The Resource Valuation and Optimisation Model

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The extraction of a resource is typically conducted in the presence of uncertainty. This uncertainty could be economic, such as price, or physical, such as the estimated quantity of extractable commodity. How a company should optimally behave in such an uncertain environment has been a very tough problem to solve, despite several attempts over the past thirty years to do so. The Resource Valuation and Optimisation Model (RVOM) provides both mathematical analysis and computer software to help answer this globally important question.

The RVOM is presently a tool to help the mining industry (which currently makes up around 4% of global GDP). When a company plans to extract from a mine, it does so by artifically separating the mine into 3-D blocks, each with its own estimated ore-grade content. The number of blocks for a large mine can be in the millions, which means that deciding the optimal order in which to extract these blocks is incredibly tough to solve. That said, modern mining companies already have techniques for deciding feasible orders of extraction. With such an order of extraction, they can couple it to the array of operational decisions available to them (such as mothballing, expansion and abandonment) to form an extraction schedule. A comparison between various feasible extraction schedules can then be conducted, to find the schedule with the highest valuation. However, this is generally conducted without full consideration of the uncertainties, meaning the optimal abandonment) to form an extraction schedule. A comparison between various feasible extraction schedules can then be conducted, to find the schedule with the highest valuation. However, this is generally conducted without full consideration of the uncertainties, meaning the optimal operational decisions and valuations cannot be calculated. To help plan and optimise this scheduling problem, the RVOM takes each of these feasible orders of extraction, and in the presence of both economic and geological uncertainty, calculates

- The net present value of the mine operation.
- The criteria for optimally exercising each operational decision,
- The probability of exercising each of the decisions.

In doing so, the RVOM applies advanced numerical techniques and probability theory, to help solve this real-world problem.

**The underpinning mathematics**

The RVOM relies upon the price and ore-grade uncertainties to be able to be written as a $\text{It}\hat{o}$ diffusion. To give a specific example, let the mine be operated under only price uncertainty, $\mathcal{S}$, which follows a log-normal process of the form,

$$ dS = \mu S dt + \sigma S \, dW, $$

where $\mu$ and $\sigma$ are the percentage drift and volatility of the price, respectively, $t$ is calendar time and $dW$ is a Wiener process. With this process, we may either employ hedging arguments, or the Feynman-Kac framework, to derive the partial differential equation relating to the valuation, $V$,

$$ \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial Q^2} + \frac{\partial V}{\partial Q} + \mu S \frac{\partial V}{\partial S} - r V + qGS - \epsilon = 0, $$

subject to the conditions

$$ V = 0 \quad \text{when} \quad \min (S - S', T - \tau, Q) = 0, $$

$$ V \sim S \quad \text{as} \quad S \to \infty. $$

Here, the variable $Q$ is the size of the resource, $\epsilon$ is the cost of extraction per unit, $q$ is the rate of extraction from the mine, $\tau$ is the discount rate, $T$ is the maximum extraction time and $G$ is the ore-grade quality. The equation relating to the probability, $P$, of changing operation states (e.g. abandonment, mothballing, expansion), can also be derived from the Feynman-Kac framework, to give,

$$ \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 P}{\partial Q^2} + \frac{\partial P}{\partial Q} + \mu S \frac{\partial P}{\partial S} - r P = 0, $$

subject to

$$ P = 0 \quad \text{on} \quad S = S', $$

$$ P = 1 \quad \text{when} \quad \min (Q, T - \tau) = 0, $$

$$ P \sim 1 \quad \text{as} \quad S \to \infty. $$

With this underpinning mathematics, we can extend the model to include further uncertainties, such as additional commodities prices or geological estimates.

**The resource valuation and optimisation model**

**Concept**

On first consideration, the uncertainty in the amount of ore contained within each block might seem a problem contained in 3-D space. But once the ore-bearing material has been extracted, and sent to a processing facility for block-by-block refinement, it can then be thought of as a one-dimensional uncertainty. This is what Figure 1 demonstrates, which is the estimated ore-grade versus the order of processing.

**Geological uncertainty**

We treat this uncertainty to follow a CIR process,

$$ dQ = -k(Q - G)dt + a\sqrt{Q} \, dW, $$

where $Q$ is the (local) mean value of the ores grade, and the parameter values $k$ and $a\sqrt{Q}$ can be determined from the data via maximum likelihood estimates.

The ore-grade process can be turned into a time varying CIR diffusion via the substitution, $dQ = -\eta Q dt$, meaning we can add this uncertainty into the Feynman-Kac framework to write the two-factor governing valuation equation as,

$$ \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial Q^2} + \frac{\partial V}{\partial Q} + \mu S \frac{\partial V}{\partial S} - r V + a \sqrt{Q} \frac{\partial V}{\partial Q} + \frac{1}{2} a^2 \frac{\partial^2 V}{\partial Q^2} = 0, $$

subject to

$$ V = 0 \quad \text{when} \quad \min (S - S', T - \tau, Q) = 0, $$

$$ V \sim S \quad \text{as} \quad S \to \infty. $$

**Outputs**

The results show the three keys outputs from the RVOM: the optimal price of abandonment for a mine, Figure 3; the valuation of the mine, Figure 4 (right); and the probability of never reaching the abandonment surface, Figure 4 (left).

**References**


- Geoff Evatt, Paul Johnson, John Moriarty, Peter Duck & Syd Howell.