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Colour change sensitivity of dye recipes

D P Oulton and P Chen

Dept of Textiles, UMIST, PO Box 88, Manchester M60 1QD, UK

Relationships are investigated between concentration changes in multi-dye recipes and colour changes. Two methods are examined for predicting colour changes associated with small changes in dye concentration around ten colour centres. A synthetic reflectance curve generator and commercial match prediction system are used to generate the recipes. This simulates a dyeing process of zero variability and isolates the variables associated with the dyes. A matrix method is introduced to calculate ΔE for a given recipe concentration change in one or more dyes, possibly moving in opposite directions. The results are related to dye standardisations of $\pm 2.5\%$, 5% and 10% strength. The matrix method is confirmed using a manual match prediction program.

INTRODUCTION

A reproducible colour appearance and a high probability of being 'right first time' in a coloration process are always major objectives of dyers and colourists. The penalties for not achieving right-first-time processing in coloration are severe [1]. It has been reported that weighing and measuring errors are major sources of off-shade dyeings [2,3]. Inaccuracy of colorant dispensing is one of the major problems in the coloration process, especially in blind dyeing. The present investigation is concerned with the relationship of small colour changes and the recipe changes associated with them. The investigation also relates colorant standardisation to colour changes.

A model system has been established to isolate the variables concerned. The coloration process itself is assumed to be constant. In other words, the same recipe always produces an identical colour.

The model uses a CMC(2:1) ΔE value of one to define a small volume of colour space in which to establish relationships between changes in colour and concentration. This in turn establishes the smallest recipe change that will produce a given colour difference. Changes along the semi-axes are analysed, i.e. chroma differences when hue and lightness differences are zero, with equivalent pure hue and pure lightness differences. We reported in a previous paper that $\Delta E = 1$ hue change results from much smaller recipe changes than $\Delta E = 1$ lightness or chroma changes [4]. In this further investigation our aim was:

- To establish the relationship between small colour changes and recipe changes
- To relate the colour change sensitivity to a colorant standardisation limit [5].

In each case the results refer to the model system described above. This includes just one set of variables, i.e. colorant concentrations, which may affect colour uncertainty. All other variables were assumed constant. In a real coloration process additional sources of uncertainty in colour would be present.

EXPERIMENTAL

The UMIST Shademaster and Macbeth colour match prediction system were used for the generation of experimental data [6,7].

Generation of colour difference pairs

The Shademaster system was designed to allow change of chroma, hue and lightness independently. Thus a colour A was selected, on Shademaster as a target colour in CIE colour space, then it was changed to colour A' making the difference ΔE equal to 1. This gives a pair of colours precisely one ΔE unit apart, with a specific direction of change, e.g. in hue only. Synthetic reflectance curves for these colours were then generated by the Shademaster system. The curves were generated using the CMC(2:1) colour-difference formula. Three different reflectance curves were generated for samples changed in the hue, chroma and lightness directions from the target colour to $\Delta E = 1$. The data from these reflectance curves were entered into the Matchmaker system to make recipe predictions. Pairs of reflectance curves representing colours one ΔE CMC(2:1) unit apart were generated.

Table 1 Colorants (all BASF) used with Matchmaker

No.	Colorant name
N2	Helizarin Yellow G Conc
N3	Helizarin Brilliant Yellow RRT Conc
N4	Helizarin Brilliant Orange GT Conc
N5	Helizarin Scarlet B Conc
N6	Helizarin Brilliant Red BBT
N7	Helizarin Brilliant Violet BT
N8	Helizarin Blue RT
N9	Helizarin Blue Conc
N10	Helizarin Green BT Conc
N11	Helizarin Brown RRT Conc
N12	Helizarin Dark Brown TT
N13	Helizarin Black HDT

Table 2 Target shades, recipes and alterations for one ΔE unit change in lightness, chroma and hue

Target	Recipe no.	Colorant	Concn	C _l	C _{ch}	C _h	Target	Recipe no.	Colorant	Concn	C _l	C _{ch}	C _h
A	1	N2	0.7216	-0.0652	0.1065	0.0005	F	11	N2	2.566	-0.6388	0.5608	-0.2471
		N5	0.0723	-0.0089	-0.0004	-0.0006			N4	0.034	-0.0077	-0.008	-0.0042
		N10	0.0283	-0.0047	-0.0005	0.0028			N10	0.3111	-0.0583	0.0154	0.0166
	2	N3	0.2781	-0.0253	0.0374	-0.0008	12	N2	2.717	-0.6801	0.5592	-0.2764	
		N5	0.0412	-0.0054	-0.0021	-0.0025		N6	0.0236	-0.0056	-0.0054	-0.0025	
		N10	0.0397	-0.0057	0.0001	0.0035		N10	0.2795	-0.0499	0.0216	0.0186	
	3	N2	0.7648	-0.0711	0.1101	-0.0009	G	13	N3	0.4969	-0.2145	-0.372	-0.2242
		N6	0.0607	-0.0068	-0.0005	-0.0021			N6	6.7554	-0.9495	1.2205	0.1708
		N10	0.021	-0.0038	-0.0003	0.0026			N7	0.0527	-0.0128	-0.0369	-0.0046
	B	4	N3	1.4692	-0.1584	0.3609	0.0172	14	N4	1.4876	-0.4964	-0.9966	-0.6048
N5			0.4926	-0.0388	0.0115	-0.0643	N6		5.9308	-0.8726	1.7264	0.4407	
N10			0.0344	-0.0045	-0.0038	-0.0018	N7		0.0681	-0.0147	-0.0473	-0.0098	
C	5	N2	2.891	-0.6936	0.6534	-0.0649	H	15	N3	0.3962	-0.0487	0.1249	0.0918
		N10	0.1715	-0.0274	0.0048	0.0157			N4	2.0323	-0.4386	0.1556	-0.0858
		N11	0.4903	-0.066	-0.0093	-0.0137			N10	0.0681	-0.0137	-0.005	0.0025
	6	N3	1.1952	-0.3016	0.2231	-0.0252		16	N2	4.101	-1.4457	0.4586	0.3179
		N10	0.2698	-0.0487	0.0137	0.0208			N4	1.0799	-0.1511	0.1301	-0.0536
D	7	N11	0.303	-0.034	-0.023	-0.0186	17	N7	0.0884	-0.0176	-0.005	0.002	
		N5	0.2085	-0.0345	-0.0471	0.0156		I	N3	2.6797	-0.6775	0.4992	0.292
		N7	0.2662	-0.0297	0.0682	0.0103			N5	1.1091	-0.2	0.0625	-0.0419
	8	N8	0.089	-0.0184	-0.0483	-0.0218	18		N10	0.0133	-0.0047	-0.0038	0.0019
		N6	0.3129	-0.0511	-0.0843	0.0243		N2	7.1022	-1.156	0.8906	0.5803	
		N7	0.1753	-0.0232	-0.089	0.0059		N5	1.516	-0.3507	0.1421	-0.0557	
	E	9	N8	0.1277	-0.0209	0.1293	-0.0197	19	N10	0.0096	-0.004	-0.0031	0.0013
			N2	1.3091	-0.2552	0.3305	-0.0649		N3	2.62	-0.6634	0.511	0.2809
			N7	0.0465	-0.0098	-0.0028	-0.0019		N5	0.9832	-0.1588	0.0949	-0.0629
10		N10	0.0911	-0.0164	0.0074	0.0097	J	N11	0.1446	-0.0473	-0.0379	0.0194	
		N3	0.4916	-0.1062	0.0762	-0.03		N2	6.2769	-1.1944	0.8453	0.3545	
		N7	0.0125	-0.0035	-0.0038	-0.0014		N4	0.5827	-0.0655	0.0625	-0.045	
		N10	0.1678	-0.0321	0.0139	0.0126		N7	0.0936	-0.019	-0.0045	0.003	

Recipe prediction

Standard recipe prediction procedures were used to generate recipes using a Macbeth Matchmaker system. There are twelve colorants available in the system, listed in Table 1. Three-colorant recipes were predicted. Typical results of the base recipes are listed in Table 2.

The resulting recipes were analysed and compared for critical variables that might have affected colorant standardisation and colorant dispensing accuracy. All had a predicted $\Delta E < 0.05$ to the input curve under illuminant D_{65} , although metamerism was allowed under other illuminants.

Colour variation with colorant concentration

To reveal the variation of recipe ΔE , ΔL , ΔC and ΔH (where ΔL , ΔC and ΔH are the lightness, chroma and hue changes) with colorant concentration, a second experiment was carried out. For a given target, a predicted recipe can be generated using the Matchmaker system. If a colorant concentration in this recipe is changed, the resulting colour will shift from the target. For a given concentration change of a colorant, there is a corresponding ΔE , ΔL , ΔC and ΔH . The manual match program allows specific colorant concentration changes to be explored. This experiment was carried out for two targets with concentration changes of each colorant in increasing 1% steps until $\Delta E = 1$ was reached. The plots of ΔE , ΔL , ΔC and ΔH versus Δc are shown in Figure 1 (where Δc is change in

colorant concentration). This shows the effect of changing a single components of the three-colorant recipe.

It has been found that rate of change of colour with concentration varies substantially, thus the x -axis and y -axis scales in Figure 1 are varied. The slopes for colorant 1 are twice as steep as those for colorant 3 in this recipe. It follows that in this recipe colorant 1 is the major contributor to colour change sensitivity.

RESULTS AND DISCUSSION

Determining recipe sensitivity

In general, a CMC unit hue change corresponds to the subjectively smallest curve changes, while CMC unit lightness changes result in largest curve shifts. From Figure 1, it can be seen that there are nearly linear relationships between colorant concentration change and colour within a small volume of colour space, e.g. $\Delta E < 1$.

If a linear relationship is assumed for all nine variables (three colorants and three dimensions of colour), it is possible to construct a set of nine equations linking concentration change to colour change in a given recipe. R_l , R_{ch} and R_h are defined as lightness, chroma and hue unit rates of change for unit colorant concentration change, i.e. $R_l = \Delta L/\Delta c$, $R_{ch} = \Delta C/\Delta c$ and $R_h = \Delta H/\Delta c$ (Eqn 1):

$$\Delta L = \Delta c R_l \quad \Delta C = \Delta c R_{ch} \quad \Delta H = \Delta c R_h \quad (1)$$

Figure 1 Target change corresponding dye concentration change

R_l , R_{ch} and R_h are characteristic constant rates of change for a given colorant in a given recipe, expressing the linear relationships.

Using the values from Table 2 giving the concentration changes of colorants, for one ΔE unit change in lightness, chroma and hue separately, the values of R_l , R_{ch} and R_h defined above can be calculated for each colorant.

It is now possible to establish the nine R values R_{l1} to R_{h3} in the matrix which relates the colorant concentration changes Δc_1 , Δc_2 and Δc_3 to colour change ΔL , ΔC and ΔH , for all colorants in a given three-colorant recipe. In the following example, the three cases of lightness, chroma and hue changes only are used to generate the full matrix.

Table 2 shows recipe changes for CMC unit changes in lightness (Δc_l), chroma (Δc_{ch}) and hue (Δc_h). Three-colorant recipes are given for a target colour using different colorant combinations (column headed 'Colorant') at

different concentrations (c). In recipe 8, target colour D , which has CIELAB coordinates $L^* = 46.0$, $C^* = 34.91$ and $H^* = 314.34$, as an example, recipe changes are given in Table 3 for moving the target colour along each of the colour-difference component vectors in CIELAB space corresponding to one CMC unit of colour difference.

The values in the Δc_l column in Table 2 are the concentration changes of colorants for CMC unit lightness change only. For the chosen recipe 8, if colorant concentration changes are $\Delta c_6 = -0.0511$, $\Delta c_7 = -0.0232$ and $\Delta c_8 = -0.0209$, they will result in a shade change of one CMC unit difference, with CIELAB changes $\Delta L = 2.1$, $\Delta C = 0$ and $\Delta H = 0$. Thus the changes due to individual colorants add up to give Eqn 2:

$$\begin{aligned}\Delta L &= \Delta L_6 + \Delta L_7 + \Delta L_8 = 2.1 \\ \Delta C &= \Delta C_6 + \Delta C_7 + \Delta C_8 = 0 \\ \Delta H &= \Delta H_6 + \Delta H_7 + \Delta H_8 = 0\end{aligned}\quad (2)$$

where subscripts 6, 7 and 8 refer to colorants N6, N7 and N8, and ΔL etc., are those used in Eqn 1. These can be related to concentration by Eqn 1, giving Eqn 3:

$$\begin{aligned}\Delta L &= \Delta C_6 * R_{l6} + \Delta C_7 * R_{l7} + \Delta C_8 * R_{l8} = 2.1 \\ \Delta C &= \Delta C_6 * R_{ch6} + \Delta C_7 * R_{ch7} + \Delta C_8 * R_{ch8} = 0 \\ \Delta H &= \Delta C_6 * R_{h6} + \Delta C_7 * R_{h7} + \Delta C_8 * R_{h8} = 0\end{aligned}\quad (3)$$

Substituting the concentration change Δc_6 , Δc_7 and Δc_8 with the appropriate values from Table 2 (-0.0511 , -0.0232 and -0.0209 ; recipe 8, lightness change only) gives Eqn 4:

$$\begin{aligned}-0.0511 R_{l6} - 0.0232 R_{l7} - 0.0209 R_{l8} &= 2.1 \\ -0.0511 R_{ch6} - 0.0232 R_{ch7} - 0.0209 R_{ch8} &= 0 \\ -0.0511 R_{h6} - 0.0232 R_{h7} - 0.0209 R_{h8} &= 0\end{aligned}\quad (4)$$

If in the same way the columns Δc_{ch} and Δc_h (Table 2) are analysed, then for chroma change only (Eqn 5):

$$\begin{aligned}-0.0843 R_{l6} - 0.0890 R_{l7} + 0.1293 R_{l8} &= 0 \\ -0.0843 R_{ch6} - 0.0890 R_{ch7} + 0.1293 R_{ch8} &= 2.2 \\ -0.0843 R_{h6} - 0.0890 R_{h7} + 0.1293 R_{h8} &= 0\end{aligned}\quad (5)$$

and for hue change only (Eqn 6):

$$\begin{aligned}0.0243 R_{l6} + 0.0059 R_{l7} - 0.0197 R_{l8} &= 0 \\ 0.0243 R_{ch6} + 0.0059 R_{ch7} - 0.0197 R_{ch8} &= 0 \\ 0.0243 R_{h6} + 0.0059 R_{h7} - 0.0197 R_{h8} &= 1.45\end{aligned}\quad (6)$$

These nine equations can be expressed as a matrix equation (Eqn 7):

$$\begin{pmatrix} -0.0511 & -0.0232 & -0.0209 \\ -0.0843 & -0.0890 & 0.1293 \\ 0.0243 & 0.0059 & -0.0197 \end{pmatrix} \begin{pmatrix} R_{l6} & R_{ch6} & R_{h6} \\ R_{l7} & R_{ch7} & R_{h7} \\ R_{l8} & R_{ch8} & R_{h8} \end{pmatrix} = \begin{pmatrix} 2.1 & 0 & 0 \\ 0 & 2.2 & 0 \\ 0 & 0 & 1.45 \end{pmatrix}\quad (7)$$

Table 3 Recipe changes for moving target colour along each colour-difference component vector in CIELAB space corresponding to one CMC unit of colour difference

Vector	ΔL^*	ΔC^*	ΔH^*
Lightness only	2.1	0	0
Chroma only	0	2.2	0
Hue only	0	0	1.45

The solution of this matrix equation is Eqn 8:

$$\begin{pmatrix} R_{l6} & R_{ch6} & R_{h6} \\ R_{l7} & R_{ch7} & R_{h7} \\ R_{l8} & R_{ch8} & R_{h8} \end{pmatrix} = \begin{pmatrix} -17.36 & 10.66 & 58.83 \\ -25.97 & -27.82 & -101.3 \\ -29.20 & 4.817 & -31.38 \end{pmatrix} \quad (8)$$

The numeric subscripts indicating values for the individual colorants N6, N7 and N8.

The generalised form of Eqn 7 is the matrix equation (Eqn 9):

$$\begin{pmatrix} \Delta C_{l1} & \Delta C_{l2} & \Delta C_{l3} \\ \Delta C_{ch1} & \Delta C_{ch2} & \Delta C_{ch3} \\ \Delta C_{h1} & \Delta C_{h2} & \Delta C_{h3} \end{pmatrix} \begin{pmatrix} R_{l1} & R_{ch1} & R_{h1} \\ R_{l2} & R_{ch2} & R_{h2} \\ R_{l3} & R_{ch3} & R_{h3} \end{pmatrix} = \begin{pmatrix} \Delta L^* & 0 & 0 \\ 0 & \Delta C^* & 0 \\ 0 & 0 & \Delta H^* \end{pmatrix} \quad (9)$$

where Δc is concentration change, subscripts 1, 2 and 3 are the recipe components, subscripts l, ch and h mean that the Δc is taken for lightness change ΔL^* , chroma change ΔC^* and hue change ΔH^* respectively. This makes it possible to establish the nine R values R_{l1} to R_{h3} in the matrix which relate the colorant concentration changes to colour change for all colorants in a given three-colorant recipe.

Table 4 shows the results of the application of Eqn 9 to Table 2 for six recipes. It contains the rates of change data

for each colorant in each recipe. This assumes that rate of change is linear over the appropriate small volume of colour space, as shown in Figure 1. The data can now be used to examine the effect of individual concentration changes.

The variations in individual rates must now be combined to give an overall recipe sensitivity δ , i.e. percentage recipe change for one ΔE unit of colour change.

For given R_l , R_{ch} and R_h percentage concentration (c) change of each colorant the δ value is defined by Eqn 10:

$$\delta = \frac{\Delta c}{c} \times 100 \quad (10)$$

From Eqn 1 for a single colorant, the changes in lightness, chroma and hue are thus (Eqn 11):

$$\Delta L = \frac{\delta c R_l}{100}, \quad \Delta C = \frac{\delta c R_{ch}}{100}, \quad \Delta H = \frac{\delta c R_h}{100} \quad (11)$$

Then, for a given recipe with three components, if the concentrations of three colorants are c_1 , c_2 and c_3 , the equations above become Eqn 12:

$$\begin{aligned} \Delta L &= \Delta L_1 + \Delta L_2 + \Delta L_3 \\ &= \frac{\delta c_1 R_{l1}}{100} + \frac{\delta c_2 R_{l2}}{100} + \frac{\delta c_3 R_{l3}}{100} \\ &= \frac{\delta}{100} \sum_{i=1}^3 (c_i R_{li}) \end{aligned} \quad (12)$$

Similarly, for chroma change and hue change (Eqns 13 and 14):

$$\Delta C = \frac{\delta}{100} \sum_{i=1}^3 (c_i R_{chi}) \quad (13)$$

$$\Delta H = \frac{\delta}{100} \sum_{i=1}^3 (c_i R_{hi}) \quad (14)$$

Table 4 Ratios of colour change to colorant concentration change with different recipe numbers from Table 2

	1			2			3		
	R_l	R_{ch}	R_h	R_l	R_{ch}	R_h	R_l	R_{ch}	R_h
1	0.888376	21.69844	-1.41385	-12.4273	55.65785	-11.9207	0.067111	20.78951	-1.56367
2	-183.828	-100.868	-174.365	-209.258	-156.329	-219.911	-210.781	-152.543	-231.707
3	336.2866	-97.5378	-161.657	253.404	-98.9417	-159.804	375.931	-116.011	-187.689
	20			7			8		
	R_l	R_{ch}	R_h	R_l	R_{ch}	R_h	R_l	R_{ch}	R_h
1	1.138436	1.945582	-0.02409	33.76113	-15.9416	-17.9858	58.83015	10.65911	-17.3641
2	-16.4634	5.832675	-6.63427	-10.0391	19.7921	-32.3667	-101.311	-27.8171	-25.9696
3	-14.8101	-142.413	-96.6676	-47.0977	-2.05646	-28.1631	-31.3787	4.817032	-29.1963

The CMC(2:1) colour-difference formula is (Eqn 15):

$$\Delta E = \left[\left(\frac{\Delta L^*}{2S_L} \right)^2 + \left(\frac{\Delta C^*}{S_C} \right)^2 + \left(\frac{\Delta H^*}{S_H} \right)^2 \right]^{1/2} \quad (15)$$

Hence from Eqns 13–15 the percentage concentration change of each colorant may be calculated by substitution for ΔL , ΔC and ΔH to give an overall recipe sensitivity δ (Eqn 16):

$$\delta = \frac{100}{\left\{ \left[\sum \frac{c_i R_{li}}{2S_L} \right]^2 + \left[\sum \frac{c_i R_{chi}}{S_C} \right]^2 + \left[\sum \frac{c_i R_{hi}}{S_H} \right]^2 \right\}^{1/2}} \quad (16)$$

where the tolerance values S_L , S_C and S_H are those of the CMC(l:c) difference formula.

Effects of sensitivity in various combinations

The effect of the individual colorants reaching their tolerance limits is now examined. Table 5 shows the full permutation of possible recipe deviations, + indicating a maximum upward strength deviation, – indicating a downward maximum deviation. Permutations 21–26 show worst-case scenarios where individual colorants move to the limit in opposite directions. Fortunately Table 5 is symmetrical: + 0 0 produces an effect equal and opposite to – 0 0 (Figure 1), so only 13 variations are examined in Tables 6 and 7, i.e. 13 combinations. A four-colorant recipe would have $3^4 = 81$ combinations.

For each recipe, nine simultaneous equations are solved (Eqn 9), giving the values illustrated in Table 4. For a chosen recipe and for each combination of concentration changes, a set of δ values are then derived using Eqn 16. Their column average expresses the overall sensitivity of the chosen recipe (Tables 6 and 7). Table 6 shows the sensitivities of these recipes, as percentage colorant concentration changes (δ values). Table 7 shows these transformed into ΔE values, assuming $\pm 2.5\%$ strength

changes. Averaging along a row shows the effect of a given combination across all 20 recipes. Thus a row average indicates the severity of a given combination of changes.

Recipe sensitivity

From the results shown in Table 6, it is clear that recipe sensitivities are different in different situations. For a given recipe, the sensitivity varies for different directions of change and combinations of direction. When concentrations of colorants are changed in the same direction, or just a single colorant concentration is changed, a small colour change may be expected (combinations 1–6 and 10). When colorants are changed in opposite directions, there may be a large change of colour (combinations 7–9 and 11–13).

In Table 6, combinations 1–3 involve an increase in strength of either colorant 1, 2 or 3 in the recipe. Column 1 shows that for this recipe the concentration of colorant 1 must be increased by 14.88% to give a colour change of one ΔE unit. Colorant 2 only requires a 9.18% change and colorant 3 requires a 14.38% change.

Recipe 8 is highly sensitive to combination 13. If colorant 1 is 3.53% under strength and colorants 2 and 3 3.53% over strength, a ΔE 1 shade change results. By contrast, it would take a 49.37% increase in strength in both colorants 1 and 3 (the minor components) to produce a ΔE 1 change in recipe 13 (combination 5). Combination 8 involves the same two colorants in the same recipe, this time moving in opposite directions. With this combination much smaller concentration changes (14.71%) cause a ΔE 1 shift, for this recipe.

Selection of colorants in a recipe is thus quite important. For example, recipes 7 and 8 are two recipes for the same target colour. Their sensitivities are 17.45% and 11.13% (column average Table 6), indicating the latter is significantly more sensitive to dispensing or standardisation error. These values translate into ΔE values in Table 7 which shows a double ΔE uncertainty for recipe 8 against recipe 7.

Relating recipe sensitivity to colorant standardisation

Since the results in Table 6 relate to a colour change of one ΔE unit, and the relationship between ΔE and Δc is linear, it is possible to relate the recipe sensitivity to colorant standardisation. For a certain combination and a given δ value in Table 6, colorant standardisation of $\pm 2.5\%$ strength would be expected to produce $\Delta E(\text{CMC})$ of $2.5/\delta$. From Table 6, the results shown in Table 7 can be calculated. The effects of limiting colorant strength variation to $\pm 2.5\%$ are clear. Even the worst-case scenario of combination 13, with all three colorants moving to their standardisation limits in opposite directions, produces few problems. However, recipes 8 (at ΔE 0.71) and 20 (at ΔE 0.47) might be troublesome.

It should, however, be emphasised that colorant strength variation is only one possible cause of colour uncertainty.

Table 5 Full permutation of possible recipe deviations of individual dyes reaching their tolerance limits^a

Recipe change				Recipe change				Recipe change			
A	B	C		A	B	C		A	B	C	
1	+	0	0	2	–	0	0	3	0	+	0
4	0	–	0	5	0	0	+	6	0	0	–
7	0	+	+	8	0	–	–	9	0	+	–
10	0	–	+	11	+	0	–	12	–	0	–
13	+	0	–	14	–	0	+	15	+	+	0
16	–	–	0	17	+	–	0	18	–	+	0
19	+	+	+	20	–	–	–	21	+	+	–
22	–	–	+	23	+	–	+	24	–	+	–
25	–	+	+	26	+	–	–	27	0	0	0

^a A, B and C are the recipe components

+ signifies a positive change of dye concentration, – a negative change of dye concentration and 0 no change of dye concentration

Analysis of 260 recipes

Figure 2 gives a cumulative frequency analysis of Table 7 ($\pm 2.5\%$ strength variation), taking each recipe change combination as an independent case (260 recipes). Similarly, cumulative frequency analysis can be calculated from Table 6 at colorant standardisation values of $\pm 5\%$ and 10% ; the results are shown in Figures 3 and 4.

Over all the 260 recipes in Table 7, an average $\Delta E(\text{CMC})$ uncertainty of 0.23, 0.46 and 0.92 (with colorant standardising limits of $\pm 2.5\%$, 5% and 10% respectively) is expected, due solely to colorant strength variation.

Confirmation of calculated sensitivity by straight match prediction

The prediction of sensitivity by the matrix method given above is based on ΔE changes of one unit in hue, chroma and lightness. It relies on a substantially linear relationship, over a small volume of colour space, between

concentration change and colour change. It is possible, but laborious, to achieve the same result using a match prediction system. In this alternative approach, a given recipe is varied manually by, say, $\pm 2.5\%$ in each component, and in varying combinations to produce results similar to those in Table 7.

The matrix method was used as a research tool to reveal the underlying characteristics of recipe sensitivity. The match prediction approach (based on systematic recipe variation by known percentage change) is more practicable for general use, as it does not depend on a specialised synthetic reflectance curve generator.

In order to confirm the validity of the results given by the matrix method of Eqns 9 and 16, match prediction was used. Thirteen combinations and two levels of difference (i.e. 26 variants of recipe 20 in Table 6, target colour J) were examined. Concentration changes predicted by the matrix method to produce ΔE values of 1.0 and 0.5 were used to

Table 6 Colorant concentration change (%) for $\Delta E = 1^a$

Colorant combination	Colorant no.			Recipe no.									
	1	2	3	1	2	3	4	5	6	7	8	9	10
1	+	0	0	14.88	13.98	14.68	24.24	24.26	17.92	18.47	7.49	22.38	14.78
2	0	+	0	9.18	13.4	8.97	14.4	11.96	11.1	19.43	7.89	17.59	40.95
3	0	0	+	14.38	13.16	17.26	16.87	9.64	14.12	31.84	30.19	13.43	11.81
4	0	+	+	12.04	13.39	10.87	13	10.38	9.79	13.29	6.37	14.55	11.8
5	+	0	+	10.85	12.99	12.16	12.33	9.95	10.99	25.36	8.85	17.63	20.94
6	+	+	0	9.22	9.61	9.07	15.16	13.42	17.99	14.86	20.2	13.42	13.74
7	0	+	-	6.14	7.64	6.58	9.64	6.17	7.95	24.95	10.15	8.83	10.94
8	+	0	-	9.9	7.94	10.41	16.12	8.32	11.2	12.62	6.32	9.18	6.86
9	+	-	0	6.9	9.73	6.74	10.72	9.2	7.19	12.28	3.91	14.28	14.07
10	+	+	+	12.96	13.07	12.03	13.06	13.41	13.9	14.35	19.14	20.38	
11	+	+	-	6.06	6.05	6.45	10.59	6.11	9.69	13.8	21.14	7.17	6.66
12	+	-	+	5.36	7.64	5.64	7.91	5.86	6.06	17.6	4.35	9.92	17.29
13	-	+	+	7.7	8.02	7.2	10.89	7.88	6.82	9.13	3.53	9.66	6.88
$\bar{\sigma}$				9.66	10.51	9.85	13.38	10.48	11.09	17.45	11.13	13.63	15.16

Colorant combination	Dye no.			Recipe no.										$\bar{\sigma}$ row
	1	2	3	11	12	13	14	15	16	17	18	19	20	
1	+	0	0	21.96	21.41	35.85	26.61	34.77	11.33	19.49	20.28	27.37	12.83	20.25
2	0	+	0	31.75	33.09	17.63	21.63	14.05	9.02	10.65	12.33	10.48	11.45	16.35
3	0	0	+	12.18	12.85	23.99	19.56	14.47	9.51	41.98	41.49	32.7	15.53	19.85
4	0	+	+	13.09	12.66	11.85	11.9	22.41	5.24	11.29	11.65	10.06	8.49	11.71
5	+	0	+	17.86	19.18	49.37	44.11	12.09	18.94	17.21	20.29	21.85	15.47	18.92
6	+	+	0	16.67	18.12	18.55	19.68	16.22	16.48	15.97	18.75	13.82	15.49	15.32
7	0	+	-	10.19	11.4	18.95	20.26	7.52	9.88	9.57	11.99	9.91	10.17	10.94
8	+	0	-	8.31	8.53	14.71	11.52	15.15	5.35	18.17	16.67	20.22	7.84	11.27
9	+	-	0	19.86	17.83	14.03	14.88	11.2	5.23	7.26	8.12	8	6.56	10.4
10	+	+	+	19.03	18.72	14.57	15.81	22.49	9.21	17.89	18.11	13.09	15.1	15.42
11	+	+	-	7.47	8.07	14.77	12.51	8.12	7.52	13.07	16.22	12.4	9.03	10.15
12	+	-	+	13.49	15.06	19.83	27.71	6.77	7.38	6.86	8.17	7.76	7.21	10.39
13	-	+	+	8.75	8.46	9.49	8.78	16.53	3.61	7.49	7.77	12.17	5.31	8.3
$\bar{\sigma}$				15.43	16.8	20.28	19.61	15.52	9.13	15.14	16.3	15.37	10.81	13.84

a Percentage dye concentration change is applied to each component (dyes 1-3) in the direction indicated

Table 7 ΔE values expected with colorant standardisation of $\pm 2.5\%$ applied in the pattern indicated by each combination

Colorant combination	Recipe no.									
	1	2	3	4	5	6	7	8	9	10
1	0.17	0.18	0.17	0.10	0.10	0.14	0.14	0.33	0.11	0.17
2	0.27	0.19	0.28	0.17	0.21	0.23	0.13	0.32	0.14	0.06
3	0.17	0.19	0.14	0.15	0.26	0.18	0.08	0.08	0.19	0.21
4	0.21	0.19	0.23	0.19	0.24	0.26	0.19	0.39	0.17	0.21
5	0.23	0.19	0.21	0.20	0.25	0.23	0.10	0.28	0.14	0.12
6	0.27	0.26	0.28	0.16	0.19	0.14	0.17	0.12	0.19	0.18
7	0.41	0.33	0.38	0.26	0.41	0.31	0.10	0.25	0.28	0.23
8	0.25	0.31	0.24	0.16	0.30	0.22	0.20	0.40	0.27	0.36
9	0.36	0.26	0.37	0.23	0.27	0.35	0.20	0.64	0.18	0.18
10	0.19	0.19	0.21	0.21	0.19	0.19	0.18	0.17	0.13	0.12
11	0.41	0.41	0.39	0.24	0.41	0.26	0.18	0.12	0.35	0.38
12	0.47	0.33	0.44	0.32	0.43	0.41	0.14	0.57	0.25	0.14
13	0.32	0.31	0.35	0.23	0.32	0.37	0.27	0.71	0.26	0.36
Avg ΔE	0.29	0.26	0.28	0.20	0.27	0.25	0.16	0.34	0.20	0.21

Colorant combination											Row ΔE
	11	12	13	14	15	16	17	18	19	20	
1	0.11	0.12	0.07	0.09	0.07	0.22	0.13	0.12	0.09	0.19	0.14
2	0.08	0.08	0.14	0.12	0.18	0.28	0.23	0.20	0.24	0.22	0.19
3	0.21	0.19	0.10	0.13	0.17	0.26	0.06	0.06	0.08	0.16	0.15
4	0.19	0.20	0.21	0.21	0.11	0.48	0.22	0.21	0.25	0.29	0.23
5	0.14	0.13	0.05	0.06	0.21	0.13	0.15	0.12	0.11	0.16	0.16
6	0.15	0.14	0.13	0.13	0.15	0.15	0.16	0.13	0.18	0.16	0.17
7	0.25	0.22	0.13	0.12	0.33	0.25	0.26	0.21	0.25	0.25	0.26
8	0.30	0.29	0.17	0.22	0.17	0.47	0.14	0.15	0.12	0.32	0.25
9	0.13	0.14	0.18	0.17	0.22	0.48	0.34	0.31	0.31	0.38	0.28
10	0.13	0.13	0.17	0.16	0.11	0.27	0.14	0.14	0.19	0.17	0.17
11	0.33	0.31	0.17	0.20	0.31	0.33	0.19	0.15	0.20	0.28	0.28
12	0.19	0.17	0.13	0.09	0.37	0.34	0.36	0.31	0.32	0.34	0.31
13	0.29	0.30	0.26	0.28	0.15	0.69	0.33	0.32	0.21	0.47	0.34
Avg ΔE	0.19	0.19	0.15	0.15	0.20	0.33	0.21	0.19	0.20	0.26	0.23

alter recipes manually in the Matchmaker system. The sample-standard ΔE values produced by the match prediction system were then compared with those used in the matrix method (ΔE 0.5 for row 1 and ΔE 1.0 for row 2, Table 8).

Table 8 row 1 shows the sample-standard colour difference from the Matchmaker system for each of 13 combinations predicted to be ΔE 0.5 by the matrix method. Row 2 shows Matchmaker values predicted by matrix calculation to be ΔE 1.0. The results in Table 8 show close correlation between the two methods, and confirm the validity of the conclusions based on the matrix method.

CONCLUSIONS

1. It is possible to predict sensitivity of recipes to colour change.
2. Colour shows almost a linear change with concentration change along all semi-axes, for small concentration changes.
3. A matrix method can be used to calculate potential colour differences from colorant concentration changes.

4. A match prediction program can generate the following matrix coefficients:

$$\begin{matrix} R_{l1} & R_{ch1} & R_{h1} \\ R_{l2} & R_{ch2} & R_{h2} \\ R_{l3} & R_{ch3} & R_{h3} \end{matrix}$$

Figure 2 Effect on resulting colour with dye standardisation of $\pm 2.5\%$

Figure 3 Effect on resulting colour with dye standardisation of $\pm 5\%$

Figure 4 Effect on resulting colour with dye standardisation of $\pm 10\%$

Table 8 ΔE values by standard match prediction for recipe 20^a

C	1	2	3	4	5	6	7	8	9	10	11	12	13	Avg
ΔE	0.49	0.46	0.49	0.45	0.46	0.51	0.52	0.50	0.48	0.46	0.55	0.45	0.49	0.49
ΔE	0.91	0.88	0.99	0.89	0.92	1.03	1.02	0.99	0.95	0.89	1.08	0.95	0.94	0.96

a For key see Table 6

by systematic alteration of recipe concentration. The 13 combinations of plus and minus movements described are sufficient for a three-colorant recipe. The full matrix method, involving synthetic reflectance curve generation is thus unnecessary for general use in calculating recipe sensitivity.

- Average shade uncertainties of 0.23, 0.46 and 0.92 ΔE units at $\pm 2.5\%$, $\pm 5\%$ and 10% colorant strength standardisations respectively would be expected.
- From cumulative frequency analysis (based on 260 recipes), 98% of all coloured samples are within ΔE 0.5 (attributable to colorant strength variation) with $\pm 2.5\%$ colorant standardisation. The figures for 5% and 10% are ΔE 1.0 and 2.0 respectively. For target shade D, recipe 8 was twice as sensitive as recipe 7.
- High sensitivity is shown in worst-case scenarios, with colorants moving to their standardisation limits in opposite directions (combinations 11–13), an event with 0.23 probability if the colorants are at or near their standardisation limit. In recipe 8, this combines

with general sensitivity to produce a ΔE of 0.71 with strength changes of $\pm 2.5\%$ (combination 13).

- Given that colorant standardisation $\pm 2.5\%$ alone can produce an average ΔE uncertainty of 0.25, it is important to keep dosing accuracy as high as possible. Dosing to $\pm 0.5\%$ would add a further uncertainty of ΔE 0.05 to the average.
- It would be relatively easy to modify match prediction programs to test individual recipe sensitivity to possible colorant strength variations, and provide practical warning of sensitive combinations.

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