Group Inequality and Conflict: A Simple Model

ABSTRACT

This paper presents a theoretical model to show how distributional concerns can engender social conflict. We have a two period model, where the cost of conflict is endogenous in the sense that parties involved have full control over the level of conflict they can create. Our analysis highlights the crucial role of future inequality plays. Thus equality of assets or income in the current period does not stop conflict from taking place if the anticipated future inequality is significant. Further we find that the impact of inequality on conflict is not straightforward. Since conflict is costly for both groups, societies with low levels of inequality, in our model, show no conflict. It is only when inequality increases beyond a threshold level, that groups engage in conflict. Additionally the model shows that the link between inequality and conflict may be non-monotonic.

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1 Introduction

This paper presents a simple model showing how distributional concerns can engender social conflict. We focus on the phenomena of intra-state conflict that has become common in recent years (Stewart et al. 2001). It is usually manifested in terms of widescale demonstrations, protests, strikes and sometimes violent rebellions, leading to severe disruption of economic activity.¹ This can weaken a country’s institutions and severely impede its economic progress. In fact, many of the states in the poorest regions of the world, have gone through serious intra-state conflict in the recent past. While it may be plausible that conflict may exacerbate the existing levels of poverty and inequality, a number of studies have demonstrated the opposite. Nafziger and Auvinen (2000) using an improved inequality data set and a broader definition of conflict find a strong link between inequality and war. Other studies such as Alesina and Perotti (1996), Cramer (2003), World Bank (2003), point to economic inequality as an important cause of conflict. Based on the World Values Survey, MacCullouch (2005), finds a robust positive link between higher inequality and the potential for conflict.²

There is a growing body of evidence which implies that more than inequality among households (or individuals), what matters for conflict is the inequality among groups. Using national surveys for developing countries, Ostby (2007, 2008) finds strong evidence that countries with high levels of systematic between-group inequalities in terms of household assets and education, does have a higher probability of an outbreak of civil war. More detailed case studies have also established the importance of group inequality in fostering conflict (Nafziger et al 2000; Stewart 2001, Stewart 2008). The emphasis on asset inequality does not in any way reduce the importance

¹See Nafziger et al. (2000) and Sachs (1989).
²Collier and Hoefler (2004, 2009) does not find any significant impact of inequality on conflict. However, they do not analyse group inequality, which is our main focus here. For further issues with the data and methodology in these papers refer to Cramer (2003), Nafziger and Auvinen (2002, p.156), Nathan (2005), Ward et al (2010).
of other factors, historical, ethnic or religious, in creating conflict. In fact our analysis presumes the polarization of a society into rival groups.\(^3\) How these groups are formed and the ensuing tensions between them are essential part of any description of conflict. We take these group formations as given.\(^4\)

In essence, therefore, this paper models the impact of group inequality, and in particular asset based group inequality on conflict. In mainly agrarian economies, for instance, land inequality closely reflects asset inequality and the distribution of land can be a source of discontent. In Central American countries, such as El Salvador and Guatemala, strong reliance on agro based exports led to an extremely disproportionate amount of land in the hands of a few rich and powerful interests. This resulted in serious conflict with those who have been dispossessed (Brockett, 1988). But inequality in assets is not just limited to land inequality. One of the important reasons for conflict in Angola and the D.R. Congo was for the control of the natural resources.\(^5\) The share (or the lack of share) of the different groups in these resources can be seen as the source of asset inequality.

To demonstrate how group inequality and conflict are interlinked, we use a two period game framework which is similar to Garfinkel and Skaperdas (2000) and Skaperdas and Syropoulous (1996).\(^6\) However, unlike those models, the groups here directly choose the level of conflict, rather than choosing between productive and defensive activities.\(^7\) Another difference

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\(^3\)Esteban and Ray (1999) discuss how the distribution of the population across different groups effect conflict. They find that conflict is the highest under a symmetric bimodal distribution, i.e. when the society is polarized. Empirical evidence of polarization (based on ethnic lines) leading to conflict has been reported by Matlova and Reynal-Querol, (2005).

\(^4\)For the dynamics of group formations see Garfinkel (2004a, 2004b).


\(^6\)We, therefore, broadly follow the choice theoretic approach. For other approaches to modelling conflict and inequality see Benabou (1996) and Somanathan (2002) among others.

\(^7\)Addison et al.(2003) and Benhabib and Rustichini (1996) also take a similar approach as ours.
with the previous papers lie in how the joint output is distributed. In standard choice theoretic models, the share of each group depends on the amount of resources the groups invest in enhancing their relative capability to capture a larger share of the output. In contrast, we presume an underlying social contract between the groups when it comes to the distribution of joint outputs. This contract may be arrived at through some bargaining process between the groups. In this sense our model is closer to Bannerjee and Duflo (2000) and Rodrik (1998). The shares of the groups, in our model, depend on the relative levels of wealth. If a group is relatively wealthy, then presumably it can have more leverage in the bargaining and thus be able to appropriate a larger share of the output. The current level of group wealth inequality is then reflected in a more skewed distribution of income between the groups in the future. Whilst Skaperdas and Syropoulos (1997) discusses distributional issues in the context of conflict, it is in a static framework. Also, unlike their model, ours does not allow conflict in the absence of inequality. In addition, one of the features of their model is that, groups with higher appropriative capabilities enjoy a larger share of the output. By specifying a stable social contract through the distribution rule, our model refrains from such an anarchic situation.

Yet we are able to demonstrate how group asset (wealth) inequality can tip a peaceful society to conflict. Since higher inequality leads to a more skewed distribution of the joint output, beyond a certain level of inequality the costs of engaging in conflict are less than the benefits of a higher share of the output resulting from the conflict. We proceed to show that even if wealth and income were equally shared, conflict may still arise, so long as there is a possibility of future inequality. Taking the analysis further, we argue that conflict-inequality link may not be linear and the

\footnote{In a similar context, Benhabib and Rustichini (1996) presents a dynamic model, but they also allow for conflict under perfect equality. Further, unlike ours, the groups in their paper do not incur any cost in the current period to initiate conflict.}
disadvantaged groups are not the only one to engage in conflict. At higher levels of inequality both the advantaged and the disadvantaged groups may engage in conflict which is what we often see when repressive measures are undertaken by the advantaged group (and in many case by governments aligned to the advantaged group). We also find that as inequality rises the potential increase in conflict may be high enough to act as an disincentive for groups to participate in production processes, the sharing of the output of which is the main source of conflict. We show that the link between inequality and conflict is non-monotonic.\footnote{Milante (2004) also finds a non-monotonic relation between wealth inequality and conflict. However the structure of the model and the general result differ significantly from ours.}

The plan of the paper is as follows. In the next section, we describe the basic structure of the model used in the paper including the production technology, the consumption decisions made by the groups, the social contract and the stages of the game between the groups. Section 3, we analyze in detail how future group inequality and current levels of conflict may be related. The following section discusses some extensions of the model and Section 5 concludes the paper with some discussion about the policy implications of our results. For the rest of the paper, inequality will imply wealth inequality between groups.

2 Model: Basic Framework

2.1 Production

Consider two groups, $i$ and $j$, involved in production of an output over two time periods, 1 and 2. The groups either decide to produce the output jointly or to produce on their own. In the beginning of period $t = 1, 2$, groups $i$ and $j$ have wealth $w_i^t$ and $w_j^t$ respectively and are also endowed with one unit of indivisible human capital. Let $h_i^n \in \{0, 1\}$ represent the
level of human capital used for joint production by any group \( m = i, j \).

Output under own production for group \( m \) is,

\[
Y^m_t = u^m_t (1 - h^m_t).
\]

Thus when effort in joint production \( h^m_t = 0 \), own output for group \( m \), will be \( u^m_t \). On the other hand if \( h^m_t = 1 \), \( Y^m_t = 0 \).

For the joint production case, we assume that the groups divide an exogenously given level of output say \( R_t \) in each period. Further, \( R_{t+1} \geq R_t \), that is in each period the joint production is at least as great as the previous period. The joint output is given by

\[
Y_t = R_t h^i_t h^j_t.
\]

When either \( h^i_t = 0 \) or \( h^j_t = 0 \), \( Y_t = 0 \). If joint output, if produced, is equal to \( R_t \). We would assume that \( R_t \gg w^i_t + w^j_t \), that is, the joint output is far greater than the combined total of each group’s own production. Wealth levels do not affect the joint output, but it does affect the level of own production.\(^{10} \) Both groups receive a part of the joint output according to some distribution rule, which is discussed next.

### 2.2 Social Contract

Social contract or the sharing rule is of crucial importance in any conflict model. This paper, will not be an exception in that regard. In the literature, the exogenous distribution rules (known as ‘contest success functions’) are represented by proportional sharing rules, with an emphasis on a winner-
takes-all feature.\textsuperscript{11} This type of sharing rules are appropriate in analyzing situations of war, where there is an element that the victor commands all the resources. However, most conflict that we see today, are intra-state conflict, be it peaceful protests or civil war. For such cases the winner-takes-all feature may not be appropriate, since the loser may still be receiving some share of the resources, albeit, a very small one. This feature is particularly desirable for conflict situations and not all distribution rules share that property (Hirshleifer, 1989).

In our model, similar to Banerjee and Duflo (2003), we propose an exogenous sharing rule for the joint output based on the fact that groups can choose to take part in the joint production, however, if any of them decide not to take part in the joint production, their fall back option is their own production. Keeping this aspect in mind, we propose the ‘split-the-difference’ sharing rule,

\begin{align}
    d_i^t &= Y_i^t + (1/2)(R_t - Y_i^t - Y_j^t), \\
    d_j^t &= Y_j^t + (1/2)(R_t - Y_i^t - Y_j^t),
\end{align}

where \(i\) and \(j\) is share of the joint output, given by \(d_i^t\) and \(d_j^t\), depends on the difference in the outputs from own production between the two groups (which in turn depends on the wealth levels).\textsuperscript{12} Equal levels of wealth, will result in equal distribution of the pie. We would assume that the share of the joint output that each groups receives is greater than their respective level of own production, that is, \(d_i^t > Y_i^t\) and \(d_j^t > Y_j^t\) for \(t = 1, 2\), which would incline the groups towards joint production. Note that both groups have equal bargaining power under this sharing rule, but more general rules


\textsuperscript{12}This is the same as the Nash Bargaining Solution with equal bargaining power, which has easy intuitive interpretations and strong axiomatic foundation (Muthoo, 1999). The own output levels act as the outside options.
can be used.

2.3 Conflict

While both the groups have some control over the production aspect (in the sense that they can choose between joint and own production), they have little control over the sharing rule of the joint output. In such a case, if group $i$ is unhappy with its share of the joint output, $d^i_t$, it can resort to conflict. This takes the form of destruction of the other groups share of the output.\footnote{It is important to note that conflict does not affect the sharing rule (as in Bannerjee and Duflo 2000).} There is, however, no direct appropriation of the opponents share. Our model, therefore, does not discuss looting.\footnote{Refer to Azam (2002) for a model that includes looting.} When one group indulges in conflict, it not only harms their opponent, but also adversely effects it’s own income, albeit not to the same extent.

Let $n^i_t$ and $n^j_t$ represent the level of conflict that group $i$ and $j$ respectively chooses in time $t$. In particular, $n^i_t$ is the proportion of destruction of group $j$’s share by group $i$ and similarly $n^j_t$ is the proportion of destruction of group $i$’s share by group $j$. The net income of the groups will be

\begin{align*}
y^i_t &= (1 - kn^i_t)(1 - n^j_t)d^i_t, \quad (5) \\
y^j_t &= (1 - n^i_t)(1 - kn^j_t)d^j_t, \quad (6)
\end{align*}

where $k < 1$ reflects limited self damage. For simplicity, for rest of the analysis, we assume the proportion of ‘self-damage’ $k = (1/2)$. Further, we assume that no group has the ability to destroy each others initial level of wealth. It may be that initial levels of wealth are better protected than their respective shares from the joint output. Hence if own production takes
place then net income of each group will be

\[ y^i_t = w^i_t \quad \text{and} \quad y^j_t = w^j_t. \]

(7)

To keep the focus on the future we assume that there is no wealth inequality to begin with. Thus \( w^i_1 = w^j_1 = w^0 \) hence

\[ y^i_1 = w^0 \quad \text{and} \quad y^j_1 = w^0 \]

The total amount of conflict in period \( t \) in the society, denoted by \( n_t \), should involve some aggregation of the level of conflict by both groups. Although, different aggregation rules are possible, in this paper we consider the ‘additive’ aggregation rule, where the total conflict is the sum of the level of conflict engaged in by each group.

\[ n_t = n^i_t + n^j_t. \]

(8)

2.4 Consumption and Savings

Both groups choose a level of consumption (and therefore a certain level of savings) and a level of conflict each period, to maximize the group’s lifetime utility. Since period 2 is the final period, there will be no savings and hence both groups will consume their total income in that period. The groups, however, have to incur a mobilization cost for engaging in conflict. Similar to Dixit (2004, p 41) we assume that the cost of mobilization increases at an increasing rate with the level of conflict. Any group \( m \), would maximize
the following,

\[ V^m(c^m_1, n_1, n_2) = V^m_1 + \rho V^m_2 = c^m_1 - \frac{1}{2}(n^m_1)^2d^m_1 + \rho[y^m_2 - \frac{1}{2}(n^m_2)^2d^m_2], \quad (9) \]

s.t. \( c^m_1 + s^m_1 = y^m_1, \)
\( c^m_1, n^m_1, n^m_2 \geq 0, \)

where \( c^m_1 \) and \( s^m_1 \) are the level of consumption and savings for group \( m \) in period 1, \( \rho < 1 \) is the discount factor. \( \left( \frac{1}{2}(n^m_i)^2d^m_i \right) \) captures the mobilization cost of conflict in period \( t \). For analytical tractability we will also assume that for both groups savings is proportional to the level of income, i.e. \( s^m_1 = \alpha y^m_1 \), where \( \alpha \leq (1/2) \).

2.5 Inequality

We define wealth \( I_t \) as the difference in wealth levels in period \( t \)

\[ I_t = \left| w^j_t - w^i_t \right| \]

Since \( w^j_1 = w^i_1 \) there is no wealth inequality in the initial period. Thus \( I_1 = 0 \).

Inequality in period 2 is \( I_2 = \left| w^j_2 - w^i_2 \right| \) where

\[ w^m_2 = \begin{cases} 
  r^m s^m_1 & \text{when } h^m_1 = 0 \\
  r^m(s^m_1 + w^0) & \text{otherwise}
\end{cases} \]

where \( r^m \) is the interest opportunities factor on the gross savings in period \( t \). These \( r^m \)'s refer to differential opportunities each group faces. For example, the interest

\[ \text{margin of future gain from saving outweighs the marginal loss of current consumption} \]
factors may well depend on people’s talents and abilities, or differential access to asset markets, or sheer good fortune. This heterogeneity will be the crucial element which will drive the conflict in this paper.

For most of the paper we will assume, without loss of generality, that group \( j \) is the fortunate (or the advantaged) group and group \( i \) is the unfortunate (or the disadvantaged) group i.e. \( r^j > r^i \). For sake of simplifying the analysis we shall assume \( r^i = 1 \), which means that the disadvantaged groups get no return on their savings.\(^{16}\)

Our interest in this paper is with the level of future inequality that groups anticipate before they engage in conflict. In other words, the ‘anticipated future inequality’ conveys the level of inequality when the status-quo is maintained. Therefore under the anticipated future inequality, \( n^i_1 = n^i_1 = 0 \) which implies \( s^1_i = s^1_j = \alpha(R_1/2) = s \) and thus

\[
I_2^n = (r^j - r^i)w_2, \text{ where } w = (s + w^0) \text{ and } r^i = 1.
\]

It is this notion of ‘anticipated future inequality’, based on which groups decide whether to undertake conflict or not.

### 2.6 The Game

We represent the interaction between the two groups as a game \( G \). Given that the distribution rule is fixed, \( G \) is a two period game with each period consisting of the following two stages:

Stage 1: Knowing the distribution, the groups can decide either to produce on their own (\( h^i_t = 0 \), or \( h^j_t = 0 \)), or to produce jointly (\( h^i_t = 1 \), and \( h^j_t = 1 \)).

Stage 2: If they decide to produce jointly, then each party decides on the

\(^{16}\)However, this is not a severe restriction. All the analysis below will go through so long as \( r^i \in [1, 2\gamma] \) where \( \gamma = \frac{(1-a_i)}{2} > 1 \). In otherwords, the return the disadvantaged group receives should be less than 100%.
level of conflict, that is, \((n_1^i, n_2^i)\) for group \(i\) and \((n_1^j, n_2^j)\) for group \(j\).

The strategy for each group is to choose in both periods whether to take part in the joint production and the level of conflict. Let \((n_1^{is}, n_2^{is})\) represent the equilibrium level of conflict and \(h_i^{is}\), and \(h_j^{is}\) represent the equilibrium human capital input of group \(i\) and \(j\) respectively for the joint output.

**Definition 1** A sub game perfect equilibrium is given by the quadruplet \((n_1^{is}, n_2^{is}, h_i^{is}(n_1^{is}, n_2^{is}), h_j^{is}(n_1^{is}, n_2^{is}))\), \(t = 1, 2\) such that each players choice is a best response to the other player and satisfies sequential rationality.

We shall use the backward induction approach to find the subgame perfect equilibrium of the game.\(^{17}\)

### 3 Future Inequality and Equilibrium Level of Current Conflict

In this section we demonstrate the role of future inequality in engendering conflict and investigate how conflict evolves with the changes in future inequality. We find that under certain restrictions on the parameters, for low levels of inequality, only the disadvantaged group engages in conflict in equilibrium. However, when levels of inequality are high, both groups engage in the conflict. Later we use these results to uncover the link between inequality and conflict.

As groups engage in conflict, the realised level of future inequality will differ from the anticipated level of future inequality prior to any conflict. This is because conflict will bring down the level of inequality by reducing the overall level of income and thus savings. Our interest in this section is with the level of future inequality that groups anticipate before they engage

\(^{17}\)We show in the Appendix (Proposition A1) the existence of a pure strategy equilibrium for game \(G\).
in conflict. First we show that in the most general case the groups will not engage in conflict in the final period irrespective of the level of inequality.

**Proposition 1** No group will engage in conflict in the last period.

Proof: Suppose both groups are engaged in joint production. Using (5) and (9), for group $j$, $V_j^2(n_j^i, n_j^j) = \max\{ (1 - k.n_j^i).d_j^i - \frac{1}{2}(n_j^i)^2.d_j^i \}$. Since any increase in $n_j^i$ will reduce $V_j^2$, group $j$ will not engage in conflict. Same will hold true for group $i$. Hence, $n_i^j = n_j^i = 0$. If one of the group decides to engage in own production, then by definition $n_i^j = n_j^i = 0$.

As there are no benefits from conflict in the last period, none of the groups engage in conflict. What about conflict in period 1? Consider group $i$. Given (5), (9) and $c_1^i = (1 - \alpha)y_1^i$, in period 1, group $i$ will choose $n_1^i$ such that it maximizes the following:

$$V^i = (1 - \alpha)(1 - kn_1^i)(1 - n_1^i)d_1^i - \frac{1}{2}(n_1^i)^2.d_1^i + \rho_1^i(R_2 + (s_1^i + w^0) - r^j(s_1^j + w^0))$$

where $d_1^i = d_1^j = \frac{R_1}{2}$ and $s_1^i = \alpha y_1^i$, $s_1^j = \alpha y_1^j$. The first order condition for group $i$ will be

$$\frac{\partial V_i^1}{\partial n_1^i} = -(1 - \alpha)(1 - n_1^i)kd_1^i - n_1^i d_1^i + \frac{\rho_1^i}{2}(r^j(1 - kn_1^i)d_1^i - (1 - n_1^i)kd_1^i) = 0.$$  

(11)

Similarly the first order condition for group $j$ will

$$\frac{\partial V_j^1}{\partial n_1^j} = -(1 - \alpha)(1 - n_1^j)kd_1^j - n_1^j d_1^j + \frac{\rho_1^j}{2}((1 - kn_1^j)d_1^j - r^j(1 - n_1^j)kd_1^j) = 0.$$  

(12)

The best-response functions of each group can be derived from their first order conditions. For group $i$, it will be (from (11)),

$$n_1^i = \frac{\rho_1^i}{2}[r^j(1 - kn_1^j) - k(1 - n_1^j)] - (1 - \alpha)k(1 - n_1^j).$$
This can be written as,

\[ n_i = A + Bn_j, \]  

(13)

where \( A = \left[ \frac{\rho \alpha}{2} \left( \frac{I_a}{w} + \frac{1}{2} - \gamma \right) \right] \) \( B = \left[ \frac{\rho \alpha}{2} \left( \gamma - \frac{I_a}{2w} \right) \right] \) and \( \gamma = \frac{(1-\alpha)}{\rho \alpha} > 1 \). \( A \) represents the amount of conflict group \( i \) will engage in when it initiates the conflict and \( B \) is the change in \( i \)'s level of conflict when group \( j \) changes its level of conflict.

Clearly whether \( B \leq 0 \), will depend on \( 2\gamma w \leq I_2 \). The intuition for the change in slope is the following. \( V^i \) is affected by \( n^i \) in mainly three ways: a negative effect on present consumption, a negative effect on future income through own savings and future wealth, and a positive effect on future income through other group's low savings and low future wealth. In addition, there is the direct cost of engaging in conflict. When inequality is sufficiently high \( (I_a > (2w\gamma - w)/2) \), the third effect can be sufficient to induce the disadvantaged group to initiate conflict. This is the one which depends on the level of inequality, the other two does not. Moreover, as \( n^j \) changes the marginal effect (first) is lower, that is, the marginal loss to current consumption is likely to be lower. The third positive effect also depends on \( n^j \) but because of the self damage factor, \( k \), the rate at which the marginal benefit depends is given by \( (1 - (n^j/2))I_2/w \). Hence when \( I_2 \) is not too large \( (I_2 < 2\gamma w) \), the first effect dominates in marginal terms and a high \( n^j \) leads to a high \( n^i \) (positive slope). For large values of \( I_2 \), the opposite is true, and a high \( n^j \) makes conflict less attractive to group \( i \).

Group \( j \)'s best-response function (from (12)) is,

\[ n_j = \frac{\rho \alpha}{2} [r^i(1 - kn_1^i) - r^j k(1 - n_1^i)] - (1 - \alpha)k(1 - n_1^i), \]

\[ n_j = C + Dn_1^i, \]  

(14)
where $C = \left[ \frac{\alpha}{2} \left( \frac{1}{2} - (\gamma + \frac{I_2}{2w}) \right) \right]$ and $D = \left[ \frac{\alpha}{2} \left( \gamma + \frac{I_2}{2w} \right) \right]$. Since group $j$ is the advantaged group, one can show $C < 0$ and $D > 0$.

In the following analysis, group $i$ and $j$’s best-response functions are depicted in Figures 1 to 4, for various parameter values.

### 3.1 Low Inequality and Conflict

When inequality level is low, such that $I_2^a \leq (1/2)(2\gamma w - w)$, then none of the groups will engage in conflict. Group $i$, the disadvantaged group, would not initiate conflict since the difference in inequality is not high enough to merit engaging in conflict, a part of the cost of which it has to bear. Since the disadvantaged group does not initiate conflict, the advantaged group does not engage in conflict in equilibrium either. When inequality is low, the best-response functions of each of the group (as in (13) and (14)) can be depicted as in the figure below:

Insert Figure 1

Since $I_2^a/w \leq (\gamma - 1/2)$ this implies that $A \leq 0$ from (13). From the best-response function of group $i$ we know when $n_i^1 = -A/B$, $n_i^1 = 0$. Hence we see a positive intercept for the best-response function of group $i$. The best-response function of group $j$ shows that for $n_j^1 = -C/D$, $n_j^1 = 0$. Thus the only equilibrium is at the point where $n_i^1 = 0$ and $n_j^1 = 0$. Therefore, total conflict under low inequality is $n^*_1 = n_{i^*} + n_{j^*} = 0$

To see whether groups will engage in joint or own production, first consider period 1. Since $n_{i^*} = n_{j^*} = 0$, the share of each group from the joint output will be $R_1/2$. On the other hand under own production they will get $w^0$. Thus groups will engage in joint production if $R_1/2 \geq w^0$. In period 2, from Proposition 1 we know there will be no conflict. The disadvantaged group will receive $(R_2 - I_2^a)/2$ where $I_2^a = (r^j - 1)w_2$, and own production for
both groups in the second period is \( w_2 = w^0 + \alpha(R_1/2) \). For joint production, therefore, \( (R_3 - I^a_2) > 2w_2 \), which inturn implies \( (R_2 - r^1 \alpha(R_1/2)) > w_2 \). Similarly, for the advantaged group the condition for joint production turns out to be \( (R_2 + I^a_2) > 2w_2 \).

The subgame perfect equilibrium of the game is given by the following proposition.

**Proposition 2** For the level of inequality \( I^a_2 \leq (1/2)(2\gamma w - w) \), if \( R_1/2 \geq w^0 \), and \( (R_2 - r^j \alpha(R_1/2)) > w_2 \), the subgame perfect equilibrium is \( (n^i_1 = 0, n^j = 0, h^i_1(n^i_1, n^j_1) = 1, h^j_1(n^i_1, n^j_1) = 1) \) and \( (n^i_2 = 0, n^j_2 = 0, h^i_2(n^i_2, n^j_2) = 1, h^j_2(n^i_2, n^j_2) = 1) \).

Note that \( (R_2 - r^j \alpha(R_1/2)) > w_2 \) ensures that in the second period the groups will always opt for joint production and we assume that it holds for the rest of the analysis. This is because the worst second period distribution that the disadvantaged group can expect will be when the joint production is divided based on wealth in period 2 \( (w_2) \) for each of the groups. If under such situation the disadvantaged groups still decides to engage in joint production in period 2, then the group will obviously engage in joint production when the distribution improves in the second period, which will be the case under conflict, dealt below.

### 3.2 Medium Inequality and Conflict

Next consider the level of inequality, \( I^a_2 \), such that \( (1/2)(2\gamma w - w) < I^a_2 < \hat{I}^a_2 \). (We define \( \hat{I}^a_2 \) later; \( I^a_2 > \hat{I}^a_2 \) would represent high inequality.) We split the discussion of medium inequality in to two cases: (a) \( (1/2)(2\gamma w - w) < I^a_2 \leq 2\gamma w \), and (b) \( 2\gamma w < I^a_2 \leq \hat{I}^a_2 \).

When \( (1/2)(2\gamma w - w) < I^a_2 < 2\gamma w \), the best response functions of the groups are shown in the diagram below.

Insert Figure 2.
The best response function of group $i$ (13) translates to an intercept $A$ with gradient $(1/B)$ in Figure 2. Given the bounds on the level of inequality, it is easy to establish that $0 < A \leq 1$ and $0 < B < 1$. Similarly, the best response function of group $j$ (14), has intercept $0 < (-C/D) < 1$ where $C < 0$ and gradient $D < 1$. Notice that in the presence of non-negativity constraints on levels of conflict, $C < 0$ implies that the best response function for group $j$ extends to the origin, with a kink at $n_1^j = (-C/D)$. Further one can show that, given $\gamma > 1$, $(-C/D) > A$.\textsuperscript{18}

Next consider the case where $2\gamma w < I_a^2 \leq \hat{I}_a^2$. The implication of $I_a^2 > 2\gamma w$ is that the slope of the group $i$ reaction function now becomes negative. So beyond this point, if the advantaged group engages in conflict, the disadvantaged group will reduce its level of conflict. Figure 3 shows the reaction functions of the two groups under this situation.

Insert Figure 3.

From Figure 2 and 3, it becomes clear, that in case of joint production, $(n_1^{j*} = A, n_1^{i*} = 0)$ is the equilibrium level of conflict. Group $j$, the advantaged group, does not engage in conflict. The intuition is simple. $(-C/D)$ reflects the level of conflict engaged by group $i$ that will be tolerated by group $j$. Hence, so long as the level of conflict (which is group $i$ intercept term $A$) is less than $(-C/D)$, group $j$ shall not engage in conflict.

If there is joint production, the overall level of conflict will be

$$ n_1^i = n_1^{i*} + n_1^{j*} = \left[ \frac{\rho \alpha}{2} \left( \frac{I_a^2}{w} + \frac{1}{2} - \gamma \right) \right]. \quad (15) $$

Differentiating with respect to $I_a^2$ we get, $\partial n_1^i / \partial I_a^2 = \rho \alpha / 2w > 0$, i.e. as the level of future inequality increases, overall conflict will also be on the rise.

\textsuperscript{18}Also note that $D < 1 < (1/B)$ i.e. group is reaction function is steeper than group $j$s. This reflects the fact that group $j$ has more to loose by escalating the conflict and hence would increase its own level of conflict at a lower rate than group $i$. 

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However, whether both the groups will decide for joint production or not depends on their initial level of wealth. We continue to assume that in the second period groups engage in joint production. In period 1, the disadvantaged group under own production will consume \((1 - \alpha)w^0\) and under joint production in equilibrium will engage in conflict i.e. \(n_1^* = A\) and thus from (9) will consume \((1 - \alpha)(1 - (A/2) - (A^2/2))(R_1/2)\). Therefore, the sufficient condition under which the disadvantaged group will participate in joint production is \((1 - (A/2) - (A^2/2))R_1 > 2w\). For the advantaged the condition will be \((1 - A)R_1 > 2w\). Since \(A < 1\), the equilibrium can be characterized as follows:

**Proposition 3** Given \((1/2)(2\gamma w - w) < I_2^s \leq \hat{I}_2^s\), and \((1 - A)R_1 > 2w\), the subgame perfect equilibrium is \((n_1^{s*} = A, n_1^{j*} = 0, h_1^{s*}(n_1^{is}, n_1^{js}) = 1, h_1^{j*}(n_1^{is}, n_1^{js}) = 1)\).

In the second period there will be no conflict as earlier we continue to assume that the conditions of both groups engaging in joint production are met. Here while one of the group engages in conflict, the other refrains from conflict. This is unlike Benhabib and Rustichini (1996) and Skaperdas and Syropoulous (1997), where both groups always end up engaging in conflict, although only one group might have started it.

### 3.3 High Inequality and Conflict.

Now consider the case where \(\hat{I}_2^s < I_2^s \leq \hat{T}_2\).\(^{19}\) \(\hat{I}_2^s\) represents the level of inequality such that \((-C/D) = A\).\(^{20}\) The best response functions for both groups would now be the following:

> Insert Figure 4.

\(^{19}\hat{T}_2\) (the maximum level of inequality) is the level of inequality such that \(\max(n_1^{is}, n_1^{js}) = 1\).

\(^{20}\)It is shown in the Appendix (Proposition A2), there exists a level of inequality \(\hat{I}_2\) such that \((-C/D) = A\).
If there is joint production, then the equilibrium levels of conflict for both groups (from the best response functions (13) and (14)) are

\begin{align*}
n_i^* & = \frac{A + BC}{(1 - BD)}; \\
n_j^* & = \frac{AD + C}{(1 - BD)}; \tag{16} \tag{17}
\end{align*}

where \(C < 0\) and \(B < 0\). Since \((-C/D) < A\) and \(B < A\), we can be sure that \(n_i^* > 0\) and \(n_j^* > 0\). At higher levels of inequality, the level of conflict initiated by group \(i\), is greater than what group \(j\) can tolerate, that is, \(A > (-C/D)\). Hence group \(j\) engages in conflict to counter the conflict initiated by group \(i\). One can easily check that \(0 < n_i^* \leq A \leq 1\) and \(0 < n_j^* \leq 1\). The overall level of conflict will be the total of (16) and (17) i.e.

\[ n^*_1 = n_i^* + n_j^* = \frac{A + (-B)(-C) + AD - (-C)}{1 + (-B)D}. \]

In the Appendix (Proposition A3) we show that \((\partial n^*_1 / \partial I^a_2) > 0\). This means that as inequality increases further, the level of conflict also increases. Note, here the disadvantaged group reduces its own level of conflict. Since in this case \((-B) < 1\), the decrease of conflict by the disadvantaged group is more than made up by the increase in the advantaged groups conflict. Therefore, the overall level of conflict increases, by more than it would have, under the increased level of inequality if the advantaged group did not join in.

On the question of joint or own production under high inequality, it can be shown that when group reaches \(I^a_2\), they prefer own production (see Appendix, Proposition A4). This is because the excessive level of inequality leads to such a high level of conflict thereby reducing the net income of the groups from joint production to such a level that own production becomes a better alternative. Since both groups engage in joint production at \(\tilde{I}^a_2\) but decide for own production at \(\tilde{T}^a_2\), there must exist some \(I^a_2 \in (\tilde{I}^a_2, \tilde{T}^a_2)\) such that
min \left[V_S^j - V_S^i, V_S^j - V_S^i\right] = 0,

where for any group \(m\), \(V_S^m\) and \(V_J^m\) represents its total benefit from own production and joint production respectively. This condition shows the level of inequality in which at least one of the group will be indifferent between joint production and own production.

We therefore discuss the possibility of two cases: (a) \(\tilde{I}_2^i < I_2^o < \tilde{I}_2^i\) and (b) \(\tilde{I}_2^j \leq I_2^o \leq \tilde{I}_2^j\). When \(\tilde{I}_2^i < I_2^o < \tilde{I}_2^i\) groups will continue to be in joint production and the equilibrium will be as given next.

**Proposition 4** Given \(\tilde{I}_2^i < I_2^o < \tilde{I}_2^i\), the subgame perfect equilibrium is 
\[(n_1^i > 0, n_1^j > 0, h_1^i(n_1^i, n_1^j) = 1, h_1^j(n_1^i, n_1^j) = 1)\].

However, when \(\tilde{I}_2^j \leq I_2^o \leq \tilde{I}_2^j\), clearly either group \(i\) or group \(j\) drops out of joint production and since in our model own wealth is indestructible, we get the following equilibrium.

**Proposition 5** Given \(\tilde{I}_2^j \leq I_2^o \leq \tilde{I}_2^j\), the subgame perfect equilibrium is 
\[(n_1^i = 0, n_1^j = 0, h_1^i(n_1^i, n_1^j) = 0, h_1^j(n_1^i, n_1^j) = 0)\].

The above proposition shows that under some circumstances there will be no joint production. Hence, unlike other cases, although ex-ante there is a possibility of conflict, ex-post no conflict will take place. As earlier, in both these cases, in the second period there is no conflict and groups engage in joint production.

### 3.4 Inequality and Total Conflict

So where does all this leave us when it comes to the question about the link between inequality and conflict? As is clear from the above discussion that until \(I_2^o\), there will be no conflict, since inequality is low. However, beyond \(I_2^o\), we know there is a positive amount of conflict since the disadvantaged
group engages in conflict. Conflict now increases steadily with increase with inequality until $I_2^a$. Then from $I_2^a$ onwards both groups are engaged in conflict and the overall level of conflict also increases. Now as inequality increases, conflict again steadily rises until it reaches $I_2^a$. At $I_2^a$, for group $i$, high levels of conflict makes joint production inviable. This is captured in the diagram below.

Insert Figure 5.

Therefore one can state the following proposition.

**Proposition 6** *The relationship between inequality and conflict is non-monotonic.*

We would like to emphasize that the non-monotonicity in our model results from a sharp change in the level of conflict arising out groups preferring own production beyond a certain level of inequality. Although, Milante (2004) also finds a non-monotonic relationship, unlike ours, this is reflected in an inverted-U relationship between inequality and conflict. Hence, in his model, over a certain level of inequality, there is a gradual decrease of conflict as inequality rises.

4 Discussions

In this section we discuss changes to some assumptions so far made in this model and how they impact the results. In particular we deal with four of the assumptions: (a) the rate of savings are the same for both the groups, (b) the proportion of ‘self damage’ is equal for both groups, (c) that groups have perfect foresight and (d) the absence of any fixed costs.

*Rate of savings.* Suppose instead of having the same savings rate, consider without loss of generality, that $\alpha^i < \alpha^j$. Further assume that $r^j = r^i = 1$. This would mean that $w_2^j < w_2^i$, and therefore from the distribution rule it would be obvious that $y_2^j < y_2^i$. Group $i$ again is the
disadvantaged group. The rest of the analysis will follow through, so long as now our inequality measured the difference between the two savings rate, i.e. $I_2 = (\alpha^j - \alpha^i)w$ where $w = R_1/2$. Along with this if we had assumed that $r^j > 1$ the results in the previous sections will only be amplified. However, if $\alpha^i > \alpha^j$ and at the same time $r^j > 1$, the results derived in the earlier sections will now depend on which of these has greater impact. Obviously, since the relative rate of return and the relative rate of savings are going in opposite direction, the results in the earlier sections will be dampened. Since we were interested in understanding the impact of inequality on conflict, distilling all else, we had assumed $\alpha^i = \alpha^j$.

Proportion of ‘self damage’. Thus far we have assumed that the proportion of self damage, $k$, is the same for all the groups and $k = (1/2)$. As mentioned earlier, for $0 < k < 1$, all the results derived earlier will hold. Here we shall discuss a few cases when $k$ takes extreme values and when the $k$ varies between groups.

First, when $k = 0$ for both groups, the reaction function of group $i$ and $j$ are, respectively, (derived from (13) and (14)) $n_1^i = (\rho\alpha/2)r^j > 0$ and $n_1^j = (\rho\alpha/2) > 0$. Clearly, now both groups will engage in conflict irrespective of the level of inequality and the level of conflict will depend on the rate of return of the rival group. This is not surprising, since $k > 0$ makes it costly for groups to engage in conflict by reducing both their current and future levels of consumption. The overall level of conflict will be higher now.

Next, let $k = 1$ for both the groups. Recall that the way conflict works in this model is that under high inequality, the disadvantaged group wants to reduce the amount of income devoted to savings by the advantaged group so that even with a relatively higher return, the advantaged group does not receive a higher level of the output in the future. Now with $k = 1$, this will be extremely costly. Under this assumption, so long as $r^j > 1$, from (13)
and (14) the reactions functions of group \( i \) and \( j \) will be

\[
\begin{align*}
n^i_1 &= \frac{\rho \alpha}{2w} (I^i_2 - 2\gamma w) \left( 1 - n^j_1 \right), \\
n^j_1 &= -\frac{\rho \alpha}{2w} (I^j_2 + 2\gamma w) \left( 1 - n^i_1 \right).
\end{align*}
\]

Thus, group \( i \), the disadvantaged group will be the only group involved in conflict and that too when \( I^i_2 > 2\gamma w \). Group \( j \), irrespective of the level on inequality and group \( i \) level of conflict, will not engage in conflict. It is easy to see if the level of self damage of group \( i \) is, \( k^i = 0 \) and of group \( j \) is, \( k^j = 1 \), then the earlier result will be just amplified in the sense that now group \( i \) will engage in conflict irrespective of the level of inequality and group \( j \) will never engage in conflict. On the other hand, if \( k^i = 1 \) and \( k^j = 0 \), group \( j \) will always engage in conflict and group \( i \) will engage in conflict only when inequality is high, i.e. \( I^i_2 > 2\gamma w \). In this situation, unlike the standard results, it will be the advantaged group which will engage in conflict.

**Information.** Our model assumes that groups have perfect foresight. Hence they can anticipate future inequalities perfectly. This, however, is not very realistic. One way to bring in imperfect information in the model would be to assume that both the groups know the distributions of \( r^j \) and \( r^i \). In that case the anticipated future inequality will then be given by \( I^a_2 = (E(r^j) - E(r^i))w \), where \( E(r) \) is the expected rate of return. Thus the conditions under which groups will initiate conflict, will remain the same except for inequality being interpreted as expected anticipated future inequality. Hence, all the results that we have discussed earlier will also go through for a case of imperfect foresight. In the event of complete uncertainty, however, the analysis will be more complex and will depend on the groups behaviour. If, for instance, the groups presume that the rate of returns are going to be the same, then obviously there will be no reason for
conflict arising from future inequality.

*Fixed Cost.* Cost of mobilization plays an important role in this model. Without it, groups would always engage in conflict. We have considered cost of conflict entering the model in two ways. First, a group engaging in conflict will also inflict some damage to their own share of output and second that there is a mobilization cost of conflict. Both of these costs are contingent on the level of conflict. Groups, however, will usually face fixed costs if they decide to engage in conflict. These costs may reflect among others the costs involved in forming the groups, the minimum physical infrastructure that may be needed to run a conflict. Boix et al (2006) argues that any group engaging in conflict will face both fixed and variable costs. Suppose that the mobilization of the cost of conflict includes a fixed cost \( F \). Therefore, group \( i \), will maximise the following

\[
\tilde{V}^i = \begin{cases} 
V^i - F & \text{if } n_i^i > 0 \\
V^i & \text{otherwise}
\end{cases}
\]

where \( V^i \) is based on equation (10). Group \( j \)'s objective function will be similarly changed in the presence of fixed costs.

Fixed costs will not change any of the equilibrium condition, hence the threshold inequality levels at which the groups start engaging in conflict remains unchanged. The level of conflict, however, will increase taking in to account the fixed cost. It is clear from Figure 5 above, that until \( I^a_2 \), there will be no conflict due to low inequality. Beyond \( I^a_2 \), however, there is a positive amount of conflict by the disadvantaged group. Since to engage in conflict the groups have to incur a fixed cost, we will find a discontinuous jump in the level of conflict at \( I^a_2 \). Similarly we will find another discontinuous jump at \( \tilde{I}^a_2 \), this time due the advantaged group engaging in conflict. The discontinuity between inequality and conflict will now be at three levels of inequality: \( I^a_2, \tilde{I}^a_2 \) and \( \tilde{I}^a_2 \). Thus around each of these levels, there will be
sharp changes in the level of conflict. Thus, there may be cases with similar levels of anticipated future inequality but very different levels of conflict.

5 Conclusion

The purpose of the paper was to analyze the interlinkages between group inequality and conflict. In our analysis we find that although inequality may cause conflict, the impact of inequality on conflict is not straightforward. Since conflict is costly for both groups, societies with low levels of inequality, in our model, show no conflict. It is only when inequality increases beyond a threshold, that the disadvantaged group engages in conflict. At higher levels of inequality both groups engage in conflict. Thus, our model is able to capture both rebellion by the disadvantaged group and also the suppression by the advantaged group. El Salvador and Guatemala are examples where the state acting on behalf of the advantaged group unleashed severe repression to curb insurgencies. When inequality reaches extreme levels, the economy goes back to subsistence levels as the high output joint production sector is not developed for fear of severe rebellion. For instance, the Bougainville rebellion, arising out of a concern for the local environment and the lack of benefits to the local populace, led to the closure of copper mines, thus leading to a decline in the income of the region.²¹ It is important to note that the traditional sense of ‘greivance’ is absent in this model since both groups have same level of income and wealth in the period in which conflict occurs. Groups, however, anticipate future levels of inequality which may precipitate conflict in the current period.

Our analysis demonstrates the crucial role future inequality plays. Current inequality will not necessarily lead to conflict if in the future there is less inequality. On the other hand, current equality does not stop conflict

from taking place if the future inequality is significant. In Sri Lanka, only when the government failed to guarantee the rights of Tamils (and also curtailed their access to higher education), did the Tamil insurgency begin in earnest. The government policies were seen as a potential source of future inequality where the Tamils would loose out significantly.

This brings us to the policy implications of our results. Since the future plays an important role in fostering conflict, one has to put in place policies that will reduce future inequality. For example, the warring factions in Sudan have now decided to split future profits from the oil wells equally. If such egalitarian rules can be institutionalized and implemented, then reasons for conflict will definitely diminish. However, typically if one of the groups becomes ‘weaker’ (maybe due to exogenous shocks) in terms of bargaining, the stronger groups tend to capture a higher share of the joint output and that is when the problems start again. This may explain why so many peace agreements fail. What is implicit here is that enforceable contracts are not viable and therefore parties cannot forge some kind of ex-ante contract to avoid conflict. If, however, we allow for long term interaction between the groups, there may be a possibility of overcoming the incomplete contract problem. What the structure will be of such long term contracts under uncertainty is an issue for future research.

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24Infact the current hostilities in Sudan started after the discovery of oil in the south, which none of the parties were aware of when signing the Addis Ababa peace deal in 1972 (Human Rights Watch, 2003).

25For a very interesting application of contract theory to conflict refer to Azam and Mesnard (2003).
References


A Appendix

First we show the existence of equilibrium in the game $G$. The proof is constructed using standard arguments found in game theory texts such as Fudenberg and Tirole (1991).

**Proposition A.1** There exists a pure strategy equilibrium in $G$.

Proof: First let us start with stage 2 game in period 2. The strategies for group $i$ and $j$, $n^i_t \in [0,1]$ and $n^j_t \in [0,1]$ are compact for $t = 1,2$. Since period 1 payoff is already known for group $j$, and the total income is consumed in the last period, the period 2 payoff, from (9) is

$$V^j_2(n^i_2, n^j_2) = \begin{cases} 
\max\{(1-k.n^i_2).d^m_2 - \frac{1}{2}.(n^j_2)^2.d^l_2\} & \text{if } h^i_2 = 1, h^j_2 = 1, \\
w^j_2 & \text{otherwise.}
\end{cases}$$

$V^j_2(n^j_2)$ is continuous and quasi-concave in $n^j_2$. Similarly $V^i_2(n^i_2, n^j_2)$ is continuous and quasi-concave in $n^i_2$. From Theorem 1.2 of Fudenberg and Tirole (1991), we know there exists a pure strategy equilibrium $(n^{i*}_2, n^{j*}_2)$ in stage 2 of period 2. In Stage 1, since $Y_t = 0$, when $h^i_1 = 0$ or $h^j_1 = 0$, the groups will either both choose own production, i.e. $(h^i_1 = 0$ and $h^j_1 = 0)$, or both will choose joint production i.e. $(h^i_1 = 1$ and $h^j_1 = 1$). Whether $(h^i_2 = 0, h^j_2 = 0)$ or $(h^i_2 = 1, h^j_2 = 1)$ will depend on $(n^{i*}_2, n^{j*}_2)$. Therefore the subgame perfect equilibrium would be $(h^{i*}_2 = 0, h^{j*}_2 = 0, n^{i*}_2, n^{j*}_2)$ or $(h^{j*}_1 = 1, h^{j*}_1 = 1, n^{i*}_2, n^{j*}_2)$. Hence, there exists a pure strategy equilibrium in period 2. Let that equilibrium level of payoff for group $j$ in period 2 be $V^j_2$. Payoff in stage 2 of period 1 for group $j$ can be then written as

$$V^j(n^i_1, n^j_1) = \begin{cases} 
(1-\alpha)(1-\alpha k^j_1)(1-n^i_1)d^m_1 - \frac{1}{2} (n^j_1)^2.d^l_1 + \rho V^j_2 & \text{if } h^i_1 = 1, h^j_1 = 1, \\
w^j_1 & \text{otherwise,}
\end{cases}$$

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which is continuous and quasi-concave in \( n_1^i \). Through similar argument as above we can show \( V^i(n_1^i, n_1^j) \) is continuous and quasi-concave in \( n_1^i \). Therefore, from Theorem 1.2 of Fudenberg and Tirole (1991), there exists a pure strategy equilibrium in \((n_1^i, n_1^j)\) stage 2 of period 1. Using similar arguments as earlier, we can deduce that the subgame perfect equilibrium in period 1 would be \((h_1^i = 0, h_1^j = 0, n_1^i, n_1^j)\) or \((h_1^i = 1, h_1^j = 1, n_1^i, n_1^j)\). Since both period 1 and 2 have pure strategy equilibriums, the game \( G \) will also have a pure strategy equilibrium. ■

Next we formally show the existence of a level of inequality, which clearly demarcates high inequality from medium or low inequality in our model. Referring back to Figure 2, \( A \) is the intercept term of group \( i \)’s reaction function and \((-C/D)\) is group \( j \)’s, both of which are dependent on the level of inequality. We define the lower bound of the high inequality interval as the level of inequality at which \((-C/D) = A\).

**Proposition A.2** There exists a level of inequality, \( \hat{I}_2 \), where \((-C/D) = A\).

Proof: Let \( f = ((-C/D) - A) \). Further,

\[
\frac{\partial(-C/D)}{\partial I_2} = \frac{w}{(2\gamma w + I_2^a)^2} \quad \text{and} \quad \frac{\partial A}{\partial I_2} = \frac{\rho \alpha}{2w} = \frac{1 - \alpha}{2\gamma w}. \tag{A1}
\]

Hence,

\[
\frac{\partial f}{\partial I_2} = w \left[ \frac{1}{(2\gamma w + I_2^a)^2} - \frac{1 - \alpha}{2\gamma w^2} \right] < 0 \text{ for } I_2 \geq 0 \text{ and } 0 < \alpha < (1/2).
\]

We know that for \( I_2^a \leq 2\gamma w \), \((-C/D) > A\), which implies that at \( I_2 = 2\gamma w, \ f > 0 \). Now consider the level of inequality \( I_2^a \) such that \( A = 1 \). At this level \( I_2^a > 2\gamma w \), and \( A = 1 > (-C/D) \) (since \( D > (-C) \) for all \( I_2^a \)). Hence for \( I_2^a = \hat{I}_2^a, \ f < 0 \). Therefore, by the Intermediate Value Theorem we can find an \( \hat{I}_2 \in (2\gamma w, \hat{I}_2^a) \) such that at \( \hat{I}_2, \ f = 0 \), implying \((-C/D) = A\). Further since \( \partial f/\partial I_2^a < 0 \) for all \( I_2^a \geq 2\gamma w \), \( \hat{I}_2 \) will be unique. ■
Third, we demonstrate that under high inequality, the total level of conflict will increase with inequality. Recall that in this case the disadvantaged group reduces its level of conflict and the advantaged group increases its level of conflict, with increase in inequality.

**Proposition A.3** For all $I_a^2 > \hat{I}_2^a$, $(\partial n_1 / \partial I_2^a) > 0$.

Proof: Differentiating both group’s best response functions (i.e. (13) and (14)) with respect to $I$ we get

\[
\frac{\partial n_1^i}{\partial I_2^a} = \frac{\partial A}{\partial I_2^a} - \frac{\partial(-B)}{\partial I_2^a} n_1^j - (-B) \frac{\partial n_1^j}{\partial I_2^a},
\]

\[
\frac{\partial n_1^j}{\partial I_2^a} = - \frac{\partial(-C)}{\partial I_2^a} + \frac{\partial D}{\partial I_2^a} n_1^i + D \frac{\partial n_1^i}{\partial I_2^a}.
\]

Solving these for group $i$ we get,

\[
(1 + (-B)D) \frac{\partial n_1^i}{\partial I_2^a} = \frac{\partial A}{\partial I_2^a} - \frac{\partial(-B)}{\partial I_2^a} n_1^j - (-B) \frac{\partial(-C)}{\partial I_2^a} + (-B) \frac{\partial D}{\partial I_2^a} n_1^i.
\]

Noting that $n_1^j \leq 1$; $\frac{\partial A}{\partial I_2^a} > \frac{\partial(-B)}{\partial I_2^a} > 0$ and $\frac{\partial(-C)}{\partial I_2^a} > 0$; $\frac{\partial D}{\partial I_2^a} > 0$, the above equation implies $\frac{\partial n_1^i}{\partial I_2^a} > 0$. Similarly the result will hold for group $j$. Since both $\frac{\partial n_1^i}{\partial I_2^a} > 0$ and $\frac{\partial n_1^j}{\partial I_2^a} > 0$, we can conclude $\frac{\partial n_1}{\partial I_2^a} > 0$. □

Finally, we show that when inequality level becomes excessive, this would lead to own production instead of joint production.

**Proposition A.4** When inequality is $\bar{T}_2^a$, groups will choose own production over joint production.

Proof: For the high inequality case, whether $n_1^i* > n_1^i$ or $n_1^j* < n_1^i*$ depends on parametric specifications. Let us consider the case where $n_1^i* > n_1^j*$. Since by definition, at $\bar{T}_2^a$, $\max(n_1^i*, n_1^j*) = 1$, this implies that at $\bar{T}_2^a$, $n_1^i* = 1$ and from (17), $n_1^i* = (\rho, \alpha/4)$. Using (5) and (9), group is payoff
from joint production then will be

$$(1 - \alpha)(1 - \frac{\rho \alpha}{4}) \frac{R_1}{4} - \frac{R_1}{4} + (\rho/2)(R_2 + \alpha(1 - \frac{\rho \alpha}{4}) \frac{R_1}{4} + (1 - r^j)w^0). \quad (A2)$$

On the other hand group is payoff under own production will be

$$(1 - \alpha)w^0 + (\rho/2)(R_2 + (1 - r^j)\alpha w^0). \quad (A3)$$

Subtracting (A3) from (A2) and rearranging terms we get

$$-\left(1 - (1 - \frac{\rho \alpha}{4})((1 - \alpha) + \frac{\rho \alpha}{2}) \frac{R_1}{4} - \left((1 - \alpha) - \frac{\rho \alpha}{2}\right) w^0 - r^j P \frac{1 - \alpha \alpha}{2} w^0 < 0,$$

since, $(1 - \alpha) \geq \alpha > 0$ and $0 < \rho < 1$. Therefore group $i$ will drop out of joint production before inequality reaches $T^i_2$, which would also imply from (2) that group $j$ will also not engage in joint production. Similarly one can also show that when $n_1^j > n_i^*$, and at $T^j$, $n_1^j = 1$, group $j$ will prefer own production to joint production and therefore group $i$ will also not engage in joint production. ■
Figure 1: Reaction functions of both groups under low inequality where $A<0$ and $B>0$. 
Figure 2: Reaction functions of both groups under medium inequality where $A > 0$ and $B > 0$. 
Figure 3: Reaction functions of both groups under medium inequality where group $i$’s reaction function has a negative slope ($B < 0$).
Figure 4: Pure strategy equilibrium under high inequality.
Figure 5: Link between inequality and conflict under the additive aggregation rule.