Millimeter Wave Communications over Relay Networks

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Abstract—Relays are expected to be an important factor in the implementation of millimeter wave (mmWave) based cellular networks due to the limited coverage of mmWave communications. Adding a single or multiple relays in the link between two nodes is an inevitable step towards high capacity wireless access and/or backhaul links. In this paper we suggest employing two-hop multiple relay networks as a solution to link two points using mmWave frequencies. Hybrid (Analog and Digital) precoding is used at the two terminal nodes with analog beamformers/combiners at the relays. The proposed design algorithm requires no channel information with relatively limited training overhead. The performance of the system is examined under different channel and system layouts. The results highlight the positive impact of employing relays in mmWave link as well as the effect of array size on the overall system performance.

Index Terms—Millimeter wave, Hybrid beamforming, Relay Networks, Channel Sounding, 5G.

I. INTRODUCTION

In a few years time, cellular networks will be required to provide 1000 times the throughput provided today [1]. In order to meet such high demands, three scenarios are suggested: increasing the frequency bandwidth, enhancing the spectral efficiency, and more geographical re-usability [2], [3]. The relatively wide bandwidth and the introduction of the cost efficient CMOS technology make mmWave a perfect fit to be the core of the next generation communication. Despite its many advantages, 60 GHz frequencies suffer from relatively high propagation losses. Several researchers investigated ways to overcome these losses [4]–[6]. One of the proposed solutions is to increase the directivity gain of the systems using analog, digital, or hybrid (analog and digital) beamformers [7]. Others include changing the layout of the cellular system by introducing dense networks, and using cooperative networks [8], [9].

The complexity of the digital beamformers (DBF) and the limitations of the analog beamformers (ABF) make hybrid beamformers (HBF) an interesting solution to provide the required directivity. The number of the RF chains in hybrid systems is less than the number of antennas which makes hybrid systems cost efficient [10]. One of the popular methods to jointly design the analog and digital processors in HBF systems was introduced by [14]. The proposed design algorithm uses the sparsity of the mmWave channel and orthogonal match pursuit (OMP) [13] principle while assuming full CSI. This method gained importance because of the low complexity of the design [15].

Furthermore, relay networks attract a great deal of research interest in recent years as an architecture to increase the system performance [16]. The study of two-hop multiple single-antenna relay networks was introduced in [17], and the optimization of the MMSE solution for MIMO relay networks has been investigated in [18]. In millimeter waves, there is a shortage of the research in single relay networks, rather than multiple relays. In a recent study, half duplex (HD) relay network was introduced in mmWave environment where the sparsity principle is used to jointly design the hybrid beamforming processors at the relay [19]. However, multiple relay networks have not been studied thoroughly.

Combining the benefits of the wide mmWave spectrum with the flexibility provided by the multiple relay networks is a key solution in designing small dense cells architecture that secures the high throughput needed. In this paper, we study the performance of hybrid beamformers in the two-hop multiple relay networks in mmWave. A split domain algorithm is proposed inspired by the performance of the well known sparsity based algorithm for HBF design [14], [20] and the channel sounding techniques for ABF [21], [22].

The main contribution of this work is the study of multi-relay networks in mmWave and developing the beamformers design algorithm. The work will be extended to include comparing different search techniques providing power and time saving. The rest of the paper organized as follows. The system and channel models with the main assumptions used in the paper are presented in Section II. The design of the hybrid and analog beamformers is presented in Section III. Simulation results are presented in Section IV before concluding the paper in Section V.

We use the following notation throughout this paper: $\mathbf{A}$ is a matrix; $\mathbf{a}$ is a vector; $a$ is a scalar. $\mathbf{A}^{(i)}$ is the $i$-th column of $\mathbf{A}$, $\mathbf{A}_{(i,j)}$ is the $(i,j)$-th entry of $\mathbf{A}$. $||\mathbf{A}||_F$ is the Frobenius norm of $\mathbf{A}$, whereas $\mathbf{A}^H, (\mathbf{A})^{-1}$ denote transpose, Hermitian, and inverse respectively. $||\mathbf{a}||_p$ is the $p$-norm of $\mathbf{a}$. $\mathbf{I}_N$ is the $N \times N$ identity matrix; $\mathbf{0}_{M \times N}$ is the $M \times N$ all-zeros matrix. $\mathcal{CN}(\mu; \mathbf{A})$ is a complex Gaussian vector with mean $\mu$ and covariance matrix $\mathbf{A}$. 
II. SYSTEM AND CHANNEL MODELS

A. Network Model

Consider a multi relay network consists of one source, one destination, and K relays, as illustrated in Fig. 1. The source \{destination\} is equipped with \(N_S\) \{\(N_R\)\} antennas and \(M_T\) \{\(M_R\)\} RF chains. Each relay has \(N_R\)/\(N_T\) receive/transmit antennas and a single RF chain. In this paper, we consider source-destination connection via one stream through each relay (i.e. \(N_S = K\)). We assume limited number of relays where \(K \leq \min\{M_R, M_T\}\). It is also assumed the source uses only K out of the available RF chains.

B. Beamforming System Setup

Using the available multiple RF chains, both the source and the destination process the signal in the analog and digital domains. During the transmission, the source applies \(N_S\) \(\times M_T\) digital precoder \(F_D = [\mathbf{f}_1, \mathbf{f}_2, \ldots, \mathbf{f}_{N_T}]\) followed by an \(N_T\) \(\times M_T\) analog precoder \(F_A = [\mathbf{f}_1, \mathbf{f}_2, \ldots, \mathbf{f}_{N_T}]\) on the transmit data. Therefore, the \(k\)-th relay received signal will be

\[
y_{r,k} = H_k \sum_{i=1}^{K} F_{A_k} s_i + n_{1,k} \tag{1}
\]

where \(s = [s_1 \ldots s_K]\) is the \(N_S \times 1\) transmit signal with \(E[ss^H] = \frac{\rho}{N_S} \mathbf{I}_{N_S}\), where \(\rho\) is the total transmitted power. \(H_k\) is the mmWave MIMO channel between the source and the \(k\)th relay, \(n_{1,k}\) is the additive noise vector with covariance matrix \(\mathcal{R}_{n_{1,k}} = \sigma_n^2 \mathbf{I}_{N_R}\). Each relay has an analog combiner \(c_k\) to receive the signal, and an analog beamformer \(b_k\) for transmission. The \(k\)th relay transmitted signal can be described as:

\[
x_{r,k} = b_k c_k^H y_{r,k} \tag{2}
\]

the destination observes the signals transmitted by the relays and the received signal can be written as

\[
y = \sum_{k=1}^{K} (G_k b_k c_k^H H_k) \sum_{i=1}^{K} F_{A_i} s_i + G_k b_k c_k^H n_{1,k} + n_2 \tag{3}
\]

where \(G_k\) is the mmWave channel between the \(k\)th relay and the destination, and \(n_2\) is the additive white noise at the destination with covariance matrix \(\mathcal{R}_{n_2} = \sigma_n^2 \mathbf{I}_{N_R}\). Finally, the destination uses hybrid beamforming to process the received signal. The estimated signal can be written as

\[
s = W_G B_G H_F D s + W_H B_H n_1 \tag{4}
\]

where \(W = W_A W_D\), with \(W_A\) being the analog combiner, and \(W_D\) is the digital processor. \(H = [H^T_1, H^T_2, \ldots, H^T_K]\) is the \(K N_T \times N_T\) channel matrix between the source and the relays, \(n_1 = [n_{1,1}^T, n_{1,2}^T, \ldots, n_{1,K}^T]\) is the \(K N_R \times 1\) noise matrix at the relays, \(G = [G_1, G_2, \ldots, G_K]\) is the \(N_R \times K N_T\) channel matrix between the all the relay nodes and the destination, \(C = [c_1^T, c_2^T, \ldots, c_K^T]^T\) is the \(K N_T \times 1\) equivalent all relays combiner matrix, and \(B = [b_1^T, b_2^T, \ldots, b_K^T]^T\) is the \(K N_T \times 1\) equivalent all relays beamformer matrix. The analog beamformers are implemented using analog phase shifters, therefore, their entries have equal norm, i.e. at the source \((F_A^H F_A^{\epsilon(H)})_{(\ell,\ell')} = N_T^{-1}\). On the other hand, digital beamformer at the source, \(F_D\), has no hardware constraints except the total power constraint. With the limitation on the analog beamformers hardware, such as choosing the phase shifters angles out of quantized sets, the analog beamformer vectors need to be chosen from finite size codebooks. The codebook size will effect the duration of the channel training as will be explained shortly in Section III.

C. Channel Model

The measurements of outdoor mmWave 28 GHz and the 72 GHz channels showed that they normally have limited scattering [23], [24] where the multipath components are mainly generated by reflection. To incorporate this effect, we adopt a geometric channel model based on the extended Saleh-Valenzuela model in which the channel is assumed to be a sum of \(L\) scattering clusters, each of which contributes \(P\) propagation paths to the channel [25], [26]. With half-wave spaced uniform linear arrays (ULAs) used at the transmitter and the receiver, the channel matrix can be expressed as

\[
H_k \{G_k\} = \frac{1}{\sqrt{P L}} \sum_{\ell=1}^{L} \sum_{p=1}^{P} \alpha_{p,\ell} b_k \mathbf{a}_{\ell}(\theta^p_{p,\ell})^* \tag{5}
\]

where \(\alpha_{p,\ell}\) is the complex small-scale fading gain of the \(p\)-th sub-path in the \(\ell\)-th cluster, \(\theta^p_{p,\ell}\) and \(\theta^p_{p,\ell} \in \{0, 2\pi\}\) are the angles of arrival and departure (AoA/AoD), respectively. The gains \(\alpha_{p,\ell}\) are complex Gaussian random variables with zero mean and variance \(\sigma_n^2\). The mean angle associated with each cluster is uniformly distributed over \([-\pi, \pi]\), and the distribution of the difference between an AoA \{AoD\} and its mean is Laplacian with angular standard deviation \(\gamma\). The vectors \(\mathbf{a}_{\ell}(\theta_{p,\ell})^*\) and \(\mathbf{a}_{\ell}^H(\theta_{p,\ell})^*\) are the array response vectors at the receiver and transmitter, respectively.

Without loss of generality, we are using ULA to get the results.
in the simulation in Section IV. In this case, the array response vector \( \mathbf{a}(\theta) \) is defined as
\[
\mathbf{a}(\theta) = \frac{1}{\sqrt{M}} \left[ 1, e^{j2\pi \frac{d}{\lambda} \sin(\theta)}, \ldots, e^{j2\pi (M-1) \frac{d}{\lambda} \sin(\theta)} \right]^T
\]
where \( d \) is the space between the antennas, \( M \) is the number of antennas, and \( \lambda \) is the wavelength.

### III. BEAMFORMERS DESIGN

This work aims to effectively design the hybrid beamformer at the source and the destination as well as the analog beamformers at the relays in order to maximize the achievable rate at the destination. Assuming Gaussian transmission, the achievable rate could be written as described in (7), at the top of the next page.

Maximizing the rate requires joint design of the \((2K + 4)\) beamforming matrices. However, the constraints on the analog beamformers make finding the joint solution untraceable [14], [15]. In this work, we propose splitting the design into two domains: analog and digital. The design will be performed over two phases, the first phase involves designing the analog beamforming matrices. Then, during the second phase, the digital processors are designed.

#### A. Phase One: Analog Domain

The analog beamforming matrices are designed by choosing the best beamforming vectors from sets of predefined vectors known as codebooks. The process includes repeatedly transmitting training signal and search for the vector pair that maximizes a measuring factor (we use beamforming gain as the measuring factor in this work). This process is known as channel sounding (CS) and it is performed without the need for CSI using relatively low overhead [15], [21]. Exhaustive search (ES) where the all the possible beamforming/combining vectors combinations are tested is used for search for the best pair. ES is considered the most straightforward search technique, yet it has higher overhead as compared to other search techniques. Optimistically, if the analog beamformers have continuous angles capabilities and the infinite codebook size, the selected vectors will have the same angles as the AoA/AoD. Comparing different adaptive and non-adaptive search techniques to provide time and power saving is a natural extension to this work.

Each node of the system has its own codebook to use during the channel sounding. \( B, C \) represent the transmitting and receiving codebooks at the relay, respectively. \( F, W \) represent source, destination codebooks. We also propose using sectored codebooks where the source and destination sectorize the available spectrum over the available relays, assuming the relays are distributed uniformly and equally. \( F_k \) and \( W_k \) represent the sectored codebook assigned to the \( k \)th relay at the source and destination, respectively. The duration of the sounding period depends on the size of the codebook, for instance, the duration of the sounding between the source and the \( k \)th relay will be \( T = N_{F_k} \times N_{C_k} \), where \( N_{a} \) is the number of columns in \( a \), or codebook size.

At each iteration of phase one, a relay combiner \( c_k \), beamformer \( b_k \), and the related source \{destination\} beamformer \{combiner\} \( F_k^a \{w_k^a\} \) are found. After \( K \) iterations, \( F_A, W_A, b_i, c_i \) (1 \( \leq i \leq K \)) are all constructed. The first phase is concluded by calculating the effective channels \( \mathbf{h}_k = e^{j\theta} \mathbf{H}_k \mathbf{F}_A \) and \( \mathbf{g}_k = \mathbf{W}_A^T \mathbf{G}_b \mathbf{b}_k \) (1 \( \leq k \leq K \)) to be used in the digital processor design.

#### B. Phase Two: Digital Domain

During the second phase, the source and destination design their zero-forcing (ZF) digital processor using the effective channel calculated in the previous stage. Due to the sparse nature of the mmWave channel and the sectored beamforming, the effective channels are expected to be well-conditioned to be used in designing the digital processors using ZF technique. The design procedure is outlined in Algorithm 1. The design is concluded by satisfying the power condition for the digital processor at the source.

### IV. SIMULATION RESULTS

The performance of the proposed algorithm is examined by measuring the achievable rate through computer simulations. All the relays are assumed to have the same distance from the source and destination so that all the channels have the same statistics. Each of the propagation channels is modeled with \( L \) clusters channel and each cluster contributes with single ray with uniformly distributed AoAs and AoDs. The complex path gains are assumed to be Gaussian distributed with equal variances. We assume a ULA with inter-elements spacing \( d = \lambda/2 \). The noise variance \( \sigma_{n_1}^2 = \sigma_{n_2}^2 \), and the signal to noise ratio is defined as \( SNR = \frac{\rho}{\sigma_n^2} \). The coverage of each node is specified with \( \theta \in \left\{ -\\frac{\pi}{2}, \frac{\pi}{2} \right\} \).

Fig. 2 shows the performance of the system under single path channel as compared to the optimistic results where the CS uses continuous angles. Two different size codebooks are used, one with 7 vectors and the other with 11 vectors. Adding vectors to the codebook improve the performance and eventually the system performance will approach the optimistic results. However, this comes on the price of the time as the sounding duration for the larger codebook is 3 times the duration of the other. ULAs with 16 antennas are used at the source, destination, and each of the relays.

The effect of the number of the relays is examined in Fig. 3. The results are obtained using channels with \( L = 20, 40 \) paths. Having more relays in the network increases the probability of reaching the destination, thus higher spectral efficiency. Fig. 4 compares the system performance for different array size at source/destination with fixed relay arrays at 8 antennas on each side. Increasing the number of antennas in the array will improve the directivity, which leads to better performance. However, the capacity to improve the performance is limited by the size of the relays arrays.

Finally, the performance of the proposed algorithm is examined using channels with more than single path as shown in Fig. 5. The results indicate a similar behavior to that with
Fig. 2: Spectral efficiency versus SNR with \( L = 1, K = 4 \) relays, and each node of the network has 16 antennas.

Fig. 3: Spectral efficiency versus number of relays when \( L \in \{20, 40\} \), and SNR=5 dB.

\[
R = \log_2 \left( 1 + \frac{\rho}{\rho} \left| W_W^H W_D^H \sum_{k=1}^K G_k b_k c_k^H H_k F_A F_D^H \right|^2 + \left| W_D^H W_W^H \right|^2 + 1 \right) 
\]

(7)

**Algorithm 1** Analog and digital beamformers design by splitting into analog and digital design phases

**Input:** analog sets:

\[ \mathcal{F} = [f_1, f_2, \ldots, f_{N_F}], \mathcal{W} = [w_1, w_2, \ldots, w_{N_W}], \mathcal{C} = [u_1, u_2, \ldots, u_{N_C}], \mathcal{B} = [v_1, v_2, \ldots, v_{N_B}] \]

**Output:** \( F_A, W_A, b_i, c_i (1 \leq i \leq K) \)

**Initializing:** Sectorize \( \mathcal{F} \) and \( \mathcal{W} \) based on the number of relays

- **Phase One:** Design the analog processors.
  - Set the analog sets for each relay
  - Search for the optimum pair that maximize the gain:
    * source-relay
    \[
    (f_k^A, c_k) = \arg\max_{\forall f \in F, \forall u \in C} |u^H H_k f|^2 \quad (8)
    \]
    * relay-destination
    \[
    (w_k^A, b_k) = \arg\max_{\forall w \in W, \forall v \in B} |w^H H_k v|^2 \quad (9)
    \]
  - Compute the effective channels at both sides as \( \tilde{h}_k = c_k^H H_k F_A, \tilde{g}_k = W_k^H G_k b_k \)

- **Phase Two:** Design the digital processor at the source and destination.
  - Define the effective first hop and second hop channels as \( \tilde{H} = [\tilde{h}_1^T, \tilde{h}_2^T, \ldots, \tilde{h}_K^T] \) and \( \tilde{G} = [\tilde{g}_1, \tilde{g}_2, \ldots, \tilde{g}_K] \), respectively.
  - Design \( F_D \) and \( W_D \) using ZF:
    \[
    F_D = \tilde{H}^{H} \left( \tilde{H} \tilde{H}^{H} \right)^{-1}, \quad W_D = \tilde{G}^{H} \left( \tilde{G} \tilde{G}^{H} \right)^{-1} \quad (10)
    \]
  - Compute the final digital processors that satisfy the power constraints.
    \[
    f_k^{D} = \frac{f_k^D}{\| F_A f_k^D \|_F} \quad \forall k
    \]

single path channel. Adding more relays have a positive impact on the performance.

**V. CONCLUSION**

In this paper, we propose a two phase split algorithm for two-hop multiple relays networks in millimeter waves. The source and the destination are equipped with hybrid beamformers and multiple RF chains, while each of the relays is equipped with single RF chain and an analog beamformer.
Fig. 4: Spectral efficiency versus array size at the source and destination. Number of antennas at each relay fixed to $N_R^f = N_R^t = 8$, $K \in \{2, 4\}$, SNR = 5dB, and $L = 10$.

Fig. 5: Spectral efficiency versus training duration for the analog beamforming, array size $N_T = N_R = 16$, $N_T^f = N_R^f = 8$, $K \in \{2, 4\}$, and $L = 10$.

The beamformers work on directing the signal to obtain higher beamforming gains help overcoming the path losses in outdoors mmWave environments. The performance of the system is compared to the optimistic results where the analog beamformers have continuous angles capabilities. The analog beamformers/combines are designed using the modified joint channel sounding by sectoring the coverage spectrum based on the number of relays. The results showed that the array size, the codebook size and the number of the relays in the system have positive impact on the system performance.

REFERENCES


