Routing Algorithms for Recursively-Defined Data Centre Networks

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Abstract—The server-centric data centre network architecture can accommodate a wide variety of network topologies. Newly proposed topologies in this arena often require several rounds of analysis and experimentation in order that they might achieve their full potential as data centre networks. We propose a family of novel routing algorithms on two well-known data centre networks of this type, (Generalized) DCell and FiConn, using techniques that can be applied more generally to the class of networks we call completely connected recursively-defined networks. In doing so, we develop a classification of all possible routes from server-node to server-node on these networks, called networks. In doing so, we develop a classification of all possible routes from server-node to server-node on these networks, called Generalized DCell (CDPRDN), which we use to develop a classification (which, to our knowledge, is novel) of all possible routes from server-node to server-node in the DCNs (Generalized) DCell and FiConn. Our main result pertains to a specific family of routing algorithms, called PR or ProxyRoute, which we develop with the primary aim of improving upon the originally proposed (and best known) routing algorithms, as regards hop-length. This goal is achieved with improvements as high as 16% for certain topologies and paths that are comparable, in length, to shortest paths. In addition, we give empirical evidence that the path diversity provided by PR does a better job of balancing load than DCellRouting. Hitherto, the only algorithms for balancing communication load in (Generalized) DCell and FiConn are the adaptive routing algorithms DFR and TAR presented in [5,6], so PR is also novel in this respect.

Two of our instances of PR called GP_I and GP_0, exploit the topological structure of (Generalized) DCell and FiConn in order to find short paths efficiently by means of an intelligent search (see Section V-A) of sub-structures called “proxies”. We then empirically compare the results of our intelligent versions of PR with a shortest path algorithm, a brute force version of PR and the routing algorithms that were originally proposed in [5,6,12].

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I. INTRODUCTION

The explosive growth of online services powered by data centres (web search, cloud computing, etc.) has motivated intense research into data centre network (DCN) design over the past decade and brought about major breakthroughs. For example, fat-tree DCNs, introduced in [1], use commodity off-the-shelf (COTS) servers and switches in a fat-tree (topology), and have resulted in an evolutionary shift in production data centres towards leaf-spine topologies, built from COTS hardware. COTS fat-tree DCNs are not a panacea, however; for example, fat-trees are difficult to scale.

Research on DCN architecture is ongoing and each new architecture invites the use of certain classes of topologies. Indirect networks, where servers are the terminals connected to a switching fabric, are the prevailing example. Fat-trees are among the topologies that can be implemented in indirect network architectures. A host of alternative topologies can be implemented as indirect networks, including random regular graphs ([2]) and butterfly networks ([3]). Likewise, the optical-switch hybrid DCN Helios ([4]) can be seen as an architecture with the capacity to accommodate a variety of topologies (both in the wired links as well as in the optical switch itself). Each architecture sets constraints on the topology in a variety of ways; for example, by the separation of switching nodes from server nodes or the number of ports in the available hardware.

The server-centric DCN (SCDCN) architecture, introduced in [5], accommodates a great variety of network topologies and has resulted in a number of new DCN designs, both derived from existing and well-understood topologies in interconnection networks as well as topologies geared explicitly towards DCNs (e.g., [5]–[10]).

Only dumb crossbar-like switches are used in an SCDCN and the servers are responsible for routing packets through the network. Therefore, the switches have no knowledge of the network topology and are only connected to servers. Servers, on the other hand, may be connected to both switches and servers. These parameters, which make up part of the SCDCN architecture, invite sophisticated topologies from abstractions as graphs, along with accompanying analyses. We are concerned primarily with routing algorithms for two well-known SCDCNs, DCell ([5]) and FiConn ([6]), and the topologies called Generalized DCell ([11,12]),

We characterise (Generalized) DCell and FiConn as a special case of completely connected recursively-defined networks (CCRDN), which we use to develop a classification (which, to our knowledge, is novel) of all possible routes from server-node to server-node in the DCNs (Generalized) DCell and FiConn. Our main result pertains to a specific family of routing algorithms, called PR or ProxyRoute, which we develop with the primary aim of improving upon the originally proposed (and best known) routing algorithms, as regards hop-length. This goal is achieved with improvements as high as 16% for certain topologies and paths that are comparable, in length, to shortest paths. In addition, we give empirical evidence that the path diversity provided by PR does a better job of balancing load than DCellRouting. Hitherto, the only algorithms for balancing communication load in (Generalized) DCell and FiConn are the adaptive routing algorithms DFR and TAR presented in [5,6], so PR is also novel in this respect.

Two of our instances of PR called GP_I and GP_0, exploit the topological structure of (Generalized) DCell and FiConn in order to find short paths efficiently by means of an intelligent search (see Section V-A) of sub-structures called “proxies”. We then empirically compare the results of our intelligent versions of PR with a shortest path algorithm, a brute force version of PR and the routing algorithms that were originally proposed in [5,6,12].

We give definitions in Sections II–III, where we abstract the DCNs (Generalized) DCell and FiConn as graphs which can
be characterised as CCRDNs. Section IV describes previously known routing algorithms for these DCNs in the context of CCRDNs, and our classification of routes in CCRDNs is given in Section IV-B, as general routes of order t. We present our main contribution in Section V: the design of PR. Our empirical work is described and evaluated in Section VI and future avenues for research are identified in the conclusion.

II. SERVER-CENTRIC DCNs

Our results and experiments are concentrated on graph theoretical abstractions of certain SCDCNs. Therefore, it is appropriate that we define this abstraction precisely.

An SCDCN consists of switches, which act only as crossbars and have no routing intelligence, and servers. These components are linked together, with the only restriction being that a switch cannot be linked directly to another switch; we assume all links are bidirectional. As such, an SCDCN is abstracted here by an undirected graph \( G = (W \cup S, E) \), with two types of nodes called switch-nodes, \( W \), and server-nodes, \( S \). Naturally, each switch of the SCDCN corresponds to a switch-node, \( w \in W \), and each server corresponds to a server-node, \( x \in S \). Each link of the SCDCN corresponds to an edge \( e \) of \( E \), which, for convenience, we shall also call a link. The condition that switch-to-switch links are not allowed implies that \( \exists(u, v) \in E \) such that \( u, v \in W \). See [13] for undefined graph-theoretic terms.

Also relevant to our discussion of routing algorithms in SCDCNs is the fact that (1) packets are sent and received only by servers, and (2) packets endure a negligible amount of processing time in each switch, compared to the time spent in each server. The reason for (2) is that we assume the packet is routed in the server’s operating system, either via a table lookup or computation. This could be done, e.g., by a dedicated virtual machine or a specialised hypervisor with the capability to route packets. In any case, we may assume that with today’s COTS servers, a packet spends much more time at servers than in switches.

The outcome of (1) is that we need only discuss routing algorithms that construct paths whose endpoints are server-nodes. That is, a route on \( G \) is a path whose endpoints are server-nodes. The outcome of (2) is that a hop from server-node to server-node is indistinguishable from one that also passes through a switch-node.

III. RECURSIVELY-DEFINED NETWORKS

Our results are concerned with network topologies of a certain form that have arisen frequently in the area of interconnection networks, and recently as SCDCNs.

**Definition III.1.** A family \( \mathcal{X} = \{ X(h) : h = 0, 1, \ldots \} \) of interconnection networks is recursively-defined if \( X(h) \), where \( h > 0 \), is the disjoint union of copies of \( X(h - 1) \) with the addition of extra links joining nodes in the different copies. We call a member of \( \mathcal{X} \) a recursively-defined network (RDN). A family of RDNs \( \mathcal{X} \) is a completely-connected RDN (CCRDN) (see, e.g., [14]) if there is at least one link joining every copy of \( X(h - 1) \) within \( X(h) \) to every other copy.

A. The DCNs DCell

The DCNs DCell ([5]) were the first family of SCDCNs to be proposed, and their graphs form the family of CCRDNs described below.

Fix some \( n > 2 \). The graph DCell\(_{0,n} \) consists of one switch-node connected to \( n \) server-nodes. For \( k \geq 0 \), let \( t_k \) be the number of server-nodes in DCell\(_{k,n} \). For \( k > 0 \), the graph DCell\(_{k,n} \) consists of \( t_{k-1} + 1 \) disjoint copies of DCell\(_{k-1,n} \), labelled \( D_{k-1}^i \), for \( 0 \leq i \leq t_{k-1} \). Each pair of distinct DCell\(_{k,n} \), for \( 0 \leq i \leq t_{k-1} \), is joined by exactly one link, called a level-\( k \) link, whose exact definition is given below, in terms of the labels of the server-nodes.

Label a server-node of a DCell\(_{k,n} \), for some \( k > 0 \), by \( x = x_kx_{k-1} \cdots x_0 \), where \( x_{k-1}x_{k-2} \cdots x_0 \) is the label of a server-node in DCell\(_{k-1,n} \), and \( 0 \leq x_0 < n \) and \( 0 \leq x_i < g_k \) for \( i > 0 \), where \( g_k = t_{k-1} + 1 \). The labels of DCell\(_{k,n} \), for \( 0 \leq i \leq t_{k-1} \), are mapped bijectively to the set \( \{0, 1, \ldots, t_{k-1} - 1\} \) by \( \text{uid}_k(x) = x_kt_{k-1} + x_{k-1}t_{k-2} + \cdots + x_1t_0 + x_0 \). Label and wid are combined in the notation \( [x_k, \text{uid}_k(x)] \).

Let \( 0 \leq k < y_k < t_k + 1 \) be the indices of the DCell\(_{k,n} \) labelled \( D_{k-1}^y \) and \( D_{k-1}^k \). A level-\( k \) link connects node \( y_k - 1 \) in DCell\(_{k-1,n} \) to node \( x_k \) in DCell\(_{k,n} \). This is the link \( (y_k - 1 + x_k)t_{k-1} + x_k + y_kt_{k-1} \).

1) Generalized DCell: The definition of the DCNs DCell generalises readily; see [11,12]. The key observation is that the level-\( k \) links are a perfect matching of the server nodes in the disjoint copies of the DCell\(_{k-1,n} \), where every pair of distinct DCell\(_{k-1,n} \) is connected by a link. Many such matchings are possible. A given matching \( \rho_k \) which satisfies the stated properties defines the level-\( k \) links and is called a \( \rho_k \)-connection rule ([12]).

A Generalized DCell\(_{k,n} \) inherits the definition of DCell\(_{k,n} \), for \( k \geq 0 \), except that the level-\( k \) links may satisfy an arbitrary \( \rho_k \)-connection rule. Note that we insist that there be only one connection rule for each level \( k \), so that a given family of Generalized DCNs can be specified by a set of connection rules \( \{\rho_1, \rho_2, \rho_3, \ldots\} \).

This is in accordance with Definition 1 in [12], with two exceptions. We model Generalized DCell\(_{k,n} \) as a switch-node connected to \( n \) server-nodes, rather than modelling it as \( K_n \), and we require \( n > 2 \).

In order to demonstrate the impact of different connection rules on the routing algorithms presented in Section IV, it suffices to consider just one connection rule besides the one for DCell. For this purpose, we use \( \beta \)-DCell, defined by the \( \beta \)-connection rule given in [12].

The \( \beta \)-connection rule is (perhaps not obviously) as follows:

Let \( 0 \leq x_k < y_k < t_k - 1 + 1 \) be the indices of the \( \beta \)-DCell\(_{k-1,n} \) labelled \( B_{k-1}^{x_k} \) and \( B_{k-1}^{y_k} \). A level-\( k \) link connects node \( y_k - x_k - 1 \) in DCell\(_{k-1,n} \) to node \( t_k - 1 - y_k + x_k \) in DCell\(_{k-1,n} \). This is the link \( (y_k - x_k - 1 + x_k)t_{k-1} + t_k - 1 - y_k + x_k + y_kt_{k-1} \).

B. The DCNs FiConn

One of the issues with (Generalized) DCell\(_{k,n} \) is that each server-node has degree \( k + 1 \). This requires that each server
has \( k+1 \) NIC ports, which is not typically the case for COTS servers when \( k > 1 \).

FiConn, proposed in [6], is a CCRDN that requires at most two ports per server; it uses only half of the available server-nodes (those of degree one) in each copy of FiConn_{k-1,n} when building FiConn_{k,n}. This, in turn, leaves server-nodes of degree one available to build the next level. We describe FiConn below.

Fix some even \( n > 3 \). FiConn_{k,n} is the network consisting of one switch-node connected to \( n \) server-nodes. Let \( b \) be the number of available server-nodes in FiConn_{k-1,n} for \( k > 0 \). Build FiConn_{k,n} from \( b/2 + 1 \) copies of FiConn_{k-1,n}, labelled \( F^i_{k-1} \), for \( 0 \leq i \leq b/2 \). From [6] we have that \( b/2 + 1 = \frac{t_{k-1}}{2^k} + 1 \), so that the label of a server-node \( x \) of a FiConn_{k,n} is expressed as the \((k+1)\)-tuple \( x = x_k x_{k-1} \cdots x_0 \), where \( x_k x_{k-1} \cdots x_0 \) is a server-node in \( F^x_{k-1} \) and we have \( 0 \leq x_0 < n \), but \( 0 \leq x_i < g_k \), where \( g_k = b/2 + 1 = \frac{t_{k-1}}{2^k} + 1 \) (diverging slightly from the labels in DCell). We have \( \text{uid}(x) = x_k x_{k-1} + x_{k-1} x_{k-2} + \cdots + x_1 + 0 \) and \( \{x_k, \text{uid}(x_k x_{k-1} x_{k-2} \cdots x_0)\} \) to label server-nodes, once more.

Let \( 0 \leq x_k < y_k < t_{k-1}/2^k + 1 \) be the indices of the FiConn_{k-1,n}s \( F^x_{k-1} \) and \( F^{y}_{k-1} \). A level-k link connects a server-node \((y_k - 1)2^k + 2^k + \cdots + 1 \) in \( D^x_{k-1} \) to server-node \( x_k 2^k + 2^k - 1 \) in \( D^{y}_{k-1} \). This is the link \(((y_k - 1)2^k + 2^k + \cdots + 1 + y_k x_{k-1}, x_k 2^k + 2^k - 1 + 1 + y_k x_{k-1})\).

IV. ROUTING

CCRDNs feature a class of routing algorithms that emerges naturally from their definition, called dimensional routing.

A. Dimensional routing

**Definition IV.1.** Let \( \mathcal{X} = \{X(h) : h = 0, 1, \ldots\} \) be a family of CCRDNs, and let \( X_h \) be a copy of \( X(h) \), for some fixed \( h > 0 \). Let \( X^a_{h-1} \) and \( X^b_{h-1} \) be disjoint copies of \( X(h-1) \) in \( X_h \), and let src and dst be nodes of \( X^a_{h-1} \) and \( X^b_{h-1} \), respectively. Since \( X_h \) is completely connected, there is a level-\( h \) link in \( X_h \) incident with a node dst' in \( X^a_{h-1} \) and a node src' in \( X^b_{h-1} \). If \( h = 0 \) then either \( \text{src} = \text{src}' \) or \((\text{src}, \text{dst}')\) is a link, and otherwise a path \( P_a \) from src to dst' can be recursively computed in \( X^a_{h-1} \). This same method provides a path \( P_b \) from src' to dst in \( X^b_{h-1} \). A dimensional routing algorithm on \( \mathcal{X} \) is one which computes paths of the form \( P_a + (\text{dst}', \text{src}') + P_b \), between any source-destination pair of nodes in a member of \( \mathcal{X} \), and is denoted \( \mathcal{D} \mathcal{R} \mathcal{X} \). A dimensional route is one that can be computed by a dimensional routing algorithm.

Remarkably (and, perhaps, unfortunately), there are topologies and source-destination pairs for which no dimensional routing algorithm computes a shortest path; a notable example is the family of WK-recursive networks ([15]), for which a shortest path algorithm is developed in [16].

1) Dimensional routing in (Generalized) DCell and FiConn: (Generalized) DCell and FiConn are CCRDNs in which each pair of disjoint copies of DCell_{k-1,n} within DCell_{k,n} is joined by exactly one edge. As such, there is only one choice for the edge \((\text{dst}', \text{src}')\), which is computed by the connection rule for level-\( h \) links. Therefore, the connection rules in Sections III-A--III-B suffice to describe dimensional routing for these DCNs.

The dimensional routing algorithms for each of these networks serves as a basis for fault-tolerant and load-balancing routing algorithms \( \mathcal{D} \mathcal{F} \mathcal{R} \) in [5], and \( \mathcal{T} \mathcal{A} \mathcal{R} \) in [6], and it is precisely the algorithm called Generalized DCellRouting, given in [12]. The former two are fault and congestion-tolerant routing algorithms that compute significantly longer paths, on average, than the dimensional routing algorithms.

B. Proxy Routing

A general routing algorithm on a family \( \mathcal{X} = \{X(h) : h = 0, 1, \ldots\} \) of CCRDNs is of the following form. Let \( X_h \) be a copy of \( X(h) \), for some fixed \( h > 0 \). Let \( X^a_{h-1} \) and \( X^b_{h-1} \) be disjoint copies of \( X(h-1) \) in \( X_h \), with src_{ca} and dst_{ca} of nodes of \( X^a_{h-1} \) and \( X^b_{h-1} \), respectively. Let \( X^a_{h-1}, X^b_{h-1}, \ldots, X^r_{h-1} \) be a sequence of copies of \( X(h-1) \), where \( c_0 = a \), \( c_{t-1} = b \), \( c_t \neq c_{t+1} \), for \( 0 \leq i \leq t < \tau \); and \( X^r_{h-1} \) is disjoint from \( X^c_{h-1} \) whenever \( c_t \neq c_{t+1} \). Let \((\text{src}_{ca}, \text{dst}_{ca})\) be a link from \( X^c_{h-1} \) to \( X^c_{h-1} \), and let \( P_i \) be paths in each \( X^c_{h-1} \) from \( \text{src}_{ca} \) to \( \text{dst}_{ca} \).

Every routing algorithm computes a path (we shall assume that there are no repeated nodes) of the form \( P_0 + (\text{dst}_{ca}, \text{src}_{ca}) + P_1 + \ldots + (\text{dst}_{ca}, \text{src}_{ca}) + P_t \).

A general route of order \( T \) is one in which \( t \leq T \) for each \( X(h) \), with \( h = 0, 1, \ldots \) and \( T = \tau \) for at least one of these. A proxy route, computed by a proxy routing algorithm, is a general route of order 3 (and a dimensional route is of order 2).

1) \( \mathcal{D} \mathcal{F} \mathcal{R} \) for \( \text{DCell} \) and \( \mathcal{T} \mathcal{A} \mathcal{R} \) for \( \text{FiConn} \): While we do not provide full details here, we sketch the proxy-routing-like subroutine that is common to \( \mathcal{D} \mathcal{F} \mathcal{R} \) ([5]) and \( \mathcal{T} \mathcal{A} \mathcal{R} \) ([6]). Both \( \mathcal{D} \mathcal{F} \mathcal{R} \) and \( \mathcal{T} \mathcal{A} \mathcal{R} \) are adaptive routing algorithms which compute paths in a distributed manner, making decisions on the fly, based on information that is local to the current location of the packet being routed.

This subroutine computes a part of a proxy route to replace a sub-path of the intended route. In particular, a packet may bypass a level \( m \) link, \( e \), from sub-structure \( D^a_{m-1} \) to \( D^b_{m-1} \) by re-routing through a proxy, \( D^c_{m-1} \), with \( a, b, c \) distinct. The decision to bypass is made when the packet arrives at \( e \) (or near \( e \), as determined by a parameter in \( \mathcal{D} \mathcal{F} \mathcal{R} \)), and upon its arrival in \( D^b_{m-1} \), the packet is routed directly to its final destination.

The algorithms \( \mathcal{D} \mathcal{F} \mathcal{R} \) and \( \mathcal{T} \mathcal{A} \mathcal{R} \) produce much longer than \( \text{DR} \), on average. The simulations in [5] show that \( \mathcal{D} \mathcal{F} \mathcal{R} \), although fault-tolerant, computes paths that are over 10% longer than the shortest paths, on average, even with as little as 2% failures. The maximum length of a route computed by the implementation of \( \mathcal{T} \mathcal{A} \mathcal{R} \) in [6] (Theorem 7) is \( 2^3 - 1 \), whilst it is \( 2^k - 1 \) for \( \text{DR} \) (called \( \text{TOR} \) in [6]). This is reflected in their simulations of random and burst traffic, where \( \mathcal{T} \mathcal{A} \mathcal{R} \) computes paths that are 15-30% longer, on average, than those computed by \( \text{DR} \).
V. Proxy routing in DCell and FiConn

We propose that proxy routing be used more broadly than it is in DFR and TAR, and with the primary goal of efficiently computing short paths, rather than fault-tolerance and balancing load, by applying it in a fundamentally different manner: firstly, we seek to compute a proxy route at the outset, rather than building the route piecemeal; secondly, we use this pre-planning in order to find a proxy route that offers a high degree of savings over the dimensional route.

One reason for focusing on \( t \leq 3 \) is that visiting each \( X^c_j \) for \( 0 < j < t - 1 \), has an associated cost, and when \( m \) is small, as it is when our graphs represent DCNs with a realistically deployable number of servers, it becomes less likely that general routes with \( t > 3 \) will be useful. Furthermore, the methods of searching for a “good” proxy that we explore here may become impractical for \( t > 3 \), because the search space of potential (multiple) proxies is much larger.

Henceforth we use \( \mathcal{G} \text{-Cell} \) in place of (Generalized) DCell and FiConn whenever we make statements or arguments that apply to all of these.

The following lower bound on the hop-length of a general route of order \( t \) is obvious.

**Lemma V.1.** Let \( \text{src} \) and \( \text{dst} \) be server-nodes in a \( \mathcal{G} \text{-Cell}_{k,n} \), with \( k > 0 \), such that \( \text{src} \) is in \( D^k_{k-1} \) and \( \text{dst} \) is in \( D^k_{k-1} \), with \( a \neq b \). A general route of order \( t \) has length at least \( 2t - 3 \). In particular, a dimensional route has length at least 1 and a proxy route has length at least 3.

The remainder of our paper is a comparative empirical analysis of several versions of \( \text{PR} \), given in Algorithm 1.

**Algorithm 1** \( \text{PR} \) for \( \mathcal{G} \text{-Cell} \) returns a proxy route if it finds one that is shorter than the corresponding dimensional route.

**Require:** \( \text{src} \) and \( \text{dst} \) are server-nodes in a \( \mathcal{G} \text{-Cell}_{k,n} \).

**function** \( \text{PR}(\text{src}, \text{dst}, m) \)

if \( m > 0 \) and both \( \text{src} \) and \( \text{dst} \) are in the same copy of \( \mathcal{G} \text{-Cell}_{m-1,n} \)

return \( \text{PR}(\text{src}, \text{dst}, m - 1) \)

end if

\( D^m_{m-1} \leftarrow \text{GP}(\text{src}, \text{dst}, m) \).

if \( D^m_{m-1} = \text{null} \)

return \( \text{DR}(\text{src}, \text{dst}) \).

else

\( D^n_{m-1} \leftarrow \) the \( \mathcal{G} \text{-Cell}_{m-1,n} \) containing \( \text{src} \).

\( D^b_{m-1} \leftarrow \) the \( \mathcal{G} \text{-Cell}_{m-1,n} \) containing \( \text{dst} \).

\( (a^c, c^e) \leftarrow \) the link from \( D^n_{m-1} \) to \( D^e_{m-1} \).

\( (b^d, c^f) \leftarrow \) the link from \( D^b_{m-1} \) to \( D^c_{m-1} \).

\[ \text{PR}(\text{src}, a^c, m - 1) + (a^c, c^e) + \]

\[ \text{PR}(c^e, c^f, m - 1) + (b^d, c^f) + \]

\[ \text{PR}(b^d, \text{dst}, m - 1) \].

end if

end function

A. GP: GetProxy

GP is the subroutine of \( \text{PR} \) that computes the proxy used in Expression (1), if a proxy is to be used. That is, GP returns either a proxy sub-\( \mathcal{G} \text{-Cell} \), \( D^e_{m-1} \), or it returns \text{null}. Obviously, the performance of \( \text{PR} \) (and its success in producing a shorter route than \( \text{DR} \)) depends on the proxy returned by GP and how GP is implemented.

Ideally GP would instantly compute a unique proxy sub-\( \mathcal{G} \text{-Cell} \) \( D^e_{m-1} \), if it exists, such that the proxy route through \( D^e_{m-1} \) is the shortest one possible. Such an algorithm is unknown to us.

Our strategy, however, is widely applicable, as regards different connection rules and path diversity. Every version of GP that we explore is of the following form. Let \( (\text{src}, \text{dst}, m) \) be the inputs to GP. If \( m = 0 \), GP outputs \text{null}; otherwise, let \( m > 0 \), so that \( \text{src} \) is in \( D^n_{m-1} \) and \( \text{dst} \) is in \( D^b_{m-1} \), for some \( a \) not equal to \( b \). GP computes a set of candidate proxies, \( \{D^a_{m-1}, D^b_{m-1}, \ldots, D^{n-1}_{m-1} \} \) (taken from the set of all potential proxy \( \mathcal{G} \text{-Cell}_{m-1,n} \)) and then finds a \( c \) for which the path in Expression (1) is shortest (replacing \( c \) by \( c^i \)), by constructing the paths explicitly. If the set of candidate proxies is empty, then GP returns \text{null}.

The key observation is that we must minimise the number of candidate \( D^m_{m-1} \) in order to reduce the search space. Our goal is to identify and evaluate general techniques towards this end, and not to catalogue all of the ways to tune GP. Some more complicated techniques are avoided because there is no room to discuss them in this paper; for example when routing in a \( \mathcal{G} \text{-Cell}_{k,n} \) we only apply \( \text{PR} \) at the top level, whereas slightly shorter paths can be obtained, on average, by using proxy routes in the recursive calls to \( \text{PR} \) at Expression (1). Other techniques are avoided because they are evidently unprofitable; for example, a much larger search is encountered if GP computes proxy paths for each proxy candidate. We describe three strategies for generating the candidate proxies below.

1) \( \text{GP}_E \) as an exhaustive search: A proxy \( \mathcal{G} \text{-Cell}_{m-1,n} \) \( D^m_{m-1} \) can be obtained, naively, if GP is implemented as an exhaustive search; that is, we perform the steps described in Section V-A for every \( c \) in \( \{0, 1, \ldots, m-1\} \setminus \{a, b\} \). Measuring the length of each proxy route has an associated cost, but \( \text{GP}_E \) provides the optimal proxy route with top-level proxies only against which to test the two strategies given below.

2) \( \text{GP}_I \) as an intelligent search: We propose a general method for reducing the proxy search space, based on the labels of \( \text{src} \) and \( \text{dst} \). In particular, we look at proxies \( D^i_{k-1} \) whose relationship to \( D^n_{k-1} \) and \( D^b_{k-1} \) is such that at least one of the routes computed by the recursive calls to \( \text{PR} \) is confined to a \( \mathcal{G} \text{-Cell}_{k-2,n} \) (see Fig. 1).

We first give some notation. Henceforth, let \( D_k \) be an instance of \( \mathcal{G} \text{-Cell}_{k,n} \), and let \( \text{DR} \) be the dimensional routing algorithm on \( \mathcal{G} \text{-Cell} \). For clarity of exposition we describe a method for selecting a proxy \( D^j_k \) when routing in a \( \mathcal{G} \text{-Cell}_{k,n} \), with \( k = 3 \), but the notation extends to all \( k > 1 \).

Let \( \text{src} \) and \( \text{dst} \) be nodes in a \( \mathcal{G} \text{-Cell}_{3,n} \), with \( \text{src} = a_3a_2a_1a_0 \) and \( \text{dst} = b_3b_2b_1b_0 \), so that \( \text{uid}_3(\text{src}) = t_2a_3 + \)
Fig. 1. Strategy for $\text{GP}_\text{I}$, where $k = h - 2$, and for $\text{GP}_\text{0}$ where $h = 0$; select $c$ such that at least one sub-path is contained in a $G$-$\text{Cell}_{h,n}$. Solid arcs represent links, and dashed or dotted curves represent paths.

t_1a_2 + t_0a_1 + a_0$ and $uid_3(dst) = t_2b_3 + t_1b_2 + t_0b_1 + b_0$. Let $a_3 \neq b_3$, and note that without loss of generality, we may assume $a_3 < b_3$.

Our convention for denoting the link between two sub-$G$-Cells is as follows: let $D_2^a$ and $D_2^b$ be $G$-$\text{Cell}_{2,n}$'s and recall that we may write $[a, uid_2(v)]$ for a node $v = a_2v_1v_0$ in $D_2^a$, where $uid_2(v) = t_1v_2 + t_0v_1 + v_0$. Let $([\alpha, \alpha^2], [\beta, \beta^2])$ be the link from $D_2^a$ to $D_2^b$, with $\alpha^2 = \alpha^2_1\alpha^2_0\alpha^2_3$, and similarly for $\beta^2 = \beta^2_2\beta^2_1\beta^2_3$.

$\text{GP}_\text{I}$ builds its set of proxy candidates on the condition that the source and destination are not near to each other. Let $a = a_3$ and let $b = b_3$. $\text{GP}_\text{I}$ outputs null if $[a_3, \alpha^3]$ is a server-node of $D_1^{a_2}$ or $[b_3, \beta^3]$ is a server-node of $D_1^{b_3}$. That is, when $a_2 = a_2^b$ and $b_2 = b_2^b$.

Provided the above condition is avoided, we then select a proxy $D_2^k$ to be a candidate, where $c$ is such that one of the three sub-paths, $\text{PR}(src, [a, \alpha^c])$ or $\text{PR}([c, \alpha^c], [c, \beta^c])$ or $\text{PR}([b, \beta^c], dst)$, is short; specifically, if at least one of the three sub-paths is contained inside a single $G$-$\text{Cell}_{1,n}$. That is, $c$ satisfies at least one of the following three properties (in a non-trivial way; see discussion below):

\begin{align*}
\text{src} \text{ and } [a, \alpha^c] & \text{ are in the same } D_1: a_2 = a_2^c \ (2) \\
[c, \alpha^c] \text{ and } [c, \beta^c] & \text{ are in the same } D_1: c_2 = c_2^b \ (3) \\
[b, \beta^c] \text{ and } dst & \text{ are in the same } D_1: b_2 = b_2^c \ (4)
\end{align*}

where $a_2^c = [a^c_1/t_1]$ and similarly for $c_2^b$ and $b_2^c$. Clearly for any $G$-$\text{Cell}$ we can verify whether a proxy candidate $D_2^k$ satisfies one (or more) of the Properties (2)–(4), since the numerators are computed directly from the various connection rules of each $G$-$\text{Cell}$. However, we wish to compute the set of values $c$ which satisfy Properties (2)–(4) in constant time.

The floor function yields that $\lfloor a^c_1/t_1 \rfloor = a_2$ if, and only if, $a_2t_1 \leq a^c < (a_2 + 1)t_1$. It happens that for our connection rules (see Sections III), $a^c$ is piecewise linear (as a function of $c$), and similarly for $b^c$, $c^b$, and $\beta^c$, with exactly three cases: namely, $c_3 < a_3 < b_3$; $a_3 < c_3 < b_3$; and, $a_3 < b_3 < c_3$ (where the case $b_3 < a_3$ is treated by swapping $\text{src}$ and $\text{dst}$). As a result of this, the set of values $c$ which satisfy Properties (2)–(4) can be computed very efficiently for our connection rules as the union of, at most, a constant number of intervals (see Table I). Note that for the connection rules explored in this paper Property (3) is redundant because it does not narrow the search space; for certain pairs $(a, b)$, all $c$ satisfy Property (3), while no $c$ satisfies it for other pairs.

For the case $k = 3$ and the connection rules for DCell, $\beta$-DCell, and FiConn, $\text{GP}_\text{I}$ considers a small set with around $t_1$ or $2t_1$ candidate proxies. More generally, a close inspection of Properties (2) and (4) reveals that they each yield exactly $t_2$ (possibly disjoint) candidate proxies for Generalized DCell and at most $t_1$ candidate proxies for FiConn. Due to space constraints we omit a full discussion of this, but we remark that a better understanding of this aspect of proxy routes may shed light on the sophisticated relationship between the connection rule and various distance metrics on $G$-$\text{Cell}$.

3) $\text{GP}_\text{0}$ level-0 proxy search: We note that for a $G$-$\text{Cell}_{k,n}$, with $k = 2$, the proxy candidates $D_1^k$ computed by $\text{GP}_\text{I}$ are simply those for which $a^c$ is in the same copy of $G$-$\text{Cell}_{2,n}$ as src or $b^c$ is in the same copy of $G$-$\text{Cell}_{1,n}$ as dst or $c^b$ and $b^c$ are in the same copy of $G$-$\text{Cell}_{2,n}$. $\text{GP}_\text{0}$ mimics $\text{GP}_\text{I}$, but computes the set of proxies that satisfy at least one of the aforementioned properties, in place of Properties (2)–(4). It is applied only to $G$-$\text{Cell}_{k,n}$ with $k > 2$.

4) Implementation notes: The savings in hop-length and the benefit to load-balancing come at the cost of searching proxy candidates, whose number is given by $p$ in Fig. 3. For each proxy candidate $c$, the lengths of sub-paths $\text{PR}(src, [a, \alpha^c])$ or $\text{PR}([c, \alpha^c], [c, \beta^c])$ or $\text{PR}([b, \beta^c], dst)$ must be computed; hence the reason for devising $\text{GP}_\text{I}$ and $\text{GP}_\text{0}$ with the object of minimising $c$. Once $\text{GP}*$ is “tuned” to suit a certain application and network size, however, there are several choices for how it can be implemented. How exactly this is done depends on the size of the network and the nature of the application, but we shall remind ourselves of some of the available tools.

The most naïve method is to compute the route at the source-node, by computing the candidate paths explicitly, and measuring their length, however, other methods such as table look-ups must be considered.

$\text{GP}_\text{I}$, in particular, leverages the fact that $G$-$\text{Cell}_{k,n}$ grow double-exponentially in $k$ in order to find proxy candidates $D_{k-1}^k$ that are linked to the same copy of $G$-$\text{Cell}_{k-2,n}$ as src or dst. This has a secondary benefit; namely, $G$-$\text{Cell}_{k-2,n}$ (and even $G$-$\text{Cell}_{k-1,n}$) is small, relative to $G$-$\text{Cell}_{k,n}$, and this makes table look-ups feasible for storing the lengths of paths within each copy of $G$-$\text{Cell}_{k-2,n}$, and possibly within each copy of $G$-$\text{Cell}_{k-1,n}$. The whole table must be replicated at each server-node to be used this way, but this is still much smaller than storing every $(\text{src}, \text{dst})$-pair. For example, there are 24,492 = 599, 830, 064 such pairs in DCell_{1,3}, and $g_3t_2 = 157*150^2 = 3, 820, 752$ pairs confined to sub-DCell_{2,3,8}, and $g_3g_2t_2^2 = 157*13*12^2 = 293, 904$ pairs confined to sub-DCell_{1,3,8} (see Table II).

In addition to table look-ups, we also leverage the fact that paths are computed for flows, rather than packets, and in
certain applications may be re-used for multiple flows among a set of server-nodes that is small, relative to the entire network. In addition, each time we compute a proxy path, we may identify multiple viable proxies (the context of the application and network size defines what this means), and hence, path diversity comes at no extra cost. We may choose from several paths at random, send a probe packet to explore the loads and possible faults on each path before sending a larger flow, or remember proxies for common and recent destinations.

VI. EXPERIMENTS

A. Experimental setup

We compare up to five different routing algorithms for various G-Cells. They are: DR; shortest paths, computed by a breadth first search (BFS); PR with GP_E; PR with GP_I; and, PR with GP_0. Each routing algorithm (for a given DCN) is tested with the same 10,000 input pairs, (src, dst). The estimated standard error of the mean is computed by $s_x / \sqrt{\text{trials}}$, where $s_x$ is the sample standard deviation and trials = 10,000. For our purposes of surveying the effects of different instances of GP, this value is negligible, and we therefore omit error bars in Figs. 2–3.

For each algorithm we plot 100($\bar{x}_{DB} - \bar{x}$)/$\bar{x}_{DB}$ in Fig. 2, where $\bar{x}$ is the mean hop-length in the sample of computed routes. In other words, we plot the percent savings in hop-length over DR. Note that GP_0 is implicitly plotted for $k = 1$ because it is equivalent to GP_I in this case.

We also plot, in Fig. 3, the mean number of proxies considered by GP_I and GP_0 denoted $\bar{p}_1$ and $\bar{p}_0$, respectively, and the mean number of routes PR(src, dst) found to be no longer than DR(src, dst), denoted $\bar{r}_1$ and $\bar{r}_0$, respectively. Note that $\bar{p}_1 = \bar{p}_0$ for $k = 2$ and, as such, this value is implicitly plotted for $k = 2$ in Fig. 3.

The two histograms in Fig. 4 show the proportion of links with a given load (number of flows) in $\beta$-DCell_3,3, under 1 million one-to-one communications, generated uniformly at random; one histogram is for DR and the other one is for PR with GP_I.

The networks we tested are given with their basic properties in Table II, and the details of each version of GP* are given in Section V-A.

B. Evaluation

The plots in Fig. 2 show that for many G-Cell topologies, significant savings in hop-length can be made over dimensional routes by using proxy routes, depending on the connection rule, network size, and the parameters $k$ and $n$. It is immediate that GP_I and GP_0 retain some good proxies, in relation to GP_E, which tries all of them. Furthermore, GP_E is comparable to BFS. Fig. 3 tells us how much searching each of the methods GP_I and GP_0 must do, and how much path diversity they create, on average.

Note that the means plotted in Figs. 2–3 hide the success rate of PR in finding a good proxy path; as a typical example, PR(src, dst) is shorter than DR(src, dst) for approximately 30% of input pairs when using GP_I in DCell_3,6.

We highlight (and explain, where possible) some of the trends observable in the plot of Fig. 2: In general, proxy routes are more effective in $\beta$-DCell_k,n than in DCell_k,n and FiConn_k,n of comparable size, with fixed $k$, however, even FiConn_k,n still sees up to a 6–7% improvement.

The apparent weakness of PR in FiConn is partly explained by the fact that for given $k$ and $n$, there are fewer proxy FiConn_m,n,S to consider at level $m$. On the other hand we find that GP_0 considers fewer than $g_1 = 6$ proxies for FiConn_3,10, while it considers more than $g_1 = 7$ proxies for DCell_3,6 and $\beta$-DCell_3,6. In addition, there are an equal number of potential proxy candidates in $\beta$-DCell_k,n and DCell_k,n in general, yet GP_E, GP_I, and GP_0 invariably consider more proxy candidates for DCell_k,n, only to produce proxy paths that perform better in $\beta$-DCell_k,n. We must conclude that the connection rule and topology (FiConn vs Generalised DCell) profoundly impacts the performance of our proxy routing algorithms. This is somewhat unsurprising, however, since the connection rule and topology also affect the shortest paths; for example, the mean distance in $\beta$-DCell_3,3 is far shorter than in DCell_3,3 (see also [12]).

Proxy paths in larger networks (when increasing $n$) are worse than those in smaller networks, for each DCN with fixed $k$; for example DCell_3,3 and DCell_3,6, and also FiConn_3,10 and FiConn3,16.

A related trend appears to be that for each family of DCNs, proxy-path-savings increase with $k$, in every version of GP*; for example, FiConn3,10 and FiConn4,6. The main reason for this is that the performance of BFS, relative to DR, also increases with $k$, thus providing a greater margin for improvement by using PR.

The difference between GP_I and GP_0 grows with $k$ (note that for $k = 2$, they are the same, and hence GP_0 is not plotted for $k = 2$). This is because GP_I looks for sub-paths within a copy of $G$Cell_k-2,n, whereas GP_0 looks for sub-paths within a copy of $G$Cell_0,n, and as the gap between 0 and $k - 2$ increases, GP_I considers a larger set of proxy candidates. Similarly, we explain how the difference between GP_E and GP_I grows with $k$, but here it is the double exponential growth of G-Cell that contributes extra proxy candidates to GP_E, since the search space for GP_I is proportional to $g_{k-1}$, whereas, GP_E considers exactly $g_k$. 

<table>
<thead>
<tr>
<th>route \ c</th>
<th>a_3 &lt; a_2 &lt; b_4</th>
<th>a_3 &lt; c_3 &lt; b_4</th>
<th>a_3 &lt; b_3 &lt; c_3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[a, a']$ to $[b, c']$</td>
<td>$[a_3 = a_2]$</td>
<td>$[a_3 = c_3]$</td>
<td>$[a_3 = b_3]$</td>
</tr>
<tr>
<td>$[b, c']$ to $[b, c']$</td>
<td>$[a_3 = b_3]$</td>
<td>$[a_3 = c_3]$</td>
<td>$[a_3 = b_3]$</td>
</tr>
</tbody>
</table>

TABLE I PROPERTIES (2)-(4) APPLIED TO DCell_3,n.
proxy candidates (see Table II). Most notably, however, is the fact that for \( G_{\text{Cell}_{2,3}} \), the performance of \( \text{GP}_E \) is almost identical to the performance of \( \text{GP}_I \); whereas \( \text{DCell}_{2,4,3} \) has \( g_1 = 44 \), and \( g_2 = 1893 \), our results show that optimal proxies are nevertheless considered by \( \text{GP}_I \) (and hence, \( \text{GP}_0 \)).

Although \( \text{GP}^* \) is effective in computing shorter paths and comes fairly close to BFS (typically over 80% of the savings are obtained with PR), we can confirm that the shortest paths for these topologies are not, in general, a proxy route of the form we are considering in this paper as sometimes (e.g. (\( \beta \)-)\( \text{DCell}_{3,3} \)) this difference is considerable. This was expected, and provides motivation to explore novel general routing algorithms of order 3 and higher in future work.

Another benefit of proxy routing is that it also yields some path diversity which can be exploited for load balancing and fault-tolerance purposes. This can be seen in Fig. 3, where \( r \) is the number of distinct (but not necessarily disjoint) paths considered by \( \text{PR}(\text{src}, \text{dst}) \) that are no longer than \( \text{DR}(\text{src}, \text{dst}) \). Additional data must be studied, however, to determine exactly how \( r \) affects the load-balancing properties of the network.

We computed histograms that show the proportion of links with a given load, under 1 million one-to-one communications, plotted in Fig. 4. The histogram for \( \text{GP}_I \) is shifted left relative to the histogram for \( \text{DR} \), meaning that many links carry less load than in the same scenario for \( \text{DR} \). In addition, the maximum load is reduced (in our sample), suggesting many \( G_{\text{Cell}} \) uses have a higher aggregate bottleneck throughput (ABT, introduced in [10], and closely related to the most heavily loaded link in the network) with \( \text{PR} \) than with \( \text{DR} \).

Note that our primary focus is to reduce hop-length and implementation overheads of \( \text{GP} \), and that we could increase path diversity even more if we were willing to route on longer paths than \( \text{DR}(\text{src}, \text{dst}) \); we do not do this here, but will explore this possibility in future research.

### C. Significance

Various aspects of routing in a DCN depend heavily on the availability of short one-to-one paths. For example, minimising latency and energy usage, and building fault-tolerant and load balancing routing algorithms.

While there are inherent trade-offs in computing short proxy routes, there are also multiple benefits: using shorter one-to-one paths in a DCN reduces the average latency of communications, the aggregate load, and thereby the energy usage; and, we obtain a non-deterministic path diversity at no
extra cost while computing these paths, which can be used both adaptively or randomly to deal with faults and congestion, in addition to forming the building blocks of other fault-tolerant and load balancing routing algorithms (such as the way DR is used in DFR and TAR). As such, proxy routes are not only a good candidate for replacing DR in (Generalized) DCell and FiConn, they are also effective at performing some of the functions of the known adaptive routing algorithms for these networks, namely DFR and TAR, while simultaneously producing short paths.

VII. CONCLUSIONS AND FUTURE RESEARCH

In this paper we have shown that the topologies of the DCNs Generalized DCell and FiConn are completely connected recursively-defined networks. As such, we characterised all possible routes (with no repeated nodes) on these networks and then proposed the family of routing algorithms PR to compute proxy routes; that is, general routes of order 3. We detailed three instances of this family, GP_E, GP_I, and GP_0, where each one considers a number of candidate proxy sub-structures, and selects the optimal proxy to route through. We performed an analytical and empirical comparison between these, shortest paths, and the previously known dimensional routes, as regards mean hop-length; The main results of our experiments are that significant savings in hop-length can be made over dimensional routes by using proxy routes, even with only a relatively small set of candidate proxies, and that the amount of savings depends on connection rule, network size, and the parameters $k$ and $n$.

In future research we will perform a deeper analysis of the DCNs in question, with two major goals. The first one, motivated by the fact that GP_I sometimes discards the optimal proxy candidate, calls for a closer inspection of the topologies. We want to both find the optimal proxy candidates, and reduce the size of the search space. Furthermore, whereas this paper is focused on dimensional and proxy routing, there may be cases where no shortest path between two server-nodes is a dimensional route or a proxy route. Note that whilst a given shortest path may be found not to be a dimensional or proxy route, this does not preclude other paths with the same terminal nodes from being dimensional or proxy routes. A deeper mathematical analysis of the DCNs in question may shed light on (1) whether or not higher-order routing algorithms are needed, and (2) how to compute optimal routes of this type efficiently.

REFERENCES