Effective short-range interaction for spin-singlet P -wave nucleon-nucleon scattering

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Distorted-wave methods are used to remove the effects of one- and two-pion exchange up to order Q^3 from the empirical 1P_1 phase shift. The one divergence that arises can be renormalized using an order- Q^2 counterterm which is provided by the (Weinberg) power counting appropriate to the effective field theory for this channel. The residual interaction is used to estimate the scale of the underlying physics.

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Effective field theories (EFTs) provide important tools for constructing nuclear forces within a systematic framework.¹ They respect the symmetries of underlying theory, in this case quantum chromodynamics, and offer model-independent descriptions of the dynamics at low-energy scales. They rely on the existence of clear separation of scales between the low-energy physics of interest and the underlying, high-energy (or short-distance) physics. This makes it possible to use a perturbation theory where the expansion parameters are ratios of low- to high-energy scales.

To construct such a theory one begins by writing down the most general Lagrangian or Hamiltonian for the relevant low-energy degrees of freedom, containing all the interaction terms allowed by the symmetries of the system. The parameters of the theory are the (initially unknown) constants multiplying these terms. There are in general an infinite number of these but they can be organised in terms of a “power counting” of low-energy scales. To any given order in this counting, only a finite number of terms are needed.

Since Weinberg first suggested that these ideas could be applied to nuclear forces [4], there has been considerable debate about the appropriate power counting to use.² Most of these arguments have been about channels where scattering is strong and some pieces of the potential may need to be treated nonperturbatively, namely the S waves and the lower spin-triplet waves. In the higher partial waves, especially the spin-singlet ones, the scattering is weak and the picture is clearer. For these channels, Weinberg’s original power counting is valid. This is based on simple dimensional analysis, counting powers of momenta and the pion mass. These low-energy scales are generically denoted by Q .

Many applications of EFTs test convergence by presenting theoretical nucleon-nucleon phase shifts, calculated at leading and higher orders, alongside empirical data. At each order, short-range interactions, expanded as polynomials in energy or momenta, have to be refitted to the data. A powerful alternative is to use distorted-wave (DW) methods to remove the effects of known long-range forces from the observed phase shifts to leave residual short-range interactions. The sizes and energy

dependences of these can then be used to test the convergence of the EFT and to estimate the scales of the underlying physics. Such an approach was proposed in Ref. [7] and applied to the peripheral spin-singlet waves. More recently it has been applied to spin-triplet waves [8] and the 1S_0 channel [9] and closely related methods have been used by other groups to study the S waves [10,11].

In this Brief Report, we extend the method of Ref. [7] by applying it to the 1P_1 channel. We first construct DWs for the leading-order one-pion-exchange (OPE) potential. Iterating this potential to all orders is not essential in a spin-singlet wave with $L \neq 0$ since Weinberg’s power counting is expected to hold. However doing so avoids the need to calculate terms to fourth order in perturbation theory. Since only the central part of OPE contributes and its $1/r$ singularity does not alter the power-law behaviour of the wave functions near the origin, this treatment does not affect the power counting for the short-range pieces of the potential.

From the K matrix that describes scattering between these DWs, we can define a residual interaction. This still contains effects arising from long-range potentials of orders Q^2 and above. We then use the distorted-wave Born approximation (DWBA) to subtract the matrix elements of the order- Q^2 and Q^3 two-pion-exchange (TPE) potentials [12–14]. This leaves a residual interaction that should represent purely short-range physics to this order.

One issue that arises here, but not for the channels studied in Ref. [7], is that the matrix elements of TPE between the DWs are divergent. This divergence must be removed by renormalization so that only finite quantities are being treated in perturbation theory. Weinberg’s power counting provides one energy-independent counterterm, of order Q^2 , in this channel and this is sufficient to cancel the divergence. This is similar to what has been found in other waves where different counting schemes apply [8–11].³

We start by outlining the main features of the method from Ref. [7]. The DWs $\psi_{\text{OPE}}(p, r)$ are obtained by solving the radial Schrödinger equation with the OPE and centrifugal

¹For reviews, see Refs. [1–3].²For recent summaries of two of these points of view, see Refs. [5] and [6].³Note that here we follow the power counting by treating the order- $Q^{2,3}$ potentials as perturbations. Other approaches, which treat the whole potential to all orders, can lead to different conclusions [16,17].

potentials,

$$\frac{d^2\psi_{\text{OPE}}}{dr^2} + \frac{2}{r} \frac{d\psi_{\text{OPE}}}{dr} - \left(\frac{L(L+1)}{r^2} + M_N V_{\text{OPE}}^{(0)}(r) \right) \psi_{\text{OPE}}(p, r) = p^2 \psi_{\text{OPE}}(p, r). \quad (1)$$

Here $L = 1$ is the orbital angular momentum, M_N is the nucleon mass and p is the on-shell relative momentum in the centre-of-mass frame. We write the leading-order OPE potential in the same form as used in the Nijmegen analyses and take their preferred value for the πN coupling, $f_{\pi NN}^2 = 0.075$ [18,19].

From the large- r forms of these waves we can extract the OPE phase shift, $\delta_{\text{OPE}}(p)$. Taking the difference between this and the empirical phase shift, $\delta(p)$, we can define a residual K matrix,

$$\tilde{K}(p) = -\frac{4\pi}{M_N p} \tan[\delta(p) - \delta_{\text{OPE}}(p)]. \quad (2)$$

This describes the additional scattering between the DWs, produced by short-range interactions and long-range forces of order Q^2 and higher. If we take a δ -shell form for the potential responsible,

$$V_S(p, r) = \frac{9}{4\pi R_0^4} \tilde{V}(p) \delta(r - R_0), \quad (3)$$

then we can extract its strength directly from $\tilde{K}(p)$,

$$\tilde{V}^{(2)}(p) = \frac{R_0^2}{9 \psi_{\text{OPE}}(p, R_0)^2} \tilde{K}(p), \quad (4)$$

where the superscript (2) indicates that long-range effects of order Q^2 and higher are still present.

Finally, we use the DWBA to subtract the effects of long-range potentials to order Q^3 . This leaves a residual short-range potential whose strength is given by

$$\tilde{V}^{(4)}(p) = \frac{R_0^2}{9 \psi_{\text{OPE}}(p, R_0)^2} \times [\tilde{K}(p) - \langle \psi_{\text{OPE}}(p) | V_{\text{OPE}}^{(2)} + V_{\text{TPE}}^{(2,3)} | \psi_{\text{OPE}}(p) \rangle], \quad (5)$$

where the order- $Q^{2,3}$ TPE potentials can be found in Refs. [12,13] and the corresponding order- Q^2 recoil correction to OPE is given by Friar [14]. In the order- Q^3 potential we have used the πN low-energy constants (LECs) from Ref. [15].

The TPE potential has terms with $1/r^6$ and $1/r^5$ singularities. Consequently, the radial integrals in its matrix elements between P waves, which have the form $\psi(r) \propto r$ for small r , contain $1/r$ and $\ln r$ divergences. To rectify this, we regularise the integrals by imposing a radial cutoff at the same value of R_0 as used in the δ -shell potential. We can then renormalize the matrix elements by subtracting the sum of these divergent pieces (a constant which is independent of energy).

We show first, in Fig. 1, the effective short-range potential $\tilde{V}^{(2)}(p)$ in the 1P_1 partial wave after the removal of OPE only. The cut-off radius of the δ shell was taken to be $R_0 = 0.1$ fm. Results are shown for PWA93 and three different Nijmegen potentials [18–20], all of which provide high-quality fits to the NN scattering data. One can see that significant energy dependence is present at low energies, like that found in

FIG. 1. The short-range effective potential $\tilde{V}^{(2)}(p)$, in fm^{-4} , plotted against the laboratory kinetic energy T in MeV. The four curves correspond to different Nijmegen PWAs. A cut-off radius of $R_0 = 0.1$ fm was used.

the higher partial waves [7]. This suggests that there may still be important contributions from long-range forces in this interaction. However we should note that the different Nijmegen analyses diverge for energies below about 50 MeV and so this region should not be regarded as well constrained by the data. This is because the centrifugal barrier prevents the nucleons from approaching each other closely at these energies and so the 1P_1 phase shift is completely dominated by OPE.

Using the DWBA to remove the effects of the order- $Q^{2,3}$ long-range potentials, we obtain the results shown in Figs. 2 and 3. The matrix elements of the TPE potential have been renormalized by simply subtracting their values at some very low energy, $T = 5$ MeV. Figure 2 shows that the renormalized potential does indeed converge to a cut-off-independent form as $R_0 \rightarrow 0$. For numerical convenience we present our results for $R_0 = 0.1$ fm, for which any cut-off artifacts (terms in the potential proportional to positive powers of R_0) are very small.

The resulting renormalized potentials $\tilde{V}^{(4)}(p)$ obtained from several Nijmegen PWAs are shown in Fig. 3. The “turnover” at low energies that was visible in Fig. 1 has been significantly reduced by subtracting the higher-order long-range forces. However we should point out that much of this effect lies in the region where uncertainties in the PWAs make it hard to draw very strong conclusions.

To try to quantify the energy dependence of the residual potential, we made a least squares fit of a quadratic in the energy to $\tilde{V}^{(4)}(p)$. We fitted this over the energy range $T =$

FIG. 2. The renormalized short-range effective potential $\tilde{V}^{(4)}(p)$, in fm^{-4} , plotted against the laboratory kinetic energy T in MeV. The different curves show the potentials obtained with cut-off radii $R_0 = 0.8$ fm (dotted), 0.4 fm, 0.2 fm, 0.1 fm, and 0.05 fm (solid).

FIG. 3. The short-range effective potential $\tilde{V}^{(4)}(p)$ for four different Nijmegen PWAs. A cut-off radius of $R_0 = 0.1$ fm was used.

100–200 MeV, which was chosen to avoid both the low-energy region where the data are too inaccurate to determine the short-range potential reliably, and the high-energy region where our EFT is expected to converge slowly if at all. A quadratic was chosen because attempts to fit higher order polynomials to this range did not lead to stable values for the coefficients.

Writing the polynomial in the form

$$\tilde{V}^{(4)}(p) = a_0 + a_1 p^2 + a_2 p^4, \quad (6)$$

we obtain $a_0 = 0.28 \text{ fm}^4$, $a_1 = -0.20 \text{ fm}^6$, and $a_2 = 0.0056 \text{ fm}^8$. This is dominated by a linear dependence on the energy (p^2), consistent with the visual impression of Fig. 3. Without knowing the uncertainties on the phase shifts that are used to determine the potential, it is difficult to assign definite errors to these coefficients. However simple estimates suggest that the values for the first two are reasonably accurate but a_2 is not.

For present purposes, our main interest in these coefficients is to estimate the scale of the short-distance physics represented by $\tilde{V}^{(4)}(p)$. This is the scale that controls the convergence of the expansion of our EFT. Here, we cannot use a_0 for this because it depends on our (somewhat ad hoc) choice of renormalization scheme. Also, as just noted, the coefficient a_2 is not accurately determined and so we are left with only a_1 , the coefficient of the term linear in energy.

If our theory has a “natural” scale dependence, a_1 should be of the form $a_1 = \hat{a}_1/\Lambda_0^6$, where Λ_0 is the scale of the

underlying physics and $\hat{a}_1$ is a dimensionless number of order unity. The higher-order pion-exchange forces that have not been subtracted out can contribute to this coefficient but, since these are at least of order Q^4 , their contributions are suppressed by $(m_\pi/\Lambda_0)^2$. If we set $\hat{a}_1 = -1$, then our value of a_1 implies a scale of approximately 260 MeV. This is similar to the breakdown scale that was recently estimated for the 1S_0 wave [9]. Such low values for the scales suggest the presence of additional low-energy physics that has not been considered in the present EFT.

One important example of such physics is the Δ resonance. Including this as an explicit degree of freedom as in Refs. [21,22], could lead to more natural short-range forces. To gain some insight into what might happen in such an extended theory, we have tried adding the potential arising from single- Δ excitation, as in Ref. [21]. On its own, this has relatively little effect on the residual interaction but, when the Δ is added, the πN LECs also need to be corrected accordingly.

The adjusted constants depend on the value taken for the $N\Delta$ axial coupling and so we have examined two possible sets from Ref. [22]: “fit 1” and “fit 2”, both at order Q^2 in the small-scale expansion. These lead to residual interactions with much smaller slopes for energies between 100 and 200 MeV. The fact that the main effects of adding the Δ arise from changes in the LECs agrees with the observations of Epelbaum [23] for higher partial waves.

However, the resulting residual interactions now show much stronger energy dependences below about 100 MeV, implying that not all the important long-range forces have been subtracted. Therefore, although the effects of adjusting the LECs are suggestive, full calculations with the Δ at order Q^3 will be needed before one could conclude definitively that the Δ is responsible for the low values of the scales found above.

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