Multiple-Access Capabilities of a Common Gateway

DOI:

[10.1109/TVT.2016.2627810](https://doi.org/10.1109/TVT.2016.2627810)

Document Version

Accepted author manuscript

[Link to publication record in Manchester Research Explorer](#)

Citation for published version (APA):

Alfitouri, A., & Hamdi, K. (2016). Multiple-Access Capabilities of a Common Gateway. *IEEE Transactions on Vehicular Technology*. <https://doi.org/10.1109/TVT.2016.2627810>

Published in:

IEEE Transactions on Vehicular Technology

Citing this paper

Please note that where the full-text provided on Manchester Research Explorer is the Author Accepted Manuscript or Proof version this may differ from the final Published version. If citing, it is advised that you check and use the publisher's definitive version.

General rights

Copyright and moral rights for the publications made accessible in the Research Explorer are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

Takedown policy

If you believe that this document breaches copyright please refer to the University of Manchester's Takedown Procedures [<http://man.ac.uk/04Y6Bo>] or contact uml.scholarlycommunications@manchester.ac.uk providing relevant details, so we can investigate your claim.

Multiple-Access Capabilities of a Common Gateway

Abdurrahman Alfitouri, Student Member, IEEE, and Khairi Ashour Hamdi, Senior Member, IEEE

Abstract—This paper is concerned with the performance analysis of a dual-hop random access network where a single gateway is employed to connect a random number of independent users which are hidden from their destination(s). The gateway is an amplify and forward relay, and is the only mean to connect users to their destinations. We develop accurate mathematical models for the multiple access interference which take into account, in addition to the distribution of the number of active users and their random locations, the effects of fading and thermal noise at both the relay and destinations. A new unified mathematical method is developed, which leads to the derivation of new analytical expressions for the overall spectral-efficiency of the dual-hop random access network over Rayleigh and Nakagami- m fading channels. The accuracy of the new mathematical results is confirmed by Monte Carlo simulation. The results of this paper can be used to estimate the aggregate throughput of a dual-hop system employing the slotted-Aloha type protocols.

Index Terms—Amplify and Forward, multiple-access capabilities, Nakagami- m fading, Rayleigh fading, Slotted-ALOHA protocol.

I. INTRODUCTION

MULTIPLE access techniques are used to allow a large number of users to share the allocated frequency band in the most efficient way. Random access protocols provide a simple solution for a group of independent users to share a common communication channel. Slotted-ALOHA is one of the multiple access protocols that was developed about a half-century ago and, due to its simplicity and low cost in implementation, it is still widely utilized in modern wireless communication systems such as satellite communication and mobile communication systems. In the Long Term Evolution (LTE) and LTE-Advanced (LTE-A), Slotted-ALOHA is adopted as the channel access technique for the random access channel. Slotted-ALOHA type protocols are employed recently in vehicular ad-hoc networks (VANET) [1]–[3].

This paper investigates the efficiency of a single amplify-and-forward (AF) relay serving a group of independent users that are hidden from their destinations and employing an Aloha type random multiple access protocol. This model of an AF gateway system applying Slotted-ALOHA protocol is widely used in many different practical scenarios. For instance, in underwater acoustic sensor networks used in pollution monitoring, undersea oilfields or military uses [4]–[6]. Another

Fig. 1: System Model

application is in satellite communications where a satellite relays the uplink carrier into a downlink [7]. This concept has become more popular in cooperative wireless communication systems where a gateway relays a signal between the source and the destination when the channel between them is not reliable due to shadowing, deep fading or large separation distances [8]–[10].

Different types of relaying systems have been reported in the literature, such as AF and decode-and-forward (DF). AF is a simple scheme where the relay receives the signal, amplifies it and resends it to the destination. However, in a DF scheme, the relay receives a signal from the source, decodes the message, and then re-encodes it before transmission to its destination [11]–[13]. Fig. 1, shows an example where a number of users and their destinations are randomly located around the gateway, and cannot communicate with their destination directly. We assume that there is no direct links between the users and their destinations and the communication in this network occurs via the gateway.

A. Related Work

AF repeater-enhanced random access in single cell wireless communications is reported in [14] where the author investigates the use of AF repeaters to enhance Slotted-ALOHA-based random access in a single cell with multiple relays. The

Copyright (c) 2015 IEEE. Personal use of this material is permitted. However, permission to use this material for any other purposes must be obtained from the IEEE by sending a request to pubs-permissions@ieee.org.

Abdurrahman. Alfitouri is with the school of Electrical and Electronic Engineering, The University of Manchester, Manchester, M13 9PL, UK. He is also with the school of Electrical and Electronic Engineering, Engineering Academy Tajoura, Tripoli, Libya. (E-mail: Abdurrahman.alfitouri@manchester.ac.uk).

K. A. Hamdi is with the school of Electrical and Electronic Engineering, The University of Manchester, Manchester, M13 9PL, UK. (E-mail: K.Hamdi@manchester.ac.uk)

effect of various channel state information (CSI) mismatches under multi-user two-hop relay is investigated in [15] where the authors derived upper bounds on the achievable rate. The sum rate of multi-pair AF relaying is proposed in [16] where each user only has a single antenna, while the relay is equipped with a very large antenna array. However, this model considers multiple antennas at the relay, and the random number and location of the users is not taken into account. The end-to-end performance of a two-hop system with non-regenerative relays over flat Rayleigh-fading channels is proposed in [17] where the outage probability formulas for noise-limited systems are derived. However, the authors considered fixed-gain relays instead of relays assisted by channel state information (CSI), and only considered one source communicating with a destination, thus ignoring the multiple access interference.

The ergodic capacity of a AF multi-input-multi-output (MIMO) relay channel is studied in Rayleigh fading channels in [18] where the relay is equipped with multiple antennas. However, the authors only consider one source and a destination equipped with multiple antennas. A multiple-access relay channel with network coding and non-ideal source-relay channels are proposed in [19] where multiple sources communicate with one destination. However, the code is applied between source signals in order to avoid multiple access interference. The amplify-and-forward relay transmission with end-to-end antenna selection is proposed in [20] where the exact closed-form for outage probability is investigated when only one source communicates with one destination thus ignoring the the impact of interference on the system performance.

B. Main Contributions and Outcomes

In this paper, we analyse the performance of a gateway connecting a group of independent users to their destinations, employing a random Slotted-ALOHA type protocol. The users are isolated from their destinations, and the only way they can reach them is through the common gateway which acts as a blind AF relay. Users are randomly distributed around the gateway and experience Rayleigh or Nakagami- m fading. The main contributions of this paper are summarized as follows:

- We develop a new non-direct and novel mathematical method which simplifies greatly performance analysis of a complicated random access network for the multiple access interference at both the gateway and destinations, taking into account (in addition to the random number of the active users and their locations) the effects of fading and thermal noise at both the relay and the destinations. To the best of the authors' knowledge, there is little prior work which considers the number of users as a random number.
- We derive a new analytical expressions for the overall spectral efficiency (SE) which can be used to estimate the throughput of the ALOHA-based gateways in Rayleigh and Nakagami- m fading channels, and to study the impact of the different system parameters on their efficiency.
- The signal-to-interference plus noise ratio (SINR) in the presented models is in fact a ratio of correlated random variables involving a random number of arbitrary non-identical random variables (that represent the random

number of users and their arbitrary relative positions). Direct analytical methods would require a huge computational (at least 2K random variables) [21]–[23]. As result, in this paper we propose a simple and useful unified mathematical tools, *Lemma1*, which simplifies greatly the evaluation of the overall spectral efficiency. We provide the proof of this simple *Lemma1* in the Appendix A.

- The proposed analytical method provides a new unified framework for efficient performance evaluation of the practical and realistic random multiple access schemes where a single AF relay is shared by a random number of arbitrary users having non-identical locations; such as ALOHA type protocols where users can be active/idle with a given probability and are allowed to be anywhere within the service area.
- To the best of the authors' knowledge, there is no other similar results in the open literatures that take into account accurately the arbitrary locations of non-identical random number of users. Most previous work assumed identical users where each user would experience the same average SNR (which implies that all users are distributed on the circumference of a circle around the relay so each of them is equidistant to the gateway).

C. Paper Organization and Notation

The remainder of this paper is organized as follows. In Section II, the system model and problem formulation are briefly described. The SE analysis in Rayleigh fading channel, and the exact-form are presented in Section III. In Section IV, The SE analysis for Nakagami- m fading channel, and the exact-form are presented. In Section V, the results are presented and discussed. Finally, conclusions are drawn in Section VI.

The following notations are used in the paper. Bold lower case letters denote vectors, $E[\cdot]$ denotes the expectation operator, $\Pr(\cdot)$ denotes the probability function, $|\cdot|$ denotes the absolute value, $\frac{\partial}{\partial A}$ denotes the derivation operation. Also, we use $\sim CN(0, 1)$ to stand for a circularly symmetric complex Gaussian distribution with zero mean and variance 1.

II. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we introduce the system model for multiple access gateway, and formulate the SE analysis problem.

A. System Model

The system model under consideration is shown in Fig. 1; there is a random number, K , of active users, distributed uniformly in the service area around the gateway, and the service area is considered as a circle with radius D . In this model, we will represent K by a binomial random variable with probability $\Pr(K = i) = \binom{M}{i} \rho^i (1 - \rho)^{M-i}$ [24], where M is the total number of users, and ρ is the probability that a user is active, which takes a value in the range $0 \leq \rho \leq 1$. Each user has a random location, and the distance between

each user and the gateway r is represented by a random variable with distribution ($f(r) = 2r/D^2, r \leq D$). The received composite signal at the gateway is

$$y_r = \sum_{k=1}^K \sqrt{p_k} x_k h_k r_k^{-\frac{\alpha}{2}} + n_g \quad (1)$$

where p_k is the transmitted power by user k , x_k is the transmitted symbol with unit power, h_k is the complex channel gain between user k and the gateway, r_k is the random distance between user k and the gateway, α is the path loss exponent and n_g is the additive white Gaussian noise (AWGN) at the gateway with zero mean and variance N_g . The received signal at the gateway, y_r , is then multiplied by a gain G before retransmission in the second hop. In order to satisfy the peak power concentration, the gain G is given by

$$G = \sqrt{\frac{p_g}{\sum_{k=1}^K p_k |h_k|^2 r_k^{-\alpha} + N_g}} \quad (2)$$

where p_g is the gateway transmitting power. The received signal at the destination i can be expressed as

$$y_i = G g_i l_i^{-\frac{\alpha}{2}} \times \left(\left(\sqrt{p_i} x_i h_i r_i^{-\frac{\alpha}{2}} + n_g \right) + \left(\sum_{k \neq i}^K \sqrt{p_k} h_k x_k r_k^{-\frac{\alpha}{2}} \right) \right) + n_D \quad (3)$$

where g_i is the complex channel gain between the gateway and destination i , l_i is the distance between the gateway and the destination i , and n_D is the AWGN at the destination with zero mean and variance N_D . We assume that the destinations are randomly distributed around the gateway. The SINR at destination i , SINR_i , can be written as

$$\text{SINR}_i = \frac{p_i |h_i|^2 |g_i|^2 r_i^{-\alpha} l_i^{-\alpha}}{|g_i|^2 l_i^{-\alpha} \left(N_g + \sum_{k \neq i}^K p_k |h_k|^2 r_k^{-\alpha} \right) + N_D / G^2}. \quad (4)$$

Plugging the gain of the gateway as given in (2) into the expression for SINR_i in (4), we get (5), which shown at the top of next page.

From Eq. (5), the final form of SINR_i can be expressed as

$$\text{SINR}_i = \frac{\frac{p_i |h_i|^2 r_i^{-\alpha}}{\sum_{k \neq i}^K p_k |h_k|^2 r_k^{-\alpha} + N_g} \frac{p_g |g_i|^2 l_i^{-\alpha}}{N_D}}{\frac{p_g |g_i|^2 l_i^{-\alpha}}{N_D} + \frac{p_i |h_i|^2 r_i^{-\alpha}}{\sum_{k \neq i}^K p_k |h_k|^2 r_k^{-\alpha} + N_g} + 1}. \quad (6)$$

B. Problem Formulation

In communication systems, the spectral efficiency (SE) describes the data rate that can be achieved for a specific bandwidth. In this section, we derive the achievable SE of the two-phase relay system which can be expressed as

$$\Re(\rho, M) = \frac{1}{2} \mathbb{E} \left\{ \sum_{k=1}^K [\log_2(1 + \text{SINR}_k)] \right\} \quad (7)$$

where $[\log_2(1 + \text{SINR}_k)]$ is the instantaneous SE of the user k , and the factor $\frac{1}{2}$ comes from the fact that communication

between the source and destination is performed in two phases [25]–[27]. Without any loss of generality we assume equal transmit power for all users $p_1 = p_2 = \dots = p_K$. Therefore, as we have assumed users to be statically identical, the overall SE of this model can be written as

$$\Re(\rho, M) = \frac{1}{2} \mathbb{E} \{ K [\log_2(1 + \text{SINR}_1)] \} \quad (8)$$

where SINR_1 is the instantaneous SINR experienced by an arbitrary user. The expectation in (8) is to be taken with respect to the correlated random variables K and SINR_1 . The distribution of K is known, however, SINR_1 is a ratio of a large number of random variables and it seems to be difficult to derive a closed-form expression for its distribution.

The overall SE of the proposed model is obtained from (8) which requires the evaluation of the expectation with respect to the following set of $(2K + 3)$ random variables:

$\{h_1, h_2, \dots, h_K, r_1, r_2, \dots, r_K, g_1, l_1, K\}$. The direct approach to compute this expectation is difficult to obtain in general, as it may require the computation of at least $(2K + 3)$ -fold convolution integrals. As a result, in this paper we investigate a simple and useful lemma 1 which simplifies greatly the evaluation of the averaging in (8) in terms of a single numerical integration.

Lemma 1. *Let U and V be two arbitrary non-negative independent random variables. Then*

$$\mathbb{E} \left[\ln \left(1 + \frac{UV}{1 + U + V} \right) \right] = \int_0^\infty \frac{1}{z} (1 - M_U(z)) (1 - M_V(z)) e^{-z} dz \quad (9)$$

where $M_U(z) = \mathbb{E}[e^{-zU}]$ and $M_V(z) = \mathbb{E}[e^{-zV}]$ are the moment generating functions (MGF) of U and V , respectively.

Proof: The proof of Lemma 1 is given in the Appendix A. ■

From (9), the overall SE in (8) can be obtained as follows

$$\Re(\rho, M) = \frac{1}{2} (\log_2 e) \times \mathbb{E} \left[\int_0^\infty \frac{1}{z} (K - K M_U(z|K)) (1 - M_V(z)) e^{-z} dz \right] \quad (10)$$

where $M_U(z|K) = \mathbb{E}[e^{-zV}|K]$,

$$U = \frac{|h_1|^2 \left(\frac{r_1}{D} \right)^{-\alpha}}{\sum_{k \neq 1}^K |h_k|^2 \left(\frac{r_k}{D} \right)^{-\alpha} + 1/\gamma} \quad (11)$$

and

$$V = \gamma_D |g_1|^2 \left(\frac{l_1}{D} \right)^{-\alpha}. \quad (12)$$

Here $\gamma = \frac{p_1}{N_0} D^{-\alpha}$ and $\gamma_D = \frac{p_g}{N_0} D^{-\alpha}$ are the normalized received average SNRs at the gateway and destination nodes when the user is at the edge of the service area.

It is worth mentioning that (10) can also be expressed in terms of the weights and abscissas of a Laguerre orthogonal

$$\text{SINR}_i = \frac{p_i |h_i|^2 P_g |g_i|^2 r_i^{-\alpha} l_i^{-\alpha}}{p_g |g_i|^2 \left(N_g + \sum_{k \neq i}^K p_k |h_k|^2 r_k^{-\alpha} \right) + N_D \left(\sum_{k=1}^K p_k |h_k|^2 r_k^{-\alpha} + N_g \right)}. \quad (5)$$

polynomial

$$\Re(\rho, M) = \frac{1}{2}(\log_2 e) \times \sum_{n=1}^N \xi_n \mathbb{E} \{ (K - K M_U(\beta_n)) (1 - M_V(\beta_n)) \} + R_N \quad (13)$$

where β_n and ξ_n are the sample points and the weights factors of the Laguerre polynomial, respectively, tabulated in [28, eq. (25.4.45)]. Therefore (13) provides an efficient numerical evaluation method for the required SE. The the remainder error, R_N , becomes sufficiently small when $N \geq 15$.

In the next two sections, we evaluate (13) in two different fading scenarios; namely Rayleigh and Nakagami- m models.

III. RAYLEIGH FADING CHANNELS

In the first scenario, all channels are assumed to be subjected to independent and identical distributed complex Gaussian fading, with zero mean and unit variance. As a result, $|h_i|$ and $|g_i|$ follow a Rayleigh distribution and therefore the distribution of power channel gains, $|h_i|^2$ and $|g_i|^2$ become exponential distribution. To find the SE, we need to determine firstly the MGFs $M_U(z)$ and $M_V(z)$.

A. The MGF of U

In this subsection we derive an expression for the conditional MGF $M_U(z|K) = \mathbb{E}[e^{-zU}|K]$, with U being given in 11. In this regard, we proceed to derive firstly an expression for the complementary cumulative distribution function (CCDF)

$$\Pr(U > u|K) = \Pr\left(\frac{|h_1|^2 \left(\frac{r_1}{D}\right)^{-\alpha}}{\sum_{k \neq 1}^K |h_k|^2 \left(\frac{r_k}{D}\right)^{-\alpha} + a} > u|K\right) \quad (14)$$

where $a = \frac{1}{\gamma}$. (14) can be rewritten as

$$\Pr(U > u|K) = \Pr\left(|h_1|^2 > u \left(\frac{r_1}{D}\right)^\alpha \left(\sum_{k \neq 1}^K |h_k|^2 \left(\frac{r_k}{D}\right)^{-\alpha} + a\right) | K\right). \quad (15)$$

Equation (15) has more than one random variable, therefore we condition firstly on the set of random variables $(r_2, \dots, r_K, h_2, \dots, h_K, K)$ to get the conditional CCDF

$$\Pr(U > u | \mathbf{h}, \mathbf{r}, K) = \Pr\left(|h_1|^2 > u \left(\frac{r_1}{D}\right)^\alpha \left(\sum_{k \neq 1}^K |h_k|^2 \left(\frac{r_k}{D}\right)^{-\alpha} + a\right) | \mathbf{h}, \mathbf{r}, K\right) \quad (16)$$

where $\mathbf{h} = \{h_2, h_3, \dots, h_K\}$, $\mathbf{r} = \{r_2, r_3, \dots, r_K\}$.

Now, since $|h_1|^2$ have an exponential distribution, then

$$\Pr(U > u | \mathbf{h}, \mathbf{r}, K) = e^{-u \left(\frac{r_1}{D}\right)^\alpha \left(\sum_{k \neq 1}^K |h_k|^2 \left(\frac{r_k}{D}\right)^{-\alpha} + a\right)}. \quad (17)$$

We obtain when we average out the independent random variables $\{|h_k|^2, r_k, k = 2, \dots, K, k \neq 1\}$ from (17)

$$\begin{aligned} \Pr(U > u | r_1, K) &= \mathbb{E} \left[e^{-u \left(\frac{r_1}{D}\right)^\alpha \left(\sum_{k \neq 1}^K |h_k|^2 \left(\frac{r_k}{D}\right)^{-\alpha} + a\right)} \right] \\ &= \prod_{k \neq 1}^K \mathbb{E} \left[e^{-u \left(\frac{r_1}{D}\right)^\alpha |h_k|^2 \left(\frac{r_k}{D}\right)^{-\alpha}} | r_1 \right] e^{-ua \left(\frac{r_1}{D}\right)^\alpha} \\ &= \prod_{k \neq 1}^K \mathbb{E} \left[\frac{1}{1 + u r_1^\alpha r_k^{-\alpha}} | r_1 \right] e^{-ua \left(\frac{r_1}{D}\right)^\alpha} \quad (18) \end{aligned}$$

where we have used the fact that $|h_k|^2$ are exponentially distributed random variables and the last expectation in (18) is with respect to the random distance r_k . As far as the distribution of r_k is concerned, it can be shown that their probability density function (PDF) is $f(r) = 2r/D^2, r \leq D$. Equation (18) can be written as

$$\Pr(U > u | r_1, K) = \left(\int_0^D \frac{1}{1 + u r_1^\alpha r^{-\alpha}} \frac{2r}{D^2} dr \right)^{K-1} e^{-ua \left(\frac{r_1}{D}\right)^\alpha}. \quad (19)$$

The integral in (19) can be evaluated in closed form to get

$$\Pr(U > u | r_1, K) = \left(\left(1 - {}_2F_1\left(1; \frac{2}{\alpha}; \frac{2+\alpha}{\alpha}; -\frac{D^\alpha}{u r_1^\alpha}\right) \right)^{K-1} e^{-ua \left(\frac{r_1}{D}\right)^\alpha} \right) \quad (20)$$

where ${}_2F_1(a; b; u)$ is the confluent hypergeometric function. We obtain when we average out r_1

$$\begin{aligned} \Pr(U > u | K) &= \int_0^D \left(1 - {}_2F_1\left(1; \frac{2}{\alpha}; \frac{2+\alpha}{\alpha}; -\frac{D^\alpha}{u r^\alpha}\right) \right)^{K-1} \frac{2r}{D^2} e^{-ua \left(\frac{r}{D}\right)^\alpha} dr. \quad (21) \end{aligned}$$

The cumulative distribution function (CDF) of U is given by $1 - \Pr(U > u|K)$. Therefore, using integration by parts, the conditional MGF $M_U(z|K)$ can be found as follows

$$\begin{aligned} M_U(z|K) &= \int_0^\infty e^{-zU} f(U|K) dU \\ &= 1 - z \int_0^\infty \int_0^D \left(1 - {}_2F_1\left(1; \frac{2}{\alpha}; \frac{2+\alpha}{\alpha}; -\frac{D^\alpha}{u r^\alpha}\right) \right)^{K-1} \\ &\quad \times e^{-u(z + \frac{(r/D)^\alpha}{\gamma})} \frac{2r}{D^2} du dr. \quad (22) \end{aligned}$$

B. The MGF of V

In order to compute the MGF $M_V(z)$, we evaluate firstly the conditional MGF, conditioned on the distance l_1

$$M_V(z | l_1) = E[e^{-zV} | l_1] = \frac{1}{1 + z\gamma_D \left(\frac{l_1}{D}\right)^{-\alpha}}. \quad (23)$$

which follows since $|g_1|^2$ follows an exponential distribution in case of Rayleigh fading. We obtain when we average out l_1 from (23)

$$\begin{aligned} M_V(z) &= E[M_V(z | l_1)] = \\ &= \int_0^D \frac{1}{1 + z\gamma_D(l_1/D)^{-\alpha}} \frac{2l_1}{D^2} dl_1 \\ &= 2 \frac{{}_2F_1\left(1; \frac{2+\alpha}{\alpha}; 2 + \frac{2}{\alpha}; -\frac{1}{z\gamma_D}\right)}{\gamma_D(2+\alpha)z}. \end{aligned} \quad (24)$$

Substituting (24) and (31) into (13) leads to the following tractable explicit expression for the overall average SE in Rayleigh fading. The result shown in (13) includes only a single numerical integration. It is worth noting that a direct method to find the overall SE would require at least $(2K+3)$ -fold integrals.

As far as the evaluation of the SE in (10) is concerned, substitute (24) and 22 in (10) to get

$$\begin{aligned} E[K \log_2(1 + \text{SINR}_1)] &= \\ \log_2 e \int_0^\infty \frac{1}{z} (E[K] - E[K M_U(z|K)]) (1 - M_V(z)) e^{-z} dz \end{aligned} \quad (25)$$

where the expectations inside the integrand are with respect to the binomial random variable K . It is known that $E[K] = \rho M$. On the other hand, from (22)

$$\begin{aligned} E[K M_U(z|K)] &= \\ E\left[K \left(1 - z \int_0^\infty \int_0^D \left(1 - {}_2F_1\left(1; \frac{2}{\alpha}; \frac{2+\alpha}{\alpha}; -\frac{D^\alpha}{ur^\alpha}\right)\right)^{K-1} \right. \right. \\ &\quad \left. \left. \times e^{-u(z + \frac{(r/D)^\alpha}{\gamma})} \frac{2r}{D^2} du dr\right] \right] \end{aligned} \quad (26)$$

which simplifies into

$$\begin{aligned} E[K M_U(z)] &= E[K] \\ &- z \int_0^\infty \int_0^D E\left[K \left(1 - {}_2F_1\left(1; \frac{2}{\alpha}; \frac{2+\alpha}{\alpha}; -\frac{D^\alpha}{ur^\alpha}\right)\right)^{K-1} \right. \\ &\quad \left. \times e^{-u(z + \frac{(r/D)^\alpha}{\gamma})} \frac{2r}{D^2} du dr\right] \end{aligned} \quad (27)$$

The expectation inside the integrand can be easily evaluated by invoking the following lemma.

Lemma 2. Let K be a binomial random variable with probability $P_r(K = i) = \binom{M}{i} \rho^i (1 - \rho)^{M-i}$. Then for any non-negative variable A ,

$$E[KA^{K-1}] = \rho M(1 - \rho + \rho A)^{M-1}. \quad (28)$$

Proof: The proof is given in Appendix B. ■

Therefore using (28), with $A = 1 - {}_2F_1\left(1; \frac{2}{\alpha}; \frac{2+\alpha}{\alpha}; -\frac{D^\alpha}{ur^\alpha}\right)$, we obtain

$$\begin{aligned} &E\left[K \left(1 - {}_2F_1\left(1; \frac{2}{\alpha}; \frac{2+\alpha}{\alpha}; -\frac{D^\alpha}{ur^\alpha}\right)\right)^{K-1}\right] \\ &= \rho M \left(1 - \rho + \rho \left(1 - {}_2F_1\left(1; \frac{2}{\alpha}; \frac{2+\alpha}{\alpha}; -\frac{D^\alpha}{ur^\alpha}\right)\right)\right)^{M-1} \\ &= \rho M \left(1 - \rho {}_2F_1\left(1; \frac{2}{\alpha}; \frac{2+\alpha}{\alpha}; -\frac{D^\alpha}{ur^\alpha}\right)\right)^{M-1}. \end{aligned} \quad (29)$$

Substitute (29) into (27) to get

$$\begin{aligned} E[K M_U(z|K)] &= \\ \rho M - z \int_0^\infty \int_0^D \rho M \left(1 - \rho {}_2F_1\left(1; \frac{2}{\alpha}; \frac{2+\alpha}{\alpha}; -\frac{D^\alpha}{ur^\alpha}\right)\right)^{M-1} \\ &\quad \times e^{-u(z + \frac{(r/D)^\alpha}{\gamma})} \frac{2r}{D^2} du dr. \end{aligned} \quad (30)$$

It is worth noting that (30) can also be expressed in terms of the weights and abscissas of a Laguerre orthogonal polynomial

$$\begin{aligned} E[K M_U(z|K)] &= \rho M - z \sum_{b=1}^B \mu_b \int_0^D \gamma \left(\frac{r}{D}\right)^{-\alpha} \\ &\quad \times \rho M \left(1 - \rho {}_2F_1\left(1; \frac{2}{\alpha}; \frac{2+\alpha}{\alpha}; -\frac{1}{\lambda_b \gamma}\right)\right)^{M-1} \\ &\quad \times e^{-z\lambda_b \gamma \left(\frac{r}{D}\right)^{-\alpha}} \frac{2r}{D^2} dr + R_B \end{aligned} \quad (31)$$

where λ_b and μ_b are the sample points and the weights factors of the Laguerre polynomial, respectively, tabulated in [28, eq. (25.4.45)]. The remainder, R_B , is sufficiently small for $B \geq 15$.

IV. NAKAGAMI- m FADING CHANNELS

In Nakagami- m fading, the magnitude of the complex channel gains, $|h_i|$ and $|g_i|$, are characterised by the Nakagami- m distribution with parameter m . In this case, the distribution of the power channel gains of $|h_i|^2$ and $|g_i|^2$ follow a gamma distribution. It should be noted that Nakagami- m fading represents a wide range of multipath channels through changes in the m fading parameter; for instance, when $m = \frac{1}{2}$, it represents a one sided Gaussian distribution. A special case of Nakagami- m fading is when $m = 1$ which represent the Rayleigh fading distribution. The Rician distribution can also be closely approximated when $m > 1$. Moreover, the Nakagami- m distribution fits urban radio multipath environments and indoor applications [29].

The SE in case of Nakagami- m channels is given by the unified expression (13). However, we need to evaluate $M_U(z|K)$ and $M_V(z)$ in Nakagami- m models.

A. The MGF of U

The technique that we have just used in the case of Rayleigh fading channel to obtain $M_U(z|K)$ can not be used

straightforwardly in the case of Nakagami- m fading channel. However, we will use a different method to find $M_U(z)$. In order to simplify the presentation of this method, we rewire U in (11) as $U = \frac{x}{y+b}$ with $x = |h_1|^2$, $y = \sum_{k \neq 1}^K |h_k|^2 \left(\frac{r_k}{r_1}\right)^{-\alpha}$ and $b = \frac{(r_1/D)^\alpha}{\gamma}$.

Corollary 3. According to [22, eq. (7)], when x follows a gamma distribution with unit mean and parameter m then for an arbitrary function $g(\cdot)$

$$\mathbb{E} \left[g \left(\frac{x}{y+b} \right) \right] = g(0) + \int_0^\infty g_m(s) M_y(ms) e^{-smb} ds \quad (32)$$

where $M_y(s) = \mathbb{E}[e^{-sy}]$ is the MGF of y and $g_m(s) = \frac{1}{\Gamma(m)} \frac{d^m}{ds^m} s^{m-1} g(s)$.

Since users are assumed to be independent then

$$\begin{aligned} M_y(s) &= \mathbb{E} \left[e^{-sr_1^\alpha \sum_{k \neq 1}^K |h_k|^2 r_k^{-\alpha}} \mid r_1, K \right] \\ &= \prod_{k \neq 1}^K \mathbb{E} \left[e^{-sr_1^\alpha |h_k|^2 r_k^{-\alpha}} \mid r_1 \right] \end{aligned} \quad (33)$$

where the expectations in (33) are with respect to the bivariate $(|h_k|^2, r_k)$ which represent the channel gain and random distances of user k to the gateway. It can be shown that when $|h_k|^2$ is a gamma random variable with parameter m then

$$\begin{aligned} \mathbb{E} \left[e^{-sr_1^\alpha |h_k|^2 r_k^{-\alpha}} \mid r_1 \right] &= \\ & {}_2F_1 \left(m; -\frac{2}{\alpha}; \frac{-2+\alpha}{\alpha}; -s \left(\frac{r_1}{D} \right)^\alpha \right). \end{aligned} \quad (34)$$

Therefore (33) reduces into

$$M_y(ms \mid K) = \int_0^D \left({}_2F_1 \left(m; -\frac{2}{\alpha}; \frac{-2+\alpha}{\alpha}; -s \left(\frac{r}{D} \right)^\alpha \right) \right)^{K-1} \frac{2r}{D^2} dr. \quad (35)$$

By substituting (35) into (32) with $g(s) = \exp(-s)$, $g(0) = 1$ and $g_m(s) = -m {}_1F_1(1-m; 2; s) e^{-s}$ we get

$$\begin{aligned} \mathbb{E} \left[\exp \left(-s \frac{x}{y+b} \mid K \right) \right] &= 1 - m \int_0^\infty {}_1F_1(1-m; 2; s) e^{-s} \\ &\times \int_0^D \left({}_2F_1 \left(m; -\frac{2}{\alpha}; \frac{-2+\alpha}{\alpha}; -s \left(\frac{r}{D} \right)^\alpha \right) \right)^{K-1} \\ &\times e^{-\frac{sm(r/D)^\alpha}{\gamma}} \frac{2r}{D^2} dr ds. \end{aligned} \quad (36)$$

The conditional MGF of U , $M_U(u \mid K) = \mathbb{E} \left[e^{-u \frac{x}{y+b}} \right]$, is shown in (37)

$$\begin{aligned} \mathbb{E}[\exp(-us) \mid K] &= 1 - m \int_0^\infty {}_1F_1(1-m; 2; s) e^{-s} \\ &\times \int_0^D \left({}_2F_1 \left(m; -\frac{2}{\alpha}; \frac{-2+\alpha}{\alpha}; -s \left(\frac{r}{D} \right)^\alpha \right) \right)^{K-1} \\ &\times e^{-\frac{sm(r/D)^\alpha}{u\gamma}} \frac{2r}{D^2} dr ds. \end{aligned} \quad (37)$$

To simplify (37), let $z = sm/u$. Then $s = zu/m$, $ds = zdu/m$ and (37) can be expressed as (38)

$$\begin{aligned} M_u(z \mid K) &= 1 - z \int_0^\infty {}_1F_1(1-m; 2; \frac{zu}{m}) \\ &\times \int_0^D \left({}_2F_1 \left(m; -\frac{2}{\alpha}; \frac{-2+\alpha}{\alpha}; -\frac{zu}{m} \left(\frac{r}{D} \right)^\alpha \right) \right)^{K-1} \\ &\times e^{-z \left(\frac{u}{m} + \frac{(r/D)^\alpha}{\gamma} \right)} \frac{2r}{D^2} dr du. \end{aligned} \quad (38)$$

Applying Lemma 2 and following the same method used in the Rayleigh fading scenario, we can find $\mathbb{E}[KM_U(z)]$, which is shown in (39)

$$\begin{aligned} \mathbb{E}[KM_U(z)] &= \rho M - z \int_0^\infty \int_0^D \rho M \\ &\times \left(1 - \rho + \rho \left({}_2F_1 \left(m; -\frac{2}{\alpha}; \frac{-2+\alpha}{\alpha}; -\frac{zu}{m} \left(\frac{r}{D} \right)^\alpha \right) \right) \right)^{M-1} \\ &\times {}_1F_1(1-m; 2; \frac{zu}{m}) e^{-z \left(\frac{u}{m} + \frac{(r/D)^\alpha}{\gamma} \right)} \frac{2r}{D^2} dr du. \end{aligned} \quad (39)$$

According to [28, eq. (25.4.45)], (39) can also be expressed in terms of the weights and abscissas of a Laguerre orthogonal polynomial

$$\begin{aligned} \mathbb{E}[KM_U(z)] &= \rho M - \sum_{w=1}^W m \psi_w \int_0^D \rho M \\ &\times \left(1 - \rho + \rho \left({}_2F_1 \left(m; -\frac{2}{\alpha}; \frac{-2+\alpha}{\alpha}; -\Upsilon_w \left(\frac{r}{D} \right)^\alpha \right) \right) \right)^{M-1} \\ &\times {}_1F_1(1-m; 2; \Upsilon_w) e^{-z \frac{(r/D)^\alpha}{\gamma}} \frac{2r}{D^2} dr + R_W \end{aligned} \quad (40)$$

where Υ_w and ψ_w are the sample points and the weights factors of the Laguerre polynomial, respectively, tabulated in [28, eq. (25.4.45)]. The remainder, R_W , is sufficiently small for $W \geq 15$.

B. The MGF of V

The second MGF in (13) in the case of Nakagami- m fading channel $M_V(z)$ can be obtained as

$$\begin{aligned} M_V(z \mid l_1^{-\alpha}) &= \mathbb{E} \left[e^{-zV} l_1^{-\alpha} \right] = \\ &\int_0^\infty e^{-z\gamma_D V} \frac{1}{\Gamma(m)} V^{m-1} m^m e^{-mV} dV \\ &= \frac{1}{\left(\frac{z(l_1/D)^{-\alpha}}{m} \gamma_D + 1 \right)^m}. \end{aligned} \quad (41)$$

Therefore

$$\begin{aligned} M_V(z) &= \int_0^D \frac{1}{\left(\frac{z(l_1/D)^{-\alpha}}{m} \gamma_D + 1 \right)^m} \frac{2l_1}{D^2} dl_1 = \\ &2 \left(\frac{m}{z\gamma_D} \right)^m \frac{{}_2F_1 \left(m, m + \frac{2}{\alpha}, 1 + m + \frac{2}{\alpha}, -\frac{m}{z\gamma_D} \right)}{2 + m\alpha}. \end{aligned} \quad (42)$$

Substituting (40) and (42) into (13), leads to the tractable explicit expression for the overall average SE over a Nakagami- m fading channel.

Fig. 2: SE as function of M when $\gamma = 10$ dB, $\rho = 0.1, 0.3$ and 0.5 , $\alpha = 3$.

Fig. 3: SE as function of ρ when $\gamma = -1, 0$ and 1 dB, $M = 10$ users, $\alpha = 3$.

V. NUMERICAL AND SIMULATION RESULTS

In this section, the SE achieved by the multiple-access AF gateway is evaluated using Monte-Carlo simulations, and compared to the derived asymptotic results. In addition, the complex channels gains are modelled as Rayleigh and Nakagami- m distributions. It can be assumed that $\gamma_D = \gamma$, without loss of generality. Different graphical plots of SE are presented below, corresponding to various numbers of users, γ values, path loss exponents, and user statuses. The user status factor (ρ), path loss exponent (α), and the γ are given different values, and the user number is set at $M = 10$ users, with the radius of the area served by the gateway (D) set as 1000 m.

A. Rayleigh Fading Channel

In Fig. 2, the SE is plotted as function of the number of users for different values of ρ . From this figure, it is clear that as the number of users increases, the SE increases correspondingly until it attains a stable level. In general, when $\rho = 0.5$, better performance is achieved compared to the case when $\rho = 0.3$, which is, in turn, better than $\rho = 0.1$; this is due to the sources of the useful signal in the first case being larger than for the latter two, which leads to a better SINR value. However, in case of $\rho = 0.5$, which means the probability of active users is 50 %, the effect of the interference increases with increase the number of users, which leads to drop in the system performance.

In Fig. 3, the SE is plotted as function of the user state for different values of γ . The SE is initially low but increases with increase in either ρ or γ . It can be seen that as ρ increases above $\rho = 0.5$, the value of SE tends to level off, this is because the interference becomes high and tends to limit the performance of the system. It can also be seen that the SE increases with increase in γ from -1 to 0 to 1 dB.

Fig. 4, illustrates what happen to the SE for the case of high values of γ (15, 20, and 30 dB), where the effect of

Fig. 4: SE as function of ρ when $\gamma = 15, 20$ and 30 dB, $M = 10$ users, $\alpha = 3$.

the noise is limited, and the initial value of the SE is high. Initially, the performance of the system increases dramatically as ρ increases, until it reaches a maximum value at $\rho = 0.2$. As ρ increases further - which affects the random number of active users - the interference increases and, as a result, the SE decreases until it reaches a stable level. Moreover, the performance of the system decreases as the γ decreases.

It is clear in Fig. 5, that in noise-limited, or low γ region, as the γ increases, the system performance dramatically improves. Moreover, at low γ , the value of the SE is greater when $\rho = 1$ and 0.5 than when $\rho = 0.1$. However, at higher levels of γ , the SE for $\rho = 0.1$ exceeds the SE for $\rho = 1$ and 0.5 by 0.4 bits/s/Hz. This can be interpreted as showing that, in interference limited systems, the noise contribution becomes minimal as ($\frac{1}{\gamma} \rightarrow 0$), which means that as ρ decreases, the interference in the system also decreases.

In Fig. 6, we consider the effect of the path loss exponent

Fig. 5: SE as function of γ when $\rho = 1, 0.5$ and 0.1 , $M = 10$ users, $\alpha = 3$.

Fig. 7: SE as function of M when $\gamma = 1\text{dB}$, $\rho = 0.1, 0.3$ and 0.5 , $\alpha = 3$.

Fig. 6: SE as function of M when $\gamma = 10\text{ dB}$, $\rho = 0.5$, $\alpha = 2.5, 3, 3.5$, and 4 .

Fig. 8: SE as function of ρ when $\gamma = -1, 0$ and 1dB , $M = 10$ users, $\alpha = 3$.

(α), for four different values of α . From this figure, it is clear that as α increases, the performance of the system significantly and consistently improved, since the effect of α on the interference is greater. This is particularly true for values of M from 1 to 4, whereas for $M > 5$ users, the SE start to decrease and maintains a stable level; the increase in the number of users results in increased interference, which leads to a stable SE level.

B. Nakagami-m Fading Channel

The results obtained for the case of the Nakagami-m channel show similar trends to those obtained for the Rayleigh fading channel, i.e. the SE increases as the number of users increases. In Fig. 7, when $m = 5$, the SE increases until the performance is saturated, when the SE achieved is between 1.1 to 1.3 bits/s/Hz. In general, the performance of the three cases

($\rho = 0.5, 0.3$ and 0.1) was the same as for the Rayleigh fading channel, for the same reasons as given earlier.

In Fig. 8, the SE is plotted as function of user status for three different values of γ (-1, 0, and 1 dB). As ρ increases, the SE also increases. This increase in SE continues until ρ reaches values greater than about 0.4, where the curves become increasingly saturated.

In case of high values of γ , as shown in Fig. 9, the initial value of the SE is high and, initially, it increases dramatically as ρ increases until it reaches a maximum values, when $\rho = 0.2$. Moreover, the effect of the noise is limited, which helps to improve the system performance; this performance continues to increase until it reaches a maximum value, when $\rho = 0.2$. As ρ increases further, the random number of active users is increased, the interference is increased and the SE decreases until it reaches a stable level. The effect of γ on performance of the system is positive, with higher γ leading

Fig. 9: SE as function of ρ when $\gamma = 15, 20$ and 30 dB, $M = 10$ users, $\alpha = 3$.

Fig. 10: SE as function of γ when $\rho = 1, 0.5$ and 0.1 , $M = 10$ users, $\alpha = 3$.

to better performance.

In Fig. 10, the SE is plotted as a function of the γ , when the user status is $\rho = 1, 0.5$ and 0.1 . The results shows that the SE increases with increasing γ . Moreover, the value of ρ has a different effect on the SE depending on the value of γ . Interestingly, for low values of γ , the SE is higher when $\rho = 1$ and 0.5 than when $\rho = 0.1$. However, at higher values of γ the SE is lower when $\rho = 1$ and 0.5 and higher when $\rho = 0.1$ by around 0.6 bits/s/Hz; this is due to the same reasons of interference limitation as mentioned earlier for the Rayleigh fading case.

Fig. 11, illustrates the effect of the path loss exponent (α) on the system performance, where four different values of α were considered ($\alpha = 2.5, 3, 3.5$, and 4). From this figure, it clear that as α increases, the SE increases substantially, which confirms that the interfering signals are more attenuated. This is notable from $M = 1$ to 3 , whereas from $M > 4$ users, the

Fig. 11: SE as function of M when $\gamma = 10$ dB, $\rho = 1$, $\alpha = 2.5, 3, 3.5$, and 4 .

SE start to decrease and is increasingly saturated; the reason for this is an increase in the interference effect, as previously mentioned.

C. Asymptotic Spectral Efficiency

It is of interest to investigate the asymptotic spectral efficiency when $M \rightarrow \infty$ and $\rho \rightarrow 0$ such that their product is kept finite $0 < \rho M < \infty$. We prove in Appendix C the following lemma

Lemma 4. *In a large population regime when $M \rightarrow \infty$, $\rho \rightarrow 0$ such that $M\rho = \eta$.*

$$\lim_{\substack{M \rightarrow \infty \\ \rho \rightarrow 0 \\ M\rho = \eta}} \Re(\eta) = \eta \gamma (\log_2 e) \sum_{n=1}^N \sum_{b=1}^B \xi_n \mu_b \int_0^D e^{-\eta \left({}_2F_1\left(1; \frac{2}{\alpha}; \frac{2+\alpha}{\alpha}; -\frac{D^\alpha}{\lambda_b r^\alpha} \right) \right)} \times \left(1 - 2 \frac{{}_2F_1\left(1; \frac{2+\alpha}{\alpha}; 2 + \frac{2}{\alpha}; -\frac{1}{\beta_n \gamma_D} \right)}{\gamma_D (2 + \alpha) \beta_n} \right) \times e^{-\lambda_b (\beta_n + \frac{(r/D)^\alpha}{\gamma})} \left(\frac{r}{D} \right)^{-\alpha} \frac{r}{D^2} dr. \quad (43)$$

Proof: The proof is given in Appendix C. ■

Fig. 12, the asymptotic SE is plotted as function of the SNR (γ) and users intensity (η). From this figure, it is clear that as the γ increases, the system performance dramatically improves. This is more clear at small value of η (1, and 2). As η increase, the performance of the system decrease correspondingly until it attains a stable level. This can be explained as, increase η leads to increase the intensity on the users, which means increase the interference. As a result, the SE decreases until it reaches a stable level.

VI. CONCLUSIONS.

In this paper, we have developed accurate analytical models for the performance analysis of a multiple access network

Asymptotic Analysis in High Number of Users

Fig. 12: SE as function of γ and η .

served by a single gateway which connects a group of independent users employing a random Slotted-ALOHA-type protocol. The users are randomly distributed around the gateway and are hidden from their destinations. We developed a unified analytical framework which can be used to evaluate the performance of such random access networks over different fading channels. This led to the derivation of new exact expressions for the overall spectral efficiency over Rayleigh and Nakagami-m fading channels. Compared with the classical methods which would require at least 2K-fold numerical integrations, with K being the number of active users, the new approach is computationally efficient and requires only one single numerical integration which can be expressed in terms of the coefficients of a Laguerre polynomial. The new results are then used to investigate the asymptotic SE and the impact of different system parameters into the overall performance. Numerical results showed that increasing the number of users beyond a given limit saturates the resultant SE.

APPENDIX A PROOF OF Lemma 1

In this appendix we prove Lemma 1, which is presented in (9), where two independent random variables U and V in this equation are considered. The left hand side of (9) can be simplified as

$$\left(1 + \frac{UV}{1+U+V}\right) = \frac{1+U+V+UV}{1+U+V} = \frac{(1+U)(1+V)}{1+U+V}. \quad (44)$$

Applying the rules of $\ln$ for both sides of (44) gives

$$\ln\left(1 + \frac{UV}{1+U+V}\right) = \ln(1+U) + \ln(1+V) - \ln(1+U+V). \quad (45)$$

From [21, eq. (6)] we see that

$$\ln(1+U) = \int_0^\infty \frac{1}{z} (1 - e^{-zU}) e^{-z} dz. \quad (46)$$

Thus (45) can be written as

$$\ln\left(1 + \frac{UV}{1+U+V}\right) = \int_0^\infty \frac{1}{z} (1 - e^{-zU}) e^{-z} dz + \int_0^\infty \frac{1}{z} (1 - e^{-zV}) e^{-z} dz - \int_0^\infty \frac{1}{z} (1 - e^{-zU} e^{-zV}) e^{-z} dz. \quad (47)$$

After algebraic manipulation, (47) can be written as

$$\ln\left(1 + \frac{UV}{1+U+V}\right) = \int_0^\infty \frac{1}{z} (1 - e^{-zU} + 1 - e^{-zV} - 1 - e^{-zU} e^{-zV}) e^{-z} dz. \quad (48)$$

Which can be expressed as

$$\ln\left(1 + \frac{UV}{1+U+V}\right) = \int_0^\infty \frac{1}{z} (1 - e^{-zU}) (1 - e^{-zV}) e^{-z} dz. \quad (49)$$

This leads to the desired result in (9).

APPENDIX B PROOF OF Lemma 2

In this appendix we prove Lemma 2, which is presented in (28). The right hand side of (28) can be simplified as

$$\mathbb{E}[K(A)^{K-1}] = \mathbb{E}\left\{\frac{\partial}{\partial A} A^K\right\} \quad (50)$$

where $\frac{\partial}{\partial A}$ is stands for the derivative with respect to A . Therefore, (50) can be simplify as

$$\mathbb{E}[K(A)^{K-1}] = \frac{\partial}{\partial A} \mathbb{E}\{A^K\}. \quad (51)$$

As mentioned earlier, K is a binomial random variable with probability $P_r(K=i) = \binom{M}{i} \rho^i (1-\rho)^{M-i}$, this means (51) can be written as

$$\mathbb{E}[K(A)^{K-1}] = \frac{\partial}{\partial A} (1-\rho + \rho A)^M. \quad (52)$$

Taking the derivation of the right hand side in (52), this leads to the results that shown in (28), which can be written as

$$\mathbb{E}[K(A)^{K-1}] = \rho M (1-\rho + \rho A)^{M-1}. \quad (53)$$

APPENDIX C
PROOF OF Lemma 4

In this appendix we prove Lemma 4, which is presented in (43). The limit of (43) can be found as

$$\begin{aligned} \lim_{\substack{M \rightarrow \infty \\ \rho \rightarrow 0 \\ M\rho = \eta}} \Re(\rho, M) &= (\rho M) (\log_2 e) \sum_{n=1}^N \sum_{b=1}^B \gamma \xi_n \mu_b \\ &\times \int_0^D \lim_{\substack{M \rightarrow \infty \\ \rho \rightarrow 0 \\ M\rho = \eta}} \left(1 - \rho {}_2F_1\left(1; \frac{2}{\alpha}; \frac{2+\alpha}{\alpha}; -\frac{D^\alpha}{\lambda_b r^\alpha}\right) \right)^{M-1} \\ &\times \left(1 - 2 \frac{{}_2F_1\left(1; \frac{2+\alpha}{\alpha}; 2 + \frac{2}{\alpha}; -\frac{1}{\beta_n \gamma_D}\right)}{\gamma_D(2+\alpha)\beta_n} \right) \\ &\times e^{-\lambda_b(\beta_n + \frac{(r/D)^\alpha}{\gamma})} \left(\frac{r}{D}\right)^{-\alpha} \frac{r}{D^2} dr. \quad (54) \end{aligned}$$

We invoke the identity [30, Eq. (1.2.3)]

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x} \right)^x = e. \quad (55)$$

To get

$$\begin{aligned} \lim_{M \rightarrow \infty} \left(1 - \frac{\eta}{M} {}_2F_1\left(1; \frac{2}{\alpha}; \frac{2+\alpha}{\alpha}; -\frac{D^\alpha}{\lambda_b r^\alpha}\right) \right)^{M-1} \\ = e^{-\eta \left({}_2F_1\left(1; \frac{2}{\alpha}; \frac{2+\alpha}{\alpha}; -\frac{D^\alpha}{\lambda_b r^\alpha}\right) \right)}. \quad (56) \end{aligned}$$

Therefore, (54) can be written as function of η

$$\begin{aligned} \Re(\eta) &= \eta \gamma (\log_2 e) \sum_{n=1}^N \sum_{b=1}^B \xi_n \mu_b \\ &\times \int_0^D e^{-\eta \left({}_2F_1\left(1; \frac{2}{\alpha}; \frac{2+\alpha}{\alpha}; -\frac{D^\alpha}{\lambda_b r^\alpha}\right) \right)} e^{-\lambda_b(\beta_n + \frac{(r/D)^\alpha}{\gamma})} \\ &\times \left(1 - 2 \frac{{}_2F_1\left(1; \frac{2+\alpha}{\alpha}; 2 + \frac{2}{\alpha}; -\frac{1}{\beta_n \gamma_D}\right)}{\gamma_D(2+\alpha)\beta_n} \right) \left(\frac{r}{D}\right)^{-\alpha} \frac{r}{D^2} dr. \quad (57) \end{aligned}$$

Which is (43).

REFERENCES

- [1] Y. Li and L. Dai, "Maximum sum rate of slotted aloha with capture," *IEEE Trans. Commun.*, vol. 64, no. 2, pp. 690–705, 2016.
- [2] W. S. Jeon and D. G. Jeong, "Combined channel access and sensing in cognitive radio slotted-aloha networks," *IEEE Trans. Veh. Technol.*, vol. 64, no. 5, pp. 2128–2133, 2015.
- [3] Y. H. Bae, B. D. Choi, and A. S. Alfa, "Achieving maximum throughput in random access protocols with multipacket reception," *IEEE Trans. Mobile Comput.*, vol. 13, no. 3, pp. 497–511, 2014.
- [4] J. Heidemann, W. Ye, J. Wills, A. Syed, and Y. Li, "Research challenges and applications for underwater sensor networking," in *IEEE Wireless Commun. Netw. Conf. (WCNC)*, vol. 1, Apr. 2006, pp. 228–235.
- [5] E. Sozer, M. Stojanovic, and J. Proakis, "Underwater acoustic networks," *IEEE J. Oceanic Eng.*, vol. 25, no. 1, pp. 72–83, Jan. 2000.
- [6] S. Ibrahim, J.-H. Cui, and R. Ammar, "Surface-level gateway deployment for underwater sensor networks," in *IEEE Mil. Commun. Conf. (MILCOM)*, Oct. 2007, pp. 1–7.
- [7] R. M. Gagliardi, *Introduction to Communications Engineering*. New York: Wiley, 1998.
- [8] J. Laneman and G. Wornell, "Energy-efficient antenna sharing and relaying for wireless networks," in *IEEE Wireless Commun. Netw. Conf. (WCNC)*, vol. 1, 2000, pp. 7–12 vol.1.
- [9] J. Laneman, G. W. Wornell, and D. Tse, "An efficient protocol for realizing cooperative diversity in wireless networks," in *Int. Symp. Inf. Theory. (ISIT)*, 2001, pp. 294–.
- [10] A. Sendonaris, E. Erkip, and B. Aazhang, "Increasing uplink capacity via user cooperation diversity," in *Int. Symp. Inf. Theory. (ISIT)*, Aug 1998, pp. 156–.
- [11] S. Shrestha and K. Chang, "Analysis of outage capacity performance for cooperative DF and AF relaying in dissimilar Rayleigh fading channels," in *Int. Symp. Inf. Theory. (ISIT)*, Jul. 2008, pp. 494–498.
- [12] P. Zhang, J. Yuan, J. Chen, J. Wang, and J. Yang, "Analyzing amplify-and-forward and decode-and-forward cooperative strategies in wyner's channel model," in *IEEE Wireless Commun. Netw. Conf. (WCNC)*, Apr. 2009, pp. 1–5.
- [13] M. Souryal and B. Vojcic, "Performance of amplify-and-forward and decode-and-forward relaying in Rayleigh fading with turbo codes," in *Int. Conf. Acoustics Speech Signal Process. (ICASSP)*, vol. 4, May 2006, pp. IV–IV.
- [14] Y. Xue, "Amplify-and-forward repeater enhanced random access in single-cell wireless communications," in *IEEE Int. Symp. Personal, Indoor and Mobile Radio Commun. (PIMRC)*, Sep. 2008, pp. 1–5.
- [15] Z. Peng, W. Xu, L. C. Wang, and C. Zhao, "Achievable rate analysis and feedback design for multiuser MIMO relay with imperfect csi," *IEEE Trans. Wireless Commun.*, vol. 13, no. 2, pp. 780–793, 2014.
- [16] H. Suraweera, H. Q. Ngo, T. Duong, C. Yuen, and E. Larsson, "Multi-pair amplify-and-forward relaying with very large antenna arrays," in *IEEE Int. Commun. Conf. (ICC)*, Jun. 2013, pp. 4635–4640.
- [17] M. Hasna and M.-S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 2, no. 6, pp. 1126–1131, Nov. 2003.
- [18] S. Chen, W. Wang, X. Zhang, M. Peng, and D. Zhao, "Capacity performance of amplify-and-forward MIMO relay with transmit antenna selection and maximal-ratio combining," in *IEEE Global Telecommun. Conf. (GLOBECOM)*, Nov. 2009, pp. 1–6.
- [19] D. Woldegebreal and H. Karl, "Multiple-access relay channel with network coding and non-ideal source-relay channels," in *Int. Symp. Wireless Commun. Syst. (ISWCS)*, Oct. 2007, pp. 732–736.
- [20] H. Suraweera, G. Karagiannidis, Y. Li, H. Garg, A. Nallanathan, and B. Vucetic, "Amplify-and-forward relay transmission with end-to-end antenna selection," in *IEEE Wireless Commun. Netw. Conf. (WCNC)*, April 2010, pp. 1–6.
- [21] K. Hamdi, "Capacity of MRC on correlated rician fading channels," *IEEE Trans. Commun.*, vol. 56, no. 5, pp. 708–711, May 2008.
- [22] —, "A useful technique for interference analysis in nakagami fading," *IEEE Trans. Commun.*, vol. 55, no. 6, pp. 1120–1124, Jun. 2007.
- [23] —, "A useful lemma for capacity analysis of fading interference channels," *IEEE Trans. Commun.*, vol. 58, no. 2, pp. 411–416, Feb. 2010.
- [24] S. M. Ross, *Introduction to Probability Models*. 6th Ed. Academic Press, USA, 1997.
- [25] B. Zhong, X. Zhang, Y. Li, Z. Zhang, and K. Long, "Impact of partial relay selection on the capacity of communications systems with outdated CSI and adaptive transmission techniques," in *IEEE Wireless Commun. Netw. Conf. (WCNC)*, Apr. 2013, pp. 3720–3725.
- [26] R. Nabar, H. Bolcskei, and F. Kneubuhler, "Fading relay channels: performance limits and space-time signal design," *IEEE J. Select. Areas Commun.*, vol. 22, no. 6, pp. 1099–1109, Aug. 2004.
- [27] H. Chen, J. Liu, Z. Dong, Y. Zhou, and W. Guo, "Exact capacity analysis of partial relay selection under outdated CSI over Rayleigh fading channels," *IEEE Trans. Veh. Technol.*, vol. 60, no. 8, pp. 4014–4018, Oct. 2011.
- [28] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*. New York, NY, USA: Dover, 1965.
- [29] M. K. Simon and M.-S. Alouini, *Digital Communication Over Fading Channels*. 2nd ed. New York: Wiley, 2005.
- [30] D. Zwillinger, *CRC Standard Mathematical Tables and Formulae*. 31st edition, A CRC Press Company, 2003.

Abdurrahman Alfitouri (S'16) received the B.Sc. degree in electrical and electronic engineering from the Engineering Academy Tajoura, Tripoli, Libya, in 2004. He then received the M.Sc. degree in mobile communication engineering from Lancaster University, Lancaster, UK, in 2012. He is currently pursuing a Ph.D degree with the microwave and communications systems (MACS) group of the University of Manchester. His current research interests in the wireless communications field including modelling and performance analysis of wireless communication

systems and networks, as well as Relay systems.

Khairi Ashour Hamdi (M'99-SM'02) received the B.Sc. degree in electrical engineering from the University of Tripoli, Tripoli, Libya, in 1981; the M.Sc degree (with distinction) from the Technical University of Budapest, Budapest, Hungary, in 1998; and the Ph.D. degree in telecommunication engineering from Hungarian Academy of Sciences, Budapest, in 1993. He was with the University of Essex, Colchester, U.K. He is currently with the School of Electrical and Electronic Engineering, The University of Manchester, Manchester, U.K. His current

research interests include modelling and performance analysis of wireless communication systems and networks, green communication systems, and heterogeneous mobile networks.