Inversion of the Magnetic Polarisability Tensor at a Single Frequency from Walk-through Metal Detector Measurements

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ABSTRACT

A tomographic metal detection and characterisation system has been designed and built with the intention of recovering detailed information about magnetic and/or conductive objects within the detector space. This information is gathered as a result of a “walk-through” scan of a candidate in the same manner as would be expected at a typical security metal detector archway. Following the passage of the candidate, the system uses measurements from an array of coils to calculate the polarisability tensor, which is considered to describe the low frequency electromagnetic characteristic of a small metallic object when it interacts with a magnetic field. The tensor is known to vary according to the excitation frequency of the applied magnetic field, however in this system only a single frequency measurement is considered. In addition to the magnetic polarisability dyadic tensor the position of the perturbation is also determined as a product of the inversion algorithm.

The inversion algorithm used in this paper is based on the implementation of a model which is calculated by the multiplication of the transmit and the receive magnetic field strength vectors with the magnetic polarisability dyadic. The inversion algorithm seeks to minimise the error between the actual measurements and the simulated measurements through the modification of an estimated object tensor and the variation in estimated object location.

The system has been tested and is capable of inverting object tensors with <20% typical parameter variation, and determines three-dimensional object location with a typical error of less than ±3 cm. In this paper results are shown for two test objects, both with a different magnetic polarisability tensor. The objects used are a ferrite sphere and a steel NIJ handgun shape.

Keywords Magnetic polarisability tensor, Inversion, Walk-through metal detection, Electromagnetic security, Screening systems

1 INTRODUCTION

Walk-through metal detection (WTMD) has proven to be a fundamental part of the screening of large numbers of personnel for potential threat objects and WTMD use is widespread in locations such as airports, prisons, embassies, ports and government buildings. In these scenarios their primary purpose is to keep prohibited metallic items such as knives and guns out of a secured area.

Due to the importance of maintaining a high level of security, it is a natural assumption that walk-through metal detectors should be as sensitive as possible. This is not the case for all scenarios, because there are a variety of operating requirements which depend upon the location in which the detector is being used. For example, at an airport it is conceivable that passengers will be in possession of a variety of legitimate metallic items such as keys, mobile telephones, wristwatches and glasses, whereas in a prison it is possible to control the environment more strictly because inmates should not be in possession of any metallic items. Although the detector should be as sensitive as practically possible, there will usually be a large number of alarms triggered by the presence of innocuous items, where these are allowed. Security policies at airports in many countries state that all metal detector alarms require further investigation, which can lead to long delays for passengers. Some restricted sites such as airports and government buildings have sought to combat this by requiring that candidates remove all metallic items which they are in possession of before they are scanned. This is a fairly trivial and quick process for many people, however it can be time consuming for those carrying a large amount of innocuous metallic items or shoes and clothing accessories containing metallic components, which may need to be either completely removed or searched by hand. In a large-scale environment such as a busy airport this generates the need for a large number of parallel screening channels requiring both equipment and staff.
The current technology of walkthrough metal detectors provides very basic location estimations for detected objects, which typically refer to a horizontal band within the detector. This basic localisation typically restricts the object to be within a particular ‘horizontal band’ of the detector space (typically 1/5 to 1/10 of the overall detector volume). As this is a relatively large area it can take a significant amount of time for an operator to search manually, and when this effect is multiplied across hundreds of candidates it can lead to significant delays. Delays such as these are often inconvenient to the individuals being scanned and can lead to the incursion of expenses to organisations e.g. more staff must be employed to operate a sufficient number of detectors to ensure a particular throughput. As it is often innocuous objects which trigger walk-through metal detector alarms in the airport environment, any screening system which is able to provide the operator with information regarding the likelihood that a particular object poses a threat e.g. is it shaped like a knife or gun rather than a mobile phone or wristwatch, would be of benefit.

It is clear that future walkthrough metal detector systems should focus upon the use of technology which is both intelligent and sensitive in equal measures. It is necessary to go beyond the straightforward detection of metallic items by recovering detailed information about potential threat objects alongside fulfilling the aim of reducing the overall time required to process a single candidate. This information may include the specific location, approximate dimensions and orientation of the threat object as well as additional material properties. Much of the object-specific information is contained in a quantity known as the ‘magnetic polarisability dyadic tensor’, referred to as \( \mathbf{\tilde{M}} \). The double-arrowed accent is used to denote a dyadic quantity rather than a matrix or vector quantity. With these considerations in mind a prototype system has been constructed which allows for the inversion of the magnetic polarisability dyadic tensor, in addition to the 3D localisation of a threat object. The system can also operate with multiple metallic objects, however only the results for single objects will be reported in this paper.

The magnetic polarisability dyadic is a complex 3x3 matrix which defines the contribution of individual field components to the induced voltage at the receive coil. Section 2 demonstrates that it is the only part of the system which contains any information about the object interacting with the field, and hence can be treated as a solo entity which can define the characteristics of the object in question (Bell, 2001). A magnetic polarisability tensor will exist for any object which may alter the magnetic properties of the system e.g. a conductive or magnetic material. It is dependent upon excitation frequency, object conductivity and permeability (Norton, 2001). It should also be noted that although the tensor contains 18 values (9 real and 9 imaginary), due to electromagnetic reciprocity between transmitted and received signals (Harrington, 2001), the tensor is symmetric, and consequently only contains 12 values (6 real and 6 imaginary). The structure of the tensor is shown below:

\[
\mathbf{\tilde{M}} = \begin{bmatrix}
M_{11} + jN_{11} & M_{12} + jN_{12} & M_{13} + jN_{13} \\
M_{12} + jN_{12} & M_{22} + jN_{22} & M_{23} + jN_{23} \\
M_{13} + jN_{13} & M_{23} + jN_{23} & M_{33} + jN_{33}
\end{bmatrix}
\]

This tensor is used to relate the orthogonal components of one 3D vector field with those of another, independent vector field. In the example provided in this paper these two fields represent the magnetic fields produced by two coils, with one acting as a transmit coil and another acting as a receive coil. In this instance the two vector quantities are combined in a manner which allows for cross-multiplication between dimensions. Consequently, the tensor is capable of weighting complex contributions of a particular object across a six-dimensional field – \( H_{xx}, H_{xy}, H_{xz}, H_{yx}, H_{yz}, H_{zz} \). The magnitudes of these contributions are determined by the aspect ratio of an arbitrary 3D object. The complex nature of the tensor relates to the conductive and magnetic properties of the object. For a magnetic, non-conducting object the tensor components are entirely real, as the object concentrates the primary (incident) magnetic field along its axis. For a conductive, non-magnetic object, the primary field induces eddy currents which circulate within the object, thereby producing a secondary magnetic field. This secondary field is represented in the complex components of the tensor. Similarly, for an object which is both magnetic and conductive, e.g. ferritic steel, the tensor will also contain complex components.

As a final introductory comment, the tensor contains information relating to object composition and aspect ratio, and therefore can be considered to adequately describe a target object. However, the tensor varies as a function of frequency, and consequently it is necessary to capture the full spectroscopic contents of the tensor for a more complete representation of the target object. Such spectral information is not considered in this paper, and the tensors produced are point values of this...
spectroscopic information at a single frequency, although clearly the procedures described in this paper could be repeated at several frequencies to obtain a full complex, spectral tensor description of the metallic object.

2 THEORY

The theory presented here assumes that the target object may be represented as a magnetic dipole (see ‘Practical Limitations’ below). It is possible to relate this equivalent magnetic dipole moment to the incident magnetic field through the use of an object tensor, \( \mathbf{M} \), (Dooley, 2010).

Practical Limitations

The derivation contains a number of important assumptions which, although not prohibitively restrictive in many cases, it is important to clarify before suitable applications may be considered. The first two of these assumptions are derived from those described in the Biot-Savart law; these being that the conductors are filamentary and exist in vacuo. It is practically impossible to fully satisfy these conditions, however in order to approximate these requirements the conductor bunches used to form each coil have been tightly packed together to minimise unwanted air gaps, and care has been taken to ensure that unnecessary objects are not placed in the vicinity of the coils. It should be noted that although the Biot-Savart law has been used in this paper, largely for computational convenience, other techniques for electromagnetic field calculation, such as the finite element method could be employed.

It is also assumed that the incident (primary) field is parallel across the target object, and that the secondary field (as experienced by the receive coil) can be accurately approximated by the field of a magnetic dipole. To satisfy the requirements for a parallel field, then the object dimensions must be much smaller than the dimensions of the coils producing the field, and that the object distance from the coils is sufficiently large that the direction of the field lines does not change significantly across the object volume. Additionally, a magnetic dipole approximation may be used to represent a current loop, where the loop has dimensions which are much smaller than the distance to the point under observation (Ulaby, 2010).

Derivation of the Theory

The magnetic field produced by a conductor can be calculated at a point relative to the conductor as a function of conductor current, \( I \), a linear segment of the conductor, \( \mathbf{dl} \), and the distance between the point of observation and the conductor midpoint \( \mathbf{r} \). This relationship is known as Biot-Savart law and is shown as equation (1).

\[
\mathbf{B} = \frac{\mu_0 I}{4\pi} \int \frac{\mathbf{dl} \times \mathbf{r}}{r^2}
\]  

(1)

Figure 1 defines a system consisting of a conductor, \( \mathbf{dl} \), and a point of observation, \( \mathbf{P} \). The angles \( \alpha_1 \) and \( \alpha_2 \) are defined relative to the Euclidean distance to the line formed by an infinitely long \( \mathbf{dl} \), and the distance between \( \mathbf{P} \) and the points at both limits of \( \mathbf{dl} \) (\( P_1 \) and \( P_2 \) respectively).

![Fig 1. Geometry of parameters for magnetic field calculation](image-url)
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It is possible to calculate the magnetic field that is experienced at point \( P \) as a result of current flowing through conductor \( L_d \) by using an adapted version Biot-Savart law as indicated in equation (2). Note that this equation has been altered to calculate \( H \) rather than \( B \); a factor of \( \mu_0 \) has been removed to produce this change.

\[
\mathbf{H}_{P(x,y,z)} = \frac{I}{4\pi} \left( \sin \alpha_x - \sin \alpha_y \right) \mathbf{a}_k
\]

This magnetic field can be calculated for a designated transmit coil, \( T \), and also for a designated receive coil, \( R \). These can then be related to the system response through the specification of the object tensor, \( \mathbf{M} \), as shown in equation (3), (Norton, 2001).

\[
V_{ind} = K \mathbf{M} \cdot \mathbf{H}_T \cdot \mathbf{H}_R
\]

where \( K \propto \frac{j\omega L_0}{L_k} \).

As the values of magnetic field can be pre-computed for a particular coil array, it is possible to combine this information to produce a system matrix, \( A_P \). The subscript ‘P’ is used to denote the fact that the sensitivity matrix is dependent upon location relative to the coil network. A system measurement, \( b \), can be partially expressed as the product of the magnetic polarisability tensor and the system matrix. This is shown in equation (4). This relationship is defined to be the forward model for the system, denoted as \( f(P, M) \).

\[
b = \mathbf{M} \cdot A_P
\]

To account for the effect of \( K \) it is necessary to multiply \( b \) by a calibration constant in order to represent measurable values.

For the inversion the magnetic field information contained within the system matrix \( A_P \) is augmented with positional information to form a new matrix, \( J \). This facilitates the solution of both object tensor, \( \mathbf{M} \), and object location, \( \mathbf{P} \). The inversion algorithm uses a modified Levenberg-Marquardt method to find the least squares solution to equation (5). In this equation the quantity \( \mathbf{V} \) represents a vector of known sequential measurements, and \( f(P, M) \) represents the results of the forward model (equation (4)) as a function of the unknown values of object location and magnetic polarisability tensor.

\[
\arg\min \left\{ \| \mathbf{V} - f(P, M) \| \right\}
\]

The inversion algorithm is described in equation (6) below.

\[
[\mathbf{M} \; \mathbf{P}] = (J^T J + \lambda I \; L) \left( J^T \right)^{-1} \mathbf{R}
\]

In this equation \( \lambda \) refers the regularisation parameter and \( L \) refers to a regularisation matrix. The value of \( \lambda \) is altered during the iteration of the algorithm according to whether the inverted location is within the detector space. The term \( \mathbf{R} \) represents the residual value, as defined in equation (7).

\[
\mathbf{R} = \mathbf{V} - f(P, M)
\]

### 3 EXPERIMENTAL SETUP

Two experiments are presented in this paper. These experiments investigate the consistency and accuracy of the inverted tensors for a variety of object types.

**Experiment 1 – Object Testing at Fixed Orientations**

In this experiment, two objects are reported; these objects are shown below in figure 2(a) and (b), on the background of a 1 cm × 1 cm grid. These objects are a ferrite sphere and a steel NIJ handgun (Paulter, 2009). A greater number of objects were tested, however only these two are reported here.
The sphere has uniform contributions in all directions. The expected tensor consists of values in the diagonal components only i.e. the ‘xx’, ‘yy’ and ‘zz’ components. To represent the uniformity of the sphere it is expected that these diagonal components should be equal in magnitude. The object is filled with ferrite powder, so that it is magnetic and non-conductive. In order to reflect this the tensor should consist of real parts only.

![Image](a) Ferrite sphere, (b) Steel NIJ handgun, and (c) gun orientation

The tensor for the NIJ gun is expected to have a very different value to that of the sphere according to both the alignment of the object, and the nature of the conductive and/or magnetic nature of the object. Steel is a combination of conductive metals and, due to its iron content, it is also magnetic. As a consequence of this it is expected that the resulting tensor should have both real and imaginary parts, and the real parts could contain any conceivable value, which would be determined by its conductive and magnetic make-up.

![Graph](a) Tensor Verification Test Locations - Experiment 1  
(b) Tensor Verification Test Locations - Experiment 2

Each scan involving the NIJ gun was conducted with the dominant component aligned with the x-axis as shown in figure 2(c), and each object was carried through the detector spread along a number of different trajectories as shown in figure 3(a). The sphere was scanned five times at each of the five trajectories shown – a total of 25 scans. The other objects were scanned 15 times at the trajectories labelled ‘Left Shoulder’, ‘Waist’ and ‘Right Pocket’ in figure 3(a). Markers were positioned on the floor to allow for a consistent walk-through profile, and a laser measuring tool with a precision of ±1 mm was used to initially position the test objects. During the walk-through the objects were kept as close as possible to a constant position and orientation. It is assumed that the objects will not have moved by more than 2 or 3 cm in the y or z-direction during the walk-through.
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**Experiment 2 – Comparison of Results for Objects Both Aligned and Not Aligned with Principle Axes**

In this experiment the steel gun shown in figure 2(b) was carried at the trajectory shown in figure 3(b). A total of 40 walk-through scans were conducted at each of the orientations shown in figure 4 below.

![Fig 4. Scan orientations (a) Barrel aligned with x-axis, (b) Barrel aligned with y-axis, (c) Barrel aligned with z-axis](image)

Three different rotations were then applied so that the object was no longer aligned along the three principal axes. These rotations are described below and shown on figure 5. At each orientation 15 walk-through scans were performed.

(a) The initial ‘x’ orientation for each object in figure 4(a) rotated 45° clockwise about the z-axis.
(b) The initial ‘y’ orientation for each object in figure 4(b) rotated 45° counterclockwise about the x-axis.
(c) The initial ‘z’ orientation for each object in figure 4(c) rotated 45° clockwise about the y-axis.

![Fig 5. Object rotations not aligned with the principle axes.](image)

The objects were attached to a block of wood, on which angles had been measured using a protractor with an estimated degree of accuracy of 0.5°. A test conducted using a laser spirit level indicated that the error in angle caused due to the walk-through action was rarely greater than 10°, with a more typical value of <5°.

The inverted tensors were rotated using equation (15) below, to align the angular offsets in figure 5 so that the orientations correspond to those shown in figure 4. This allows for direct comparison of the two sets of results. In (15) the values \( \hat{M} \) and \( \tilde{M} \) represent the un-rotated and rotated tensors respectively.

\[
\tilde{M}_{(i)} = \sum_{j=1}^{3} a_{(i,j)} \hat{M}_{(j)} \quad i = 1, 2, 3
\]  

The value \( a \) in (15) represents the dyadic rotation matrix (Tai, 1997) as shown below in equation (16).

\[
a(\theta_x, \theta_y, \theta_z) = \begin{bmatrix}
\cos(\theta_x) \cos(\theta_y) & -\sin(\theta_x) \sin(\theta_z) & \sin(\theta_x) \cos(\theta_y) + \cos(\theta_x) \sin(\theta_z) \\
\sin(\theta_x) \cos(\theta_y) & \cos(\theta_x) \cos(\theta_y) & -\sin(\theta_x) \sin(\theta_y) \\
-\sin(\theta_z) & \cos(\theta_z) & \sin(\theta_z)
\end{bmatrix}
\]  

**Presentation of Inverted Tensors**

All of the tensors presented in the results section have been averaged from the full set of inverted tensors. In the case of experiment 1 the average tensor is calculated per object, and for experiment 2 the average tensor is calculated per orientation.
4 RESULTS

Experiment 1

The results for the testing using the ferrite sphere and steel gun, as shown in figure 2(a) and (b) respectively, are as follows:

(a) Ferrite sphere

\[
\mathbf{M} = \begin{bmatrix}
0.9444 - 0.0144j & -0.0153 - 0.0076j & 0.0284 + 0.0107j \\
-0.0153 - 0.0076j & 0.812 + 0.0445j & 0.0152 - 0.0092j \\
0.0284 + 0.0107j & 0.0152 - 0.0092j & 0.8472 - 0.0271j
\end{bmatrix}
\]

Note that this tensor is as close to what is expected for an object of this type, i.e. the unity matrix. The magnetic, non-conductive nature of the object is shown in the fact that the dominant components are real values, and the similarity in the diagonal components implies a comparable aspect ratio in all dimensions i.e. a sphere. It is possible to see that these components are not equal, as would be expected for an ideal spherical case. However, the values are sufficiently close to one another to determine that this object has approximately equal contributions in the three orthogonal components.

(b) Steel gun

\[
\mathbf{M} = \begin{bmatrix}
0.8584 - 1.4056j & 0.0065 - 0.0026j & 0.0204 - 0.0227j \\
0.0065 - 0.0026j & 0.743 - 1.1386j & -0.0408 + 0.0143j \\
0.0204 - 0.0227j & -0.0408 + 0.0143j & 0.4257 - 0.8105j
\end{bmatrix}
\]

It is possible to see that this tensor contains significant imaginary values. As discussed previously, this can be attributed to the variety of magnetic and conductive materials used to produce the steel for this item. The comparison between real and imaginary contributions from the object is one of the properties contained within the tensor which may be used to classify an item. When the magnitude of each tensor element is calculated it is possible to determine that the largest value is contained in the ‘xx’ component (element $M_{11}$ in the tensor), the second largest value is in the ‘yy’ component (element $M_{22}$ in the tensor), and the smallest value is in the ‘zz’ component (element $M_{33}$ in the tensor). These correspond with the largest preferential direction of the gun (the barrel) being aligned along the x-axis; the second largest direction (the handle), being aligned in the y-direction, and the smallest direction being aligned in the z-direction.

Experiment 2

Some results have been presented showing the spread of inverted tensors for each set of 40 walk-through scans. In such instances the tensor elements are plotted as scatter graphs with the label ‘tensor component’ along the x-axis. The x-axis of each graph is scaled with integer values which correspond to tensor elements in the following way:

\[
\begin{bmatrix}
M_{11} & M_{12} & M_{13} \\
M_{12} & M_{22} & M_{23} \\
M_{13} & M_{23} & M_{33}
\end{bmatrix} \Rightarrow \begin{bmatrix}1 & 2 & 3 \\
2 & 4 & 5 \\
3 & 5 & 6
\end{bmatrix}
\]

The inverted locations for all 40 walk-through scans at the orientation shown in figure 4(a) are shown in figure 2(a). This figure shows that all the inverted locations lie approximately in the same region in the x-z plane, and follow a linear trajectory in the x-axis. This correctly reflects the walkthrough profile, and is an encouraging result. The averaged inverted tensor for this orientation is shown below, followed by the spread of tensor components for each walk-through.

\[
\bar{\mathbf{M}} = \begin{bmatrix}
0.9000 - 1.4437j & 0.0566 + 0.0733j & 0.0355 - 0.0264j \\
0.0566 + 0.0733j & 0.7139 + 1.3510j & -0.1768 + 0.0062j \\
0.0355 - 0.0264j & -0.1768 + 0.0062j & 0.3465 + 0.7055j
\end{bmatrix}
\]

In this instance the gun is in the same orientation as for the testing in 'Experiment 1'; consequently the results are directly comparable. When examining both tensors it is possible to see that the values follow similar trends in terms of the sign of the dominant components, and the relative magnitudes with respect to the principal directions of the object. The numerical differences are shown in table 1. This table shows that the difference in the largest component (corresponding to the preferential direction of the gun) is 4.62% and 2.64% for real and imaginary parts respectively. The differences for the
remaining values are larger, typically in the region of 15% to 23%. However, it should be noted that the largest value of nearly 23% corresponds to a considerably smaller value than for the other diagonal components.

<table>
<thead>
<tr>
<th>XX</th>
<th>YY</th>
<th>ZZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re(xx)</td>
<td>Im(xx)</td>
<td>Re(yy)</td>
</tr>
<tr>
<td>4.62%</td>
<td>2.64%</td>
<td>4.08%</td>
</tr>
</tbody>
</table>

Table 1. Differences between averaged tensor components for experiments 1 and 2

Figure 7 displays a plot of the range of inverted tensor components for each of the 40 walk-through tests. It is possible to see that the spread of points is typically fairly small, and that all values have an even distribution within the overall range. It should be noted that the spread of tensor values is affected with rotation of the object, and a small degree of rotation (previously defined as ~5°) is within the limits of experimental uncertainty.

The inverted locations for all 40 walk-through scans at the orientation shown in figure 4(b) are shown in figure 6(b). As with the previous orientation it is possible to see that the inverted range of data points is in agreement with the known trajectory.

The averaged inverted tensor for this orientation is shown below:

\[
\mathbf{M} = \begin{bmatrix}
0.0815 - 1.2259j & 0.0129 - 0.0596j & 0.0061 - 0.0187j \\
0.0129 - 0.0596j & 0.8548 - 1.5924j & -0.1606 - 0.0283j \\
0.0061 - 0.0187j & -0.1606 - 0.0283j & 0.3589 - 0.7107j
\end{bmatrix}
\]

By transposing the first two diagonal components it is possible to mimic the effect of rotating the object from the previous rotation (with respect to the orthogonal field components). This allows for some comparison of the two tensors. Upon comparison of the two tensors it is possible to see that the characteristics of the tensor values correspond to expected values. Figure 8 shows the spread of inverted tensor components for all walk-through tests. As for the previous orientation it is possible to see that in each case the ranges of inverted values are well concentrated.

![Inverted positional information for all walk-through scans as shown in figure 6](image)

The inverted locations for all 40 walk-through scans at the orientation shown in figure 4(c) are shown in figure 6(c). The results are consistent with those from the testing at the other two orientations. The averaged inverted tensor for this orientation is shown below, and the spread of tensor components for each walk-through is shown in figure 10.

\[
\mathbf{M} = \begin{bmatrix}
0.7469 - 1.2161j & 0.0621 - 0.0025j & 0.0171 + 0.0841j \\
0.0621 - 0.0025j & 0.4282 - 1.0632j & -0.1233 - 0.0484j \\
0.0171 - 0.0841j & -0.1233 - 0.0484j & 0.9184 - 1.3753j
\end{bmatrix}
\]

These results show that the inversion has again yielded consistent results for this orientation. In addition to this, the relative magnitudes of the tensor components, as well as their signs, show consistency with the previous orientations.
By performing a series of scans at the orientation shown in figure 5(a) an average tensor was produced. This tensor was then rotated using (15) and (16) to produce an equivalent tensor shown below. Table 2, index (a), shows the error between the diagonal tensor components following this rotation, and that the error from the un-rotated orientation shown in figure 4(a). This shows an agreement of results of typically <8% for the dominant tensor components.

\[
\bar{M}(a)_{\text{rotated}} = \begin{bmatrix}
0.8501 - 1.4568j & 0.0017 - 0.1094j & -0.0399 + 0.0083j \\
0.0017 - 0.1094j & 0.6728 - 1.2531j & -0.0902 + 0.0021j \\
-0.0399 + 0.0083j & -0.0902 + 0.0021j & 0.3193 - 0.6522j \\
\end{bmatrix}
\]

A tensor was inverted for scans at the orientation shown in figure 5(b), this tensor was then rotated to produce the tensor shown below. Table 2, index (b), shows that the difference between the rotated tensor, and that from the orientation shown in figure 4(b) is larger than for the previous orientation, with difference this time up to almost 23%.

\[
\bar{\bar{M}}(b)_{\text{rotated}} = \begin{bmatrix}
0.6934 - 0.9999j & 0.0139 - 0.0269j & 0.0154 - 0.0425j \\
0.0139 - 0.0269j & 0.6605 - 1.5234j & 0.0144 + 0.1831j \\
0.0154 - 0.0425j & 0.0144 + 0.1831j & 0.4030 - 0.8572j \\
\end{bmatrix}
\]

For the orientation shown in figure 5(c) a tensor was produced. This tensor was then rotated to produce the following result:

\[
\bar{\bar{M}}(c)_{\text{rotated}} = \begin{bmatrix}
0.6880 - 1.0144j & -0.0615 + 0.0144j & 0.0137 + 0.0546j \\
-0.0615 + 0.0144j & 0.3444 - 1.0569j & -0.0875 - 0.0044j \\
0.0137 + 0.0546j & -0.0875 - 0.0044j & 0.7361 - 1.2048j \\
\end{bmatrix}
\]
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Table 2, index (c), shows that the comparison with this rotated tensor, and the tensor produced from testing at the orientation shown in figure 4(c) had a larger range of difference than for previous measurements. In this instance the difference is up to 26.24%. However, the difference in the dominant component of the tensor, the ‘zz’ component, is relatively small at 1.45% and 0.93% for real and imaginary parts respectively.

Table 2. Difference between tensors for aligned and rotated tests

<table>
<thead>
<tr>
<th>Orientation</th>
<th>xx</th>
<th>yy</th>
<th>zz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Re(xx)</td>
<td>Im(xx)</td>
<td>Re(yy)</td>
</tr>
<tr>
<td>(a)</td>
<td>5.55%</td>
<td>0.91%</td>
<td>5.75%</td>
</tr>
<tr>
<td>(b)</td>
<td>13.49%</td>
<td>18.43%</td>
<td>22.73%</td>
</tr>
<tr>
<td>(c)</td>
<td>25.09%</td>
<td>26.24%</td>
<td>19.57%</td>
</tr>
</tbody>
</table>

The results presented in this paper show that it is possible to reliably reconstruct the magnetic polarisability dyadic tensor from a single walk-through scan. This has also been shown to be the case for instances where objects are rotated at arbitrary angles, which is a common scenario. Typically the difference between sets of results has been shown to vary from near-zero up to about 20%. The research also shows that the information contained within the tensor reflects the ratio of dimensions, as well as the information regarding the conductivity and permeability of an object. This provides the basis of a method for object classification and possible discrimination between objects which are considered to be a threat, and those which are innocuous.

The reliability in inverting positional information for a detectable object has also been shown to yield consistent and accurate results. This demonstrates that it is possible to reconstruct the location of an object to a standard which would potentially allow for walk-through metal detector operators to carry out a quick, highly constrained physical search, rather than the time-consuming, wide-scale search that the current generation of detectors require.

5 CONCLUSION

6 ACKNOWLEDGEMENTS

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