An Investigation of Value Modelling for Commercial Aircraft

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One of the issues with commercial aircraft design is that most of the methods of developing and propagating requirements allow for nominally satisfying but less than ideal designs to be produced. There is a significant body of work that illustrates the issues that are inherent with the standard requirements flow-down approach. As an attempt to address this, a series of approaches variously named Value-Driven and Value-Centric Design have been proposed. In looking at commercial aircraft and engine design and operations several authors have suggested using a simplified Value model known as Surplus Value. This paper investigates the derivation of and assumptions that are inherent to the standard surplus value formulation. Further, several interesting and potentially useful outcomes of the standard model are investigated including the relationship with discount rate and maximum used programme duration or product life. Lastly, this paper attempts to remove one of the most restrictive assumptions in the basic surplus value formulation. This results in a more general formulation that incorporates basic sales and operating leases plus the ability to provide managed services. The resulting model is more general, but does require more user information to use.

Nomenclature

\( C \) = Cost
\( N \) = Total Market Size/Number of Aircraft
\( R \) = Revenue
\( U \) = Annual Utilization
\( D \) = Discount Multiplier
\( P \) = Profit
\( SV \) = Surplus value
\( t \) = Programme duration or product life
\( \bar{X} \) = Design variable vector
\( acq \) = acquisition
\( c \) = customer
\( dev \) = development
\( man \) = manufacturing
\( mtc \) = maintenance
\( p \) = producer
\( p_{tc} \) = producer for product life
\( svc \) = service, service contract
\( u/\text{use} \) = per use
\( \alpha \) = percentage of maximum discount factor
\( \beta \) = manage serviced fraction
\( \gamma \) = lease rate ratio
\( \delta \) = producer discount rate ratio
\( \sigma \) = discount rate

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I. Introduction

Commercial aircraft programmes, both new designs, derivatives often suffer from slipped schedules and degraded performance as the programme develops. Further, once these aircraft are in service it is common for design flaws to make themselves known and lead to at least temporary further decreases in performance or increases in cost. For the modern commercial aircraft operator these decreases in performance or increases in cost can quickly shift an aircraft or fleet from profitability to unprofitability. This is especially true when coupled with the often difficult financing environment that is faced by many modern airlines. In order to counter this trend more innovative and flexible financing and service arrangements have been proposed by the airframe and engine manufacturers. These programmes, often referred to as manage services or ‘power-by-the-hour’, can effectively shift risk and potential cost burdens from the operator. In the ideal sense these arrangement create a net value for both the operator and the manufacturer, eliminating what would otherwise be a deadweight loss for one or both parties. However, what is not clear is whether these changes in arrangements shift the ‘optimal’ design point for the aircraft or engine. That is should the manufacturer produce a different product under these conditions than they would under the previous regime. The purpose of this paper is to present straightforward methods to allow this to occur.

This paper builds upon the previous work in Value Driven Design (VDD) and specifically the Surplus Value (SV) approach to investigate how differences in organizational structure and contract conditions affect the ‘best’ design an aircraft. This is the first step in a broader investigation on how an engineering or engineering organization should create a value model to represent any design programme. While this paper focuses on a discounted financial basis for value, using revenue as the utility and capital and operating costs as the cost aspect, it will provide the foundation for further investigations that might not be quite as straightforward as most civil commercial contracts.

II. Background

Previous work in value driven design for commercial aircraft has focused on a relatively simple value model, often called surplus value, to estimate the benefit or detriment to design choices at the aircraft or engine level. The SV approach is one that simplifies programme research, development, testing and certification (RDT&C) expenditures along with future revenues into a simple easy to digest form. In its most basic form the SV approach makes the following assumptions.

1. All RDT&C costs are pre-discounted brought to the date of initial deliveries
2. Market size is independent of the design or designs being offered
3. Future revenue potential remains the same from one year to the next
4. The cost of capital or chosen discount rate for both the producer and consumer are not affected by the design
5. Single producer
6. Single customer
7. No product upgrades through its life
8. Contracts are simple sales propositions

The SV approach can be represented in its simplest form in Eq. 1.

\[ SV = D_p \times N_{market} \times \left[ D_c \times U \times (R_{flight} - C_{flight}) - C_{manufacture} \right] - C_{dev} \]  

where,

- \( D_p \) = Producer Discount Multiplier
- \( D_c \) = Customer Discount Multiplier
- \( N_{market} \) = Total Market Size
- \( U \) = Annual utilization in flights
- \( R_{flight} \) = Revenue per flight
- \( C_{flight} \) = Cost per flight
- \( C_{manufacture} \) = Cost per unit to manufacturer
- \( C_{dev} \) = Programme development costs

The producer and customer discount multipliers are functions of the respective discount rates (\( \sigma \)) and either the programme duration or the product life. The means of determining the discount multipliers is given in Eq. 2.
While the SV equation is relatively easy to implement, requiring only that the engineering organization be able to determine the effect of design choices on revenue and costs, the basic assumptions rarely hold in the real world. As a consequence it is necessary to investigate whether or not these assumptions can be expanded to a more realistic set of cases. Each of these will be addressed in turn.

1. **RDT&C Costs**: Real research and development costs vary from year to year and often continue after the first delivery. However, the discounting method is relatively straightforward and the final cost only offsets the final value of the SV equation. Pre-computing prior to entry into the SV equation is straightforward.

2. **Market Size**: Provided the designs are substantially similar, i.e. not something like a CTOL vs STOL or of significantly different sizes, the design decisions will have relatively small effects on the overall market size. This assumption is beyond the scope of this paper.

3. **Future Revenue Trends**: This is a challenge as there is projected growth in total air traffic and a decrease in real fare yields. In most cases the design choices made will not affect this trajectory and the results of any comparison will not be affected. While this is of interest it is beyond the scope of this paper.

4. **Discount Rate**: This assumes that one is not ‘betting the company’ with a design. As Collopy showed, for most mature aerospace companies this is rarely the case.

5. **Single producer**: The semi-competitive duopoly market of the civil aerospace industry is fairly close to a monopoly. Further, the single customer, monopsony, condition means that the design that maximizes the total surplus value is independent of the competition.

6. **Single customer**: In theory this would be a single type of customer. In reality there are many different airlines, which negate this assumption. However, if there is only one producer, monopoly, then a mean airline can be represented. The interesting case is where there a multiples of both. This is specifically beyond the scope of this paper, but is of great research interest.

7. **No upgrades**: For many aircraft operators the effective time horizon of a new aircraft purchase is actually short enough to preclude the inclusion of specific upgrades in the analysis. This is also of research interest but beyond the scope of this paper.

8. **Contracting**: The primary focus of this paper.

In addressing assumption 8 the paper performs and expands the derivation of a more generic form of the SV equation and demonstrates the potential differences in aircraft design choices, through a simplified example. Obviously the use of real data is needed to fully verify and potentially validate the approach, but that is anticipated for future efforts.

### III. Developing a Financial Based Model

The basics of a financial based value approach is the idea that some form of revenue can be achieved through each use of the engineering product and that each usage incurs some cost. This is straightforward for the cases of commercial, civil products as there is a clear and identifiable revenue chain. In the case of commercial aircraft this would be the fare charges to carry either a passenger or a unit of cargo, i.e. the fare yield. For commercial other products, specifically those that are sold directly to the consumer this is harder to determine. However, even in these cases estimating revenue is much easier than for non-commercial, civil and military engineering products.

The reader should keep in mind that the revenue point can be entered at any stage in the usage cycle. In the commercial aircraft example it is entered at the point where the ultimate customer purchases a seat or cargo capacity on a certain flight. It could also be entered where the customer decides that he or she wants to travel and is making a judgment on mode. The issue here is that while it is very easy to back-up to a point where the engineer is comfortable, say the sale of an aircraft or engine to an airline. The risk is that it is easy to make a value model where the critical engineering decisions become exogenous, i.e. they cannot be measured by the model, under which case the results could easily be erroneous.

Starting from the revenue and cost per use for a single product it is possible to calculate the marginal profit for such a use. This is given in Eq. 3.

\[
D_i = \frac{1}{\sigma} - \frac{1}{\sigma(1 + \sigma)^i}
\] (2)
\[ P_{\text{marginal,use}} = R_{\text{use}} - C_{\text{use}} \]  

(3)

Note: this does not contain any capital or acquisition costs, as these are fixed and not marginal, we will address this shortly. If a product gets used many times we can figure out the total marginal profit simply by multiplying \( P_{\text{marginal}} \) by the number of times the product is used (\( U \)), shown in Eq. 4.

\[ P_{\text{marginal}} = U(R_{\text{use}} - C_{\text{use}}) \]  

(4)

To determine the total profit for the entity operating the engineering product all we have to do is subtract the fixed costs (\( P_{\text{fixed}} \)) from \( P_{\text{marginal}} \), shown in Eq. 5.

\[ P_{\text{net,unit}} = U(R_{\text{use}} - C_{\text{use}}) - C_{\text{fixed}} \]  

(5)

This is a really simple and straightforward way to look at a single engineering product’s financial value. However, most engineering products are used over a number of years. In these cases the time value of money needs to be incorporated. Therefore, it is customary to look at the total net profit per unit (\( P_{\text{net,unit}} \)) as a sum of a series of discounted cash flows, or discounted profits, shown in Eq. 6.

\[ P_{\text{net,unit}} = \sum_{i=1}^{n} \left[ U_i (R_{\text{use},i} - C_{\text{use},i}) - C_{\text{fixed},i} \right] \frac{1}{(1 + \sigma)^i} - C_{\text{fixed,initial}} \]  

(6)

where \( \sigma \) represents the discount rate on money, and \( i \) is the time unit of accounting, typically annual for durable products. If a company operates a number of these units the total net profit (\( P_{\text{net}} \)) is simply the multiple of all of the units operated, given in Eq. 7.

\[ P_{\text{net}} = N \left\{ \sum_{i=1}^{n} \left[ U_i (R_{\text{use},i} - C_{\text{use},i}) - C_{\text{fixed},i} \right] \frac{1}{(1 + \sigma)^i} - C_{\text{fixed,initial}} \right\} \]  

(7)

A common assumption to make at this point is that some if not all of the reoccurring costs are fundamentally the same for each time period being calculated. The simplest of this is making the discount rate a constant over time. This would slightly simplify Eq. 7. Additionally, in many cases for engineering projects it is possible to represent both the marginal use and annual fixed costs as independent with time. This would allow the simplification of Eq. 7 to the form shown in Eq. 8.

\[ P_{\text{net}} = N \left\{ \sum_{i=1}^{t} \left[ U(R_{\text{use}} - C_{\text{use}}) - C_{\text{fixed}} \right] \frac{1}{(1 + \sigma)^i} - C_{\text{fixed,initial}} \right\} \]  

(8)

There are many methods of doing this simplification, each dependent upon the assumptions being made in the study. The simplest of these is that the design choices being made do not affect the future revenue or cost trends for the product. That is in any comparison the annual net profit profile for each of the choices being compared will behave similarly. In the case of most commercial aircraft programmes the revenue profile is probably independent of the choices being made. However, other items like maintenance costs may not follow this assumption.

Looking at Eq. 8 it is possible to see that another simplification can be made, shown in Eq. 9.

\[ P_{\text{net}} = N \left\{ \sum_{i=1}^{t} \left[ \frac{1}{(1 + \sigma)^i} \right] \times \left[ U(R_{\text{use}} - C_{\text{use}}) - C_{\text{fixed}} \right] - C_{\text{fixed,initial}} \right\} \]  

(9)

Further, portion of Eq. 9 given by \( \sum_{i=1}^{t} \left[ \frac{1}{(1 + \sigma)^i} \right] \) can easily be represented as a simple multiplier (\( D \)) given in Eq. 10.

\[ D = \sum_{i=1}^{t} \left[ \frac{1}{(1 + \sigma)^i} \right] = \frac{1}{\sigma} - \frac{1}{\sigma(1 + \sigma)^t} \]  

(10)

This means that we can modify Eq. 9 with Eq 10, giving Eq. 11.

\[ P_{\text{net}} = N \left\{ D \left[ U(R_{\text{use}} - C_{\text{use}}) - C_{\text{fixed}} \right] - C_{\text{fixed,initial}} \right\} \]  

(11)

This is the simplest form of the net profit equation for the customer. We can apply the same approach to the producer. In this case the marginal, per unit profit would be given by Eq. 12.
\[ P_{\text{marginal unit}} = R_{\text{unit}} - C_{\text{unit}} \]  

(12)

Following the same principles as the customer case we get Eq. 12 to represent the net profit for the programme, shown in Eq. 13.

\[ P_{\text{net}} = \{D \left[ N(R_{\text{unit}} - C_{\text{unit}}) - C_{\text{fixed}} \right] - C_{\text{fixed, initial}} \} \]  

(13)

To get the total net profile for the entire programme sum Eqs. 11 and 13. This is shown in Eq. 14.

\[ P_{\text{net}} = \{D_p \left[ N(R_{\text{unit}} - C_{\text{unit}}) - C_{\text{fixed, p}} \right] - C_{\text{fixed, initial, p}} \} \]

+ \[ N \{D_c \left[ U(R_{\text{use}} - C_{\text{use}}) - C_{\text{fixed, c}} \right] - C_{\text{fixed, initial, c}} \} \]

(14)

In order to simplify Eq. 14. we can make the assumption that the only fixed initial costs to the customer \((C_{\text{fixed, initial}})\) is the acquisition cost. This would make the revenue per unit \((R_{\text{unit}})\) for the producer to be the same as the fixed initial cost for the customer \((C_{\text{fixed, initial}} = R_{\text{unit}})\). This would give us Eq. 15.

\[ P_{\text{net}} = \{D_p \left[ N(R_{\text{unit}} - C_{\text{unit}}) - C_{\text{fixed, p}} \right] - C_{\text{fixed, initial, p}} \}

+ \[ N \{D_c \left[ U(R_{\text{use}} - C_{\text{use}}) - C_{\text{fixed, c}} \right] - R_{\text{unit}} \} \]

(15)

This can be further simplified by introducing the concept of reservation price. Which is defined as:

Reservation Price: The maximum price that a customer is willing to pay for a given item. In a simplified discounted cashflow analysis this is the price which sets the total of future net profit equal to zero.

This would mean the reservation price, given here as \(R_{\text{unit, reservation}}\), could be represented using Eq. 16.

\[ R_{\text{unit, reservation}} = D_c \left[ U(R_{\text{use}} - C_{\text{use}}) - C_{\text{fixed, c}} \right] \]

(16)

Using the concept of reservation price we can greatly simplify Eq. 15, giving use Eq. 16.

\[ P_{\text{net}} = D_p \left\{ N \{D_c \left[ U(R_{\text{use}} - C_{\text{use}}) - C_{\text{fixed, c}} \right] - C_{\text{unit}} \} - C_{\text{fixed, p}} \right\} \]

(17)

This is equivalent to the basic surplus value equation, given in Eq. 1, for a single customer.

Looking back at the assumptions that were made we have a monopoly-monopsony market, identical temporal trends in real revenue and costs for both operation and production, discount rates that are independent of engineering choices, no difference in upgradability of the product being produce and a simple sales contract. The purpose of this paper is to investigate how different contracting arrangements can affect the future balance of these profits. This will be done in two parts. The first will look at the discount multiplier and the effect on effective life of either the engineering programme or the product it produces. The second will be to look at how a change in the formulation can affect the mix of fixed vs. marginal costs. From this it should be possible to give insights into how more complicated changes in the institution and contract structure effect the ‘value’ of the programme.

IV. Effects of the Discount Rate

One of the outcomes of the development of the surplus value, or other financial model, is that it becomes possible to quickly look at the effect of non-engineering factors on the engineering decisions that are made during the design and development of a product. These include many factors that are normally exogenous to the engineering process such as finance and market limitation, but also those that pertain directly to the engineering process, e.g. the opportunity cost of committing engineering and financial resources on a project. For the purposes of this model these are all lumped in the discount rate, and while endogenous to the surplus value model they inputs are still exogenous to the engineering process. Two of the areas that are of particular interest are the effect on programme and product life, and the direct effect on surplus value and the resulting engineering trades.

A. Discount Rate and Effective Programme Life
One of the most significant factors of the long-term ‘value’ of an engineering programme is the discount rate selected by the firms involved. The problem is what discount rate should be used. Discount rates are inherently a value judgement about the future prospects of the company. It is common, in engineering cash-flow analyses to use weighted average cost of capital (WACC) as the basis for the discount rate. There are many issues with using WACC as a basis for calculating discount rate the least of which is the issues with the WACC method itself. Independent of this is the fact that WACC will not necessarily represent the chosen discount rate for the company. One option is to, temporarily, set aside the issues with WACC and use the WACC as a lower boundary on the discount rate. The assumptions behind this are that WACC is close enough for a quick estimate of the actual cost of debt and equity to the company, and that while each programme or product is assumed to not be large enough that engineering decisions will change the future costs it is assumed to be large enough to require a companywide investment to be made.

Regardless of the method used to estimate the discount rate it is possible to investigate the effect it has on effective product life and the utility trade on fixed vs. variable costs. To do this let’s look first at the effect of time on the discount multiplier ($D$). Taking Eq. 10 and looking at the limit as $t$ approaches $\infty$. This is shown in Eq. 18.

$$\lim_{t \to \infty} \left( \frac{1}{\sigma} - \frac{1}{\sigma \times (1 + \sigma)^t} \right) = \frac{1}{\sigma} - \frac{1}{\sigma + \infty} = \frac{1}{\sigma}$$ (18)

The result is that the maximum multiplier on the future cashflow is just the multiplicative inverse of the discount rate. The effect of this is shown in Table 1.

<table>
<thead>
<tr>
<th>Discount Rate</th>
<th>Maximum Discount Rate Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>20</td>
</tr>
<tr>
<td>10%</td>
<td>10</td>
</tr>
<tr>
<td>12%</td>
<td>8.3</td>
</tr>
<tr>
<td>15%</td>
<td>6.7</td>
</tr>
<tr>
<td>20%</td>
<td>5</td>
</tr>
<tr>
<td>25%</td>
<td>4</td>
</tr>
</tbody>
</table>

Going along with this is the asymptotic trend of multiplier with time for projects that for durations short of infinity. This was shown by Sutcliffe and Hollingsworth $^4$, and is reprinted here in Figure 1.

Figure 1: Discount Rate Multiplier as a Function of Time, from 4

Air Transport and Operations Symposium 2011
The impact of this is that the effective life of a programme or product is limited, i.e. it does the company no good for a programme to last any longer, at least at the design phase, as it provides no additional value. There are potentially significant implications of this for the design of products. Further, it helps explain why some customers hold off buying new equipment for much longer than others. Additionally, the other fallout is that if you reduce the discount rate the value of any given programme will increase without and engineering design changes.

One of the benefits of the realization of the affect on discount rate on effective life is that we can determine an upper boundary on the useful life of a programme or product. Going back to Eqs. 10 and 18 we can create a means of determining the effective life by using Eq. 18.

\[ \frac{1}{\sigma} - \frac{1}{\sigma (1 + \sigma)^t} = \alpha \left( \frac{1}{\sigma} \right) \]  

(19)

where \( \alpha \) is the percentage of the maximum discount multiplier that is achieved at the end of the effective product life. Solving for \( t \) gives the maximum effective product life or investment horizon. The resulting form is given in Eq. 20.

\[ t = -\frac{\ln|1 - \alpha|}{\ln|1 + \sigma|} \]  

(20)

Conversely Eq. 19 can be solved for discount rate. This is shown in Eq. 21.

\[ \sigma = \left( \frac{1}{1 - \alpha} \right)^\frac{1}{t} - 1 \]  

(21)

Instead of solving for the time horizon this gives that maximum allowable discount rate for a product with a specific use or market life. For example the Apple iPhone is effectively replaced every year. This means that \( t = 1 \) for this product. This would give a discount rate significantly higher than the basic rate that Apple would use for its product developments. Some examples of the results from Eqs. 20 & 21 are shown in Table 2.

<table>
<thead>
<tr>
<th>Discount Rate</th>
<th>Approx. Effective Product Life (yrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>46</td>
</tr>
<tr>
<td>10%</td>
<td>23</td>
</tr>
<tr>
<td>12%</td>
<td>19</td>
</tr>
<tr>
<td>15%</td>
<td>15</td>
</tr>
<tr>
<td>20%</td>
<td>12</td>
</tr>
<tr>
<td>25%</td>
<td>9</td>
</tr>
<tr>
<td>30%</td>
<td>8</td>
</tr>
</tbody>
</table>

Note that the choice of \( \alpha \) is inherently a value judgement by the engineer or manager. However, it should be possible to make reasonable guesses.

The outcome of this is that for many of the world’s commercial airlines the current costs of debt and equity mean that their minimum discount rates are quite high, often higher than 25%. This is somewhat controversial given than the WACC methods often give values below 20%; however, these estimates are based upon decreasing forward debt to equity ratios. While this has been achieved over the short-term in most cases the long-term trend for airline debt to equity ratios has only increased. Taking this trend into account the resulting minimum discount rates would be much higher than those in the literature. This trend seems to be supported by the increasing tendency of airlines to move to shorter-term operating leases and managed service contracts. The consequences of this are that there is the potential for both significant value addition and engineering design changes introduced by identifying and exploiting this.

B. Effect of Discount Rate on Surplus Value

Given that most modern airlines most likely have real discount rates north of 20%, maybe even approaching 30% and most manufacturers having discount rates that are much lower there exists a significant value premium potential that can be achieved by shifting some costs and revenues from the current customer to the producer. This is pretty obvious by looking at Eqs. 16 & 18. Let us
consider two cases one where the customer has a 25% discount rate and the producer a 12% 
discount rate and another where both the customer and producer have a 12% discount rate for a 20 
year program and a product life of 5 years and there are no initial or annual unit costs. In this case 
the percent difference in value between the second and the first case is given by Eq. 22.

\[
\frac{SV_2}{SV_1} - 1 = \frac{D_p N \{D_p \{U (P_{use} - C_{use})]\}}{D_p N \{D_c \{U (R_{use} - C_{use})]\}} - 1
\]  
(22)

The resulting value difference is now down to solely the ratio between the producer and the 
customers effective discount rate, \(\frac{D_p}{D_c}\). Of course this is a trivial condition as this requires a system that 
costs nothing to develop and nothing to produce, but does cost something to operate. However, it 
gives insight into another aspect, the trades between discount rate and the reservation 
price/allowable manufacturing cost and ultimately the allowable development cost.

1. Development Cost

In reality the development and certification cost of a new engineering programme is generally 
considered not insignificant to the organization undertaking the programme. This is because a new 
programme will consume monetary, human and temporal resources that might otherwise be available 
if no programme were undertaken. There are a number of ways to handle this opportunity cost, but 
the simplest is in the discount rate, i.e. the cost of undertaking the programme. However, in most 
engineering design decisions the relative opportunity cost difference between the two choices is small 
enough that the discount rate may be assumed constant, i.e. \(D_p\) can be considered constant. Note: 
this assumption needs to be tested for each programme, but looking at this case helps us understand 
the relationship between development costs and surplus value.

To look into this it is useful to bring back the concept of a reservation price for the producer. To 
do this one should think about the case where the producer has the ability to purchase a wholly 
developed and certified product which they can manufacture and sell. Note: This is nearly the same 
as looking back on a completed development and certification programme for an existing product. In 
this case the maximum price that a rational, risk neutral firm would be willing to pay would be that 
which makes the expected present value of all future earnings from this programme zero. This is the 
same relationship the customer has with the product. Revisiting Eq. 17 and setting \(P_{net}\) equal to zero, 
the limit on development cost, \(C_{fixed, initial, p}\), can be viewed as a function of future revenue, costs and 
discount factors. This is shown in Eq. 23.

\[
C_{fixed, initial, p} = D_p \{N \{D_c \{U (P_{use} - C_{fixed, c}) - C_{unit}\}\} - C_{fixed, p}\}
\]  
(23)

Looking at this formulation it is straightforward to investigate the relative power of each of the 
factors, especially those for which design choices have a significant effect. The two factors with the 
highest leverage are \(U\) and \(P_{use}\). This means that small changes here tend to be significantly amplified 
and can easily offset larger changes in producer costs, e.g. annual fixed costs associated with the 
programme and development costs.

As an example, consider the case where a producer is making a design decision on an engineering 
programme, where an increase in development cost will lead to an increase in the per-use operating 
profit. Taking the partial derivative Eq. 23, with \(D_p\), \(N\), \(D_c\), \(U\), \(C_{fixed, c}\), \(C_{unit}\), and \(C_{fixed, p}\) assumed 
constant it is possible to see the leverage of this factor has on allowable costs, Eq. 24.

\[
\frac{\partial (C_{fixed, initial, p})}{\partial P_{use}} = D_p N D_c U
\]  
(24)

The same partials can be created for other each of the other factors that are directly related to the 
engineering design choices. These are shown in Eqs. 25 to 29.

\[
\frac{\partial (C_{fixed, initial, p})}{\partial U} = D_p N D_c P_{use}
\]  
(25)

\[
\frac{\partial (C_{fixed, initial, p})}{\partial N} = D_p D_c (U P_{use} - C_{fixed, c})
\]  
(26)

\[
\frac{\partial (C_{fixed, initial, p})}{\partial C_{unit}} = -D_p N
\]  
(27)
\[
\frac{\partial (C_{\text{fixed, initial, p}})}{\partial C_{\text{fixed, p}}} = -D_p
\]  

(28)

\[
\frac{\partial (C_{\text{fixed, initial, p}})}{\partial D_c} = D_p (U P_{\text{use}} - C_{\text{fixed, c}})
\]  

(29)

Note: the partials for \(D_p\) and \(C_{\text{fixed, c}}\) are not give as these are likely to be significantly less sensitive to engineering design choices and more sensitive to external influences than the other factors. Now consider that each of the design factors are themselves function of a vector of design choices or variables, \(\bar{X}\). In this case we can look at the total derivative of the above in Eq. 30.

\[
d(C_{\text{fixed, initial, p}}) = D_p \left\{ D_c \left[ N \left( U \frac{\partial (P_{\text{use}})}{\partial \bar{X}} + P_{\text{use}} \frac{\partial (U)}{\partial \bar{X}} \right) + \left( U P_{\text{use}} - C_{\text{fixed, c}} \right) \frac{\partial (N)}{\partial \bar{X}} \right] \\
+ N \left[ (U P_{\text{use}} - C_{\text{fixed, c}}) \frac{\partial (D_c)}{\partial \bar{X}} - \left( N \frac{\partial (C_{\text{unit}})}{\partial \bar{X}} + \frac{\partial (C_{\text{fixed, p}})}{\partial \bar{X}} \right) - C_{\text{unit}} \frac{\partial (N)}{\partial \bar{X}} \right] \right\}
\]  

(30)

The totality of the above equations gives insight into the relative power of small changes in other costs. For instance using a simplistic example of a 20 year programme and a product that has a 15 year operational life, with a market size of 250 units, 1250 uses per year and with producer and consumer discount rates of 12.5% and 20% respectively a design decision that increases operating profit by 1% makes sense, all else remaining equal, if the percent change in development cost is less than \(1.1 \times 10^5\) times the original ratio between operating profit per use and the development cost. For instance if the typical ratio, \(\frac{P_{\text{use}}}{C_{\text{fixed, initial, p}}}\), is on the order of \(1.0 \times 10^6\), the allowable change in development cost is about 10%, then doubling the number of units sold doubles the allowable percent change in development cost, as does a doubling of utilization.

The outcome of this is that it is very often the case that significant changes in development and certification costs are perfectly reasonable for only small changes in operating cost or revenue potential.

2. Product Life, Discount Rate and Development Cost

Another aspect of consideration is the effective life of the product once it enters service. This includes regular maintenance, but does not take into account any upgrades that go beyond restoring functionality, i.e. the addition of winglets or engine performance improvement package. To look into this take the time derivative of Eq. 10. This is shown in Eq. 31.

\[
\frac{d(D_c)}{dt} = \frac{(\sigma + 1)^{-2} \ln(\sigma + 1)}{\sigma}
\]  

(31)

Combining Eqs. 29 and 31 we get a formulation for the relationship between \(C_{\text{fixed, initial, p}}\) and product life \((t)\), this is shown in Eq. 32.

\[
\frac{d(C_{\text{fixed, initial, p}})}{dt} = \frac{\partial (C_{\text{fixed, initial, p}})}{\partial D_c} \frac{d(D_c)}{dt} = \frac{(\sigma_c + 1)^{-2} \ln(\sigma_c + 1)}{\sigma_c} \left( D_p (U P_{\text{use}} - C_{\text{fixed, c}}) \right)
\]  

(32)

The result is that depending on the discount rate that is assumed for the customer and the original product life the slope of the reservation development cost can be very small.
of the lease. Of more interest is the effect of managed services on the design and design behaviour.

The operating lease is relatively straightforward as it shifts the customer relationship from that of the airline to that of the lessor. Of more interest is the effect of managed services on the design and design behaviour.

To develop a basic value approach for a managed services environment it is necessary to return to the basic formulation of the SV equation, shown in Eq. 14, and revisit the distribution of revenues and expenses. There are at least two basic ways of incorporating a managed service contract into the basic value model. The first is to transfer the expected future costs into the fixed initial cost that the customer pays. This can be represented in Eqs. 33-35.

\[ C_{use} = C_{use, non-mtc} + C_{use, mtc} \]  
\[ C_{mtc} = D_cU C_{use, mtc} = C_{initial, svc} \]  
\[ R_{unit, reservation} = D_c \left[ U (R_{use} - C_{use, non-mtc}) - C_{fixed, c} \right] \]

The result is that the surplus value equation, Eq. 17, can be rewritten as follows, Eqs. 36 and 37,

\[ P_{net} = D_{p, t} \left[ N \left[ D_c \left[ U (R_{use} - C_{use, non-mtc}) - C_{fixed, c} \right] - C_{unit} \right] - C_{fixed, initial, p} \right] - D_{p, t} N U S U C_{use, mtc} - C_{fixed, initial, p} \]

\[ P_{net} = D_{p} \left[ N \left[ D_c \left[ U (R_{use} - C_{use, non-mtc}) - C_{fixed, c} \right] - C_{unit} - SU C_{use, mtc} - C_{fixed, p} \right] \right. \]

where \( \delta \) is the ratio of discount factors for the producer if the expected service life of the product is different from the program. A simplified version of this ratio can be expressed in Eq. 38,

\[ \delta = \frac{D_{p, t}}{D_{p, t, p}} = \frac{(1 + \sigma)^{t_c} - 1}{(1 + \sigma)^{t_p} - 1} \]

where \( t_c \) is the product service life and \( t_p \) is the programme duration. The result is that depending on the expected life of the product and the program the present value cost of maintenance can be substantially lower. To take a look at this consider the relationship between \( \delta \), \( t_c \) and \( t_p \). This is shown in Figure 3.

V. Alternative Contract Options

One of the factors of modern commercial aerospace systems is the more and more the contracts between the producers and customers are no longer simple sales. The most common of these new contract vehicles are the operating lease and the newer managed service approach. The operating lease is relatively straightforward as it shifts the customer relationship from that of the airline to that of the lessor. Of more interest is the effect of managed services on the design and design behaviour.

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\[ R_{unit, reservation} = D_c \left[ U (R_{use} - C_{use, non-mtc}) - C_{fixed, c} \right] \]

The result is that the surplus value equation, Eq. 17, can be rewritten as follows, Eqs. 36 and 37,

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\[ P_{net} = D_{p} \left[ N \left[ D_c \left[ U (R_{use} - C_{use, non-mtc}) - C_{fixed, c} \right] - C_{unit} - SU C_{use, mtc} - C_{fixed, p} \right] \right. \]

where \( \delta \) is the ratio of discount factors for the producer if the expected service life of the product is different from the program. A simplified version of this ratio can be expressed in Eq. 38,

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Figure 3: Discount Multiplier Factor as a function of Product Service Life for Different Program Durations, in years

The result is that instead of the customer paying for maintenance throughout the life of each aircraft, it is purchased upfront in the ‘sales’ transaction. This has the effect of shifting the time and cost burden of maintenance. The upside of this is that longer-life products make more sense, but it constrains the cash-flows for longer into the future.

Consider the case where all operating costs are purely marginal, \( C_{\text{fixed,c}} = 0 \), and maintenance costs are a fraction (\( \beta \)) of the total per use cost. In this case the difference in surplus values between the two options can be represented by Eq. 39.

\[
P_{\text{net}} = D_p N \left\{ D_c \left[ U(R_{\text{use}} - [1 - \beta] C_{\text{use}}) - C_{\text{fixed,c}} - C_{\text{unit}} - \delta \beta U C_{\text{use}} - C_{\text{fixed,p}} \right] \right\}
\]

A simplified way of looking at it is how do the relationships between product life, maintenance cost, and manufacturing cost relate if the surplus values in the two formulations, Eqs. 1 and 39, are directly equated. The resulting relationship is given in Eq. 40.

\[
D_p N \left\{ U [\Delta C_u + \beta (\delta - D_c) C_{\text{use,2}}] + \Delta C_{\text{man}} \right\} + \Delta C_{\text{dev}} = 0
\]

where the \( \Delta \) values are the differences between two different designs. The resulting equation is relatively straightforward and resembles a simple total derivative equation, such as that shown in Eq. 30. This gives us a complete derivative represented in Eq. 41.

\[
\frac{d (SV)}{dX} = D_p N \left\{ U \left[ \frac{d(C_u)}{dX} + \beta (\delta - D_c) C_u \right] + \frac{d(C_{\text{man}})}{dX} + \frac{d(C_{\text{dev}})}{dX} \right\}
\]

Taking \( \beta \) to be about twelve percent of operating cost\(^1\), the same programme and product spans and discount rates as before, and assuming that \( \frac{dC_{\text{dev}}}{dX} = 0 \), it is possible to explore the value trade between changes in maintenance cost and manufacturing cost. This is given in Eq. 42.

\[
\frac{d(C_u)}{dX} = 0.23 C_u - \frac{1}{1250} \frac{d(C_{\text{man}})}{dX}
\]

\(^1\) Using US Bureau of Transportation Statistics data from the first three quarters of 2010\(^6\) for airline operating costs, Schedule P-6 of Form 41, the sum of maintenance labour, benefits, material, and maintenance services was approximately 12% of the operating cost for each airline. These values vary quite significantly from airline to airline, from a low of 3.8% to a high of 32.1%.
An alternative method for managed services is basically a full operating lease, where the operator pays the producer for each flight undertaken. There are a variety of ways of structuring such a deal; however, the two simplest forms are the ones where the customer pays all of his revenue to the lessee and the lessee receives none of the revenue. The three versions are given in Eqs. 43 to 45.

\[
SV = N\{D_cU[R_u - (1 - \beta)C_u - C_{lease}] - C_{acq}\} + D_pN(C_{acq} - C_{man})
+ D_{p,t}NU(C_{lease} - \beta C_u) - C_{dev}
\]

(43)

\[
SV = -D_pN(C_{man}) + D_{p,t}NU(R_u - C_u) - C_{dev}
\]

(44)

\[
SV = N\{D_cU[R_u - (1 - \beta)C_u]\} - D_pN(C_{man}) - D_{p,t}NU(\beta C_u) - C_{dev}
\]

(45)

In both Eqs. 44 & 45 it is assumed that the acquisition cost \(C_{acq}\) is zero and \(\beta\) is a constant value. Obviously Eq. 45 presents a situation where the producer would not sell the aircraft and Eq. 44 represents a situation where the airline would be unable to raise any capital. A more general form, where the lease rate fraction is a variable \((\gamma)\) is shown in Eq. 46,

\[
SV = ND_cU(1 - \gamma)[R_u - (1 - \beta)C_u] - D_pN(C_{man})
+ D_{p,t}NU(\gamma[R_u - (1 - \beta)C_u] - \beta C_u) - C_{dev}
\]

(46)

Again, \(C_{acq}\) is zero and \(\beta\) is a constant. One further modification can be made, by returning to the concept of reservation price. In this case the revised form of reservation price is given in Eq. 47 and the resulting surplus value equation in Eq. 48.

\[
R_{unit, reservation} = D_cU(1 - \gamma)[R_u - (1 - \beta)C_u]
\]

(47)

\[
SV = D_pN(D_cU(1 - \gamma)[R_u - (1 - \beta)C_u] - (C_{man}))
+ D_{p,t}NU(\gamma[R_u - (1 - \beta)C_u] - \beta C_u) - C_{dev}
\]

(48)

Reintroducing the producer discount factor ratio, \(\delta\) from Eq. 38, results in Eq. 49.

\[
SV = D_pN(D_cU(1 - \gamma)[R_u - (1 - \beta)C_u] - (C_{man}) + \delta U(\gamma[R_u - (1 - \beta)C_u] - \beta C_u))
- C_{dev}
\]

(49)

This form is a more general form of the surplus value equation, taking into account a variety of contract options. By setting \(\gamma = 0\) and \(\beta = 0\) the result is Eq. 1, with all of the associated assumptions. Conversely by setting \(\gamma = 1\) an operating lease is approximated. For \(\beta \neq 0\) some form of managed services are approximated.

As a result the total derivative for a typical design vector can be represented in Eq. 50.

\[
\frac{dSV}{dX} = D_p[NU(1 - \gamma)D_c + NU\delta]\frac{d(R_u)}{dX}
+ D_p[NU(\beta - 1)(1 - \gamma)D_c + NU(-\beta + (\beta - 1)\gamma)\delta]\frac{d(C_u)}{dX}
- ND_p\frac{d(C_{man})}{dX} - \frac{d(C_{dev})}{dX}
+ (N(1 - \gamma)\delta)D_p[R_u - (1 - \beta)C_u]
+ ND_p(- \beta C_u + \gamma[R_u - (1 - \beta)C_u])\delta\frac{d(U)}{dX}
+ \delta U D_p(-\beta C_u + \gamma(-1 - \beta)C_u + R_u)\delta\frac{d(N)}{dX}
+ NU(1 - \gamma)D_c[R_u - (1 - \beta)C_u]\frac{d(D_c)}{dX}
+ N(U(1 - \gamma)D_c[R_u - (1 - \beta)C_u - C_{man}]\frac{d(D_p)}{dX}
\]

(50)

This form is quite useful as it represents a basic gradient formulation that could be used to create the design 'scorecards' as represented by Figure 8 in Collopy and Hollingsworth's paper on value driven design 2.
The form presented in Eqs. 49 and 50 are still based upon assumptions 1 through 7 as listed in Section II. In each of these cases the user has to evaluate whether or not the working conditions meet or are close to the working assumptions.

VI. Final Comments

The formulations presented in this paper investigate the use of the surplus value equation in commercial aircraft and engine design. Further, it shows the starting assumptions and derivation of the basic form of the surplus value equation as used by Cheung et al. 1 and Sutcliffe and Hollingsworth † amongst others. This paper also explores some of the implications with respect to design choices and design life with respect to the formulation of the surplus value equation. Additionally, this paper has looked at the effect of changing one of the critical assumptions in the basics surplus value equation, that of the simple sales contract. The results is that a more general formulation and its total derivative were presented allowing for a range of contracts from simple sales to operating leases to managed services formulations to be investigated.

What this paper did not present is how the design of an aircraft or engine would change based upon the change of formulation. This requires that relationships between revenues and costs in terms of design variables, e.g. \( R_a = f(\vec{X}) \), \( C_a = f(\vec{X}) \), etc, be understood. These relationships are functions of the physics of the problem, the design and technology architectures and the specifics of the businesses involved. Because of this complexity the further evaluation of designs is left to future papers and activities. Further, this paper did not explore the effects of opening up of assumptions 5, 6 or 7, all of which could create significant changes in the design philosophy. Again these investigations are left to future activities.

However, while this seems like only a small opening up of the flexibility of the surplus value equation, it is the opinion of the author that this small increase in flexibility presents significant additional opportunity to the end user.

Notwithstanding all of the benefits and limitations of the surplus value formulation from a financial analysis standpoint one other significant limitation is that the formulations presented within this paper do not allow for or incorporate the effect of hard contractual or regulatory limits. These are very real considerations. Other authors and this author are undertaking further work on how these limitations might be incorporated into the model presented here-in; however, at this time no specific insights can be provided.

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References

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