Generalised t-V model in one dimension

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The generalised t-V model [2] of fermions distributed on a chain of L sites:
\[ \hat{H} = -t \sum_{i=1}^{L} (c_i^\dagger c_{i+1} + \text{h.c.}) + \sum_{i=1}^{L} \sum_{m=1}^{p} U_m \tilde{c}_i \tilde{c}_{i+m} \]

Kinetic energy, i.e. the hopping term, is much smaller than the potential: 
\( t \ll U_m \).

Potential energy makes sure the particles are not closer than \( p \) sites; otherwise energy cost is 
\( U_m \). Example for \( p = 2 \):

\[ E_{\text{pot}} = 0 \quad \text{if} \quad U_m > U_{m+1} \]

Depending on fermion density \( Q = N/L \) we have different phases:
- Critical density \( Q_c = \frac{2}{p+1} \) \( q = 1 \ldots p \)
  - Mott insulator
  - Simple unperturbed ground state
- Away from critical density
  - Luttinger liquid
  - Highly degenerate ground state of \( \hat{H}_0 \)

Using SCE for near-critical densities, the Hamiltonian is small enough to calculate approximate solution to a very high precision.

Example: \( p = 3 \), \( Q = 1/4 \), step "2" in SCE:
\[ \hat{H} = \begin{pmatrix}
-\sqrt{7/2} t & u_1 & -\sqrt{6t} & 0 & 0 \\
-\sqrt{7/2} t & u_3 & -\sqrt{6t} & u_2 & 0 \\
0 & -\sqrt{6t} & u_1 & u_2 & u_4 \\
0 & 0 & -\sqrt{6t} & u_3 & 0 \\
0 & 0 & 0 & -\sqrt{6t} & u_3
\end{pmatrix} \]

This simple 5x5 Hamiltonian gives the ground state energy of the system up to order \( (t/U_3)^2 \).

The generalised t-V model \( \hat{H} \) is constructed using SCF. The method starts similarly to the perturbation theory. Assume:
\[ \hat{H} = \hat{H}_0 + \lambda \hat{V} \]
where \( \lambda \ll 1 \), so we can treat \( \hat{V} \) as perturbation. Eigenstates \( \{ \alpha_n \} \) of \( \hat{H}_0 \) are known. Now, we want to create a new truncated basis \( \{ \beta_n \} \) using \( \{ \alpha_n \} \):
- Include in your basis the desired subspace of unperturbed states that you want to approximate.
- They are of step “0” in SCE.
  - Example: using state \( |1\uparrow 1\uparrow 1\uparrow \rangle \).
- Act with \( \hat{V} \) on states from previous SCE step (“n”), creating set of states \( S \).
- States in \( S \) are linear combinations of the unperturbed Hamiltonian eigenstates.
  - Example: \( |1\uparrow 1\uparrow 1\uparrow \rangle + |1\uparrow 1\uparrow 1\uparrow \rangle \).
- Separate every state in \( S \) according to their unperturbed energy.
  - Example: \( |1\uparrow 1\uparrow 1\uparrow \rangle \).
- Orthonormalise the states in set \( S \), so they would be orthonormal to each other and the basis.
  - Include them in the basis.
- They are of step “0” in SCE.
  - Example The basis is now: \( |1\uparrow 1\uparrow 1\uparrow \rangle, |\uparrow \uparrow \rangle, |\uparrow \uparrow \rangle \).
- Repeat from step II until you achieve desired SCE step.

All the information about the desired states (e.g. ground states) will be encoded in the truncated \( \hat{H} \) in the new basis [4,5].