Abstract—In this paper, we study the performance of a wireless communication system, where an arbitrary number of amplify-and-forward relays cooperate to enhance the signal between source and destination. The source and destination are hidden from each other, and there is no direct link between them. Optimal relay power control (ORPC) is introduced as an effective scheme to achieve cooperative diversity. This paper develops a new and more accurate model for the ORPC scheme in multiple antenna relays scenario, and takes into account the effects of fading and thermal noise at both the relays and destination. This leads to the derivation of new analytical expressions for the overall spectral efficiency which can be used to estimate the throughput, and to study the impact of different system parameters on efficiency. The accuracy of the new mathematical results is confirmed by Monte Carlo simulation.

Index Terms—amplify-and-forward, moment generating function, multiple input multiple output, Optimal Relay Power Control, spectral efficiency, relay.

I. INTRODUCTION

The performance of wireless communication system suffers from several attenuation in signal strength, due to the effect of the fading in wireless channel. Cooperative diversity network technology, see [1]–[4], is known to be an effective means of high data-rate coverage and a means of coping with the fading of wireless channels, both of which are necessary for future cellular and ad-hoc wireless communication systems. Fig. 1, below, shows the basic idea of cooperative diversity networks, which can be summarize as using relay nodes to enhance the source signal at the destination. However, this necessarily implies the receiver node will receive multiple individual copies of the same signal and should be able to achieve diversity without the need to use multiple antennas at the transmitter or receiver nodes [5].

Various cooperation diversity protocols have been proposed; these include amplify-and-forward (AF) and decode-and-forward (DF) [6]–[8]. In the first protocol, the relay re-sends the received signals after amplifying it, whereas in the second protocol, the relay detects the signals and then retransmits a new signal. Multiple antennas have been considered as cooperative diversity protocols which uses increased diversity to combat channel fading [9]. When each antenna at the transmitter sends the same signal information to another antenna at the receiver, multiple copies of the data symbols can arrive at the receiver node and this can lead to an improvement in system performance.

In cooperative networks, maximum ratio transmission (MRT) is considered as the optimum technique to combine multiple relays signals and to achieve maximum data rate. However, the complexity of the MRT is greater than for any other technique, and each relay in the network needs to know every channel between the source, relays, and destination. On the other hand, the best relay selection (BRS) scheme is cooperation scheme that attracted considerable interests because it can achieve full diversity with fewer synchronization or feedback requirements. However, the achievable data rate for BRS is less than the MRT, because only one relay carries the data between the source and destination. In this regard, Optimal relay power control (ORPC) is introduced as an effective means of achieving cooperative diversity. ORPC is a trade-off between complexity and system performance. The performance of a ORPC cooperative diversity system is significantly better than for BRS, but less than for MRT. On the other hand, ORPC has less complexity at both the relays and the destination since the relay needs to know only its own channels [10], [11].

The idea of ORPC is to optimize the transmitted power for each relay, which leads to optimize the system performance, where ORPC scheme is used when all the elements of a beamforming vector have the same magnitude. However, this is not the case in MRT, which uses the right singular vector equivalent to the largest singular value of the channel matrix. Moreover, MRT requires feedback of both magnitude and phase information for the beamforming vector, whereas ORPC needs only the phase information vector. As result, ORPC has many advantages such as eliminating half of the feedback redundancy [12], [13].

The distributed space-time coding for multiple relay antennas is proposed in [14], where the authors achieve a degree of cooperative diversity in wireless relay network. The investigation of distributed decode and forward fixed relays is discussed in [15]. The authors examined the impact of multiple antennas on the system performance. The trade-off between the diversity and multiplexing in multiple antenna network is studied in [16], where the authors investigate the outage probability of multiple antenna relays system. The analyses of cooperative protocol in home power line communication is reported in [17], where the authors investigate the maximum data rate using hermitian symmetric orthogonal frequency division multiplexing together with equal gain combining,
selection combining and maximal ratio combining techniques. The performance of cooperative diversity using equal gain combinations over Nakagami-m fading channels is discussed in [18], where the authors determined closed-form expressions for the probability density function (PDF), and the moment generating function (MGF) of the instantaneous signal to noise ratio (SNR) of the indirect link. The single and multiple relay selection schemes and their achievable diversity orders are reported in [19], where the authors generalize the idea of single relay selection to multiple relay selection in parallel AF relays, and derive the achievable diversity of some existing single and multiple relay selection schemes.

The end-to-end performance of a two-hop system with non-regenerative relays over flat Rayleigh-fading channels is proposed in [20], where the outage probability formulas for noise limited systems are derived. For a Rayleigh-fading environment, analysis of the average symbol error rate for a distributed spatial diversity wireless system with amplifying relays is presented in [21].

Most previous work assumes that the destination has perfect knowledge of the channel for all links. This is not a practical assumption, because the destination cannot obtain the $(S \rightarrow \text{Relay})$ channel information perfectly without noise amplification. Moreover, the complexity overhead for the system increases with the number of relay nodes [22].

In this paper, we analyse the performance of cooperative diversity networks, where the effect of relays that cooperate in enhancing the source signal at the destination on the system performance is considered as a randomly occurring factor. There is one source, one destination, and $M$ total number of relays applying amplify-and-forward (AF) protocol. The source and destination are equipped with a single antenna, which can be used for both transmission and reception, while the gateway equipped with $N$ antennas. There is no direct link between the source and destination nodes, which are out of range of each other. The communication between the source and the destination is divided into two phases; in the first phase, the source sends its signal to all relays, which will receive the signal and amplify it. In the second phase, random number of cooperative relays forward the signal to the destination.

We analyse the performance of ORPC cooperative diversity networks over Rayleigh and Nakagami-m fading channels, taking into account (in addition to the random number of cooperative relays) the effects of fading and thermal noise at both the relays and the destination. To the best of the authors’ knowledge, system where the number of the cooperative relays is treated as random have not been considered in the literature but is related to many practical scenarios. However, this assumption makes the proposed model more practical than previous models. The proposed formula reveals the implication of source activity on the overall spectral efficiency (SE) of the system.

We derive the lower bound approximation expressions for the SE of the ORPC model, which can be used to show the effect of important parameters on the system performance. The lower bound of SE of ORPC has not previously been addressed in the literature. We validate our analysis by comparing it with exact simulation results, where we found that as the number of relays, number of relays antenna, and relay status increase, the our lower bound approximation become tight closed to exact simulation. Moreover, the new results are expressed in terms of the weights and abscissas of a Laguerre orthogonal polynomial.

The SNR in this model is a ratio of a large number of random variables. However, the direct approach to finding the overall SE may require at least $(3M)$-fold numerical integrations, which is difficult to obtain in general. As result, we investigate the use of a simple and useful Lemma 1 to evaluate this averaging. Moreover, the numerator of the SNR is not a linear sum, since it is the square of the sum of multiple random variables, which cannot be directly solved. Therefore, we use corollary 2 to simplify this equation, where the result includes a simple mathematical equation that can be easily evaluated.

We compare our scheme with some popular schemes, such as the BRS, and MRT. Furthermore, simulation results are provided to validate our analysis. We found that the ORPC can benefit from increasing the number of relays, and the reduction of noise power at the relays and destination. Also, our results show that the performance of the ORPC is comparable to the BRS cooperative scheme, however the performance of the MRT was better.

The remainder of this paper is organized as follows. Section II introduces the system model for the ORPC. The SE analysis for ORPC is explained in section III. The SE analysis in Rayleigh fading channels, and the exact-form are presented in Section IV. In Section V, the SE analysis for Nakagami-m fading channels, and the exact-form are presented. Results are presented and discussed in Section VI. Finally, concluding remarks are given in Section VII.

**Notations:** The following notations are used in the paper. Lower case boldface denote vectors, $E[.]$ denotes the expectation operator, $B^*$ denotes the conjugate transpose of channel $B$, $|.|$ denote the Euclidean norm of a vector, $B|$ denotes the conjugates of channel $B$, $\Re(B)$ denotes the absolute value, $\Re(B)$ denotes the real part of $B$. Also, we use $\sim CN(0,1)$ for a circularly symmetric complex Gaussian distribution with zero mean and variance 1.

## II. SYSTEM MODEL

The system model under consideration is shown in Fig. 1. There is one source, one destination, and each of these nodes is equipped with a single antenna, which can be used for both transmission and reception. There are $M$ number of relays each equipped with $N$ antennas, distributed in the service area between the source and the destination. The source and destination are hidden from each other with no direct link between them. The communication between the source and the destination is divided into two phases. During the first phase, also known also as the Broadcast Channel (BC) phase, the source sends its signal to the relays simultaneously. In the
second phase, know as the Multiple Access Channel (MAC) phase, a random number of relays forward the amplified signal to the destination. The channel from the source to the \( i \)th relay is denoted as \( h_i \) and the channel from the \( i \)th relay to the destination as \( g_i \). All channels are assumed to be subject to complex Gaussian fading with zero mean and unit variance, e.g., \( h_i, g_i \sim CN(0, 1) \). We assume the relays non-identical, therefore the noise power generated at each relay is unique. All the additive white Gaussian noise (AWGN) terms related to all links (Source \( S \to \) Relays \( \to D \)), are assumed to have zero mean and variance \((\sigma_{h_i}^2, \sigma_{g_i}^2) \), respectively. Without loss of generality, in the first phase, the received signal at the \( i \)th relay is given by

\[ y_i = h_i x + n_i \] (1)

where \( h_i \) is the complex channel gain vector between the source and \( N \) antennas of relay \( i \). The received composite signal at the destination in the BC phase is given by

\[ y_D = \sum_{i=1}^{M} g_i \delta_i w_i (h_i x + n_i) + n_d \] (2)

where \( g_i \) is the complex channel gain between the destination and relay \( i \). The weight of the relay \( i \) can be expressed as

\[ w_i = \frac{1}{\sqrt{M}} \frac{h_i^\dagger}{\|h_i\|} \] (3)

Therefore, the received signal at the destination can be written as

\[ y_D = \frac{1}{\sqrt{M}} \sum_{i=1}^{M} \left( \delta_i \|h_i\| \|g_i\| x + \delta_i \|g_i\| \frac{h_i^\dagger}{\|h_i\|} n_i \right) + n_d. \] (4)

Thus, the SNR at the destination node is given by

\[ \text{SNR} = \frac{\left( \sum_{i=1}^{M} \delta_i \|h_i\| \|g_i\| \right)^2}{\sum_{i=1}^{M} \delta_i \|g_i\|^2 \sigma_{g_i}^2 + M \sigma_{d}^2}. \] (5)

### III. LOWER BOUND OF SE ANALYSIS

The SE can be found from equation (6), which can be written as

\[ \Re = \frac{1}{2} E[\log_2 (1 + \text{SNR})]. \] (6)

However, the exact SE analysis is difficult to obtain for such model, where the sum of the useful signal is correlated with sum of the relays noise. Moreover, the distribution of \( \|h_i\| \) and \( \|g_i\| \) is not Rayleigh fading any more. Therefore, in this paper we aim to approximate the exact SE in (6) to the approximation formula know as lower bound of (6), which can be found as

\[ \Re = \frac{1}{2} \log_2 (1 + E[\text{SNR}]). \] (7)

Therefore, the lower bound of SE of this model can be obtained as [7]

\[ \Re = \frac{1}{2} \log_2 \left( 1 + \frac{E \left\{ \left( \sum_{i=1}^{M} \delta_i \|h_i\| \|g_i\| \right)^2 \right\} }{\sum_{i=1}^{M} E \left\{ \delta_i \|g_i\|^2 \sigma_{g_i}^2 \right\} + M \sigma_{d}^2} \right). \] (8)

The expectation in (8) for the term \( E \left\{ \left( \sum_{i=1}^{M} \delta_i \|h_i\| \|g_i\| \right)^2 \right\} \) can not be obtained directly, where the square includes all the summation. Therefore, it is need further simplification to be obtained. Let's assume \( X_i = \delta_i \|h_i\| \|g_i\| \), therefore, \( E \left\{ \left( \sum_{i=1}^{M} \delta_i \|h_i\| \|g_i\| \right)^2 \right\} = E \left\{ \left( \sum_{i=1}^{M} X_i \right)^2 \right\} \). It is well know that the random variable \( X \) with mean \( \nu \) and variance \( \sigma^2 \) has expected value, which can be expressed as

\[ E \left\{ X^2 \right\} = \nu^2 + \sigma^2. \] (9)

Which is leads to

\[ E \left\{ \left( \sum_{i=1}^{M} X_i \right)^2 \right\} = M^2 \sigma^2 + M \nu^2. \] (10)

As we assumed earlier, \( X_i = \delta_i \|h_i\| \|g_i\| \), where \( \|h_i\| \) and \( \|g_i\| \) are random variables represented by Chi Square random variable with mean \( N \) and variance \( 2N \), where \( N \) is the number of relay antennas, and \( \delta_i \) modelled by independent binomial random variable with mean \( M \rho \) and variance \( M \rho(1 - \rho) \), where \( \rho \) is the probability of the relay is cooperated which take values between \( 0 < \rho < 1 \). Therefore, the expectation in the nominator of (8) can be obtained as

\[ E \left\{ \left( \sum_{i=1}^{M} \delta_i \|h_i\| \|g_i\| \right)^2 \right\} = M^2 (N^2 M \rho)^2 + M (4N^2 M \rho (1 - \rho)). \] (11)
The second unknown in (8) is the expectation in denominator \( \sum_{i=1}^{M} \mathbb{E}[\delta_i [g_i]^2 \sigma_r^2] \), which can be found as

\[
\mathbb{E}[\delta_i [g_i]^2 \sigma_r^2] = M \sigma_r^2 \rho N (2 + N) .
\] (12)

Substitute (11) and (12) in (8), this lead to the simple lower bound formula, which can be written as

\[
\Re = \frac{1}{2} \log_2 \left( 1 + \frac{M^2 (N^2 M \rho)^2 + M (4N^2 M \rho (1-\rho))}{M \sigma_r^2 \rho N (2 + N) + M \sigma_d^2} \right) .
\] (13)

The result in (13) is a simple close form approximation result, which can be evaluated easily. The result shows the powerful of our mathematical analysis and how we can simplify the complicated system models and introduce simple mathematical results.

IV. NUMERICAL AND SIMULATION RESULTS

In this section, the exact SE achieved by the ORPC scheme is evaluated using Monte-Carlo simulation, and compared to the derived asymptotic results. Different graphical plots of SE are presented below, corresponding to various numbers of relays, number of relay antennas, and relay statuses. For simplicity we assume \( \rho_1 = \ldots = \rho_{10} = \rho \), and \( \sigma_d^2 = \ldots = \sigma_{d10}^2 = \sigma_d^2 \). The simulation parameters used to obtain these results are given in Table (I).

![Fig. 2: SE as function of number of cooperating relay nodes (M)](image)

![Fig. 3: SE as function of number of relay antennas (N)](image)

In Fig. 2, shows the SE (in bits/s/Hz) of ORPC schemes as function of the number of relays, \( M \), when \( \sigma_d^2 = \sigma_r^2 = 1 \)watt/Hz. We can see that the theoretical approximation of ORPC curve is an excellent match to the exact simulated ORPC performance when \( \rho = 1 \), and 0.8. However, as the \( \rho \) decrease from 0.5 to 0.1, the gap between the exact simulation and theoretical approximation increase. This due to the increase of \( \rho \) leads to increase number of cooperated relays, which leads to better system performance. Fig. 2 also confirms that the (8) is no longer a lower bound for the average SNR when the useful signal becomes correlated with the sum of relays noise. We also see that an increase in the number of cooperating relay nodes (M) beneficially enhances the SE performance, and the achieved diversity order for the different schemes.

In Fig. 3, the SE is plotted as function of the number of relays antennas (N) for different values of \( \rho \) (0.1, 0.3, 0.5, 0.8, 1) with \( \sigma_d^2 = \sigma_r^2 = 1 \)watt/Hz, and \( M = 10 \). It can be seen that as N has positive effect of the system performance, where as N increases, the performance of the system increases. Moreover, increase \( \rho \) leads to enhance the system performance, where when \( \rho = 1 \) is better performance than \( \rho = 0.5 \) and 0.1.

In Fig. 4, the SE is plotted as function of the relay status (\( \rho \)) for different values of N (10, 20, 30, 80, 100 antennas) with \( \sigma_d^2 = \sigma_r^2 = 1 \)watt/Hz, and \( M = 10 \). It can be seen that as \( \rho \) increases, the performance of the system increases. However, as would be expected, increasing the number of relays antennas has positive effect on the system performance, for example, increasing N from 10 to 20 antennas gives marked improve in system performance. However, the when \( \rho = 0.1 \), the exact simulation of ORPC and the analytical approximation is tidy matching, which is not as shown in Fig. 2. This is due to the remarkable increase in the number of relays antennas, which has positive effect of the system performance.

Fig. 5, illustrates the SE as function of the noise power at

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of relays (( M ))</td>
<td>10</td>
</tr>
<tr>
<td>Number of relay antennas (( N ))</td>
<td>10</td>
</tr>
<tr>
<td>Noise power at relay (( \sigma_d^2 ))</td>
<td>1 watt/Hz</td>
</tr>
<tr>
<td>Noise power at destination (( \sigma_r^2 ))</td>
<td>1 watt/Hz</td>
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In this paper, we analysed in detail the performance of a cooperative diversity network using ORPC scheme, with $M$ cooperating relays used to enhance data transmission when there is no direct link between source and destination. The relays are located between the source and destination, and status of each relay is randomly considered. All the nodes (source, and destination) are equipped with a single antenna, where the relays are equipped with multiple antenna. We derive analytical expressions for the overall SE, where the performance analysis of this model is difficult to obtain in general using the direct approach, as it may require the computation of a $(3Jf)$ - fold convolution integral to find the distribution of the SNR. As result, we used a useful Lemma 1 to evaluate the performance of the model. Our result reflects the fact that the mathematical analysis and Lemma 1 are valid for different types of channel fading, and can be used in different system models. We validated the accuracy of our results with Monte Carlo simulations. We examined the impact of increasing the number of relays on the performance of such a system; we show that, as the number of relays increases, the SE also increases. In addition, we used the model to evaluate the effect of important factors such as relay status, and noise power at relays on system performance.

**REFERENCES**


