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Statistical field theory

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Essay review Statistical field theory

R. F. BISHOP

A review of *Statistical Field Theory*. (Frontiers in Physics, Volume 66.) By GIORGIO PARISI. (Addison-Wesley, 1988.) [Pp. xvi + 352.] \$48.50, hardback. ISBN 0 201 05985 1.

There is undoubted pressure on any rapidly developing subject for it to fragment into ever more narrow fields of specialisation. By itself this form of intellectual evolution is both natural and possibly even inevitable. Yet its usual consequence is that the lines of communication across these sub-divisions become increasingly difficult, and indeed can become so tenuous and stretched as to hinder the future healthy development of the erstwhile parent subject. This trend is (almost) as true of theoretical physics as it is of mathematics or any other of the sciences. Except for the most romantically optimistic, most of us have accepted, possibly with regret, that the ideal of the unity of science is nowadays nought but a pipe-dream. Even in a single discipline such as theoretical physics, where unity in the ultimate form of some unified and universal theory is still a vision cherished by many, most practitioners accept the diversity of the subject as part of its intellectual richness and of its enormous range of practical applicability.

Nevertheless, the increasing fragmentation of physics is a source of concern. If we accept that the ideal of unity is beyond our immediate reach, we are certainly still entitled to nurture its cousin, connectivity. Without question there are often rich rewards and insights to be had by bringing otherwise disparate areas into relation. Three sets of concepts in particular have come to the forefront in recent years in providing links across, and thereby means of skewering together, the more traditional subdivisions of physics, sliced largely according to energy scale. These three modes of thinking about physical systems may be denoted and loosely differentiated by the terms statistical mechanics, quantum field theory and quantum many-body theory, although the boundaries between them are themselves becoming increasingly blurred, as is discussed below.

Thus, in the last thirty years or so there has slowly emerged from various fields of physics a set of common and very powerful ideas. Theorists working in such diverse fields as high-energy particle physics and low-energy condensed matter, have come to realise that there are certain key concepts which span these fields and enable them to share and mutually enrich a rather fertile common ground. Most of these shared ideas first fell more or less squarely into one of the above areas of statistical mechanics, quantum field theory or quantum many-body theory, which burgeoned as major fields of research in their own right to accommodate them. More recently, the last decade has seen the further rapid development of this trend across the mutual boundaries of these three areas, to provide even greater areas of commonality. The at least partial dissolution of one of these boundaries has created the subject of modern statistical field theory. Its discussion as a set of unifying concepts is the topic of the present book.

In many ways this book is a natural successor to such earlier classics as that by Kadanoff and Baym (1962) in the same *Frontiers in Physics* series, and that of Abrikosov, Gorkov and Dzyaloshinski (1963). However, where these predecessors focused particularly on quantum field theory as applied to statistical physics, Parisi provides an overview of a much greater synthesis between the two fields, and which now considerably transcends the employment of field theoretic methodology. This genealogy of the literature neatly reflects the changing emphasis that has taken place in these fields. Whereas in the early days those features of condensed matter systems which were mostly studied at the microscopic level, may be largely characterised as *local* properties (for example, the local hydrodynamical densities, and order parameters), increasing attention is nowadays focused on such *global* properties as phase transitions and critical phenomena, spontaneous symmetry breaking, and topological excitations.

Even within statistical physics itself, this shift from a local to a global description may be seen at the most fundamental level. The traditional, and intrinsically local, approach to statistical mechanics at finite temperatures

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has hinged on the ensemble approach of Gibbs and the use of the statistical density operator. On the other hand, we can imagine the Hilbert space for an arbitrary quantum many-body problem as being characterised in terms of some complete set of many-body amplitudes (or generalised order parameters or generalised mean fields), which exactly describe an arbitrary state of the system. This set of quantities may be thought of as defining some representative point in an appropriately defined phase space. The Schrödinger equation of motion for the system may then, in principle, be exactly decomposed into a set of microscopic equations for these amplitudes. One may regard it as a major task of quantum many-body theory to define this complete set of amplitudes in an appropriate fashion, and to give exact or approximate prescriptions to extract from them a proper local hydrodynamical description of the system. This description should cover such general features as non-equilibrium behaviour, and the possibility of such phenomena as phase transitions, states of topological excitation or deformation (for example, vortices, solitons), and broken symmetry.

From the above viewpoint, the temporal behaviour of the system is described by some trajectory in the phase space along which the chosen representative point moves. Alternatively, according to a more modern view of statistical mechanics, we should focus attention not on single trajectories in this phase space but on the totality of all possible trajectories, namely the so-called *phase portrait*. This stance clearly leads to a global description of the system. Mathematically, one can see that one is focusing more on the large-scale geometrical, and especially topological, properties of the entire phase space, rather than on local features at given points within it. One is led rather naturally to think in terms of the language of *fibre bundles* of trajectories, rather than of individual trajectories.

This overall shift of attention in our description of condensed matter systems is naturally mirrored elsewhere in the techniques used. Most of the early work hinged on the application of Green function and allied techniques of quantum field theory. Although such phases as the superconducting phase were certainly studied, this was generally done from the starting-point of some model for that phase, rather than for the phase transition itself. Conversely, the newer viewpoint has brought to the fore both more recent techniques in field theory, especially those emanating from functional or path-integral techniques, and such novel techniques as the renormalisation group. It is to these later developments and their applications that Parisi devotes most of his attention.

This book focuses at every opportunity on the common concerns of quantum and statistical mechanics, and

every attempt is made to draw connections between the two fields. One of the running themes used to illustrate these connections is the technique of functional integration, which is introduced at an early stage and then used as a common thread to provide continuity. On the one hand, in this context it is not surprising to see a discussion of the way in which the functional integral (say, for the statistical density operator) of statistical mechanics may be regarded as the analytic continuation to imaginary times of the real-time Feynman path-integral representation (say, for the time evolution operator) of quantum mechanics, and vice versa. Nevertheless, Parisi draws this equivalence between the corresponding imaginary-time and real-time theories in a most lucid fashion, particularly in terms of the correspondence between the correlation functions of statistical mechanics and the Green functions or ground-state expectation values for the time-ordered products of quantum mechanics. On the other hand, at the same time the author also describes other less known bridges between the two fields, such as the transfer matrix. This particular technique is perhaps best known as a tool for obtaining exact solutions of such one-dimensional and two-dimensional models in statistical mechanics as the Ising model. The discussion of how the technique can also be used to draw parallels between statistical and quantum mechanics is but one example of the many masterful insights offered by this book.

It is a characteristic of this book that even where it discusses material which has received a great deal of recent attention, it still manages a real freshness and originality of approach. Parisi has the lightness of touch to get to the heart of the matter in a very compact yet rigorous fashion. A good example of this is provided by the exposition and application of path integration, where his approach may be compared, for example, with that in the three excellent recent books by Schulman (1981), Popov (1987) and Rivers (1987), each of which is devoted wholly to this material. The latter two authors are concerned with the separate applications of path integral methods in statistical mechanics and quantum field theory respectively, whereas Schulman covers a wide variety of applications including some drawn from both of these fields. With much less space at his disposal, Parisi manages to convey not only the essence of the method but also both its universality and some of its distinctive features. To say that the present book complements these other fine works with regard to this material, is to offer it high praise indeed.

One example of many that could be cited, that arises from treating statistical field theory as a unified subject, is the very nice discussion of the advantages of working with the imaginary-time formalism over the real-time formalism in view of the oscillatory integrals that arise in

the latter case. The associated mathematical problems in connection with the lack of absolute convergence, particularly with regard to the taking of various physical limits, are highlighted. Indeed it is a remarkable feature throughout this book that although the physics is never allowed to be buried by the formal mathematical manipulations, the exposition of the mathematical techniques and where problems may occur, is handled with great regard to rigour. It is a sure mark of the high level of both the physical intuition and the mathematical skills of the author that this juxtaposition of the details of technique with physical applications is always handled with great delicacy.

Beginning with traditional classical equilibrium statistical mechanics, the order of presentation of the material is first aimed at reaching the functional integral representation of Euclidean field theory. The intervening route proceeds via magnetic systems (and especially the Ising model), the Landau–Ginzburg model, and second-order phase transitions and critical phenomena. In keeping with the discussion cited above of the relative merits of introducing the functional integration approach at imaginary-times first, it is only at a much later stage that the Feynman path-integral representation for quantum mechanics is introduced. It is then finally left almost to the end of the book to complete the process for relativistic quantum field theory. This approach may be contrasted with the historical introduction of path integrals in quantum mechanics originally by Feynman in 1948 (although the seed of the idea was sown in a 1933 paper by Dirac), and with their comparable first introduction in the book by Schulman (1981) in the context of quantum mechanics for real time. The remainder of the book is devoted to various applications of the functional integral approach, and to such other topics as the approach to equilibrium, particle–field duality, stochastic descriptions of many-body systems, and computer simulations thereof.

Not only are the scope of the book and the range of material covered very imaginatively conceived, but the amount of detail and the depth of thought exhibited on each topic are also extremely impressive for a book of this size. In part this feat is achieved by a careful pruning of topics which do not illustrate the connectedness of the overall subject matter. An example is afforded by such quantum-field theoretic techniques or concepts as second quantisation, Fock space, and scattering theory, which are special to the real-time formalism. Since most of these concepts are adequately discussed in the older textbooks, Parisi amply succeeds in making a virtue out of their scant treatment or total omission.

The author leads the reader with consummate skill over a very wide terrain of material. The choice of illustrative material treads the very careful and skilfully

chosen path between condensed matter and particle physics that one would expect from someone as uniquely qualified as Giorgio Parisi to speak authoritatively on behalf of both fields. Although the coverage is often brief to the point of sketchiness on many topics, I regard this as ultimately being one of the strengths of the book. The resulting thumbnail sketches are so painstakingly and masterfully drawn as to amply repay their close study. They provide a superb introduction for the beginner into areas which are otherwise difficult to find described in such an integrated fashion, or, indeed, sometimes even at all. At the same time even the seasoned expert will be constantly enlightened or challenged by the rigour and depth of the descriptions. Thus the author is always at pains to point out the possible pitfalls or sources of error or confusion. Furthermore, he is usually able either to resolve them or to indicate the means of their resolution that might be sought elsewhere. Alternatively, where there are still unresolved problems in the formalism, this fact is stated very clearly.

There are so many fine thumbnail sketches included that it is difficult to choose examples. The book opens with a short chapter on the basic axiom of statistical mechanics and a description of entropy and absolute temperature that would be hard to better in such a short space. Similar beautifully crafted miniature portraits follow on the essentials of such diverse topics as functionals and Borel summation; and such aspects of critical phenomena as scaling behaviour, non-integral spatial dimensionality D , and why the theorist's life becomes simpler near $D = 4$. With regard to spontaneous symmetry breaking, the related aspects of the so-called no-go theorems, the appearance of Goldstone bosons, and the corresponding importance of the region around $D = 2$, are particularly well described. In the latter portion of the book, specially noteworthy are the descriptions of the transfer matrix technique, tunnelling phenomena, Levinson's theorem and the more general trace identities of quantum scattering theory, and the particle–field duality of quantum mechanics. The last four chapters contain a large number of gems. These include brief but very lucid descriptions of the essentials of both such old topics as quantum statistical mechanics, the microcanonical and canonical thermodynamic ensembles, ergodicity, mixing, irreversibility, and the Kolmogoroff–Arnold–Moser theorem, as well as such newer topics as various stochastic approaches and the very powerful computer simulation tools of molecular dynamics and the Monte Carlo method.

In conclusion, the fact that Giorgio Parisi is one of the deepest and most lucid thinkers in the fields covered by the umbrella title of static field theory, shines through on almost every page. Each chapter provides original insights or new angles on old problems. The entire book

is thought-provoking. It is a first-rate addition to a venerable series which, under the continuing editorship of David Pines since its inception, has amply fulfilled its stated aim of communicating in a coherent fashion recent developments in the most exciting and active areas of physics. It brings together a remarkable variety of material and viewpoints which are difficult to find in any of the more specialised texts which are devoted to only a portion of the subject matter. The book is highly recommended to all whose interests lie within the broad purview of its field of discourse.

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