Lattice Hamiltonian approach to the Schwinger model

Link to publication record in Manchester Research Explorer

Citation for published version (APA):

Published in:
host publication

Citing this paper
Please note that where the full-text provided on Manchester Research Explorer is the Author Accepted Manuscript or Proof version this may differ from the final Published version. If citing, it is advised that you check and use the publisher's definitive version.

General rights
Copyright and moral rights for the publications made accessible in the Research Explorer are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

Takedown policy
If you believe that this document breaches copyright please refer to the University of Manchester's Takedown Procedures [http://man.ac.uk/04Y6Bo] or contact uml.scholarlycommunications@manchester.ac.uk providing relevant details, so we can investigate your claim.

Download date:21. Jan. 2020
Lattice Hamiltonian approach to the Schwinger model

K. Cichy\textsuperscript{1,2}, A. Kujawa-Cichy\textsuperscript{3}, M. Szyniszewski\textsuperscript{4,5}

\textsuperscript{1}NIC, DESY, Platanenallee 6, D-15738 Zeuthen, Germany
\textsuperscript{2}Department of Physics, Adam Mickiewicz University, 61-614 Poznań, Poland
\textsuperscript{3}Institut für Theoretische Physik, Goethe-Universität, 60438 Frankfurt am Main, Germany
\textsuperscript{4}Physics Department, Lancaster University, Lancaster, LA1 4YB, UK
\textsuperscript{5}NoWNano DTC, University of Manchester, Manchester, M13 9PL, UK
Outline

1. The Schwinger model on lattice
2. Strong coupling expansion (SCE)
3. Ground state energy
4. Mass gaps
5. Chiral condensate
6. Oscillations of chiral condensate
7. Summary & outlook
The Schwinger model

Hamiltonian of the Schwinger model in the Kogut-Susskind staggered discretization [1,2]:

\[ H = -\frac{i}{2a} \sum_{n=1}^{M} \left( \phi^+(n)e^{i\theta(n)}\phi(n+1) - \phi^+(n+1)e^{-i\theta(n)}\phi(n) \right) \]

\[ + m \sum_{n=1}^{M} (-1)^n \phi^+(n)\phi(n) + \frac{ag^2}{2} \sum_{n=1}^{M} L^2(n) \]

- \( \phi(n) \) — single-component fermion field on a circle with \( M \) sites
- \( \theta(n) = agA_1(n) \) — gauge field variable related to the Abelian vector potential
- \( L(n) = E(n)/g \) — variable related directly to the electric field
- \( m \) — fermion mass
- \( a \) — lattice spacing
- \( g \) — gauge coupling constant

The Schwinger model

Hamiltonian of the Schwinger model in lattice representation after the Jordan-Wigner transformation [3]:

\[
\mathcal{H}_{jw} = -\frac{1}{2a} \sum_{n=1}^{M} (\sigma^+(n)e^{i\theta(n)}\sigma^-(n + 1) + \text{h.c.}) \\
+ \frac{m}{2} \sum_{n=1}^{M} (1 + (-1)^n\sigma^3(n)) + \frac{ag^2}{2} \sum_{n=1}^{M} L^2(n)
\]

- \(\sigma^i(n)\) — Pauli matrices residing on the sites
- \(L(n)\) — gauge field excitations defined between sites \(n\) and \(n + 1\)
- \(e^{\pm i\theta(n)}\) — ladder operators for gauge field excitations

Strong coupling expansion on the Schwinger model

Rewrite the Hamiltonian in a dimensionless form:

\[ W = \frac{2}{a g^2} \mathcal{H}_J W = W_0 + xV \]

\[ \sum_{n=1}^{M} (\sigma_n^+ e^{i\theta(n)} \sigma_{n+1}^- + \text{h.c.}) + \frac{m}{a g^2} \sum_{n=1}^{M} (1 + (-1)^n \sigma_n^3) + \sum_{n=1}^{M} L^2(n) \]

- If \( x \equiv \beta = \frac{1}{a^2 g^2} \) is small, we can treat \( W_0 \) as an unperturbed Hamiltonian and \( V \) as a perturbation.
- SCE creates the truncated basis of \( W \)

Start with ground state of \( W_0, |0\rangle \)
Create states by acting with \( V \)
Separate states according to their unperturbed energy
Ortho-normalize
Include in the truncated basis

\[ |0\rangle \rightarrow V |0\rangle \rightarrow |\tilde{1}\rangle \rightarrow |1\rangle \]
\[ |1\rangle \rightarrow |1'\rangle \]
Observables

- **Ground state energy:**
  \[ E_0 = \frac{\omega_0}{2Mx} \quad \text{as } a \to 0 \quad \text{as } M \to \infty \]
  \[ - \frac{1}{\pi} \]

- **Scalar mass gap** \((m = 0):\)
  \[ M_S = \frac{\omega_1 - \omega_0}{2\sqrt{x}} \quad \text{as } a \to 0 \quad \text{as } M \to \infty \]
  \[ 2 \quad \frac{2}{\sqrt{\pi}} \]

- **Vector mass gap** \((m = 0):\)
  \[ M_V = \frac{\omega_V^0 - \omega_0}{2\sqrt{x}} \quad \text{as } a \to 0 \quad \text{as } M \to \infty \]
  \[ 1 \quad \frac{1}{\sqrt{\pi}} \]

- **Chiral condensate** (chiral order parameter):
  \[ \langle \bar{\psi}\psi \rangle_0 = \frac{\sqrt{x}}{2M} \langle 0 | \sum_{n=1}^{M} (-1)^n \sigma^3(n) | 0 \rangle \]

\(\omega_i\) — eigenvalues of \(W_0\)
\(\omega_i^V\) — eigenvalues of vector Hamiltonian created using SCE with first state \(V^-|0\rangle\).
Eigenvalue flow with the order of strong coupling expansion, $N$

Example for: $M = 8$, $x \equiv \beta = 2500$

Lattice Hamiltonian approach to the Schwinger model saturated!
Ground state energy

\[ E_0 = \frac{\omega_0}{2Mx} \quad m = 0 \]
Scalar mass gap

\[
\frac{M_S}{g} = \frac{\omega_1 - \omega_0}{2\sqrt{x}}
\]

\( m = 0 \)
Vector mass gap

\[
\frac{M_V}{g} = \frac{\omega_0^V - \omega_0}{2\sqrt{x}}
\]

\(m = 0\)
Chiral condensate

Lattice Hamiltonian approach to the Schwinger model

$m = 0$
Comparison with MPS results

- Ground state energy and mass gaps – massless model:

<table>
<thead>
<tr>
<th>Observable</th>
<th>SCE+ED</th>
<th>MPS [4]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_0$</td>
<td>-0.318309886(2)</td>
<td>-0.318338(24)</td>
</tr>
<tr>
<td>$M_{S/g}$</td>
<td>1.12837916711(1)</td>
<td>1.1279(12)</td>
</tr>
<tr>
<td>$M_{V/g}$</td>
<td>0.5641895845(9)</td>
<td>0.56421(9)</td>
</tr>
</tbody>
</table>

- Chiral condensate - massless case:

<table>
<thead>
<tr>
<th>$x$</th>
<th>SCE+ED</th>
<th>MPS [5]</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>-0.189878819389204</td>
<td>-0.19025255847009401</td>
<td>0.00037</td>
</tr>
<tr>
<td>25</td>
<td>-0.187519020840406</td>
<td>-0.18796879340592226</td>
<td>0.00045</td>
</tr>
<tr>
<td>30</td>
<td>-0.185829589660617</td>
<td>-0.18620821935803569</td>
<td>0.00038</td>
</tr>
<tr>
<td>cont.</td>
<td>-0.16(1)</td>
<td>-0.159930(8)</td>
<td></td>
</tr>
</tbody>
</table>

- C.c. - massive $m = 0.125$ (this is after subtracting log divergence [6]):

<table>
<thead>
<tr>
<th>$x$</th>
<th>SCE+ED</th>
<th>MPS [5]</th>
</tr>
</thead>
<tbody>
<tr>
<td>cont.</td>
<td>-0.091(5)</td>
<td>-0.092023(4)</td>
</tr>
</tbody>
</table>

Oscillations of chiral condensate while changing the SCE order $N$

$Lattice Hamiltonian approach to the Schwinger model$
Oscillations: fitting ansatz

• We have chosen the following fitting function:

\[
\Sigma(N) = \Sigma(N \to \infty) + a \left( \frac{b}{N^3} + e^{-\alpha N} \right) \sin \left( \frac{2\pi}{T} N + \varphi \right)
\]

  for huge \( x \)

  for small \( x \)

• If we can guess the fitting ansatz correctly, we can use small number of points to approximate saturated values, \( \Sigma(N \to \infty) \).
Oscillations: fitting parameters

Phase and period:

\[ T = M \]
\[ \phi = \frac{2\pi}{M} \]
Fitting function: $\Sigma(N) = \Sigma(N \to \infty) + a \left( \frac{b}{N^3} + e^{-\alpha N} \right) \sin \left( \frac{2\pi}{T} N + \phi \right)$

Lattice Hamiltonian approach to the Schwinger model
Fitting function: \( \Sigma(N) = \Sigma(N \to \infty) + a \left( \frac{b}{N^3} + e^{-\alpha N} \right) \sin \left( \frac{2\pi}{T} N + \varphi \right) \)

- High \( x \): errors due to huge oscillations
- \( \Sigma(N \to \infty) \) seems to go to zero – because of huge finite volume effects.
Oscillations of the chiral condensate and flux loops

- Every time $N = kM$, we reach the next flux loop in the system
- Period must reflect presence of the flux loops

$$
\Sigma(N, M, x) = \Sigma(N \to \infty, M, x) + \left( A(M) \frac{\sqrt{x}}{N^3} + B(M, x)e^{-\alpha(M,x)N} \right) \sin \frac{2\pi}{M} (N + 1)
$$

Example:

$x = 10^{10}$
$M = 24$
Summary and outlook

Lattice Hamiltonian results for massless Schwinger model:
- Almost machine precision for GS energy and mass gaps
- Chiral condensate, but we have a problem with FVE

Future:

- How to deal with finite volume effects in the chiral condensate?
- Fitting functions?
- Chiral condensate oscillations – further investigation
- Fitting function?
- Damped harmonic oscillator for small x?
Thanks for listening!

Acknowledgements:

• Co-authors:
  - dr Krzysztof Cichy
  - dr Agnieszka Kujawa-Cichy

• I’m funded by:

- EPSRC
- Lancaster University
- Manchester 1824
- NowNano