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Essay review

Path integrals and condensed matter physics

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A review of *Functional Integrals and Collective Excitations*. By V. N. POPOV. (Cambridge University Press, 1988.) [Pp. viii + 216.] £30.00, hardback. ISBN 0 521 30777 5.

Path integrals were introduced to theoretical physics by Feynman (1948), and that paper still remains one of the most lucid, intuitive and physically appealing descriptions of what a path integral is. Feynman has emphasized that path-integral techniques may be used as a fundamental means of passing from classical to quantum mechanics, and he showed that they provide a powerful framework on which to hone one's basic understanding of quantum mechanics and with which to supplement the more usual canonical approach. The aesthetic appeal of the path-integral formulation of quantum mechanics was very quickly recognized, and Feynman himself also showed how the formalism could very profitably be exploited practically to attack particular problems or to formulate new approximations. Much of this early work is summarised in his book with Hibbs (Feynman and Hibbs 1965).

In the last fifteen years or so, the application of path integrals—or functional integration as the subject has increasingly become known after the intervention of the mathematicians—to several particularly active areas of theoretical physics, has again seen a tremendous increase. For example, path integration has been widely applied to the very important class of physical systems which essentially contain some element of randomness. Examples include systems with random impurities, random potentials and polymers. In the last case, a typical polymeric molecule such as rubber is often modelled as some form of random walk. The ease with which the path-integral approach makes contact either with the classical limit or with suitable semiclassical approximations also makes it very suitable in connection with the quantum-mechanical states of minimum uncertainty or coherent states. For this reason, applications to such fields as quantum optics abound.

The techniques of path integration have also been applied at very broad and fundamental levels to quantum-mechanical systems. An example is scattering theory, where it is well known that for many physical systems the semiclassical approximations to the scattering process work rather well. Path-integral techniques are again ideal in this regime, and many applications have been made in such fields as chemical physics, atomic physics and nuclear physics, and particularly with regard to the last of these in the domain of heavy-ion scattering, which has become a major experimental tool over the last decade or so. Particularly important applications in elementary-particle physics have been made to the so-called gauge field theories which are believed to provide the correct basis for all of the fundamental physical interactions, including the still missing theory of quantum gravity. Special attention has also focused on the particle-like or spatially extended solutions of these field theories, with non-trivial

topological properties—the ‘solitons’ and ‘instantons’ of so much interest in the 1970s. A typical soliton solution of such an archetypal field theory as the Yang–Mills type is the magnetic monopole first hypothesized by Dirac in 1931. Finally, path integration has by now become a well established and major tool for both formal developments and practical applications in the loosely allied topics of quantum field theory and statistical mechanics. In statistical mechanics, for example, it provided the original framework in which to formulate the very important renormalization group techniques that have been so successfully applied to the general theory of phase transitions and critical phenomena.

Many of these later applications are covered in the excellent book by Schulman (1981), in the preface of which he makes the observation that: ‘Statistical mechanical applications of path integration could easily take up a book of their own’. The present book seems to have been designed partially to fill that perceived niche. A related book (Rivers 1987) in the same admirable series of Cambridge Monographs on Mathematical Physics has also recently appeared, which addresses more fully some of the more recent applications of path integrals to quantum field theory.

So what exactly is a path integral? The original impetus behind its formal development and the later techniques of functional integration, was expressed by Feynman in his so-called ‘sum over histories’ formulation of quantum mechanics. In the older original version of quantum mechanics, the classical notion of a definite trajectory or path for a particle moving in space-time was superseded by the evolution of the quantum-mechanical wavefunction and its interpretation as a probability wave. The usual Heisenberg uncertainty relations thus arise naturally from this canonical picture, and the idea of a definite classical trajectory is thereby replaced by its more ‘fuzzy’ quantum-mechanical counterpart. Feynman took this basic novel feature quite literally and both made it more visual and intuitive, and somehow took it to its logical conclusion. He imagined that the quantum particle could thus behave however it pleased by following *any* permissible path. Roughly speaking, when these various paths or possible ‘histories’ are added together in some suitably weighted fashion, he envisaged that the Schrödinger wavefunction should arise. It is this sum over histories which was formalised as the path integral or, speaking more mathematically, as an integral over an infinite-dimensional function space.

More precisely, the time-evolution operator, which plays a very fundamental role in quantum mechanics, is given as $\exp(-iHt/\hbar)$ in terms of the time t , the Dirac constant $\hbar$, and the Hamiltonian operator H . Path integrals provide us with a very convenient procedure to calculate matrix elements of this evolution operator between arbitrary states of the system. If H describes a single-particle system for example, and the states are chosen to be position eigenstates, the resulting matrix element is simply the quantum-mechanical propagator or one-body Green function, from a knowledge of which all physical observables of the system may be calculated.

In many-body or field-theoretical problems, the Hamiltonian H either describes many interacting particles or an infinite number of field modes. Such basic quantities as the various Green functions and vertex functions may then be similarly expressed in a comparable fashion by appropriate functional averages or path integrals. By contrast, statistical mechanics has at its heart the (un-normalized) statistical density operator, $\exp(-H/k_B T)$, where k_B is the Boltzmann constant and T is the temperature of the system. The trace of this operator is precisely the partition function, a knowledge of which suffices to determine completely the statistical-mechanical properties of the system. The fundamental relationship between quantum field theory and quantum

statistical mechanics is thus just(!) one of an analytic continuation from real to imaginary times of the quantities calculated in the former to their counterparts in the latter. It probably comes as no surprise however that the word 'just' here may cover a multitude of technical problems. It is partly for this reason and partly because the physical applications are often so different in character, that although very similar path-integral techniques can be applied to both quantum-field/many-body theory and to statistical mechanics and condensed matter systems, the present book, for example, deals only with statistical physics applications.

For practical applications of these functional integration techniques, once the path integrals have been properly formulated one naturally needs to approximate them. It is at this stage that their usefulness becomes apparent. The present book by Popov devotes very little space to attempts to justify the formal aspects of the path-integral formalism in a rigorous fashion. Rather, it almost wholly concentrates on the various types of approximation and their applications to a range of physical problems drawn from condensed matter physics interpreted reasonably broadly. Two particular approximation schemes have become associated with path integrals, and both are described and widely applied here. In the first place is the perturbation theory expansion and the related diagrammatic techniques. The second scheme does not comparably rely fundamentally on the basic weakness of the perturbation or the smallness of some parameter in the theory. It is based instead on the assumption of some well defined collective motion of the system, about which it may be hoped that the intrinsic quantum-mechanical fluctuations are small. This basic concept of collective excitations has proven to be an extremely rich device for describing much of the interesting behaviour in many-body systems of interest in condensed matter physics. Examples include plasma oscillations in Coulombic systems, quantum vortices and long-wavelength phonons in superfluids and superconductors, and the collective modes in both the pure helium isotope liquids and their mixtures. The application of path-integral techniques to such collective behaviour provides the subject of this book.

The basic idea of collective excitations is well established from a large corpus of earlier work in many-body and condensed matter theory. In the present context it may be expressed as a reformulation of the relevant functional integral given in terms of some original basic set of variables (functions), to a new set of amplitudes which include as the most important contributors the relevant collective degrees of freedom. The reformulated integral is then usually calculated by the methods of saddle point and background fields. In the case of the original Feynman path integral, the saddle-point or stationary phase approximation leads to the usual semiclassical (or WKB) approximation for quantum mechanics. Similarly, if we consider matrix elements of the time-evolution or statistical density operator between some suitable set of (generalized) coherent states, the resulting equations of motion describe orbits or trajectories through this set of states. Such orbits play a very comparable role to the particle orbits in coordinate space of the allied problem in classical mechanics, and can provide much information on the many-body systems of interest.

We have already seen that path-integral techniques are particularly appropriate to describe both standard perturbation theory and the modifications to it that are necessitated by the presence of collective excitations in the system. The author thus describes in detail how to establish the relevant perturbation theory, and how to pass from one such perturbation theory to another suitably modified or renormalized version of it. Such modifications are necessary for superfluids, for example, and indeed for all of the other systems of condensed matter at non-zero temperature which are discussed in this book.

The book is divided into four main parts, the first of which accounts for about 10% of the book and which sets up the basic framework. It discusses functional integrals and diagrammatic techniques both for systems of bosons and systems of fermions. The quantization is performed using functional integration respectively over Grassmann-even variables (ordinary commuting complex numbers) and Grassmann-odd (anticommuting) variables. The first part ends with a discussion of the important method of successive integration over fast and slow variables. In statistical physics, as in field theory, many of the interesting problems are connected with the existence of so-called infrared singularities which make every (unmodified) perturbation-theory diagram blow up at small momenta or energies. It is precisely this sort of singularity that necessitates the modification of the original perturbative expansion. Typical physical examples are thus again provided by superfluidity, superconductivity, and the general theory of critical phenomena and phase transitions.

The remaining 90% of the book is devoted to applications. The second part deals with superfluid Bose systems, including quantum vortices, while the third part deals with the comparable plasma and superfluid Fermi systems. Particularly full discussions are given here of the collective excitations in both the A- and B-phases of liquid ^3He . This discussion is also very nicely extended to a model of the ^3He film as a two-dimensional analogue of the previous three-dimensional model. Finally, the last part deals with applications to crystals, many-electron atoms, and to the two model Hamiltonian problems provided by the BCS model for superconductivity and the Dicke model, which is used widely in quantum optics, of a single bosonic mode (of the electromagnetic radiation field) interacting with a number N of two-level fermions (atoms).

As may be gathered from the above list of topics, the coverage is illustrative rather than comprehensive. The applications, however, are very well chosen to exemplify the main points. The text is very tersely written, but in such a way that it is still both complete and easy to read. This is no mean feat, and is certainly in the best Soviet tradition exemplified by such authors as Landau. The book draws together a considerable amount of diverse material and presents it in a uniform style that sheds much new light on even the best known of the examples chosen. For these reasons it is highly recommended to both the relative newcomer to statistical physics and condensed matter theory and the established expert in these fields. It will amply repay careful study by such a wide audience.

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