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Using AC Optimal Power Flow for DG Planning and Optimisation

Luis F. Ochoa, *Member, IEEE*, and Gareth P. Harrison, *Member, IEEE*

Abstract—While it is difficult to estimate how much distributed generation (DG) capacity will be connected to distribution systems in the coming years, it is certain that increasing penetration levels require robust tools that help assess the capabilities and requirements of the networks in order to produce the best planning strategies. This work presents an overview of some of the uses to which a tailored AC Optimal Power Flow (OPF) can be applied to DG planning and optimisation. Its application to maximising connectable capacity and energy loss minimisation are presented and discussed. The incorporation of a wide range of network constraints, including fault levels, voltage step change and N-1 security are also outlined. The AC OPF technique is extended to a multi-period approach to cater for the variability of demand and renewable generation. Advanced smart grid-like control strategies are also incorporated into this planning tool in order to evaluate the potential benefits. Simple test feeders as well as generic UK distribution systems are adopted to demonstrate the methodology.

Index Terms—Distributed generation, optimal power flow, active network management, wind power, distribution networks.

I. INTRODUCTION

FOR Distribution Network Operators (DNOs) in fully liberalised electricity markets (e.g., UK), planning the siting and sizing of Distributed Generation (DG) units is, in many respects, not possible. Due to unbundling rules, DNOs cannot invest in generation facilities and are meant to provide DG owners with cost-effective connection means, irrespective of the technology or geographical location. In this context, uncertainties due to, for instance, planning consents or financial support surrounding DG investments pose DNOs with major challenges as to what, where and when to reinforce the system in order to deliver timely connections without the risk of stranded assets. This lack of certainty and planning coordination translates into distribution networks that connect DG units by adopting a ‘fit and forget’, case-by-case approach where only traditional reinforcements (e.g., new lines or transformers) are carried out. Thus, any sophisticated solution – albeit potentially more cost-effective for society in the long

term – is left behind.

Depending on the particular circumstances of a DG development, such as resource availability, planning consents or declared net capacity, it might be possible to have more than one network integration scheme (i.e., connection point and/or operation strategy) that is economically sound for the DG owner. The economics of different locations becomes even more relevant if not only are infrastructure costs involved but also distribution connection charges. Indeed, DNOs could tune the latter to steer DG projects towards specific areas where the technical and economic impacts on the system are less onerous or even beneficial. Alternatively, bilateral commercial arrangements between DNOs and DG owners could also provide win-win situations. However, for DNOs to determine appropriate locational signals or commercial arrangements they need to investigate how capable their networks are for integrating renewable or conventional DG.

While it is difficult to estimate how much distributed generation capacity will be connected to distribution systems in the coming years, it is certain that increasing penetration levels require robust tools that help assess the capabilities and requirements of the networks in order to produce the best planning strategies. Whatever the particular driver for a given DNO, e.g., to allow the connection of more DG capacity or to reduce energy losses, these planning tools must take into account network constraints such as voltage and thermal limits. The inherent variability of demand and renewable generation (e.g., wind power) is an aspect that must also be considered. In addition, envisaging actively managed networks (as opposed to the current ‘fit and forget’ approach), where Smart Grid-like control schemes employing real-time control and communication systems allow more effective management of different network participants, including DG units and voltage regulation devices, need also to be accounted for.

Based on the authors’ previous and ongoing work, this paper compiles the different DG planning and optimisation applications to which Optimal Power Flow (OPF) techniques can be applied. The flexibility provided by a tailored AC OPF makes it possible to extend the analysis to cater for a number of complex aspects. It can handle multi-periods to deal with the variability of demand and renewable generation. Advanced control strategies such as coordinated voltage control, adaptive power factor and generation curtailment can also be incorporated to evaluate potential benefits. More

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complex problems resulting from other network constraints commonly overlooked by DG studies, such as fault levels, N-1 security and voltage step change, are also viable within this approach.

This paper, composed mainly by adapted extracts from [1-5], is structured as follows: First, the maximum connectable DG capacity problem [1, 6] is presented in section II. Then, section III addresses the energy loss minimisation problem [2, 6]. Section IV discusses the incorporation of further network constraints such as fault levels [3, 6], N-1 security [4, 7], and voltage step change [5]. Finally, section V concludes the work. For the benefit of the reader, the framework for handling variable resources and demand, as well as the basic mathematical structure for the multi-period AC OPF, including the incorporation of active network management schemes, are briefly included in the Appendix. In all the studies the method was coded in the AIMMS optimisation modelling environment [8] and solved using the CONOPT 3.14A NLP solver.

II. DISTRIBUTION NETWORK CAPACITY ASSESSMENT [1]

Distribution network capacity assessment studies, i.e., the evaluation of the spare connection capacity, have been approached in the literature from different angles, taking account of technical, economic and environmental issues. In [9-11], the siting and sizing of DG units was investigated using impact indices. A number of studies have adopted metaheuristics techniques, such as genetic algorithms [12-14], but analytical approaches have also been proposed [15, 16]. The use of linear programming was demonstrated in [17, 18], whereas the non-linear formulation based on the AC Optimal Power Flow (OPF) problem was adopted in [1, 4-6, 19-21].

The approach proposed by the authors in [1, 6], uses the non-linear programming (NLP) formulation of a multi-period AC OPF adapted to determine the maximum DG capacity able to be connected to a given network. In addition to effectively handling the time-variation of multiple renewable sites and demand, it also considers a range of active network techniques to allow maximum absorption of renewable generation capacity while respecting voltage statutory limits and thermal constraints. Active network management control algorithms including coordinated voltage management control of transformers and voltage regulators, adaptive power factor control and energy curtailment are embedded within the formulation.

The objective function corresponding to the maximisation of the total active DG capacity p of a set of generators G (indexed by g) across the set of periods M (indexed by m), is given by ($\forall m \in M$):

$$\max \sum_{g \in G} p_g, \quad (1)$$

It is subject to the constraints presented in the Appendix.

A. Case Study - Full EHV1 Network

Fig. 1 shows the EHV1 Network, a 61-bus 33/11-kV radial distribution system available in [22]. The interconnector, treated as a PV bus, has a target voltage of 1.00 pu and is able

to provide/absorb 15 MVA. In the original demand-only case, the OLTC at the substation has a target voltage of 1.045 pu at the secondary bus. The voltage regulator (VR) and OLTCs on the 33/11-kV rural distribution transformers have a target voltage of 1.03 pu. Voltage limits are $\pm 6\%$ of nominal, reflecting UK practice. The total peak demand is 38 MW.

Hourly demand for central Scotland in 2003, as well as two different wind sites (named here, WP1 and WP2) derived from UK Meteorological Office measured wind speed data [23], are adopted. Six wind generation sites are available. Buses 1105, 1106, 1108) are considered to all use the WP1 profile. The network contains a subsea cable connecting the 'mainland' with an island on which the other three sites lie (1113, 1114, 1115). The island enjoys approximately the same wind resource as the first group but is sufficiently distinct to require the use of WP2.

Fig. 1 UK GDS EHV1 Network [22] and potential locations for distributed wind power generation.

Fig. 2 Full EHV 1 Network: Connectable DG capacity (in MW) with ANM strategies (c: capacitive, i: inductive, and PFC: adaptive power factor control).

The aggregate DG capacity that can be connected to the six sites has been evaluated for all control modes and the results

are shown in Fig. 2. Due to the number of potential locations and their corresponding proximity to loads, relatively high values of connectable DG capacity were found. It is clear that the greater flexibility offered by adaptive power flow control (PFC) leads to the largest wind power generation capacities in all cases. Without coordinated voltage control ('no CVC' case), PFC alone allows a DG penetration of 85% relative to peak demand (an increase of around a fifth over passive management). The allowable penetration reaches 103% when applying CVC, while curtailment, permits progressively greater integration of wind capacity, reaching 118% penetration for a limit of 2% and 143% at the 10% curtailment level (i.e., doubling capacity).

Given the detailed modeling of each DG site, it is possible to have a breakdown of the connectable generation capacity across the six sites (Fig. 3). The impact of using coordinated voltage control and adaptive power factor control on the available capacity of each location is evident. In the passive management case with fixed 0.98 capacitive power factor, more than 73% of the total capacity is sited on the mainland (1108, 1106 and 1105). Adopting the CVC and PFC schemes, however, it is possible to make much more capacity available on the island, increasing it by almost three-fold to a 46% share of the total capacity.

Fig. 3 Locational breakdown of available DG capacity across the sites. Except for *, 33/11kV transformers at the DG connection nodes reached their thermal limits.

III. MINIMISING ENERGY LOSSES [2]

Capturing the effects that the variability of both demand and (renewable) generation has on total energy losses for a given horizon is essential as it considers the actual metrics used by DNOs. Modeling DG plants as firm generation (to some extent a less complex optimisation problem) has been adopted for loss analyses using Tabu Search [24] or Genetic Algorithm (GA)-based multiobjective approaches [13]. As for variable (renewable) generation, the optimal allocation of DG plants based on impact indices (including losses) was previously proposed by the authors in [11], and extended to a GA-based multiobjective formulation in [14]. Energy losses were also considered in [25], where it was presented a multi-resource GA-based multiobjective technique that catered for some aspects of active network management through the use of a linearised OPF. Energy loss minimisation was also studied in [26] through the optimal mix of statistically-modelled renewable sources considering a passive approach to manage the network. An AC OPF-like (reduced gradient) method applied to a (power) loss minimisation problem was proposed in [19]. However, in this and other OPF-based

approaches [20, 21], only peak demand and passive operation of the network were considered.

Here, the AC OPF technique used in the previous section is tailored to minimise energy losses across a given time horizon. The objective function corresponding to the minimisation of the total energy losses of the network over a time horizon comprising m periods, $m \in M$, is given by:

$$\min \sum_{m \in M} \left(\sum_{l \in L} f_{l,m}^{1,P} + f_{l,m}^{2,P} \right) \cdot \tau_m \quad (2)$$

where $f_{l,m}^{1,P}$ and $f_{l,m}^{2,P}$ are the active power injections at each end (denoted 1 and 2) of branch l , $l \in L$; and, τ_m is the duration of period m . The difference between the net injections at each end of the branch defines the energy loss.

A. Case Study - Full EHV1 Network

The previously described EHV1 Network (Fig. 2), together with the same data for demand and wind profiles (WP1 and WP2), is considered in this analysis. Using the segmented demand scenarios (similar to those applied in section II) for the original configuration (no DG) results in an annual consumption of 214 GWh with energy losses of 4.7% (comparable with typical UK rural networks).

Fig. 4 presents the minimum percentage energy losses that can be achieved if wind power generation is optimally accommodated under each operating strategy and without exceeding voltage or thermal limits. Compared to the original losses significant gains are achieved when optimally accommodated. Assuming a business as usual (BAU) management of the network unity power factor operation of the DG units sees energy losses reduced by 40%. If coordinated voltage control is incorporated, then losses are cut by more than a half. From all the studied cases, the adoption of both CVC and adaptive power factor control lead to the lowest losses. Nonetheless, it is clear that, for this particular network, the largest benefits are brought about by the CVC scheme, raising the question of the cost effectiveness of using further control mechanisms.

Fig. 4 Percentage of energy losses considering wind power generation – business as usual operation and two different Smart Grid strategies.

In terms of installed capacity, Fig. 5 shows the total values found for each of the analysed cases. Due to the variable wind availability for the different demand levels, critical scenarios

such as minimum and peak demand do not normally coincide with maximum wind potential. For this reason, more capacity than for firm generation can be connected to the network. It can also be seen that, again, generation capacity is strongly related to the reduction of losses. It is worth pointing out that while in most cases the CVC scheme only allows a marginal increase in capacity, when DG units are operated at 0.98 capacitive power factor, the gain is much more significant. This is primarily due to the ability of the CVC scheme to alleviate voltage rise problems. As for the PFC scheme, whilst it does provide lower losses, it is also clear that, for this network, similar gains can easily be achieved by setting the operation of the generators to unity or capacitive power factor.

Fig. 5 Total DG capacity – business as usual operation and two different Smart Grid strategies.

IV. INCORPORATING MORE NETWORK CONSTRAINTS

This section presents the underlying concepts behind the incorporation of more complex network constraints (irrespective of the objective function), such as fault levels, N-1 security and voltage step change. Although, they can be implemented within a multi-period framework, for simplicity and the fact that these constraints are more important during worst-case scenarios, the formulation is presented for the single-period analysis (index m is not used).

A. Fault Levels [3, 6]

Only a few capacity assessment studies have considered fault level contributions from DG [10, 11, 13, 18]. Fault level constraints were formally included in the OPF approach presented in [21], where the corresponding derivatives with respect to the OPF variables were obtained by expressing the Z_{bus} as a function of new capacity. It used a highly complex series of matrix operations to enact this and works well in meshed networks. Its formulation does, however, make it very difficult to integrate with other analyses such as security constraints [4, 27], voltage step change [5] or multiple time periods [1, 28].

Here, a simplified approach to cater for fault contributions that avoids the use of derivatives is adopted, allowing better integration within optimisation modelling environments. The three-phase fault at the substation is calculated according to the IEC 60909 standard with the calculation of fault level contributions from DG based on the corresponding rated currents and the generation technology. The design short-circuit capacity of the network provides the constraint.

With distribution networks typically radial in configuration, the maximum fault level will be obtained when

considering a three-phase fault at the MV busbar of the substation. The initial symmetrical short-circuit power at the MV busbar resulting from the contributions of both the upstream grid and the DG capacity connected to each feeder has to be limited to the design short-circuit capacity of the network, SC_{max} . The IEC 60909 standard is adopted for the corresponding calculations.

The fault level constraint incorporated into the single-period AC OPF formulation is as follows:

$$\frac{\overbrace{c_{max} \cdot V_{b_1}^2}^{\text{upstream grid}}}{\left| \bar{Z}_{SYS\ MV} + \bar{Z}_T \right|} + \sum_{g \in G} \frac{\overbrace{c_{max} \cdot V_{b_1}^2}^{\text{DG fault contribution}}}{\left| \bar{Z}_{(b_g, b_1)} + \bar{Z}_g \right|} \leq SC_{max} \quad (3)$$

where $\bar{Z}_{SYS\ MV}$ is the equivalent impedance of the upstream grid (at the secondary side) and $\bar{Z}_T$ is the short-circuit impedance of the substation transformer(s). The second term on the left hand side of (3) corresponds to the fault level contributions from the distributed generators. $\bar{Z}_{(b_g, b_1)}$ is the line impedance from the DG connection point to the MV busbar, while $\bar{Z}_g$ represents the (equivalent) impedance of the generator(s) g . The simplified formulation of (3) implies only one DG connection per feeder; however, it can be tailored to cater for multiple DG configurations.

For planning and indicative studies it is common to consider the fault current contribution at the network terminals of a given DG plant, I_{k, b_g}^* , to be a multiple of the corresponding rated current, I_{rg} , called contribution factor ($CF = I_{k, b_g}^* / I_{rg}$). To explore the effects that different fault level contributions from DG have on the total generation capacity that can be accommodated, this (linear) dependency is taken into account. Consequently, $\bar{Z}_g$ will be a function of the contribution factor CF .

B. N-1 Security [4, 7]

The analysis of N-1 contingencies (e.g., loss of a line) can also be incorporated in the AC OPF (also known as Security Constrained OPF). It ensures that no limits are exceeded even during contingencies that might occur in the system. In practical terms, multiple network topologies are simultaneously analysed.

Apart from the multiple periods (if taken into account), multiple topologies, K , due to N-1 contingencies require different sets of power flow variables. Thus, all the affected equations need to be adapted to cater for N-1 security. For example, the voltage limits at each bus (see Appendix) will be re-formulated considering the index k : $V_b^- \leq V_{b, m, k} \leq V_b^+$.

C. Voltage Step Change [5]

Most DG studies do not consider a particular requirement of the distribution networks: voltage step constraints on loss of a generator, which is a quite distinct issue from voltage rise. Voltage step changes occur when a DG is started up or suddenly disconnected from the network, and limits are

typically placed on the maximum step change allowed.

Voltage step constraints are incorporated using a security constrained OPF-like formulation (see previous subsection), where the contingencies considered are outages of generators rather than lines. Thus, the voltage step constraint to be incorporated into the single-period AC OPF is:

$$V_b - V_S^+ \leq V_{b,g'} \leq V_b + V_S^+ \quad (4)$$

where for an outage of generator g , indexed by g' , the contingency voltage $V_{b,g'}$ at bus b must differ by no more than V_S^+ from the pre-outage voltage V_b . In practical terms, extra topologies – similar to the N-1 security constraint – are evaluated considering (4) on top of other network constraints.

V. CONCLUSIONS

The AC Optimal Power Flow technique is widely accepted and mainly used to solve the economic dispatch problem. It has been demonstrated by a number of studies that it provides a platform where other network analyses can be carried out. This paper has compiled the authors' previous and ongoing work on DG Planning and Optimisation using the AC OPF technique. The way this approach can handle real-life complexities such as variable demand and generation, different network constraints, and Smart Grid-like control schemes, proves its versatility and robustness.

It is important to point out that increasing the level of complexity will result in longer computing times. Ultimately, solving extremely large problems, in terms of the number of variables and constraints, will depend on the capability of the commercial or non-commercial solvers available.

VI. APPENDIX [1, 2]

A. Framework for Handling Variable Resources and Demand

To diminish the number of periods to be evaluated whilst preserving the behaviour and inter-relationships between resource and demand, it is used a process of discretisation and then aggregation according to the characteristics of 'similar' periods. To illustrate this, Fig. 6 (top) presents a week-long snapshot of hourly demand and wind power data for central Scotland in 2003 [23].

Fig. 6 (Top) Winter week hourly demand and wind power for central Scotland, 2003 [36]. (Bottom) Discretised data processed before aggregating the coincident hours of each demand-generation scenario.

Fig. 6 (bottom) shows the discrete values following the allocation of the original data into a series of 7 bins covering specific ranges ($\{0\}$, $(0,0.2pu]$, $(0.2pu,0.4pu]$, ..., $(0.8pu,1.0pu]$, $\{1.0\}$) in which the mean values (e.g., 0.3pu for the $(0.2pu,0.4pu]$ range) characterised each new hour. The aggregation process groups hours in which the same combination of demand and generation occur. For instance, the arrows point to hours where demand is 0.7pu and wind is zero; these conditions occur for a total of 18 hours in this particular week. This will constitute a period to be evaluated along with other combinations of different total duration in the optimisation problem. References [1, 2] provide a more detailed treatment of the framework.

B. Multi-Period AC Optimal Power Flow

Although the objective function will vary according to the study, i.e., either DG capacity maximisation or energy loss minimisation, the mathematical structure to consider basic network constraints, such as voltage and thermal limits, remains the same, as well as the multi-periodicity, in terms of demand/generation combinations. This multi-periodicity is achieved by providing each combination, m , with a different set of power flow variables with a unique, *inter-period* set of generation capacity variables is used throughout the analysis.

Given an objective function, the basic multi-period AC OPF formulation (single topology) will be subject to a range of constraints (more details in [1, 2]). Voltages at each bus b (B , set of buses) are constrained by maximum/minimum levels $V_b^{(+,-)}$:

$$V_b^- \leq V_{b,m} \leq V_b^+ \quad \forall b \in B \quad (5)$$

Thermal constraints on the flow at each end of lines and transformers, l (L , set of lines):

$$\left(f_{l,m}^{(1,2),P}\right)^2 + \left(f_{l,m}^{(1,2),Q}\right)^2 = \left(f_l^+\right)^2 \quad \forall l \in L \quad (6)$$

where $f_{l,m}^{(1,2),P}$ and $f_{l,m}^{(1,2),Q}$ are the active and reactive power injections at each end of the branch and f_l^+ is the branch apparent power flow limit.

Active and reactive power injections into each end of the lines are governed by Kirchhoff's voltage law:

$$f_{l,m}^{(1,2),(P,Q)} = f_{l,(1,2),m}^{KVL(P,Q)}(\mathbf{V}_m, \delta_m) \quad \forall l \in L \quad (7)$$

where $f_{l,(1,2),m}^{KVL P}(\mathbf{V}_m, \delta_m)$ and $f_{l,(1,2),m}^{KVL Q}(\mathbf{V}_m, \delta_m)$ are standard Kirchhoff voltage law expressions.

Kirchhoff's current law describes the active and reactive nodal power balance, $\forall b \in B$:

$$\sum_{l \in L | \beta_l^1 = b} p_{b,m}^L + d_b^P \eta_m = \sum_{g \in G_b | \beta_g = b} p_g \omega_m + \sum_{x \in X | \beta_x = b} p_{x,m} \quad (8)$$

$$\sum_{l \in L | \beta_l^1 = b} q_{b,m}^L + d_b^Q \eta_m = \sum_{g \in G_b | \beta_g = b} p_g \omega_m \tan(\phi_{g,m}) + \sum_{x \in X | \beta_x = b} q_{x,m} \quad (9)$$

Here $(p, q)_{b,m}^L$ are the total power injections into lines at b ; and $d_b^{(P,Q)}$ are the peak active or reactive demands at the same bus. In period m , η_m is the demand level relative to peak and

ω_m is the generation level relative to nominal capacity as dictated by the variable (renewable) resource in that period.

The distribution network has external connections x (X , set of external sources) at the GSP (and interconnectors) through which power can be exported/imported subject to flow constraints :

$$\left. \begin{aligned} p_x^- &\leq p_{x,m} \leq p_x^+ \\ q_x^- &\leq q_{x,m} \leq q_x^+ \end{aligned} \right\} \quad \forall x \in X \quad (10)$$

The GSP is taken as the reference (slack) bus b_0 with the voltage angle set at zero, i.e., $\delta_{b_0,m} = 0$.

C. Incorporating Active Network Management

In addition to voltage and thermal constraints, variables and constraints derived from ANM schemes must also be incorporated in the method:

1) Coordinated Voltage Control (CVC)

In each period the secondary voltage of the OLTC will be treated as a variable, rather than a fixed parameter, while maintaining its value within the statutory range:

$$V_{b_{OLTC}}^- \leq V_{b_{OLTC},m} \leq V_{b_{OLTC}}^+ \quad (11)$$

2) Adaptive Power Factor Control (PFC)

Here, it is envisaged that DG provides a *dispatchable* power factor scheme, hence the power angle of each generator, $\phi_{g,m}$, is considered as a variable. In practice DG will be required to operate within a certain range of power factors ($\phi_g^{(+,-)}$); the following constraint applies:

$$\phi_g^- \leq \phi_{g,m} \leq \phi_g^+ \quad (12)$$

3) Energy Curtailment

Power curtailment is formulated here by adding a negative generation (or positive demand) variable ($p_{g,m}^{curt}$) at the same location of each DG unit; solely affecting the constraints related to active and reactive nodal power balance. Thus, (8) and (9) are, adapted by adding terms $\sum_{g \in G_b} p_{g,m}^{curt}$ and

$\sum_{g \in G_b} p_{g,m}^{curt} \tan(\phi_{g,m})$, respectively.

To examine the impact of different allowed levels of curtailment on overall DG capacity, the total amount of curtailed energy from each DG will be restricted to a curtailment factor λ_{curt} , a percentage of the potential energy that could have otherwise been delivered by each DG. The following constraint follows:

$$\sum_{m \in M} p_{g,m}^{curt} \tau_m \leq \lambda_{curt} \left[\sum_{m \in M} p_g \omega_m \tau_m \right] \quad \forall g \in G \quad (13)$$

where τ_m is the duration of period m . The curtailment variables $p_{g,m}^{curt}$ need to be limited to the output of g at the corresponding period:

$$p_{g,m}^{curt} \leq \omega_m p_g \quad \forall g \in G \quad (14)$$

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