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Calculations of Breakdown Voltage of Rod-plane Air Gaps in the Presence of Water Streams

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ABSTRACT

A breakdown model of rod-plane air gaps in the presence of water streams has been established. Water streams which do not fully cross air-gaps are considered. This model involves the breakup length and surface conductivity of a water stream, the stream's diameter, the number of the water droplets in the remaining air gap and the distance between two adjacent water droplets. The breakdown voltage is considered in two parts: one being the voltage drop along the water stream surface, and the other the voltage drop across the remaining air gap. To study the two components of voltage drop, the equivalent surface conductivity of a water stream and the voltage-current characteristics between two water droplets were investigated. Additionally, theoretical and experimental results are compared and it has been found that under the conditions considered, the breakdown voltages are predictable to within 13%. Evidence regarding the impact of radial dispersion of droplets and their distortion on the breakdown process is considered.

Index Terms — Water stream, breakdown, air gap, rod-plane, water jet breakup, electrified jets

1 INTRODUCTION

THE ac breakdown performance and the breakdown process of short rod-plane air gaps in the presence of water streams have been investigated in [1]. This is of interest because water streams can naturally occur on high voltage outdoor insulators, partially bridging the gap between the electrodes. In the recent past, a high number of trip faults associated with adverse weather conditions have been reported in China [1]. One cause of the trip faults is thought to be the formation of water streams along outdoor insulators. IEC/TS 60815 provides suggestions for the selection of insulators; however it does not give any advice on the selection of air gap clearances in heavy rain conditions.

According to the test results in [1], the ac breakdown voltage of the air gap is affected by the velocity of the water stream, the diameter of the stream and water's conductivity. For a better understanding of the breakdown mechanism across water stream-air gap insulation, a simple model has been developed by the authors to predict the breakdown voltage.

In previous work [2]-[5], researchers have mainly concentrated their efforts on investigating the propagation of a discharge along a polluted insulating surface. However, on the subject of the breakdown or flashover along a water stream little is published. Rumeli [6][7] established a flashover model along a water column based on the well-known Obenaus theory by considering the heating effect of leakage current and the constriction of the water jet. Although the results obtained by this model were in a good agreement with experiments, it

cannot be employed to calculate the breakdown voltage in our case. This is because firstly, the model proposed by *Rumeli* was mainly used to investigate the arc propagation along a water column or a conductive surface. Secondly, the length of the air gap from the high voltage electrode to the water column was only 2 mm, which was originally to initiate the arc along the water column. However, the air gap in our model is much longer than this. Lastly, in *Rumeli*'s model, the water column was not charged, but according to results in [1], the length of water stream decreases with the increase of applied voltage.

In practice, many configurations of air insulation such as that between a conductor and tower are regarded as simple rod-plane structures. On the other hand, experience suggests that heavy rain can pose a significant threat to the reliable operation of transmission lines (in addition to lightning, pollution and icing).

It is proposed that the formation of water streams along the conductors, fittings, and insulators can effectively shorten the insulating gaps and so pose a threat to the safe operation of transmission line systems. Therefore, establishment of a breakdown model of rod-plane air gaps in the presence of water streams is required to better predict breakdown of practical air gaps in wet environments.

2 THE MODEL

2.1 WATER STREAM BREAKDOWN EQUATION

The physical behavior of a water stream and its relationship to an applied voltage and the breakdown process must be considered before developing a mathematical model for the breakdown process. Figure 1 illustrates the situation for a 30 cm gap.

Figure 1. The behaviour of a water stream under the effect of an applied voltage and the breakdown process of the rod-plane air gap (U_0 is the applied voltage, U_b is the breakdown voltage).

Figure 1a shows the profile of a water stream and the distribution of water droplets in the air gap when the applied voltage is 80 kV. Under the effect of the applied voltage, a strong oscillatory motion at the tip of the water stream occurs, so that the droplets formed fall away from the vertical axis. In addition, it can also be seen that the water stream does not flow smoothly. Its surface undulates like a wave with a characteristic wavelength. However, such wave-like phenomenon appears to be less obvious as the applied voltage

is increased.

One of the breakdown images is illustrated in Figure 1b. It can be seen that the spark which originates from the nozzle electrode, progresses along the water stream surface first, and then traverses the remaining air gap. In addition, it can also be observed that the spark passes over the surface of the water droplets which are formed due to the breakup of the water stream because of the effect of surface tension [1]. Based on this process, a sketch of the physical model on the breakdown of a rod-plane air gap in the presence of a water stream is established and shown as Figure 2.

For the breakdown model, three assumptions are made to simplify the calculation. These are:

- (1) The shape of the water stream is considered to be a cylinder with the same radius as the orifice of the nozzle electrode.
- (2) The water droplets fall downward with constant speed along the symmetric axis and with no radial displacement.
- (3) The spark passes along the water stream surface first, and then develops in the air-droplet gaps to the ground plate.

Figure 2. The sketch of the physical breakdown model.

According to Figure 2, we can establish the water stream breakdown voltage as:

$$U_b = U_w + U_a \quad (1)$$

where, U_b is the breakdown voltage, kV; U_w is the voltage drop across the water stream surface, kV; U_a is the voltage drop across the air-droplet gap. In following, the details of U_w and U_a will be discussed.

2.2 DETERMINATION OF THE VOLTAGE DROP ACROSS THE WATER STREAM SURFACE

A first approximation would be to treat the water stream as so conductive compared to the air-gap that U_w can be ignored. Here the water stream is not treated as an ideal conductor and its presence is taken into account. This is necessary because the previous test results [6]-[9] showed that the resistance of the water has a significant impact on the voltage drop along the water column. Additionally, experiments, conducted by the authors [1], also suggest that increasing the water conductivity from 80 $\mu\text{S}/\text{cm}$ to 1000 $\mu\text{S}/\text{cm}$ decreases the breakdown voltage of such a system by around 10%. In [6] and [7],

Rumeli built a mathematical model to calculate the flashover voltage along a water column, in which an equation for solving the resistance of a water column was proposed, and results were in a good agreement with the test results. However, it is found that a big difference occurs when employing this equation to calculate the breakdown voltage under test conditions employed here. For instance, for a gap of 30 cm and a water stream diameter of 2 mm and velocity 1.4 m/s, the model predicts the breakdown voltage decreases by 30 kV as the water conductivity increases from 80 $\mu\text{S}/\text{cm}$ to 1000 $\mu\text{S}/\text{cm}$. In comparison, the decreased voltage difference is 13 kV according to the authors' results in [1]. In [1], tap water with conductivity between 80 and 1000 $\mu\text{S}/\text{cm}$ was used to carry out tests because of the high conductivity of rain in China: In Chongqing, the rain conductivity is 350 $\mu\text{S}/\text{cm}$ and it reaches to 700 $\mu\text{S}/\text{cm}$ in Beijing.

Figure 3 Breakdown voltage variation with water conductivity (d is the gap separation, v is the flow velocity, D is the radius of the water stream). Taken from [1].

The error between the Rumeli's model and our test results may be attributed to Rumeli's model using the volume resistance to calculate the voltage drop along the water column. But, as can be seen in Figure 1, the spark only progresses on the surface of the water stream. It may therefore be more reasonable to consider the surface conductivity rather than the volume resistance. In [10-12], the relation between the equivalent surface conductance γ_e (μS) and the applied water conductivity γ_w ($\mu\text{S}/\text{cm}$) was expressed as:

$$\gamma_e = h \cdot \gamma_w \quad (2)$$

where h is the thickness of the surface layer, cm. As the water stream is shaped as a cylinder in our model, the value of h is taken as the radius of the water stream r .

According to observation, the spark propagates along the surface of the water stream as the breakdown occurs, which is very like the arc propagation along a conductive electrolyte solution. Therefore, the Wilkin's equation [10] is employed to calculate the surface resistance of a water stream:

$$R = \frac{10^3}{2\pi\gamma_e} \left[\frac{\pi L_b}{a} + \log \frac{a}{2\pi r_d} \right] \quad (3)$$

where R is resistance of the water stream, k Ω ; L_b is the length of the water stream, cm; $a = \pi D/10$ is the circumference of the

cross section of the water stream, cm; D is the diameter of the water stream, mm; r_d is the spark root radius (cm) which is a function of current [10]:

$$r_d = \sqrt{\frac{I_b}{1.45\pi}} \quad (4)$$

and I_b is the breakdown current, A. The voltage drop across the surface of a water stream U_w can be obtained:

$$U_w = I_b \cdot R \quad (5)$$

2.3 DETERMINATION OF THE VOLTAGE DROP ACROSS THE REMAINING GAP

2.3.1 DISTANCE BETWEEN TWO DROPLETS

According to the proposed model in Figure 2, the remaining gap is composed of a number of droplet-droplet gaps. The distance S between two droplets can be obtained as long as the average velocity of the droplet v_d and the average breakup time interval Δt are known. To determine v_d and Δt , image process software was used to measure the two parameters by analyzing high speed images.

Figure 4. Determination of the average velocity of a water droplet.

As shown in Figure 4, v_d is determined by measuring the distance from the water droplet (marked with red circle) to the nozzle electrode and the time intervals between the two images above. In each case, hundreds of droplets are used to improve the accuracy of the measurement. Similarly, the average breakup time interval Δt can also be obtained. The results in each condition are given in Table 1.

Table 1 Measured results of v_d and Δt .

D (mm)	v (m/s)	v_d (m/s)	Δt (ms)	Δt_{ave} (ms)	S (cm)
3.0	0.75	1.72	7.1	7.1	1.22
	1.00	1.93	7.4		1.37
	1.40	2.11	6.7		1.50
	1.70	2.50	7.1		1.78
2.0	1.57	2.45	5.8	5.7	1.39
	1.93	3.04	5.6		1.73

Note: the applied voltage is 80 kV for all the results above.

It can be observed that Δt for a fixed diameter of water stream appears to be independent of the velocity of the water stream v . Thus, it can be said that the average breakup time interval is only determined by the diameter of the water stream. Then, the average distance between two water droplets

can be obtained with the measured v_d and Δt_{ave} .

2.3.2 VOLTAGE-CURRENT CHARACTERISTICS BETWEEN TWO WATER DROPLETS

As suggested in the model in Figure 2, it is assumed that the spark progresses through the air gaps between water droplets (and over their surfaces). To investigate the voltage - current characteristics of the air gaps between water droplets, the authors have carried out the experiments according to the test method in [13].

Figure 5. Experiment setup [13].

The experimental setup is shown in Figure 5. The ratio of the voltage divider is 10000:1, and the capacitance is 25 pF. The rated voltage of the transformer is 30kV/30kVA. The rated output of the variac is 410V/72.5A. The leakage current was measured with a 2 kΩ shunt resistor. To form the discharge between water droplets, two syringes were introduced to create water electrodes. Syringes were filled with tap water (80 μS/cm) and connected to the circuit by wire through a sealed hole. Small water droplets (1 mL) were formed in front of the syringes' openings. In [13], the voltage-current relations for the stable arc between water droplets were investigated. In contrast, considering the fact in our model that the spark formed at the moment of the breakdown of the air gap only lasts several micro-seconds, this test mainly focused on the formation of the transient arc.

By adjusting the gap length between the two syringes, the breakdown voltage or the average breakdown electric gradient and breakdown current were obtained. In our tests, the gap lengths were 2, 4, 6, 8 and 10 mm.

Figure 6 Relationship between electrical field and transient breakdown current of a water droplet gap.

The line of the best fit in Figure 6 shows the relationship

between electrical field, E , and transient breakdown current, I_t of a water droplet gap:

$$E = 0.219I_t^{-0.906} \quad (6)$$

with a regression coefficient R^2 of 0.9939.

Hence, together with the distance between water droplets, S in Table 1, the voltage drop across the remaining gap can be expressed as:

$$U_a = L_0 \cdot E = 0.219(d - L_b - N \cdot D_w / 10) \cdot I_t^{-0.906} \quad (7)$$

where L_0 is the length of the remaining air gap, cm; d is the electrode separation, cm; L_b is the length of the water stream, cm; N is the number of water droplets gaps, D_w is the diameter of the water droplets, mm; E is the voltage gradient, kV/cm. Typical experimental values of these parameters are provided in Table 2.

Table 2. The breakup length of the water stream L_b and the diameter and number of water droplets D_w in the gap ($U_0=80$ kV).

D (mm)	v_d (m/s)	Δt_{ave} (ms)	L_b (cm)	D_w (mm)	N
3.0	1.7	7.1	5.5	2.2	20
3.0	1.9	7.1	6.1	2.4	17
3.0	2.1	7.1	6.8	2.6	15
3.0	2.5	7.1	8.5	2.7	12
2.0	2.5	5.7	4.4	1.8	18
2.0	3.0	5.7	4.7	1.7	15

Note: $N=(30-L_b)/(v_d \times \Delta t_{ave})$

2.4 DETERMINATION OF THE CURRENT IN THE SPARK

Based on the work in [14] and [15], a mathematical expression showing the relation between the current in a spark channel developing in air and the streamer length has been deduced by Waters, which can be written as:

$$I = 278L_s \quad (8)$$

where L_s is the streamer length, cm. To verify this equation, a comparison to Rumeli's model [6] is made, and shown in Figure 7 and Figure 8, in which the separation of the rod-plane air gap is 30 cm and L_s is assumed equal to S .

Figure 7 shows the variation of the breakdown current I_b with the water conductivity γ_w ranging from 80 to 1000 μS/cm. It is shown that I_b , calculated by Rumeli's model, increases with an increase of γ_w linearly. However, according to Waters's model, I_b is independent of γ_w and only determined by the gap length S . In addition, as the water conductivity increases, the variation of the breakdown current is by an order of magnitude, from 9 to 80 mA, according to the Rumeli's theory.

On the other hand, as illustrated in Figure 8, the breakdown currents obtained by the two models both increase with the increase of the water stream velocity. However, there is a difference between the two models which lies in the values of I_b . It can be seen that the current obtained by the Waters's model varies from about 30 to 50 mA within the range of the water conductivity considered. In contrast, a small increase

and absolute values of around 9 mA are determined by the Rumeli's model.

To determine which theory is more suitable for the calculation of the breakdown voltage of an air gap, a group of experiments was carried out to measure the breakdown current. The details of the experiment are given in [1].

The authors measured the breakdown current in the conditions of different nozzle diameter D , velocity of water stream v and water conductivity γ_w . However, it was found that the relationships between the current and these parameters were not obvious and appeared to be random. This suggests a fairly chaotic process. Therefore, we only recorded the variation range of the breakdown current corresponding to the test conditions in Table 1. In these conditions the minimum and maximum value of I_b were 28.0 mA and 61.3 mA respectively.

Figure 7 Comparison between Rumeli and Waters models (variation with water conductivity).

Figure 8. Comparison between *Rumeli* and *Waters* model (variation with the velocity of the water stream).

In Figure 7 and 8, it is shown that the breakdown current calculated by Rumeli is independent of the velocity of a water stream, however, according to Waters model, the current is not affected by the water conductivity. According to our experiments [1], the water velocity can decrease the breakdown voltage by 40%, which is more significant than the effect of water conductivity. Thus, after considering Figure 7

and Figure 8 in comparison with the measured breakdown current results, Waters's model is employed to determine the breakdown voltage of air gap.

By combining equations (1), (3), (4), (7) and (8), we can write:

$$U_b = \frac{10^3 I}{2\pi\gamma_e} \left[\frac{10L_b}{D} + \log\left(\frac{D}{20\sqrt{\frac{I}{1.45\pi}}}\right) \right] + 0.219L_0 I^{-0.906} \quad (9)$$

3 COMPARISON WITH EXPERIMENTAL RESULTS

Test results which have been previously carried out by the authors [1] will now be introduced and applied to the model proposed in this paper. The ac voltage was generated by an 800 kV/0.5 A transformer and measured to within 2%. In all of the experiments a uniform increase in voltage of 5 kV/s was used. At least 10 experiments were performed for each condition.

The test conditions and associated results are given in Figure 9, Figure 10 and Table 3, where U_{b1} is the calculated breakdown voltage, and U_{b2} is the test result, $\Delta U = 100 \times (U_{b2} - U_{b1}) / U_{b2}$. The rod-plane air gap is 30 cm.

Figure 9. Variation of breakdown voltage with the velocity of water stream for water conductivity of 80 $\mu\text{S/cm}$.

Figure 9 shows the comparison of U_b vs. v characteristics for the rod-plane air gap between experimental and theoretical data, for water conductivity of 80 $\mu\text{S/cm}$. It can be seen that the agreement between the model and experimental results is good except for the point in the condition of $D=2$ mm, $v=1.57$ m/s, for which the difference is 9.5%. However, for other cases, the calculated errors are all within 5%.

Figure 10 illustrates another comparison with respect to U_b vs. γ_w between the model and the experimental results. It can be seen that the model presented in this paper has given satisfactory agreement with the test data for the 2 mm diameter water stream. In comparison, the agreement between the model

and experimental results is not so good in the condition of $D=3$ mm and as can be seen the maximum relative variation is 13%.

Figure 10. Variation of breakdown voltage with the water conductivity.

Additionally, it can also be observed that, according to Figure 10, the results given by the authors' model are within the top and bottom envelopes of the experimental lines. Considering the range of experimental results, it can be concluded that the model is capable of simulating breakdown voltage of rod-plane air gap in the presence of a water stream at least with the same order of accuracy as that the experiment.

The relatively good agreement between experiment and model suggest also that the three initial assumptions listed in Section 2.1 are appropriate.

Table 3. Comparison between experimental and theoretical results.

D (mm)	v (m/s)	L_b (cm)	γ_w (μ S/cm)	U_{b1} (kV)	U_{b2} (kV)	ΔU (%)
3.0	0.75	5.48	80	102.5	101.7	-0.7
	1.00	6.05	80	94.2	96.9	2.8
	1.40	6.81	80	87.6	89.4	2.0
	1.70	8.50	80	79.7	82.3	3.1
	1.40	6.81	220	79.7	91.2	12.7
	1.40	6.81	520	77.0	87.5	11.9
	1.40	6.81	1000	76.1	84.4	9.8
2.0	1.57	4.37	80	110.0	100.4	-9.5
	1.93	4.74	80	100.6	98.8	-1.8
	1.57	4.37	220	99.2	95.6	-3.8
	1.57	4.37	520	95.7	92.3	-3.6
	1.57	4.37	1000	94.4	87.5	-7.9

Note: the breakup length of the water stream is measured with an applied voltage of 80 kV.

4 DISCUSSION

4.1 TEMPERATURE RISE OF THE WATER STREAM

According to Rumeli's theory [6], the variation of the temperature along the water stream due to the heating effect of the leakage current can be considered in the process of the establishment of the flashover model along a water jet. By assuming that the heat dissipated in a water stream element δ is

only used for raising its temperature, a heat balance equation can then be obtained [6]:

$$i^2 \left(\frac{\delta}{\gamma A} \right) \Delta t = (\delta A \rho) c \Delta T \quad (10)$$

where $\gamma = \gamma_0(1 + \beta\tau)$ is the specific conductivity of water, A is the area of the cross section of the water stream, ρ is the water density, c is the specific heat of water.

In contrast to the assumption made in this paper, Rumeli assumed that the diameter of the water stream D is not constant along the jet due to acceleration, and a relationship between D , flow rate V and jet velocity v is approximated as:

$$V = A_0 v_0 = Av \quad (11)$$

Where A_0 , v_0 are the initial area and velocity of the jet respectively.

To express the constriction of the water stream, Rumeli defined a constriction ratio α :

$$\alpha = \frac{v}{v_0} = \sqrt{1 + \frac{2g}{v_0^2} l} \quad (12)$$

where $g=981 \text{ cm/s}^2$, l is the distance from the element δ to the nozzle electrode.

By combining equations (10) - (12), the temperature distribution along the water stream can be obtained:

$$\tau = T - T_0 = \frac{1}{\beta} [-1 + \sqrt{1 + Ki^2 (\alpha^3 - 1)}] \quad (13)$$

where $K=32\beta v_0/(3\pi^2 g \rho c \cdot D_0^4 \cdot \gamma_0)$.

With the use of equation (13), the temperature rise of the water stream can be calculated, and a 20°C rise in temperature was found according to Rumeli's results.

However, the effect of such a temperature rise on water conductivity is not considered in our model. The reason for that is principally that in reality, the temperature rise along the water stream will be small (less than 20 °C) because the flow of the water stream reduces the heating effect of the leakage current. In addition, according to the calculation function in [6], a 20°C rise in temperature only leads to a 44 μ S/cm increase in water conductivity from the initial value is 80 μ S/cm. It can be seen in Figure 3 that the increase in water conductivity from 80 to 220 μ S/cm decreases the breakdown voltage by less than 5 kV. This indicates that the temperature rise has little effect on the variation of the water conductivity.

4.2 EFFECT OF THE CONSTRICTION OF THE WATER STREAM

One of the assumptions underlying both the model presented here and Rumeli's model is that the profile of the water stream is approximated as a cylinder, which is quite different with the real one that is believed to behave as a wave-like shape.

Figure 11a shows the profile of the water stream without an applied voltage. It can be seen that the diameter of the water stream does not keep constant in the process of the falling of the jet but varies with the vertical axis distance z . This variation is less obvious than in the case that with the applied

voltage, as shown in Figure 11b. This phenomenon was first investigated by Plateau [17], and later Lord Rayleigh gave an analytical explanation [18-19] to this observation based on Plateau's work.

According to the observed images and the theoretical model above, the radius of the water stream can be written as [18]:

$$r = R_0 + \varepsilon e^{\omega t + ikz} \quad (14)$$

where $R_0 = D/2$; the initial perturbation amplitude $\varepsilon \ll R_0$; ω is the growth rate of the instability; k is the wave number and $k = 2\pi/\lambda$; λ is the wavelength shown in Figure 11.

Figure 11. Variation of the profile of the water stream with applied voltage and the theoretical model [16].

In our model, the calculation of the equivalent surface conductivity γ_e is linked with the radius of the water stream r . In this case of an undulating surface, γ_e is no longer a constant after substituting equation (14) into equation (2), and this substitution will decrease the value of γ_e . In other words, assuming the radius of the water stream as a constant reduces the resistance of the jet and decreases the breakdown voltage as the breakdown current is only dependent of the separation between two water droplets.

One way to improve the calculation accuracy is to determine r by solving the unknown parameters ε , ω and k in equation (14). Accordingly, many researchers [20-24] have contributed their work to this subject but few of them focused on the effect of the applied voltage [25], [26]]. Unfortunately, the models proposed in [25], [26]] only agree with the test results at lower applied voltage and the calculation error becomes larger as the voltage increases to 80 kV according to the authors' calculations. (e.g. when the radius of nozzle electrode is 1.5 mm, the applied voltage is 80 kV and the velocity of the water stream is 1 m/s, the diameter of the breakup droplets is 1.3 mm according to the proposed model. However, according to our test results, the diameter of the breakup droplets is about 2.35 mm: a relative error of 45%).

4.3 RADIAL DISPLACEMENT OF WATER DROPLETS

To simplify our model, the radial displacement of water droplets r_r is set to zero which is in contrast to the fact that the disintegrated water droplets are characterized by a radial component of velocity.

The radial displacement of droplets is mainly due to the electrostatic force which can provide a pressure difference ΔP_e

across the water stream surface.

As a result of the applied voltage, the interface between the water stream and surrounding air accumulates a density of surface charge ρ_s . In this case, the normal field components involving surface charge is:

$$\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s \quad (15)$$

where "1" and "2" corresponds to the water and air domain, respectively.

According to the Korteweg-Helmholtz Polarization equations [27]:

$$\begin{cases} T_{i,j} = \varepsilon E_i E_j - \frac{1}{2} \delta_{i,j} \varepsilon E_k E_k & \text{(stress tensor)} \\ \delta_{i,j} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases} \\ E_k E_k = E_i E_i + E_j E_j \end{cases} \quad (16)$$

The stress tensor $T_{i,j}$ represents the stress in the i^{th} direction for an interface with a normal in the j^{th} direction. The subscript n and t represent the normal and tangential components at the interface. Then $T_{n,n}$ is the normal stress tensor and $T_{t,n}$ is the tangential stress tensor.

Figure 12. Distribution of water droplets as the voltage is applied.

By combining equations (15) and (16) and applying the tangential boundary condition, the pressure difference ΔP_e can be written as:

$$\Delta P_e = T_{n2n2} - T_{n1n1} = \frac{-\varepsilon_0(\varepsilon_r - 1)}{2} (\varepsilon_r E_{1n}^2 + E_{1t}^2) + \rho_s (\varepsilon_r E_{1n} - \frac{\rho_s}{2\varepsilon_0}) \quad (17)$$

This pressure difference ΔP_e can provide the water stream with an electrostatic force, leading to the radial movement of the water stream.

Under the effect of the electrostatic force, it is found that the tip of a water stream is characterized by helix motion, as shown in Figure 13.

Through the analysis of the helix motion of a water stream, the radial velocity of water droplets can be calculated by simply considering water as an ideal conductor. In this way, equation (17) can be simplified as:

$$\Delta P_e = T_{n2n2} = \frac{1}{2} \varepsilon_2 E_{2n}^2 \quad (18)$$

Figure 13. Helix motion of a water stream under the effect of applied voltage of 80 kV.

In the following we assume the equivalent radius of each helix of a water stream and the angular velocity in the radial direction is constant (r_h and ω_a). The opposing radial components of electric field for a helical water stream are shown in Figure 14.

Figure 14. Distribution of Electric field of a unit water stream.

By applying the Newton's second law of motion, we can obtain:

$$\frac{1}{2} \varepsilon_2 (E_A^2 - E_B^2) \cdot \Delta S = 2 \Delta m \cdot \omega_a^2 \cdot r_h \quad (19)$$

Where ΔS and Δm are the surface area and weight of this unit.

According to our observations, the breakup of a water stream, occurs at a regular time interval of Δt . over half a helical pitch. Therefore, the angular velocity can be expressed as:

$$\omega_a = \frac{\pi}{\Delta t} \quad (20)$$

The mechanical equilibrium between surface tension, the electric field and gravity can be described by the augmented Young-Laplace equation:

$$\begin{cases} \frac{2T_s}{r_h} = \Delta P + \rho gh + \frac{1}{2} \varepsilon_0 E_A^2 \\ \frac{2T_s}{r_h + D} = \Delta P + \rho gh + \frac{1}{2} \varepsilon_0 E_B^2 \end{cases} \quad (21)$$

where T_s is the surface tension, ΔP is the pressure difference across the interface. ρ is the density difference across the interface, g is the gravitational acceleration, h is the vertical distance of any point on the stream surface from the reference.

By considering the constriction of the water jet according to [6], the radius of the tip of the water stream can be induced as:

$$R = R_0 / \left(1 + \frac{2g}{v_0^2} L_b \right)^{\frac{1}{4}} \quad (22)$$

where R_0 is the initial diameter of the water stream, and v_0 is the initial velocity. In this paper, v_0 is assumed to be equal to the average velocity of a water stream v .

By combining equations (19)-(22), we can obtain the equivalent radius r_h :

$$r_h^3 + 2Rr_h^2 - \frac{T_s \Delta t^2}{R \rho \pi^2} r_h = 0 \quad (23)$$

The radial velocity of water droplets can now be calculated from equation (23). In addition, by analyzing the recorded images, the radial velocity of water droplets can also be measured. The results are shown in Table 4, where v_{r1} are the calculated results and v_{r2} the measured ones.

Table 4. Radial velocity of water droplets.

$D(\text{mm})$	$U_0(\text{kV})$	$v(\text{m/s})$	$L_b(\text{mm})$	$\Delta t(\text{ms})$	$v_{r1}(\text{m/s})$	$v_{r2}(\text{m/s})$
3	80	0.75	54.8	7.1	0.24	0.29
3	60	0.75	60.7	7.8	0.22	0.25
3	40	0.75	65.3	8.5	0.21	0.20
3	80	1.00	60.5	7.4	0.23	0.24
3	60	1.00	61.9	8.1	0.21	0.21
3	40	1.00	74.8	8.1	0.20	0.18
3	80	1.40	68.1	6.7	0.22	0.19
3	80	1.70	85.0	7.1	0.20	0.16

According to Table 4, it can be seen that v_{r2} increases with an increase of the applied voltage and decreases with the increase of the velocity of a water stream as the applied voltage keeps constant.

4.4 DISTORTION OF DROPLETS

Based on the breakdown process shown in Figure 1b, it can be observed that the spark passes over the surface of individual water droplets. This is believed to result from the distortion of droplets due to the local increase of the electric field strength. To make it clear, the process of the distortion of droplets is illustrated Figure 15.

As shown in Figure 15, before the breakdown occurs, droplet 'A' elongates and forms a cone-like tip. The formation of the tip increases the local E-field around the droplet due to the very small radii curvature, and thereby attracts the spark toward to the droplet. However, the formation of the cone-like tip is only observed on the droplet in vicinity of the water stream. For other water droplets in the remaining gap, most of them are ellipsoidal because of the effect of electric field.

Figure 16 shows the electric field distribution calculated by FEA across a 10 cm rod-plane air gap in the presence of spherical water droplets of a range of radii. As illustrated in

Figure 16, without droplets it can be seen that the field distribution from plane to rod is very smooth as there are no discontinuities in the air gap, and the maximum field located at the tip of rod is about 22.5 kV/cm. Even spherical water droplets affect the field distribution in the air gap significantly. Moreover, it can also be observed that the field distortion resulting from droplets which are close to the rod is more intense than that caused by droplets near the plane. That is to say, the effect exerted by the water droplets is determined by the distance from the droplet to the water source. Clearly the distortion of droplets by the local field, and in particular cone formation as shown in Figure 15 would accentuate this further.

In section 4.3, the radial displacement of water droplets is analysed. To consider the effect of the radial distribution of water droplets on the breakdown voltage, other numerical simulations can be carried out, as shown in Figure 17. Here the impact of different distributions of droplets on $E_{\max}$, the maximum field in the air gap, is considered.

Figure 15. Instability of water droplet before discharge ($t=0$ ms represents the moment of breakdown).

The size of the drops and their proximity to the line of symmetry impact $E_{\max}$, which is at the needle tip. In Figure 17a the maximum field is 19.9 kV/cm. After increasing the size of water droplets the maximum field increases to 22.3 kV/cm.

According to Table 4 and [1], the increase of water stream velocity decreases the radial displacement and increases the size of water droplets. In other words, the increase of water

stream velocity not only increases the breakup length of a water stream but also affects the radial distribution of water droplets, and both these factors will increase the maximum electric field at a given voltage.

Figure 16. Electric field within a rod-plane air gap with and without spherical water droplets (FEA simulation conditions: length of air gap 10 cm; the tip of the rod is a hemisphere of radius of 2 mm; various droplets with radii 1, 2, 3 mm; the voltage applied on the rod electrode is 30 kV).

Figure 17. Maximum field of rod-plane air gap with rain droplets of different sizes (length of air gap is 10 cm; the tip of the rod is a cone shape with a radius of 1 mm; the droplet is not charged and with an equivalent radius of (a) 1 mm, and (b) 2 mm. The voltage applied on the rod electrode is 30 kV)

5 CONCLUSIONS

The breakdown voltage and current across an air gap of 30 cm into which a water stream is flowing have been calculated. In the model presented, the breakdown voltage is a function of the breakup length of the water stream, its surface conductivity and diameter, the number of the water droplets and the distance between consecutive water droplets. It has been found that for the given experimental conditions, the model is in good agreement with the experimental results for which measurements are available: air gaps in the range of 20 to 50 cm. For the case of diameter of nozzle electrode D of 3 mm, the predicted breakdown voltage ranges from 103 to 76 kV in comparison with the test results changing between 102 and 84

kV. For a D of 2 mm, the calculated voltages vary between 110 and 94 kV, all higher than the experimental results. It is shown that the relative error ranges from 1-13%.

The difference between the theoretical and experimental results can be explained by the limitations of the assumption that the shape of the water stream is cylindrical. In addition, the displacement of the water droplets from the line original line of propagation of the stream has not been considered in the quantitative model but is seen to be physically significant.

Under the effect of the electrostatic force, it is found that the tip of a water stream is characterized by helix motion, which is believed to be the reason for the radial displacement of water droplets. According to observations, it is found that the radial velocity of water droplets increases with an increase of the applied voltage and decreases with the increase of the velocity of a water stream as the applied voltage keeps constant.

A cone-like tip is formed on the surface of water droplets in the vicinity of the water stream. This increases the local electric field around the droplet and influences the propagation path of the spark.

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