STOCHASTIC FINITE ELEMENT SLOPE STABILITY ANALYSIS

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Symbols and Notation

**Superscripts**
- $e$: Elastic
- $p$: Plastic
- \(\cdot\): Denotes increment or first derivative
- \(\cdot\cdot\cdot\): Denotes second derivative
- \(\cdot\cdot\cdot\cdot\cdot\): Denotes effective stress conditions

**Subscripts**
- 0: Initial Conditions
- 1, 2, 3: Principal directions of stress or strain
- ave: Average
- c: Critical state conditions
- c, d: Compressive/deviatoric components (for Monot parameters)
- ET: End of test conditions
- f: Failure
- i: Image conditions (occurs when $D_p \geq 0$ for all $D_p \geq 0$)
- MC: Maximum contraction conditions
- res: Residual
- SD: Stress-dilatancy
- ss: Steady state conditions
- tc: Triaxial compression conditions ($\theta = \pi/6$)
- te: Triaxial extension conditions ($\theta = -\pi/6$)
- v: Vertical, Volumetric
- y: Yield

**Geometry Variables**
- $A(I,3)$ [L]: Depth below soil surface for each point
- $d$ [L]: Depth above the sea water level for each CPT profile
- $Z$ [L]: Depth below PWD +10m; PWD = Public Works Department datum in Bangladesh
Notation

Stress Variables (dash or bar over denotes effective)

\( \eta \) [-] Dimensionless shear measure as ratio of stress invariants \( \frac{\bar{\sigma}_q}{\bar{\sigma}_m} \)

\( \theta \) [Rad] Lode angle, \( \sin(3\theta) = -13.5 \frac{\bar{\sigma}_1 \bar{\sigma}_2 \bar{\sigma}_3}{\bar{\sigma}_q^3} \)

\( \sigma_{i,2,3} \) [FL\(^{-2}\)] Principal stresses

\( \bar{\sigma}_m \) [FL\(^{-2}\)] Mean effective stress \( = (\bar{\sigma}_1 + \bar{\sigma}_2 + \bar{\sigma}_3)/3 \)

\( \bar{\sigma}_q \) [FL\(^{-2}\)] Deviatoric stress invariant

\( I_{1,2,3} \) [FL\(^{-2}\)] Effective stress invariants

\( p_0 \) [FL\(^{-2}\)] Initial mean total stress

\( p_a \) [FL\(^{-2}\)] Atmospheric pressure =100 kPa

\( p' \) [FL\(^{-2}\)] Mean effective stress \( = \bar{\sigma}_m \)

\( q \) [FL\(^{-2}\)] Triaxial deviator stress \( = \sigma_1 - \sigma_3 = \bar{\sigma}_q \)

\( s' \) [FL\(^{-2}\)] Effective volumetric stress invariant \( = \sqrt{3}p' \)

\( t \) [FL\(^{-2}\)] Deviatoric stress invariant \( = \sqrt{\frac{2}{3}}q \)

\( u \) [FL\(^{-2}\)] Pore pressure

Strain Variables (dash or bar over denotes effective)

\( \gamma \) [-] Deviatoric strain invariant

\( \varepsilon_{i,2,3} \) [-] Principal strains (assumed to be coaxial with principal stresses)

\( \dot{\varepsilon}_v \) [-] Volumetric strain rate

\( \dot{\varepsilon}_q \) [-] Shear strain rate measure work conjugate with \( \bar{\sigma}_q \)

\( \nu \) [-] Volumetric strain invariant

\( D^p \) [-] Plastic dilatancy, as strain rate ratio \( \dot{\varepsilon}_v^p / \dot{\varepsilon}_q^p \)

State Variables

\( \gamma \) [FL\(^{-3}\)] Total unit weight

\( \gamma' \) [FL\(^{-3}\)] Submerged unit weight
### Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_d$</td>
<td>[FL(^{-3})]</td>
<td>Dry unit weight</td>
</tr>
<tr>
<td>$\gamma_{sat}$</td>
<td>[FL(^{-3})]</td>
<td>Saturated unit weight</td>
</tr>
<tr>
<td>$\gamma_{sw}$</td>
<td>[FL(^{-3})]</td>
<td>Salt-water unit weight</td>
</tr>
<tr>
<td>$\psi$</td>
<td>[-]</td>
<td>State parameter</td>
</tr>
<tr>
<td>$e$</td>
<td>[-]</td>
<td>Void ratio</td>
</tr>
<tr>
<td>$g_{new}$</td>
<td>[LT(^{-2})]</td>
<td>Additional gravitational loading ($g = 9.81 \text{m/s}^2$)</td>
</tr>
<tr>
<td>$K_0$</td>
<td>[-]</td>
<td>Geostatic stress ratio, $\frac{\sigma_h}{\sigma_v}$</td>
</tr>
</tbody>
</table>

### Testing Parameters and Variables (excluding CPT test)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0, n_v$</td>
<td>[-]</td>
<td>Elastic material constants relating to shear modulus</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>[FL(^{-2})]</td>
<td>Applied axial stress of triaxial tests</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>[FL(^{-2})]</td>
<td>Applied radial stress of triaxial tests</td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>[deg]</td>
<td>Mobilised friction angle corresponding to no-volume-change</td>
</tr>
<tr>
<td>$\phi_{cv}$</td>
<td>[deg]</td>
<td>Constant-volume-change friction angle</td>
</tr>
<tr>
<td>$\phi_{peak}$</td>
<td>[deg]</td>
<td>Peak friction angle</td>
</tr>
<tr>
<td>$C_u$</td>
<td>[-]</td>
<td>Coefficient of uniformity of soil</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>[-]</td>
<td>Median grain size</td>
</tr>
<tr>
<td>$D_{10}$</td>
<td>[-]</td>
<td>Diameter of the 10 percentile grain size</td>
</tr>
<tr>
<td>$f(%)$</td>
<td>[-]</td>
<td>Fines contents</td>
</tr>
<tr>
<td>$G_{CPM}$</td>
<td>[FL(^{-2})]</td>
<td>Shear modulus based on the cone pressuremeter test</td>
</tr>
<tr>
<td>$G_i$</td>
<td>[FL(^{-2})]</td>
<td>Initial shear modulus</td>
</tr>
<tr>
<td>$G_{max}$</td>
<td>[FL(^{-2})]</td>
<td>Shear modulus based on the seismic body wave test</td>
</tr>
<tr>
<td>$G_{ru}$</td>
<td>[FL(^{-2})]</td>
<td>Reload/unload shear modulus</td>
</tr>
<tr>
<td>$G_{ur}$</td>
<td>[FL(^{-2})]</td>
<td>Unload/reload shear modulus</td>
</tr>
<tr>
<td>$K_G, n, b$</td>
<td>[-]</td>
<td>Modulus number and modulus exponent of the cone-pressuremeter test</td>
</tr>
<tr>
<td>$V_s$</td>
<td>[LT(^{-1})]</td>
<td>Shear wave velocity passed through the soil</td>
</tr>
</tbody>
</table>
Notation

CPT Parameters and Variables

\( f_s \) [FL\(^{-2}\)]  CPT sleeve friction stress measurement

\( F \) [-]  Stress normalised CPT friction ratio, \( = f_s/(q_t - \sigma_{v0}) \)

\( k, m \) [-]  Soil and rigidity specific coefficients in equation relating \( Q_p \) to \( \psi \)

\( q_c \) [FL\(^{-2}\)]  CPT tip resistance, as measured

\( q_t \) [FL\(^{-2}\)]  CPT tip resistance after correction for unequal area effect

\( Q \) [-]  Dimensionless CPT resistance based on vertical stress

\( Q_p \) [-]  Dimensionless CPT resistance based on mean stress

\( u \) [FL\(^{-2}\)]  CPT pore pressure measurement

Elasticity

\( \nu \) [-]  Poisson’s ratio

\( E, E_Y \) [FL\(^{-2}\)]  Young’s modulus

\( G \) [FL\(^{-2}\)]  Shear modulus

\( I_r \) [-]  Soil shear rigidity, \( = G/\bar{\sigma}_m \)

\( K \) [FL\(^{-2}\)]  Bulk modulus

Plasticity

\( \kappa_c, \kappa_d \) [-]  Compressive and deviatoric hardening parameters

\( D_{\text{min}} \) [-]  Minimum dilatancy

\( F_c \) [-]  Compressive yield surface

\( G_c \) [-]  Compressive plastic potential surface

\( G_d \) [-]  Deviatoric plastic potential surface

\( M \) [-]  Critical friction ratio

\( M^* \) [-]  Stress ratio corresponding to no volume change surface

\( M_{MC} \) [-]  Mohr-Coulumb critical friction ratio

\( M_{MN} \) [-]  Matsuoka-Nakai critical friction ratio

\( N \) [-]  Volumetric coupling parameter
Notation

Critical State

\( \Gamma \) [-] Altitude of the critical state line, defined at 1kPa
\( \lambda \) [-] Slope of the critical state line, defined on base \( e \)

NorSand Model Parameters

\( \chi \) [-] Dilatancy constant
\( H \) [-] Plastic hardening modulus
\( H_r \) [-] Plastic softening modulus under principal stress rotation
\( M_i \) [-] Current value of \( \eta \) at \( D^p = 0 \) (used in the flow rule)

Monot Model Parameters

\( A, AP \) [-] Non-linear elastic stiffness scale and curvature
\( B, BP \) [-] Plastic compressive hardening scalar and curvature
\( C, CP \) [-] Plastic deviatoric failure surface scalar and curvature
\( CG, CV \) [-] Plastic deviatoric failure surface adjustment to \( dv/d\gamma \)
\( E, EP, LB \) [-] Plastic deviatoric failure surface scalar and curvatures
\( FICV \) [-] Plastic deviatoric failure surface constant volume friction angle
\( FIMU \) [-] Plastic deviatoric failure surface interparticale friction angle
\( N \) [-] Plastic deviatoric failure surface transition to failure
\( SCV \) [-] Plastic deviatoric failure surface transition angle
\( RT \) [-] Plastic deviatoric failure surface potential
\( V \) [-] Poisson’s ratio
\( VGC, VGP, NU \) [-] Plastic deviatoric failure surface adjustment for low \( t/s' \)

Statistical Parameter and Functions

\( \beta \) [-] Beta function
\( \gamma \) [-] Spectral exponent/Semivariogram
\( \xi \) [-] Degree of anisotropy of the heterogeneity
\( \theta \) [-] Scale parameter
\( \theta_h \) [L] Horizontal scale of fluctuation
### Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Units</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_v$</td>
<td>[L]</td>
<td>Vertical scale of fluctuation</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>[-]</td>
<td>Shape parameter</td>
</tr>
<tr>
<td>$\mu$</td>
<td>[-]</td>
<td>Mean</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>[-]</td>
<td>Standard deviation</td>
</tr>
<tr>
<td>$\chi_k^2$</td>
<td>[-]</td>
<td>Chi-square function; $k = \text{degree of freedom for chi-square}$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>[-]</td>
<td>Digamma function</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>[-]</td>
<td>Gamma function</td>
</tr>
<tr>
<td>$\Gamma^2(T)$</td>
<td>[-]</td>
<td>Variance function</td>
</tr>
<tr>
<td>$\psi_j^m$</td>
<td>[-]</td>
<td>Mother wavelet</td>
</tr>
<tr>
<td>$C_2(\delta)$</td>
<td>[-]</td>
<td>Autocovariance function</td>
</tr>
<tr>
<td>$dy$</td>
<td>[L]</td>
<td>Distance between data points</td>
</tr>
<tr>
<td>$E$</td>
<td>[-]</td>
<td>Expected frequency</td>
</tr>
<tr>
<td>$E(x^n)$</td>
<td>[-]</td>
<td>$n^{th}$ moment of a probability distribution function</td>
</tr>
<tr>
<td>$G_0$</td>
<td>[-]</td>
<td>Spectral intensity</td>
</tr>
<tr>
<td>$G(\omega)$</td>
<td>[-]</td>
<td>One-sided spectral density function</td>
</tr>
<tr>
<td>$H$</td>
<td>[-]</td>
<td>Hurst/Self-similarity coefficient</td>
</tr>
<tr>
<td>$L(\theta)$</td>
<td>[-]</td>
<td>Likelihood function</td>
</tr>
<tr>
<td>$O$</td>
<td>[-]</td>
<td>Observed frequency</td>
</tr>
<tr>
<td>$P[A]$</td>
<td>[-]</td>
<td>Prior probability of $A$</td>
</tr>
<tr>
<td>$P[A</td>
<td>B]$</td>
<td>[-]</td>
</tr>
<tr>
<td>$R_z(\delta)$</td>
<td>[-]</td>
<td>Autocorrelation function</td>
</tr>
<tr>
<td>$S_e(\hat{\theta})$</td>
<td>[-]</td>
<td>Standard error of $\hat{\theta}$</td>
</tr>
<tr>
<td>$S(\omega)$</td>
<td>[-]</td>
<td>Spectral density function</td>
</tr>
<tr>
<td>$t$</td>
<td>[-]</td>
<td>Any mathematical variable</td>
</tr>
<tr>
<td>$T$</td>
<td>[L]</td>
<td>Averaging distance</td>
</tr>
</tbody>
</table>

### Random field and Local Average Subdivision Variables and Functions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Units</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta(t,t')$</td>
<td>[-]</td>
<td>Covariance function</td>
</tr>
<tr>
<td>$\tau$</td>
<td>[L]</td>
<td>Lag distance</td>
</tr>
<tr>
<td>$\varphi(t,t')$</td>
<td>[-]</td>
<td>Correlation function</td>
</tr>
<tr>
<td>$a$</td>
<td>[-]</td>
<td>Weighting coefficients</td>
</tr>
<tr>
<td>$D$</td>
<td>[L]</td>
<td>Cell width; Domain size</td>
</tr>
</tbody>
</table>
Notation

\( i, m \) [-] Stage, cell number
\( I_r \) [-] Local integral process
\( m(p), n(p) \) [-] Traversing vectors
\( M \) [-] Estimated mean for a new cell
\( S_q \) [-] Amount of squashing
\( S_t \) [-] Amount of stretching
\( U \) [-] Gaussian white noise
\( X_A \) [-] Local average
\( X_r(t) \) [-] Moving average of property \( X \)
\( Z_1^0 \) [-] Global mean of the process \( Z(t) \)

Finite Element Vectors and Matrices

\( \{ \sigma_0 \} \) Stress state at the beginning of an increment
\( \{ \sigma_i^* \} \) Illegal normal stress state
\( \{ \sigma_n \} \) Stress state at the end of an increment
\( [B] \) Derivatives of Gauss point solid shape functions
\( [D_{ep}]_\text{mod} \) Modified elastoplastic stress-strain matrix
\( \{ d\varepsilon \} \) Strain increment
\( \{ d\varepsilon_{\text{sub}} \} \) Strain subincrement
\( \{ dF_{\text{err}} \} \) The error computed in the body loads
\( \{ dF_{\text{ext}} \} \) Externally applied loads vector
\( \{ dF_n \} \) Body loads vector
\( [K] \) Global elastoplastic stiffness matrix
\( S_i'' \) Isotropic stress invariant for the illegal stress state
Abstract

In this thesis, the failures that occurred during the construction of the Jamuna Bridge Abutment in Bangladesh have been investigated. In particular, the influence of heterogeneity on slope stability has been studied using statistical methods, random field theory and the finite element method. The research is divided into three main parts: the statistical characterization of the Jamuna River Sand, based on an extensive in-situ and laboratory database available for the site; calibration of the laboratory data against a double-hardening elastoplastic soil model; and stochastic finite element slope stability analyses, using a Monte Carlo simulation, to analyse the slope failures accounting for heterogeneity.

The sand state has been characterised in terms of state parameter, a meaningful quantity which can fully represent the mechanical behaviour of the soil. It was found that the site consists of predominantly loose to mildly dilative material and is very variable. Also, a Normal distribution was found to best represent the state parameter and a Lognormal distribution was found to best represent the tip resistance.

The calibration of the constitutive model parameters was found to be challenging, as alternative approaches had to be adopted due to lack of appropriate test results available for the site. Single-variate random fields of state parameter were then linked to the constitutive model parameters based on the relationships found between them, and a parametric study of the abutment was then carried out by linking finite elements and random field theory within a Monte Carlo framework.

It was found that, as the degree of anisotropy of the heterogeneity increases, the range of structural responses increases as well. For the isotropic cases, the range of responses was relatively smaller and tended to result in more localised failures. For the anisotropic cases, it was found that there are two different types of deformation mechanism. It was also found that, as the vertical scale of fluctuation becomes bigger, the range of possible structural responses increases and failure is more likely. Finally, it was found that the failed zones observed during the excavation of the West Guide Bund of the Jamuna Bridge Abutment could be closely predicted if heterogeneity was considered in the finite element analyses. In particular, it was found that, for such a natural deposit, a large degree of anisotropy (in the range of 20) could account for the deformation mechanisms observed on site.
Declaration

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Last but not the least I would like to thank my parents and my family for supporting me during all this time, financially and emotionally. I believe my greatest asset in life is my family and for that I feel so blessed and am so thankful.

The research in this thesis has been carried out as part of the European Union FP7 Project “IRIS: Integrated European Industrial Risk Reduction System”, Project Number 213968.
Dedication

To my parents.

Siamak Bakhtiari
January 2011
Chapter 1

Introduction
Chapter 1

Introduction

1.1 Background to the research

Soil structures are generally idealised as layers (or zones) of uniform material, in which each layer is assumed to be homogeneous and is therefore given a single set of property values. This simplification leads to the use of mean property values (or other single representative values) for each layer and enables a deterministic analysis of the structure leading to a single factor of safety. However, in-situ testing techniques, such as the cone penetration test (CPT) from which continuous soil profiles can be obtained, have revealed that soil property values are not the same within a stratum and differ from one point to another; therefore, soil layers are not homogeneous. Heterogeneity is not only limited to soil strength, but includes other properties such as deformability and permeability. Hence, to account for soil variability, soil structures need to be analysed stochastically; in this thesis, this means that random fields of the material properties are first generated and the structure is then analysed repeatedly for various random fields as part of a Monte Carlo simulation. This leads to a ‘range’ of results instead of a single factor of safety. The results can then be represented by statistical tools, from which a more meaningful definition of structural performance based on reliability (the probability of failure not occurring) may be obtained.

Natural variability can be categorized as Temporal and Spatial. When a phenomenon happens at one location over time, it is classed as temporal variability; whereas, if the phenomenon is dependent on space, or on both space and time, it is considered to be spatial variability. Spatially varying soil properties can be modelled by the theory of random fields (Vanmarcke, 1984), in which fields can be either uni-variate, multi-variate or reduced-variate. A uni-variate random field comprises only one spatially varying material property, while multi-variate random fields comprise more than one spatially varying property. Reduced-variate random fields use fewer spatially varying material properties, from which the spatial variability of other properties can be subsequently backfigured. In this thesis, spatial variability is reduced to a single spatially varying property, the state parameter (Been & Jeffries, 1985). By deriving the statistics of this parameter and generating its corresponding random fields, it is then possible to backfigure spatially variable material properties which can be used in stochastic finite element analyses of geo-structural performance.
The importance of considering heterogeneity of material properties in analysing soil structures has been highlighted in recent research, including a number of studies in which random field theory has been combined with the finite element method. These studies include steady state seepage (e.g. Griffiths & Fenton, 1997 and Fenton & Griffiths, 1996), static liquefaction (e.g. Hicks & Onisiphorou, 2005), seismic liquefaction (e.g. Popescu et al., 2005), soil-structure interaction (e.g. Breysse et al., 2005), foundations (e.g. Fenton & Griffiths, 2003) and settlement problems (e.g. Houy et al., 2005). These investigations also vary with respect to being either 2D or 3D. In addition, most codes of practice, including the European Union’s Eurocode 7, have recently begun to address the issue of soil variability by introducing the idea of reliability-based characteristic values for soil properties; i.e. single values that can produce the same response in a deterministic analysis as the equivalent stochastic analysis for a certain level of reliability.

The research in this thesis is a continuation of previous research done in the Geotechnics Research Group at the University of Manchester. In particular, Hicks & Boughrarou (1998), Onisiphorou (2004) and Wong (2004) carried out detailed investigations of the liquefaction of the Nerlerk Berm in the Canadian Beaufort Sea, by using deterministic and stochastic methods and incorporating the Monot constitutive soil model (Molenkamp, 1981). Also, Bakhtiari (2006) investigated the statistical characterization of spatial variability by employing a new method for interpreting the cone penetration test data in terms of state parameter based on Shuttle & Jefferies (1998).

This research is a case study based on data from the Jamuna Bridge Site in Bangladesh. The data are used to perform stochastic analyses of the Jamuna Bridge Abutment, which experienced slope failures many times during its construction, to investigate how soil variability influences geo-structural response and how failure mechanisms can be predicted more realistically using this method. More specifically, finite elements and Monot soil model are employed to investigate the failure mechanisms of this dredge-excavated slope, with the spatial variability of property values being modelled using random field theory.
1.2 Study objectives

This research aims to use the Jamuna Bridge case history (Fugro reports, 1986 & 1996) to investigate the influence of spatial variability of soil properties on the failure mechanisms of a natural soil deposit; in particular, to evaluate the slides at the Jamuna Bridge Abutment in the light of the evidence provided by the computations. In order to carry out the investigation, the site has been characterized with respect to a single varying property; the state parameter. Then, random fields of this parameter are generated and assigned to a finite element mesh. Finally, stochastic analyses of the problem have been carried out using finite elements and a realistic constitutive model, as part of a parametric study to investigate geo-structural responses and possible failure mechanisms, and to evaluate the failures that occurred on-site. Figure 1.1 summarizes the different stages followed in carrying out the research in flowchart format. The stages are as follow:

- Calibration of the NorSand soil model (Jefferies, 1993; Jefferies & Shuttle, 2005) against an extensive laboratory database in order to derive the parameters which are required for interpreting the CPT data in terms of state parameter.
- Determination of state parameter profiles from the CPT data, based on the new framework of interpretation suggested by Shuttle & Jefferies (1998), and characterisation of the spatial variability of state parameter across the site by deriving its point statistics (i.e. mean and standard deviation), scale of fluctuation and the type of probability distribution function.
- Calibration of the double-hardening soil model Monot against laboratory triaxial data, in order to derive the model parameters that are needed for the finite element analysis of the structure. Part of this process involves the derivation of relationships between the Monot parameters and state parameter; these are needed for backfiguring Monot parameter values from the random fields of state parameter.
- Generation of uni-variate random fields of state parameter using Local Average Subdivision (Fenton & Vanmarcke, 1990) from which Monot parameter values can be backfigured.
- Performing stochastic finite element analyses of the Jamuna Bridge Abutment, using the computer algorithm MONICA (Hicks, 1995a & 1995b) and the random fields generated in the previous stage, to study the influence of heterogeneity on possible structural responses and failure mechanisms, and to evaluate the slides that occurred on-site.
1.3 Outline of the thesis

This thesis comprises eight chapters:

Chapter 2 reviews previous studies related to the research. It is subdivided into three main sections, focusing on the previous research in characterization, stochastic finite element analysis and the Jamuna Bridge case history.

Chapter 3 describes the NorSand soil model and the different methods which may be used to derive the model parameters. It then describes the calibration of the model against laboratory triaxial data available in the Fugro reports (1986 & 1996).

Chapter 4 introduces the old and new methods for interpreting CPT data in terms of state parameter. It also describes the different theories which are used in deriving the statistics of the data and reviews the different methods by which variability can be quantified. Then, a brief summary of the Jamuna Bridge Site, together with its history of construction and the location of the slope failures, is given. Finally, the numerical interpretation and statistical characterisation of the CPT data are carried out using the NorSand results found in Chapter 3.

Chapter 5 describes the Monot soil model and the methods by which its parameters are calibrated. It then continues by numerically deriving the different model parameters and finding their relationships with respect to state parameter.

Chapter 6 starts by describing the method of Local Average Subdivision (L.A.S) in different dimensions (i.e. 1D and 2D). Then, using the numerical results in Chapter 4, first uni-variate random fields of state parameter are generated and then, based on the relationships derived in Chapter 5, these are related to the Monot parameter values. Finally, these random fields are mapped onto the finite element mesh representing the Jamuna Bridge Abutment.

Chapter 7 starts by describing MONICA and its application to the current case study. Then stochastic analyses of the slope are carried out and the computed results are used to evaluate the slides that occurred at the Jamuna Bridge Site.

Chapter 8 summarizes the final conclusions of this thesis and gives recommendations for further research.
Extensive laboratory database

Calibration of Monot

Find relationships between Monot parameter values and state parameter

Extensive in-situ database (i.e. CPT tests)

Calibration of NorSand

Interpretation of CPT data in terms of state parameter

Probability distribution

Point statistics

Spatial correlation

Random field generation of state parameter

Spatial variability of Monot parameter values

Perform stochastic slope stability finite element analyses using MONICA as part of a parametric study

- Interpretation of the results
- Evaluating the failure mechanisms

Figure 1.1 Flowchart representation of the research.
Chapter 2

Literature Review
2 Literature review

2.1 Introduction

This chapter begins by looking at the definitions of some of the fundamental terms that are widely used in this thesis and traces back their origins, in order to better understand their relationships to the current research. In particular, terms such as deterministic, stochastic, randomness and uncertainty need to be defined.

Traditionally, scientists used to think of nature as being deterministic (Baecher & Christian, 2003). That is, they used to seek a cause for any effect to develop direct relationships between natural events. But this opinion was hopeless in describing events with unknown causes, or in cases where one cause could have many effects and one was interested in predicting the future outcome.

Two schools of philosophy have been developed since early times to remedy the question of fate and fortune. One considers events to happen by necessity and the other relates the occurrence of events to chance.

The main idea of the first school of philosophy was introduced by Leucippus and Democritus (5th Century BC), who believed that every event in this world is a consequence of a cause. To them, ‘chance’ was only an event with an unknown cause, neglecting whether or not the event had been knowable beforehand. Centuries later, the Doctrine of Necessity was developed by Laplace (1814), who believed that all history, the past and in retrospect the future, can be explained and realized through natural laws. This is analogues to what is nowadays adopted as boundary and initial conditions.

In the 3rd Century BC Epicurus rejected the first opinion, declaring that people have free will and hence their decisions are not controlled. In this doctrine, things only happen by necessity (i.e. without free will) due to people’s responsibility. Later, in the 19th century, Maxwell and Pierce took this idea and initiated the new view of the world, stochastic.

The Probabilism doctrine brings these two extreme ideas together, by describing the world as being random. That is, events are unpredictable, due to our knowledge being insufficient about their initial state or about the governing natural laws, whether or not that event happens by necessity.
Randomness is a property of nature which refers to something caused by chance and/or not predictable. This is in contrast to Uncertainty which is something unknown, unverified, but not unpredictable. Uncertainty can be categorized as either aleatory or epistemic. Aleatory uncertainty is equivalent to randomness, which means that, no matter how many experiments are done, there is always a random chance of getting a result which is different from what has been estimated (e.g. dice games in which the sequential outcomes are assumed to be random). On the other hand, epistemic uncertainties are those which relate to a degree of belief one has got about the truth or falsity of some uncertain process. The uncertainty in this case will decrease (i.e. confidence will increase) as more experiments are done – although sometimes the process may be unique and unrepeatable (Baecher & Christian, 2003).

From a geotechnical engineering perspective, there are four sources of uncertainty (Baecher & Christian, 2003):

a) Natural variability: This can be treated as the major source of uncertainty in geotechnical engineering, which is due to the inherent randomness of natural processes.

b) Knowledge uncertainty: This is what can be attributed to one’s lack of measuring and modelling of the world and hence is subjective. Site characterization comes into this category.

c) Operational uncertainties: Which are due to human-induced parameters and are not considered in mathematical models of engineering performance. These include issues such as construction, manufacturing, deterioration, maintenance and so on.

d) Decision model uncertainties: These include issues which are considered to be socially-contributed, involving social objectives, social hatred of risk or desirable temporal consumption-investment trade-offs.

In recent years, geotechnical engineers have tried to address these uncertainties in various ways and hence many methods have been either used or developed to tackle the uncertainties involved in site characterization, stochastic modelling and decision makings. Although these subjects have only attracted interest in very recent years in geotechnical engineering, the recognition of considering a heterogeneous nature for soil properties can be traced back to the famous Terzaghi (1936) statement that: “…the earth in its natural state is never uniform…”.
In this chapter of the thesis, recent studies related to the current research are reviewed. In particular, those relevant to site characterization, stochastic finite element analysis and the Jamuna Bridge case history are reviewed. Hence, the chapter is divided into three sections: Section 2.2, reviews the literature related to probabilistic and statistical methods in site characterization with focus being more on the type of statistical probability distributions and the point statistics which have been observed for different soil properties. Section 2.3, reviews the literature related to stochastic finite element analysis which have been carried out by different people for different problems with focus being mostly on liquefaction and slope stability problems. In Section 2.4, previous research related to the Jamuna Bridge case history is reviewed; and finally, in Section 2.5 the advancements that will be made by the present work from the previous investigations is given.

Note that, to keep this chapter concise, further reviews of the literature related to different parts of this research are given, where necessary, in Chapters 3 to 7. For instance, different methods of data interpretation and also characterization of the spatial variability are reviewed in detail in Chapter 4.

### 2.2 Probabilistic and statistical site characterization

Site characterization can be defined as a set of activities (i.e. processes) which will lead to information about site geology and from which one can get estimates of parameters to be used in modelling engineering performance (Baecher & Christian, 2003). This has been previously thought of as an intuitive process based on engineering judgment and without any analytical considerations. However, analytical approaches to site characterization have been developed in industries such as oil, gas and minerals industries. In such industries the importance of accurate estimates of oil pools or mineral deposits are of greater economic considerations than, for example, finding weaker seams beneath an earth dam.

Baecher & Christian (2003) proposed a site characterisation programme consisting of three stages: *reconnaissance, preliminary investigation* and *detailed investigation*.

Reconnaissance – also sometimes called a desk study – involves gathering the existing information on site geology from different sources and making qualitative hypotheses out from them. It is in the preliminary investigation stage that one can obtain
quantitative estimates of properties to be modelled in structural performance in a way that not too many tests are performed.

Detailed investigation aims to verify the hypotheses made in the preliminary investigation stage, mostly in cases where they govern the facility performance. Hence, a broad testing program is required for accurate estimation of geometry and material properties. Rowe (1972) categorized site investigation programmes into three classes based on the importance of the projects: Class A projects are those which are both risky and important – e.g. dams, tunnels and major sensitive structures – where either the ground geology is so complex or the design decision is so challenging that it requires a great deal of site investigation effort. Class B projects are those in which the risk involved in the project is so that deciding about the amount of effort required for site investigation is a hard job. Finally, Class C projects are those projects of routine use and low risk and where the cost of exploration of the ground is not justified by the value of the project. In fact, these three classes are comparable with what is now referred to as geotechnical categories in Eurocode 7 (BS EN 1997-1, 2004).

Since, the measurements of soil properties are at discrete points in space, any continuous shape for the strata and zones within the site can be only inferred. Therefore, in a site characterization program, one needs to first make hypotheses about site geology and then based on them generate random processes in order to describe the parameters one is looking for. Having made the measurements on site or in laboratory, one then needs to perform statistical analysis of the observed data to draw inferences about the random process model. Finally, by applying decision analysis one can optimize the type, number and location of the observations (i.e. measurements).

There are two ways of thinking about random models of spatial variability: either, to think of these random processes as frequency models of natural variation and hence think of them as aleatory uncertainties; or, to think of them as uncertainties in the information about a fixed but spatially varying realization, i.e. think of them as knowledge uncertainty and try to model them as epistemic uncertainties. However, Baecher & Christian (2003) suggest that the best way of modelling uncertainties is neither to think of them as aleatory nor as epistemic in particular, but to think of them as a combination of these two types of uncertainties.
Hence, site characterization studies in the literature can be categorized based on how researchers have employed probabilistic and statistical methods in their attempt to model uncertainties and drawing inferences based on the hypotheses they have made. Thus, in what follows in this section these different aspects of site investigation are reviewed with focus being on the previous studies carried out in the author’s research group and how these findings can be related to the current research.

Uzielli et al. (2005) used the data from 70 homogeneous CPT profiles taken from 304 soundings at Turkish and North American sites and characterized the spatial variability of normalised cone tip resistance ($q_{c1N}$) and friction ratio ($F_R$). They employed the modified Bartlett test, which is a statistical tool enabling rejection of the null hypothesis of weak stationarity for spatially correlated data. They found that only 40 $q_{c1N}$ profiles and 25 $F_R$ profiles could be considered as to be both physically and statistically homogeneous. Moreover, they found that $q_{c1N}$ profiles are more variable than $F_R$ profiles, with the vertical scales of fluctuation being in the range of 0.1-1.2m and 0.1-0.6m, respectively. They concluded that the procedure applied in their study would lead to better drawing of inferences about the uncertainties involved in site characterization.

Phoon & Kulhawy (2005) attempted to characterize model uncertainties for rigid long laterally loaded drilled shafts. They conducted their study on full-scale field tests and small-scale laboratory tests on rigid drilled shafts which were subjected to lateral moment loading. Among the field tests, undrained data were collected from 14 sites which mainly comprised clayey soils and drained field load test data were collected from 10 sites on sandy soils. They applied different soil stress distribution models including those proposed by:

(a) Broms (1964b), Hansen (1961) and Reese et al. (1974), for drained data
(b) Broms (1964a), Hansen (1961), Reese (1958), Randolph & Houlsby (1984) and Steven & Audibert (1979), for undrained data

and in order to evaluate the influence of model statistics, considered moment limit and hyperbolic capacity methods for the interpretation of lateral capacities from load tests. They concluded that, unlike what is generally believed, there is no well-defined or obvious relationship between the measured and theoretical capacities. However, if one adopts an interpretation which is consistent with what they did in their study, the coefficient of variation of the lateral capacities would stay relatively constant (between
30% and 40%) and a simple log-normal distribution can be used for similar reliability analyses. They also concluded that model factors from undrained analysis are less variable than those from drained analysis.

Juang et al. (2002) assessed some of the probabilistic methods for evaluating the liquefaction potential. In particular, they compared the logistic regression approach and the Bayesian mapping approach. They employed three different deterministic methods to evaluate the liquefaction potential: the Seed – Idriss method as presented in Youd et al. (2001) was used for evaluating the results of standard penetration test; the Robertson – Wride (1998) method was used for evaluation of cone penetration test results; and the Andrus – Stokoe (2000) method was used for evaluating the shear wave velocity test results. It was found that due to the Bayesian mapping approach being method-independent, it would preserve the characteristics of a particular deterministic approach and thus can be more useful for reliability-based design decisions. They also found that, in case of assessing the liquefaction potential based on only a factor of safety, the Andrus – Stokoe method would lead to the most conservative results and the Robertson – Wride method would lead to the least conservative results.

In addition, Moss et al. (2006) assessed both probabilistic and deterministic seismic soil liquefaction potential based on an extensive worldwide CPT data set gathered from different case histories and presented correlations which could be employed to estimate the liquefaction potential. They applied cavity expansion methods to normalise the measured sleeve and tip resistances, in order to avoid the influence of effective overburden stress; and also, the Bayesian framework to consider different types of uncertainties associated with various soil resistance variables and seismic demands. In this method, knowing the mean and variance of the penetration resistance that is needed for liquefaction, can lead to assessment of the probability of the seismic demand. They concluded that, the entire range of potentially liquefiable materials can be accurately captured using their relationships and knowing the CPT measurements on site.

It should be noted that the spatial and statistical characterisations of soils are often carried out by using the results of cone penetration tests and/or other tests such as the standard penetration test (as would be seen in Chapter 4). However, Cho et al. (2004) developed an electrical needle-size probe in order to characterize the spatial variability of sandy and clayey soils. Their probe could be used both in the laboratory and in the
field. It is particularly useful for measuring the spatial variability of either fluid resistivity or porosity (or both) in the soil.

Baecher & Christian (2003) summarized the statistical properties of different material properties including their means, standard deviations and best-fit distribution functions. Their reported information have been based on the findings of other researchers including Lacasse & Nadim (1996), Lumb (1974), Phoon & Kulhaway (1999), Baecher et al. (1983), etc. In particular, they reported that for the cone penetration tests on sands, the range of mean for the tip resistance, $q_c$, was 0.5 to 30MPa and the coefficient of variations were ranging from 20 to 60%. They also reported that the best distribution function for $q_c$ is Lognormal distribution. Moreover, the mean and standard deviation for the relative density, $D_r$, of sands was reported to range between 30 to 70% and 10 to 40%, respectively. Gitman & Hicks (2006) also examined four different probability distribution functions, which are very common in representing soil properties (i.e. Normal, Lognormal, Beta and Gamma), and applied the chi-square goodness-of-fit test to find out which one would best represent the distribution of state parameter for a hydraulically-place sand fill. They found that a Normal distribution can best characterize the variability of state parameter; while a Lognormal distribution best represents the variability for tip resistance. They also found that a Normal distribution can best represent the state parameter distribution at the looser end of the spectrum which according to Hicks & Onisiphorou (2005) has a greater influence on the overall stability of a structure. Moreover, they found that removing the depth-trend will not change the best-fit-distribution for state parameter.

Finally, it should be noted that in reality soil properties do not always exhibit a well-defined mono-modal distribution. Hicks & Onisiphorou (2005), for example, pointed out that a multi-modal distribution for state parameter can be an indication of distinct loose and dense zones. Their study demonstrated that a predominantly dilative sand fill can liquefy due to the presence of pockets of loose materials and a high degree of spatial variability.
2.3 Stochastic finite element analysis

The advancement of computers in recent years has enabled researchers to model random variability in the material properties by performing stochastic analysis, usually using Monte Carlo simulations. In this context, geotechnical engineering has been on the forefront of the innovations by combining the theory of random fields with the finite element method. This combination will lead to more realistic results and enable researchers to quantify the uncertainties and risks involved when considering the inherent spatial variability of soil properties; especially, for important structures where the consequences of performing just a deterministic analysis are high (e.g. the failure of the Nerlerk artificial island in the Canadian Beaufort Sea in 1983 led to millions of Pounds of damage due to neglecting the influence of heterogeneity of the soil properties during its design as later investigated by Hicks & Onisiphorou, 2005).

Hence, this section reviews the literature related to finite element analysis using the theory of random fields with focus mostly being on liquefaction and slope stability analysis and in particular the findings within the author’s research group. However, it should be mentioned that there are a number of different other methods and techniques to incorporate the random nature of soil properties in geotechnical analyses: The Bayesian method for example, which is a statistical method in which by observing the previous events one can infer the probability of a future event, was used in a case study by Cheung & Tang (2005) to assess the reliability of slopes to estimate the total number of slope failures within the whole domain, knowing the amount of rainfall; Fuzzy set theory for example is another method which is an extension of the set theory and is usually used when there is not enough information about the mean and variance of the material properties, is studied by Peschl & Schweiger (2003) in an attempt to compare the fuzzy finite element method with stochastic finite element method. Their results indicated that the fuzzy sets led to an overestimation in the range of possible solutions and hence more studies needed to be done in order to develop the method so that it can become useful in the area of geotechnical engineering.

It should be noted that, there are two different methods by which finite elements are incorporated with the variability of soil properties: one is the Stochastic Finite Element Method (SFEM) in which the intrinsic uncertainties of the variable soil properties would be replaced with the absolute definitions in the stiffness matrix. The solution of this method would be in the form of a probability density function; the second, which is used in this research and described in more detail Chapters 6 and 7, is the Random
field Finite Element Method (RFEM) in which the probability of an event is determined by multiple realisations of the problem using random fields of the material properties as part of a Monte Carlo simulation. The RFEM is computationally slower than the SFEM; however, it requires less memory during the calculations (Andres & Hori, 2001). The SFEM has found less attraction in geotechnics as compared to the RFEM and has been limited to applications such as 2D slope stability (e.g. Mellah et al., 2000) and settlement (e.g. Andres & Hori, 1999). However, the RFEM method has been applied to a wide variety of problems, including settlement and bearing capacity (e.g. Paice et al., 1996 and Fenton & Griffiths, 2002 and 2003), seepage and permeability problems (e.g. Griffiths & Fenton, 1997 and De Marsily et al., 2005), static liquefaction (e.g. Hicks & Samy, 2002a & 200b and Hicks & Onisiphorou, 2005) and dynamic liquefaction (e.g. Popescu et al., 2005). The method has been applied to both 2D and 3D problems and hence has been the method of choice for the current research.

Paice et al. (1996) studied the effect of the coefficient of variation of shear strength, $V$, by considering a heterogeneous soil and performing finite element analysis on a strip footing. They found that within the typical range of $V < 0.48$, stochastic results indicated only a small increase of 12% in the expected mean displacement. Fenton & Griffiths (2002) generated 2D random fields of elastic modulus, represented by a lognormal distribution, and performed multiple realisations of single and double footings using finite elements to work out the statistics and density functions of total and differential settlements under the footings. They found that for both cases, a lognormal distribution best represents the total settlements and a normal distribution can best represent the differential settlements. Fenton & Griffiths (2003) later analysed the same problem, this time by considering a random field which cross-correlated shear strength and friction angle. The bearing capacities of the footings were then computed by RFEM and compared with the traditional deterministic results (based on Prandtl, 1921). They concluded that the traditional method was in good agreement with the stochastic method. Kuo et al. (2004) considered a 3D pad footing on a layered spatially varying soil and computed the settlement under the footing using RFEM. They concluded that the soil layer which was the closest to the footing had the greatest influence on the settlement variance and that the variation in the settlement of the layered soil was reduced compared to a randomly spatially varying soil.
Griffiths & Fenton (1997 & 1998) studied seepage through spatially random soils in both 2D and 3D. They found that considering the 3\textsuperscript{rd} dimension in seepage RFEM would lead to an increase in the mean flow rate due to the ability for the flow to move easier in 3D and avoid less permeable areas. However, they pointed out that there is not much difference between the 2D and 3D responses and therefore for most practical purposes a 2D seepage RFEM can prove to be more efficient and cost effective. De Marsily \textit{et al.} (2005) also did a 3D stochastic analysis of seepage through a spatially random soil. However, the manner by which they modelled the spatial variability was different from the previous research. They introduced a \textit{genesis} (or genetic) method of random field generation in which heterogeneity is not modelled as what is observed at a particular point during the deposition history (i.e. now); rather, the method is more comprehensive and incorporates the past history of the deposition (i.e. through tens of thousands of years of deposition, erosion, tectonic shifts, river action and climate change) and generates random fields which are statistically representative of all the changes to the soil properties during the years of deposition. Although, this method could better represent the heterogeneity within a soil deposit, it was found to be impractical (at least for small sites).

Popescu \textit{et al.} (1995) performed 2D slope stability RFEM based on data taken from the Akita Harbour in Japan. Their study assumed a standard normal distribution for the cone tip resistance and a simple triangular correlation structure for the scales of fluctuation, $\theta$. They found that stochastic analyses resulted in larger overall pore pressures compared to deterministic results. Also, the results would have had significant differences if different probability distributions have been assumed for the cone tip resistance, when generating the random fields (i.e. a 25% decrease in the liquefaction index if a lognormal distribution was considered for the cone tip resistance). Finally, they found that there was no significant difference in the excess pore pressures when different values of the scale of fluctuation were assumed for generating the random fields. Popescu \textit{et al.} (1997) later performed another 2D slope stability RFEM based on the data taken from the sandfill core of the Tarsuit island site in the Canadian Beaufort Sea. The vertical and horizontal scales of fluctuation were found to be 0.95m and 12.1m, respectively, based on the data from eight soil CPT profiles. In this study, they used a bivariate random field in which the cone tip resistance, $q_c$, and the soil classification index, $I_c$, were cross-correlated and were both represented by Beta distribution. Several other soil parameters were also back-figured.
from them, including dilation angle and peak friction angle. They concluded that stochastic analyses resulted in higher excessive pore pressures as compared with the deterministic results. Also, the areas with the higher pore pressures corresponded to areas where the material was looser. They also found a characteristic percentile value of 80% for the soil strength (i.e. the percentile of the cone tip resistance in the deterministic analysis which was equivalent to the upper limit of the stochastic responses). Griffiths & Fenton (2004) performed 2D slope stability RFEM on cohesive slopes using a lognormal distribution for the shear strength, $C$. In their study they investigated the influence of the vertical scales of fluctuation and compared the stochastic results with the deterministic results. Their study also focused on the effect of local averaging on the random fields and the corresponding structural behaviour. Their study indicated that stochastic analysis led to more conservative results compared to the simplified probabilistic analysis, especially when the factor of safety was low or the coefficient of variation was high. Moreover, their study indicated that both the mean and standard deviation were reduced by the averaging method used.

Hicks & Samy (2004) studied the effect of considering different degrees of anisotropy of the heterogeneity and also the influence of varying slope angles on 2D slopes in which the undrained shear strength, $c_u$, was modelled by a normal distribution. Their parametric study showed that the degree of anisotropy of the heterogeneity of the soil properties can have a significant effect on the range of stochastic responses. Also, they found that considering the spatial variability of the soil properties would make the results problem-dependent (i.e. dependent on the slope angle); whereas, the solution would be problem-independent for the deterministic case in which the soil properties are assumed to be homogeneous. Hicks & Onisiphorou (2005) studied 2D slope stability RFEM of a predominantly dilative sand fill of the Nerlerk underwater berm in the Canadian Beaufort Sea based on an extensive database. They employed the theory of Local Average Subdivision (Fenton & Vanmarcke, 1990) in order to generate univariate random fields of the state parameter assuming a normal probability distribution and incorporated it with finite elements based on a double-hardening elastoplastic soil model called Monot (Molenkamp, 1981). They then backfigured other soil parameters from the univariate random fields and did multiple realisations of the problem by gradually increasing the gravity loading in undrained conditions to resemble static liquefaction. Their parametric study revealed that for large scales of fluctuations, which in turn are a result of deposition-induced anisotropy, it is possible for pockets of loose material to trigger the failure and lead to global instability of the
structure. Spencer (2007) performed 3D slope stability RFEM of a cohesive soil in which random fields of the undrained shear strength, $c_u$, were modelled assuming a normal distribution. Multiple realisations of the problem were performed based on a simple Mohr-Coulomb soil model and parallel computation was employed in order to enable analysing bigger slopes (i.e. 100m in length) with less computational time. The results were then compared with 2D case. His study was focused on investigating the influence of slope length, scale of fluctuation and degree of anisotropy of heterogeneity on the failure mechanism and reliability of the structure. It was concluded that considering the 3rd dimension in the analyses will lead to three distinct failure mechanisms to be observed: one at single or multiple discrete points along the length of the slope and two along the whole length of the slope. It was also found that the 2D and 3D stochastic responses are different in the way that the mean is increased and the variance is reduced. This is due to the fact that in 3D a larger failure surface is averaged for the shear strength values, as compared to 2D. It was also found that the horizontal scale of fluctuation has a significant influence on the failure mode.

The RFEM has also been applied to seismic liquefaction of soil structures. Tantella et al. (2001) analysed a 2D slope subject to seismic ground motion and investigated the influence of spatial variability of soil strength. Structural response was then represented in the form of fragility curves, which indicate a cumulative distribution function for the probably of damage to the structure. They concluded that this representation would lead to better estimations of the structural response corresponding to different earthquake intensities. Assimaki et al. (2003) estimated the ground motion of a spatially random soil subject to earthquake waves. Their study showed that there is little variation in the stochastic response of cohesionless soils as compared with the deterministic response; whereas, in cohesive soils, there is an increase in the range of responses by an order magnitude. Popescu et al. (2005) performed RFEM of a spatially varying soil deposit in 3D which was subject to earthquake loading with varying densities and incorporated multivariate, multi-dimensional random fields with non-linear finite element analyses. Their results, which were presented in the form of fragility curves, suggested that for liquefaction strength assessments in which differential settlements are important, only 3D stochastic analysis can accurately capture the behaviour of the structure; whereas, for cases in which the differential settlements are not important, 2D plane strain stochastic analysis can capture the structural response to a sufficiently good degree of accuracy.
2.4 Jamuna Bridge literature review

Detailed description of the Jamuna Bridge Abutment, including the laboratory and in-situ tests, history of construction and location of slides, are given in Chapters 3 and 4 of this thesis based on the information provided to the author (Fugro reports, 1986 & 1996). However, in this section, a brief summary of the work carried out by other researchers involving the Jamuna River Sand is given.

Hight et al. (1999) carried out extensive studies on the Jamuna River Sand on laboratory and addressed various characteristics such as its anisotropic mode of decomposition and its high contents of mica which would have led to the soil behave in an abnormally collapsible manner.

Moreover, Yasin & Tatsuoka (2006) studied the stress-strain behaviour of the Jamuna River Sand using plane strain compression tests and concluded that the Jamuna sand is very contractive and its behaviour is considerably different from non-mica sands. They also confirmed the anisotropic nature of this sand by demonstrating that the stress-strain behaviour and the peak angle of friction for this sand are completely dependent on the direction of loading. Finally, they concluded that the stress-strain behaviour of the sand is not dependent on the strain rate for shearing at monotonic strain rates; whereas, the material shows a viscous behaviour (including creep and relaxation) for change in the monotonic strain rates during shearing.

Ishihara (2008) attempted to assess the flow slides of the Jamuna Bridge Abutment by taking into account the micaceous characteristic of the material and performing some simple slope stability analysis in which slip planes are assumed and factors of safety are calculated for the slope using basic equilibrium equations. He echoed the previous findings of other researchers by concluding that, over a wide range of void ratios, the Jamuna River Sand exhibits a contractive (i.e. strain softening) behaviour.

To the knowledge of the author, no attempt has ever been made yet to review the Jamuna Bridge Abutment’s failures using stochastic finite element analysis.
2.5 Conclusion

This chapter reviewed the literature related to the current research by separately reviewing the main subjects of this study (i.e. probabilistic and stochastic methods). Then a brief summary of the literature regarding the case history analysed in this study was given. It should be noted that probabilistic and stochastic methods are very useful tools to provide an insight into the structural response of the soil structures in a more realistic manner by considering the heterogeneity in the soil properties. This is what has been also suggested in the new Eurocodes (BS EN 1997-1, 2004). Many of the important characteristics of structural behaviour would be hidden if only a deterministic analysis is carried out.

The focus of this study is put on analysing the static liquefaction of the Jamuna Bridge Abutment and it is a continuation of the previous research in the author’s research group. Hence, some of the papers mentioned in this chapter would be used as benchmarks for the current study. These include the studies by Hicks (1995a & b), Hicks (2000), Wong (2004) and Hicks & Onisiphorou (2005), Spencer (2007). However, it is noted that there would be other subjects related to the current study which are not referred to in this chapter and would be addressed in the following chapters where necessary.

The study carried out in this thesis has not been published in other literature ever before and it aims to provide a further insight into better understanding of the behaviour of soil structures when heterogeneity is implemented into the problem.
Chapter 3

NorSand Calibration
Chapter 3
NorSand Calibration

3 NorSand Calibration of Jamuna Bridge Sand

3.1 Introduction

Extensive tests on sands have shown that their mechanical behaviour cannot be characterized based only on their bulk characteristics such as void ratio or relative density (Lade, 1972; Lee, 1965; Been & Jefferies, 1985). First of all, it is known that the mean stress can suppress dilatancy. Also, if a sand contains even a few percent of silt, its behaviour can change depending on its relative density. For example, a mixture with a 60% relative density can be contractive while another mixture with a 40% relative density can be dilative. Hence, an alternative approach which avoids such problems needs to be used to describe sand behaviour. Two such approaches are the ’state parameter’ and ‘relative density index’.

The state parameter approach, which was first introduced by Been & Jefferies (1985), has been the more widely used of the two aforementioned alternatives and it can uniquely characterize the main behavioural properties of sands irrespective of their silt content, mineralogy, median grain size, and stress level.

One of the main advantages in the state parameter approach is that state parameter can be measured using the in-situ data from cone penetration tests across a site, using a general framework of interpretation, and it therefore avoids the difficulties involved in sampling sands in anything like an undisturbed condition. In other words, because other methods of measuring sand states require laboratory tests which are performed on samples that are usually disturbed when they are taken from a site, the use of an in-situ test instead of a laboratory test makes the results more reliable, since there is less disturbance issues involved in the CPT measurements for sandy soils. Also, since the CPT test can be performed at relatively low cost and give a continuous soil profile while maintaining excellent accuracy and repeatability, the state parameter approach for characterizing sands can be very advantageous and straightforward. However, in contrast to its practical advantages, the method can be numerically difficult because CPT test results give an inverse boundary value problem which requires the interpretation of data in order to find the parameters of interest.

Based on the above overviews, the state parameter approach has been chosen in this research to characterize the behaviour of Jamuna Bridge Sand. To do so, a soil model called NorSand (developed by Jefferies, 1993) needs to be employed and its model parameters need to be calibrated against laboratory data on sand obtained from the site.
Hence, this chapter begins by looking at the state parameter concept in more detail and is followed by the basic theories in critical state soil mechanics (CSSM), which are needed to understand the NorSand soil model. The chapter then describes how the different parameters of NorSand are derived using various calibration methods. Later, in Chapter 4, the derived parameters will be used alongside the CPT data, in order to characterise the spatial variability of state parameter across the Jamuna Bridge Site.

3.2 State parameter

The state parameter is a physical property that takes account of the influence of void ratio and ambient stress level, with respect to an ultimate stress level called the steady state, in order to describe soil behaviour. It is represented by the symbol $\psi$, and is the difference between the initial and the steady state void ratios:

$$\psi = e_0 - e_{ss}$$  \hspace{1cm} (3.1)

The state parameter describes a physical condition rather than the properties of a material or substance and therefore it is an important parameter because many material properties can be directly related to it. Moreover, sand behaviour is also controlled by the sand matrix structure, which links properties such as void ratio, fabric and composition (Mitchell, 1976). Hence, Been & Jeffries (1985) assumed that the two following variables can be used to characterize sand behaviour:

(a) A fabric parameter accounting for the arrangement of sand grains.
(b) A state parameter contributing to void ratio and stress level.

If a reference state is chosen which can give a unique sand matrix structure that is not influenced by the original test condition, then the above two variables can be reduced to one. This unique structure is achieved at the steady state and that is why the state parameter is the difference between the current void ratio and the steady state void ratio. However, it should be mentioned that some people (e.g. Rowe, 1962 and Schofield & Wroth, 1968) assume there is no structure for sands at the steady state, while others (e.g. Poulos, 1981 and Cassagrande, 1975) assume a ‘flow’ structure at the steady state condition. Nevertheless, the state parameter approach is not dependant
on the nature of the sand structure at the steady state; rather, is based on the existence of a repeatable, unique particle arrangement at the steady state condition.

Figure 3.1 illustrates the state parameter concept in void ratio-effective stress space. It is seen that the sand’s state is measured by considering a reference steady state and calculating the difference between the initial condition of the sand and this steady state condition. A positive $\psi$ value indicates contractive (liquefiable) behaviour, whereas a negative $\psi$ value indicates dilative behaviour. Important design parameters for sands with negative $\psi$ values are the drained angle of shearing resistance and volumetric response (Been & Jefferies, 1985). Samples with negative $\psi$ will reach a clear peak during shearing, whereas, as $\psi$ becomes less negative, the peak becomes less marked until it disappears when it becomes positive. Also, for high negative values of $\psi$ a strong dilative behaviour is apparent, while for positive $\psi$ less dilation is observed.

There are many advantages to the state parameter approach which make it preferable to its most frequently used rival, the relative density approach. Firstly, the relative density approach does not consider the influence of stress level and silt content in its reference relative densities (i.e. $e_{\text{max}}$ and $e_{\text{min}}$). In contrast, the state parameter approach takes account of the influence of stress level, density, $\phi_{ss}$, compressibility, grain size/shape and uniformity in a unique representation by employing the steady state as its reference level. Secondly, it is easy to measure a reference state, which can be repeated without any difficulties. In contrast, finding the maximum and minimum densities has proved to be difficult to measure repeatedly (Selig & Ladd, 1973). Finally, due to combining the two variables (a) and (b) –listed in the previous page – the state parameter approach only requires one correlation line to characterize sand behaviour; in contrast, the relative density approach needs more variables and may need up to twelve lines in order to fully characterize sand behaviour (Been & Jefferies, 1985). Hence, due to the importance of the steady state concept, the next section of this chapter is dedicated to explaining this concept in more detail.
3.3 Steady/Critical State

The term *state* is used to describe the condition under which sand exists and includes primarily variables such as void ratio and stress level, as well as fabric which is a variable of secondary importance. Poulos (1981) defines the *steady state* of deformation for any mass of particles as a state in which the mass continues to deform at constant volume, constant normal effective stress, constant shear stress and constant velocity. However, his definition does not address what the constant velocity should be or what value it should have. Nevertheless, according to this definition, the steady state of deformation is achieved after all particle breakage is stopped and the orientation of particles has reached a static steady state condition, so that the velocity of deformation and the shear stress needed to carry on the deformation become constant.

In practice, steady state is defined in void ratio-effective stress space as the locus of points which is unique and repeatable and not affected by testing conditions. Figure 3.2, illustrates the steady state line as a function of stress level. As can be seen, for stress levels below 1MPa, the steady state can be represented by a straight line. However, for higher stress levels the line suddenly curves, which is indicative of the change in shearing mechanism at higher stress levels as a consequence of grain crushing effect. At very low stress levels (less than 10kPa) the determination of the steady state line has proved to be difficult and Tatsuoka *et al.* (1986) found that the critical state line becomes flatter at these stress levels.

Another important state for soils is the *critical state*, which has been described by Roscoe *et al.* (1958) as the state at which the soil continues to deform at constant void ratio and constant stress. Note that the critical state is measured from drained, strain-controlled tests on dense (dilative) samples, whereas the steady state is measured from undrained tests on loose (contractive) samples. Some people (e.g. Castro, 1969 and Alarcon-Guzman & Leonards, 1988) assume that the steady state and critical state lines are different from each other, while others (e.g. Been *et al.*, 1991 and Poulos *et al.*, 1988) have shown that the two lines are the same and coincide. People who believe these two states to be different cite various reasons. For instance, Casagrande (1975) believes that the difference is due to the effect of strain rate. Poulos (1981) has indicated that, by definition, an associated flow structure and a requirement for constant velocity at the steady state cause this difference in the two lines. However, he has not defined the particular flow structure and the applicable strain rates which can be used for a clear distinction of the steady and critical states. Therefore, it is
justifiable to assume that the critical state and the steady state are identical. In this thesis, the steady and critical states are treated as interchangeable.

### 3.4 NorSand Soil Model

NorSand is a generalized Cambridge-type constitutive model for soils developed by Jefferies (1993), based on the state parameter $\psi$ and incorporating the fundamental axioms of critical state theory which are stated below:

**Axiom 1.** There is a unique locus in $q$, $p'$, $e$ space such that the soil can deform infinitely at a constant void ratio and constant stress level. This locus is called the critical state locus (CSL) and can be formally expressed as:

$$\exists C(e, q, p') | p' = 0 \exists \dot{\varepsilon}_p' \equiv 0 \land \dot{\varepsilon}_p' \equiv 0 \forall e_q$$

(3.2)

in which the locus of the critical state is described by function $C(e, q, p') = 0$ which is single-valued; $q$ and $p'$ are the deviatoric and mean effective stresses, respectively; and $\varepsilon_q$ is the deviatoric strain. The two conditions appearing on the right-hand side of equation (3.2) require no change in the void ratio of the soil, not only instantaneously due to $\dot{\varepsilon}_p'$ being equal to zero, but also to continue to be unchanged because $\dot{\varepsilon}_p'$ as well is equal to zero.

**Axiom 2.** The CSL establishes the ultimate condition of all distortional processes in the soil, in such a way that all monotonic distortional stress state paths tend to this locus. By choosing the state parameter as a divergence measurement, this axiom can be formally expressed as:

$$\psi \rightarrow 0 \text{ as } \varepsilon_q \rightarrow \infty$$

(3.3)

meaning that the sample will tend to a critical condition with shear.

As well as the above critical state axioms, NorSand considers four more requirements which are stated below (Jefferies & Shuttle, 2005):

a) The state parameter tends to zero as the shear strains accumulate.

b) The maximum possible dilation rate is linearly related to $\psi$.

c) The yield surface will always soften with the rotation of the principal stress.
d) There are infinite possible yield surfaces in $e - \bar{\sigma}_m$ space (in which $\bar{\sigma}_m$ is the mean effective stress, which is equal to $p'$ under triaxial conditions). It is not necessary for any yield surface to intersect the CSL, and the current yield surface position in $e - \bar{\sigma}_m$ space is defined by $\psi$.

Like any Cam Clay variant, NorSand idealizes plastic work dissipation. Therefore, for self-consistency the deviatoric strain invariant which is commonly used in finite element analysis cannot be used in this soil model; instead, the strain invariant introduced by Resende & Martin (1985) is used in the formulation of NorSand which is given by:

$$\dot{\varepsilon}_q = \frac{1}{3} \left( (\sin \theta + \sqrt{3} \cos \theta) \dot{\varepsilon}_1 - 2 \sin \theta \dot{\varepsilon}_2 + (\sin \theta - \sqrt{3} \cos \theta) \dot{\varepsilon}_3 \right)$$  \hspace{1cm} (3.4)

in which $\dot{\varepsilon}_1$, $\dot{\varepsilon}_2$ and $\dot{\varepsilon}_3$ are the incremental principal strains and $\theta$ is the current Lode angle. Note that equation (3.4) is linear with respect to the incremental strains. Hence, it is possible to decompose the strains into plastic and elastic components within the invariant. It should be noted that modifications have been made to the NorSand soil model since Jefferies (1993) first introduced it and the description presented here is based on the latest version of the model (Jefferies & Shuttle, 2005). NorSand comprises three main components: a yield surface, an associated flow rule and a hardening law.

As with Cam Clay, NorSand has got the familiar bullet-shaped yield surface. However, a significant difference is that it has also got an internal flat-plane cap which prevents the soil from unloading to very low mean stresses without yielding and the position of this cap is controlled by the current state parameter of the soil. The NorSand yield surface has been illustrated in Figure 3.3 for (a) very loose and (b) very dense sands. It can be seen that the location of the internal cap controls the limiting stress ratio $\eta_L$ sustained by the soil.

The mean effective stress at image condition, $\bar{\sigma}_{m,i}$ is also introduced in the model to control the size of the yield surface. The hardening law in the model is derived from this image condition. The image condition is described as the state at which one of the two conditions of equation (3.2) is satisfied and can be formally expressed by the single equation $\dot{\varepsilon}_{p'} \equiv 0$, which, due to convexity and normality, occurs at only one point on each admissible yield surface. The second condition of equation (3.2) is satisfied if, and only if, this image state coincides with the CSL. In $p' - q$ space, admissible yield surfaces are those which intersect the $p'$-axis at non-zero values due
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NorSand Calibration

to the existence of normal consolidation loci (NCL). Hardening or softening of the yield surface is a function of the current state parameter and also the loading direction. If the stresses pass the internal cap or if the principal stresses are rotated, then softening will occur. The most significant difference between NorSand and Cam Clay is that, in NorSand, the critical state does not usually intersect the yield surface and hence the hardening law in NorSand is based on the divergence of the yield surface from the critical state in a way that plastic shear strains cause the hardening law to move the yield surface towards the critical state.

NorSand also considers elasticity by introducing a simple constant shear rigidity $I_r$ and a constant Poisson’s ratio. All the formulae which are used to describe NorSand mathematically are summarized in Table 3.1. Finally, it should be mentioned that one of the other advantages of NorSand compared to most constitutive soil models is that it is a sparse model and only requires eight parameters to fully model a soil’s behaviour. Table 3.2 summarizes these dimensionless parameters, together with typical values. Section 3.5 will describe how these parameters are calibrated against laboratory data.

3.5 Calibration of Jamuna Bridge Data

In this section, laboratory test data on Jamuna River Sand are used to calibrate the NorSand soil model parameters. These are required later for calculating the state parameter from CPT data across the site, and will be the subject of Chapter 4. Each sub-section starts by explaining the different procedures available for deriving each NorSand parameter and then goes on to calibrate the particular parameter based on the data provided for Jamuna River Sand. The complete laboratory data available to the author are tabulated in Table 3.3, although some of these data were not suitable for calibration purposes. Jefferies & Shuttle (2005) have recommended that the critical state parameters of NorSand (i.e. $M_{tc}$, $\Gamma$, $\lambda_{cc}$) should be measured from test results on reconstituted samples of loose soil. Reconstituted samples are sufficient because the soil structure does not have any influence on the critical state properties and the loose soil state is adopted because it ensures contractive behaviour which can provide a clear indication of the critical state without any localization issues.

Jefferies & Shuttle (2005) also recommended that the testing program should include two samples tested drained and two samples tested undrained, with a fifth sample in
reserve depending on the results from the first four tests. They recommended that the samples be prepared using a moist tamping technique and to test them using modern triaxial equipment which have internal load transducers and digital data acquisitions. They also recommended freezing the samples after shearing and on completion of the tests, in order to accurately measure the void ratios at the end of each test.

For the two undrained tests on loose samples, the recommendation was to isotropically consolidate the samples to 50 kPa and 500 kPa effective confining pressures, followed by undrained shearing to produce substantially contractive behaviour, so that a clear critical mean effective stress for that sample’s void ratio at the start of shearing can be established. However, this condition is only feasible if the critical state is achievable within the strain limit of the testing equipment and that is why they recommended the tests be performed on loose samples; that is, for dense samples, a much higher initial cell pressure would be needed and thus it would be difficult to ascertain the critical state void ratio.

Jefferies & Shuttle (2005) recommended that the two drained triaxial tests be performed on loose samples at initial effective confining pressures of 200 kPa and 800 kPa; such samples will be contractive and can reach the critical state within the strain limit of the testing equipment. After the CSL is established based on one of the above approaches, one more test on a sample which has been prepared as dense as possible should be performed in a drained condition at an initial effective confining pressure of 100 kPa, in order to estimate the maximum dilatancy of the soil. However, in practice it is preferable to perform at least two tests on dense samples in order to get some redundancy on the parameter estimate.

Despite the above recommendations, the obvious question is: in practice, will there always be appropriate test results for a particular project? Is the situation always as predefined as the guidelines given by the experts? This has been proved to be the most challenging part of the calibration procedure for the author and, as mentioned before, for this project not all the data proved to be appropriate. Therefore, considerable effort in the calibration process has concentrated on estimating the parameters using different methods and by comparison of the results, in order to find the most confident estimates. The following sections describe how each NorSand parameter needs to be calibrated, what data were available in this investigation and how the model parameters have been determined. As mentioned in Section 3.4, NorSand only needs eight parameters to be calibrated. Among these, Poisson’s ratio has been previously
determined as 0.23, as recorded in the Fugro reports (1986 & 1996) provided to the author. Also, the dilatancy constant, \( \chi_{tc} \), has been assumed to be 3.5, as recommended by Jefferies & Shuttle (2005). However, the remaining six parameters have been calibrated in the following sections.

### 3.5.1 Determination of the Critical State parameters (\( M \) and \( N \))

Section 3.5 mentioned that, in practice, the critical state of the soil is often achieved at very high strains. Hence, some of the tests may not have reached the critical state within the strain limit of the testing equipment. Also, after the stresses reach their peak, shear banding may take place which will negate the measured stresses. Jefferies & Shuttle (2005) suggested that the ratio of shear stress invariant to mean effective stress (\( \bar{\sigma}_{qc}/\bar{\sigma}_{m,e} \)) at the critical state conditions is a constant equal to \( M \), which is a property of the soil which is a fraction of the Lode angle. If the triaxial compression test is taken as the reference condition from which soil properties are determined, \( M_{tc} \) can be defined as the critical state friction ratio corresponding to this condition. Jefferies & Shuttle (2002) have found that \( M_{tc} \) in NorSand, which is represented in Table 3.1 (note that subscript \( tc \) stands for the triaxial compression condition), is the average of the Matsuouka-Nakai (1974) and Mohr Coulomb failure criteria, as illustrated in Figure 3.4. If triaxial compression is taken as the reference condition and \( M_{tc} \) as the reference soil property, then the Mohr Coulomb and Matsuouka-Nakai criteria for \( M(\theta) \) can be represented by equations (3.5) and (3.6), respectively.

\[
M_{MC} = (3\sqrt{3})\left[ \cos \theta (1 + 6/M_{tc}) - \sqrt{3}\sin \theta \right]
\]

\[
\frac{27 - 3M_{MN}^2}{3 - M_{MN}^2 + \frac{8}{9} M_{MN}^3 \sin(\theta)[\frac{1}{4} - \sin^2 \theta]} = \frac{27 - 3M_{tc}^2}{3 - M_{tc}^2 + \frac{2}{9} M_{tc}^3}
\]

In which \( M_{MN} \) and \( M_{MC} \) are the Matsuouka-Nakai and Mohr Coulomb critical state parameters, respectively.

In this section, four methods for estimating \( M \) and two methods for estimating \( N \) are presented, along with the corresponding calibrations against isotropically consolidated drained triaxial tests on Jamuna River Sand. The author has been provided with 18 test results performed isotropically on samples taken from different depths and locations across the site. However, since the focus is on characterizing the top sand layer which
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extends to about 35m deep (and that is as deep as the CPT data are available), only the 9 tests belonging to this top layer have been considered in the calibrations. This so-called Upper Sand Layer has been considered based on the different test results on the site and as reported in the Fugro reports (1986 & 1996) provided to the author. The material below the top sand layer exhibits different properties, and since the slip zones in the Jamuna Bridge Abutment do not pass through this bottom layer and since there are no CPT data available for this bottom layer, this layer has not been considered in the calibration and characterisation process.

Before describing the calibration, it is essential to give a brief explanation of stress–dilatancy which is widely used in this context. The stress–dilatancy plot gives a very useful representation of soil behaviour by showing stress ratio, \( \eta = q/p' \), versus plastic dilatancy, \( D^p = \dot{\varepsilon}_v^p/\dot{\varepsilon}_q^p \). Usually, when plotting data from triaxial tests – in order to reduce the noise in raw data – a central difference approach is employed to compute the plastic dilatancy, and so:

\[
D_j = \frac{\varepsilon_{v,j+1} - \varepsilon_{v,j-1}}{\varepsilon_{q,j+1} - \varepsilon_{q,j-1}} \quad (3.7)
\]

in which the subscript \( j \) refers to the value of the \( j^{th} \) data point and \( \varepsilon_v \) and \( \varepsilon_q \) refer to the volumetric and deviatoric strains, respectively. Since stress–dilatancy is a concept which relates to plastic strain rates, equation (3.7) is in itself insufficient to be representative. Thus, it should be modified to:

\[
D_j^p = \frac{\dot{\varepsilon}_v^p}{\dot{\varepsilon}_q^p} = \frac{(\varepsilon_{v,j+1} - \varepsilon_{v,j-1}) - (p'_{j+1} - p'_{j-1})/K}{(\varepsilon_{q,j+1} - \varepsilon_{q,j-1}) - (q_{j+1} - q_{j-1})/(3G)} \quad (3.8)
\]

where \( K \) and \( G \) are the elastic bulk and shear moduli, respectively. In order to plot the stress–dilatancy graphs from the triaxial data, the graphical results were digitized and then \( \varepsilon_q \) derived from \( \varepsilon_1 \) and \( \varepsilon_v \). Specifically, combining \( \varepsilon_q = 2/3 (\varepsilon_1 - \varepsilon_3) \) and \( \varepsilon_v = \varepsilon_1 + 2\varepsilon_3 \) leads to \( \varepsilon_q = \varepsilon_1 - \varepsilon_v/3 \). The stress–dilatancy graphs were then plotted and the model parameters calibrated using the following methods.
(a) End of Test (ET) Method

In this method, graphs of stress ratio ($\eta$) against deviatoric strain ($\varepsilon_q$) are plotted for drained tests on loose soil samples sheared up to about 20% of strain and the stress ratio at the end of the test is taken as ($M_{tc}$)$_{ET}$. The problem with this method is that samples often do not reach their critical conditions by the end of the test, or 20% of strain. Moreover, for samples which are sheared to very large strains, localization can occur which will affect the results. Figure 3.5 illustrates a typical graph of stress ratio versus deviatoric strain for a dilatant soil and the corresponding ($M_{tc}$)$_{ET}$. The first column in Table 3.4 represents the measured ($M_{tc}$)$_{ET}$ values for the Jamuna Bridge data based on the 9 isotropically consolidated drained tests belonging to the Upper Sand Layer. These test responses are illustrated in Figures 3.20 to 3.28 (which also include the final calibrated response) and for what follows in this section these figures can be referenced as the soil responses for the 9 drained tests. The average value for ($M_{tc}$)$_{ET}$ based on this method has been found to be 1.38.

(b) Maximum Contraction (MC) Method

The point at which a soil sample reaches its minimum volume during shearing will correspond to the maximum contraction condition as illustrated in Figure 3.5. At this point there would be no volume change, that is $\dot{\varepsilon}_v = 0$, and so one of the conditions of critical state soil mechanics in equation (3.2) is satisfied. Negussey et al. (1988) suggested that the critical state friction angle $\phi_c$ can be taken as the mobilized friction angle at maximum contraction. The value of $M_{tc}$ obtained from this method is here represented by ($M_{tc}$)$_{MC}$ and is shown in Figure 3.5. The second column of Table 3.4 represents the values of ($M_{tc}$)$_{MC}$ obtained from the Jamuna Bridge data. Since, only 3 of the 9 drained test results reached the condition of maximum contraction within the strain limits of the testing apparatus (i.e. samples B231-31, B221-U19A and B221-U19B), only these 3 tests have been used to derive the average value of ($M_{tc}$)$_{MC}$ as being 1.31. Clearly, this is an example of when only a few test results have been acceptable and it shows the importance of estimating parameters through different methods.
This method was proposed by Bishop (1971) and uses the results of drained tests on dense samples with different densities in order to obtain the critical state coefficients. It involves finding out the value of the minimum dilatancy $D_{\text{min}}$ at the peak strength $\phi_{\text{peak}}$ and takes account of the fact that the incremental elastic strains are zero at the peak strength, which means that the total and plastic incremental strains are the same. Note that, in practice, dilatancy and peak friction angle will increase with density and that, for a sand which reaches the critical state without any dilation, the peak stress ratio $\eta_{\text{max}}$ will respond to the critical state condition and therefore $M_{\text{tc}}=\eta_{\text{max}}$ and $D_{\text{min}}=0$. However, if the soil dilates before reaching the critical state condition, $D_{\text{min}}$ would not be zero and it would have a negative value due to the compression-positive sign convention. After finding $\eta_{\text{max}}$ and the corresponding $D_{\text{min}}$ values for each test, the results are plotted in $\eta_{\text{max}} - D_{\text{min}}$ space and the best-fit line identified. The flow rule adopted in the NorSand model is:

$$\eta_{\text{max}} = M - (1 - N)D_{\text{min}}$$

(3.9)

and so the critical state parameters, $M$ and $N$, can be easily found. Table 3.5 lists the $\eta_{\text{max}}$ and $D_{\text{min}}$ values found for the Jamuna Bridge drained tests. These results have been plotted in $\eta_{\text{max}} - D_{\text{min}}$ space, as shown in Figure 3.6, and the best-fit line passing through them is the thick black line which gives the values of $M$ and $N$ as 1.37 and 0.47, respectively, with $R^2$ being 0.51. However, as indicated in Table 3.5, for those tests in which the sample reaches the peak without dilation, the $D_{\text{min}}$ values have been found to be zero, which is reasonable according to the description given above. Here is another instance of when the not all the test results are suitable for calibration purposes: firstly, because the scatter of data is relatively big; secondly, because some of the samples do not dilate before reaching the peak; and finally, because the value of $N$ measured by all these data points is out of the recommended range given in Table 3.2. Hence, in order to make more accurate estimates of the critical state parameters, those tests which were not suitable have been eliminated and then another best-fit line derived for the remaining tests (as indicated in Figure 3.6 by the thin blue line). This results in a reduced scatter in the data, as indicated by $R^2 = 0.83$, and also in more reasonable values for $M$ and $N$ (1.33 and 0.19, respectively) which are now in the recommended range.
(d) Stress – Dilatancy (SD) Method

Recalling the conditions of the critical state in equation (3.2), the critical state stress ratio can be defined as the stress ratio at which not only the volume of the sample does not change instantaneously, but also it remains the same with continued shearing. In mathematical terms these two conditions mean that $D = 0$ and $\dot{D} = 0$, respectively. It has already been mentioned that most triaxial tests do not reach their critical state conditions within the strain limit of the testing equipment and also that, due to localization which can take place after the peak, measurements of the stress ratio may become unreliable. However, by extrapolating the post-peak portion of the $\eta$ – dilatancy plot to zero dilatancy, it is possible to obtain reliable values of $M_{tc}$ (Ghazanfari & Shuttle, 2006). This post-peak portion looks like a hook in a $\eta$ – $\varepsilon_q$ space and is usually linear if it is plotted in $\eta$ – dilatancy space. As illustrated in Figure 3.7, fitting a linear relationship which passes through this post-peak response can give $M$ and $(N - I)$ values as its intercept and slope, respectively. The $M_{tc}$ obtained from this method is called $(M_{tc})_{SD}$, where SD stands for the stress – dilatancy method. Ghazanfari & Shuttle (2006) suggest that this is the most accurate method in measuring the critical state parameters of NorSand, but when applying this method caution should be taken because of the fact that, in most cases, localization may occur when strains within the sample exceed those associated with the peak stress state. Also, at higher strains, problems such as tilting or bulging of the sample or membrane penetration can occur. However, if these problems do not occur, the results become more accurate as the sample reaches its critical conditions. It is also suggested that, in cases where dilatancy paths show relatively large deviations from the linear trend on their way to reach the end of the test, the second part of the hook-shaped dilatancy path be removed from the calibrations and higher weight be given to the initial part of the hook.

In the case of Jamuna Bridge data, it was found that only 2 of the 9 tests had a recognizable post-peak portion in the $\eta$ – $\varepsilon_q$ space; their corresponding $M$ and $N$ values are given in the third and fourth columns of Table 3.4. Figure 3.8 illustrates the post-peak portion of these two tests in $\eta$ – dilatancy space. It should be mentioned that, before plotting the $\eta$ – dilatancy graphs for these two tests, the digitized data have first been smoothed to remove scatter in the digitised graphs. The mean $M$ and $N$ values for these two tests have been found as 1.39 and 0.20, respectively. These results closely match the ones which had been found previously.
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The maximum and minimum values of $M$ and $N$ found by all the aforementioned methods, have only 6% and 5% difference, respectively which indicate a good level of confidence in the estimations. To a good level of precision, $M$ and $N$ can be taken as 1.35 and 0.2, respectively. Based on the gradation of the Jamuna Bridge sand which is #250/8 (explained in part (b) of Section 3.5.3) and comparing to the values reported by Jefferies (1993) it can be seen that the results are similar to Hilton Mine sand which has a gradation of #200/2.5 and has $M = 1.39$ and $N = 0.2$. Gradation is an indication of the median grain size and the fraction of the soil passing the #200 sieve.

3.5.2 Determination of Shear Modulus, $G$

Shear modulus is an elasticity parameter used in the NorSand formulation and can be derived from data obtained from seismic body wave, pressuremeter and drained isotropic triaxial tests. Note that the shear modulus is a function of stress level and that it may be expressed in the form:

$$\frac{G}{p_a} = K_G \left(\frac{p'}{p_a}\right)^b$$

(3.10)

in which $K_G$ is called the modulus number and $b$ is called the modulus exponent. Also, $p_a$ is the atmospheric pressure, usually taken as 100kPa, and $p'$ is the mean effective stress. A brief description of each of the above testing methods is given below, with most emphasis being on the pressuremeter and seismic body wave tests which were performed for the Jamuna Bridge project.

(a) Pressuremeter Test

The pressuremeter test is an in-situ test which can be conducted in a borehole, either as a self-boring pressuremeter (SBP) type or cone pressuremeter (CPM) type. The two methods are different with respect to their installation methods and more information about the installation procedures can be found in soil mechanics textbooks such as Das (2004). However, it should be mentioned that among these two types of tests, the SBP is better for determining the in-situ shear modulus because it causes less disturbance in its installation compared to the CPM method.
In order to evaluate $G$, unload/reload cycles need to be performed during the test at different depths in a borehole and the results then plotted in total applied pressure – displacement space, as illustrated in Figure 3.9 which shows three different unload/reload loops. For a material which behaves perfectly elastically, the unload/reload loop will be a straight line with a gradient of $2G$. In this figure, $G_i$, $G_{ur}$ and $G_{ru}$ are the initial, unload/reload and reload/unload shear moduli, respectively. The initial modulus $G_i$ is the least reliable value due to the disturbance produced during the installation of the device. As can be seen in the figure, the three unload/reload loops are nonlinear and non-symmetrical, and so a strategy is needed to calculate the shear modulus. Houlsby & Schnaid (1994) recommended that either a least square line could be fitted between all the points enclosed between the two apexes of the loop, or simply a straight line connecting the two apexes be drawn and then the gradient calculated.

For the Jamuna Bridge case, CPM tests were performed at 7 locations and at different depths relative to PWD, which is the public works department datum in Bangladesh (where the site is located) and is 0.46m higher than mean sea level (MSL). The results of the CPM tests have been plotted in terms of modulus number, modulus exponent and shear modulus for depths of 10m above PWD to almost 50m below PWD, with the average values also plotted as illustrated in Figures 3.10 and 3.11. Thus, in order to express shear modulus in the form of equation (3.10), the equations of these average lines (indicated in the original Fugro reports, 1986 & 1996) need to be found. So, from the regression lines in the two figures:

\[ G = Z + 4.5 \]  
\[ K_G = 6Z + 80 \]  
\[ b = -0.005Z + 0.75 \]

in which $Z$ is the depth below PWD + 10m. If $G$, $K_G$ and $b$ are replaced with the above functions in equation (3.10), it leads to:

\[ \ln \left( \frac{Z+4.5}{-0.005Z + 0.75} \right) = \ln \left( \frac{\sigma'}{0.1} \right) \]

in which the unit for $\sigma'$ is MPa. In order to solve the above equation for $G$ and $\sigma'$ a graphical solution should be sought. Since the left-hand side of equation (3.14) is equal to $\ln (\sigma'/\sigma_a)$ as a function of depth and since the shear modulus is known from equation (3.11), it is possible to calculate $G$ and $\sigma'/\sigma_a$ at every depth, $Z$. If these values are
plotted in logarithmic $G - p'/p_a$ space, as illustrated in Figure 3.12, the best-fit line which passes through these points may be found. This leads to:

$$G_{CPM} = 14.106 \left( \frac{p'}{p_a} \right)^{1.8518}$$  \hspace{1cm} (3.15)

(b) Seismic Body Wave Test

This test can be performed either in the laboratory or in-situ. The in-situ test is a very rapid test that is performed in combination with a cone penetration test, and it is therefore a very reliable and economical method for measuring the shear modulus. The laboratory test is performed either in a resonant column device or using piezoelectric transducers. If the soil has a density of $\rho$ and the shear wave velocity passed through the soil is $V_s$, then the dynamic shear modulus is calculated directly as:

$$G_{max} = \rho V_s^2$$  \hspace{1cm} (3.16)

More information about the testing procedures can be found in standard soil mechanics textbooks.

At the Jamuna Bridge Site the shear modulus of the soil at low strains was obtained using a seismic cone in order to establish the shear wave profile with depth, as illustrated in Figure 3.13. The average shear wave velocity increases from 95 m/sec at the ground surface to 300 m/sec at 50 to 80m depths. As can be seen from the figure, the relationship between dynamic shear modulus and depth for the Upper Sand Layer is:

$$G_{max} = 20 + 2.75Z$$  \hspace{1cm} (3.17)

in which $Z$ is again the depth below PWD +10m. However the shear modulus still needs to be expressed in the form of equation (3.10). To do so, the mean effective stress $p'$ needs to be measured at every depth $Z$ and related to $G_{max}$ in the form of equation (3.10). The mean effective stress can be calculated as:

$$p' = \frac{\sigma_v' + 2\sigma_h'}{3} = (\gamma - \gamma_{sw})Z \cdot \frac{1 + 2K_0}{3}$$  \hspace{1cm} (3.18)

In the above equation, the coefficient of earth pressure at rest, $K_0$, was found to be 0.44 (based on the Fugro reports, 1986 & 1996) and the unit weight of salt water was taken
to be 10.03 kN/m³. However, the unit weight of the soil $\gamma$ is not constant with depth, as indicated by the results of the nuclear density test in Figure 3.14. The line which represents the average unit weight of the soil with depth has been found to follow the following equations:

$$
\gamma(Z) = \begin{cases} 
17 \text{kN/m}^3, & Z < 5m \\
0.0229Z + 19.19746 \text{kN/m}^3, & Z \geq 5m 
\end{cases}
$$  \hspace{1cm} (3.19)

in which $Z$ is the depth below Ground Level and is equal to PWD + 10m. Hence, equation (3.18) can be rearranged as:

$$
p'(Z) = \begin{cases} 
4.368Z, & Z < 5m \\
0.0143Z^2 + 6.088Z, & Z \geq 5m 
\end{cases}
$$  \hspace{1cm} (3.20)

A graphical solution was again sought for rearranging the relationship between equations (3.17) and (3.20) in the form of equation (3.10). For every depth $Z$, values of $G_{max}$ and $p'$ were found and plotted in logarithmic $G_{max} - p'/p_a$ space, as illustrated in Figure 3.15. The best-fit lines passing through the points were found to be:

$$
G_{max} = \begin{cases} 
48.304 \left(\frac{p'}{p_a}\right)^{0.231}, & Z < 5m \\
63.494 \left(\frac{p'}{p_a}\right)^{0.6875}, & Z \geq 5m 
\end{cases}
$$  \hspace{1cm} (3.21)

However, as will be mentioned later in Chapter 4, for site characterization and calculations of the state parameter, not all sections of the soil profiles have been considered and, for shear modulus calculations, only the second part of equation (3.21) has been used. Figure 3.14 indicates that the first 5m of the soil profile does not belong to the Upper Sand Layer and the first part of equation (3.21) has been omitted from the calculations.
(c) Isotropically consolidated drained triaxial compression (CID) Test

In this approach, the unload/reload cycles during drained triaxial compression make it possible to separate the elastic and plastic behaviours in the sample to enable an estimation of the shear modulus of the soil. The shear modulus of the soil is obtained from (Scott, 1980):

\[ 3G = \frac{dq'}{d\varepsilon_q} \]  

(3.22)

in which \( \varepsilon_q = \frac{2}{3} (\varepsilon_1 - \varepsilon_3) \) is the local shear strain and \( q' = \sigma_1' - \sigma_3' \) is the Mohr Circle diameter, with \( \sigma_1' \) and \( \sigma_3' \) being the axial and radial stresses and \( \varepsilon_1 \) and \( \varepsilon_3 \) their corresponding strains, respectively. Since the Jamuna Bridge data available to the author did not include CID tests involving unload/reload loops, no calculations have been carried out here.

From one of the relationships obtained in equations (3.15) and (3.21), it is necessary to choose a representative relationship for the shear modulus estimations in what will follow after this section. As can be clearly seen from Figure 3.13, the average shear modulus measured from the shear wave test is higher than the average shear modulus measured from the CPM test, i.e.:

\[ 2.9 \leq \frac{G_{\text{max}}}{G_{\text{CPM}}} \leq 4.1 \]  

(3.23)

(which is based on the average lines on the graph). This difference is due to the fact that the two test methods are performed at different strain levels. For the shear wave test the shear strain levels are around 0.0001%, whereas, for the CPM test, the shear strains typically range from 0.1 to 0.2%. Wong (2004) has compared the different methods in detail and has suggested that the values of shear modulus estimated from the shear wave test are the most accurate. Also, the NorSand formulae derived by Shuttle & Jefferies (1998) are based on the shear moduli derived from seismic body wave tests. Moreover, the shear strain values remain constant during the seismic test, whereas the shear strains in unload/reload loops of the CPM test decrease as the shear strain levels increase. Besides, Campanella et al. (1986) have also found that the shear wave tests are more reliable, as they can be repeated without much change in \( G_{\text{max}} \). For these reasons, the second part of equation (3.21) has been chosen for the shear modulus estimations in this thesis.
3.5.3 Determination of $\lambda_{ss}$ & $\Gamma$

As mentioned earlier in section 3.3, the void ratio at the critical state conditions for a soil sample, $e_c$, is dependent on the mean effective stress, $\bar{\sigma}_m$. Various relationships have been proposed to relate these two variables and the choice of such a relationship is only a matter of calibration; that is, choosing one relationship over another does not change the basic framework for NorSand as long as it is capable of fitting the behaviour of that particular soil. However, Jefferies (1993) proposed to use the following simple relationship:

$$e_c = \Gamma - \lambda \ln(\bar{\sigma}_m) \quad (3.24)$$

This equation represents a straight line in a semi-log plot of $e_c - \ln(\bar{\sigma}_m)$. However, it has been found that the critical state line is not a straight line in $e_c - \ln(\bar{\sigma}_m)$ space, as illustrated in Figure 3.2, but bends downward at higher stress levels. Therefore, Jefferies & Been (2000) suggested separating the steady state line into two distinct lines at a stress level $\sigma_\chi$, which is the apparent grain crushing stress during shearing of the sample, and replacing equation (3.24) with the following two equations:

$$e_c = \Gamma - \lambda \ln(\bar{\sigma}_m) \quad \forall \bar{\sigma}_m \leq \sigma_\chi \quad (3.25)$$

$$e_c = g - l \ln(\bar{\sigma}_m) \quad \forall \bar{\sigma}_m > \sigma_\chi \quad (3.26)$$

For cone penetration tests, the stress levels during the test are very high and would tend to follow equation (3.26). However, the stress levels in the available triaxial tests do not exceed the grain crushing stress and, therefore, for the purposes of triaxial calibrations it would be adequate to use equation (3.25) as the governing equation for the critical state line.

As can be seen in Table 3.2, the slope of the critical state line, $\lambda_{ss}$, ranges between 0.01 and 0.07 and it can be deduced that approximate estimations of this value may not have that much effect on the whole calibration process, due to its small variation range. Nevertheless, in this section four methods for estimating the $\lambda_{ss}$ and $\Gamma$ parameters are given. Estimation of these parameters using different methods and different test results is essential, since, as mentioned earlier, not all the triaxial tests have reached their steady states within the strain range of the testing apparatus and, therefore, relying only on triaxial test results may be insufficient in estimating the critical state line parameters. Finally, the estimated values for the Jamuna Bridge data will be compared at the end of this section and reliable values will be chosen for further calculations.
(a) Triaxial Test Method

The results of both drained and undrained triaxial tests may be used to derive the CSL. For each triaxial test the steady state condition is investigated: if the test has not reached the critical state condition it is eliminated from the next step of the calibration; but, if it has reached the critical state conditions within the strain limit of the testing apparatus, then the void ratio and mean effective stress corresponding to this condition are transferred onto \( e - \ln p' \) space. The best-fit line which passes through the critical state points is plotted and the slope and intercept of this line give \( \lambda_{ss} \) and \( \Gamma \), respectively. The crucial aspect of this method is selecting the critical/steady state points, or, in other words, determining whether the sample has reached its steady state or not. Traditionally, the terms ‘S line’ and ‘F line’ have been used by professionals to represent the steady state paths for ‘drained dense’ and ‘undrained loose’ soil samples, respectively (after Casagrande, 1975). For drained tests, the void ratio of the sample will change during the second phase of the test (i.e. when the deviator stress is applied). In contrast, the void ratio will remain constant during the second phase of an undrained test.

No matter what kind of stress path one is dealing with, caution is required in choosing what values of \( e \) and \( p' \) should be transferred in to \( e - \ln p' \) space. Are all the triaxial tests suitable for identifying the critical state conditions? Are the end-of-test conditions representative of the critical state? These two questions were considered by Been et al. (1991) by comparing some experimental examples. According to them, only those test results which reached the critical state conditions and then continued in this condition indefinitely are suitable for the calibration of the critical state line.

However, Poulos et al. (1988) showed that, in the laboratory, the critical state can only be achieved for uniform clean sands which are highly contractive. Therefore, even for samples that appear to have reached a continuous steady state within a 20% strain range, a different behaviour at larger strains may be possible, which means that the end-of-test conditions may not be a good representation of the critical state conditions. In contrast, for many samples, the sample may tend to dilate at the end of the test due to non-uniformities and other testing effects which are not related to the real behaviour of the sample; in such cases, the end of test conditions can be taken as representative of the steady state conditions.
For the Jamuna Bridge investigation, 51 triaxial test results have been made available to the author: 31 undrained tests and 20 drained tests, as summarized in Table 3.6. The table indicates whether the tests have reached their critical/steady states and also gives information regarding the initial and end of test conditions. Figure 3.16 shows three undrained test results to illustrate how the \( e - \ln p' \) results in Figure 3.17 have been obtained. Figures 3.16(a) and (b) illustrate two samples which have not reached the steady state conditions. In both samples, the excess pore water pressures are initially positive and then, after around 10% strain, they become negative; both samples seem to be trying to dilate at the end of the test. However, due to the slope of the \( q - \varepsilon_1 \) graphs for these two samples it can be deduced that the samples have not reached the steady state conditions. On the other hand, Figure 3.16(c) illustrates a sample which has reached the steady state conditions according to the \( q - \varepsilon_1 \) graph. As can be seen, the stress has reached a constant level and the rate of change in the stress level is also not changing after it has reached that constant level, which totally satisfies the steady state conditions. It was found that only 7 of the 51 tests reached their critical state (which is a very conservative conclusion, since determining the steady state conditions requires a lot of experience). These results have been transferred into \( e - \ln p' \) space as illustrated in Figure 3.17 and the best-fit line passing through these points was found to have an equation of \( e = -0.014 \ln (p') + 0.8981 \), which gives \( \lambda_{ss} = 0.014 \) and \( \Gamma = 0.8981 \).

Note that this method has some deficiencies which may make the results less reliable. First of all, one needs to have enough information about the testing condition, a good understanding of the definition of the steady state in practice and enough experience in determining whether a test has reached its steady state conditions or not. Secondly, this method will become less reliable if the silt content varies from one sample to another. Finally, in most tests the end-of-test condition is a result of the back pressures which have been used and is not the real behaviour of the soil sample. However, the author has tried to pick only those results which seem to be more reliable.
(b) Fines Content Approach

Bouckovalas et al. (2002) found the relationship between the location of the critical state line and the fines content of the samples by conducting statistical analyses. They suggested that the $\lambda$ and $\Gamma$ values in equation (3.24), for a soil which has got a fines content of $f (%)$, can be found from the following equations:

$$\lambda = 0.018 + 0.0027 f(\%)$$  \hfill (3.27)

$$\Gamma = 0.863 + 0.011 f(\%)$$  \hfill (3.28)

Although they affirmed that the above relationships should not be used to replace experiments in the practical definition of the critical state line, they nevertheless provide guideline values of $\lambda$ and $\Gamma$ against which the derived values from various methods may be compared.

For the Jamuna Bridge site, extensive particle size distribution tests have been performed. In particular, graphs of $D_{50}$ and $f(\%)$ varying with depth at 10 locations across the site have been provided. After digitizing these data and plotting the results of all 10 locations in one graph, as illustrated in Figure 3.18(a) and (b), it is possible to work out the average $D_{50}$ and $f(\%)$ across the site. As can be seen from Figure 3.18(a), the fines content of the first 10m of the soil profile is higher than the rest of the soil profile, indicating a silty soil near the surface. Since the later evaluations of state parameter will not consider this top silty layer, the layer has not been considered for calculating the gradation of the sand under consideration. According to these figures, and as mentioned in section 3.5.1, the gradation of the sand under consideration is #250/8 which means that the average fines content across the site for the sand layer is 8%. Using this value in equations (3.27) and (3.28), the values of $\lambda$ and $\Gamma$ are 0.04 and 0.951, respectively.

(c) Oedometer test results

The oedometer test can be performed on sands in with a constant rate of strain (CRS) and the results from this test can be plotted in $e - \ln p'$ space and look similar to traditional oedometer test results for clays. Jefferies (2006) recommended using the results of this test as a starting point in estimating $\lambda$ and $\Gamma$ values (which are the slope and intercept of the linear part of the oedometer response, respectively) and using the estimated values for further comparisons. However, he stated that the values derived
for the critical state line using this method would be smaller than the actual $\lambda$ and $\Gamma$ for the soil. For the Jamuna Bridge site, the author has been provided with 15 oedometric test results, 9 of which have been performed in a triaxial cell with isotropic compression and the rest of them in an oedometer apparatus with incremental loading. However, only 9 of these tests were performed on samples taken from the sand layer under consideration and their results are presented in Table 3.7. On average it was found that $\lambda_{ss} = 0.017$ and $\Gamma = 0.92$.

(d) Assumed – Chi Method (Intersection Method)

This method was proposed by Jefferies (2006) and has been further developed by the author. It employs the simple equation,

$$D_{\text{min}} = \chi \psi_0$$  \hspace{1cm} (3.29)

which is a part of the NorSand formulation, and combines it with equations (3.1) and (3.24) to give:

$$e_0 - \frac{D_{\text{min}}}{\chi} = \Gamma - \lambda \ln p'$$  \hspace{1cm} (3.30)

In this equation, three parameters are unknown, i.e $\chi$, $\Gamma$ and $\lambda$. Therefore, by assuming a value for $\chi$, equation (3.30) will reduce to only two unknowns for each triaxial test and these are the parameters we are looking for. As can be seen from Table 3.2, $\chi$ values generally range between 2.5 and 4.5. By taking $\chi = 3.5$, which is based on the findings of Jefferies (1993) for tests on different types of sands, and finding $e_0$, $D_{\text{min}}$ and the average $\ln p'$ for each triaxial test, equation (3.30) then represents a straight line in $\Gamma - \lambda$ space. By plotting these lines for each triaxial test and finding the point at which these lines intersect the values of $\lambda$ and $\Gamma$ can be estimated.

This procedure has been applied for the drained triaxial data of the Jamuna Bridge site and the results are summarized in Table 3.8 and illustrated in Figure 3.19. As can be seen in the figure, the lines intersect each other at more than one location and therefore an average value of the intersection points which are within the acceptable range of $\Gamma$ and $\lambda$ (a condition that the author found more reasonable due to more reliable estimates) has been taken as the estimation. This average location was found to be at $\lambda_{ss} = 0.047$ and $\Gamma = 1.15$. 
(e) **Forward Iterating Method**

This method was suggested by Jefferies & Shuttle (2005) and is mainly used for estimating the hardening modulus, $H$, by formal modelling of the triaxial test results. Formal modelling is a forward iterative method that is used to model the stress-strain behaviour of the soil. It involves first choosing a set of material parameters and then computing the stress-strain behaviour corresponding to these parameters; the next step is to compare the computed stress-strain response with the actual behaviour of the material, and then to revise the parameters by further iterations until the computed behaviour matches the actual behaviour of the soil sample. However, aside from estimating the hardening modulus, this method can also be used to simultaneously estimate the $\lambda$ and $\Gamma$ values as the parameters are optimized to completely fit the actual behaviour of the soil sample. Hence, since for every triaxial test $M_{tc}, I_r, p_0, \nu$ and $D_{min}$ are known, one can assume an initial state parameter $\psi_0$ (usually using equation (3.29) and by assuming $\chi = 3.5$) and, by choosing different values for $\lambda, \Gamma$ and $H$, try to get a model behaviour based on the NorSand formulation which can fit the stress-strain behaviour of that particular triaxial test under consideration.

An advantage with such a method is that, by a process of iterations, the model can be optimized to fit the overall data. However, the disadvantage is that there is no assurance of the reconstituted samples in the laboratory test result being good representations of the in-situ conditions (Jefferies & Shuttle, 2005).

For the Jamuna Bridge site, the same 9 drained triaxial tests have been used for the formal modelling. A spreadsheet\(^1\) with an open VBA source code implementation of NorSand was employed for this method. The known parameters for each triaxial test were then put into the code and different values for $\lambda, \Gamma$ and $H$ were chosen and the model was updated to numerically integrate the NorSand equations based on Table 3.1. This procedure was iterated until the model fitted the data to a good degree. The fitted results for each test are illustrated in Figures 3.20 to 3.28 and the calibrated values are presented in Table 3.9. In this table the initial values of the state parameter (which are indicated in the $7^{th}$ column) should have been obtained from the initial void ratio (indicated in the $3^{rd}$ column) and by using equation (3.29). This is the case for the first 6 samples on the table (i.e. samples B231-13, B231-19, B231-31, B104-17, B231-23 and B231-29). However, during the formal modelling of the last 3 samples in Table

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\(^1\) Developed by the University of British Columbia and previously used by Bakhtiari (2006).
3.9, it was found that choosing values for \( \psi_0 \) other than those obtained by equation (3.29) gave better fits to the actual soil response and hence better optimized the estimated parameters. Looking at Figures 3.20 to 3.28, it can be seen that the computed responses closely match the results for samples B231-13, B231-19, B231-31 and B221-U19. However, for other samples, although the computed results closely fit the \( \sigma_1 - \varepsilon_1 \) and other responses, there are deviations in the \( \varepsilon_v - \varepsilon_1 \) curves although it has been best tried to get these responses as close as possible to the soil behaviour. The average values of \( \lambda \) and \( \Gamma \) from this method were found to be 1.17 and 0.06, respectively.

Figure 3.29 illustrates the critical/steady state lines obtained using the 5 calibration methods for \( \lambda \) and \( \Gamma \). It is seen that the intersection method gives the upper bound solution for the Jamuna Bridge data, whereas the fines content approach gives the lower bound. The line which passes through the middle of the lower and upper bounds was taken as the average of all the results and hence was taken as a more reliable estimate for the critical state line. This line has \( \lambda \) and \( \Gamma \) values of 0.04 and 1.03, respectively.

### 3.5.4 Determination of the Plastic Hardening Modulus, \( H \)

As mentioned in section 3.5.3, Jefferies & Shuttle (2005) suggested using the forward iterative method for estimating the plastic hardening modulus. It was also mentioned that a potential disadvantage with this method is that the reconstituted triaxial samples may not be good representations of the in-situ conditions. However, in recent years the results of SBP tests have proved to be useful in determining the in-situ \( H \) (Shuttle, 2004) and more work is in progress in this regard (which is not the subject of this thesis).

For the Jamuna Bridge site, the plastic hardening modulus has already been estimated in the previous section and the results were presented in Table 3.9. As can be seen in this table, the hardening modulus varies with state parameter and therefore a relationship between \( H \) and \( \psi_0 \) needs to be established by transferring these points into \( H - \psi_0 \) space as illustrated in Figure 3.30. This relationship is expressed as:

\[
H = -903.46\psi_0 + 91.758
\]  

(3.31)
However, in Chapter 4 it will be seen that, in order to interpret the CPT data in terms of state parameter, a constant value of $H$ is needed. Therefore, what needs to be done in that chapter is, first, a value for $H$ would be chosen (let’s say $H_1$) and then the average state parameter for all the CPT data would be calculated (let’s say $\psi_1$). By putting $\psi_1$ in equation (3.31), one can get a new plastic modulus (let’s say $H_2$). If $H_1=H_2$, then the chosen value can be taken as acceptable. Otherwise, another value for $H$ should be chosen and this procedure repeated until the condition $H_1=H_2$ is satisfied. This process is presented later in Chapter 4.

### 3.6 Summary

In this chapter of the thesis it was described that in order to determine the mechanical behaviour of sands, the best approach would be to characterize its behaviour by the state parameter. It was mentioned that the state parameter can be interpreted from CPT data based on a framework which employs the NorSand soil model. Since this interpretation framework will be the subject of the next chapter, the focus of this chapter was on explaining the NorSand soil model and the calibration of its parameters from laboratory tests. It was mentioned that out of the 8 parameters in NorSand, only 6 need to be calibrated (i.e. $M$, $N$, $\Gamma$, $\lambda$, $G$ and $H$). Then the various methodologies for estimating these parameters were described and numerical evaluations of the Jamuna Bridge Site data were performed. The data which the author has been provided with were in hardcopy format and, in the first stage, these data were digitized into computer format. A significant obstacle in the calibration process was found to be the inappropriateness of many laboratory test results. However, much effort was expanded to estimate the parameter values using the different methods and not to rely on just one method of calibration, in order to get more accurate estimates with a higher degree of reliability. Table 3.10 summarizes the values/relationships which were found to be more reliable. These results will be used in Chapter 4, in combination with the CPT data, in order to derive and statistically characterize state parameter across the Jamuna Bridge site. The statistics of state parameter will be used to generate random fields of state parameter in Chapter 6 and to do stochastic slope stability analyses in Chapter 7.
Internal Model Variables
\[ \psi_i = \psi + \lambda \ln(\bar{\sigma}_{m,i} / \bar{\sigma}_m) \quad \text{where} \quad \psi = e - e_c \]
\[ M_i = M - |\psi_i| \]

Critical State
\[ e_c = \Gamma - \lambda \ln(\bar{\sigma}_m) \quad \text{AND} \quad \eta_c = M = (M_{MC} + M_{MN}) / 2 \]
where \( M_{MC} = (3\sqrt{3}) / (\cos(\theta(1 + 6 / M_{nc}) - \sqrt{3}\sin(\theta)) \]
and \( 27 - 3M_{MN}^2 \)
\[ 3 - M_{MN}^2 + \frac{8}{9} M_{MN}^3 \sin(\theta)[\frac{1}{2} - \sin^2(\theta)] = \frac{27 - 3M_{ic}^2}{3 - M_{ic}^2 + \frac{2}{3} M_{ic}^3} \]

Yield Surface & Internal Cap
\[ \eta_i = 1 - \ln \left( \frac{\bar{\sigma}_{m,i}}{\bar{\sigma}_m} \right) \quad \text{with} \quad \left( \frac{\bar{\sigma}_{m,i}}{\bar{\sigma}_m} \right) = \exp(-\chi_c \psi_i / M_{ic}) \]

Hardening Rule
On outer yield surface:
\[ \frac{\dot{\sigma}_{m,i}}{\sigma_{m,i}} = H \frac{M_i}{M_{i,ic}} \left( \frac{\sigma_m}{\sigma_{m,i}} \right)^2 \left[ \left( \frac{\sigma_{m,i}}{\sigma_m} \right)_{\text{max}} - \frac{\sigma_{m,i}}{\sigma_m} \right]^p \]
On internal cap:
\[ \frac{\dot{\sigma}_{m,i}}{\sigma_{m,i}} = -\frac{H}{2} \frac{M_i}{M_{i,ic}} \left( \dot{\varepsilon}_q^p \right) \]
During principal stress rotation:
\[ \frac{\dot{\sigma}_{m,i}}{\sigma_{m,i}} = -H \frac{\dot{\varepsilon}}{\pi} \]

Stress Dilatancy & Plastic Strain Rate Ratios
\[ D^p = \dot{\varepsilon}_q^p \left/ \dot{\varepsilon}_q^p \right|_{M_i} - \eta \Rightarrow D^p_{ic} = D^p_{i,ic} / M_i \quad \text{and} \quad D^\eta_{ic} = M_{i,ic} / M_i \]
\[ \text{define} \ z_{3,ic} = - \frac{2D^p_{ic} - 3}{6 + 2D^p_{ic}} \quad \text{and} \quad z_{3,ic} = \frac{2D^p_{ic} - 6}{3 + 2D^p_{ic}} \]
\[ \Rightarrow \dot{\varepsilon}_3 = z_{3,ic} - (z_{3,ic} - z_{3,ic})\cos^2(\frac{3\theta + 90}{2}) \]
\[ \text{define} \ a = (\sin \theta + \sqrt{3}\cos \theta) / 3, b = -2\sin \theta / 3, \]
\[ c = (\sin \theta - \sqrt{3}\cos \theta) / 3 \]
\[ \Rightarrow \dot{\varepsilon}_2 = (aD^p - 1 + \dot{\varepsilon}_3 (cD^p - 1))/ (1 - bD^p) \]

Elasticity
\[ I = \frac{G}{\sigma_m} \quad \text{with} \quad K = \frac{2(1 + \nu)}{3(1 - 2\nu)} G \]

**Table 3.1** Summary of NorSand (Jefferies & Shuttle, 2005)
Chapter 3

NorSand Calibration

<table>
<thead>
<tr>
<th>Property</th>
<th>Typical Range</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSL $I$</td>
<td>0.9 – 1.4</td>
<td>‘Altitude’ of CSL, defined at 1kPa</td>
</tr>
<tr>
<td>CSL $\lambda$</td>
<td>0.01 – 0.07</td>
<td>Slope of CSL, defined on base $e$</td>
</tr>
</tbody>
</table>

**Plasticity**

<table>
<thead>
<tr>
<th>Property</th>
<th>Typical Range</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{tc}$</td>
<td>1.2 – 1.5</td>
<td>Critical friction ratio, triaxial compression as reference condition</td>
</tr>
<tr>
<td>$N$</td>
<td>0.0 – 0.4</td>
<td>Volumetric coupling parameter</td>
</tr>
<tr>
<td>$H$</td>
<td>50 – 500</td>
<td>Plastic hardening modulus for loading, often $f(\psi)$</td>
</tr>
<tr>
<td>$H_r$</td>
<td>15 – 30</td>
<td>Plastic softening modulus for principal stress rotation</td>
</tr>
<tr>
<td>$\chi_{tc}$</td>
<td>2.5 – 4.5</td>
<td>Relates minimum dilatancy to $\psi$. Often taken as 3.5. Triaxial compression as reference condition</td>
</tr>
</tbody>
</table>

**Elasticity**

<table>
<thead>
<tr>
<th>Property</th>
<th>Typical Range</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_r$</td>
<td>100 – 800</td>
<td>Dimensionless shear rigidity</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.1 – 0.3</td>
<td>Poisson’s ratio, commonly 0.2 adopted</td>
</tr>
</tbody>
</table>

Table 3.2 NorSand soil properties with typical ranges for sands (Jefferies & Shuttle, 2005)

<table>
<thead>
<tr>
<th>Test</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nuclear density test</td>
<td>1 location</td>
</tr>
<tr>
<td>Isotropic consolidated drained triaxial test</td>
<td>18 samples</td>
</tr>
<tr>
<td>Anisotropic consolidated drained triaxial test</td>
<td>2 samples</td>
</tr>
<tr>
<td>Isotropic consolidated undrained triaxial test</td>
<td>3 samples</td>
</tr>
<tr>
<td>Anisotropic consolidated undrained triaxial test</td>
<td>28 samples</td>
</tr>
<tr>
<td>SPT</td>
<td>17 locations</td>
</tr>
<tr>
<td>CPM</td>
<td>7 locations</td>
</tr>
<tr>
<td>Shear wave velocity</td>
<td>1 location</td>
</tr>
<tr>
<td>$D_{10}, D_{50}, C_u, f(%)$</td>
<td>10 locations</td>
</tr>
<tr>
<td>$K_0$ test</td>
<td>4 samples</td>
</tr>
<tr>
<td>Particle size distribution</td>
<td>Extensive</td>
</tr>
<tr>
<td>Mica content</td>
<td>Extensive</td>
</tr>
<tr>
<td>Oedometric incremental loading test</td>
<td>8 samples</td>
</tr>
<tr>
<td>Isotropic compression oedometer test</td>
<td>9 samples</td>
</tr>
<tr>
<td>Constant head permeability test</td>
<td>16 samples</td>
</tr>
</tbody>
</table>

Table 3.3 Summary of available test results for Jamuna Bridge site.
Table 3.4 $M$ & $N$ values for triaxial tests based on different methods.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$(M_{tc})_{ET}$</th>
<th>$(M_{tc})_{MC}$</th>
<th>$(M_{tc})_{SD}$</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>B231-13</td>
<td>1.437</td>
<td>1.437</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B231-19</td>
<td>1.373</td>
<td>1.373</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B231-31</td>
<td>1.448</td>
<td>1.372</td>
<td>1.452</td>
<td>0.201</td>
</tr>
<tr>
<td>B104-17</td>
<td>1.333</td>
<td>1.333</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B231-23</td>
<td>1.358</td>
<td>1.358</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B231-28</td>
<td>1.398</td>
<td>1.398</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B221-U19</td>
<td>1.342</td>
<td>1.342</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B221-U19A</td>
<td>1.444</td>
<td>1.435</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B221-U19B</td>
<td>1.324</td>
<td>1.128</td>
<td>1.333</td>
<td>0.196</td>
</tr>
</tbody>
</table>

Table 3.5 $\eta_{max}$ – $D_{min}$ values for drained triaxial tests.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\eta_{max}$</th>
<th>$D_{min}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B231-13</td>
<td>1.440</td>
<td>0.000</td>
</tr>
<tr>
<td>B231-19</td>
<td>1.375</td>
<td>0.000</td>
</tr>
<tr>
<td>B231-31</td>
<td>1.475</td>
<td>-0.188</td>
</tr>
<tr>
<td>B104-17</td>
<td>1.333</td>
<td>0.000</td>
</tr>
<tr>
<td>B231-23</td>
<td>1.358</td>
<td>0.000</td>
</tr>
<tr>
<td>B231-28</td>
<td>1.398</td>
<td>0.000</td>
</tr>
<tr>
<td>B221-U19</td>
<td>1.351</td>
<td>-0.015</td>
</tr>
<tr>
<td>B221-U19A</td>
<td>1.472</td>
<td>-0.134</td>
</tr>
<tr>
<td>B221-U19B</td>
<td>1.372</td>
<td>-0.088</td>
</tr>
</tbody>
</table>
Chapter 3

Sample

NorSand Calibration
Initial
Conditions

Test Conditions†

End of Test

Void
ratio (%)

p’
(kPa)

Drainage

Strain
rate
(%hr)

Stress
Path

Steady
‡
state

p’ (kPa)

q (kPa)

e (%)

cs’ (◦)

A1AA

0.990

67

U

4

C

Con

160

193

0.776

30

A2

0.824

67

U

4

C

Yes

260

350

0.754

33

A3

0.925

67

U

4

C

Yes

161

233

0.821

36

A4

0.969

67

U

4

C

Dil

68

50

0.809

19

A6

0.844

67

U

4

C

Dil

68

50

0.780

19

A7

0.921

67

U

3

C

Mdil

67

50

0.802

19

A101

1.000

275

D

4

C

Maybe

486

600

0.809

31

A102

0.936

275

U

3

C

Yes

54

75

0.847

34

A103

0.857

75

U

4

C

Yes

7

10

0.848

35

A104

0.938

67

U

4

C

Con

26

43

0.865

40

A105

1.048

67

U

3

C

Con

37

48

0.931

32

B2

0.994

67

U

4

C

Yes

31

41

0.893

33

B3

1.003

67

U

4

C

Con

66

51

0.906

20

B4

0.944

67

U

4

C

Con

17

28

0.948

40

B6A

1.065

67

U

3

C

Dil

68

50

0.825

19

B7

0.786

67

U

4

C

Dil

68

50

0.719

19

B8A

0.868

67

U

4

C

Mdil

68

50

0.776

19

B10

0.872

67

U

4

C

Yes

332

463

0.867

34

B10A

0.919

67

U

4

C

Dil

147

-95

0.826

21

B20A

0.980

67

U

4

C

Mdil

87

103

0.855

30

B21

1.010

67

U

4

C

Con

70

88

0.850

31

B22

0.999

67

U

3

C

Mdil

98

140

0.822

35

B23A

0.975

67

U

3

C

Mdil

122

167

0.825

34

B23B

0.890

67

U

3

C

Mdil

133

193

0.774

36

B100

1.097

67

U

3

C

Dil

113

150

0.827

33

B101

1.072

67

U

3

C

Con

68

82

0.888

30

B102

1.000

67

U

4

C

Dil

68

49

0.860

19

B201

0.925

67

U

4

C

Con

67

48

0.841

19

B202

0.870

267

U

4

C

Yes

128

175

0.818

34

B203

0.993

68

D

4

C

Con

85

112

0.869

33

B204

0.902

268

D

4

C

Mdil

354

473

0.792

33

B205

0.890

67

U

4

C

Mdil

68

50

0.786

19

B206

0.821

93

U

4

C

Mdil

32

70

0.819

33

B101A-70

0.656

550

D

4

C

Mdil

1067

1521

0.567

35

B101A-73

0.743

575

D

4

C

Con

1005

1357

0.550

33

B101A-75

0.879

600

D

4

C

Con

1043

1359

0.496

32

B102-47

0.677

500

D

4

C

Con

904

1221

0.501

33

B102-49

0.698

515

D

6

C

Con

957

1331

0.542

34

B102-57

0.767

585

D

4

C

Con

1033

1367

0.541

33

B102-59

0.709

600

D

4

C

Dil

1108

1572

0.582

35

B103-38

0.778

350

D

4

C

Con

621

840

0.648

33

B103-70

0.577

550

D

4

C

Maybe

1041

1506

0.533

36

B104-17

0.934

280

D

4

C

Con

484

645

0.747

33

B221-U19

0.963

140

D

6

C

Mdil

140

188

0.963

33

B221-U19A

0.906

140

D

6

C

Mdil

138

199

0.906

36

B221-U19B

0.853

140

D

6

C

Maybe

144

190

0.872

33

B231-13

0.840

165

D

4

C

Mdil

306

440

0.793

35

B231-19

0.866

215

D

4

C

Con

388

533

0.798

34

B231-23

0.893

250

D

4

C

Con

450

610

0.730

34

B231-28

0.866

275

D

4

C

Con

500

695

0.761

34

B231-31

0.700

300

D

4

C

Mdil

572

828

0.659

36

Table 3.6 Summary of testing program.
†

U, undrained condition; D, drained condition; C, strain-controlled compression; L, load-controlled condition.
Yes = steady state apparently reached; Dil/Con = sample still dilating or contracting at end of test;
Mdil = small amount of dilation, sample close to steady state at end of test.
‡

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Table 3.7 CSL derived from isotropic and oedometric tests.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Elev. PWD (m)</th>
<th>λ</th>
<th>e₀</th>
<th>p₀ (kPa)</th>
<th>Test Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>B231-13A</td>
<td>-8.1</td>
<td>0.014</td>
<td>0.9036</td>
<td>0.81</td>
<td>800 OEDOMETRIC INCREMENTAL LOADING</td>
</tr>
<tr>
<td>B231-19</td>
<td>-18.6</td>
<td>0.014</td>
<td>0.9236</td>
<td>0.83</td>
<td>800 OEDOMETRIC INCREMENTAL LOADING</td>
</tr>
<tr>
<td>B231-23A</td>
<td>-24.6</td>
<td>0.020</td>
<td>0.9937</td>
<td>0.86</td>
<td>800 OEDOMETRIC INCREMENTAL LOADING</td>
</tr>
<tr>
<td>B231-26</td>
<td>-29.1</td>
<td>0.016</td>
<td>0.9370</td>
<td>0.83</td>
<td>800 OEDOMETRIC INCREMENTAL LOADING</td>
</tr>
<tr>
<td>B231-31</td>
<td>-33.6</td>
<td>0.016</td>
<td>0.7770</td>
<td>0.67</td>
<td>800 OEDOMETRIC INCREMENTAL LOADING</td>
</tr>
<tr>
<td>B231-13</td>
<td>-7.9</td>
<td>0.018</td>
<td>0.9303</td>
<td>0.81</td>
<td>800 ISOTROPIC COMPRESSION</td>
</tr>
<tr>
<td>B231-23</td>
<td>-24.4</td>
<td>0.023</td>
<td>1.0137</td>
<td>0.86</td>
<td>800 ISOTROPIC COMPRESSION</td>
</tr>
<tr>
<td>B231-29</td>
<td>-30.5</td>
<td>0.017</td>
<td>0.8736</td>
<td>0.76</td>
<td>800 ISOTROPIC COMPRESSION</td>
</tr>
</tbody>
</table>

Table 3.8 Values used in assumed-chi (intersection) method.

<table>
<thead>
<tr>
<th>Sample</th>
<th>D_min</th>
<th>e₀</th>
<th>e_cs</th>
<th>ΣInp'/n **</th>
</tr>
</thead>
<tbody>
<tr>
<td>B231-13</td>
<td>0.000</td>
<td>0.840</td>
<td>0.8400</td>
<td>5.682</td>
</tr>
<tr>
<td>B231-19</td>
<td>0.000</td>
<td>0.866</td>
<td>0.8660</td>
<td>5.914</td>
</tr>
<tr>
<td>B231-31</td>
<td>-0.188</td>
<td>0.700</td>
<td>0.7537</td>
<td>6.327</td>
</tr>
<tr>
<td>B104-17</td>
<td>0.000</td>
<td>0.934</td>
<td>0.9340</td>
<td>6.071</td>
</tr>
<tr>
<td>B231-23</td>
<td>0.000</td>
<td>0.893</td>
<td>0.8930</td>
<td>6.008</td>
</tr>
<tr>
<td>B231-28</td>
<td>0.000</td>
<td>0.866</td>
<td>0.8660</td>
<td>6.133</td>
</tr>
<tr>
<td>B221-U19</td>
<td>-0.015</td>
<td>0.963</td>
<td>0.9673</td>
<td>4.942</td>
</tr>
<tr>
<td>B221-U19A</td>
<td>-0.134</td>
<td>0.906</td>
<td>0.9443</td>
<td>4.938</td>
</tr>
<tr>
<td>B221-U19B</td>
<td>-0.088</td>
<td>0.853</td>
<td>0.8782</td>
<td>4.968</td>
</tr>
</tbody>
</table>

Table 3.9 Forward iterative method results.

<table>
<thead>
<tr>
<th>Sample</th>
<th>G (MPa)</th>
<th>e₀</th>
<th>p₀′ (kPa)</th>
<th>D_min</th>
<th>I</th>
<th>ψ₀′′</th>
<th>M</th>
<th>H</th>
<th>†</th>
<th>‡</th>
</tr>
</thead>
<tbody>
<tr>
<td>B231-13</td>
<td>89.59</td>
<td>0.840</td>
<td>165</td>
<td>0.000</td>
<td>542.97</td>
<td>0.000</td>
<td>1.437</td>
<td>160</td>
<td>1.03</td>
<td>0.05</td>
</tr>
<tr>
<td>B231-19</td>
<td>107.47</td>
<td>0.866</td>
<td>215</td>
<td>0.000</td>
<td>499.86</td>
<td>0.000</td>
<td>1.373</td>
<td>100</td>
<td>1.15</td>
<td>0.06</td>
</tr>
<tr>
<td>B231-31</td>
<td>135.13</td>
<td>0.700</td>
<td>300</td>
<td>-0.188</td>
<td>450.43</td>
<td>-0.054</td>
<td>1.372</td>
<td>220</td>
<td>0.98</td>
<td>0.05</td>
</tr>
<tr>
<td>B104-17</td>
<td>128.87</td>
<td>0.934</td>
<td>280</td>
<td>0.000</td>
<td>460.25</td>
<td>0.000</td>
<td>1.333</td>
<td>25</td>
<td>1.40</td>
<td>0.10</td>
</tr>
<tr>
<td>B231-23</td>
<td>119.21</td>
<td>0.893</td>
<td>250</td>
<td>0.000</td>
<td>476.84</td>
<td>0.000</td>
<td>1.358</td>
<td>33</td>
<td>1.28</td>
<td>0.09</td>
</tr>
<tr>
<td>B231-29</td>
<td>127.28</td>
<td>0.866</td>
<td>275</td>
<td>0.000</td>
<td>462.84</td>
<td>0.000</td>
<td>1.398</td>
<td>50</td>
<td>1.38</td>
<td>0.10</td>
</tr>
<tr>
<td>B221-U19</td>
<td>80.02</td>
<td>0.963</td>
<td>140</td>
<td>-0.015</td>
<td>571.57</td>
<td>0.025</td>
<td>1.342</td>
<td>110</td>
<td>1.10</td>
<td>0.03</td>
</tr>
<tr>
<td>B221-U19A</td>
<td>80.02</td>
<td>0.906</td>
<td>140</td>
<td>-0.134</td>
<td>571.57</td>
<td>-0.065</td>
<td>1.435</td>
<td>75</td>
<td>1.32</td>
<td>0.08</td>
</tr>
<tr>
<td>B221-U19B</td>
<td>80.02</td>
<td>0.853</td>
<td>140</td>
<td>-0.088</td>
<td>571.57</td>
<td>-0.036</td>
<td>1.128</td>
<td>170</td>
<td>0.92</td>
<td>0.01</td>
</tr>
</tbody>
</table>

† Based on e₀ and using e_cs = e₀ - D_min/λ.

** During the entire test.

†† Based on equation (3.29) apart from the last three values as described in section 3.5.3.
### Table 3.10 Summary of the Calibration results for the Jamuna Bridge data.

<table>
<thead>
<tr>
<th>NorSand Parameter</th>
<th>Calibrated Value/Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{lc}$</td>
<td>1.35</td>
</tr>
<tr>
<td>$N$</td>
<td>0.2</td>
</tr>
<tr>
<td>$G$</td>
<td>$63.494 \left( \frac{p'}{p_a} \right)^{0.6875}$</td>
</tr>
<tr>
<td>$\lambda_{cr}$</td>
<td>0.04</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>1.03</td>
</tr>
<tr>
<td>$H$</td>
<td>$-903.46 \psi_0 + 91.758$</td>
</tr>
</tbody>
</table>
Figure 3.1 Graphical definition of the State Parameter (Jefferies & Shuttle, 2005).

Figure 3.2 Steady state line for undrained compression tests on contractive samples of Erksak 330/0.7 sand with initial state above the critical state line (Been et al., 1991).
Figure 3.3 Illustration of NorSand yield surface and limiting stress ratios (Jefferies & Shuttle, 2005).
Figure 3.4 Comparison of functions for $M(\theta)$ - (Jefferies & Shuttle, 2005).

Figure 3.5 Typical stress ratio and volumetric strain versus deviatoric strain (Ghazanfari & Shuttle, 2006).
Figure 3.6 Bishop’s Method applied to the Jamuna Bridge data.

Figure 3.7 Typical stress-dilatancy plot around peak (Ghazanfari & Shuttle, 2006).
Figure 3.8 Stress – Dilatancy graphs for the post-peak part of the drained triaxial test results of the Jamuna Bridge site. (only two of the samples have identifiable post-peak part as illustrated above).

Figure 3.9 Illustration of the determination of three shear moduli from a pressuremeter test (Bellotti et al., 1986).
Figure 3.10 Modulus number and modulus exponent measured at the Jamuna Bridge site using CPM. The thick line indicates the average and the dashed lines indicate 90% confidence limits (Fugro reports, 1986 & 1996).
Figure 3.11 Shear modulus profile of the Jamuna Bridge Site measured from CPM test (results and the average line as in the Fugro reports, 1986 & 1996).
Figure 3.12 Graphical solution of the shear modulus relationship measured from CPM tests for the Jamuna bridge site.

\[ G_{CPM} = 14.106(p'/p_a)^{1.8518} \]
\[ R^2 = 0.9954 \]

Figure 3.13 G\text{max} measured at the Jamuna Bridge Site (Fugro reports, 1986 & 1996).
Figure 3.14 Nuclear Density test result at the Jamuna Bridge Site (Fugro reports, 1986 & 1996).

Figure 3.15 Graphical solution of $G_{\text{max}}$ relationship with $p'$ measured from body wave test for the Jamuna Bridge Site.
Figure 3.16 Examples of undrained triaxial data for determining the critical state conditions.
Figure 3.17 CSL based on triaxial test results.

Figure 3.18 $f/\%$ and $D_{50}$ measurements for the Jamuna Bridge Site.
**Figure 3.19** Intersection of CSLs based on $\chi = 3.5$.

**Figure 3.20** NorSand Calibration using the Forward Iterating Method for Sample B231-13.
Figure 3.21 NorSand Calibration using the Forward Iterating Method for Sample B231-19.

Figure 3.22 NorSand Calibration using the Forward Iterating Method for Sample B231-31.
Figure 3.23 NorSand Calibration using the Forward Iterating Method for Sample B104-17.

Figure 3.24 NorSand Calibration using the Forward Iterating Method for Sample B231-23.
Figure 3.25 NorSand Calibration using the Forward Iterating Method for Sample B231-28.

Figure 3.26 NorSand Calibration using the Forward Iterating Method for Sample 221-U19.
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Figure 3.28 NorSand Calibration using the Forward Iterating Method for Sample 221-U19B.
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Figure 3.30 $H - \varphi_0$ relationship for the Jamuna Bridge Site.
Chapter 4

Statistical Characterization of State Parameter
Chapter 4

4 Statistical Characterization of the State Parameter

4.1 Introduction

In this chapter, the statistical characterization of state parameter for the Jamuna Bridge data is presented based on the NorSand calibration obtained in Chapter 3. The chapter starts by describing the different methods of interpreting CPT data in terms of state parameter in Section 4.2 and this is followed, in Section 4.3, by the pre-processing steps required prior to commencement of the interpretation procedure. Then, in Sections 4.4 and 4.5, the methodology for working out the point statistics and scale of fluctuation of the interpreted data is described. Finally, in Section 4.6, numerical calculations for the Jamuna Bridge data are described and the results are presented in Appendix A. These results are used in Chapter 6, in order to generate random fields of the state parameter and later perform stochastic analyses as described in Chapter 7.

4.2 Methods of Interpretation

As mentioned in Chapter 3, the in-situ state parameter across the site can be measured using the results of CPT tests. It should be mentioned that, in this context, CPT is referred to as a cone penetration test in which pore water pressures are measured during the test and hence it is taken to be the same as a CPTu test unless stated otherwise. Two methods of interpreting the CPT data into state parameter are described in this chapter; i.e. the Original Method, which is a unified framework developed by Been et al. (1987b) and which is described in Section 4.2.1, and the Revised Method, which was developed by Shuttle & Jefferies (1998) based on the NorSand soil model. The latter method uses the results obtained in Chapter 3 and is described in Section 4.2.3. The reason for choosing the Shuttle & Jefferies’ (1998) method for interpreting the CPT data is due to the deficiencies which exist in the Original Method. These deficiencies, which have been pointed out by Sladen (1989a & 1989b), are briefly presented in Section 4.2.2.
4.2.1 Original Method of Interpretation

This method is based on the results collected from large chamber tests for six different normally and over-consolidated sands with different ranges of \( K_0 \) (i.e. Reid Bedford Sand, Ottawa Sand, Hokksund Sand, Ticino Sand, Hilton Mines Sand and Monterey No.0 Sand). Been et al. (1987b) derived a relationship between the cone tip resistance \( (q_c) \) and the state parameter \( (\psi) \) using the results from these sands and knowing the steady state lines corresponding to each of them. Since this framework takes into account the fundamental properties of different types of sands by considering the slope of the steady state line, it is claimed to be applicable to all types of sands. However, the relationship they derived between \( q_c \) and \( \psi \) is only applicable to the evaluation of data obtained from modern electronic right cylindrical cones.

In this framework, the first stage is to determine the steady state line of the sand under consideration (using the methods described in Chapter 3) and then the laboratory \( q_c - \psi \) relationship needs to be obtained from a calibration chamber test. Having taken these two preliminary steps, it is then possible to calculate the state parameter via the relationship:

\[
Q_c = k \exp (-m\psi) \tag{4.1}
\]

in which

\[
k = 8 + \frac{0.55}{\lambda_{ss} - 0.01} \tag{4.2}
\]

\[
m = 8.1 - \ln (\lambda_{ss}) \tag{4.3}
\]

and \( Q_c \) is the normalized tip resistance, given by

\[
Q_c = \frac{q_t - p}{p'} \approx \frac{q_t}{p'} \tag{4.4}
\]

In equation (4.4), \( q_t \) is the tip resistance corrected for the effect of cone area reduction, that is

\[
q_t = q_c - (1 - \alpha)u \tag{4.5}
\]

where \( q_c \) is the measured CPT tip resistance and \( \alpha = A_{cr}/A_c \) is the tip area correction ratio, which is the ratio of the reduced cross-sectional area near the cone \( (A_{cr}) \) and the its actual cross-sectional area \( (A_c) \) as illustrated in Figure 4.1. \( p \) and \( p' \) are the mean total and effective stresses, respectively.
Since the two constants $m$ and $k$ in equation (4.1) are unique values for any particular sand, Been et al. (1987b) plotted these values against their corresponding $\lambda_{ss}$ values for the six sands they had considered (as illustrated in Figures 4.2 and 4.3) and by fitting appropriate curves through these data points derived equations (4.2) and (4.3). By substituting equations (4.2) and (4.3) into (4.1) and rearranging for $\psi$, one gets:

$$\psi = -\frac{1}{(8.1 - \ln \lambda_{ss})} \times \ln \left[ \frac{q_c - p}{p'} \left( 8 + \frac{0.55}{\lambda_{ss} - 0.01} \right)^{-1} \right]$$  \hspace{1cm} (4.6)

Therefore, in this method, after finding the slope of the steady state line ($\lambda_{ss}$) for any sand, values of $p'$ for each point in the CPT profile need to be calculated and then put into equation (4.6) in order to find their corresponding state parameter and subsequently the state parameter profile with depth. The mean total and effective stresses $p$ and $p'$ are calculated as follows:

$$p = \frac{\sigma_v + 2\sigma_h}{3}$$  \hspace{1cm} (4.7)

$$p' = \frac{\sigma_v' + 2\sigma_h'}{3}$$  \hspace{1cm} (4.8)

where $\sigma_v$ and $\sigma_v'$ are the total and effective vertical stresses and $\sigma_h$ and $\sigma_h'$ are the total and effective horizontal stresses at any point in the soil profile, respectively. The vertical and horizontal stresses are related to each other through the coefficient of earth pressure at rest $K_0 = \sigma_h' / \sigma_v'$. This method of interpretation is very straightforward and requires only one parameter to be calibrated (i.e. only $\lambda_{ss}$ in equation (4.6)). However, as described in the next section, it has got deficiencies which make the estimated values less accurate and which have prompted the need for a more accurate approach.

Before moving on to the next section, it is worth giving a brief description of the calibration chamber test used by Been et al. (1987c). Figure 4.4 illustrates the apparatus, which consists of a chamber capable to accommodate soil samples of typically 1m height and 1.2m diameter mounted on three 18kN load cells. The calibration chamber is in fact a large triaxial cell, in which the sample is prepared using moist tamping technique at a known density and which will then be subjected to a desired stress regime with the applied vertical and radial stresses independently controlled. Then a 4-channel, 60° standard electrical cone penetrometer is pushed into the soil similar to the field, and data such as tip resistance, pore pressure, sleeve friction and horizontal stresses are measured. Finally, the SSL is determined by
knowing the void ratio and stress level of the test and is related to the tip resistance in order to get the $q_c - \psi$ relationship in laboratory.

### 4.2.2 Deficiencies in the Original Method

Been et al. (1987b) declared that their original framework of interpretation was incomplete and biased due to neglecting several factors such as shear modulus ($G$), angle of friction at steady state ($\phi_{ss}$), constrained modulus ($M_c$), stress history, fabric, chamber size and boundary condition effects. Nevertheless, Sladen (1989a & b) carried out further studies on the data used by Been et al. (1989) in developing their original framework of interpretation, and question them on a number of issues.

The results presented in the original paper by Been et al. (1987c) were obtained from 14 calibration chamber tests on Erksak sand from a Beaufort Sea borrow source. These samples were compacted to varying states from dense to very dense and had fines contents varying between 3% and 6%, and the test results were completely consistent with the relationship Been et al. (1987b) had previously found to exist between the state parameter and cone tip resistance as presented in equation (4.1). Hence, they concluded that the determination of the in-situ state parameter requires only knowledge of the steady state line and the in-situ stress conditions such as $\sigma_v$, $\sigma_{vv}$ and $u$.

However, Sladen (1989b) questioned how relevant such hydraulically placed sand fills were to those 14 chamber tests on dense to very dense sands, or how valid is that unique relationship between $Q_c$ and $\psi$ for the Erksak sand or even all sands, or how relevant can the chamber test data be to in-situ conditions and also if the state parameter interpreted from CPT data are valid at all. In what follows in this section, these issues are reviewed and discussed in more detail.

- As mentioned above, Been et al. (1987c) presented the results of 14 chamber tests on Erksak sand with relative densities varying between 69% to 99% (i.e. dense to very dense). However, these samples had been taken from hydraulically placed sand fills in the Canadian Beaufort Sea with states varying from loose to medium dense. Hence, for the relationship derived from the chamber tests to be used for the in-situ material, data extrapolation was needed and Sladen (1989b) questioned if such an extrapolation was justifiable.
 Been et al. (1989) responded to this issue by recommending that comparison cannot be made between variables which cannot measured unarguably and it is better to make such comparisons through variables such as $q_c$, $\sigma_v$, grain size and fines contents. They also recommended that interpolation between closely spaced data would be sufficient and not lead to any potential errors.

- Sladen (1989b) questioned whether the unique relationship between $Q_c$ and $\psi$ derived for Erksak sand was valid. Unlike the available data for Ticino sand, no clear increase in density by increasing normalised tip resistance could be observed in the Erksak sand data, and so he was concerned if the unique relationship between the normalised tip resistance and either the state parameter or the void ratio could be representative for the Erksak sand. Sladen (1989b) was also concerned whether the general $Q_c - \psi$ relationship could be valid for all types of sands. Sladen used the results of Ticino sand, a normally-consolidated sand, previously studied in detail by other researchers, and plotted $Q_c$ vs. $\psi$ values to investigate a more general $Q_c - \psi$ relationship. The data obtained, although being highly scattered, seemed not to be too random. He therefore, grouped the data based on their mean stress levels into quite narrow ranges and fitted linear regression lines for each group. This revealed a systematic variation in the $Q_c - \psi$ relationship with mean effective stress level, which contradicted the unique relationship of Been et al. (1987b) for all stress levels. His study indicated that using Been $Q_c - \psi$ relationship would overestimate the state parameter for higher stress levels and underestimate it for lower stress levels.

Been et al.’s (1989) response to this issue was to highlight that the data used by Sladen (1989b) had comprised three types of Ticino sand, which were all similar with respect to gradation and mineralogy, but different with respect to grain shape, thereby leading to arguable conclusions. They agreed that there was a stress level dependency in their method which needed to be further investigated, but they attributed this dependency to neglected factors such as grain crushing effect, chamber boundary effect, shear modulus and elastic behaviour in their interpretation framework.

- Another issue raised by Sladen (1989b) was the relevance of chamber test results to the in-situ conditions. Since factors such as fabric, ageing and mode of deposition
could have a significant effect on CPT field performance and were not considered in Been’s framework of interpretation, he questioned whether it was possible to compare the results of chamber tests with in-situ CPT tests. Moreover, he questioned whether deriving the steady state lines based only on triaxial test data could be reliable, since he had found that sand gradation could have an effect on the location of the steady state line (Sladen et al. 1989b).

In response, Been et al. (1989) highlighted three points. Firstly, their studies had been based on samples which had been prepared by the same method of preparation and therefore the fabric effect was kept small. Secondly, Jefferies et al. (1988) had performed full-scale testing on Erksak sand over a period of 10 months and found that only 2% degradation in \( q_c \) during that period had occurred and that therefore there was no significant ageing effect in their studies. Finally, they pointed out that the influence of deposition mode had been considered in their method by including unbiased \( K_0 \) values in their laboratory \( Q_c - \psi \) plot.

- Sladen (1989b) also compared the values of normalised tip resistance of Erksak sand with those of Ticino sand. They found that they were higher for Erksak sand and concluded that interpreting the state parameter based on CPT tests could be unreliable and invalid.

In response, Been et al. (1989) referred to the theory of cavity expansion by Vesic (1972) which, in mathematical terms, is expressed as:

\[
\frac{q}{p'} = f(\phi', G)
\]

(4.9)

in which \( q \) is the cavity expansion pressure, \( p' \) is the mean effective stress, \( \phi' \) is the effective friction angle and \( G \) is the shear modulus of elasticity. They combined this relationship with one which had previously been developed by Been & Jefferies (1985 and 1986), that is:

\[
\phi' = g(\psi, \phi_{ss})
\]

(4.10)

to give the following relationship:

\[
\frac{q}{p'} = f(\psi, \phi_{ss}, G)
\]

(4.11)

They concluded that the cavity expansion pressure would be related to the state parameter and hence, further studies were necessary in order to develop their
method of interpretation based on this relationship. Therefore, the development of a new method of interpretation proved to be inevitable.

After these issues were raised by Sladen, many people tried to develop other methods from which the state parameter could be interpreted in a manner consistent with Salden’s arguments. Collins et al. (1992) developed a numerical method which could use the original method of Been et al. (1987b) and include the effect of the stress level to interpret the state parameter in the manner advocated by Sladen. However, they did not present a framework which was applicable to all sand types. Later, Konrad (1998) proposed another method which was applicable to all sand types. This method used Ticino sand as its reference sand type and could transform any type of sand into this reference type. No calibration was required using this method and one of its advantages was that it was able to interpret CPT data with very limited stress ranges.

Shuttle & Jefferies (1998) eventually developed another method of interpreting the state parameter, which was unbiased with respect to the stress level. It also resolved the deficiencies in Been’s method, by introducing a function $C$ which was based on comprehensive numerical studies and expressed as follows:

$$
\psi_0 = C(q_c, G, p, ...) \pm 0.01
$$

They found that the issue of stress level effect, which had gained a lot of attention in the past, was not a major deficiency in the original method and another factor plays a more important role in the original method. They indicated that the weakest point in the original method’s $Q_c - \psi$ relationship, as described by equation (4.1), is the dependency of the $m$ and $k$ parameters on only a single parameter $\lambda_{ss}$, which has not been the result of numerical or theoretical assumptions, but has instead been based on the simple CSSM assumption that $\lambda_{ss}$ controls plastic hardening. Instead, they linked the estimation of state parameter to more than one parameter, as seen in equation (4.12). These parameters, which are the variables in function $C$, are either those which can be easily determined by routine laboratory tests on reconstituted samples or those which can be easily and directly measured in-situ (e.g. shear modulus and tip resistance). Thus, this new method does not have the deficiencies of the original method of Been et al. (1987b) and is therefore the method which has been adopted in this thesis for estimating the state parameter for the Jamuna Bridge CPT data. It is described in more detail in Section 4.2.3.
4.2.3 Adopted Method of Interpretation

In an attempt to directly address the effect of stress level postulated by Sladen (1989a and 1989b), Shuttle & Jefferies (1998) performed finite element analysis of cavity expansion using the NorSand soil model. They found that, although equation (4.1) was a realistic representation of the relationship between tip resistance and state parameter, because the parameters \( k \) and \( m \) in this equation are treated as constants, the effect of stress level had become biased. Their detailed numerical simulations indicated that these two parameters have to be functions of \( G/\bar{\sigma}_m \), which was not taken into account in the original work of Been et al. (1987b). Therefore, they did not make any change to equation (4.1), but instead replaced equations (4.2) and (4.3) with equations which could take account of most of soil properties and were obtained by fitting trend lines to the numerical results to give a closed form method of approximate inversion, that is:

\[
\begin{align*}
  k &= (f_1(^{s}(I_r)) f_2(M) f_3(N) f_4(H) f_5(\lambda_{ss}) f_6(\nu))^{1.45} \\
  m &= 1.45((f_7(^{s}(I_r)) f_8(M) f_9(N) f_{10}(H) f_{11}(\lambda_{ss}) f_{12}(\nu))
\end{align*}
\]

(4.13)  
(4.14)

where functions \( f_1 \) to \( f_{12} \) are simple algebraic expressions fitted to the numerical results and are presented in Table 4.1. The dimensionless variables in equations (4.13) and (4.14) have been previously introduced in Chapter 3; i.e. \( I_r = G/\bar{\sigma}_m \) is the soil rigidity, \( M \) is the critical state coefficient, \( N \) is the volumetric coupling parameter, \( H \) is the plastic hardening modulus, \( \lambda_{ss} \) is the slope of the steady state line and \( \nu \) is Poisson’s ratio which is always an independent value and hence will give \( f(\nu) = 1 \). Therefore, as described in Chapter 3, only five out of eight parameters in the NorSand soil model need to be calibrated (i.e. \( M, N, G, \lambda_{ss}, H \)). The calibration procedure was described in detail in Chapter 3 and numerical estimations for the Jamuna Bridge data were also presented.

Finally, it should be mentioned that this method is not suitable when dealing with silts, due to the sampling problems which are a result of these soils being too soft to be set up in a triaxial test without collapsing under their self-weight. Plewes et al. (1992) have proposed a first order empirical approach for evaluating the in-situ state parameter in silts, but, because this thesis is only considering the top sand layer at the Jamuna Bridge Site, further details of their empirical approach is not given herein.
4.3 Pre-Processing of the Data

Pre-processing of the data is referred to as all the procedures that the CPT data have to go through prior to converting into state parameter and working out the point statistics and the spatial variability. There are three main pre-processing stages: data filtering, data shortening and trend removal of non-stationary data.

4.3.1 Data Filtering

Data Filtering is the process in which some data are eliminated because they do not represent the true variation of the soil profile. This may be because the CPT cone has entered a completely different material, or because some thin layers of other materials are contained within the soil stratum being studied (Wong, 2004). It is crucial to distinguish between these two possibilities in order to make accurate estimates of the statistics. A change in soil properties in all CPT profiles at a certain depth indicates that the soil stratification has changed at that depth; in this case one needs to consider all strata separately when estimating the state parameter and its statistical properties. That means that for each layer, the NorSand parameters have to be estimated. But, if sudden changes in soil properties are observed at random depths and over very small distances in one or a limited number of profiles, it may be deduced that these sudden changes are due to the presence of other materials contained within the soil layer; such data should be eliminated from the raw data.

Data Filtering is generally carried out in two stages. The first stage is before transforming \( q_c \) values into state parameter and includes identifying thin gravel or clay layers within the sand profile. This may be done by looking at the CPT pore pressure profiles and knowing that readings with \( u \approx 0 \) represent thin gravel layers and readings with sudden difference (more than 30%) with respect to their previous and/or following readings represent thin clay layers; these data need to be eliminated from the sand profile. The second stage is after transforming \( q_c \) values into state parameter and includes eliminating those parts of the state parameter profile which seem to belong to another stratum and which can occur at different depths for each of the profiles due to uneven soil surfaces. However, in practice, such inaccuracies are avoided by eliminating some certain depths of the soil profile from the top and bottom of the layer. After these zones are identified within each profile, the removed data will be replaced
by the interpolated data which are an interpolation between the two adjacent data points that bound the eliminated data. Figure 4.5 illustrates a typical CPT profile before and after data filtering. The filtering method explained in this section is implemented in such a way that it does not induce any adverse effect on the general statistics of the CPT profiles.

4.3.2 Data Shortening

Data shortening is the process performed when dealing with large amounts of CPT data and involves reducing the number of data points to lessen the computational time. Popescu (1995) reported that keeping the averaging of the cone data to small enough depth intervals, of the order of 5 to 10cm, would have no effect on the estimations of the statistics.

4.3.3 Trend Removal of Non-Stationary Data

Like other soil properties, state parameter is generally not constant with depth; its depth dependency can affect the accuracy of the statistical evaluations as well as the estimation of the scale of fluctuation, because the evaluation of the scale of fluctuation is based on the assumption that the data are stationary. Hence, before proceeding into statistical evaluations and calculations of the scale of fluctuation, the non-stationary data should be de-trended. The simplest way to do so is by employing the least square method to find a linear regression line for the $\psi$ profile and then normalizing the state parameter profile, so that:

$$\psi_N = \frac{\psi_i - \psi_{ave}}{\sigma_{res}}$$

(4.15)

where, for a given depth, $\psi_i$ is the measured state parameter, and $\psi_{ave}$ is the mean state parameter estimated from the regression line and $\sigma_{res}$ is the standard deviation of the residual state parameter $\psi_i - \psi_{ave}$. The normalised state parameter profile will have a mean which is equal to zero and a standard deviation which is equal to one.
4.4   Estimation of the Point Statistics

Section 4.2.3 described how to interpret CPT data in terms of state parameter, based on NorSand parameter values calibrated using procedures described in the previous chapter. However, in order to perform stochastic analyses, random fields of the state parameter need to be generated (as is the subject of Chapter 6) and for this it is needed to characterize the soil by calculating its point statistics (i.e. mean and standard deviation), the scale of fluctuation and the type of statistical distribution that the data follow. Therefore, this section briefly describes four probability distribution functions that are common in soil mechanics (i.e. Normal, Lognormal, Beta and Gamma) along with various statistical methods which are useful in finding the best probability distribution for a set of data (including the maximum likelihood estimator concept and the Chi-square goodness-of-fit test). Later, Section 4.5 will describe how to measure the scale of fluctuation.

4.4.1   Examples of Probability Distribution Functions

A probability distribution function is a function that contains most of the important properties of any random variable and can assign a probability to every interval of that variable. For example, soils often follow distribution functions such as Normal, Lognormal, Beta and Gamma.

If a random variable \( x \in (-\infty, +\infty) \), with mean \( \mu \) and standard deviation \( \sigma \), is represented by a Normal distribution function, the governing equation is,

\[
f(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) = \frac{1}{\sigma} \rho \left(\frac{x-\mu}{\sigma}\right)
\]  

(4.16)

in which \( \rho \) is given by

\[
\rho(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}
\]  

(4.17)

and is called the density function of a standard normal distribution, which has \( \mu = 0 \) and \( \sigma = 1 \). Equation (4.16) represents a bell-shaped curve which is not skewed and has a variance of \( \sigma^2 \) and extends over \( \pm\infty \).
A probability distribution function of any random variable \( x \) whose logarithm is normally distributed is called a Lognormal probability distribution function. Thus, if \( x \) is a random variable with a Normal distribution, then \( \exp(x) \) will have a Lognormal distribution expressed, that is

\[
f(x; \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-(\ln x - \mu)^2/2\sigma^2} ; x > 0
d\]

which represents a skewed bell-shaped curve which can only have positive values. Knowing that state parameter can have both positive and negative values, it would be unlikely for the data to be characterized by such a distribution. The variance of Lognormal distribution is given by:

\[
\text{var}(x) = (e^{\sigma^2} - 1)e^{2\mu + \rho^2}
\]

The Gamma distribution function for a variable \( x \) can also only take positive values and is skewed and is expressed as:

\[
f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} ; x > 0
\]

in which \( k > 0 \) is the shape parameter and \( \theta > 0 \) is the scale parameter of the Gamma distribution, and in which \( \Gamma(k) \) is the gamma function which is expressed as

\[
\Gamma(k) = \int_0^\infty x^{k-1}e^{-x}dx
\]

In this case, because \( k \) is a real non-complex variable, equation (4.21) can be simplified to:

\[
\Gamma(k + 1) = k!
\]

The variance of the Gamma distribution is calculated by \( k\theta \).

Finally, the Beta distribution and its variance are expressed by equations (4.23) and (4.24), respectively:

\[
f(x; \alpha, \beta) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1} ; x \in [0,1]
\]

\[
\text{var}(x) = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}
\]
and where $B$ is the beta function expressed as

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1} (1 - t)^{\beta-1} dt$$  \hspace{1cm} (4.25)

where $\alpha$ and $\beta$ are parameters which must be greater than zero and $t$ is a variable.

### 4.4.2 Maximum Likelihood Estimator (MLE)

A statistical concept which may be used to help find the best-fit distribution function for a set of data is the Maximum Likelihood Estimator concept. If a parameterized family of distributions, $D_\theta$, is associated with a known distribution function, $f_\theta$, and a sample of $n$ values $x_1, x_2, ..., x_n$ is drawn from this distribution function, then it might be possible to use $f_\theta$ to compute the probability distribution associated with the observed data $f_\theta(x_1, x_2, ..., x_n|\theta)$ as a function of $\theta$, in which $x_1, x_2, ..., x_n$ are fixed. The likelihood function for such an observation is expressed as:

$$L(\theta) = f_\theta(x_1, x_2, ..., x_n|\theta)$$ \hspace{1cm} (4.26)

The maximum likelihood estimator of $\theta$, which is denoted by $\hat{\theta}$, is that value of $\theta$ which maximizes $L(\theta)$ in equation (4.26). It should be noted that the maximum likelihood estimator may not be unique or even exist. Also, for the case where $n$ is unknown, the bias of the MLE method is very significant. Nevertheless, the method has proved useful in characterising state parameter and the following discussion briefly reviews its application for the distributions considered in this research.

In order to form the likelihood function $L(\theta)$ for the Normal distribution, equation (4.16) needs to be written in the form of equation (4.26). To do so, the function $f_\theta(x; \mu, \theta)$ needs to be written for $x_1, x_2, ..., x_n$ and summed up to give:

$$L(\mu, \sigma) = f_\theta(x_1, x_2, ..., x_n|\mu, \sigma) = \left(\frac{1}{2\pi\sigma^2}\right)^{n/2} \exp\left(-\frac{\sum_{i=1}^{n}(x_i - \mu)^2}{2\sigma^2}\right)$$ \hspace{1cm} (4.27)

However, as can be seen in equation (4.27), the likelihood function contains two variables, $\mu$ and $\sigma$, and therefore needs to be maximized with respect to both by differentiating equation (4.27) and equating it to zero, giving:

$$\hat{\theta} = (\hat{\mu}, \hat{\sigma}^2) = \left(\mu, \sum_{i=1}^{n}(x_i - \mu)^2 / n\right)$$ \hspace{1cm} (4.28)
If the same procedure is followed for the Lognormal distribution, the following MLE is obtained:

\[
\hat{\theta} = (\hat{\mu}, \hat{\sigma}^2) = \left( \frac{\sum_k \ln x_k}{n}, \frac{\sum_k (\ln x_k - \hat{\mu})^2}{n} \right)
\]  \hspace{2cm} (4.29)

The MLE for the Gamma distribution, with respect to the shape parameter \( k \) and scale parameter \( \theta \), are given by equations (4.30) and (4.31), respectively:

\[
\ln(k) - \psi(k) = \ln \left( \frac{1}{N} \sum_{i=1}^{N} x_i \right) - \frac{1}{N} \sum_{i=1}^{N} \ln(x_i)
\]  \hspace{2cm} (4.30)

\[
\theta = \frac{1}{kN} \sum_{i=1}^{N} x_i
\]  \hspace{2cm} (4.31)

where \( \psi(k) = \Gamma'(k)/\Gamma(k) \) is the digamma function. The MLE for the Beta distribution requires the solution of the following simultaneous equations:

\[
\begin{align*}
\psi(\alpha) - \psi(\alpha + \beta) &= \frac{1}{n} \sum_{i=1}^{n} \log \left( \frac{Y_i - a}{b - a} \right) \\
\psi(\beta) - \psi(\alpha + \beta) &= \frac{1}{n} \sum_{i=1}^{n} \log \left( \frac{b - Y_i}{b - a} \right)
\end{align*}
\]  \hspace{2cm} (4.32)

where \( a \) is the lower bound and \( b \) is the upper bound, which, based on equation (4.23) should be taken as 0 and 1, respectively.

### 4.4.3 The Chi-square goodness-of-fit test

Chi-square is a theoretical probability distribution function which has many applications in statistics. It is expressed by:

\[
f(x; k) = \frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{k/2-1} e^{-x/2}
\]  \hspace{2cm} (4.33)

in which \( k \) is called the degree of freedom for the distribution and \( x \geq 0 \). However, for \( x \leq 0, f(x;k) = 0 \). 
An important application of the Chi-square distribution is the goodness-of-fit test. It has been proven that, under reasonable assumptions and if the null hypothesis is true, easily calculated quantities can have distributions that can approximate the Chi-square distribution. Thus, it is possible to calculate the chi-square for different distributions and, by comparison the values of chi-square for the fitted distributions, find the distribution which best fits a given set of data.

If \( O \) and \( E \) represent the observed and expected frequencies of a data set, respectively (e.g. the calculated values of the state parameter and its expected value if a particular distribution is adopted for the data), then the chi-square can be calculated from:

\[
\text{chi-square, } \chi^2_k = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}
\]

(4.34)

in which \( k \) is called the degree of freedom and is equal to the number of cells (i.e. classes or bins) minus the number of linear constraints on the expected frequencies. It can be seen from equations (4.34) that, the larger the value of \( \chi^2 \), the larger the deviation of the observed distribution is from the expected distribution, and vice versa.

Hence, in order to find the probability distribution function which best fits a set of data (e.g. the state parameter profile), the data first need to be grouped into bins (or classes) and the frequencies (number of observations) found for each bin. Then, for an assumed distribution function (e.g. the Normal distribution), the expected frequencies for each bin are found based on the maximum likelihood estimator and then the chi-square value corresponding to fitting a Normal distribution to the data using equation (4.34). The same procedure may be repeated for other distribution types (i.e. Lognormal, Beta and Gamma). The chi-square values for different distribution functions are compared and the distribution function with the smallest chi-square value is taken as the best-fit distribution function. However, one more check that should be done is to compare the value of the smallest chi-square to table values given in textbooks (e.g. Lindley & Scott, 1984) for significance level testing purposes. In order to fully characterize the state parameter across the site, this procedure is followed for each profile individually and also for the assembly of all profiles. A detailed description of how this is done for the Jamuna Bridge data is given in Section 4.6.
4.5 Spatial Variability

As mentioned earlier, the variability in soil data can be represented by the statistics of the data, which are usually taken to be the mean and standard deviation. But, these statistics say nothing about the spatial characteristics of the variability. Representing spatial variation needs additional tools which are the subject of this section. Understanding the spatial variation of the soil properties in a site will lead to site characterization. This has been previously thought of as an intuitive process, based only on engineering judgment and without any analytical considerations. Indeed, the first person who recognized the importance of site characterization as an analytical method was Terzaghi (1936), who pointed out the effect of minor geological details that are not easily verified and can govern the engineering performance of the structure. However, analytical approaches to site characterization have been developed in industries such as the oil, gas and minerals industries, because in such industries the importance of an accurate estimation of natural resources is more economical than, say, finding weaker seams beneath an earth dam.

Two schools of statistical thoughts were developed in the early days to answer the question of random variation in a rational way; i.e. the frequentists and the Bayesians. Each of them have got their own view of spatial variations and offer a different method for characterizing the spatial variability. Hence, before going into any detail of how to characterize the spatial variability, it is worth describing these two schools of thoughts (Baecher & Christian, 2003).

Frequentists are themselves divided into two groups of thought, the likelihoodists and the non-likelihoodists. The first group, follow the concept of estimators and believe that any unknown value can be estimated through maximizing the likelihood function which was described in Section 4.4.2. On the other hand, unlike the first group, the non-likelihoodists believe that, not only the likelihood function carries information about an experiment, but also the whole experimental frame of reference contains information about the unknown value, \( \theta \) (as in equation (4.26)). Hence, this group of thought bases their inferences on hypothesis testing. In this method, probability is not used as a tool for inference, but is used as a tool to characterize the sample space for future hypothesis testing.
Bayesian methods are based on Bayes’ Theorem and the likelihood function, in which Bayes’ theorem is described mathematically as:

\[
P[A|B] = \frac{P[A \text{ and } B]}{P[B]} = \frac{P[A] \cdot P[B|A]}{P[B]} \tag{4.35}
\]

in which \(P[A]\) is the prior probability of \(A\), and is called so, because it does not take into account any information about \(B\); \(P[A|B]\) is called the posterior probability, and is called so, because it is dependant on the value of \(B\); \(P[B|A]\) is called the likelihood and \(P[B]\) which acts as a normalising constant is called the prior probability of \(B\).

In the Bayesian method a prior probability before observing the data is taken and then by applying the Bayes’ theorem and the likelihood function a posterior probability is reached, which can then be used for drawing inferences. Specifying a prior distribution is the key step in this method and thus having “experience” in this method is important. In this respect, prior probabilities can be either informative or non-informative, which correspond to the cases where there is some or no information about the uncertainty involved in the initial data.

The first stage in statistical analysis is to plot the data. Having plotted the data, one then needs to describe the spatial variability within the data by a model. For simplicity, the model will be divided into a known deterministic trend and a residual variability about that trend, and thus can be written as:

\[
z(x) = t(x) + u(x) \tag{4.36}
\]

in which \(x\) is the location where the soil property has been measured – which can be a vector, for example in 3D space – \(z(x)\) is the value of the soil property at that location, \(t(x)\) is the deterministic trend at \(x\) and \(u(x)\) is the residual variation at \(x\). The trend can be deterministically characterized by an equation, whereas the residuals will be characterized statistically as random variables having a zero mean and a non-zero variance represented by

\[
Var[u] = E_z[\{z(x) - t(x)\}^2] \tag{4.37}
\]

The residuals can be considered as the aleatory part of uncertainty; this means that the soil properties are not random themselves, but are treated as being random because there are too few data at hand.
The second stage in statistical analysis is the trend analysis. Trends are mathematical functions (i.e. lines, curves or surfaces) that can fit the data points in space. Theoretically, the best polynomial which can fit \( n \) data points has an order of \( n - 1 \), which will result in the variance of residuals to be zero. But the problem with such a polynomial is that, as the order of the polynomial becomes larger and larger, the uncertainty in the parameters of the estimated trend will increase, and finally when the order reaches \( n - 1 \), the uncertainty reaches infinity. A simple method for trend analysis is regression analysis. Due to the uncertainty involved in finding the flexibility – i.e. the order – of a trend, one needs to treat the uncertainty by statistical methods. As usual, it is investigated how the two schools of thought – i.e. the frequentists and the Bayesians – deal with the problem of trend analysis. Expressing equation (4.36) in another form, such as:

\[
\mathbf{z} = \mathbf{X}\mathbf{\beta} + \mathbf{u}
\]

in which \( \mathbf{z} \) is the vector of \( n \) observations \( \mathbf{z} = \{z_1, z_2, \ldots, z_n\} \), \( \mathbf{X} \) is the matrix of location coordinates, \( \mathbf{\beta} \) is the vector of trend parameters with a dimension equal to the order of the fitted polynomial, and \( \mathbf{u} \) is the vector of residuals corresponding to the observations. The frequentists approach to the problem of trend analysis is based on repeated sampling and the least squares method. That is, they first employ the least squares method to estimate the behaviour of the trend surface. Next, they assume that, because a slight change in the location of samples or method of sampling can lead to a different trend surface and this will lead to different probability distributions of the trend parameters as the sampling is repeated, minimizing the variance of the residuals \( \mathbf{u} \) over \( \mathbf{\beta} \) gives the best fit trend surface. The Bayesian approach specifies a probability distribution function on the coefficients of the model \( (\mathbf{\beta}, \sigma) \), and uses Bayes’ Theorem to update that distribution function in the light of the observed data. In both methods, the variations of the residuals about the trend are often assumed to be Normal. The problem in the Bayesian approach will then be maximizing the likelihood function. Two assumptions have been made in trend analysis: the first is that the deviations of the residuals are of same importance and are independent of their location in space; the second is that the variations of the residuals are independent of one another, which means that knowing the residual variations at one location does not give any information about the residual variations at other locations.
The third and final stage in statistical analysis is the estimation of the autocorrelation and autocovariance, which are used to describe the spatial variation of the residuals. Baecher & Christian (2003) define autocorrelation as a property that residuals off the mean trend are not probabilistically independent, but display a degree of association among themselves which is a function of their separation in space. If \( i \) and \( j \) are two separate locations in space that are close together, then their residuals \( u(x_i) \) and \( u(x_j) \) should be expected to be similar. Increasing the separation distance between the two locations will in return decrease the correlation between \( u(x_i) \) and \( u(x_j) \). If the separation distance is \( \delta \), then the mathematical expression for the autocorrelation function is:

\[
R_z(\delta) = \frac{1}{\text{Var}[u(x)]} E[u(x_i) \ u(x_{i+\delta})] \tag{4.39}
\]

in which \( R_z(\delta) \) is called the autocorrelation function, \( \text{Var}[u(x)] \) is the variance of the residuals and \( E[u(x_i) \ u(x_{i+\delta})] = \text{Cov}[u(x_i), u(x_{i+\delta})] \) is the covariance of the residuals. Multiplying \( R_z(\delta) \) by the variance \( \text{Var}[u(x)] \) leads to the autocovariance function, which is mathematically expressed as

\[
C_z(\delta) = E[u(x_i) \ u(x_{i+\delta})] \tag{4.40}
\]

When the separation distance is zero, then the autocorrelation function will obtain its maximum value, which is unity, and as the separation distance increases, in general, the autocorrelation function will decrease. The autocorrelation has smaller values in vertical directions within a soil deposit than in the horizontal direction and a ratio of about 1 to 10 may be found between the autocorrelation in the two directions (Baecher & Christian, 2003). However, the autocorrelation in the horizontal direction may be isotropic, that is the autocorrelation is the same in different direction. Isotropy or anisotropy of a soil deposit is a consequence of its geological formation.

In the remainder of Section 4.5, different methods of statistical characterization of the spatial variability are discussed. These include the estimation of the autocorrelation and autocovariance functions, the concept of variograms, scale of fluctuation, wavelet coefficient variance, periodograms and resampling techniques. Finally, the adopted method of estimating the spatial variability based on the scale of fluctuation and using the numerical method proposed by Campanella et al. (1986) will be given. All numerical calculations for the Jamuna Bridge Site are presented in Section 4.6.
4.5.1 Estimation of the Autocovariance and Autocorrelation Functions

The estimation of the autocorrelation and autocovariance functions, like any other statistical estimation, can be tackled using the two schools of inference: the frequentists and the Bayesians. The two methods of approaching the problem in a frequentist sense are: moment estimation and maximum likelihood estimation. The $n^{th}$ moment of a probability distribution about the origin is defined as (Baecher & Christian, 2003):

$$E(x^n) = \int_{-\infty}^{\infty} x^n f_x(x) \, dx$$

(4.41)

in which $f_x(x)$ is the probability density function. The mean is the first moment of the probability distribution function about the origin. The $n^{th}$ central moment is the $n^{th}$ moment about the mean and is defined as:

$$E[x - E(x)]^n = \int_{-\infty}^{\infty} [x - E(x)]^n f_x(x) \, dx$$

(4.42)

The second central moment is the variance, the third is called the skew and the forth is the kurtosis. If there are $n$ observations $\{z_1,\ldots,z_n\}$ at locations $\{x_1,\ldots,x_n\}$ and one needs to estimate the value of an unknown parameter $\theta$ within this population by $\hat{\theta} = g\{z_1,\ldots,z_n\}$, he/she can use statistics of the observations as estimates of $\theta$.

Since, $\theta$ is unknown, its statistics are also unknown and therefore a function that can give estimates as close as possible to the true value of $\theta$ is sought.

Methods of moment estimation take the statistical moments of a set of observations (e.g. mean, variance or covariance) as estimators (i.e. $\hat{\theta}$). If $\{x_1,\ldots,x_n\}$ are independent and identically distributed (IID) samples from a sample space, $S$, having a $k$-dimensional parameter vector $\theta$, then, based on the method of moment estimation (MOME), $\hat{\theta}$ will be the solution to the following system of equations:

$$E_{\hat{\theta}}(x^k) = \frac{1}{n} \sum_{j=1}^{n} x^k_j \quad ; k = 1,\ldots,k$$

(4.43)

where $x^k_j$ is the $j^{th}$ moment of the observations, $\{x_1,\ldots,x_n\}$. Keeping this equation in mind and knowing that the autocovariance function is calculated by equation (4.40), its moment estimation will be expressed as:
\[
\hat{C}_z = \frac{1}{n - \delta} \sum_{i=1}^{n-\delta} [u(x_i) u(x_{i+\delta})]
\]

\[
= \frac{1}{n - \delta} \sum_{i=1}^{n-\delta} \{z(x_i) - t(x_i)\} \{z(x_{i+\delta}) - t(x_{i+\delta})\}
\] (4.44)

in which \( n - \delta \) is the number of data pairs with a separation distance of \( \delta \), and \( z(x_i) - t(x_i) \) is the trend-removed data. The corresponding moment estimator of the autocorrelation function can be calculated by dividing both sides of the above equation by the sample variance, \( s_z^2 \) giving:

\[
\hat{R}_z(\delta) = \frac{1}{(n - \delta)s_z^2} \sum_{i=1}^{n-\delta} \{z(x_i) - t(x_i)\} \{z(x_{i+\delta}) - t(x_{i+\delta})\}
\] (4.45)

The solution of equations (4.44) and (4.45) is done by first finding the separation distance of each set of data and then calculating their corresponding estimates of the autocorrelation function, \( \hat{R}_z(\delta) \). Finally, these data are plotted in a separation distance vs. autocorrelation graph. Some properties of this method of estimating the autocorrelation and autocovariance functions are:

a) The method is non-parametric, which means that no assumptions are needed about the mathematical shape of the autocovariance function; there is only a need to assume that the second moment exists. This property is valuable, since choosing a priori functional shape for the autocovariance is hardly justified.

b) The moment estimator is consistent and asymptotically unbiased and hence it is a desirable method of estimation.

Maximum likelihood estimation is another frequentist approach for estimating the autocorrelation function. The likelihood function for a set of observations \( \{z_1, \ldots, z_n\} \) is given by:

\[
L(\theta|z_1, \ldots, z_n) = f_{z_1, \ldots, z_n}(z_1, \ldots, z_n)
\] (4.46)

which, in the particular case where \( z_1, \ldots, z_n \) are independent of one another, will become:

\[
L(\theta|z_1, \ldots, z_n) = \prod_{i=1}^{n} f_z(z_i|\theta)
\] (4.47)
However, since many of the most important probability distributions involve exponential terms, it is easier to use the log-likelihood function rather than the likelihood function itself, which can be expressed as (Baecher & Christian, 2003):

\[
LL(\theta | z_1, \ldots, z_n) = \sum_{i=1}^{n} \log f_{z_i}(z_i | \theta) dz_i
\]  

(4.48)

The likelihood function will be maximized if:

\[
\frac{\partial L(\theta | z_1, \ldots, z_n)}{\partial \theta} = 0 \quad \text{such that} \quad \frac{\partial^2 L(\theta | z_1, \ldots, z_n)}{\partial \theta^2} < 0
\]  

(4.49)

In this method, the trend surface and the autocorrelation of the residuals can be estimated at the same time. If the trend is represented by a regression surface in the form \(T(x) = x\beta\), in which \(x\) is the matrix of location coordinates, and \(\beta = \{\beta_1, \ldots, \beta_n\}\) is a vector of regression coefficients, the random field of soil properties can be assumed to be isotropically Gaussian and modelled by:

\[
z(x) = x\beta + \varepsilon
\]  

(4.50)

where \(\varepsilon\) is a vector of correlated residuals defined by a parametric autocovariance function, \(C_z(\delta)\), which is twice differentiable with respect to \(\theta = \{\theta_1, \theta_2, \ldots, \theta_p\}\), the parameters of the autocovariance function. If \(z = \{z(x_1), z(x_2), \ldots, z(x_n)\}\) is a set of measurements and the unknown parameters are described by a vector \(\phi = \{\beta, \theta\}\), then the log-likelihood of \(\phi\) is:

\[
LL(\phi | Z) = -\frac{n}{2} \ln C - \frac{1}{2} (Z - \beta X)' C^{-1} (Z - \beta X)
\]  

(4.51)

in which \(C = \text{Cov}[z_i, z_j]\) is the autocovariance matrix and is positive definite and \(X\) is an \(n \times q\) matrix of locations. The maximum likelihood estimates of \(\beta\) and \(C_z(\delta)\) are then found by maximizing \(LL(\phi | Z)\) with respect to \(\phi\).

The maximum likelihood method of estimating the autocorrelation has the following properties:

a) It is a parametric method, which means that the distributional form \(f_{z_i}(z_i | \theta)\) is assumed to be known.

b) The method is consistent and asymptotically Normal.
Before going to the Bayesian approach of autocovariance estimation, it is worth mentioning that a good way of estimating the autocovariance in a frequentist manner is to use both moment and maximum likelihood estimations in the analysis. That is, a graphical representation of the data can be obtained by the moment method, in the form of separation distance vs. autocovariance, and this can be used as a means of determining what type of autocovariance model should be used when using the maximum likelihood method for estimating the autocovariance parameters and trend coefficients; for example, it is better to use an exponential model or a squared-exponential one.

Bayesian methods of estimating the autocovariance function are very rare in the literature. This is mainly because the posterior PDFs calculated through the Bayes’ theorem, for many of the common non-informative distributions, do not converge to zero at infinity and hence are improper. However, this problem can be overcome by choosing an appropriate non-informative reference prior-distribution such as the one suggested by Berger et al. (2001), which is expressed as

\[
f(\beta, \sigma, \theta) \propto \frac{1}{\sigma^2} \left( |W_{\theta}^2| - \frac{|W_{\theta}^2|}{(n-k)} \right)^{\frac{1}{2}}
\]  

(4.52)

where

\[
W_{\theta}^2 = \frac{\partial R_{\theta}}{\partial \theta} R_{\theta}^{-1} \left\{ I - X(X'R_{\theta}^{-1}X)^{-1} X' R_{\theta}^{-1} \right\}
\]

(4.53)

in which $\theta$ is the vector of unknown parameters, $\sigma$ is the standard deviation and other symbols are as before. Moreover, Berger (Berger 1993, Berger et al. 2001) and Kitanidis (1985, 1997) suggest the use of a multi-Normal spatial random field of the form:

\[
z(x) = \sum_{i=1}^{k} f_i(x) \beta + \varepsilon(x)
\]

(4.54)

The posterior distribution function, which can be calculated by Bayes’ theorem, is then numerically calculated. However, based on the autocovariance function model used and one’s knowledge of the parameters $\theta$, a closed-form solution can be sought (Baecher & Christian, 2003).
4.5.2 Variograms

Geostatistics is a theory of random fields, which is probabilistic rather than statistical and was first developed in the mining industry. In mining geostatistics, it is more common to express the spatial structure of data by a function called the variogram, rather than by the autocovariance. The advantage of using a variogram is that it requires less statistical assumptions regarding stationarity than does the autocovariance function. However, it is more difficult to use in spatial interpolation and engineering analysis than the autocovariance, and this disadvantage makes its application restricted. Nevertheless, in practice, the two methods of characterizing the spatial structure are closely related. The variogram function is mathematically expressed as:

$$2\gamma = E[(z(x_i) - z(x_j))^2] = Var[z(x_i) - z(x_j)]$$  \hspace{1cm} (4.55)

in which, $z(x_i)$ and $z(x_j)$ are two random properties at points $i$ and $j$ in the space and $\gamma$ is the semivariogram. As can be seen, the variogram function is the result of the expected value of the squared difference of two observations, unlike the autocovariance function which, as seen in equation (4.45), is the expected value of the product of two observations. Hence, the variogram function is a function of the increments of the spatial properties, whereas the autocovariance function is a function of their absolute values.

For the special case where the means and autocovariances of the spatial variables are stationary, the variogram and the autocovariance functions are directly related, that is,

$$2\gamma = Var[z(x_i) - z(x_j)] = Var[z(x_i)] - Var[z(x_j)] - 2Cov[z(x_i), z(x_j)]$$  \hspace{1cm} (4.56)

so that,

$$2\gamma(\delta) = 2\{C_z(0) - C_z(\delta)\}$$  \hspace{1cm} (4.57)

$$\gamma(\delta) = C_z(0) - C_z(\delta)$$  \hspace{1cm} (4.58)

As with the autocovariance function, there are common analytical variogram models. In a stationary process, as $|\delta| \to \infty$, $C_z(\delta) \to 0$ and hence, $\gamma(\delta) \to C_z(0) = Var[z(x)]$. The value at which the variogram levels off, $2C_z(\delta)$, is called the still value and its corresponding distance is called the range.
4.5.3 Concept of the Scale of Fluctuation

In random field theory, a great range of problems can be solved by knowledge of the covariance function. However, due to the practical and economical limitations which exist in geotechnical engineering, such knowledge is difficult to obtain. Thus, two options are possible for obtaining the form of the covariance function:

a) Assume a form for the covariance function, try to adjust it to the limited data and use the resulting covariance function together with the exact formulae of the theory of random fields to arrive at the required results.

b) Make reasonable assumptions about the results themselves, taking into account any data that are available.

If the first option is chosen, the resulting calculations may be quite complicated, possibly leading to a false impression of accuracy. However, choosing the second option avoids such problems.

If $X(t)$ is a zero-mean stationary random process, with covariance function $\text{Cov}(\tau)$ and with $Y_T$ being the average of $X(t)$ over the length $T$, which is a random variable expressed as:

$$Y_T = \frac{1}{T} \int_0^T X(t) \, dt$$

(4.59)

the variance of averages of such a random process will then be

$$\text{Var}[Y_T] = \frac{1}{T^2} \int_0^T \int_0^T \text{Cov}(\tau_1 - \tau_2) \, d\tau_1 \, d\tau_2$$

$$= \frac{2\sigma^2}{T} \int_0^T (1 - \frac{\tau}{T}) \rho(\tau) \, d\tau \quad \text{where} \quad \text{Cov}(\tau) = \sigma^2 \rho(\tau)$$

(4.60)

$$= \sigma^2 \Gamma^2(T) \quad \text{where} \quad \Gamma^2(T) = \frac{2}{T} \int_0^T (1 - \frac{\tau}{T}) \rho(\tau) \, d\tau$$

The above results can also be obtained using the frequency domain approach. If $S(\omega)$ is the spectral density function of $X(t)$, then the variance of the averages can be expressed as:

$$\text{Var}[Y_T] = \int_{-\infty}^{\infty} \left( \frac{\text{Sin}(\omega T/2)}{\omega T/2} \right)^2 S(\omega) \, d\omega$$

(4.61)

or:

$$\text{Var}[Y_T] = \int_{-\infty}^{\infty} \left( \frac{\text{Sin}(\omega T/2)}{\omega T/2} \right)^2 S(\omega) \, d\omega$$

(4.61)
\[ \Gamma^2(T) = \int_{-\infty}^{\infty} \left( \frac{\sin(\omega T/2)}{\omega T/2} \right)^2 s(\omega) d\omega \]  \hspace{1cm} (4.62)

in which:

\[ s(\omega) = \frac{S(\omega)}{\sigma^2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \cos(\omega \tau) \rho(\tau) d\tau \]  \hspace{1cm} (4.63)

Using the approximation

\[ \left( \frac{\sin(\omega T/2)}{\omega T/2} \right)^2 \approx \begin{cases} 1 & \text{if } T \to 0 \\ \frac{2\pi}{T} \delta(\omega) & \text{if } T \to \infty \end{cases} \]  \hspace{1cm} (4.64)

equation (4.62) can be rewritten in the form

\[ \Gamma^2(T) \approx \begin{cases} \int_{-\infty}^{\infty} s(\omega) d\omega = 1 & \text{if } T \to 0 \\ \frac{2\pi}{T} s(0) & \text{if } T \to \infty \end{cases} \]  \hspace{1cm} (4.65)

From equation (4.63),

\[ s(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \rho(\tau) d\tau \]  \hspace{1cm} (4.66)

so that equation (4.65) can be written as:

\[ \Gamma^2(T) \approx \begin{cases} 1 & \text{if } T \to 0 \\ \frac{2\pi}{T} \int_{-\infty}^{\infty} \rho(\tau) d\tau = \frac{\theta}{T} & \text{if } T \to \infty \end{cases} \]  \hspace{1cm} (4.67)

in which, \( \theta \) is called the scale of fluctuation, which is given by:

\[ \theta = \int_{-\infty}^{\infty} \rho(\tau) d\tau \]  \hspace{1cm} (4.68)

The above approximation can be modified to:

\[ \Gamma^2(T) \approx \begin{cases} 1 & ;T \leq \theta \\ \frac{\theta}{T} & ;T \geq \theta \end{cases} \]  \hspace{1cm} (4.69)

This relationship may be used as an approximation to \( \Gamma^2(T) \) with very good results, even if the exact form of \( \rho(\tau) \) is unknown. With such an approximation, only a knowledge of \( \theta \) is required. Two points that are more than a distance of \( \theta \) apart, are expected to have poorly correlated values of the random process, whereas, for two
points that are less than a distance of $\theta$ from each other, the values of the random process are expected to be highly correlated.

### 4.5.4 Concept of Fractals

A fractal is usually defined as a rough or fragmented geometric shape that can be subdivided into parts, each of which is (at least approximately) a reduced-sized copy of the whole (Vanmarcke, 1984). From a geotechnical standpoint, however, fractional behaviour means that the characteristics of soil properties are the same (or self-similar) irrespective of the observation scale. Fractal sets or processes can be used as alternative techniques in the modelling of irregular data.

Consider a composite random process $X(t)$, which consists of two independent components whose scales of fluctuation are completely different and assume that $\theta_1 < \theta_2$. If the correlation function of the two components is assumed to be triangular with base $2\theta_1$, as illustrated in Figure 4.6, the correlation function of $X(t)$ is then a composite of these two triangular shapes. Focusing on the behaviour of the composite variance function, this becomes inversely proportional to the interval $T$ if $T > \theta_2$. On the other hand, if $T \leq \theta_2$ the variance function will be proportional to $T^{-b}$ in which $b < 1$. The presence of a slowly varying random trend makes it appear that the process is not obeying the basic $b = 1$ decay law for the variance function. Many natural phenomena exhibit this kind of behaviour. Such phenomena can be modelled by fractional noise which has a spectral density function proportional to $\omega^{-1}$.

The fractional noise is said to be self-similar because patterns of fluctuation appear similar regardless of the scale at which they are observed. The scale of fluctuation for an ideal fractional noise is infinite and is physically unrealizable; hence the area under the correlation function (which can be used to determine the scale of fluctuation) is infinite. There are obviously practical limits to real observation times/distances and the time interval/distance between observations. This will mean that there is a frequency range outside which the theoretical fractional noise model is unsupported by real data. Inside this frequency range, there is always the possibility of constructing a composite process model, like the one illustrated in Figure 4.7 whose spectral density function (s.d.f.) will decay approximately in accordance with the $\omega^{-1}$ law. Hence it is possible
to model natural phenomena which seem to obey the fractional noise model as a composite random process model. Such a model provides a simpler tool for stochastic modelling of the real data.

The term fractal can sometimes be described in literature as a long-memory random process, statistically self-similar process and \( / / \) noise. What these terms mean is that there is a significant correlation between a soil’s properties at two points, even when the points are very widely separated. This is in contrast with short-memory or finite-scale random processes. One of the reasons that makes it important to determine whether soil properties are fractal in nature or finite-scale, is that maximum likelihood estimates of the scale of fluctuation in fractal processes are dependent on the size of the sampling domain. This means that if a researcher reports, for example a scale of fluctuation of 0.6m over a domain size of 5m, the same person will obtain a much larger scale of fluctuation for the same deposit if he/she reports it over a domain size which is 10 times larger. Hence this makes the obtained results questionable. Fractal models are often described by the one-sided spectral density function:

\[
G(\omega) = \frac{G_0}{\omega^\gamma}
\]

in which \( G_0 \) is the spectral intensity and \( \gamma \) is a parameter that controls how the spectral power is partitioned from low-to-high frequencies. \( \gamma = 0 \) corresponds to white noise, \( 0 \leq \gamma < 1 \) corresponds to fractional Gaussian noise, which is stationary, and \( \gamma > 1 \) corresponds to fractional Brownian motion which is non-stationary. The last two processes are infinite-variance processes that are physically unrealizable. Mandelbrot and Ness (1968), in an attempt to render fractional Gaussian noise physically realizable, locally averaged the fractional Gaussian noise process over the small distance \( \delta \) and obtained the finite-variance process whose correlation function is:

\[
\rho(\tau) = \frac{1}{2\delta^{2H}}[|\tau + \delta|^{2H} - 2|\tau|^{2H} + |\tau - \delta|^{2H}]
\]

in which:

\[
H = (1/2) (\gamma + 1)
\]
is called the Hurst or self-similarity coefficient with $\frac{1}{2} \leq H < 1$. $H = \frac{1}{2}$ corresponds to white noise, whereas $H = 1$ means perfect correlation. The process variance can also be described as:

$$\sigma_x^2 = G_0 \delta^{2H-2} \Gamma(1-2H) \cos(\pi H)/(\pi H)$$

(4.73)

Hence, the variance can be fully determined by knowing $G_0$, $H$ and $\delta$. Self-similarity for fractional Gaussian noise is expressed by saying that the process $X(z)$ has the same distribution as the scaled process $a^{1-H} X(az)$ for some positive $a$. On the other hand, for fractional Brownian motion self-similarity means that $X(z)$ has the same distribution as $a^{-H} X(az)$, in which the different exponent on $a$ is due to the fact that fractional Gaussian noise is the derivative of fractional Brownian motion.

### 4.5.5 Wavelet Coefficient Variance

A wavelet is a mathematical function which is used to divide a given function into different frequency components, so that each component can be studied with a resolution that matches its scale. A wavelet transform is a representation of a function by wavelets. Wavelet transforms, in contrast to traditional Fourier transforms, have the ability to represent functions that have discontinuities and sharp peaks. Hence, they can be viewed as an alternative to Fourier decomposition, except that sinusoids are placed by wavelets that act only over a limited domain. The wavelets are scaled and translated copies (known as daughter wavelets) of a finite-length or fast-decaying oscillating waveform (known as the mother wavelet). In the 1D case, wavelets are usually defined as translations along the real axis and dilations (scalings) of a mother wavelet, $\psi$, as in:

$$\psi_j^m(z) = 2^{m/2} \psi(2^m z - j)$$

(4.74)

in which $m$ and $j$ are the dilation and translation indices, respectively. Wavelets have attracted so much attention in recent years in areas such as signal analysis, image compression and in special fractal process modelling because of having the property of being self-similar in nature. If random process $X(z)$ is expressed as a linear combination of various scalings, translations and dilations of a common shape, it can be written in the following form:
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\[ X(z) = \sum_m \sum_j X_j^m \psi_j^m(z) \]  \hspace{1cm} (4.75)

If the wavelets are selected to be orthonormal, then the coefficients can be found through the inversion:

\[ X_j^m = \int_{-\infty}^{\infty} X(z) \psi_j^m(z) \, dz \]  \hspace{1cm} (4.76)

which can be solved by a number of highly efficient numerical algorithms. Details of the wavelet decomposition can be found in the literature (Strang & Nguyen, 1996 and Fliege, 1994). Wornel (1996) stated that, under reasonably general conditions, if the coefficients \( X_j^m \) are mutually uncorrelated, zero-mean random variables with variances,

\[ \sigma_m^2 = \text{Var}[X_j^m] = \sigma^2 2^{-m} \]  \hspace{1cm} (4.77)

then \( X(z) \) obtained through equation (4.75) will have a spectrum that is very nearly fractal. In addition, he made theoretical and simulation-based arguments indicating that the reverse is also approximately true; that is, if \( X(z) \) is fractal with a spectral density proportional to \( \omega^{-\gamma} \), then the coefficients \( X_j^m \) will be approximately uncorrelated with the variance given by equation (4.77). This means that a plot of \( \ln(\text{Var}[X_j^m]) \) versus the scale index \( m \) will be a straight line. Fenton (1999) used a fifth-order Daubechies wavelet and plotted the estimated wavelet coefficient variances \( \hat{\sigma}_m^2 \), in which:

\[ \hat{\sigma}_m^2 = \frac{1}{2^{m-1}} \sum_{j=1}^{2^{m-1}} (x_j^m)^2 \]  \hspace{1cm} (4.78)

against the scale index \( m \), for both the finite-scale and fractal cases. It was found that the plots for a fractal simulation led to a straight line, as expected through Wornel’s theorem, whereas the plots for finite-scale simulations showed a slight flattening of the variance at lower values of \( m \) (larger scales). Fenton (1999) concluded that the wavelet coefficient variance plot had some potential in identifying an appropriate stochastic model (i.e. finite-scale or fractal).
4.5.6 Periodograms

The periodogram is an estimate of the true spectral density function of a random process and is obtained by first computing the Fourier transform of the data,

\[ \chi(\omega_j) = \frac{1}{n} \sum_{k=0}^{n-1} X(t_k) e^{-j\omega_j k} \]  

(4.79)

at each Fourier frequency \( \omega_j = 2\pi j / D, j = 0, 1, \ldots, (n - 1)/2 \). It is then given by the squared magnitude of the complex Fourier coefficients according to:

\[ \hat{G}(\omega_j) = \frac{D}{\pi} |\chi(\omega_j)|^2 \]  

(4.80)

in which \( D = n\Delta z \). Beran (1994) concluded that, for stationary processes which have finite variance, the above periodogram estimates are independent and exponentially distributed with means equal to the true one-sided spectral density \( G(\omega_j) \). Vanmarcke (1984) also showed that, if \( D = n\Delta z \) is finite, then the periodogram has a non-zero scale of fluctuation equal to \( 2\pi / D \), which implies a serial correlation between periodogram estimators. However, since the periodogram estimates at Fourier frequencies are separated by \( 2\pi / D \), they are approximately independent according to the physical interpretation of the scale of fluctuation. Moreover, Yajima (1986) has shown that this dependence and distribution is true for both fractal and finite-scale cases. Knowing the distribution of the periodogram estimates, it is possible to carry out both maximum likelihood estimates and hypothesis tests on the data.

One way to determine whether the data are fractal in nature or not, is to look directly at the plot of the periodogram. For fractal processes, spectral density functions of the form \( G(\omega) \propto \omega^{-\gamma} \) exist in which \( \gamma > 0 \). Hence, \[ \ln G(\omega) = c - \gamma \ln \omega, \] for some constant \( c \), and therefore the log-log plot of the sample spectral density function of the fractal processes will be a straight line with a slope of \( -\gamma \). Having determined whether the data are fractal or finite-scale, the related parameters may then be calculated. In the case of finite-scale processes, the parameter of interest is the scale of fluctuation, whereas, for fractal processes, the parameter of interest will be the spectral exponent \( \gamma \), or, equivalently for \( 0 \leq \gamma < 1 \), the self-similarity parameter \( H = (1/2)(\gamma + 1) \).
4.5.7 Resampling Techniques

The basic idea in resampling techniques is to generate new data sets numerically by randomly sub-sampling the available data, and then computing estimators by summarising these samples’ variability. Among the many resampling techniques, the bootstrap method has found many attractions in various fields ranging from time-series modelling to seismic hazard analysis and recently in geotechnical engineering, due to its accuracy and the fact that it can be applied to both parametric and non-parametric statistical analyses of data.

Ever since the application of statistical methods for characterizing soil variability, the problem of encountering small sample sizes has been evident. Resampling techniques are very useful in overcoming this problem. Uncertainty attached to statistical estimators of soil properties such as the mean, standard deviation, cross-correlation and spatial auto-correlation lowers the confidence level in reliability computations. On the other hand, if the uncertainty in the estimators is not assessed, the confidence levels remain unknown. Attempts have been made earlier than 2005, in order to address this issue: for example, by formulating optimal estimators (Baecher, 1981), by estimating maximum likelihood autocovariance function (DeGroot & Baecher, 1993) and by using Bayesian updating technique (Zhang et al., 2004).

The bootstrap resampling technique was first proposed by Efron (1979a, 1979b) as a versatile computational method that could be combined with traditional deterministic equations to perform fast and accurate calculations of typical statistics, such as the mean, standard error and confidence limits. One of the most important properties of the bootstrap method is its applicability to both parametric and non-parametric problems. Bourdeau & Amundaray (2005) have applied non-parametric bootstrapping to investigate how it can affect the uncertainty associated with statistical characterisation of the data. One of the benefits of non-parametric methods is that they do not need any assumption about the population distribution. Hence, these methods eliminate the inaccuracies caused by losing valuable information about a population when a specific distribution is assumed.
Here a brief summary of non-parametric bootstrapping is given, with reference to Bourdeau & Amundaray (2005).

The term *sample* is defined as a collection of *n* measurements without regard to order, drawn randomly from a population with an unknown distribution *F* and is written as

\[ A = \{ y_1, y_2, \ldots, y_n \} \]

The *y* <sub>i</sub> s can be regarded as *n* IID measurements of the *n* independent components of the random vector \{ *Y*<sub>1</sub>, *Y*<sub>2</sub>, . . . *Y*<sub>n</sub> \}. The resample or bootstrap \( A^* = \{ y_1^*, y_2^*, \ldots, y_n^* \} \) is an un-ordered collection of *n* items drawn randomly from *A* with replacement, so that each \( y_i^* \) has the same probability, 1/\( n \), of being equal to any of the \( y_j \) measurements, that is

\[
P(y_i^* = y_j | A) = 1/n, \quad 1 \leq i, j \leq n
\]

(4.81)

The \( y_i^* \) s are also IID, but conditional on *A*. \( \hat{F} \) is the *sample distribution* from which the \( y_i^* \) s are drawn. It can be noticed that, from an original sample *A*, there are a large number of independent resamples \( A^* \) of the same size that can be drawn repeatedly. For each individual resample, statistics can be computed in order to obtain estimates of the unknown population parameters. Then, by summarizing these simulation results, bootstrap approximations are obtained empirically for the parameter estimators, as well as for their standard errors and confidence intervals. Increasing the number of resamples will increase the accuracy of the estimates.

As an example, let’s consider \( \theta(Y) \) as a parameter of interest, which has an estimate of \( \hat{\theta}(y) \). In order to see how accurate this estimate is, the standard error of \( \hat{\theta}(y) \), denoted by \( Se[\hat{\theta}, \hat{F}] \), needs to be found. In most cases it is not possible to simply determine \( Se \). However, by means of a Monte Carlo algorithm, it is relatively easy to evaluate the bootstrap estimate, \( Se[\hat{\theta}, \hat{F}] \), of the standard error for a large number of resamples. Bourdeau & Amundaray (2005) suggested an algorithm which performs the following sequence of tasks:

(a) From the original sample *A*, draw a large number, *M*, of resamples \( A^*(1), A^*(2), \ldots, A^*(M) \) using a random number generator. The random number generator should follow a uniform distribution in order to ensure that, for any
draw, every data measurement \( y_i \) has the same probability of being chosen with repetition being allowed.

(b) For each bootstrap sample \( A^* (j) \), where \( j = 1, 2, \ldots, M \), the statistics of interest \( \hat{\theta}^* (j) = \hat{\theta}^* [A^* (j)] \) are estimated.

(c) The expected value and standard error of \( \hat{\theta}^* (j) \) are computed, for \( j = 1, 2, \ldots, M \), as:

\[
E[\hat{\theta}^*] \approx \frac{\sum_{j=1}^{M} \hat{\theta}^* (j)}{M} \quad (4.82)
\]

\[
Se[\hat{\theta}^*] \approx \sqrt{\frac{\sum_{j=1}^{M} (\hat{\theta}^* (j) - E[\hat{\theta}^*])^2}{M-1}} \quad (4.83)
\]

As the number of bootstrap simulations increases (\( M \to \infty \)), the above two equations become more accurate and hence the bootstrap results will improve. Standard errors can be used to obtain approximate confidence intervals of the unknown parameter \( \theta \), that is

\[
\theta \in \hat{\theta} \pm Se[\hat{\theta}^*].z^{(1-\alpha)} \quad (4.84)
\]

in which, assuming a normal distribution for the bootstrap estimator \( \hat{\theta}^* (j) \), \( z^{(1-\alpha)} \) would be the (100.\( \alpha \)) percentile point of the standard normal variate. For example, for a confidence level of 95%, \( z^{(0.95)} \) would be 1.96. Using the idea of quantiles, confidence intervals can be determined without assuming bootstrap estimators following a particular distribution function. Quantiles are estimated by finding the values that bound \((\frac{1}{2} \alpha.100\%)\) and \(((1-\frac{1}{2}).100\%\) of the data. Efron & Tibshirani (1986) found that, in practice, the number of bootstrap simulations which are adequate for the estimation of the standard error falls in the range \( M = 50 \) to 200, whereas reasonable confidence intervals can be estimated for a number of simulations in the range \( M = 1000 \) to 2000.

There are several potential applications of bootstrapping, in addition to the analysis of moment estimators of soil properties, such as the mean and variance. These include: finding the correlation between soil properties, application to the estimation of the autocorrelation functions, and engineering design and reliability analysis.
4.5.8 Adopted Method of Characterizing the Spatial Variability

It was mentioned earlier in this section that characterizing the spatial variability within a domain needs not only knowledge about the point statistics of the material properties, but also knowledge of how the data within the domain are correlated in space. It was also mentioned that estimating the autocovariance and autocorrelation functions can be helpful in gaining such knowledge for the domain being dealt with. Moreover, alternative methods and concepts regarding the spatial characterization of the data were given, each having their own advantages and disadvantages. For example: variograms need less assumptions regarding the stationarity of the data, whereas autocovariance estimation requires the data to be stationary; for domains which are large enough and seem to have properties which repeat themselves over the domain, the domain can be subdivided and the concept of fractals be used in order to express the spatial variability within that domain; if there is not enough data available about the domain, resampling techniques can be used to generate new data sets that can reasonably be regarded as extra data for the domain.

The different techniques used for spatial characterization were summarised in Sections 4.5.1 to 4.5.7 and comparing their performance could be the subject of additional research. However, in this research it is needed to adopt a method which can characterize the spatial variability of the site being studied to a good degree of approximation while maintaining accuracy. Therefore, due to the domain properties (which does not repeat itself to apply the concept of fractals) and the data set available (which is large enough not to require resampling techniques) in this case study, the spatial variability of the site can reasonably be estimated by employing the concept of the Scale of Fluctuation. This is useful, because the scale of fluctuation can be linked to random field theory later in Chapter 6.

The scale of fluctuation can be different in every direction in space. However, estimation of the vertical scale of fluctuation has proved to be easier than estimating the horizontal scales of fluctuation, since the latter requires closely-spaced CPT profiles and the process cannot be automated (whereas it can be for the vertical scale of fluctuation). Note that the horizontal scales of fluctuation are usually greater in value than the vertical scale of fluctuation (which usually varies between 0.3m to 3.0m) due to the process of deposition. Moreover, if the scales of fluctuation are large in comparison to the domain size, the domain can be regarded as more uniform due to lower degrees of spatial variability, and vice versa.
In order to calculate the scale of fluctuation for the Jamuna River CPT data, equation (4.67) needs to be recalled and the second part of that equation be rearranged to give

\[ \theta \equiv T \Gamma^2(T) \quad \text{for} \quad T \to \infty \] (4.85)

in which, as mentioned earlier in Section 4.5.3, \( T \) is the averaging distance and \( \Gamma^2(T) \) is the variance function which can be numerically calculated as follows:

\[ T = (n - 1)dy \] (4.86)
\[ \Gamma^2(T) = (\sigma_T/\sigma)^2 \] (4.87)

In these equations, \( dy \) is the distance between the data points, \( \sigma \) is the standard deviation of all data points in the CPT profile and \( \sigma_T \) is the standard deviation of the local averages based on the averaging distance \( T \). Wong (2004) suggested adding a constant value to equation (4.86) in order to account for the lag produced by the measurement method and recommended a value of 0.1m for this purpose. For a total of \( N \) data measurements in a state parameter profile, then their local average (\( \psi_n \)) and variance (\( \sigma_{n}^2 \)) for every set of \( n \) data points can be calculated through the equations below:

\[ \psi_n(k) = \frac{1}{n} \sum_{i=k \cdot \frac{n-1}{2}}^{k \cdot \frac{n-1}{2} + \frac{n-1}{2}} \psi_i \] (4.88)

\[ \sigma_{n}^2 = \frac{1}{N - n + 1} \sum_{k=1+\frac{n-1}{2}}^{N-\frac{n-1}{2}} \psi_n^2(k) \] (4.89)

\[ \psi_n(k) = \frac{1}{n} \sum_{i=k \cdot \frac{n}{2}}^{k \cdot \frac{n}{2} + \frac{n-1}{2}} \psi_i \] (4.90)

\[ \sigma_{n}^2 = \frac{1}{N - n + 1} \sum_{k=1+\frac{n}{2}}^{N \cdot \frac{n}{2} + \frac{n}{2} + 1} \psi_n^2(k) \] (4.91)

in which equations (4.88) and (4.89) are used when \( n \) is an odd number and equations (4.90) and (4.91) are used when \( n \) is an even number.
Campanella et al. (1986) proposed a method for evaluating the scale of fluctuation and this has been adopted in this thesis. In this method, the scale of fluctuation is evaluated for every profile by first finding the standard deviation $\sigma$ for all $N$ data points in the profile. Then, using equations (4.88) to (4.91), $\psi_n$ and $\sigma_n^2$ are computed for $n = 1, 2, 3, \ldots$ keeping in mind which equations should be used, depending on whether $n$ is an even number or an odd number. Then, for any $n$ value, $T$ is calculated using equation (4.86) and accounting for the lag effect. The variance function and corresponding $\theta$ can then be calculated using equations (4.87) and (4.85), respectively. Finally, a graph of $T$ vs. $\theta$ is plotted (which is usually an arch-shaped curve) and the peak point of the curve will be taken as the scale of fluctuation for that particular state parameter profile, keeping in mind that by definition of the scale of fluctuation given by equation (4.85).

Having described all the concepts and methods which will be used to first interpret the CPT data into state parameter and then characterizing the site, Section 4.6 describes how the Jamuna Bridge data have been interpreted and the spatial variability worked out.
4.6 Data Interpretation and Characterization

Using the NorSand parameters calibrated in Chapter 3, the method of CPT interpretation described in Section 4.2.3 can be used to obtain the values of state parameter for each soil profile. The statistics of the interpreted data may then be determined using the techniques described in Section 4.4 and the method described in Section 4.5.8 may be employed to derive the scale of fluctuation for each CPT profile (and also for the whole site). These procedures are carried out using a computer code written in MATLAB (developed by Wong (2004) and Gitman (2006), and further developed by the author). Before describing the code, it is necessary to give some details of the Jamuna Bridge site and the history of its construction, as this information will be helpful for understanding the calculations.

The Jamuna River, together with the Ganges and Padma Rivers, constitute a system of rivers which divide Bangladesh into East and West zones, and the west part of the country into North and South parts. Hence, in order to connect the East and West parts of the country, the construction of a multi-purpose 4.8km bridge at the middle reaches of the Jamuna River, 110km northwest of Dhaka (as illustrated in Figure 4.8), was planned and constructed during 1995 to 1999. A major consideration in the bridge design was that the Jamuna River is a shifting braided river, in which the river bed consists of several channels which change their course and width significantly during the seasons. Therefore, in order to train the river to flow under the bridge corridor, it was necessary to construct guide bunds on both sides of the river to act as bridge abutments and provide some countermeasure against the effects of scouring.

Figure 4.9 illustrates the horseshoe-shaped guide bunds which are located on both sides of the river. The construction of these guide bunds involved excavating the riverbed, by dredging the sand which had been deposited by the river in the past, and then placing erosion-protecting materials such as stones or geotextiles over the underwater permanent slopes, as illustrated in Figure 4.10 which is a typical cross-section through the West Guide Bund. It can be seen from this figure that the trench, which was dug by means of cutter-suction dredgers operating from the water above, has a depth varying from 22m to 30m. The focus of the site characterization in this research is the West Guide Bund, which was constructed first and experienced slides at different locations during its construction. Many in-situ and laboratory tests were carried out after the slides occurred in order to investigate the cause of the slope instability and also to modify the design of the East Guide Bund.
As can be seen in Figure 4.9, the West Guide Bund was constructed on a recently formed sand island of young, rapidly deposited sediments. Figures 4.11 and 4.12 show a detailed plan view of the excavation and a typical cross-section in the EW direction through the dredged channel. Figure 4.11 also contains the locations of the slides and the CPT tests which were carried out on the West Guide Bund. When designing this structure, it was decided to treat the western slope as a permanent slope, due to the position of the bridge abutment being on its west side, and to use protection materials on the slope surface to prevent scouring and erosion; it was also designed to have a gentle slope of 1:5.0 in the middle as illustrated in Figure 4.12. In contrast, the slope on the east side of the trench was designed to be a temporary slope, with no protection on its surface and free to be washed away over time with no effect on the west side of the trench or the bridge abutment. Hence, the temporary slope was designed to be steeper than the permanent slope, with an angle of 1:3.0 as seen in Figure 4.12.

In October 1995, the dredging work started from the southern rim of the sand bar in a northwards direction and proceeded until the first slide occurred in the permanent slope on November 19th at cross-section 1270. This was followed by another slide on November 22nd at cross-section 1410; the two slides are shown in Figure 4.11 by W1 and W2, respectively. However, the largest slide occurred on December 3rd at the location of chainage 1550, denoted by W3 in Figure 4.11, and covered an area that was 150m wide and 150m long. Cross-sections through this slide at different locations are given in Figures 4.13 to 4.16. After these slides, many other slides occurred in the temporary slope during 1996. These are denoted by WT6, 7, 9E, 13, 15, 16, 17, 18, 22 and 24, in Figure 4.11. As the construction continued, it was decided to avoid slope failures by changing the angles of the slopes from the initial design. Hence, the angle of the bottom part of the permanent slope was changed from 1:3.5 to 1:6.0 and the angle of the temporary slope was changed from 1:3.0 to 1:5.0, as illustrated in Figure 4.12. After this change in design, the excavation process continued to full depth. Although more slides took place after this stage, they only occurred in the temporary slope and mostly during the period of March to June of 1996. The locations of the slides, before and after this change of design, are illustrated in Figure 4.11.

After the occurrence of the slides during the excavation of the West Guide Bund, extensive in-situ and laboratory tests were performed in order to investigate the nature of the slides. Some of these tests were previously studied in Chapter 3 and summarized
in Table 3.3. In particular, the triaxial tests that were looked at in Chapter 3 were all performed after the slope failures using tube sampling techniques. However, due to these samples being highly disturbed, the results of the triaxial tests could not have been truly representative of the in-situ conditions and therefore more in-situ tests needed to be carried out. It should be mentioned that, during the feasibility study stage of the design, CPT tests at 13 locations along the river bed had been carried out, as indicated in Figure 4.17 by C1 to C13. However, these data were not available to the author and hence were not considered in the site characterization. However, after the failures took place, 22 more CPT tests were conducted along the shoulders of the West guide bund. These included C330W, C310W, C270WRD, C230W, C210W, C190W, C170W, C150WRD, C140W, C130W, C110W and C070W, which were in the permanent sand bar, and C330E, C310E, C290E, C250E, C230E, C190E, C150E, C130E, C110E and C090E, which were in the temporary sand bar and are illustrated in Figure 4.11. These data were made available to the author and have been used in order to perform statistical analyses for working out the state parameter and its characterization.

Before numerically interpreting the CPT data and working out the statistics and variability, it should be mentioned that extensive mica-content tests at the Jamuna Bridge Site indicated that the sand consists of high amounts of mica (20-30%), as illustrated in Figure 4.18. Therefore, many investigators have tried to relate the slope failures to the mica-contents of the soil, including Hight et al. (1999) who compared the behaviour of clean silica sand with the same sand containing 1% mica. They concluded that the presence of mica in the sands would change the sand behaviour from ductile to brittle, that there would be a potential for samples with 1% mica to collapse at medium strains and that this would lead to smaller residual strengths at large strains, as illustrated in Figure 4.19. Moreover, Ishihara (2008) also studied the slides, by considering the triggering causes of the failures and their consequences, separately and concluded that the presence of mica in the Jamuna River sand would lead to a contractive behaviour which would eventually cause slope instabilities. However, for the research in this thesis, the effect of mica-content has not considered when investigating the slope failures; rather, the link between soil behaviour (i.e. contractive or dilative) and state parameter (including its spatial variation across the site) has been investigated. Section 4.6.1 explains the results of characterizing the Jamuna River Site in terms of state parameter.
4.6.1 Jamuna River Sand Statistical Characterization

It was mentioned in the previous section that a MATLAB code has been used to interpret the CPT data and calculate the point statistics and scale of fluctuation. The code, which consists of 1 parent file and 4 function-files called from inside the parent file, involves the following calculation steps:

1. The soil parameters are input (i.e. the NorSand parameters and soil properties such as $\gamma_{sat}$, $\gamma_d$, $K_0$, etc.).
2. For each CPT profile, the total and effective vertical stresses, $\sigma_v$ and $\sigma'_v$ are computed at every measured point in the soil profile, accounting for the geometry of the problem (i.e. free water surface).
3. The mean total and effective stresses, $p$ and $p'$, for each data point are calculated using equations (4.7) and (4.8), respectively.
4. $q_t$ and $Q_c$ are calculated using equations (4.5) and (4.4), respectively.
5. The shear modulus for each data point is calculated using equation (3.21).
6. Functions $f_1$ to $f_{12}$ in Table 4.1 are calculated for each data point, and $k$ and $m$ are formed using equations (4.13) and (4.14).
7. State parameter is calculated for each data point using equation (4.1).
8. Data shortening is applied for every CPT profile, if required.
9. The mean and standard deviation of the state parameter are calculated for every CPT profile.
10. The data points are divided into bins or classes for every CPT profile and histograms for the data are formed. Then, using the maximum likelihood estimation method, 4 different distributions are fitted to the data and, using the chi-square goodness-of-fit test, it is worked out which distribution function best represents the data. The number of bins is here chosen to be 30, which, based on the author’s previous experience, leads to smoother PDFs and also makes it easier to distinguish the presence of layering (i.e. the occurrence of multi-modal distributions).
11. The data are normalised for every CPT profile and the depth trend is removed using equation (4.15).
12. The scale of fluctuation is worked out for each trend-removed profile using the method described in Section 4.5.8.
13. Finally, all the CPT profiles are assembled into just one file and steps (8) to (10) are repeated.
Starting with the first step in the process, the NorSand parameters are those which were calibrated in the previous chapter and presented in Table 3.10. However, in that table (and as mentioned in Section 3.5.4) the plastic hardening modulus $H$ was not measured as a constant value and, therefore, some iterations were needed to work out the best value. Specifically, an initial estimate for $H$ was chosen and the mean state parameter value for all the data was then calculated using the computer. This value of the mean state parameter was then compared with the value which could be found using equation (3.31) and, after some iterations, it was found that by choosing $H=142$ the calculated mean state parameter would be the same as that calculated by equation (3.31). These iterations are presented in Table 4.2. Apart from this parameter, the rest of the parameters have the same values as presented in Table 3.10. Other parameters which were derived from the Fugro reports (1986 & 1996) are related to the soil, i.e. $\gamma_{sat}=18.9$ kN/m$^3$, $\gamma=18.5$ kN/m$^3$ and $K_0=0.44$. One last parameter is the tip area correction factor, $\alpha$, which, from the Fugro reports (1986 & 1996), was found to be 0.6 when using Dutch cones.

In the second step of the process, the vertical stresses at every point in the soil profile need to be calculated. As mentioned in Section 4.6, the 22 CPT profiles have been obtained from the shoulders of each side of the excavation and, therefore, they were located on the sand bars. This means that, for each soil profile, there is a section above the free water surface and a section below the water surface as shown schematically in Figure 4.12. Furthermore, the figure shows that the water surface has a rise and fall varying between $+13$ m PWD and $+6.2$ m PWD, which gives an average water level of $+9.6$ m PWD. The figure also shows that the sand bar level for the permanent slope is $+16.5$ m PWD, whereas for the temporary slope it varies between $+10$ m and $+12$ m PWD. Hence, on average the amount of the soil profile above the water surface is 6.9 m and 1.4 m for the permanent and the temporary slopes, respectively. Denoting this amount by $d$ and the depth below the soil surface for each data point by $A(I,3)$, the total vertical stress can be calculated as follows (in which the matrix $A$ contains the measurements in each CPT profile):

$$\sigma_v = \begin{cases} \gamma . A(I,3) & , A(I,3) < d \\ \gamma . d + \gamma_{sat} . (A(I,3) - d) & , A(I,3) \geq d \end{cases}$$

(4.92)

Also, because the data with which the author had been provided, included total pore pressure measurements (i.e. hydrostatic plus excess pore pressures), the vertical effective stresses can be easily calculated by deducting $u_{total}$ from $\sigma_v$. The rest of the
analysis steps are automatically done within the code and no further comments are needed. Therefore, the next section moves on to discuss the results.

### 4.6.2 Discussion of the results

The MATLAB code was run for the 22 CPT profiles of the West Guide Bund and the results are summarized in Appendix A. For each CPT profile, a single-sheet representation of the evaluated data has been prepared, as shown in Figures A.3 to A.24. Each sheet includes the tip resistance, pore pressure, sleeve friction, friction ratio and state parameter profiles for a particular cone penetration test. Also included is a graph of the vertical scale of fluctuation versus averaging distance, calculated using the method described in Section 4.5.8, as well as histograms and fitted distribution functions of state parameter and tip resistance for de-trended and trended data. Each sheet also includes a table summarising the state parameter statistics. Note that the statistics and scales of fluctuation represented on these sheets are not for the whole profile depth. For each profile, in order to analyse a more uniform soil layer, some data near the soil surface have been omitted. The sections for which the statistics have been derived are represented by the lengths of the trend lines shown on the state parameter profiles.

The appendix also includes histograms of state parameter and tip resistance for the data from all the 22 CPTs combined, as shown in Figure A.1 (with exclusion of the omitted section of each layer). For each histogram, the fitted PDFs are also illustrated along with their respective chi-square values. In addition, histograms of the best-fit distributions for state parameter and tip resistance for each CPT are shown in Figure A.1(c). Finally, a histogram of the vertical scales of fluctuation across the site is shown in Figure A.2.

The results in Appendix A are summarised in Table 4.3. In this table, the point statistics for each CPT profile (for both before and after removing the depth trend) are presented, together with the vertical scales of fluctuation. As mentioned in Section 4.5.3, the concept of scale of fluctuation can only be applied to stationary data. Hence, for calculating the scales of fluctuation, any depth-trend should first be removed from the data by using the method described in Section 4.3.1.
As can be seen from this table, the mean state parameter for all the 22 CPT profiles is $\mu_\psi = -0.055$, with the standard deviation being $\sigma_\psi = 0.07$. Note that this mean value has been used for the comparisons in Table 4.2 to determine the value of the plastic hardening modulus $H$, described in the previous section. The mean value for the tip resistance was found to be $\mu_{Q_c} = 8.292 \, MPa$ with a standard deviation of $\sigma_{Q_c} = 4. \, MPa$. Removing the depth trend from the data will lead to the mean values to become zero and the standard deviations to decreasing to 0.06 and 2.8MPa for the state parameter and the tip resistance, respectively. The mean value of the state parameter indicates a loose to mildly dilating sand which is consistent with the assumption made in the Fugro reports (1986 & 1996) for the Upper Sand Layer. However, this layer is very heterogeneous, as indicated by the large standard deviation and also by the vertical scales of fluctuation being small (varying between 0.5m to 2.4m) relative to the Upper Sand Layer domain size of about 30m in depth.

Figures 4.20 and 4.21 show plan views of the West Guide Bund with the mean state parameter and scale of fluctuation shown at the location of each CPT test, respectively. As can be seen, the mean state parameter values across the site all indicate a loose to mildly dilating sand (i.e. $0.0 < \mu_\psi < -0.1$) and there are only relatively small variations in the values of $\mu_\psi$. In contrast, the scales of fluctuation show more variation over the site. In particular, the horseshoe shaped guide bund can be divided into two areas, based on the values of the scale of fluctuation. The northern part of the horseshoe has values which are predominantly around 1.0m (except at location C310W) with an average of about 0.8m. The middle straight part of the horseshoe has values which are predominantly bigger than 1.0m and have an average of about 1.6m. Keeping in mind that smaller values of the scale of fluctuation are beneficial to stability, due to reducing the probability of semi-continuous loose zones being formed (Hicks & Onisiphorou, 2005), and remembering that the largest slide during the excavation occurred in the middle part of the horseshoe-shaped guide bund (at W3), it can be clearly understood why the largest slide has occurred at that location.

As can be seen from the chi-square values in Figures A.1(a) and (b), and also from the summary in Table 4.3, the state parameter for the whole site can best be represented by a Normal distribution, both before and after removing the depth trend, which is consistent with what has previously been found for other sands (Gitman & Hicks, 2006; Bakhtiari, 2006). On the other hand, the tip resistance for the whole data can best be represented by a Beta distribution, both before and after trend removal.
Note that the best-fit distribution function is not only the one with the smallest chi-square value, but is also the one which can pass the statistical significance test of the null hypothesis (also indicated by $h_0$). The null hypothesis in this case is the suitability of a distribution function to represent the state parameter and/or tip resistance, and the significance test is to either accept or reject $h_0$. As can be seen from the figures, the accuracy of the goodness-of-fit test has been chosen to be 95% which is very reasonable for most practical purposes. This level of accuracy will lead to a significance level of 5% and since the degree of freedom, when choosing 30 bins to classify the data, is 27, the corresponding target chi-square value is easily found from statistical textbooks to be $\chi^2_{0.05,27} = 16.151$. Comparing this value with the calculated chi-square values, one can tell whether the distribution function with the smallest $\chi^2$ value can pass the $h_0$ test and be a suitable representation of the data or not. As it can be seen from Figures A.1 (a) and (b), most of the calculated $\chi^2$ values are smaller than $\chi^2_{0.05,27}$ and therefore pass the test and can be regarded as suitable. However, for the rejected cases, the test can be only passed if the accuracy level is reduced from 95% to 89% because $\chi^2_{0.11,27} = 18.425$ and the calculated $\chi^2$ for the Normal distribution is 18.451. In contrast to the above findings, Figure A.1 (c) illustrates that most of the individual CPT profiles follow a Normal distribution for the state parameter before and after trend removal. For tip resistance, most profiles follow a Lognormal distribution both before and after trend removal (as also seen from the last 4 columns in Table 4.3). Thus, due to the logarithmic relationship between $\psi$ and $q_c$ (i.e. equation 4.1), it is more reasonable to assume that a Normal distribution best represents the variations in state parameter and a Lognormal distribution can best represent tip resistance variations.

Figure A.2 illustrates how the vertical scales of fluctuation are distributed across the site. It can be seen that the average scale of fluctuation across the site is 1.18m with the standard deviation being 0.6m.
The discussion around Figures A.3 to A.24 is now the focus of this section and the issues relating to these figures are presented in the bullet points below:

- Due to the large number of data measurements for all the CPT profiles, data shortening was found to be necessary in order to reduce the analysis time. As mentioned earlier in Section 4.3.2, Popescu (1995) suggested that shortening the data over an interval of 5 to 10cm does not affect the statistics and hence it was chosen to shorten the data by 10cm intervals. However, for calculating the scales of fluctuation in this thesis, data shortening was not applied, in order to get the most accurate values for each profile.

- The figures show that most of the state parameter profiles have depth trends indicating a decrease in the value of the state parameter with depth (i.e. it becomes more negative as the depth is increased). This is indicative of a greater tendency for the sand to dilate at higher stress levels.

- Since the state parameter follows a Normal distribution before removing the depth trend and knowing that, for a Normal distribution, 95% of data lie between \( \mu \pm 2\sigma \), it can be deduced that the state parameter ranges between \(-0.175 \leq \mu_{\psi} \leq 0.065\) across the site, which is consistent with material ranging from very dense to highly contractive. The range may be compared with the range of \(-0.3 \leq \psi \leq 0.05\) previously observed by Shuttle & Jefferies (1998) for different types of sands.

- Data filtering was not performed before analysing the data and the presence of some very thin layers of other materials can be observed in some of the profiles: in particular, profiles C230W, C110W, C330E, C250E and C190E. Hence, these profiles have been filtered and reanalysed. Figure 4.22 illustrates these profiles, before and after filtration, and Table 4.4 summarizes the results of the reanalysed profiles. It is observed that, although the mean values for the individual profiles have decreased (i.e. have become more negative), the mean state parameter for all profiles has not changed a lot (within an absolute tolerance of 0.001). On the other hand, the values of the scales of fluctuation have become bigger for most of the profiles, which is reasonable because filtration will lead to more gradually varying profiles.
• As mentioned earlier in this section, the layering phenomenon was not considered in presenting the results in Appendix A; rather, these results have been obtained by assuming that the Upper Sand Layer does not include any sub-layers and only the top part of each profile was omitted in order to get more uniform results. However, the histograms of state parameter and tip resistance indicate that most of the profiles have multi-modal characteristics (i.e. there is more than one apex in the histogram). Hence, there is a need to divide the Upper Sand Layer into sub-layers which are more uniform and which have a more mono-modal behaviour. Figure 4.23 shows the state parameter profiles for all the 22 profiles combined together. It is seen that the Upper Sand Layer can be divided into 4 sub-layers which are presented in Table 4.5 together with their point statistics. These layers are not identifiable at those depths stated in Table 4.5, when looking at each profile individually. However, by considering all the profiles and looking at the whole picture, one can conclude that such layering exists at these given depths.

• An important characteristic of the data, which would be needed for generating random fields in Chapter 6, is the depth dependency of mean and standard deviation. As is seen from Figure 4.23, the mean state parameter of all profiles is almost constant with depth (i.e. varying from -0.058 to -0.052). However, the standard deviation is not constant with depth and seems to be decreasing with depth. On the figure, four distinct zones are illustrated within the Upper Sand Layer and the $\mu \pm 2\sigma$ ranges are illustrated for them.

• As is seen from Figure A.2 in Appendix A, the distribution of the scales of fluctuation across the site suggests that those profiles which have scales of fluctuation of less than 0.8m, have distributions which are predominantly monomodal (e.g. CPT C190W, C330E, C250E and C230E). However, as the scales of fluctuation increase, the distributions will become multi-modal.

• It should be mentioned that each soil profile could have been divided into loose and dense zones as in Hicks & Onisiphorou (2005), in order to later investigate the effect of loose zones. However, this division in here seemed unnessary as the sand layer predominantly consists of loose to mildly dilative material, whereas in the case history investigated by Hicks & Onisophorou, the sand was dilative but it contained pockets of loose material.
4.7 Summary

In this chapter, different methods of interpreting the CPT data in terms of state parameter have been described and the reasons why the new method of state parameter interpretation has been chosen in this research have been given. Also, the different statistical concepts which are needed to statistically evaluate the interpreted data were presented, together with the different methods commonly used to evaluate the variability within the soil. It was also mentioned why the concept of scale of fluctuation has been chosen in this research to express variability. A brief description of the history of the Jamuna Bridge Site and its construction was given and the 22 CPT profiles which were related to the West Guide Bund that were used in the characterization of the site were introduced. Finally, the 22 CPT profiles were statistically interpreted using the calibrated results of Chapter 3.

It was found that the Upper Sand Layer of the site is predominantly loose to mildly dilative, and is also very variable as indicated by the high coefficients of variation and small vertical scales of fluctuation relative to the domain size. It was found that the data can be best characterized by a Normal distribution for the state parameter and a Lognormal distribution for the tip resistance. Based on the mean values of state parameter and the vertical scales of fluctuation, a possible reason was identified for the largest slide occurring in the middle part of the horseshoe-shaped West Guide Bund. It was also found that the Upper Sand Layer can itself be divided into sub-layers, which are less variable and are predominantly loose to mildly dilative, except for a thick layer of about 6.0m deep on the top surface which consists of very dense sand.
### Function Approximation

<table>
<thead>
<tr>
<th>Function</th>
<th>Approximation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1(G/p_a)$</td>
<td>$3.79 + 1.12 \ln(G/p_a)$</td>
</tr>
<tr>
<td>$f_2(M)$</td>
<td>$1 + 1.06(M - 1.25)$</td>
</tr>
<tr>
<td>$f_3(N)$</td>
<td>$1 - 0.30(N - 0.2)$</td>
</tr>
<tr>
<td>$f_4(H)$</td>
<td>$(H/100)^{0.326}$</td>
</tr>
<tr>
<td>$f_5(\lambda)$</td>
<td>$1 - 1.55(\lambda - 0.01)$</td>
</tr>
<tr>
<td>$f_6(\nu)$</td>
<td>$\text{Unity}$</td>
</tr>
<tr>
<td>$f_7(G/p_a)$</td>
<td>$1.04 + 0.46\ln(G/p_a)$</td>
</tr>
<tr>
<td>$f_8(M)$</td>
<td>$1 - 0.40(M - 1.25)$</td>
</tr>
<tr>
<td>$f_9(N)$</td>
<td>$1 - 0.30(N - 0.2)$</td>
</tr>
<tr>
<td>$f_{10}(H)$</td>
<td>$(H/100)^{0.15}$</td>
</tr>
<tr>
<td>$f_{11}(\lambda)$</td>
<td>$1 - 2.21(\lambda - 0.01)$</td>
</tr>
<tr>
<td>$f_{12}(\nu)$</td>
<td>$\text{Unity}$</td>
</tr>
</tbody>
</table>

Table 4.1 Approximate expressions for $k$ and $m$ in the new method of interpretation (Shuttle & Jeffries, 1998).

<table>
<thead>
<tr>
<th>$H$</th>
<th>$(\Psi_0)_{\text{expected}}^*$</th>
<th>$(\Psi_0)_{\text{calculated}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>-0.1751</td>
<td>-0.0262</td>
</tr>
<tr>
<td>200</td>
<td>-0.1198</td>
<td>-0.0385</td>
</tr>
<tr>
<td>150</td>
<td>-0.0755</td>
<td>-0.0467</td>
</tr>
<tr>
<td>142</td>
<td>-0.0556</td>
<td>-0.0554</td>
</tr>
</tbody>
</table>

Table 4.2 Iterations carried out to calculate the plastic hardening modulus, $H$.

* Based on equation (3.31).
<table>
<thead>
<tr>
<th>CPT Name</th>
<th>No-Trend-Removed</th>
<th>Trend-removed</th>
<th>Best-fit Distribution(^\dagger)</th>
<th>(\theta_v) (m)(^\ddagger)</th>
<th>(\psi_{\text{NTR}})</th>
<th>(\psi_{\text{TR}})</th>
<th>(\phi_{\text{NTR}})</th>
<th>(\phi_{\text{TR}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>C330W</td>
<td>-0.068</td>
<td>0.06</td>
<td>0.0</td>
<td>0.05</td>
<td>1.01</td>
<td>N</td>
<td>N</td>
<td>LN</td>
</tr>
<tr>
<td>C310W</td>
<td>-0.062</td>
<td>0.07</td>
<td>0.0</td>
<td>0.06</td>
<td>2.15</td>
<td>N</td>
<td>N</td>
<td>LN</td>
</tr>
<tr>
<td>C270WRD</td>
<td>-0.055</td>
<td>0.07</td>
<td>0.0</td>
<td>0.07</td>
<td>0.68</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>C230W</td>
<td>-0.009</td>
<td>0.09</td>
<td>0.0</td>
<td>0.08</td>
<td>1.54</td>
<td>N</td>
<td>N</td>
<td>LN</td>
</tr>
<tr>
<td>C210W</td>
<td>-0.043</td>
<td>0.07</td>
<td>0.0</td>
<td>0.06</td>
<td>0.86</td>
<td>(\beta)</td>
<td>(\beta)</td>
<td>N</td>
</tr>
<tr>
<td>C190W</td>
<td>-0.021</td>
<td>0.06</td>
<td>0.0</td>
<td>0.05</td>
<td>0.46</td>
<td>(\beta)</td>
<td>(\beta)</td>
<td>N</td>
</tr>
<tr>
<td>C170W</td>
<td>-0.030</td>
<td>0.06</td>
<td>0.0</td>
<td>0.06</td>
<td>0.63</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>C150WRD</td>
<td>-0.014</td>
<td>0.08</td>
<td>0.0</td>
<td>0.07</td>
<td>1.51</td>
<td>(\beta)</td>
<td>(\beta)</td>
<td>(\Gamma)</td>
</tr>
<tr>
<td>C140W</td>
<td>-0.036</td>
<td>0.07</td>
<td>0.0</td>
<td>0.07</td>
<td>1.64</td>
<td>N</td>
<td>N</td>
<td>LN</td>
</tr>
<tr>
<td>C130W</td>
<td>-0.056</td>
<td>0.08</td>
<td>0.0</td>
<td>0.07</td>
<td>2.00</td>
<td>(\beta)</td>
<td>(\beta)</td>
<td>(\Gamma)</td>
</tr>
<tr>
<td>C110W</td>
<td>-0.070</td>
<td>0.07</td>
<td>0.0</td>
<td>0.07</td>
<td>2.38</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>C070W</td>
<td>-0.049</td>
<td>0.06</td>
<td>0.0</td>
<td>0.06</td>
<td>1.05</td>
<td>N</td>
<td>N</td>
<td>LN</td>
</tr>
<tr>
<td>C330E</td>
<td>-0.073</td>
<td>0.07</td>
<td>0.0</td>
<td>0.07</td>
<td>0.62</td>
<td>(\Gamma)</td>
<td>(\beta)</td>
<td>LN</td>
</tr>
<tr>
<td>C310E</td>
<td>-0.094</td>
<td>0.05</td>
<td>0.0</td>
<td>0.05</td>
<td>0.95</td>
<td>(\beta)</td>
<td>(\Gamma)</td>
<td>N</td>
</tr>
<tr>
<td>C290E</td>
<td>-0.068</td>
<td>0.05</td>
<td>0.0</td>
<td>0.05</td>
<td>0.83</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>C250E</td>
<td>-0.053</td>
<td>0.06</td>
<td>0.0</td>
<td>0.06</td>
<td>0.56</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>C230E</td>
<td>-0.074</td>
<td>0.05</td>
<td>0.0</td>
<td>0.05</td>
<td>0.41</td>
<td>N</td>
<td>N</td>
<td>(\Gamma)</td>
</tr>
<tr>
<td>C190E</td>
<td>-0.069</td>
<td>0.06</td>
<td>0.0</td>
<td>0.06</td>
<td>1.28</td>
<td>N</td>
<td>N</td>
<td>LN</td>
</tr>
<tr>
<td>C150E</td>
<td>-0.044</td>
<td>0.06</td>
<td>0.0</td>
<td>0.05</td>
<td>2.15</td>
<td>N</td>
<td>N</td>
<td>LN</td>
</tr>
<tr>
<td>C130E</td>
<td>-0.085</td>
<td>0.06</td>
<td>0.0</td>
<td>0.06</td>
<td>1.11</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>C110E</td>
<td>-0.057</td>
<td>0.07</td>
<td>0.0</td>
<td>0.06</td>
<td>0.97</td>
<td>(\beta)</td>
<td>(\Gamma)</td>
<td>(\Gamma)</td>
</tr>
<tr>
<td>C090E</td>
<td>-0.073</td>
<td>0.08</td>
<td>0.0</td>
<td>0.07</td>
<td>1.20</td>
<td>N</td>
<td>(\Gamma)</td>
<td>LN</td>
</tr>
<tr>
<td>All CPTs</td>
<td>-0.055</td>
<td>0.07</td>
<td>0.0</td>
<td>0.06</td>
<td>1.18</td>
<td>N</td>
<td>N</td>
<td>LN</td>
</tr>
</tbody>
</table>

Table 4.3 Summary of the interpretation results for the West Guide Bund of the Jamuna Bridge CPT data (unfiltered).

\(^\dagger\) The vertical scales of fluctuation can only be calculated from stationary data and hence these values are obtained after removing the depth-trend.

\(^\ddagger\) N = Normal distribution, LN = Lognormal distribution, \(\Gamma\) = Gamma distribution, \(\beta\) = Beta distribution.

\(\psi_{\text{NTR}}\), refers to before depth-trend removal.

\(\psi_{\text{TR}}\), refers to after depth-trend removal.
### Table 4.4 Summary of the reanalysed data after filtration.

<table>
<thead>
<tr>
<th>CPT Name</th>
<th>No-Trend-Removed</th>
<th>Trend-removed</th>
<th>Best-fit Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu_v$</td>
<td>$\sigma_v$</td>
<td>$\mu_v$</td>
</tr>
<tr>
<td>C230W</td>
<td>-0.0408</td>
<td>0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>C110W</td>
<td>-0.1038</td>
<td>0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>C330E</td>
<td>-0.0713</td>
<td>0.08</td>
<td>0.0</td>
</tr>
<tr>
<td>C250E</td>
<td>-0.0607</td>
<td>0.07</td>
<td>0.0</td>
</tr>
<tr>
<td>C190E</td>
<td>-0.0711</td>
<td>0.08</td>
<td>0.0</td>
</tr>
</tbody>
</table>

### Table 4.5 Point statistics for the four sub-layers identified within the Upper Sand Layer of the Site.

<table>
<thead>
<tr>
<th>Sub-Layer</th>
<th>Depth Below Soil Surface (m)</th>
<th>$\mu_v$</th>
<th>$\sigma_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sub 1</td>
<td>approximately from 6.0~12.0</td>
<td>-0.0543</td>
<td>0.075</td>
</tr>
<tr>
<td>Sub 2</td>
<td>approximately from 12.0~26.0</td>
<td>-0.0545</td>
<td>0.063</td>
</tr>
<tr>
<td>Sub 3</td>
<td>approximately from 26.0~30.0</td>
<td>-0.0450</td>
<td>0.044</td>
</tr>
<tr>
<td>Sub 4</td>
<td>approximately from 30.0~35.0</td>
<td>-0.0522</td>
<td>0.036</td>
</tr>
</tbody>
</table>

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Chapter 5
Monot Calibration of the Jamuna River Sand
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Monot Calibration of the Jamuna River Sand

5.1 Introduction

In the previous two chapters, it was described how the mechanical behaviour of the Jamuna Bridge Sand can be described in terms of a more meaningful parameter, the state parameter, and how this parameter can be derived using the results of laboratory and CPT tests; it was also shown how the spatial variability of the state parameter can be characterized statistically. Chapter 4 also included a brief summary of the slides which occurred during the construction of the slopes for the West Guide Bund of the Jamuna Bridge. However, in order to analyse the slope failures by the finite element stochastic analyses, one needs to choose a constitutive soil model which is realistic and can accurately mimic sand behaviour ranging from liquefiable to strongly dilative (due to the range of state parameters derived from cone penetration tests in Chapter 4 being $-0.175 \leq \psi \leq 0.065$).

The double-hardening soil model Monot (Molenkamp, 1981) has therefore been employed in this research to model the soil behaviour in finite element computations of slope stability using random field theory (Vanmarcke, 1984) to model the soil variability. This chapter starts by describing the soil model in Section 5.2. It is then calibrated against the Jamuna Bridge data in order to derive the model parameters.

Although random field generation is the subject of the next chapter, it should be mentioned that, for modelling the spatial variability of soil, two different strategies can be employed. Note that Monot needs several material parameters to be fully defined and that, for each random field analysis (i.e. realisation), a different set of parameter values needs to be assigned to every sampling point within each element of the finite element mesh. For this purpose, one can either generate a separate random field for each parameter and then cross-correlate between the fields to account for parameter inter-dependency (e.g. Fenton & Vanmarcke, 1998); or one can generate random fields of smaller number of parameters and then backfigure other material parameters from these random fields (e.g. Popescu et al., 1997 and Hicks & Onisiphorou, 2005). In this research, the second strategy has been employed and to take it even one step further; it was decided to generate uni-variate random fields of the state parameter and then backfigure all the other Monot parameters from this single parameter.
5.2 Review of the Monot Soil Model

Monot is a double-hardening elastoplastic constitutive soil model which was developed by Molenkamp (1981) and which, as implied by its name, can be applied to problems involving monotonic loading. The model is expressed in terms of the effective volumetric and deviatoric stress invariants $s'$ and $t$, and their corresponding strain invariants $v$ and $\gamma$, defined as follows:

$$s' = \frac{\sigma'_1 + \sigma'_2 + \sigma'_3}{\sqrt{3}}$$  \hspace{1cm} (5.1)

$$t = \frac{1}{\sqrt{3}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$$  \hspace{1cm} (5.2)

$$v = \frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_3}{\sqrt{3}}$$  \hspace{1cm} (5.3)

$$\gamma = \frac{1}{\sqrt{3}}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2]$$  \hspace{1cm} (5.4)

Figure 5.1 shows the yield and failure surfaces of Monot in the $s' - t$ and $\pi$-planes. It can be seen from Figure 5.1(a) that the failure surface can have a curved shape to enable reduction in the peak friction angle for higher confining pressures. The model possesses a nonlinear elastic component and two isotropically hardening yield surfaces: a deviatoric yield surface, which has a non-associated flow-rule and is used for modelling plastic strains during shearing, with the $M'$-line indicating the transition from contraction to dilation; and a compressive cap yield surface, which has an associated flow-rule and is used for modelling plastic strains caused by an increase in confining pressure. In the following sub-sections, the basic features and formulations of the three Monot components (i.e. non-linear elastic, plastic compressive and plastic deviatoric) are briefly explained, in particular, with respect to details necessary for model calibration. However, the interested reader can find more detailed descriptions in the works of Molenkamp (1981) and Hicks (2003).

Figure 5.1(a) shows that Monot has two yield surfaces which subdivide the stress space into four distinct regions. Therefore, for a stress increment which starts from the intersection of the two surfaces, defined by point P, four different stress-strain responses are possible depending on how many yield surfaces are activated. If the stress path is into region 1, the response is purely elastic, because neither of the yield surfaces is activated. If the stress path is into region 4 will activate both yield surfaces, whereas a stress path into regions 2 and 3 will only activate the
deviatoric and cap yield surfaces, respectively. These different possible stress paths are illustrated in Figure 5.2.

Figure 5.1(b) shows that, for a given friction angle in triaxial compression, the Mohr-Coulumb failure surface lies inside the Monot failure surface. The latter failure surface is that proposed by Lade & Duncan (1975); it is based on the results obtained from cubical triaxial tests on sands (Lade & Duncan, 1973) and is therefore capable of simulating the effects of the intermediate principal stress $\sigma_2$.

Table 5.1 summarizes the 21 parameters that are used to determine Monot, based on the model component to which they apply: the plastic deviatoric component needs 16 material parameters, the elastic and plastic compressive components only need 3 and 2 parameters, respectively. Although these parameters will be discussed later in the forthcoming sections, but the table indicates the importance of the plastic deviatoric component compared to the other components. In fact, the most important feature of Monot is its ability to accurately reproduce dilative and/or contractive behaviour during shearing, particularly for the case of undrained or partially drained problems in which the excess pore pressures generated during shearing can have a significant effect on the soil strength. For very loose soils, undrained deformations result in low peak strengths which are followed by a reduction in strength that can lead to liquefaction. However, for dense soils, they will lead to increased strengths. Note that for practical purposes, many of the 21 parameters can be given default values (as will be discussed presently).
5.2.1 Non-linear elastic component

Monot uses the isotropic non-linear elastic model proposed by Vermeer (1980) which is mathematically expressed as:

\[
\{e^e\} = \frac{1}{4\mu} \left(f_{\text{function}}(V)\right)\{\sigma\}
\]  
(5.5)

in which, \(\{\sigma\}\) is the stress field and \(\{e^e\}\) the elastic strain field. \(V\) is a constant similar to Poisson’s ratio; however, it is different from the conventional Poisson’s ratio \(\nu\), due to the lateral strains in this model being a result of both uniaxial compression (i.e. \(d\sigma_x > 0 \) and \(d\sigma_y = d\sigma_z = 0\)) and the shear modulus \(\mu\), which is expressed as:

\[
\mu = \mu_0 \left[\frac{\sigma}{P_a}\right]^{n_v}
\]  
(5.6)

in which \(\mu_0\) and \(n_v\) are material constants and \(\sigma\) is a measure of the stress state and is given by:

\[
\sigma = \frac{s'}{\sqrt{3}} \left[1 + \frac{\left(1 + V\right)\left[t\right]}{\left(1 - 2V\right)s'}\right]^{0.5}
\]  
(5.7)

For the special case of isotropic stress paths, the elastic strain formulation simplifies to:

\[
\nu^e = A \left[\frac{s'}{P_a}\right]^{AP}
\]  
(5.8)

in which \(\nu^e\) is the isotropic elastic strain invariant and \(AP\) is a curvature parameter related to \(n_v\) by:

\[
n_v = 1 - AP
\]  
(5.9)

\(A\) is a scalar parameter analogous to the inverse of stiffness and is related to \(\mu_0\) by:

\[
\mu_0 = \frac{3^{(n_v/2)} \left[1 - 2V\right] P_a}{2 \left[1 + V\right] \cdot A}
\]  
(5.10)

Taking \(AP\) as 1.0 leads to a linear elastic component, in which \(V = \nu\) and \(A\) is related to Young’s modulus \(E_Y\) by:

\[
A = \frac{1 - 2\nu}{E_Y P_a}
\]  
(5.11)
5.2.2 Plastic compressive components

This component uses the spherical cap yield surface proposed by Lade (1977) to model plastic strains due to an increase in confining pressures. The cap has an associated flow rule and is only allowed to expand. It is mathematically expressed as:

\[ F_C = G_C = I_1^2 + 2I_2 - \kappa_C = 0 \]  \hspace{1cm} (5.12)

in which \( F_C \) and \( G_C \) are the yield and potential surfaces, respectively, \( I_1 \) and \( I_2 \) are the stress invariants and \( \kappa_C \) is a hardening parameter which is dependant on the radius of the cap. For the special case of isotropic loading, the plastic strains will be purely volumetric and the plastic compressive component can then be expressed in terms of the simple volumetric relationship:

\[ v^C = B \left( \frac{S{'}}{P_a} \right)^{BP} \]  \hspace{1cm} (5.13)

in which \( v^C \) is the isotropic plastic compressive strain invariant, and \( B \) and \( BP \) are scalar and curvature parameters, respectively. These two parameters are analogous to \( A \) and \( AP \) in equation (5.8).

5.2.3 Plastic deviatoric component

The plastic deviatoric component of Monot uses a non-associated flow rule which is based on Rowe’s (1962) stress-dilatancy theory and enables modelling of both contractive and dilative behaviours during shearing. Note that most of the plastic strains which develop during shearing are deviatoric and, therefore, this yield surface is the more relevant surface for modelling shearing (cf. the compressive cap yield surface which mainly models strains due to change in mean stress). Note that, unlike the cap yield surface which can expand with no limit, the deviatoric yield surface expands to a limit equal to the failure surface of the model. However, as with the compressive cap, this yield surface can only expand and thus stress paths which are within this surface are purely elastic and only those stress paths which cause it to expand lead to plastic strains.
The formulation of both the deviatoric yield and failure surfaces in the \( s' - t \) plane for triaxial compression is given by equation (5.14), which is based on the data of Tatsuoka and Ishihara (1974):

\[
\frac{t_f}{p_a} = C \left[ \frac{s'}{p_a} \right]^{CP} \tag{5.14}
\]

in which, for a given \( s' \), \( t_f \) is the value of \( t \) at shear failure, and \( C \) and \( CP \) are the scaling and curvature parameters defining the frictional strength of the soil. For very loose soils, \( CP \) becomes equal to 1.0 and equation (5.14) then represents a straight line in the \( s' - t \) plane which is analogous to the Mohr-Coulomb failure surface; \( C \) is then directly related to the peak friction angle \( \phi_{peak} \) through:

\[
C = \frac{2\sqrt{2}}{3 - \sin \phi_{peak}} \tag{5.15}
\]

or

\[
\phi_{peak} = \sin^{-1} \left[ \frac{3C}{2\sqrt{2} + C} \right] \tag{5.16}
\]

The failure surface in equation (5.14) is itself the bounding surface of a family of yield surfaces of similar shape, which are given by:

\[
\frac{t_d}{p_a} = \frac{C \cdot D \left[ \frac{s'}{p_a} \right]^{(CP+DP)}}{\left\{ C^N \left[ \frac{s'}{p_a} \right]^{CP,N} + D^N \left[ \frac{s'}{p_a} \right]^{DP,N} \right\}^{1/N}} \tag{5.17}
\]

in which \( t_d \) is the deviatoric stress invariant corresponding to the yield surface, and \( D \) and \( DP \) represent the extent and the curvature of the yield surface, respectively. However, the lower bound of equation (5.17) at very low stress levels can be expressed by:

\[
\frac{t_y}{p_a} = D \left[ \frac{s'}{p_a} \right]^{DP} \tag{5.18}
\]

and, therefore, the transition between \( t_y \) to \( t_f \), requires another yield surface parameter, \( N \), which relates \( t_y \) and \( t_f \) as follows:

\[
t_d = \frac{t_y \cdot t_f}{\left[ t_y^N + t_f^N \right]^{1/N}} \tag{5.19}
\]
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The formulation of the deviatoric yield surface in the \( \pi \)-plane, however, is identical to the relationship proposed by Lade & Duncan (1975) and is expressed as:

\[
F_d = \frac{I_1^3}{I_3} - 27 - \kappa_d = 0
\]  

(5.20)

in which \( I_1 \) and \( I_3 \) are the conventional stress invariants and \( \kappa_d \) is the deviatoric hardening parameter which is related to \( D \) and controls the size of the yield surface in the \( \pi \)-plane. Parameter \( D \) can be derived by rearranging equation (5.17) to give:

\[
D = \frac{C \left( \frac{S'}{P_a} \right)^{(CP-DP)} \frac{t}{s'}}{\left[ C^N \left( \frac{S'}{P_a} \right)^{(CP-1)N} - \left( \frac{t}{s'} \right)^N \right]^\frac{1}{N}}
\]  

(5.21)

and it varies from zero on the isotropic axis to infinity on the failure surface.

The deviatoric plastic potential surface is mathematically expressed as:

\[
G_d = G_d\{\{\sigma}\} = 0
\]  

(5.22)

in which \( \{\sigma\} \) is the stress field. Hence the plastic deviatoric strain increment \( \{d\varepsilon^d\} \), due to an increment in stress, is given by:

\[
\{d\varepsilon^d\} = d\lambda_d \left( \frac{\partial G_d}{\partial \sigma} \right)
\]  

(5.23)

in which \( d\lambda_d \) defines the magnitude of the plastic strains and \( \{\partial G_d / \partial \sigma\} \) is a vector perpendicular to the deviatoric plastic potential surface and defines the direction of the plastic strains. The dilatancy ratio is based on Rowe’s (1962) Stress-Dilatancy theory and can be expressed as:

\[
\frac{d\nu^d}{d\gamma^d} = \frac{2\sqrt{2} \sin \phi_0 - (3 - \sin \phi_0) \frac{t}{s'}}{(3 + \sin \phi_0) - 2\sqrt{2} \sin \phi_0 \frac{t}{s'}}
\]  

(5.24)

in which \( \nu^d \) and \( \gamma^d \) are the volumetric and deviatoric strain invariants for the deviatoric component of the model and \( \phi_0 \) is the mobilised friction angle corresponding to no volume change. For a constant mean triaxial stress path, \( \gamma^d \) can be simplified to:

\[
\gamma^d = E \left[ D \left( \frac{S'}{P_a} \right)^{(CP-DP-1)EP} \right] \left( \frac{S'}{P_a} \right)^{LB}
\]  

(5.25)
in which $E$ is a scalar, and $LB$ and $EP$ are curvature parameters related to isotropic stress and shear stress levels, respectively.

For loose sands and at high isotropic stress levels, $\phi_0$ is higher and tends to the no-volume-change friction angle, $FICV$. However, for dense sands and at low stress levels, the value is lower and tends to the interparticle friction angle, $FIMU$. Hence, it can be said that:

$$FIMU \leq \phi_0 \leq FICV \quad (5.26)$$

The rate at which $\phi_0$ changes between these two extremes, is defined by parameter $SCV$ through the relationship:

$$\phi_0 = FICV - (FICV - FIMU).\exp\left[\frac{s'}{p_0 \cdot SCV}\right] \quad (5.27)$$

Section 5.2 and Figure 5.1 introduced the $M^*$ line as indicating the transition from contraction to dilation. The equation of this line can be found by equating $d\nu^d/dy^d$ to zero, because, for stress states located on this line, no volume change due to deviatoric stresses will occur and thus equation (5.24) can be simplified to:

$$\frac{t}{s'} = \frac{2\sqrt{2}}{3} \sin \phi_0 = M^* \quad (5.28)$$

Those stress states which are above the $M^*$ line will indicate dilation, whereas those which are below this line will indicate contraction.

The plastic potential surface in the $\pi$-plane has a shape which can be derived from the deviatoric yield surface by utilizing parameter $RT$, which varies from zero to unity as it changes the surface from a circular shape to a shape identical to the deviatoric yield surface.
5.3 Monot calibration of the Jamuna River Sand

It was mentioned earlier in this chapter that Monot requires 21 parameters to be fully defined. However, based on the review made by Hicks (2000) on 30 previous Monot calibrations, many of these parameters are refinements which are difficult to calibrate and rarely needed for modelling soil behaviour. Hence, he suggested categorizing Monot parameters into four groups, as shown in Table 5.2. The so-called discarded parameters are never calibrated, whereas the essential parameters are always calibrated. Fringe parameters, on the other hand, are those which are hardly ever calibrated and luxury parameters are those which can easily be given default values but which are often calibrated.

For the purpose of the current research only the essential and luxury parameters have been calibrated, with the remaining parameters being given the default values from Tables 5.2(a) and (b). It is therefore assumed that $CP=1.0$ and $\phi_0 = FIMU = FICV$ and thus, for a given relative density, the failure surface and $M'$-line are assumed to be straight in the $s'$- $t$ plane. Hence, the parameters the parameters that have been calibrated in this research are the scalar stiffnesses $A$, $B$ and $E$; the peak friction angle, $C$; no-volume-change angle, $FICV$; and stiffness curvatures $AP$, $BP$ and $EP$. The last three parameters, the so-called luxury parameters, have only been calibrated to compare their values with the default values suggested by Hicks (2000).

This section starts by explaining the calibration sequence and is followed by the calibration of the required parameters against the Jamuna Bridge data. Note that a flashback to the results used in Chapter 3 for the NorSand calibration is given from time to time during this section, since both model calibrations are state-parameter-dependant; therefore, the results of one model can be useful to better understand another model and also to avoid repeating the same calculations in this chapter. The results will be discussed at the end of this section.

What should be noticed here, is that, as for the NorSand calibration in Chapter 3, the data available in the Fugro reports (1986 & 1996) do not include all the appropriate tests which are needed in the calibration process; it has therefore, been necessary to find alternative approaches to the conventional calibration methods suggested by Hicks (2003) in the upcoming sections.
5.3.1 Calibration sequence

Table 5.3 summarizes the calibration sequence proposed by Hicks (1990), in which one or two parameters are isolated at every step, then calibrated using well defined stress paths and then eliminated from the calibration process. It is easier to carry out the calibration based on the results of drained triaxial tests; but, for accurate simulations of the pore pressures, which are important in the current research, it is recommended to use undrained triaxial test data as well. Therefore, drained triaxial data were first used to calibrate the model parameters using stages 1 to 5, as in Table 5.3, and then the calibrated parameters were checked against undrained data for further adjustments if needed.

5.3.2 Calibration of $A$ and $AP$

This section starts with a description of the calibration method proposed by Hicks (1990), in which the results of unload/reload loops on isotropically consolidated triaxial specimens are used. However, since none of the triaxial data in the Fugro reports (1986 & 1996) contained these unload/reload loops, an alternative method for estimating $A$ and $AP$ was adopted. This method makes use of the basic formulation for the non-linear elastic component of Monot and the shear modulus relationship derived earlier in Chapter 3.

Hicks (1990) proposed two methods for deriving parameters $A$ and $AP$. Both methods use the unload/reload loops of isotropically consolidated triaxial tests, and employ the basic relationship for the elastic strains in equation (5.8) to estimate $A$ and $AP$. However, since for unload/reload loops the experimental data do not generally start from zero strains, the following equation is used instead of equation (5.8):

$$\Delta v^e = v^e - v^e_0 = A \left[ \frac{S^*}{p_a} \right]^{AP} - \left[ \frac{s'_e}{p_a} \right]^{AP}$$

in which $s'_e$ is the reference isotropic effective stress for the experimental data and is taken to be the value at which unloading stops and reloading begins. Equation (5.29) can be rearranged as:

$$\ln[\Delta v^e + v^e_0] = AP \ln\left[ \frac{S^*}{p_a} \right] + \ln[A]$$

(5.30)
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The first approach (as summarized in Figure 5.3) is an automated approach in which a value for \( v_e^c \) is first assumed and then, using equation (5.30), the unloading part of the experimental data are plotted in \( \ln[\Delta v^e + v_e^c] - \ln\left[\frac{s}{p_a}\right] \) space to give \( A \) and \( AP \) as the intercept and slope of the best-fit line passing through the data, respectively. These values are then put in equation (5.8) to get a new estimate of \( v_e^c \) assuming \( s' = s'_e \), and the two previous steps are then repeated to calculate new values for \( A \) and \( AP \). The iteration is carried on until \( v_e^c \) is converged.

The second approach, also called the manual approach, follows the steps summarized in Figure 5.4, in which a value for \( AP \) is first assumed and then, using equation (5.29), \( A \) is back-figured by comparing the computed and experimental unloading curves. If the computed and experimental curves match to a good degree, then the assumed \( AP \) and its corresponding \( A \) value are accepted. Otherwise, a new value for \( AP \) is assumed and the method is iterated until the two curves converge.

However, as there are no unload/reload loops available in the Jamuna Bridge triaxial data, the above two approaches cannot be used. Therefore, four different methods have been adopted in order to estimate the elastic components \( A \) and \( AP \):

Referring to Section 5.2.1, \( n_v \) (equation (5.6)), which is related to the elastic shear modulus relationship of the model, is related to \( AP \) through equation (5.9). Moreover, the second part of equation (3.21) represents the shear modulus of the Jamuna River Sand in a similar form to equation (5.6). The only difference is that the shear modulus in equation (3.21) is expressed in terms of the mean effective stress \( p' \), but, for equation (5.6) it is expressed in terms of a different stress state \( \sigma \), which is calculated through equation (5.7). Therefore, by converting the stress state in equation (3.21) from \( p' \) to \( \sigma \), it is possible to find \( AP \) using equation (5.9).

It should be noted that \( t \) and \( s' \) in equation (5.7) are related to \( q \) and \( p' \) introduced in Chapter 3. Therefore, considering that the shear modulus in equation (3.21) is calculated for the beginning of every triaxial test, \( t \) is equal to zero and hence equation (5.7) may be simplified to:

\[
\sigma = \frac{s'}{\sqrt{3}} = p'
\]  

which leads to equation (3.21) being the same as before, so:

\[
\mu = G = 63.494 \left(\frac{\sigma}{p_a}\right)^{0.6875}
\]  

(5.32)
which means that $AP = 0.31$; a value which is in close agreement with the suggested value of Hicks (2000) in Table 5.2.

Parameter $A$ can be evaluated using equation (5.10) and taking the default value of $V = 0.1$, as given in Table 5.2, and by knowing that, from equation (5.32), $\mu_0 = 63.494$. This leads to $A = 0.00083$, which is just slightly bigger than the minimum value of 0.0008 observed by Hicks (2000) from different sand calibrations.

As mentioned in Table 3.3, the Fugro reports (1986 & 1996) contain 9 isotropic compression test data. However, as illustrated in Figures 5.5a to 5.5c, only three of these test results belong to the Upper Sand Layer. These tests results can be used as an alternative to the isotropic unload/reload loops which are usually used for calibrating $A$ and $AP$. That is, the linear unloading part of the graphs have been used in combination with the automated method, to derive $A$ and $AP$ in the way illustrated in Figures 5.6 to 5.8. It is seen from the figures that a value of $AP = 0.31$ can be taken as a good estimate. However, parameter $A$ varies between tests, although its values are within the range of about 0.004 found by Hicks (2000) for different types of sands (e.g. Nerlerk & Kogyok Sands).

A relationship between parameter $A$ and the state parameter may be also established if one knows the initial state parameter for each test. But, since these tests have been carried out at initial void ratios different from those in Figures 3.20 to 3.28, and their $\Gamma$ and $\lambda$ values would not be similar to those presented in Table 3.9, it would be impossible to use equations (3.1) and (3.24) to derive their initial state parameters. However, by considering the values of the initial state parameters for each test from equations (3.24) and (3.1) and assuming $\lambda_{ss} = 0.04$ and $\Gamma = 1.03$, as derived in Chapter 3, then, $E_Y$ for each test is calculated via $E_Y = 2G(1 + \nu)$ and by recalling from Chapter 3 that $\nu = 0.23$; then, $A$ for each test can be calculated using equation (5.11) to produce Table (5.4). By plotting these values against their corresponding state parameters, it is then possible to find the $\psi - A$ relationship as in Figure 5.9, giving:

$$A = 0.0002\psi + 0.0002$$ (5.33)
which, as can be seen from the figure, gives scattered results in this case. Hence, it was decided not to use the above relationship for estimating parameter $A$. The reason that the values of $A$ found via this method are so small (i.e. a stiffer response) is that, these values have been derived using the results of the seismic body wave test and not isotropic compression test; and as mentioned in Chapter 3, the seismic body wave test represents higher strain rates compared to other methods by which the elastic shear modulus can be derived.

However, as will be described later in Section 5.3.5, formal modelling of the drained triaxial tests using a forward iterative approach similar to that in Chapter 3, showed that a value of $A = 0.004$ and $AP = 0.31$ can better reproduce the soil behaviour, since they give computed responses which fit the experimental responses to a very good degree of accuracy. Hence, it was decided to take $A$ as a constant value equal to 0.004 and therefore, all the calculations which follow from now on are based on this constant value for $A$.

### 5.3.3 Calibration of $B$ and $BP$

Calibration of $B$ and $BP$ can be done by isolating the plastic compressive component during isotropic loading. The plastic compressive strains of the experimental data can be approximated by:

\[
\Delta v^c = \Delta v - \Delta v^e
\]

(5.34)

in which $\Delta v$ represents the changes in the volumetric strains and superscripts $c$ and $e$ refer to plastic compressive and elastic components of the model, respectively. Having already calibrated $A$ and $AP$, $\Delta v^e$ can be easily computed from equation (5.8) and so:

\[
\Delta v^e = A \left\{ \frac{s'}{P_a}^{AP} - \left[ \frac{s^{'c}}{P_a} \right]^{AP} \right\}
\]

(5.35)

which is similar to equation (5.29), with the exception that $s^{'e}$ is now replaced by $s^{'c}$ which is the isotropic stress at the beginning of the triaxial test. A similar relationship to equation (5.35) can be established for $\Delta v^c$ which is expressed as:
Two approaches have been sought in this section to derive these two parameters. The first one is based on the loading part of the isotropic compression test data, earlier used in calibrating $A$ and $AP$, and applying the manual method (as illustrated in Figures 5.10 to 5.12). It is seen from these figures that, apart from sample B231-23, a value of $BP = 0.69$ can be taken as a good estimate for the curvature parameter. However, parameter $B$ varies for every sample, suggesting that there should be a relationship between $B$ and state parameter. This relationship, however, cannot be established through these test results due to reasons given in the previous section for deriving the $\psi - A$ relationship.

Another approach, suggested by the author, is to use the results of isotropically consolidated triaxial tests (presented in Figures 3.20 to 3.28). In this approach, it is assumed that the linear part of the $q - \varepsilon_1$ graphs consist mostly of elastic and plastic compressive strains with the deviatoric plastic strains being negligible. Then, a single element is modelled using MONICA, for which only the plastic compressive yield surface can be activated. The computed response is then compared with the experimental data and the process iterated until the computed and experimental data well match each other (as seen in Figures 5.13 to 5.18). As seen from the figures, the value of $BP$ is constant for each triaxial sample and is equal to 0.68, whereas the value of $B$ varies for each sample. However, knowing the state parameter value for each triaxial sample, it is possible to derive a relationship between $B$ and $\psi$, as illustrated on Figure 5.19, leading to:

$$B = 0.0043 \exp(7.5\psi)$$  \hspace{1cm} (5.37)

Note that it is this relationship which has been used later in Section 5.3.5 for formal modelling of the data. It was found that, using this relationship can lead to better fits of the computed and experimental curves presented in Section 5.3.5 and thus it can be taken as a good estimate for the current research.
Chapter 5  Monot Calibration

5.3.4 Calibration of C and FICV

It was mentioned in Section 3.5.1 that \((M_{tc})_{MC}\) in the NorSand soil model can be related to \(\phi_{cv}\). This relationship is given by (Ghazanfari & Shuttle, 2006):

\[
M_{tc} = \frac{6 \sin \phi_{cv}}{3 - \sin \phi_{cv}}
\]  \hspace{1cm} (5.38)

Hence, recalling from Chapter 3 that the average \((M_{tc})_{MC}\) is 1.35, FICV = 33.4° is obtained from equation (5.38).

It was also mentioned earlier in Section 5.2.3 that, for a linear failure surface in which \(CP = 1.0\), parameter \(C\) can be related to \(\phi_{peak}\) using equation (5.15). Hence, by estimating the value of \(\phi_{peak}\) for drained triaxial tests and employing equation (5.15), this parameter can be easily calibrated. Table 5.5 represents the values of the peak friction angle for each test, obtained by the following expression for cohesionless materials:

\[
\phi_{peak} = 2 \left[ \tan^{-1} \left( \frac{\sigma_a}{\sigma_r} \right) - 45^\circ \right]
\]  \hspace{1cm} (5.39)

in which \(\sigma_a/\sigma_r\) represents the principal stress ratio at failure and is calculated using the values of \(p'\) and \(q\) at the end of the test for each sample and knowing that \(\sigma_a/\sigma_r = (2q + 3p')/(3p' - q)\). Note that, all the 9 triaxial samples have been used for the calibration of the peak friction angle. Also, in order to avoid low estimates of this parameter for loose state parameter values (i.e. \(\psi \geq 0.0\)), it was decided to take the peak friction angle equal to FICV for positive state parameter values (as seen in Figure 5.20). This leads to:

\[
\phi_{peak} = \begin{cases} 
-15.365\psi + 33.4 & ; \psi < 0.0 \\
33.4 & ; \psi \geq 0.0 
\end{cases}
\]  \hspace{1cm} (5.40)

Hence, to calculate \(C\), the above equation should be used in conjunction with equation (5.15). Also, in order to avoid unrealistic values for \(\phi_{peak}\), \(C\) should be limited to 0.63 < \(C\) < 0.87, since \(\phi_{cv} = 33.4^\circ\) and so that \(\phi_{cv} < \phi_{peak} < 45^\circ\).
5.3.5 **Calibration of $E$ and $EP$**

A forward iterative approach, similar to that explained in Chapter 3 for calibrating the plastic hardening modulus of NorSand, was adopted in order to calibrate $E$ and $EP$. This method, which was suggested by Hicks (1990), optimizes the model so that it can closely fit the actual stress-strain behaviour of the experimental data, leading to better estimates of the model parameters. Like the rest of the parameters, the results of isotropically consolidated drained triaxial tests are used for calibrating these two parameters. However, as will be discussed later in Section 5.3.6, the calibrated parameters are also checked against undrained test results in order to make further improvements to the model parameters, where needed.

Hence, for each test, first an estimate of $EP$ is made and then, by trial and error, a value of $E$ to give the best fit of the computed response to the experimental data. $EP$ can then be changed and the process repeated until the computed and experimental responses match each other well. However, as was mentioned in Section 5.3.2, it was decided to calibrate parameter $A$ at the same time as $E$ and $EP$, in order to get better fits to the data and also due to not being able to calibrate $A$ using the approaches described in Section 5.3.2 (due to the absence of unload/reload loops in the triaxial data).

Hicks (2003) suggested comparing the $v - \gamma$ and $t/s' - \gamma$ curves, in order to compare the stress-strain behaviour of the experimental data with the computed response. Hence, the incremental computer algorithm which incorporates Monot (called MONICA and developed by Hicks, 1995a & b), was employed to model the stress-strain behaviour of a single element under isotropically consolidated drained triaxial conditions. The drained response was achieved by assuming a zero bulk modulus for the pore fluid.

Figures 5.21 to 5.29 illustrate the experimental and computed responses of all of the 9 drained triaxial tests. It should be noted that, in order to plot the experimental responses, $p', q, \epsilon_1$ and $\epsilon_v$ (which are the form of the stresses and strains presented in the actual experimental data) needed to be converted to $s', t$ and $\gamma$ using the following equations:

$$s' = \sqrt{3} p'$$  \hspace{1cm} (5.41)
As seen from Table 5.6, all tests have a constant $A$ and $EP$ value, while $E$ is different for each test. The value of $A = 0.004$ seems to be very close to the values of $A$ reported by Hicks (2003) for different types of sands in the Canadian Beaufort Sea. $EP = 3.4$ also well matches the default value of $EP$ suggested by Hicks (2000). However, $E$ can be plotted against the state parameter (as in Figure 5.30) and can be expressed as a function of the state parameter, so that:

$$E = 0.09 \exp(9.5\psi)$$  \hspace{1cm} (5.45)

Note that, a minimum of 0.005 for $E$ has also been defined in order to avoid getting an over-stiff response when calibrating the undrained data. This is based on the previous experience in calibrating Monot (Hicks, 2000). This minimum limit applies when $\psi > -0.30$ in equation 5.45, which is bigger than the minimum state parameter value derived in the previous chapter (i.e. $-0.175 \leq \psi \leq 0.065$), meaning that the above equation can be used without any risk of unreasonably small values for $E$. 

\[
t = \frac{2}{\sqrt{3}} q \hspace{1cm} (5.42)
\]

\[
\gamma = \frac{1}{\sqrt{6}} (3\varepsilon_1 - \varepsilon_v) \hspace{1cm} (5.43)
\]

\[
v = \sqrt{3}\varepsilon_q \hspace{1cm} (5.44)
\]
5.3.6 Validity of the calibration

It was mentioned in Section 5.3.1 that the parameters derived using the results of drained triaxial tests need to be checked against undrained test results, in order to examine the accuracy of the calibration and to make further adjustments if needed. As indicated in Table 3.3, the Fugro reports (1986 & 1996) contain 33 undrained triaxial test results of which 30 tests belong to the Upper Sand Layer which is the focus of this study. Figure 5.31 illustrates the stress paths of these tests in the $s' - t$ plane. As can be seen from this figure, and also from Tables 3.3 and 3.6, most of the stress paths start from anisotropic stress conditions at very low confining pressures (mostly around $p_0' = 67 \text{kPa}$). What is clear from this figure is that all the stress paths are contained within an almost linear envelope, which is indicative of the failure surface of the material. Hence, using the information about the steady state line, obtained in Chapter 3, it is possible to calculate the state parameter for each undrained test and then, based on the relationships derived earlier in this section, calculate the model parameters corresponding to each test, in order to check the validity of the calibration.

Table 5.7 represents the calculated state parameters for the 30 undrained tests on the Upper Sand Layer. The initial mean effective stresses and void ratios are the same as those reported in Table 3.6. Using this information, together with equations (3.24) and (3.1) and taking $\lambda_{ss} = 0.04$ and $\Gamma = 1.03$ (as derived in Section 3.5.3), the state parameter for each test has been derived. Then, one of the isotropic undrained samples (i.e. sample A103) was chosen as a typical example to investigate how close the calibrated parameters can model the soil behaviour. The reason for choosing an isotropic stress path was due to the simplicity involved in modelling isotropic stress paths, as compared to other anisotropic stress paths in which the confining pressures change during the test. Hence, by assuming a big value for the bulk modulus of the pore fluid in MONICA, the undrained behaviour was modelled for a single finite element in the same way as for the drained data in Section 5.3.5. It should be noted that parameter $A$ was taken as 0.004 again and was not calculated through equation (5.33). However, parameters $B$, $C$ and $E$ were calculated using equations (5.37), (5.40) and (5.45), respectively. The reason for assuming $A = 0.004$ was firstly, because during the calibration of $E$ and $EP$ by formal modelling, it was found that estimating $A$ through equation (5.33) will lead to less stiff responses whereas taking it as this constant value will produce more accurate responses that better match the experimental data. Secondly, since $A$ was not calibrated using the conventional unload/reload
method of Hicks (1990) and was instead measured based on the results of the seismic body wave test and since, as was described in Section 3.5.2, the triaxial test and the seismic body wave test are performed at two different strain levels, it was decided not to use equation (5.33).

Figure 5.32 illustrates the experimental and computed responses for sample A103. Figures 5.32(b) and (c) illustrate the responses relative to $\varepsilon_1$ instead of $\gamma$, and that is because volumetric strains are zero for the undrained tests. It can be seen from these figures that the computed responses well match the experimental data. However, as seen in Figure 5.32(c), the computed response has a lower maximum compressive pore pressures compared to the experiment, although the rate of change in the excess pore pressures seem to perfectly match each other.

Figure 5.33 illustrates the undrained stress paths computed for different state parameters using the parameters calibrated for drained tests for a confining pressure of 67kPa. It can be clearly seen that the stress paths are again contained within an envelope which has got a slope very close to that in Figure 5.31 (i.e. both having a slope around 0.67). Hence, according to this agreement and also examining the example sample mentioned above, it can be deduced that the calibrated parameters can well model the undrained behaviour as well as the drained responses and are valid for both drained and undrained situations.
5.4 Summary

In this chapter the double-hardening elastoplastic constitutive soil model, Monot, was introduced and calibrated against the laboratory data for future stochastic slope stability analysis of the Jamuna Bridge abutment. The model has 21 parameters, many of which can be given default values. The remaining parameters have to be calibrated.

Table 5.8 presents a summary of the calibrated parameters. It can be seen that the stiffness curvature parameters, \( AP \), \( BP \) and \( EP \), are close to the values recommended by Hicks (2000) and given in Table 5.2. Parameters \( B \), \( C \) and \( E \) were also found to be related to the state parameter \( \psi \) through equations (5.37), (5.40) and (5.45), respectively. However, since the available laboratory data did not include any unload/reload loops, parameter \( A \) was estimated using the results of the seismic body wave test, isotropic compression tests and also formal modelling. Although a relationship between parameter \( A \) and the state parameter was derived from the seismic body wave test, using equation (5.33), more accurate computed responses were achieved using a constant value of \( A \) in the forward iterative approach. Also, due to the difference in the strain levels of the seismic body wave test and the triaxial unload/reload test, it was decided to take \( A \) as a constant value. Hence, in order to perform the stochastic slope stability analysis of the Jamuna Bridge abutment, one needs only to generate uni-variate random fields of the state parameter, and to backfigure parameters \( B \), \( C \) and \( E \) using the relationships in Table 5.8 and taking the rest of the parameters as constant values.

Finally, since the results in Table 5.8 were derived using the results of drained triaxial tests, it was necessary to check these results with undrained triaxial tests in order to make sure that the calibration is valid for both drained and undrained behaviours. This was investigated in Section 5.3.6 and it was observed that the calibrated parameters can well reproduce the undrained responses as well as the drained responses.

In the next chapter of the thesis, these results will be used together with the results in Chapter 4 to generate random fields of the state parameter and finally analyzing the structure in Chapter 7.
<table>
<thead>
<tr>
<th>model component</th>
<th>general</th>
<th>further comments</th>
<th>parameter</th>
<th>range</th>
<th>equation</th>
</tr>
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<tbody>
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<td>non-linear elastic</td>
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<td>Poisson’s ratio</td>
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<td>-</td>
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<td>hardening</td>
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</tr>
<tr>
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</tr>
<tr>
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<td>interparticle $\phi$</td>
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<td>$FIMU$</td>
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<td>5.27</td>
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<td>constant volume $\phi$</td>
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<td>$FICV$</td>
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</tr>
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<td></td>
<td>transition (FIMU-FICV)</td>
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<td>$SCV$</td>
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<td>adjustment for low $t/s'$</td>
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<td>$VGC$</td>
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<td></td>
<td>adjustment for low $t/s'$</td>
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<td></td>
<td>adjustment for low $t/s'$</td>
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<td>$NU$</td>
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<td>-</td>
</tr>
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<td>scalar</td>
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<td>$E$</td>
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<td>5.25</td>
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<td>curvature</td>
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<td>$EP$</td>
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<td>5.25</td>
</tr>
<tr>
<td></td>
<td>curvature</td>
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</tr>
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<td></td>
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<td>$CG$</td>
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<td>adjustment to $dv/d\gamma$</td>
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<td>$CV$</td>
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<td>-</td>
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<td>shape of plastic potential</td>
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<td>-</td>
</tr>
</tbody>
</table>

Table 5.1 Monot material properties (Onisiphorou, 2004).
Table 5.2 Relative importance of Monot parameters (Hicks, 2000).

Table 5.3 Summary of calibration sequence (Hicks, 1990).
### Table 5.4 Values of $A$ for drained triaxial tests on Jamuna River Sand.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$e_0$</th>
<th>$p'_0$ (kPa)</th>
<th>$\psi$</th>
<th>$G$ (MPa)</th>
<th>$E_Y$ (MPa)</th>
<th>$A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B231-13</td>
<td>0.840</td>
<td>165</td>
<td>-0.057</td>
<td>89.59</td>
<td>220.39</td>
<td>0.00025</td>
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<td>107.47</td>
<td>264.38</td>
<td>0.00020</td>
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<td>B231-31</td>
<td>0.700</td>
<td>300</td>
<td>-0.182</td>
<td>135.13</td>
<td>332.42</td>
<td>0.00016</td>
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<tr>
<td>B104-17</td>
<td>0.934</td>
<td>280</td>
<td>0.051</td>
<td>128.87</td>
<td>317.02</td>
<td>0.00017</td>
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<tr>
<td>B231-23</td>
<td>0.893</td>
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<td>0.007</td>
<td>119.21</td>
<td>293.26</td>
<td>0.00018</td>
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<td>B231-29</td>
<td>0.866</td>
<td>275</td>
<td>-0.018</td>
<td>127.28</td>
<td>313.11</td>
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<td>B221-U19</td>
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<td>196.85</td>
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<td>0.004</td>
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<td>196.85</td>
<td>0.00027</td>
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<tr>
<td>B221-U19B</td>
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<td>-0.049</td>
<td>80.02</td>
<td>196.85</td>
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</table>

### Table 5.5 Values of $\phi_{peak}$ for drained triaxial tests on Jamuna River Sand.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$e_0$</th>
<th>$p'_0$ (kPa)</th>
<th>$\psi$</th>
<th>$p'$ (kPa)</th>
<th>$q_f$ (kPa)</th>
<th>$(\sigma_f/\sigma_r)_f$</th>
<th>$\phi_{peak}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B231-13</td>
<td>0.840</td>
<td>165</td>
<td>-0.057</td>
<td>306.4</td>
<td>440.2</td>
<td>3.757</td>
<td>35.4</td>
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<td>B231-19</td>
<td>0.866</td>
<td>215</td>
<td>-0.024</td>
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<td>533.2</td>
<td>3.531</td>
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<td>3.483</td>
<td>33.6</td>
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</table>

### Table 5.6 Values of $E$ and $EP$ for drained triaxial tests on Jamuna River Sand.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$e_0$</th>
<th>$p'_0$ (kPa)</th>
<th>$\psi$</th>
<th>$A$</th>
<th>$E$</th>
<th>$EP$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B231-13</td>
<td>0.840</td>
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<td>-0.057</td>
<td>0.004</td>
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<td>3.4</td>
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<td>215</td>
<td>-0.024</td>
<td>0.004</td>
<td>0.048</td>
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</tr>
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<td>B231-31</td>
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<td>300</td>
<td>-0.182</td>
<td>0.004</td>
<td>0.025</td>
<td>3.4</td>
</tr>
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<td>0.051</td>
<td>0.004</td>
<td>0.190</td>
<td>3.4</td>
</tr>
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<td>250</td>
<td>0.007</td>
<td>0.004</td>
<td>0.150</td>
<td>3.4</td>
</tr>
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<td>-0.018</td>
<td>0.004</td>
<td>0.090</td>
<td>3.4</td>
</tr>
<tr>
<td>B221-U19</td>
<td>0.963</td>
<td>140</td>
<td>0.061</td>
<td>0.004</td>
<td>0.110</td>
<td>3.4</td>
</tr>
<tr>
<td>B221-U19A</td>
<td>0.906</td>
<td>140</td>
<td>0.004</td>
<td>0.004</td>
<td>0.092</td>
<td>3.4</td>
</tr>
<tr>
<td>B221-U19B</td>
<td>0.853</td>
<td>140</td>
<td>-0.049</td>
<td>0.004</td>
<td>0.085</td>
<td>3.4</td>
</tr>
</tbody>
</table>
### Table 5.7 Values of the state parameter for undrained triaxial tests on the Upper Sand Layer.

(based on equation 3.24 and 3.1)

<table>
<thead>
<tr>
<th>Sample</th>
<th>$e_0$</th>
<th>$p_0$ (kPa)</th>
<th>$\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1AA</td>
<td>0.990</td>
<td>67</td>
<td>0.1280</td>
</tr>
<tr>
<td>A2</td>
<td>0.824</td>
<td>67</td>
<td>-0.0380</td>
</tr>
<tr>
<td>A3</td>
<td>0.925</td>
<td>67</td>
<td>0.0630</td>
</tr>
<tr>
<td>A4</td>
<td>0.969</td>
<td>67</td>
<td>0.1070</td>
</tr>
<tr>
<td>A6</td>
<td>0.844</td>
<td>67</td>
<td>-0.0180</td>
</tr>
<tr>
<td>A7</td>
<td>0.921</td>
<td>67</td>
<td>0.0590</td>
</tr>
<tr>
<td>A102</td>
<td>0.936</td>
<td>275</td>
<td>0.1307</td>
</tr>
<tr>
<td>A103</td>
<td>0.857</td>
<td>75</td>
<td>-0.0003</td>
</tr>
<tr>
<td>A104</td>
<td>0.938</td>
<td>67</td>
<td>0.0760</td>
</tr>
<tr>
<td>A105</td>
<td>1.048</td>
<td>67</td>
<td>0.1860</td>
</tr>
<tr>
<td>B2</td>
<td>0.994</td>
<td>67</td>
<td>0.1320</td>
</tr>
<tr>
<td>B3</td>
<td>1.003</td>
<td>67</td>
<td>0.1410</td>
</tr>
<tr>
<td>B4</td>
<td>0.944</td>
<td>67</td>
<td>0.0820</td>
</tr>
<tr>
<td>B6A</td>
<td>1.065</td>
<td>67</td>
<td>0.2030</td>
</tr>
<tr>
<td>B7</td>
<td>0.786</td>
<td>67</td>
<td>-0.0760</td>
</tr>
<tr>
<td>B8A</td>
<td>0.868</td>
<td>67</td>
<td>0.0060</td>
</tr>
<tr>
<td>B10</td>
<td>0.872</td>
<td>67</td>
<td>0.0100</td>
</tr>
<tr>
<td>B10A</td>
<td>0.919</td>
<td>67</td>
<td>0.0570</td>
</tr>
<tr>
<td>B20A</td>
<td>0.980</td>
<td>67</td>
<td>0.1180</td>
</tr>
<tr>
<td>B21</td>
<td>1.010</td>
<td>67</td>
<td>0.1480</td>
</tr>
<tr>
<td>B22</td>
<td>0.999</td>
<td>67</td>
<td>0.1370</td>
</tr>
<tr>
<td>B23A</td>
<td>0.975</td>
<td>67</td>
<td>0.1130</td>
</tr>
<tr>
<td>B23B</td>
<td>0.890</td>
<td>67</td>
<td>0.0280</td>
</tr>
<tr>
<td>B100</td>
<td>1.097</td>
<td>67</td>
<td>0.2350</td>
</tr>
<tr>
<td>B101</td>
<td>1.072</td>
<td>67</td>
<td>0.2100</td>
</tr>
<tr>
<td>B102</td>
<td>1.000</td>
<td>67</td>
<td>0.1380</td>
</tr>
<tr>
<td>B201</td>
<td>0.925</td>
<td>67</td>
<td>0.0630</td>
</tr>
<tr>
<td>B202</td>
<td>0.870</td>
<td>267</td>
<td>0.0634</td>
</tr>
<tr>
<td>B205</td>
<td>0.890</td>
<td>67</td>
<td>0.0280</td>
</tr>
<tr>
<td>B206</td>
<td>0.821</td>
<td>93</td>
<td>-0.0276</td>
</tr>
</tbody>
</table>
Table 5.8 Summary of the calibrated parameters for the Jamuna River Sand.

<table>
<thead>
<tr>
<th>Monot parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$0.004$</td>
</tr>
<tr>
<td>$AP$</td>
<td>$0.31$</td>
</tr>
<tr>
<td>$B$</td>
<td>$0.0043 \exp(7.5\psi)$</td>
</tr>
<tr>
<td>$BP$</td>
<td>$0.69$</td>
</tr>
<tr>
<td>$\phi_{peak}$</td>
<td>$-15.4\psi + 33.4^\circ$</td>
</tr>
<tr>
<td>$FICV$</td>
<td>$33.4^\circ$</td>
</tr>
<tr>
<td>$E$</td>
<td>$0.091 \exp(9.5\psi)$</td>
</tr>
<tr>
<td>$EP$</td>
<td>$3.4$</td>
</tr>
</tbody>
</table>
Figure 5.1 Basic features of Monot (Hicks, 2003).
Figure 5.2 Possible stress paths and yield surface expansions in the $s' - t$ plane: (a) deviatoric yield surface activated; (b) compressive yield surface activated; (c) both yield surfaces activated; (d) purely elastic response (Hicks, 1995a).
Using least squares solve
\[ \ln(s'/p_a) = AP \ln(s'/p_a) + \ln(A) \]
to give \( A \) and \( AP \)

Compute new estimate for offset
\[ v_e^c = A(s_e'/p_a)^{AP} \]

Figure 5.3 Automated approach for determining \( A \) and \( AP \) (Hicks, 1990).
Figure 5.4 Manual approach for determining $A$ and $AP$ (Hicks, 1990).
Figure 5.5 Isotropic compression test results (Fugro reports, 1986 & 1996).
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**Figure 5.6** Calibration of $A$ and $AP$ based on the isotropic compression tests (Sample 231-13).

**Figure 5.7** Calibration of $A$ and $AP$ based on the isotropic compression tests (Sample 231-23).

**Figure 5.8** Calibration of $A$ and $AP$ based on the isotropic compression tests (Sample 231-29).
Figure 5.9  Relationship between $A$ and $\psi$ for drained triaxial tests of the Upper Layer Sand.

Figure 5.10  Calibration of $B$ and $BP$ based on the oedometric compression tests (Sample 231-13).
Figure 5.11 Calibration of $B$ and $BP$ based on the oedometric compression tests (Sample 231-23).

Figure 5.12 Calibration of $B$ and $BP$ based on the oedometric compression tests (Sample 231-29).
Figure 5.13 Deriving $B$ and $BP$ for sample B231-13.

Figure 5.14 Deriving $B$ and $BP$ for sample B231-19.
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Figure 5.15 Deriving $B$ and $BP$ for sample B231-31.

Figure 5.16 Deriving $B$ and $BP$ for sample B104-17.
Figure 5.17 Deriving $B$ and $BP$ for sample B231-23.

Figure 5.18 Deriving $B$ and $BP$ for sample B231-28.
**Figure 5.19** Relationship between $B$ and $\psi$ for drained triaxial tests on the Upper Layer Sand.

$B = 0.0043e^{7.4929\psi}$

$R^2 = 0.9611$

---

**Figure 5.20** Relationship between $\phi_{peak}$ and $\psi$ for drained triaxial tests on the Upper Layer Sand.

$\phi_{peak} = -15.365\psi + 33.4$

$R^2 = 0.4939$
Figure 5.21 Deriving $E$ and $EP$ for sample B231-13.

Figure 5.22 Deriving $E$ and $EP$ for sample B231-19.
Figure 5.23 Deriving $E$ and $EP$ for sample B231-31.

Figure 5.24 Deriving $E$ and $EP$ for sample B104-17.
Figure 5.25 Deriving $E$ and $EP$ for sample B231-23.

Figure 5.26 Deriving $E$ and $EP$ for sample B231-28.
Figure 5.27 Deriving $E$ and $EP$ for sample B221-U19.

Figure 5.28 Deriving $E$ and $EP$ for sample B221-U19A.
Figure 5.29 Deriving $E$ and $EP$ for sample B221-U19B.
Figure 5.30 Relationship between $E$ and $\psi$ for drained triaxial tests of the Upper Layer Sand.

Figure 5.31 Stress paths for the undrained triaxial data of the Jamuna River Sand.
Validity check of the drained calibration results using isotropic undrained sample A103.

Figure 5.32
Figure 5.33 Computed undrained stress paths corresponding to different values of the state parameter.
Chapter 6

Random Field Generation of the Jamuna Bridge Abutment
6 Random Field Generation of the Jamuna Bridge Abutment

6.1 Introduction

In this chapter of the thesis, the random field theory is described in more detail and random fields for the Jamuna Bridge Abutment are generated based on the statistical characterization results of the site, carried out in chapter 4. These random fields will then be used in Chapter 7 for the stochastic analysis of the Jamuna Bridge Abutment as part of a Monte Carlo simulation.

Due to the previous experience of the author’s research group in developing random field generators (Samy, 2003 and Spencer, 2007) using the Local Average Subdivision (LAS) method, as described by Fenton & Vanmarcke (1990), the author has decided to adopt the LAS method throughout the rest of this thesis for generating random fields.

However, Section 6.2 gives a brief summary of different types of random field generators and then more detailed descriptions of the LAS method in 1D and 2D are given in Section 6.3. Section 6.4 describes the application of LAS to the Jamuna Bridge Abutment, including the essential theories and assumptions regarding the methods by which the random field is cropped and mapped onto the finite element mesh. Finally, Section 6.5 gives concluding comments and describes further steps in the final stochastic analyses.

Before moving on to the next section, it should be remembered that all the random fields generated in this chapter are for the state parameter and are based on the statistics that were derived earlier. Hence, uni-variate random fields are first generated and then, based on the relationships derived in Chapter 5 between state parameter and some of the Monot model parameters, random fields of other material properties will be backfigured and mapped on to the finite element mesh for further analysis of the structure.
6.2 Methods of random field generation

Random field generation can be described as the post-processing of the outcomes of a random number generator in such a way that an array of random numbers, representing particular characteristics, is formed. Random numbers have been traditionally generated by actual random processes such as the spin of a roulette wheel or the role of a die. However, with the widespread use of computers, algorithmic pseudo random number generators have been developed in recent years to reproduce the outcomes of a random process. Hence, in this section, a brief description of some of the random field generation methods which can produce random values in one or more dimensions is given; then, in the next section, the LAS method is discussed in more detail.

In the Moving Average method (Gersch & Yonemoto, 1977), random field values at every point are generated using a weighted moving average of a white noise function. From a computational perspective, the solution for this method involves the inversion of a matrix with a size proportional to the square of the number of dimensions. Therefore, as the number of dimensions increases, the method becomes increasingly inefficient, since it would be an order of magnitude more time-consuming than the fastest generation method (Fenton, 1994).

The Fast Fourier Transform method (Cooley & Tukey, 1965), which is also called the spectral method, employs the spectral density function directly. It is very flexible, since it can be easily adapted to higher dimensions, non-Gaussian distributions and anisotropic fields. However, the method is computationally inefficient, since due to symmetry it requires discarding half the field in 1D, a quarter of the field in 2D and so on.

In the Turning Bands method (Matheron, 1973), 1D random processes are superimposed in the form of lines, until the required multi-dimensional domain is built up. Hence, the method is simple to implement and provides a good random field structure, if the number of lines used to build up the domain are adequate and also if the quality of the 1D random generator is good. However, as seen in Figure 6.1, if the number of the lines used in building up the domain is not adequate, the patterning in the random field would reduce the accuracy of the analyses.
The Local Average Subdivision method (Fenton & Vanmarcke, 1990), which will be discussed in more detail in the next section, employs the variance function for representing the field. It is one of the methods which have been implemented more often than other methods, due to its computational efficiency, especially for higher dimensions. Figure 6.2 illustrates a random field generated using the LAS method. By close inspection, a fractal square pattern can be seen to exist in the field (even if it is trivial).

The Generalised Stochastic Subdivision method (Lewis, 1987) is the point process equivalent of the LAS method, in which the domain is subdivided many times and each time the new midpoint value is interpolated from the neighbouring values in the vicinity. This repeated subdivision will result in a regular lattice structure, as seen in Figure 6.3.

Fenton (1994) compared three of the most commonly used methods of random field generation (i.e. the Fast Fourier Transformation method, the LAS method and the Turning Bands method) and gave guidelines on how these methods should be implemented. He concluded that each method can produce a good field structure and can well represent the mean and covariance of the domain. He found that the Fast Fourier Transformation method was the slowest of the three methods of generating random fields and that patterning plays an important role in all methods of random field generation, and needs to be well considered in order to obtain more representative results in subsequent analyses.
6.3 Local Average Subdivision method

6.3.1 Local Average theory and implementation in 1D

As mentioned in the previous section, the LAS method employs the variance function for representing the field. Although some of the concepts presented in this section (e.g. variance function, covariance function and scale of fluctuation) have already been discussed in Chapter 4, a more structured review of mathematical concepts involved in the method will now be discussed. Note that, in what follows, it is assumed that a continuous stationary function is averaged over a finite domain. Other complicated conditions are discussed broadly by Vanmarcke (1984).

The mean, \( \mu \), and variance, \( \sigma^2 \), of the property \( X(t) \) between two points \( t \) and \( t' \) (Figure 6.4a) are defined as

\[
\mu = E[X(t)] = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} X(t) \, dt \quad (6.1)
\]

\[
\sigma^2 = E[(X(t) - \mu)^2] = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [(X(t) - \mu)^2] \, dt \quad (6.2)
\]

Also, the covariance function, \( \beta(t,t') \), and correlation function, \( \rho(t,t') \), are defined as

\[
\beta_X(t,t') = \text{Cov}[X(t), X(t')] = E[X(t)X(t')] - \mu(t)\mu(t') \quad (6.3)
\]

\[
\rho_X(t,t') = \frac{\beta(t,t')}{\sigma(t)\sigma(t')} \quad (6.4)
\]

in which subscript \( X \) stands for the property \( X \). The moving average, \( X_T(t) \), of the continuous stationary random process \( X(t) \), over a finite domain of length \( T = t_2 - t_1 \), is then calculated by:

\[
X_T(t) = \frac{1}{T} \int_{\frac{t-T}{2}}^{\frac{t+T}{2}} X(u) \, du \quad (6.5)
\]

where \( u \) is a point within the range of the process \( X \) and the integral on the right hand side of the equation is called the “local integral process” and is referred to as \( I_T \). Hence, equation (6.5) can be rewritten as:

\[
X_T(t) = \left( \frac{1}{T} \right) I_T(t) \quad (6.6)
\]
or by:

\[ I_T(t) = TX_T(t) \]  \hspace{1cm} (6.7)

Figures 6.4b and 6.4c illustrate the processes \( X_T(t) \) and \( I_T(t) \), respectively. It can be seen that these functions are much smoother than the original process \( X(t) \), as expected. \( X_T(t) \) has a mean value of \( \mu_T \) which is equal to \( \mu \); whereas, its variance, \( \sigma_T^2 \), is less than the variance of the original process, \( \sigma^2 \). Samy (2003) indicated that, by taking the expectations of these two variances over a window of length \( T \), they can be related by the relationship:

\[ \sigma = \gamma(T) \sigma^2 \]  \hspace{1cm} (6.8)

where \( \gamma(T) \) is the variance function over the window \( T \) (similar to \( \Gamma^2(T) \) in equation (4.67)) and represents the reduction relative to the point variance, \( \sigma^2 \). This is found by averaging over the length \( T \) and is given by:

\[ \gamma(T) = \frac{2}{T} \int_0^T \rho(\tau) d\tau - \frac{2}{T^2} \int_0^T \rho(\tau) d\tau \]  \hspace{1cm} (6.9)

where \( \tau \) is the lag distance between the points at the boundaries of the integral window and \( \rho(\tau) \) is the correlation function. In order to make sure that \( \sigma_T^2 \) is less than \( \sigma^2 \), the variance function \( \gamma \) should be kept between 0 and 1; this means that as \( T \rightarrow 0 \), \( X_T(t) \rightarrow X(t) \), \( \sigma_T^2 \rightarrow \sigma^2 \) and therefore, \( \gamma(T \rightarrow 0) \rightarrow 1 \); and as \( T \rightarrow \infty \), \( X_T(t) \rightarrow \mu \), \( \sigma_T^2 \rightarrow 0 \) and therefore, \( \gamma(T \rightarrow \infty) \rightarrow 0 \).

Since the correlation function is generally not known, it is necessary to obtain a constant scale of fluctuation which is independent of the size of the integral window. It is seen from equation (6.9) that, as \( T \) becomes very large, the second term on the right hand side of the equation becomes very small and so can be neglected; hence, equation (6.9) reduces to

\[ T \gamma(T) \approx 2 \int_0^\infty \rho(\tau) d\tau = \frac{2}{\sigma^2} \int_0^\infty \beta(\tau) d\tau \]  \hspace{1cm} (6.10)

in which \( T \gamma(T) = \theta \) is the scale of fluctuation described by Vanmarcke (1984) and is derived based on the method described in Section 4.5.8.
Note that equation (6.10) is an approximate function and that the condition $\gamma(T) = 0$ as $T \to \infty$ is a very important condition for this relationship to be valid. Figure 6.5 shows the approximate and exact functions with respect to $T$. The approximate function is therefore often defined by a condition, in a similar way to equation (4.67). Note that, the variance and covariance functions are defined in a similar way to that which was described in Section 4.5.3 and presented by equation (4.60).

Having reviewed the basic concepts of random field theory, it is now possible to describe the general process and mathematical implementation of the LAS method as proposed by Fenton (1990) and Fenton and Vanmarcke (1990). It should be noted that the LAS method is an internally consistent method, which means that a constant mean is maintained over all levels of subdivision; also, the final field can easily give information about the previous levels of subdivision. The method is also advantageous, since it allows refining parts of the field which are of interest more than others. The 1D algorithm, as described by Spencer (2007), is governed by an Ornstein-Uhlenbeck process, a fractional Gaussian noise proposed by Mandelbrot & Ness (1968) and an exponential correlation function.

The method advances by recursion, which means that the initial domain is progressively subdivided into smaller cells until a series of local averages can represent the random process being modelled. In order to start the LAS process, one needs to know the mean of the whole domain. The aim is to ensure that the global mean follows a standard Gaussian distribution, bearing in mind that the expected variance of the global means need to be reduced to account for the domain size ($D$) with respect to the scale of fluctuation ($\theta$). Therefore, it is required for the global mean to be assigned a random value from a Gaussian distribution which has a variance of,

$$\sigma^2_0 = \sigma^2 \gamma(D)$$  \hspace{1cm} (6.11)

Due to internal consistency, the global mean is maintained as a constant value throughout the process.

In this method, the initial stage is defined as stage 0, and the number of stages increases as the level of subdivisions is increased. Beyond stage 0, each cell has a parent cell, from which it is subdivided. Figure 6.6 shows how the subdivision process is progressed. As can be seen, each local average is denoted by $Z$, with the subscripts and the superscripts referring to the cell and stage numbers, respectively. The recursive division progression follows the steps below (Spencer, 2007):
1) Assigning a global mean to the initial domain.
2) Subdividing the parent cell, \( Z_{j}^{i-1} \), into two equal-sized cells.
3) Assigning the local average, \( Z_{2j}^{i} \), a weighted random value.
4) Calculating \( Z_{j-1}^{i} = 2Z_{j}^{i-1} - Z_{2j}^{i} \) by upward averaging.
5) Increasing the stage number and going to step 2.

Note that the global mean is maintained during the subdivision process and that the weighting of the random value in step (3) guarantees the correct variance and correlation structure. Also, note that the mean of the final field is the same as that of the first step and that its variance would tend to the target value of 1. However, the standard Gaussian random field can be transformed into other desirable distribution functions or statistical properties by the methods described in Section 6.4.1.2.

If \( Z_{1}^{0} \) is the global mean of the process \( Z(t) \) at stage zero, which is defined by

\[
Z_{1}^{0} = \frac{1}{D} \int_{0}^{D} Z(t) dt \tag{6.12}
\]

in which a standard normal distribution defines the target mean and variance; then, based on the local average theory, the expected mean would be zero and the expected variance (using equation (6.2)) would be

\[
E[(Z_{1}^{0})^2] = \sigma^2 \gamma(D) \tag{6.13}
\]

The covariance function, which is based on the zero mean Ornstein-Uhlenbeck process, would then be

\[
\beta(r) = \sigma^2 \exp \left( -\frac{2|\tau|}{\theta} \right) \tag{6.14}
\]

Hence, if the right-hand side of equation (6.10) is replaced by the above relationship, \( \gamma(D) \) is given by

\[
\gamma(D) = \frac{\theta^2}{2D^2} \left[ \frac{2|D|}{\theta} + \exp \left( -\frac{2|D|}{\theta} \right) - 1 \right] \tag{6.15}
\]

which gives the initial global mean as

\[
Z_{1}^{0} = U\gamma(D) \tag{6.16}
\]
in which $U$ represents a white Gaussian noise with a mean of zero and a variance of unity; the pseudo random number generator (PRNG), which is employed to compute $U$, is described in more detail in Section 6.4.1.1. However, an important point to mention is that PRNG is reset with an arbitrary seed for each realisation of the random field; note that, the first random number generated by PRNG has a particular importance, as it determines the global mean of the field.

The LAS method is mathematically implemented by considering an arbitrary cell at stage $i$ and position $j$ (as illustrated in Figure 6.7), which has a local average of $Z_j^i$, and then working out the value of the cell which is subdivided from this parent cell at stage $i + 1$; that is $Z_{2j}^{i+1}$. Note that, in order to converge to the correct covariance structure, the process requires information about those cells neighbouring the parent cell being subdivided. Fenton (1990) suggested that a neighbourhood size of 3 cells would be sufficient for a monotonic covariance function. In addition, Samy (1998) found no significant improvement when considering a larger neighbourhood. Hence, in this research the 3 cell neighbourhood assumption has been adopted, meaning that the value of $Z_{2j}^{i+1}$ depends not only on a random variable, but also on the values of $Z_{j-1}^i$, $Z_j^i$ and $Z_{j+1}^i$. This can be stated as:

$$Z_{2j}^{i+1} = M_{2j}^{i+1} + c_j^{i+1}U_j^{i+1} \tag{6.17}$$

in which: $c_j^{i+1}$ is the standard deviation of the white Gaussian noise term, which varies if the correct scale of fluctuation is reached (i.e. it is a maximum at $\theta \approx D^i$ and becomes very small for $\theta \ll D^i$ and $\theta \gg D^i$); $U_j^{i+1}$ is the white Gaussian noise term, generated from a standard normal Gaussian distribution; and $M_{2j}^{i+1}$ is the estimated mean for the new cell, using the parent cells within the neighbourhood, and is given by:

$$M_{2j}^{i+1} = a_{-1}^iZ_{j-1}^i + a_0^iZ_j^i + a_1^iZ_{j+1}^i \tag{6.18}$$

where the weighting coefficients are the $a$ terms, and in which the subscripts refer to the stage number and the superscripts indicate to which side of the parent cell the weighting is applied. The expected value of $Z_{2j}^{i+1}$ is strongly correlated with respect to its immediate parent cell, $Z_j^i$; if is less correlated with respect to $Z_{j+1}^i$ and the correlation reduces even further with respect to $Z_{j-1}^i$. 


Putting $M_{2j}^{i+1}$ from equation (6.18) into equation (6.17) gives

$$Z_{2j}^{i+1} = \{ a_{i-1} Z_{j-1}^i + a_0 Z_j^i + a_1 Z_{j+1}^i \} + c^{i+1} U_{j+1}$$ \hspace{1cm} (6.19)

in which the weighting coefficients $a$ and $c$ are unknown and need to be derived.

If $Z_m^i$ is the value of any cell $m$ at stage $i$, and both sides of equation (6.19) are multiplied by this value, applying the expectation operator gives,

$$E[Z_{2j}^{i+1} Z_m^i] = \sum_{k=j-1}^{k=j+1} a_{k-j} E[Z_k^i Z_m^i] + c^{i+1} E[Z_m^i U_{j+1}^{i+1}]$$ \hspace{1cm} (6.20)

Note that $E[Z_m^i U_{j+1}^{i+1}] = 0$, due to the white Gaussian noise term being independent of any value in the field. Therefore, equation (6.20) reduces to

$$E[Z_{2j}^{i+1} Z_m^i] = \sum_{k=j-1}^{k=j+1} a_{k-j} E[Z_k^i Z_m^i]$$ \hspace{1cm} (6.21)

By substituting $Z_m^i$ with the 3 neighbouring cells of interest, equation (6.21) becomes

$$\begin{pmatrix} E[Z_{2j}^{i+1} Z_{j-1}^i] \\ E[Z_{2j}^{i+1} Z_j^i] \\ E[Z_{2j}^{i+1} Z_{j+1}^i] \end{pmatrix} = \begin{pmatrix} E[Z_j^{i-1} Z_{j-1}^i] E[Z_j^i Z_{j-1}^i] E[Z_{j+1}^i Z_{j-1}^i] \\ E[Z_j^{i-1} Z_j^i] E[Z_j^i Z_j^i] E[Z_{j+1}^i Z_j^i] \\ E[Z_j^{i-1} Z_{j+1}^i] E[Z_j^i Z_{j+1}^i] E[Z_{j+1}^i Z_{j+1}^i] \end{pmatrix} \begin{pmatrix} a_{i-1} \\ a_0 \\ a_i \end{pmatrix}$$ \hspace{1cm} (6.22)

in which the square matrix on the right-hand side is symmetric and its elements along each diagonal are equal; the vector on the left-hand side is the expectation between cells at different stages and needs to be determined; and the $a$ coefficients, can be evaluated based on the cell sizes at stages $i$ and $i+1$ and knowing the variance structure over the domain.

Note that, since these coefficients are independent of the actual cell values, they only need to be evaluated once at each stage. Also, as mentioned before, preserving the upward averaging between stages means that the left-hand side of equation (6.22) should be rewritten as

$$\begin{pmatrix} E[Z_{2j}^{i+1} Z_{j-1}^i] \\ E[Z_{2j}^{i+1} Z_j^i] \\ E[Z_{2j}^{i+1} Z_{j+1}^i] \end{pmatrix} = \frac{1}{2} \begin{pmatrix} E[Z_{2j}^{i+1} Z_{2j-3}^i] + E[Z_{2j}^{i+1} Z_{2j-2}^i] \\ E[Z_{2j}^{i+1} Z_{2j}^i] + E[Z_{2j}^{i+1} Z_{2j+1}^i] \\ E[Z_{2j}^{i+1} Z_{2j+1}^i] + E[Z_{2j}^{i+1} Z_{2j+2}^i] \end{pmatrix}$$ \hspace{1cm} (6.23)

In the above equation, the expectation terms can be evaluated using the general relationship.
\[ E[Z_k^i Z_{k+m}^i] = \frac{\sigma^2}{2} \left[ (m - 1)^2 \gamma (m - 1)D^i - 2m^2 \gamma (mD^i) + (m + 1)^2 \gamma (m + 1)D^i \right] \]

(6.24)

in which: \( i, k \) and \( m \) are the stage number, cell number and the number of cells by which the two cells are being separated from each other, respectively; \( D \) is the cell width; and \( \sigma \) and \( \gamma \) are the standard deviation and variance function, respectively.

Another term which needs to be estimated in equation (6.17) is the standard deviation of the Gaussian noise, \( c^{i+1} \). This term is calculated by squaring equation (6.17) and taking expectations of both hands of the relationship to give

\[ E[(Z_{2j}^{i+1})^2] = E[(M_{2j}^{i+1})^2] + (c^{i+1})^2 \]

(6.25)

in which: \( c^{i+1} \) is taken out from the expectation operator, since for every stage \( i \) it is a constant; and, \( E[(U_{j}^{i+1})^2] = Var[U_{j}^{i+1}] = 1 \). On the other hand, squaring equation (6.18) and applying the expectation operator to its both sides, gives

\[ E[(M_{2j}^{i+1})^2] = \sum_{k=j-1}^{j+1} a_k Z_{2j}^{i+1} Z_k \]

(6.26)

Substituting equation (6.25) into equation (6.26) and evaluating the cross-stage terms in a similar way to equation (6.22), then leads to

\[ (c^{i+1})^2 = E[(Z_{2j}^{i+1})^2] - \{a_{i-1}^i a_0^i a_j^i \} \left\{ \begin{array}{c} E[Z_{2j}^{i+1} Z_{j-1}^{i+1}] \\ E[Z_{2j}^{i+1} Z_j^{i+1}] \\ E[Z_{2j}^{i+1} Z_{j+1}^{i+1}] \end{array} \right\} \]

(6.27)

The \( c \) coefficients only need to be calculated once at every stage (similar to the \( a \) coefficients). This makes it convenient to calculate \( c \) and \( a \) at every stage and store them before starting the subdivision process. The local average, \( Z_{2j}^{i+1} \), can then be calculated using equations (6.17), (6.22) and (6.27), making it then possible to calculate the average value for the cell \( Z_{2j-1}^{i+1} \) by upward averaging, i.e.:

\[ Z_{2j-1}^{i+1} = 2Z_j^i - Z_{2j}^{i+1} \]

(6.28)

Subdividing cells which are located on the boundaries of the domain requires consideration of boundary effects. This is because, as mentioned earlier, the value of the subdivided cell not only depends on the immediate parent cell, but also on the neighbouring cells. Hence, for those cells which are located on the boundaries of the
domain, some of these parent cells will lie outside the domain. This will, in turn, affect
the generation of the $\alpha$ coefficients in equation (6.22). Therefore, equation (6.22) needs
to be reformulated for the case where the parent cells are missing.

Referring to Figure 6.7, for generating the cell value $Z_{2j-1}^{i+1}$, if only one neighbour is
missing to the left (i.e. $Z_{j-1}^i$ lies outside the domain),

$$
\begin{pmatrix}
E[Z_{2j}^{i+1}Z_{j}^i] \\
E[Z_{2j}^{i+1}Z_{j+1}^i]
\end{pmatrix}
= \begin{pmatrix}
E[Z_{j}^{i+1}Z_{j}^i] & E[Z_{j+1}^{i+1}Z_{j}^i] \\
E[Z_{j}^{i}Z_{j+1}^i] & E[Z_{j+1}^{i}Z_{j+1}^i]
\end{pmatrix}
\begin{pmatrix}
a_0^i \\
\end{pmatrix}
$$

(6.29)

whereas if only one neighbour is missing to the right (i.e. $Z_{j+1}^i$ lies outside the
domain),

$$
\begin{pmatrix}
E[Z_{2j}^{i+1}Z_{j-1}^i] \\
E[Z_{2j}^{i+1}Z_{j}^i]
\end{pmatrix}
= \begin{pmatrix}
E[Z_{j-1}^{i+1}Z_{j-1}^i] & E[Z_{j}^{i+1}Z_{j-1}^i] \\
E[Z_{j-1}^{i}Z_{j}^i] & E[Z_{j}^{i}Z_{j}^i]
\end{pmatrix}
\begin{pmatrix}
a_{-1}^i \\
\end{pmatrix}
$$

(6.30)

and if two neighbours are missing (i.e. $Z_{j}^i$ is only available),

$$
\begin{pmatrix}
E[Z_{2j}^{i+1}Z_{j}^i]
\end{pmatrix}
= \begin{pmatrix}
E[Z_{j}^{i+1}Z_{j}^i]
\end{pmatrix}\begin{pmatrix}
a_0^i
\end{pmatrix}
$$

(6.31)

Therefore, in general terms, equation (6.19) can be written as:

$$
Z_{2j}^{i+1} = c^{i+1}U_{j}^{i+1} + \sum_{k=j-p}^{j+q} a_{k-j}^i Z_{k}^j
$$

(6.32)

in which $p = \min(n, j - 1)$, $q = \min(n, 2^i - j)$ and $n$ is the number of missing parent
cells.
6.3.2 Local Average theory and implementation in 2D

The application of the LAS method for higher dimensions can be done in the same way as the 1D case. Vanmarcke (1984) gives a comprehensive review of how local average subdivision is carried out in 2D, 3D, 4D and generic multi-dimensional cases. Hence, in this section, only a brief summary of the most important features and formulations for the 2D case are given.

Figure 6.8 shows a rectangular area of $A = T_1 T_2$, for which the 2D local averaging process will be explained. The local average, $X_A(t_1, t_2)$, for this case is given by

$$X_A(t_1, t_2) = X_{T_1 T_2}(t_1, t_2) = \frac{1}{A} \int_{T_1}^{T_1 + T_1} \int_{T_2}^{T_2 + T_2} X_A(t_1, t_2) dt_1 dt_2$$

(6.33)

The variance function in 2D is given by

$$\gamma(T_1, T_2) = \frac{1}{(T_1 T_2)^2} \int_{-T_1}^{+T_1} \int_{-T_2}^{+T_2} (1 - \frac{|\tau_1|}{T_1})(1 - \frac{|\tau_2|}{T_2}) \rho(\tau_1, \tau_2) d\tau_1 d\tau_2$$

(6.34)

in which $\tau_1$ and $\tau_2$ are the lag distances in each direction, and $\rho(\tau_1, \tau_2)$ is the correlation function. When axes $t_1$ and $t_2$ are considered in such a way that $X(t_1, t_2)$ becomes quadrant symmetric and the field is isotropic, as shown in Figure 6.9, it is possible to evaluate the above relationship by first integrating over the distance $T_1$ along the $t_1$ axis and then integrating the subsequent relationship over $T_2$ and along the $t_2$ axis. This leads to the conditional variance function below (skipping through all the equations which have led to this final relationship):

$$\gamma(T_1, T_2) = \gamma(T_1)\gamma(T_2|T_1)$$

(6.35)

This different form of the variance function, compared to the 1D form in equation (6.9), means that the scale of fluctuation is evaluated differently in the 2D case. It is given by

$$\theta_t^{(2)} = \frac{c_\alpha \theta_t^{(1)}}{T_1 \gamma(T_1)} \theta_t^{(1)}$$

(6.36)

where: $\theta_t^{(2)}$ is the conditional scale of fluctuation over the length $T_1$ and along the $t_2$ axis; $\theta_t^{(1)}$ is the directional scale of fluctuation along the $t_1$ axis; and $c_\alpha$ is given by

$$c_\alpha = \frac{\alpha}{\theta_t^{(1)}\theta_t^{(2)}}$$

(6.37)
in which $\theta^{(2)}$ is the directional scale of fluctuation along the $t_2$ axis, and $\alpha$ is the characteristic area given by

$$\alpha = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \rho(\tau_1, \tau_2) d\tau_1 d\tau_2$$  \hspace{1cm} (6.38)

For the special case where the two areas shown in Figure (6.9) are of the same size (i.e. $T_1 = T'_1$ and $T_2 = T'_2$, as in Figure 6.10), the covariance function can be estimated from:

$$\beta_A(\tau_1, \tau_2) = \frac{\sigma^2}{4T_1^2 T_2^2} [\Delta(T_1 + \tau_1, T_2 + \tau_2) + \Delta(T_1 - \tau_1, T_2 + \tau_2) + \Delta(T_1 + \tau_1, T_2 - \tau_2) + \Delta(T_1 - \tau_1, T_2 - \tau_2) - 2\Delta(\tau_1, T_2 + \tau_2) - 2\Delta(\tau_1, T_2 - \tau_2) - 2\Delta(T_1 + \tau_1 - \tau_2) + 4\Delta(\tau_1, \tau_2)]$$  \hspace{1cm} (6.39)

in which the general term,

$$\Delta(T_1, T_2) = A^2 \gamma(T_1, T_2)$$  \hspace{1cm} (6.40)

is the variance function associated with the local integral process.

There are a number of different forms that the covariance function can take, as described by Vanmarcke (1984). However, in this research, for the 2D case a Gauss-Markov process with an exponential covariance function has been implemented. The correlation function for the Gauss-Markov process is expressed as

$$\rho(\tau_1, \tau_2) = \exp \left\{ -\frac{(2\frac{\tau_1}{\theta_1})^2 + (2\frac{\tau_2}{\theta_2})^2}{\theta_1 \theta_2} \right\}$$  \hspace{1cm} (6.41)

and its covariance function is therefore given by

$$\beta(\tau_1, \tau_2) = \sigma^2 \exp \left\{ -\frac{(2\frac{\tau_1}{\theta_1})^2 + (2\frac{\tau_2}{\theta_2})^2}{\theta_1 \theta_2} \right\}$$  \hspace{1cm} (6.42)

Hence, its variance function is given by

$$\gamma(T_1, T_2) = \frac{1}{2} [\gamma(T_1) \gamma(T_2 | T_1) + \gamma(T_2) \gamma(T_1 | T_2)]$$  \hspace{1cm} (6.43)

where
and

\[ \gamma(T_i) = \left[ 1 + \left( \frac{T_i}{\theta_i} \right)^{2 \frac{3}{2}} \right] \]  \hspace{1cm} (6.44)

and

\[ \gamma(T_i|T_j) = \left[ 1 + \left( \frac{T_i}{\theta_j} \right)^{2 \frac{3}{2}} \right] \]  \hspace{1cm} (6.45)

and in which

\[ \theta_j^i = \theta_i \left[ c_\alpha + (1 - c_\alpha) \exp \left\{ - \left( \frac{T_j}{c_\alpha \theta_j} \right)^2 \right\} \right] \]  \hspace{1cm} (6.46)

Fenton (1990) suggested a value of \( c_\alpha = \pi/2 \) should be used in evaluating the above equation which is a result of assuming the directional scales of fluctuation being measured in two planes separated by a 90 degree angle.

Having presented the important features of 2D LAS theory, it is now possible to see how the method is implemented. Figure 6.11 illustrates the process by which recursive subdivision of an initial domain, with a local average of \( Z_{1,1}^0 \), is carried out. The subdivision process for a general cell, \( Z_{j,k}^i \), located at position \((j,k)\) of stage \(i\) within the domain (as illustrated in Figure 6.12), leads to four new cells at stage \(i+1\), which are evaluated as follows:

\[ Z_{1}^{i+1} = Z_{2j,2k}^{i+1} = c_{11} U_{1jk}^{i+1} + \sum_{l=1}^{n_{xy}} a_{1l}^i Z_{m(l),n(l)}^i \] \hspace{1cm} (6.47a)

\[ Z_{2}^{i+1} = Z_{2j,2k-1}^{i+1} = c_{21} U_{1jk}^{i+1} + c_{22} U_{2jk}^{i+1} + \sum_{l=1}^{n_{xy}} a_{2l}^i Z_{m(l),n(l)}^i \] \hspace{1cm} (6.47b)

\[ Z_{3}^{i+1} = Z_{2j-1,2k}^{i+1} = c_{31} U_{1jk}^{i+1} + c_{32} U_{2jk}^{i+1} + c_{33} U_{3jk}^{i+1} + \sum_{l=1}^{n_{xy}} a_{3l}^i Z_{m(l),n(l)}^i \] \hspace{1cm} (6.47c)

\[ Z_{4}^{i+1} = Z_{2j-1,2k-1}^{i+1} = 4Z_{jk}^i - Z_{2j,2k}^{i+1} - Z_{2j,2k-1}^{i+1} - Z_{2j-1,2k}^{i+1} \] \hspace{1cm} (6.47d)
in which: the white Gaussian noise term is represented by $U$ and has a mean of zero, variance of unity and a standard deviation which is defined by coefficients $c$; $m(l)$ and $n(l)$ are the indexing functions which follow a fixed pattern of traversing, as illustrated in Figure 6.13; $n_{xy}$, in this method of implementation, is taken as 9 (which means that 9 parent cells are required in this method); $\{a_{ir}^i\}$ are the weighting coefficients relating to the neighbourhood correlation, and their splitting and subscript numberings are illustrated in Figures 6.14 and 6.15, respectively.

Coefficients $a$ can be estimated in exactly the same manner as in the 1D case: i.e. by multiplying equations (6.47) by $Z_{m(p),n(p)}^i$ on both sides (where $m(p)$ and $n(p)$ are traversing vectors) and then taking the expectations of the results. The final relationships, which are in the form of equation (6.22), are not presented in this section; however, they can be found in Vanmarcke (1984).

The standard deviation of the white Gaussian noise is taken as the matrix $c^{i+1}$, which should satisfy the relationship

$$c^{i+1}(c^{i+1})^T = R$$

in which,

$$R_{rs} = E[Z_{r}^{i+1}Z_{s}^{i+1}] - \sum_{l=1}^{n_{xy}} a_{ir}^i E[Z_{m(l),n(l)}^{i}Z_{s}^{i+1}] ; r, s = 1, 2, 3$$

Note that, in the special case of a homogeneous field, coefficients $a$ and $c$ are evaluated only once at each stage of the LAS process, because they are independent of the field values. This reduces the computation time needed in 2D LAS.

Spencer (2007) considered two kinds of boundary conditions for those cells which are located at the boundaries of the domain. The first method was that proposed by Fenton & Vanmarcke (1990), which assumes that what happens outside the domain will have no effect on what happens inside; this leads to a series of modified coefficients, in the same way as for the 1D case. The second method was proposed by Samy (2003) and considered imaginary boundary cells located immediately outside the domain, leading to no significant modifications to the coefficients. However, Spencer (2007) concluded that neither method could accurately account for the boundaries and thereby suggested another method. His method is what has been used in this thesis; it is a compromise between complexity and accuracy, and is based on the assumption that the ultimate field should represent a small portion of an infinitely large continuous random field.
with a scale of fluctuation of $\theta$ and in which the size of the final domain can be altered to give a desired $\theta/D$. This is based on the correlation between cells inside the domain and those imaginary boundary cells immediately outside the domain being exactly the same. Hence, good estimates can be gained for those imaginary boundary cells, based on the values of the near neighbours inside the domain.

As illustrated in Figure 6.16, for imaginary boundary cells which are located on the edges of the domain, four neighbours need to be considered in 2D; while, according to Figure 6.17, for those located on the corners, only three neighbours must be used. The edge boundary cells at the left-hand side of the field can be estimated by

$$I_{kj}^{l} = \frac{\sum_{l=1}^{4} \left( E[Z_{0,j}^{i}, Z_{p(l),q(l)}^{i}] \times Z_{p(l),q(l)}^{i} \right)}{\sum_{l=1}^{4} E[Z_{0,j}^{i} Z_{p(l),q(l)}^{i}]}$$  (6.50)

in which the index functions traversing the neighbourhood cells within the domain (i.e. $p(l)$ and $q(l)$) are given in Figure 6.18. The above equation can be extended to other edges of the domain by appropriately switching the axes and mirroring the correlation directions. In a similar way, the corner boundary cell at the left-hand side of the field can be estimated by

$$I_{G}^{l} = \frac{\sum_{l=1}^{3} \left( E[Z_{m,n}^{i}, Z_{p(l),q(l)}^{i}] \times Z_{p(l),q(l)}^{i} \right)}{\sum_{l=1}^{3} E[Z_{m,n}^{i} Z_{p(l),q(l)}^{i}]}$$  (6.51)

in which: the index functions traversing the neighbourhood cells within the domain (i.e. $p(l)$ and $q(l)$) are given in Figure 6.19; and $m$ and $n$ represent the positions of the imaginary boundary cell, $I_{G}$, given by

- $G = R1$  \hspace{1cm} $m = 1$  \hspace{1cm} $n = 0$
- $G = X$  \hspace{1cm} $m = 1$  \hspace{1cm} $n = 0$
- $G = X1$  \hspace{1cm} $m = 1$  \hspace{1cm} $n = 0$

as shown in Figure 6.17. It should be noted that, in a similar way as for the edge cells, equation (6.51) can be easily transformed to deal with other corner cells.
6.4 Application to the Jamuna Bridge Abutment

6.4.1 Implementation and Utilisation

In the previous section, LAS theory was reviewed in detail. This section describes how the random fields of state parameter are generated to be used as part of the stochastic analysis of the Jamuna Bridge Abutment. It is therefore divided into 4 main parts, which will allow the reader to understand the sequential process. First, standard Gaussian random fields are generated; they are then transformed to represent the desired mean and standard deviation of state parameter (as derived in Chapter 5); and then further transformed to represent the anisotropy of the material properties; and finally, cropped and mapped on to the finite element mesh at the Gauss point level. In Section 6.4.2.2, example random fields of state parameter for the permanent slope of Jamuna Bridge Abutment will be presented. The results of this chapter are used in Chapter 7 to perform stochastic slope stability analyses.

Note that the methods utilised in this research for random field generation are based on the work of Spencer (2007). In particular, he developed and validated random field generators which could be employed for a number of different geotechnical problems, including the current study. This code was based on the previous work in the author’s research group (e.g. Samy, 2003). His method uses an array-based, rather than a tree-based, LAS random field generation technique. In the array-based method, two arrays are needed to allocate the information about the parent cells and the subdivided cells at any division level during the process. This method was shown to be computationally more efficient than the tree-based method. A blackbox module was therefore developed by Spencer (2007) which carries out the random field generation and its required transformations automatically. The blackbox includes different routines and functions which are capable of post-processing the standard Gaussian random field. The post-processing steps included in the blackbox module are described in more detail in Sections 6.4.1.2 to 6.4.1.6.
6.4.1.1 Random Number Generation

Pseudo random number generators (PRNG’s), as compared to truly random processes, have the advantage of creating random numbers which can be reproduced if the same initiating seed is used. PRNG’s have been widely studied in mathematics and are not described in detail in this thesis. However, the interested reader is advised to look at mathematical books, such as Luby (1996).

The random number generator used for this study is based on previous research in the author’s research group. The author inherited two different codes: one developed by Samy (2003) and another by Spencer (2007). However, the latter was found to be more robust and more efficient with respect to the required amount of memory (Spencer, 2007). Therefore, it has been used in this research to generate random fields. Spencer found that Samy’s PRNG was not random enough when the initial number from consecutive seeds was used. This resulted in a negative bias in the mean of the initial means.

Note that the pseudo random number generator first used by Samy (2003) was based on the ‘ran1’ routine given in the ‘Numerical Recipes’ series of books (Press et al., 1990). This PRNG is from the Lehmer family and uses Park & Miller (1988) criteria and, in order to improve its randomness, its generated numbers are processed by a Bays-Burham shuffle (Mills, 2003).

However, Spencer (2007) developed a new random number generator routine based on Press et al., (2002). His routine used two random number generators: a Park-Miller sequence by Scharge’s method, which has a period of $2^{32} - 2$; and a Marsaglia shift sequence, with a period of $2^{32} - 2$, combined using a logical bit operation. Although this new random number generator was found to have no significant effect on the time taken to generate the field, it effectively reduced the amount of memory needed for the generation process. He further suggested that the first 50 generated values should be discarded, to ensure the independence of the first number used from progressive seeds.
6.4.1.2 Mean and Standard Deviation Transformation

The first post-processing step, after generating a random field, is to transform the field from that characterised by a standard Gaussian distribution (with \( \mu = 0 \) and \( \sigma = 1 \)) into another characterised by a Gaussian distribution which has a mean and standard deviation consistent with that obtained for the material property across the site. If the desired mean and standard deviation are denoted by \( \mu_x \) and \( \sigma_x \), respectively, then

\[
Z_x = \mu_x + \sigma_x Z
\]  

(6.52)

is the transformed cell value of the standard Gaussian random field cell \( Z \) with the desired mean and standard deviation.

However, if the material property that random fields are to be generated for is depth-dependent (as is the case with state parameter for the Jamuna Bridge Site), then the transformation is done by a modified form of equation (6.52), i.e.

\[
Z_x = \mu_{top} + \frac{d}{H} (\mu_{top} - \mu_{bot}) + \left( \sigma_{top} + \frac{d}{H} (\sigma_{top} - \sigma_{bot}) \right) Z
\]  

(6.53)

in which: \( d \) refers to the depth of the centre of the cell, \( Z \), with respect to the total depth of the domain, \( H \); and, the subscripts \( bot \) and \( top \) refer to the desired values at the bottom and top of the field, respectively. Note that the above equation is only applicable to a linear depth-trend. As mentioned earlier, this transformation has been implemented in Spencer’s blackbox routine.

6.4.1.3 Random Field Anisotropy

The second post-processing step, after transforming the mean and standard deviation, is to transform an isotropic field into an anisotropic field with respect to any orthogonal direction. Anisotropy is a consequence of soil deposition and, in the case of natural deposits, is a consequence of natural processes; hence it is dependent on many factors and is therefore a very random process itself. Fenton (1990) stated that the overall statistics of anisotropic processes are poorly preserved by LAS, even though the within-cell covariance structure is reflected closely. Therefore, in order to overcome this problem, a post-processing method was developed by Hicks & Samy (2002b) based on the method suggested by Vanmarcke (1984). This method takes an
isotropic field and then, by squashing or stretching the field, transforms it into an anisotropic field which has the desired scales of fluctuation in each direction.

For the 2D conditions, the horizontal scale of fluctuation, $\theta_h$, is generally larger than the vertical one, $\theta_v$. Hence, the degree of anisotropy, $\xi$, is given by

$$\xi = \frac{\theta_h}{\theta_v}$$  \hspace{1cm} (6.54)

which is assumed to have integer values throughout this study, due to the simplicity of generating such anisotropic random fields.

Figure 6.20 illustrates the process by which the field is squashed. As can be seen, first an isotropic field is generated and then the values of $\xi$ cells are averaged vertically to produce an anisotropic field with a degree of anisotropy of $\xi$. In contrast, stretching of the field is done by first generating a random field with the vertical scale of fluctuation of $\theta_v$; and then widening the field by inserting $\xi - 1$ cells in between the cells, as shown in Figure 6.21. The random values of these new cells can be estimated by two different methods: Samy (2003) suggested that these new cells should be given values which are copied laterally across the desired number of cells (as in Figure 6.21c); while Spencer (2007) suggested that these new cells should be given values which are found by linear interpolation between the field values (as in Figure 6.21d). Note that the method adopted in the current research is based on Spencer’s criterion. This is because it produces a smoother field and can hence better represent the natural variability. Nevertheless, squashing is preferable over stretching, because it avoids the disjointed or interpolated issues of the resulting anisotropic field; on the other hand, squashing has the limitation of being dependant on the final size of the domain (i.e. the squashed field should not be smaller than the size of the domain, in the vertical direction).

Note that squashing and stretching methods can be combined: if the amount of squashing is denoted by $S_q$, and the amount of stretching is denoted by $S_t$, then $\xi = S_q \times S_t$. In this case, the scale of fluctuation for generating the random field should be equal to $S_q \times \theta_v$.

The aforementioned methods for applying the anisotropy to the random field have been implemented in Spencer’s blackbox routine and have been used in this study in combination with the Jamuna Bridge Abutment geometry and its estimated degree of anisotropy. Further details are given in Section 6.4.2.2.
6.4.1.4 Cropping the Field with Random Movement

Usually, problems involving random fields do not comprise rectangular or cuboidal domains that have a number of cells, in any direction, that are equal to a power of 2. Rather, the problems vary with respect to their geometry and degree of soil anisotropy. So, the last post-processing step, before mapping the random field onto the finite element mesh, is to crop the field. Samy (2003) suggested that generating a random field with enough cells and taking the desired domain from the centre of the field would be adequate. However, Spencer (2007) found that, over many realisations, this would result in considerable spatial variation in the point variance of the random field. In order to overcome this problem, he suggested generating a random field which is bigger than the problem size and complies with the required degree of anisotropy; and then, taking the mesh from a random location within the larger field. He showed that this would lead to more uniform fields which have much smaller spatial variations in their point variance over many realisations. It is this method which has been implemented in the blackbox routine and used in the subsequent analyses in this thesis.

Examples of random fields for a square domain of $256 \times 256$ cells are shown in Figures 6.22 and 6.23. The first figure shows the influence of the degree of anisotropy, whereas the second figure shows the influence of varying scales of fluctuation.
6.4.2 Random fields of the Jamuna Bridge Abutment

After cropping a random field to meet the domain size and statistical characteristics of a problem, two more steps are needed to setup all the random fields for the final stochastic finite element analysis of the structure. These are generating the finite element mesh and mapping the material properties onto the mesh. These two steps are described below and example random fields of the Jamuna Bridge Abutment are presented.

6.4.2.1 Mesh Generation

The problem geometry, for which the finite element mesh has to be generated, is based on the general cross-section of the permanent slope of the West Guide Bund of the Jamuna Bridge, as shown in Figure 4.12. Mesh generation was carried out using a code previously developed in the author’s research group (Williams, 1993).

Figure 6.24 illustrates the finite element mesh of the problem, which has element sizes varying within the different regions of the mesh (i.e. $2\text{m} \times 1\text{m}, 0 \leq x \leq 20\text{m}$; $1.7857\text{m} \times 1\text{m}$, $20 \leq x \leq 45\text{m}$; $1.6667\text{m} \times 1\text{m}$, $45 \leq x \leq 120\text{m}$; $1.7857\text{m} \times 1\text{m}$, $120 \leq x \leq 145\text{m}$; and $1.6667\text{m} \times 1\text{m}$, $145 \leq x \leq 160\text{m}$; where $x$ is the distance from the left hand boundary). The reason for having a mesh with varying element sizes is that, due to varying slope angles (i.e. 1:3.5, 1:5.0 and 1:3.5 from left to right), using equal-sized elements would have led to elements other than triangular and trapezoidal shapes at the sloping surface; since the finite element code at the moment is only able to deal with elements of rectangular, triangular and trapezoidal shapes, the author found it easier to vary the mesh size to fit the code restrictions). The variation in the mesh size would not have any significant effect on the finite element analysis of the structure.

Overall, the mesh comprises 2305 8-noded plane strain elements which have $2 \times 2$ Gaussian integration points, meaning that 4 sets of material properties are assigned to each element (as suggested by Hicks & Onisiphorou, 2005). Hence, the in-situ data can be better reproduced without excessive local averaging of the random field. The total number of nodes in the mesh is 7178 and the total number of degrees of freedom is 13888 (N.B. the boundary conditions are fixed at the bottom and rollers – restrained against horizontal displacements – at the sides).
6.4.2.2 Mapping the Random Field onto the Finite Element Mesh

The last step to be taken before performing the stochastic analysis of the structure is to map the cropped field onto the finite element mesh. This has been done by first generating a random field with square cells of $0.5\text{m} \times 0.5\text{m}$, and a total dimension of $160\text{m} \times 39\text{m}$, using the statistics estimated in Chapter 4; then every 2 cells are averaged in the $x$ direction to produce a random field of $1.0\text{m} \times 0.5\text{m}$ cells. In this way, it can be ensured that every Gauss point in the finite element mesh is assigned a cell from the random field. Finally, by placing the resulting random field on top of the mesh and working out the position of the cells relative to the Gauss points, the random values are mapped onto the mesh. This process has been implemented in a separate code written by the author. It uses the finite element mesh and the random field results of Spencer’s blackbox to work out the mapping process.

It should be noted that, although this method is not as accurate as having cells which are exactly half the elements size in each direction at each region within the mesh, nevertheless, this procedure is not too far out and can produce reasonable random fields for the problem (i.e. without significant deviations from the original statistics of the field). Figures 6.25 and 6.26 show examples of the mapped fields and the influence of $\xi$ and $\theta_v$, respectively.
6.5 Conclusion

In this chapter, random field theory was introduced and different methods for generating random fields were briefly described. The LAS method, as proposed by Fenton & Vanmarcke (1990), was chosen as the method of random field generation and the mathematical concepts of local average theory were described in more detail. It was also described how considering different boundary conditions can affect the generated field and why Spencer’s (2007) method was an improvement over Samy’s method. Then, it was described how a random field can be generated and mapped onto the finite element mesh. This process involved: generating a standard Gaussian random field by the PRNG suggested by Spencer (2007); transforming the generated field to produce the target mean and standard deviation; squashing/stretching of the field to produce anisotropy; cropping a field equal in size to the problem domain, from a bigger random field; generating the finite element mesh; and finally, mapping the field values onto the element Gaussian integration points. The first four steps were implemented within a blackbox routine developed by Spencer (2007) and the mapping step has been implemented by the author. Hence, random fields of state parameter can be generated based on the statistics derived in Chapter 4. These random fields can then be back-figured to produce Monot material properties that are dependant on state parameter. These parameters are \( B, C \) and \( E \) and their relationship with state parameter has already been derived in the previous chapter. The resulting multi-variate random fields will then be used, later in Chapter 7, to perform stochastic analysis of the Jamuna Bridge Abutment.
Figure 6.1 Random field generated by the Turning Bands method (Fenton, 1994).

Figure 6.2 Random field generated by the Local Average Subdivision method (Fenton, 1994).

Figure 6.3 Random field generated by the Generalised Stochastic Subdivision method (Lewis, 1987).
Figure 6.4 Sample functions of a local average process (Samy, 2003).
Figure 6.5 Variance function (Vanmarcke, 1984).

Figure 6.6 Progression of field generation in 1D LAS (Spencer, 2007).
### Figure 6.7 General cell arrangement (Spencer, 2007).

![Cell Arrangement Diagram]

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<th>Stage $i$</th>
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### Figure 6.8 Local averaging in 2D over a rectangle (Samy, 2003).

![Local Averaging Diagram]
Figure 6.9 Areas under consideration in covariance analysis (Spencer, 2007).

Figure 6.10 Special case of similar rectangles.
Figure 6.11 Local Average Subdivision process in 2D (Spencer, 2007).

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Figure 6.13 Traversing pattern of index functions (Spencer, 2007).

Figure 6.14 Splitting of a typical parent cell for the next level down (Spencer, 2007).

Figure 6.15 Weighting coefficients associated with their respective parent cell (Spencer, 2007).
Figure 6.16 Neighbourhood for weighting of imaginary edge cells (Spencer, 2007).

Figure 6.17 Neighbourhood for weighting of imaginary corner cells (Spencer, 2007).
Figure 6.18 Traversing pattern and values for index functions $p(l)$ and $q(l)$ for edge cells (Spencer, 2007).

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Figure 6.19 Traversing pattern and values for index functions $p(l)$ and $q(l)$ for corner cells (Spencer, 2007).

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Figure 6.20 Squashing an isotropic field to produce an anisotropic field ($\xi = 6$).

Figure 6.21 Methods of stretching an isotropic field ($\xi = 3$).
Figure 6.22 Influence of the degree of anisotropy on a random field of 256 × 256 cells of of 0.25m × 0.25m ($\mu = -0.055, \sigma = 0.07, \theta_o = 1.0m$).
Figure 6.23 Influence of the vertical scale of fluctuation on a random field of $256 \times 256$ cells of $0.25\text{m} \times 0.25\text{m}$ ($\mu = -0.055, \sigma = 0.07, \xi = 8$).
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Figure 6.26 Influence of the vertical scale of fluctuation on random fields of the Jamuna Bridge Abutment $(\mu = -0.055, \sigma = 0.07, \xi = 8)$. 
Chapter 7

Stochastic Finite Element Analysis of the Jamuna Bridge Abutment
Chapter 7

Stochastic FEA of the Jamuna Bridge Abutment

7.1 Introduction

In this chapter, stochastic finite element analyses of the Jamuna Bridge Abutment are carried out, based on the results of the previous chapters and using the incremental finite element algorithm MONICA (Hicks, 1995a & 1995b) which incorporates the constitutive soil model Monot (Molenkamp, 1981).

The objective of the chapter is to investigate the influence of material heterogeneity on the structural response of the Jamuna Bridge Abutment, by performing parametric studies in which different factors are varied; that is, the degree of anisotropy of the heterogeneity, the vertical scale of fluctuation and the depth-dependency of the material properties statistics. The slope failures observed during the construction of the Jamuna Bridge Abutment are then evaluated, in the light of failure mechanisms computed during the finite element investigation. In particular, the computed results are discussed in detail.

This chapter is divided into four further sections: Section 7.2 describes the finite element algorithm MONICA; Section 7.3 explains the parametric stochastic finite element analysis of the Jamuna Bridge Abutment; Section 7.4 discusses the results in more detail and explains the influence of different factors on the structural response; and Section 7.5 summarises the chapter and its main conclusions.

7.2 Description of the finite element algorithm MONICA

MONICA (short for “MONot Incremental Computer Algorithm”) is a general purpose finite element computer code for solving boundary value problems, incorporating the double-hardening constitutive soil model Monot (Hicks, 1990). It is based on the initial stress algorithm of Zienkiewicz et al. (1969), in which loads or displacements are applied incrementally and in which the stiffness matrix is assumed to be constant within an increment. For each increment, those stresses which violate the yield criterion are redistributed as internally applied excess nodal loads and an iterative procedure is performed, until the change of displacement in two consecutive iterations becomes less than a specified tolerance thereby indicating convergence. MONICA has five distinctive features as described below (Hicks, 1990):
(a) Elastoplastic stiffness matrix

The highly nonlinear nature of Monot requires the global elastoplastic stiffness matrix [K] to be updated at the beginning of every increment. This is done by estimating the stress state and stress region (refer to Section 5.2) at the end of the previous increment. Since the plastic deviatoric component of Monot follows a non-associated flow-rule, using a tangent stiffness matrix would lead to a non-symmetric [K]. Therefore, to overcome this problem and also to avoid the algorithm becoming unstable for liquefiable behaviour, Hicks (1990) modified the integration of the elastoplastic stress-strain relationship, by ignoring the plastic deviatoric model component during the stiffness matrix integration. This led to the forming of $[D^{ep}]_{mod}$, which is the modified elastoplastic stress-strain relationship based only on the elastic and the associated plastic compressive model component. Table 7.1 shows how, at the start of every increment, $[D^{ep}]_{mod}$ is calculated using either the nonlinear elastic component, or a combination of the nonlinear elastic component with the associated plastic compressive model component. Note that this modified stress-strain matrix is symmetric and therefore requires only half the memory of the full stress-strain matrix. Figure 7.1 illustrates how the global elastoplastic stiffness matrix is subsequently formed after deriving $[D^{ep}]_{mod}$.

(b) Subincrements of strain

In order to reduce deviations of the computed response from the actual Monot stress-strain response, Hicks (1990) suggested subdividing the strain increments into smaller subincrements. Subdivision provides the possibility of using larger increments, as well as enabling the better modelling of stress region transitions during a load increment. Moreover, it makes up for errors, due to erroneous stress region estimates, by averaging $[D^{ep}]$ over an increment to achieve a smoother response. Subdividing the strain increment $\{d\varepsilon\}$ into $n$ equal subincrements $\{d\varepsilon_{sub}\}$ means that the stress state $\{\sigma_n\}$ at the end of the increment is calculated by:

$$\{\sigma_n\} = \{\sigma_0\} + \sum_{i=1}^{i=n} [D^{ep}]_{i-1} \{d\varepsilon_{sub}\}$$ \hspace{1cm} (7.1)

where $\{\sigma_0\}$ is the stress state at the beginning of the increment. Figure 7.2 shows an example in which an increment of strain is subdivided into 4 equal subincrements.
(c) Stress region estimate

The most important feature of MONICA is its ability to correctly estimate the stress region at the beginning of every increment. The method by which stress regions are estimated follows a robust, but simple algorithm, in which the stress region at the beginning of each increment/subincrement is considered to be the stress region in which the stress state was located at the end of the previous increment/subincrement (Hicks, 1990). However, the first increment does not follow this approach, because there is no available previous increment. Hence, it is assumed that the first increment lies within the elastic stress region (refer to Figure 5.2). Note that the stiffness matrix is based only on the stress region at the beginning of the increment and is not updated during the iteration process (Hicks, 1990).

(d) Error loads

One of the differences between MONICA and most initial stress algorithms is that a cruder tolerance is often needed for the algorithm to “converge”. Hicks (1990) suggested that a tolerance of about 0.02 would be sufficient for most problems. However, using a crude tolerance means that the computed response could significantly deviate from the actual, fully converged, solution. To tackle this problem, Hicks (1990) suggested estimating the error in the computed body loads at convergence, \( \{dF_{\text{err}}\} \), such that

\[
\{dF_{\text{err}}\} = \{dF_n\} - \{dF_{n-1}\}
\]  

(7.2)

in which “body loads” refers to those loads which are redistributed as internally applied nodal loads due to excess stresses violating the yield criterion. \( \{dF_{n-1}\} \) is the body loads vector at the end of the iteration prior to convergence and \( \{dF_n\} \) is the body loads vector at convergence. \( \{dF_{\text{err}}\} \) therefore represents the unconverged body loads at the end of an increment and may be added to the externally applied loads vector \( \{dF_{\text{ext}}\} \) in the next increment in order to correct errors and achieve reliable estimates.
(e) An improved no-tension correction

The singularity of Monot’s failure surface at its apex can cause numerical instability and so, to avoid this problem, Hicks (1990) suggested using a deviatoric plane at \( s' = 0.006 \text{ kPa} \) to act as a no-tension surface. As seen in Figure 7.3, this correction means that those stress points which drift into the tensile stress space need to be corrected, such that they are returned onto or just inside the tensile area. This is done by linearly interpolating between the final and initial stress states so that,

\[
\{\sigma_I\} = \{\sigma_{i-1}\} + A[\{\sigma_i^*\} - \{\sigma_{i-1}\}]
\]

in which \( \{\sigma_i^*\} \) represents the illegal normal stress state, and \( \{\sigma_I\} \) and \( \{\sigma_{i-1}\} \) are the legal stress states at the end and beginning of the increment, respectively. \( A \) is a factor which is defined as

\[
A = \left[ \frac{s_i' - s_{i-1}'}{s_i'' - s_{i-1}'} \right] \times 0.9999
\]

where \( s_i'' \) is the isotropic stress invariant for the illegal stress state, and \( s_i' \) and \( s_{i-1}' \) are isotropic stress invariants corresponding to the legal stress states at the end and beginning of the increment. Note that the factor 0.9999 is used to ensure that the corrected stress state lies just inside tension cutoff.

7.3 Finite Element Analysis of the Jamuna Bridge Abutment

As mentioned in Chapter 6 and illustrated in Figure 6.24, the finite element mesh for the Jamuna Bridge Abutment has 2305 elements, 7178 nodes and 13888 degrees of freedom. The initial stresses have been prescribed assuming a buoyant unit weight of \( \gamma' = 8.9 \text{ kN/m}^3 \) and a coefficient of earth pressure at rest of \( K_0 = 0.44 \) (as presented in Chapter 3). Note that, as seen in Figure 4.12, the slope is not fully submerged in the water and so in computing the initial stresses, it has been assumed that the sea water level is at +9.5m PWD (as illustrated in Figure 4.12). The Monot soil model parameters are based on the calibrations to laboratory data in Chapter 5, the random field generation methods described in Chapter 6 and the state parameter statistics presented in Chapter 4. The structure has then been subjected to a sudden increase in gravitational loading in an attempt to trigger static liquefaction. Undrained effective stress conditions have been simulated by assuming a large bulk modulus to represent
the incompressible pore water. Note that there are other methods of loading such as cyclic loading that can trigger liquefaction (i.e. catastrophic slides). However, rapid undrained monotonic loading due to increasing gravity is used in this study because Monot, by definition, is derived for such loading conditions.

The geo-structural responses have been presented in terms of the average sloping surface settlement versus the additional gravitational loading (relative to in-situ conditions) applied to the structure (i.e. $g_{\text{new}}/g$ where $g = 9.81 \text{ m/s}^2$), as illustrated in Figures 7.4, 7.6, 7.9, 7.12, 7.15, 7.18, 7.21, 7.24 and 7.27. Note that the average slope surface settlement has been used because the failure mechanisms for the realisations of the stochastic analyses are variable in nature and often localised within the slope; hence, crest settlement would not always be a good indicator of slope failure.

Figure 7.4 illustrates the deterministic responses for different state parameter values ranging from +0.06 to -0.18. For each case, the Monot model parameters are estimated using the relationships presented in Table 5.8. Note that the gravitational loading has been applied in increments of 0.008 times the acceleration due to gravity (as suggested by Wong, 2004).

Table 7.2 shows the 8 cases considered for the stochastic parametric study: Cases 1 to 3 are based on the average statistical properties across the site (i.e. $\mu = -0.055$, $\sigma = 0.07$, $\theta_v = 1.18 \text{ m}$); they don’t account for depth-dependency, but consider different degrees of anisotropy of heterogeneity in the random fields (i.e. $\xi = 1, 8$ and 20). Cases 4 to 6 are also based on the average statistical characteristics of the site; however, in contrast to the first three cases, they do account for depth dependency of the statistics in the random field generation. Cases 7 and 8 compare depth-dependent and depth-independent assumptions for the largest vertical scale of fluctuation (i.e. 2.4m) recorded at the site, assuming a degree of anisotropy of 20.

Note that depth dependency has been incorporated by using the results of Chapter 4; specifically, by using the slope of the mean trend line for state parameter and the variations of standard deviation with depth illustrated in Figure 4.23. However, in order to ensure that the statistics used in the random field generation do not deviate from the average statistical characteristics of the site, the top and bottom values for the mean and standard deviation of the state parameter have been interpolated. This means that, first, the results of Table 4.5 have been used to work out the linear trend line for the mean and standard deviation; then, based on the equation of this line, $\mu_{\text{top}}, \mu_{\text{bot}}, \sigma_{\text{top}}$
and \( \sigma_{\text{bot}} \) have been calculated such that their average values are equal to the average values of the mean and standard deviation across the site. This leads to \( \mu_{\text{top}} = -0.058 \), \( \mu_{\text{bot}} = -0.052 \), \( \sigma_{\text{top}} = 0.088 \) and \( \sigma_{\text{bot}} = 0.048 \). Note that this interpolation ensures that the state parameter statistics used in generating the random fields for Cases 4 to 6 are consistent with those used in Cases 1 to 3, i.e. they all have the same mean and standard deviation.

For each stochastic case, 20 realisations are analysed and the structural responses of all realisations are plotted, together with the deterministic response based on the mean state parameter for comparative purposes. The loading increment for the stochastic cases is taken to be 0.006 times the gravitational acceleration (Wong, 2004). Hence, as the maximum gravitational loading applied to the structure is taken to be \( g_{\text{new}}/g = 2.2 \), for each realisation the maximum number of increments applied is 200.

Figures 7.5, 7.7, 7.8, 7.10, 7.11, 7.13, 7.14, 7.16, 7.17, 7.20, 7.22, 7.23, 7.25, 7.26, 7.28 and 7.29 show contours of accumulated shear strain invariant for the deterministic case and for representative weaker and stronger responses of each stochastic case. These contours help in studying the failure mechanisms of the Jamuna Bridge Abutment. Note that, even though for each case there are 20 computed responses, only those deformation mechanisms which the author has found to have occurred most often are illustrated in these figures. In each figure, automatic scaling has been used; the blue areas represent the minimum shear strain invariant values and the red areas represent the maximum shear strain invariant values. Note that the mobilised gravitational loading at which these responses have been observed are also presented in each figure. The figures are grouped so that the contours are presented under the structural response curves. This makes it easier for the reader to compare the results of each case and better understand the effect of material heterogeneity.

### 7.4 Discussion of the results

Figure 7.4 shows the results of deterministic analyses for various state parameter values and indicates that, for \( \psi \geq -0.035 \), the structure clearly fails; in contrast, for \( \psi < -0.035 \), the structure becomes more stable due to the suppression of dilation at higher shear stress levels. It is seen in the figure that, for \(-0.035 > \psi > -0.05 \), this tendency for strength increase is modest, whereas for \( \psi < -0.05 \) the strengthening is
more obvious. This response is consistent with the definition of state parameter; i.e. negative values represent a dilative mechanical behaviour and positive values represent a liquefiable behaviour. Note that the maximum gravitational load that can be sustained by the structure varies significantly as the state parameter varies from positive to increasingly negative. Also note that, for the average state parameter at the Jamuna Bridge Site (i.e. \( \psi = -0.055 \)) the maximum \( g_{\text{new}}/g \) for which the structure can withstand is around 2.2.

The accumulated shear strain invariant contour for this case is illustrated in Figure 7.13. As can be seen, the failure mechanism consists of two almost separate failed zones that extend from the toe of the slope to the slope crest. This mechanism differs from the actual failures observed during the excavation of the Jamuna Bridge Abutment (Figures 4.13 to 4.16). Specifically, the actual failures were global (rather than local) and took the form of shallow failed zones that either cut through the top half of the slope (as in Figure 4.16) or along the surface of the slope (as in Figures 4.13 to 4.14). Hence, a single deterministic analysis, based on the mean state parameter for the site, is not adequate to explain the failure mechanisms of the Jamuna Bridge Abutment. Note that the accumulated shear strain invariant contours are better representative to describe the failure mechanisms, as compared to the excess pore pressures which are usually used to study liquefactions, because, due to the rapid undrained loading nature of the problem, the whole load will be taken by the pore waters in the first instance. The pore pressures will therefore change instantaneously from hydrostatic to something bigger which have a distribution similar to the strong and weak zones. This was previously studied by Bakhtiari & Hicks (2008), where it was found that the excess pore pressure distributions have a similar distribution to the accumulated shear strain invariants.

The stochastic responses for Cases 1 to 3, presented in Figures 7.6, 7.9 and 7.12, indicate that, as the degree of anisotropy of the heterogeneity is increased, the range of possible structural responses increases. It is also seen for the isotropic case, presented in Figure 7.6, that all realisations involve structural failure at a load less than that for the deterministic analysis based on the mean value of state parameter; that is, the deterministic analysis based on the mean gives an upper bound solution. However, as can be seen from Figures 7.7 and 7.8, in this investigation, isotropic heterogeneity leads to failures that occur locally at the top and bottom of the slope; that is, where the gradient of the slope changes.
This is in contrast to the findings of Wong (2004), who concluded that pockets of loose and dense materials co-existing in an isotropic field average their effects out leading to the average stochastic response being close to the deterministic response based on the average material property values. However, the geometry of the Jamuna Bridge Abutment seems to be an important factor which may account for this contradiction. Because of the gradient of the slope changing at two points along the slope surface (as seen from the finite element mesh in Figure 6.24), there is a stress concentration at the points where the gradient of the slope change. This leads to an increased possibility of local failures at these locations. Note that this stress concentration is also present in the deterministic case (Figure 7.5). However, due to the highly heterogeneous nature of the random field for the stochastic isotropic case (as seen from Figure 6.25a), local failures can take place independently at any of these points of stress concentration (as compared to the homogeneous case in which the structure acts as a unified body), which then leads to a weaker global response. Note that, as the degree of anisotropy increases and the correlation between material properties becomes stronger in the horizontal direction, the effect of these points of stress concentration also changes, as will be described later in this section. The mean stochastic response for the isotropic case is approximately equivalent to the deterministic response for an average state parameter of $\psi = -0.035$, while the weakest isotropic stochastic response is equivalent to a deterministic response for $\psi = -0.30$.

Figures 7.9 and 7.12 show that, for higher degrees of anisotropy, not only does the range of structural responses increase, but also the type of response changes. Specifically, two distinct structural responses can be observed from these figures: one is a weaker (i.e. looser) response in which the structure fails at a lower mobilised gravitational loading; the other is a stronger response, in which the structure can take higher gravitational loads. It is seen from these figures that, as the degree of anisotropy becomes larger, there are a greater proportion of stronger responses; that is, stronger responses than the deterministic response based on the average state parameter are observed. Moreover, the deformation mechanisms for these two cases differ from those of the isotropic case. As can be seen from Figures 7.10 and 7.13, the failure zones for the looser responses seem to be localised near the locations of changing slope geometry, in a similar manner to the isotropic case. However, as the degree of anisotropy increases, the failure zones become deeper and more extensive, as observed in Figures 7.13. This is reasonable, because, as the degree of anisotropy increases, the
random field becomes more layered (as seen in Figure 6.25), so that failure is attracted to semi-continuous looser zones, even if these occur at greater depths. Note that this explanation is consistent with the observations of Hicks & Onisiphorou (2005) relating to the Nerlerk berm stochastic study. They concluded that, as the scale of fluctuation increases, even if only in one direction, continuous liquefied zones can be observed in semi-continuous weaker “layers”, leading to overall (i.e. global) instability. Figures 7.11 and 7.14 indicate that equivalent conclusions can be made for the stronger structural responses; i.e. as the degree of anisotropy increases the larger deformations become deeper and more extensive. However, for the stronger responses, the deformation mechanisms tend to be global and not localised around the locations of changing slope geometry.

Comparing the deformation mechanisms in Figures 7.13 and 7.14 to the actual failures that occurred during the excavation of the Jamuna Bridge Abutment (Figures 4.13 to 4.16), it is seen that the mechanisms observed for the case where $\xi = 20$ better match site observations. Hence, it may be concluded that the Jamuna River Site has a degree of anisotropy of the heterogeneity of the order of $\xi = 20$ which seems reasonable for a natural deposit. Once again, it is clear that a deterministic analysis based upon the mean property values is completely inadequate for representing the possible structural responses and failure mechanisms for a naturally deposited soil structure. Figures 7.9 and 7.12 show that the lower and upper bounds of the anisotropic structural responses are equivalent to deterministic responses varying from $0.0 > \psi > -0.12$. However, while the loose responses are associated with local failures leading eventually to global instability, the stronger responses are associated with a stable structure, as indicated by the significantly lower average settlements.

The stochastic results for Cases 4 to 6, presented in Figures 7.15, 7.18 and 7.21, indicate that the structural response is not affected much by incorporating the depth-dependency of the state parameter in the analyses. As can be seen, for the isotropic case, the structural responses are still all loose and on the weak side of the deterministic response based on the mean property values. Also, for the anisotropic analyses, the range of solutions is very similar to those of Cases 2 and 3, and there are stronger responses as the degree of anisotropy is increased. Note that the depth-dependency of the standard deviation of state parameter is large, whereas the variation of the mean state parameter with depth is much smaller. However, since, in these three cases, the average of the top and bottom mean and standard deviation of the random
field are the same as the overall mean and standard deviations of the site, it seems reasonable for the range of structural responses to be similar (in these cases) to the first three stochastic cases in which the depth-trends are not implemented.

Figures 7.16, 7.17, 7.19, 7.20, 7.22 and 7.23 show that, although incorporating the depth-dependency of the state parameter statistics in the stochastic analyses of the structure has not affected the range of structural responses to a great extent, it nevertheless has affected the deformation mechanisms and their evolution. It is seen from these figures that, for the isotropic case, the failure mechanism is more spider-web shaped than for the first stochastic case (Case 1); but, as the degree of anisotropy increases, the mechanisms tend to become less spider-web shaped and more similar in appearance to those of Cases 2 and 3. Hence, it can be concluded that, for this natural deposit (where the degree of anisotropy is large), incorporating the depth-dependency of the material properties statistics does not have a significant effect on the range of possible structural responses and the way in which deformation mechanisms develop.

Figures 7.24 and 7.27 show that, as the vertical scale of fluctuation increases, the range of possible structural responses increases as well. It is seen from these figures that, when $\theta_v = 2.4m$ and $\xi = 20$, the lower and upper bounds of the structural responses are equivalent to deterministic responses in the range of $0.05 > \psi > -0.12$. Hence, as for Cases 3 and 6, the structural responses vary from very weak to relatively strong behaviour. Figures 7.25 to 7.29 indicate that, as for Cases 3 and 6, there are two deformation mechanisms that dominate the structural behaviour and the way in which potential failure zones are formed: the weaker responses have failure mechanisms that are smaller and mainly near the top of the slope (as in Figures 7.25 and 7.28); whereas the stronger responses show mechanisms that are deeper and more extensive (as in Figures 7.26 and 7.29). Figure 7.26 shows that the shear strain invariant contours for the case in which $\theta_v = 2.4m$ are deeper than that for $\theta_v = 1.18m$ (i.e. Figure 7.14). The results may explain why the largest failures, during the excavation of the Jamuna Bridge, occurred in the middle of the West Guide Bund Abutment where the vertical scales of fluctuation were larger (as explained in Section 4.6.2).
7.5 Summary

In this chapter, the finite element computer algorithm MONICA was briefly described and then used to perform a series of parametric studies for the Jamuna Bridge Abutment. The finite element mesh used in these analyses was the same as that introduced in Chapter 6. The structure was subject to undrained gravitational loading to trigger static liquefaction. Note that one could have considered the effect of pore pressure migration in these analyses by implementing the Biot formulation available in MONICA. However, for the purpose of the current research, the analyses have been simplified to undrained effective stress conditions.

Deterministic analyses of the slope for different state parameter values indicated that, for $\psi > -0.035$, the structure clearly fails (i.e. liquefaction) and that, for higher state parameter values, the structure can withstand higher gravitational loads (due to it behaving in a more dilative manner; i.e. tensile pore pressures hold the structure together).

Eight different stochastic cases, each consisting of 20 realisations, were considered to study the influence of heterogeneity on the structural response and the possible failure mechanisms. It was found that, as the degree of anisotropy of the heterogeneity increases, the range of structural responses increases as well. For the isotropic cases, the range of responses was relatively smaller and tended to result in more localised failures. It was found that, for the isotropic case, a single deterministic analysis based on the mean state parameter value of -0.030 was equivalent to the weakest stochastic response. For the anisotropic cases, it was found that there are two different types of deformation mechanism. It was also found that, as the vertical scale of fluctuation becomes bigger, the range of possible structural responses increases and failure is more likely. Moreover, it was found that considering the depth-dependency of the material property statistics for this case history has not had a significant effect on the range of structural responses, although some difference in the possible failure mechanisms have been observed.

Finally, it was found that the failed zones observed during the excavation of the West Guide Bund of the Jamuna Bridge Abutment could be closely predicted if heterogeneity was considered in the finite element analyses. In particular, it was found that, for such a natural deposit, a large degree of anisotropy (in the range of 20) could account for the deformation mechanisms observed on site.
### Table 7.1
Formulation of the modified elastoplastic stress-strain matrix, $[D_{\text{mod}}^{\text{op}}]$ (Williams, 1993).

<table>
<thead>
<tr>
<th>Stress Region</th>
<th>Modified elastoplastic stress-strain matrix $[D_{\text{mod}}^{\text{op}}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 1</td>
<td>$[D^e]$</td>
</tr>
<tr>
<td>Region 2</td>
<td>$[D^e]$</td>
</tr>
<tr>
<td>Region 3</td>
<td>$[D^e]-[D^c]$</td>
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<tr>
<td>Region 4</td>
<td>$[D^c]-[D^c]$</td>
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### Table 7.2
Different state parameter stochastic cases considered in the parametric study.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\mu_{\text{bot}}$</th>
<th>$\mu_{\text{top}}$</th>
<th>$\sigma_{\text{bot}}$</th>
<th>$\sigma_{\text{top}}$</th>
<th>$\theta_{\nu}(m)$</th>
<th>$\xi$</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>-0.055</td>
<td>-0.055</td>
<td>0.07</td>
<td>0.07</td>
<td>1.18</td>
<td>1</td>
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<tr>
<td>2</td>
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<td>0.07</td>
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<td>8</td>
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<td>-0.055</td>
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<td>0.07</td>
<td>1.18</td>
<td>20</td>
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<tr>
<td>4</td>
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<td>-0.058</td>
<td>0.048</td>
<td>0.088</td>
<td>1.18</td>
<td>1</td>
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<td>5</td>
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<td>-0.058</td>
<td>0.048</td>
<td>0.088</td>
<td>1.18</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>-0.052</td>
<td>-0.058</td>
<td>0.048</td>
<td>0.088</td>
<td>1.18</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>-0.055</td>
<td>-0.055</td>
<td>0.07</td>
<td>0.07</td>
<td>2.4</td>
<td>20</td>
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<tr>
<td>8</td>
<td>-0.052</td>
<td>-0.058</td>
<td>0.048</td>
<td>0.088</td>
<td>2.4</td>
<td>20</td>
</tr>
</tbody>
</table>
Figure 7.1 Structure chart showing the derivation of the global stiffness matrix (Williams, 1993).
Figure 7.2 Illustrating the use of subincrements to reduce numerical drift from the correct solution (Hicks, 1995a).

Figure 7.3 Illustration of MONICA’s original no-tension correction (Williams, 1993).
Figure 7.4 Deterministic responses for different state parameter values.

Figure 7.5 Accumulated shear strain invariant contours for the deterministic case, $\psi = -0.055$, at $g_{new}/g = 2.225$. 
Figure 7.6 Stochastic responses for $\mu=-0.055$, $\sigma_{\text{top}}=\sigma_{\text{bot}}=0.07$, $\theta_c=1.18\text{m}$, $\xi=1$.  

Figure 7.7 Accumulated shear strain invariant contours for a weaker stochastic response at $g_{\text{new}}/g = 1.89$ ($\mu=-0.055$, $\sigma_{\text{top}}=\sigma_{\text{bot}}=0.07$, $\theta_c=1.18\text{m}$, $\xi=1$).  

Figure 7.8 Accumulated shear strain invariant contours for a stronger stochastic response at $g_{\text{new}}/g = 2.08$ ($\mu=-0.055$, $\sigma_{\text{top}}=\sigma_{\text{bot}}=0.07$, $\theta_c=1.18\text{m}$, $\xi=1$).
Figure 7.9 Stochastic responses for $\mu = -0.055$, $\sigma_{\text{top}} = \sigma_{\text{bot}} = 0.07$, $\theta_c = 1.18\text{m}$, $\xi = 8$.

Figure 7.10 Accumulated shear strain invariant contours for a weaker stochastic response at $g_{\text{new}}/g = 1.59$ ($\mu = -0.055$, $\sigma_{\text{top}} = \sigma_{\text{bot}} = 0.07$, $\theta_c = 1.18\text{m}$, $\xi = 8$).

Figure 7.11 Accumulated shear strain invariant contours for a stronger stochastic response at $g_{\text{new}}/g = 2.2$ ($\mu = -0.055$, $\sigma_{\text{top}} = \sigma_{\text{bot}} = 0.07$, $\theta_c = 1.18\text{m}$, $\xi = 8$).
Figure 7.12 Stochastic responses for $\mu = -0.055$, $\sigma_{\text{top}} = \sigma_{\text{bot}} = 0.07$, $\theta = 1.18\text{m}$, $\xi = 20$.

Figure 7.13 Accumulated shear strain invariant contours for a weaker stochastic response at $g_{\text{new}}/g = 1.68$ ($\mu = -0.055$, $\sigma_{\text{top}} = \sigma_{\text{bot}} = 0.07$, $\theta = 1.18\text{m}$, $\xi = 20$).

Figure 7.14 Accumulated shear strain invariant contours for a stronger stochastic response at $g_{\text{new}}/g = 2.2$ ($\mu = -0.055$, $\sigma_{\text{top}} = \sigma_{\text{bot}} = 0.07$, $\theta = 1.18\text{m}$, $\xi = 20$).
Figure 7.15 Stochastic responses for $\mu_{\text{top}}=-0.058$, $\mu_{\text{bot}}=-0.052$, $\sigma_{\text{top}}=0.088$, $\sigma_{\text{bot}}=0.048$, $\theta=1.18\text{m, } \xi=1$.

Figure 7.16 Accumulated shear strain invariant contours for a weaker stochastic response at $g_{\text{new}}/g = 1.76$ ($\mu_{\text{top}}=-0.058$, $\mu_{\text{bot}}=-0.052$, $\sigma_{\text{top}}=0.088$, $\sigma_{\text{bot}}=0.048$, $\theta=1.18\text{m, } \xi=1$).

Figure 7.17 Accumulated shear strain invariant contours for a stronger stochastic response at $g_{\text{new}}/g = 1.98$ ($\mu_{\text{top}}=-0.058$, $\mu_{\text{bot}}=-0.052$, $\sigma_{\text{top}}=0.088$, $\sigma_{\text{bot}}=0.048$, $\theta=1.18\text{m, } \xi=1$).
Figure 7.18 Stochastic responses for $\mu_{\text{top}}=-0.058$, $\mu_{\text{bot}}=-0.052$, $\sigma_{\text{top}}=0.088$, $\sigma_{\text{bot}}=0.048$, $\theta=1.18m$, $\zeta=8$.

Figure 7.19 Accumulated shear strain invariant contours for a weaker stochastic response at $g_{\text{new}}/g = 1.62$ ($\mu_{\text{top}}=-0.058$, $\mu_{\text{bot}}=-0.052$, $\sigma_{\text{top}}=0.088$, $\sigma_{\text{bot}}=0.048$, $\theta=1.18m$, $\zeta=8$).

Figure 7.20 Accumulated shear strain invariant contours for a stronger stochastic response at $g_{\text{new}}/g = 2.2$ ($\mu_{\text{top}}=-0.058$, $\mu_{\text{bot}}=-0.052$, $\sigma_{\text{top}}=0.088$, $\sigma_{\text{bot}}=0.048$, $\theta=1.18m$, $\zeta=8$).
Figure 7.21 Stochastic responses for $\mu_{\text{top}}=-0.058$, $\mu_{\text{bot}}=-0.052$, $\sigma_{\text{top}}=0.088$, $\sigma_{\text{bot}}=0.048$, $\theta=1.18\text{m}$, $\xi=20$.

Figure 7.22 Accumulated shear strain invariant contours for a weaker stochastic response at $g_{\text{new}}/g = 1.68$ ($\mu_{\text{top}}=-0.058$, $\mu_{\text{bot}}=-0.052$, $\sigma_{\text{top}}=0.088$, $\sigma_{\text{bot}}=0.048$, $\theta=1.18\text{m}$, $\xi=20$).

Figure 7.23 Accumulated shear strain invariant contours for a stronger stochastic response at $g_{\text{new}}/g = 2.2$ ($\mu_{\text{top}}=-0.058$, $\mu_{\text{bot}}=-0.052$, $\sigma_{\text{top}}=0.088$, $\sigma_{\text{bot}}=0.048$, $\theta=1.18\text{m}$, $\xi=20$).
Figure 7.24 Stochastic responses for $\mu_{\text{top}} = \mu_{\text{bot}} = -0.055$, $\sigma_{\text{top}} = \sigma_{\text{bot}} = 0.07$, $h = 2.4\text{m}$, $\xi = 20$.

Figure 7.25 Accumulated shear strain invariant contours for a weaker stochastic response at $g_{\text{new}}/g = 1.76$
($\mu_{\text{top}} = \mu_{\text{bot}} = -0.055$, $\sigma_{\text{top}} = \sigma_{\text{bot}} = 0.07$, $h = 2.4\text{m}$, $\xi = 20$).

Figure 7.26 Accumulated shear strain invariant contours for a stronger stochastic response at $g_{\text{new}}/g = 2.2$
($\mu_{\text{top}} = \mu_{\text{bot}} = -0.055$, $\sigma_{\text{top}} = \sigma_{\text{bot}} = 0.07$, $h = 2.4\text{m}$, $\xi = 20$).
Figure 7.27 Stochastic responses for $\mu_{\text{top}} = -0.058$, $\mu_{\text{bot}} = -0.052$, $\sigma_{\text{top}} = 0.088$, $\sigma_{\text{bot}} = 0.048$, $\theta = 2.4\text{m}, \zeta = 20$.

Figure 7.28 Accumulated shear strain invariant contours for a weaker stochastic response at $g_{\text{new}}/g = 1.62$ ($\mu_{\text{top}} = -0.058$, $\mu_{\text{bot}} = -0.052$, $\sigma_{\text{top}} = 0.088$, $\sigma_{\text{bot}} = 0.048$, $\theta = 2.4\text{m}, \zeta = 20$).

Figure 7.29 Accumulated shear strain invariant contours for a stronger stochastic response at $g_{\text{new}}/g = 2.2$ ($\mu_{\text{top}} = -0.058$, $\mu_{\text{bot}} = -0.052$, $\sigma_{\text{top}} = 0.088$, $\sigma_{\text{bot}} = 0.048$, $\theta = 2.4\text{m}, \zeta = 20$).
Chapter 8

Conclusion
Chapter 8

8 Conclusion

8.1 Introduction

In this chapter of the thesis, the results and conclusions from the previous chapters are summarized and recommendations for future studies are given. Some of the comments are problem-specific and belong to the Jamuna Bridge case study, while other comments are general and are applicable to a variety of problems.

8.2 Research conclusions

The outcome of this study can be concluded in the bullet points below (dot bullet-points indicate site specific conclusions and square bullet-points indicate conclusions that can be applicable to other geo-structures as well as the Jamuna Bridge Abutment):

- The state parameter for the Jamuna River Sand follows a Normal distribution both before and after removing the depth trend. It varies between \(-0.175 \leq \mu_\psi \leq 0.065\) across the site, which is consistent with material ranging from very dense to highly contractive as reported in the original Fugro reports (1986 & 1996). Also, the Upper Sand Layer is very variable, as indicated by the high standard deviation and the small vertical scales of fluctuation.

- The Lognormal distribution best represents the variations in the tip resistance for the Jamuna River Sand.

- The histograms of state parameter and tip resistance, presented in the Appendix, indicated that most of the profiles had multi-modal characteristics, which is indicative of material zoning.

- The Jamuna Bridge Site has state parameter profiles in which the value of the state parameter decreases with depth (i.e. it becomes more negative as the depth increases). This is indicative of a greater tendency for the sand to dilate at higher stress levels, which is opposite to what one would normally be expected for uniform sands.

- Based on the mean values of state parameter and the vertical scales of fluctuation, a possible reason for the slides occurred during the excavation of the Jamuna Bridge West Guide Bund can be related to the heterogeneity of the material properties. In particular, the largest slides occurred in the middle part of the
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horseshoe-shaped West Guide Bund. This could have been predicted from the larger vertical scales of fluctuation at this location, compared to other locations within the West Guide Bund.

- It was found that the values of the calibrated stiffness curvature parameters, $AP$, $BP$ and $EP$ for the Jamuna River Sand, were close to the values recommended by Hicks (2003), thereby supporting the findings of previous research that these parameters need not to be calibrated. Also, parameters $B$, $C$ and $E$ were found to be related to state parameter, $\psi$, in agreement with the findings of Hicks & Onisiphorou (2005).

- Deterministic analyses of the slope for different state parameter values indicated that, for $\psi > -0.035$, the structure clearly fails (i.e. liquefaction), while for higher negative state parameter values, the structure is able to withstand larger gravitational loads.

- As the degree of anisotropy of heterogeneity increases, the range of structural responses increases. For isotropic heterogeneity, the range of responses was much smaller and local failures were computed in the slope (as opposed to global failure of the structure). It was found that, for the isotropic case, a single deterministic analysis based on the mean state parameter value of -0.035 can be equivalent to the average stochastic response.

- It was found for the anisotropic stochastic cases that there are two different types of dominant failure mechanisms which are dominant and thus a single deterministic analysis, which can only lead to one failure mechanism, can not be used to fully understand the geo-structural behaviour.

- Increasing the vertical scale of fluctuation leads to a bigger range of possible structural responses. This is in agreement with the previous findings of Hicks & Onisiphorou (2005).

- The slight depth-dependency of the material statistics has not had a significant effect on the range of structural responses and possible failure mechanisms.

- The failure zones observed during excavation of the West Guide Bund of the Jamuna Bridge Abutment could be closely predicted if the heterogeneity was considered. In particular, it was found that, for such a natural deposit, a large degree of anisotropy (in the range of 20) could closely explain the failure mechanisms observed on site. Hence a stochastic analysis is recommended when dealing with soil structures as the influence of the heterogeneity of the material properties can significantly change the geo-structural behaviour.
8.3 Recommendations for further work

The main recommendations for further studies can be prioritised as follow:

- The uncertainties in the calibrated NorSand parameters and their influence on the derived state parameter values should be studied; in particular, the impact of these uncertainties on the prediction of structural performance needs to be investigated.

- The different methods for characterising the spatial variability presented in Chapter 4 (e.g. the concept of fractals, variograms, resampling techniques, etc.) may be applied to the current case history and their influence on structural performance studied and compared.

- The random field generator used in this research should be developed further so that it would be possible to consider different element sizes throughout the finite element mesh. At the moment the code is only capable of generating random fields in which the dimensions of the elements are integer multipliers of the cell sizes used in the generation of the random field.

- Random fields can be conditioned to the CPT data to reduce the uncertainties involved in assuming a mono-modal distribution throughout the soil layer and their impact on the structural performance needs to be investigated.

- The effect of pore fluid migration due to soil heterogeneity can be also investigated by performing coupled analyses of the structure in the same manner carried out by Bakhtiari & Hicks (2008).

- More detailed stochastic analyses of the slope, involving a larger number of realisations, can be carried out and the results can be presented in terms of statistical distributions to work out the reliability of the structure (as in Hicks & Onisiphorou, 2005; Spencer, 2007).

- 3D stochastic analyses of the structure needs also to be performed, in order to investigate the effect of heterogeneity in the 3rd dimension on the performance of the Jamuna Bridge Abutment. 3D version of the computer algorithm MONICA was first developed by Williams (1993) and then further developed by the author to enable the application of heterogeneity and gravitational loading (Bakhtiari, 2008). However, due to the size of the application, the code needs to be parallelised for effective use in 3D computations. After parallelisation, the possible failure mechanisms along the slope’s length need to be investigated.


References


References


References


References


References

8th International Conference on Structural Safety and Reliability, Newport Beach, California, USA, 287.


APPENDIX A

Statistical Characterization Results
Figure A. 1

(a) Distribution functions and their corresponding chi-square values for all state parameter data: left-hand side is for before trend removal and right-hand side for after trend removal and normalization.

(b) Distribution functions and their corresponding chi-square values for all tip-resistance data: left-hand side is for before trend removal and right-hand side is for after trend removal and normalization.

(c) Best distribution function classification for all data: top left is for state parameter before trend removal, top right is for state parameter after trend removal and normalization, bottom left is for tip-resistance before trend removal and bottom right is for tip-resistance after trend removal and normalization.
Figure A.2 Distribution function for trend-removed vertical scale of fluctuation across the site
(a) Output results

(b) Determination of the vertical scale of fluctuation for the trend-removed case

(c) State parameter statistics

<table>
<thead>
<tr>
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<td>σ</td>
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<td>θ_v (m)</td>
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(d) Distribution function fitting

Figure A. 3 Summary of evaluated CPT data for C330W.
Appendix A

(a) Output results

(b) Determination of the vertical scale of fluctuation for the trend-removed case

(c) State parameter statistics

<table>
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(d) Distribution function fitting

Figure A. 4 Summary of evaluated CPT data for C310W.
Appendix A

(a) Output results

(b) Determination of the vertical scale of fluctuation for the trend-removed case

(c) State parameter statistics

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<td>( \sigma )</td>
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<td>( \theta_v ) (m)</td>
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(d) Distribution function fitting

Figure A.5 Summary of evaluated CPT data for C270WRD.
Appendix A

295

(a) Output results

(b) Determination of the vertical scale of fluctuation for the trend-removed case

(c) State parameter statistics

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<th>Point Statistics</th>
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(d) Distribution function fitting

**Figure A. 6** Summary of evaluated CPT data for C230W.
Appendix A

(a) Output results

(b) Determination of the vertical scale of fluctuation for the trend-removed case

(c) State parameter statistics

<table>
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(d) Distribution function fitting

Figure A. 7 Summary of evaluated CPT data for C210W.
Appendix A

Figure A. 8 Summary of evaluated CPT data for C190W.

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(a) Output results

(b) Determination of the vertical scale of fluctuation for the trend-removed case

(c) State parameter statistics

(d) Distribution function fitting

(i) No depth trend removed

(ii) Depth trend removed
Appendix A

(a) Output results

(b) Determination of the vertical scale of fluctuation for the trend-removed case

(c) State parameter statistics

<table>
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(d) Distribution function fitting

Figure A. 9 Summary of evaluated CPT data for C170W.
Appendix A

(a) Output results

(b) Determination of the vertical scale of fluctuation for the trend-removed case

(c) State parameter statistics

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(d) Distribution function fitting

Figure A.10 Summary of evaluated CPT data for C150WRD.
**Appendix A**

### (a) Output results

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### (b) Determination of the vertical scale of fluctuation for the trend-removed case

### (c) State parameter statistics

<table>
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<tr>
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<th>Trend Removed</th>
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### (d) Distribution function fitting

**Figure A.11** Summary of evaluated CPT data for C140W.
Figure A.12 Summary of evaluated CPT data for C130W.

<table>
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<td>( \theta_v )</td>
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Appendix A

(a) Output results

(b) Determination of the vertical scale of fluctuation for the trend-removed case

(c) State parameter statistics

<table>
<thead>
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Figure A.13 Summary of evaluated CPT data for C110W.

(d) Distribution function fitting
Appendix A

(a) Output results

(b) Determination of the vertical scale of fluctuation for the trend-removed case

(c) State parameter statistics

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(d) Distribution function fitting

Figure A.14 Summary of evaluated CPT data for C070W.
(a) Output results

(b) Determination of the vertical scale of fluctuation for the trend-removed case

(c) State parameter statistics

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(d) Distribution function fitting

Figure A.15 Summary of evaluated CPT data for C330E.
Appendix A

(a) Output results

(b) Determination of the vertical scale of fluctuation for the trend-removed case

(c) State parameter statistics

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(i) No depth trend removed

(ii) Depth trend removed

(d) Distribution function fitting

Figure A.16 Summary of evaluated CPT data for C310E.
Appendix A

Figure A.17 Summary of evaluated CPT data for C290E.

(a) Output results

(b) Determination of the vertical scale of fluctuation for the trend-removed case

(c) State parameter statistics

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(d) Distribution function fitting

(i) No depth trend removed

(ii) Depth trend removed
Appendix A

(a) Output results

(b) Determination of the vertical scale of fluctuation for the trend-removed case

(c) State parameter statistics

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(d) Distribution function fitting

Figure A.18 Summary of evaluated CPT data for C250E.
Appendix A

308

(a) Output results

(b) Determination of the vertical scale of fluctuation for the trend-removed case

(c) State parameter statistics

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(d) Distribution function fitting

Figure A.19 Summary of evaluated CPT data for C230E.
Appendix A

309

(a) Output results

(b) Determination of the vertical scale of fluctuation for the trend-removed case

(c) State parameter statistics

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(d) Distribution function fitting

Figure A.20 Summary of evaluated CPT data for C190E.
Appendix A

Summary of evaluated CPT data for C150E.

**Figure A.21**

(a) Output results

(b) Determination of the vertical scale of fluctuation for the trend-removed case

(c) State parameter statistics

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(d) Distribution function fitting

(i) No depth trend removed

(ii) Depth trend removed
Appendix A

(a) Output results

(b) Determination of the vertical scale of fluctuation for the trend-removed case

(c) State parameter statistics

<table>
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(d) Distribution function fitting

Figure A.22 Summary of evaluated CPT data for C130E.
Appendix A

(a) Output results

(b) Determination of the vertical scale of fluctuation for the trend-removed case

(c) State parameter statistics

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(d) Distribution function fitting

Figure A.23 Summary of evaluated CPT data for C110E.
Figure A.24 Summary of evaluated CPT data for C090E.