Numerical and Experimental investigations into Electrochemical Machining

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<td>$A$</td>
<td>Atomic weight</td>
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<tr>
<td>$b$</td>
<td>Bare length (mm)</td>
</tr>
<tr>
<td>$E$</td>
<td>Effective voltage (V)</td>
</tr>
<tr>
<td>$e$</td>
<td>Euler’s number</td>
</tr>
<tr>
<td>$F$</td>
<td>Faraday’s constant (96,500 coulombs)</td>
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<td>$f$</td>
<td>Tool feed rate (mm/min)</td>
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<td>$f_a$</td>
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<td>$k_{eo}$</td>
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<td>Number of element</td>
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<td>$V$</td>
<td>Applied voltage (V)</td>
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<td>$V_{g}$</td>
<td>Gas velocity (mm/min)</td>
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<td>$V_s$</td>
<td>Average velocity of sludge (mm/min)</td>
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<td>$V_m$</td>
<td>Machining vector</td>
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<td>$Y_g$</td>
<td>Thickness of gas layer (mm)</td>
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<tr>
<td>$Y_s$</td>
<td>Thickness of sludge layer (mm)</td>
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<tr>
<td>$z$</td>
<td>Valency electron</td>
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<tr>
<td>$\Delta t$</td>
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<td>Total over-potential (V)</td>
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<tr>
<td>$\rho$</td>
<td>Density of workpiece material (g/cm³)</td>
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<tr>
<td>$\delta$</td>
<td>Thickness of mixture gas and liquid layer (mm)</td>
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Abstract

This thesis presents numerical and experimental investigations into Electrochemical Machining (ECM). The aim is to develop a computer program to predict the shape of a workpiece machined by the ECM process. The program is able to simulate various applications of ECM machining which are drilling, milling, turning and shaped tube electrochemical drilling (STED). The program has been developed in a MATLAB environment. In this present work, EC-drilling, EC-milling and EC-turning are analysed as three-dimensional problems whereas STED is simulated in two-dimensions. Experiments have been carried out to verify the accuracy of the predicted results in the cases of EC-milling and EC-turning.

The ECM modeller is based on the boundary element method (BEM) and uses Laplace’s equation to determine the current distribution at nodes on the workpiece surface. In 3D, the surfaces of the tool and the workpiece are discretised into continuous linear triangular element types whereas in 2D, the boundaries of the tool and workpiece are discretised into linear elements. The ECM modeller is completely self-contained, i.e. it does not rely on any other commercial package. The program contains modules to automatically discretize the surfaces/boundaries of the tool and workpiece. Since the simulation of the ECM process is a temporal problem, several time steps are required to obtain the final workpiece shape. At the end of each time step, the shape of the workpiece is calculated using Faraday’s laws. However, the workpiece’s shape changes with progressing time steps causing the elements to become stretched and distorted. Mesh refinement techniques are built in the ECM modeller, and these subdivide the mesh automatically when necessary.

The effect of time step on the predicted 3D shape of a hole in EC-drilling is investigated. The effect of discontinuity in the slope between neighbouring elements is also studied. Results obtained from the ECM modeller are compared with 2D analytical results to verify the accuracy that can be obtained from the ECM modeller. Milling features ranging from a simple slot to a pocket with a complex protrusion were machined in order to determine the feasibility of the EC milling process. These features were machined on a 3-axes CNC machine converted to permit EC milling. The effect of tool geometry, tool feed rate, applied voltage and step-over distances on the dimensions, shape and surface finish of the machined features were investigated. A pocket with a human shape protrusion was machined using two different types of tool paths, namely contour-parallel and zig-zag. Both types resulted in the base surface of the pocket being concave and the final dimensions of the pockets are compared with the design drawing to determine the effect of tool path type on the accuracy of machining.

The ECM modeller was used to simulate the machining of a thin-walled turned component. The machining parameters, i.e. initial gap, rotational speed, and applied voltage, were specified by the collaborating company. Since only a small amount of material had to be removed from the thin-walled component, the tool was held stationary i.e. a feed in the radial or longitudinal direction was not required. By taking advantage of the axi-symmetric nature of a turned component, only a sector of the component was analysed thereby reducing the computing time considerably. The accuracy of the modeller was verified by comparing the predicted time to machine the thin-walled component with the actual machining time.

The initial investigations in STED were both experimental and numerical in nature and they studied the effect of applied voltage, tool feed rate and electrolyte pressure on the dimensions of the holes. Later investigations were numerical and an iterative methodology has been developed to calculate a set of feed rates which could machine a specified turbulator shape.
Declaration

That no portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.
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Publication

Chapter 1
Introduction

The demand for using components with complex shapes is increasing in many fields such as automotive parts, toys and medical parts. Thus, new and unconventional manufacturing techniques to produce these components are required. There are many techniques to manufacture a workpiece. Some of the more popular ones are additive and removal processes.

In additive manufacturing, the new layer of material is applied on top of the previous layer. If the material added is in form of small solid particles, it is melted by a high power heat source such as a laser beam. If it is in liquid form, it is solidified by ultra-violet light. Additive layered manufacturing can be used to create complex parts but the production rate is low and the cost of the required equipment is high.

In the removal method, the unwanted stock is removed by processes such as grinding, turning or milling. Conventionally, these removal processes use a cutting tool to remove the stock from the billet through abrasion, shearing, etc. Hence, the cutting tools have to be made of a material that is harder and stronger than that of the workpiece; moreover, these cutting tools are exposed to cutting forces and thermal stresses and therefore, their design and the cutting parameters used, should be such that their cutting life is maximized. Recently, several techniques to improve material properties have been widely researched. Consequently many high strength, heat and corrosion resistant materials such as super-alloys have been developed. It is difficult to machine these materials using conventional machining methods.

Therefore non-conventional processes are required. Non-conventional processes that are usually used in machining are: electrical discharge machining (EDM), laser beam machining and electrochemical machining (ECM). In the first two processes, the material is removed by using high thermal energy to melt the material (by electrically sparking on the workpiece in EDM, or by laser beam). Although in these processes the material is removed without the tool coming into contact with the workpiece, heat-affected zone, residual stresses, and sometimes even surface cracks, are present in the
workpiece. In the ECM process, the material is removed by attracting atoms of material by means of electrolysis but not by the thermal effect or by mechanical contact.

To machine a workpiece using ECM process, the tool is placed close to the workpiece surface and electrolyte is pumped through the gap as shown in the exaggerated diagram of Fig. 1.1.

![Schematic of electrochemical machining](image)

**Fig 1.1: Schematic of electrochemical machining**

The tool in ECM can be made from any material that can conduct electricity. Typically it is made of copper because copper has a high electrical conductivity. The Electrolyte can be an aqueous solution of salts such as sodium chloride (NaCl) or sodium nitrate (NaNO₃). In some case, it is acid diluted with water. When salts are dissolved, or the acid is diluted, the electrolyte will contain ions of salt or acid and water. For example, when sodium chloride is dissolved into water, it will be dissociated into sodium ion (Na⁺) and chloride ion (Cl⁻) and water (H₂O) will be dissociated to hydrogen ion (H⁺) and hydroxyl ion (OH⁻).

To remove material from a workpiece, a potential difference (or applied voltage is applied between workpiece (positive) and the tool (negative) by connecting the positive pole at the workpiece and the negative pole at the tool. Consequently, the positive ions move to the tool surface and negative ions move to workpiece surface.

Since hydrogen ions have a higher ionic mobility than sodium ions, hydrogen ions will move towards the tool surface and produce hydrogen gas (H₂) whereas sodium ions will combine with hydroxyl ions in the electrolyte to produce sodium hydroxide.
\[
Na^+ + OH^- \rightarrow NaOH
\]

On the workpiece surface, metal ions from the workpiece will be released as

\[
Fe \rightarrow Fe^{++} + 2e
\]

In practice the metal ions will combine with chloride ions or hydroxyl ions to form \(FeCl_2\) and \(Fe(OH)_2\) which will be precipitated in the form of sludge.

Fig 1.2: A schematic view of the reaction in an electrolytic cell

Therefore, ECM offers the opportunity of removing material regardless of its physical properties such as hardness and toughness. Thus, this process can be used to machine hard materials such as super-alloys which are difficult to machine using conventional machining processes. Because the tool is not in contact with the workpiece surface and no heat energy is applied, the problems of tool wear and residual stresses in the workpiece do not arise.

Today, ECM is used in many fields of machining such as deep hole drilling, turning, grinding, milling and cavity sinking to manufacture components with complex geometry such as turbine blades, gears and medical implants., More recently, the success of using this process for machining at the micro-scale level has been reported.

Although ECM offers many advantages, a major problem with the process is the difficulty of obtaining the required dimensions of the workpiece at the first one or two attempts. ECM involves passing current through the electrolyte in the inter-electrode gap. However, the direction of current within electrolyte cannot be precisely controlled or, in other words,
it cannot be completely focused over the region of the workpiece surface required to be machined. Instead, because the electrolyte is spread all over the exposed workpiece surface, material which is not directly opposite the tool face is also machined. Consequently, the tool geometry is not exactly replicated on the workpiece surface. Therefore, several attempts with varying tool dimensions may be necessary before the workpiece shape is obtained to the required accuracy. The number of trials performed experimentally can be reduced if the shape of the workpiece with reasonably accuracy can be predicted for the given tool shape before actual machining takes place. This is the main motivation for the present work. The main purpose of this research is to develop a program to accurately predict the shape of workpiece for different types of ECM processes (turning, milling and drilling) based on computed values of the current density at the workpiece surface for a given tool shape and given process parameters.
Chapter 2
Literature review

Since the workpiece, which is the anode in electrochemical machining, is machined by electro-chemical reaction, the shape produced on the workpiece surface does not have the same dimensions as those of the tool. To machine the workpiece precisely to a given shape, the tool is designed using a trial-and-error approach using appropriate machining parameters (applied voltage, feed rate, etc.).

Therefore the problems in ECM can be divided into two categories which are: (i) predicting the shape of the workpiece for a given set of machining parameters when the tool shape is specified and (ii) determining the shape of the tool for a given shape of the workpiece and specified machining parameters. The latter is often referred to as the ‘inverse problem’.

Only published literature concerned with the prediction of the workpiece shape is considered in this thesis. The effects of using different values of machining parameters on the shape of workpiece can be investigated either experimentally or by analytical/numerical models. Although the results obtained experimentally tend to be more accurate than those obtained analytically, the experimental approach is rather limited as the results are valid only in the range investigated. On the other hand, analytical/numerical models offer more flexibility.

In this chapter, analytical and numerical techniques/methods for predicting the changing shape of the workpiece (anode) profile are considered. These techniques are reviewed according to their application i.e. type of operation modelled (drilling and cavity die sinking, milling and turning).

2.1 Modelling of EC cavity sinking and drilling process

ECM is widely used in cavity die sinking and drilling. In these processes, the tool is fed towards the workpiece or vice versa. In cavity die sinking, a 3-D shape is produced on the workpiece surface (Fig 2.1 (a)), whereas in drilling, the final shape is a cylindrical
hole (Fig 2.1(b)). There are several techniques that are able to predict the shape of the workpiece in EC sinking and drilling.

2.1.1 Experimental and Semi-analytical methods

Although the anodic dissolution process can be explained simply from chemical reaction as stated in Chapter 1, in practice, ideal machining hardly occurs since, when dissolution occurs, the process involves not only transfer of ions, causing an ion film on the working surface and hydrogen bubble generation but also heat generation which causes the electrolyte conductivity across the gap between cathode (tool) and anode (workpiece) to vary. This complex physico-chemical process causes uneven flow of machining current through the machining zone. Many researchers have tried to take into account the effect of machining parameters such as applied voltage, tool feed rate and electrolyte concentration on the workpiece surface quality and its material removal rate (MRR) and to find the optimal value of these parameters for actual machining.

The effect of electrolyte flow rate and electrolyte concentration on the surface roughness was investigated by Sokhel et al. [3]. Their experimental results showed that with a sodium chloride (NaCl) electrolyte, the surface roughness of the machined surface is strongly dependent on the electrolyte flow rate. The surface roughness of the surface machined with a low flow rate is greater than that machined with a higher flow rate. This is because at low electrolyte flow rates, the electrolyte is not able to flush the sludge from the chemical reaction that adheres to the workpiece. As a result, a poor surface finish (high surface roughness) is obtained. On the other hand, if the flow rate is
high enough, all of the sludge is removed from the surface resulting in a better surface finish.

Bhattacharyya et al. [4] studied the effect of applied voltage and electrolyte concentration of sodium nitrate (NaNO₃) on material removal rate (MRR) and over-cut in micro electrochemical drilling with a stationary tool system. They found that, for small gaps, the MRR increases non-proportionally with an increase in the machining voltage. This differs from Faraday’s laws. Unlike large inter-electrode gaps, the current distribution within a small gap is non-uniform and the over-cut increases with applied voltage. The reason for this is that when the applied voltage increases, the localization effect of current flux flow decreases which causes the stray current to increase resulting in greater overcut and more material being removed. They also observed that, with a higher concentration of the electrolyte, the MRR and over-cut values are higher as well. This is understandable because at higher concentrations, there are more ions in the electrolyte resulting in an increase in the current applied on the anode. For a non-stationary tool, Cirilo et al. [5] observed that the MRR is influenced by the tool feed rate and the flow of electrolyte has no affect on it. Comparing the use of NaCl and NaNO₃, they found that NaCl gives a higher MRR value than NaNO₃ because the NaCl solution is a non-passivator electrolyte and has a constant current efficiency during machining. However, NaNO₃ gives a smoother surface and a smaller over-cut.

Konig et al. [6] proposed equations to predict the over-cut in EC drilling for a cylindrical tool with blend radii based on their experiments which can be calculated as:

\[
h_o = r_c^{0.35} \cdot 0.35 \cdot (10e)^{0.5h_e}
\]  (2.1)

where \(h_o\) is the over-cut (see Fig 2.2), \(r_c\) the blend radius, \(h_e\) the equilibrium gap and \(e\) Euler’s number. This relation is valid when \(0.15 < h_e < 0.6\) mm and \(0.5 < r_c < 5\) mm.

When the tool is partially insulated, Konig et al. [7] related the over-cut value according to the bare length \(b\) as follows [7] (see Fig 2.2).

\[
h_o = (0.1 + h_e) \cdot (0.314 \cdot r_c + 1.17) \quad \text{when } b = 0
\]  (2.2)

\[
h_o = 2h_e + 0.1 \cdot [6.283 \cdot (r_c - 1)]^{0.5} \quad \text{when } b > 1
\]  (2.3)
The values of the side gap $h_s$ and $h_s'$ can be calculated from eq. (2.4) and (2.5).

$$h_s' = (2bh_e + h_e^2)^{0.5} \quad (2.4)$$

$$h_s = [2 \cdot b \cdot h_e + r_e^{0.7} \cdot 0.123 \cdot (10e)^h]^{0.5} + 0.65 \cdot h_e \quad (2.5)$$

Fig. 2.2: Variables defining the front and side gaps in EC drilling [6]

Ippolito et al. [8] have investigated the effect of electrolyte conductivity ($k_e$), the length of the bare tool ($b$), and the ratio between effective voltage ($E$) and tool feed rate ($f$) on the size of the over-cut in drilling. By using a multiple regression technique, the over-cut ($h_s'$) at the end of the bare part of the tool at a feed rate of 1.16 mm/min can be calculated by the following equation:

$$h_s' = [2.58 \cdot h_e \cdot b + (1.29 \cdot h_e \cdot f)^2 - 0.29]^{0.5} \quad (2.6)$$

From their experimental results, the over-cut at the beginning of the hole ($h_s$) is independent of the bare length ($b$) and is given by:

$$h_s = h_s' + \frac{2 \cdot B' \cdot h_e}{D}[\arctan\left(\frac{2 \cdot X + B'}{D}\right) - \arctan\left(\frac{B'}{D}\right)] \quad (2.7)$$
where \( B' = \frac{v \cdot k_e \cdot E}{\rho \cdot f} \) and \( D = \sqrt{4 \cdot f^2 (a_0)^2 - B'^2} \), \( v \) is a correction factor for the electrolyte conductivity \((k_e)\) due to the presence of metallic oxides and hydroxides, \( X \) the distance along electrolyte flow direction, and \( \rho \) the density of workpiece material.

Although the equations derived from experimental results can be employed to predict the workpiece shape, they are valid only when the machining parameters such as feed rate, tool shape, electrolyte and material of tool and workpiece are within the corresponding range used in the experiments. Thus, if the value of a machining parameter is outside the range investigated, there may be an error in determining the side/frontal gap from the above equations.

Since, in actual practice, it is not feasible to investigate the effect of all the process parameters in ECM experimentally due to time and cost limitations, the use of mathematical models has begun to play an important role in investigating the process. In recent times, many researchers have developed analytical methods and modelling techniques to predict the phenomenon occurring within the inter-electrode gap.

### 2.2.2 Analytical methods

A one-dimensional analysis [1] of ECM allows one to calculate the thickness of the material machined. The analysis assumes the electrodes to be planar, parallel to each other and normal to the feed direction, with a voltage \((V)\) applied between them, and the cathode moved at a constant feed rate \((f)\) (see Fig 2.3).

![Fig 2.3: Tool and workpiece considered as plane-parallel electrodes](image-url)
Assuming that the electrolyte properties such as electrical conductivity remain constant when flowing through the gap and ignoring the effect which laminar or turbulent flow may have, the rate of erosion, from Faraday’s laws, is given by [1]:

\[
\frac{dh}{dt} = \frac{M \cdot (V - \eta)}{h} - f
\]  

(2.8)

where \( M \) is the machining parameter which is a function of the electrolyte conductivity \( (k_e) \), atomic weight \( (A) \), valency electron \( (z) \) of workpiece material, Faraday’s constant \( (F) \) and density of workpiece material \( (\rho) \), \( V \) the applied voltage, \( \eta \) the over-potential and \( h \) the inter-electrode gap. In the case of a stationary tool system, \( f = 0 \) and the solution of eq. (2.8) is

\[
h^2(t) = h^2(0) + 2 \cdot M \cdot (V - \eta) \cdot \Delta t
\]

(2.9)

In the case of a moving tool, the gap thickness at steady state will be constant because \( \frac{dh}{dt} = 0 \). Thus the equilibrium gap, \( h_e \) can be calculated as

\[
h_e = \frac{M \cdot (V - \eta)}{f}
\]

(2.10)

However in ECM, the chemical reaction generates hydrogen gas on the tool face, leading to the formation of bubbles. These bubbles act as insulators and affect the electrical conductivity of the electrolyte. In addition to this, the temperature of the electrolyte increases due to the electrical current in the gap. This temperature increase also affects the conductivity of the electrolyte, however small. The above equation is therefore approximate as it does not take into account these changes in conductivity.

Therefore to predict the equilibrium gap more accurately, Thorpe et al. [9] presented a 1-D analytical model that considers the effect of the void fraction and temperature dependent resistance of the electrolyte. Their model is shown in Fig. 2.4. The upper surface is the tool which moves towards the workpiece (lower surface), with a feed rate of \( f \) which results in a dissolution rate \( f_a \) of the workpiece. When the current flows across the gap, hydrogen gas bubbles are generated on the tool surface and they flow in the same direction as the electrolyte with an average velocity \( V_g \) in an equivalent area \( Y_g \). The bubbles are contained within an electrolyte layer near the tool surface which has a thickness \( \delta \). The machined workpiece material will flow into the electrolyte as sludge.
with an average velocity $V_s$ and thickness $Y_s$. The linearly-varied electrical conductivity of the electrolyte is given by [9]

$$k_e = k_{eo}[1 + \gamma(T - T_o)]$$

(2.11)

where $k_{eo}$ is the electrolyte conductivity at inlet, $\gamma$ the conductivity constant, $T_o$ the electrolyte inlet temperature and $T$ the temperature at a point within the gap. The effect of bubble fraction is represented by the following equation [9].

$$f\left(\frac{Y}{\delta} \alpha\right) = \left[1 - \frac{Y}{\delta} \alpha\right]^n$$

(2.12)

where $Y$ is the exit gap, $\delta$ the thickness of the layer of gas and electrolyte liquid, $\alpha$ void fraction and $n$ constant. By balancing the transport of mass, momentum, energy and charge in a controlled volume within the inter-electrode gap and using numerical integration, the analytical solution for the one-dimensional ECM model can be obtained.

Fig 2.4: Portion of the inter-electrode gap occupied by sludge and gas bubbles [9]

The plane-parallel and Thorpe-enhanced models are useful in determining the removal thickness of the material in case the tool and workpiece faces are parallel to each other which makes it a one-dimensional analysis. However, in actual EC machining, the shape of the tool can be complex resulting in it being not parallel to workpiece face. Hence, the shape of the workpiece requires a two or three-dimensional analysis.

The simplest mathematical technique used to predict the workpiece shape is the cos$\theta$ method first introduced by Tipton [10]. In this method, the tool profile is divided into
several segments and the equilibrium gap between corresponding segments of the tool profile and workpiece (Fig 2.5) is calculated as

\[ h_{e,i} = \frac{h_e}{\cos \theta} \]  

(2.13)

where \( h_{e,i} \) is the equilibrium gap at a segment of the tool profile, \( h_e \) the equilibrium gap computed from eq. (2.13) and \( \theta \) the angle measured between the normal vector of a segment on the tool profile and the tool feed direction. The workpiece shape is created by joining the line segments which are offset from the tool segments by the equilibrium gap (\( h_{e,i} \)).

Since the calculation of the equilibrium gap in eq. 2.6 is based on two parallel planes (eq.(2.10)), the predicted profiles will only be obtained at steady-state conditions and the effects of temperature variation, bubble generation and flow conditions in the gap are not considered.

However Jain et al. [11] found that the \( \cos \theta \) method is not reliable when the tool has sharp corners. It is accurate when the angle (\( \theta \)) is not greater than 45° although from a mathematical viewpoint, it should be applicable even if the angle is near 90°. Therefore this method is not to be recommended when the tool has a complex shape.
Hocheng et al. [12] combined the fundamental laws of electrolysis, Faraday’s laws, and their “integral of finite width tool” method to predict the machined profile of the workpiece as a function of time. Thus the evolution of the profile at each time step can be determined. In their model which is illustrated by Fig 2.6, the tool boundary is divided into several point charge sources (e.g. point \(a\) in Fig 2.6). A point on the workpiece (anode) surface is influenced by all the point sources on the tool. To calculate the anode profile for a particular time step, an integral of the source point over the tool edge is carried out. The amount of material removed from the workpiece at time \(t\) from a single charge is represented by the following equation.

\[
m_i = -\int_0^d \frac{C}{(y-a)^2 + y_i^2} da
\]  

(2.14)

where \(C\) is a constant depending on the machining (they used a value of 6.6), \(d\) the tool diameter and \(a\) the distance of the source point on the tool measured from tool centre. Therefore the actual depth after each time increment can be calculated from the following equation:

\[
y_{i+1} = y_i + \Delta t \cdot m_i
\]  

(2.15)

![Fig 2.6: Two-dimensional analysis](image)

The variation of current density across the gap in using the mathematical models presented above is assumed to be linear. Actually the electrical current distribution in the electrolyte between the electrode surfaces is governed by Laplace’s equation. Thus for steady state conditions the distribution current can be expressed as

\[
\nabla^2 V = 0
\]  

(2.16)
Collett *et al.* [13] applied the complex variable method to find the shape of the workpiece under steady-state condition and they, like most other researchers, ignored the effects of electrolyte flow and sum of over-potential. In their work, the workpiece is machined by stepped tools and the shape of the workpiece is represented by the ratio of the over-cut at an edge of the tool to the equilibrium gap. Their results show that the values of the ratio were 0.731 and 1.159 when the workpiece was machined with and without insulation on the side of the tool, respectively. Hewson-Brown [14] extended the work from Collett *et al.* to find the shape of the workpiece when machined by a tool with rounded corners and by a tool that was partially insulated on its side.

Nilson *et al.* [15] applied an inverted formulation to find the workpiece shape. In their technique, the spatial coordinates \((x, y)\) were selected as dependent variables on the plane of complex potential \((\phi, \iota)\). The inverted Laplace equations are given below.

\[
\frac{\partial^2 x}{\partial \phi^2} + \frac{\partial^2 x}{\partial \iota^2} = 0 \quad \text{and} \quad \frac{\partial^2 y}{\partial \phi^2} + \frac{\partial^2 y}{\partial \iota^2} = 0
\]

(2.17)

The potential plane is assumed to be a rectangular domain (Fig 2.7). The physical boundary conditions (insulation or no insulation of the tool and applied voltage on the workpiece) are transformed to a complex plane. The unknowns, \(x\) and \(y\), in eq. (2.17) are solved by using the finite difference method (FDM).

Zhitnikov *et al.* [16] developed a numerical technique based on the complex variable method to simulate the evolution of workpiece shape as the tool is moving towards the workpiece face. They used the following procedure.

(i) Specify the initial shape of the workpiece and tool.

(ii) Specify the points where the current density is to be calculated.
(iii) Calculate the current density at specified points on the workpiece.
(iv) Move the points in a direction perpendicular to the current shape of the workpiece according to Faraday’s law. Thus the new shape of the workpiece is created.
(v) Move the tool boundary.

Steps (ii)-(v) are repeated several times until the required workpiece shape is obtained.

Although the complex variable method can be used to predict the workpiece shape at steady-state and transient conditions, it is not widely used for ECM analysis because this method requires strong knowledge in mathematics and it is limited in two-dimensional analysis. The effect of bubble and temperature variation on machining cannot be applied in analysis.

2.2.3 Analogue method

The early techniques to solve the field distribution in ECM are the conducting paper analogue [1-2] and electrolytic tank methods [1-2]. In the conducting paper analogue method, profiles representing the tool and workpiece have to be drawn on a conducting paper to scale. A D.C. power supply is used to supply constant current to the paper. A volt meter is used to measure the potential gradient \( \frac{dV}{dn} \) along the workpiece profile on the paper (see Fig 2.8). At points on the workpiece profile, the machining vector \( V_m = k_e \frac{dV}{dn} \cdot \Delta t \) and feed rate vector \( V_f = f \cdot \Delta t \cdot \cos \theta \) have to be determined (Fig 2.9). By adding both vectors at a point, a new position that represent the machined location of this point after time step, \( \Delta t \) is obtained (for example, point A has to move to position B in Fig. 2.8). The paper is cut through a series of the points to obtain a new profile of the workpiece and this process is repeated until the updated profile coincides with that of the required workpiece.
The electrolytic tank method is similar to the conducting paper analogue method but instead of using conducting paper, flexible electrodes (thin copper strips) are immersed in an electrolyte solution in a tank (see Fig 2.10). The procedure to find the workpiece shape is the same as in the conducting paper analogue method except that the strip representing the workpiece has to be bent instead of cut.

Although these methods are able to predict the evolution of the workpiece machined by shaped tools, the accuracy of the predicted shape is strongly dependent on the skill of the operator and they are time-consuming.
2.2.4. Numerical Methods

The current distribution governed by Laplace’s equation (eq. (2.16)) on the workpiece surface can be solved using numerical methods. There are three techniques that are often used to model/simulate the ECM problem. These are the finite difference method (FDM), finite element method (FEM) and the boundary element method (BEM).

2.2.4.1 Finite Difference Method (FDM)

Kozak [17] proposed a model using the FDM to determine the shape of the workpiece under steady state conditions for machining with a shaped tool (ECM cavity die sinking). Figure 2.11 shows the system of machining with a shaped tool.

Fig 2.10: Equipment for electrolytic tank analogue [1]

Fig 2.11: Conceptual model for ECM die sinking [17]
The tool moves up with a feed rate of \( f \) towards the work piece and the dissolution rate at point A on the work piece is given by \( f_n \) (see Fig 2.11). The equilibrium gap at steady state is given by:

\[
h_e = M \frac{(V - \eta)}{f}
\]  
(2.18)

In his model, Kozak divided the flow areas into 3 sub-regions. The electrolyte enters through the first sub-region and it is assumed that, in this zone only, the electrolyte, in its pure form, flows. In the second zone, bubbles, resulting from the chemical reaction, are formed on the tool surface. In the third sub-region, the bubbles and electrolyte are mixed together resulting in homogeneous flow. In this case, the equation for two-phase flow due to Bruggeman [17] is used to approximate the electrical conductivity of the electrolyte.

\[
\kappa = \kappa_o [1 + \alpha_T \theta] (1 - \beta)^3
\]  
(2.19)

where \( \theta = T - T_o \), \( T_o \) being the inlet temperature, \( \alpha_T \), a temperature coefficient of the electrolyte conductivity, \( \kappa_o \) the electrolyte conductivity at \( T_o \), respectively and \( \beta \) the void fraction (they assumed that \( \beta = 0 \)). To calculate the conductivity of the electrolyte in equation (2.19), it is necessary to determine the type of flow i.e. laminar or turbulent, this is determined so that the temperature distributions can be calculated. Although Kozak defines the physics of the problem through several relationships, very few details of the actual implementation, especially the adaptive finite difference grids, are provided.

Hourng et al. [18] simulated the ECM drilling process as a two-dimensional problem under quasi-equilibrium conditions and studied the effect of varying the electric potential and thermal-fluid properties using one-dimensional two-phase flow equations. After every time step, the shape of the workpiece and tool are transformed into a natural co-ordinate system (\( \xi - \eta \) plane) using Poisson’s equation. The grid in the \( \xi - \eta \) domain is controlled by interpolation between the workpiece and tool boundaries. The finite difference method, with successive over-relaxation, is employed to solve the electrical potential, using a forward difference of the time derivatives. The Euler implicit difference method is used to determine the thermal properties and flow velocity of the electrolyte.
Although FDM is an efficient technique to deal with ECM, its main limitation is that only rectangular grids can be used to approximate the domain. This means that the boundaries of the domain, i.e. tool and workpiece shapes, cannot be matched accurately especially for complex shapes. In order to increase the accuracy of the simulation, other superior numerical techniques such as the finite element method (FEM) or boundary element method (BEM) have been deployed with increased meshing capabilities.

2.2.4.2 Finite Element Method: FEM

Hardisty et al. [19] used the FEM to predict the evolution of the workpiece shape when it is machined by flat and stepped tools. They used rectangular meshes to discretise the domain but neglected the effect of conductivity variation and void fraction. The gap sizes when the flat tool is moved towards the work piece computed from their FEM result showed good agreement with the values calculated from mathematical model (eqs. 2.8 and 2.9).

Jain et al. [20] attempted to predict the workpiece profile in ECM drilling using both an ordinary tool and a partially insulated tool. They used the finite element method to analyze the process as a one- and two-dimensional problem taking into account the influence of thermal distribution and bubbles dispersion in the electrolyte. They subdivided the machining area into four zones i.e. stagnation, front, transition and side zones as shown in Fig 2.12(a).

![Fig 2.12: (a) Schematic diagram with electrolyte flow zone, (b) and (c) one and two-dimensional discretization of the gap][20]

Fig 2.12: (a) Schematic diagram with electrolyte flow zone, (b) and (c) one and two-dimensional discretization of the gap [20]
They considered only the front and transition zones to determine the workpiece profile. In the case of one-dimensional analysis, the gap is divided into several segments (Fig. 2.12(b)) and each segment is represented by linear elements. For a two-dimensional analysis, they used linear triangular elements to mesh the domain (see Fig 2.12(c)).

Heat transfer and fluid flow equations in each zone (transition and front zone) were considered separately because of the difference in the type of coordinate system used for the zones. In the one-dimensional analysis, a Cartesian coordinate system was used for the front zone and a cylindrical co-ordinate system for the transition zone whereas a cylindrical co-ordinate system was used for all the zones in the two-dimensional analysis. However, the values of the variables in all the elements were combined together to create a global matrix, which was solved using the Gaussian elimination technique.

After each time step $\Delta t$, the element length changes and, in order to predict the shape of the workpiece correctly in the next iteration, the feed vector on each node on the workpiece face has to be determined using the $\cos \theta$ method. In the front zone, the shape becomes tapered as machining proceeds in which case, $\theta$ is measured relative to the tool feed direction and in the transition zone, on the other hand, the profile is assumed to be a quarter part of a circle and is approximated by line segments. Thus $\theta$ is approximated as the angle between adjacent segments.

Since the electrolyte flow rate is considered in their model, it is approximated using a function of the pressure and the distance along electrolyte flow. Hence, although their technique can be used in 2-dimensional analysis, the flow rate is still approximated using a one-dimensional approach.

Hourng et al. [21] used the CFD technique to investigate two-dimensional fluid properties in each zone within the gap when drilling a pre-drilled hole. In their model, Laplace’s equation and the two-dimensional flow equation are used under quasi-steady analysis. The influence of hydrogen gas and slag resulting from the chemical reaction during machining are neglected. Therefore, the momentum and energy equations are applied to the electrolyte which is assumed to have incompressible laminar flow in an axisymmetric coordinate system. A body-fitted coordinate transformation technique is
used to reduce the numerical error caused by changing the physical domain during machining time. The simulation results show that in the transition zone where the gap between the electrodes changes rapidly, the electrolyte pressure near the workpiece is higher than that near the tool because the electrolyte velocity near the workpiece is lower than that near the tool. The accuracy of the ECM drilling process can be improved at higher tool feed rates and lower applied voltage conditions.

2.2.4.3 Boundary Element Method: BEM

Although the finite difference and finite element methods have been used extensively by researchers to solve the ECM problem, they have a major limitation. Since both methods are domain-based techniques, it means that the entire domain including the interior space within the domain has to be discretised into small elements. However, in ECM, since the value of the variables are not required throughout the domain but only on a certain part of the boundary, it appears that the boundary element method is ideally suited to solve this problem.

Narayanan et al. [22] used the BEM to predict the shape of the workpiece under idealized conditions in a two-dimensional system. Two types of isoparametric elements, linear and quadratic types, were used together with Laplace’s equation for the determination of the field distribution within the gap. The distance resulting from erosion on the workpiece boundary is calculated from the material removal rate according to Faraday’s law. As machining proceeds, some of the elements representing the workpiece boundary get stretched and others compressed leading to an unacceptable error in the computed results; hence the parabolic blending technique was used for curve fitting and re-meshing purposes. The simulation results show that the accuracy of the results computed mainly depends on the mesh size and time step or in other words, the coarser the mesh and larger the time step, the greater is the error; also the quadratic element yields more accurate results than the linear element because it can represent the changing shape of the workpiece more accurately.

Marius et al. [23] and Leslie et al.[24] developed software to analyse the ECM problem using the BEM technique. They, of course, used Laplace’s equation expressed in a three-dimensional coordinate system. The effect of polarization (over-potential) is taken into account using a linear relation approximation together with current efficiency. To
discretize the three-dimensional domain, they used a hybrid triangular grid generator to create unstructured and structured triangular meshes on the workpiece surface. The structured meshes were created in the region near the tool surface whereas unstructured meshes were created in the region further away from the tool surface.

2.2 Modelling of shaped tube electrochemical drilling (STED)

The main difference between conventional EC-drilling (ECD) and STED is that the aspect ratio of the hole (i.e. the ratio between the hole diameter and its depth) is around 40 to 250 and the hole diameter can be as small as 0.8 mm [25]. Because the hole is long, the electrolyte cannot be supplied from an external nozzle and therefore, solid cylindrical tools cannot be used to drill the holes. Typically the tools for STED are thin-walled hollow cylinders (see Fig 2.13). The electrolyte flows down the central hole emerging from the bottom of the tool end face.

Fig 2.13: Different views of a STEM drill [25]

In STED, some of the machining parameters have their magnitude changed during the drilling process. For example, the flow rate of the electrolyte keeps decreasing with increasing hole depth. As the hole depth increases, the electrolyte has to flow past a longer length of the drilled hole, causing the pressure drop to increase due to friction between the electrolyte and the wall of the drilled hole. The electrolyte also experiences a throttling effect when it emerges from the tool end face. A decrease in the electrolyte flow rate causes a corresponding decrease in the material removal rate. The reduced material removal rate causes the over-cut value to decrease causing the diameter to decrease, making the hole convergent.

In order to predict the hole shape accurately, the effect that the change in flow rate has on the process should be included in the simulation model. However, there is no
published literature wherein the STED process has been modelled as a coupled flow and field problem; therefore, this effect can only be determined by conducting experiments. Therefore, it is not surprising that most of the published research in STED has been experimental.

Sharma et al. [26] and Bilgi et al. [27-28] studied the effect of different feed rates and D.C. voltages on the variation in diameter with the depth of holes. The holes were drilled in a super alloy material by a partially insulated tool. A mixture of salt solution and acid was used as their electrolyte. Their experimental results show that the machining conditions that resulted in a low value of electrode gap reduced the variation in the diameter of the holes. By using regression analysis, they found that the average hole diameter along the depth of the hole depends on the feed rate, voltage, concentration of salt and acid and bare length, while the material removal rate strongly depends on the voltage and bare length. Bilgi et al. [29] extended their experimental work in [27-28] by using pulsed voltage instead of direct voltage to drill the holes.

The work reported [26-29] by the above researchers was done in a laboratory environment with the length of their holes being around 20-30 mm and a salt solution for the electrolyte. However in industry, the holes are much longer and diluted acid, and not a salt solution, is used as the electrolyte. Although the workpiece surface is cleaned during pulsed machining, the tool surface is still covered by undissolved material from the chemical reaction. To remove the material adhering on the tool surface, the polarity has to be reversed, albeit for a very short time.

Ali et al. [25] used forward-reverse voltage in STED experiments and they studied the effect of operating parameters such as electrolyte pressure, voltage, feed rate and acid concentration on the quality of deep holes drilled in turbine blades. They also discussed the different types of defects occurring in STED.

More recently, there has emerged a need for the cooling channels to have turbulators which can be defined as profiled depressions superimposed on the basic cylindrical shape of the cooling channel at regular intervals. There is a very limited amount of work which investigates the machining of these turbulators.
Recently Wang et al. [30-31] investigated the feasibility of machining spiral-shaped turbulators. In their work, a blind hole was first machined using electro-discharge machining (EDM) after which the spiral turbulator was machined using ECM with a tool that had spiral insulation on its external surface (see Fig 2.14).

![Fig 2.14: (a) Contour pattern [30], (b) Spiral pattern of insulation on tool [31]](image)

Although turbulators can be machined using a tool with insulation at regular intervals (as in Fig. 2.14(a)) or a spirally-insulated tool as in Fig. 2.14(b), the technique presented by Wang et al. is impractical because:

1. it requires a special tool to be made, and
2. drilling the hole with turbulators has to be done in two steps, first drilling a blind hole and then with the partially-insulated tool to machine the turbulators. As a result, the production time and cost of manufacturing are increased.

These disadvantages can be overcome by machining the turbulated hole with one tool by continuously varying the feed rate. Jain et al. [32] experimentally studied the STEM drilling process for producing turbulated holes. In their work, profiled turbulators were created at a specified applied voltage by changing the feed rate from a higher to a slower feed rate to increase the hole diameter and then changing the feed rate back to
the higher value to decrease its diameter. They studied the effects of material type, voltage and feed rate on the profile of the machined holes. However, only very simple turbulator shapes can be produced using only two feed rates.

Noot *et al.* [33] and Wang *et al.* [30] used the finite element method to study how the turbulator shape evolves. However, in the work of Noot *et al.*, very few details of the procedure to produce the turbulator are given, whereas Wang *et al.* [30] used FEM to determine how the shape evolves when it was machined by a partially insulated tool (Fig 2.14 (a)).

### 2.3 Modelling of Electrochemical milling (EC-milling)

In EC cavity die sinking, a complex 3D shape is generated by an axial movement of the tool. This process requires a considerable amount of effort in designing the tool shape as its shape is not identical to that of the workpiece but similar. Usually it is obtained by trial and error and therefore its cost depends on the dimensions and complexity of the workpiece.

The difficulties of designing the tool in EC cavity die sinking can be eliminated by using EC milling in which the required shape is obtained by moving a tool with a relatively simple shape (rectangular, spherical or cylindrical), as in conventional milling, parallel to the workpiece surface.

Kim *et al.* [34] produced micro milling features such as simple grooves and cavities (see Fig 2.15) using ultra-short pulsed voltage. They used two different tools geometries (i.e. cylindrical and disc-shaped (Fig 2.16)) to machine the features. They found that the side walls of the feature were tapered and the taper depended on the tool geometry and not on the pulse on/off time. Features machined by disc-shaped tool had a smaller wall taper than those machined by the cylindrical tool.

To model the EC-milling process, methods that can calculate the current distribution in the three-dimensional domain are required. Therefore analytical models based on the complex variable method cannot be used. Although FDM or FEM methods can be used to calculate the current distribution in a 3-dimensional domain, they do not lend themselves because the entire interior domain has to be re-generated with 3D elements.
after each time step due to the workpiece being machined. Hence, using a simple mathematical relation to calculate current density would be preferable.

Kozak et al. [35] presented their mathematical model to simulate the workpiece machined by a spherical tool. They estimated the current density by assuming a linear variation of the electric potential variation between tool centre and the corresponding point on the workpiece surface which is expressed as

$$i_a = k_e \frac{V - \eta}{\sqrt{(x - x_e)^2 + (y - y_e)^2 + (z - z_e)^2} - R} \quad (2.20)$$

where R is the radius of the tool, and x, y, z and x_e, y_e and z_e the coordinates of points on the workpiece and tool centre respectively (see Fig 2.17). Thus the rate of machining of the workpiece surface can be calculated from

$$\frac{\partial h}{\partial t} = M \cdot i_a \cdot \sqrt{1 + \left(\frac{\partial Z}{\partial x}\right)^2 + \left(\frac{\partial Z}{\partial y}\right)^2} \quad (2.21)$$
Ruszaj et al. [36] derived a mathematical model to calculate the inter-electrode gap size when a flat-ended tool is moved parallel to the workpiece as shown in Fig. 2.18. Their model considers that current flows only in the region directly below the tool end face and stray currents are not considered. Their mathematical results show that, for sculptured surface machining, the tool feed rate plays a major role in the material removal rate and erosion thickness.
2.4 Modelling of Electrochemical turning (ECT)

Axi-symmetric components can also be turned using the ECM process and the tools are said to be either shaped or un-shaped. A shaped tool is similar to a formed tool in conventional turning although, unlike conventional turning, its shape is not reproduced on the workpiece surface. When a shaped tool is used to machine the workpiece, the tool is either stationary or fed towards the workpiece surface in the radial direction. Use of a shaped tool affects not only the final shape of the workpiece but also its surface finish.

Pa [37] investigated the effect of using different tool geometries on the machined surface of the workpiece. His experimental results show that, at the same rotational speed, tool geometry that results in a higher current density such as a tool with a small wedge angle and a small edge radius produces a better surface finish. Hence when shaped tools are used in ECT, tools have to be designed carefully which increases their cost.

To avoid the effort required for designing the tools, tools with end faces having simple analytical shapes i.e. spherical, cylindrical or planar are used in EC-turning. These tools are referred to as being unshaped. When using an unshaped tool, it is moved along the workpiece axis (negative Z-axis direction in Fig 2.19) and/or in the radial direction (negative X-axis). To machine a workpiece accurately, the machining should be carried out on a CNC machine; hence this machining process is sometimes called CNC-ECT.

Using a CNC machine makes it possible to machine a greater variety of workpiece shapes by adjusting the machining parameters such as the tool feed rates in X and Z direction and rotational speed.
Kang et al. [38] developed their CNC-ECT equipment to study the effect of rotational speed and applied voltage on the gap size. They used a tool with a rectangular cross section. Their experiments showed that the material removal rate at higher rotational speeds is less than that at lower speeds and that it increases linearly with the applied voltage.

The subsequent work by Kang et al. [39] used a CNC-ECT machine to make the components shown in figure 2.20 both of which contain curved surfaces. They created the cam surface (Fig. 2.20(a)) by adjusting the rotational speed and feed rate of the tool in the X-direction. To create a curved profile (Fig. 2.20(b)), the feed rate in the Z direction and rotation speed of the workpiece were varied depending on the curvature required, but the tool was not moved in the X-direction.
NC-ECT can be used to reduce the surface roughness of a turned component. Hochen et al. [40 and 41] investigated the surface roughness of a workpiece which was machined with different tool geometries and machining parameters. In their experiments, the use of conventional turning tools (Fig 2.21(a)) and disc-shaped tools (Fig 2.21(b)) were compared; if only the surface roughness is considered, their results show that tool geometry is more significant than machining parameters and disc-shape tools produce better a surface finish than conventional turning tools.

![Diagram of EC turning using a turning tool](a)

![Diagram of EC turning using a disc tool](b)

Fig 2.21: EC turning using a (a) using turning tool [40] and (b) disc tool [41]

Recently, Kunieda et al. [42] have designed a new type of electrode tool (see Fig. 2.22 (a)) for use in micro EC turning. The tool has a rectangular section and has a rectangular slot from which the electrolyte flows. The electrolyte is pumped at high pressure through the channel creating a rectangular sheet of electrolyte which meets the workpiece. By using a constant rotational speed and by varying the longitudinal feed rate of the tool, a profiled turned component as shown in Fig. 2.22(b) was machined.
In contrast to experimental work, a very limited amount of work has been done on modelling the EC-turning process. Ma et al. [43] proposed a mathematical model developed from Faraday’s and Ohm’s laws to calculate the amount of material removed in the pulsed EC-turning process. Their model is a function of the rotational speed and width of the tool, and pulse on/off time. However, their model has limited use as it is one dimensional in nature.

### 2.5 Aim and objectives of the project

The main aim of this project is to develop, test and validate a BE model which can simulate EC-drilling, milling and turning processes. The program should be able to take into account parameters such as tool feed rate, initial gap size and applied voltage.

As stated previously, the finite element and finite difference methods are efficient techniques to simulate the ECM process. However, both these methods are limited to simulating the ECM process in two dimensions. Extending finite elements to three-dimensions would require meshing the entire domain with solid hexahedron or triangular prism elements repeatedly after each time step and since hundreds of time steps are involved before equilibrium conditions are reached, FE and FD methods rule themselves out. Therefore it is not surprising that no research has been reported with a 3D FE model of the ECM process.

Moreover, in ECM, the values of the potential at points in the gap between the tool and workpiece are not required to compute the changing shape of the workpiece.
Calculating the potential at points in the gap only serves to increase the computational time.

Therefore, the boundary element method has been employed to model the ECM process as it reduces the size of the problem by one-dimension i.e. only the surfaces of the tool, workpiece and the bounding surfaces have to be discretised using triangular meshes. Another advantage of the BEM is that the values of the current density are determined directly. This is likely to lead to more accurate results than the FDM and FEM.

In practice, the tool is positioned very close to the workpiece surface (usually around 0.5 mm) but from a BEM viewpoint, it poses a problem in the sense that a node on the workpiece may be very close to another on the tool surface. This proximity introduces errors when the coefficients in the element matrix are calculated using numerical integration. (More details of this are given in Chapter 3). To obtain an acceptable error, the meshes are refined with smaller element sizes. But this increases the computational time. Therefore, to reduce the error without refining the mesh, the linear triangular element developed by Davey et al. [47] wherein the coefficients are evaluated analytically, is used herein.

As the simulation proceeds, the surface of the workpiece also changes at each time step thus it is unavoidable that the shape of the elements on workpiece surface will be deformed which leads to poor quality meshes. Recently, the commercial software for ECM simulation namely “Elsyca” [24] was released and the mathematical models used in the software were explained [23]. However, although unstructured and structured triangular meshes are used in the software, the logic as to when to change the mesh size is not explained. In this report, a mesh refinement method presented by Jose et al. [53-54] and Rivera et al. [55] is employed to divide the element into smaller elements.

In turbulator machining, although Jain et al. [32] have investigated the shape of turbulator by using two feed rates, in practice, it may required several feed rates to produce required shape of turbulators. Therefore, there is need to investigate whether it is feasible to machine a profiled turbulator with one tool using a set of feed rates.
The main objectives of the project are as follows:

(1) To develop and test 2D and 3D BE models for determining the current distribution on the workpiece surface using Laplace’s equation, and hence be able to calculate the new position of the workpiece.

(2) To develop mesh refinement schemes which would detect when elements become distorted and replace with a refined mesh.

(3) To investigate the feasibility of EC milling with an unshaped tool and investigate the effect of machining parameters (tool feed rate, initial gap size and applied voltage) on the shape of workpiece, material removal rate and surface finish.

(4) To test the capability of the model in predicting the workpiece shape in EC milling and EC turning.

(5) To enhance the model so that it can not only simulate STED but also be able to determine a set of feed rates which when used will result in a profiled turbulator.
Chapter 3
Electrochemical Machining and Boundary Element Theory

This chapter describes the fundamental theory of electrochemical machining, the boundary element method and the determination of the element coefficients using analytical integration. This determination of the elements coefficients for a linear triangular plate element was first suggested by Davey et al. [47] but is described herein for the sake of completeness. It will also give the reader an appreciation of the effort and complexity involved in developing the software.

3.1 Determination of current distribution in ECM

The distribution of the current in the gap between the tool and workpiece surfaces is governed by Laplace’s equation. Thus, under steady state conditions the voltage distribution can be expressed as

\[ \nabla^2 V = 0 \]  \hspace{1cm} (3.1)

The solution of equation (3.1) yields the voltage \( V \) at any point in the domain. The current density \( J \) can be calculated from the potential determined from eq. (3.1) and the known electrolyte conductivity \( \kappa_e \).

\[ J = \kappa_e \nabla V = \kappa_e \frac{dV}{dn} \]  \hspace{1cm} (3.2)

The local variation of anode dissolution the eroded distance \( \Delta h \) for a certain time step \( \Delta t \) is given by

\[ \Delta h = N_{\text{eff}} \frac{AJ\Delta t}{zF\rho_a} \]  \hspace{1cm} (3.3)

where \( N_{\text{eff}} \) is current efficiency, \( z \) the valency, \( A \) the atomic weight of the workpiece material, \( \rho_a \) the density of the anode material and \( F \) Faraday’s constant. For a more detailed explanation of ECM theory, see McGeough [1] and De Barr [2].
3.2 Boundary element formulation

In the boundary element method (BEM), the governing equations (i.e. Laplace’s equation (eq. 3.1)) which contain differential terms, is transformed into a system of boundary integral equations. These equations contain integral terms involving variables (i.e. \( V \) and \( \frac{\partial V}{\partial n} \)) which are determined at points (i.e. nodes) on the boundary of the problem domain.

There are some techniques to create boundary integral equation from governing equation, such as the reciprocal theorem, Somigliana’s identity or the weight-residual method. However, in the first two techniques, some assumptions are made for transformation governing equations whereas those assumptions are not required when weight-residual method is used. Hence, weight-residual method can be used to form integral equations in various problems and it is usually used in derivating the integral equation in several numerical techniques such as FEM and BEM. The complete detail of using weight-residual method to form boundary integral equation for BEM can be found in Brebbia [44], EL-Zafrany [45], and F. Paris [46].

By using the weighted residual method for Laplace’s equation (Eq. 3.1), the boundary integral equation with respect to a source point \( i \) can be expressed as follows [44]:

\[
C_i V_i + \int_V V \cdot q^* \, d\Gamma = \int_V q \cdot V^* \, d\Gamma
\]  

(3.4)

where \( q = \frac{\partial V}{\partial n} \) is the gradient of the voltage in the outward normal direction, caused by applying voltage \( V \) on the electrodes. \( C \) is an integration constant for source point \( i \) lying on the boundary, \( V^* \) the fundamental solution of eq. 3.4 and \( q^* = \frac{\partial V^*}{\partial n} \).

The variables \( V \) and \( q \) in eq. (3.4) are the unknown variable and have to be solved. To perform integration in eq. (3.4) the equations representing distribution of value \( V \) and \( q \) along the boundary of the domain \( (V=V(d\Gamma) \text{ and } q=q(d\Gamma)) \) are required. However it is very difficult to find the equations that can be used to calculate the unknown variables on the entire domain.

Hence, in order to apply the BEM, the boundaries of the domain must be discretised into small elements \( (\Gamma e) \); and the distribution of unknown variables is approximated by
shape functions of element (i.e. \( V = V(\Gamma_e) \) and \( q = q(\Gamma_e) \)). Thus eq. (3.4) is transformed into

\[
C_i V_i + \sum_{e=1}^{N_e} \left[ \int_{\Gamma_e} V(\Gamma_e) \cdot q(\Gamma_e, i) \cdot d\Gamma \right] = \sum_{e=1}^{N_e} \left[ \int_{\Gamma_e} q(\Gamma_e) \cdot V^*(\Gamma_e, i) \cdot d\Gamma \right]
\]

(3.5)

### 3.3 Continuous linear elements [45, 46]

In the present work, continuous linear elements are used to model the ECM process in a 2D analysis as in EC-drilling. When this type of element is used, the boundaries of the domain are represented by segments of connected straight lines (see Fig 3.1). An element \( \Gamma_e \) is defined by its two end nodes, \( a \) and \( b \).

With this kind of element, the value of potential and its derivative (value of \( V(\Gamma_e) \) and \( q(\Gamma_e) \) in eq. (3.5)) have a linear distribution in the element. To overcome the difficulty of integrating these variables in eq. (3.5), they are expressed as a function of their values at the end-points (point \( a \) and \( b \) in Fig. 3.1) of the element, using natural co-ordinates.

By using parametric equations together with element definition, the values of those variables are approximated as follows:

\[
V_{\Gamma_e}(\varepsilon) = N_a(\varepsilon) \cdot V_{\Gamma_e,a} + N_b(\varepsilon) \cdot V_{\Gamma_e,b} = (1 - \varepsilon) \cdot V_{\Gamma_e,a} + \varepsilon \cdot V_{\Gamma_e,b}
\]

(3.6)

\[
q_{\Gamma_e}(\varepsilon) = N_a(\varepsilon) \cdot q_{\Gamma_e,a} + N_b(\varepsilon) \cdot q_{\Gamma_e,b} = (1 - \varepsilon) \cdot q_{\Gamma_e,a} + \varepsilon \cdot q_{\Gamma_e,b}
\]

(3.7)

where \( N_a \) and \( N_b \) are the shape functions at the end nodes \( a \) and \( b \), respectively, \( \varepsilon \) the natural coordinate along the elements having values in the range [0,1]
Substituting eqs. (3.6) and (3.7) into eq. (3.5), the boundary integral equation can be re-written in the following:

\[ C_i V_i + \sum_{e=1}^{Ne} \sum_{j=a}^{h} h_{Te,j} \cdot V_{Te,j} = \sum_{e=1}^{Ne} \sum_{j=a}^{h} g_{Te,j} \cdot q_{Te,j} \]  

(3.8)

Hence, the element matrices \( h \) and \( g \) can be expressed as:

\[ h_{Te,j} = \int_{\Gamma_e} N_j(\xi) \cdot q^*(\Gamma_e,i) \cdot d\Gamma \]  

(3.9)

\[ g_{Te,j} = \int_{\Gamma_e} N_j(\xi) \cdot V^*(\Gamma_e,i) \cdot d\Gamma \]  

(3.10)

The length of an element \( (d\Gamma_e) \) in eq. (3.9) and (3.10) can be defined as:

\[ d\Gamma = \sqrt{(dx)^2 + (dy)^2} \]  

(3.11)

The coordinates \( x \) and \( y \) can also be expressed in shape function form similar to eqs. (3.6) and (3.7) as follows:

\[ x(\xi) = (1 - \xi) \cdot x_a + \xi \cdot x_b \]  

(3.12)

\[ y(\xi) = (1 - \xi) \cdot y_a + \xi \cdot y_b \]  

(3.13)

Hence, \( d\Gamma_e \) from eq. (3.11) can be calculated as:

\[ d\Gamma = \frac{dx}{d\xi} \cdot d\xi + \frac{dy}{d\xi} \cdot d\xi = J \cdot d\xi \]  

(3.14)

where

\[ J = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2} \]  

(3.15)

The unit normal vector \( \vec{n} \) is a vector normal to the element computed from the cross product between unit tangent vector of element and the unit vector in the positive z direction (see Fig 3.2):

\[ \vec{n} = \begin{vmatrix} i & j & k \\ (x_b - x_a) & (y_b - y_a) & 0 \\ J & 0 & 1 \end{vmatrix} = \frac{(y_b - y_a)}{J} \hat{j} + \frac{(x_b - x_a)}{J} \hat{j} \]  

(3.16)
In two-dimensions, the fundamental solutions are:
\[ V^* = \frac{1}{2\pi} \ln \left( \frac{1}{r} \right) \]  
(3.17)

\[ q^* = \frac{\partial V^*}{\partial n} = -\frac{1}{2\pi r} \frac{\partial r}{\partial n} \]  
(3.18)

where
\[ r = \sqrt{[x(\varepsilon) - x_i]^2 + [y(\varepsilon) - y_i]^2} \]  
(3.19)

Hence, the rate of change of r with respect to the normal direction is:
\[ \frac{\partial r}{\partial n} = \nabla r \cdot \hat{n} = \frac{(y_b - y_a)}{J} \frac{\partial r}{\partial x} + \frac{(x_b - x_a)}{J} \frac{\partial r}{\partial y} \]  
(3.20)

Substituting and differentiating r partially with respect to x and y in eq. (3.18) to eq. (3.20)
\[ \frac{\partial r}{\partial n} = \frac{1}{J \cdot r} \left[ (y_b - y_a) \cdot (x(\varepsilon) - x_i) + (x_b - x_a) \cdot (y(\varepsilon) - y_i) \right] \]  
(3.21)

Fig 3.2: Tangent and normal vectors for a linear element

Hence, by substituting the fundamental solutions (eq. (3.16) and (3.17)) and the length of element \(d\Gamma_e\) from eqs. (3.14) to (3.9) and (3.10), the element matrices \(h\) and \(g\) can be expressed in terms of \(\varepsilon\) and collocation point i as follows:
\[ h_{\varepsilon,i,j} = \int_{\varepsilon_i}^{\varepsilon_j} N_j(\varepsilon) \cdot q^*(\varepsilon, i) \cdot d\Gamma = \int_{\varepsilon_i}^{\varepsilon_j} N_j(\varepsilon) \cdot q^*(\varepsilon, i) \cdot J \cdot d\varepsilon \quad (j = a, b) \]  
(3.22)

\[ g_{\varepsilon,i,j} = \int_{\varepsilon_i}^{\varepsilon_j} N_j(\varepsilon) \cdot V^*(\varepsilon, i) \cdot d\Gamma = \int_{\varepsilon_i}^{\varepsilon_j} N_j(\varepsilon) \cdot V^*(\varepsilon, i) \cdot J \cdot d\varepsilon \quad (j = a, b) \]  
(3.23)
Therefore the integration of element matrices defined by eq. (3.22) and (3.23) can be numerically evaluated by Gaussian quadrature method stated that:

\[ \int_0^1 F(\varepsilon) \cdot d\varepsilon = \sum_{q=1}^{N_q} W_q \cdot f(\varepsilon_q) \]  \hspace{1cm} (3.24)

where \( W_q \) is the weighting value at \( \varepsilon_q \). When linear elements are used in this 2D analysis, the coefficient of the element matrices are approximated by 4-point Gaussian quadrature as:

\[ h_{i,e,j}(i) = \sum_{q=1}^{4} N_j(\varepsilon_q) \cdot q^*(\varepsilon_q,i) \cdot J \cdot W_q \] \hspace{1cm} (j = a, b) \hspace{1cm} (3.25a)

\[ g_{i,e,j}(i) = \sum_{q=1}^{4} N_j(\varepsilon_q) \cdot V^*(\varepsilon_q,i) \cdot J \cdot W_q \] \hspace{1cm} (j = a, b) \hspace{1cm} (3.26a)

Hence eq. (3.8) can be re-arranged as:

\[ C_i V_i + \sum_{e=1}^{N_e} \left[ \begin{array}{c} h_{e,a}(i) \\ h_{e,b}(i) \end{array} \right] \cdot \left[ \begin{array}{c} V_{e,a} \\ V_{e,b} \end{array} \right] = \sum_{e=1}^{N_e} \left[ \begin{array}{c} g_{e,a}(i) \\ g_{e,b} \end{array} \right] \cdot \left[ q_{e,a} \right] \]  \hspace{1cm} (3.27)

Where \( C_i \) is a constant and equal to solid angle at node \( i \). Equation (3.27) is used to form a system of equations, each equation pertaining to one collocation node in the domain; solution of these equations gives the values of \( V \) and \( q \) at each node.

**3.4 Linear Triangular Element**

In 3D ECM modelling, the domains are discretised into small triangular elements with each element having nodes at the three vertices of the triangle (Fig 3.3). To evaluate the \( h \) and \( g \) coefficients, both numerical and analytical integration is used.

![Fig 3.3: Three-node triangular element](image-url)
3.4.1 Numerical integration of linear triangular elements [46]

A typical linear triangular element (see Fig.3.3) in a three-dimensional Cartesian coordinate system is defined by three nodes. Therefore, the variations of dependent variables, $V$ and $q$ are defined as

$$V_{\Gamma e} = L_1 \cdot V_{\Gamma e,1} + L_2 \cdot V_{\Gamma e,2} + L_3 (\epsilon, \eta) \cdot V_{\Gamma e,3}$$

$$q_{\Gamma e} = L_1 \cdot q_{\Gamma e,1} + L_2 \cdot q_{\Gamma e,2} + L_3 \cdot q_{\Gamma e,3}$$

where $L_1$, $L_2$ and $L_3$ are the shape functions for nodes 1, 2 and 3 respectively. The shape functions are defined in a non-orthogonal coordinate system $(u, v)$ as illustrated in Fig 3.3. The $u$ and $v$ axes are along the vectors $\overrightarrow{21}$ and $\overrightarrow{31}$ respectively. Therefore, the shape functions for the element can be defined as

$$L_1 = 1 - \frac{u}{b} - \frac{v}{a}$$

$$L_2 = \frac{u}{b}$$

$$L_3 = \frac{v}{a}$$

where $a$, $b$ and $c$ are the lengths of each side of triangular and are given by

$$a = |P3 - P1|, \quad b = |P2 - P1| \quad \text{and} \quad c = |P3 - P2|$$

By defining $\epsilon = \frac{u}{b}$, and $\eta = \frac{v}{a}$, and substituting them into eqs. (3.30a) to (3.30c), $V$ and $q$ are rewritten as:

$$V_{\Gamma e}(\epsilon, \eta) = L_1(\epsilon, \eta) \cdot V_{\Gamma e,1} + L_2(\epsilon, \eta) \cdot V_{\Gamma e,2} + L_3(\epsilon, \eta) \cdot V_{\Gamma e,3}$$

$$q_{\Gamma e}(\epsilon, \eta) = L_1(\epsilon, \eta) \cdot q_{\Gamma e,1} + L_2(\epsilon, \eta) \cdot q_{\Gamma e,2} + L_3(\epsilon, \eta) \cdot q_{\Gamma e,3}$$

where $L_1 = 1 - \epsilon - \eta$, $L_2 = \epsilon$ and $L_3 = \eta$

Differentiation of the global coordinate $(x, y, z)$ with respect to $\epsilon$ and $\eta$ axes is given by:

$$d\vec{X} = \left( \frac{\partial x}{\partial \epsilon} i + \frac{\partial y}{\partial \epsilon} j + \frac{\partial z}{\partial \epsilon} k \right) \cdot d\epsilon$$

$$d\vec{X} = \left( \frac{\partial x}{\partial \eta} i + \frac{\partial y}{\partial \eta} j + \frac{\partial z}{\partial \eta} k \right) \cdot d\eta$$
The unit normal vector to the plane of the element can be obtained by applying the cross product of vectors in eq. (3.33) and (3.34) as follows:

\[ \hat{n} = \frac{d\vec{X} \times d\vec{X}}{d\epsilon \times d\eta} = \frac{(J_x \hat{i} + J_y \hat{j} + J_z \hat{k})}{J} = \hat{l} + m\hat{j} + n\hat{k} \]  

(3.35)

where \( J = \sqrt{J_x^2 + J_y^2 + J_z^2} \)

\[
J_x = \begin{vmatrix} \frac{\partial y}{\partial \epsilon} & \frac{\partial z}{\partial \epsilon} \\ \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \end{vmatrix}, \quad J_y = \begin{vmatrix} \frac{\partial z}{\partial \epsilon} & \frac{\partial x}{\partial \epsilon} \\ \frac{\partial z}{\partial \eta} & \frac{\partial x}{\partial \eta} \end{vmatrix}, \quad J_z = \begin{vmatrix} \frac{\partial x}{\partial \epsilon} & \frac{\partial y}{\partial \epsilon} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{vmatrix}
\]

Then, the area of an element can be expressed as:

\[ d\Gamma_e = J \cdot d\epsilon \cdot d\eta \]  

(3.36)

In three-dimensions, integration of the fundamental solution is given by:

\[ V^* = \frac{1}{4\pi r} \]  

(3.37)

\[ q^* = \frac{\partial V^*}{\partial n} = -\frac{1}{4\pi r^2} \frac{\partial r}{\partial n} \]  

(3.38)

\[ r = \sqrt{[x(\epsilon, \eta) - x_i]^2 + [y(\epsilon, \eta) - y_i]^2 + [z(\epsilon, \eta) - z_i]^2} \]  

(3.39)

Hence, the rate of change of \( r \) with respect to the normal direction is:

\[ \frac{\partial r}{\partial n} = \nabla r \cdot n = \hat{l} \cdot \frac{\partial r}{\partial \epsilon} + m \cdot \frac{\partial r}{\partial \eta} + n \cdot \frac{\partial r}{\partial \eta} \]  

(3.40)

By substituting the fundamental solutions (eqs. (3.37) and (3.38)) and the area of element \( d\Gamma_e \) from eq. (3.36), to eq. (3.9) and (3.10), the element matrices \( h \) and \( g \) can be expressed in term of \( \epsilon, \eta \) and collocation point \( i \) as follows:

\[ h_{\epsilon,i,j} = \int_{\Gamma_e} L_j(\epsilon, \eta) \cdot q^*(\Gamma_e, i) \cdot d\Gamma = \int_{0}^{1} \int_{0}^{1} L_j(\epsilon, \eta) \cdot q^*(\epsilon, \eta, i) \cdot J \cdot d\epsilon \cdot d\eta \]  

(3.41)
\[ g_{\varepsilon,i} = \int_{\Gamma} L_j(\varepsilon,\eta) \cdot V^*(\varepsilon,\eta) \cdot J \cdot d\varepsilon \cdot d\eta \] (3.42)

Using Gaussian quadrature, integration of the element matrices in eq. (3.41) and (3.42) can be calculated as:

\[ h_{e,j}(i) = \sum_{s=1}^{N_s} \sum_{r=1}^{N_r} (1-\eta_s) \cdot L_j(\varepsilon_{r,s},\eta_s) \cdot q^*(\varepsilon_{r,s},\eta_s,i) \cdot J \cdot W_r \cdot W_s \quad (j=1, 2, 3) \] (3.43)

\[ g_{e,j}(i) = \sum_{s=1}^{N_s} \sum_{r=1}^{N_r} (1-\eta_s) \cdot L_j(\varepsilon_{r,s},\eta_s) \cdot V^*(\varepsilon_{r,s},\eta_s,i) \cdot J \cdot W_r \cdot W_s \quad (j=1, 2, 3) \] (3.44)

When the triangle is numerically integrated, Gaussian quadrature is used with 8 integrating points (Ns and Nr = 8). Equation (3.8) can be re-arranged as:

\[
\begin{bmatrix}
C V_r \\
+ \sum_{e=1}^{N_e} \begin{bmatrix} h_{e,1}(i) & h_{e,2}(i) & h_{e,3}(i) \end{bmatrix} \begin{bmatrix} V_{e,1} \\
V_{e,2} \\
V_{e,3} \end{bmatrix}
\end{bmatrix} = \sum_{e=1}^{N_e} \begin{bmatrix} g_{e,1}(i) & g_{e,2}(i) & g_{e,3}(i) \end{bmatrix} \begin{bmatrix} q_{e,1} \\
q_{e,2} \\
q_{e,3} \end{bmatrix}
\] (3.45)

3.4.2 Analytical integration of linear triangular elements [47]

To integrate analytically over the triangular element, the boundary integral equation (eq. (3.4)) including the 3D fundamental solution (eqs. (3.37) and (3.38)) can be re-arranged as:

\[
4\pi C V_r \int_{\Gamma} V \cdot \frac{\partial}{\partial n} \left( \frac{1}{r} \right) d\Gamma = \int_{\Gamma} q \cdot \left( \frac{1}{r} \right) d\Gamma
\] (3.46)

For the analytical integration, the non-orthogonal 2-D coordinate system (u,v) is transformed to a polar coordinate system (r,θ). Thus the element area, dΓ, is defined as:

\[ d\Gamma = r \, dr \, d\theta \] (3.47)

Referring to Fig. 3.4, by using trigonometry, u and v can be expressed in terms of r and θ as:

\[ u = \frac{r \cdot \sin(\psi - \theta)}{\sin(\psi)} \] (3.48)

\[ v = \frac{r \cdot \sin(\theta)}{\sin(\psi)} \] (3.49)

60
To express $r$ in eqs. (3.48) and (3.49) in terms of $\theta$, the distance between point 1 and line 2-3 for any angle $\theta$ will be calculated by using the relation:

$$1 - \frac{u}{b} - \frac{v}{a} = 0 \quad (3.50)$$

Substituting for $u$ and $v$ into eq. (3.50):

$$1 - \frac{R(\theta) \cdot \sin(\psi-\theta)}{b \cdot \sin(\psi)} - \frac{R(\theta) \cdot \sin(\theta)}{a \cdot \sin(\psi)} = 0 \quad (3.51)$$

Rearranging eq. (3.51), $R(\theta)$ will be expressed as:

$$R(\theta) = \frac{a \cdot b \cdot \sin(\psi)}{c \cdot \sin(\theta) + \sin^{-1} \left[ \frac{a \cdot \sin(\psi)}{c} \right]} \quad (3.52)$$

To determine shape function $L_j$ in term of $r$ and $\theta$, $u$ and $v$ from eqs. (3.48) and (3.49) have to be substituted into eqs. (3.30a) to (3.30c) to obtain

$$L_1 = 1 - \frac{r}{R(\theta)}, \quad L_2 = \frac{r \cdot \sin(\psi-\theta)}{b \cdot \sin(\psi)}, \quad L_3 = \frac{r \cdot \sin(\theta)}{a \cdot \sin(\psi)} \quad (3.53)$$

From eq (3.46) the term of $\frac{\partial}{\partial n} \left( \frac{1}{r} \right)$ can be expressed as
\[
\frac{\partial}{\partial n} \left( \frac{1}{r} \right) = -\frac{r(i) \cdot \vec{n}}{r^3(i)}
\] 

(3.54)

Note that \( r(i) \cdot \vec{n} \) is the perpendicular distance from the collocation (source) point to the plane of the element.

Consider eq 3.46, the terms to be integrated are

\[
\int_{\Gamma} V \cdot \frac{\partial}{\partial n} \left( \frac{1}{r} \right) d\Gamma \tag{3.55}
\]

and

\[
\int_{\Gamma} q \cdot \left( \frac{1}{r} \right) d\Gamma \tag{3.56}
\]

Substitution of eqs. (3.28), (3.29), (3.47) and (3.54) into eqs. (3.55) and (3.56), the integral terms in eq. (3.46) become:

\[
\int_{\Gamma} V \cdot \frac{\partial}{\partial n} \left( \frac{1}{r} \right) d\Gamma = \int_{\Gamma} L_j \cdot \frac{\partial}{\partial n} \left( \frac{1}{r_j} \right) d\Gamma = -r_j \cdot \vec{n} \int_{0}^{\varphi}(\theta) \int_{r_j}^{r} \frac{L_j}{r} dr d\theta \quad j=1, 2, 3 \tag{3.57}
\]

and

\[
\int_{\Gamma} q \cdot \left( \frac{1}{r} \right) d\Gamma = \int_{\Gamma} L_j \left( \frac{1}{r_j} \right) d\Gamma = \int_{0}^{\varphi}(\theta) \int_{r_j}^{r} L_j r dr d\theta \quad j=1, 2, 3 \tag{3.58}
\]

To integrate eqs. (3.57) and (3.58), the relative position of the source point compared with the element plane must be determined. There are three possible cases for the relative position:

(i) the source point coincides with one of the nodes of the element;
(ii) the source point is outside the element but still lies in the same plane;
(iii) the source point is not in the same plane as the element and is positioned at an arbitrary point in three dimensional space.

For the first two cases, the dot product \( r_i \cdot n \) is zero. This means \( r \) is laid on the element plane. Thus, in these cases only eq.(3.58) has to be integrated. However, in the second case, to obtain the integration over the element, the triangular element is divided into three sub-triangles as shown in Fig 3.6 and the integral obtained as follows.
The above equation can also be used for the first case i.e. when the source point is at a node of each sub-triangle. Thus, for example, if the source point \( i \) is at node \( l \), the second and third terms vanish.

In the case when the source point is at an arbitrary position, the dot product \( (r_j \cdot n) \) is not equal to zero, requiring eqs. (3.57) and (3.58) to be integrated. To carry out the integration, firstly the source point is projected onto the element plane. Hence either the integral technique used in the first or the second case is applied to integrate both equations.

![Fig 3.5: Domain of integral \( \Delta i12, \Delta i13 \) and \( \Delta i23 \)](image)

The final system of equation is still similar to eq 3.45 which is:

\[
4\pi C_i V_i + \sum_{e=1}^{N_e} \begin{bmatrix} h_{e,1}(i) & h_{e,2}(i) & h_{e,3}(i) \\ V_{e,1} & V_{e,2} & V_{e,3} \end{bmatrix} \begin{bmatrix} \mathbf{V}_e \\ \mathbf{q}_e \end{bmatrix} = \sum_{e=1}^{N_e} \begin{bmatrix} g_{e,1}(i) & g_{e,2}(i) & g_{e,3}(i) \\ \mathbf{g}_{e,1} & \mathbf{g}_{e,2} & \mathbf{g}_{e,3} \end{bmatrix} \begin{bmatrix} \mathbf{V}_e \\ \mathbf{q}_e \end{bmatrix}
\]

where \( H_j \) and \( G_j \) (\( j=1,2,3 \)) are calculated from eqs. (3.57) and (3.58), respectively.

From the above, it is clear that the analytical method of the coefficients would take longer to compute than the numerical technique. However, when the source point is located very close to the element, analytical integration is used because the integration is exact [47]. Therefore both integration schemes are used in the program. Analytical
integration is used when the minimum distance from a source point to one of the nodes of the element is less than the maximum length of the element edges.

3.5 Assembly of the system of equation

To solve for the unknown at each node which may be either $V$ or its gradient $q$, equation (3.60) formed at every source point $i$ has to be assembled and then the boundary conditions are applied. The assembled matrices are then re-arranged into a form so that they can be solved by the Guassian elimination technique.

3.5.1 Assembly of the system of equations

Consider a 2D domain which is discretised into three linear elements, as shown in Fig 3.6.

Fig 3.6: Linear elements representing a triangular domain.

The system of equations (eq. (3.27)) when $i=1$ is expressed as:

$$
C_1 V_1 + (h_{v1,1} V_1 + h_{c1,2} V_2) + (h_{c2,1} V_1 + h_{c2,2} V_2) + (h_{c3,1} V_1 + h_{c3,2} V_2) = (g_{c1,1} q_1 + g_{c1,2} q_2) + (g_{c2,1} q_2 + g_{c2,2} q_3) + (g_{c3,1} q_3 + g_{c3,2} q_1)
$$

(3.61)

Hence, eq 3.61 can be re-arranged as:

$$(C_1 + h_{v1,1} + h_{c3,2}) V_1 + (h_{v1,1} + h_{c2,2}) V_2 + (h_{v1,1} + h_{c3,1}) V_3 = (g_{c1,1} + g_{c3,2}) q_1 + (g_{c1,2} + g_{c2,2}) q_2 + (g_{c2,2} + g_{c3,1}) q_3
$$

(3.61a)

Eq (3.61a) can be presented in matrix form as:
\[
(C_1 + h_{e,1} + h_{e,3,2}) (h_{e,2,1} + h_{e,2,2} + h_{e,3,1}) \begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix}
= \begin{bmatrix}
g_{e,1} + g_{e,3,2} \\
g_{e,2,1} + g_{e,2,2} \\
g_{e,2,2} + g_{e,3,1}
\end{bmatrix} \begin{bmatrix}
q_1 \\
q_2 \\
q_3
\end{bmatrix}
\]  
(3.62)

or

\[
\begin{bmatrix}
H(1,1) & H(1,2) & H(1,3) \\
H(2,1) & H(2,2) & H(2,3) \\
H(3,1) & H(3,2) & H(3,3)
\end{bmatrix} \begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} = \begin{bmatrix}
G(1,1) & G(1,2) & G(1,3) \\
G(2,1) & G(2,2) & G(2,3) \\
G(3,1) & G(3,2) & G(3,3)
\end{bmatrix} \begin{bmatrix}
q_1 \\
q_2 \\
q_3
\end{bmatrix}
\]  
(3.62a)

If eq. 3.27 is applied to every source point of the domain in Fig 3.7 (i=1 to 3), the resulting system of equations in matrix form become:

\[
\begin{bmatrix}
H(1,1) & H(1,2) & H(1,3) \\
H(2,1) & H(2,2) & H(2,3) \\
H(3,1) & H(3,2) & H(3,3)
\end{bmatrix} \begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} = \begin{bmatrix}
G(1,1) & G(1,2) & G(1,3) \\
G(2,1) & G(2,2) & G(2,3) \\
G(3,1) & G(3,2) & G(3,3)
\end{bmatrix} \begin{bmatrix}
q_1 \\
q_2 \\
q_3
\end{bmatrix}
\]  
(3.63)

In general, when the domain boundary is discretised into K nodes, the number of unknown variables is also equal to K and the system of K equations can be represented as:

\[
[H]_{(K\times K)} [V]_{(K\times 1)} = [G]_{(K\times K)} [q]_{(K\times 1)}
\]  
(3.64)

### 3.5.2 Application of boundary conditions

The boundary conditions that are imposed are illustrated in Fig 3.7. Part of the tool (cathode) surface may be insulated, i.e. \( \frac{dV}{dn} = 0 \). Other parts of the tool i.e. bare tool, have a voltage applied to them which is the difference between the actual voltage applied and the sum of over potentials and electrode potentials. The voltage on the workpiece surface (anode) is set to be zero (anode). The remaining boundaries which are virtual and relatively far away from the tool and workpiece are assumed to be insulated, i.e. \( \frac{dV}{dn} = 0 \).
The values of $V$ and $q$ in eq (3.64) are determined after the boundary conditions are applied to the equation. Consequently, the system of equation will be in the form:

$$[A]_{(KxK)}[x]_{(Kx1)} = [B]_{(Kx1)} \quad (3.65)$$

where $[A]$ is coefficient matrix, $[x]$ unknown matrix and $[B]$ constant matrix.

The system of equations (eq. (3.65)) is solved for the unknown variables using the Gaussian elimination technique.

### 3.6 Corner problem

The variables that are calculated are the voltage $V$ and its gradient $\frac{dV}{dn}$. A field variable is expressed by only its value at the node while the flux is expressed by its value and its direction which is normal to the surface. However, when continuous elements are used to discretise the domain, the flux is calculated at the node of the element. Therefore, to use the method discussed earlier to calculate the flux variable correctly, neighbouring elements (e.g. elements e1 and e2 in Fig 3.8) should also share the same normal which means that the two elements must have continuity of slope at the common node k.
If the boundary is not smooth as at node k in Fig. 3.9, the normals to elements e1 and e2 will not be in the same direction. This problem is referred to as the corner problem. When there is a corner problem, there is an error in the computed value of the flux.

To deal with the corner problem, discontinuous or partially discontinuous elements can be used [48]. These elements still require the domain to be discretised into straight lines (in the case of 2D analysis) or triangles (in the case of 3D analysis) but the nodes are not located at the ends of the edges. Instead they can be located anywhere along the edges or even within the element (see Fig. 3.10). As before, the field and flux variables will be calculated at the nodes. This requires, of course, different shape functions. Whilst this is not difficult, analytical integration will certainly present difficulties.

Another technique to cope with the corner problem without changing the element type is to use the double node technique introduced by Brebbia [49]. Using this technique,
continuous elements can be used to discretize the domain but the elements are not rigidly connected to other adjacent elements (see Fig 3.11). Thus, each node will have two values for the flux variable, one for each element. This approach means that the same functions and analytical integration can be used. However, if double nodes are used to mesh the entire domain boundary, the number of equations is increased by 2 times in the case of linear elements and by 3 times in the case of linear triangular elements. Consequently, the computing time to solve these equations is increased.

In ECM analysis, the current density is required to determine the amount of machining that takes place at the workpiece surface. During simulation, the workpiece surface has to be checked to see if it is smooth; if it is not, then a mesh refinement technique, which is discussed in Chapter 4, is used to refine the mesh further. Therefore, it is not necessary to use the double node technique for elements on the workpiece surface. However, there are sharp corners on the tool where the end face meets the side face as shown in Fig 3.12a. Hence, the double node technique will only be used at the tool corners (see Fig 3.12b).
3.7 Workpiece shape change calculation

After each time step, the workpiece shape is re-generated by moving each node on the workpiece surface through a specified distance ($\Delta h$) along the normal vector at the node. This distance is the amount that the node has been machined by and is given by eq. (3.3).

Since the workpiece surface is modelled by linear plate-like triangular elements, the workpiece is not completely smooth – it will be faceted which means there will be several normal vectors at each node – each element that this node is common to will contribute one normal vector. Therefore, the unit normal vector ($\vec{n}_{nd}$) at a node is given by the average of the normal vectors to the elements sharing the node calculated by summation of the element normal vector ($\vec{n}_e$) of elements connected to the node as:

$$\vec{n}_{nd} = \frac{1}{Ne} \left( \sum_{e=1}^{Ne} \vec{n}_e \right)$$  \hspace{1cm} (3.66)

![Fig 3.13: Normal vector on a node surrounded by (a) linear and (b) triangular element](image)
Chapter 4  
Mesh Generation and Refinement

To model the ECM process using the BEM, the bounding faces of the analysis domain have to be discretised into elements. In the present work, for a two-dimensional domain, all the edges are discretised into line elements and for a three-dimensional domain, all the bounding faces are discretised into triangular elements.

During the simulation of the ECM process, the shape of the workpiece surface changes after each time step. As the result, some elements become distorted and the quality of the mesh becomes poor. This requires the existing mesh to be modified which usually involves the distorted elements being refined into several smaller elements.

This chapter explains the mesh generation and refinement techniques which have been developed to discretise a 2D domain into linear elements and a 3D domain into linear triangular elements.

4.1 Mesh generation techniques

4.1.1 Line element

If the problem can be analyzed as a 2-D problem (as in EC drilling), then the boundary of the domain is represented by a combination of lines, arcs and/or free-form curves. The bounding lines are discretised into line elements which can be either linear or quadratic (Fig 4.1(a) and (b)). A linear element is represented by its two end points whereas a quadratic element is represented by three points, i.e. two end points and one mid-point. Since quadratic elements allow a second degree variation of the governing variable, they are more accurate than linear elements. However, using quadratic elements requires a much longer computing time although fewer of them may be required. The other problem of using quadratic elements is the additional book keeping required to re-arrange the connectivity between node and element when it is refined. Therefore, in this work, linear elements are used to approximate the boundary of the domain.
4.1.2 Triangular element

When meshing the bounding faces of a 3D domain, quadrilateral elements can be used. When compared to triangular elements, fewer elements are required to mesh a given domain. For example, consider the square face in Fig 4.2(a), only 9 quadrilateral elements (Fig 4.2(b)) are required as opposed to 18 triangular elements (Fig 4.2(c)). It is known that quadrilateral elements, for the same mesh density, result in a better accuracy. However, triangular elements have the big advantage that curved surfaces can be represented more accurately and therefore in this research they are used exclusively.

There are various techniques to create triangular elements on surfaces, such as Delaunay triangulation, advancing front technique, quadtree technique and the Coring technique.
In the first two techniques, triangular meshes are created directly on the surface whereas in the remaining techniques, square elements are first created and then they are split into triangular elements. Hence, to reduce the time in the triangulation process, Delaunay triangulation and the advancing front technique are commonly used to create triangular elements.

The mesh created using Delaunay’s method satisfies the empty-circle property which means that the circle circumscribing each triangle does not contain any other node (see triangle A in Fig 4.3). The triangulation is initiated by constructing triangles from the nodes lying on the surface. The Delauney’s triangulation tries to maximize the smallest angle and minimizes the maximum angle, i.e. it tries to minimize the angle between the difference between the maximum and minimum angles in a triangle. The ideal solution is, of course, when all the angles are equal, i.e. as in a equilateral triangle. But the Delauney algorithm requires the grid of points to start the meshing points and no new point are inserted during the meshing procedure. The quality of the mesh is dependent on the density and distribution of the grid points presented to the algorithm.

A better control is obtained with the advancing front technique wherein the user can control the mesh density user input. The advancing front technique constructs a new triangle directly in the region that has not been meshed. Unlike the Delauney algorithm, it does not require an initial grid of points, mesh of triangles or techniques to modify/delete elements. Because of these advantages, the advancing front technique has been used herein to generate unstructured triangular meshes on planar and curved surfaces.

Fig 4.3: Empty circle property of Delauney’s algorithm.
The procedure to create a mesh is as follows [51]:

(i) The boundaries of the domain are initially discretized resulting in a union of simple straight line segments called the front (see Fig 4.4(b)).

(ii) The triangulation process is commenced by selecting the shortest line segment from the front (for example line AB in Fig 4.5(a)). A new point or front node (node C in Fig 4.5(a)) is created. The distance (h) of the point from the segment AB depends upon the length of the current front (AB) and the required mesh density as input by the user. A new triangle is generated by either creating a new interior point (see ΔABC Fig. 4.5(b)) or by creating a triangle from an existing point on the front (see ΔBCD Fig. 4.5(c)).
(iii) Insertion of a new triangle causes the front to change. Some new segments have to be inserted and others deleted. For example, formation of triangle ABC means that the original front AB should be deleted and segments $AC$ and $BC$ added to the front (see Fig.4.5 (b)).

The above generation process is repeated until there are no more line segments in the front (see Fig 4.6(b)).

For meshing a curved surface the procedure is similar to that described above but in order to create a new interior point, the “point to surface projection” step is required to bring the new node back on to the surface [52].

In order to determine the quality of a triangle, a criterion based on the shape and size of element is defined [51]. To judge the quality of a triangle (say ABC), the parameter $\alpha$ is calculated.

$$\alpha(ABC) = 2\sqrt{3} \frac{\|AB \times AC\|}{\|AB\|^2 + \|BC\|^2 + \|CA\|^2} \quad (4.1)$$

The value $2\sqrt{3}$ is a normalizing factor so that an equilateral triangle will have the maximum value of $\alpha$ as 1. However, in a graded mesh in which the element size changes progressively, the deviation of the element size from the required value should also be taken into account. In view of the element size effect, another parameter, $\beta$, is proposed to judge the quality of triangular element which is
\[ \beta = \lambda \alpha \]  \hspace{1cm} (4.2)

where \( \lambda = \frac{s}{\rho} \left( 2 - \frac{s}{\rho} \right) \) is a factor accounting for the deviation between the required size \( \rho \) and the actual element size \( s \). The target for lambda is 1.

4.2 Mesh refinement

Since the shape of the workpiece surface changes after each time step, it causes some of the line elements on the workpiece to shrink and others to be stretched; in the case of triangular elements, the elements become distorted. To deal with this problem, the distorted or stretched elements on the workpiece have to be modified using mesh refinement techniques.

In this section, a procedure which is based on a geometrical criterion has been developed for mesh refinement. This criterion is used to decide whether the elements on the workpiece surface have to be refined or not. The mesh refinement techniques based on sub-dividing the distorted elements are discussed in this chapter.

4.2.1. Geometrical criterion for refinement

The criterion used in the program is based on the included angle \( \theta \) between the unit vectors \( \vec{N}_1 \) and \( \vec{N}_2 \) which are normal to adjacent line elements \( (L_1, L_2) \) as shown in Fig 4.7. The normal vector of a linear element defined by vertices \( P_1, P_2 \) is a unit vector calculated by the cross product between a unit vector in positive z direction \( (0i+0j+1k) \) and the tangent vector \( (\overrightarrow{PP_2}) \):

\[ \vec{N} = \frac{\overrightarrow{PP_2} \times Z}{\|\overrightarrow{PP_2} \times Z\|} \]  \hspace{1cm} (4.3)

![Fig 4.7: Angle \( \theta \) between the normal vectors of adjacent line elements](image)
For a triangular element, \( \theta \) is the angle between unit vectors \((N_1, N_2)\) which are normal to the planes of the adjacent triangles \((T_1, T_2)\) as shown in Fig 4.8.

![Fig 4.8: Angle \( \theta \) between the two normal vectors of adjacent triangles](image)

The vectors \( N_1 \) and \( N_2 \) are normal to the planes defined by vertices \( P_1, P_2, P_3 \) and \( P_3, P_2, P_1 \) respectively. For example, \( N_1 \) is given by:

\[
N_1 = \frac{P_1P_2 \times P_1P_3}{\|P_1P_2 \times P_1P_3\|}
\]  

(4.4)

Thus, two adjacent line elements or triangles have to be sub-divided if \( \theta \) is greater than the maximum allowable pre-set angle \((\theta_{\text{max}})\). Thus, subdivision/refinement is performed if the following geometrical criterion is satisfied.

\[
\arccos(N_1.N_2) > \text{maximum allowable angle} \,(\theta_{\text{max}})
\]  

(4.5)

**4.2.2 Mesh refinement: Linear element**

In the case of linear elements, each of the two elements is refined by inserting a new point at the centre of each element and creating two new elements. For example, consider element \( L_1 \), defined by point \( P_1 \) and \( P_2 \) (see Fig 4.9(a)). When point \( m_1 \) is inserted at its centre, element \( L_1 \) is defined by nodes \( P_1 \) and \( m_1 \), and the new element \( L_3 \), by nodes \( m_1 \) and \( P_2 \) as shown in Fig 4.8(b).
4.2.3 Mesh refinement: Triangular element

4.2.3.1. Primary and secondary elements

The triangles to be subdivided are classified into two types, primary and secondary. When the angle ($\theta$) is greater than the preset angle, then the two adjacent triangles are said to be primary. If a primary triangle is connected to another triangle through an edge and not a node and if the angle between them is less than $\theta_{\text{max}}$, then the other triangle is said to be secondary. Fig.4.10 shows two secondary triangles, $t_1$ and $t_2$ which are connected to the primary triangle $t_0$. Note that the secondary triangles have only one mid-side node which is referred to as non-conforming. Since secondary triangles contain only one or two non-conforming nodes, they are also referred to as non-conforming triangles.
4.2.3.2. Mesh refinement techniques

Subdividing techniques based on bisecting the triangular edges are used for mesh refinement in the program. Three subdividing techniques have been used: the four-triangle longest-edge (4T-LE); the four-triangle self similar (4T-SS) and the longest-edge partition (LE) [53-55]. The first two techniques, 4T-LE and 4T-SS, are employed for refining primary triangles while the third (LE) is used for secondary triangles.

In the 4T-LE technique[53], all the edges of two adjacent triangles are first bisected by introducing a new mid-point on each edge and then new sub-triangles are created by joining the midpoint on the longest edge with the vertex directly opposite it (Fig. 4.11(a)) and then with the other two midpoints (see Fig 4.11(b)). Therefore, by using this technique, two sub-triangles, \( t_o \), similar to the original triangle and two new similar sub-triangles, \( t_n \), called second generation triangles, are created at each refinement step. In the program, the 4T-LE refinement technique is used for refinement when the largest internal angle of the triangle is obtuse.

Similarly, in the 4T-SS technique [54] mid-side nodes are first created on each of the three edges. However, new edges are created by joining the mid-side nodes and these edges are parallel to the original edges (see Fig 4.12). Obviously, this technique also yields four sub-triangles, \( t_o \), similar to the original one. The 4T-SS technique is used to refine a triangle when all the internal angles in the original triangle are less than 90º.

In the longest-edge (LE) [55] partition technique, a triangle is subdivided into two sub-triangles by simply joining the midpoint of the longest edge of the original triangle with the vertex directly opposite it as depicted in Fig 4.11(a).
4.2.3.3. Progressive mesh refinement (Secondary refinement)

Since secondary triangles have a non-conforming node, the 4T-SS and 4T-LE cannot be deployed but the LE technique can be used to subdivide the triangle into two. Figure 4.13(a) shows secondary triangles A and B subdivided into two triangles each. Unfortunately not every instance of a secondary triangle can be directly refined by the LE technique. Difficulties arise when the non-conforming node does not lie on the longest edge. One such example (node m) is triangle C in Fig.4.13 (a). In such cases, another non-conforming node (node n in Figure 4.13(a)) is created at the mid-position of the longest edge. The secondary triangle is now sub-divided into three triangles by firstly joining the new non-conforming node with the original non-conforming one and then by joining the new non-conforming node with the vertex opposite it as shown in Fig 4.13 (c). The new non-conforming node causes the adjacent triangle (labeled D) to be subdivided into two; triangle D is also referred to as a propagation triangle (see Fig 4.13 (b). After that the first original secondary triangle, C, is sub-divided into 3 triangles by firstly joining the point on its longest edge with the non-conforming one and then by joining the node on longest edge with the opposite vertex as shown in Fig 4.13 (c).
4.3. Example of using mesh refinement technique

This example demonstrates the improvement that can be obtained in the shape of the workpiece when mesh refinement techniques are used. Fig. 4.14(a1) shows the current shape of the workpiece surface. Most of these elements have stretched and become distorted and these distorted elements are shown in Fig. 4.14(b1) in red colour. Similarly, Figs. 4.14(a2) and (b2) show linear elements which have stretched.

By applying the geometrical criterion for refinement as discussed in section 4.2.1, the distorted triangular elements are subdivided into smaller elements and the refined mesh on the workpiece surface is shown in Fig 4.14(c1). Obviously the refinement process results in new nodes being inserted on the workpiece surface. Comparing the meshes in Figs. 4.14(b2) and Fig. 4.14(c2), it is clear that the refinement process has resulted in an increased number of smaller elements. Although there are more elements on the surface or boundary, the geometry of the surface or boundary has not changed as yet.

However, in the next time step, the new nodes will improve the predicted values of current density and as a result the surface (or boundary) will be smoother. The workpiece shape predicted in the next time step with the refined mesh is shown in Figs. Fig 4.14 (d1) and (d2). Note that the surface shown in Fig 4.14(d1) is much smoother than that shown in Fig 4.14(c1) (compare the two insets). The same is true for Fig 4.14(c2) and (d2).
Fig. 4.14: Shape of workpiece when some elements on workpiece are refined
Chapter 5
The Boundary Element-ECM Program

In this chapter details are given of the in-house boundary element ECM program that has been developed in a MATLAB environment. The program developed can be divided into three main subprograms. The first deals with creation of the domain geometry for analysis, the second with the actual analysis, and the third with mesh refinement and re-meshing.

A typical example in EC milling which is modelled by the program is illustrated in Fig 5.1. The following sections explain how the domain required for analysis is created and current density is determined.

![Fig 5.1: Schematic view of the workpiece, cutting tool and feature to be machined in EC milling](image)

5.1 Geometry subprogram

The main tasks performed by the geometry subprogram are to create the domain and to discretise the domain. The domain is created by defining planar and cylindrical faces which bound it. The faces themselves are defined by edges. For example, the rectangular face of the workpiece S1 (see Fig 5.2), is formed by four edges (E1 to E4). The cylindrical face (S2) of the tool is defined by edges E5 and E6. The tool end face, S3, is represented by a circular edge E5.
Virtual faces are created on the sides and top so that a closed shell of faces is formed (Fig. 5.3(a)). These virtual faces are sufficiently far away from the machined region on the workpiece face so as not to influence the computed results. Different views of the domain are shown in Fig. 5.3(a) and (b).

Once the domain is created, it is discretised into triangular elements using an in-house mesh generation program. The mesh generation program uses the advancing front technique to create linear triangular elements over each surface of the domain. The program offers two options: structured and unstructured meshes. In this example, structured meshes are created on the planar workpiece face and the cylindrical face of the tool (see Fig 5.4(a)) and unstructured meshes on the remaining surfaces of the domain (Fig. 5.4(b)) whereas Fig. 5.4(c) shows the entire domain discretised.
In the geometry subprogram, the boundary conditions are also defined. The condition \((V - \eta)\) is imposed the bottom face of the tool. If the tool is to be insulated, then the condition \(\frac{dV}{dn} = 0\) is imposed on the cylindrical face of the tool. On the other hand, if it is not insulated, the condition \((V - \eta)\) is also imposed on the cylindrical face. The workpiece surface is assumed to be at zero voltage. Because the virtual surfaces are far away from the tool faces, the flux value on these surfaces will be small. Therefore the virtual faces are assumed to be insulated.

5.2 Analysis subprogram

The electrical and physical data are required as input by this program. An example of the input data required to carry out the analysis of the EC milling process as shown in Fig.5.1, is shown below.

(i) The tool is fed in the X-axis direction and the tool feed rate has to be specified.
(ii) The voltage applied on the tool surface \((V - \eta)\).
(iii) The conductivity of the electrolyte ($\kappa_e$)
(iv) The over potential ($\eta$)
(v) Current efficiency ($N_{eff}$)
(vi) The workpiece material and
(vii) Workpiece density

Note that the chemical equivalents (the ration of atomic weight to valency electron, $A/z$) of each element in workpiece material is calculated by percentage by weight method which is explained in [1].

With the data provided by the geometry subprogram, the analysis program calculates the current flux at each of the nodes on the workpiece surface and on the bottom surface of the tool. To solve for the current flux, a system of equations has to be assembled using the boundary element method, with each source point (i.e. node) giving rise to one equation. The equations can be expressed in matrix form as:

$$[H][V] = [G][\phi]$$  \hfill (5.1)

where $[H]$ and $[G]$ are matrices depending on the geometry and boundary conditions of the domain. $[V]$ and $[Q]$ are column matrices for values of voltage and flux at each of the nodes on the domain surfaces. Analytical and numerical integration was used to calculate the coefficients of the $H$ and $G$ matrices to prevent any errors creeping into the analysis (see Chapter 3).

The standard Gaussian elimination technique is employed to solve the system of equations. Solution of these equations yields either the voltage or the current flux at a node. In the case of nodes lying on the workpiece, the values of the current flux would be determined. These values of current flux are then used to determine the amount by which the workpiece surface has been machined according to Faraday’s laws (equation 3.3). Each node on the workpiece surface is moved in a direction of its normal by an amount proportional to the current density at the node.

After every time step it was noticed that the current density on the workpiece surface had positive values not only in the region directly under or close to the tool but one had to traverse some distance away from the tool before its magnitude became zero. The
blue region on the workpiece surface in Fig. 5.5 shows where the current flux is virtually zero.

Fig 5.5: Distribution of current density on the workpiece (Ampere/mm²)

Therefore, nodes on the workpiece can be divided into two groups i.e. active and passive nodes. Nodes that have a current density value greater than zero are said to be ‘active’ whereas nodes that have a very small value of current density (less than 0.01 Ampere/mm²) are said to be ‘passive’ (Fig 5.6).

Fig 5.6: Active and passive nodes

When a passive node is considered as a source node and its equation is being formed, both the parameters associated with it are known because the voltage is zero since it lies
on the workpiece surface and the current density is also almost zero. Therefore, there is no unknown to be solved at this node and no equation should be formed i.e. it should not be considered as a source point. Consequently, when the number of source points is decreased, the number of equations to be solved is also decreased and with it the computing time.

In the first iteration, all the nodes on the workpiece are used as source points and the current density is computed at each node on the workpiece surface. Before the second iteration, active nodes have to be identified i.e. nodes which have a current density greater than 0.01 Ampere/mm². But this process of identification should not be repeated in the subsequent iterations. Therefore, at the end of the first iteration, distances from the tool centre to each active node are calculated (r₁ to rₙ in Fig 5.7) and the maximum value ‘‘scanning distance (r_max)’’ determined. This distance is used to find the active nodes in the subsequent iterations.

To reduce the time to find the active nodes on the workpiece surface, a set of the nodes close to the tool is formed. The nodes in this set contain the currently active nodes (pink nodes in Fig 5.8) and the passive nodes that are neighbours to the outermost ring of pink nodes (green nodes).
When the tool moves to a new position, a node whose distance is less than $r_{\text{max}}$ is considered to be an active node. For example, the blue nodes in Fig 5.9 are considered active nodes in the next time step. Hence of all the nodes on the workpiece surface, only the blue nodes are used as source points.
By classifying nodes as being active or passive, it was possible to reduce the computing time in a time step by as much as 40% (e.g. from 183 s to 108 s for a time step).

5.3 Mesh refinement and re-meshing subprogram

Since the workpiece surface changes shape after each time step, some regions of the surface become stretched and distorted (see Fig 5.10(a)). Therefore, the mesh in these distorted regions has to be refined otherwise the surface will be poorly represented. Hence, some elements in the distorted region must be subdivided into smaller elements if the geometrical criterion mentioned in Chapter 4 is to be satisfied. The mesh refinement method is used for this purpose and the refined mesh is shown in Fig 5.10(b).

Another problem in the modelling of EC milling or turning process is that the elements on the virtual surface that are connected to the tool edges become distorted. This is because the tool must be moved in the feed direction (EC-milling) or the workpiece rotated (EC-turning). For example, consider the meshes on the top virtual face of the domain (Fig 5.11(a)). As the tool is fed in the direction of the arrow, some elements connected to the tool edge are compressed and others stretched (see Fig. 5.11(b)).
Subsequent tool movements will cause the compressed elements to become even smaller whilst the ratio of the longest to shortest side of the stretched elements becomes larger. The distortion of these elements will affect the accuracy of the computed results. This problem was solved by deleting and re-generating at each time step the mesh on the virtual face connected to the tool edge. For example, the mesh on the virtual surface after re-generation is shown in Fig 5.11(c).

Fig 5.11: (a) Original mesh (b) deformed meshes (c) mesh after re-generation

Apart from the electrical and chemical constants, a word address program is input which is then decoded and the movements of the tool derived. Each movement of the tool is then subdivided into several time steps depending upon the feed rate. The program is terminated when all the motion blocks have been executed.
5.4 Conclusion

The highlights of the program developed are as follows:

(1) It uses both analytical and numerical integration to calculate the element matrices.
(2) Computing time is reduced by classifying the nodes as being active or passive.
(3) The mesh on the workpiece surface is automatically refined as the workpiece is progressively machined.
(4) The mesh on some of the virtual faces is automatically re-generated when the mesh quality deteriorates.

From the highlights, as discussed above, the developed program is sufficiently flexible to model different ECM operations such as EC-drilling, milling and turning. The overall program structure is shown in Fig 5.12.
Fig 5.12: Structure of the analysis program

1. Geometry formation
   Define the surfaces of the model

2. Mesh generation
   Mesh the surfaces of the domain

3. Geometry definition
   Define the boundary conditions and specify the electrical and physical data

4. t=0 s or N=1

5. Solve the equations to calculate the current flux

6. Determine the new position of workpiece surface

7. Refine the distorted elements on workpiece surface

8. Move the tool to new position

9. Re-mesh workpiece surface and re-define boundary conditions and mesh topology

10. t=t+Δt or N=N+1

Start
Fig 5.12: Structure of the analysis program (continue)

1. t < process time or N < total movement
   - Yes: Final shape of workpiece
   - No: 2

2. End
Chapter 6

Three Dimensional EC drilling modelling

The main objectives of this chapter are to model the 3D EC drilling process and to verify the modelling results by comparing them with known analytical solutions.

In this chapter, the 3D model for EC drilling, the boundary conditions and the assumptions made in modelling are described and the results for the insulated and un-insulated tool are presented. The shape of the holes predicted with and without using mesh refinement techniques are compared.

At this moment, there is no known analytical method that can be used for solving a 3D potential problem such as that encountered in EC drilling. However, in 2D, Collettt et al. [13] and Hewson-Browne [14] have used a method called the complex variable method for analytically solving the 2D EC drilling problem. Thus, the solutions derived by these researchers will be assumed to represent the exact solution and used to evaluate the accuracy of the boundary elements results.

In their work, the results are given in the form of ratios of \( h_c/h_g \) and \( h_o/h_g \), where the variables \( h_g \) and \( h_c \) represent the gap size measured from the tool center and tool edge, respectively, to the workpiece in the vertical direction whereas \( h_o \) is the overcut value measured from tool edge horizontally to the workpiece as illustrated in Fig 6.1.

![Diagram of variables](image)

Fig 6.1: Variables \( h_g, h_c \) and \( h_o \)
6.1 Three dimensional Boundary element analysis

In the analytical solution (Fig 6.1), the tool and workpiece are assumed to be of cylindrical shape. The 3D model was created by forming a closed shell of interconnected faces. The tool faces consist of a cylindrical face (i.e. side face of the tool) and a circular face (end face of the tool) as shown in orange colour in Fig 6.2(a). The workpiece is represented by a circular face (grey face in Fig 6.2(a)). In order to close the domain, virtual faces (as shown in green) are formed to connect the tool and workpiece faces. These virtual faces were created sufficiently far away from the tool and workpiece surfaces so as not to influence the computed results. The complete 3D domain is shown in Fig 6.2(b).

When the domain has been created, all the faces are discretised by triangular elements as shown in Fig 6.3.
Before the system of equations is formed and the current density on each node is calculated, boundary conditions have to be applied on each face of the domain. Typically, the condition voltages of 0 and $U-\eta$ are imposed on the workpiece and tool end faces respectively. The virtual faces in the model have the condition $\frac{dV}{dn} = 0$ applied on them. In the case that the tool is insulated, the condition $\frac{dV}{dn} = 0$ is also imposed on the tool side face; if is not insulated, a voltage of $U-\eta$ is also applied on the side face of the tool.

The assumptions made in the simulation are listed below.

1. The electrical conductivity of the electrolyte is constant across the entire gap.
2. The effects of heat generated, hydrogen bubbles or metal film formed during the actual machining are not considered in the simulation.
3. The simulation considers that the gap is always full of the electrolyte, i.e. ideal flow conditions are assumed. Also, the electrolyte flow condition, be it laminar or turbulent, is ignored.
4. At each time step of the analysis, the phenomena occurring in the domain are assumed to be in steady state condition.

The results computed by using analytical technique used the machining parameters were: tool feed rate of 1 mm/s, a voltage difference of 1 volt is applied between the electrodes and an initial gap of 1 mm. These machining parameters are used in this simulation.

The evolution of workpiece shape can be represented by 2D profiles obtained by taking a section of the workpiece surface. To obtain this sectional view requires calculating the intersection points between the intersection plane and the edges of triangles (see Fig.6.4). By joining all the intersection points together, the required 2D profile is obtained. Note that the intersecting plane must pass through the centre of the workpiece as shown in Fig 6.4.
6.2 Optimal time step

When modelling the ECM process, the total machining time is divided into small time steps ($\Delta t$). During each time step, the tool is moved towards the workpiece, and the current density and amount of machining taking place at each node on the workpiece surface are calculated. In the simulation, the value of time step has to be chosen carefully. If the value of time step is too high, it may cause the tool to move too close to the workpiece surface and sometimes the tool profile could even intersect it. Therefore, the value of time step has to be optimised. The shape of the workpiece predicted for different values of time step is presented in Fig 6.5.
Fig 6.5 shows that when time steps of 0.15s and 0.12s are used, the workpiece profile is not smooth – there are fluctuations in the profiles obtained after nine and twenty steps (as shown in the red and dark blue curves in Fig 6.5). This is because the nodes on the workpiece, especially those directly opposite to the end face of the tool, are not machined uniformly. Consequently the workpiece surface becomes faceted as show in Fig 6.6.

A smooth profile is obtained when the time step is reduced to 0.1s or less as shown by the green and brown curves (both of them overlap). Hence, it is found that the accuracy of shape prediction is improved when smaller time steps are used. However, using too small a time step will affect the computing time. From Fig 6.5, although the profiles calculated using time steps of 0.1s and 0.08s are almost identical, the program had to perform 75 analyses when a time step of 0.08s was used whereas the computing is reduced to 60 analyses when time steps of 0.1s were used. Therefore the optimum time step is 0.1s and this time step is used to generate all the other results in this chapter.

![Fig 6.6: A spiky workpiece surface resulting from too high a value of time step](image)

6.3 Investigation of optimal criterion angle for mesh refinement

As stated earlier, the shape of the workpiece surface changes after each time step. As a result, some elements on the surface become distorted and the quality of the mesh becomes poor.

In order to maintain a smooth surface during simulation, the distorted elements need to be sub-divided into smaller elements using the mesh refinement techniques described in Chapter 4. As mentioned in that chapter, elements will be refined when the angle ($\theta$) included by the normals is greater than the maximum allowable pre-set angle ($\theta_{\text{max}}$).
Therefore $\theta_{\text{max}}$ is the criterion angle to decide whether the elements have to be subdivided or not.

Fig 6.7 shows the effect of using different values for $\theta_{\text{max}}$ on the profile at the bottom of the hole. From the figure it is clear that a smoother profile is obtained for the smaller values of $\theta_{\text{max}}$. For example, the profile obtained with $\theta_{\text{max}} = 8^\circ$ is smoother than that obtained with $10^\circ$. The profiles obtained with $3^\circ$ and $6^\circ$ are very similar. Although using small values for $\theta_{\text{max}}$ can improve the smoothness of the surface, the number of elements on the refined surface increase rapidly. For example, the number of elements increases from 1,288 to 2,056 elements when $\theta_{\text{max}}$ is reduced from $6^\circ$ to $3^\circ$ degrees as shown in Fig 6.8. Therefore, the value of $\theta_{\text{max}}$ should not be greater than $6^\circ$.

Therefore in the simulation, the computing parameters are: time step = 0.1 s; $\theta_{\text{max}} = 5$ degrees; and convergence criterion for ratios $h_c/h_g$ and $h_o/h_g = 1E-04$.

Fig 6.7: Profiles at the bottom of the hole predicted with different values of $\theta_{\text{max}}$
6.4. Three dimensional EC drilling simulation results

6.4.1. Insulated tool condition

The cylindrical surface of the tool was insulated and the end face was assumed to be at 1 V whereas the workpiece surface was at 0 V. The ratios of $h_e/h_g$ and $h_o/h_g$ calculated from 3D BEM and 2D analytical method are compared and shown in Figs. 6.9 and 6.10 respectively.

The results show that, at the beginning of simulation, the values of $h_e/h_g$ and $h_o/h_g$ are 0.76 and 0.96 respectively and they decrease rapidly and converge, within 2.8 s to values of 0.739 and 0.738 respectively. The analytical solution is 0.731 for both ratios. Therefore the errors of $h_e/h_g$ and $h_o/h_g$ are 1.01% and 0.96% respectively.
Fig 6.9: Convergence of $h_c/h_g$ for an insulated tool

Fig 6.10: Convergence of $h_o/h_g$ for an insulated tool
6.4.2. Bare tool

The bare tool is simulated by assuming that all the surfaces of the tool are at 1 V. The values of $h_c/h_g$ and $h_o/h_g$ obtained from the BEM program and the analytical solution are shown in Fig 6.11 and 6.12 respectively. Similar to the insulated tool, the starting values of $h_c/h_g$ and $h_o/h_g$ are 0.865 and 3.37 respectively and they rapidly decrease to a steady value.

The simulation results from Fig 6.11 and 6.12 show that the value of $h_c/h_g$ has converged to 0.812 after 3.4 s whereas the value of $h_o/h_g$ converges to the value 1.161 after 4.2 s whereas the analytical solution for $h_c/h_g$ and $h_o/h_g$ are 0.80 and 1.150 respectively. Hence, the error in the calculated (BEM) values for $h_c/h_g$ and $h_o/h_g$ are 1.5% and 0.95% respectively.

![Fig 6.11: Convergence of $h_c/h_g$ for a bare tool](image_url)
6.5. Effect of mesh refinement

To confirm that refining the mesh improves not only the smoothness of the surface but also the simulation results, the values of $h_c/h_g$ and $h_o/h_g$ with no refinement are shown in Fig 6.13 and 6.14. From the figures, it is clear that $h_c/h_g$ and $h_o/h_g$ converge to 0.859 and 0.848 resulting in errors of 17.5% and 16% respectively.

Fig 6.12: Convergence of $h_o/h_g$ for a bare tool

Fig 6.13: Convergence of $h_c/h_g$ with and without mesh refinement

Fig 6.14: Convergence of $h_o/h_g$ with and without mesh refinement
6.6. Conclusion

From comparisons with the analytical solutions, the values of \( h_c/h_g \) and \( h_o/h_g \) calculated from the program showed that the maximum errors, for the insulated and bare tools were 1.01\% and is 1.5\% respectively. In addition, using mesh refinement techniques to sub-divide the distorted element during simulation can improve the simulation results and workpiece surface smoothness. As shown in section 6.3 and 6.5, the final workpiece surface involving the use of refinement techniques is visually smoother than that obtained without refining the mesh. The error in the converged results reduces from 17.5\% to 1.5\% when the mesh is refined continuously.
Chapter 7
Experimental and numerical investigations into electrochemical milling

The common applications of ECM are EC drilling and die sinking. In the latter, a complex 3D shape is generated by an axial movement of the tool whereas in the former, the shape is axisymmetric. Die sinking requires a considerable effort to design the tool shape as its shape is not identical to that of the workpiece but similar and it is usually obtained by trial and error. Thus the cost of the workpiece depends on the dimensions and complexity of the tool.

The difficulties of tool design in EC sinking can be eliminated by using EC milling in which the required shape is obtained by moving a simple tool shape (rectangular, spherical or cylindrical), as in conventional milling, parallel to the workpiece surface (Fig. 7.1)

One of the problems in electrochemical machining is stray machining which has a considerable effect on the final workpiece shape. Hence in this chapter, the effect of process parameters such as tool feed rate and applied voltage on the workpiece shape are studied; this chapter also investigates the modelling of the EC milling process by the boundary element method and the accuracy of the predicted workpiece shape.

Fig 7.1: Symbolic representation of EC milling
7.2. Experimental setup

The experimental set-up, as shown in Fig. 7.2, consisted of a three-axis CNC machine which was adapted for ECM, a DC power supply and pumping system. The machine was equipped with a data acquisition system to record the voltage and current.

![3-axis CNC machine and DC power supply](image)

Fig 7.2: Three-axis ECM milling machine and DC power supply.

The tools were made from copper and three different cross-sectional shapes i.e. rectangular, square and cylindrical have been investigated. The rectangular tool had a cross section of 5x2 mm, the square tool was of size 1x1 mm and the cylindrical tool was of 1 mm diameter. The workpiece material was stainless steel (SS-316). The workpiece was placed on a plastic block (Fig. 7.3) which provided insulation and protected the machine from electrical sparking that could occur during the tests. The slots of varying widths and depths were machined using the above-mentioned tools.

The electrolyte was sodium nitrate (NaNO₃) (10% by weight in water. A flexible nozzle was positioned in front of the tool as shown in Fig. 7.3 to flush the sludge from chemical reaction. The workpiece shapes were measured using a laser profiler machine.
7.3. Three dimensional domain configuration for BE modelling.

Although there are three elements i.e. tool, workpiece and insulating block (see Fig.7.4), the boundary element method requires only the tool and the workpiece to be modelled. The domain for modelling the tool and workpiece is created by initially forming open shells from some of the appropriate external faces of the tool and workpiece. Figure 7.5 shows the rectangular tool and workpiece represented as a hollow open-top box (orange faces) and an inverted U-shaped channel (grey faces), respectively. To form a set of interconnected faces and a closed domain, a closed shell of virtual surfaces (light green faces) is created at positions that are sufficiently far away from the workpiece surface so as not to influence the predicted results.
Since the top face on the workpiece and the bottom face of the tool will affect the accuracy of the predicted results, finely structured meshes are created on these surfaces as illustrated in Fig. 7.6.

Fig 7.6: Meshes created on the base face of the tool and top surface of the workpiece.

The side surfaces of the tool and workpiece have a graded mesh, the mesh being fine near the top/bottom face of the workpiece/tool. Since the virtual surfaces will not have
a significant effect on the predicted shape of the slots, they have been discretised with coarse unstructured meshes (Fig. 7.7). The complete mesh is shown in Fig. 7.8.

Fig 7.7: Virtual surfaces with coarse unstructured meshes.

Fig 7.8: The complete discretised 3D domain

In EC milling, the tool is moved continuously in the feed direction. To simulate this temporal problem, the tool is moved every $\Delta t$ seconds (in the X-direction) and for every discrete position of the tool, the new workpiece shape is calculated. If the slot is machined in more than one axial pass, then at the end of its traverse in the X-direction, it is lowered by a small amount (in the $-Z$ direction) and the analysis is repeated. The
tool movements in the X-direction will necessitate the mesh on the top virtual surface and the workpiece to be continuously modified and at the end of every feed movement, the tool surfaces and top virtual surfaces have to be lowered necessitating the mesh on the side virtual surfaces to be modified as well. Whilst the mesh on the tool surfaces needs a translation at the end of every time step and feed traverse, the triangular elements on the workpiece surface get stretched due to the machining action, resulting in the mesh becoming distorted. Therefore, even though the initial mesh on the workpiece surface may be very fine, it quickly deteriorates and its quality and density have to be improved after every few time steps.

At the end of every time step, the workpiece shape is updated by moving each node on its surface by $\Delta d$ in a direction which is normal to the surface at that node. This distance is based on Faraday’s law which is mentioned in Chapter 3.

Since the side surfaces of the tools used in the experiments were bare, a voltage of $V-\eta$ was imposed on the side and base surfaces of the tool while the workpiece surface, a condition of 0 V was assumed. Since the virtual surfaces are far away from the tool surface, they were assumed to be completely insulated (i.e. $\frac{dV}{dn} = 0$).

The assumptions made in the boundary element analysis are the same as those stated in Chapter 6 for modelling the EC drilling process.

### 7.4. Deeper and wide slot machining

This section discusses the experimental and predicted results for deep and wide slots which were machined using a tool with a width of 5 mm and a thickness of 2 mm.
The following values were assumed in the BE model.

1. A conductivity of 0.023 S.mm\(^{-1}\) for the electrolyte (k\(_e\)) which was a solution of Sodium Nitrate (NaNO\(_3\)), 10% concentration by weight.
2. The machining constant (which is for the ratio of the average atomic mass and valency (M/zF)) for 316 stainless steel.
3. An initial gap of 0.2 mm.
4. Over-potential (\(\eta\)) of 1.5 V, which was determined experimentally in a pre-test.
5. A current efficiency of 80% for the EC milling process.

As mentioned earlier, in the EC milling process the tool moves at a constant feed rate, but in the simulation the total machining time is divided into several time steps (\(\Delta t\)) which is one of the factors that affects the accuracy of the BE results. To investigate the effect of \(\Delta t\), the depths of a single slot as predicted by the BEM for different values of \(\Delta t\) are compared with the depth of a single slot experimentally measured.

The single slot was machined by traversing the tool from point A to B, as illustrated in Fig. 7.10 at a feed rate of 5 mm/min. The applied voltage was 10 volts. The depth of the slot obtained from the BE model for different values of \(\Delta t\) is shown in Fig. 7.11.

![Fig 7.10: Tool path for machining a single slot.](image-url)
Fig 7.11: The depth of slot computed by the BE model for different time steps.

The predicted value of depth of cut decreases when the time step is decreased and it converges to 0.0260 mm when $\Delta t$ is less than 4.5s. However, when $\Delta t$ is 5s, the depth of cut is 0.0261 mm which is 0.0001 mm greater than the converged value. This error is very small. Therefore the time step of 5 s will be used in BE model to predict the other milling features.

7.4.1. Deeper slot

To machine a deeper slot, the tool travels from point A to B (see Fig 7.12(a)), then moves rapidly back to point A (Fig 7.12(b)). It is then moved in the axial direction by a certain amount (Fig.7.12(c)). These sets of movements are repeated several until the required slot depth is obtained. The slots discussed in this section were machined in eight passes and each time the axial depth was 0.05 mm. The other parameters were as follows:

(i) applied voltage = 10 V,
(ii) feed rate = 5 mm/min in both the transverse and axial directions, and
(iii) initial gap = 0.2 mm.

In the BE analysis, the machining is uni-directional, i.e. the machining takes place only when the tool travels from A to B. The return path from B to A is done at a relatively high feed rate and therefore it can be assumed that no machining takes place.
The experimental and predicted cross-sectional shapes of the slot after the first, fourth and eight passes are shown in Fig 7.13, and the values for the slot depth and width are given in Table 7.1.
The results show that, although the tool is not moved in the radial direction, the width of the slot increases progressively, albeit by a small amount, as the depth of the slot is increased.

Also, the depth and width of the slot as predicted by the BEM are greater than those obtained experimentally. The average difference between the two sets of results is 1.49% for the slot width and 4.6% for the depth.

### 7.4.2. Wide slot

Since most features will be wider than the width of the tool, the tool will be required to move not only in the axial direction but also in the radial direction, hence the machining of a wide slot is investigated. The wide slot is machined by travelling the tool from one end (point A in Fig.7.14) of the slot to the other end (point B) at a specified feed rate (Fig 7.14(a)) and rapidly travelling back to the starting point (point A see Fig 7.14(b))). After this the tool is offset by a certain distance in the radial direction (travel to point C, Fig 7.14(c)). This distance will here afterwards be referred to the “step-over” distance. The effect of step-over on the surface finish of the slot is also examined.

The slots were machined using step-over values of 0.1, 0.3 and 0.5 mm and 40, 15 and 10 radial passes respectively. The cross-section profiles obtained experimentally at these step-over distances are shown in Fig 7.15.

<table>
<thead>
<tr>
<th>Number of tool passes</th>
<th>Width of slot (mm)</th>
<th>Depth of slot (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Experiment</td>
<td>5.591</td>
<td>5.864</td>
</tr>
<tr>
<td>BE model</td>
<td>5.724</td>
<td>5.903</td>
</tr>
<tr>
<td>Error</td>
<td>2.38%</td>
<td>0.67%</td>
</tr>
</tbody>
</table>

Table 7.1: BE and experimental values for width and depth of deeper slot.
Fig 7.14: The tool path for wider slot machining

Fig 7.15: Cross-sectional profile of the slot for different step-over values.
Each of the cross-section profiles in Fig 7.15. can be subdivided into three distinct regions, i.e. trailing side wall (A), base surface (B) and leading side wall (C). The tool transverse feed direction is from A to C. The base surface is flat and the leading side wall smooth. However, the trailing side wall exhibits serrations the size of which depend on the step-over distance. The serrations obviously are very prominent when the step-over distance is 0.5 mm and they disappear when the step-over distance is reduced to 0.1 mm. The actual surfaces obtained with 0.5, 0.3 and 0.1 mm step-over distances are shown in Fig 7.16. Although the step-over distance is equal to the width of the tool (there is no overlap), there was still a serration (see Fig 7.17(a)). The serration also appeared when the step-over distance was greater than the width of tool (Fig 7.17(b)).

![Fig 7.16: Actual surface finish obtained on the trailing surface with varying step-over values](image)

![Fig 7.17: Actual surface finish obtained with step-over (a) 100% tool width (b) 105% of tool width](image)

In EC milling, even if there no overlap between two neighbouring passes, the tool will machine some material directly under the previous pass because of the stray currents. It is not possible to do away with the stray currents as this would require the electrolyte to flow over only the area where the machining is required, something difficult to achieve as the electrolyte usually flows over the entire workpiece surface.
When the step-over is 0.5 mm, the trailing edge of the tool is only 0.5 mm far away from the previous pass (Fig 7.18(b)), and therefore some of the material machined in the previous pass will be further machined due to the stray current (Fig 7.18(c)). Therefore, parts of the surface that are re-machined will have greater depth of cut than those that are not re-machined.

![Fig 7.18: Workpiece profile machined with a step-over of 0.5 mm.](image)

On the other hand, when the step-over = 0.1 mm, most of the material that was machined in the previous pass is re-machined (Fig 7.19.). Therefore, the change of depth is more gradual, resulting in a smoother surface.

![Fig 7.19: Workpiece profile machined with a step-over of 0.1 mm.](image)

The cross-section profiles of the slots machined with 0.1, 0.3 and 0.5 mm step-over distances as predicted by the BE model are shown in Fig 7.20 and they are compared with those obtained experimentally. The measured and predicted values of the slot depth and width are given in Table 7.2. Again, there is good agreement between the BE and experimental values with the average differences in the width and depth of the wide slot being 1.91% and 3.22% respectively.
Table 7.2: BE and experimental values for width and depth of wider slot

<table>
<thead>
<tr>
<th>Step-over distance (mm)</th>
<th>Width of slot (mm)</th>
<th>Depth of slot (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>Experiment</td>
<td>8.122</td>
<td>7.738</td>
</tr>
<tr>
<td>BE model</td>
<td>8.301</td>
<td>7.876</td>
</tr>
<tr>
<td>Error</td>
<td>2.20%</td>
<td>1.78%</td>
</tr>
</tbody>
</table>

Fig 7.20: The profile of slot from BEM and experiment at various axial depths

7.5. Pocket machining

In conventional milling, a 2 ½ D pocket is usually machined using contour-parallel tool paths rather than zig-zag or zig tool paths. This type of tool path, also referred to as a spiral-in or spiral-out tool paths will be investigated for its suitability in electro-chemical milling. This would require, for example, when machining the rectangular pocket shown in Fig. 7.21, the tool having to travel and machine both in the x and y directions. Since in ECM the tool machines the material that lies not only directly under it but also that around it due to stray current flux, the cross-sectional shape and size of the tool is a factor that affects the final shape of the workpiece.
To confirm that the tool geometry affects the machining rate of the material, consider a tool with a rectangular cross-section. The tool has a depth of $L$, a width of $W$ and travels at a feed rate of $f$. When the tool is travelling from position 1 to 2 (Fig 7.22.), the time taken by the tool will be equal to $W/f$ seconds. However the machining time at point $J$ will be $L/f$ seconds as the tool travels from position 3 to 4. Since $L$ is greater than $W$, point $J$ will see the tool for a greater time than point $I$. Therefore, more material will be machined at point $J$ than at $I$ or, in other words, the machining depth, is different at $J$ than that at $I$. Therefore, in this section, the shape of the tool with simple geometrical shape i.e. square and cylindrical tool shapes are investigated in machining a simple feature.
The square tool was of size 1x1 mm and the cylindrical tool was 1 mm in diameter as shown in Fig 7.23.

![Fig 7.23: Square tool (1x1 mm) and cylindrical tool (Ø1 mm)](image)

Since these tool shapes have not been used before, pre-tests had to be performed to obtain the optimal machining parameters. In these tests, the square tool was used to machine a simple slot at different values of feed rates and applied voltages which were 3, 6 and 9 mm/min and 6, 10 and 14V respectively. To find the machining condition that yields the best accuracy in machining, a full factorial analysis was performed to determine the machining conditions which would result in the minimum over-cut. Once the machining conditions were obtained, the effect of tool geometry on the final shape of the feature was investigated. The cross-sectional profiles of slots obtained experimentally are shown in Figs. 7.24 to 7.26 and the values of the depth and width of these slots are shown in Tables 7.3 and 7.4 respectively.

<table>
<thead>
<tr>
<th>Feed rate (mm/min)</th>
<th>14</th>
<th>10</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.137</td>
<td>0.101</td>
<td>0.067</td>
</tr>
<tr>
<td>6</td>
<td>0.107</td>
<td>0.069</td>
<td>0.041</td>
</tr>
<tr>
<td>9</td>
<td>0.058</td>
<td>0.038</td>
<td>0.021</td>
</tr>
</tbody>
</table>

Table 7.3: Depths of slot machined at different conditions.

<table>
<thead>
<tr>
<th>Feed rate (mm/min)</th>
<th>14</th>
<th>10</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2.24</td>
<td>1.86</td>
<td>1.47</td>
</tr>
<tr>
<td>6</td>
<td>2.03</td>
<td>1.66</td>
<td>1.32</td>
</tr>
<tr>
<td>9</td>
<td>1.75</td>
<td>1.47</td>
<td>1.17</td>
</tr>
</tbody>
</table>

Table 7.4: Width of slots machined at different conditions.
Fig 7.24: Slot profiles machined at different voltages and at a feed rate of 3 mm/min

Fig 7.25: Slot profiles machined at different voltages and at a feed rate of 6 mm/min
The results show that for a specified applied voltage, the slot depth and width increase when the tool feed rate decreases. For example, at 10 V applied voltage, the depth and width of slot machined with a 9 mm/min tool feed rate are 0.038 mm and 1.47 mm respectively and they increase to 0.101 mm and 1.86 mm when the feed rate is reduced to 3 mm/min.

On the other hand, at a specified feed rate, both the width and depth increase with increasing applied voltage. For example, at 6 mm/min tool feed rate, when the applied voltage changes from 6 to 14V, the slot depth increases from 0.041 to 0.107 mm and the slot width increases from 1.32 to 2.03 mm.

This is reasonable because when the tool feed rate decreases, a unit area of the workpiece sees the tool for a longer time, and as a result more material is removed and when the applied voltage increases there will be a corresponding increase in the current density value, causing a greater amount of material to be removed.

The maximum values of slot depth and slot width are 0.137 mm and 2.24 mm obtained with an applied voltage of 14V and tool feed rate of 3 mm/min whereas the condition of 6V applied voltage and 9 mm/min tool feed rate yields the minimum slot depth, 0.021
mm, and slot width, 1.17 mm. Therefore the condition of 6V and 9 mm/min were selected as the optimal machining parameters.

To investigate the effect of the tool geometry on the shape of feature, tools with a cylindrical and rectangular cross-section were used to machine deep slots and pockets. The deep slots, as defined in section 7.4.1, are machined using tools with square and cylindrical cross-sections. The machining parameters are as follows:

1. applied voltage = 6 V;
2. feed rate = 9 mm/min;
3. axial depth = 0.02 mm, and
4. Initial gap = 0.2 mm.

The cross-sectional profiles of the slots machined by the square and cylindrical tools after the first, fourteenth and twenty-fifth axial passes are presented in Fig 7.27. and the depths and widths of the slot are given in Table 7.5.

![Profiles of the slot after the 1st, 14th and 25th axial passes](image_url)

Fig 7.27: Profiles of the slot after the 1st, 14th and 25th axial passes
Fig 7.28 Slot profiles machined by (a) square and (b) cylindrical tools

<table>
<thead>
<tr>
<th>Width of slot (mm)</th>
<th>Depth of slot (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of tool passes</td>
<td>1</td>
</tr>
<tr>
<td>Square tool shape</td>
<td></td>
</tr>
<tr>
<td>1.17</td>
<td>1.54</td>
</tr>
<tr>
<td>Cylindrical tool shape</td>
<td></td>
</tr>
<tr>
<td>1.15</td>
<td>1.30</td>
</tr>
</tbody>
</table>

Table 7.5: The width and depth of slot obtained experimentally

The profiles shown in Figs. 7.27 and 7.28 show that the base surface of the slot machined by the square tool is flat whereas it was concavely curved when machined by the cylindrical tool. The side walls of the slot are not vertical and the junction between the side wall and the base surface is not sharp but curved. The depth and width of slots machined by the circular tool are less than those obtained by the square tool. This is true for all the passes.

To investigate the effect of tool geometry on the shape of the slot, consider how the cross-sectional shape changes as the tools travel. This is best studied by considering a transverse profile of the slot and seeing how this profile evolves as the square and cylindrical tools travel along the length of the slot. Consider a section across the middle of the workpiece (see Fig. 7.29). The left side of the figure shows the tool at various instances in time and the right side shows the changing shape of the profile A-A, which for the sake of simplicity, is characterised by five points on this section i.e. 1-2-3-4-5.

In the case of the cylindrical tool, the material on the section A-A starts to be machined when the tool is tangent to it. Hence point 3 in Fig 7.29(b) is the first point to be...
machined. The tool continues to machine material at different rates (Fig 7.29(c)) until it reaches the position where the tool is tangent to the exit edge (Fig 7.29.).

Since the tool is curved, each point on the section will see the tool for different amounts of time. For example, point 3 has a machining time of $L_1/f$ seconds whereas machining time at point 2 and 4 is $L_2/f$ seconds and point 1 and 5 is $L_3/f$ seconds. Since distance $L_1$ is the longest and $L_2$ is longer than $L_3$ as shown in Fig 7.29(a), material at point 3 has the maximum machining time and points 2 and 4 have more machining time than points 1 and 5. Since the amount machined is proportional to the machining time, there will be more machining at points such as 3 than at points 1 and 5. As a result, the bottom surface of the slot is not flat but concavely shaped.

Fig 7.29: Evolution of profile A-A when machining with a cylindrical tool
The square tool has no curved surface and therefore the distance of points on the tool edge to material on section A-A (Fig 1.30(a)) will be equal i.e. the machining time will be the same which is \( \frac{L}{f} \) seconds resulting in a constant the depth of cut.

Fig 7.30: Evolution of cross-sectional profile A-A when machined by a square tool
Fig 7.30: Evolution of cross-sectional profile A-A when machined by a square tool (cont).

Since in ECM, tools will machine not only the material which is directly under the tool but also that around it due to stray currents, tool shape is a factor that affects the size of the machined workpiece. In the case of machining a slot by ECM, the width of the slot will depend on how long the material around the tool is machined. Consider once again, the time for which points 1 and 5 see the cylindrical tool. It is $L_3/f$ seconds (Fig 7.29) whereas it is $L/f$ seconds when using a square tool (Fig 7.30). Because $L_3$ is shorter than $L$, assuming that the feed rate does not change, points on the workpiece are machined by cylindrical tool for less machining time than that when machined by a square tool. This results in the width of slot machined by a square tool to be greater than that machined by a cylindrical tool.

A square tool has a bigger cross-sectional area than a cylindrical tool shape. For example, a square tool with dimensions $L \times L$ mm has a cross-sectional area of $L^2$ mm$^2$ whereas a cylindrical tool with diameter $L$ mm has an area of $0.25 \pi L^2 (0.79L^2)$ mm$^2$. The tool which has a greater cross-section area also has a greater current. Increasing the current causes more material to be machined. Therefore, for a given number of tool
passes, the slots machined by a square tool will be deeper than those machined by a cylindrical tool shape.

Since the tools are not insulated, current will flow not only from the end face of the tool but also from the side, front and back faces of the tool to the workpiece. As a result, additional material from the side faces of slots will be machined as the tool travels deeper in every pass. If the slot is machined in 25 axial depths, at the end of the machining, material at the top of the side surfaces would have seen the tool 25 times, whereas the material at the very bottom of the side surface would have seen the tool only once and this causes the side faces of slots machined with a non-insulated tool to be inclined and not vertical.

The slot profiles obtained experimentally and from the BE model, using square and cylindrical tools are compared in Figs. 7.31 and 7.32 and the values of width and depth of the slot at various axial depths are given in Tables 7.6 and 7.7 respectively.

<table>
<thead>
<tr>
<th>Number of tool passes</th>
<th>Width of slot (mm)</th>
<th>Depth of slot (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>Experiment</td>
<td>1.17</td>
<td>1.54</td>
</tr>
<tr>
<td>BE model</td>
<td>1.21</td>
<td>1.60</td>
</tr>
<tr>
<td>Error</td>
<td>3.41%</td>
<td>3.89%</td>
</tr>
</tbody>
</table>

Table 7.6: BE and experimental values for width and depth of a wide slot

<table>
<thead>
<tr>
<th>Number of tool passes</th>
<th>Width of slot (mm)</th>
<th>Depth of slot (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>Experiment</td>
<td>1.15</td>
<td>1.30</td>
</tr>
<tr>
<td>BE model</td>
<td>1.19</td>
<td>1.35</td>
</tr>
<tr>
<td>Error</td>
<td>3.47%</td>
<td>3.85%</td>
</tr>
</tbody>
</table>

Table 7.7: BE and experimental values for width and depth of a wide slot
Fig 7.31: Slot profiles obtained experimentally and from the BEM at various axial depths

Fig 7.32: The profile of slot from BEM and experiment at various axial depths
The average difference between the experimental and BE values for the width and depth of the slots is 4.05% and 1.37% respectively for the square tool and 4.03% and 1.4% for the cylindrical tool shape.

7.5.1. Square pocket

The effect of machining a square pocket with square and cylindrical tools is investigated in this section. The square pocket was of size 5x5 mm and the tool paths are shown in Fig 7.21. The pocket was machined using twenty four axial passes using the following machining parameters.

1. Applied voltage = 6 V.
2. Feed rate = 9 mm/min.
3. Axial depth = 0.02 mm.
4. Step-over distance = 0.1 mm.
5. Initial gap = 0.2 mm.

Two views of the pocket machined by the cylindrical tool are shown in Fig 7.33 whereas Fig.7.34 shows the same when machined by the square tool. The cross-section profiles of the pockets machined by the tools are shown in Fig 7.35; the shapes are similar with the slot machined by the square tool deeper and wider. The bottom surfaces are flat and, as expected, the side walls are not vertical. The junction between the side walls and the base surface is rounded. The width and depth are 6.06 mm and 0.63 mm for slot machined by the square tool shape and 5.86 mm and 0.6 mm for cylindrical tool shape. Hence, the square tool machines a bigger pocket than the cylindrical tool. Although the square tool has sharp corners, the corners of the pocket are not (Fig 7.34). This is because there are current flux lines emanating from the tool corner to the workpiece.

Fig 7.33: Pocket machined by a cylindrical tool
To model the machining of the pocket by the BEM, the tool path shown in Fig 7.21, must be divided into small segments. The length of each segment depends on the time step required to be used for BEM which is 0.1s. Hence, from the time step required and the tool feed rate used in the experiments (9 mm/min), the maximum length of the segment is 0.015 mm. The tool path for BEM is illustrated in Fig 7.36.
The cross-section profiles of the pockets obtained from the BEM and experiments with the square and cylindrical tools are compared in Fig 7.37. Figures 7.38 and 7.39 show the actual pockets machined by the square and cylindrical tools as well as 3D views of the pocket machined by the BEM respectively.

The depth and width value of pockets obtained from the BEM are 6.22 mm and 0.65 mm for machining by the square tool (Fig 7.37) whereas they are 6.04 mm and 0.61 mm for the cylindrical tool. The average difference between the two profiles is 2.91% for the square tool and 2.37% for the cylindrical tool.
Fig 7.37: Profiles of pockets machined by different tools

Fig 7.38: Pockets machined by a square tool and predicted by the BEM

Fig 7.39: Pocket machined by a cylindrical tool and predicted by the BEM
7.5.2. Human figure

In the previous sections, the machining of simple features such as a deep slot, a wide slot and a square pocket were simulated and the predictions experimentally verified. Since the geometrical shapes of these features were rather straightforward, the tool paths were not complex. For example, in the case of the square pocket, the tool paths consisted of straight lines and were obtained by successively offsetting the boundary shape. To further investigate whether EC milling is capable of machining more complex features, the machining of a pocket with a protrusion in the shape of a human being, is simulated. This pocket (see Fig 7.40) has been considerably modified from the original milling feature proposed by Held [56] and the optimum milling tool paths for which were generated by Hinduja et al. [57]. The principal modifications include scaling down the feature and converting the rectangular shape of the pocket to a near-diamond shape. The tool shape which is suitable to machine curved profile feature is chosen. The required depth of the pocket was 0.5 mm. The effect of tool path on final shape of the pockets is considered.

![Fig 7.40: Pocket with a human-shaped protrusion](image)

The choice of the tool shape is dictated by the geometry of the pocket. Consider the machining of the groove formed by one of the armpits (Fig. 7.41). The tool path to machine this groove is obtained by offsetting the groove shape by the tool radius.
(assuming that the side over-cut when machining is zero). This is rather straightforward in the case of the circular tool, the tool edge being tangent to the protrusion boundary (Fig.7.41(a)). However, in the case of the square tool, a similar offset curve as the tool path is not acceptable since a corner of the square tool will invade the required profile (Fig. 7.41(b)). Therefore, it was decided to abandon the square tool for machining the human-shaped pocket.

Fig 7.41: Suggested tool paths for machining the armpit with cylindrical and square tools.

In conventional milling, there are several types of tool paths to machine features. They are:

1. zig machining;
2. zig-zag machining;
3. spiral-in or contour machining, and
4. spiral-out or contour machining.

Since there is no commercial software specially developed to generate tool paths for EC milling, they will be generated using conventional CAM systems for milling. However, not all the types of tool paths mentioned will be tried. Only the zig-zag and spiral-in tool paths will be used in the experiments.

When machining the square pocket with the cylindrical tool (section 7.5.1), twenty-four axial passes were required to achieve a pocket depth of 0.6 mm. Therefore, twenty axial
passes were used to machine the pockets, with each axial depth being 0.02 mm per
depth and a step-over of 0.1 mm. The other machining parameters were as follows.

1. Applied voltage = 6 V.
2. Feed rate = 9 mm/min.
3. Initial gap = 0.2 mm.

To machine the human-figure protrusion precisely, the profile of the human shape (Fig 7.40) has to be offset by over-cut of machining. This over-cut distance is also calculated from the experimental result in square pocket machining in section 7.5.1. Consider the square pocket machined by a cylindrical tool. The width of the pocket required to be produced was 5 mm but after machining it with twenty-four passes, the width of the pocket was 5.86 mm (see Fig 7.35) which yielded the over-cut of 0.43 mm. Thus, it can be assumed that the rate of over-cut is 0.018 mm/axial pass. Hence if the pocket is machined in twenty axial passes, the over-cut will be around 0.36 mm. Therefore the over-cut of 0.36 mm is used as offset distance for the human shape. The offset human figure is shown in Fig 7.42 and the tool paths were created using this offset profile.

![Diagram of offset profile](image)

Fig 7.42: The offset profile of human-shaped protrusion

Since the gap between the tool and workpiece face affects the thickness of material removed, the tool was not lifted (i.e. moved in the Z-direction) when machining an axial pass. The zig-zag and spiral tool paths used to machine the pocket are shown in Figs. 7.43(a) and 7.43(b), respectively.
The total lengths of the zig-zag and contour-parallel tool paths were 553.4 mm and 608.3 mm respectively. Thus, the machining times, at feed rate of 9 mm/min, for each axial pass were approximately 62 minutes and 68 minutes for these two types of tool paths.

The cross-section profiles at sections A-A and B-B (Fig 7.44(a)) and lengths L_1, L_2 and L_3 (Fig 7.44(b)) are used to assess the accuracy of the predicted BEM values by comparing them with experimental values.
The profiles at A-A and B-B obtained from actual machining are shown in Figs. 7.45 and 7.46. The actual and required values of L1, L2 and L3 are shown in Table 7.8. The average difference between the required profile and actual profiles are 2.1% and 1.7% for the pocket machined by zigzag and spiral tool patterns respectively.

<table>
<thead>
<tr>
<th></th>
<th>Required profile</th>
<th>Zigzag path</th>
<th>Error (%)</th>
<th>Contour path</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 (mm)</td>
<td>1.80</td>
<td>1.76</td>
<td>2.06</td>
<td>1.78</td>
<td>1.39</td>
</tr>
<tr>
<td>L2 (mm)</td>
<td>1.50</td>
<td>1.46</td>
<td>2.73</td>
<td>1.48</td>
<td>1.60</td>
</tr>
<tr>
<td>L3 (mm)</td>
<td>2.00</td>
<td>2.03</td>
<td>1.50</td>
<td>2.04</td>
<td>2.00</td>
</tr>
</tbody>
</table>

Table 7.8: L1, L2 and L3 values of profile machined by zigzag and contour tool paths

From Figs. 7.45 and 7.46, it is clear that in both cases, i.e. with zig-zag and contour-parallel type of tool paths, the base surface is not flat. Instead, it is concave.

As explained in section 7.4.1 when machining the deep slot, the flatness of the base surface of the slots depends on the tool shape. However, when machining a pocket, the
flatness of the base surface depends not only on tool shape but also on the type of tool paths.

![Fig 7.46: Profile of the pockets at section B-B](image)

To demonstrate that the type of tool path can affect the flatness of the base surface of the pocket, consider the removal of the material lying between the inner and outer profiles along section B-B. This material is represented by points 1 to 5 in Fig. 7.47.

![Fig. 7.47: Points along section B-B machined using (a) contour and (b) zig-zag paths.](image)

Figure 7.48 shows the progressive machining of the region. The grey shaded area represents the area machined by the cutter and the tool paths in red are those paths that
have been traversed by the tool. The left and right sides of the figure refer to the removal of the material using contour and zigzag tool paths respectively.

Fig 7.48: Removal of material across B-B using zig-zag and contour parallel paths
Figures 7.48 (a) to (e) show the number of times points 1 to 5 are machined respectively. From the figure, in both types of tool paths, the tool travels past points 1 and 5 only once (Fig 7.48(a) and (e)) whereas it travels past point 2 six times (Fig 7.48 (b)). Point 3 is machined for eleven times (Fig 7.48 (c)) and point 4 is machined eight and nine times with contour-parallel and zigzag tool paths respectively. Since point 3 is crossed by the tool more times than the other points, the machining depth is a maximum there resulting in a concave surface as shown in Fig 7.49 and 7.50.
Fig 7.49: Cross-section of the profile machined using contour-parallel tool paths

Fig 7.50: Cross-section of the profile machined using zig-zag tool paths
Referring to Table 7.8, the experimental profiles are not the same as the required profile in spite of taking the over-cut into consideration when calculating the tool paths. This is because of the different types of tool paths used to machine the pocket. They affect the dimensions of the machined shape of the pocket. With the CAM software used in this research, each axial pass was machined in two stages.

In the first stage, tool paths were created to remove as much of the stock as is possible. In this step, two different types of tool paths were created, the first of which was created by repeatedly offsetting the outer profile (see Fig.7.51 (a)). The tool travelled either along curved paths that were obtained by offsetting the outer and inner boundaries (Fig 7.51(a)). In the second type, zig-zag tool paths were created as shown Fig.7.51 (b); in this case the paths were linear with the end points of the lines lying on the outer and inner boundaries of the pocket.

In the first stage, the stock removed by the tool is the material between the outer profile (black) and the orange profile i.e. the tool will be tangent to these two profiles in the first stage. Note that in the case of zig-zag tool paths, the tool will not be able to machine certain parts such as between the arm pits. Hence only parts of the protrusion boundary are in orange.

Fig 7.51: Tool paths generated in the first stage of (a) contour and (b) zigzag path type

At the end of the first machining stage, there were four regions (labelled as I, II, III and IV in Fig 7.52(a)) that had not been machined. These were machined in the second stage.
and the tool paths to machine these regions were generated using Mastercam, which was used also for the first stage.

The paths for connecting the un-machined regions are curves obtained by offsetting the inner profiles by the tool radius; these paths are labelled as A-B, C-D, E-F and G-H and the tool was moved along them when moving from one region to the next as shown in Fig 7.52 (b) to (e). Alternatively, the tool could have been lifted at, say A, to a safe height, moved to a position directly above B and then lowered to the appropriate height.

The contour-parallel and zig-zag tool paths to machine each of the four regions are shown in the left and right columns of Fig 7.52 (b) to (e) respectively. When the tool travels along the connecting segments A-B, C-D, etc, the material on the inner profile is removed again.

Therefore, in each axial pass of machining the pocket, the material in the inner profile (human figure) is removed twice (i.e. in the first and second stage) which is unlike machining the square pocket where the material on the outer profile was removed only once in each axial pass. This is probably why the human-shaped protrusion is smaller than what it should have been.
Fig 7.52: Unmachined regions and the tool paths to link them
Fig 7.52: Unmachined regions and the tool paths to link them (cont)
The effect of different applied voltages and step-over distances on the accuracy of the machined pocket has also been investigated. The pocket is machined using a zigzag path because it required less machining time than contour-parallel paths.

When the pocket was machined with an applied voltage of 10V, dimensions of the inner profile were much smaller than those obtained with 6V as shown in Table 7.9.

<table>
<thead>
<tr>
<th>Required profile</th>
<th>Zigzag path (6V)</th>
<th>Error (%)</th>
<th>Zigzag path (10V)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 (mm)</td>
<td>1.80</td>
<td>2.06</td>
<td>0.94</td>
<td>47.8</td>
</tr>
<tr>
<td>L2 (mm)</td>
<td>1.50</td>
<td>2.73</td>
<td>0.53</td>
<td>64.7</td>
</tr>
<tr>
<td>L3 (mm)</td>
<td>2.00</td>
<td>1.50</td>
<td>3.08</td>
<td>54.0</td>
</tr>
</tbody>
</table>

Table 7.9: Comparison of values L1, L2 and L3 using applied voltages of 6 and 10 V

This is understandable since the over-cut in the case of 10V would have been greater. The pockets machined with the two different voltages are shown in Fig 7.53.

Fig 7.53. Pockets machined with (a) 6V and (b) 10V

The step-over distance has a profound effect on the surface finish and the dimensions of the pocket. When machining the pocket using contour-parallel tool paths, when the step-over distance was changed from 0.1 mm to 0.3 mm, the base surface of the pocket was not as smooth as shown in Fig 7.54(b). Measuring the surface roughness (Ra) with a laser profile scanning machine, the average Ra values on the base surface of pocket surface using 0.3 mm and 0.1 mm step-over distances were 0.36 and 0.089 respectively.
Fig 7.54: Pockets machined using contour-parallel tool paths and different step-over distances (a) 0.1 mm and (b) 0.3 mm

When using zig-zag tool paths, changing the step-over distance affects not only the base surface but also the profiles of the pocket. Fig 7.55 (b) shows the scallops that are present on the inner and outer boundaries of the pocket when using a step-over distance of 0.3 mm.

Fig 7.55: Pockets machined using zig-zag tool paths and different step-over distances (a) 0.1 mm and (b) 0.3 mm

To predict the shape that would be generated by the BEM, the contour and zig-zag tool paths from Fig 7.43 were divided into small segments as illustrated in Fig 7.56, the maximum length of a segment being 0.015 mm. When modelling, in each simulation step, the tool was moved from the start point of a segment to its end point. The tool was made to travel each segment in 0.1 s which corresponds to the actual machining time for each segment.
The shape of the pocket obtained experimentally and that by computer modelling are shown in Fig 7.57. Next the profiles obtained by taking sections A-A and B-B are compared and these are shown in Figs.7.58 to 7.61. In these figures, the experimental results obtained both with zig-zag and contour-parallel are compared with the BE results.
Fig 7.57: Experimental and predicted shape of pockets.
Fig 7.58: Comparison of profiles obtained experimentally and from the BE model at section A-A using contour-parallel tool paths

Fig 7.59: Comparison of profiles obtained experimentally and from the BE model at section B-B using contour-parallel tool paths
Fig 7.60: Comparison of profiles obtained experimentally and from the BE model at section AA using zig-zag tool paths

Fig 7.61: Comparison of profiles obtained experimentally and from the BE model at section BB using zig-zag tool paths
The values of L1, L2 and L3 from the BE model and actual machining are compared in Table 7.10

<table>
<thead>
<tr>
<th></th>
<th>Contour pattern</th>
<th>Zig-zag pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L1</td>
<td>L2</td>
</tr>
<tr>
<td>BE model</td>
<td>2.07</td>
<td>1.81</td>
</tr>
<tr>
<td>Experiment</td>
<td>2.23</td>
<td>1.88</td>
</tr>
<tr>
<td>Error (%)</td>
<td>7.17</td>
<td>3.72</td>
</tr>
</tbody>
</table>

Table 7.10: Comparison of value L1, L2 and L3 obtained experimentally and from BE model.

From Table 7.10, the average difference between the experimental and BE model is 7.1% for the contour-parallel tool paths and 5.75% for the zig-zag tool paths. Comparisons between the profiles obtained experimentally and from the BE model show that in all cases the BE profiles are deeper and have a greater overcut than those obtained experimentally. This is because ideal flow conditions have been assumed for the electrolyte solution as it flows past the tool. Normally when a liquid flows past an object, separation of flow will occur at the edge of the object and a vortex of the liquid is generated. This separation will affect the current flow patterns and the current densities at the nodes on the workpiece.

In the experimental set-up, the electrolyte was supplied to the gap using an external nozzle as illustrated in Fig 7.3. Therefore, vortices could have been generated during machining especially when machining a pocket where the end of the tool is below the top surface of workpiece. It was observed that the position of the vortex was at the tool edge where the electrolyte first encountered the tool. The vortex region is shown in Fig 7.62. Vortices generated in the electrode gap will reduce not only the flow rate of the electrolyte but also reduce the flushing rate of the electrolyte to remove heat, bubbles and the sludge generated by the chemical reaction.

In modelling, the effect of flow separation is not taken into account; no account is also taken of the products generated from chemical reactions such as the release of hydrogen and the formation of bubbles; these bubbles will increase the electrical resistance between the tool and workpiece which in turn will reduce the current flux density.
Therefore, it is not surprising that the profiles at sections A-A and B-B are deeper and fuller than those measured experimentally.

7.6 Conclusion

In electrochemical milling, the accuracy of the workpiece and its surface finish depend not only on machining parameters but also on the tool geometry. Using a low applied voltage and a high feed rate generally results in more accurate machining (section 7.5).

When machining deep slots, the base surface of the slot will be flatter with a square tool than with a cylindrical tool. If a cylindrical tool is used to machine the slot, the shape of the base surface of the slot is more concave. However, the slots machined by a cylindrical tool are more accurate. When machining features which are wider than the width of the tool such as a pocket, the surface finish depends strongly on the step-over distance. The depth of the scallops is sensitive to the step-over distance and the scallops disappeared with a step-over of 0.1 mm.

A comparison between the boundary element predictions and the experimental results showed that the depth of cut and overcut predicted by the boundary element model were greater by as much as 10.40%.
Chapter 8
Modelling the Electrochemical Turning Process

The electrochemical turning process is used to machine axi-symmetric components. The workpiece is machined by clamping it on a workholding device mounted on a rotating spindle. The tool (cathode) is stationary and it is fed radially inwards towards the centre-line of the rotating component (negative X-axis, see Figs 8.1 and 8.2). The electrolyte is supplied to the gap by using an external nozzle or by ejecting the electrolyte from the tool. To machine profiled surfaces, a formed tool is used with a radial feed (negative X-axis, see Fig. 8.1), otherwise, as shown in Fig.8.2, the end face of the tool can be planar with an axial feed (Z-axis). As in other electro-chemical machining processes, material is removed by electro-chemical reaction when a voltage is applied across the workpiece and tool, the two being separated by a very small gap containing electrolyte solution.

Since no cutting force is involved and there is no physical contact between the tool and workpiece, the hardness of the workpiece has no influence in the process and furthermore, no residual stresses occur in the workpiece and neither does the tool wear during electro-chemical turning (ECT). Therefore, this process is ideally suited for machining high-strength and thin-walled rotational parts.
When a shaped tool is used to machine the workpiece, the tool is stationary or it is fed towards the workpiece surface in the radial direction (negative X-direction in Fig 8.1). However, when shaped tools are used in ECT, tools have to be designed carefully which increases their cost. To avoid the effort required for designing the tools, often tools with end faces having simple analytical shapes (i.e. spherical, cylindrical or planar) are used in ECT. These tools are referred to as being unshaped.

When using an unshaped tool, it is moved along the workpiece axis (Z-axis direction in Fig 8.2) and/or along the radial direction (-X-axis). To machine a workpiece accurately, the machining would have to be carried out on a CNC machine; hence this machining process is sometimes called CNC-ECT. Using a CNC machine makes it possible to machine a greater variety of workpiece shapes by adjusting the machining parameters such as tool feed rates in X and Z directions and the rotational speed.

The main aim of this chapter is to describe the development of a boundary element model for machining turned components. The model will be verified by simulating the machining of a thin-walled component.
8.1 The thin-wall component

The purpose of modelling the thin-walled ring shown in Fig 8.3 was to determine the time that would be required to reduce the radius of the cylindrical part labelled AB by 50 µm. The length of cylinder is 12 mm and its external diameter is 95.5 mm. The thin-walled ring was made from stainless steel and has a wall thickness of 90 µm. Because the wall is very thin, it is unsuitable for machining on a CNC turning centre – the cutting force would probably cause the component to become distorted. Therefore, the ECT process is suitable to machine this component because the tool will not contact workpiece surface.

The cutting tool was made from copper and was unshaped with a planar end face. The width of the tool was 20 mm and its depth 12 mm. The initial gap was 0.2 mm. The electrolyte was sodium nitrite (NaNO₂) with a concentration of 15% by weight. The tool was kept stationary during machining. A schematic arrangement of the experimental set-up is shown in Fig.8.4.
8.2 Three-dimensional domain configuration for ECT simulation

Referring to the experimental setup shown in Fig 8.4, a 3D BE model is formed by creating a closed shell of interconnected faces (see Fig 8.5 and Fig 8.6). The outer surface of the component to be machined is represented by a cylindrical surface (shown in grey colour, Fig. 8.5(a)). The radius of the cylinder is 47.8 mm. The tool is represented by its end face i.e. a rectangular face (shown in orange colour in Fig.8.4), the dimensions of which are 20x12 mm. The initial gap between the tool and workpiece faces (i.e. orange face and the grey cylindrical surface) was 0.2 mm.

Virtual faces (shown in green) are required to connect the tool and workpiece surface in order to close the domain. To eliminate any influence that the virtual surfaces may have on the computational results, the outer surface of the domain is formed by a virtual cylindrical surface with the radius of 55 mm (see Fig 8.5). Thus, the distance between this surface and the workpiece surface is 7.2 mm which should not have any effect on the computed flux values on the workpiece surface. The top and bottom faces (see Fig 8.6(a)) are added to form the closed shell. The complete 3D domain is shown in Fig 8.6 (b).
However, the 3D domain in Fig 8.6 is still not suitable for use by the BEM. Consider the top face of the domain (see Fig 8.7); its radial width varies from 0.2 to 7.2 mm. This will require very small elements where the width is small and larger elements elsewhere. Using small elements will increase the number of elements and with it, the computing time. Furthermore, as the workpiece is rotated, all the faces in the domain will have to be re-meshed. If the shape of the top and bottom faces is complicated, it increases the complexity of generating the mesh and hence the computing time.
Therefore to reduce the difficulty in the re-meshing process and to reduce the computational time, the domain has to be modified. To eliminate the variation in the radial distance on the top and bottom faces, a set of virtual external rib surfaces is formed at the top and bottom of the domain (see Fig 8.8 (a)). The rib has 5 mm of thickness and 60 mm of outer diameter.

When the external rib surfaces are formed on the top of the previous domain, the workpiece surface has to be extended by 5 mm (equal to the rib height see Fig 8.8(b)) and the top of the domain can be closed by using a hollow disc face which has uniform radial thickness (see Fig 8.9(a)). When the external rib surfaces are formed on the bottom, only the circular face will be used as the face to close the bottom of the domain (see Fig 8.9(a)).

Since the bottom of the workpiece surface is not connected with the bottom surface of the domain, a circular virtual surface is created as the bottom face of the workpiece (see the red face in Fig 8.8(b))
Therefore, compared with the previous domain, the top face of the new domain is also a disc but has a uniform radial width. Thus very small elements are not required on the face when it is meshed. Since it is the top face that connects the virtual outer surface to the workpiece surface, it is necessary to re-mesh only the elements on this face (apart from the workpiece surface) after each analysis.
8.3 BEM calculation

To perform a BE analysis, the domain has to be discretised and for this, 2-D linear triangular elements were used and the mesh was created using the advancing front technique as described in Chapter 4. The discretised 3-D domain is shown in Fig 8.10. After every time step, the mesh on the workpiece surface has to be rotated by a small angle and some nodes have to be moved radially inwards. This necessitated the existing elements on the top face to be deleted and the face re-meshed.

![Fig 8.10: Triangular elements created on the surfaces of the domain](image)

The planar end face of the tool was assumed to be at the applied voltage (V-η) and the workpiece surface had the condition V=0 imposed on it. The condition of \( \frac{dV}{dn} = 0 \) was imposed on all the virtual surfaces. All the assumptions discussed in the previous chapter are also valid for modelling the ECT process. The above component was machined using the applied voltage of 20V and a rotational speed of 25 rpm. The conductivity of the electrolyte (NaNO₂, 15% by weight) was measured as 0.02 S.mm⁻¹, the over-potential as 1.5V and a current efficiency of 80%. The actual machining time was 90s. The BE analysis was performed using a time step of 0.005s. After each time step, the nodes on the workpiece surface had to be moved in the radial direction and the amount by which they were moved was calculated using Faraday's law (Chapter 3) and the flux values calculated in the current step by the BEM. However, nodes lying on the top and bottom edges of the workpiece are singular because, at these nodes, both flux \( \frac{dV}{dn} = 0 \) and V=0. This means that these nodes do not have any unknown variables associated with them and these nodes are therefore assumed to be fixed in position. The
reduction in the radius as computed by the BEM is shown in Fig. 8.11 and the 3-D workpiece in Fig. 8.12.

Fig 8.11: Changing radius of the workpiece with time calculated by BEM

The BEM result in Fig. 8.11 shows that the radius of the workpiece is reduced by 51 µm after 90s of machining. When the component was actually machined, it was found that the radius could be reduced by 50 µm in 90 s. Therefore the result from BEM agrees very well with experimental result and error in calculation is 2%. The computing time is 18 hours.
8.4 A technique to reduce the computational time

From the BE analysis, it was noticed that, at the end of each time step, only nodes lying in front of the tool surface were machined, causing the radius of the workpiece at these machined nodes to become smaller than that of the un-machined nodes. For example, consider node A in Fig 8.13. Before the analysis, when node A is far away from the tool surface (as in Fig 8.13(a)), it has a zero current density. Therefore, node A will not be machined and its radius ($r_A$) will remain unchanged (see Fig 8.13(b)).

Fig 8.13: (a) Distribution of the current density (Ampere/mm$^2$) on workpiece surface (b) radius of the workpiece at points A

However, as the workpiece is rotated clockwise, point A approaches the tool and its current density increases gradually (see Fig 8.14). Consequently the radius of the workpiece at point A decreases (see Fig 8.15). The current density value at point A reaches a maximum value when it is directly opposite the tool face (i.e. when it has rotated by 12.5° from its initial position (see Fig 8.14)).

When the workpiece is rotated further, point A moves away from the tool face and its current density begins to reduce. When point A has rotated by 22 degrees from its initial position, there is no further reduction in its radius (see Fig 8.15).
Fig 8.14: Current density varied on point A as the workpiece is rotating

Fig 8.15: Decreasing of workpiece radius at point A as the workpiece is rotating
Fig 8.16: (a) Current density distribution (Ampere/mm²) (b) Radius at point A as it is rotated by 90°

Figure 8.16(b) shows the workpiece when point A is 90° degrees from the tool edge. It begins to be machined once again when it has completed one revolution (see Fig 8.17 (b)).

Fig 8.17: (a) Current density distribution (b) Workpiece shape after one revolution

From observation, when the workpiece rotates by one revolution, the average radius of the workpiece is equal to $r_A$ and node A will be machined when it is close to the tool surface. Therefore, the radius of the workpiece machined in a single revolution can be approximate by rotating the workpiece at specified angular distance.
A technique to calculate the machining of an axi-symmetric turned component is as follows:

Step 1: Calculate the current density at all nodes on the workpiece surface.

Step 2: Select the node on the workpiece surface which is nearest to the tool edge and has a current density of zero. This node must be travelling towards the tool edge. In Fig. 8.18 (b) this point is labelled as A and its radius is \( r_A \).

![Fig 8.18: (a) Reference node (node A) on the workpiece surface (b) Point A approaching the tool](image)

Step 3: Rotate the workpiece through angle \( \Delta \theta \) where \( \Delta \theta = \omega \cdot \Delta t \). \( \omega \) is the angular velocity (rad/sec) and \( \Delta t \) the time step. For this position of the workpiece, a BE analysis is performed and the current density is calculated at each of the nodes.

Step 4: Step 3 is repeated until point A has travelled past the tool and reached an extreme position at which its current density is virtually zero (see Fig 8.19(a) and (b)).
Fig 8.19: (a) Current density distribution on workpiece surface (b) Workpiece shape when point A has reached the right extreme position

Step 5: Let the radius of A when it has reached the right extreme position be $r_{A_{\text{new}}}$ as shown in Fig 8.19(b). This would be the radius of all the other nodes on the workpiece surface if the workpiece were to be rotated by one complete revolution. Therefore, after this step, the radius of workpiece is reduced to $r_{A_{\text{new}}}$. To reduce the radius of the workpiece in 3D, nodes lying on workpiece surface are moved in the radial direction until their radii are equal $r_{A_{\text{new}}}$. The grey surface in Fig. 8.20 shows the workpiece surface which is to be analysed in the next revolution.

Fig 8.20: The workpiece surface used for calculation at the current and the next revolution
If the entire cylindrical surface of the workpiece was analysed through one revolution, it was found that the workpiece radius after one revolution was slightly different from that when only a segment of the workpiece is analysed as described above. Both these results are shown in Fig 8.21 and the maximum difference is 0.003% which can be attributed due to computing errors creeping into the analyses due to the several time steps. However, this difference is very small and can be ignored. The main advantage of the method suggested herein is the drastic saving in computing time which was reduced from 18 hours to 4 hours.

![Graph showing radius vs. revolution](image)

Fig 8.21: The radius of the workpiece computed by BEM and developed technique

### 8.5 Conclusion

A program has been developed which can simulate the EC turning process using a 3-dimensional representation of the workpiece and tool. Results from this BE model are in good agreement with the experimental result with an error of 2%. Since the process involves an axi-symmetric solid of revolution, the program has shown that a reduction in radius of a short section of the workpiece surface as it passes in front of the tool is the same as the reduction in radius of the whole workpiece as it passes the tool in a complete revolution. By not without the need to computing the reduction for the whole rotation, the computing time can be reduced by as much as 75%.
Chapter 9
Experimental and numerical investigations into Shaped Tube Electrolytic Drilling (STED)

At the present time, designers and researchers are striving to design combustion engines to operate at a higher efficiency. Most of the increases in efficiency are achieved using techniques to improve the combustion process in the chamber by optimizing parameters such as the amount and pressure of air and fuel injected into the chamber. When the fuel is completely burnt, a large amount of heat energy is generated causing the engines having to operate at very high temperatures. To protect the engine from damage at these very high temperatures, cooling holes are machined in the blades in order to reduce the temperature in the engine. For example, in gas turbines, many small holes are drilled in the turbine blade as indicated in Fig 9.1.

![Fig 9.1: Cooling holes on turbine blade](image)

The diameter of the cooling holes is usually between 1 and 4 mm and the aspect ratio (i.e. the ratio between hole diameter and the depth of the hole) is 40 to 200 (Dayanand et al. [27]). However, holes with a diameter as small as 0.8 mm and an aspect ratio as large as 250 are often required on some turbine blades (Shiraz et al. [25]).

Engines and turbine blades are made from materials with a high thermal resistance such as super alloys (Inconel) which are difficult to machine. Moreover, the aspect ratio of the holes is vary large. These factors make it difficult to machine the holes using conventional drilling. Therefore, advanced machining techniques such as laser beam machining, electrical discharge machining (EDM) and electrochemical machining
(ECM) are preferred. In laser beam or electrical discharge machining, the material is removed by heating it to a high temperature until it melts and vaporises. These drilling processes cause internal cracks and residual stresses (when material cools). However, these problems do not occur with electro-chemical drilling, thus making it very suitable for drilling high aspect ratio holes in difficult-to-machine materials. The process that uses ECM to produce deep holes is referred to as shaped tube electrolytic machining (STEM) or, more appropriately, as shaped tube electrolytic drilling (STED).

The aim of this chapter is to model the STED process using the boundary element method and to experimentally verify the numerical predictions for various machining conditions. Details of the experimental set-up, the tool, workpiece and the experimental results are given in Section 9.1. The BE model is described in Section 9.2 as are the computed results and their comparison with experimental results. The application of the STED process to produce turbulated cooling channels (holes with periodic increases in diameter) is introduced in section 9.3. A turbulator is machined by stepped changes in the feedrates and Section 9.4 describes a generic procedure to determine the set of tool feed rates for machining a given turbulator shape. Two turbulator shapes are investigated and the set of feed rates recommended by the boundary element methods are verified experimentally.

9.1 Experimental investigation

9.1.1 Experimental set-up

The equipment used for STEM drilling consisted of a lab-scale ECM machine, which was numerically controlled in the Z-direction, i.e. tool axis direction (Fig 9.2); an AC electrical power supply; and a pumping system to supply the electrolyte to the machine. The machine was fitted with sensors to record the voltage, current, inlet pressure and volume flow rate of the electrolyte. The experiment set-up is shown in Fig 9.2
A STEM drill consists of a hollow titanium cylinder the outside surface of which is coated with a non-conductive material (the dark orange part in Fig 9.3(a)). STEM drills are available in various sizes. The experiments reported in this thesis were conducted with a tool of diameter 1.73 mm. This tool has an inner (bore) diameter of 1.17 mm and the thickness of the insulation is 0.075 mm (see Fig 9.3(b)).

Since the diameter of the tool is smaller than the tube used to supply electrolyte, a tube connector was used to connect the electrolyte supply tube to the tool (Fig 9.4). To guide
the tool to workpiece, the tool was clamped in a tool holder which was connected with the moving part of the Z-slide. The tool was guided through a hole on a guide block (Fig 9.5). It prevented the tool from wandering, thus ensuring that the hole was drilled at the required position; it also suppressed any vibrations at the tip of the tool caused by the flow of electrolyte. The workpiece was clamped in a holder and placed on the table of the machine (Fig 9.5).

Fig 9.4: Tool holder and tool-tube connector (see insert).
The electrolyte was Sodium Chloride (NaCl) diluted to 15% by weight in water and the workpiece was Stainless Steel (SS 316). Pulsed D.C. voltage of 12 and 14 volts was applied, with each cycle consisting of 6s of on-time and 0.2s of off-time interval. The holes were 48 mm deep. In the experiments, the effect of applied voltage, tool feed rate and pressure of the electrolyte on the diameter of the machined hole was investigated.

9.1.2 Experimental results

The holes drilled at different voltages, tool feed rates and inlet pressures of the electrolyte are shown in Fig 9.6 and their diameters are given in the Table 9.1
Table 9.1: Diameter of the holes

<table>
<thead>
<tr>
<th>case</th>
<th>Tool feed rate (mm/min)</th>
<th>Applied Voltage (V)</th>
<th>Inlet pressure of electrolyte (bars)</th>
<th>Diameter of the hole (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5</td>
<td>14</td>
<td>2.2</td>
<td>2.048</td>
</tr>
<tr>
<td>2</td>
<td>1.2</td>
<td>14</td>
<td>2.2</td>
<td>2.146</td>
</tr>
<tr>
<td>3</td>
<td>1.2</td>
<td>12</td>
<td>2.2</td>
<td>2.100</td>
</tr>
<tr>
<td>4</td>
<td>1.2</td>
<td>14</td>
<td>1.4</td>
<td>2.139</td>
</tr>
<tr>
<td>5</td>
<td>1.2</td>
<td>12</td>
<td>1.4</td>
<td>2.093</td>
</tr>
</tbody>
</table>

From Table 9.1, it is clear that drilling the holes at a lower tool feed rate (compare test 1 with test 2) and higher applied voltage (compare test 2 and test 3) increases the diameter of the drilled hole. In the former case, a unit area of the workpiece sees the tool for a longer time resulting in an increased hole diameter. In the latter case, the higher voltage increases the current density resulting in more material being removed. Also, the diameter of the hole drilled at a higher inlet pressure of 2.2 bars is slightly bigger than that drilled at a lower pressure (compare test 2 with 4, and 3 with 5). This is because when the pressure increases, the flow velocity of the electrolyte increases and, as a result, more of the sludge and bubbles are flushed away by the electrolyte, thus reducing the electrical resistance between the tool and workpiece.

9.2 Modelling STEM by BEM

9.2.1 BE domain

Since the tool and the hole to be machined are completely axi-symmetric (see Fig.9.7), it becomes unnecessary to represent the STED process using a 3D BE model. A 2D boundary element model is adequate (see Fig 9.7). Not only is a 2D model required but advantage can be taken of symmetry, which means that only one half of the tool and workpiece need to be modelled. Furthermore, since the boundary element method is being deployed, the tool and workpiece boundary are discretised using 1-D line elements (see Fig 9.8). The boundary of the tool, insulation and the workpiece are represented by blue, orange and red line elements respectively. The green line represents the tool axis. Since the BEM requires a closed domain, virtual elements (shown in black) are introduced to bridge the gaps between the tool and workpiece.
The boundary conditions are:

1. $V - \eta$ is imposed on the workpiece elements;
2. $V=0$ is imposed on the tool; and
3. $\frac{dV}{dn}=0$ is imposed on the virtual, symmetrical and insulation elements.

Continuous linear elements (as described in Chapter 3) were used to approximate the value of the gradient ($\frac{dV}{dn}$) at each node. To compute the coefficient of the G and H matrices (Chapter 3), 4-point Guassian quadrature was used.
At the start of the analysis, the length of each element on the workpiece was 0.1 mm. However, after each time step, the shape of the workpiece changes, especially the region in front of the tool. As a result, some elements are shrunk and others stretched as shown in Fig 9.9. When elements are shrunk several times, their length can become very small, thus introducing errors in the element coefficients since numerical integration is used. This error, of course, causes the wrong shape of workpiece to be predicted as shown in the profile of the workpiece after 54s of machining (see Fig 9.9). Decreasing the element size beyond a certain amount, leads to an unacceptable solution as shown in Fig 9.10 where the workpiece profile predicted with an element size of 0.07 mm self-intersects.

Fig 9.9: The evolution of workpiece profile after each time step.
Fig 9.10: The effect on minimum length of element on predicted profile.

When the elements are longer, numerical integration of the coefficients is exact but another error is introduced. Because linear elements are being used to represent the curved shape of the workpiece, the angle between the normal vectors to two neighbouring elements at the common node becomes large. To predict the workpiece profile accurately, in this research, if stretching an element causes the angle to be greater than a pre-set angle, the mesh is refined by splitting the stretched element into two elements as explained in Chapter 4.

To obtain the optimum value of the pre-set angle, workpiece profiles were computed and compared for different pre-set angles (see Fig 9.11). It is clear that there is not much difference between the profiles predicted with pre-set angles of 7° and 8°. Therefore, when generating the BE mesh, if an element is shorter than 0.08 mm then it is merged with one of its neighbouring elements. On the other hand if the angle between the two normal vectors at a node is greater than 8°, then the element is refined by splitting it into two.
9.2.2 BE results

The assumptions made in Chapter 6 regarding the 3D BE model are also valid for the 2D STED process model. Also, the procedure to determine the current efficiency, as explained in [1], was also used for the STED process. With an applied voltage of 12V and 0.3 mm of initial gap, the current efficiency for the STED process was found to be 86.8% and 84.3% at inlet pressures of 2.2 and 1.4 bars respectively. The diameters of the holes predicted from BEM are compared with the experimental results shown in Table 9.2

![Fig 9.11: Workpiece profiles computed with different pre-set angles](image-url)
Even though some machining parameters such as the sum of over potentials and electrode potentials and current efficiency have been experimentally determined and fed into the BE model, there are errors between the experimental and predicted values. However, the average error is only 1.08% with the maximum error being 1.957%. The BE results are always greater than the experimental values because the flux density is overestimated. The BE model assumes that ideal flow conditions exist and the gap is completely full of electrolyte i.e. there are no hydrogen bubbles or sludge all of which will tend to increase the electrical resistance. Since the feed rate and voltage are the same for tests 2 and 4, (the electrolyte pressures are different, which the BE model does not consider) the hole diameters predicted are nearly the same (i.e. 2.188 and 2.177 mm). For the same reason the BE model should have predicted the same diameter in tests 3 and 5, which it almost does.

### 9.3 Modelling of turbulated holes by BEM

In gas turbines, although passing cool air through the several cooling channels in the blades can reduce the temperature of the engine, the cooling rate can be increased if the cooling channels contain turbulators which are local increases in diameter superimposed, at periodic intervals, on the basic cylindrical shape of the cooling channel. These turbulators not only increase the heat transfer area but also induce turbulent flow as the cooling air flows past them.

There are some techniques available to produce turbulators. Some use tools that have a pattern of insulation on the side of them and they are inserted into holes already drilled (Wang et al. [30-31]). However this technique is not is not often used in practice due to the difficulty of producing the pattern of insulation on the tool surface which increases...
the cost and time of machining. Another technique is to produce turbulators by changing the tool feed rate as the tool is drilling the holes (Jain et al. [32]). However, these investigators have experimentally produced simple turbulator shapes as the feed rate changes from faster to slower values. But there technique was not capable of producing the more complex shapes of turbulators.

Therefore, in this section, a computational methodology is developed to determine a set of tool feed rates which, when used, will result in a given turbulator shape. The methodology is iterative in nature and uses the BEM to compute the current distribution at the workpiece boundary. The methodology will be tested computationally on a hole with turbulators (see Fig.9.12) which a local company was asked to produce by one of its clients. The turbulator has a maximum diameter of 3.30 mm and a throat diameter of 3.05 mm (see Fig 9.12). The other dimensions of the turbulator are given in Fig 9.13.

![Fig 9.12: 3D picture of the turbulator](image)
Initially, a STEM drill with an external diameter of 2.74 mm was used to produce the turbulator shape. This drill had a bore diameter of 2.03 mm and the thickness of the insulation was 0.1 mm. The workpiece material was super alloy Inconel 738 and the electrolyte was nitric acid (HNO₃) diluted to 20% by weight in water.

The acceptable tolerances on the turbulated hole is 10% of the nominal diameter and Fig 9.14 shows the inner and outer limits of the hole shape i.e. the hole must lie between the red and blue profiles.
Since the modelling is to be carried out by the BE M the domain for analysis is shown in Fig 9.15 which is similar to that described in Section 9.2. The blue lines represent the bare part of the drill; the red line represents the workpiece; the green line is the line of symmetry; the brown line is the insulated surface of the tool; and finally, the black lines represent the virtual boundary where the current flux density can be assumed to be zero.

The boundary condition and the assumptions made to model the machining of the turbulator are identical to those used in section 9.2. The machining parameters to produce the turbulator were:

1. applied voltage =16V;
2. over potential=1.2 V;
3. initial gap=0.4 mm.
Since Nitric acid (20% by wt) was used as the electrolyte instead of Sodium Chloride (15% by wt), a higher value of current efficiency was used, i.e. 90% and the time step was 0.1s.

During machining, it will become necessary to increase the tool feed rate, as for example, when machining the section along AB (see Fig 9.14) where the diameter of the hole becomes smaller. An increase in the tool feed rate will result in the frontal gap size become smaller. When the gap is too small, electrical sparking can occur resulting in the tool becoming damaged. Therefore, the gap size measured from the tool edge to the workpiece in the vertical direction (hc) and the over-cut value (ho) measured from the tool edge horizontally to the workpiece, as illustrated in Fig 9.16, were monitored after each time step.

Therefore, a constraint was imposed on the gap size which should not be smaller than 0.140 mm. This value was suggested by the collaborating company, ELE Advanced Technologies Ltd. Therefore, during any iteration, when a particular feed rate resulted in either hc or ho becoming less than 0.140 mm, it was considered unacceptable and had to be modified.
Before the profile of the turbulator was machined, the feed rate for producing a hole of diameter 3.30 mm was determined using the BEM. Using the machining parameters given above, the hole sizes that were generated using different feed rates are shown in Table 9.3.

<table>
<thead>
<tr>
<th>Feed rate (mm/min)</th>
<th>1.6</th>
<th>1.8</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hole diameter (mm)</td>
<td>3.40</td>
<td>3.29</td>
<td>3.13</td>
</tr>
</tbody>
</table>

Table 9.3: Diameter of holes obtained at different feed rates.

From Table 9.3, using a 1.8 mm/min feed rate can produce a hole whose diameter is closest to the required diameter of 3.30 mm and therefore it was chosen as the initial feed rate for producing the required shape of the turbulator. The turbulator shape was divided into 32 sections (see Fig 9.17), the width of each section being 0.1 mm (see Fig 9.17). Within each section, the feed rate was maintained at a constant value. It was felt that a smaller width would not only make the number of feed changes large and the control system would also find it difficult to respond to changes every few seconds.
In the first iteration, the inner shape of the turbulator (the blue curve in Fig 9.17) is chosen as the target shape curve because as the tool is fed through the hole, the hole will be enlarged due to back machining. The machining of the hole is simulated until the end of the tool reaches Section 1. As mentioned earlier, a feedrate of 1.8 mm/min is
required to machine a hole of 3.30 mm. The profile of the workpiece at this instant is shown in Fig 9.18.

Next, intersection points between a horizontal line drawn at section 1 (S1) through the current workpiece profile and the target curve are determined and they are labelled as A1 and B1 respectively (see Fig 9.18). Typically, point A1 will be to the left of point B1; however, there will be some instances where A1 will be to the right of B1.

The value of the feed rate has now to be computed which would move the tool end from section 1 to section 2. This feed rate should cause point A1 on the workpiece to move to point B1 on the target curve. This feed rate cannot be computed directly but requires
several attempts with each attempt deploying a trial feed rate. The only clue is that, for A1 to move to B1, more material should be machined which means that the feed rate should be smaller. It was found that the tool should move by 1 mm/min from section 1 to 2. The resulting workpiece shape, with the tool end face at level 2, is shown in Fig 9.19. Note that points A1 and B1 are now coincident.

As before, intersection points of the horizontal line thru section 2 with the current workpiece profile and the target curve are determined, i.e. A2 and B2 (see Fig.9.19). Next, as before, a feed rate is determined using a trial and error approach to move the tool from section 2 to 3 such that A2 coincides with B2. This procedure of machining in
stages with the tool moving from one section to the next is continued until the end of the
tool is at the end of section 7, with A7 almost coincident with B7. Figure 9.20 shows
how the workpiece has evolved until then.

![Fig 9.20: Evolution of workpiece profile when the tool moves from section 1 to 7](image)

The inner diameter of the hole decreases from section 5 to 9 suggesting that the feed
rate should be increased to avoid removing excessive material. However, increasing the
tool feed rate results in a smaller front and side gap. Therefore the highest feed rate that
can be used is that which will not result in the gaps becoming less than a specified
value, which from experience was taken to be 0.14 mm. For example, consider the
workpiece profile when the tool is at section 7; point A7 is almost coincident with point
B7 (see Fig9.20). At this stage, it was found that $h_c$ and $h_o$ were still far greater than
0.14 mm – they were 0.1746 and 0.2339 mm, respectively, and therefore the maximum feed rate of 10 mm/min was used to move the tool from section 7 to 8. The evolution of the workpiece profile from section 1 to section 20 is shown in Fig 9.21 and the predicted profile computed at the first iteration is shown in Fig 9.22. This shape is not acceptable because the diameter at some sections is greater than that of the required turbulator. The reason for it exceeding the turbulator shape is that points Ai, which were moved to the right to coincide with Bi, have moved further to the right as the tool moves down because of secondary machining.

In the second iteration, the predicted shape has to be corrected and for this a new reference curve is required. This curve is obtained by determining the difference between the curve predicted in the first iteration and the required shape of the turbulator (see Fig 9.23); the differences (i.e. errors) between the two shapes, measured at each level are then added to the original reference curve to give a new reference curve. The errors are shown in Fig 9.23 and the new reference curve in Fig 9.24.

Using the procedure explained above, a second iteration is performed with the tool being moved from section to section, and the feed rate computed by trial and error using the BEM, such that points A_i and B_i coincide. The points B_i, it must be emphasised, are on the new reference curve. The computed set of feed rates is shown in Table 9.4 and the evolution of the profile is shown in Fig 9.25. The profiles predicted from the first and the second iterations are shown in Fig 9.26. The predicted profile calculated from the second iteration shows good agreement with the required profile and the diameter at each section lies between the outer and inner turbulator shapes. Therefore, the second iteration results are acceptable and the procedure is terminated.

The flow chart in Appendix I the procedure to calculate the feed rates at the different sections.
Fig 9.21: Evolution of predicted profile from section 1 to section 20
Fig 9.22: Workpiece profile predicted from the first iteration.
Fig 9.23: Difference between the predicted and required profiles
Fig 9.24: A new reference curve

New reference curve

Previous reference curve

X (mm)

Z (mm)
Fig 9.25: Evolution of workpiece profile calculated from the second iteration
Table 9.4: Tool feed rates at each section

Although the feed rate was calculated at 31 sections, the first 16 sections are repeated (see Table 9.4). The values of tool feed rate, $h_s$ and $h_o$ at each section are also given in Table 9.5.

<table>
<thead>
<tr>
<th>Section</th>
<th>Feed rate (mm/min)</th>
<th>Distance (mm)</th>
<th>$h_s$ (mm)</th>
<th>$h_o$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.8</td>
<td>0.1</td>
<td>0.257</td>
<td>0.305</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.1</td>
<td>0.266</td>
<td>0.328</td>
</tr>
<tr>
<td>3</td>
<td>1.6</td>
<td>0.1</td>
<td>0.269</td>
<td>0.327</td>
</tr>
<tr>
<td>4</td>
<td>1.6</td>
<td>0.1</td>
<td>0.287</td>
<td>0.357</td>
</tr>
<tr>
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<td>0.295</td>
<td>0.358</td>
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<td>0.238</td>
</tr>
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<td>8</td>
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<tr>
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<td>0.1</td>
<td>0.150</td>
<td>0.198</td>
</tr>
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<td>0.1</td>
<td>0.158</td>
<td>0.224</td>
</tr>
<tr>
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<td>0.1</td>
<td>0.150</td>
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</tr>
<tr>
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<td>0.147</td>
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<td>0.208</td>
<td>0.308</td>
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<td>0.1</td>
<td>0.269</td>
<td>0.375</td>
</tr>
<tr>
<td>15</td>
<td>4</td>
<td>0.1</td>
<td>0.264</td>
<td>0.317</td>
</tr>
<tr>
<td>16</td>
<td>2</td>
<td>0.1</td>
<td>0.185</td>
<td>0.249</td>
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</tbody>
</table>

Table 9.5: Set of feed rates for producing the required turbulator shape
Fig 9.26: Required profile and profiles computed from the 1st and 2nd iterations
Although the predicted profile shown in Fig 9.26 is within the acceptable error of required profile (between red and blue colour profiles), it was predicted by using the maximum feed rate of 10 mm/min. In practice, changing feed rate from a low value to very high value (changing from 3 mm/min to 10 mm/min, from step 6 to 7 as shown in Table 9.5) within a short distance (0.1 mm) may not be possible in actual machining because this requires the use of a high accuracy machine and it may result in tool damage when the tool contacts the workpiece. The suggestion from the collaborating company is to use the maximum feed rate of 3.5 mm/min instead of 10 mm/min.

From using the procedure presented earlier to predict the turbulator shape with the maximum feed rate of 3.5 mm/min, the predicted shape is calculated until the profile converges after the third iteration (see Fig 9.27) and the final profile, compared with using 10 mm/min for the maximum feed rate, is shown in Fig 9.28. The values of tool feed rate, $h_s$ and $h_o$, at each section are also given in Table 9.6.

<table>
<thead>
<tr>
<th>STEP</th>
<th>Feed rate (mm/min)</th>
<th>Distance (mm)</th>
<th>$h_s$ (mm)</th>
<th>$h_o$ (mm)</th>
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<tbody>
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<td>0.284</td>
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<td>16</td>
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<td>0.1</td>
<td>0.265</td>
<td>0.305</td>
</tr>
</tbody>
</table>

Table 9.6: Set of feed rates for producing turbulator shape with 3.5 mm/min maximum feed rate
Fig 9.27: Profiles computed from 1\textsuperscript{st} to 4\textsuperscript{th} iteration with a maximum feed rate of 3.5 mm/min
Fig 9.28: Comparison of profiles computed using maximum feed rates of 3.5 and 10 mm/min.

Profiles predicted by using the maximum feed rate of 10 mm/min.

Profiles predicted by using the maximum feed rate of 3.5 mm/min.

Required profile.
From Fig 9.28, it is clear that by reducing the maximum feed rate the, the predicted profile is not acceptable as the predicted shape, especially at the throat, as it is beyond the outer limits (point A in Fig 9.28). More material has been removed than required because of the reduced feed rate. The maximum error (the error being defined as the distance measured between the predicted and required profiles at each section), at 10 and 3.5 mm/min of maximum feed rates, was 0.008 mm and 0.019 mm respectively. In practice, however, a slower feed rate is preferred because it can reduces the possibility of a short circuit occurring and with tool damage.

To reduce the number of steps used to produce the turbulator, a solution is to increase the distance moved between each section. Fig 9.29 shows the profiles calculated with distances of 0.1 and 0.2 mm between two sections. Both profiles have their maximum error of 0.019 mm at the throat and the average errors for two profiles are 0.007 mm and 0.012 mm respectively. The set of feed rates for 0.2 mm sections is given in Table 9.7.

<table>
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<tr>
<th>STEP</th>
<th>Feed rate (mm/min)</th>
<th>Distance (mm)</th>
<th>$h_s$ (mm)</th>
<th>$h_o$ (mm)</th>
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<td>3.5</td>
<td>0.2</td>
<td>0.2579</td>
<td>0.2832</td>
</tr>
<tr>
<td>8</td>
<td>1.5</td>
<td>0.2</td>
<td>0.2688</td>
<td>0.3339</td>
</tr>
</tbody>
</table>

Table 9.7: Set of feed rates to machine required turbulator shape with 0.2 mm section distance

The predicted shape for the turbulator using the 0.2 mm section distance was verified experimentally by drilling several turbulated holes in an Inconel block with the freed rates shown in Table 9.7. The hole shapes were measured by filling the machined hole with a resin. The cast specimens were then measured using a microscope and one of these shapes is shown in Fig.9.30.
Fig 9.29: Profiles calculated with section distances of 0.1 and 0.2 mm

Fig 9.30: The cast specimen
From Fig 9.31, there is some difference between the experimental and predicted profile, with the maximum error of 0.022 mm occurring at section A-A and the minimum error of 0.008 at section B-B (see Fig 9.30). These errors could be due to the several assumptions made in the BE model such as assuming the current efficiency to be constant at 90%. Another possible reason is a drop in pressure accompanied by a
smaller flow rate. This may result in the sludge and bubbles not being carried away, leading to a change in conductivity. A flow analysis would have been useful to ensure that there is no cavitation at these reduced pressures.

9.4 Conclusion

The developed program can be used to model the STED process. The predicted hole diameters, using different machining parameters, show good agreement with experimental results. The maximum error is 1.957%. A procedure to determine a set of feed rates to produce a given turbulator shape has been developed and the effect of the maximum rapid feed rate and the rate at which the feed rate should be changed have been investigated.
Chapter 10

Conclusions and Future work

10.1 Conclusions

10.1.1 Boundary element model

In summary, a 3-D boundary element model has been developed to simulate the electrochemical machining process. The boundary element method is ideally suited to model the ECM process because the solution to the unknown variable, i.e. the current density, is not required at points inside the domain but only at discrete points on the workpiece surface. This model has been applied to EC drilling, turning and milling and from the results obtained, the following conclusions can be made.

(1) The accuracy of prediction is strongly dependent on the magnitude of the time step. If the value of the time step is high, the predicted profile exhibits fluctuations.

(2) The predicted shape of the workpiece also depends upon the mesh density. The workpiece shape becomes spiky if the mesh is not fine enough. The mesh should be refined when the angle subtended between two neighbouring elements is greater than 5° to 6°.

(3) Because of the small time steps, simulation of the ECM process is very time consuming, e.g. 15 minutes per time step on a single processor PC with 3000 nodes in the model. If each time step is 0.1s and if several minutes of machining has to be simulated, the simulation can take hours.

(4) A novel method of reducing the computing time has been implemented by classifying the nodes as active or passive.

(5) Numerical results obtained from the ECM modeller were generally in good agreement with the experimental results, the maximum error in the predicted results being 11%.

10.1.2 Experiments in EC-milling and STEM drilling

Experimental work has also been performed in EC-milling and STEM drilling and the main conclusions that can be made are as follows.
(1) Low feed rates and high voltages increase the diameter of the STEM drilled hole.

(2) Holes drilled with the electrolyte at a higher pressure have a slightly bigger diameter than those drilled with electrolyte at a lower pressure.

(3) The cross-sectional shape of the tool affects the geometry of the machined slot in EC-milling. The slot machined by a square tool is wider than that machined by a cylindrical tool. The base surface of a wide slot is flatter when it is machined by a square tool than with a cylindrical tool. However, when a square pocket is machined, both tool shapes yield similar results.

(4) The profile of the base surface in a pocket or a wide slot depends not only on the tool shape but also very strongly on the step-over distance. A good surface is obtained if the step-over distance is as small as 0.1 mm.

(5) In the case of EC-milling, when machining a pocket with a complex internal feature, it was found that the pocket machined using contour-parallel path type results in a slightly more accurate feature than that machined with zig-zag tool paths.

(6) Both types of tool path are not suitable for EC milling because there were instances when certain parts of the pockets were machined more than once causing the base surface to become concave.

10.2 Future work

In modelling of EC-sinking or drilling, it is essential to model the flow of the electrolyte within the electrode gap. But this may not be straightforward because modelling the flow would require the domain to be discretised, a requirement which is in direct conflict with the boundary element method wherein only the boundary surfaces are discretised. Modelling the flow at every time step would be unnecessary. A flow analysis would identify regions where there is little or no flow and these could be represented in the boundary element method by regions of high electrical resistance, thus forcing no current to flow through these regions.

Similarly, the accuracy of the BE-ECM model can be improved by taking into consideration the presence of hydrogen bubbles and sludge, both of which increase the effective electrical resistance of the electrolyte. These effects are probably best incorporated by using empirical equations rather than modelling two-phase flow.
The modelling of flow is more essential in STED because pressure and flow have a more pronounced effect on the quality of deep holes than in other EC machining processes.

In EC-milling, although the effect of products generated by chemical reaction on machining can be omitted, the vortices formed in the gap, due to the electrolyte flowing past the tool body, will affect the material removal process. When vortices appear within the gap, the flow rate of the electrolyte is reduced. Consequently, the material removal rate is reduced. Therefore, the effect of vortices occurring during the machining process has to be considered when modelling EC-milling.

As mentioned earlier, a special CAM system is required for generating special tool paths for EC-milling. Whilst it may not be possible to generate tool paths where every point on the surface has the same machining time, tool paths with varying feed rates may be a feasible solution.

Whilst the current BE-EC model can simulate reversed polarity, it would be interesting to predict tool wear and hence tool life. Machining with a worn tool leads to defects in the drilled hole.
References


Appendix I

1. Input the shape required to produce

2. Input value of constrain variable such as the acceptable error, maximum allowable feed rate, minimum value of \( h_o \) and \( h_c \) and error value to check convergent.

3. Define a reference curve

4. Split the reference curve into sections

5. Position the end of the tool and the profile of workpiece machined by using initial feed rate at the first section.

6. Compute the intersection point between current section line and profile of the workpiece (point A) and between current section line and reference curve (point B)

7. Check position of point A referring to point B
1. Calculate the feed rate for moving the tool to the next section. (This feed rate results the material of the workpiece at point A is removed to point B)
   Note: at the end of this step, the tool will be moved to the next section line.

2. Calculate the highest feed rate that the tool can move to the next section. This feed rate must not be greater than the limited feed rate and it results the value of $h_0$ and $h_c$ are less than their minimum value.
   Note: at the end of this step, the tool will be moved to the next section line.

3. Check whether the current section line is the last section line or not

4. Calculate the error of diameter between the required profile and the predicted profile at every section

5. Check whether error value at all sections is in acceptable range or not
   Yes

6. If the current section line is not the last section line, go back to step 3.

7. If the current section line is the last section line, stop the process.
Calculate the difference value of diameter between predicted profile computed at the latest iteration and the one computed at the previous iteration.

Check whether the current computation is in the 1st iteration or not.

No

Calculate the difference value of diameter between predicted profile computed at the latest iteration and the one computed at the previous iteration.

Check whether the different value at all sections is less than the convergent error.

No

Yes

End