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Simulation of the Effectiveness of Dynamic Cooling for Controlling Residual Stresses in Friction Stir Welds

D.G. Richards¹,ᵃ, P.B. Prangnell¹,ᵇ, P.J. Withers¹,ᶜ S.W. Williams²,ᵈ, S. Morgan³,ᵉ

¹Manchester Materials Science Centre, Manchester University, Grosvenor St. Manchester, M1 7HS, UK.
²Cranfield University, Welding Engineering Research Centre, Bedfordshire, MK43 OAL, UK.
³BAE SYSTEMS, Optics and Laser Technology Department, Advanced Technology Centre, P.O. Box 5, FPC 267, Bristol, BS12 7QW, UK.

ᵃdavid.g.richards@postgrad.manchester.ac.uk, ⁣ᵇphilip.prangnell@manchester.ac.uk*, ⁣ᶜphilip.withers@manchester.ac.uk, ⁣ᵈs.williams@cranfield.ac.uk, ⁣ᵉstephen.morgan@baesystems.com

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ABSTRACT
An FE model has been used to study the effect of localised dynamic cooling on the residual stresses developed during friction stir welding. The main aim of the work was to see if the cooling power and source positions required, to achieve significant residual stress reductions in friction stir welds, were compatible with the FSW process and recent developments in CO₂ cooling systems. Comparisons were made between welds produced with a single cold spot placed over the weld line, either ahead or behind the tool, or with cold spots both leading and trailing the tool. Simulations revealed that a large reduction in the residual stresses can be obtained, particularly at the weld line, but the benefits are very dependent on the size and positioning of the cooling sinks. Cold spots placed behind the heat source have the greatest effect in reducing the build up of trailing tensile stresses, by counteracting the compressive plastic misfit of the hot soft weld metal. Overall, it is shown that the application of local dynamic cooling has potential as a practical solution for controlling residual stresses in industry.

1.0 INTRODUCTION
Friction stir welding (FSW) is being used increasingly in a wide range of applications, particularly for joining aluminium alloys. However, although FSW is a solid state welding method, welds can still suffer from significant levels of residual stress, which are often similar in magnitude to those seen in fusion welds [1-3]. The residual stresses arise from plastic misfit strains introduced as a result of the steep gradients in temperature that are generated local to the heat source as the tool advances [4]. A large number of techniques have been proposed for controlling residual stresses in welding, including both thermal and mechanical tensioning methods (e.g. [5-7]). Thermal tensioning can involve heating or cooling the plate either globally [8] or dynamically, with the sources/sinks moving with the welding head. This latter approach is easier to apply in practice and is known as Dynamically Controlled Low Stress no Distortion Welding (DC-LSND) [9-11]. In the case of DC-LSND the aim is to introduce further local thermal fields that counteract the plastic misfit strains and subsequent residual stresses formed by the welding heat source. Although this can be achieved by heating, for thermally sensitive materials like high strength Al-alloys it is more sensible to use cooling sinks. Various cooling medium can be used including water, liquid nitrogen and semi-solid (snow) or liquid CO₂. In this context FSW has a considerable advantage over fusion processes in that the coolant can interfere with welding arcs and shielding/plasma gasses associated with other heat source. In previous work on arc welding [10] and laser welding [11] low conductivity stainless steels notable improvements have been found using this approach. Van der Aa, et al [12] has shown that with the cooling sink placed behind the heat source it should be as intense, as small, and as close to the melt pool as possible to obtain the greatest improvement. With more conductive alloys like aluminium the benefits are less certain. Furthermore, little has previously been published on the use of such techniques with FSW processes, nor have there been any attempts to model DC-LSND when applied to friction stir welding. However, in a preliminary experiment by Williams, et al on a 2024 FSWs it has been shown that the application of liquid CO₂ cooling behind the tool can give a significant reduction in residual stress at the weld line, with the peak stresses at the edge of the TMAZ also being reduced [11,13].

*Corresponding author.
In previous work we have used a simplified finite element modelling approach to predict the residual stress development in friction stir welds and study the effect of global mechanical tensioning on how the welding stresses develop and can be controlled [14,15]. This model was compared to x-ray synchrotron and neutron diffraction data showing good agreement with experimental results. Here we have used this same model to analyse the DC-LSND technique for mitigating residual stress development in FSWs, with a view to understanding how the interactions between the welding heat source and local cooling affect the development of the residual stress field. A key aim of the work was to simulate the effect of the main variables involved in order to ‘intelligently’ optimise the process and see if it is a viable method with aluminium FSWs, given the physical limitations on the placement of the coolant system and the high thermal diffusivity of aluminium. Following this analysis, more experimental data from a parallel experimental programme will be published at a future date, which will trial the technique in earnest.

2.0 METHODOLOGY

In order to examine the effectiveness of dynamic thermal tensioning two approaches were used. Firstly, to check the validity of the approach, the FE model was used to simulate the only currently available residual stress measurements made on FSWs where a liquid CO$_2$ spray/snow had been applied. Secondly, experiments were conducted to calibrate the cooling power of an advanced cooling nozzle developed by BAE Systems for DC-LSND welding. This cooling system was then included in a previously developed and validated model for residual stress prediction in friction stir welding, to investigate the effect of; cooling source placement, size, and the relative effectiveness of dual and single nozzles.

**FE modelling**

The FE model used to simulate the FSW process has been described in detail elsewhere [14,15]. It is comprised of two sequentially linked 3D simulations – a calibrated heat transfer model to imitate the weld thermal cycle, followed by a stress model in which the thermal cycle and boundary constraints are applied. In the model the welding process was represented purely as a heat source, with the metal flow and mechanical effects of the tool neglected. As a result, the findings have relevance to a wide range of welding processes. The extensive material flow that occurs in friction stir welding means that it is not practical to develop a fully coupled thermo-plastic model for this analysis. Furthermore, previous work suggests that the residual stresses are primarily a function of the thermal excursion and that the mechanical deformation of the tool has relatively little effect besides that relating to the localised heat it generates [1,15]. A kinetically dependent material softening model was also incorporated to capture the yield stress behaviour at a given node during welding. This aspect of the model is very important for obtaining reliable results and simulates the effect of material softening due to heating and microstructural changes that occur during the welding thermal cycle, caused by effects such as overageing in the TMAZ and HAZ. The level of material softening is particularly important as it is one of the main factors that controls the retained residual stresses after relaxation. To simulate the welds used in the above BAE experiments the only change required to the model was to re-calibrate the heat source for the new welding conditions and to calibrate and apply the cooling sinks.

**Heat Source Representation**

The heat source in the FE model was represented by a circular surface source for the shoulder and a cylindrical volume source for the pin using the actual tool dimensions. Based on work of Chao, et al [16,17], the distribution of the heat flux under the tool shoulder, $q_s$, was assumed to vary linearly with angular velocity, given by:

$$q_s(r) = \frac{3 Q_s r}{(2\pi) (R_s^3 - R_p^3)}$$

for $R_p \leq r \leq R_s$  \hspace{1cm} (1)

Where $R_s$ and $R_p$ are the dimensions of the shoulder and pin and $r$ is the radial distance. The heat input from the pin was assumed to be uniform with the pin depth. As in the original model [15], the power for the tool heat source was first estimated by using a computational fluid dynamics (CFD) model, developed by Colegrove and Shercliff [18-21], which is capable of predicting the heat input from the tool geometry and welding conditions. Results obtained from the FE model were then compared to measured thermocouple results with minor alterations made until a good fit was obtained.

**CO$_2$ Cooling Nozzle**

The cooling system replicated in the simulations was originally developed by BAE Systems and uses a semisolid CO$_2$ snow that is applied via a spray to the plate surface (Figure 1). Cooling occurs by heat...
conduction across the plate interface into the cold snow and due to the latent heat of sublimation. Application of the coolant is constrained within a circular inner 25 mm nozzle, which is surrounded by a concentric sleeve used to extract the gas generated for safety reasons and to avoid interference with other welding processes (e.g. welding arcs). The sleeve has a rubber seal with the plate. Early experimental trails have given an indicative cooling power of ~ -800 W for this nozzle. The cooling source was applied to the FE model in the same way as the heat source, but in this case it was represented by a simple circular surface with a uniform convective loss, \(q_c\) [22]:

\[
q_c = h_c(T - T_{min})
\]  

Where \(T_{min}\) was set to -72°C for simplicity (the boiling temperature for CO\(_2\)) and \(h_c\) is the interfacial heat transfer coefficient. It should be noted that these values will depend on the amount of gas evolved which can provide an insulation layer at the interface.

![Diagram](image)

**Figure 1.** The self extracting cryogenic cooling system used in the DC-LSND simulations; (a) image of the nozzle with a screen to shield the arc for fusion welding applications and (b) a schematic diagram.

Calibration of the cooling nozzle was achieved by experiments carried out on a cylindrical AA2024-T3 test billet of dimensions 150mm \(\phi\) x 25mm thick with a 50 mm \(\phi\) x 3.2mm thin central web. The billet was preheated to different temperatures in a furnace after which the cooling nozzle was directed at the centre of the top surface. The cooling rate was measured using thermocouples positioned in the billet. The experimental set up was then simulated using a separate FE model, to obtain the best fit for the interfacial heat transfer coefficient. An example comparison of measured and simulated results is shown in Figure 2. The best fit value found for \(h_c\) was \(~2000\) W m\(^{-2}\) K\(^{-1}\), which is similar to the maximum values obtained by Van der Aa using a simple unconstrained liquid CO\(_2\) jet [12].

![Graph](image)

**Figure 2.** Measured and simulated results for cooling applied to a preheated AA2024-T3 test billet, in (a) the thermal profile across the surface of the billet after 50 seconds and (b) the cooling rate at the centre of the top surface. Measured data is shown as points with model results given by continuous lines.

Experiments with traversing cold sinks were also undertaken with two CO\(_2\) sprays placed in tandem along the weld line 90mm apart (centre to centre) applied to a plate using an 85AMAL jet and traversed at 195mm/min (Figure 3(a)). Thermocouples placed at various distances from the weld line were used to measure the thermal profile. This arrangement was then simulated with the FE Model using the value of \(2000\) W m\(^{-2}\) K\(^{-1}\) for \(h_c\) found above. The results are compared in Figure 3(b) where the simulated thermal profile shows a similar response to the measured results using the calibrated power.
Although not perfect, the method described above for representing the cooling nozzle was deemed realistic enough to be used to explore the effectiveness of dynamic local cooling on residual stress development as a function of the process variables. Furthermore, van der Aa has found little difference between the application of a more sophisticated distributed cooling sink compared to a simple uniform sink on the residual stresses predicted [23].

Validation

The only available residual stress data for DC-LSND on aluminium FSWs was obtained as a result of welding trials undertaken by BAE Systems [11,13]. In these original experiments the liquid CO\textsubscript{2} was simply sprayed onto the plate through two pipes positioned close to the rear of the tool. This approach, although crude, provided sufficient cooling to the plate to show measurable differences in the subsequent results (see Figure 4). The welding arrangements comprised two 380mm x 125mm x 6.3 mm plates of 2024 alloy in the T3 condition that were butt welded with and without local cooling. The welding conditions were 355 rpm and 95.2 mm/min. Residual stress measurements were made at the Geesthacht Neutron Faculty. The FE model was used to attempt to reproduce these results, using a tool power predicted from the CFD model developed by Colegrove [18], by placing a cold spot 40 mm behind the tool. The results are given in Figure 4 and in the case of the conventional weld produced without active cooling a typical ‘M’ shaped residual stress profile, widely observed in FSWs [1,24] and explained previously [15], can be seen. Given the number of unknowns, surprisingly good agreement was found between the measured and simulated results, despite the simple representation of the cooling source. It can be noted that with dynamic cooling there is a large reduction in the longitudinal residual stresses at the weld line (~ 60% within the nugget), but a more modest effect on the peak stresses within the HAZ (~15%).

3.0 RESULTS AND DISCUSSION

For the investigation described here, the same previously validated geometry and welding conditions used to investigate global mechanical tensioning [15] were adopted. The welding conditions were 770 rpm and 195 mm/min, with a 14 mm diameter tool shoulder and plate geometry of 350mm x 125mm x 3.2mm, in the same AA2024-T3 material. The cryogenic nozzle was applied in various locations along the weld line for each test, either singularly in front, or behind, the weld tool, or in dual application - both leading and trailing the weld tool. Centre to centre distances between the tool and round cooling nozzles are quoted below.
The effect of position and distance of the cold spots

Figure 5a shows the weld line thermal profiles produced with the 25 mm diameter nozzle at a distance of 20 mm behind the tool. It can clearly be seen that the trailing cold spot greatly reduces the material temperature behind the weld and has a dramatic effect on the residual stress development in the still hot and soft material emerging from behind the weld tool. For this condition, (Figure 5b) the original centre line longitudinal tensile stresses of ~ 200MPa have been completely reversed and replaced by a compressive stress of -50MPa. Further out on the edge of the HAZ the effect is lessened as the material here is stronger ahead of where the cooling is applied, due to its lower temperature (see below). However, there has still been a reduction of approximately 60% in the maximum stresses. In reality such a close placement of a cooling nozzle would be impossible to achieve unless the cooling system was integrated into the design of the welding head. Nevertheless, it demonstrates the maximum theoretical improvement that could be achieved via a local cryogenic cooling system.

In Figure 5(b) further stress profiles are shown for 40 and 75 mm centre to centre distances from the tool and the data is summarised in Figure 6, where the weld line and peak longitudinal residual stresses are plotted as a function of nozzle distance trailing the tool. From these results it can clearly be seen that the placement is extremely important. In general the reduction in residual stresses is still most significant at the weld line, but falls off very rapidly with distance. For example, with a more realistically achievable nozzle position at a stand off distance of 60 mm the maximum residual stresses are reduced by 30% and the weld line stresses by ~ 40%. For distances greater than ~ 140 mm there is no improvement.

In Figure 7 a comparison is made with a larger 100 mm diameter cold spot placed 75mm behind the tool. The results for this condition are better than those for a 25mm cold spot placed at the same distance, but this is mainly because the edge of the larger cold spot was nearer to the weld tool, and...
therefore had a greater effect (see below). In fact this larger spot is slightly less good than a smaller spot placed at the same edge to edge distance form the tool, which is equivalent to a 25 mm nozzle at a centre to centre distance of 38 mm. It should be noted that the behaviour of a cold spot has been claimed to change depending on whether it is intense enough to split the rear of the thermal field generated by the tool [23].

In Figure 8a residual stress profiles are shown from simulations with the cold spots placed ahead of the weld tool at 20mm and 40mm distances along the weld line. From these modelling results it is evident that this has minimal effect on the residual stresses, which are only marginally reduced, particularly if the more realistic distance of 40 mm is used. This small stress reduction is matched by a slight narrowing of the profiles and caused by the pre-cooling of the plate which reduces the thermal footprint (see below). Furthermore, the tests using dual cold spots were found to produce results very close to those for a single trailing cold spot placed at the same distance (Figure 8b), with the addition of a similar slight reduction in width of the stress profiles caused by the leading cooling nozzle being the only difference between the two data sets.

**Figure 8.** Comparison of predicted longitudinal stress profiles, at the mid thickness halfway along welded AA2024-T3 sheets, in (a) showing the effect of a 25 mm diameter cooling nozzle placed ahead of the tool at a distance of 20 mm (line and squares) and 40 mm’s (line and triangles) and in (b) the effect of combining leading and trailing cold spots, both at a distance of 20 mm (line and squares) and 40 mm (line and triangles) form the tool. (In (b) data for a single trailing spot at 40 mm distance is shown as a dashed line for comparison.)

**The effect of local thermal tensioning**

The above results have shown that the application of trailing cold spots, placed as close to the tool as possible, can result in a significant reduction in the weld residual stresses. Similar observations have also been found in studies of dynamic cryogenic cooling in GTA welding [12]. The application of dual cold spots can marginally increase this effect, while a leading single cold spot on its own has a very small influence on the residual stress distribution. In all cases the reduction in residual stresses was reduced as the distance between the cold sink and the weld tool was increased.

The reasons for this behaviour are investigated in Figures 9 to 12. It should be noted that the situation is extremely complex because of the overlapping thermal fields, steep thermal gradients, and the very important influence of the material flow stress, which is extremely dependent on temperature and by softening at high temperatures causes a larger plastic misfit as well as dictating the level of stress that the material can support. In common with all welding processes involving a travelling heat source, the expanding hot material under the tool results in a compressive bow wave (Figures 9a & 10a). Because of the low flow stress at high temperature, this misfit is plastically relaxed under the tool reducing the stresses to ~ 0. As a result, on cooling a large negative misfit develops due to thermal contraction, which generates tensile residual stresses in the strengthening weld metal that build up at the weld line behind the tool as the yield stress increases and the temperature falls [4]. These stresses are balanced by far field compressive stresses near the plate edges, which result in the buckling of thin sheets [25, 26].

It should also be noted that due to the forward motion of the heat source the thermal gradient is very steep ahead of the tool and shallower in its wake. Therefore, if a cold sink is placed ahead of the tool it has to be extremely close to have any impact on the shape of the thermal field (Figure 10a). For example, only a slight change in the distribution of the isotherms is noted for a distance as close as 20 mm, where the nozzle is touching the tool and none for 40 mm in Figure 10a. However, the tensile field generated by the cold sink might still be expected to counteract the heat sources’ compressive bow wave. Placing the cold spot ahead of the tool also pre-cools the plate and slightly reduces the subsequent weld temperature. The effect of this is equivalent to reducing the tool power, which moderately reduces the width of the HAZ and moves the peak stresses slightly inwards. Both of these
effects can be seen in the simulations, with the compressive bow wave being concentrated somewhat closer to the tool, when the cold sink is present (Figure 10b), and reaching slightly higher peak stress levels due to the fact that the colder metal can support larger stresses (Figure 9b). Overall, however, the benefits to the final residual stress distribution are small because the plastic misfit and stress field emerging from behind the tool are not greatly affected (as seen in Figure 8a).

Just behind the tool longitudinal tensile stresses begin to generate as the weld material begins to contract as it cools. Initially stress development near to the weld line is limited by the very low tensile yield stress of the hot metal. Greater local tensile plastic relaxation in the hotter softer material at the weld line results in the formation of the ‘M’ shaped residual stress profiles typically observed in measured results (see Figure 2a), as the wider region previously plastically deformed in compression ahead of the tool now becomes tensilely stressed as it cools. Cold spots applied in this region cause a sudden quenching of the weld material. This has several effects. Firstly, a small intense cold sink penetrates the original heat source thermal field modifying its shape from an ellipse to a crescent (Figure 11a). This results from the material on the weld line being rapidly cooled, which generates a large tensile misfit and reverses the thermal path; i.e. from the point after the cold sink the weld material is warming up rather than cooling down and therefore undergoes expansion, although the temperature rise is modest (Figure 5a). In comparison the thermal path of the material either side of the cold spot is more similar to in the original weld, although the cooling rate is increased. Evidence of this effect can also be seen in Figure 9c, where under the cold spot the harder weld metal carries a substantial tensile stress. The local thermal contraction from the cold spot counteracts the compressive plastic misfit developing from the hot material emerging from behind the tool resulting in a smaller plastic misfit and reducing the weld line residual stresses.

When the cold sink is very close to the heat source a tensile misfit is actually created in the material on the weld line, which then becomes compressive in the final residual stress distribution when the material warms back up after welding. Further out from the weld line the material level with the cold spot is now actually slightly hotter and thus softer, and will have a compressive misfit relative to the weld line. However, in this position the strain development is mainly elastic and the main reason the residual stresses are reduced less either side of the weld line is similar to the reason for the development of the original M shaped profile, in that when the tensile misfit from the cold spot is applied to the material plastically contracted due to the compressive bow wave it is already too cold (and hard) to relax as greatly as at the weld line. This is also why the cold spot has to be as close as possible to the tool, i.e. it works best when the tensile misfit it generates can act on the material under the heat source that has a near zero flow stress.

Fig. 9. Development of the longitudinal (lines), transverse (lines and squares), and Von Mises (dashed line) stresses with distance along the weld line relative to the tool centre, predicted from the model, in (a) a standard weld with no cooling, in (b) a weld with cooling applied ahead of the tool at 40mm distance, in (c), a weld with the cold spot placed at the same distance behind the tool, and (d) for a much larger diameter 100mm trailing cold spot at 75 mm behind the tool.
Figure 10. Example thermal (a) and longitudinal stress (b) distributions form the mid sheet plane for 25 mm cold sinks placed at different distances ahead of the welding tool and for dual cold spots.

Figure 11. Example of predicted 2D thermal (a) and longitudinal stress (b) distributions form the mid plane of a welded sheet for cold spots placed at different distances behind the welding tool.
With local dynamic cooling the final residual stress state after welding is a result of the net local misfit strains generated from the interaction of the heating and cooling sources during the weld thermal cycle, which with ideal positioning and intensity are partially balanced. In comparison the previously studied technique of global mechanical tensioning [5,15] uses brute force to impose a far field background stress state upon the material during welding, and opposes the stress development in each phase of the thermal cycle by counteracting the compressive bow wave and over tensioning the material behind the heat source. The different behaviours of these two stress engineering methods are compared in Figure 12. Although the mechanically tensioned plate is at an elevated background tensile stress, the similarities in the longitudinal stress contours when compared to those for a trailing cold spot are evident. Both methods have reduced the compressive bow wave ahead of the tool, although this occurs more effectively with global tensioning, and both show a reduction in the levels of tensile stresses developing behind the tool. However, the application of local dynamic cooling has more potential for practical application particularly when you consider the magnitude of the tensile loads required in global mechanical tensioning and the restriction of the technique to straight weld seams.

![Figure 12. 2D longitudinal stress distributions produced for a standard weld and for a 25 mm and 100 mm cold spot, compared to that for a global mechanically tensioned weld - preloaded in the welding direction to 35% of its room temperature yield stress.](image)

### 4.0 CONCLUSIONS

In a standard weld, a large compressive thermally induced misfit strain field ahead of the heat source is formed, which causes compressive plastic straining where the material is hot and thus soft near the tool. On cooling, a tensile stress then develops behind the tool as a large misfit strain develops with the surrounding colder material due to thermal contraction. This misfit is plastically relaxed to the greatest extent near to the weld line, due to the rapidly falling load bearing capacity of the material with temperature, and results in the ‘M’ shaped transverse tensile longitudinal residual stress profile conventionally seen after welding.

Both global mechanical tensioning and local thermal tensioning attempt to balance this misfit by reducing the compressive bow wave ahead of the travelling heat source and increasing the tensile plastic strain developed in the hot zone trailing the weld. Global mechanical tensioning does this by brute force and is therefore limited by the equipment capable of delivering the required loads. In comparison local thermal tensioning operates in a more subtle manner, in that it attempts to apply an artificial thermal state on the material that balances out the misfit strains. It is also more versatile and not limited to straight welds.

The studies carried out using the FE model have shown that local thermal tensioning by the application of an advanced CO₂ cooling system can produce a significant reduction in the residual stress state after welding, particularly when dual or trailing single cold spots are used. However, the benefits are very dependent on the positioning of the cooling sources, and fall off rapidly with distance behind the tool. With realistically achievable cooling powers, the application of dual spots at a distance of 20mm from the weld tool was found to reduce the peak residual stresses by a maximum of ~ 60%, but this reduction fell to 20% with the same cooling nozzle applied at a 75mm distance behind the tool. The stress reduction from a trailing cold spot is magnified at the weld line, due to the softer nature of the material when subjected to tensile straining resulting in the tensile stresses being completely reversed and compressive stresses generated when the cold sink is very close to the tool. Examination of the stress states developed during welding in the FE models revealed that the cold spots placed behind
the heat source produced the primary effect, which was to increase the level of thermal contraction and offset the plastic misfit of the hot soft weld material emerging from behind the tool, thereby reducing the build up of trailing tensile stresses. Overall the studies have shown that the dynamic application of local cooling can produce a significant reduction in the residual stresses formed during the friction stir welding process and that the method has potential for practical application in industry.

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