Generalised t-V model in one dimension

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The generalised t-V model [2] of fermions distributed on a chain of $L$ sites:

$$\mathcal{H} = -t \sum_{i=1}^{L} \left( \hat{c}_{i}^{\dagger} \hat{c}_{i+1} + \text{h.c.} \right) + \sum_{i=1}^{L} \sum_{m=1}^{p} U_{m} \hat{n}_{i+m}$$

The hopping term, i.e. the kinetic energy, is much smaller than the potential.$t \ll U_{m}$.  

**Kinetic energy** makes sure the particles are not closer than $p$ sites; otherwise energy cost is $U_{m}$. Example for $p = 2$:

<table>
<thead>
<tr>
<th>$E_{\text{pot}}$</th>
<th>$U_{1}$</th>
<th>$U_{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{pot}} = 0$</td>
<td>$U_{1} &gt; U_{2}$</td>
<td>$U_{2} &gt; U_{1}$</td>
</tr>
</tbody>
</table>

Depending on fermion density $Q = N/L$ we have different phases:

**Critical density** $Q_{C} = \frac{1}{p+1} q = 1$.

- **Mott insulator**
- **Simple unperturbed ground state**

Avoiding from critical density

- **Luttinger liquid**
- **Highly degenerate ground state of $H_{B}$**

Using SCE for near-critical densities, the Hamiltonian is small in the new basis of the ground state energy of the system up to order $(t/U)^{2}$.

Example: $p = 3$, $Q = 1/4$, step “$2^{nd}$” in SCE:

$$\mathcal{H} = \begin{pmatrix} -\sqrt{2/2} t & u_{a} & -\sqrt{4t} & -2t & \cdots \\
-\sqrt{4t} & u_{a} & -\sqrt{4t} & u_{a} & \cdots \\
-2t & -\sqrt{4t} & \cdots & \cdots & u_{a} \\
\cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

This simple $5 \times 5$ Hamiltonian gives the ground state energy of the system up to order $(t/U)^{2}$.

Below we present results for a system with $p = 3$. Similar results have been obtained for $p = 1$ (integrable) and $p = 2$ systems. $Q_{C} = 1/4$. This is step “$3^{rd}$” in SCE.

**Ground state energy:**

$$E_{0} = \frac{-t}{2U} \left[ \frac{1}{3} U_{0}^2 \frac{t}{2U} \right]^{4} + \frac{4U_{0}^2}{5U} + \frac{56U_{0}^2}{25U} + \frac{512U_{0}^2}{125U} + \cdots + \frac{256}{39} \left( \frac{t}{2U} \right)^{6}$$

**Current density:**

$$\frac{J}{L} = \frac{t}{2U} \left( \frac{1}{3} U_{0}^2 \frac{t}{2U} \right) \frac{1}{4} t^{4} + \frac{4U_{0}^2}{5U} + \frac{56U_{0}^2}{25U} + \frac{512U_{0}^2}{125U} + \cdots + \frac{256}{39} \left( \frac{t}{2U} \right)^{6}$$

**Density-density correlations:**

$\langle \hat{n}_{i} \hat{n}_{i+3} \rangle$ were also obtained. Leading order is cyclic in $\delta$, which is consistent with expectations.

**Leading order of $t/U$**

$$\langle \hat{n}_{i} \hat{n}_{i+3} \rangle \sim \frac{1}{2} \left( \frac{t}{U} \right)^{2}$$

Obtained accuracy was $O(t^{6}) + O(t^{8})$.

Summary:

- High precision results for both integrable and non-integrable models in Mott insulating phases.
- Results are fully consistent with other works [1,2,3].

**Further work:**
- Phase transition investigation
- Temperature dependence
- More observables
- Time dependence

**REFERENCES**