On the Equivalence of Quantity Competition and Supply Function Competition with Sunk Costs

DOI:
10.1007/s00182-016-0542-x

Document Version
Accepted author manuscript

On the Equivalence of Quantity Competition and Supply Function Competition with Sunk Costs*

Michał Król†

November 26, 2015

Abstract

This paper considers competition in supply functions in a homogeneous goods market in the absence of cost or demand uncertainty. In order to commit to a supply schedule, firms are required to build sufficient capacity to produce any quantity that may be prescribed by that schedule. When the cost of extra capacity (given the level of sales) is strictly positive, any Nash Equilibrium outcome of supply function competition is also a Nash Equilibrium outcome of the corresponding Cournot game, and vice-versa. Conversely, when the cost-savings from reducing output (given the capacity level) are sufficiently small, any outcome of iterated elimination of weakly dominated strategies in the supply function game is also an outcome of the same process in Cournot, and vice-versa.

KEYWORDS: Cournot game, sunk costs, supply functions

1 Introduction

A common argument against the Cournot model of oligopolistic competition is that it leads to plausible comparative statics results, but does so, unrealistically, without allowing the firms to make any pricing decisions. In the words of Shapiro [1989], 'A common view is that pricing competition more accurately reflects actual behavior, but the predictions of Cournot's theory are closer to matching the evidence'.

Existing literature attempts to resolve the difficulty by demonstrating that 'price setting models may boil down to Cournot outcomes' (Vives [1989]). A seminal study by Kreps and Scheinkman [1983] considered capacity pre-commitment at stage one, followed by competition in prices. It turned out that the Cournot equilibrium outcome is also the unique subgame-perfect equilibrium outcome of the two-stage game. More recently, Moreno and Ubeda [2006] extend this to situations where firms set reservation rather than exact prices.

*I would like to thank Igor Evstigneev, Rachel Griffith, Roger Hartley, Leo Kaas and Paul Madden for helpful comments and suggestions, as well as the participants of my talks at the University of East Anglia, University of Nottingham, University of St Andrews and City University London, particularly Marco Mariotti and Klaus Zauner.

†Department of Economics, The University of Manchester, michal.krol@manchester.ac.uk

An alternative is to consider static models encompassing both Cournot and Bertrand strategies. For instance, Klemperer and Meyer [1989] allow for 'supply-function' competition (henceforth, SFC), where each firm simultaneously specifies the quantity of output it would be ready to provide at every possible price. While SFC typically results in a large range of potential equilibria, it turns out that their number is reduced in the presence of demand uncertainty. When firms are few and production costs steeply rising, the surviving equilibria exhibit supply functions that are steep, i.e. close to the 'vertical' Cournot-like commitment to a given quantity regardless of the price.

Another way of reducing the number of equilibria under SFC has been to impose the equilibrium refinement condition of coalition-proofness. When the number of firms is large (Delgado and Moreno [2004]), and their capacities are not too asymmetric (Delgado [2006]), the Cournot outcome is the only coalition-proof equilibrium of SFC.

This paper explores a new link between Cournot and SFC, by formalizing the suggestion by Klemperer and Meyer [1989] that 'different supply functions may have different costs of implementation: for example, choosing a vertical supply function (fixed quantity) may offer a cost advantage relative to [...] a flexible production technology'. Thus, we introduce a cost function depending not only on the output sold at market clearing, but also on the maximum quantity the firm declared itself ready to supply (at some price) by virtue of its supply schedule. The rationale is that in order to credibly commit to providing a given quantity of output at a specified price, the firm needs to first build sufficient capacity to produce this quantity. Even if some of this capacity proves surplus to requirements at market clearing, its cost may be at least partly impossible to recover ('sunk').

Within this framework, two results are obtained. First, assuming excess capacity is costly is sufficient to ensure not only that any Nash Equilibrium outcome of Cournot is a Nash Equilibrium outcome of SFC, but also that the converse is true as well. Intuitively, the advantage of Cournot competition is that firms know in advance the quantity they will sell, so they can only build the capacity they need. At the same time, they lose the flexibility to adjust output depending on market prices. The nature of the (pure-strategy) Nash Equilibrium concept makes the last motive irrelevant, as each firm expects rivals to choose specific actions, and so can also infer the market price.

The second result is that a one-to-one correspondence also holds, under certain conditions, between iterated elimination of weakly dominated strategies in the Cournot and SFC games. Specifically, any restricted game obtained via iterated elimination of weakly dominated strategies in the SFC game is equivalent to some restricted game being an outcome of the same process in Cournot, and vice-versa. Intuitively, weak dominance can capture strategic uncertainty, since different strategies of the counterparts are jointly considered. Thus, the motive of reduced flexibility of Cournot strategies comes into play, as firms face uncertainty about the price prior to market-clearing. However, when the number of players is small and capacity constraints are 'tight' (i.e. uncertainty about others' output / market price is small) relative to the proportion of the cost that is 'sunk', the motive of excess capacity reduction is once more stronger then that of maintaining flexibility.

2 The Model

2.1 Industry

There are $n \geq 2$ identical firms operating in a market for a homogeneous good with an inverse demand schedule $P : \mathbb{R}_+ \to \mathbb{R}_+$. We assume that $P(\cdot)$ satisfies $P(Q) \geq 0$ for all $Q \geq 0$, and that for all $Q \geq 0$ such that $P(Q) > 0$, $P(\cdot)$ is twice continuously differentiable, strictly decreasing, and satisfies the Hahn [1962] condition, i.e.:

$$P'(Q) + QP''(Q) < 0, \text{ for all } Q > 0 \tag{1}$$

The firms' production technology is represented by a cost function $C(x, q)$, where q is the 'installed production capacity' and x is the output actually being produced. We assume that $x, q \in [0, q^c]$, where $q^c > 0$ is a parameter representing maximum production / capacity. We also use $C(q)$ to denote $C(q, q)$ for short, and assume that this function is twice continuously differentiable and non-decreasing convex, i.e. $C'(q) \geqslant 0$ and $C''(q) \geqslant 0$ for all $q \in [0, q^c)$.
In addition to the above assumptions, imposed throughout the paper, two further assumptions will be used interchangeably:

$$C(x, q) < C(x, q') \text{ for all } x, q, q' \text{ such that } x \leq q < q' \leq q^c \tag{A1}$$

The second assumption is that the derivative $\partial C / \partial x = C_x(x, q)$ exists, and:

$$C_x(x, q) \leq C_x(q^c, q^c) \in (0, \infty) \text{ for all } x, q \text{ such that } x \leq q \leq q^c \tag{A2}$$

2.1.1 Discussion of Assumptions

Let $\mathcal{G}_C$ denote the associated Cournot game, i.e. one in which firms simultaneously select quantities $q_i \in [0, q^c]$, $i \in N = \{1, 2, ..., n\}$, and receive payoffs:

$$\pi_i^c(q_i, Q_{-i}) = q_i P(q_i + Q_{-i}) - C(q_i), \; Q_{-i} = \Sigma_{j \in N/\{i\}} q_j$$

Condition (1) is then equivalent to the firms' individual marginal revenues being decreasing in the rivals' aggregate output Q_{-i}. For the production technology specified above, this ensures that the firms' reaction functions are continuous and downward sloping, so that a Cournot equilibrium exists (see Novshek [1985]). A sufficient condition for (1) to hold is $P''(Q) \leq 0$, i.e. that the demand is concave.

Leaving the Cournot game aside, one may think of $C(x, q)$ as a 'short-run' cost function, in the sense that q, the capacity level, is built in the long-run, using the lowest-cost combination of inputs at the firm's disposal. The firm may then adjust the actual level of output x, by revising the chosen amounts of *some* of the inputs, whilst others are fixed in the short-run and constitute a fixed cost. The resulting (total) cost of producing x is given by $C(x,q)$.

Assumption (A1) states that increasing excess capacity is always costly. Assumption (A2) states that the marginal cost $C_x(x,q)$ of an increase in 'short-run' output reducing excess capacity (i.e. of a ceteris paribus increase of x while $x \leq q$) is at its highest when the capacity q is maximized but excess capacity vanishes ($x = q$). This would happen when the (ceteris paribus) returns from variable inputs are diminishing, even when the amounts of *all* inputs are increased in the long-run to allow for a higher capacity. For instance, (A2) holds for a Cobb-Douglas production function exhibiting non-increasing returns to scale.

We now proceed to describe the game of supply-function competition, denoted $\mathcal{G}$, as opposed to $\mathcal{G}_C$ used to denote the Cournot game.

2.2 Firm Strategies and the Market Clearing Process

Each firm $i \in N = \{1, 2, ..., n\}$ simultaneously sets a non-decreasing, right-continuous supply schedule $s_i : [0, \bar{p}] \to [0, q^c]$, specifying the quantity of output it offers to provide at every possible market price, where $\bar{p} = P(0)$. We will refer to $s_i(\bar{p})$ as the 'maximum quantity' of a given schedule. We also restrict attention to pure strategies.

For any supply function profile $\mathbf{s} = \{s_i(\cdot)\}_{i \in N}$, a market price p and a subset of players $A \subseteq N$, define the following 'aggregate supply' functions:

$$S_A(\mathbf{s}, p) = \sum_{i \in A} s_i(p), \;\; S_A^-(\mathbf{s}, p) = \begin{cases} \lim_{p' \to p^-} S_A(\mathbf{s}, p') & \text{for } p \in (0, \bar{p}] \\ 0 & \text{for } p = 0 \end{cases}$$

Note that for $p > 0$ the existence of the left-sided limit involved in the above is guaranteed by the monotonicity of each $s_i(\cdot)$, and hence the aggregate supply schedule of any subset of players as well. A market-clearing price $p^*(\mathbf{s})$ must then satisfy:

$$D(p^*) \in \left[S_N^-(\mathbf{s}, p^*), S_N(\mathbf{s}, p^*)\right], \;\; D(\cdot) = P^{-1}(\cdot) \tag{2}$$

Due to the monotonicity of $S_N(\mathbf{s}, \cdot)$ and $D(\cdot)$, a unique price $p^* \in [0, \bar{p}]$ satisfying (2) always exists. Indeed, consider an alternative price $p \in [0, \bar{p}]$ such that $p > p^*$, which implies $S_N^-(\mathbf{s}, p) \geq S_N(\mathbf{s}, p^*)$ and $D(p) < D(p^*)$. Thus, it cannot be the case that $D(p^*) \leq S_N(\mathbf{s}, p^*)$ and $D(p) \geq S_N^-(\mathbf{s}, p)$, so p and p^* cannot both satisfy (2).

As there is a possibility of excess supply at the market-clearing price p^*, some firms may not be able to sell as much as $s_i(p^*)$. Let $x_i(\mathbf{s}) \leq s_i(p^*)$ denote the quantity of output that firm i is actually able to sell at market clearing, and note that $x_i(\mathbf{s})$ must lie within the following interval:

$$\left[\max\left\{S^-_{\{i\}}(\mathbf{s},p^*),D(p^*)-S_{N\backslash\{i\}}(\mathbf{s},p^*)\right\},\min\left\{s_i(p^*),D(p^*)-S^-_{N\backslash\{i\}}(\mathbf{s},p^*)\right\}\right] \tag{3}$$

Observe that when all supply schedules are continuous at p^*, we have $x_i(\mathbf{s}) = s_i(p^*)$, since:

$$S^-_{\{i\}}(\mathbf{s},p^*) = s_i(p^*) = D(p^*) - S_{N\backslash\{i\}}(\mathbf{s},p^*) = D(p^*) - S^-_{N\backslash\{i\}}(\mathbf{s},p^*)$$

When the supply schedule of at most one firm (i) is discontinuous at p^*, it follows from (2) that:

$$D(p^*) - S_{N\backslash\{i\}}(\mathbf{s},p^*) = D(p^*) - S^-_{N\backslash\{i\}}(\mathbf{s},p^*) \in \left[S^-_{\{i\}}(\mathbf{s},p^*), s_i(p^*)\right]$$

which implies that $x_i(\mathbf{s}) = D(p^*) - S_{N\backslash\{i\}}(\mathbf{s},p^*)$, i.e. firm i only gets what remains after all counterparts have sold their entire outputs.

Finally, suppose the supply schedule of more than one firm is discontinuous at p^*, leading to excess supply at p^*. Let N_0 denote the set of those firms. Then (3) is a proper interval for all $i \in N_0$ (while $x_i(\mathbf{s}) = s_i(p^*)$ for all $i \in N\backslash N_0$). The exact value of $x_i(\mathbf{s})$ for $i \in N_0$ is then determined by a sharing rule, specifying how the $D(p^*) - S_{N\backslash N_0}(\mathbf{s},p^*)$ part of the demand is to be distributed among the firms in N_0. Since the results of this paper turn out to hold for any such sharing rule, it is left unspecified throughout the remainder of the text.

2.3 Costs and Payoffs

With the firms' sales $x_i(\mathbf{s})$ determined, the resulting profits are:

$$\pi_i(\mathbf{s}) = p^*(\mathbf{s})\,x_i(\mathbf{s}) - C(x_i(\mathbf{s}), s_i(\bar{p})) \tag{4}$$

Thus, we assume that, in order to commit to the chosen supply schedule prior to getting to know the market clearing price, the firm needs to build the capacity required to produce the maximum quantity that may be prescribed by the schedule, $s_i(\bar{p})$. Upon discovering that it only needs to produce $x_i(\mathbf{s}) \leq s_i(\bar{p})$, the firm may recoup some of the cost of its excess capacity $s_i(\bar{p}) - x_i(\mathbf{s})$, but the remainder of that cost is sunk. As was suggested in Section 2.1, we may also think of this in terms of 'long-run' (prior to setting the supply schedule) vs. 'short-run' (at market-clearing), where in the latter case some of the input factors are fixed and constitute a sunk cost.

Note that this cost specification generalizes the one normally used in the SFC literature, which restricts to cost functions that depend only on actual sales, i.e. ones that satisfy $C(x,q) = C(x,q')$ for all x, q and q'.

3 Results

The first step is to specify a definition to help formalize the relationships between the two games: the Cournot game (denoted $\mathcal{G}_C$) and the SFC game (denoted $\mathcal{G}$), both as described in Section 2.

Definition 3.1 *The outcome of a profile of quantities* $\mathbf{q} = \{q_j\}_{j \in N}$ *in* $\mathcal{G}_C$ *is equivalent to the outcome of a profile of supply functions* $\mathbf{s} = \{s_j(\cdot)\}_{j \in N}$ *in* $\mathcal{G}$*, when for all* $i \in N$ *we have* $x_i(\mathbf{s}) = q_i$ *and* $\pi_i(\mathbf{s}) = \pi_i^c(\mathbf{q})$.

Note that the fact that $x_i(\mathbf{s}) = q_i$ for all $i \in N$ also implies that the Cournot market clearing price given $\mathbf{q}$ is the same as the SFC market clearing price given $\mathbf{s}$.

Proposition 3.1 *Under assumption* ($A1$)*, and for any profile of quantities* $\mathbf{q} = \{q_j\}_{j \in N}$, $\mathbf{q}$ *is a Nash Equilibrium of* $\mathcal{G}_C$ *if and only if there exists a profile of supply-functions* $\mathbf{s} = \{s_j(\cdot)\}_{j \in N}$ *such that* $\mathbf{s}$ *is a Nash Equilibrium of* $\mathcal{G}$ *and the outcome of* $\mathbf{s}$ *in* $\mathcal{G}$ *is equivalent to the outcome of* $\mathbf{q}$ *in* $\mathcal{G}_C$.

In other words, for every equilibrium of the SFC game there is a corresponding equilibrium of the Cournot game that yields the same outcome (in terms of market prices, outputs and profits), and the converse is also true.

Let us compare this result with those of Delgado and Moreno [2004] (DM). They show that requiring the supply-functions to be non-decreasing means only prices not greater than the Cournot equilibrium price may be sustained (where any SFC equilibrium that yields the Cournot price also exhibits the symmetric Cournot outputs). The assumption of non-decreasing supply-functions has the same effect here, even though, unlike in DM, the supply schedules are not required to be continuous.

The main difference is that in order to eliminate the equilibria that support prices strictly below the Cournot level, DM impose an additional requirement of 'coalition-proofness' on the set of equilibria (which must be invulnerable to improving deviations by any coalition of players). Intuitively, this requirement becomes stronger when the number of players is greater, and when it is sufficiently large the only equilibrium outcome that remains is the Cournot outcome.

In contrast, the present paper does not use an equilibrium refinement criterion (or assume firms communicate prior to taking actions, which coalition-proofness necessitates). Instead, we consider a generalization of the usual cost specification, allowing for sunk costs of excess capacity. It turns out that when additional capacity is always costly ($A1$), only Cournot prices (and outputs) may occur in SFC equilibria.

The intuition behind this result is simple. Given a profile of supply-schedules which a firm expects its rivals to choose, it can always alter its own supply-schedule so as to keep the same market price and individual sales as before, but eliminate any excess capacity, thereby reducing costs and increasing profits. Thus, profit-maximizing supply schedules must equalize sales and the capacity level, which ensures that the best-response dynamics of the SFC game is analogous to the Cournot one (the role of Cournot quantity played by the SFC sales / capacity level).

The result also mirrors those of Moreno and Ubeda [2006] (MU), who consider a choice of capacity at stage one followed by setting a reservation price, which amounts to constructing a discontinuous one-step supply schedule over the two stages of the game. However, unlike in both DM and MU, here the supply-schedules can be multi-step discontinuous, which permits additional applications of the model. For instance, sellers in on-line auctions can set several one-unit auctions with different reservation prices, which is equivalent to setting a step-wise supply schedule with several steps. In addition, unlike DM and MU, the present model will apply to situations in which there is no prior communication between players, and no knowledge of others' prior capacity decisions.

We now turn to the other main result of the paper, namely extending the correspondence between SFC and Cournot by means of iterated weak dominance.

To this end, let X_i denote the strategy set of player i in $\mathcal{G}_C$, and let Y_i denote the strategy set of player i in $\mathcal{G}$. Let $\mathcal{G}'_C$ and $\mathcal{G}'$ denote the corresponding *restricted games*, obtained by eliminating some of the strategies in $\mathcal{G}_C$ and $\mathcal{G}$, so that the set of strategies X'_i of player i in $\mathcal{G}'_C$ satisfies $X'_i \subseteq X_i$, and the set of strategies Y'_i of player i in $\mathcal{G}'$ satisfies $Y'_i \subseteq Y_i$ for every $i \in N$.

Definition 3.2 *Restricted games $\mathcal{G}'_C$ and $\mathcal{G}'$ are equivalent ($\mathcal{G}'_C \equiv \mathcal{G}'$) when there exists a collection of surjective functions $\{\varphi_i : Y'_i \to X'_i\}_{i\in N}$ such that, for any $\mathbf{s} = \{s_j(\cdot)\}_{j\in N} \in \times_{j\in N} Y'_j$, the outcome of $\mathbf{s}$ in $\mathcal{G}$ is equivalent to the outcome of $\mathbf{q} = \left\{\varphi_j(s_j(\cdot))\right\}_{j\in N}$ in $\mathcal{G}_C$.*

In other words, every supply schedule in $\mathcal{G}'$ corresponds to some quantity in $\mathcal{G}'_C$, and vice-versa, in the sense that the firms' sales, profits and the market clearing price for a given profile of strategies in $\mathcal{G}'$ (respectively $\mathcal{G}'_C$) are the same as they would have been for a corresponding profile of strategies in $\mathcal{G}'_C$ (resp. $\mathcal{G}'$). A player's choice in $\mathcal{G}'$ is the same as in $\mathcal{G}'_C$, except for a possible duplication of strategies, when multiple supply schedules correspond to the same Cournot strategy, and so always yield the same payoffs. Hence, the two games are strategically equivalent.

We will use the term 'iterated elimination of weakly dominated strategies' (IEWDS) in the usual sense, and say that $\mathcal{G}'$ (or $\mathcal{G}'_C$) is an *outcome* of IEWDS from $\mathcal{G}$ (resp. $\mathcal{G}_C$), when it is obtained by IEWDS from $\mathcal{G}$ (resp. $\mathcal{G}_C$), but no further strategies are weakly dominated in $\mathcal{G}'$ (resp. $\mathcal{G}'_C$)

Proposition 3.2 *Suppose assumption (A2) holds and, in addition:*

$$P'(nq^c)\,q^c + P(nq^c) \geqslant C_x(q^c, q^c) \tag{5}$$

We then have:

1. *For any $\mathcal{G}'$ being an outcome of IEWDS from $\mathcal{G}$, there exists a $\mathcal{G}'_C$ being an outcome of IEWDS from $\mathcal{G}_C$, such that $\mathcal{G}'_C \equiv \mathcal{G}'$.*

2. *Conversely, for any $\mathcal{G}'_C$ being an outcome of IEWDS from $\mathcal{G}_C$, there exists a $\mathcal{G}'$ being an outcome of IEWDS from $\mathcal{G}$, such that $\mathcal{G}'_C \equiv \mathcal{G}'$.*

In other words, for every outcome of IEWDS from the SFC game, there is a corresponding equivalent outcome of IEWDS from Cournot, and vice-versa. Thus, under the conditions specified in Proposition 3.2, given avoidance of weakly dominated strategies and common knowledge of rationality, the two games ($\mathcal{G}$ and $\mathcal{G}_C$) are strategically equivalent.

Under assumption (1), the LHS of (5) is the lowest possible marginal revenue of an individual firm from selling more output (as it corresponds to a case where everyone already sell up to q^c, and are willing to do so even as the firm's increase in output leads to a reduced market price). Likewise, under ($A2$) the RHS is the highest possible marginal cost of selling more output without increasing the maximum quantity $s_i(\bar{p})$, or the required capacity, of the supply schedule. Hence, condition (5) implies that even when rivals want to sell their output regardless of the price, it pays off to be equally competitive, making fixed-quantity schedules optimal regardless of what others do.

Condition (5) holds when n, q^c and $C_x(\cdot)$, the marginal cost of extra sales (or cost-saving from a marginal sales reduction), are not too big. In particular, $C_x(\cdot)$ is small when costs depend mostly on the required capacity level $s_i(\bar{p})$, and not on actual sales, i.e. when the capacity costs are 'sunk'.

The intuition for this is that, on the one hand, the main advantage of quantity competition à la Cournot is that it eliminates the cost of excess capacity. This is because players fix the exact amounts they will sell, and so are able to build only the capacity that they actually need, which is particularly important when the capacity costs are 'sunk'. On the other hand, a greater number of players and higher maximum capacity add to strategic uncertainty about the other players' total output. Hence, this makes Cournot strategies less advantageous compared to more flexible supply-functions, because the latter can prepare the player for various aggregate supply / market price scenarios. Thus, more firms and relaxed capacity constraints make the equivalence between Cournot and SFC in terms of IEWDS more difficult to obtain.

4 Concluding Remarks

The paper provided a new link between the Cournot model and competition in supply functions, by considering a possibility that firms must build sufficient capacity to produce any quantity that may be prescribed by the chosen supply schedule. In contrast with existing literature, there is no assumption of prior communication between players or knowledge of others' prior capacity decisions. Moreover, the firms' supply schedules need not be continuous, which makes the model applicable to a wider range of real-world situations. Lastly, the paper established a one-to-one correspondence not only between the sets of Nash Equilibria of the two games in question, but also, under certain conditions, between the sets of strategies that survive the process of iterated elimination of weakly dominated strategies. This provides support not only for the Cournot equilibrium outcome, but also for quantity competition in general, as a reduced form of competition in supply functions.

Appendix: Proofs

Proof of Proposition 3.1. Suppose $\mathbf{s}^* = \left\{s_j^*(\cdot)\right\}_{j\in N}$ is a profile of supply functions, and p^* is the associated market-clearing price.

We will first prove that for any $i \in N$ such that $x_i(\mathbf{s}^*) < s_i^*(\bar{p})$, it is possible for player i to profitably deviate from her strategy $s_i^*(\cdot)$. Suppose that, in addition to $x_i(\mathbf{s}^*) < s_i^*(\bar{p})$ for some $i \in N$, we have $S_{\{i\}}^-(\mathbf{s}^*, p^*) < s_i^*(p^*)$, which implies $S_N^-(\mathbf{s}^*, p^*) < D(p^*) < S_N(\mathbf{s}^*, p^*)$. Suppose now player i changes her supply schedule to a $s_i'(\cdot)$ such that $s_i'(p) = x_i(\mathbf{s}^*)$ for $p \leq p^*$, and $s_i'(p) = s_i^*(p)$ for $p > p^*$. From (3), we know that:

$$x_i(\mathbf{s}^*) \in \left[D(p^*) - S_{N\backslash\{i\}}(\mathbf{s}^*, p^*), D(p^*) - S_{N\backslash\{i\}}^-(\mathbf{s}^*, p^*)\right]$$

Hence, given the new profile of supply functions $\mathbf{s}' = s_i'(\cdot) \cup \left\{s_j^*(\cdot)\right\}_{j\in N\backslash\{i\}}$, and from the fact that $S_{N\backslash\{i\}}^-(\mathbf{s}^*, p^*) = S_{N\backslash\{i\}}^-(\mathbf{s}', p^*)$ and $S_{N\backslash\{i\}}(\mathbf{s}^*, p^*) = S_{N\backslash\{i\}}(\mathbf{s}', p^*)$ we have:

$$\begin{aligned} S_N^-(\mathbf{s}', p^*) &= S_{N\backslash\{i\}}^-(\mathbf{s}^*, p^*) + x_i(\mathbf{s}^*) \leq D(p^*) \\ S_N(\mathbf{s}', p^*) &= S_{N\backslash\{i\}}(\mathbf{s}^*, p^*) + x_i(\mathbf{s}^*) \geqslant D(p^*) \end{aligned}$$

Hence, p^* is still the market-clearing price under $\mathbf{s}'$, and $x_i(\mathbf{s}') = x_i(\mathbf{s}^*)$. Since $s_i'(\bar{p}) = s_i^*(\bar{p})$, the costs incurred by firm i are also the same under $s_i'(\cdot)$ as under $s_i^*(\cdot)$, and so are the profits. Hence, to show that i can profitably deviate from $s_i^*(\cdot)$, we now assume that $s_i^*(\cdot)$ is such that $S_{\{i\}}^-(\mathbf{s}^*, p^*) = s_i^*(p^*)$, because one can always alter $s_i^*(\cdot)$ to satisfy this property without changing the profit of firm i. Hence, it remains to consider a situation where:

$$S_{\{i\}}^-(\mathbf{s}^*, p^*) = s_i^*(p^*) = x_i(\mathbf{s}^*) < s_i^*(\bar{p})$$

Suppose then player i switches to an alternative supply schedule $s_i'(\cdot)$ such that $s_i'(p) = s_i^*(p)$ for $p \leq p^*$, and $s_i'(p) = s_i^*(p^*) = x_i(\mathbf{s}^*)$ for $p > p^*$. Once again, let $\mathbf{s}' = s_i'(\cdot) \cup \left\{s_j^*(\cdot)\right\}_{j\in N\backslash\{i\}}$. We then have $S_N^-(\mathbf{s}^*, p^*) = S_N^-(\mathbf{s}', p^*)$ and $S_N(\mathbf{s}^*, p^*) = S_N(\mathbf{s}', p^*)$, which means p^* is still the market-clearing price under $\mathbf{s}'$, and $x_i(\mathbf{s}') = x_i(\mathbf{s}^*)$. However, we have:

$$C(x_i(\mathbf{s}'), s_i'(\bar{p})) = C(x_i(\mathbf{s}^*), x_i(\mathbf{s}^*)) < C(x_i(\mathbf{s}^*), s_i^*(\bar{p}))$$

Given assumption $(A1)$ holds and using $x_i(\mathbf{s}^*) < s_i^*(\bar{p})$, this means costs are lower under $s_i'(\cdot)$ than under $s_i^*(\cdot)$, making the profit of firm i larger in the former case. Hence, any player $i \in N$ who sets a supply schedule $s_i^*(\cdot)$ such that $x_i(\mathbf{s}^*) < s_i^*(\bar{p})$, can benefit by unilaterally deviating to an alternative supply schedule $s_i'(\cdot)$ such that $x_i(\mathbf{s}') = s_i'(\bar{p})$. Consequently, any Nash Equilibrium strategy profile $\mathbf{s}^*$ must satisfy $x_i(\mathbf{s}^*) = s_i^*(\bar{p})$ for all $i \in N$.

Suppose then q_{NE}^* is a Cournot Nash Equilibrium quantity, i.e. we have $q_{NE}^* = q_{BR}((n-1)\, q_{NE}^*)$, where $q_{BR}(Q)$ is the Cournot best-response to an aggregate quantity Q produced by all other players. Consider a profile of supply schedules $\mathbf{s}^* = \{s_j^*(\cdot)\}_{j\in N}$ such that for every $j \in N$ we have $s_j^*(p) = q_{NE}^*$ for all $p \geqslant 0$. This means $\pi_j(\mathbf{s}^*) = P(n\, q_{NE}^*)\, q_{NE}^* - C(q_{NE}^*)$ for all $j \in N$, i.e. profits are equal to the Cournot Nash Equilibrium ones. Thus, an optimal deviation by player i from $s_i^*(\cdot)$ must entail a supply schedule $s_i'(\cdot)$ such that $x_i(\mathbf{s}') = s_i'(\bar{p})$, where again $\mathbf{s}' = s_i'(\cdot) \cup \{s_j^*(\cdot)\}_{j\in N\setminus\{i\}}$. The resulting profit would be $\pi_i(\mathbf{s}') = P((n-1)\, q_{NE}^*)\, s_i'(\bar{p}) - C(s_i'(\bar{p}))$, i.e. it would equal the Cournot profit given quantity $s_i'(\bar{p})$ when others produce $(n-1)\, q_{NE}^*$ in total. Thus, it cannot exceed the profit resulting from $s_i^*(\cdot)$ due to $s_i^*(\bar{p}) = q_{BR}((n-1)\, q_{NE}^*)$ being the Cournot best-response quantity. As a result, the Nash Equilibrium outcome of the Cournot game is also a NE outcome of the supply-function competition game.

Conversely, suppose a profile of supply schedules $\mathbf{s}^* = \{s_j^*(\cdot)\}_{j\in N}$ is a NE of the supply-function competition game, which means it must satisfy

$$x_i(\mathbf{s}^*) = s_i^*(\bar{p}) \quad \text{and} \quad \pi_i(\mathbf{s}') = P\left(\textstyle\sum_{j\in N} s_j^*(\bar{p})\right) s_i^*(\bar{p}) - C(s_i^*(\bar{p})) \quad \text{for all } i \in N$$

Suppose further that we do not have $s_i^*(\bar{p}) = q_{NE}^*$ for all $i \in N$, where q_{NE}^* is a Cournot Nash Equilibrium quantity. It must then be the case that for some $i \in N$ and some $q' \neq s_i^*(\bar{p})$ we have:

$$P\left(q' + \textstyle\sum_{j\in N/\{i\}} s_j^*(\bar{p})\right) q' - C(q') > P\left(\textstyle\sum_{j\in N} s_j^*(\bar{p})\right) s_i^*(\bar{p}) - C(s_i^*(\bar{p}))$$

Hence, by deviating from $s_i^*(\cdot)$ to a $s_i'(\cdot)$ such that $s_i'(p) = q'$ for all $p \geqslant 0$, player i can increase its payoff in the supply-function game, i.e. $\mathbf{s}^*$ is not a Nash Equilibrium if it does not implement a Cournot Nash Equilibrium outcome. ■

Proof of Proposition 3.2. Consider a strategy profile $\mathbf{s}^* = \{s_j^*(\cdot)\}_{j\in N}$ such that for some $i \in N$ we have $x_i(\mathbf{s}^*) < s_i^*(\bar{p})$, and a strategy $s_i'(\cdot)$ such that $s_i'(p) = s_i^*(\bar{p})$ for all $p \geqslant 0$. In the first part of the proof, we will show that under the condition stated in the proposition we then have $\pi_i(\mathbf{s}') > \pi_i(\mathbf{s}^*)$, where $\mathbf{s}' = s_i'(\cdot) \cup \{s_j^*(\cdot)\}_{j\in N\setminus\{i\}}$.

Observe first that $x_i(\mathbf{s}') = s_i^*(\bar{p})$ based on (3), since $S_{\{i\}}^-(\mathbf{s}', p) = S_{\{i\}}(\mathbf{s}', p) = s_i^*(\bar{p})$ for any $p > 0$ (note that we must have $p^*(\mathbf{s}') > 0$, since condition (5) together with assumption $(A2)$ imply $P'(nq^c)\, q^c + P(nq^c) > 0$, and so $P(nq^c) > 0$). Let $q_i = s_i^*(\bar{p})$ and $Q_{-i} = D(p^*) - s_i^*(\bar{p})$, where p^* is the market-clearing price associated with $\mathbf{s}'$, i.e. one that satisfies $D(p^*) \in [S_N^-(\mathbf{s}', p^*), S_N(\mathbf{s}', p^*)]$. We then have:

$$\pi_i(\mathbf{s}') = \pi_i^c(q_i, Q_{-i}) = P(q_i + Q_{-i})\, q_i - C(q_i)$$

The market-clearing price under $\mathbf{s}^*$ cannot be smaller than p^*, since $S_N^-(\mathbf{s}^*, p) \leq S_N^-(\mathbf{s}', p)$ and $S_N(\mathbf{s}^*, p) \leq S_N(\mathbf{s}', p)$ for all $p \geqslant 0$. Hence, as supply schedules are non-decreasing, the demand allocated to other players cannot be smaller than Q_{-i}. This means:

$$\pi_i(\mathbf{s}^*) \leq \hat{\pi}_i(\hat{q}_i, Q_{-i}) = P(\hat{q}_i + Q_{-i})\, \hat{q}_i - C(\hat{q}_i, q_i) \text{ for } \hat{q}_i = x_i(\mathbf{s}^*)$$

We proceed to show that the RHS of the above inequality is strictly smaller than $\pi_i^c(q_i, Q_{-i})$. A sufficient condition for this is that $\partial\hat{\pi}_i/\partial\hat{q}_i$ is non-negative for all and strictly positive for some $\hat{q}_i \in [x_i(\mathbf{s}^*), q_i]$. We have:

$$\partial\hat{\pi}_i/\partial\hat{q}_i = P'(\hat{q}_i + Q_{-i})\hat{q}_i + P(\hat{q}_i + Q_{-i}) - C_x(\hat{q}_i, q_i)$$

and, by virtue of assumptions (1) and $(A2)$:

$$P'(\hat{q}_i + Q_{-i})\hat{q}_i + P(\hat{q}_i + Q_{-i}) - C_x(\hat{q}_i, q_i) \geqslant P'(nq^c)q^c + P(nq^c) - C_x(q^c, q^c)$$

where the inequality is strict for $\hat{q}_i \in [x_i(\mathbf{s}^*), q_i)$, and the RHS of the inequality is non-negative by virtue of the condition imposed in the proposition. Thus, we have shown that $\pi_i(\mathbf{s}') > \pi_i(\mathbf{s}^*)$.

As a result, observe that we can conduct IEWDS in $\mathcal{G}$ until we end up with a restricted game $\mathcal{G}'$ such that:

1. for any strategy profile $\mathbf{s}^*$ that is part of $\mathcal{G}'$ we have $x_i(\mathbf{s}^*) = s_i(\bar{p})$ for all $i \in N$
2. for any $q \in [0, q^c], i \in N$ there exists a $s_i(\cdot)$ in $\mathcal{G}'$ such that $s_i(\bar{p}) = q$

In other words, we then have $\mathcal{G}' \equiv \mathcal{G}_C$. Suppose then a quantity q_i' weakly dominates q_i in $\mathcal{G}_C$ for some player i, and as such q_i can be eliminated, resulting in a restricted game $\mathcal{G}_C'$. This means we can equally eliminate all such $s_i(\cdot)$ in $\mathcal{G}'$ that satisfy $s_i(\bar{p}) = q_i$, as each of them is weakly dominated in $\mathcal{G}'$ by any $s_i'(\cdot)$ in $\mathcal{G}'$ that satisfies $s_i'(\bar{p}) = q_i'$. Eliminating all such $s_i(\cdot)$ results in a further restricted game $\mathcal{G}''$, where $\mathcal{G}'' \equiv \mathcal{G}_C'$. Thus, we can continue to apply the same reasoning to eliminate any further strategies from $\mathcal{G}_C'$, and, correspondingly, from $\mathcal{G}''$. In the end of the IEWDS process, we end up with two restricted games, $\mathcal{G}_C''$ and $\mathcal{G}'''$, such that $\mathcal{G}''' \equiv \mathcal{G}_C''$. Consequently, for any restricted game obtained from $\mathcal{G}_C$ by IEWDS, an equivalent game can be obtained from $\mathcal{G}$ by the same process.

We proceed to show the converse, i.e. that for any restricted game obtained from $\mathcal{G}$ by IEWDS, there exists an equivalent restricted game obtained from $\mathcal{G}_C$ by the same process. By virtue of what was shown above, any restricted game $\mathcal{G}^*$ that remains after completing the process of IEWDS in $\mathcal{G}$, must satisfy $x_i(\mathbf{s}^*) = s_i^*(\bar{p})$ for all $i \in N$ and any strategy profile $\mathbf{s}^*$ in $\mathcal{G}^*$.

Consider then the first round of IEWDS in $\mathcal{G}$ after which for some $q \in [0, q^c], i \in N$ there exists no $s_i(\cdot)$ in the resulting restricted game $\mathcal{G}'$ such that $s_i(\bar{p}) = q$. In particular, this means strategy $s_i(\cdot)$ such that $s_i(p) = q$ for all $p \geqslant 0$ must have already been eliminated, being weakly dominated by some other strategy $s_i'(\cdot)$. Thus, it must also have been weakly dominated by a strategy $s_i''(\cdot)$ such that $s_i''(p) = s_i'(\bar{p})$ for all $p \geqslant 0$. This implies $\pi_i(s_i''(\cdot), \mathbf{s}_{-i}) \geqslant \pi_i(s_i(\cdot), \mathbf{s}_{-i})$ for all $\mathbf{s}_{-i} = \{s_j(\cdot)\}_{j \in N/\{i\}}$ such that for every $j \in N/\{i\}$ we have $s_j(p) = q'$ for all $p \geqslant 0$ and some $q' \in [0, q^c]$. This in turn means that $\pi_i^c(s_i'(\bar{p}), Q_{-i}) \geqslant \pi_i^c(q, Q_{-i})$ for all $Q_{-i} \in [0, (n-1)q^c]$.

Under condition (1), the last inequality must be strict for at least some $Q_{-i} \in [0, (n-1)q^c]$, i.e.

quantity q must be weakly dominated by quantity $s'_i(\bar{p})$ in $\mathcal{G}_C$. Thus, one could eliminate q from $\mathcal{G}_C$ to obtain a restricted game $\mathcal{G}'_C$ such that $\mathcal{G}'_C \equiv \mathcal{G}'$.

One could then apply the same reasoning again, and consider the first round of IEWDS in $\mathcal{G}$ after which for some $i \in N, q' \in [0, q^c], q' \neq q$ there exists no $s_i(\cdot)$ in the resulting restricted game $\mathcal{G}''$ such that $s_i(\bar{p}) = q'$. Repeating the same steps would show that it is then possible to remove q' from the set of strategies available to player i in $\mathcal{G}'_C$, to obtain a further restricted game $\mathcal{G}''_C$ such that $\mathcal{G}''_C \equiv \mathcal{G}''$.

The process could be repeated until such time that it is impossible to eliminate any further strategies from $\mathcal{G}$, and $\mathcal{G}^*$ is the restricted game that remains. Correspondingly, there will then exist a $\mathcal{G}^*_C$, obtained from $\mathcal{G}_C$ by IEWDS, such that $\mathcal{G}^* \equiv \mathcal{G}^*_C$. Clearly, there may not exist two quantities q, q' in $\mathcal{G}^*_C$ such that q weakly dominates q' in $\mathcal{G}^*_C$. If this was the case, then a strategy $s_i(\cdot)$ such that $s_i(p) = q$ for all $p \geqslant 0$ would weakly dominate a strategy $s'_i(\cdot)$ such that $s'_i(p) = q'$ for all $p \geqslant 0$. As both $s_i(\cdot)$ and $s'_i(\cdot)$ would be part of $\mathcal{G}^*$ by virtue of $\mathcal{G}^* \equiv \mathcal{G}^*_C$, this would then contradict the fact that $\mathcal{G}^*$ is what remains after the IEWDS process is complete. Thus, we have shown that for any restricted game obtained from $\mathcal{G}$ by IEWDS, there exists an equivalent restricted game obtained from $\mathcal{G}_C$ by the same process. ■

References

Delgado, J. (2006): "Coalition-proof supply function equilibria under capacity constraints," *Economic Theory*, 29, 219–229.

Delgado, J. and D. Moreno (2004): "Coalition-proof supply function equilibria in oligopoly," *Journal of Economic Theory*, 114, 231 – 254.

Klemperer, P. D. and M. A. Meyer (1989): "Supply function equilibria in oligopoly under uncertainty," *Econometrica*, 57, 1243–1277.

Kreps, D. M. and J. A. Scheinkman (1983): "Quantity precommitment and bertrand competition yield cournot outcomes," *The Bell Journal of Economics*, 14, 326–337.

Moreno, D. and L. Ubeda (2006): "Capacity precommitment and price competition yield the cournot outcome," *Games and Economic Behavior*, 56, 323–332.

Novshek, W. (1985): "On the existence of cournot equilibrium," *The Review of Economic Studies*, 52, 85–98.

Shapiro, C. (1989): *Chapter 6: Theories of oligopoly behavior*, *Handbook of Industrial Organization*, volume 1, Elsevier.

Vives, X. (1989): "Cournot and the oligopoly problem," *European Economic Review*, 33, 503–514.