(x) \geq 0$ is the desired damping matrix. \cite{41}

Applying IDA-PBC to a port-controlled Hamiltonian system, and enables this Hamiltonian system to satisfy the following energy-balancing equation:

\[
H[x(t)] - H[x(0)] = \int_0^t u^T(s)y(s)ds - d(t) \tag{2.46}
\]

where $H[x(t)] - H[x(0)]$ is the stored energy, $x \in \mathbb{R}^n$ is the state vector, $\int_0^t u^T(s)y(s)ds$ is the energy supplied by the system input, and $d$ is the non-negative dissipation energy. \cite{43, 44}

Choosing a function $\beta(x)$ satisfies:

\[
-\int_0^t \beta^T[x(s)]y(s)ds = H_a[x(t)] - \kappa \tag{2.47}
\]
where $H_a$ is the closed-loop energy, which is equal to the difference between stored energy and supplied energy, $\kappa$ is constant of system. [43], [44]

The control law $u = \beta(x) + v$ ensures that the system is passive by a new total energy equation as follows:

$$H_d(x) = H(x) + H_a(x)$$  \hspace{1cm} (2.48)

This new energy function still satisfies:

$$H_d[x(t)] - H_d[x(0)] = \int_0^t v^T(s)y(s)ds - d_d(t)$$  \hspace{1cm} (2.49)

For Hamiltonian systems:

$$\dot{x} = f(x) + g(x)u$$  \hspace{1cm} (2.50)

$$y = g^T(x)\frac{\partial H}{\partial x}$$  \hspace{1cm} (2.51)

a function $\beta(x)$ can be found, a new designed Hamiltonian system can be obtained:
2.3.3 EB-based control for Hamiltonian systems

\[\dot{x} = f_d(x) + g(x)v \tag{2.52}\]
\[z = g^T(x) \frac{\partial H_d}{\partial x} \tag{2.53}\]

where \(f_d(x) = f(x) + g(x)\beta(x)\), and \(H_d(x) = H(x) + H_a(x)\) is the new energy function of the closed-loop system, and \(z\) is the output of the new system. From equations (2.50) to (2.53),

\[\frac{\partial H_a^T(x)}{\partial x} g(x) = (z - y)^T \tag{2.54}\]

Combining equations (2.54) and (2.49), gives:

\[\frac{\partial H_a^T(x)}{\partial x} f_d(x) = -\beta^T y \tag{2.55}\]

Multiplying equation (2.54) with input \(v\):

\[\frac{\partial H_a^T(x)}{\partial x} g(x)v = (z - y)^T v \tag{2.56}\]

Combining equations (2.54) and (2.56),

\[\frac{\partial H_a^T(x)}{\partial x} [f_d(x) + g(x)v] = -\beta^T y + v^T (z - y) \tag{2.57}\]
Integrating both sides of equation (2.57), the following is obtained:

\[- \int_0^t \beta^T[x(s)]y(s)ds + \int_0^t v^T(z - y)ds = H_a[x(t)] - \kappa \quad (2.58)\]

Equation (2.58) shows that the difference between stored and supplied energies is no longer the closed-loop energy. The system is passive with the Hamiltonian function \(H_d(x)\). Thus, the control law can be derived as:

\[u = \beta(x) + v \quad (2.59)\]

This theory is based on generalized canonical transformations of time-varying port controlled Hamiltonian systems, and the designed controller is based on only the storage equation which is a function of the tracking error. \([43], [44]\)

In this chapter, firstly, analyze power systems, then, according to the structure and character of power systems, build state space equations. The state space equations are the foundations of control design in this thesis. These first two parts are reviews of control object, the last part of this chapter introduces control methods for power system, which are
2.3.3 EB-based control for Hamiltonian systems

Hamiltonian system, adaptive and energy-balancing-based control designs based on Hamiltonian system.
Chapter 3

Adaptive Control for Hamiltonian Systems

As introduced in previous chapter, adaptive control is used to adapt the unknown parameters in energy function. In this chapter, adaptive control design for Hamiltonian systems is discussed.

3.1 Adaptive control design for Hamiltonian systems

Taking a basic Hamiltonian system as an example, which is given by:

\[
\dot{x} = J(x) \frac{\partial H}{\partial x}(x) + g(x)u \\
y = g^T(x) \frac{\partial H}{\partial x}(x)
\]  

(3.1)  

(3.2)

According to adaptive control method, Hamiltonian function of above system can be divided into two differentiable terms in the following
3.1 Adaptive control design for Hamiltonian systems

form:

\[ H(x) = L_0(x) + \sum_{i=1}^{m} \theta_i L_i(x) \]  \hspace{1cm} (3.3)

where \( \theta = (\theta_1, \theta_2, \ldots, \theta_m)^T \) is the unknown constant vector, and \( L_0(x) \) and \( L_i(x) \) are the known differentiable functions. \[35\]

The original Hamiltonian control system with dissipation can now be expressed as the following form:

\[
\dot{x} = J(x)\left( \frac{\partial L_0}{\partial x} + \sum_{i=1}^{m} \theta_i \frac{\partial L_i}{\partial x} \right) + g(x)u
\]  \hspace{1cm} (3.4)

In this case, Hamiltonian system as equations (3.1) and (3.2) can be rewritten as:

\[
\dot{x} = J(x)\left( \frac{\partial L_0}{\partial x}(x) + \sum_{i=1}^{m} \theta_i \frac{\partial L_i}{\partial x}(x) \right) + g(x)u
\]  \hspace{1cm} (3.5)

\[
y = g^T(x)\left( \frac{\partial L_0}{\partial x}(x) + \sum_{i=1}^{m} \theta_i \frac{\partial L_i}{\partial x}(x) \right)
\]  \hspace{1cm} (3.6)

and the control law is expressed as: \[35\], \[38\], \[39\]
Taking $V(x, \hat{\theta})$ as Lyapunov function, like equations (2.41) and (2.42), the stability of above adaptive controlled system can be proved.

### 3.2 Simulation result for an adaptive control design

The LC circuit shown in Figure 3.1 consists of two inductors and a capacitor was chosen for the Hamiltonian system. In this circuit, $V$ is the voltage source, $\psi_1$ and $\psi_2$ are the magnetic flux linkages of inductors, $H_1(\psi_1)$ and $H_2(\psi_2)$ are the magnetic energies of two inductors, $Q$ is the charge of capacitor, and $H_3(Q)$ is the electric energy of the capacitor. Considering the linear relationship of the elements above:

\[
H_1(\psi_1) = \frac{1}{2L_1} \psi_1^2, \\
H_2(\psi_2) = \frac{1}{2L_2} \psi_2^2, \\
H_3(Q) = \frac{1}{2C} Q^2
\]  

The total energy of this LC circuit is $H(\psi_1, \psi_2, Q) = H_1(\psi_1) + H_2(\psi_2) + H_3(Q)$. The dynamic equation of the system can be obtained
3.2 Simulation result for an adaptive control design

Figure 3.1: LC circuit

as:

\[
\begin{bmatrix}
\dot{\psi}_1 \\
\dot{\psi}_2 \\
\dot{Q}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & -1 \\
0 & 0 & 1 \\
1 & -1 & 0
\end{bmatrix}
\begin{bmatrix}
\frac{\partial H}{\partial \psi_1} \\
\frac{\partial H}{\partial \psi_2} \\
\frac{\partial H}{\partial Q}
\end{bmatrix} +
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} u \quad (3.9)
\]

Rewriting this dynamic equation \((3.9)\) as a port-controlled Hamiltonian system:
In this case, adaptive control is used to design this Hamiltonian system of LC circuit. Let $a = \frac{1}{C}$, $b = \frac{1}{L_1}$, $c = \frac{1}{L_2}$.

Using adaptive control design, above Hamiltonian system can be represented in the following form:
3.2 Simulation result for an adaptive control design

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & -1 \\
-1 & 0 & 0 \\
1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\hat{a}} \\
0 \\
0
\end{bmatrix} + \begin{bmatrix}
\hat{b} \\
x_2 \\
0
\end{bmatrix}
\]
\[+ \hat{c} \begin{bmatrix}
0 \\
0 \\
x_3
\end{bmatrix} + \begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix} u
\]

(3.13)

Let \( \theta = [a, b, c] \) and \( \hat{\theta} = [\hat{a}, \hat{b}, \hat{c}] \), the adaptive control law can be obtained from the following equation:

\[
\frac{\partial \hat{\theta}}{\partial t} = -k \begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} \begin{bmatrix}
\hat{a}x_1 + \hat{b}x_2 + \hat{c}x_3
\end{bmatrix}
\]

(3.14)

\[
= -k \begin{bmatrix}
x_1^2 & x_1x_2 & x_1x_3 \\
x_1x_2 & x_2^2 & x_2x_3 \\
x_1x_3 & x_2x_3 & x_3^2
\end{bmatrix} \begin{bmatrix}
\hat{a} \\
\hat{b} \\
\hat{c}
\end{bmatrix}
\]

\[
u = -k[\hat{a}x_1 + \hat{b}x_2 + \hat{c}x_3]
\]

(3.15)

The values of this LC circuit are used for both the port-controlled Hamiltonian system for LC circuit and the adaptive controlled Hamiltonian system. The simulation results are shown in Figure 3.2 and Figure
3.2 Simulation result for an adaptive control design

Figure 3.2: Simulation result of Hamiltonian system for LC circuit

Figure 3.3: Simulation result of adaptive control for Hamiltonian model of LC circuit
3.2 Simulation result for an adaptive control design

3.3 Figure 3.2 is the simulation result of Hamiltonian system for LC circuit, and Figure 3.3 is the performance of adaptive control design for above basic Hamiltonian system.

In Figure 3.2 and 3.3, $Q$ is the capacitor, $\psi_1$ and $\psi_2$ are the inductors, respectively. $\hat{a}$, $\hat{b}$, and $\hat{c}$ in Figure 3.3 are the adaptive parameters of the adaptive control system.

LC circuit is an oscillation circuit, in Figure 3.2, without dissipation, parameters keep persistent oscillation. Using adaptive control design for this Hamiltonian expression of LC circuit, in Figure 3.3, parameters can be stable.

In this chapter, adaptive control is applied to port-controlled Hamiltonian system, as an example, LC circuit is expressed as Hamiltonian system. Compared the simulation results of Hamiltonian system and adaptive controlled Hamiltonian system, the stability in adaptive control design is better than that in Hamiltonian system.
Chapter 4

EB-Based Control for Hamiltonian Systems

As discussed in Chapter 2, IDA-PBC is a kind of energy-balancing-based control that can be used to design the Euler-Lagrange system, and from where the Hamiltonian equation is derived. In this section, IDA-PBC is also applicable to port-controlled Hamiltonian systems.

4.1 EB-based design for the Hamiltonian systems

Take the port-controlled Hamiltonian system with dissipation as system $\sum$, which is expressed as:

$$\dot{x} = [J(x) - R(x)]\frac{\partial H}{\partial x}(x) + g(x)u \quad (4.1)$$
$$y = g^T(x)\frac{\partial H}{\partial x}(x) \quad (4.2)$$
The EB-based control design is expressed in $f_d(x) = f(x) + g(x)\beta(x)$, and, the new system $\sum_d$ can be obtained as: \[40\] – \[44\]

\[
\begin{align*}
\dot{x} &= [J(x) - R(x)] \frac{\partial H_d}{\partial x}(x) + g(x)\beta(x) + g(x)u \\
y &= g^T(x) \frac{\partial H_d}{\partial x}(x)
\end{align*}
\] (4.3)

(4.4)

Taking this energy function $H_d$ as Lyapunov function, the stability of designed Hamiltonian system $\sum_d$ can be proved.

### 4.2 Simulation result for an EB-based design

In this section, EB-based control design requires a port-controlled Hamiltonian system with dissipation (PCHD). Take the Hamiltonian system for LC circuit from last chapter as an example. Considering dissipation, a port-controlled Hamiltonian system with dissipation for LC circuit is obtained as follows:
4.2 Simulation result for an EB-based design

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\end{bmatrix}
= \begin{bmatrix}
0 & 1 & 0 \\
-1 & -1 & 0 \\
0 & 0 & -1 \\
\end{bmatrix}
\begin{bmatrix}
ax_1 \\
bx_2 \\
cx_3 \\
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
1 \\
\end{bmatrix} u 
\] 

(4.5)

\[
y = \begin{bmatrix}
0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
ax_1 \\
bx_2 \\
cx_3 \\
\end{bmatrix} 
\] 

(4.6)

where, Hamiltonian function is \( H(x) = \frac{1}{2} ax_1^2 + \frac{1}{2} bx_2^2 + \frac{1}{2} cx_3^2 \), substituting it as Lyapunov function.

\[
\frac{dH}{dt}(x) = ax_1 \dot{x}_1 + bx_2 \dot{x}_2 + cx_3 \dot{x}_3 \\
= ax_1 bx_2 + bx_2(-ax_1 - bx_2) + cx_3(-cx_3 + u) \\
= -b^2 x_2^2 - c^2 x_3^2 + cx_3 u \\
\leq yu 
\] 

(4.7)

In this case, the Hamiltonian system is asymptotically stable. The new system which is given by EB-based control design can be expressed as:
4.2 Simulation result for an EB-based design

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
-1 & -1 & 0 \\
0 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
ax_1 \\
0 \\
0
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
bx_2 \\
2cx_3
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
\]

(4.8)

\[
y = 
\begin{bmatrix}
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
ax_1 \\
bx_2 \\
2cx_3
\end{bmatrix}
\]

(4.9)

From the above Hamiltonian system, a new Hamiltonian function can be obtained as 
\(H_d(x) = \frac{1}{2}ax_1^2 + \frac{1}{2}bx_2^2 + cx_3^2\), and substituting this new function as the Lyapunov function:

\[
\frac{dH_d}{dt}(x) = ax_1\dot{x}_1 + bx_2\dot{x}_2 + 2cx_3\dot{x}_3
\]

\[
= ax_1bx_2 + bx_2(-ax_1 - bx_2) + 2cx_3(-cx_3 + u)
\]

\[
= -b^2x_2^2 - 2c^2x_3^2 + 2cx_3u
\]

\[
\leq yu
\]

(4.10)

This new system is also asymptotically stable, and the control law is:

\[
u = -2cx_3
\]

(4.11)
4.2 Simulation result for an EB-based design

To compare the results, both the PCHD for LC circuit and the EB-based controlled Hamiltonian system were simulated by using the same parameters’ values.

Figure 4.1 shows the performance of PCHD, and the simulation result of EB-based control design for above PCHD is shown in Figure 4.2.

In Figure 4.1 and 4.2, results essentially have no difference except in parameter $x_3$. In EB-based control design for PCHD, $x_3$ is faster to get into stable state.
In this section, EB-based control design is used to port-controlled Hamiltonian system with dissipation. The PCHD is achieved from LC circuit which is discussed in previous chapter. Compared the simulation results of PCHD and EB-based controlled PCHD, EB-based control design is more effective on reducing damping.
Chapter 5

Adaptive and EB-Based Control for Hamiltonian Systems

Adaptive control design can solve the problem of unknown constant parameters, and EB-based control design enhances the stability of the Hamiltonian system. To obtain a better running result of Hamiltonian systems, adaptive control and the energy-balancing-based control are combined.

5.1 Adaptive and EB-based control for Hamiltonian systems

From EB-based controlled system in the previous chapter, using the PCHD as the foundation of control design, the system with EB-based controller can be obtained as equation (5.1):
5.1 Adaptive and EB-based control for Hamiltonian systems

\[ \begin{align*}
\dot{x}_1 &= bx_2 \\
\dot{x}_2 &= -ax_1 - bx_2 \\
\dot{x}_3 &= -2cx_3 + u
\end{align*} \tag{5.1} \]

Using adaptive control to design the Hamiltonian system which is expressed as equation (5.1), the new function with adaptive control parameters \([\hat{a}, \hat{b}, \hat{c}]\) can be obtained as:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix}
= \begin{bmatrix}
0 & 1 & 0 \\
-1 & -1 & 0 \\
0 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
\hat{a}x_1 \\
\hat{b}x_2 \\
2\hat{c}x_3
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} u
\]

\[
= \begin{bmatrix}
0 & 1 & 0 \\
-1 & -1 & 0 \\
0 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
0 \\
0
\end{bmatrix}
+ \hat{a} \begin{bmatrix}
x_1 \\
0 \\
0
\end{bmatrix}
+ \hat{b} \begin{bmatrix}
x_2 \\
0 \\
0
\end{bmatrix}
+ \hat{c} \begin{bmatrix}
x_3 \\
0 \\
2x_3
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} u \tag{5.2}
\]

Let \(\hat{\theta} = [\hat{a}, \hat{b}, \hat{c}]\):
\[
\frac{d\hat{\theta}}{dt} = -\Gamma \frac{\partial L}{\partial x} gPg^T \left[ \frac{\partial L_0}{\partial x} + \left( \frac{\partial \tilde{L}}{\partial x} \right) \hat{\theta} \right] = -k \begin{bmatrix}
\hat{a}x_1^2 + \hat{b}x_1x_2 + 2\hat{c}x_1x_3 \\
\hat{a}x_1x_2 + \hat{b}x_2^2 + 2\hat{c}x_2x_3 \\
2\hat{a}x_1x_3 + 2\hat{b}x_2x_3 + 4\hat{c}x_3^2
\end{bmatrix} \text{(5.3)}
\]

The adaptive control law can be obtained as:

\[
u = -Pg^T \left[ \frac{\partial L_0}{\partial x} + \sum_{i=1}^{m} \theta_i \frac{\partial L_i}{\partial x} \right] = -k \begin{bmatrix}
\hat{a}x_1 + \hat{b}x_2 + 2\hat{c}x_3
\end{bmatrix} \text{(5.4)}
\]

Taking

\[
V(x, \tilde{\theta}) = \sum_{i=1}^{m} \theta_i L_i(x) + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \text{ (5.5)}
\]

as a Lyapunov function, where \( \tilde{\theta} = \hat{\theta} - \theta \), we can obtain:
\[ \dot{V} = -\left( \sum_{i=1}^{m} \theta_i \frac{\partial L_i}{\partial x} \right)^T R \sum_{i=1}^{m} \theta_i \frac{\partial L_i}{\partial x} - \left( \sum_{i=1}^{m} \theta_i \frac{\partial L_i}{\partial x} \right)^T g P g^T \sum_{i=1}^{m} \hat{\theta}_i \frac{\partial L_i}{\partial x} \leq 0 \] (5.6)

The stability of this adaptive and EB-based controlled Hamiltonian system can be proved.

### 5.2 Simulation result of adaptive and EB-based control design

Under the same operating conditions with last chapter, the simulation results of adaptive and energy-balancing-based control design for PCHD system is given by Figure 5.1.

In Figure 5.1, \( x_1, x_2, x_3 \) are control parameters in Hamiltonian system, and \( \hat{a}, \hat{b}, \hat{c} \) are adaptive control parameters of system.

Compared Figure 5.1 with Figure 4.1, Figure 4.1 is the simulation result of port-controlled Hamiltonian system with dissipation (PCHD), all parameters in adaptive and energy-balancing-based control are quicker to get into stable state.
In this chapter, adaptive and EB-based control designs are compounded together. Use this combination to design PCHD which is mentioned in previous chapter. This control method has the advantages of both adaptive and EB-based control designs.
Chapter 6

Control Designs for Power systems

In previous chapters, adaptive and EB-based control designs were discussed by applying to Hamiltonian systems for LC circuit. However, the present thesis focuses on the improved performance of power systems. In this chapter, all control methods which were discussed in Chapter 3 through Chapter 5 apply to same power system and the simulation results under the same running condition are compared.

6.1 The Hamiltonian system for power systems

The state space equation of power systems can be described via electrical equation and mechanical equations in the following form [8], [23] – [28]:

\[
8, 23, 28:
\]
6.1 The Hamiltonian system for power systems

\[
\begin{align*}
\dot{\theta} &= \omega - \omega_0 \quad (6.1) \\
\dot{\omega} &= \frac{\omega_0}{M} P_m - \frac{D}{M} (\omega - \omega_0) - \frac{\omega_0}{M} \frac{V_s}{x_{ds}} E_q' \sin \theta \quad (6.2) \\
\dot{E}_q' &= -\frac{1}{T_{d0}} E_q' + \frac{1}{T_{d0}} \frac{x_d - x_{d}'}{x_{ds}} V_s \cos \theta + \frac{1}{T_{d0}} V_f \quad (6.3)
\end{align*}
\]

where

\( \theta \) is the rotor angle,

\( \omega \) is the rotor speed,

\( E_q' \) is the internal transient voltage,

\( P_m \) is the mechanical power,

\( M \) is the inertia coefficient of the generator,

\( D \) is the damping constant,

\( P_e \) is equal to \( E_q' V_s / x_{ds} \sin \theta \), is the active electrical power,

\( T_{d} \) is the stator closed loop time constant,

\( T_{d0} \) is the excitation circuit time constant,

\( x_d \) is the \( d \)-axis synchronous reactance of a generator,

\( x_{d}' \) is the \( d \)-axis transient reactance,

\( V_f \) is the voltage of the field circuit of a generator.

Consider \( u = \frac{1}{T_{d0}} V_f \) as the control law, set \( x_1 = \theta \), \( x_2 = \omega - \omega_0 \), \( x_3 = E_q' \) as the state, and denote \( a = \frac{\omega_0}{M} \), \( b = \frac{D}{M} \), \( c = \frac{\omega_0 V_s}{M} \), \( e_1 = \frac{M}{T_{d0} \omega_0} \), these are classified as known constants, because they represent the phys-
The Hamiltonian system for power systems

ica parameters of the power system. Since some constants are concerned with network, let \( f = \frac{x_d - x_d'}{x_{ds}} \), \( d = \frac{1}{T_d} \), \( P_m \) and \( e_2 = x_d - x_d' \), these are unknown constants. [8], [23] – [28]

The state space equations of above power system can be rewritten as:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 
\end{bmatrix} =
\begin{bmatrix}
o & \frac{1}{e_2} & 0 \\
-\frac{1}{e_2} & -\frac{b}{e_2} & 0 \\
0 & 0 & -e_1 
\end{bmatrix}
\begin{bmatrix}
cf x_3 \sin x_1 - ae_2 P_m \\
e_2 x_2 \\
d \frac{x_3}{e_1} - cf \cos x_1 
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
1 
\end{bmatrix} u
\]

(6.4)

By choosing energy function of the Hamiltonian system as:

\[
H(x) = \frac{1}{2} e_2 x_2^2 - cf x_3 \cos x_1 + \frac{d}{2e_1} x_3^2 - ae_2 P_m x_1
\]

(6.5)

the structure matrix \( M \) is obtained:

\[
M = \begin{bmatrix}
o & \frac{1}{e_2} & 0 \\
-\frac{1}{e_2} & -\frac{b}{e_2} & 0 \\
0 & 0 & -e_1 
\end{bmatrix}
\]

(6.6)
where \(-(M + M^T)\) is semi-positive, and \(M = J - R\), \(J\) is a skew-symmetric matrix which is the internal interconnection structure, and \(R\) is a positive semi-definite symmetric matrix which is the additional resistive structure. \[8\], \[23\] – \[28\]

Thus, this power system can be represented by PCHD in the following form:

\[
\begin{align*}
\dot{x} &= [J(x) - R(x)] \frac{\partial H}{\partial x} + g(x)u \\
y &= g^T(x) \frac{\partial H}{\partial x}
\end{align*}
\]  
\tag{6.7}

where,

\[
M = \begin{bmatrix}
o & \frac{1}{e_2} & 0 \\
-\frac{1}{e_2} & \frac{b}{e_2} & 0 \\
0 & 0 & -e_1
\end{bmatrix}
\]  
\tag{6.8}

\[
J = \frac{M - M^T}{2}, R = \frac{M + M^T}{2}
\]  
\tag{6.9}

\[
H(x) = \frac{1}{2} e_2 x_2^2 - c f x_3 \cos x_1 + \frac{d}{2e_1} x_3^2 - a e_2 P_m x_1
\]  
\tag{6.10}
6.1 The Hamiltonian system for power systems

\[
\frac{\partial H(x)}{\partial x} = \begin{bmatrix}
    cfx_3 \sin x_1 - ae_2 P_m \\
    e_2 x_2 \\
    d - c \cos x_1
\end{bmatrix}
\]

(6.11)

\[
H(x) = \frac{1}{2} e_2 x_2^2 - cfx_3 \cos x_1 + \frac{d}{2e_1} x_3^2 - ae_2 P_m x_1
\]

\[
= \frac{1}{2} e_2 x_2^2 + \frac{d}{2e_1} [x_3 - \frac{ce_1 f}{d} \cos x_1]^2
\]

\[
- ae_2 P_m x_1 - \frac{c^2 e_1 f^2}{2d} (\cos x_1)^2
\]

(6.12)

where \( H \) is the Hamiltonian function which is the total energy of system. When \( x_1 \in [-\pi, \pi] \), this Hamiltonian function has a minimum value. [8], [23] – [28]

This Hamiltonian function can be substituted as the Lyapunov function,

\[
\frac{dH(x)}{dt} = 2 \frac{1}{2} e_2 x_2 \dot{x}_2 - ae_2 P_m \dot{x}_1 - cf \cos x_1 \dot{x}_3
\]

\[
+ cf x_3 \sin x_1 \dot{x}_1 + 2 \frac{d}{2e_1} x_3 \dot{x}_3
\]

\[
= -be_2 x_2^2 - e_1 [cf \cos x_1 - \frac{d}{e_1} x_3]^2 + \left[ \frac{d}{e_1} x_3 - cf \cos x_1 \right] u
\]

\[
\leq y u
\]

(6.13)

Hamiltonian system is stable at its equilibrium point. The derivative of this Hamiltonian function has a maximum value, and PCHD for power
system is asymptotically stable.

The condition signifies that the increased energy of system is equal to the difference between the work supplied to system and the dissipated energy of system.

After ensuring the stability of the Hamiltonian systems, simulation was done, based on other power system researches, using the following parameters values: $D = 5$, $M = 6$, $\omega_0 = 314$, $V_s = 1$, $x_{ds} = 0.36$, $P_m = 0.9$, $T_d = 5$, $T_{d0} = 7.4$, $x_d = 1.875$, $x_d' = 0.257$, $x_L = 0.04$. [8]

The original PCHD is given as:

\[
\begin{align*}
\dot{\theta} &= \omega - \omega_0 \quad (6.14) \\
\dot{\omega} &= \frac{\omega_0}{M} P_m - \frac{D}{M} (\omega - \omega_0) - \frac{\omega_0}{M} \frac{V_s}{x_{ds}} E_q' \sin \theta \quad (6.15) \\
\dot{E}_q' &= -\frac{1}{T_{d0}} E_q' + \frac{1}{T_{d0}} \frac{x_d - x_d'}{x_{ds}} V_s \cos \theta + \frac{1}{T_{d0}} V_f \quad (6.16)
\end{align*}
\]

Substituting the parameters into above Hamiltonian system results in:
6.1 The Hamiltonian system for power systems

Figure 6.1: Simulation result of Hamiltonian system for power system

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} = 
\begin{bmatrix}
0 & \frac{1}{e_2} & 0 \\
-\frac{1}{e_2} & -b & 0 \\
0 & 0 & -e_1
\end{bmatrix}
\begin{bmatrix}
c_3 x_3 \sin x_1 - a e_2 P_m \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
e_2 x_2 \\
\frac{d}{e_1} x_3 - c f \cos x_1
\end{bmatrix} + 
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} u
\] (6.17)

Figure 6.1 shows the operating result of the original Hamiltonian system at equilibrium point \([x_1 = 6.2, x_2 = 0, x_3 = 3]\).

As shown in Figure 6.1, \(x_1\) is the power angle of the generator, \(x_2\) is
6.2 Adaptive control design for the Hamiltonian system

the relative speed of the generator, and $x_3$ acts as the transient electromagnetic force in the quadrature axis of the generator.

6.2 Adaptive control design for the Hamiltonian system

Using the same parameter value of system and the coefficient $k = 0.05$, the following result of the adaptive control for Hamiltonian system can be derived:

$$
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\end{bmatrix} = 
\begin{bmatrix}
o & \frac{1}{e_2} & 0 \\
-\frac{1}{e_2} & -\frac{b}{e_2} & 0 \\
0 & 0 & -e_1 \\
\end{bmatrix}
\begin{bmatrix}
-c e_2 P_m \\
e_2 x_2 \\
0 \\
\end{bmatrix} + \dot{f} 
\begin{bmatrix}
c x_3 \sin x_1 \\
0 \\
-c \cos x_1 \\
\end{bmatrix}
$$

(6.18)

and the control law is expressed as:

$$
u = k \left( c \dot{f} \cos x_1 - \frac{\dot{d} x_3}{e_1} \right)
$$

(6.20)
6.2 Adaptive control design for the Hamiltonian system

\[ \dot{V} = -\left( \frac{\partial L_0}{\partial x} + f \frac{\partial L_1}{\partial x} + d \frac{\partial L_2}{\partial x} \right)^T R \left( \frac{\partial L_0}{\partial x} + f \frac{\partial L_1}{\partial x} + d \frac{\partial L_2}{\partial x} \right) 
\] 
\[ - \left( \frac{\partial L_0}{\partial x} + \hat{f} \frac{\partial L_1}{\partial x} + \hat{d} \frac{\partial L_2}{\partial x} \right)^T g P g^T \left( \frac{\partial L_0}{\partial x} + \hat{f} \frac{\partial L_1}{\partial x} + \hat{d} \frac{\partial L_2}{\partial x} \right) \]

\[ \leq 0 \]

Taking \( V(x, \hat{\theta}) \) as Lyapunov function, where \( \hat{\theta} = [\hat{f}, \hat{d}] \) is the adaptive control vector, system is asymptotically stable at equilibrium point.

Figure 6.2 shows the operation of the adaptive control for the Hamiltonian system at equilibrium point \([x_1 = 6.2, x_2 = 0, x_3 = 3, \hat{f} = 5.4, \hat{d} = 1]\).

As shown in Figure 6.2, \( x_1 \) is the power angle of the generator, \( x_2 \) is the relative speed of the generator, \( x_3 \) acts as the transient electromagnetic force in the quadrature axis of the generator, and \( \hat{f}, \hat{d} \) are adaptive control parameters of the system.

Compared Figure 6.2 with Figure 6.1, in adaptive control design for Hamiltonian system, \( x_1, \text{ and } x_3 \) could be stable faster. The adaptive control system achieves better performance than PCHD.
6.3 EB-based control design for the Hamiltonian system

Applying the same condition of the Hamiltonian system, in EB-based control, the system can be expressed as:

![Simulation result of adaptive control for Hamiltonian system](image)

Figure 6.2: Simulation result of adaptive control for Hamiltonian system
6.3 EB-based control design for the Hamiltonian system

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
0 & \frac{1}{e_2} & 0 \\
-\frac{1}{e_2} & -\frac{b}{e_2} & 0 \\
0 & 0 & -e_1
\end{bmatrix}
\begin{bmatrix}
-xe_2P_m + cf x_3 \sin x_1 \\
e_2 x_2 \\
\frac{2d}{e_1} x_3 - cf \cos x_1
\end{bmatrix}
\]

\[6.22\]

\[
\begin{bmatrix}
0 \\
0 \\
dx_3
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} v
\]

and the control law is expressed as:

\[
u = cf \cos x_1 - \frac{2d}{e_1} x_3
\]

\[6.23\]

New Hamiltonian function is:

\[
H_d = \frac{1}{2} e_2 x_2^2 - a e_2 P_m x_1 - cf x_2 \cos x_1 + \frac{d}{e_1} x_3^2
\]

\[6.24\]

Taking above Hamiltonian function as Lyapunov function:
6.4 Adaptive and EB-based control design for the Hamiltonian system

\[
\frac{dH(x)}{dt} = e_2 x_2 \dot{x}_2 - a e_2 P_m \dot{x}_1 + c f x_3 \sin x_1 x_1 - c f \cos x_1 \dot{x}_3 + \frac{2d}{e_1} x_3 \dot{x}_3
= -b e_2 x_2^2 - e_1 (c f \cos x_1 - \frac{2d}{e_1} x_3)^2 + \left(\frac{2d}{e_1} x_3 - c f \cos x_1\right)
\leq y u
\] (6.25)

the stability of EB-based controlled Hamiltonian system can be proved.

Figure [6.3] shows the operation of the EB-based control design for Hamiltonian system at equilibrium point \([x_1 = 12.5, x_2 = 0, x_3 = 1.5]\).

As shown in Figure [6.3], \(x_1, x_2, x_3\) have the same physical meanings with the parameters in Figure [6.1], respectively. Compared with Figure [6.1] in EB-based control, there is almost no change in stability of parameters, however, in vibration frequency of parameters, there are decrease, especially in \(x_2\).

6.4 Adaptive and EB-based control design for the Hamiltonian system

Considering both adaptive control and energy-balancing-based control, under the same operating condition in the original Hamiltonian system, the following result can be derived as:
6.4 Adaptive and EB-based control design for the Hamiltonian system

Figure 6.3: Simulation result of EB-based control for Hamiltonian system

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix}
= \begin{bmatrix}
aP_m - \frac{c\hat{f}}{e_2}x_3 \sin x_1 - bx_2 \\
-2\hat{d}x_3 + c\hat{f}e_1 \cos x_1
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} u
\]

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix}
= \begin{bmatrix}
o & \frac{1}{e_2} & 0 \\
-\frac{1}{e_2} & -\frac{b}{e_2} & 0 \\
0 & 0 & -e_1
\end{bmatrix}
\begin{bmatrix}
-a e_2 P_m \\
e_2 x_2 \\
0
\end{bmatrix}
+ \hat{f} \begin{bmatrix}
c x_3 \sin x_1 \\
0 \\
-c \cos x_1
\end{bmatrix}
+ \hat{d} \begin{bmatrix}
0 \\
0 \\
\frac{2}{e_1} x_3
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} u
\] (6.26)
6.4 Adaptive and EB-based control design for the Hamiltonian system

\[
\begin{bmatrix}
\dot{\hat{f}} \\
\dot{\hat{d}}
\end{bmatrix} = k \begin{bmatrix}
-c^2\cos x_1^2 & -\frac{2c}{e_1}x_3 \cos x_1 \\
\frac{2c}{e_1}x_3 \cos x_1 & -\frac{4}{e_1^2}x_3^2
\end{bmatrix} \begin{bmatrix}
\hat{f} \\
\hat{d}
\end{bmatrix}
\]  
(6.27)

and the control law is expressed as:

\[
u = -k \left[ \frac{2\hat{d}}{e_1}x_3 - c\hat{f} \cos x_1 \right]
\]  
(6.28)

Taking \(V(x, \hat{\theta})\) as Lyapunov function just like equation (6.21), where \(\hat{\theta} = [\hat{f}, \hat{d}]\) is the adaptive control vector, system is asymptotically stable at equilibrium point.

Figure 6.4 shows the operation of Hamiltonian system at equilibrium point \([x_1 = 12.8, x_2 = 0, x_3 = 1.5, \hat{f} = 5.4, \hat{d} = 1]\).

As shown in Figure 6.4, \(x_1, x_2, x_3\) and \(\hat{f}, \hat{d}\) have the same physical significance with the parameters in Figure 6.2, respectively. Compared with Figure 6.1, the performance of adaptive and EB-based control system is better in terms of stability or vibration frequency and amplitude.

In this chapter, apply adaptive and EB-based control design to Hamiltonian equations of power system. The simulation results in this chapter is more clearer than those results in the chapter 3, 4 and 5. Adaptive
control design is more effective on stability of parameters. EB-based control design is better at reducing damping. Combining adaptive and EB-based control, the performance of adaptive and EB-based controlled Hamiltonian system has the excellences of those two control methods.
Chapter 7

Conclusion

The present MPhil thesis first made allusion to Hamiltonian systems, built a Hamiltonian model for power system, and then discussed adaptive control and EB-based control which were applied to design controller for this Hamiltonian system. Finally, under the same running conditions, and using different control designs, the simulation results are shown in the accompanying figures.

In Chapter 1 and Chapter 2, general background of power systems and control strategies were introduced. In these chapters, the mechanical and electrical structures of power systems were discussed, and state space equations for power systems were obtained. For sequent chapters, the properties of Hamiltonian systems, port-controlled Hamiltonian systems and port-controlled Hamiltonian systems with dissipation, as well as both adaptive control and EB-based control designs for Hamiltonian systems were discussed.
In Chapter 3, port-controlled Hamiltonian system and adaptive control design for this Hamiltonian system were applied to LC circuit. The simulation results of both Hamiltonian system and adaptive controlled Hamiltonian system for LC circuit were shown in the accompanying figures.

In Chapter 4, a kind of EB-based control, specifically, IDA-PBC was attributed to Hamiltonian system. Considering dissipation, this Hamiltonian system was derived from the LC circuit which is discussed in the Chapter 3. Using the same data from LC circuit, the simulation result was given.

In Chapter 5, adaptive control and EB-based control were combined together, and simulated under the same series of data from LC circuit in the Chapter 3, the simulation result of Hamiltonian system with the new control design was obtained.

In Chapter 6, the focus of thesis back to power system, and PCHD was used to modelling the power system discussed in chapter 2. To obtain the improved performance of power system, control strategies which were discussed in Chapter 3, Chapter 4 and Chapter 5 were applied to this PCHD for power system. Using the data from other’s work,
simulation results under Hamiltonian system and different control designs for this Hamiltonian system were derived by figures.
Bibliography


$w_1, w_2$—are disturbances of the system.

$\delta$—is the power angle of the generator, in radian.

$\omega$—is the relative speed of the generator, in rad/s.

$P_m$—is the mechanical input power, in p.u., which is assumed to be constant.

$P_e$—is the active electrical power delivered by the generator, in p.u.

$\omega_0$—is the synchronous machine speed, in rad/s.

$D$—is the per unit damping constant.

$H$—is the per unit inertia constant, in second.

$V_a$—is the voltage of the rest of power system.

$E_a$—is the open circuit voltage.

$r$—is the resistance of open circuit.

$I_a$—is the current of open circuit.

$jX_s$—is the inductance of open circuit.

$x'_{ds}$—is the mutual reactance between the excitation coin and the stator coil of generator.

$\theta_e$—is the rotor angular position of generator with respect to a station-
APPENDIX A VARIABLES

ary axis.

\( Q_e \)—is the reactive power, in p.u..

\( E_q' \)—is the transient electromagnetic force in the quadrature axis of the generator, in p.u..

\( E_q \)—is the electromagnetic force in the quadrature axis of generator, in p.u..

\( E_f \)—is the equivalent electromagnetic force in the excitation coil, in p.u..

\( T_{d0}' \)—is the direct axis transient short circuit time constant, in second.

\( I_f \)—is the excitation current.

\( I_q \)—is the quadrature axis current.

\( k_c \)—is the gain of the excitation amplifier.

\( u_f \)—is the input of SCR amplifier of the generator.

\( x_{ad} \)—is the mutual reactance between the excitation coil and the stator coil of the generator.

\( V_s \)—is the infinite bus voltage.

\( x_L \)—is the transmission line reactance.

\( x_T \)—is the transformer reactance.

\( x_d \)—is the direct axis reactance of the generator.

\( x_d' \)—is the direct axis transient reactance of the generator.
Appendix B Simulation

function xdot=Work(t,x)
%Test a response of a first order system, from a nonlinear exosystem

%original x: x1 x2 x3
x1=x(1); x2=x(2); x3=x(3);

%adaptive control
%x1=x(1); x2=x(2); x3=x(3); fh=x(4); dh=x(5);
%moniter the time
if(mod(t,1)>0.999) t end

%constant parameters used in the controller
a=314/6;Pm=0.9;b=5/6;c=314/6;e1=6/(7.4*314);
f=(1.867-0.257)/0.36;d=1/5;e2=(1.867-0.257);
k=0.01;omega=3;

%Nonlinear functions
control design
Hamiltonian
\[ u = -d \cdot x_3/e_1 + c \cdot f \cdot \cos(x_1); \]

adaptive control
\[ u = k \cdot (c \cdot f_h \cdot \cos(x_1) - dh \cdot x_3/e_1); \]

energy balancing control
\[ u = -(\omega + 1) \cdot d \cdot x_3/e_1 + c \cdot f \cdot \cos(x_1); \]

EB adaptive control law
\[ u = k \cdot (c \cdot f_h \cdot \cos(x_1) - (\omega + 1) \cdot dh \cdot x_3/e_1); \]

Dynamic systems
original
\[ f_1 = x_2; \]
\[ f_2 = a \cdot P_m - c \cdot f \cdot x_3 \cdot \sin(x_1)/e_2 - x_2 \cdot b; \]
\[ f_3 = c \cdot f \cdot \cos(x_1) \cdot e_1 - d \cdot x_3 + u; \]

adaptive control \( f_h, dh \)
\[ &f_1 = x_2; \]
\[ %f_2 = a \cdot P_m - c \cdot f_h \cdot x_3 \cdot \sin(x_1)/e_2 - x_2 \cdot b; \]
\%f3 = c*fh*cos(x1)*e1-dh*x3+u;
\%f4 = k*[-c*c*cos(x1)*cos(x1), c*x3*cos(x1)/e1; c*x3*cos(x1)/e1,-x3*x3/e1/e1]*[fh;dh];

\text{%energy balancing control}
\%f1 = x2;
\%f2 = a*Pm-c*f*x3*sin(x1)/e2-x2*b;
\%f3 = c*f*cos(x1)*e1-(omega+1)*d*x3+u;

\text{%EB adaptive control}
\%f1 = x2;
\%f2 = a*Pm-c*fh*x3*sin(x1)/e2-x2*b;
\%f3 = c*fh*cos(x1)*e1-(omega+1)*dh*x3+u;
\%f4 = k*[-c*c*cos(x1)*cos(x1), (omega+1)*c*x3*cos(x1)/e1; (omega+1)*c*x3*cos(x1)/e1,-(omega+1)*(omega+1)*x3*x3/e1/e1]*[fh;dh];

\text{%Observer}
\text{%original EB}
xdot=[f1;f2;f3];

\text{%adaptive EB adaptive}
%xdot=[f1;f2;f3;f4];