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1D Filter for Ring Artifact Suppression

Valeriy Titarenko

Abstract—Reconstruction algorithms in x-ray tomography provide a user with a set of images. For many scientific problems these images should be segmented. Ring artifacts are the most noticeable artifacts which do not allow a scientist to achieve acceptable segmentation especially in case of noisy input data. Several ring artifact suppression methods based on theory of ill-posed problems exist. In this letter the basic ring artifact suppression algorithm is rewritten as a 1D filter, so it does not require separate data processing and can be combined with standard filtering used in filtered backprojection algorithms.

Index Terms—Ring artifacts, Tikhonov regularization, filter, convolution, x-ray tomography

I. INTRODUCTION

X-ray tomography is a non-destructive procedure to see an internal structure of a sample. A lot of applications of this technique exist in biology, medicine, engineering, earth, environmental and material sciences. In some areas, e.g. in cancer research, a user is mostly interested in small variations of attenuation coefficients, while other problems require proper segmentation, so further mathematical simulations can provide information of physical/chemical properties of a specimen. The second group of problems can be very sensitive with respect to errors during segmentation process. While x-ray tomography as an experimental technique is prone to various artifacts (limited number of projections, noise, beam hardening, motion), the most critical artifacts for segmentation are ring artifacts. In this letter we consider suppression of ring artifacts for data from synchrotron radiation facilities and laboratory based x-ray imagers used for material science applications.

A standard data collection includes acquisition of projections I_{proj} of an object rotating in an x-ray beam and a set of flat/white I_{flat} and dark I_{dark} field images when there is no specimen and the beam is switched on and off respectively. Thus an optical path along an x-ray through the object can be found as

$$P = \ln \left(\frac{I_{\text{flat}} - I_{\text{dark}}}{I_{\text{proj}} - I_{\text{dark}}} \right) \quad (1)$$

and a 2D/3D reconstruction algorithm is then used to find the internal structure. Due to various instabilities in the x-ray beam [1], [2], defects in a scintillator, optical system or image-recording sensor, the optical path images obtained by applying formula (1) may still have additional systematic peaks. A flat field image is shown in Fig. 1: hard-to remove dirt on various optical surfaces causes decrease in intensity of the beam (darker areas), while scratches and defects on a scintillator increase it (white areas). Considering only one

Fig. 1. An example of flat field image acquired at beamline I13, Diamond Light Source: (a) original and (b) zoomed-in areas.

Fig. 2. A example of sinogram for a graphite sample: (a) original and (b) zoomed-in and high contrast areas.

row of a sensor (usually perpendicular to the sample's axis of rotation), the cross-section slice, a so-called sinogram, has lines of higher/lower values caused by these peaks, see Fig. 2. After reconstruction the slice will contain extra circular structures called ring artifacts. In case of half-rotation scans or cone-beam geometry, these artifacts look like arches of different strengths.

Several groups of methods have already been developed to suppress ring artifacts. Some of them are based on “hardware” solution, for instance in [3] the camera is moving during the scan, so each pixel in the recorded image is made up from contributions from all detector elements. In other methods

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various filters are applied to a sinogram or a reconstructed slice taking into account a linear structure of artifacts on sinogram images or their circular structure on reconstructed images. Details of these approaches can be found in [2], [4]–[21].

In this letter we propose an approach to modify a group of ring artifact suppression methods based on the core results obtained in [7]. These methods described in [2], [8], [10], [11] are implemented as routine procedures in a reconstruction software used at imaging beamlines I12 and I13 of Diamond Light Source [22], [23], have also applied to data from beamline 2-BM of Advanced Photon Source, Argonne National Laboratory and have already proved their quality and robustness. The aim of the new algorithm is to increase algorithms' performance by reducing amount of data required to be used during ring artifact suppression and combining them with other filters built in standard reconstruction algorithms.

II. ORIGINAL METHOD

The idea behind the results from [7] is the following. If there are no artifacts, then two neighboring pixels of a sensor should have similar values. In case of a regular ring artifact it can be described as a constant but unknown value added to the ideal value of the optical path. This unknown value is the same for all projections. As the result the original problem can be written as a problem to minimize a so-called Tikonov functional [24]

$$M^\alpha = \frac{1}{2} \sum_{k=1}^{n-1} (p_k - p_{k+1})^2 + \frac{\alpha}{2} \sum_{k=1}^n (r_k - p_k)^2, \quad (2)$$

where r_k is the average recorded optical path for the k -th pixel, p_k is the unknown optical path, $\alpha > 0$ is a regularization parameter and should in principle depend on level of noise in input data. The first term in (2) means p_k should not vary much from r_k , the second term implies restrictions on neighboring values of $p \equiv \{p_k\}_1^n$. For a given α a solution of (2) exists, is unique and can be found by solving the system of n linear equations

$$\frac{\partial M^\alpha}{\partial z_k} = \sum_{l=1}^n A_{kl} p_l - r_k = 0, \quad (3)$$

where matrix A is three-diagonal and defined as

$$\begin{pmatrix} 1+\alpha & -1 & 0 & \dots & 0 & 0 \\ -1 & 2+\alpha & -1 & \dots & 0 & 0 \\ 0 & -1 & 2+\alpha & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & -1 & 2+\alpha & -1 \\ 0 & 0 & 0 & 0 & -1 & 1+\alpha \end{pmatrix}. \quad (4)$$

In [7] the inverse matrix A^{-1} is found, so the vector p can be found from the vector $r \equiv \{r_k\}_1^n$. However for practical reasons it is better to have a filter $G \equiv \{G_k\}_1^n$ such that the solution p is just the convolution $G * r$. Therefore instead of $(n \times n)$ -matrix A^{-1} we need to store n -vector G and the convolution can be performed by Fast Fourier Transform algorithms.

III. FILTER

We intend to find the value of the central element of vector p , therefore we suppose n to be an odd number. Let $(n \times n)$ -matrix B_n be defined as

$$B_n = \begin{pmatrix} 2\sigma - 1 & -1 & 0 & \dots & 0 & 0 \\ -1 & 2\sigma & -1 & \dots & 0 & 0 \\ 0 & -1 & 2\sigma & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 2\sigma & -1 \\ 0 & 0 & 0 & \dots & -1 & 2\sigma - 1 \end{pmatrix}. \quad (5)$$

Then $A = B_n$ if $\sigma = \alpha/2 + 1$. Let us replace the element $(1, 1)$ of B_n by χ and denote the corresponding matrix as $F_n(\chi)$, $F_n(2\sigma - 1) = B_n$. We use the first row of $F_n(\chi)$ to find its determinant, then $\det F_n(\chi) = \chi \det F_{n-1}(2\sigma) - \det F_{n-2}(2\sigma)$. Considering cases of $\chi = 2\sigma$ and $\chi = 2\sigma - 1$ we get $\det B_n = \det F_n(2\sigma) - \det F_{n-1}(2\sigma)$ and $\det F_n(2\sigma) + \det F_{n-2}(2\sigma) = 2\sigma \det F_{n-1}(2\sigma)$, so $\det B_n + \det B_{n-2} = 2\sigma \det B_{n-1}$. Therefore we need to find a sequence $\{b_n\}$ such that

$$b_n + b_{n-2} = 2\sigma b_{n-1} \quad (6)$$

and $b_n = \det B_n$. Note that matrices B_n are formally defined for $n \geq 3$. One can find their determinants explicitly for $n = 3$ and 4, so $b_3 = 8\sigma^3 - 8\sigma^2 - 2\sigma + 2$, $b_4 = 16\sigma^4 - 16\sigma^3 - 8\sigma^2 + 8\sigma$ and using (6) we extrapolate values b_n for $n < 3$: $b_2 = 4\sigma^2 - 4\sigma$, $b_1 = 2\sigma - 2$, $b_0 = 0$.

If $\sigma \in [-1, 1]$ and we set $b_n = \tau \sin(n\omega + \psi)$ and $\sigma = \cos \omega$, then (6) is met. Taking into account b_0 and b_1 we get $\psi = 0$ and

$$\tau = \frac{2\sigma - 2}{\sin \omega} = \frac{2(\cos \omega - 1)}{\sin \omega}, \quad (7)$$

$$\det B_n = 2(\cos \omega - 1) \frac{\sin n\omega}{\sin \omega}. \quad (8)$$

Matrix B_n is an $(n \times n)$ -matrix, so $\det B_n$ can not have more than n roots. Below we show that there are n roots for $\sigma \in [-1, 1]$, thus there is no need to consider cases $|\sigma| > 1$.

The first root can be found from $\cos \omega = 1$. The other roots can be found from $\sin(n\omega) = 0$, so $n \arccos \sigma = k\pi$ and $\sigma = \cos(k\pi/n)$, $k = 0, \dots, n-1$. All eigenvalues of matrix B_n can be written as $\lambda_k = 2(\sigma + \rho_k)$, where $\rho_k = \cos(k\pi/n)$, $k = 0, \dots, n-1$.

Denote $\theta = k\pi/n$, then for a given eigenvalue $\lambda = 2(\sigma + \rho_k)$ the corresponding eigenvector y_q , $q = 1, \dots, n$:

$$y_q = (-1)^{q-1} \left[1 + 2 \sum_{s=1}^{q-1} \cos(s\theta) \right]. \quad (9)$$

In [25] the following formula is given

$$\sum_{s=0}^{N-1} \cos(a + sb) = \cos\left(a + \frac{N-1}{2}b\right) \frac{\sin Nb/2}{\sin b/2}, \quad (10)$$

thus if set $a = 0$, then

$$y_q = (-1)^{q-1} \frac{\sin(2q-1)\theta/2}{\sin \theta/2}. \quad (11)$$

Now we normalize vector y and therefore rewrite $\sin^2 \frac{2q-1}{2}\theta$ as $(1 - \cos(2q-1)\theta)/2$. Substituting $a = \theta$ and $b = 2\theta$ into (10) we find

$$\sum_{q=1}^n \cos(2q-1)\theta = \sum_{s=0}^{n-1} \cos(\theta + 2s\theta) = \frac{\sin 2n\theta}{2 \sin \theta}, \quad (12)$$

$$\sum_{q=1}^n \sin^2 \frac{2q-1}{2}\theta = \frac{1}{4} \left(2n - \frac{\sin 2n\theta}{\sin \theta} \right). \quad (13)$$

Since $\theta = k\pi/n$, then $\sin 2n\theta = \sin 2\pi k = 0$ for any integer k . Note that $\sin \theta \neq 0$ for $k = 1, \dots, n-1$ and $\sin \theta = 0$ for $k = n$. In the last case we get $y_q = 1$, so its normalization is trivial. Finally, we get the normalized vector y with elements

$$y_q = \begin{cases} \frac{(-1)^{q-1}}{\sqrt{n/2}} \sin \frac{(2q-1)k\pi}{2n}, & k = 1, \dots, n-1, \\ \frac{1}{\sqrt{n}}, & k = n. \end{cases} \quad (14)$$

The original matrix $B = B_n$ from (5) can be written as

$$B = YDY^T, \quad (15)$$

where Y is an orthogonal matrix, i.e. $Y^{-1} = Y^T$, with elements

$$Y_{ij} = \begin{cases} \frac{(-1)^{i-1}}{\sqrt{n/2}} \sin \frac{(2i-1)j\pi}{2n}, & j = 1, \dots, n-1, \\ \frac{1}{\sqrt{n}}, & j = n, \end{cases} \quad (16)$$

D is a diagonal matrix with elements $D_{ii} = 2(\sigma + \cos(i\pi/n))$, see [26].

The inverse of matrix B_n can then be written as

$$B^{-1} = YD^{-1}Y^T, \quad (17)$$

where D^{-1} is a diagonal matrix with elements $D_{ii}^{-1} = 1/[2(\sigma + \cos(i\pi/n))]$. Therefore

$$D^{-1}Y^T|_{kj} = \sum_{q=1}^n D_{kq}^{-1}Y_{qj}^T = D_{kk}^{-1}Y_{kj}^T = D_{kk}^{-1}Y_{jk} \quad (18)$$

and the elements

$$B_{ij}^{-1} = \sum_{k=1}^n Y_{ik} D_{kk}^{-1} Y_{jk} \quad (19)$$

of B^{-1} can be written as

$$B_{ij}^{-1} = \frac{1}{n} \left[\sum_{k=1}^{n-1} \frac{(-1)^{i+j}}{\sigma + \cos \frac{k\pi}{n}} \sin \frac{(2i-1)k\pi}{2n} \sin \frac{(2j-1)k\pi}{2n} + \frac{1}{2(\sigma-1)} \right].$$

Taking in to account that

$$\sin \frac{(2i-1)(2n-k)\pi}{2n} = \sin \frac{(2i-1)k\pi}{2n} \quad (20)$$

and $\cos \frac{n\pi}{n} = -1$ we get

$$B_{ij}^{-1} = \frac{1}{2n} \sum_{k=1}^{2n-1} \frac{(-1)^{i+j}}{\sigma + \cos \frac{k\pi}{n}} \sin \frac{(2i-1)k\pi}{2n} \sin \frac{(2j-1)k\pi}{2n}. \quad (21)$$

All matrices mentioned above have odd width/height and indexes starting from 1. Our aim is to find a filter, so it is more convenient to rewrite matrices/vectors as ones centered with respect to the central element. If a standard matrix Q

is given, then by $\check{Q}$ we denote the centred one. So instead of matrix B_{ij}^{-1} defined for $i = 1, \dots, n, j = 1, \dots, n$ we subtract n from k and $(\check{n} + 1)$, where $\check{n} = (n-1)/2$, from the other indexes of the above formula and get

$$\begin{aligned} \check{B}_{ij}^{-1} &= \frac{1}{2n} \sum_{k=-(n-1)}^{n-1} \frac{(-1)^{i+j}}{\sigma + \cos \frac{(k+n)\pi}{n}} \\ &\times \sin \frac{(2(i+\check{n})+1)(k+n)\pi}{2n} \sin \frac{(2(j+\check{n})+1)(k+n)\pi}{2n}, \\ &= \frac{1}{4n} \sum_{k=-(n-1)}^{n-1} \frac{\cos \frac{(i-j)k\pi}{n} + \cos \frac{(i+j+n)k\pi}{n}}{\sigma - \cos \frac{k\pi}{n}}. \end{aligned}$$

For the central row of $\check{B}^{-1}$ we get

$$\check{B}_{0j}^{-1} = \frac{1}{4n} \sum_{k=-(n-1)}^{n-1} \frac{\cos \frac{jk\pi}{n} (1 + (-1)^k)}{\sigma - \cos \frac{k\pi}{n}}. \quad (22)$$

For odd k the value of $(1 + (-1)^k) = 0$, so we need to consider only even k and use $s = k/2$:

$$\check{B}_{0j}^{-1} = \frac{1}{2n} \sum_{s=-\check{n}}^{\check{n}} \frac{\cos \frac{2js\pi}{n}}{\sigma - \cos \frac{2s\pi}{n}}. \quad (23)$$

For large values of n we may replace the sum by the integral

$$\frac{1}{2n} \int_{-n/2}^{n/2} \frac{\cos \frac{2js\pi}{n}}{\sigma - \cos \frac{2s\pi}{n}} ds. \quad (24)$$

Taking into account $\sigma = \alpha/2 + 1$ we get values of filter $\check{G}$

$$\check{G}_j = \frac{2}{(2+\alpha)\pi} \int_0^\pi \frac{\cos j\xi}{1 - \frac{2}{2+\alpha} \cos \xi} d\xi. \quad (25)$$

Let us denote

$$\begin{aligned} \sin \phi &= \frac{2}{2+\alpha}, \quad \cos \phi = \frac{\sqrt{\alpha(4+\alpha)}}{2+\alpha}, \\ I_m &\equiv \int_0^\pi \frac{\cos mx}{1 - \cos x \sin \phi} dx, \end{aligned} \quad (26)$$

$\phi \in (0, \frac{\pi}{2})$ since $\alpha > 0$. Using the formula $\cos mx + \cos(m-2)x = 2 \cos x \cos(m-1)x$ we obtain for $m > 1$

$$\begin{aligned} I_m + I_{m-2} &= \int_0^\pi \frac{2 \cos x \cos(m-1)x}{1 - \cos x \sin \phi} dx \\ &= \frac{2}{\sin \phi} \int_0^\pi \left(\frac{1}{1 - \cos x \sin \phi} - 1 \right) \cos(m-1)x dx \\ &= \frac{2}{\sin \phi} I_{m-1}, \end{aligned}$$

since $\int_0^\pi \cos(m-1)x dx = 0$. So we get the recurrent formula

$$I_m = \frac{2}{\sin \phi} I_{m-1} - I_{m-2}. \quad (27)$$

We may explicitly find that

$$I_0 = \frac{\pi}{\cos \phi}, \quad I_1 = \frac{\pi}{\cos \phi} \frac{\sin \phi}{1 + \cos \phi} \quad (28)$$

and check that I_m is a geometric sequence, i.e. $I_{m+1} = \gamma I_m$, where $\gamma = \sin \phi / (1 + \cos \phi)$. So the values of filter $\check{G}$ can be written as

$$\check{G}_j = \frac{2}{\sqrt{\alpha(4+\alpha)}} \left(\frac{2+\alpha - \sqrt{\alpha(4+\alpha)}}{2} \right)^{|j|}. \quad (29)$$

Fig. 3. A horizontal slice of the graphite after the standard flat-field correction procedure is applied.

Fig. 4. The same slice reconstructed after the ring artifact suppression method is applied, $\alpha = 500$.

It is easy to check that $\gamma \in (0, 1)$, the sequence is a decreasing one, i.e. $\tilde{G}_{|j+1|} < \tilde{G}_{|j|}$, and $\sum_{j=-\infty}^{\infty} \tilde{G}_j = 1$.

Above the classical approach used in the theory of inverse and ill-posed problems is used. In this case values of regularization parameter α belong to the positive half-line, i.e. $\alpha \in (0, +\infty)$. It may be more convenient to normalize the range. Let us introduce parameter $\beta = 1 - \hat{G}_0$, then

$$\alpha = \frac{2(\sqrt{(1-\beta)^2 + 1} - (1-\beta))}{1-\beta}, \quad (30)$$

$$\tilde{G}_j = (1-\beta) \left(\frac{\sqrt{(1-\beta)^2 + 1} - 1}{1-\beta} \right)^{|j|}. \quad (31)$$

So $\beta \in [0, 1)$, $\beta = 0$ corresponds to the case of no filtering, i.e. $\tilde{G}_0 = 1$ and $\tilde{G}_j = 0$ for $j \neq 0$. The value of β cannot reach 1 but in case of approaching to it the filter tends to be the mean filter.

IV. DISCUSSION

The formula (29) proposed in this letter is applied to suppress ring artifacts for a graphite sample. The data was acquired at beamline I13 of Diamond Light Source, UK [23]. There were 4001 x-ray projections acquired, each projection was a 16-bit 2560×2160 image. As the beamline provides users with almost parallel geometry of the x-ray beam, reconstruction of one horizontal section of a volume requires only one sinogram, i.e. one data from one row of a sensor. For a given slice the standard flat-field correction procedure defined in (1) is applied. In Figure 3 one can see that ring artifacts have regular structure, so their strengths are almost the same along half-circles. The reconstruction based on the filter (29) for $\alpha = 500$ is performed, see Figure 4.

Of course, the choice of parameter α should depend on level on noise/strength of ring artifacts. The smaller parameter α is, the closer the slice is to the one after the standard flat-field correction, i.e. to Figure 3. If the value of α is too high, then additional wave structures may appear on the image. See examples in [27]. For ring artifacts dependant on angular position of a specimen the method presented in [11] can be used.

Standard fast reconstruction algorithms like filtered back-projection (FBP) or Feldkamp–Davis–Kress (FDK) for parallel and cone geometry of the x-ray beam consist of two steps: filtering and backprojection [28]. The algorithm presented in this letter is designed to be performed before reconstruction. Having it written in the form of filtering operation allows us to combine preprocessing and reconstruction steps by using the modified filter for FBP and FDK which can be defined as the convolution of the ring artefact filter with the standard one.

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