Customers’ Complaints and Quality Regulation

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Abstract

This paper studies the informativeness of customer complaints and their potential as a regulatory tool in contexts in which quality is not verifiable and consumers cannot (fully) appropriate the benefits of their complaints. The conventional wisdom about complaints is that the smaller the number of customers that complain, the better the market is performing. However, its theoretical foundations are unclear. On one hand, consumers may be well informed about the quality of the service they received and hence, more complaints may be indicative of lower quality. On the other hand, empirical evidence suggests that complaints are driven by expectations as well as by actual quality. Hence, consumers’ complaint decisions are based on the difference between the quality they received and some reference point. Combined with the existence of different degrees of free riding, this implies that consumers’ incentives to complain may vary across markets and along time. As a result, more complaints may reflect higher expectations or lower free riding incentives and not lower quality. This paper identifies conditions under which complaints may help overcome the regulators’ lack of information about the firm’s investment. It is shown that consumer complaints are not always informative and that this lack of informativeness can be worsened by the repeated interaction between the firm and the consumers. Furthermore, the paper shows that the absence of a reference point results in the proportion of complaints being independent of the realised level of quality (and hence, even less informative).

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1 Introduction

Customers’ complaints constitute an important source of consumer-generated information. For instance, the European Union’s (2011) report “Monitoring Consumer Markets in the European Union” states that customers’ complaints constitute “a key metric to evaluate the functioning of a market” (page 12); as a result, complaints are one of the elements the study takes into consideration in order to derive conclusions about the market’s performance. Despite its relevance and generalised use for policy purposes, very little is known about the

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informational content of customers’ complaints. The conventional wisdom about the role of complaints in a market is that the smaller the number of customers that complain, the better the market performs — i.e., complaints are informative about the distortions existing in a market. However, its theoretical foundations are not clear.

Consider, for example, a firm’s decision to invest in improving its customer service department (hereafter, “quality”). Even though the consumers do not observe the firm’s investment, it is likely that they are better informed than the regulator about the quality of the service. Then, everything else constant, more complaints may be indicative of lower quality. However, there is an important caveat because consumers’ incentives to complain may vary across markets and along time for at least two reasons. First, suppose, as the empirical evidence suggests, that complaints are driven by expectations as well as by actual quality and so, that they depend (at least partially) on consumers’ “disappointment” with the quality they received (Forbes 2008). As a result, the definition of what constitutes an “appropriate service level” is likely to change with the context; a higher number of complaints may then be the result not of lower quality but of higher expectations. Second, complaining is a costly action the benefits of which cannot be always fully appropriated. For instance, if future investment increases with current complaints but all the consumers benefit from the resulting higher quality, then each individual consumer would prefer others to face the cost of complaining. Hence, a smaller number of complaints may reflect a significant degree of free riding incentives and not a higher quality.

This paper studies the informativeness of customers complaints about a firm’s investment and their potential as a regulatory tool. The starting point is the assumption that, as suggested by the empirical evidence, customers’ complaints may be the result of either low quality or high expectations. As in the customer service example, the paper considers some contexts in which quality is not verifiable and consumers cannot (fully) appropriate the benefits of their complaints. It is shown that, while a regulation based on complaints may induce a higher investment, those complaints are not systematically informative about the firm’s behaviour. As a result, the firm may be punished more frequently when it invests than when it does not. Furthermore, the lack of informativeness may be worsen by a repeated interaction between the firm and consumers, because it creates incentives for the firm to try to “keep expectations low”.

The paper also identifies conditions under which complaints may help overcome the regulator’s lack of information about the firm’s investment behaviour. In particular, the results challenge the conventional wisdom that the easier it is for consumers to complain the more

1 The relationship between complaints and “disappointment” or “dissatisfaction” seems to be generally accepted in Marketing Literature — see Oliver (1977), Singh (1988) and Boulding, Kalra, Staelin, and Zeithaml (1993), among others. It also seems to be an accepted relationship among regulatory agencies. For example, OFGEM defines complaints as “any expression of dissatisfaction made to an organisation, related to any one or more of its products, its services or the manner in which it has dealt with any such expression of dissatisfaction” OFGEM (2008).

2 Quality is verifiable when it can be (costlessly) described ex ante in a contract and ascertained ex post by a court (Laffont and Tirole 1993). When quality is verifiable, the regulator can reward or punish the firm directly as a function of the level of quality. It can, for example, dictate the heating value of gas or punish an electric utility on the basis of the number and intensity of outages (Laffont and Tirole 1993). On the contrary, when quality is not verifiable it is not possible to write contracts contingent on outcomes.

3 When the consumer expects to receive a direct benefit out of his complaint (like monetary compensations because of electricity shortcuts or reimbursements of incorrectly high bills), his complaining decision can be perfectly explained using standard microeconomic theory: the consumer lodges a complaint as long as the (expected) cost is below the (expected) benefit.
information is contained in these complaints. When the cost of lodging a complaint is zero, the amount of complaints becomes independent of the quality received by the consumers and so they convey no information about the firm’s investment. Finally, the paper delivers comparative static results on consumers’ complaining decisions that explain Forbes’s (2008) empirical findings –namely, that the number of complaints decreases with actual quality and that, after controlling for actual quality, consumers complain more often when they would have expected to receive higher quality.

The paper proposes a psychological game between a monopoly firm and the consumers. A regulated monopoly decides whether to make a costly investment that increases quality. The consumers do not observe the firm’s investment, but they observe a realisation of quality that is related to investment in a first order stochastic dominance sense. After observing quality, consumers decide whether to complain by comparing the realised quality with the one they were expecting to receive. If a high proportion of consumers complains, the firm is fined. Consumers’ reference point is determined by their rational expectations. In this way, the model captures the idea that “disappointment” and “poor performance” are endogenously defined and depend on the context. The presence of a reference point in consumers’ complaining decisions implies that the payoff functions of both the consumers and the firm depend not only on what they do but also on what consumers were expecting from the firm, which reflects the psychological aspect of the game.

The combination of a reference point with a fine that depends on the number of complaints implies that “disappointed” consumers consider lodging a complaint only if by complaining they increase the probability that the firm is “punished” for its “poor performance”. However, complaining is a costly action the benefits of which the consumer cannot (fully) appropriate. Even more, since I make the simplifying assumption that there is a continuum of consumers, the model suffers from an extreme version of free riding and so, without additional assumptions, there would be no complaints in equilibrium. A similar result holds when studying consumers’ incentives to participate in a large election. To tackle this difficulty I borrow from the voting literature the assumption that a fraction of the consumers are group-utilitarians, i.e., they receive a positive payoff for acting according to a strategy that maximises consumers’ aggregate utility. Given their disappointment, consumers have preferences about the probability with which the firm should be punished and the cost of complaining. These preferences are not identical across consumers because complaining costs are heterogeneous. The complaining rule maximises their (expected) aggregate utility, given their disappointment and the regulatory rule. This rule consists of a cut off cost of complaining below which a consumer lodges a complaint.

An equilibrium of the complaining game satisfies three requirements. First, the firm chooses the investment level that maximises its expected profits given its beliefs about the consumers’ strategy and expected quality. Second, consumers choose their complaining strat-

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4Forbes (2008) assumes that consumers form an unbiased expectation of the quality they will receive. With rational expectations, her empirical results imply that an increase in quality decreases the (expected) proportion of complaints only when the higher quality was not anticipated by consumers. The same is true in the model of this paper.

5See, for example, Geanakoplos, Pearce, and Stacchetti (1989) and Battigalli and Dufwenberg (2009).

6In the context of a voting game, Feddersen and Sandroni (2006a) show that a behaviour rule profile that defines rules such that each agent decides he must follow given a proper anticipation of the behaviour of other agents can be described by cutoff points. Their result extends to the application in this paper.

7See Geanakoplos, Pearce, and Stacchetti (1989)
egy optimally given their disappointment with the quality they received and their payoff for following the complaining rule. And third, the firm correctly anticipates consumers’ expected quality, which is in turn consistent with the firm’s strategy and the consumers’ prior beliefs. Using this definition, the one-shot version of the model has two different equilibria: a “high quality equilibrium” in which consumers expect the firm to invest and the firm optimally invests, and a “low quality equilibrium” in which consumers do not expect the firm to invest and the firm optimally fulfils those expectations.

The paper makes a methodological contribution because it proposes a novel way of dealing with the well-known free riding problem that lies at the very root of the generation of information by consumers. The notion of utilitarian agents has been proposed by Harsanyi (1980) and formalised by Feddersen and Sandroni (2006a, 2006b), as a solution to the so-called “paradox of no-voting”. The key insight of the utilitarian model is that the optimal probability of voting is between zero and one: not everybody should stay at home, but not everybody should vote either. This assumption is useful in the context of this paper because it constitutes a plausible explanation for consumers’ behaviour in settings in which tangible benefits accrue only if aggregate participation is high, and no one can be excluded from the benefits of group success. The application in this paper is, to the best of my knowledge, the first formal application of Harsanyi’s ideas outside the area of Political Economy.

The model assumes the existence of a formal regulation based on complaints. However, all that is needed for the results is that consumers’ complaining decision is driven by their disappointment with the quality they received and that the firm is somehow “punished” when consumers complain (and the consumers are aware of this possibility). This situation is more general than the regulatory context I use for presentation purposes. Consider, for instance, a hotel chain that tries to verify that each of its members delivers an appropriate level of service. Clearly in this context what constitutes an “appropriate” service depends on the consumers’ preferences. Thus, the chain may want to rely in customers’ feedback to learn how much effort each of its members is exerting. When doing this, the chain is assuming that the feedback given by consumers can be compared across hotels and along time. The results in this paper suggest that this is not always the case.

There exist many other examples of the type of situation considered in this paper. For example, Amazon keeps record of customers’ complaints about the various companies that use the platform and may prevent them from continuing to use it if the number of customers’ complaints is high enough. Amazon behaves in this case as a sort of “regulatory agency” that punishes the firm based on the amount of complaints. Another clear example of customers’ “dissatisfaction” that was followed by a firm being punished is the decision of some major retailers to stop using the delivery services of Youdel—the biggest delivery service in the United Kingdom, outside of Royal Mail.

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8If voting is costly then, since the likelihood of a vote being pivotal is very small, standard game-theoretic models predict low levels of turnout (Downs 1957).

9If everybody stays at home, the policy will not pass (or the favourite candidate has no opportunity of winning the election), but everybody voting would result in a surfeit of votes, imposing unnecessary costs on society. In this way, the logic of rule-utilitarianism yields an elegant theory of turnout. Harsanyi (1980) assumes that everyone does their duty, but rejects the implicit assumption that doing one’s duty always involves voting.

10This feature is independent of the well-known reviewing system that allows consumers and buyers to rate each other (or among them).

11This includes major retailers like John Lewis, Mothercare and Matalan. (The Guardian 2012).

12According to The Guardian (2012) “about 5,000 customers posted messages in Amazon’s online forums
The rest of the paper is organised as follows. The reminder of this section briefly revises the existing literature and discusses the contributions made by this paper. Section 2 presents the details of the model and describes the players action spaces and payoff functions. It also discusses how consumers’ quality expectations are formed. Section 3 analyses the implications of complaints for the firm’s investment decision in a one shot game. This exercise is useful because it highlights most of the strategic considerations that will shape the equilibrium when the game is repeated. This section also analyses how the equilibrium proportion of complaints is affected by changes in the various parameters of the model and how informative is that proportion about the firm’s investment. Section 4 studies a repeated version of the complaining game. Finally, section 5 concludes.

Related Literature

The model in this paper contributes to the extensive literature on quality provision by a monopoly. Starting with the seminal papers of Spence (1975) and Shesinski (1982), the literature suggests that an unregulated monopoly will over or under supply quality according to whether the marginal consumer values additional quality more or less highly than do infra-marginal consumers on average.\footnote{The difference depends on whether quantity and quality are seen as substitutes or as complements by consumers. In the former case, consumers willingness to pay a higher price for an increase in quality decreases with the quantity that he buys (i.e., the demand curve becomes less elastic as quality increases), while in the latter the elasticity of the demand increases with quality. Thus, the monopoly is more likely to undersupply quality if quality and quantity are substitutes, and to oversupply it if they are complements.} It has further been shown that regulation of service prices can compound, ameliorate or otherwise complicate the already existing market failure (see for example, Spence (1975), Mussa and Rosen (1978) and Besanko, Donnenfeld, and White (1987); Sappington (2005) surveys the literature).\footnote{Price ceilings that are independent of the firm’s realised costs limits its incentives to supply quality, because they prevent the firm from capturing any of the incremental consumer’s surpluses that would result from the higher service quality. However, as noted by Laffont and Tirole (1993), even under pure cost-of-service regulation, the regulated firm does not gain from providing costly services either, so a low perceived cost of supplying quality does not imply a high incentive to provide quality.} The main conclusion of much of the existing literature is that when quality is not verifiable the regulator needs to have a great deal of information before even knowing in which direction he should intervene. By studying the informational content of complaints, this paper considers whether the “policing power” could be moved from the regulatory agency to consumers.

This paper is also related to the literature on reference dependence utility\footnote{Kahneman and Tversky (2001), Koszegi and Rabin (2006), among others.} and with some models in marketing research on customer satisfaction.\footnote{See for example, Singh (1988), Zithaml, Berry, and Parasuraman (1996), Boulding, Kalra, Staelin, and Zeithaml (1993), Oliver (1977), Oliver (1980).} In both cases, it is suggested that consumers utility depends not only on the quality he actually received but also on whether that quality was above or below some reference level. This paper adds to the first branch of literature because, in spite of being widely accepted, the effect of that reference point on consumers’ complaint decisions and on the firm’s incentives to invest have not been previously studied.\footnote{In a different context, Akerlof (2010) shows that norms may be followed because a failure to do so provokes anger and (potentially) punishment.} It differs from the second branch in that they do not require consumers’ expectations to be rational and, as a result, they are not able to make clear predictions about the firm’s strategic response to consumers’ complaints.

calling for the online retailing giant to stop using the parcel delivery company".\footnote{In a different context, Akerlof (2010) shows that norms may be followed because a failure to do so provokes anger and (potentially) punishment.}
Apart from the application in this paper and the voting literature, Harsanyi (1980)-type arguments have also been used to explain household responses to conservation appeals during the California’s energy crisis in 2000 and 2001. Reiss and White (2008) find empirical evidence that consumers do respond to voluntary appeals provided the costs of a collective action failure are tangible and that the public is well aware of it. In this case, each household faces private costs of reducing consumption, a virtually zero possibility of bringing about any tangible benefit with respect to the crisis through individual effort, and a considerable incentive to free-ride on whatever efforts are made by others. The nature of individual free-rider problems here and the lack of private incentives for electricity conservation leave largely “moral suasion”-type arguments to explain their behaviour: consumers individually wanting to “do their part” to mitigate the crisis.

2 The Model

This section presents a static game of quality regulation based on customers’ complaints. A regulated monopoly decides whether to make a costly investment that increases the level of quality received by the consumers in a first order stochastic dominance sense. After observing a quality realisation, the consumers may file a complaint to “inform” the regulator they received a low quality realisation. The regulatory agency is not an strategic player, it observes the proportion of customers that complained ($\delta$) and fines the firm if that proportion is above a threshold $\bar{\delta}$. The fine equals $m$ times the firm’s revenues, with a probability that is proportional to the level of complaints. Hence, a regulatory rule consists of a pair $(\delta, m) \in [0, 1]^2$ of parameters that are public information.

The model assumes that customers complain if they feel disappointed with the quality they received and consider the firm should be punished for its “poor performance”. Consumers’ disappointment is defined as the difference between the level of quality they were expecting to receive ($\hat{z}$) and the one they actually received ($q$). If they are disappointed, the consumers consider the regulator should fine the firm and so they complain in order to increase the probability with which the firm is punished. The fact that consumers’ prior expectations affect their complaining behaviour implies that the complaining game belongs to the class of psychological games.\footnote{Psychological games differ from standard games in that the domain of the utility function includes explicitly the beliefs a player holds about the other players’ strategies. As a result, payoffs at a given endnode are endogenous: beliefs determine the player’s utility and they are explained/predicted via some solution concept (Battigalli and Dufwenberg (2009), Geanakoplos, Pearce, and Stacchetti (1989)). In the context of this paper, this means that a given level of investment may lead to different final payoffs for different pre-play beliefs (consumers expected quality). The standard assumption is that beliefs are correct in equilibrium, and that is the condition I impose in the equilibrium definitions of sections 3 and 4.}

The firm faces a unit demand for its product and an exogenously given price, $p$ (a binding price cap). Thus, its revenues are deterministic and independent of its investment decision.\footnote{This implies that the firm’s investment in quality is not aimed at increasing future demand; see Shapiro (1982) for model a in which the firm’s incentives to investment are related with future demand.}

As a result, its only incentive to invest in quality is to reduce the (expected) proportion of complaints and, hence, the expected value of the fine.

The section proceeds as follows. Section 2.1 presents the payoff function of the consumers
and discusses their complaining decision, while section 2.2 considers the firm’s investment decision. Finally, section 2.3 explains how consumers’ expectations are formed.

2.1 The Consumers

There is a continuum of consumers normalised to size one. After receiving a quality draw, each consumer decides whether to lodge a complaint. Hence, his action space is $C_i \in \{0, 1\}$, where $C_i = 1$ means consumer $i$ files a complaint. Each consumer’s utility is the sum of the consumption utility he derives from the quality he received and, if he makes a complaint, his payoff from complaining. The consumer’s payoff from complaining depends on his disappointment and on his cost of complaining, but also on whether the firm is punished for its “poor performance”. Each consumer $i$ faces a cost of complaining $\sigma_i c$, where $\sigma_i$ is the realisation of a random variable uniformly distributed over $[0, 1]$, and $c$ is a positive constant. $\sigma_i$ is independent of any other random variable in the model. Consumers do not observe the cost of other consumers, but do know the distribution from which they are drawn. The utility of an individual consumer $i$ with cost $c\sigma_i$, who was expecting $\hat{z}$ and received $q$ is:

$$U_i(C_i; q, \hat{z}, \sigma_i) = q + \theta(\hat{z} - q)1_{\{\delta \geq \bar{\delta}\}}(\delta) - C_i c\sigma_i$$

where $\theta \in (0, 1)$ can be interpreted as the consumer’s marginal utility per unit of punished-disappointment and the indicator function $1_{\{\delta \geq \bar{\delta}\}}(\delta) \in \{0, 1\}$ takes the value 1 if the firm is fined (i.e., if $\delta \geq \bar{\delta}$) and zero otherwise. Implicit in the utility function is the additional assumption that consumers heterogeneity is restricted to individual costs of complaining ($\sigma_i$); this means that all the consumers have the same willingness to complain and the same intensity of preferences over quality. The assumption is relevant in that it sidesteps the question of how the burden of complaining should be shared among consumers with different intensities of preferences.

The utility function reflects the assumption that the consumer complains in order to “punish the firm’s poor performance”. The consumer receives a positive payoff only if $q < \hat{z}$ and the firm is fined. The implications in terms of complaining behaviour are twofold. First, if the realised quality is above the quality the consumer was expecting to receive he will not lodge a complaint. Second, a disappointed consumer is willing to face the cost of complaining if by doing so he increases the probability with which the firm is punished.

However, individual consumers cannot appropriate the benefits of their complaints. If the firm is fined, every consumer receives a payoff $\theta(\hat{z} - q)$ independently of whether he made a complaint or not. Only those agents who actually filed a complaint ($C_i = 1$) face the costs. As there is continuum of consumers, the model as defined so far suffers from an extreme version of free-riding. Hence, without additional assumptions there would be no complaints in equilibrium. To overcome this limitation, I borrow from the voting literature the assumption that consumers are “group - utilitarian”$: they receive a positive payoff for acting according to a strategy that maximises consumers’ aggregate utility.\footnote{The qualitative results would not change if only a proportion $\gamma \in (0, 1)$ of consumers were group utilitarians, as long as either $\gamma$ or the distribution from which it is drawn, is public information.} Formally, the utilitarian assumption implies that the group’s problem is strategically equivalent to a
one person decision problem with payoff function defined as consumers’ aggregate (expected) utility.\textsuperscript{22}

Let a social rule $\sigma'$ be a cut off that specifies a critical cost level below which a consumer makes a complaint.\textsuperscript{23} By the law of large numbers, the proportion of complaints equals the cut off cost, i.e. $\delta = \sigma'$. The utilitarian assumption implies that the group’s expected utility from following a rule $\sigma'$, when they received quality $q$ and were expecting $\hat{z}$, is:\textsuperscript{24}

$$
\mathbb{E}U(\sigma'; q, \hat{z}) = \begin{cases} 
q + \theta(\hat{z} - q)\sigma' - \frac{\xi}{\sigma} - \frac{\xi}{2}\sigma'^2 & \text{if } \sigma' \geq \delta \\
q - \frac{\xi}{2}\sigma'^2 & \text{if } \sigma' < \delta 
\end{cases}
$$

(2)

Consumers’ complaining decision is made after they observed quality. Given their disappointment, the consumers problem is to choose the cut off rule $\sigma^*$ that maximises (2). Thus, consumers’ strategy is a mapping from their disappointment $(\hat{z} - q)$ into a cutoff point between zero and one: $\sigma(q; \hat{z}) : [0, 1]^2 \rightarrow [0, 1]$. The cut off rule that maximises consumers’ expected utility, given a realisation of quality and consumers’ expectations, is:\textsuperscript{25}

$$
\sigma^*(q; \hat{z}) = \begin{cases} 
1 & \text{if } q \leq \hat{z} - \psi \\
\frac{\theta(\hat{z} - q)}{\psi} & \text{if } \hat{z} - \psi < q \leq \hat{z} - \frac{\delta \psi}{\sigma'} \\
\frac{\delta}{\psi} & \text{if } \hat{z} - \frac{\delta \psi}{\sigma'} < q \leq \hat{z} - \frac{\delta \psi}{\sigma'} \\
0 & \text{Otherwise}
\end{cases}
$$

(3)

where $\psi = \frac{\xi}{\sigma}$. Given $\hat{z}$, the proportion of complaints induced by $\sigma^*$ is decreasing in the realised quality: the smaller is $q$ the higher is consumers’ disappointment and so is the cost they are willing to face in order to have the firm punished. The optimal cut-off rule is shown in Figure 1. The flat regions for very low quality realisations and for $q \in (\hat{z} - \delta \psi, \hat{z} - \frac{\delta \psi}{\sigma'})$ are due to the restrictions that the proportion of complaints cannot be higher than 1 in the first case, and that the probability of fine becomes zero for $\sigma^* < \delta$ in the latter.\textsuperscript{26} Finally, note that there is a “region of tolerance” in which consumers do not complain despite the

\textsuperscript{22}As discussed by Feddersen and Sandroni (2006a), one possible intuition is that, if a consumer believes that all the other utilitarian agents will use the same strategy as he does himself, he will independently decide that the right strategy is the one that maximises aggregate utility. In this way, a consumer will be willing to face the cost of complaining even though he understands that his single complaint has no effect on the final outcome. However, the mathematical structure of the model is equivalent to the one in elite driven turnout models. In the complaining game of this paper, this second interpretation could mean, for example, that consumers follow the directions of some sort of Consumers’ Associations.

\textsuperscript{23}Given a social rule $\sigma'$, a (utilitarian) consumer’s action is:

$$
C_i(\sigma_i, \sigma') = \begin{cases} 
1 & \text{if } \sigma_i < \sigma' \\
0 & \text{otherwise}
\end{cases}
$$

\textsuperscript{24}Expectation is taken with respect to the rule $\sigma'$. The probability that an agent makes a complaint is $\text{Prob}(\sigma_i \leq \sigma') = \sigma'$. The expected cost of complaining, conditional on the consumer effectively making a complaint is $E(\sigma_i|\sigma_i \leq \sigma') = (1/\sigma') \int_0^\sigma dx = \sigma'/2$.

\textsuperscript{25}$\sigma^*(q; \hat{z})$ could in principle take any value in the interval $[0, 1]$: however, it is clear from (2) that values of $\sigma'$ different from zero but smaller than $\delta$ cannot be optimal. Consumers’ optimisation problem can then be written as: $\max_{\sigma'} \{EU(q, 0); \max_{\sigma \in [0, 1]} \mathbb{E}U(q, \sigma; q; z)\}$.

\textsuperscript{26}Consumers’ optimal strategy in (3) implicitly assumes that when the realised quality is exactly $\hat{z} - \frac{\delta \psi}{\sigma'}$, consumers do complaint, even though they are indifferent between complaining in a proportion $\delta$ and not complaining at all. The exact way in which this indifference is broken does not affect the results.
realised quality being below \( \hat{z} \). Within this region, the group’s disappointment is not high enough to compensate the cost of a proportion of complaints equal to or greater than \( \bar{\delta} \). This result supports some arguments made in the marketing literature that define a “zone of tolerance” within which “the company is meeting customer expectations” (Singh 1988).

2.2 The Firm’s Investment Decision

The firm is risk neutral and seeks to maximise expected profits. It can be of any of two types, “bad” (\( B \)) or “good” (\( G \)). The good firm’s investment is a binary decision \( I_G \in \{L, H\} \), where \( H \) means the firm invests and delivers higher (expected) quality and \( L \) means it does not invest (and hence, it delivers low quality). The bad firm has a singleton action space \( I_B \in \{L\} \). In order to keep the model tractable, it is assumed that quality is uniformly distributed over \([0, 1/2]\) if the firm is bad or if it is of the good type but it does not invest, and \( q \sim U[0, 1] \) if the good firm invests. The firm has private information about its type. To simplify notation, \( I \) is used as a shorthand notation for \( I_G \). Investing in increasing quality costs \( h > 0 \). This investment cost is independent of any other cost faced by the company and it is public information. The firm also faces a (potential) cost derived from the fine. If the proportion of complaints, \( \delta \), is above the threshold \( \bar{\delta} \), the firm is fined with a probability equal to \( \delta \). Then, given a proportion of complaints \( \delta \), the good firm’s profits are:

\[
\Pi(I, \delta) = \begin{cases} 
p - 1_{I=H}(I) \cdot h - mp & \text{if } \delta \geq \bar{\delta} \text{ and the firm is fined} \\
p - 1_{I=H}(I) \cdot h & \text{otherwise} 
\end{cases}
\]

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27 Not even utilitarian consumers would follow a social rule that directs a positive proportion of consumers to complain within this region of quality realisations.

28 According to (Singh 1988), this region is delimited by the desired service level and the adequate service level (i.e., the level of service the customer will accept).

29 The uniform distributions simplify the exposition by allowing closed form solution for the expected proportion of complaints. However, all my findings remain true for a more general class of quality distributions as long as \( F(q; I = H) \leq F(q; I = L) \forall q \) (with strict inequality for some \( q \)).

30 Results would not change if we assume that the regulator cannot observe the firm’s costs. All that is required is that it is able to observe the proportion of complaints and the firm’s revenues.
where \(1_H \{ I \} \) is an indicator function that takes the value of 1 if the firm invests \( I = H \) and zero otherwise; \( p \) denotes the firm’s revenues and \( mp\delta \) is the fine paid by the firm.

The firm’s expected cost depends on the observed proportion of complaints, which in turn depends on the realised quality, the quality the consumers were expecting to receive and their complaining strategy. When the firm makes its investment decision, it does not observe the level of quality consumers expect to receive, but it does have some beliefs about it, \( \hat{z} \). Given those beliefs, the good firm’s expected payoff is the expectation of (4) with respect to the probability measure over quality induced by its investment strategy. The expected proportion of complaints when the firm invests \( I \) is \( E_{qI} \delta(qI, \hat{z}, \sigma) \). The firm’s optimal action depends on the trade-off between the cost of investment and the expected fine: by not investing, the firm reduces its costs by \( h \), but it also makes it less likely that quality meets consumers’ expectations, increasing the expected value of the fine. To simplify notation, denote \( \pi_H(\hat{z}, \sigma) = E_{qH} \Pi(H, \delta(qH, \hat{z}, \sigma)) \) and \( \pi_L(\hat{z}, \sigma) = E_{qL} \Pi(L, \delta(qL, \hat{z}, \sigma)) \). Given its beliefs about the level of quality consumers’ expect to receive, the firm invests if and only if:

\[
\pi_H(\hat{z}, \sigma) \geq \pi_L(\hat{z}, \sigma) \iff \left[ E_{qL} \delta(qL, \hat{z}, \sigma) - E_{qH} \delta(qH, \hat{z}, \sigma) \right] \geq \frac{h}{mp} \tag{5}
\]

The firm’s investment strategy is a function from the level of quality the firm believes consumers expect to receive (\( \hat{z} \)) to an investment level: \( I : \hat{z} \rightarrow \{ L, H \} \). Denote by \( \hat{z}^* \) the level of \( \hat{z} \) at which the firm is indifferent between investing and not investing; \( \frac{h}{mp} \) is constant and independent of consumers’ expectations, but the change in the expected proportion of complaints when the firm’s investment changes is an increasing function of \( \hat{z} \). Then, the firm’s optimal strategy is a cut-off of the form:

\[
I^* = \begin{cases} 
H & \text{if } \hat{z} \geq \hat{z}^* \\
L & \text{if } \hat{z} < \hat{z}^*
\end{cases}
\]

The firm’s strategy is increasing in consumers’ expectations. When consumers expect too much from the firm, the firm’s best reply is to fulfil those expectations, as otherwise the fine becomes too heavy. However the firm also fulfils consumers prior expectations when they are low, because if consumers do not expect much, their disappointment is not very high and the (expected) proportion of complaints is not enough to compensate the cost of investment (\( h \)).

### 2.3 Consumers’ Beliefs and Expectations

The level of quality consumers expect to receive is determined by their beliefs about the type and strategy of the firm, and the equilibrium condition requires those beliefs to be correct.\(^{32}\) Denote by \( \bar{q}_B = \int_B q \cdot f(q; B) dq \) the average quality that is delivered by the bad

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\(^{31}\) Consumers’ complaining decision depends on their prior expectations, so the firm needs to form some beliefs about them in order to decide the level of investment that maximises its profits; \( \hat{z} \) denotes the firm’s belief about \( \hat{z} \).

\(^{32}\) This assumption rules out beliefs structures in which, for example, the consumer reduces his prior expectations so that he does not feel disappointed if the quality realisation is low. For models of belief-dependent preferences in which the agents can choose beliefs see Akerlof and Dickens (1982) or Brunnermeier and Parker (2005), for example.
type of the firm, and by \( \bar{q}_{G,I} = \int_{G,I} q \cdot f(q; G, I) dq \) the average quality that is delivered by the good firm if it invests \( I \):

\[
\bar{q}_{G,I} = \begin{cases} 
\bar{q}_{G,H} & \text{if } I = H \\
\bar{q}_{G,L} & \text{if } I = L
\end{cases}
\]

Then, the level of quality consumers expect to receive is:

\[
\hat{z}_I(\tau) = \begin{cases} 
\tau \bar{q}_{G,H} + (1 - \tau) \bar{q}_B & \text{if } I = H \\
\tau \bar{q}_{G,L} + (1 - \tau) \bar{q}_B & \text{if } I = L
\end{cases}
\]

where \( \tau \) is the probability consumers assign to the firm being good.

3 Equilibrium

The firm and the consumers choose their actions according to their prior beliefs without observing each other’s action, and the consumers do not observe the type of the firm neither. The equilibrium concept I use is, therefore, Bayesian Nash Equilibrium. In this application, such an equilibrium needs to satisfy three requirements. First, the firm chooses the investment level that maximises its expected profits given its beliefs about consumers’ cut off rule and expected quality. Second, consumers choose the complaining rule optimally given their disappointment with the quality they received (i.e., given \( \hat{z} \) and \( q \)). And third, the firm correctly anticipates consumers’ expected quality, which is in turn consistent with the firm’s strategy and consumers’ prior about its type, \( \tau \). Definition 1 formalises the equilibrium requirements.

Definition 1. Equilibrium in the Static Game. An equilibrium of the complaining game when \( T = 1 \) is a pair of strategies \((I^*, \sigma^*)\) and expected qualities \((\hat{z}, \hat{\hat{z}})\) for which the following conditions are satisfied:

1. \( I^* \) maximises the firm’s expected profits given \( \hat{z} \) and \( \sigma^* \)
2. \( \sigma^* \) maximizes consumers’ expected utility given \( \hat{z} \)
3. \( \hat{\hat{z}} = \hat{z} = \hat{z}_{I^*}(\tau) \)

The expected proportion of complaints when the good type of the firm invests is \( E_{qH}(\sigma^*(q; \hat{z})) = \hat{z} - \hat{\psi} \), and when it does not invest (or when the firm is bad) is \( E_{qL}(\sigma^*(q; \hat{z})) = 2\hat{z} - \psi \). Then, the cut off point in the firm’s strategy is \( \hat{z}^* = \frac{h}{mp} + \frac{\hat{\psi}}{2} \). \( \hat{z}^* \) is determined by the magnitude of the “punishment” \((mp)\) relative to the investment cost \((h)\), and by consumers’ relative cost of complaining \((\psi = \frac{c}{\theta})\). The less harsh the punishment or the more difficult it is for consumers to complain, the higher is \( \hat{z}^* \) and thus the higher is the \( \hat{z} \) required for the firm’s optimal action to be \( I = H \).

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\(^{33}\) Consumers’ beliefs about the firm’s strategy are they “first order beliefs”, defined as a probability distribution over the firm’s action space (Battigalli and Dufwenberg 2009). As I consider only pure strategies, consumers’ first order beliefs assign probability one or zero to the event in which the good firm invests.

\(^{34}\) Geanakoplos, Pearce, and Stacchetti (1989)

\(^{35}\) \( E_{qH}(\sigma^*(q; \hat{z})) \) is the expectation of consumers’ optimal strategy when \( q \sim U[0, 1] \), and \( E_{qL}(\sigma^*(q; \hat{z})) \) is the expectation when \( q \sim U[0, 1/2] \). See Appendix ?? for details.
The static game has a separating and a pooling equilibrium. In the first case, the good type of the firm invests and differentiates itself from the other type with a positive probability. In the second case, the firm does not invest and so it camouflages itself with the bad type. Define a “High Quality Equilibrium” (HQE) as one in which the good type of the firm invests, and a “Low Quality Equilibrium” (LQE) as one in which it does not. Given the equilibrium definition above, a HQE exists if and only if \( \pi_H(\hat{z}_H, \sigma^*) \geq \pi_L(\hat{z}_H, \sigma^*) \), and while a LQE exists if and only if \( \pi_L(\hat{z}_L, \sigma^*) \geq \pi_H(\hat{z}_L, \sigma^*) \).

Proposition 1. Equilibria of the Static Game. Given \( \psi < \frac{1}{4} \) and \( \tau \in (0, 1) \):

1. If \( \hat{z}^* \geq \frac{1}{2} \) a unique low quality equilibrium exists.

2. If \( \hat{z}^* \in (\frac{1}{4}, \frac{1}{2}) \) there exists \( \tau^* \in (0, 1) \) such that a unique low quality equilibrium exists for \( \tau \in (0, \tau^*) \), but high and low quality equilibria coexist for \( \tau \in [\tau^*, 1) \).

3. If \( \hat{z}^* \leq \frac{1}{4} \) a unique high quality equilibrium exists.

Proof. In equilibrium the firm has correct beliefs about the level of quality consumers expect to receive, so \( \hat{\pi} = \hat{\pi} \). Given that the distributions of quality are public information, \( \hat{\tau}_H(\tau) = \frac{1}{4} + \frac{1}{4} \tau \) and \( \hat{\tau}_L = \frac{1}{4} \). The firm’s optimal strategy depends on whether \( \hat{z} \) is greater than or smaller than \( \hat{z}^* \). There are three possibilities:

- When \( \hat{z}^* \geq \frac{1}{2} \), the cost of investing in quality is high relative to the (expected) punishment, \( \hat{z}_L < \hat{\tau}_H(\tau) < \hat{z}^* \), and as a result \( \pi_L(\hat{z}_L, \sigma^*) > \pi_H(\hat{z}_L, \sigma^*) \) and \( \pi_L(\hat{z}_H(\tau), \sigma^*) > \pi_H(\hat{z}_H(\tau), \sigma^*) \). The firm’s optimal strategy is \( I = L \), independently of consumers expectations, and so rational consumers do not expect something different from low quality. There is a unique low quality equilibrium.

- If \( \hat{z}^* \in (\frac{1}{4}, \frac{1}{2}) \), there exists a unique \( \tau^* \) such that \( \hat{\tau}_H(\tau^*) = \hat{z}^* \). Uniqueness is given by the fact that \( \hat{\tau}_H(\tau) \in [\frac{1}{4}, \frac{1}{2}] \) and is a monotone and increasing function of \( \tau \), while \( \hat{\tau}^* \) belongs to the same interval but is exogenous and independent of \( \tau \). For \( \tau < \tau^* \), \( \hat{\tau}_L < \hat{\tau}_H(\tau) < \hat{z}_L \), implying that \( \pi_L(\hat{z}_L, \sigma^*) > \pi_H(\hat{z}_L, \sigma^*) \) and \( \pi_L(\hat{z}_H(\tau), \sigma^*) > \pi_H(\hat{z}_H(\tau), \sigma^*) \). The firm’s optimal strategy is \( I = L \) if \( \hat{z} \) and there exists a unique low quality equilibrium. As \( \tau \) increases so does the level of quality consumers expect to receive if they anticipate \( I^* = H \). For \( \tau \geq \tau^* \), \( \hat{\tau}_H(\tau) \geq \hat{z}_L \) and \( \pi_L(\hat{z}_L, \sigma^*) > \pi_H(\hat{z}_L, \sigma^*) \) and \( \pi_L(\hat{z}_H(\tau), \sigma^*) > \pi_H(\hat{z}_H(\tau), \sigma^*) \) and so there are two equilibria: the firm optimally invests if consumers expect high quality (HQE) and the firm does not invest if consumers’ expected quality is \( \hat{z}_L \) (LQE).

- If \( \hat{z}^* \leq \frac{1}{4} \), the (expected) punishment is harsh relative to \( h \) and \( \hat{z}^* \leq \hat{z}_L < \hat{\tau}_H(\tau) \) \( \forall \tau \in (0, 1) \). In this case \( \pi_H(\hat{z}_H(\tau), \sigma^*) > \pi_L(\hat{z}_H(\tau), \sigma^*) \) and \( \pi_H(\hat{z}_H(\tau), \sigma^*) > \pi_L(\hat{z}_L, \sigma^*) \). Then, investing is the firm’s optimal strategy. As consumers anticipate this, they expect high quality (\( \hat{z}_H(\tau) \)) and there is a unique high quality equilibrium.

36 \( \pi_L(\hat{z}_H, \sigma^*) \) and \( \pi_H(\hat{z}_L, \sigma^*) \) cannot be the firm’s profits in any equilibrium of the game, as they both fail to comply with the “correct beliefs” requirement of Definition 1. In both cases, the firm’s actual investment differs from its beliefs about \( \hat{z} \), meaning that either the firm has incorrect beliefs about the level of quality consumers expect to receive or that the firm’s beliefs are correct but consumers expectations are not consistent with the firm’s investment strategy.

37 When \( I = L \), the good firm does not differentiate itself from the bad one. Thus, consumers’ expected quality, \( \hat{z}_L \), is independent of \( \tau \) (and so is \( \hat{z}_L \)).
As shown in Appendix ??, the condition $\psi < 1/4$ is sufficient but not necessary for the results in this section and in the next ones. This condition guarantees that there exists a positive probability of complain for every quality realisation.

Proposition 1 shows how a regulatory rule based on customers complaints affects the monopoly’s investment behaviour. Such a rule induces a higher investment in quality as long as the punishment is “harsh enough”. In the context of this paper this requires not only that the size of the fine -the proportion of revenues lost in case of a fine ($mp$)- is high relative to the cost of investment ($h$) but also that the consumers do transmit their dissatisfaction to the regulator. Both conditions are summarised by the parameter $\hat{z}^*$: a high value of $\hat{z}^*$ reflects a reduced effectiveness of the punishment, either because the cost of investment is high relative to the fine or because consumers’ relative cost of complaining is high ($\psi = 0$). Therefore, as $\hat{z}^*$ increases the game moves towards a low quality equilibrium.

The quality consumers expect to receive is higher when they believe the firm is investing, but also when they assign a higher probability to the firm being of the good type -i.e., consumers expect more from a good firm. As a result, the firm’s payoff in a HQE is a decreasing function of $\tau$: the more convinced consumers are that they are facing a good firm, the more they expect and so the higher is the (expected) proportion of complaints and the lower are the firm’s (expected) profits. Furthermore, the closer is $\tau$ to one, the smaller is the size of the fine required to induce investment ($m$).

It is worth noting, however, that the change in the set of equilibria resulting from the introduction of the fine does not necessarily imply an increase in total welfare. The (average) quality in a low quality equilibrium is the same that would be delivered without the regulatory rule. The firm’s expected profits, however, are smaller after the introduction of the regulation because it faces a positive probability of fine. An equivalent statement about the change in consumers’ welfare with and without the regulatory rule is less clear because it is assumed that they derive some positive utility from complaining.

In a high quality equilibrium the firm optimally invests because the cost of investing is smaller than the fine it would have to pay if a low realisation of quality results in a high proportion of complaints. Even though the level of quality consumers receive in this case is higher than without the regulation, the cost of that quality exceeds the cost of investing by the expected fine (because there is a positive probability of fine). As this creates an inefficiency, the result is only a “second best” result. Furthermore, the cost of delivering a higher quality is increasing in consumers’ expectations and so the more convinced are consumers that the firm is of the good type, the higher is the cost of the quality increase.

### 3.1 Informativeness of Complaints

The regulatory agency may be interested in punishing the firm more harshly when it is not investing. However, the proportion of complaints observed by the regulator reflects consumers’ disappointment with the quality they received and not necessarily the quality itself. This section identifies conditions under which a higher proportion of complaints reflects both a higher disappointment and a lower investment - i.e., when $E_{qL}(\sigma^*(q; \hat{z}_L)) \geq E_{qH}(\sigma^*(q; \hat{z}_H))$. 
If this is the case, I say that complaints are informative about the equilibrium being played.

Given \( \hat{\tau} \), the expected proportion of complaints is lower in a separating than in a pooling equilibrium because the probability of high quality realizations is higher when the firm invests -i.e., \( E_{qL}(\sigma^*(q; \hat{\tau})) \geq E_{qH}(\sigma^*(q; \hat{\tau})) \). However, in equilibrium consumers have correct beliefs about the firm’s strategy and they modify their expectations accordingly. Because consumers expectations are higher in a high quality equilibrium, it is not clear whether they will complain more when the firm is not investing. Lemma 1 presents the conditions under which complaints are informative in the one shot game. In order to study the informativeness of complaints, I focus on the set of parameters for which a high quality and a low quality equilibria coexist.

Lemma 1. Given \( \hat{\tau} \in (\frac{1}{4}, \frac{1}{2}) \), complaints are informative about the equilibrium being played if and only if the following conditions hold:

1. \( \tau \in (\tau^*, 1 - 2\psi) \), were \( \tau^* \) is such that \( z_H(\tau^*) = \hat{\tau} \)
2. \( \psi \in (0, \frac{1}{2} - \frac{h}{mp}) \)

Proof. First, note that the definition of informativeness of complaints is based in the existence of multiple equilibria, so the analysis is restricted to \( \tau > \tau^* \) and \( \hat{\tau} \in (\frac{1}{4}, \frac{1}{2}) \). \( E_{qL}(\sigma^*(q; \hat{\tau})) > E_{qH}(\sigma^*(q; \hat{\tau}) \) if and only if \( 2\hat{\tau}L - \hat{\tau}H(\tau) > \frac{\psi}{\tau} \) (where \( \psi = \frac{c}{\theta} \)).

The second part of the Lemma implies that for complaints to be informative, complaining must be “neither too cheap nor too costly”. When the relative cost of complaining, \( \psi \), is zero the expected utility in (2) is maximised when every consumers complains if \( \hat{\tau} > q \) (because \( \sigma^* = 1 \) maximises the probability that the firm is fined) and when nobody complaints if \( \hat{\tau} \leq q \). Then, the proportion of complaints becomes constant and independent of consumers’ disappointment. Finally, when complaining is very costly, complaints are not informative because the set of \( \tau \)’s determined in the previous paragraph is empty. \( \tau \in (\tau^*, 1 - 2\psi) \) is not an empty set if \( 1 - 2\psi \geq \tau^* \). From Proposition 1, \( \tau^* = 4\hat{\tau}^* - 1 \), where \( \hat{\tau}^* = \frac{h}{mp} + \frac{\psi}{\tau^*} \).

Then, the second condition in the Lemma implies that complaints are informative only for values of \( \psi \) in the set \( (0, \frac{1}{2} - \frac{h}{mp}) \)-which is smaller than the set induced by the condition that \( \hat{\tau}^* \leq 1/2 \).

The Lemma shows that complaints are not always a good signal of the firm’s investment. Consumers complaints are not informative of the equilibrium being played whenever the change in their disappointment between the low and the high quality equilibria is driven by a change in their expectations and not by a change in the (average) quality being delivered by the firm. When consumers are reasonably convinced that the firm is “good” (high \( \tau \)), \( \hat{\tau} \) increases more than the (average) realisations of quality and so complaints are (on expectations) higher in an equilibrium in which the firm invests. In this case, complaints are informative about how disappointed consumers are with the quality they received but not about the firm’s investment. As a result, the firm might be punished more harshly when it

\[ \text{Note that } \psi = 0 \text{ may be the result of either } c = 0 \text{ or } \theta \rightarrow \infty. \]
is investing than when it is not.

The condition \( \tau \leq 1 - 2\psi \) means that, given \( \tau \), the informativeness of complaints decreases if \( \psi \) increases -i.e., if the cost of making a complaint is higher relative to consumers’ willingness to complain. Hence, the informativeness of complaints depends also on how easy it is for consumers to complain. The result in the Lemma shows that if complaining is too costly, the level of disappointment required for consumers to be willing to face the cost of “informing” the regulator is too high and so the regulator observes only a small proportion of complaints -i.e., consumers do not complain enough so as to transmit information to the regulator. On the other extreme, if \( c = 0 \) the optimal social norm becomes independent of the size of the difference between expected and realised quality, and the (expected) proportion of complaints is the same in both equilibria.\(^{39}\) When complaining is very cheap, the proportion of consumers that lodge a complain is so high that complaints become meaningless.

The limited informativeness of complaints is due to the fact that consumers’ complaining decision does not depend solely on the realisation of quality but also on their prior expectations. As a result, there is not a unique relationship between the proportion of complaints and the firm’s investing behaviour. However, the existence of a reference point is a necessary condition for the existence of a positive proportion of complaints in any equilibrium of the game.

### 3.2 Comparative Static of the Optimal Complaining Rule

The optimal rule in (3) can be used to derive predictions about the way in which complaints depend on the exogenous variables in both, consumers’ individual utility and the regulatory rule. Those predictions are summarised in the following properties. Figure 2 presents the changes in the optimal complaining rule resulting from the properties below. In all the cases, the continuous line represents consumers’ optimal strategy before the change and the dashed line is their optimal strategy after it.

**Property 1.** The (expected) proportion of complaints is increasing in consumers’ prior expectations.

A higher \( \hat{z} \) increases consumers’ disappointment with every realisation of quality \( (q) \), and so consumers are willing to accept the higher social cost that results from an increase in the cutoff point. This effect is showed in Figure 2a.

**Property 2.** Given consumers’ disappointment, the optimal social rule decreases when \( \psi = \frac{c}{\theta} \) increases.

An increase in \( c \) or a decrease in \( \theta \) implies that the relative utility consumers derive from complaining is reduced. As a result, the optimal proportion of complaints for any given quality realisation is smaller. The only exception are very low realisations for which it is still optimal to direct every ethical agent to complaint.

\(^{39}\)When \( c = 0, \hat{z}^* = \frac{h}{\eta p} \) and the firm’s incentives to invest depend solely on the relative magnitude of the investment cost and the fine.
Figure 2: Changes in the Optimal Cutoff as the Parameters Change

(a) Increase in $\tilde{z}$

(b) Increase in $\psi$

(c) Increase in $\tilde{\delta}$
Property 3. An increase in $\delta$ makes it more costly for consumers to punish the firm when quality realisations are relatively high.

A higher $\delta$ makes it more difficult for consumers to meet the regulator’s requirements and so it increases the group’s cost of punishing the firm for high levels of quality, when the payoff of complaining is relatively low. For realisations of quality that induce a $\sigma^* > \delta$, the change in $\delta$ does not affect the optimal cut off rule.

4 The Repeated Game

In this section I explore how the repetition of the game affects the firm’s incentives to invest in quality and the informativeness of consumers’ complaints. When the complaining game is played repeatedly, consumers’ beliefs are updated at the end of every period and so their reference point changes over time. The firm’s strategy depends on consumers current expectations, but also on how today’s investment affects the level of quality they expect to receive in the future: higher investment in a given period reduces that period’s expected fine but it increases consumers’ future expectations and so it increases the probability of being fined in the future. In this way, the repetition of the game generates incentives for the good firm to induce particular beliefs in the consumers.\footnote{The psychological aspect of the game means that current actions affect future play like in standard dynamic games, but they also affect players’ beliefs and, because beliefs affect payoffs, current actions affect future payoff for any possible action.}

The main result is that a repeated interaction between the consumers and the firm may reduce both the firm’s incentives to invest and the informativeness of complaints. The later is due to the fact that the adverse effect on investment is stronger for values of the parameters for which complaints would have been informative in the one shot game: by not investing in any of the $T$ periods, the firm keeps consumers’ expectations (and the proportion of complaints) low, and so the regulator can not infer the lack of investment from the proportion of consumers that complain.

The public information in period $t$ is the history of past quality realisations and the proportion of consumers’ complaints, $h^t = (q_1, \sigma_1; q_2, \sigma_2; \ldots; q_{t-1}, \sigma_{t-1})$ for $t \geq 2$. The set of public histories is then $\mathcal{H} = [0, 1]^{2(T-1)}$. When making a complaining decision, consumers know the public history up to that moment but they also have private information about the level of quality they expect to receive (and the level they expected in any previous period). The set of their private histories is then defined as $\mathcal{H}_C = \bigcup_{t=1}^{T} H^t_C$, where $H^t_C = [0, 1]^{3(t-1)}$. Consumers’ optimal rule maximises the present value of their expected utility, $\sigma^*_k \in \arg \max \sum_{k=1}^{T} \beta^{k-1} EU(\sigma_k; q_k, \hat{z}_k)$ and so, their strategy in the repeated game is a sequence of complaining decisions $\{\sigma_k\}_{k=1}^{T}$, each of which maps their private history and the current period quality into a cutoff between zero and one, $\sigma_t : H^t_C \times [0, 1] \rightarrow [0, 1]$.\footnote{Consumers make their period-$t$ complaining decision after observing the realisation of $q_t$, so this last quality realisation also forms part of the information they have when deciding how strongly to complain. Also note that individual consumers have private information about their costs of complaining and actions. However, the group utilitarian assumption implies that their strategy is the same that would result if there were only one “big” consumer. As a result, the only relevant information is the distribution of the costs of complaining (which is public information). The latter holds because, as there is a continuous of anonymous consumers, each of them can do no better than myopically follow the complaining rule. See Maliath and Samuelson (2006).}

The firm has information about past quality and complaints, but also about its own past actions and its beliefs about consumers’ expectations. Then, a private history for the good type of the firm includes both the public history and the history of its investment decisions
and beliefs. The firm’s private history up to period \( t \) is \( H^t_G = \{L, H\}^{t-1} \times [0, 1]^{3(t-1)} \) and the set of all possible private histories for the firm is \( \mathcal{H}_G = \bigcup_{t=1}^T H^t_G \). The strategy of the good type of the firm in period \( t \) is a sequence \( \{I_t\}_{t=1}^T \) that assigns, in each period, an investment level for any possible private history, \( I_t : H^t_G \rightarrow \{L, H\} \). The firm’s private history up to period \( t \) is \( H^t_G = \{L, H\}^{t-1} \times [0, 1]^{3(t-1)} \) and the set of all possible private histories for the firm is \( \mathcal{H}_G = \bigcup_{t=1}^T H^t_G \). The strategy of the good type of the firm in period \( t \) is a sequence \( \{I_t\}_{t=1}^T \) that assigns, in each period, an investment level for any possible private history, \( I_t : H^t_G \rightarrow \{L, H\} \). The firm’s investment decision in period \( t (I^*_t) \) maximises the present value of its profits, which is the expectation with respect to the probability distribution induced by the current investment; this implies that given a history \( H^t_G, I^*_t \) solves:

\[
\max_{I_t \in \{L, H\}} \pi_{I_t}(\hat{z}_t, \sigma^*_t) + \mathbb{E}_{I_t}[\sum_{k=t+1}^T 3^{k-t} \pi_{I_k}(\hat{z}_k, \sigma^*_k)] \quad (6)
\]

The equilibrium of the repeated game requires players behaviour to be optimal in every period given their beliefs about the other players’ type and strategy but also their understanding of the way in which current behaviour affects future payoffs. At the end of every period consumers update their beliefs about the type of the firm and form some expectations about the level of quality they should receive in the following period. The equilibrium requires consumers’ beliefs to be correct in the sense of being consistent with the firm’s strategy in the repeated game. Beliefs about the type of the firm are required to be consistent in a bayesian way. At the beginning of the game consumers assign a probability \( \tau_1 \) to the firm being good. At the end of each period they update that probability using Bayes’ rule (and the firm’s strategy). It is worth mentioning that the probability distributions I am assuming imply that quality realisations between zero and \( 1/2 \) can not be off the equilibrium path, while realisations higher than \( 1/2 \) can be off the equilibrium path but they are fully revealing of the firm’s type. Definition 2 formalises the requirements for an equilibrium.

**Definition 2.** An equilibrium of the complaining game when \( T > 1 \) is a sequence of strategies \( \{I^*_t, \sigma^*_t\}_{t=1}^T \) and expected qualities \( \{\hat{z}_t, \tilde{z}_t\}_{t=1}^T \) such that:

1. For each \( h^t_G \in H^t_G, I^*_t \) maximises the present value of the firm’s expected profits.
2. For each \( h^t_C \in H^t_C, \sigma^*_t \) maximises the present value of consumers expected utility.
3. For each \( h^t_G \in H^t_G \), the firm has correct beliefs about the level of quality consumers expect \( \hat{z}_t = \hat{z}_t \) and consumers expectations are consistent with the equilibrium strategies in the repeated game and Bayes’ Rule.
4. For each \( h^t_C \), in which every \( q_k \leq 1/2 \), consumers’ beliefs about the type of the firm are updated according to Bayes’ Rule and the firm’s strategy; otherwise, \( \tau_{t+1} = 1 \) and consumers expect the firm to invest.

The definition states that consumers’ optimal strategy in the repeated game maximises the sum of their current and future expected utility. However, the specific regulatory rule I am studying implies that the firm is punished in the same period in which complaints occur. Hence, if the consumers complaining strategy is a function only of their beliefs (i.e., if consumers use Markov strategies), current complaints do not affect the firm’s future behaviour. As rational consumers anticipate this, \( \mathbb{E}(q_{t+1}; \sigma_t) = \mathbb{E}(q_{t+1}) \) and \( \mathbb{E}(\hat{z}_{t+1}; \sigma_t) = \mathbb{E}(\hat{z}_{t+1}) \). This

\[ \text{Recall that } \pi_t(\hat{z}, \sigma) = \mathbb{E}_{q_t, \Pi}(I, \delta(q_t, \hat{z}, \sigma)) \]
results in consumers behaving as if they were myopic: consumers complain only to punish the firm’s current poor performance and so \( \sigma^*_t(q_t; \hat{z}_t) = \sigma^*(q_t; \hat{z}_t) \). This result is presented in Lemma 2.

**Lemma 2.** In any Markovian Equilibrium the consumers behave as if they were myopic.

**Proof.** Consider the firm’s optimal action in the last period. In period \( T \) its optimal strategy is a cut off analogous to the one in the one-shot game: the firm invests as long as \( \hat{z}_T > \hat{z}^* \) and it does not invest if the inequality is reversed. In equilibrium, \( \hat{z}_T = \hat{z}_T \), which depends on consumers’ beliefs about the firm’s type (\( \tau_T \)) and investment strategy. \( \tau_T \) is a function of past quality realisations (through Bayesian updating) and consumers beliefs about the firm’s strategy reflect common knowledge of the strategy profile. Therefore, the firm’s investment in period \( T \) is determined by past realisations of quality but it is not affected by the fact that the firm was fined in previous periods. A similar argument explains why past fines (and hence past complaints) do not affect current or future investment in periods before the last one. As consumers anticipate the firm’s best response, they do not expect current complaints to affect future quality. Consumers’ problem in the repeated game, \( \max \sigma \sum_{k=1}^{T} \beta^{k-1} \mathbb{E} U(\sigma_k; q_k, \hat{z}_k) \) is then equivalent to \( \sum_{k=1}^{T} \beta^{k-1} \max \sigma \mathbb{E} U(\sigma_k; q_k, \hat{z}_k) \) and so \( \sigma^*_t(q_t; \hat{z}_t) = \sigma^*(q_t; \hat{z}_t) \).

The intuition behind this result is that, as the firm is punished in the same period in which complaints occur, past fines (and hence past complaints) become a sunk cost when the firm decides its current (and future) investment. Rational consumers understand that future quality (and future firm’s behaviour) is not affected by current complaints and so their complaining strategy is the myopic best response to the quality realisation they received, given their prior expectations -i.e., they complain in order to punish the firm’s current “poor performance”, and not to affect its future behaviour. An important consequence of Lemma 2 is that without the presence of the reference point in consumers’ utility function, the optimal complaining strategy would be \( \sigma^*_t = 0 \) for every \( t = 1, 2, ... T \), because not even utilitarian consumers would receive a positive payoff from complaining.

The formal implication of consumers’ myopic behaviour is that the repeated game is strategically equivalent to a game in which a long-lived firm faces a sequence of short-lived consumers, each of which plays only once but observes all previous realisations of quality and complaints.

### 4.1 Equilibrium

The definition of informativeness of complaints I introduced in section 3 is based on the existence of multiple equilibria, as it compares the (expected) proportion of complaints in a low quality equilibrium with that in a high quality equilibrium. Therefore, in this section I consider only the set of parameters for which high and low quality equilibria coexist in the stage game and I study how that set is affected by the repetition of the game. This means that I focus on values of \( \hat{z}^* \in [1/4, 1/2] \) and \( \tau_1 \geq \tau^* \) (see Proposition 1). Recall that \( \hat{z}^* = \frac{h}{mp} + \frac{\psi}{2} \) summarises the main parameters determining the strength of the punishment, namely, the relative size of the fine and consumers’ cost of complaining. In a way analogous to the one in section 3, I say that there is a high quality equilibrium (HQE) when the firm invests in every period, and a low quality equilibrium (LQE) when it does not invest in any period. Furthermore, as two periods are enough to prove the main results, this section presents only the case in which \( T = 2 \). The case in which \( T \to \infty \) is presented in Appendix.
C. Proposition 2 summarises the main result of the repeated game.

**Proposition 2.** Let $\tau^*$ be as defined in Proposition 1. Given $\hat{\tau}^* \in [1/4, 1/2]$, $\psi < \frac{1}{4}$ and $T = 2$, there exist $\tau^{**} \leq 1 - 2\psi$ such that for every $\tau_1 \in (\tau^*, \tau^{**})$, the one shot game has a high quality and a low quality equilibrium, but the repeated game has only a low quality equilibrium. As a result, complaints are less informative in the repeated game than in the one shot game.

The proof of this proposition is divided in three parts. Lemmas 3 and 4 below characterise the set of parameters for which a high and a low quality equilibria exist in the two-period game, while Lemma 5 relates those results to the degree of informativeness of complaints.

**Lemma 3.** Given $\hat{\tau}^* \in (1/4, 1/2)$ and $T = 2$, there exists an equilibrium in which the firm invests in both periods for every $\tau_1 > \tau^{**}$, for some $\tau^{**} \in (0, 1)$.

**Proof.** Denote by $s$ a strategy in which the firm invests in the first period and it invest in the second period only if $\tau_2 > \tau^*$. The value for the firm from following strategy $s$ in the first period is:

$$V^s(\tau_1) = \pi_{H,1}(\hat{\tau}_{H,1}(\tau_1), \sigma^*) + \beta \left[ \frac{1}{2} \pi_{H,2}(\hat{\tau}_{H,2}(1), \sigma^*) + \frac{1}{2} \left[ \Pr(\tau_2 > \tau^*) \pi_{H,2}(\hat{\tau}_{H,2}(\tau_2), \sigma^*) + \Pr(\tau_2 \leq \tau^*) \pi_{L,2}(\hat{\tau}_{L,2}, \sigma^*) \right] \right]$$

Under the equilibrium strategy, $\tau_2 = \frac{q_1 - 1}{2}$, after $q_1 \leq 1/2$ and $\tau_2 = 1$ after $q_1 > 1/2$. For $\tau_2 > \tau^*$, the consumers expect the firm to invest in the second period and so the level of quality they expect is either $\hat{\tau}_{H,2}(1)$ or $\hat{\tau}_{H,2}(\tau_2)$; for $\tau_2 < \tau^*$, their expected quality is $\hat{\tau}_{L,2}$.

If the firm deviates in the first period, the realisation of quality is smaller than $1/2$ for sure; consumers do not detect the deviation, but the (low) quality realisation induces them to reduce the probability they assign to the firm being good and to lower their second period expectations accordingly. Hence, the value of deviating in the first period but following strategy $s$ in the second one is:

$$V^d(\tau_1) = \pi_{L,1}(\hat{\tau}_{H,1}(\tau_1), \sigma^*) + \beta \left[ \Pr(\tau_2 > \tau^*) \pi_{H,2}(\hat{\tau}_{H,2}(\tau_2), \sigma^*) + \Pr(\tau_2 \leq \tau^*) \pi_{L,2}(\hat{\tau}_{L,2}, \sigma^*) \right]$$

By looking at the game backwards, it can be shown that for $\tau_1$ high enough the investment strategy $s$, together with expected qualities $\hat{\tau}_{H,1}(\tau_1)$, $\hat{\tau}_{H,2}(\tau_2)$ for $\tau_2 > \tau^*$ and $\hat{\tau}_{L,2}$ for $\tau_2 \leq \tau^*$, constitute an equilibrium of the two-period game. Consider the second period. As this is the last period of the game it is equivalent to a one shot game with prior $\tau_2$ and so, from Proposition 1, the firm has no incentives to deviate from $s$. The firm follows $s$ in the first period if $V^s(\tau_1) \geq V^d(\tau_1)$. The difference $[V^s(\tau_1) - V^d(\tau_1)]$ is monotone, increasing and continuous in $\tau_1$, $V^s(0) < V^d(0)$ and $V^s(1) > V^d(1)$. Then, there exists a unique prior belief

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43The infinite horizon game shows that the results of this Section do not depend upon the existence of a final period.

44From Lemma 2, the consumers’ complaining strategy is the same in every period. Therefore, I denote it by $\sigma^*$ and not by $\sigma^T$. 

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\[ \tilde{\sigma} \] such that \( V^s(\tilde{\sigma}) = V^d(\tilde{\sigma}) \), and so for any \( \tau_1 \geq \tilde{\sigma} \) and \( t \in \{1, 2\} \) the firm has no incentives to deviate from \( s \).

The fact that the firm follows strategy \( s \) may or may not result in \( I_2 = H \). The firm invests in the second period if and only if consumers expect so and \( \tau_2 \geq \tau^* \). This second condition holds for \( \tau_1 \geq \tilde{\sigma} = \frac{2\tilde{\sigma}}{1+\tilde{\sigma}} \). In this case, \( \Pr(\tau_2 > \tau^*) = 1 \) and the firm invests in the first period too if \( \tau_1 \geq \tilde{\sigma} = 4\tilde{\sigma}^* - 1 + \frac{\beta}{2}(1 - \frac{\tilde{\sigma}}{2}) = \tau^* + \frac{\beta}{2}(1 - \tau_2) \). Then, there exists an equilibrium in which the firm invests in both periods if and only if \( \tau_1 \geq \tau^{**} = \max\{\tilde{\sigma}, \tilde{\sigma}\} \).

Consumers’ expected qualities in equilibrium are \( \tilde{z}_1 = \tilde{z}_{H,1}(\tau_1) \) and \( \tilde{z}_2 = \tilde{z}_{H,2}(1) \) if \( \tau_1 > 1/2 \) and \( \tilde{z}_2 = \tilde{z}_{H,2}(\tau_2) \) if \( \tau_1 \leq 1/2 \).

**Lemma 4.** Given \( \tilde{z}^* \in [1/4, 1/2] \) and \( T = 2 \), there exists an equilibrium in which the firm does not invest in any of the two periods.

**Proof.** The second period is the last period of the game. From Proposition 1, if consumers expected quality in the second period is \( \tilde{z}_L \), the firm’s best reply is \( I_2 = L \) as long as \( \tilde{z}^* \geq 1/4 \). On the equilibrium path, \( I_1 = L \) and so a low quality realisation in the first period is not informative about the firm’s type. Hence, \( \tau_2 = \tau_1 \) and consumers expect low quality in the second period too.

Given \( \tilde{z}^* \geq 1/4 \), the firm has no incentives to deviate either in the first or in the second period. If the firm deviates in the first period (i.e., it invests when consumers were expecting \( \tilde{z}_L \)), that period’s (expected) profits are smaller because the smaller fine does not compensate the higher cost of investment. The (expected) second period profits do not increase with the deviation either. After \( I_1 = 1 \), there exists a positive probability that \( \tau_1 \leq 1/2 \). In this case, the consumers do not detect the firm’s deviation and second period profits are not affected by the deviation. There also exists a positive probability of a high quality realisation that reveals the firm’s type. In this case, \( \tilde{z}_{2,H}(1) \) and the firm’s second period profits are smaller under the deviation than under the equilibrium path. Then, \((I^*_1, I^*_2) = (0, 0)\) together with \( \tilde{z}_1 = \tilde{z}_2 = \tilde{z}_L \) and \( \tilde{z}^* \in (1/4, 1/2) \) constitute an equilibrium when \( \tilde{z}^* \geq 1/4 \), and the Lemma is proved.

It can be further shown that the low quality equilibrium holds for a wider set of parameters. In particular, it can be shown that the firm’s expected profits under the deviation are smaller than under the equilibrium strategy as long as \( \tilde{z}^* > \frac{1}{2(2+\beta)} \).

**Lemma 5.** Customers complaints are less informative in the repeated game than in the one shot game.

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45Substituting the firm’s expected profits, this expression becomes \( V^s(\tau_1) - V^d(\tau_1) = -h + mp(\tilde{z}_{H,1}(\tau_1) - \tilde{\sigma}) - \frac{\beta}{2}mp(\tilde{z}_{H,2}(1) - \tilde{z}_{H,2}(\tau_2)) > 0 \). As both \( \tilde{z}_{H,1}(\tau_1) \) and \( \tilde{z}_{H,2}(\tau_2) \) are increasing in \( \tau_1 \), while \( \tilde{z}_{H,2}(1) \) and \( \tilde{z}_{2,L} \) are independent of \( \tau_1 \), the difference is increasing in \( \tau_1 \).

46Note that while \( \tilde{\sigma} > \tau^* \) and \( \tilde{\sigma} \geq \tau^* \) depending on how patient is the firm. In particular, there exists \( \beta \) such that when \( \beta > \tilde{\beta} > \tilde{\sigma} \) while if \( \beta \leq \tilde{\beta} \) the opposite is true. To see that this is the case, note that \( \tau^{**} = \tau^* \frac{2\tilde{\beta}}{1+\tilde{\beta}} \) if \( \beta = 0 \), \( \tau^{**} \) is strictly increasing in \( \beta \) and \( \tau^{**} > \frac{2\tilde{\beta}}{1+\tilde{\beta}} > \tau^* \) when \( \beta = 1 \).

47The value for the firm of following the non-investment strategy is \( \pi_{L,1}((\tilde{z}_1, \sigma^*) - (1 + \beta)) \), and the value of deviating in the first period is \( \pi_{H,1}(\tilde{z}_1, \sigma^*) + \frac{1}{2}[\pi_{H,2}(\tilde{z}_1, \sigma^*) + \pi_{L,2}(\tilde{z}_1, \sigma^*)] \). The first expression is greater than the latter if and only if \( \tilde{z}^* > \frac{1}{2(2+\beta)} \). Thus, if the firm is very impatient (\( \beta = 0 \)), we are back in the one shot game and the LQE holds for \( \tilde{z}^* > \frac{1}{2} \), but if \( \beta \to 1 \), the firm will not invest in any of the two periods as long as \( \tilde{z}^* > \frac{1}{2} \). Therefore, the region of parameters in which the firm does not invest is greater in the repeated game.
Proof. Lemmas 3 and 4 show that when $T = 2$ and $\hat{z}^* \in (1/4, 1/2)$, there exists an equilibrium in which the firm does not invest in any of the two periods for any $\tau_1 \in (0, 1)$, but an equilibrium in which the firm invests in $t = 1, 2$ exists for $\tau_1 \geq \tau^{**}$. Proposition 1 shows that, for the same set of values of $\tau^*$, the static game also has a low quality equilibrium for any $\tau_1$, and a high quality equilibrium for $\tau_1 \geq \tau^*$. As long as $\tau_1 < 1$, $\tau^{**} > \tau^*$ and so there exists a set $\tau_1 \in (\tau^*, \tau^{**})$, for which the HQE exists in the one shot game but not in the repeated game. $\tau^{**} - \tau^* = \frac{\beta}{2}(1 - \tau_2)$, which implies that the more patient is the firm and the smaller is the initial $\tau_1$, the greater is the set of values of $\tau_1$ for which the repetition of the game eliminates the high quality equilibrium.

In the one shot game, complaints are informative if $\tau_1 \in (\tau^*, 1 - 2\psi)$. As shown in Lemma 1, the lower bound on $\tau_1$ is the minimum level at which the firm is willing to invest, while the upper bound results from imposing the condition that the (expected) proportion of complaints is higher in an equilibrium in which the firm is not investing than in one in which $I = 1$. Imposing the same condition to each period of the repeated game results in complaints being informative about the firm’s investment only if $\tau_1 \in (\tau^{**}, 1 - 2\psi)$. This guarantees that $E_{q_{L}}(\sigma^*(q; \hat{z}_{1, H}(\tau_1))) < E_{q_{H}}(\sigma^*(q; \hat{z}_{1, H}(\tau_1)))$ and that $E_{q_{H}}(\sigma^*(q; \hat{z}_{2, H}(\tau_2))) < E_{q_{L}}(\sigma^*(q; \hat{z}_{2, L}))$. Therefore, the set $(\tau^*, \tau^{**})$ for which the equilibrium with high quality ceases to exist in the repeated game contains values of $\tau_1$ for which complaints would have been informative if $T = 1$. Hence, the repetition of the game also reduces the degree of informativeness of complaints.

Lemma 5, together with Lemmas 3 and 4, shows that the set of $\tau_1$’s for which the firm invests in equilibrium is reduced by the repetition of the game, which in turn reduces the of informativeness of complaints. The intuition behind this result is that when the firm invests it faces the risk that consumers find out its type. When that happens in the last period (or when the game is played only once) it does not affect the firm’s continuation value and so it optimally invests if the reduction of the (expected) fine compensates the cost of investment. However, when consumers find out the type of the firm before the end of the game, they increase the level of quality they expect to receive in the future, reducing the firm’s future profits. The cost of reducing the current fine is higher in the repeated game as it includes not only the cost of a higher (current) investment but also the cost of smaller (expected) future profits. The change in the firm’s expected profits due to higher consumers expectations is a decreasing function of $\tau_1$ and so the firm’s incentives to keep consumers’ expectations low are higher for smaller values of $\tau_1$. When $\tau_1$ is small, the inter temporal trade-off is more relevant for its investment decision than the intra temporal trade-off and the firm’s optimal action is to keep future expected quality low by not investing today. On the contrary, when the probability consumers assign to the firm being of the good type is very high, there is only a small scope to “manage” consumers expectations, what makes the intra temporal trade-off more relevant. In this case, the firm’s profit maximisation strategy is to invest if consumers expect high quality and not to invest if their expect so. The relevance of the inter temporal trade off also depends on how patient is the firm: the higher $\beta$ the more value the firm assigns to future profits and the more it cares about keeping consumers expectations low.$^{49}$

The appendix C shows that, the same as in the finitely repeated game, when $T \rightarrow \infty$, the existence of a high quality equilibrium depends on the scope of the firm to manage consumers’ future expectation, which is positively related to $\beta$ and inversely related to $\tau_1$.$^{48}$

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$^{48}$\(\tau_2 < \tau_1\), so \(\tau_1 < 1 - 2\psi\) implies \(\tau_2 < 1 - 2\psi\).

$^{49}$In the case in which the game is repeated two periods, the lower bound of $\tau$ goes up from $4\hat{z}^* - 1$ to $4\hat{z}^* - 1 + \frac{\beta}{2}(1 - \tau_2)$. 

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Furthermore, as discussed in this section, the less scope to manage consumers expectations the less informative are complaints about the equilibrium being played (because the higher is the proportion of complaints in a high quality equilibrium).

5 Conclusions

The model in this paper considers the role of customer complains as a regulatory tool in contexts in which quality is relatively well-perceived by the consumers, but it is very costly for the regulator to observe it. In line with some empirical evidence, it is assumed that consumers complain because they feel “disappointed” with the level of quality they received and consider the firm should be punished for its “poor performance”. The paper studies the firm’s incentives to invest and the informativeness of customers complaints in such a context.

It is shown that complaints are informative when complaining is neither too cheap nor too costly and when consumers assign a relatively low probability to the firm being good. The presence of a rational reference point in consumer’s complaining decision implies that “disappointment” and “poor performance” are endogenously determined. As a result, the proportion of complaints may be higher in a high quality equilibrium than in a low quality one if consumers believe they are facing a good firm. Hence, the regulator may observe more complaints when the firm is investing than when it is not and may punish the firm more harshly in the first case. The paper further shows that the degree in which complaints are informative is reduced by the repetition of the game. It is shown that when the agency uses a regulatory rule as the one analysed in this paper, consumers’ optimal behaviour in the repeated game is the myopic best response to the quality they received, given they prior expectations. This behaviour creates the conditions for the firm to try to “keep expectations low” making complaints less informative in the repeated game than in the one shot game. As a result, the set of parameters for which complaints are informative about the firm’s behaviour is reduced when the game is played repeatedly.

The results in this paper provide insights that may be useful in interpreting consumers’ complaints (or other type of feedback) in a variety of settings. The context considered here is one in which consumers receive no direct benefit out of their complaints and thus their only reason to lodge a complaint is to transmit their dissatisfaction. However, it is likely that in another settings consumers do appropriate at least partially the benefit of their complaints. In this case, complaints may be explained by a combination of reasons, one of which could be disappointment. Therefore, a note of care should be taken when interpreting consumers’ complaints also in those settings.
A Expected Proportion of Complaints and Updating

The expected proportion of complaints depends on the realisation of quality, which depends on the firm’s type and investment. Taking expectations of the consumers’ optimal complaining rule with respect to the quality distribution induced by investment \( I \), the expected proportion of complaints is:

\[
E_{q_1}(\sigma^*(q; \hat{z})) = [P_{q_1}(\sigma^*(q; \hat{z}) = 1)*1 + P_{q_1}(\sigma^*(q; \hat{z}) = \frac{\theta}{c}(\hat{z} - q))E_{q_1}(\sigma^*(q; \hat{z}))|\sigma^*(q; \hat{z}) \in (\hat{\delta}, 1)] + P_{q_1}(\sigma^*(q; \hat{z}) = \hat{\delta})*\hat{\delta}
\]

Using consumer’s optimal strategy in (3), the expectation can be rewritten in terms of the realised level of quality:

\[
E_{q_1}(\sigma^*(q; \hat{z})) = [P_{q_1}(q \leq \hat{z} - \frac{c}{\theta}) + P_{q_1}(\hat{z} - \frac{c}{\theta} \leq q \leq \hat{z} - \frac{\delta c}{\theta})*\
* E_{q_1}[\sigma^*_t(q; \hat{z})|\hat{z} - \frac{c}{\theta} \leq q \leq \hat{z} - \frac{\delta c}{\theta}] + P_{q_1}(\hat{z} - \frac{\delta c}{\theta} \leq q \leq \hat{z} - \frac{\delta c}{2\theta})*\frac{\hat{\delta}}{2}]
\]

where:

\[
E_{q_1}[\sigma^*_t(q; \hat{z})|\hat{z} - \frac{c}{\theta} \leq q \leq \hat{z} - \frac{\delta c}{\theta}] = \frac{1}{P_{q_1}(\hat{z} - \frac{c}{\theta} \leq q \leq \hat{z} - \frac{\delta c}{\theta})} \int_{\hat{z} - \frac{c}{\theta}}^{\hat{z} - \frac{\delta c}{\theta}} xf(x)dx
\]

The expected proportion of complaints is then a function of the distribution of quality, which in turns depend on the type and investment decision of the firm. Denote \( \psi = \frac{c}{\theta} \). For a good firm which invests \( q \sim U(0, 1) \). The probabilities in the above expressions, become:

\[
P_{t=H}(q \leq \hat{z} - \psi) = \hat{z} - \psi
\]

\[
P_{q_H}(\hat{z} - \hat{\psi} \leq q \leq \hat{z} - \delta \hat{\psi}) = \psi(1 - \hat{\delta})
\]

\[
P_{q_H}(\hat{z} - \delta \hat{\psi} \leq q \leq \hat{z} - \delta \hat{\psi}) = \frac{\delta \psi}{2}
\]

\[
E_{q_H}(q|\hat{z} - \psi < q < \hat{z} - \delta \hat{\psi}) = \hat{z} - \frac{c}{2\theta}(1 + \delta)
\]

\[
E_{q_H}[\sigma^*_t(q; \hat{z})|\hat{z} - \psi \leq q \leq \hat{z} - \delta \psi] = \frac{1+\delta}{2}
\]

All the above probabilities are equal than or greater than zero if \( \tau > 4\psi - 1 \). Substituting these results in (7), the expected proportion of complaints when the good type of the firm invests becomes \( E_{q_H}(\sigma^*(q; \hat{z})) = \hat{z} - \frac{\psi}{\tau} \).

When the good type of the firm does not invest, or when the firm is bad, \( q \sim U(0, 1/2) \). The above probabilities in this case are:
The expected proportion of complaints when a good firm does not invest or when the firm is of the bad type is
\[ \psi = \frac{c}{\theta} \geq \frac{1}{4}, \]
guarantees that all the above probabilities are greater than zero. Then, the condition in Propositions 1 and 2 is sufficient but not necessary.

B Bayesian Updating

When consumers expect the good type of the firm to invest, a low realisation of quality increases the evidence in favour of the firm being bad. Bayesian updating implies:

\[ \tau_1 = P(G|q_1 < \frac{1}{2}) = \frac{P(q_1 < \frac{1}{2}|G)P(G)}{P(q_1 < \frac{1}{2}|G)P(G) + P(q_1 < \frac{1}{2}|B)P(B)} \]
\[ = \frac{\tau}{\frac{7}{2} - (1 - \tau)} \]
\[ = \frac{\tau}{2 - \tau} \] (8)

If consumers do not expect the good type of the firm to invest, a low realisation of quality does not give them any additional information about the firm’s type and so there is no updating of beliefs.

In both cases, a high (expected or unexpected) realisation of quality is fully revealing of the firm type, and implies \( \tau_1 = 1. \)

C Equilibria of the Repeated Game when \( T \to \infty \)

This Appendix studies how the results of Section 4 extend to the case in which the number of periods goes to infinity. It is shown that, even when \( T \to \infty, \) a regulation based on customers complaints induces an equilibrium with positive probability of investment as long as the firm is not extremely patient, consumers’ prior about the firm being good is relatively high and the punishment is harsh enough. The informativeness of customers complaints is also analysed. The main results of the Appendix are summarised in the following Proposition:
Proposition 3. Given $\hat{z}^* < 1/2$ and $\beta \in (0,1)$ there exists $\tau^{**}$ such that if $\tau_1 > \tau^{**}$ a high quality and low quality equilibria coexist for all $T$. Customers complaints become less informative as $T$ increases.

In order to prove this Proposition, I first show the existence of a high quality (separating) and a low quality (pooling) equilibrium. The following Lemma will be useful in proving the existence of those equilibria. It shows that, once consumers know for sure the firm is good, they expect the firm to invest in every period and the firm’s best reply is to invest.\footnote{See Definition 2.}

Lemma 6. Given $\hat{z}_t = \hat{z}_H(1)$ and $\hat{z}^* < 1/2$, there exists a unique subgame perfect equilibrium in which $I_t = 1$ for all $t$.

Proof. Let $\tau_t = 1$. The firm’s best reply in the one-shot game is $I_t = H$ (see sub-section 2.2). Furthermore, $\tau_t = 1$ implies that $\tau_k = 1$ for any $k \geq t$. Then, the game becomes a repetition of the one shot game in which consumers expect to receive $\hat{z}_{H,k}(1)$ in every period $k \geq t$, and thus the firm optimally invests in any period $k \geq t$.

Consider the firm’s incentives to deviate to $L$ in a period $s \geq t$. Because consumers’ already know the type of the firm, the deviation does not affect the level of quality consumers’ expect to receive in any period after $s$, and thus it does not affect the firm’s future profits. However, the deviation reduces the firm’s current profits as it induces a lower quality realisation (on expectations). Then, given $\tau_t = 1$, there is no profitable one-period deviation. \qed

Claim 1. Investment in Equilibrium. Given $\hat{z}^* \leq 1/2$ and $\beta \in (0,1)$, there exists $\tau^{***}$ such that there is an equilibrium in which the firm invests in every period $t$ in which $\tau_t$ is above $\tau^{***}$ and it does not invest otherwise.

Proof. Denote by $s(\tau)$ a firm’s investment strategy in which:

$$ I = \begin{cases} H & \text{if } \tau \geq \tau \\ L & \text{otherwise} \end{cases} $$

The level of quality consumers expect to receive when the firm follows this strategy is:

$$ \hat{z} = \begin{cases} \hat{z}_H(\tau) & \text{if } \tau \geq \tau \\ \hat{z}_L & \text{otherwise} \end{cases} $$

The value for the firm of following the strategy $s(\tau)$ depends only on $\tau$. For $\tau < \tau$, this value is:

$$ V_{s(\tau)}(\tau) = \frac{\pi_L(\hat{z}_L, \sigma^*)}{1 - \beta} \quad (9) $$

while for $\tau \geq \tau$ this value is:

$$ V_{s(\tau)}(\tau) = \pi_H(\hat{z}_H(\tau), \sigma^*) + \left[ \frac{\pi_H(\hat{z}_H(1), \sigma^*)}{2(1 - \beta)} + \frac{1}{2} \Pr(\tau' \geq \tau) V_{s(\tau)}(\tau') + \frac{1}{2} \Pr(\tau' < \tau) \frac{\pi_L(\hat{z}_L, \sigma^*)}{(1 - \beta)} \right] \quad (10) $$

\footnotetext[50]{See Definition 2.}
where \( \Pr(\tau' > \tau) = 1 \) if consumers’ beliefs about the type of the firm after a low quality realisation is greater than \( \tau \): \( \tau' = \frac{\tau}{2-\tau} \geq \tau \).

There exists a unique function, \( V^*_s(\tau) \), that solves (10). Consider the set \( C([\tau, 1]) \) of bounded, continuous and weakly decreasing functions of \( \tau \), and define an operator \( T \) on \( C([\tau, 1]) \) as:

\[
TV_s(\tau) = \pi_H(\hat{z}_H(\tau), \sigma^*) + \beta \left[ \frac{\pi_H(\hat{z}_H(1), \sigma^*)}{2(1-\beta)} + \frac{1}{2} \Pr(\tau' > Z) V_s(\tau') + \frac{1}{2} \Pr(\tau' < Z) \pi_L(\hat{z}_L, \sigma^*) \right]
\]

Standard arguments can be used to show that a function \( V \in C([\tau, 1]) \), solves (10) if and only if it is a fixed point of \( T \). The (expected) fine in any period \( t \) is a weakly increasing function of consumers’ expected quality and so the firm’s current profits are weakly decreasing in \( \tau \).\(^{51}\) Furthermore, the per period return function is bounded and continuous in \( \tau \).\(^{52}\) Then, \( TV_s(\tau) \) is also bounded, continuous and weakly decreasing in \( \tau \), and so \( T \) maps \( C([\tau, 1]) \) into itself. It is straightforward to show that Blackwell’s sufficient conditions of monotonicity and discounting apply to the operator \( T \). Hence, \( T \) is a contraction mapping and so it has a unique fixed point, \( V^*_s(\tau) \),\(^{53}\) which is a bounded, continuous and weakly decreasing function of \( \tau \).

If the firm deviates in the first period, but it attaches to the original strategy thereafter, its expected payoff is:

\[
V^d_s(\tau) = \begin{cases} 
\pi_L(\hat{z}_H(\tau), \sigma^*) + \beta \left[ \Pr(\tau' > Z) V^*_s(\tau') + \Pr(\tau' \leq Z) \frac{\pi_L(\hat{z}_L, \sigma^*)}{(1-\beta)} \right] & \text{if } \tau \geq \tau \\
\pi_H(\hat{z}_L, \sigma^*) + \frac{\beta}{2(1-\beta)} [\pi_L(\hat{z}_L, \sigma^*) + \pi_H(\hat{z}_H(1), \sigma^*)] & \text{otherwise}
\end{cases}
\]

Denote by \( f_s(\tau) \) the difference between the firm’s expected payoffs when it follows strategy \( s(\tau) \) and when it deviates. When \( \tau \geq \tau \), \( f_s(\tau) \) is a bounded and continuous function of \( \tau \) because both \( V^*_s(\tau) \) and \( V^d_s(\tau) \) are bounded and continuous. Furthermore, for all \( \tau \geq \tau \), \( f_s(\tau) \) is weakly increasing in \( \tau \) and it is given by:

\[
f_s(\tau) = V^*_s(\tau) - V^d_s(\tau) = -h + mp\left( \hat{z}_H(\tau) - \frac{\psi}{2} \right) + \beta \left[ \frac{\pi_H(\hat{z}_H(1), \sigma^*)}{2(1-\beta)} - \frac{1}{2} \Pr\left( \frac{\tau}{2-\tau} > Z \right) V^*_s(\tau) - \frac{1}{2} \Pr\left( \frac{\tau}{2-\tau} \leq Z \right) \frac{\pi_L(\hat{z}_L, \sigma^*)}{1-\beta} \right]
\]

The first term of (13) is an increasing function of \( \tau \) because \( \hat{z}_H(\tau) \) is increasing in \( \tau \). The second term is also weakly increasing in \( \tau \). When \( \tau > \frac{\tau}{2-\tau} > \tau \), \( \Pr(\frac{\tau}{2-\tau} > Z) = 1 \) and the second term is weakly increasing in \( \tau \) because \( V^*_s(\tau) \) is weakly decreasing in \( \tau \). If \( \tau \) is such that \( \tau \geq \tau \geq \frac{\tau}{2-\tau}, \Pr(\frac{\tau}{2-\tau} \leq Z) = 1 \) and the second term of (13) becomes independent of \( \tau \).

\(^{51}\) For any \( I \in \{L, H\} \), \( \pi_I(\hat{z}_H(\tau), \sigma^*) \) is a weakly decreasing function of \( \tau \) (strictly decreasing for \( \tau < 1 \)), and \( \pi_I(\hat{z}_L, \sigma^*) \) is constant in \( \tau \).

\(^{52}\) \( V^*_s(\tau) = \frac{\pi_L(\hat{z}_L, \sigma^*)}{(1-\beta)} \) if \( \tau = \tau \). Then, \( V^*_s(\tau) \) is continuous in \( \tau \).

\(^{53}\) By the Contraction Mapping Theorem.
Recall that when $\tau' < \underline{\tau}$, $V_s^{\tau}(\tau') = \frac{H(L, \underline{\tau})}{2(1 - \beta)}$. Then, equation (13) can be written as:

$$f_s^{\tau}(\tau) = -h + mp \left( \hat{z}_H(\tau) - \frac{\psi}{2} \right) + \beta \left[ \frac{\pi_H(\hat{z}_H(1), \sigma^*)}{2(1 - \beta)} - \frac{1}{2} V_s^{\tau}(\tau') \right]$$

The strategy $s(\tau)$ constitutes an equilibrium strategy if and only if $f_s^{\tau}(\tau) \geq 0$ for every $\tau \in [0, 1]$. Since $f_s^{\tau}(\tau)$ is weakly increasing in $\tau$ for every $\tau \geq \underline{\tau}$, it suffices to show that (1) $f_s^{\tau}(\tau) \geq 0$ and (2) $f_s^{\tau}(\tau) > 0$ for every $\tau < \underline{\tau}$. Consider the first case:

$$f_s^{\tau}(\tau) = -h + mp \left( \hat{z}_H(\tau) - \frac{\psi}{2} \right) - \frac{\beta}{2(1 - \beta)} \left[ \pi_H(\hat{z}_H(1), \sigma^*) - \pi_L(\hat{z}_L, \sigma^*) \right]$$

which is positive as long as $\hat{z}_H(\tau) \geq \hat{z}^*(1 + \frac{\beta}{2(1 - \beta)})$. Denote by $\tau^{***}$ the value of $\tau$ for which $f_s^{\tau}(\tau) = 0$ -i.e., the $\tau$ for which $\hat{z}_H(\tau) = \hat{z}^*(1 + \frac{\beta}{2(1 - \beta)})$. Since $\hat{z}_H(\tau)$ is increasing in $\tau$, $\hat{z}_H(\tau) \geq \hat{z}^*(1 + \frac{\beta}{2(1 - \beta)})$ for all $\tau \geq \tau^{***}$.

Let $\tau \geq \tau^{***}$; given strategy $s(\tau)$, the firm does not have a profitable deviation. Suppose, to the contrary, that for some $\tau \geq \tau^{***}$, $f_s^{\tau}(\tau) < 0$; then $\hat{z}_H(\tau) < \hat{z}^*(1 + \frac{\beta}{2(1 - \beta)})$, and so $\tau < \tau^{***}$ which is a contradiction. Hence (1) holds.

Finally, consider (2). If $\tau < \underline{\tau}$, $f_s^{\tau}(\tau)$ is defined as the difference between (9) and the second line of (12), and is equal to:

$$f_s^{\tau}(\tau) = h - mp \left( \hat{z}_L - \frac{\psi}{2} \right) + \frac{\beta}{2(1 - \beta)} \left( h + mp \frac{\psi}{2} \right)$$

which is positive as long as $\hat{z}_L \leq \hat{z}^*(1 + \frac{\beta}{2(1 - \beta)})$.\footnote{Recall that $\hat{z}^* = \frac{h}{mp} + \frac{\psi}{2}$.} Furthermore, this condition is independent of $\tau$ and so it implies that for every $\tau \in [0, \underline{\tau})$, $f_s^{\tau}(\tau) \geq 0$ and so the firm is not willing to deviate from strategy $s(\tau)$.

Thus, given any $\tau \geq \tau^{***}$, there exists an equilibrium in which the firm follows strategy $s(\tau)$, and the Claim is proved. \qed

**Claim 2.** Pooling equilibrium *Given $\tau \in [0, 1)$, there exists an equilibrium in which $I_t = 0$, for all $t$.*

**Proof.** In a pooling equilibrium the good type of the firm does not invest and consumers anticipate this behaviour; therefore, quality realisations below 1/2 are not informative about the firm’s type. On equilibrium path, consumers’ beliefs about the type of the firm are:

$$\tau_t = \begin{cases} \tau_0 & \text{if } q_k \leq 1/2 \text{ for every } k \leq t - 1 \\ 1 & \text{if } q_k > 1/2 \text{ for some } k \leq t - 1 \end{cases}$$

where $\tau_0$ is the probability consumers assign to the firm being good at the beginning of the game. Denote by $s(1)$ an strategy in which the firm invests only when $\tau = 1$. The firm’s value from following this strategy is:

$$V_s^{(1)}(\tau) = \frac{\pi_L(\hat{z}_L, \sigma^*)}{1 - \beta} = \frac{p - mpE_{q_L}(\sigma^*(q; \hat{z}_L))}{1 - \beta}$$

(14)
The firm’s incentives to deviate depend on the value of $\hat{z}^*$. Consider the case in which $\hat{z}^* > 1/4$. In this case, the firm has neither short run nor long run incentives to deviate from the non-investing strategy. If in any period $t$ consumers expect to receive low quality ($\hat{z}_L$) and the firm deviates, there exists a positive probability that a high quality realisation reduces the (expected) proportion of complaints and the expected fine. However, this reduction in the fine does not compensate the cost of investment ($h$) and so period-$t$ profits are reduced too, implying that the firm has a short run incentive to fulfil consumers’ low quality expectations.\(^{55}\) Long run considerations also prevent the firm from deviating. If the firm invests in period $t$ there exists a positive probability that consumers do not find out the deviation ($q_t < 1/2$); in this case the firm’s current profits are smaller and future profits remain unchanged because $\tau_{t+1} = \tau_t = \tau_0$. There is also a positive probability that a high quality realisation reveals the firm’s type: if $q_t > 1/2$, $\tau_{t+1} = 1$ and $\hat{z}_k = \hat{z}_H(1) \forall k > t$. As shown in Lemma 6, the firm’s best response is to invest in every period after $k$, which induces a continuation value smaller than $V_{4(1)}(\tau)$.\(^{56}\) Therefore, when $\hat{z} > 1/4$ and $\tau < 1$ there exists a pooling equilibrium in which the firm never invests and consumers expect low quality.

When $\hat{z} < 1/4$, the firm has short run incentives to invest. In this case, the punishment is harsh relative to the cost of investment and so current profits increase if the firm deviates. The same as before, if the firm deviates in period $t$ there is a positive probability that consumers find out its type (and hence expect high quality from $t+1$ onwards) and a positive probability that they do not detect the firm’s deviation. The value for the firm if it deviates from $s(1)$ in the current period, but follows it since tomorrow onwards is:

$$V_{4(1)}(\tau) = \pi_H(\hat{z}_L, \sigma^*) + \frac{\beta}{2} \left[ \frac{\pi_L(\hat{z}_L, \sigma^*)}{1 - \beta} + \frac{\pi_H(\hat{z}_H(1), \sigma^*)}{1 - \beta} \right]$$  \hspace{1cm} (15)

The firm will attach to the non-investment strategy as long as the continuation value in (14) is greater than the one above. A necessary and sufficient condition is:

$$\hat{z}_L \leq \hat{z}^* \left[ 1 + \frac{\beta}{2(1 - \beta)} \right]$$  \hspace{1cm} (16)

**Claim 3.** Given $\hat{z}^* < 1/2$ and $\beta \in (0, 1)$, there exist $\tau \in (\tau^*, \tau^{***})$ such that the game has a HQE and a LQE when $T = 1$ but there exists a unique LQE when $T \to \infty$. As a result, complaints are less informative when $T \to \infty$.

**Proof.** From Proposition 1, there exist $\tau^* = 4\hat{z}^* - 1$ such that for every $\tau \geq \tau^*$ the one shot game has a high quality equilibrium. Analogously, from Claim 1, there exist $\tau^{***} = 4\hat{z}^*(1 + \frac{\beta}{2(1 - \beta)}) - 1$ such that for $\tau > \tau^{***}$ and $T \to \infty$ there exists an equilibrium in which the firm invests in every period. As $\frac{\beta}{2(1 - \beta)} > 0$, $\tau^{***} \geq \tau^*$, and so for $\tau \in (\tau^*, \tau^{***})$ the firm invests in equilibrium when $T = 1$ but it does not when $T \to \infty$.

The same as when the game is repeated an infinite number of times, the set of $\tau \in (\tau^*, \tau^{***})$ corresponds to values of the parameters for which complaints are informative in

\(^{55}\)Recall from Section 3 that when $\hat{z}^* > 1/4$, low quality is an equilibrium of the stage game - i.e., the stage game has an equilibrium in which $I = 0$ and consumers’ expect low quality ($\hat{z}_L$).

\(^{56}\)The firm’s expected profits in a low quality equilibrium of the stage game are $\pi_L(\hat{z}_L, \sigma^*) = p - mp(2\hat{z}_L - \psi)$, while its profits in a high quality equilibrium if $\tau = 1$ are $\pi_H(\hat{z}_H(1), \sigma^*) = p - h - mp(\hat{z}_H(1) - \frac{\psi}{2})$. There exists no value of $\hat{z}^* > 0$ for which the latter profits are higher than the former ones. As there is no updating of beliefs, the expected profits of the repeated game are $\frac{\pi_L(\hat{z}_L, \sigma^*)}{1 - \beta} > \frac{\pi_H(\hat{z}_H(1), \sigma^*)}{1 - \beta}$. 

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the one shot game. When $T = 1$, complaints are informative if $1 - 2\psi \geq \tau \geq 4\hat{z}^* - 1$, which implies $1 - 2\psi \geq \tau \geq \tau^*$. In each period of the infinitely repeated game this condition becomes $1 - 2\psi \geq \tau_t \geq \tau^{***}$. The set of values of $\tau$ for which the second condition holds is smaller because $\tau^{***} \geq \tau^*$. \qed
References


