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Modelling Thomson Coils with Axis-symmetric Problems: Practical Accuracy Considerations

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Abstract—This paper assesses factors that impact the simulation accuracy of a 2D finite element (FE) transient axis-symmetric model of a Thomson Coil actuator. 3D FE analysis (FEA) is used to show that geometric differences between a planar coil and the intrinsic assumptions of a 2D axial model may result in important differences in coil inductance. It is then shown that inductance and resistance compensation of the 2D model can be used to produce an accurate prediction of the TC performance. Detailed parameters of a prototype TC test system are used as inputs in the compensated 2D axial model and excellent agreement is observed between 2D FE simulations and experimental results. It is also shown that armature vibration modes explain the presence of apparent speed oscillations in the experimental results not present in numerical simulations. The compensated 2D axis-symmetric model shows good accuracy compared with experimental results for the investigated scenarios, even when armature flexing is considered.

Index Terms— High speed switches, HVDC breaker, magnetic repulsion, Thomson Coil, ultra-high speed actuator.

I. INTRODUCTION

FOR applications that require ultra-fast actuation, the preferred actuator design is based on the Thomson Coil (TC) [1-5]. The TC comprises a planar coil and an electrically conductive disc armature, placed in close proximity and parallel to the coil, Fig. 1. In operation the coil is excited by a time varying current and produces a time varying magnetic field, inducing eddy currents in the disc armature. Due to the direction of the induced eddy currents, a repulsive magnetic force appears between coil and armature, enabling rapid armature reaction and high speed displacement. Given the relevance of the design in the operation of ultra-high speed mechanical switches, such as MV [1-4], HV DC breakers[5] and traction applications [6], considerable effort has been put into its modelling, design and optimization [6-16]. From the existing modelling tools, transient 2D FEA has been shown to provide accurate results for armature speeds around 10 m/s [14, 15]. However, in [15] predicted armature velocity was shown to diverge from measured values as armature velocity

increased beyond 7 m/s. Speed oscillations in some experimental results were not present in the numerical simulation. It was argued in [15] that the differences in speed between simulations and experimental results were due to armature bending, not considered in the model. Given that a TC actuator is normally required to operate at high speed, an analysis of the factors that impact on FE simulation accuracy compared to measured values is necessary in order to reduce discrepancies.

Due to the TC actuator symmetry, a 2D axial model of the coil is generally considered to be a good approximation for FE modeling and analysis [14-16]. However, geometric differences between a 2D model and the physical planar coil and disc armature may give rise to disparities in inductance and resistance values, resulting in misleading results. For more accurate calculation of coil inductance and resistance a 3D FEA model should be used. However a full 3D FE transient simulation of a multi-physics system is taxing in terms of simulation time and computer resources. Therefore a 2D model is usually preferred for fast transient assessment, particularly at an early stage in the design process. In this paper a detailed analysis of the components present in a typical TC system is conducted. The parameters of a purpose built test-rig were used as inputs to a 2D FE axis-symmetric model implemented using COMSOL 5.2 multi-physics software. Measured inductance values predicted using alternative methods illustrate limitations to the capabilities of the 2D axis-symmetric FE model. In order to reduce error in the 2D model, a steady state 3D FEA model of the coil was used to calculate coil inductance and resistance. The 2D simulation (which predicts lower coil inductance and resistance values than the 3D model) was compensated accordingly by adding passive inductor and resistor components in series with its coil model to correct for differences. The effect of armature bending on the accuracy of the model predictions is also investigated. The compensated 2D FE axial model results exhibit excellent agreement with experimental measurements for the investigated scenarios.

II. FE MODEL IMPLEMENTATION AND EXPERIMENTAL SET UP

Figure 2 shows a schematic diagram of a typical TC test arrangement. During operation, the capacitor bank CB_1 is charged to a predefined voltage level V_0 , and then isolated from the power supply by opening switch S_1 . To operate the actuator, thyristor T_1 is triggered to excite the spiral coil and

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initiate armature displacement. The thyristor conducts until the TC current falls below the thyristor holding current. Diode D_1 protects the capacitor bank against reverse voltage. For accurate simulation of the system correct characterization of the capacitor bank, diode, thyristor, coil resistance, coil inductance and connection lead inductance and resistance is required. Although capacitors of the same rating and manufacturer were used during this research, these were found to have different capacitance values. Therefore in the simulations, each capacitor was modelled individually in terms of its capacitance and equivalent series resistance (ESR). In a TC actuator very high peak currents flow through the system components, therefore, the high temperature V-I characteristic of the diode and thyristor were used during simulations.

A. Experimental test-rig

A test-rig based on the system described above was constructed for model validation purposes. The planar coil comprises 28 turns of circular cross section 3mm diameter, enamelled copper wire. The coil possesses an inner diameter of 4cm and is embedded in a 25mm Tufnol sheet and fixed with epoxy resin. The wire insulation thickness was measured as 0.055 mm, therefore the separation gap between consecutive turns is assumed as 0.11 mm. The capacitor bank comprises three parallel connected FELSIC 85/s electrolytic capacitors, each rated at 450V, 8250 μ F. A high power diode, type VISHAY VS-240U80D, was used for capacitor reverse voltage protection and a SEMIKRON SKKT 162/12E Thyristor to initiate capacitor bank discharge. The capacitance of each capacitor was measured and the ESR obtained from the manufacturer datasheet; maximum ESR value was used in the simulations. Connection lead resistance and inductance were measured using an Agilent 34420A Micro Ohm Meter and an Agilent 4294A impedance analyzer, respectively. The circuit connections comprised several interconnected leads. Consequently the resistance and inductance may be slightly different when connected in the circuit, due to changes in contact resistance and geometry, respectively. Table I summarizes the geometric and electric parameters of the TC test rig and Fig. 3 shows a picture of the assembled test system. Coil lead outs, the short conductor segments at both ends of the spiral coil that are part of the coil wire but do not follow the planar spiral path (see Figs. 1-2), are considered separately to the connection leads values listed in Table I. The total length of the coil lead outs was measured as 47cm, equivalent to 1.13 m Ω , considering nominal conductor resistivity. Coil lead outs inductance was assumed negligible.

Fig. 1. TC 3D diagram (left) and 2D axis-symmetric representation (right).

Fig. 2. Schematic diagram of a typical TC system

TABLE I

TC TEST SYSTEM PARAMETERS

Characteristic	Value
Turn number	28
Conductor diameter	3 mm
Insulation thickness	0.055mm
Conductor resistivity	2.42 m Ω /m @ 20C
Coil inner diameter (nominal)	4 cm
Coil outer diameter (nominal)	22 cm
Capacitor C_1	10.03 mF
Capacitor C_2	9.95 mF
Capacitor C_3	10.37 mF
ESR max.	20 m Ω
Thyristor T_1 Vdrop	0.83 V
Thyristor T_1 linear resistance	1.60 m Ω
Diode D_1 Vdrop	1.10 V
Diode D_1 linear resistance	3.71 m Ω
Connection leads resistance	2.98 m Ω
Connection leads inductance	1.0 μ H
Coil leads outs resistance	1.13 m Ω

Fig. 3. TC test-rig

B. FE Model

Based on the parameters in Table I a 2D axis-symmetric FE model was implemented using COMSOL 5.2 multi-physics software. Although not strictly axis-symmetric this assumption about the coil geometry has been commonly used in the research literature for the modelling and analysis of the TC using FEA software [14-17]. COMSOL magnetic fields, solid mechanics and electrical circuit interfaces are used in the model implementation. In the multi-physics implementation a lumped representation of the excitation circuit was interfaced with a FE representation of the coil centered in a large air domain. FE and electric circuit equations were solved simultaneously using a fully coupled solver. In the 2D FE model each turn of the coil is considered separately and represented by an array of contiguous circles in the model

geometry, where the armature is shown as a rectangular region above the coil turns (Fig. 1 right). The armature mechanical behavior was modeled using COMSOL solid mechanics interface, where isotropic linear elastic material is assumed. The force acting over the armature was calculated within the mechanics interface as the combination of Lorentz force contribution and gravity effects. For the armatures considered in this work the effect of aerodynamic drag is minimal and was neglected in the model. For larger surface armatures with complicated geometry the drag force may become relevant and should be considered. A deformable moving mesh was used to accurately account for armature displacement. Equations (1)-(7) are used for the FEA software for the model solution, the corresponding variables are listed in Table II, vector variables are identified with the symbol $\rightarrow$.

$$\sigma_e \frac{\partial \vec{A}}{\partial t} - \sigma_e \vec{v} \times \vec{B} + \vec{J} = \vec{J}_e \quad (1)$$

$$\vec{\nabla} \times \vec{A} = \vec{B} \quad (2)$$

$$\vec{\nabla} \times \vec{H} = \vec{J} \quad (3)$$

$$\vec{J} \times \vec{B} = \vec{F}_l \quad (4)$$

$$\rho \frac{\partial^2 u}{\partial t^2} - \vec{\nabla} \cdot \vec{S} = F \quad (5)$$

$$\sigma_{e0} [1 + \alpha(T - T_0)]^{-1} = \sigma_e \quad (6)$$

$$\rho C_p \left(\frac{\partial T}{\partial t} + \vec{v} \cdot \vec{\nabla} T \right) = \vec{\nabla} \cdot (k \vec{\nabla} T) + Q \quad (7)$$

TABLE II
FEA SOFTWARE EQUATIONS VARIABLES

Variable	Description
$\vec{B}$	Magnetic flux density
$\vec{H}$	Magnetic field intensity
$\vec{J}$	Current density
$\vec{J}_e$	External current density
$\vec{A}$	Magnetic vector potential
$\vec{F}_l$	Lorentz force
$\vec{v}$	Velocity
$\vec{S}$	Stress tensor
σ_e	Electric conductivity
σ_{e0}	Reference electric conductivity
T	Temperature
T_0	Reference temperature
α	Temperature coefficient
ρ	Density of the material
u	Displacement
F	Force acting over the solid
C_p	Heat capacity
k	Thermal conductivity
Q	Resistive losses

The mean armature velocity was calculated with (8) using the total kinetic energy (W_k) reported by the FE software.

$$v = \sqrt{\frac{2W_k}{m}} \quad \text{m/s} \quad (8)$$

where m is armature mass. W_k is automatically calculated and results from the integral of the kinetic energy density over the solid domains. Given that the only solid domain in movement in the simulations is the armature, W_k represents the armature's kinetic energy.

Fig. 4. Coil current and capacitor voltage comparison between measurements and 2D FE model.

III. EXPERIMENTAL VALIDATION

In order to assess the accuracy of the TC 2D FE axis-symmetric model, simulation results were compared against experimental measurements. In the scenarios considered in this assessment the armature moves in the vertical direction. Therefore, a rectangular air domain with a width and height of three and six times the coil outer radius, respectively, was used on the 2D FE model implementation; with the coil placed in the center of the air domain. A fine physics-controlled triangular mesh, with a minimum element size of 0.075mm, was applied to coil and air domains. The air domain was bounded by a homogeneous Dirichlet BC. For simplicity the coil response with no armature was first measured in the TC rig. In this test, coil current and capacitor bank voltage were measured using a CWT Rogowski coil and a differential voltage probe. Fig. 4 shows the coil current and capacitor bank voltage for a simulation, obtained using COMSOL circuits interface, and an experimental result. As can be seen in Fig. 4 the simulation results differ from the experimental measurements. Since all the electric circuit parameters (besides coil values) are known from Table I, differences between simulation and experimental results are most probably related to the FE part of the simulation. Factors such as meshing, BCs, solution settings and geometric discrepancies may explain the difference. The limitations of the TC 2D FE axis-symmetric representation are assessed in detail below.

A. Axis-symmetric coil model error assessment

To assess the accuracy of the 2D model predicted coil parameters against existing alternatives, the estimated coil inductance and resistance values were compared with those obtained from measurements, a 3D FE stationary solution and analytical expressions. It should be noted that given the approximate nature of a FE solution the predicted values are susceptible to model implementation and solution settings (e.g. domain geometry, mesh density, boundary conditions, tolerance error). For instance, the size of the air domain and boundary conditions (BCs) have a marked effect on the calculated inductance value, while mesh density affects the predicted coil resistance; i.e. a larger air domain improves the accuracy of the predicted inductance value while a denser mesh is beneficial for coil resistance calculation.

In this analysis the 3D FE model was only used to estimate

coil parameters, therefore, for economy of computing resources, the coil was placed in the center of a spherical air domain with a radius three times the coil outer radius. A custom tetrahedral mesh was applied to the FE model to minimize RAM requirements. The 3D spherical air domain was set as three times the coil outer diameter, since differences in calculated inductance values for further diameter increases were negligible. An infinite element domain, with a thickness of a quarter of the sphere radius, was included as the sphere outermost layer. The presence of the infinite element domain forces the FE software to stretch the coordinates within the domain, emulating an infinitely large domain (which is beneficial for coil inductance calculation) without increasing the actual domain size, thus reducing the impact that the BC has on the calculated inductance value. The same BC type was used in both 2D and 3D models. Since the 2D model cannot include the coil lead outs, these were neglected in the 3D model in order to enable a straight forward comparison between FE implementations.

Although the 3D stationary model takes a fraction of the time required to solve the 3D transient simulation, the stationary 3D solution still requires considerable computer resources and cpu time to be completed, see section IV-B. As an alternative to the 3D simulation, and for comparison, an analytical expression was used for coil inductance calculation [18-20]. Wheeler's empirical formula for planar coil inductance calculation (9) was used here for comparison.

$$L = \frac{n^2 a^2}{(8a+11c)} \mu\text{H} \quad (9)$$

where n is the number of turns, a is the coil mean radius, and c is the coil width in inches. It should be realized that many formulae for calculation of planar coil inductance rely on simplifications and approximations that limit their applicability to certain conductor cross section geometries [19]. However if suitable geometries are considered, accuracy within 5% of measured values can be expected using (9). The accuracy will be lower if for instance: there are few turns, the spacing between turns is high, or if skin effect and distributed capacitance are important factors [19]. For completeness, theoretical DC coil resistance is also included in the comparison. Theoretical coil resistance was calculated by multiplying the coil length (11.45m), obtained using a CAD software program, by the wire manufacturer Ω/m value at 20°C. Table III summarizes coil inductance and resistance values obtained from the two FE implementations, compared with the analytically calculated and measured values. It should be noted that, contrary to the FE and analytical values, measurement quantities in Table III include coil lead out effects; spiral coil and lead outs form a single continuous piece of wire. When lead outs were included in the 3D model their effect on coil inductance was negligible. On the other hand, the resistance of the 3D model with lead outs included was calculated as 29.02 m Ω , slightly higher than the theoretical coil resistance value of 28.84 m Ω and higher than the measured value of 28.77 m Ω . This difference is probably related to discrepancies between the model and the actual coil in the regions where the coil lead outs bend (see Fig. 1 left).

Nevertheless, good agreement between the 3D model predicted values and measurements exists. For instance, the relative error in inductance for the 3D FE simulation, compared with the measured value, is 2.2% while for the analytical expression it is 2.8%. In contrast the 2D simulation exhibits a higher error in inductance of 12%. However, as noted before, in an FE simulation the model settings may have an important effect on the simulation results. For instance, when an air domain similar in size and geometry to that used in the 3D model was considered in the 2D implementation the difference in predicted inductance values between models reduced significantly, although it still remained large as illustrated in Fig. 5. Fig. 5 shows the absolute and relative resistance and inductance differences of the 2D model compared to the 3D model for several coil turn numbers when using similar air domain settings and BCs. In this exercise the radius of the spherical air domain region was increased to four times the coil outer radius to compare the models under most favorable conditions. The coil outer radius was dictated by the number of turns while all other coil parameters were left unchanged with respect to the previous case. Coil lead outs effects were not considered in the comparisons.

As can be seen in Fig. 5 the absolute difference between the 2D and 3D model calculated values for resistance and inductance increases as the turn number increases, and the relative difference increases with turn number for resistance while it decreases for inductance. Thus, under similar conditions the 3D FE model should be expected to be more accurate for coil parameter prediction than the 2D model. For example, even with the improvement in accuracy for the 28 turns coil predicted inductance value, the relative error in inductance of the 2D model is 6% of the measured value. In contrast that of the 3D model was reduced to 1.5%. It should be noted that no significant further improvement in predicted inductance value was achieved in both models by increasing further the surrounding air domain size.

In published results differences in coil response have been attributed to stray inductance and resistance due to coil terminal effects not accounted for in the simulations; subsequent experimental results have been used to adjust the FE model response [14, 17]. In contrast the analysis in this paper suggests that the simplistic assumption of coil axial symmetry can be in itself a considerable source of error. A 3D model offers a more realistic representation of the physical device, however, since coil resistance and inductance are susceptible to mesh density and domain size the computing resources required for the 3D stationary analysis may become significant. Theoretical coil resistance and analytically predicted inductance (if applicable) can provide adequate reference values if the available computer resources for the 3D analysis are limited.

TABLE III
COIL INDUCTANCE AND RESISTANCE VALUES COMPARISON

Source	Inductance (μH)	Resistance (m Ω)
2D FE (stationary)	78.25	27.06
3D FE (stationary)	87.31	27.71
Analytical	86.72	27.71
Measurement	89.24	28.77

Fig. 5. Resistance (top) and inductance (bottom) differences of the 2D model compared with the 3D FE model for different coil turn number.

It is clear from this comparison that the 2D axial model significantly lacks in accuracy compared with the 3D model, thus any subsequent analysis using the 2D model must be taken with reserve. On the other hand, a full 3D FE transient simulation may be expensive in terms of computational resources, see section IV-B. Therefore a more faithful 2D implementation would be preferred for transient analysis.

In order to improve 2D model accuracy with respect to measured values, model passive compensation is investigated here as an alternative to a full 3D FE transient simulation. The 2D model is compensated by adding series resistance and inductance to account for differences between the 2D and 3D model predicted values. This approach is feasible due to the linear nature of the magnetic circuit. Given the dependence of the FE solution on model setup it is important to keep the 2D model consistent between simulations, once the degree of compensation has been established. The validity of this approach is investigated in the next section.

B. Compensated 2D FE axis-symmetric model validation

Fig. 6 shows the coil current and capacitor bank voltage for the same case as in Fig. 4, but this time the comparison is against the compensated model. As can be seen in Fig. 6 there is excellent agreement between the compensated model and the experimental results, thus validating (for coil response only) the proposed approach. Next, the accuracy of the compensated model was investigated when armature interaction occurs. For ultra-fast applications, as in HVDC breakers, typical operation times are around 2ms [1-4]. Hence the focus in this analysis lies within this time frame. To investigate armature interaction effects, the TC armature displacement was recorded using a Photron Fastcam SA-X2 high speed camera at 50,000 fps; covering a fixed visual area of 4x3.5 cm, approximately, at a resolution of 512x432 pixels. Consequently, armature tracking time will vary between tests depending on armature speed. Automatic video tracking software was used to determine the armature displacement versus time characteristic and armature speed was estimated by numerical differentiation of the armature displacement. It should be noted that numerical differentiation of experimental data is prone to sudden and large variations: discontinuities in

the measured signal (e.g. due to poor video edge resolution) are perceived as large changes in its derivative. In order to reduce jitter in the speed signal the recorded armature displacement was first conditioned using a zero-lag running average algorithm, after which a smooth noise-robust numerical differentiator was used to calculate the armature speed. In the tests two armature material types were used: aluminum alloy 6082-T6 and high conductivity oxygen free copper. Nominal material values for electrical conductivity and density were used in the compensated 2D FE simulations. In order to reduce armature flexing in this initial assessment 100mm diameter armatures with thicknesses greater than 5% of diameter were used. The armature dimensions are listed in Table IV. TC operation was investigated at voltage levels of 100, 150, 200 and 250V.

Fig. 7 shows a picture of the experimental apparatus and Fig. 8 shows an example frame of the recorded armature displacement. A comparison between the measured and predicted coil current and armature velocity for the TC operating with the aluminum armature at an initial voltage of 250V is shown in Fig. 9. As can be observed in Fig. 9, excellent agreement exists between predicted and measured values. A similar level of agreement occurred with the copper armature. This is further illustrated in Fig. 10; where measured and predicted armature speed is compared for the two armatures and for the four different charging voltages. Owing to the fixed visual area of the high speed camera, the duration of the measured speed signal in the figure varies depending on armature speed. It is clear from the measurements and analysis, that the compensated 2D FE axial model is able to predict with high fidelity the TC behavior for highly rigid armatures, even at speeds higher than 10 m/s. However due to the extreme forces to which the armature is subject, armature bending may occur. Moving mass minimization is one recommended technique for efficient TC operation [14-16]. Armature mass reduction may help to gain speed at the cost of armature rigidity and bending, which if extreme may damage the actuator armature. The ability of the 2D FE model to predict TC behavior when moderate armature bending occurs is investigated in the next section.

Fig. 6. Coil current and capacitor voltage comparison between measurements and 2D FE model with compensation.

TABLE IV
ARMATURES DIMENSIONS

Material	Thickness [mm]	Inner radius [mm]	Outer radius [mm]
Copper	6.36	20	50
Al 6082-T6	5.98	20	50

Fig. 7. TC prototype video recording arrangement

Fig. 8. High speed camera video frame

Fig. 9. Experimental vs. predicted coil current (top) and armature velocity (bottom) for a small 6mm thick Al 6082-T6 armature.

Fig. 10. Measurement versus simulation speed comparison for aluminum (top) and copper (bottom) armatures for different capacitor bank voltages.

IV. MODEL ASSESSMENT FOR ARMATURE FLEXING

With the assumed geometry a 2D FE axis-symmetric model is limited in its capability to predict armature vibration modes, and therefore the effects of armature bending in the armature's

velocity signal. The impact of armature bending in the predicted armature velocity is investigated in this section.

A. Frequency domain analysis

Limitations of the 2D FE model for the prediction of armature flexing are illustrated in Figs. 11-12, where vibration modes predicted by 3D and 2D FE eigen frequency analysis, are shown. A 3mm thick, 4cm inner diameter, 22cm outer diameter, aluminum alloy 6082-T6 armature disc was modelled for the analysis. The vibration modes shown in Fig. 11 are a subset of the modes below 2 kHz predicted by the 3D eigen frequency analysis. In contrast, the first vibration mode in Fig. 12 (fundamental mode) was the only mode below 2 kHz that was predicted by the 2D axial model. This limitation highlights one of the drawbacks of the 2D model.

Higher frequency modes usually recede faster with time than low frequency modes. Hence, low frequency modes are expected to be more relevant on the armature mechanical behavior. Nevertheless, in order to verify the correctness of the FE model predictions six disc armatures of different material grades and thickness were tested and their mechanical and electrical properties determined. All the tested disc armatures possess inner and outer diameters of 4 and 22 cm, respectively. Material density was approximated using the armature weights and geometric dimensions, assuming 1kg of weight mirrors 1kg of mass. To determine the armature material electrical conductivity a sample of known cross section and length was connected to a high precision micro ohm meter through two high conductivity electrodes of known contact resistance, Fig. 13. The material sample resistance was measured and its material conductivity calculated.

Table V summarizes the measured armature properties. For convenience the different armatures will be referred to, from here onwards, by their rounded thickness value: 3, 6 and 9mm, accordingly. It is important to mention that given its low thickness and rigidity it was impossible to determine with accuracy the conductivity of the aluminum alloy used in the 3mm armature. When pressure was applied to the electrodes to ensure good electrical contact the sample bar tended to bend, resulting in considerable variations in measured resistance, mostly due to varying contact resistance. Consequently this armature will be omitted in most of the subsequent analysis.

Fig. 11. 3D FE eigen frequency analysis, 3mm aluminum armature predicted shape modes below 2kHz.

Fig. 12. 2D FE eigen frequency analysis, 3mm aluminum armature predicted shape modes up to 12 kHz.

Fig. 13. Armatures (top left), material samples (top right) and resistance measurement (bottom).

TABLE V
ARMATURES MEASURED PROPERTIES

Armature Material	Thickness [mm]	Weight [g]	Density [kg/m ³]	Conductivity [% IACS]
Cu	3.28	1070	8845	83
Cu	6.36	2063	8828	85
Cu	9.00	2906	8811	65
Al	2.89	283	2664	--
Al	5.98	595	2706	36
Al	9.07	888	2664	25

Vibration frequencies of annular circular plates can be analytically calculated using (10) [21].

$$f = \frac{\beta}{2\pi r^2} \sqrt{\frac{D}{\rho t}} \quad (10)$$

with

$$D = \frac{Et^3}{12(1-\mu^2)} \quad (11)$$

where r is the armature radius, t is disc thickness, E is Young's modulus, μ is Poisson's ratio and β is a coefficient that varies depending on mode shape, Poisson's ratio and plate internal and external radii. For a free-free annular plate, with an inner to outer diameter ratio of 0.18 with $\mu = 0.33$, $\beta \approx 8.50$ for the fundamental shape mode (1,0), [22]. For validation purposes the analytical frequency value for the fundamental mode obtained using (10) will be compared with that predicted by 2D and 3D FE simulations and that obtained from experimental measurements (M).

In order to determine experimentally the armatures' natural frequencies, they were subjected to an impact test. In each test a Bruel&Kjær type DT4394 miniature piezoelectric accelerometer was mounted perpendicular to the armature main surface, Fig. 14, and the armature was mechanically excited using an impact hammer. The accelerometer output signal was conditioned and recorded using a Bruel&Kjær 3560-B-130DAQ system and the Pulse Vibration Analysis platform. A vibration signal FFT analysis was performed using the Pulse platform proprietary routine, at 6400 line resolution, for a 0-2kHz bandwidth. Exponential FFT averaging was used to reduce signal noise. Fig 15 shows the resulting vibration frequency spectrum for the 9mm copper armature, as an example.

Tables VI and VII compare the eigen frequencies predicted by the 2D and 3D FE models against the measured natural frequencies for the copper and aluminum armatures for frequencies below 2kHz. To be consistent with the theoretical analysis, a Poisson's ratio of 0.33 was assumed for all armatures in the FE eigen frequency analysis. Young's modulus was adjusted for each disc to better match globally the measured frequencies. In the tables the fundamental frequency, obtained with (10), is listed in the corresponding column header as reference. Predicted and measured frequencies of similar value are listed in the same row for each armature. An empty cell indicates that no similar component was found for the armature within the corresponding set of predicted or measured frequencies. The presence of duplicated frequencies in the tables implies the existence of predicted vibration modes of similar shape but displaced in phase.

From the results in Tables VI and VII it is clear that the 3D FE model is able to predict with good accuracy the expected mechanical response of the armature. On the other hand, below 2kHz, the 2D model only predicted the existence of the fundamental vibration mode. It is interesting to note that not all frequencies predicted by the 3D model were detected in the physical system. It is also interesting to see how the number of natural frequencies below 2kHz decrease with armature thickness (rigidity), pushing the fundamental vibration mode to a higher frequency, as can be predicted with (10).

Fig. 14. Accelerometer mounting for armature impact test.

Fig. 15. 9mm Copper armature impact test vibration frequency spectrum.

TABLE VI COPPER ARMATURES
PREDICTED VS MEASURED NATURAL FREQUENCIES BELOW 2 kHz

3mm, fund=404 Hz			6mm, fund=789 Hz			9mm, fund=1121 Hz		
2D	M	3D	2D	M	3D	2D	M	3D
--	240	241	--	468	468	--	662	662
--	245	241	--	--	468	--	674	662
404	434	404	784	807	784	1110	1123	1110
--	580	579	--	1119	1118	--	1594	1576
--	--	579	--	--	1118	--	--	1576
--	--	939	--	--	1804			
--	--	939	--	--	1804			
--	1022	1017	--	1960	1957			
--	--	1017						
--	1566	1558						
--	--	1558						
--	--	1622						
--	--	1622						
--	--	1920						

TABLE VII ALUMINUM ARMATURES
PREDICTED VS MEASURED NATURAL FREQUENCIES BELOW 2kHz

3mm, fund=513 Hz			6mm, fund=1040 Hz			9mm, fund=1589 Hz		
2D	M	3D	2D	M	3D	2D	M	3D
--	325	307	--	615	618	--	936	938
--	336	307	--	623	618	--	946	938
513	525	513	1035	1048	1035	1574	1589	1574
--	731	735	--	1463	1477			
--	--	735	--	--	1477			
--	--	1194						
--	--	1194						
--	1263	1293						
--	1292	1293						
--	1953	1982						
--	1992	1982						

B. Time-domain analysis

Since armature bending occurs, not all the parts in the armature move at the same speed. This is illustrated in Fig. 16, where armature displacement is modelled using the 2D FE simulation for the 3mm copper armature; armature bending is exaggerated in the figure for illustration purposes. Thus for a correct comparison, the measured speed needs to be compared against the predicted speed of a point located in a similar position to the tracked feature. Such a comparison is shown in Fig. 17, where the measured speed is compared to the relative speed of a point located on the external armature edge of the compensated FE model, the armature mean speed is included in the figure for reference.

The results in Fig. 17 show that when simulations and measurements focus on a similarly located point the 2D model is able to predict with excellent accuracy the armature behavior, even when armature bending occurs. Thus implying that the fundamental vibration mode is the dominant one and, most importantly, that the predicted mean armature velocity is correct. Furthermore, it was found during the analysis that if sensible values of Young's modulus (around 115 GPa for copper and 70 GPa for aluminum alloys) and Poisson's ratio (0.30-0.33 for both types of base material) are used, small variations in their values do not impact significantly on the model predictions. On the other hand the numerical simulation was found to be quite sensitive to the material electrical conductivity. Electrical conductivity affects mean armature velocity.

To verify frequency dominance a joint time-frequency analysis of the armature's velocity signal was conducted. The analysis is complicated by the short duration of the recorded

signal and its non-stationary nature. Due to its robust capabilities for the analysis of non-stationary signals the highly adaptive Hilbert-Huang transform (HHT) algorithm [23] was chosen for signal processing. Fig. 18 shows the time-frequency plot of the armature velocity signal obtained using the HHT algorithm for a 0-2kHz bandwidth for simulation and experimental results for the 9mm copper armature, with an initial voltage of 150V. As can be expected, the experimental result exhibits a richer and less well defined spectrum than the simulation result, with a strong frequency component sitting around 1100 Hz; which incidentally matches the armature's fundamental vibration mode frequency. Traces of a component around 1.5 kHz are also visible in the spectrum of the experimental signal. In contrast, only the 1100 Hz component is visible for the simulation results in the higher part of the spectrum. It is interesting to notice in Fig. 18 that both spectrums exhibit a strong, non-predicted, low frequency component that falls below 100 Hz with time. Such a component is related to the armature mean speed. The components relatively high value at the beginning is most probably related to HHT end point effects [24]. It should be noted that for all the cases in Fig. 17, the presence of the fundamental vibration component was clearly visible in the corresponding velocity spectrum.

This analysis confirms that the fundamental vibration mode is the dominant mode for the investigated scenarios, and explains why the compensated 2D FE model is able to predict with good accuracy the experimental armature behavior. However, in accordance with the eigen frequency analysis above, inaccuracies in the 2D FE model prediction are to be expected. Thus, even though excellent agreement is shown between the 2D model results and experimental measurements for the illustrated cases, there will be situations where this will not occur. This is because, as shown in section IV-A, the 2D model cannot predict all possible armature mode shapes. For instance, for several frequencies in Tables VI and VII, two different vibration modes are predicted. These mode pairs are similar in shape, but displaced in phase, Fig 19. Thus depending on dominant vibration modes and location of the tracked feature, the predicted speed may not coincide in phase with the measured speed or may otherwise fail to predict vibration modes present in the physical armature. This scenario is illustrated in Fig. 20, where measured and predicted speed oscillations seem to be in phase opposition in Fig. 20a, while measured high frequency oscillations are not present in the simulation results of Fig. 20b. Interestingly, the behavior shown in Fig. 20 was not observed at higher armature speed. For higher capacitor voltage and for the same armature only the fundamental vibration mode was observed. Thus the type of vibration mode depends on system excitation. Nevertheless, the predicted speed remained close to the measured speed signal in all cases analyzed.

A 3D FE transient simulation is able to predict speed variations along the armature periphery. However, the considerable computer resources and simulation time that may be required by this type of simulation may make its use impractical at an early stage in the TC design process, if many

simulations are necessary for design refinement. For example, in a Core i5 computer at 3.2 GHz with 32 GB of RAM, the 28 turns 2D axis-symmetric model with a predefined physics controlled finer mesh was solved in an average of 40 minutes for a 3.5 ms simulation period. In contrast, a reduced 10 turns 3D transient model with an optimized user-controlled non deformable mesh took 125 hours to be solved on the same computer and for an identical simulation time. The 28 turn 3D transient case could not be processed due to insufficient RAM in the reference computer, owing to the large number of mesh elements required to represent the coil geometry with sufficient detail. The 28 turn 3D stationary solution took six hours to be completed. Clearly, for accurate fast FE analysis the 2D model with compensation represents an excellent compromise between results accuracy, computing resources and simulation speed.

V. CONCLUSIONS

In this paper the accuracy of a 2D TC axis-symmetric FE model with passive compensation is assessed by comparison with experimental results obtained from a purpose built TC test-rig. A 3D FE model was used to calculate more precise coil inductance values and the 2D model was compensated accordingly. Comparison with experimental results validated the proposed approach. It was found that analytical formulae may be used as an alternative for correct coil inductance calculation, when coil geometry allows it. It is shown that when no sizeable armature flexing occurs, detailed characterization of armature and excitation circuit allows for a high fidelity representation of the TC behavior. When armature bending occurs, the assumption of symmetry limits the 2D model capabilities. Shape modes naturally present in the TC armature may not be reflected correctly or are omitted altogether from the model predictions. However, the 2D model was able to predict with good accuracy the armature behavior for speeds in excess of 10 m/s in the scenarios where the fundamental vibration mode was dominant. Good agreement between predicted mean velocity and measurements was observed for all the cases assessed. Correct armature electrical conductivity was found to be more relevant for accurate prediction of TC behavior than the material mechanical properties (Young's modulus and Poisson's ratio).

Given the important differences in computing time and computational resources that may be required for the solution of a 2D axis-symmetric model and those of a 3D FE transient model, the 2D FE axial model with compensation is an attractive alternative for fast and accurate analysis and design of ultra-fast TC actuators.

Fig. 16. FE simulation results for 3mm copper armature at 250V, field displacement.

Fig. 17. Measured versus predicted speed for a point in the armature lateral edge for copper (top) and aluminum (bottom) armatures at 250V.

Fig. 18. Armature velocity instantaneous frequency for the 9mm copper armature with 150V excitation: simulation (top), experimental (bottom).

Fig. 19. Predicted shape modes at 1477 Hz for 6mm aluminum armature.

Fig. 20. Measured versus predicted speed for a point in the armature lateral edge for 6mm aluminum (a) and copper (b) armatures at 150V.

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