Time Varying Pessimism, Rare Disasters and Stock Returns

Hening Liu*
University of Manchester
April 2013

Abstract
Recent financial literature has advocated the risk of rare disasters to explain the equity premium puzzle (Mehra and Prescott (1985)). This paper presents a model with rare consumption disasters, state uncertainty and uncertainty aversion to explain a broad set of stylized facts about stock returns including (1) high equity premium, (2) volatile returns, (3) time-varying equity premia and volatility, (4) predictability of returns, and (5) high variance risk premium. Even with a constant and extremely small ex-ante probability of a consumption disaster, uncertainty aversion substantially amplifies disaster probabilities perceived by investors and generates time-varying pessimism and risk of rare disasters. The two effects together drive high and countercyclical price of risk.

JEL Classification: D81; G11; G12.

Keywords: Equity premium puzzle, rare disasters, uncertainty aversion, volatility.

*Accounting and Finance Group, Manchester Business School, University of Manchester, Booth Street West, Manchester M15 6PB, UK. e-mail: Hening.Liu@mbs.ac.uk.
1 Introduction

Historical equity premium of the U.S. stock market in excess of Treasury bill rates has been one of the major puzzling stylized facts in financial economics. [Mehra and Prescott (1985)] show that a standard rational representative-agent model has great difficulty in reconciling small fluctuations in consumption growth and high equity premium, both observed in the data, assuming plausible levels of risk aversion. Other prominent features of the stock market that are difficult to be explained by the standard model include (1) highly volatile equity returns relative to fundamentals, (2) countercyclical variation and persistent changes in return volatility, (3) countercyclical equity premia and predictability of excess returns, and (4) high variance risk premium (defined as the difference between the risk neutral and physical expectations of stock return variance).

One explanation for high equity premium proposed by [Rietz (1988)] relies on the risk of rare disasters. Investors require a sufficiently high risk premium to compensate for the adverse impact of rare disasters on fundamentals. By calibrating disaster probabilities to international data on severe historical contractions, [Barro (2006)] finds that disasters are infrequent by nature, and in particular, the disaster probability is about 1.7 percent per year on average. Recently, [Barro et al. (2012)] use a long data set on consumption for both OECD and non-OECD countries and find that the average disaster probability is over 3 percent per year from a Bayesian perspective. [Barro (2006)] and [Barro et al. (2012)] show that their models with disaster risk can explain the high equity premium and low risk-free rate in the data. [Wacther (2013)] extends the model of [Barro (2006)] by allowing for time-varying disaster probabilities and recursive preferences. The calibrated model can further account for high return volatility and predictability of excess returns.

In this paper, rather than having time-varying disaster probabilities, I assume that the ex-ante probability of a disaster is constant, as in [Barro (2006)], and that this probability is extremely small, capturing the spirit of “rare” disasters. The assumption implies that no disastrous event ever realized during the sampling period to the economy. This hypothesis is also maintained in the calibration below. I follow [Veronesi (2004)] and assume that investors cannot observe

---

the state (either normal or disastrous) of the economy but can only learn about it from the past data on fundamentals (consumption and dividends). The key component of the model is a new class of preferences, known as “generalized recursive smooth ambiguity preferences” (Ju and Miao (2012)). Most importantly, this utility function distinguishes risk aversion from uncertainty (ambiguity) aversion toward state or model uncertainty. In particular, investors are not only averse toward pure risk, which is represented by a distribution of consumption growth conditional on either states, but also toward state uncertainty, characterized by investors’ fluctuating beliefs. These two attitudes, however, are tied together and both governed by the risk aversion coefficient in a rational expectation model with either power utility or recursive utility (Epstein and Zin (1989)). The generalized recursive smooth ambiguity utility function also allows for separation between risk aversion and the intertemporal elasticity of substitution (IES) and therefore incorporates recursive utility as a special case when investors are uncertainty neutral.

This paper uses the original model of Ju and Miao (2012) to assess quantitatively the impact of rare disasters on stock returns\footnote{Ju and Miao (2012) examine asset returns in an endowment economy based on an estimated regime-switching model of consumption growth.}. The key insight is that a small degree of uncertainty aversion can significantly amplify the impact of state uncertainty about a rare disaster on stock returns, even though the ex-ante disaster probability is tiny and constant. State uncertainty and uncertainty aversion both are necessary for the model to match well the data. Since investors do not observe the current state, their uncertainty about the disastrous state fluctuates over time. However, due to that average consumption growth rates are drastically different within the two regimes and that the ex-ante probability of entering a disaster is extremely small, investors’ beliefs only fluctuate near the ex-ante disaster probability by small amounts. For example, a series of negative consumption shocks during normal times may increase the perceived probability of a disaster, but not by a large enough amount to reach states with high uncertainty (e.g., where the perceived probability is close to 0.5). Thus, in a rational expectation framework, investors’ concern about the occurrence of a rare disaster is not sufficiently high to generate high equity premium. Neither is the amount of uncertainty substantial enough to match the volatility of returns in the data.

On the other hand, investors endowed with smooth ambiguity preferences display aversion toward distribution of consumption growth in the disastrous state. Specifically, investors dislike
any mean-preserving spread of expected continuation value induced by the posterior beliefs about
the unobservable state. This uncertainty aversion induces time-varying pessimism, which greatly
magnifies the disaster probability perceived by investors and also makes their beliefs much more
responsive to consumption shocks. This mechanism suitably describes investors’ climbing concern
that the economy may fall into a long and deep recession during the recent crises since 2008.

This paper examines the impact of the amplification mechanism driven by uncertainty aversion
on stock returns under the assumption that the disastrous state never realized during the sampling
period. The model with uncertainty aversion can generate (1) high equity premium, (2) excess
volatility of returns, (3) high variance risk premium, (4) countercyclical variation of risk premium,
equity volatility and variance premium, and (5) the predictability of excess returns and the
absence of predictability of consumption growth by the price-dividend ratio. However, the model
lacking uncertainty aversion could barely produce any of these important results.

This paper is closely related to several recent papers addressing the impact of rare disasters
on the aggregate stock market.³ Waechter (2013) studies the potential of time-varying disaster
probabilities combined with recursive preferences to explain many features of the stock market.
The main finding is that time variation in disaster risk is important to generate excess volatility.
Indeed, this paper shows how the magnitude and time variation in disaster probabilities assumed
by Waechter (2013) can arise endogenously from uncertainty aversion and learning about the
unobservable state. Gabaix (2012) examines time-varying disaster probabilities and time-varying
dividends shocks in response to a disaster in a power utility setting. Using the technique of
linearity-generating processes, Gabaix obtains closed-form solutions, and the calibrated model
can explain a rich set of empirical regularities of stock and bond returns. Gourio (2012) explores
the impact of time-varying disaster probabilities on macroeconomic quantities and asset returns.
Barro (2009) proposes a model with a constant disaster probability and recursive preferences.
In a subsequent work, Barro et al. (2012) estimate a model with disasters followed by recoveries
and study the impact on equity premium. But these two models lack a mechanism to generate
excess volatility. It is worth mentioning that in the calibration of all these paper the average
probability of a disaster is about 3–4 percent per year, which is an order of magnitude greater

³ This paper also relates to a literature on ambiguity and asset returns. See, for example, Chen and Epstein (2002),
Maenhout (2004), Leippold et al. (2008), Ju and Miao (2012), Hansen and Sargent (2010), Collard et al. (2011)
for endowment economy models, and Jahan-Parvar and Liu (2013) for a production economy model.
than the value assumed in the calibration below. In this paper, the assumption of a tiny disaster probability intends to highlight the role of uncertainty aversion in increasing the perceived disaster probability. To this end, this paper is similar to Veronesi (2004), where the model uses exponential utility and does not take into account uncertainty aversion. Veronesi shows that learning about the disastrous state can generate high risk premia, excess volatility and asymmetric volatility reaction to good and bad news. However, the implied equity premium is still rather low compared to the data.

The rest of this paper is organized as follows. Section 2 briefly describes the model of Ju and Miao (2012). Section 3 calibrates the model and discusses quantitative results from Monte Carlo simulations. Section 4 concludes. The numerical algorithm is similar to that used by Ju and Miao (2012), which is modified and adapted to the calibration of this paper.

2 The Model

I consider a representative-agent pure exchange economy model similar to Ju and Miao (2012). Aggregate consumption follows the process

\[
\ln \left( \frac{C_{t+1}}{C_t} \right) = \theta + \sigma_c \epsilon_{t+1}
\]

where \(\epsilon_t\) is an i.i.d. standard normal random variable. During the sample period \([1, 2, ..., T]\), normal times persist and mean consumption growth is assumed to be a constant \(\theta_t = \bar{\theta}\). To capture the spirit of rare disaster, I assume that there is an infinitesimal ex-ante chance that mean consumption growth switches to a disastrous state \(\theta \ll \bar{\theta}\). I further assume that \(\theta\) is not directly observable and thus in each period the agent is uncertain about whether a regime shift has occurred or not.

I assume that in each period there is probability \(p\) that the economy enters in a disaster, and there is probability \(q\) that the economy recovers back to the normal state, and that \(q \gg p\). Since the current state is unobservable, the agent must derive beliefs from historical consumption growth data. Denote \(\mu_t = \Pr(\theta_{t+1} = \bar{\theta}|\Omega_t)\) to be the agent’s belief that a disaster may happen in the next period, where \(\Omega_t\) represents the history of consumption data. Suppose that the prior

\[^4\] See “Ambiguity, Learning, And Asset Returns”: Technical Appendix. I thank Nengjiu Ju for making the Fortran code available.
belief $\mu_0$ is known. The agent updates his beliefs according to Bayes rule as

$$
\mu_{t+1} = \frac{p f_{\theta} (1 - \mu_t) + q f_{\bar{\theta}} \mu_t}{f_{\bar{\theta}} (1 - \mu_t) + f_{\theta} \mu_t}
$$

(2)

where $f_{\theta}$ is the normal density function of consumption growth described in (1).

It is common in the literature to model dividends and consumption separately. Here, I follow Ju and Miao (2012) and specify aggregate dividends to follow

$$
\ln \left( \frac{D_{t+1}}{D_t} \right) = \lambda \ln \left( \frac{C_{t+1}}{C_t} \right) + g_d + \sigma_d e_{t+1}
$$

where $\lambda$ is the leverage ratio parameter, $e_t$ is an i.i.d. standard normal random variable that is independent of $\epsilon_t$, and the parameters $g_d$ and $\sigma_d$ can be calibrated given the dividends data. For the purpose of asset pricing, I focus on the risk free asset that pays a unit of consumption good in the next period and the equity that has a claim on aggregate dividends.

The agent’s preferences are represented by the generalized recursive smooth ambiguity utility function

$$
V_t(C) = \left[ (1 - \beta) C_t^{1 - \psi} + \beta \{ R_t (V_{t+1} (C)) \}^{1 - \psi} \right]^{\frac{1}{1 - \psi}}
$$

(3)

$$
R_t (V_{t+1} (C)) = \left( E_{\mu_t} \left[ \left( E_{\theta,t} \left[ V_{t+1}^{1 - \gamma} (C) \right] \right)^{\frac{1 - \eta}{1 - \gamma}} \right] \right)^{\frac{1}{1 - \eta}}
$$

(4)

where $\beta \in (0, 1)$ is the subjective discount factor, $\psi$ is the IES parameter, $\gamma$ is the coefficient of relative risk aversion, and $\eta$ is the uncertainty aversion parameter and must satisfy $\eta \geq \gamma$. Equation (4) characterizes the certainty equivalent of future continuation value, which is the key ingredient that distinguishes this utility function from Epstein-Zin recursive utility. In Equation (4), the expectation $E_{\theta,t} [\cdot]$ is taken over the distribution of consumption conditioning on $\theta = \bar{\theta}$ or $\theta$, and the expectation $E_{\mu_t}$ is over filtered probabilities about the disastrous state.

A key property of smooth ambiguity preferences is that the model achieves separation between uncertainty and uncertainty aversion, where the latter characterizes the agent’s attitude toward state uncertainty in this paper. In particular, uncertainty aversion prescribed by $\eta > \gamma$ precludes the compound reduction between the agent’s subjective beliefs, which are generated from Bayesian updating, and the two possible distributions of consumption growth conditioning

See Klibanoff et al. (2005, 2009) for a thorough discussion on smooth ambiguity preferences.
on either the normal or disastrous state. Thus, smooth ambiguity utility implies that the agent displays different attitudes toward subjective uncertainty and consumption risk. This is achieved by endogenously imputing additional pessimism toward states with low continuation value.

**Stock returns**

Under this utility function, the stochastic discount factor (SDF) is given by

\[
M_{\theta,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-1/\psi} \left( \frac{V_{t+1}}{R_t (V_{t+1})} \right)^{1/\psi-\gamma} \left( \frac{\mathbb{E}_{\theta,t} \left[ V_{t+1}^{1-\gamma} \right]}{R_t (V_{t+1})} \right)^{1/\psi} \right)^{-(\eta-\gamma)}
\]

where \( \theta = \bar{\theta} \) or \( \bar{\theta} \) depending on the state of the economy. The price of risk crucially depends on the volatility of the SDF. More precisely, the price of risk is defined as the ratio between the standard deviation of \( M_{\theta,t+1} \) and its expectation, that is, \( \sigma_t (M_{\theta,t+1}) / \mathbb{E}_t (M_{\theta,t+1}) \). Stock returns, \( R_{e,t+1} \), are defined by

\[
R_{e,t+1} = \frac{P_{e,t+1} + D_{t+1}}{P_{e,t}} = 1 + \varphi (\mu_{t+1}) \frac{D_{t+1}}{D_t}
\]

where \( \varphi (\mu_t) \) denotes the price-dividend ratio, and stock returns satisfy the Euler equation

\[
\mathbb{E}_t [M_{\theta,t+1} R_{e,t+1}] = 1
\]

The risk-free rate, \( R_{f,t} \), is the reciprocal of the expectation of the SDF:

\[
R_{f,t} = \frac{1}{\mathbb{E}_t [M_{\theta,t+1}]}
\]

Uncertainty aversion can significantly alter how the agent perceives the probability of a disaster. To see this, I rewrite the Euler equation as

\[
0 = (1 - \bar{\mu}_t) \mathbb{E}_{\bar{\theta},t} \left[ M_{\bar{\theta},t+1}^{E} (R_{e,t+1} - R_{f,t}) \right] + \bar{\mu}_t \mathbb{E}_{\theta,t} \left[ M_{\theta,t+1}^{E} (R_{e,t+1} - R_{f,t}) \right]
\]

where \( M_{\theta,t+1}^{E} \) can be interpreted to be the SDF for recursive utility and is given by

\[
M_{\theta,t+1}^{E} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-1/\psi} \left( \frac{V_{t+1}}{R_t (V_{t+1})} \right)^{1/\psi-\gamma}
\]
It follows from (6) and (7) that the probability \( \tilde{\mu}_t \) satisfies
\[
\tilde{\mu}_t = \frac{\mu_t \left( \mathbb{E}_{\tilde{\theta},t} \left[ V_{t+1}^{1-\gamma} \right] \right)^{-\frac{\eta-\gamma}{1-\gamma}}}{\mu_t \left( \mathbb{E}_{\tilde{\theta},t} \left[ V_{t+1}^{1-\gamma} \right] \right)^{-\frac{\eta-\gamma}{1-\gamma}} + (1 - \mu_t) \left( \mathbb{E}_{\tilde{\theta},t} \left[ V_{t+1}^{1-\gamma} \right] \right)^{-\frac{\eta-\gamma}{1-\gamma}}}.
\]
(8)

Note that \( \mathbb{E}_{\tilde{\theta},t} [\cdot] \) (\( \mathbb{E}_{\tilde{\theta},t} [\cdot] \)) is period-\( t \) conditional expectation given that the state is \( \tilde{\theta} \) (\( \tilde{\theta} \)). The probability \( \tilde{\mu}_t \) can be interpreted as the distorted belief about the disastrous state, which is not equivalent to the Bayesian posterior belief when \( \eta > \gamma \). Furthermore, the distortion induced by uncertainty aversion is also time varying and depends on conditional expectation of continuation value. Since \( \theta = \tilde{\theta} \) (\( \tilde{\theta} \)) represents the normal (disastrous) state, it is straightforward to see
\[
\left( \mathbb{E}_{\tilde{\theta},t} \left[ V_{t+1}^{1-\gamma} \right] \right)^{-\frac{\eta}{1-\gamma}} > \left( \mathbb{E}_{\theta,t} \left[ V_{t+1}^{1-\gamma} \right] \right)^{-\frac{\eta}{1-\gamma}}
\]
and thus \( \tilde{\mu}_t < \mu_t \) when \( \eta > \gamma \). It can be seen that uncertainty aversion introduces time varying pessimism by magnifying the perceived disaster probability. Figure [I] shows the Bayesian belief \( \mu_t \) and uncertainty-aversion-distorted belief \( \tilde{\mu}_t \) simulated from the model, using historical consumption data and parameter values in the calibration below. Although the Bayesian disaster probability remains at a low level, the uncertainty-averse agent perceives the disaster probability to rise notably during the two recessions in mid-1970s and early-1980s and substantially higher during the recent crises. Thus, the uncertainty-aversion-distorted belief can precisely depicts investors’ recent concern about state uncertainty.

**Variance risk premium**

Variance (risk) premium is defined as the difference between the risk-neutral and objective expectations of stock return variance for a given horizon, that is
\[
VP_t = \mathbb{E}_t^Q \left[ \sigma_{e,t+1}^2 \right] - \mathbb{E}_t \left[ \sigma_{e,t+1}^2 \right]
\]
where the horizon is assumed to be one month and \( \sigma_{e,t+1}^2 \) is variance of log returns. The Chicago Board Options Exchange (CBOE) uses a “model-free” approach proposed by [Carr and Wu (2009)] to calculate the VIX as the risk-neutral expectation of the aggregate stock market variance. Since the VIX data are reported in annualized “vol” terms, it is often convenient to convert the index to the monthly quantity \( VIX^2/12 \). The physical expectation of variance can be calculated based
on measures of realized variance for a given month. Drechsler (2013) uses high frequency returns data to construct realized variance measures, and a projection method to estimate conditional variance forecasts that serve as an empirical proxy for the physical expectation of variance. Drechsler (2013) obtains time series estimates of variance premium and reports a mean of 10.55 and a standard deviation of 8.47. These statistics, however, are difficult to match in standard consumption-based asset pricing models as they are unable to produce high and volatile price of risk.

In the model, variance premium is given by

\[ VP_t = E_t \left[ \frac{M_{\theta,t+1}}{E_t(M_{\theta,t+1})} \sigma_{e,t+1}^2 \right] - E_t(\sigma_{e,t+1}^2) \]

where \( \frac{M_{\theta,t+1}}{E_t(M_{\theta,t+1})} \) characterizes the risk-neutral measure transformation, and \( \sigma_{e,t}^2 \) is conditional variance of log returns, i.e., \( \sigma_{e,t}^2 \equiv \text{Var}_t(\ln R_{e,t+1}) \).

### 3 Calibration

In this section, I calibrate the model to assess quantitatively the impact of uncertainty aversion on the perceived disaster probability and asset returns. Due to nonlinearities, the model does not admit an approximate analytical solution. Similar to Ju and Miao (2012), I solve the model using the projection method and run Monte Carlo simulations to compute moments of returns. Quadrature method is used to compute expectation over a normal distribution. Since the state variable \( \mu_t \) takes on mostly very small values, I construct an equally spaced grid with 200 points on \( \log(\mu_t) | \mu_t \in [0,1] \). Thus, the resulting grid on \( \mu_t \) is more concentrated at its lower values.

I follow Drechsler (2013) and Wachter (2013) and use the post-war data for the period 1947–2010 to calibrate the model. All nominal quantities are converted into real terms using the CPI. The model is calibrated at an annual frequency. Post-war data are ideal for the purpose of calibration because the period corresponds to no rare disasters having ever occurred, which is consistent with the assumption maintained in the model.

Table 1 reports parameter values used in the benchmark calibration. The standard deviation of consumption growth, \( \sigma_c \), is 0.022. The average consumption growth in normal times, \( \bar{\theta} \), is 0.02.

---

6 The quantitative results in this paper are based on 20,000 simulations.
The leverage ratio is set at 2.6, and the standard deviation of dividend growth at 0.13. These parameter values are in line with the literature, see Drechsler (2013), Wachter (2013) and Bansal and Yaron (2004), among others. Turning to parameters pertaining to disasters, I set the ex-ante probability of a consumption disaster at an extremely small value, 0.003. This value is even an order of magnitude less than the average disaster probability in Barro (2009) and Wachter (2013), among others, and implies that on average a disaster will occur every 333 years and is indeed “rare”. The mean consumption growth in the disastrous state is set at \( \theta = -0.06 \), close to that in Veronesi (2004) but moderate compared to the distribution of consumption declines found by Barro and Ursua (2008) and Barro et al. (2012). The probability of recovery, \( q \), is set at \( q = 0.165 \), according to the estimate provided by Barro et al. (2012). These parameter values imply that the unconditional probability to be in the normal state is around 0.98 and the unconditional expected consumption growth is 0.02. The risk aversion coefficient is set at a low value, \( \gamma = 2 \). Following the long-run risk literature (Bansal and Yaron (2004)), the IES parameter is assumed to greater than 1, and \( \psi = 1.5 \). As argued by Bansal and Yaron (2004), \( \psi < 1 \) leads to the counterfactual implication that the price-dividend ratio increases in face of higher uncertainty. Finally, the subjective discount factor \( \beta \) and uncertainty aversion parameter \( \eta \) are set to simultaneously match the mean risk-free rate and equity premium in the data. This calibration procedure results in \( \eta = 5.3 \), which is far below the value considered by other papers such as Ju and Miao (2012), Chen et al. (2011) and Collard et al. (2011). These papers use the approach of thought experiments to gauge the uncertainty aversion parameter. The Appendix of this paper explains the approach in detail.

**Quantitative results**

Table 2 reports calibration results for the benchmark model with learning and uncertainty aversion, recursive utility model with learning, and the full information model in which the state of the economy is fully observable and the agent has recursive utility. The full information model implies an unreasonably high risk-free rate, extremely low equity premium, absence of excess volatility and a variance premium close to zero, under the specified parameterization. Both the risk-free rate and variance premium have zero volatility because of the maintained assumption

---

7 In the calibration of Ju and Miao (2012), the value of \( \eta \) is about 9, while in Chen et al. (2011), the value of \( \eta \) is considered to be between 60 and 100.
that normal times always persist during the sampling period. In addition, the model with learning generates similar results. The implied equity premium, equity volatility and variance premium are marginally higher than those in the full information model. These findings reveal that only adding state uncertainty into the model is not adequate enough to produce reasonable asset pricing implications. The main reason is that without uncertainty aversion, the implied price of risk is rather low in both models ($\sigma(M)/\mathbb{E}(M) = 0.08)$.

On the other hand, the benchmark model with uncertainty aversion represents a success. The model can match well a wide array of empirical moments. The mean risk-free rate is low as in the data while the implied equity premium is over 7% per year, even with a low risk aversion. Due to an IES greater than 1, the risk-free rate volatility can be kept low. The model can also generate excessive volatility, with the volatility of excess returns being about 18%, far greater than the dividends volatility (13%) and the return volatility (14%) implied in the learning and full information models. Moreover, the model reproduces the magnitude and variation of variance premium in the data. The mean and standard deviation of variance premium in the benchmark model are, respectively, 14.12 and 7.85, close to the empirical moments reported by Drechsler (2013).

Table 3 reports long-horizon predictive regression results using the data simulated from the benchmark model (in the column “Benchmark”) and recursive utility model with learning (in the column “Epstein-Zin”). The results reported include regression slopes and the $R^2$s for different horizons (1, 2, 4 and 6 years). I study the long-run predictability of excess returns and consumption growth by the price-dividend ratio. In the asset pricing literature, it is challenging for the long-run risks model to generate predictability of excess returns without generating predictability of consumption growth (Beeler and Campbell (2012)). The simulation results below show that the present model can resolve this challenge because disasters never realized during the sampling period.

To avoid the small sample bias, simulation results are generated from 20,000 experiments, and the results in Table 3 show the average over all simulations. Panel A presents regression results when excess returns are used as the dependent variable. The regression slope is negative in both models as in the data, suggesting that high stock prices relative to dividends predict low future excess returns. Although the data feature increasing $R^2$s in the horizon, the benchmark
calibration implies decreasing $R^2$s, a finding similar to \cite{Ju and Miao 2012}. However, the predictability of excess returns becomes much stronger in the benchmark model than in the recursive utility model, and this highlights the role of uncertainty aversion in driving predictability. For instance, the $R^2$ of 1-year horizon regression is 0.155 in the benchmark model while only 0.019 in the recursive utility model. Panel B reports the results of regressing long horizon consumption growth on the price-dividend ratio in data simulated from the benchmark model and in historical data. The benchmark model does not generate long-run predictability of consumption growth, consistent with the data.

**Time varying pessimism, risk premia and volatilities**

The key to understand these results is to note the large and countercyclical variation in the price of risk driven by uncertainty aversion. When the agent has power or recursive utility, the price of risk is countercyclical but both its magnitude and response to consumption shocks are rather small. By imputing more pessimism, uncertainty aversion substantially increases countercyclical variation in the SDF as well as the overall volatility of the SDF, which enables the model to match equity premium, equity volatility and variance premium. In Figure 4, Panel A shows conditional equity premium for different values of the state disaster probability $\mu_t$, ranging from 0 to 0.02. In Monte Carlo simulations, $\mu_t$ stays close to the ex-ante probability 0.003 most of the time, with a few exceptions when consumption shocks occur and $\mu_t$ rises significantly. In the recursive utility model, conditional equity premium remains low due to small state uncertainty. That is, the agent still has strong confidence in his beliefs about the state of the economy since large consumption declines are indeed rare. On the other hand, uncertainty aversion greatly magnifies the impact of state uncertainty on equity premium. For $\mu_t$ close to 0.02, the implied equity premium can reach 40%.

Panel B of Figure 4 compares the conditional price of risk $\sigma_t (M_{\theta,t+1}) / E_t (M_{\theta,t+1})$ as a function of $\mu_t$ for the benchmark model and the recursive utility model with learning. In the benchmark calibration, the intercept of $\sigma_t (M_{\theta,t+1}) / E_t (M_{\theta,t+1})$ when $\mu_t$ is close to zero is much higher under uncertainty aversion. More importantly, when $\mu_t$ increases following a sequence of consumption shocks, the conditional price of risk rises dramatically in the benchmark model while only mod-

\cite{Jahan-Parvar and Liu 2013} show that in a production economy with uncertainty aversion the $R^2$s from regressing excess returns on price-dividend ratios are increasing in the horizon.
estly in the recursive utility model. Thus, Figure 4 highlights an amplification mechanism: a small degree of uncertainty aversion can greatly enlarge changes in the price of risk in response to bad news, and this contributes much to high risk premium.

To see the mechanism more clearly, let us focus on the impact of uncertainty aversion on the agent’s beliefs about the disastrous state. Panel A of Figure 4 displays simulated disaster probabilities perceived by the uncertainty-averse agent (Equation (8)) in comparison with the Bayesian probabilities (Equation (2)). Since no disasters ever occurred during the simulation period, the Bayesian probability is extremely low and exhibits negligible variation around the ex-ante disaster probability. This also explains the failure of the recursive utility model to reproduce high market price of risk and equity premium. Uncertainty aversion alters the scenario in two important ways. First, it effectively raises the perceived disaster probability and hence makes disaster risk more important for asset pricing, even though the ex-ante disaster probability is tiny. In particular, the uncertainty-aversion-distorted probability \( \tilde{\mu}_t \) stays around 0.03 most often during the simulation period. This level is close to that in the calibration exercises of Barro (2009) and Wachter (2013). Second, uncertainty aversion substantially increases countercyclical variation of the perceived disaster probability. Loosely speaking, this paper indeed shows how time-varying disaster risk postulated by Wachter (2013) can arise endogenously from learning and uncertainty aversion. The countercyclical variation in \( \tilde{\mu}_t \) is the key to imply other important results. In bad times, consumption shocks increase the perceived chance that a rare disaster may happen. The enlarged concern about state uncertainty reduces the price-dividend ratio and increases the volatility of returns. The heightened concern about volatility risk further implies rising variance premium. Thus, the model generates procyclical price-dividend ratios (Panel B of Figure 4), countercyclical volatilities (Panel C) and variance premia (Panel D).

**Term structure of equities**

Zero-coupon equity pays a dividend at the specific point in time. In an empirical study, Binsbergen et al. (2011) measure the prices of dividend strips to study the term structure of equity risk premium. They find that short-term dividend claims (up to three years) have a risk premium that is significantly different from zero and also higher than that of long-term dividends. These

---

9 Dividend strips are dividend payments at each point in time. Each strip has a specific time to maturity. The value of the aggregate stock market equals the sum of discounted future dividend payments.
empirical results are at odds with the implications of several well-known consumption-based asset pricing models such as Campbell and Cochrane (1999) and Bansal and Yaron (2004). In these models, short-term risk premium is close to zero, and thus high equity premium of the aggregate stock market is indeed driven by long-term dividends. In addition, the variable rare disaster model of Gabaix (2012) predicts that the term structure of equity premium is flat. Here, I investigate zero-coupon equities in the present model with uncertainty aversion.

Formally, let $F_n(\mu_t)D_t$ be the price of zero-coupon equity that matures in $n$ periods and therefore $F_n(\mu_t)$ the corresponding price-dividend ratio, that is,

$$F_n(\mu_t) = E_t \left[ \left( \prod_{j=1}^{n} M_{\theta,t+j} \right) \frac{D_{t+n}}{D_t} \right]$$

where $M_{\theta,t+j}$ is $j$–period SDF. Since this asset is a zero-coupon equity, its one-period return is given by

$$R_{n,t+1} = \frac{F_{n-1}(\mu_{t+1})D_{t+1}}{F_n(\mu_t)D_t}$$

The Euler equation implies the following recursion

$$F_n(\mu_t) = E_t \left[ M_{\theta,t+1} \frac{D_{t+n}}{D_t} F_{n-1}(\mu_{t+1}) \right]$$

This recursion can be used to solve for the term structure of risk premium, volatility and Sharpe ratio for zero-coupon equities with different maturities, given the initial condition $F_0(\mu_t) = 1$.

I simulate data from the benchmark model and compute for each zero-coupon equity with a maturity of $N$ years ($1 \leq N \leq 40$) the average excess return (risk premium), the volatility, and the Sharpe ratio. The same exercise is repeated for the recursive utility model with learning. The results are plotted in Figure 4 using solid lines for the benchmark model and dash-dot lines for the recursive utility model. In a comparison with the benchmark model, the recursive utility model implies almost flat term structure, and the short-term risk premium and Sharpe ratio are close to zero. For the benchmark model, although the term structure of risk premium is upward sloping, the short-term dividends claim still carries significantly positive risk premium. The 1-, 2- and 3-year dividends claims have risk premiums of, respectively, 2.13%, 2.93% and 3.75% per year. In addition, since the long-term dividends claims have notably higher volatilities, their
Sharpe ratios exhibit a decreasing trend, giving rise to the humped shape of the term structure of the Sharpe ratio.

4 Conclusion

This paper has shown that uncertainty aversion, characterized by recursive smooth ambiguity preferences, can reinforce investors’ concern about the occurrence of a rare disaster. Assuming an extremely small ex-ante disaster probability and no disasters ever realized during the sampling period, I show that a low degree of uncertainty aversion can greatly amplify the disaster probability perceived by investors and introduce large countercyclical variation in the perceived disaster probability. With a low value of risk aversion, time-varying pessimism induced by uncertainty aversion is able to explain many salient features of the aggregate stock market including high equity premium, low risk-free rate, excess volatility of returns, high variance premium, countercyclical equity premia and volatilities, and predictability of excess returns and absence of predictability of consumption growth.
Appendix

In this Appendix, I follow [Ju and Miao (2012), Chen et al. (2011) and Collard et al. (2011)] and use thought experiments to illustrate the value of the uncertainty aversion parameter postulated in the benchmark calibration. This approach defines ambiguity premium in the classic case of Ellsberg Paradox and relies on the ambiguity premium to gauge a plausible range for the uncertainty (ambiguity) aversion parameter. Suppose there are two urns, which contain black and white balls mixed together. Decision makers are told that one urn has 50 white and 50 black balls, and that the second urn has 100 balls, either white or black. The exact composition of the second urn is unknown to decision makers. They are about to place a bet on the color of the ball drawn from each urn. The bet could be on either black or white. Decision makers win a prize worth $d$ dollars if a bet on a specific urn is correct, and nothing otherwise. Halevy (2007) reports that most of decision makers prefer a bet on the first urn over the second urn.

The standard expected utility framework fails to explain this behavior, regardless of decision makers’ risk aversion or beliefs, since the certainty equivalents of a bet are identical for the two urns as long as decision makers have expected utility. However, if decision makers have smooth ambiguity preferences, there exists a difference between the certainty equivalents over the two urns, giving rise to the ambiguity premium. As in [Ju and Miao (2012)], the ambiguity premium is formally defined as

$$u^{-1}\left(\int_{\Theta} \int_S u(c) d\theta d\zeta(\theta)\right) - v^{-1}\left(\int_{\Theta} v\left(u^{-1}\left(\int_S u(c) d\theta\right)\right) d\zeta(\theta)\right)$$

where $u$ and $v$ have the following functional forms

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \gamma > 0, \neq 1$$
$$v(x) = \frac{x^{1-\eta}}{1-\eta}, \eta > 0, \neq 1$$

and the set of probability distributions for the bet on the second urn is specified to be (0,1) and (1,0), that is $\Theta = \{(0,1), (1,0)\}$, and the subjective prior is $\zeta = (0.5, 0.5)$. Let $w$ be the initial wealth. The ambiguity premium is defined as

$$\left(0.5(d + w)^{1-\gamma} + 0.5w^{1-\gamma}\right)\frac{1}{1-\gamma} - \left(0.5(d + w)^{1-\eta} + 0.5w^{1-\eta}\right)\frac{1}{1-\eta}.$$
for $\eta > \gamma$. The ambiguity premium depends on the size of the bet and the prize-wealth ratio $d/w$. Table A1 reports the ambiguity premium, expressed as a percentage of the expected value of the bet $d/2$, for different values of $\eta$ and the prize-wealth ratio $d/w$ ($d/w = 1\%$: Panel A, $d/w = 0.75\%$: Panel B, and $d/w = 0.5\%$: Panel C). The level of risk aversion is set at 2 throughout the calculation. Since it is common to use small bets in experimental studies, $d$ is small relative to $w$. In the light of evidence provided by Camerer (1999) and Halevy (2007), the ambiguity premium is typically about 10-20 percent of the expected value of a bet. Table A1 reveals that the implied ambiguity premium for $\eta = 5.3$ is only between 0.41 and 0.82 percent of the expected value of a bet. Thus, the level of uncertainty aversion in the benchmark calibration of this paper is extremely small, compared to other papers such as Ju and Miao (2012), Chen et al. (2011) and Collard et al. (2011).
Table A1: Ambiguity premia for alternative choices of $\eta$

<table>
<thead>
<tr>
<th>$\gamma \backslash \eta$</th>
<th>3</th>
<th>5.3</th>
<th>8</th>
<th>15</th>
<th>30</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Prize-wealth ratio $(d/w)=1%$</td>
<td>2</td>
<td>0.25</td>
<td>0.82</td>
<td>1.49</td>
<td>3.23</td>
<td>6.94</td>
</tr>
<tr>
<td>Panel B: Prize-wealth ratio $(d/w)=0.75%$</td>
<td>2</td>
<td>0.19</td>
<td>0.62</td>
<td>1.12</td>
<td>2.43</td>
<td>5.22</td>
</tr>
<tr>
<td>Panel C: Prize-wealth ratio $(d/w)=0.50%$</td>
<td>2</td>
<td>0.12</td>
<td>0.41</td>
<td>0.75</td>
<td>1.62</td>
<td>3.49</td>
</tr>
</tbody>
</table>

This table reports ambiguity premium, expressed as a percentage of the expected value of the bet $d/2$, for various values of $\eta$. The risk aversion parameter $\gamma$ is set at 2.
Table 1: **Parameter Values**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>Relative risk aversion</td>
<td>2</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Uncertainty aversion</td>
<td>5.3</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Intertemporal elasticity of substitution</td>
<td>1.5</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Time discount rate</td>
<td>0.982</td>
</tr>
<tr>
<td>$\bar{\theta}$</td>
<td>Mean consumption growth</td>
<td>0.02</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Standard deviation of consumption growth</td>
<td>0.022</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Mean consumption growth (disaster state)</td>
<td>-0.06</td>
</tr>
<tr>
<td>$p$</td>
<td>Probability of entering into a disaster</td>
<td>0.003</td>
</tr>
<tr>
<td>$q$</td>
<td>Probability of recovery</td>
<td>0.165</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Leverage</td>
<td>2.6</td>
</tr>
</tbody>
</table>
This table reports calibration results of the benchmark model in comparison with Epstein-Zin’s recursive utility model. Parameter values of the benchmark model are given in Table 1. The parameterization of the recursive utility model with learning or full information is the same as the benchmark model except that the uncertainty aversion parameter $\eta$ is set at $\eta = 2$ to imply uncertainty neutrality. The full information model is abstracted from learning. All models are simulated at an annual frequency. Empirical moments are calculated using the CRSP data from 1947 to 2010 for the U.S. stock market except for $\mathbb{E}(VRP)$ and $\sigma(VRP)$, which are taken from Drechsler (2013). The moments $\mathbb{E}(R_f)$, $\sigma(R_f)$, $\mathbb{E}(R_e - R_f)$ and $\sigma(R_e - R_f)$ are expressed in percentage terms. The mean and standard deviation of variance premium, $\mathbb{E}(VRP)$ and $\sigma(VRP)$, are rescaled to a monthly frequency by multiplying $10^4/12$.  

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Benchmark</th>
<th>Learning</th>
<th>Full info.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{E}(R_f)$ (%)</td>
<td>1.20</td>
<td>1.14</td>
<td>3.06</td>
<td>3.07</td>
</tr>
<tr>
<td>$\sigma(R_f)$ (%)</td>
<td>1.40</td>
<td>1.32</td>
<td>0.16</td>
<td>0</td>
</tr>
<tr>
<td>$\mathbb{E}(R_e - R_f)$ (%)</td>
<td>7.16</td>
<td>7.10</td>
<td>0.51</td>
<td>0.49</td>
</tr>
<tr>
<td>$\sigma(R_e - R_f)$ (%)</td>
<td>17.48</td>
<td>18.00</td>
<td>14.19</td>
<td>14.01</td>
</tr>
<tr>
<td>$\sigma(M)/\mathbb{E}(M)$</td>
<td>n.a.</td>
<td>1.53</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>$\mathbb{E}(VRP)$</td>
<td>10.55</td>
<td>14.12</td>
<td>0.65</td>
<td>0.42</td>
</tr>
<tr>
<td>$\sigma(VRP)$</td>
<td>8.47</td>
<td>7.85</td>
<td>0.94</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Table 3: **Predictability results**

<table>
<thead>
<tr>
<th>Horizon in years</th>
<th>Data</th>
<th>Benchmark</th>
<th>Epstein-Zin</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Slope</td>
<td>$R^2$</td>
<td>Slope</td>
</tr>
<tr>
<td>Panel A: Excess returns</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-0.130</td>
<td>0.090</td>
<td>-1.216</td>
</tr>
<tr>
<td>2</td>
<td>-0.230</td>
<td>0.170</td>
<td>-1.292</td>
</tr>
<tr>
<td>4</td>
<td>-0.330</td>
<td>0.230</td>
<td>-1.304</td>
</tr>
<tr>
<td>6</td>
<td>-0.480</td>
<td>0.300</td>
<td>-1.301</td>
</tr>
<tr>
<td>Panel B: Consumption growth</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-0.0010</td>
<td>0.0006</td>
<td>-0.0038</td>
</tr>
<tr>
<td>2</td>
<td>-0.0060</td>
<td>0.0137</td>
<td>-0.0055</td>
</tr>
<tr>
<td>4</td>
<td>-0.0090</td>
<td>0.0164</td>
<td>-0.0085</td>
</tr>
<tr>
<td>6</td>
<td>-0.0110</td>
<td>0.0180</td>
<td>-0.0108</td>
</tr>
</tbody>
</table>

This table reports predictive regression results for the benchmark model and Epstein-Zin’s recursive utility model with learning. Each model is simulated 20,000 times and each simulation contains 200 data in annual frequency. The results reported for the two models are computed by taking the average of estimates over all of the simulated samples. Regression slopes and the $R^2$s reported for the data (the second and third column) are taken from **Wachter (2013)**. In Panel A and B, respectively, log excess returns and log consumption growth at different horizons are regressed on the lagged (log) price-dividend ratio.
Figure 1: Simulated beliefs about the disastrous state. The grey bars indicate NBER recession periods. The “circle” line plots Bayesian posterior disaster probability ($\mu_t$). The “diamond” line plots uncertainty-aversion-distorted disaster probability ($\tilde{\mu}_t$) implied by the benchmark model (see Equation (8)).
Figure 2: Conditional equity premium and price of risk: benchmark model and recursive utility model.
Figure 3: Benchmark model simulation: beliefs, the price-dividend ratio, conditional equity volatility and conditional variance premium.
Figure 4: Term structure of risk premium, volatility and Sharpe ratio: benchmark model and recursive utility model.
References


