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Minimal Cross-Subsidies Approach for Loss Allocation in Distribution Networks with Open Access

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Abstract— Open access philosophy applied by regulatory agencies may lead to a scenario where captive consumers will solely face the responsibility on distribution network's losses even with Independent Energy Producers (also known as Distributed Generation) and Independent Energy Consumers connected to the system. This work proposes the utilization of a loss allocation method in distribution systems where open access is allowed, in which cross-subsidies, that appear due to the influence the generators have over the system losses, are minimized. Thus, guaranteeing to some extent the efficiency and transparency of the economic signals of the market. Results obtained through the Zbus loss allocation method adapted for distribution networks are processed in such a way that the corresponding allocation to the generation buses is divided among the consumer buses, while still considering consumers spatial characteristics.

Index Terms-- Loss allocation, distribution systems, open access, distributed generation.

I. INTRODUCTION

THE search for economic efficiency in transmission systems, which started after the deregulation of the electric sector in various countries, has propelled the appearance of a number of methods for loss allocation. These methods, in general, try: to reflect power or current injections at each bus; to reflect network topology; to be simple for understanding and implementation; to provide effective incentives or otherwise to generation or load buses; and, to be consistent with the power flow solution. Nonetheless, specialized literature agrees that there are difficulties when adopting a loss allocation method since there will always be a certain degree of arbitrariness due to the non-linear characteristic of losses.

As for distribution systems, currently most electric utilities do not have a policy where technical losses are allocated among the participant agents according to a strictly technical criterion, as it usually occurs with transmission networks.

Commonly, the billing due to the consumed energy by each client includes a share of the system's losses. In Brazil, for instance, this share is calculated according to the voltage level of the captive consumer, independently of its location and size relative to the other captive consumers at the same voltage level. Consequently, this share is not the reflection of a fair allocation since it is solely based on the energy cost calculation.

Moreover, given the open access philosophy applied by regulatory agencies, Independent Energy Producers (IEPs) – also known as Distributed Generation (DG) – and Independent Energy Consumers (IECs) are allowed to connect to the distribution network and negotiate energy contracts with agents outside the host distribution network. In this way, a scenario where open access is used, distribution network's losses, result of the presence of all agents, i.e., IEPs, IECs and captive consumers, will solely be responsibility of the latter. This may be considered unfair given that IEPs and IECs power flows are indeed using the host distribution network, affecting the total power losses.

Application of loss allocation methods found in the literature would result, in many cases, in negative loss allocation for generation units due to their contribution diminishing network's total losses. Such a negative allocation may create high cross-subsidies values affecting consequently the efficiency and transparency of the economic signals of the market [1], [2].

Considering the regulating framework above described, in this work it is proposed the utilization of a loss allocation method adapted for distribution systems with open access. Cross-subsidies are to be minimized by re-allocating generators' loss allocation to the consumers (IECs and captive consumers).

The paper is set out as follows: section 2 describes the Zbus loss allocation method adapted for distribution networks [3], as well as the method for the loss re-allocation based on Zhao *et al.* [4]. A small example is solved for showing the details of the calculation process. In section 3 the analysis for the IEEE-34 test feeder considering the insertion of generation units is performed. Results are presented and discussed. Conclusions are drawn in section 4.

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II. LOSS ALLOCATION IN DISTRIBUTION NETWORKS

Loss allocation method Zbus [5], that is to be applied in distribution systems, was selected mainly due to the use of exact network equations, thus not requiring approximations, and to be of easy implementation. The process used to detach the generation units from the final calculation of loss allocation, i.e., to distribute the losses allocated in the generation nodes into the consumer nodes, is based on [4].

A. The Zbus method for Distribution Networks

Proposed in 2001 by Conejo *et al.* [5], the Zbus loss allocation method is utilized in transmission systems. It firstly considers the network's admittance matrix $Y = G + jB$, typically large, sparse and non-singular. Knowing the complex values of the nodal current injections I and the nodal voltages V , system's real power losses may be expressed in terms of Y and V , or Z and I ; where $Z = Y^{-1} = R + jX$ is the network's impedance matrix.

Therefore, total power losses L are systematically distributed among the n nodes found in the system:

$$L = \sum_{i=1}^n L_i \quad (1)$$

where L is computed in the following manner:

$$L = \text{Re} \left\{ \sum_{k=1}^n I_k^* \left(\sum_{j=1}^n Z_{kj} I_j \right) \right\} \quad (2)$$

Impedance Z_{kj} may be decoupled into its resistance and reactance, nonetheless, the summation related to the reactance represents the aggregation of the power flows of those nodes connected through matrix Z to node k , which results in zero. Consequently, (2) can be re-written as:

$$L = \text{Re} \left\{ \sum_{k=1}^n I_k^* \left(\sum_{j=1}^n R_{kj} I_j \right) \right\} \quad (3)$$

Finally, total losses can be decoupled in n fractions of losses that represent the losses attributed to each node:

$$L_i = \text{Re} \left\{ I_i^* \left(\sum_{j=1}^n R_{ij} I_j \right) \right\} \quad (4)$$

At this point, two particularities of distribution systems should be considered: network's admittance matrix Y is singular since the capacitive effect of the lines is not taken into account; and, the loss allocation must be performed among the agents that utilize the network. Thus, with the aim of eliminating matrix Y 's singularity and maintaining coherence with the loss allocation results, it is necessary to exclude the participation of the reference node, i.e., the substation. It is important to remark that this consideration does not affect neither the fair allocation nor the value of the system's total losses [3].

B. Loss Re-allocation

Proposed in 2004 by Zhao *et al.* [4], the Improved Zbus method was presented to be used in transmission networks. In distribution systems, total power losses of a network with

IEPs can be divided into two parts: one caused by the generation (L_G) and other corresponding to the consumers (L_C). Considering that the economic equilibrium of the network must be kept, the extra cost created by the loss allocation should be subtracted by the levy of the generation and increased in the cost of the consumers. Assuming a scenario where all generators are paid based on a single price p_G and all consumers have to pay for the energy supply based on a single price p_C , the total payment for the generators (α) would be calculated as follows:

$$\alpha = p_G \cdot \sum_{k=1}^{N_G} (P_{Gk} - L_{Gk}) \quad (5)$$

and the total billing (β) of the consumers would be:

$$\beta = p_C \cdot \sum_{i=1}^{N_C} (P_{Ci} + L_{Ci}) \quad (6)$$

whereas N_G and N_C are the total number of generation units and consumers, respectively.

Due to the equilibrium among economic transactions, i.e. α is equal to β , it is obtained:

$$p_G = \frac{p_C \cdot \sum_{i=1}^{N_C} (P_{Ci} + L_{Ci})}{\sum_{k=1}^{N_G} (P_{Gk} - L_{Gk})} \quad (7)$$

Thus, the total cost produced by the losses allocated to generation units (α_L):

$$\alpha_L = p_G \cdot \sum_{k=1}^{N_G} L_{Gk} \quad (8)$$

can be re-written by using (7):

$$\alpha_L = p_C \cdot \frac{\sum_{i=1}^{N_C} (P_{Ci} + L_{Ci})}{\left(\sum_{k=1}^{N_G} P_{Gk} / \sum_{k=1}^{N_G} L_{Gk} \right) - 1} \quad (9)$$

The expression that is multiplied by the price p_C represents the total losses allocated to the generators. This representation allows distributing that value to each consumer j present in the network. Thus, the corresponding new loss allocation is computed by the following expression:

$$L_{Cj\text{ novo}} = L_{Cj} + \frac{P_{Cj} + L_{Cj}}{\left(\sum_{k=1}^{N_G} P_{Gk} / \sum_{k=1}^{N_G} L_{Gk} \right) - 1} \quad (10)$$

It is important to highlight that this process requires maintaining coherence among generation, demand and losses; therefore, the substation must be included as being a "generator" whose injected power is obtained through the power flow solution. Moreover, given that in the loss allocation calculation the reference node was eliminated for the network's admittance matrix, the corresponding loss allocation is equal to zero.

This methodology considers as a generation node that whose injected power exceeds its demand. Analogically, a consumer node will present demand larger than the injected

power in case there is a generation unit installed on it.

C. Application Example

Fig. 1 shows a simple single-line radial feeder with the respective load (power factor equal to unity) and voltage data. Each line section has a impedance equal to $1.1993 + j0.877 \Omega$.

Fig. 1. Single-line radial feeder.

Varying the power generation output (unity power factor) of the generation unit inserted at node C, the Zbus loss allocation method adopted for distribution systems (substation is not taken into account) was applied. Results are presented in Table I. Fig. 2 presents the behavior of the losses allocated at each power generation scenario.

TABLE I
RESULTS OF THE ADAPTED ZBUS LOSS ALLOCATION FOR FEEDER IN FIG. 1

Power Generation Output (kW)	Losses (kW)			
	A	B	C	Total
0	2.3538	3.7815	2.1300	8.2652
300	1.6395	2.3470	-0.5278	3.4587
600	0.9350	0.9371	-0.0065	1.8656
800	0.4707	0.0108	2.0585	2.5400
1000	0.0106	-0.9053	5.4660	4.5713

Fig. 2. Results of the Zbus loss allocation for the feeder in Fig. 1.

As expected, it is verified that the generator diminishes significantly the total losses in the feeder. However, this occurs until a certain scenario where this value starts to rise. When the generation unit helps effectively to decrease losses (power output of 300 and 600 kW), negative losses (cross-subsidies) are allocated to node C. A power generation output of 800 kW produces more power flows towards the substation (counter flows or reverse flows), leading to positive loss allocation for node C. As for 1000 kW power generation, the Zbus method is consistent showing that the main agent in the total losses is node C. In this case, negative losses are allocated to node B, producing again cross-subsidies.

In Fig. 2, it is visible that from a given power generation output (in this case, 300 kW) the losses allocated to node C exhibit an increasing trend. As for the load nodes, this trend is always decreasing.

Equation (11) shows the numeric calculation required to obtain the new loss allocation for node A considering a 300 kW power generation output. It should be taken into account that: the value of 703.4587 kW corresponds to the power injection from the substation; load at node C (200 kW) is subtracted from the power generation; and, loss allocation at the substation is equal to zero.

$$L_{A_{new}} = 1.6395 + \frac{400 + 1.6395}{\frac{703.4587 + 300 - 200}{-0.5278} - 1} \quad (11)$$

$$L_{A_{new}} = 1.3758$$

The results corresponding to the re-allocation of those losses obtained in Table I, are presented in Table II. The values of the total losses coincide with that computed by the power flow analysis. Also, there are no losses allocated to the generation node. Fig. 3 shows the curves produced by the new loss allocation values. It can be noticed that both load nodes, A and B, present trends similar to that presented by the generation node (C) in Fig. 2.

TABLE II
RESULTS OF THE LOSS RE-ALLOCATION FOR THE FEEDER IN FIG. 1

Power Generation Output (kW)	Losses (kW)		
	A	B	Total
300	1.3758	2.0828	3.4587
600	0.9317	0.9339	1.8656
800	1.5006	1.0394	2.5400
1000	2.7467	1.8246	4.5713

Fig. 3. Results of the loss re-allocation for the feeder in Fig. 1.

III. CASE STUDY

To evaluate the performance of the proposed methodology a medium voltage distribution network will be utilized. Two IEPs (DG units) will be installed in distant nodes in order to emphasize their impact on power losses. Initially, aimed at evaluating the influence of the generators, a number of scenarios varying their power output while keeping the same loading will be analyzed. Later, a given power generation output will be fixed and various loading scenarios analyzed.

Results raise the discussion of whether the loss allocation problem should be utilized for specific demand-generation scenarios or should consider the time-varying characteristic of such parameters.

Medium voltage IEEE-34 test feeder [6] is shown in Fig. 4. The total single-line demand considered is 590 kW, whereas 72% of the loads are located 56 km from the root node. The feeder has ACSR 1/0, 2 and 4 conductors. Transformer originally located between nodes 19 and 20 is substituted by a line section, i.e., the entire feeder is modeled at a single voltage level. Two DG units are inserted at nodes 30 and 33, which are distant from the substation and also close to the load concentration. Total power losses of the original network (without DG) are 165.97 kW.

Fig. 4. Single-line IEEE-34 test feeder [6] with two DG units.

Table III shows the results of loss allocation considering the different power generation outputs. In the first scenario, each generator produces 30 kW (unity power factor), representing 10% of the total demand. Although the loads at nodes 30 and 33 represent 52% and 79%, respectively, of the power generation, the impact on the total power losses is not negligible. Indeed, this scenario led to a reduction of 28% of total power losses, compared to the original configuration. Therefore, the adapted Zbus method allocated negative losses to the generation nodes 30 and 33, achieving 2.7% and 4.4% of the total power losses, respectively. The summation of these values represents the cross-subsidies that, in this case, the consumers would have to pay in excess. Thus, while the consumers may find reasonable to share the economic responsibility produced by 119 kW of total power losses, according to the adapted Zbus method, they should actually share the responsibility of 127 kW, i.e. 7.1% more. This situation is overcome by the loss re-allocation method which eliminates the participation of the generation nodes and, while cross-subsidies still appear (nodes 1 and 2), the procedure leads to an expressive reduction of the total cross-subsidies: 0.12% of the total power losses.

An increase on the power generation output produces a larger reduction of the system's total power losses, as it can be verified when both generators are set to 200 kW, supplying 67% of the feeder's total demand. In this case, the losses are still decreasing, i.e., the concept of "the larger the power generation output the smaller the network's total power losses" is still working, thus the consequent negative loss

allocation to nodes 30 and 33 performed by the proposed Zbus method. The total cross-subsidies here accounts for 163.3 % of the total power losses. Certainly, while this is a significant increase on the final bill of the consumers in terms of percentage, the economic implication will actually be much better than the previous generation scenario since the overall power losses decreased by 93% compared to the original configuration. Nonetheless, cross-subsidies still may represent a hassle and may not lead to an agreement of the participating agents. The application of the re-allocation method, in addition to eliminate the loss allocation to generation nodes, reduced significantly the cross-subsidies for this scenario. In fact, the summation of the negative losses now allocated to nodes 1, 2, 3 and 4 accounts for 5.7% of the total power losses (28 times smaller than that computed by the adapted Zbus method).

TABLE III
RESULTS OF THE ADAPTED ZBUS ALLOCATION AND RE-ALLOCATION
METHODS FOR THE IEEE-34 TEST FEEDER VARYING THE POWER GENERATION
OUTPUT OF BOTH DG UNITS INSERTED AT NODES 30 AND 33

Node	Each DG unit 30 kW Losses (kW)		Each DG unit 200 kW Losses (kW)		Each DG unit 300 kW Losses (kW)	
	Zbus	Re-alloc.	Zbus	Re-alloc.	Zbus	Re-alloc.
1	0.0314	-0.0833	0.0108	-0.2843	0.0022	0.2052
2	0.0522	-0.0627	0.0177	-0.2776	0.0033	0.2064
3	0.1311	0.0959	0.0411	-0.0467	0.0045	0.0642
4	0.1312	0.0960	0.0412	-0.0466	0.0046	0.0643
8	0.1174	0.1060	0.0333	0.0066	0.0013	0.0190
9	0.8573	0.7759	0.2508	0.0601	0.0219	0.1482
10	1.3512	1.2326	0.3823	0.1074	0.0204	0.2018
11	4.5039	4.0977	1.4322	0.4830	0.2794	0.9078
12	0.2964	0.2703	0.0828	0.0225	0.0029	0.0426
13	1.1055	1.0085	0.3133	0.0887	0.0176	0.1658
14	3.6227	3.2982	1.1638	0.4058	0.2418	0.7437
15	3.1425	2.9022	0.8256	0.2803	-0.0231	0.3338
16	0.1359	0.1256	0.0355	0.0120	-0.0013	0.0141
18	0.1360	0.1256	0.0355	0.0120	-0.0013	0.0142
19	0.6236	0.5848	0.1485	0.0637	-0.0189	0.0358
21	2.0869	1.9591	0.4855	0.2072	-0.0770	0.1022
22	39.6814	37.3255	10.2771	5.1430	-0.0049	3.3050
23	7.6864	7.2221	1.7142	0.7079	-0.3765	0.2706
24	0.0779	0.0732	0.0189	0.0087	-0.0018	0.0048
25	15.2451	14.3353	3.4766	1.5073	-0.6381	0.6277
26	0.3896	0.3662	0.0875	0.0367	-0.0182	0.0145
27	5.2835	4.9657	1.1558	0.4685	-0.2873	0.1543
28	35.4166	33.1881	6.7917	1.9614	-3.2471	-0.1412
29	1.2192	1.1456	0.2645	0.1054	-0.0694	0.0328
30	-3.1599	--	-7.8650	--	7.6078	--
31	2.9178	2.7412	0.6290	0.2469	-0.1723	0.0733
32	1.2197	1.1461	0.2647	0.1057	-0.0691	0.0330
33	-5.2607	--	-10.7295	--	4.4424	--
Total	119.0416	119.0416	11.3857	11.3857	7.6440	7.6440

In the last case presented in Table III occurs what was commented in subsection 2.2: the generation exceeds the network's total demand and, consequently, the total losses exhibit an increasing trend. Here the adapted Zbus method allocates positive losses to the generation nodes pointing out their responsibility on the "increase" of network's power losses. Actually, the total power losses decreased by 95% compared to the original configuration, nonetheless, in this case where the demand is fully covered by DG various load nodes are allocated with negative losses. The total value of the cross-subsidies accounts for 65% of the network's total power losses. The re-allocation procedure made this value fall to 1.8%.

Fig. 5 graphically expresses how the network's total real

power losses decreased at each generation scenario. Moreover, a comparison between the adapted Zbus and Re-allocation methods is presented in order to highlight how the final bill of those agents sharing the total loss allocation (summation of absolute values) will be affected. It is clear the need of minimizing cross-subsidies so proper market signals can be provided.

Fig. 5. Comparison of the total losses (summation of absolute values) allocated by the adapted Zbus and Re-allocation methods for the IEEE-34 test feeder varying the power generation output of both DG units inserted at nodes 30 and 33.

In order to observe the behavior of the loss allocation results obtained through the adapted Zbus method, results of generation nodes 30 and 33, as well as those of load nodes 11, 22 and 28 are plotted in Fig. 6, considering more power generation output scenarios.

Fig. 6. Results of the adapted Zbus loss allocation method for the IEEE-34 test feeder varying the power generation output of both DG units inserted at nodes 30 and 33.

It is verified that from 300 kW injected by each DG unit, the method solely allocates positive losses to the generation nodes. Nodes 28 and 22 represents the most loaded ones, whereas node 11 is the least loaded one. It can be verified that, although loss allocation depends on the loading and location of a given node, these three nodes exhibit decreasing trends when increasing the power generation output.

After applying the Re-allocation of losses, it is observed through Fig. 7 that the load nodes 11 and 22 follow the same

trend of the total network's real power losses. The curve obtained by the addition of allocated losses to nodes 22 and 28 (their loading achieve 49.8% of the network's total demand) shows that they receive at all generation scenarios more than half of the total losses. This is coherent due to both the loading and location (load concentration) of both nodes.

Fig. 7. Results of the Re-allocation method for the IEEE-34 test feeder varying the power generation output of both DG units inserted at nodes 30 and 33.

TABLE IV
LOSS RE-ALLOCATION CONSIDERING DIFFERENT LOADING FOR THE IEEE-34 TEST NETWORK – DG AT NODES 30 E 33

node	Loading 10%	Loading 50%	Loading 120%
Loss Re-allocation (kW)			
1	0.1516	-0.0679	-0.4870
2	0.1511	-0.0663	-0.4644
3	0.0420	-0.0110	-0.0116
4	0.0420	-0.0110	-0.0115
8	0.0110	0.0016	0.0802
9	0.0786	0.0143	0.5973
10	0.1107	0.0254	0.9796
11	0.3906	0.1142	3.3061
12	0.0242	0.0053	0.2142
13	0.0905	0.0210	0.8022
14	0.3119	0.0959	2.6767
15	0.2090	0.0657	2.4094
16	0.0090	0.0028	0.1042
18	0.0090	0.0028	0.1042
19	0.0297	0.0148	0.5106
21	0.0961	0.0481	1.7104
22	1.7910	1.1882	33.6071
23	0.3425	0.1645	6.2804
24	0.0035	0.0020	0.0646
25	0.6684	0.3507	12.5510
26	0.0173	0.0085	0.3190
27	0.2326	0.1089	4.3087
28	1.6424	0.4511	27.8900
29	0.0538	0.0245	0.9920
30	---	---	---
31	0.1292	0.0574	2.3676
32	0.0538	0.0246	0.9927
33	---	---	---
Total	6.6917	2.6360	101.8937

So far it has been shown the behavior of the process of loss re-allocation regarding a number of generation scenarios. However, it is important to observe that demand is also a time-varying parameter in power systems. Therefore, since power losses in transmission or distribution vary constantly, loss allocation methods should be used in such a manner that time intervals that characterize the demand (e.g. hourly analysis) are taken into account.

Considering a 100 kW power output for each generator connected at nodes 30 and 33, loss re-allocation will be

evaluated assuming 10, 50 and 120% of the IEEE-34 network's loading adopted. Results are presented in Table IV.

Losses are certainly dependent on the connection points of the generation units given the effects that different DG arrangements may have on the system's total losses. Nonetheless, losses will also depend on the system's loading. Thus, for the most loaded node (22), re-allocation accounted for 26.8% of the total losses at the first scenario, which can be considered as "flat" loading in a load curve. At "medium" loading, re-allocation is 45.1% and at "peak" loading, accounts for 32.9%. This variability is verified for all nodes, and occurs due to the non-linearities of losses.

IV. CONCLUSIONS

Independent Energy Producers (Distributed Generation) and Independent Energy Consumers, as new agents of distribution systems that allow open access, may be widespread in the near future. In this scenario, where IEPs and IECs are able to negotiate energy contracts with agents outside the host distribution network, and considering current methodologies for calculating the sharing of losses, captive consumers will probably assume the whole responsibility of the system's losses.

In this work, a process for re-allocation of losses in distribution systems with open access was presented, aimed at minimizing cross-subsidies that appear as an effect of the generators over the system losses. Therefore, consumers (IECs and captive consumers) become the only responsible agents for the network losses, with no or minimal cross-subsidies.

The proposal considers the spatial characteristics of consumers in the distribution systems, being consistent, fair and easy to implement. Results obtained show the effectiveness in the minimization of cross-subsidies, which can be translated –to some extent– in more efficient and transparent economic signals of the market, making the proposal attractive as an option of loss allocation method in nowadays distribution systems with open access, and where DG is playing an increasing role.

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VI. BIOGRAPHIES

Luis F. Ochoa (S'01, M'07) graduated from the National Engineering University (UNI) – Lima, Peru, in 2000. He obtained the M.Sc. and Ph.D. degrees from UNESP – Ilha Solteira, Brazil, in 2003 and 2006, respectively. He is also member of the Institution of Engineering and Technology (IET). His research interests are distribution system analysis and DG.

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