D-OSKIL: a New Mechanism for Suppressing Stick-Slip in Oil Well Drillstrings

Carlos Canudas-de-Wit, Miguel A. Corchero, Francisco R. Rubio and Eva Navarro-López

Abstract—Limit cycles occurring in oil well drillstrings result from the interaction between the drill bit and the rock during drilling operations. In this paper we propose to use the weight on the bit (WoB) force as an additional control variable to extinguish limit cycles if they occur. In particular we adapt the oscillation killer (OSKIL) mechanism studied in detail in the companion paper [3] to our problem at hand. An approximate analysis based on the bias describing function and completed with some simulations, provides good evidences that the rotational dynamics of the oil well drillstring display a similar behavior. Simulations applying the D-OSKIL (D stands for Drilling) mechanism show that the stick-slip oscillations can be eliminated without requiring a re-design of the velocity rotary-table control.

Index Terms—Stick-Slip Elimination, Oil Well Drillstring, OSKIL.

I. INTRODUCTION

Oilwell drillstrings are systems which present interesting features from the dynamical and control viewpoints. The presence of stick-slip self-excited oscillations at the bottom part of drillstrings has attracted the attention of the control community in the last decade. The elimination of this kind of oscillations is a challenge for drillers and scientists. The fact of reducing these oscillations can give important cost savings in drilling operations.

Different oscillations affect the drillstring behavior. This paper is focused on stick-slip oscillations given at the bottom-hole assembly (BHA), i.e., the top of the drillstring rotates with a constant rotary speed, whereas the bit (cutting device) rotary speed varies between zero and up to six times the rotary speed measured at the surface. Stick-slip oscillations are mainly due to the friction interface between the BHA and the rock formation [9]. Consequently, a model describing the drillstring behavior should include the bit-rock friction effect. A lumped parameter model is used in this paper. The drillstring is considered as a torsional pendulum with two degrees of freedom, as shown in Figure 1.

There are several alternatives when considering the problem of suppressing stick-slip oscillations. One possibility is to manipulate the different drilling parameters such as: increasing the rotary speed, decreasing the weight-on-bit (WoB), modifying the drilling mud characteristics, or introduction of an additional friction at the bit [13]. These strategies, although not supported by formal analysis, have been shown in the field to be effective to suppress stick-slip motion [14].

Another alternative has been the introduction of new control methodologies (active, passive) to compensate drill-string stick-slip vibrations. Among them we have: the soft torque rotary system (STRS) introduced in [7],[14]; the use vibration absorber at the top of the drillstring [8], [7], [14]; the design of reduced-order PID rotary speed controllers [1], [12], [13]; the design of linear $H_{\infty}$ controllers [15]. More recently, the manipulation of the normal force regarded as WoB has been shown to play an important role in reducing stick-slip oscillations [12].

In this paper, using the weight on the bit (WoB) force as an additional control variable is proposed. In particular we adapt the oscillation killer (OSKIL) mechanism studied in detail in the companion paper [3], to the oil well drillstring systems (named here D-OSKIL) which has been shown to be particulary adapted for nonlinear systems displaying a local stable region with an stable limit set outside this local domain. An approximate analysis based on the bias describing function provides good evidences that the rotational dynamics of the oil well drillstring display a similar behavior. This analysis, although approximative, also gives a good intuition in the way that the WoB needs to be modified to suppress oscillations. An important property

1D-OSKIL stands for Drilling oscillation killer mechanism.
of the proposed D-OSKIL mechanism is that allows to recover the nominal operation condition (the WoB recovers its nominal drilling value) while oscillations are suppressed. Simulations applying the D-OSKIL mechanism show that the stick-slip oscillations can be eliminated in that way without requiring a re-design of the velocity rotary-table control.

II. DRILLSTRING MODEL

Multiple kind of models have been used in literature to describe these systems (for example; [10] and [16]). Lumped parameters models have been shown valid enough to describe properly the stick-slip oscillation phenomena and easy enough to make the study not too complex ([6]). Different bit-rock friction models are presented in [11].

The model used here (see Figure 1) is a two-degree-of-freedom model with two inertial masses \( J_r \) and \( J_b \), locally damped by \( d_r \) and \( d_b \). The inertias are coupled each other by an elastic shaft with stiffness \( k \) and damping \( c \). The variables \( \varphi_r \) and \( \varphi_b \) stand for the rotary and the bit angles. The \( T_{oB} \) (Torque on Bit) represents the total friction torque over the drill bit and \( v \) is the rotary torque control signal used to regulate the rotary angular velocity \( \dot{\varphi}_r \). The model equations are:

\[
\begin{align*}
J_r \ddot{\varphi}_r + c(\dot{\varphi}_r - \dot{\varphi}_b) + k(\varphi_r - \varphi_b) + d_r \dot{\varphi}_r &= v \\
J_b \ddot{\varphi}_b + c(\dot{\varphi}_b - \dot{\varphi}_r) + k(\varphi_b - \varphi_r) + d_b \dot{\varphi}_r &= -T_{oB}
\end{align*}
\]

In constants above, the sub-script 'r', and 'b' stands for rotary and bit, respectively.

The \( T_{oB} \) is given by the product between \( \mu(\varphi_b, z) \), which describes the normalized (dimensionless) torsional bit-rock friction, and the normal force \( u \) usually named the Weight on Bit (WoB), i.e.

\[ T_{oB} = \mu(\varphi_b, z) \cdot u, \quad u = u_0 + \ddot{u} \]

The effective value of \( T_{oB} \) can be modified by controlling the tension force, \( \ddot{u} \), of the drillstring. We assume that a counterweight force \( \ddot{u} \in [-u_0, 0] \) can be applied to counteract the nominal force \( u_0 = MG \) (this is the gravitational normal force due to the total drillstring mass \( M \) time the gravity \( g \), with a trivial minimum of \( -u_0 \).

This lumped model has the following state-space representation:

\[
\begin{aligned}
\dot{x} &= Ax + Bv + H\mu(x, z)u \\
\dot{z} &= f(x, z)
\end{aligned}
\]  

(1)

(2)

with

\[
A = \begin{pmatrix}
0 & 1/i \\
-k/iJ_r & -1/d_r - (d_r + c/i^2)/J_r & c/iJ_r \\
k/J_b & c/iJ_b & -(c + d_b)/J_b
\end{pmatrix}
\]

\[
B^T = \begin{pmatrix}
0 & 1/J_r & 0
\end{pmatrix}
\]

\[
H^T = \begin{pmatrix}
0 & 0 & -1/J_b
\end{pmatrix}
\]

where the state \( x = [x_1, x_2, x_3]^T \) is defined as: \( x_1 = \varphi_r - \varphi_b \), \( x_2 = \ddot{\varphi}_r \), and \( x_3 = \dot{\varphi}_b \). \( i \) is the ratio number. In this description, the state \( z \in R \) represents the internal friction state, and Equation (2) describes the friction dynamics. One possible model for (2) is the LuGre friction model ([5]):

\[
\begin{align*}
\dot{z} &= x_3 - \sigma_0 \frac{|x_3|}{g(x_3)} z, \\
g(x_3) &= \mu_C + (\mu_S - \mu_C) e^{-(x_3/v_s)} \\
\mu(x, z) &= \sigma_0 z + \sigma_1 z
\end{align*}
\]

Note that the torsional linear friction at the drill bit side is already incorporated in the \( A \) matrix of the representation (1). \( \sigma_0, \sigma_1, v_s, \mu_C, \mu_S \) are positive constants characterizing the friction physical properties. Realistic values for parameters (see [15]) are given in the Appendix A.

III. VELOCITY-CONTROLLED DRILLSTRING

Without loss of generality, it is assumed that the state \( x \) is measurable\(^2\). A velocity control-loop (static or dynamic) is then first designed to regulate the output rotary velocity \( \dot{\varphi}_r \) to some desired value \( \omega_d \) (a typical value for \( \omega_d \) is 5 rad/s).

Although many velocity control structures are possible, oil well drilling often operates with reduced-order simple control laws.

A common control structure [6] is:

\[
v = \left[ k_1 + \frac{k_2}{s} \right] (\omega_d - \dot{\varphi}_r) - k_3 (\dot{\varphi}_r - \dot{\varphi}_b)
\]

(4)

or equivalent

\[
v = k_1(\omega_d - x_2) + k_2 x_4 - k_3 (x_2 - x_3)
\]

(5)

\[
\dot{x}_4 = \omega_d - x_2
\]

then the closed-loop equations take the form,

\[
\begin{aligned}
\dot{x} &= A_{cl} x + B_{cl} \omega_d + H_{cl} \mu(x, z) u \\
\dot{z} &= f(x, z)
\end{aligned}
\]

(6)

with the obvious observation that the state is now of dimension four, i.e. \( x = [x_1, x_2, x_3, x_4] \), and with the \( A_{cl}, B_{cl} \) and \( H_{cl} \) defined accordingly.

The gains values can be designed using pole placement. For instance, the first pair of poles are set as a pair of complex-conjugate poles with frequency \( \omega_n \) and damping ratio \( \delta \). The second pair can be placed at \( 6 \omega_n \), the drillstring system enters into sustained oscillation, and cases where the drill bit rotational velocity reaches the desired stable equilibrium point (see for instance Figure 7).

This behavior is shown in Figure 2, where the WoB parameter is perturbed away from its nominal operation value; in Figure 2-(a), the WoB changes from 40000N to 45000N and the controller is still able to correctly regulate the rotatory speed (the state does not leave its local attraction domain), whereas in Figure 2-(b) the WoB is perturbed enough (from 40000N to 50000N) so that the states are

\(^2\)The problem can be also formulated including an observer. An observer may be needed because the bit velocity is not generally sensed.
with relatively well defined period. Therefore, the SBDF (Sinusoid plus bias describing function) method can then be used as a first approximation to predict possible limit cycle and to study its stability. To simplify further the computation of the SBDF corresponding to the nonlinear dynamic friction map (3), we rather use a simpler approximation of the resulting steady-state characteristic of (3), see Figure 4. This approximation is valid because the friction dynamics is much faster than the one of the drillstring mechanism.

\[ G(j\omega_n) = \frac{-1}{N_1(A_0, \omega_n, y_0)} \quad (7) \]

\[ y_0 [1 + G(0)N_0(A_0, y_0)] = W(0)\omega_d \quad (8) \]

\[ A_0 \text{ and } \omega_0 \text{ are the particular values satisfying both equalities. They represent the amplitude and frequency of the predicted oscillations, if any. } y_0 \text{ is the output bias, and } N_1 \text{ and } N_0 \text{ are given in the report [4].} \]

Note that \( G(j\omega) \) depends on the control parameters, hence on the assigned bandwidth \( \omega_n \). The Nyquist diagram of \( G(j\omega) \) is shown in Figure 5 as a function of \( \omega_n \). The arrows indicate the sense of increasing \( \omega_n \). It is interesting to notice that \( G(j\omega) \) changes sign twice as \( \omega_n \) increases.

For small \( \omega_n \), the map \( G \) is in the right half-plane. Increasing \( \omega_n \), \( G(j\omega) \) increases its magnitude until its Nyquist goes into the left half-plane. A further increase of \( \omega_n \) makes \( G(j\omega) \) to stretch in magnitude until its sign changes again.

Table I gives the domain for \( \omega_n \), as a function of the location of the Nyquist of \( G(j\omega) \).

<table>
<thead>
<tr>
<th>( \omega_n )</th>
<th>Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>([0, 2.85])</td>
<td>Right half-plane</td>
</tr>
<tr>
<td>([2.85, 12.01])</td>
<td>Left half-plane</td>
</tr>
<tr>
<td>([12.01, \infty])</td>
<td>Right half-plane</td>
</tr>
</tbody>
</table>

Since the observed oscillations are clearly asymmetric, the SBDF is used to compute \( N_1(A, \omega, y_0) \), see [2]. \( N_1(A, y_0) \) will have only real part since the friction torque characteristic has odd symmetry.
Note that the frequency locus of $G(j\omega_n)$ can only cross the real axis in one point, then $\omega_0$ can be uniquely computed from $\exists\ N\{G(j\omega_0)\} = 0, \forall \ \omega_0 \neq 0, \ |\omega_0| < \infty$, and hence does not depend on the other two unknown variables $A_0$ and $y_0$. The introduction of a integral action in the velocity control results in $G(0) = 0$, and $W(0) = 1$. Therefore, the output bias can be straightforwardly computed from equation (8). Yielding

$$y_0 = \frac{W(0)}{1 + G(0)N_0(A_0, y_0)} \omega_d = \omega_d$$

With $y_0 = \omega_d$ known and independent of frequency and the amplitude, the prediction of the limit cycle and its associated amplitude $A_0$, can be obtained only from equation (7), i.e.

$$\text{Re} \left\{ G(j\omega) \right\} = -\frac{1}{N(A_0)}$$

with $N(A_0) = N_1(A, \omega_d)$.

Figure 6 shows how $\frac{1}{N(A)}$ changes as a function of $A$. The main feature of this amplitude locus, pertinent to oscillations prediction, is the location of points $p_0$ and $p_1$. These points derive from the maximum and minimum values of $N(A)$. In particular, it is interesting to notice that these points have the following property:

$$|p_0| \sim \frac{1}{WoB} \quad \text{and} \quad |p_1| \sim \frac{1}{WoB}$$

Then, low values of $WoB$ makes these two points bigger in magnitude (its sign remains unchanged), reducing the possibility to intersect the Nyquist of $G(j\omega)$. Inversely, when the weight on bit force increases, the probability to get an oscillation is higher.

It is also interesting to note, that for almost every realistic choice of $\omega_n$, two sets of limit cycles will be predicted; one stable set and another unstable. The stable set arises at amplitudes $A = A_s$, whereas the unstable one occurs for $A = A_u$. In all cases $A_s > A_u$. This means that for all parameter combination ($WoB - \omega_n$) where an intersection of $G(j\omega)$ and $-1/N(A)$ takes place, there exists a local (attractive) stable domain delimited by $A_u$. This property is exploited further in the proposed D-OSKIL mechanism.

It is also of interest to compute the set of possible combination between system bandwidth and weight on bit force for which stable limit cycles are predicted. Figure 7 shows the zone in the $(WoB - \omega_n)$-plane giving rise to possible stable oscillations. This analysis allows to determine the range of proper values of controller closed-loop bandwidth to avoid oscillations as a function of the drilling depth. As the figure shows, and on the basis of this analysis, oscillations may be eliminated either by changing $\omega_n$, and/or by reducing the $WoB$ magnitude.

One possible control strategy could be then to modify the rotary table bandwidth to reduce the possibility to enter into oscillation. From Figure 7, we can see that suitable choices for $\omega_n$ are either small or large values. Small values are not suited because yield to poor performance, and large values are limited by noise. Typical values for $\omega_n$ are in the range $[20, 30] \text{rad/s}$.

Another alternative will be to keep the value of $WoB$ below the oscillation zone. However, this will have two drawbacks: first, high-performance drilling operation requires magnitudes for $WoB$ larger than the ones indicated by the oscillation limits, and second this strategy will require a precise knowledge of this map, which in addition will vary as a function of the drilling depth. Keeping $WoB$ low, will thus be a too conservative strategy.

High-performance drilling operation takes place in a region in the plane $(WoB - \omega_n)$, where potential oscillations may occur. Therefore, in this paper we account for this particularity assuming that the nominal system operation parameters are taken within this zone. The control strategy presented next, is thus built under the following base-lines:

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**Fig. 5.** Nyquist diagram of $G$ for different values of $\omega_n$.

**Fig. 6.** Describing function $N(A)$ (upper), and evolution of $\frac{1}{N(A)}$ (lower). $|p_0|$ is two or three magnitude orders lower than $|p_1|$, depending on value of $WoB$. 8263
nominal values for \((WoB - \omega_0)\) are in the potential stable limit cycle zone. Then, stick-slip limit cycles are possible under a substantial perturbation.

oscillation will be eliminated by decreasing the Weight on Bit force. In other words, by reducing the \(WoB\), the local attraction domain is enlarged about the equilibrium points, until system trajectories are attracted to them. This effect is understandable from Figure 7.

after that, the \(WoB\) will have to be set to its nominal value again to continue properly with the drilling task. This is not seen directly from Figure 7. However, this is possible as long as the system trajectories are kept within the local attraction domain, by a slow variation of the \(WoB\), see [3]

V. THE D-OSKIL: DRILLING OSCILLATION KILLER MECHANISM

The OSKIL mechanism has been designed (see detail in the companion paper [3]) assuming that originally the \(WoB\) is set to its nominal value \((u = u_0)\). Further, we assume the nominal system parameter operation, in term of values for \((WoB - \omega_0)\), is due inside potentially oscillation zone shown in Figure 7, but system trajectories are at its equilibrium, i.e. \(\dot{\varphi}_r = \omega_d\). However, as it is often the case during the drill operation, variation on the rock friction characteristics may bring the closed-loop system trajectories outside the local stable zone producing stick-slip cycles.

The problem is thus to modify (temporally) the effective value of \(u\) via the additional control signal \(\tilde{u}\) as shown in the representation (1), so that the oscillations can be eliminated, while bringing back the effective value of \(u\) to its nominal operation value, \(u_0\). One possible structure for the D-OSKIL mechanism is the following one:

\[
\begin{align*}
  y &= Cx = \varphi_b \\
  y_f &= F(s)y \\
  \xi &= \int_t^{t+T} y^2_f(\tau)d\tau \\
  \frac{1}{\sigma} \dot{\tilde{u}} &= -\tilde{u} - sat_{u_0}(K_f \xi), \quad \tilde{u}(0) = 0 \\
  u &= u_0 + \tilde{u}
\end{align*}
\]

This scheme can be completed with an estimator for the main oscillation frequency, as shown in Figure 8.

We next summarize the rationality behind the OSKIL development (see [3] for further analysis).

Assume that the system under consideration has left its local attractive domain of operation, and as a consequence the state vector \(x(t)\) has reached the stable orbit (limit-cycle). If the output \(y(t)\) has been properly chosen, the oscillations are reflected on that measurement. As a consequence, we can then assume that \(y(t)\) will be of the form

\[
y(t) = y_0 + A_0 \cos(\omega_0 t + \varphi_0) + \cdots
\]

where \(y_0\) is the bias component of the oscillation and \(A_0, \omega_0 = \frac{\alpha}{\beta}, \varphi_0\) are the amplitude, frequency and phase of the main oscillatory component.

Using a filter of the form

\[
F(s) = \frac{K_f s}{(s + \omega_1)(s + \omega_2)}
\]

with \(\omega_1 = \omega_0 - \Delta \omega > 0, \) and \(\omega_2 = \omega_0 + \Delta \omega > 0, \) the output of (10), can be approximated as:

\[
y_f(t) \approx A_0 \cos(\omega_0 t + \varphi_1)
\]

The value of \(K_f\) and \(\Delta \omega\) are designed such that \(|F(j\omega_0)| \approx 1, \) and frequencies away from \(\omega_0\) are filtered out.

Equation (11) computes the RMS value of \(y_f(t), \) yielding

\[
\xi = \int_t^{t+T} A_0^2 \cos^2(\omega_0 \tau + \varphi_1) d\tau \approx \frac{A_0^2 T}{2} \geq 0
\]

This signal reflects the magnitude of the oscillation and it is suited to control oscillations. Note that in this computation,
a re-design of the velocity rotary-table control. A formal stability analysis is currently under investigation.

ACKNOWLEDGMENTS

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APPENDIX

A. Simulation Parameters

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<tr>
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<td>( \sigma_0 = 25 ) [1/rad]</td>
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<tr>
<td>( \sigma_1 = 193 ) [s/rad]</td>
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<td>( \omega_3 = 3 ) [rad/s]</td>
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<td>( T = 5.8 ) [s]</td>
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<td>( \sigma = 0.03 ) [rad/s]</td>
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<td>( K_1 = 3.16 )</td>
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REFERENCES


