Determination of sectionalising strategies for parallel power system restoration: A spectral clustering-based methodology

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Abstract: Parallel Power System Restoration (PPSR) accelerates the restoration of a system in complete blackout by restoring isolated sections (islands) of the network in parallel. These islands are defined during the preparation stage of PPSR using a sectionalising strategy that considers system information collected after the blackout and must satisfy multiple PPSR constraints. This paper introduces a methodology based on spectral clustering that, in contrast to existing approaches, uses the physical and inherent properties of the network to determine a suitable solution. An undirected edge-weighted graph is initially constructed based on the electrical distance between buses, and constraints related to transmission line availability and cranking groups are included by modifying the edge-weights of the graph and using a subspace projection. This graph is then used to define islands that have strong internal connections but weak external connections, whilst satisfying the following constraints: blackstart availability, load-generation balance, voltage stability and the ability to monitor synchronisation between adjacent islands. Simulation results for the IEEE 39- and IEEE 118-bus test systems demonstrate the ability of the methodology to define a strategy that creates strongly connected islands. Additionally, they indicate that the new approach determines solutions that have larger ratios between the inter-cluster electrical distance and the intra-cluster electrical distance for larger systems.

Keywords: Constrained spectral clustering, graph theory, parallel power system restoration, sectionalising of power systems.

1. Introduction

Power System Restoration (PSR) aims to restore systems that have been affected by a partial or complete blackout as quickly and as safely as is reasonable [1]. The availability and deployment of the blackstart units in a system is a key element of restoration, as they provide the cranking power that allows the systems non-blackstart units to be brought online. This cranking task can be accomplished by adopting a build-down strategy, which entails the use of centralised cranking sources to provide cranking power to the entire system [1-3]; however, this approach may take a long time when restoring a large system that is in complete blackout [1, 4]. Therefore, Parallel Power System Restoration (PPSR), is commonly adopted by the utilities, as it accelerates the restoration process by sectionalising the area in blackout into multiple islands (subsystems) that are restored in parallel using their own cranking sources [1-7].

PPSR can be divided into three stages [4]:

1. Preparation;
2. System Restoration; and,
3. Load Restoration.

The first task in the preparation stage is assessing the status of the post-blackout system. Then a Sectionalising Strategy (SS) is defined that considers this new system status and that satisfies multiple PPSR constraints. Finally, the blackstart units are restarted. The system restoration stage involves the energisation of the skeleton of each island to allow the blackstart units to send cranking power to the non-blackstart units in their island. Load is restored during the system restoration stage for the purpose of stabilising frequency and voltage. The load restoration stage involves the load in each island being restored as fully and as quickly as is reasonable [4]. Finally, when most of the load is restored, the islands that have been restored independently are reconnected.

The definition of a suitable SS is a critical task that must be performed by the operators in the control centre during the preparation stage of PPSR [5-7]. This strategy defines the set of lines that optimally sectionalise the entire area that is in blackout into multiple islands [1-5]. It is important that the SS is defined using updated system information that is received after the blackout has occurred; this is necessary to ensure that it represents the actual system topology and the availability of its elements [1, 4, 5]. The most important information for the design of a suitable solution includes [4, 5]: the available blackstart capability, the availability of interconnection assistance, the status of non-blackstart units, the status of lines and circuit breakers, and the predicted load levels.

A suitable SS can assist the operators in reconnecting the entire critical load – i.e., load that must be rapidly restored, e.g., hospitals – in a timely fashion [5]. Furthermore, it will aid in restoring the significant load – i.e., the non-critical load – as quickly as possible, when capacity becomes available. [4, 5]. To ensure reliable PPSR, it is important that system security is considered during the design of the SS. Therefore, the following constraints should be considered when determining a suitable strategy [1, 4, 5]:

1. Each island must have at least one blackstart unit;
2. Each island should have sufficient voltage control resources to maintain a suitable voltage profile;
3. The tie-lines between the islands should all have monitoring equipment that allows the synchronisation of the adjacent islands that they separate to be measured;
4. Each island should have sufficient capacity to maintain a satisfactory frequency by matching generation and load; and,
5. Each island should be monitored at the system control centre to ensure correct operation.

Numerous system operators have designed sectionalising strategies for the parallel restoration of their systems, e.g., the Mexican system [8], the PJM interconnection [9] and the British network [10]. The design of these strategies has been based on factors like historical asset ownership and operator experience, rather than the study of the physical and inherent properties of the electrical network, to simplify the process of

determining the SS. However, this practice raises questions regarding the quality of the restoration planning and the proper exploitation of the physical properties of the system [5-7].

Therefore, this paper proposes a methodology to determine sectionalising strategies that is based on spectral clustering. This methodology uses the physical and inherent properties of the network to determine an SS that satisfies constraints 1, 2, 3 and 4 (these constraints are referred to here as the critical PPSR constraints) with the maximum electrical cohesiveness within the islands created. It is assumed that each island can be monitored by the control centre, which satisfies constraint 5. In practice, the creation of fully-observable subsystems for the purpose of restoration is extremely complicated due to the consequences that a large-area blackout produces in the entire power network [4, 5].

The concept of electrical cohesiveness that is used in this paper is expressed in terms of the electrical distance between the buses and has previously been used to define zones for system planning and control [11-14]. Introducing electrical cohesiveness as part of the formulation of the SS-problem allows the creation of islands that not only satisfy the critical PPSR constraints but also have strong internal connections (strong cohesion, i.e., small electrical distances, between buses in the same island) and weak external connections (weak cohesion, i.e., large electrical distances, between buses in different islands). This methodology does not use network analysis to determine the optimal solution, which means that the SS can be determined more quickly, as it avoids the need for the extensive use of simulation data to validate the proposed islands.

Moreover, maximising the electrical cohesiveness of the islands serves to reduce the number of tie-lines between the islands and, consequently, increases the number of internal connections. Reducing the number of tie-lines between the islands reduces the number of time consuming tie-line reconnections that must be performed during the resynchronisation of the islands. Increasing the number of internal connections within each island increases the number of paths available for the transmission of cranking power from the blackstart units to the non-blackstart units. Furthermore, the emphasis on creating electrically cohesive islands will mean that the lines that separate these subsystems will tend to be long, weak transmission lines. Thus, the effects of the high charging currents and overvoltages that might arise when reconnecting these long, unloaded lines can be reduced, as these lines will be reconnected at the end of PPSR when the system is nearing full strength [1-3].

The nature of the SS-problem lends itself to being solved using spectral clustering. Thus, we propose the use of constrained spectral clustering to find an SS that satisfies the necessary constraints with the maximum electrical cohesiveness within the islands [15-19]. In the context of power systems, spectral clustering has previously been used to determine controlled islanding strategies [20-23] and to reveal the internal connectivity of the network [24]. This technique can directly determine an “initial SS” that ensures blackstart availability (Constraint 1), excludes critical lines that maintain system voltage stability (Constraint 2) and ensures that the synchronisation across the tie-lines that separate the islands can be measured (Constraint 3). This initial SS is then subjected to a graph theory based refinement algorithm that modifies the initial SS to form a final SS that ensures each island will possess sufficient active power generation to restore the

predicted load within the island (Constraint 4). Simulation results for the IEEE 39- and IEEE 118-bus test systems demonstrate the effectiveness of our methodology in determining the SS that maximises the internal electrical-cohesiveness of the islands, whilst satisfying the constraints of blackstart generation availability, load-generation balance, voltage stability and the ability to monitor the resynchronisation of adjacent islands.

The methodology proposed here is for use during the preparation stage of PPSR and can assist operators by quickly providing an SS that satisfies the critical PPSR constraints, which should serve to help ensure the steady-state stability of the future islands. However, whilst a suitable SS increases the likelihood of a timely and successful restoration, the inherent complexity and risk involved in restoration means that no strategy can guarantee a successful restoration [5-7]. Successful PPSR depends on the adequate use of the available resources and proper decision-making during the actual restoration of the islands. Finally, it is important that the methodology is implemented using system information that represents the true topology of the post-blackout system and the availability of its elements.

The paper is organised as follows. Section 2 presents the background material on spectral graph clustering. Section 3 introduces the proposed methodology to determine sectionalising strategies, and Section 4 we study its effectiveness in creating islands that satisfy the critical PPSR constraints. Finally, Section 5 discusses the main strengths and limitations of the new sectionalising methodology, and Section 6 summarises the conclusions drawn from this study.

2. Graph Theory and Spectral Graph Clustering

Graph theory involves the mathematical representation of a complex system as a graph, which is made up of nodes and edges. Creating this graph representation allows the application of a number of well-established and powerful techniques, e.g., spectral clustering [15]. Spectral clustering is a computationally efficient graph theory based technique that can partition networks using the eigenvalues and eigenvectors of a matrix associated with the network [15]. In principle, a graph could be used to capture only the topological structure of the system through the well-known adjacency matrix [13]. However, functional information about the system can be included in the graph using node and edge-weights, as detailed below [5, 7].

2.1 Graph Representation of a Power System in Blackout

Terminology and notation: The physical and inherent properties of a system in blackout with n buses, m generators and b blackstart units can be represented as an undirected edge-weighted graph $G=(V,E,\rho,\omega)$. The elements $v_i \in V$, $i=1,2,\dots,n$, and $e_{ij} \in E \subset V \times V$, $i,j=1,2,\dots,n$, denote the nodes and edges of G , respectively. The sets V and E represent the buses and branches of the system, respectively. Due to the nature of power systems, G can be assumed to be simple, i.e., no multiple edges and no self-loops exist.

The node subsets $V_{GN} \subset V$ and $V_B \subset V_{GN}$ are defined to represent the m generation-buses (generation-nodes) and the b blackstart units, respectively. Moreover, the subset $V_{LD} = V \setminus V_{GN}$ (where $\setminus$ denotes the set-theoretic

difference and defines V_{LD} as the set of nodes in V that do not appear in V_{GN} is defined to represent the $n-m$ load-buses (load-nodes). The subset $V_{CLD} \subset V_{LD}$ is then defined to represent the load-buses that have critical loads connected to them. Any load connected to a generation bus is considered to be a critical load as these commonly represent local services that support the generator [8].

Node-weights: The weight factor associated with the node $v_i \in V$ is denoted as $u_i = \rho(v_i)$, $i = 1, 2, \dots, n$, and is computed as follows:

$$u_i = P_{GN,i} - \alpha_i P_{LD,i}, \text{ for all } v_i \in V \quad (1)$$

where $P_{GN,i}$ and $P_{LD,i}$ are the maximum active power capability and predicted active power consumption at bus i , respectively. The value of $P_{LD,i}$ is multiplied by the scalar α_i to represent the percentage of the load at bus i that is critical load (V_{CLD}). Therefore, for the critical load nodes, $v_i \in V_{CLD}$, we define $\alpha_i = 100\%$. Moreover, as the amount of significant load to be restored before the resynchronisation of the islands usually varies from 40% to 75% of the total system load [25], we use $\alpha_i = 70\%$ for all non-critical loads, $v_i \notin V_{CLD}$, in the results presented in this paper. Changing the value of α_i for the nodes $v_i \notin V_{CLD}$ would require more accurate load estimation levels and would likely result in only slight variations in the final Sectionalising Strategy (SS). The value u_i provides a measure of the balance at each bus between the available generation and the predicted load to be restored prior to the resynchronisation of the islands.

Edge-weights: The weight factor associated with the edge $e_{ij} \in E$ is denoted as $w_{ij} = \omega(e_{ij})$, $i, j = 1, 2, \dots, n$, and is computed as follows:

$$w_{ij} = w_{ji} = Y_{ij} = 1/X_{ij}, \text{ for all } e_{ij} \in E \quad (2)$$

where X_{ij} is the reactance in per unit of the branch between bus i and bus j . In general, $R_{ij} \ll X_{ij}$ [13]; therefore, the resistance R_{ij} is neglected in (2). The value of w_{ij} is a measure of the strength of the connection between node i and node j . Therefore, when it is used as an edge-weight it will allow strongly-connected nodes (nodes connected by large w_{ij} or those that are electrically close) to be clustered together. We formalise the concept of connection strength using an electrical distance measure $1/X_{ij}$, because it associates the physical and inherent properties of the system with the susceptance matrix $\mathbf{B} = \text{Im}(\mathbf{Y}_{\text{BUS}})$ of the system. The benefits of using the physical properties of the network when solving system partitioning problems have been discussed at length in the past [11-14].

The graph representation of the power system that is used here includes the connectivity of the elements (e.g., generator locations and the terminal buses of a branch), the admittance of the branches, the maximum active power output of the generators and the predicted active power load. No other information, such as the

presence of Flexible Alternating Current Transmission System (FACTS) devices or the voltage level of a bus, is necessary. Therefore, it is far simpler to analyse the highly simplified graph representation of a system than it is to directly analyse the complex network itself. Indeed, the opportunity to omit non-relevant information from the graph representation is one of the attractive properties of employing graph theory. Figure 1 illustrates the contrast between the graph representation of a system and the system itself: Figure 1(a) shows the single line diagram of the IEEE 9-bus test system and Figure 1(b) its graph representation.

2.2 Laplacian Matrices of Graphs

Laplacian matrices are widely used in graph theory to describe and study graphs [15]. There are two main types of Laplacian matrices: the *unnormalised* Laplacian matrix $\mathbf{L}$ and the *normalised* Laplacian matrix $\mathbf{L}_N$. The matrix $\mathbf{L}$ of the graph G that describes the electrical distance between the n buses in the system is the $n \times n$ matrix that is computed as follows [15]:

$$[\mathbf{L}]_{ij} = \begin{cases} d_i & \text{if } i = j; \\ -w_{ij} & \text{if } i \neq j \text{ and } e_{ij} \in E; \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

where d_i is the weighted degree of the node v_i , which is defined as the total weight of the edges connected to that node [15]:

$$d_i = \sum_{j=1}^n w_{ij}. \quad (4)$$

Accordingly, the matrix $\mathbf{L}$ can be defined as $\mathbf{L} = \mathbf{D} - \mathbf{W}$, where $\mathbf{D}$ is a diagonal matrix with nonzero entries d_i , and $\mathbf{W}$ is the weighted adjacency matrix of G , i.e., the matrix whose ij -entry is w_{ij} . The matrix $\mathbf{L}$ is a real, symmetrical matrix with non-positive entries outside the diagonal and the sum of the elements in each row and column is zero [15]. Note that defining $\mathbf{L}$ according to (3) directly associates this matrix with the well-known susceptance matrix $\mathbf{B}$, i.e., $\mathbf{L} = \mathbf{B}$ if the shunt susceptances are neglected.

The normalised Laplacian matrix of G is the $n \times n$ matrix $\mathbf{L}_N = \mathbf{D}^{-1/2} \mathbf{L} \mathbf{D}^{-1/2}$ that is computed as follows [15]:

$$[\mathbf{L}_N]_{ij} = \begin{cases} 1 & \text{if } i = j; \\ -w_{ij} / \sqrt{d_i d_j} & \text{if } i \neq j \text{ and } e_{ij} \in E; \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

The matrix $\mathbf{L}_N$ allows the comparison of graphs with different weights, as the normalisation renders it independent of scale, which is advantageous for clustering purposes [15-17].

2.3 Eigenvalues of the Laplacian Matrices

The eigenvalues of the Laplacian matrices are nonnegative real numbers [15] that are defined as $0 = \lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n$ and $0 = \nu_1 \leq \nu_2 \leq \dots \leq \nu_n$ for $\mathbf{L}$ and $\mathbf{L}_N$, respectively. Moreover, it is well-known that $\lambda_2 > 0$ and $\nu_2 > 0$ if, and only if, G is connected [15]. Henceforth, we consider that our graph is connected.

2.4 Constrained Spectral Clustering

Clustering refers to the partition of G into r disjoint subgraphs (clusters) in which the nodes in each cluster are more strongly connected to the other nodes in the same cluster (taking edge-weights into account) than they are to the nodes in other clusters [15, 24]. Spectral clustering refers to clustering the nodes of a graph based on the eigenvalues and eigenvectors of a Laplacian matrix of that graph [15].

Constrained spectral clustering is an extension of spectral clustering that allows two types of constraints – Must-Link (ML-) and Cannot-Link (CL-) constraints – to be included in the clustering process [18, 19]. An ML-constraint between two nodes indicates that the pair of nodes must be clustered together and a CL-constraint specifies that the pair of nodes cannot be assigned to the same cluster. Applications of ML- and CL- constraints in the context of islanding can be found in [21-23].

The basis of spectral clustering is the use of the first r eigenvectors of a Laplacian matrix to provide geometric coordinates, or data points, for the nodes $v_i \in V$ in $\mathbb{R}^r$ [15] (see Figure 2 for an example). The resulting data points can then be clustered using a clustering algorithm that is suitable for use with point clouds in a Euclidean space (e.g., *k-means*) [15]. In general, spectral clustering creates balanced clusters of approximately equal volume, as explained later [15]. We introduce a quality index in the following section that can measure the quality of the partition.

2.5 Quality of a Cutset

Constrained spectral clustering finds a cutset $E_S \subset E$ that splits the graph G into r disjoint subgraphs $G_1, \dots, G_r$. Each subgraph corresponds to one of the node sets $V_1, \dots, V_r$ such that $V_1 \cup \dots \cup V_r = V$ and $V_i \cap V_j = \emptyset$ for $i \neq j$. This cutset contains the edges that represent the branches to be included in the SS. Therefore, one can measure the quality of the solution by measuring the quality of the cutset E_S . This can be done using the ratio between the cut and the volume of each subgraph (island). Indeed, this ratio is related to the so-called *Cheeger inequality* [17], and this is one of the main theoretical justifications of the spectral clustering methodology [15].

The cut of a cluster with node set V_k , denoted by $cut(V_k, \bar{V}_k)$, is the sum of the edge-weights between the nodes in V_k and the nodes not in V_k , i.e., the sum of the edge-weights linking V_k with its complement $\bar{V}_k$ [15]:

$$cut(V_k, \bar{V}_k) = \sum_{v_i \in V_k, v_j \in \bar{V}_k} w_{ij}. \quad (6)$$

In this application, the value of the cut represents the total electrical distance between V_k and the rest of the system, i.e., the inter-cluster electrical distance. Furthermore, the volume of an island with node set V_k , denoted by $vol(V_k)$, is by definition the sum of the weighted degrees of its nodes [15]:

$$vol(V_k) = \sum_{v_i \in V_k} d_i. \quad (7)$$

In this application, the volume of a subset V_k represents the total electrical distance between the nodes within V_k , i.e., the intra-cluster electrical distance.

We can then define the quality of the island V_k as follows:

$$\phi(V_k) = 1 - \frac{cut(V_k, \bar{V}_k)}{vol(V_k)}. \quad (8)$$

The range of values that $\phi(V_k)$ can take is from zero to one. The greater the value of $\phi(V_k)$ the better the island is considered to be in terms of clustering. For example, $\phi(V_k) = 0.98$ represents a poorly connected island that is a better candidate for sectionalising than a better-connected island with $\phi(V_k) = 0.65$. Indeed, a suitable cutset will create islands with nodes that are strongly connected to the other nodes in their island (i.e., they will have large $vol(V_k)$) and weakly connected to the nodes in the other islands (i.e., they will have small $cut(V_k, \bar{V}_k)$). This is physically interpreted as each island having small inter-cluster electrical distances and large intra-cluster electrical distances.

The overall quality of the SS is then defined as being equal to the quality of the lowest quality island it creates, i.e., the quality of the strategy is the minimum value of $\phi(V_k)$: $\min_{k=1, \dots, r} (\phi(V_k))$. This allows the best r -way partition to be defined as the maximum of the worst quality (9):

$$\phi(G_r) = \max_{V_1, \dots, V_r} \min_{k=1, \dots, r} (\phi(V_k)). \quad (9)$$

This is the r -way quality constant of G [17]. However, finding the optimal partition of G is not computationally feasible [15], as search problems on graphs like these are generally *NP-hard* [26]. A problem that is *NP-hard* cannot be optimally solved in *polynomial time* [26]. Fortunately, spectral clustering can produce an approximate solution in polynomial time [15]. The approximate and the optimal solution to a problem can be related via the *Cheeger inequality* [17], this comparison is presented in Section 4.1.1.

3. Constrained Spectral Clustering-Based Methodology

The objective of Parallel Power System Restoration (PPSR) is to accelerate restoration by simultaneously restoring a number of electrically isolated islands that are defined by a Sectionalising Strategy (SS). Constrained spectral clustering is used here to identify an SS that satisfies Constraints 1, 2, 3 and 4 (Section 1). Furthermore, the islands created by the SS contain nodes that are more strongly connected to those nodes in the same island than they are to those nodes in a different island. The step-by-step execution of the proposed methodology is presented in this section for the case of bisection, i.e., $r = 2$. This methodology can find a strategy for $r > 2$ through the application of *recursive bisection* [15].

3.1 Step 1: Blackstart Availability and Cranking Groups

Each island must contain at least one blackstart unit (Constraint 1) that can provide cranking power for the restoration of the island. The locations of the blackstart units are commonly known in advance [1-4] and this information is used to define the subsets V_{B1} and V_{B2} . These groups of blackstart units can then be partnered with a number of non-blackstart units to form two cranking groups, denoted by V_{GN1} and V_{GN2} . Therefore, each cranking group V_{GNk} consists of a subset V_{Bk} of blackstart units and another subset of non-blackstart units. However, defining suitable cranking groups, which will accelerate the progress of the cranking task and the restoration process [2-4], requires extensive system studies [2, 3]. Hence, it is assumed that they are separately defined using an existing approach, such as those described in [27], and as such the proper cranking groups for the systems considered here are assumed to be known.

The methodology uses a subspace manipulation [28] to satisfy the constraint that each island must contain at least one blackstart unit (Constraint 1). This subspace manipulation, which has been used for islanding in [21, 22], creates an ML-constraint between the generators in the same cranking group and a CL-constraint between the generators in different groups. In the application presented in this paper, this preserves the cranking groups, each of which contains at least one blackstart unit by definition, by assigning each group to a separate island. Assuming that $|V_{GN1}| = m_1$ and $|V_{GN2}| = m_2$, where $m = m_1 + m_2$, the subspace manipulation explained in detail in [28] creates the projection matrix $\mathbf{P}$:

$$\mathbf{P} = \begin{pmatrix} \mathbf{1}_{m_1} & \mathbf{1}_{m_1} & \mathbf{0}_{m_1 \times (n-m)} \\ \mathbf{1}_{m_2} & -\mathbf{1}_{m_2} & \mathbf{0}_{m_2 \times (n-m)} \\ \mathbf{1}_{n-m} & \mathbf{0}_{n-m} & \mathbf{I}_{(n-m) \times (n-m)} \end{pmatrix} \quad (10)$$

where $\mathbf{I}$ is the identity matrix, $\mathbf{1}$ is the all-ones column vector, and $\mathbf{0}$ is the zero matrix or zero column vector.

The matrix $\mathbf{P}$ ensures that the same geometric coordinates in $\mathbb{R}^2$ are assigned to the nodes that are in the same cranking group $v_i \in V_{GNk}$, $k = 1, 2$. This ensures that the nodes that are in the same group will be clustered together and that the nodes that are in different groups will be separated. A mathematical explanation of the subspace manipulation can be found in [28]. The spectral embedding created by the

projection matrix for the IEEE 39-bus system is show in Figure 2. It can be seen that the projection matrix has successfully assigned the members of each cranking group, assumed here to be $V_{GN1} = \{v_{30}, v_{31}, v_{32}, v_{37}, v_{39}\}$ and $V_{GN2} = \{v_{33}, v_{34}, v_{35}, v_{36}, v_{38}\}$, to the same location in $\mathbb{R}^2$.

3.2 Step 2: Exclusion of Lines

It is necessary to exclude certain lines from being considered as part of the cutset if the SS is to satisfy Constraints 2 and 3, i.e., lines critical to voltage stability and lines that lack the ability to monitor synchronisation are not used as tie-lines in the strategy.

To define critical lines that maintain system voltage stability (Constraint 2), it is common practice to assume that the tie-lines between the islands are open during the restoration process [5, 10]. Thus, to identify these critical lines we perform an $N-1$ voltage stability analysis for every line in the network. The most severe outages are then selected using the contingency analysis method available in DIgSILENT Power Factory [29]. A line is critical if disconnecting it in the nominal case produces an excessive reactive power difference between the pre-contingency and the post-contingency [5]. When the critical lines are identified, we define a subset $E_{VS} \subset E$ that consists of the edges that represent these critical lines in the graph of the power system.

These critical lines are not the only type of branch that should be excluded from consideration in the cutset. This is because the tie-lines that separate the islands must be equipped with the ability to measure the synchronisation between the islands that they separate (Constraint 3). As with the lines critical to voltage stability, we define a subset $E_C \subset E$ that consists of the edges that represent the branches that cannot measure resynchronisation. These branches are known in advance [4].

Having defined the subsets of lines to be excluded, the subset $(E_{EX} = E_{VS} \cup E_C) \subset E$ of excluded-edges can be defined. We then constrain the edges $e_{ij} \in E_{EX}$ to be excluded from the cutset by changing the associated weight factors to the largest value in $\mathbf{W}$ (i.e., $\max(\mathbf{W})$):

$$\text{for all } e_{ij} \in E_{EX}, \text{ let } w'_{ij} = w'_{ji} = \max(\mathbf{W}). \quad (11)$$

These changes create a new weighted adjacency matrix $\mathbf{W}'$. This mechanism for excluding edges from the cutset is not limited to the branch types considered here. In practice, it could be extended to any branch, or branch type, that the operator deemed unsuitable for use as a tie-line in the SS. Examples of other branch types that should not be used as tie-lines between islands are given in [2-7].

3.3 Step 3: Initial Sectionalising Strategy-Problem

Spectral clustering can only find an SS that satisfies constraints 1, 2 and 3; henceforth, this is referred to as the initial SS-problem. This initial SS can then be refined to ensure that Constraint 4 is satisfied and form the final SS, as described in Section 3.4. The initial SS-problem consists of finding an SS for PPSR that creates

islands with strong internal connections and weak external connections and that separate the cranking groups without depending on any of the excluded lines.

To solve this SS-problem, we use a constrained spectral clustering algorithm, the projection matrix $\mathbf{P}$ (10) and the normalised Laplacian matrix $\mathbf{L}_N$ (5) of the modified matrix of weights $\mathbf{W}'$. The matrix $\mathbf{L}_N$ is used as a number of studies have shown that it offers superior performance when compared to the unnormalised Laplacian matrix [15-19].

The solution to this initial SS-problem can be obtained using the following steps:

1. Compute the first two eigenvectors $\boldsymbol{\psi}_1$ and $\boldsymbol{\psi}_2$ of the generalised eigenproblem $\mathbf{P}^T \mathbf{L}_N \mathbf{P} \boldsymbol{\psi} = \nu \mathbf{P}^T \mathbf{P} \boldsymbol{\psi}$.
2. Let $\mathbf{X} \in \mathbb{R}^{n \times 2}$ be the matrix containing the vectors $\mathbf{P} \boldsymbol{\psi}_1$ and $\mathbf{P} \boldsymbol{\psi}_2$ as columns. The vector $\mathbf{x}_i$, $i = 1, \dots, n$, provides geometric coordinates for the nodes $v_i \in V$ in $\mathbb{R}^2$ (see Figure 2(a) for an example).
3. Normalise the vectors $\mathbf{x}_i$ to have length of one in $\mathbb{R}^2$ and create the vectors $\mathbf{y}_i$.

$$\mathbf{y}_i := \frac{\mathbf{x}_i}{\|\mathbf{x}_i\|}, \quad i = 1, 2, \dots, n \quad (12)$$

This normalisation improves the quality of the solution [16, 17] and projects the vectors $\mathbf{x}_i$ to the unit circle $\mathbb{S}^1 = \{\mathbf{y}_i \in \mathbb{R}^2 \text{ such that } \|\mathbf{y}_i\| = 1\}$ (see Figure 2(b) for an example). Furthermore, this allows certain nodes to be defined as belonging to the core of the island and other nodes as belonging to the periphery of the islands. For example, in Figure 2(b) the nodes v_2, v_3, v_{18} and v_{25} are in the periphery of the clusters. Nodes in the periphery will be used to determine the final SS.

4. Cluster the vectors $\mathbf{y}_i$ into the clusters V_1 and V_2 using a clustering algorithm, e.g., the *k-means* algorithm [15]. It is important to remember that this clustering must consider the length of the arc between the points on the unit circle [16] and not the Euclidean distance [15].

When more than two islands are required ($r > 2$), the set V_1 or V_2 that contains more than one blackstart unit should be selected as the node set of a new graph. The spectral clustering should then be repeated for this new graph to create two new islands. Where necessary, another new graph should then be defined from these results to perform this *recursive bisection* [15] until r islands are created. When Step 3 is finished, the initial SS will satisfy Constraints 1, 2 and 3 for each island. However, to ensure that Constraint 4 is satisfied it is necessary to perform Step 4.

3.4 Step 4: Final Sectionalising Strategy

Step 4 seeks to modify the islands in the initial SS from Step 3 in such a way as to ensure that they will have sufficient generation capacity, i.e., it aims to ensure that Constraint 4 is satisfied. When the node sets V_1 and V_2 are known, the generation capacity available within each island is compared with the predicted load level within the same island by taking the sum of the node-weights, as defined in (1). In this way, the total weight of the nodes of the subgraph V_k is defined as follows:

$$\varphi(V_k) = \sum_{v_i \in V_k} u_i. \quad (13)$$

The load-generation balance during the actual PPSR can be ensured by creating islands with sufficient active power generation capacity. If the available generation capacity in an island is larger than the predicted load level in the island then a larger portion of the load in the island can be restored to facilitate the synchronisation of the islands [5]. To ensure that it will be possible to maintain the frequency of each island within prescribed limits during the restoration process we must ensure that (13) is positive for each island.

If $\varphi(V_k) < 0$ for any island, the refining algorithm detailed in [30] is used to reduce the load consumption within any island with $\varphi(V_k) < 0$ by moving nodes to the adjacent islands, as explained below. We define a node set $N \subset V_{LD}$ that consists of the nodes that are in the periphery of the islands. Remark that the set N will not include generation-nodes; as the manipulation performed by the projection matrix (10) will ensure that the generation-nodes lie at the core of their corresponding cluster (see Figure 2(b)). For example, in the case of the IEEE 39-bus test system we define $N = \{v_2, v_3, v_{18}, v_{25}\}$. We then exchange one node $v_i \in N$ per iteration from one island to the adjacent island and calculate the quality index (8) and the total weight of the nodes (13) for each island. The process is repeated for all $v_i \in N$, $i = 1, 2, \dots, |N|$.

This process creates a list of candidate strategies and the candidate with the highest quality becomes the final SS. The solutions will all satisfy the four constraints, i.e., blackstart availability (Constraint 1), exclusion of critical lines that maintain system voltage stability (Constraint 2), exclusion of lines that cannot measure resynchronisation with adjacent islands (Constraint 3), and load-generation balance (Constraint 4).

3.5 Summary of the Data Needed

In order to determine an optimal SS, the proposed methodology requires the following data: (i) blackstart units' definition; (ii) cranking groups' definition; (iii) details of the power capacity of the machines; (iv) network connectivity data; (v) reactance of the lines; (vi) lines to be excluded.

4. Simulation results

The methodology is now tested using simulations of the IEEE 39- and IEEE 118-bus test systems, assuming that the cranking groups have been defined separately [27]. All simulations are performed in MATLAB [31], using the values of P_{GN} , P_{LD} and X_{ij} available in MATPOWER [32]. All times quoted are based upon simulations performed on a PC with a 2.93 GHz dual core CPU and 4 GB RAM.

4.1 Test case I: IEEE 39-bus test system

We initially test our methodology using the IEEE 39-bus test system and the following sets are defined for the simulation: the set of critical loads is defined as $V_{CLD} = \{v_3, v_4, v_8, v_{16}, v_{20}\}$, the subsets of blackstart units

are $V_{B1} = \{v_{37}\}$ and $V_{B2} = \{v_{33}\}$, and the cranking groups are $V_{GN1} = \{v_{30}, v_{31}, v_{32}, v_{37}, v_{39}\}$ and $V_{GN2} = \{v_{33}, v_{34}, v_{35}, v_{36}, v_{38}\}$.

The contingency screening method available in DIgSILENT Power Factory [29] identified the critical lines that maintain voltage stability in the system as those highlighted in Figure 3 (red lines). The first scenario presented assumes that all of the elements of the system are available. A second scenario is then presented that assumes the generator at bus 35 (G6) is unavailable and the loss of the ability to monitor synchronisation on line 3-18. This second scenario seeks to illustrate how the methodology can adapt to system elements being unavailable and the loss of monitoring equipment.

4.1.1 Scenario 1: All elements available

The first scenario assumes that every blackstart unit, non-blackstart unit, line and breaker is available after the blackout. Furthermore, we assume that every line can measure synchronisation with adjacent islands, i.e., $E_C = \emptyset$. The results of the spectral embedding performed at the end of Step 3 are presented in Figure 2(a) and the resulting normalised vectors (12) are shown in Figure 2(b). These normalised vectors are clustered using the *k-means* algorithm into the islands shown in Figure 3. Note that this initial solution is known to satisfy Constraints 1, 2 and 3 and Table 1 presents the key properties of this solution.

To ensure that Constraint 4 is enforced we then execute Step 4. The implementation of Step 4 reveals that the initial Sectionalising Strategy (SS) shown in Figure 3 already satisfied Constraint 4, i.e., $\phi(V_k) \geq 0$ for both islands. The quality of this strategy (the minimum value of $\phi(V_k)$ for the islands) is 94.94%.

We now compare the quality of the SS determined by the proposed methodology, which was found in a few milliseconds (see Section 4.3), with the optimal solution, which, in general, cannot be found in polynomial time [15]. The theoretical lower bound (defined by the eigenvalue $v_2/2$ of $\mathbf{L}_N$ [17]) establishes that the quality of the optimal solution (defined by the *Cheeger inequality*, see [17] for more details) is bounded to 96.89%, whereas the quality of our approximation was 94.94%.

A benefit of this methodology is that it sectionalises the system across long transmission lines. Indeed, the three lines identified as tie-lines are those that separate the cranking groups with the largest reactance X_{ij} . Hence, the early stages of the restoration process are protected from the high charging currents that may be produced by the reconnection of long lines.

4.1.2 Scenario 2: Unavailability of G6 and loss of monitoring on line 3-18

The second scenario considered for the IEEE 39-bus system assumes that the generator at bus 35 (G6) is not available after the blackout and that line 3-18 cannot measure resynchronisation. We selected G6 as it is the largest machine in the cranking group V_{GN2} and line 3-18 was previously part of the cutset. Remark that the cranking groups remain the same, other than the unavailability of G6.

Table 2 summarises the properties of the initial SS that is found by executing Steps 1 to 3. Since $\varphi(V_1)=1540$ MW and $\varphi(V_2)=-278$ MW < 0 , the refining stage (Step 4) is necessary to create satisfactory islands. For this purpose, we define the set of peripheral nodes $N = \{v_3, v_4, v_{14}, v_{18}\}$. The optimal solution to ensure $\varphi(V_k) \geq 0$ for both islands was achieved by moving bus 3 from Island 2 to Island 1 (Figure 4(a)). However, as line 3-18 is constrained to be excluded from the strategy, the methodology moves both bus 3 and bus 18 from Island 2 to Island 1, to exclude line 3-18 from the solution.

Figure 4(b) shows the final SS and Table 2 summarises the results for this scenario. The refining process reduces the quality of the strategy from 94.05 % to 92.97 %, but this is necessary if the critical constraints for parallel restoration are all to be satisfied.

4.2 Test case II: IEEE 118-bus test system

To illustrate the methodology in a larger system, we now use the IEEE 118-bus test system shown in Figure 5. We consider in this case that every element is available. The assumed critical loads are highlighted in Figure 5. For simulation purposes, we will assume that the blackstart units are located at buses 31 and 87. Thus, the cranking groups, separately defined using [27], are $V_{GN1} = \{v_{10}, v_{12}, v_{25}, v_{26}, v_{31}\}$ and $V_{GN2} = \{v_{46}, v_{49}, v_{54}, v_{59}, v_{61}, v_{65}, v_{66}, v_{69}, v_{80}, v_{87}, v_{89}, v_{100}, v_{103}, v_{111}\}$.

The contingency screening method available in DIgSILENT Power Factory [29] identified the following set of critical lines that maintain system voltage stability, $E_{VS} = \{e_{5,8}, e_{8,9}, e_{9,10}, e_{37,38}, e_{38,65}\}$. Furthermore, we assume, for simulation purposes, that the lines that cannot measure synchronisation with adjacent islands are represented by the set $E_C = \{e_{40,42}, e_{34,43}\}$. The applicable edge-weights are then changed using (11) and the initial SS shown in Figure 5 is found.

Table 3 shows the value of the cut (6), the volume (7), the quality (8) and the value of $\varphi(V_k)$ (13) for each island and it can be seen that $\varphi(V_k) \geq 0$ for both islands, so they both have more generation than load. Hence, the final SS corresponds to the initial SS. The quality of the SS is found to be 98.25%. It can be observed that the quality index of the IEEE 118-bus test system is higher than that of the IEEE 39-bus system. In general, it is expected that the quality index of highly interconnected power systems will be higher, as the number of tie-lines will be significantly smaller than the number of internal connections.

4.2.1 Comparison with Ordered Binary Decision Diagram (OBDD)-Based Method

This section compares the results of applying the proposed methodology and an OBDD-based method [5] to the IEEE 118-bus system. The OBDD-based method is divided into three steps (see Section III of [5]): (i) creation of the Boolean variables from the branches, (ii) solution of the partitioning problem, and (iii) identification of voltage critical lines to exclude them from the solutions.

To implement this complex method, it is necessary to assume a given system load (i.e., to compute the load flow solution for a given operating point). For comparison purposes, this operating level is arbitrarily defined as 70%. The number of solutions obtained with the OBDD-based method increases as the assumed loading level increases [5]. For consistency, the blackstart units, the cranking groups, and the excluded lines are defined as those described in Section 4.2. The implementation of the OBDD-based method produces 108 possible strategies. A potential disadvantage of a method that provides a large number of solutions is that it may fail to provide clear guidance to the operator on how best to sectionalise the system. Table 5 compares the strategy obtained using the proposed methodology with the one obtained using the OBDD-based method that provides the highest generation capacity within Island 2, whilst ensuring sufficient capacity within Island 1. The main features to compare are: the generation capacity of the islands (the objective function of [5]), our measure of quality (8), which represents the connectivity of an island relative to its volume, and the complexity of the approach, which may become a major concern when adapting the approaches for use with actual power systems that may contain several thousand buses.

As it can be noticed in Table 5, the proposed methodology returns an SS with a higher quality index. Additionally, the number of lines within the solution obtained using the proposed methodology is only five, whilst the solution found by OBDD-based method requires six. Even though the OBDD-based method defines Island 2 with higher generation capacity, whilst ensuring high generation capacity within Island 1, it should be noted that our methodology can also find an SS that ensures sufficient capacity within both islands. The computational time is also compared in Table 5. The proposed methodology can find a solution in a short period of time (Section 4.3 discusses in more details the efficiency of the proposed methodology), whereas the OBDD-based method requires approximately 5 minutes to find sectionalising strategies for the IEEE 118-bus test system.

4.3 Evaluating the efficiency of the methodology

The runtime of the methodology is dominated by that of the *eigendecomposition* of $\mathbf{L}_N$. Indeed, the calculation of the eigenvectors of $\mathbf{L}_N$ represents approximately 80% of the total runtime of the methodology. Therefore, the computational complexity of our methodology is approximately $O(n^3)$ [19]. The mean runtimes of the methodology for 100 trials of each test system are given in Table 4. The runtime for these systems is below a second (1 s) and, as such, is negligible in terms of the length of time necessary for the overall restoration process.

5. Discussion

The methodology proposed in this paper can effectively determine a Sectionalising Strategy (SS), whilst satisfying the critical constraints for PPSR. It creates islands that have strong (multiple) internal connections, and weak (few) external connections. However, the proposed method provides operators with only a single solution. Therefore, if the operators were to find this single solution to be unsuitable, based on their experience with the system, the method would lack the flexibility to provide further support. This

vulnerability contrasts with the problem that if the sectionalising method returns to many possible strategies it will fail to provide useful guidance for the operator, as discussed in Section 4.2.1 for the OBDD method. Therefore, in the future it would seem wise to pursue sectionalising methods that can generate a shortlist of distinct sectionalising strategies that the operator can choose between, based on their knowledge of the system and the specific nature of the restoration that is being performed.

In addition, it has been assumed that the control centre can effectively monitor the created islands. In practice, this may not be possible due to the unavoidable consequences that a large-area blackout can have in the power system, and the communication technologies that are deployed in the network. Hence, further research should seek to include the constraint of observability in to the SS-problem presented in this paper.

In order to consider Constraint 3, the proposed methodology excludes from the solution space the critical lines for voltage stability (see Section 3.2). Although this approach follows current practices in the UK [10], which has also be used in [5], further research is required to find a method that helps ensure that this constraint is satisfied. A potential approach is to perform several contingency studies to identify the lines that are necessary during the actual restoration process to maintain the voltage stability of the islands. When these lines are identified, they can be then excluded from the solution space by using (11). This approach could be very effective, but it will require multiple studies that are not undertaken in this paper.

The proposed methodology has considered a bus branch model (similar to the methodologies for determining an SS available in the literature, e.g., [5]), and the restoration practices are commonly undertaken on bus breaker models. The type of modelling used in this paper does not affect the SS, as modelling the circuit breakers of the lines is not needed to determine a suitable SS. Nevertheless, it is important to model the islands created by the proposed methodology using the latter model during the actual restoration process.

6. Conclusions

This paper presents a methodology that could assist operators in rapidly determining a suitable sectionalising strategy (SS) for Parallel Power System Restoration (PPSR). By suitable we mean a solution that ensures blackstart availability within each island, preserves the integrity of the defined cranking groups, creates islands with more generation capacity than the predicted load level and excludes from the solution critical lines that maintain system voltage stability and those that cannot measure resynchronisation with adjacent islands. These constraints aim to provide sufficient resources to ensure that the steady-state stability of the created islands will be within acceptable limits during the actual parallel restoration. The proposed methodology is intended for use during the preparation stage of the restoration process, after updated system information from the assessment of the blackout is available, and it aims to create islands that would facilitate PPSR.

The concept of electrical cohesiveness, as expressed by the electrical distances between buses, is used to formulate the SS-problem in order to create islands that, in addition to satisfying the aforementioned

constraints, have strong internal connections – taking the electrical distance into account – and weak external connections. This approach could benefit PPSR as the number of tie-lines between islands is reduced, whilst simultaneously increasing the number of internal connections. Furthermore, this approach sectionalises the system across long, weak transmission lines. This will help to reduce the effects of high charging currents, high overvoltages, and other voltage related issues that may arise when reconnecting these long and unloaded lines [1-3]. The proposed methodology was tested using the IEEE 39 bus- and IEEE 118 bus-test systems for the case of two islands. These simulation results indicate that the methodology can determine sectionalising strategies with ratios between the inter-cluster electrical distance and the intra-cluster electrical distance that are smaller than 8% for a small test system and 2% for a large test system.

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9. Tables and Figures

Figure 1. IEEE 9-bus test system (a) Single line diagram. (b) Simplified graph representation.

Figure 2. IEEE 39-bus test system (a) 2-dimensional spectral embedding in Euclidean space $\mathbb{R}^2$ (b) Normalised vectors on the unit circle $\mathbb{S}^1$. The numbers on the diagram correspond to node numbers. The red dots and the blue asterisks represent the cranking groups and load-nodes, respectively.

Figure 3. Sectionalising strategy for the case $r = 2$ for the IEEE 39-bus system when all elements are available

Figure 4. Sectionalising strategy for the IEEE 39-bus test system when G6 is unavailable and monitoring on line 3-18 is lost (a) before refining algorithm (b) after refining algorithm

Figure 5. Sectionalising strategy for IEEE 118-bus test system

Table 1: Sectionalising results for the IEEE 39-bus system when all elements are available

	<i>Cutset</i>	<i>Island No.</i>	<i>Cut</i>	<i>Volume</i>	$\phi(V_k)$ (%)	$\varphi(V_k)$ (MW)
Results using the proposed methodology	$e_{25,26}, e_{3,18}, e_{14,15}$	1	152	4610	96.70	1218
		2	152	3004	94.94	730

Table 2: Results in the IEEE 39-bus test system when generator G6 is unavailable and monitoring on line 3-18 is lost

	<i>Cutset</i>	<i>Island No.</i>	<i>Cut</i>	<i>Volume</i>	$\phi(V_k)$ (%)	$\varphi(V_k)$ (MW)
Before refining algorithm	$e_{25,26}, e_{2,3}, e_{3,4}, e_{14,15}$	1	190	4439	95.72	1540
		2	190	3217	94.09	-278
After refining algorithm	$e_{25,26}, e_{17,18}, e_{14,15}$	1	199	4824	95.87	1108
		2	199	2831	92.97	154

Table 3: Sectionalising results for the IEEE 118-bus test system

<i>Island No.</i>	<i>Cut</i>	<i>Volume</i>	$\phi(V_k)$ (%)	$\varphi(V_k)$ (MW)
1	37	2124	98.25	747
2	37	4951	99.25	2169

Table 4: Computational time of the proposed methodology

Test System	Buses	Branches	Mean Runtime (s)
IEEE 39	39	46	Scenario 1: 0.009629 Scenario 2: 0.012629
IEEE 118	118	186	0.0218

Table 5: Comparison between the proposed methodology and the available OBDD-based method

	Sectionalising strategy	$\phi(V_k)$ (%)	$\varphi(V_k)$ (Island 1)/ (Island 2) (MW)	Average time (s)
Proposed Methodology	33-37; 19-34; 30-38; 24-70; 24-72	98.25	747/2169	0.0218
OBDD-based method	15-33; 19-34; 30-38; 69-70; 70-74; 70-75	92.52	698/2182	295 (~5 min)