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Amplify-and-Forward Gateway with Zero-Forcing in Multi-User Environments

Abdurrahman Alfitouri, Student Member, IEEE, and Khairi Ashour Hamdi, Senior Member, IEEE

Abstract—In this paper, we investigate and analyze the performance of amplify and forward (AF) gateways employing zero-forcing (ZF) beamforming techniques in a space division multiple access (SDMA) scenario. A random number of users are isolated from their destination(s) and the only means to communicate with their destination(s) is through the AF gateway equipped with multiple antennas and employing ZF beamforming technique. All forward and backward channels experience small as well as large-scale fading. New exact analytical expressions are derived for the overall spectral efficiency (SE) in the collaborative and non-collaborative SDMA scenarios. In the latter case ZF is implemented at the gateway only. Whereas with regards to the collaborative scenario, ZF is applied at both, the sources and the destinations. The new results are used to investigate the impact of different system parameters on the overall efficiency of ZF AF gateways. The accuracy of the new results is confirmed through Monte Carlo simulations.

Index Terms—Amplify and Forward (AF), Multiple-input multiple-output (MIMO), spectral efficiency, space division multiple access (SDMA), zero-forcing (ZF).

I. INTRODUCTION

MULTIPLE - input multiple-output (MIMO) technology has become an integral feature of many advanced communication systems. It can provide a remarkable increase in data rate and reliability compared with single-antenna systems. A cellular system with MIMO that has been extensively explored is multi-user MIMO (MU-MIMO) in which multiple antenna simultaneously serve a multiplicity of autonomous co-channel users [1], [2]. Consider a wireless network where multiple sources aim to communicate with multiple destinations, but the channel between them is not reliable due to shadowing, deep fading or large separation distances. In this case, the gateway plays an important role, aiding the communication, by offering a clear improvement in system performance [3]. Significant research is taking place to enable relay systems to provide higher throughput and extended coverage without increasing transmitter power, especially for the cell edge user [3], [4]. Accordingly, relays apply different techniques which can be classified as: Amplify and Forward (AF), where the relay amplifies the received signal and resends it, or Decode and Forward (DF), where the relay regenerates the signal - which includes decoding the received signal - and then resends it [5]–[7].

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The perfect scenario in a wireless communication network with multiple sources, gateway, and destinations, is that the specified destination receives the useful signal from the source without it containing any interference. However, this scenario is very hard to achieve because the source signals interfere with each other at the gateway, which degrades system performance. This has led to the concept of interference-cancellation techniques applied at the source, gateway, or destination nodes, or any combinations of these.

Fig. 1, shows an example network with only one common gateway, where a number of users are uniformly and randomly distributed in served area, and their destinations uniformly and randomly distributed in different served area. The two service area are overlapping in very small area, which is the location of the gateway. Space division multiple access (SDMA) scheme is used, which provide a simple solution for a group of independent users to share a common communication channel. This kind of multiple access techniques are used to allow a large number of users to share the allocated frequency band in the most efficient way [8]–[10]. The communication between the users and their destination occurs only through a common gateway, which is equipped with multiple antennas. Each node (user or destination) is equipped with a single antenna. These users send their signals to the gateway, considered as a blind AF relay, and it transmits these signals to the desired destinations. There are two different scenarios for the application of the zero-forcing (ZF) technique to eliminate interference at the gateway, users and destination nodes. This work continues our previous work, reported in [11], where we considered the single antenna scenario in all network nodes (source, gateway, and destination), and where no kind of precoder, such as ZF, was applied at source, destination, or gateway.

A. Related Work

Two-hop networks with relay processing have been proposed by [12], who considered the zero-forcing technique was considered. However, the capacity-scaling law was derived only for the case when the number of relay nodes tends to infinity. In addition, no exact expression for spectral efficiency was derived. Moreover, important factors such as user status, shadow fading were not considered. The sum rate of multi-pair amplify and forward relaying is proposed in [13], where the authors used maximum ratio combining/maximal ratio transmission (MRC/MRT) and ZF processing at the relay. However, the authors applied the law of large numbers when, assuming the number of relay antenna $\rightarrow \infty$, instead of deriving an exact form expression. Furthermore, they represented shadow fading

Fig. 1: System Model

as constant, and did not consider a random number of sources in their analysis, which means user status was not considered. The sum rate of AF relay networks with multiple source-destination pairs has been proposed [14], for the case of two different designs of the ZF beamforming matrix. However, the authors investigate the upper bound of the sum rate loss that caused by channel state information errors, instead of deriving an exact form expression

The general ZF two hop relay network is proposed in [15], where the authors derived exact outage probability expressions for four different two hop AF relays. However, their emphasis was on showing how adding more nodes onto sources, relays, and/or destinations will benefit or detract from system performance. MIMO relaying with linear processing for multi-user transmission in fixed relay networks has been proposed in [16], where the processing was based on the Tomlinson–Harashima precoding and QR/singular value decomposition designs. However, the user selection scheme introduced in this paper is complicated and may not be realistic in mobile applications. Energy and spectral efficiency of very large multi-user MIMO systems is discussed in [17], where the authors applied MRC, ZF, and minimum mean-square error (MMSE), to detect and minimize the interference. However, the authors' intention was to show the effect of the base station antenna on the transmitted power of each user.

B. Main Contributions and Outcomes

In this paper, we analyze the performance of a two hope SDMA wireless communication system where a set of a random number of arbitrary users communicate with their destinations through a common AF gateway equipped with multiple antennas. The communication is divided into two phases; in the first phase, the users send their packets to the gateway which will receive the signals and amplify them. In the second phase the gateway resends the received signal to the desired destinations. To improve the performance of this system, we apply ZF processing at: 1) the gateway, and 2) source and destination nodes only.

The main contributions of this paper are summarized as follows:

- We develop new and more accurate models for multiple access interference at both the gateway and the destinations taking into account - in addition to the random number of the active users and their locations - the effects of fading and thermal noise at both the relay and the destinations. To the best of the authors' knowledge, there is little prior work which considers random number of users. However, this assumption makes the proposed model more practical than previous models. The proposed formula reveals the implication of user activity on the overall spectral efficiency (SE) of the system.
- We study two different scenarios; applying ZF processing at the gateway, and at the source and destination nodes. To the best of our knowledge, the system where ZF is applied at the source and destination nodes has not been considered in the literature but is related to many practical scenarios. One example of this scenario is when members of a group of mobile users individually attempt to transmit to individual members of another a group of users through a base station relay. The transmitting group of users has sufficient power to process the information signals between them, but not enough to transmit these information directly to the destination. Communication between two users does not happen directly, it occurs through the base station, which acts as a relay. User 1 (source) sends its signal to the base station (gateway), which forwards the processed signal to user 2 (destination). Another scenario is the underwater communication network, where a group of sensors aimed to communicate with their destinations of the ground through a gateway located on the water surface. Therefore, the sensors can apply ZF technique between them to eliminate the interference. Moreover, the destination nodes can apply the same idea to improve the system performance.
- We study the effect of both small and large scale fading on the wireless channel. Small scale fading is assumed to be Rayleigh fading, large scale fading is modelled as path loss attenuation, and shadow fading is modelled as a lognormal random variable. These assumptions have not been considered in the majority of the literature.
- We derive new analytical expressions for the overall SE which can be used to estimate the throughput of the ZF beamforming techniques-based gateways in Rayleigh fading channels, and to study the impact of the different system parameters on system efficiency.
- The signal-to-interference plus noise ratio (SINR) in this model is a ratio of two matrices containing a large number of random variables. However, the direct approach to find the overall SE is difficult to obtain in general [18]–[20], which makes the analysis of the performance more challenging. As a result, we use the simple and useful *Lemma 1* to evaluate this averaging, where the result includes a simple mathematical equation that can be easily evaluated.
- We derive explicit expressions for the SE of these models, where the new results are expressed in terms of the weights and abscissas of a Laguerre orthogonal polynomial. Furthermore, simulations are provided to validate

our analysis; the results of the simulations show that the SE of the proposed system is affected by factors such as increasing number of gateway antenna, number of users, users activity and path loss exponent.

C. Paper Organization and Notation

The remainder of this paper is organized as follows. In Section II, the system model is described briefly. In Section III, the problem formulation and SE analysis of collaborative and non-collaborative scenarios and the exact-form expressions are presented. Numerical and simulation results of collaborative and non-collaborative scenarios are presented and discussed in section IV. Finally, conclusions are drawn in Section V.

The following notations are used in the paper. Upper and lower case boldface denote matrices and vectors, respectively. $E[\cdot]$ denotes the expectation operator, $\Pr(\cdot)$ denotes the probability function, $\frac{\partial}{\partial A}$ denotes the derivation operation. Also, we use $\sim \mathcal{CN}(\mu, \sigma^2)$ for a circularly symmetric complex Gaussian distribution with mean μ and variance σ^2 . The transpose and conjugate transpose are denoted by $(\cdot)^T$ and $(\cdot)^H$ respectively. Matrix inversion is denoted by $(\cdot)^{-1}$, and $[\cdot]_{ik}$ represents the $(i, k)^{th}$ element of the matrix, while $[\cdot]_i$ denotes the i^{th} row of the matrix, and $\|\cdot\|$ denote the Euclidean norm of a vector.

II. THE SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we introduce the system model for a multiple access gateway, and formulate the SE analysis problem.

A. The System Model

The system model under consideration is shown in Fig. 1 where a random number of active users, K , communicate with their intended destinations through the AF relay. We assume that users and destinations are arbitrary distributed in two disjoint service areas and that each user can communicate with its destination only through the gateway. Therefore, there are no direct links between a users and its intended destination. Users and destination nodes are equipped with a single antenna, each, whilst the gateway is equipped with M antennas. This assumption (single antenna nodes) is a realistic assumption for many practical reasons, in particular because the majority of mobile handsets have a single antenna. The single antenna scenario for the source and destination is a popular scenario and used widely in MIMO networks and simplifies greatly the spectral efficiency analysis compared with the model where sources and their destinations are equipped with multiple antennas. We represent K by a binomial random variable with probability $\Pr(K = i) = \binom{L}{i} \rho^i (1 - \rho)^{L-i}$ [21], where L is the total number of users, and ρ is the probability that a user is active, which takes a value in the range $0 \leq \rho \leq 1$. The received composite signal at the gateway is

$$\mathbf{y}_r = \sqrt{p_t} \mathbf{G}_1 \mathbf{x} + \mathbf{n}_g \quad (1)$$

where p_t is the transmitted power, $\mathbf{x} = [x_1, x_2, \dots, x_K]^T$ is transmitted symbols, $\mathbf{n}_g$ is additive white Gaussian noise (AWGN) at the gateway, $\mathbf{G}_1$ represents the channel matrix

between L users and the gateway, which can be expressed as $\mathbf{G}_1 = \mathbf{H}_1 \mathbf{D}_1^{1/2}$, where $\mathbf{H}_1$ is the $M \times K$ matrix of the small-scale fading (Rayleigh fading) coefficients between M antennas of the gateway receiver and K antennas of the active users transmitters, and $\mathbf{D}_1$ is an $K \times K$ diagonal matrix, which models the large-scale fading (attenuation and log-normal shadow fading). The elements of all channels are assumed to be subject to independent and identically distributed (i.i.d) complex Gaussian fading with zero mean and unit variance $\sim \mathcal{CN}(0, 1)$.

During the second phase, the gateway will re-transmit a transformation of the received signal multiplied by a transformation matrix $\mathbf{W}$. The received signal at the destination can be expressed as

$$\begin{aligned} y_{D_i} = & \sqrt{p_t} g_{2i}^H \mathbf{W} g_{1i} x_i \\ & + \sqrt{p_t} \sum_{k=1, k \neq i}^K g_{2i}^H \mathbf{W} g_{1k} x_k + g_{2i}^H \mathbf{W} n_g + n_{D_i} \end{aligned} \quad (2)$$

where, g_{1i} and g_{1k} are, respectively, the i^{th} and k^{th} columns of $\mathbf{G}_1$, g_{2i} is the i^{th} column of $\mathbf{G}_2$ which represents the channel matrix between the gateway and destinations. n_{D_i} is AWGN at the destination i .

III. PROBLEM FORMULATION

In this section we consider the ZF processing design and we analyze the performance of two different scenarios. In particular, we consider scenarios a non-collaborative scenario where ZF processing is done at the gateway node and a collaborative scenario where ZF processing is carried out at both the source and destination nodes. We focus on the performance analysis of both scenarios and at the end of section III.D we compare the performance of these two scenarios highlighting the advantages and disadvantages of each scenario. Although, the collaborative scenario is more complicated, however it is introduced in this paper for the sake of the comparison with the non-collaborative scenario and investigate how the different scenarios can either be beneficial or detrimental to the overall system performance.

A. Non-Collaborative Scenario

At the gateway, we consider ZF linear receivers/precoders for interference cancellation and mitigation. Though there are many interference cancellation techniques in use, however the ZF is considered as a low-complexity alternative [22]–[24]. When the channel state information (CSI) of two the channels ($S \rightarrow \text{Gateway} \rightarrow D$) is available at the gateway, the transformation matrix of the gateway can be expressed as $\mathbf{W} = a_z \mathbf{G}_2 (\mathbf{G}_2^H \mathbf{G}_2)^{-1} (\mathbf{G}_1^H \mathbf{G}_1)^{-1} \mathbf{G}_1^H$, where a_z is the gateway gain which is assumed to be a constant [25].

According to the principal of ZF detector, which is intended to eliminate the users' interference, we have

$$g_{2i}^H \mathbf{W} g_{1k} = a_z \psi_{ki} \quad (3)$$

where $\psi_{ki} = 1$ when $k = i$, and 0 otherwise [13]. Therefore, the received signal at the destination i in (2) can be written as

$$\mathbf{y}_{D_i} = \mathbf{a}_z \sqrt{p_t} x_i + \mathbf{a}_z [(\mathbf{G}_1^H \mathbf{G}_1)^{-1} \mathbf{G}_1^H]_i \mathbf{n}_g + n_{D_i} \quad (4)$$

where $[\mathbf{B}]_i$ is the i^{th} row of the matrix $\mathbf{B}$. For simplicity let $\mathbf{G}_1^\dagger = (\mathbf{G}_1^H \mathbf{G}_1)^{-1} \mathbf{G}_1^H$. We obtain

$$y_{D_i} = \mathbf{a}_z \sqrt{p_t} x_i + \mathbf{a}_z [\mathbf{G}_1^\dagger]_i \mathbf{n}_g + n_{D_i}. \quad (5)$$

The signal-to-interference plus noise ratio at destination i (SINR _{i}) can be written as

$$\text{SINR}_i = \frac{\mathbf{a}_z^2 p_t}{\mathbf{a}_z^2 \left\| [\mathbf{G}_1^\dagger]_i \right\|^2 N_g + N_D} \quad (6)$$

where N_g and N_D represent the noise powers at the gateway and destination, respectively. Equation (6) can be rewritten as

$$\text{SINR}_i = \frac{\mathbf{a}_z^2 \left\| [\mathbf{G}_1^\dagger]_i \right\|^{-2}}{\frac{\mathbf{a}_z^2}{\gamma_g} + \frac{1}{\gamma_D} \left\| [\mathbf{G}_1^\dagger]_i \right\|^{-2}}. \quad (7)$$

where $\gamma_g = \frac{p_t}{N_g}$ and $\gamma_D = \frac{p_t}{N_D}$ are the received signal to noise ratio at the gateway and destination, respectively, and $\left\| [\mathbf{G}_1^\dagger]_i \right\|^{-2}$ has an Erlang distribution with shape and scale parameters $M - K + 1$ and ξ_k , respectively [26],

B. Spectral Efficiency Analysis of Non-Collaborative Scenario

In communication systems, the SE describes the data rate that can be achieved for a specific bandwidth. In this section, we derive the achievable SE of the two-phase relay system, which can be expressed as

$$\mathcal{R} = \frac{1}{2} \mathbb{E} \left\{ \sum_{k=1}^K [\log_2(1 + \text{SINR}_k)] \right\} \quad (8)$$

where $\frac{1}{2} [\log_2(1 + \text{SINR}_k)]$ is the instantaneous SE of user k . As we assume that the users statistics are i.i.d, the overall SE of this model can be written as

$$\mathcal{R} = \frac{1}{2} \mathbb{E} \{ K [\log_2(1 + \text{SINR}_1)] \} \quad (9)$$

where SINR₁ is the SINR of the first user, and the factor $\frac{1}{2}$ comes from the fact that the communications between the user and the destination is performed in two phases [27]–[29]. If we let $\left\| [\mathbf{G}_1^\dagger]_i \right\|^{-2} = \mathbf{Y}_k$, then by using the moment generating function (MGF) method we can find the exact analytical expressions for the SE as follows

Corollary 1. [20, eq. (5)] Let $x_1, \dots, x_N, y_1, \dots, y_M$ be arbitrary random variables > 0 . Then

$$\mathbb{E} \left\{ \ln \left(1 + \frac{\sum_{n=1}^N x_n}{\sum_{m=1}^M y_m + 1} \right) \right\} = \int_0^\infty \frac{1}{z} \left(1 - e^{-z \sum_{n=1}^N x_n} \right) e^{-z (\sum_{m=1}^M y_m + 1)} dz. \quad (10)$$

Thus, (9) can be simplified as

$$\mathcal{R} = \frac{1}{2} \mathbb{E} [K (\log_2 e) \int_0^\infty \frac{e^{-z \mathbf{a}_z^2 / \gamma_g}}{z} \left(1 - e^{-z \mathbf{a}_z^2 \mathbf{Y}_k} \right) e^{-z \mathbf{Y}_k / \gamma_D} dz]. \quad (11)$$

Which can be written as

$$\mathcal{R} = \frac{1}{2} \mathbb{E} [K (\log_2 e) \times \int_0^\infty \frac{e^{-z \mathbf{a}_z^2 / \gamma_g}}{z} \left(e^{-z \mathbf{Y}_k / \gamma_D} - e^{-z \mathbf{Y}_k (\mathbf{a}_z^2 + \frac{1}{\gamma_D})} \right) dz]. \quad (12)$$

Let $u = \frac{\mathbf{Y}_k}{\gamma_D}$, and $v = \mathbf{Y}_k (\mathbf{a}_z^2 + \frac{1}{\gamma_D})$. Therefore, we call $M_u(z) = \mathbb{E} [e^{-z \mathbf{Y}_k / \gamma_D}]$ and $M_v(z) = \mathbb{E} [e^{-z \mathbf{Y}_k (\mathbf{a}_z^2 + \frac{1}{\gamma_D})}]$, where $M_u(z) = \mathbb{E} [e^{-zu}]$ and $M_v(z) = \mathbb{E} [e^{-zv}]$ are the MGF of u and v , respectively. This leads to

$$M_u(z | K, \xi_k) = \mathbb{E} \left\{ e^{-z \mathbf{Y}_k / \gamma_D} | K, \xi_k \right\} = \left\{ \frac{1}{1 + \xi_k z / \gamma_D} \right\}^{M-K+1}. \quad (13)$$

And

$$M_v(z | K, \xi_k) = \mathbb{E} \left\{ e^{-z \mathbf{Y}_k (\mathbf{a}_z^2 + \frac{1}{\gamma_D})} | K, \xi_k \right\} = \left\{ \frac{1}{1 + \xi_k z (\mathbf{a}_z^2 + \frac{1}{\gamma_D})} \right\}^{M-K+1}. \quad (14)$$

Substitute (13) and (14) into (12), we obtain for the overall SE

$$\mathcal{R} = \frac{1}{2} \mathbb{E} \left\{ K (\log_2 e) \int_0^\infty \frac{e^{-z \mathbf{a}_z^2 / \gamma_g}}{z} \left\{ \left\{ \frac{1}{1 + \frac{\xi_k z}{\gamma_D}} \right\}^{M-K+1} - \left\{ \frac{1}{1 + \xi_k z (\mathbf{a}_z^2 + \frac{1}{\gamma_D})} \right\}^{M-K+1} \right\} dz | K, \xi_k \right\}. \quad (15)$$

In order to find the expected value of each part, (16) needs further simplifications as follows

$$\begin{aligned} \mathcal{R} &= \frac{1}{2} (\log_2 e) \\ &\times \int_0^\infty \frac{e^{-z \mathbf{a}_z^2 / \gamma_g}}{z} \left\{ \mathbb{E} \left\{ K \left(1 + \frac{\xi_k z}{\gamma_D} \right)^K \left\{ \frac{1}{1 + \frac{\xi_k z}{\gamma_D}} \right\}^{M+1} \right\} \right. \\ &\quad \left. - \mathbb{E} \left\{ K \left(1 + \xi_k z (\mathbf{a}_z^2 + \frac{1}{\gamma_D}) \right)^K \right. \right. \\ &\quad \left. \left. \times \left\{ \frac{1}{1 + \xi_k z (\mathbf{a}_z^2 + \frac{1}{\gamma_D})} \right\}^{M+1} \right\} \right\} dz. \quad (16) \end{aligned}$$

Lemma 2. The expected value $\mathbb{E} [K(A)^{K-1}]$, where K is a binomial random variable with probability $P_r(K = i) = \binom{L}{i} \rho^i (1 - \rho)^{L-i}$ can be found as

$$\mathbb{E} [K(A)^{K-1}] = \rho L (1 - \rho + \rho A)^{L-1}. \quad (17)$$

Proof: The proof is given in the Appendix.

Therefore, the overall SE can be written as

$$\mathcal{R} = \frac{\rho L}{2} (\log_2 e) \int_0^\infty \frac{e^{-z a_z^2 / \gamma_g}}{z} \left\{ \left\{ \frac{(1 + \rho \xi_k z / \gamma_D)^{L-1}}{(1 + \xi_k z / \gamma_D)^M} \right\} - \left\{ \frac{(1 + \rho \xi_k z (a_z^2 + \frac{1}{\gamma_D}))^{L-1}}{(1 + \xi_k z (a_z^2 + \frac{1}{\gamma_D}))^M} \right\} \mid \xi_k \right\} dz. \quad (18)$$

In the case of both small and large scale fading, the shadow fading is considered as a lognormal random variable, and the overall SE of ZF detector in a gateway system can be written as

$$\mathcal{R} = \frac{1}{2} \mathbb{E} \left\{ K \left[\log_2 \left(1 + \frac{a_z^2 \mathbf{Y}_k \xi_k}{\frac{a_z^2}{\gamma_g} + \frac{1}{\gamma_D} \mathbf{Y}_k \xi_k} \right) \right] \right\} \quad (19)$$

where $\xi_k = \eta_k r_k^{-\alpha}$ models the large-scale fading (i.e. path-loss attenuation and lognormal shadow fading), where η_k is a log-normal random variable and represents a shadow fading with probability density function (PDF) is $f_\eta(\eta) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\eta^2}{2\sigma^2}}$, and σ is a standard deviation in decibels (dB). The overall SE can be found as

$$\mathcal{R} = \frac{1}{2} \mathbb{E} \left\{ K \left[\log_2 \left(1 + \frac{a_z^2 \mathbf{Y}_k \eta_k r_k^{-\alpha}}{\frac{a_z^2}{\gamma_g} + \frac{1}{\gamma_D} \mathbf{Y}_k \eta_k r_k^{-\alpha}} \right) \right] \right\}. \quad (20)$$

As we mentioned, each user has a random location, and the distance between each user and the centre of the service area (r) is represented by a random variable with distribution ($f(r) = 2r/D^2, r \leq D$). In this regards, we assume that the distance between the centre of the users service area and the gateway is zero. Therefore, the gateway can be located at the centre of this circle. Returning to (14) and (15), the MGFs can be written as

$$M_u(z) = \int_{-\infty}^\infty \int_0^D \left\{ \frac{(1 + \rho 10^{\eta_k/10} r_k^{-\alpha} \frac{z}{\gamma_D})^{L-1}}{(1 + 10^{\eta_k/10} r_k^{-\alpha} \frac{z}{\gamma_D})^M} \right\} \times \frac{2r_k}{D^2 \sqrt{2\pi\sigma^2}} e^{-\frac{\eta_k^2}{2\sigma^2}} dr_k d\eta_k. \quad (21)$$

$$M_v(z) = \int_{-\infty}^\infty \int_0^D \left\{ \frac{(1 + \rho 10^{\eta_k/10} r_k^{-\alpha} z (a_z^2 + \frac{1}{\gamma_D}))^{L-1}}{(1 + 10^{\eta_k/10} r_k^{-\alpha} z (a_z^2 + \frac{1}{\gamma_D}))^M} \right\} \times \frac{2r_k}{D^2 \sqrt{2\pi\sigma^2}} e^{-\frac{\eta_k^2}{2\sigma^2}} dr_k d\eta_k. \quad (22)$$

Equations (21) and (22) can be efficiently computed in terms of the weights and abscissas of a Laguerre orthogonal polynomial as follows

$$M_u(z) = \sum_{a=1}^A \Upsilon_a \int_0^D \left\{ \frac{(1 + \rho 10^{\lambda_a \sqrt{2}\sigma/10} r_k^{-\alpha} \frac{z}{\gamma_D})^{L-1}}{(1 + 10^{\lambda_a \sqrt{2}\sigma/10} r_k^{-\alpha} \frac{z}{\gamma_D})^M} \right\} \times \frac{2r_k}{D^2 \sqrt{\pi}} dr_k + R_A \quad (23)$$

and

$$M_v(z) = \sum_{b=1}^B \Lambda_b \times \int_0^D \left\{ \frac{(1 + \rho 10^{\Psi_b \sqrt{2}\sigma/10} r_k^{-\alpha} z (a_z^2 + \frac{1}{\gamma_D}))^{L-1}}{(1 + 10^{\Psi_b \sqrt{2}\sigma/10} r_k^{-\alpha} z (a_z^2 + \frac{1}{\gamma_D}))^M} \right\} \times \frac{2r_k}{D^2 \sqrt{\pi}} dr_k + R_B \quad (24)$$

where λ_a , Ψ_b and Υ_a , Λ_b are the sample points and the weights factors of the Laguerre polynomial, respectively, tabulated in [30, Eq. (25.4.46)]. The remainders R_A and R_B are sufficiently small for A and $B \geq 15$.

Substituting (21) and (22) into (12), the final expression for overall SE is found, which can be expressed as

$$\mathcal{R} = \frac{\rho L}{2} (\log_2 e) \int_0^\infty \frac{e^{-z a_z^2 / \gamma_g}}{z} \{M_u(z) - M_v(z)\} dz. \quad (25)$$

It is worth mentioning that (25) can also be expressed in terms of the weights and abscissas of a Laguerre orthogonal polynomial

$$\mathcal{R} = \frac{\rho L}{2} (\log_2 e) \sum_{n=1}^N \frac{\xi_n}{\beta_n} \{\mathcal{M}_u(\mu_n) - \mathcal{M}_v(\mu_n)\} + R_N \quad (26)$$

where $\mu_n = \beta_n \frac{\gamma_g}{a_z^2}$, β_n and ξ_n are the sample points and the weights factors of the Laguerre polynomial, respectively, tabulated in [30, Eq. (25.4.45)] and the remainder R_N is also small for $N \geq 15$; therefore, (26) provides an efficient numerical evaluation result for the required SE.

C. Collaborative Scenario

In this scenario, we consider the use of ZF processing applied to two individual groups of nodes. At the user nodes (first hop at the sending side) and at the destination nodes (second hop at the receiving side). In the first hop, we assume that user nodes (transmitters) can cooperate with each other sharing the resource information and operate as a virtual antenna array. In the second hop, the same assumption will be made for the destination nodes that they can cooperate to achieve a spatial diversity. Virtual antenna arrays allows a group of single antenna users to cooperate with each other achieving the special diversity at a low cost. Comparing with MIMO system, where each node needs to have multiple antennas which may exceed the capabilities of low power nodes (e.g. mobile user, ad-hoc and sensor network), virtual antenna array can be considered as special case on MIMO system [31]–[33].

Cooperative distributed single antenna nodes with ZF processing is known in the literature as multi-user ZF protocol [34]. There are different protocols designed for virtual antenna array such as cooperative source routing (CSR) which summarized as transmit route request and route replay control packets between the nodes [35]. In [36], the authors used the

Fig. 2: System model of Collaborative Scenario

framed ALOHA technique between the cooperative nodes, where they investigated the optimum number of slots per frame in the slotted structure and the impact of the number of hops on the system performance. The multi-layer approach including physical, medium access control and routing layers for exploiting virtual MISO links in ad-hoc networks has been presented in [37], where the authors improved the end-to-end performance in terms of throughput, delay, and interference. Recently, different types of codes have been considered for cooperative transmission nodes such as complex field codes [38], Reed-Solomon codes [39], convolutional codes [40], serial concatenated convolutional codes [41], and trellis coded modulation [42]. Such cooperative diversity or multi-antenna diversity techniques are still attracting attention because when other types of diversity are unavailable users sharing their antennas and other resources leads to improving the overall system performance [43]–[47]. It is to be emphasized that the collaborative scenario is considered in this paper for the sake of the comparison with the non-collaborative scenario and to investigate how the different scenarios can be either beneficial or detrimental to system performance. Details of the protocols that governs local information exchange are out of the scope of this paper. Fig. 2 illustrates the basic system diagram used in Collaborative Scenario.

In this scenario, we assume CSI for the channels between $(S \rightarrow \text{Gateway})$, and $(\text{Gateway} \rightarrow D)$ are available at the source and destination nodes, respectively. According to the principle of ZF, the transformation matrix for the source will given by $\mathbf{W}_S = \mathbf{G}_1^H (\mathbf{G}_1 \mathbf{G}_1^H)^{-1}$, and for the destination as $\mathbf{W}_D = \mathbf{G}_2^H (\mathbf{G}_2 \mathbf{G}_2^H)^{-1}$. The received signal at the gateway can be written as

$$\mathbf{y}_R = \sqrt{p_t} \mathbf{W}_S \mathbf{g}_{1i} x_i + \sqrt{p_t} \sum_{k=1, k \neq i}^K \mathbf{W}_S \mathbf{g}_{1k} x_k + \mathbf{n}_g. \quad (27)$$

The gateway will receive this signal, amplify it with gateway gain a_z , and send it through the second channel to the desired destination. The received signal at the destination node i can be written as

$$y_{Di} = a_z \sqrt{p_t} \mathbf{g}_{2i}^H \mathbf{W}_D \mathbf{W}_S \mathbf{g}_{1i} x_i + a_z \mathbf{g}_{2i}^H \mathbf{W}_D \mathbf{n}_g + [\mathbf{G}_2^H (\mathbf{G}_2 \mathbf{G}_2^H)^{-1}]_i n_{Di}. \quad (28)$$

The SINR at the destination i (SINR_i) is equal to

$$\text{SINR}_i = \frac{a_z^2 p_t}{a_z^2 N_g + \left\| [\mathbf{G}_1^\dagger]_i \right\|^2 N_D} \quad (29)$$

where $\mathbf{G}_1^\dagger = (\mathbf{G}_1^H \mathbf{G}_1)^{-1} \mathbf{G}_1^H$, the final form of SINR_i can be expressed as

$$\text{SINR}_i = \frac{a_z^2 \left\| [\mathbf{G}_1^\dagger]_i \right\|^{-2}}{\frac{a_z^2}{\gamma_g} \left\| [\mathbf{G}_1^\dagger]_i \right\|^{-2} + \frac{1}{\gamma_D}} \quad (30)$$

where $\left\| [\mathbf{G}_1^\dagger]_i \right\|^{-2}$ has an Erlang distribution with shape and scale parameters $M-L+1$ and ξ_k , respectively. The achievable overall SE of the system is expressed as

$$\mathcal{R} = \frac{1}{2} \mathbb{E} \left\{ \sum_{k=1}^K [\log_2(1 + \text{SINR}_k)] \right\}. \quad (31)$$

which can be written as

$$\mathcal{R} = \frac{1}{2} \mathbb{E} \{ K [\log_2(1 + \text{SINR}_1)] \}. \quad (32)$$

D. Spectral Efficiency Analysis of Collaborative Scenario

The exact analytical expressions for the overall SE can be expressed as

$$\mathcal{R} = \frac{1}{2} E[K (\log_2 e)] \times \int_0^\infty \frac{e^{-z/\gamma_D}}{z} \left(1 - e^{-z a_z^2 Q} \right) e^{-z a_z^2 Q / \gamma_g} dz. \quad (33)$$

Which can be written as

$$\mathcal{R} = \frac{1}{2} E[K (\log_2 e)] \times \int_0^\infty \frac{e^{-z/\gamma_D}}{z} \left(e^{-z a_z^2 Q / \gamma_g} - e^{-z Q (a_z^2 + \frac{a_z^2}{\gamma_g})} \right) dz. \quad (34)$$

Let $q = \frac{a_z^2 Q}{\gamma_g}$, and $p = Q(a_z^2 + \frac{a_z^2}{\gamma_g})$, where $Q = \left\| [\mathbf{G}_1^\dagger]_i \right\|^{-2}$. Therefore, $\mathcal{M}_q(z) = \mathbb{E} \left[e^{-z a_z^2 Q / \gamma_g} \right]$ and $\mathcal{M}_p(z) = \mathbb{E} \left[e^{-z Q (a_z^2 + \frac{a_z^2}{\gamma_g})} \right]$, where $\mathcal{M}_q(z)$ and $\mathcal{M}_p(z)$ are the MGF of q and p , respectively. This leads to

$$\begin{aligned} \mathcal{M}_q(z) &= \mathbb{E} \left\{ e^{-z a_z^2 Q / \gamma_g} \mid K, \xi_k \right\} \\ &= \left\{ \frac{1}{1 + \xi_k z a_z^2 / \gamma_g} \right\}^{M-K+1}. \end{aligned} \quad (35)$$

And

$$\begin{aligned} \mathcal{M}_p(z) &= \mathbb{E} \left\{ e^{-z Q (a_z^2 + \frac{a_z^2}{\gamma_g})} \mid K, \xi_k \right\} \\ &= \left\{ \frac{1}{1 + \xi_k z (a_z^2 + \frac{a_z^2}{\gamma_g})} \right\}^{M-K+1}. \end{aligned} \quad (36)$$

We follow the same mathematical analysis method that we applied in III-B. Therefore (35) and (36) can be expressed as

$$\mathcal{M}_q(z) = \int_{-\infty}^{\infty} \int_0^D \left\{ \frac{\left(1 + \rho 10^{\eta_k/10} r_k^{-\alpha} \frac{z a_z^2}{\gamma_g}\right)^{L-1}}{\left(1 + 10^{\eta_k/10} r_k^{-\alpha} \frac{z a_z^2}{\gamma_g}\right)^M} \right\} \times \frac{2r_k}{D^2 \sqrt{2\pi\sigma^2}} e^{-\frac{\eta_k^2}{2\sigma^2}} dr_k d\eta_k. \quad (37)$$

$$\mathcal{M}_p(z) = \int_{-\infty}^{\infty} \int_0^D \left\{ \frac{\left(1 + \rho 10^{\eta_k/10} r_k^{-\alpha} z \left(a_z^2 + \frac{a_z^2}{\gamma_g}\right)\right)^{L-1}}{\left(1 + 10^{\eta_k/10} r_k^{-\alpha} z \left(a_z^2 + \frac{a_z^2}{\gamma_g}\right)\right)^M} \right\} \times \frac{2r_k}{D^2 \sqrt{2\pi\sigma^2}} e^{-\frac{\eta_k^2}{2\sigma^2}} dr_k d\eta_k. \quad (38)$$

Likewise, (37) and (38) can be simplified in terms of the weights and abscissas of a Laguerre orthogonal polynomial

$$\mathcal{M}_q(z) = \sum_{c=1}^C \mathbb{Q}_c \int_0^D \left\{ \frac{\left(1 + \rho 10^{\mathcal{F}_c \sqrt{2}\sigma/10} r_k^{-\alpha} \frac{z a_z^2}{\gamma_g}\right)^{L-1}}{\left(1 + 10^{\mathcal{F}_c \sqrt{2}\sigma/10} r_k^{-\alpha} \frac{z a_z^2}{\gamma_g}\right)^M} \right\} \times \frac{2r_k}{D^2 \sqrt{\pi}} dr_k + R_C \quad (39)$$

$$\mathcal{M}_p(z) = \sum_{j=1}^J \mathcal{L}_j \int_0^D \left\{ \frac{\left(1 + \rho 10^{\aleph_j \sqrt{2}\sigma/10} r_k^{-\alpha} z \left(a_z^2 + \frac{a_z^2}{\gamma_g}\right)\right)^{L-1}}{\left(1 + 10^{\aleph_j \sqrt{2}\sigma/10} r_k^{-\alpha} z \left(a_z^2 + \frac{a_z^2}{\gamma_g}\right)\right)^M} \right\} \times \frac{2r_k}{D^2 \sqrt{\pi}} dr_k + R_J \quad (40)$$

where $\mathcal{F}_c$, $\aleph_j$ and $\mathbb{Q}_c$, $\mathcal{L}_j$ are the sample points and the weights factors of the Laguerre polynomial, respectively, tabulated in [30, Eq. (25.4.46)], and the remainders R_C and R_J are small for C and $J \geq 15$. The final form of the overall SE can be written as

$$\mathcal{R} = \frac{\rho L}{2} (\log_2 e) \int_0^{\infty} \frac{e^{-z/\gamma_D}}{z} \{\mathcal{M}_q(z) - \mathcal{M}_p(z)\} dz. \quad (41)$$

Equation (41) can also be expressed in terms of the weights and abscissas of a Laguerre orthogonal polynomial, as was done with (25)

$$\mathcal{R} = \frac{\rho L}{2} (\log_2 e) \sum_{e=1}^E \frac{\varpi_e}{\delta_e} \{\mathcal{M}_q(\varphi_e) - \mathcal{M}_p(\varphi_e)\} + R_E \quad (42)$$

where $\varphi_e = \delta_e \gamma_D$, δ_e and ϖ_e are, respectively, the sample points and the weights factors of the Laguerre polynomial, tabulated in [30, eq. (25.4.45)]. The remainder R_E is sufficiently small for $E \geq 15$; therefore, (42) provides an efficient numerical evaluation method for the required SE.

Table I: Simulation Parameters

Parameter	Value
SNR (γ)	1 dB
User status (ρ)	1
Area radius (D)	1 km
Gateway gain (a_z)	1 dB
Number of users (L)	10
Path loss exponent (α)	3
Number of gateway antenna (M)	10
Standard deviation of shadow fading (σ)	8 dB

Remark 3. Equations (26) and (42) describe the behavior of the SE for two different scenarios; ZF at gateway, and ZF at source and destination nodes. The main parameters that can affect the performances of the systems are similar. The user status, number of users, gateway antennas, SNR, and standard deviation of shadow fading are the more important parameters in both two systems. Change in any one of these factors can be either beneficial or detrimental to system performance.

However, there is a clear difference in the performance of the two scenarios as can be seen in the numerical and simulation results. The main advantage of ZF is eliminating interference between users. However, it has the drawback that it amplifies noise [48]. In non-collaborative scenario the ZF amplifies the noise at the gateway only. However, in collaborative scenario the ZF amplifies the noise at both the gateway and destination nodes. This will lead to a relative degradation of system performance in second scenario, as be seen in the next section.

IV. NUMERICAL AND SIMULATION RESULTS

In this section, the overall SE achieved by multiple-access of the AF gateway with the ZF receiver at the gateway is evaluated by Monte-Carlo simulations and compared to the derived asymptotic results. Both small and large scale fading are considered, where, as stated above, small scale fading is represented by Rayleigh fading channels, and large scale fading represented by attenuation and log-normal shadow fading. For sake of simplicity, we assume $\gamma_g = \gamma_D = \gamma$. Figures of spectral efficiency are plotted corresponding to user number, γ values, user states, standard deviation and number of gateway antenna.

A. Non-Collaborative Scenario

In Fig. 3, the overall SE, in bits/s/Hz, is plotted as function of the number of user transmitting antennas, L , for different numbers of receiving antennas, M . The simulation curves are obtained by performing Monte Carlo simulations using (8), whereas the analytical are computed via (26). This figure shows the effect of increasing the number of gateway antenna on system performance, when all users are active ($\rho = 1$). As can be seen when the number of users rises from 0 to 7, the overall SE increases dramatically. However, for the number of users greater than 7 the SE value depends on the value of M , the number of gateway antennas. As can be seen, as M increases from 9 to 10 to 11, when the number of users approaches, becomes equal to, or greater than, the number of gateway antenna, the SE start to decrease. Increasing the

Fig. 3: SE as function of number of users, L ($\gamma = 1\text{dB}$, $\rho = 1$, $\sigma = 8\text{dB}$ $M = 9, 10$ and 11).

Fig. 5: SE as function of number of users, L ($\gamma = 1\text{dB}$, $M = 10$, $\sigma = 8\text{dB}$, $\rho = 0.5, 0.8$ and 1).

Fig. 4: SE as function of user activity, ρ ($\gamma = 1\text{dB}$, $M = 10$, $\sigma = 8\text{dB}$ $L = 8, 10$ and 11).

number of users to more than the number of gateway antenna will result in a rapid decrease in spatial diversity, as well as increase of the interference at the gateway, both of which will affect the performance of the system.

In Fig 4, the SE is plotted as function of user activity (ρ), for 10 receiving antennas, and for 9, 10 and 11 users. As ρ increases, the number of active users increases, which leads to an initial increase in SE. It can be seen that for 9 users the SE increases monotonically with increase in user activity. However, when the number of users is 10, the SE start to decrease after $\rho = 0.9$. When the number of users is 11 the SE starts to decrease after $\rho = 0.7$. The reason is that the performance of the system will drop sharply as the number of active users ($\sim \rho L$) approaches the number of gateway antennas.

Fig 5, contains the same data as Figs 2 and 3, but with the number of gateway antenna fixed at 10, ($M = 10$). Here, user

activity takes three different values; $\rho = 0.5, 0.8$ and 1 . The SE increases with increase in number of users, more sharply the greater the value of ρ , i.e. the greater number of users. However, for $\rho = 1$, there is a sharp drop in SE when the number of users increases to 9 and is almost equal to the number of gateway antenna.

Fig 6, presents the SE as function of the number of gateway antenna, M , when $\rho = 0.5, 0.8$ and 1 , with 10 users. Thus, for $M=1$, where the number of antenna is much less than the number of users, the SE is effectively zero. For the remaining points on the plot, $M \geq L$ so SE will increase with both usage and number of users. However, in this figure, it can be seen that when the number of antennas is significantly greater than the number of users ($M \gg L$, that is $M > 30$) the three plots increase only very slowly with further increase in M . The effect of ρ remains positive, which mean that as ρ increases the SE value of the system increases. This behavior is expected, because the spatial diversity stays high when $M > 30$ regardless of the level of activity of the users. This could be used to determine the maximum number of antenna required for a given application.

In Fig. 7, we plot the SE as a function of the standard deviation of shadow fading (σ dB), for 10, 11 and 12 gateway antennas. We can observe that σ has a significant impact on the performance. For a given value of M , as σ increases, SE decreases. On the other hand, the effect of a given value of σ increases as M increases. For example, at $\sigma = 4$, the SE values are 5.2, 7.4, and 7.6 bits/s/Hz, for $M = 10, 11$, and 12, respectively. However, the rate of increase of SE is not linear, when we increase the number of gateway antennas by one from 10 to 11, the SE increased by 2.2 (42 %) and when M increased from 11 to 12 the SE increased by 2.0 (27 %).

Fig. 8, shows the SE as function of the received signal to noise ratio SNR (γ), for 10, 20, and 80 antennas for 10 users. As expected, when γ increases, the SE increases as well. It can be seen that, with $L = 10$ users, increasing the value of M from 10 to 20 has a clear and significant effect on the SE.

Fig. 6: SE as function of number of gateway antenna, M ($\gamma = 1\text{dB}$, $L = 10$, $\sigma = 8\text{ dB}$ $\rho = 0.5, 0.8$ and 1).

Fig. 8: SE as function of SNR, γ ($L = 10$, $\rho = 1$, $\sigma = 8\text{dB}$, $M = 10, 20$ and 80).

Fig. 7: SE as function of standard deviation of shadow fading, σ dB ($\gamma = 1\text{dB}$, $L = 10$, $\rho = 1$ $M = 10, 11$ and 12).

Fig. 9: SE as function of user activity, ρ ($L = 11$, $M = 10$, $\sigma = 8\text{dB}$ $\gamma = 10, 12$ and 15dB).

However, increasing the value of M from 20 to 80 gives only a small improvement in system performance, because the key is to have $M > L$.

Fig 9. shows how SE varies with user activity for three values of SNR, $\gamma = 10, 12$, and 15 dB . Here we present the special case where the number of gateway antenna is less than the number of users, $L = 11$ and $M = 10$. All three curves exhibit the same basic shape, there is an initial increase in SE until user activity reaches about 0.8, after which, for further increases in ρ , SE decreases until at $\rho = 1$ the number of active users is greater than the number of antennas which leads to more interference and $\text{SE} = 0$. However, it can also be seen that as γ increases, the SE increases, at all values of user activity, and the performance of the system shows a clear improvement with increase in SNR, as one would expect.

From the above results, especially Figs (3, 4, 5, and 9), we see that the number of gateway antennas (M) has to be larger

than the sum of the number of active users antennas (L) [49], [50], i.e., $M > L$. This means that, the perfect ZF precoding is only possible when $M > L$ and can be achieved using channel inversion.

B. Non-Collaborative Scenario

In Fig. 10, the SE is plotted as function of the number of users for 9, 10 and 11 gateway antennas. All users are active, $\rho = 1$. As the number of users increase, the SE initially increases dramatically for each of $M = 9, 10$ and 11 . The system performance continues to increase with increase in L while the number of gateway antenna is greater than the number of users. However, there is a fall in the value of SE when the number of active users becomes close to the number of gateway antenna, and a dramatic decrease in SE when the number of active users exceeds the number of gateway antennas. This demonstrates that increasing the number of users above the number of

Fig. 10: SE as function of number of users, L ($\gamma = 1\text{dB}$, $\rho = 1$, $\sigma = 8\text{dB}$ $M = 9, 10$ and 11).

Fig. 12: SE as function of standard deviation of shadow fading, σ ($\gamma = 1\text{dB}$, $L = 10$, $\rho = 1$ $M = 10, 11$ and 12).

Fig. 11: SE as function of user activity, ρ ($\gamma = 1\text{dB}$, $M = 10$, $\sigma = 8\text{dB}$, $L = 8, 10$ and 11).

available gateway antenna will result in interference at the gateway, which will seriously adversely affect the performance of the system.

In Fig 11, the SE is plotted as function of the user activity ρ , for three different numbers of users ($L = 9, 10$ and 11). The number of gateway antenna was constant at $M = 10$. Initially, as user activity increases there is a monotonic increase in SE. However, when the number of active users approaches the value of M we see that SE stops increasing and then falls sharply. The turning points are at $L = 11$ and $\rho = 0.7$, $L = 10$ and $\rho = 0.9$, while for $L = 9$ there is no value of ρ at which the SE starts to fall. The reason is, of course, obvious, when the number of users is more than the number of gateway antenna there will be a resulting decrease in spatial diversity, as well as an increase in interference at the gateway.

In Fig. 12, we plot the SE as function of the standard deviation of shadow fading (σ dB), for different numbers of

gateway antennas ($M = 10, 11$ and 12). We observe that, as the standard deviation of shadow fading increases the SE decreases significantly. This decrease is smaller the larger the value of M . For example when $\sigma = 7$, increasing M by one antenna, from 10 to 11, increasing the SE from 4.3 to nearly 5 bits/s/Hz - an increase of 16 %. When M increases from 11 to 12 the SE increases from nearly 5 to 5.3 bits/s/Hz an increasing of about 6 %.

In Fig. 13, the SE is plotted as function of SNR (γ) for $M = 10, 11$, and 15 , it can be seen that the effect of number of gateway antenna is limited at low γ , i.e. the spread in values of the SE is very limited at negative values of γ . However, at higher levels of γ , the number of gateway antenna positively impacts on system performance. When M increases, the value of the SE also increases. However, the larger the value of M the less increase in SE is obtained for a given increase in M . For example increasing the value of M from 10 to 11 at say, $\gamma = 20\text{ dB}$, the SE improves by 5 bits/s/Hz. However, a further increase in M from 11 to 15 produces an additional increase in SE of only 1.5 bits/s/Hz.

Fig 14, shows the effect of different user activity status on the SE. Here, the number of gateway antenna ($M = 10$) is less than the number of users ($L = 11$) for high values of γ ($\gamma = 10, 12$, and 15 dB). All three curves exhibit the same basic shape, there is an initial increase in SE until user activity reaches about 0.8, For further increase in ρ , SE decreases until at $\rho = 1$ the number of active users is greater than the number of antennas which leads to more interference and $\text{SE} = 0$. However, it can also be seen that as the SNR (γ) increases, the SE increases, at all values of user activity, and the performance of the system shows a clear improvement, as one would expect.

Fig 15, shows the effect of variation in the number of gateway antenna on system performance for three levels of user activity, $\rho = 0.5, 0.8$, and 1 . The number of users is 10. In this figure, SE increases rapidly until the number of number of gateway antennas is twice the number of users, after which the rate of increase slows rapidly until about $M = 30$, after

Fig. 13: SE as function of SNR, (γ) ($L = 10$, $\rho = 1$, $\sigma = 8\text{dB}$, $M=10, 11$ and 15).

Fig. 14: SE as function of user activity, ρ ($L = 11$, $M = 10$, $\sigma = 8\text{dB}$, $\gamma = 10, 12$ and 15 dB).

which SE increases only very slowly with further increase in M . The plots also show that SE increases significantly for increases of user activity levels from $\rho = 0.5$ to 0.8 to 1.0 . This means that as ρ increases, the performance of the system improves.

V. CONCLUSIONS.

In this paper, we have analyzed in detail the system performance when a random number of active users communicate with their destinations. All the users and destinations are equipped with a single antenna, and communicate through its AF gateway, which is equipped with multiple antennas. To eliminate interference, ZF processing is performed at 1) the relay; and 2) the source and destination nodes. We derived analytical expressions for the SE, where we assumed the channels ($S \rightarrow \text{Gateway} \rightarrow D$) are affected by both small and large scale fading (Rayleigh fading, lognormal shadow

Fig. 15: SE as function of number of gateway antenna, M ($\gamma = 1\text{dB}$, $L = 10$, $\sigma=8\text{dB}$, $\rho = 0.5, 0.8$, and 1).

fading, and path loss attenuation). We examined the impact of increasing the number of users in the network, and the number of gateway antenna on the performance of such a systems. We found that as the number of users increased, the SE also increased, until the number of users came close to the number of gateway antenna. Once the number of users was greater than the number of gateway antenna, the SE decreased rapidly. The number of gateway antenna has a positive impact on the system's performance, as the number of gateway antenna increase, the SE also increases. When M becomes large, the channel vectors between the users and the gateway are pairwise asymptotically orthogonal, and hence, interference can be cancelled out with a simple linear ZF receiver. Finally, we showed the effect of important factors in this model on the system performance, such as path loss exponent, SNR, user status, and standard deviation. It should be noted that simulation and analytical results exactly matched.

APPENDIX

PROOF OF Lemma 2

In this appendix, we aim to prove the useful Lemma 2, which is presented in Equation (17). The right hand side of (17) can be simplified as

$$\mathbb{E} \{ K(A)^{K-1} \} = \mathbb{E} \left\{ \frac{\partial}{\partial A} A^K \right\} \quad (43)$$

where $\frac{\partial}{\partial A}$ is stands for the derivative with respect to A . Therefore, (43) can be simplified as

$$\mathbb{E} \{ K(A)^{K-1} \} = \frac{\partial}{\partial A} \mathbb{E} \{ A^K \}. \quad (44)$$

As mentioned earlier, K is a binomial random variable with probability $P_r(K = i) = \binom{L}{i} \rho^i (1-\rho)^{L-i}$, this means that (44) can be written as

$$\mathbb{E} \{ K(A)^{K-1} \} = \frac{\partial}{\partial A} (1 - \rho + \rho A)^L. \quad (45)$$

Taking the derivation of the left hand side of (45) leads immediately to the results that shown in (19), which can be written as

$$\mathbb{E} \{K(A)^{K-1}\} = \rho L(1 - \rho + \rho A)^{L-1}. \quad (46)$$

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