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Numerical Approach for the Sensitivity of High Frequency Magnetic Induction Tomography System based on Boundary Elements and Perturbation Method

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Short title: Sensitivity Maps of High Frequency MIT System

Abstract

Magnetic induction tomography (MIT) is an imaging technique based on the measurement of the magnetic field perturbation due to eddy currents induced in conducting objects exposed to an external magnetic excitation field. In MIT, current-carrying coils are used to induce eddy currents in the object and the induced voltages are sensed with receiving coils. When the driving frequency is significantly high relative to the frequency range that MIT normally operates, metallic targets with high conductivity between the coils can be treated as perfect electric conductors (PEC) with negligible errors. In this scenario, the penetration depth of the magnetic field into the target is extremely small and the traditional versions of the Finite Elements Method (FEM) are not efficient for the calculation of the sensitivity and the forward problem due to the requirement for large number of elements to reach an acceptable computational precision. Other versions of FEMs (such as Hp-FEM) which have higher discretization efficiency and more advanced elements to satisfy the requirement are exceptions [14]. Nevertheless, the discretization regions for all FEMs have to extend beyond the region that contains the conducting object and volumetric elements are generally required for 3D problems. In contrast, Boundary Element Method (BEM) based on integral formulations becomes an effective way to analyze this kind of scattering problems since meshes are only required on the surface of the object. By point collocation, the boundary integral equations can be transformed into linear equations. Then numerical method is used to solve the linear equations and the solution of the original integral equations can be obtained. In this paper, we computed four typical sensitivity maps between the coil pairs in high frequency MIT system due to a PEC perturbation. The magnetic scalar potential was used to improve the efficiency. Five PEC objects of different shapes were used in the simulation. The results have been compared with the experimental results and that obtained from the $H \cdot H$ formulations. We can know that the sensitivity maps derived by BEM are in good agreement with that from experiment and theoretical solution. Overall, BEM is an effective way to calculate the sensitivity distributions of high frequency MIT system.

Keywords: Magnetic induction tomography, perfect electric conductor, boundary element method, sensitivity distribution

1. Introduction

Magnetic induction tomography (MIT) is a non-invasive and non-contact imaging approach, suitable for industrial and biomedical applications [1-6]. MIT applies a magnetic field from the excitation coil to induce eddy currents in the material and the scattering field from these is then detected by the receiving coil [7-8]. The aim in MIT is to reconstruct the passive electrical / magnetic properties of the objects measured. Simulation of the forward problem in MIT is one of the key issues that have received research interest. Finite difference method (FDM), finite element method (FEM) and boundary element method (BEM) are the most common numerical methods used in the forward problem of MIT [9]. Image reconstruction algorithms are used to solve for the spatial distribution of the complex conductivity in the target from the instrumentation measurements [10-11].

For typical low-frequency MIT system, the driving frequency is always lower than 2MHz [7]. Imaging samples with low conductivities such as biological tissues or ionized water is a difficult task for the low-frequency MIT system because the eddy current changes very little in the receiving coil when the target is put into the measuring region. High-frequency system (>2MHz) emerged to solve this problem [1] [12]. But for metallic object, when the frequency is significantly high, the incident magnetic field will be totally reflected, namely the magnetic field only exists on the surface of the metal. In this case, we don't need to discretize the volume inside the object but only need to utilize discretization on the surface. Hence, the Boundary Element Method (BEM) based on integral formulations becomes an effective way to analyze this kind of scattering problems since meshes are only required on the surface of the object. Because the wavelength for the MIT system, of which the driving frequency is 10~100MHz, is in the range of some micrometers, that is, the wavelength is much smaller than the size of the object. So the MIT system can be described as an eddy current model [13].

When the driving frequency is significantly high, the conductivity of the metal in the internal field can be treated as infinite and the metal behaves essentially like a perfect electric conductor (PEC). In this case, the penetration depth of the magnetic field into the target is extremely small and the common versions of the Finite Elements Method (FEM) are not efficient for the calculation of the sensitivity. Hp-FEM which owns high discretization efficiency and high accuracy to satisfy the requirement is an exception [14]. But because the magnetic field only exists on the surface of the metal and we only need the discretization on the surface. Hence, Boundary Element Method (BEM) based on integral formulations becomes an effective way to analyse this kind of scattering problems since meshes are only required on the surface of the object. Investigations on how to deal with the computation of MIT with integral equations have been carried out by several workers [15]. In this paper, we used the magnetic scalar potential formulation to calculate the sensitivity maps of a MIT due to a small PEC perturbation. And we investigated the significance of the shape of the 3-dimensional perturbation object used.

For a simple MIT system where a PEC is placed inside the coil array, the primary field from the excitation coil induces currents in the target, which in turn radiate a scattered field. We can retain a simple integral equation formulation in scalar potential for the region outside the target, where magnetic fields were irrotational. By calculating the gradient of the scalar potential, the distributions of magnetic field outside the target are derived. Then sensitivity maps between different pairs of coils are calculated.

Moreover, experiments with high frequency MIT system were conducted. Sensitivity distributions between the excitation coil and the receiving coils were measured using the method of perturbation. By comparing sensitivity maps obtained by different approaches, we validate that BEM based on the magnetic scalar potential formulation is an effective method in the solution of the forward problem of a high frequency MIT system.

2. MIT model for the PEC problem

Sarah Engleder presented two models for the forward problem of MIT, and for most MIT system [13], the eddy current model was applicable and the reduced model was introduced to obtain a more efficient solution with a negligible error in terms of the $A-\phi$ formulation. In this paper, we describe the eddy current model with the scalar magnetic potential ϕ .

The Reference [16] has given a simple model for PEC induction problem, which is available to our research. We begin with Ampere's law, applied to the external region of the target, and Gauss's magnetic divergence law

$$\nabla \times \mathbf{H}^e = \sigma^e \mathbf{E}^e + i\omega \epsilon \mathbf{E}^e \quad (1)$$

$$\nabla \cdot \mu \mathbf{H} = \nabla \cdot \mu^e \mathbf{H}^e = 0 \quad (2)$$

Here $\mathbf{H}$ is magnetic field (A/m) and $\mathbf{E}$ is electric field (V/m). μ , ϵ and σ are magnetic permeability (H/m), permittivity (F/m) and electrical conductivity (S/m) of the media respectively. The superscript e indicates the quantities in the external environment. The total magnetic field consists of two parts: the primary magnetic field generated by the excitation coil and the scattered magnetic field radiated by the eddy current induced by the primary field, that is $\mathbf{H}^e = \mathbf{H}^{pr} + \mathbf{H}^{sc}$, where the superscripts pr and sc indicate the primary field and the scattered field respectively.

We assume that magnetic field is quasi-static in the external region and hence the displacement current can be negligible. Also, the space exterior to the target is non-conductive to make sure that electric field is near zero and the region investigated doesn't include the given current. So the right side of (1) is eliminated which means that the exterior total magnetic fields are irrotational and thus can be represented efficiently using a simple scalar potential. Together with divergence law, the magnetic scalar potential φ^e satisfies the Laplace equation

$$\nabla^2 \varphi^e = 0 \quad (3)$$

For PEC, the internal magnetic field can be negligible, so the jump condition on the normal magnetic field across the surface is $\mathbf{H}_n^e = \mu_r \mathbf{H}_n = 0$. In this case, equation (3) is equivalent of a boundary integral equation written as:

$$c\varphi^e(\mathbf{r}) + \int_{\Gamma} \left[\varphi^e(\mathbf{r}') \frac{\partial g(\mathbf{r}, \mathbf{r}')}{\partial \mathbf{n}} \right] d\Gamma = \varphi^{pr}(\mathbf{r}) \quad (4)$$

where $g(\mathbf{r}, \mathbf{r}') = \frac{1}{4\pi |\mathbf{r} - \mathbf{r}'|}$ is the Green's function for the Laplace equation, and φ^{pr} is the magnetic scalar potential of the primary magnetic field. Vectors $\mathbf{r}$ and $\mathbf{r}'$ are observation point and source point respectively. Let Γ denotes the surface of the object with unit normal vector $\mathbf{n}$, and the parameter c depends on the location of observation point. c is 1/2 on a smooth boundary of Γ , 1 in Γ and 0 elsewhere [15].

3. Numerical solution

3.1. Numerical implementation of the BEM

Solving (4) involves evaluating several integrals which impose an enormous computational burden if the number of mesh is too large. As the complex computation of BEM, some researches were carried out and fast multi-pole method and hierarchical matrices were two representatives [17-18]. They were introduced for convenience of calculation of the non-singular matrix obtained by the BEM. At the same time, there are some methods emerged to facilitate the computation by building a local coordinate system on the basis of which the parameter transformation is accomplished to make the integral equations more easier [19-23].

For numerical purposes, the object was modeled using planar triangular patches. The surface of the target could be subdivided into NE patches with the e th patch known as Γ_e . Then the integration could be transformed into equations in discrete collocation points and NP was the number of discrete points.

$$c\varphi^e(\mathbf{r}_i) + \sum_{e=1}^{NE} \int_{\Gamma_e} \left[\varphi^e(\mathbf{r}') \frac{\partial g(\mathbf{r}, \mathbf{r}')}{\partial \mathbf{n}} \right] d\Gamma_e = \varphi^{pr}(\mathbf{r}_i) \quad i=1,2..NP \quad (5)$$

In [20], a local coordinate system was introduced to facilitate the calculation of 3-D numerical integrations based on Green's function or its gradient on a plane triangle. The new coordinate system paralleled with triangular element, and had origin on the first vertex of the triangle. One edge of the triangle was set as the abscissa.

Figure 1. (a) Relationship between the local coordinate system and the global coordinate system. (b) Parameters of the local coordinate system.

We used the similar parameters and functions introduced in [20] and here a triangle patch was taken as an example.

V_1, V_2 and V_3 are three vertexes of the triangular patch in terms of global coordinates. ∂S_i , which is opposite to V_i , is the i th edge of triangular element Γ_e . l_i, s_i and $\mathbf{m}_i$ ($i=1,2,3$) are the length, the unit tangent vector and the unit normal vector of ∂S_i respectively. Definitions of the local coordinate system in terms of the global coordinates are $\zeta = \frac{\mathbf{OV}_2 - \mathbf{OV}_1}{l_3}$, $\eta = \hat{\mathbf{n}} \times \zeta$, $\xi = \hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ is the normal vector of Γ_e .

Over the surface, we interpolated $\varphi^e(\mathbf{r}')$ with simple linear interpolation

$$\varphi^e(\mathbf{r}') = \sum_{k=1}^3 \varphi^k N_k(u, v)$$

$$N = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & \zeta_3/l_3 - 1 \\ 0 & 1 & -\zeta_3/l_3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ \zeta/l_3 \\ \eta/\eta_3 \end{bmatrix} = \begin{bmatrix} N_{10} \\ N_{20} \\ N_{30} \end{bmatrix} + \begin{bmatrix} -1 & \zeta_3/l_3 - 1 \\ 1 & -\zeta_3/l_3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \zeta'_a/l_3 \\ \eta'_a/\eta_3 \end{bmatrix} \quad (6)$$

where N is the linear nodal function and φ^k is the magnetic scalar potential on the k th vertex of Γ_e .

On this occasion, a transformation matrix $\beta = [\delta_{kj}]_{3 \times NP}$ was introduced to transform the local number into the global number:

$$\varphi^k = \sum_{j=1}^{NP} \delta_{kj} \varphi_j \quad (7)$$

where φ_j is the magnetic scalar potential of the j th point in the global coordinate system.

So integral equation was given by:

$$c\varphi^e(r_i) + \sum_{j=1}^{NP} \left(\sum_{e=1}^{NE} \sum_{k=1}^3 \left(\int_{\Gamma_e} \frac{\partial g(r_i, r')}{\partial n} N^k d\Gamma_e \right) \delta_{kj} \right) \varphi_j = \varphi^{pr}(r_i) \quad (8)$$

Let $A_{ij} = c\delta_{ij} + \sum_{e=1}^{NE} \sum_{k=1}^3 \left(\int_{\Gamma_e} \frac{\partial g(r_i, r')}{\partial n} N^k d\Gamma_e \right) \delta_{kj}$ be the coefficient matrix, thus the integral equation could be simplified to matrix form:

$$A_{ij} \varphi^e(r_i) = \varphi^{pr}(r_i) \quad i, j = 1, 2, \dots, NP \quad (9)$$

Here $\varphi_{NP \times 1}^e$ are magnetic scalar potentials required on the boundary, and $\varphi_{NP \times 1}^{pr}$ are magnetic scalar potentials of discrete points on the boundary generated by a given magnetic field.

Let $R = |\mathbf{R}| = |\mathbf{r} - \mathbf{r}'|$ be the distance between an arbitrarily located observation point r and a source point r' on Γ_e . Then

$$\frac{\partial g(\mathbf{r} - \mathbf{r}')}{\partial \mathbf{n}} = \frac{1}{4\pi} \frac{\partial}{\partial \mathbf{n}} \frac{1}{R} = \frac{1}{4\pi} \left(\nabla \frac{1}{R} \right) \cdot \hat{\mathbf{n}} = \frac{1}{4\pi} \left(\nabla \frac{1}{R} \right). \quad (10)$$

For a given triangular element

$$\nabla \frac{1}{R} = -\nabla' \frac{1}{R} = -\left(\xi \frac{\xi_0}{R^3} + \nabla'_s \frac{1}{R} \right) \quad (11)$$

So

$$\frac{\partial g(\mathbf{r} - \mathbf{r}')}{\partial \mathbf{n}} = -\frac{1}{4\pi} \frac{\xi_0}{R^3} \quad (12)$$

where ∇ operates on the unprimed coordinates while ∇' operates on the primed coordinates. ∇'_s operates on the $\xi\eta$ plane. So

$$\int_{\Gamma_e} \frac{\partial g(\mathbf{r} - \mathbf{r}')}{\partial \mathbf{n}} N^k d\Gamma_e = -\frac{1}{4\pi} \int_{\Gamma_e} \frac{\xi_0}{R^3} N^k d\Gamma_e \quad (13)$$

The relationships of parameters are defined as follows:

$$\zeta'_a = \zeta' - \zeta_0; \quad \eta'_a = \eta' - \eta_0; \quad lf_i = \ln \left(\frac{R_i^+ + s_i^+}{R_i^- + s_i^-} \right) \quad (14)$$

$$\gamma = \sum_{i=1}^3 \gamma_i = \sum_{i=1}^3 \left(\tan^{-1} \frac{d_i^0 s_i^+}{(R_i^0)^2 + |\xi_0| R_i^+} - \tan^{-1} \frac{d_i^0 s_i^-}{(R_i^0)^2 + |\xi_0| R_i^-} \right) \quad (15)$$

$$I = \begin{bmatrix} I_1^* \\ I_2^* \\ I_3^* \end{bmatrix} = \begin{bmatrix} \int_{\Gamma_e} \frac{\xi_0}{R^3} N_1 d\Gamma_e \\ \int_{\Gamma_e} \frac{\xi_0}{R^3} N_2 d\Gamma_e \\ \int_{\Gamma_e} \frac{\xi_0}{R^3} N_3 d\Gamma_e \end{bmatrix} = \begin{bmatrix} N_{10} \\ N_{20} \\ N_{30} \end{bmatrix} I^* + \begin{bmatrix} -1 & \zeta_3/l_3 - 1 \\ 1 & -\zeta_3/l_3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_{ua}^* \\ l_3 \\ I_{va}^* \\ \eta_3 \end{bmatrix} \quad (16)$$

The integrals of the function $\frac{1}{R^3}$ and $\left[\frac{\zeta'_a}{\eta'_a} \right] \frac{1}{R^3}$ are now easily obtained from [20, eq.26, 27]:

$$I^* = \xi_0 \int_{\Gamma_e} \frac{d\Gamma_e}{R^3} = \text{sgn}(\xi_0) \gamma \quad (17)$$

$$\begin{bmatrix} I_{ua}^* \\ I_{va}^* \end{bmatrix} = \xi_0 \int_{\Gamma_e} \left[\frac{\zeta'_a}{\eta'_a} \right] \frac{d\Gamma_e}{R^3} = -\xi_0 \sum_{i=1}^3 m_i \cdot \begin{bmatrix} \tilde{\zeta} \\ \tilde{\eta} \end{bmatrix} lf_i \quad (18)$$

With the aid of (17)-(18), it is convenient to computer coefficient matrix $\mathbf{A}$. After that, if we know a given additional magnetic field, then we apply Gauss elimination method to calculate $\mathbf{A}_{ij} \varphi^e(r_i) = \varphi^{pr}(r_i)$ to obtain magnetic potentials of discrete points on the surface of the target.

3.2. Computation of the scattered magnetic field

Through the similar transformation and then take the gradient of both sides of (8), magnetic field outside the target can be derived:

$$\mathbf{H}^e(r) = \mathbf{H}^{pr} + \mathbf{H}^{sc} = \mathbf{H}^{pr}(r) + \sum_{e=1}^{NE} \sum_{k=1}^3 \left(\int_{\Gamma_e} \frac{\partial(\nabla g(r, r'))}{\partial n} N^k d\Gamma_e \right) \varphi^k \quad (19)$$

Here $\mathbf{H}^{pr}$ is the additional magnetic field and φ^k is the magnetic potentials of the discrete points on surface of target, so the integral on the e th triangular element will be of the form

$$\Pi_e = \sum_{k=1}^3 \left(\int_{\Gamma_e} \frac{\partial(\nabla g(r, r'))}{\partial n} N^k d\Gamma_e \right) \varphi^k \quad e = 1, 2, \dots, NE \quad (20)$$

According to the equations above, we simply noted that all integrations were solvable. By calculating the linear simultaneous equations $\mathbf{A}_{ij} \varphi^e(r_i) = \varphi^{pr}(r_i)$, magnetic potentials and magnetic field could be obtained.

3.3. Scalar magnetic field due to the excitation coil and sensitivity maps by perturbation

If the current on circular excitation coil is I , the magnetic field generated by the coil can be easily derived from the Biot-Savart law:

$$\mathbf{H} = \int d\mathbf{H} = \frac{\mathbf{B}}{\mu_0} = \frac{I}{4\pi} \oint_l \frac{d\mathbf{l} \times \mathbf{R}}{|\mathbf{R}|^3} = \frac{I}{4\pi} \int_0^{2\pi} \frac{\mathbf{t} \times \mathbf{R}}{|\mathbf{R}|^3} r d\theta \quad (21)$$

Note that $\mathbf{H}^{pr} = -\nabla \varphi^{pr}$, the potential of the primary magnetic field is written as [24]:

$$\varphi^{pr} = \frac{I}{4\pi} \iint_s \frac{\mathbf{R}}{|\mathbf{R}|^3} \cdot \mathbf{n}_0 ds \quad (22)$$

Here $\mathbf{B}$ is magnetic flux density and s denotes the plane enclosed by the excitation coil l . $\mathbf{R} = \mathbf{r} - \mathbf{r}'$, $\mathbf{t}$ is the unit tangent vector of l and $\mathbf{n}_0$ is the unit normal vector of s . μ_0 is the permeability of the free space.

According to Maxwell's equations, we have

$$U = \oint_l \mathbf{E} \cdot d\mathbf{l} = - \iint_s \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} = - \iint_s \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{n}_0 ds = \sum_{i=1}^N \frac{\partial \mathbf{B}_i}{\partial t} \cdot \mathbf{n}_0 s_i = 2\pi f \sum_{i=1}^N \mathbf{B}_i \cdot \mathbf{n}_0 s_i = 2\pi f \mu_0 \sum_{i=1}^N \mathbf{H}_i \cdot \mathbf{n}_0 s_i \quad (23)$$

Where U is the voltage on the receiving coil and s_i is the area of the i th element of s . f is the driving frequency and N is the number of mesh of s .

Sensitivity distributions were calculated using the formula:

$$S_{en} = - \frac{U^e - U^{pr}}{U^{pr}} = - \frac{U^{sc}}{U^{pr}} \quad (24)$$

where S_{en} is the sensitivity distribution of the measured space. U^e , U^{sc} and U^{pr} are the total voltage, voltage produced by the scattering field and the voltage produced by the primary field respectively.

The induced voltage of the receiving coil could be obtained by moving the target object in the space between the coils with small steps each time. Sensitivity maps could finally be drawn.

4. Results of the simulations

Assume the excitation coil and receiving coils have the same radius of $ra=1m$ and a sinusoidal current $I = 1 \cdot \sin(\omega t)$ is applied to the excitation coil, where $\omega=2\pi f$ and t is time. Here the frequency is $f=10MHz$ and the unit normal vector of the excitation coil $\mathbf{n}_0$ is directed along the z axis. The direction of the current and $\mathbf{n}_0$ satisfy the right-handed corkscrew rule.

Simulations were carried out with one excitation coil and four receiving coils. The five coils are of the same size and their radius is $1m$. The distance from the center of the coils to the center of the measured space is $6m$. Figure 2 shows the positions of the coils. The measured space which was divided into a number of rectangular elements is a $4m$ -diameter circle. The target moved in the region of $z \in (-4m, 4m)$, $y \in (-4m, 4m)$, in yz plane where $x=0$. A PEC was scanned across the sensing space and the voltages of the receiving coils were recorded. $\mathbf{r}$ and $\mathbf{r}'$ are the distance vectors from the origin to the arbitrary point on the surface of PEC and on the excitation coil respectively. With the help of the equations above, we can obtain four typical sensitivity maps.

Comment [WY1]: What equations???
More specific !!

Figure 2. Positions of the coils and the target.

Initial targets investigated are five different shapes which will be discussed respectively with the same area of $4/3\pi$.

Comment [WY2]: 体积还是面积!!!

- 1) Sphere: the radius is $ra=1m$ with the surface given by $x^2 + y^2 + z^2 = 1$.
- 2) Cube: the length of each side is $l=(4\pi/3)^{\frac{1}{3}}m$.
- 3) Ellipsoid: the surface is given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, where $a=2m$, $b=1m$, $c=0.5m$.
- 4) Rectangle: the lengths of three sides are $l1=1m$, $l2=2m$, and $l3=2\pi/3m$ in x , y and z axis respectively.
- 5) Cylinder: the height is $h=4/3m$, the radius of the bottom is $r=1m$.

Figure 3. Four typical sensitivity maps between the excitation coil and receiving coils 1-4 obtained by BEM.

(a) for a sphere perturbation (b) for a cube perturbation (c) for a rectangle perturbation (d) for an ellipsoid perturbation (e) for a cylinder perturbation.

Figure 4. Sensitivity maps between the excitation coil and receiving coils of sphere using the H dot H formulation.

With the aid of MATLAB, the sensitivity maps were shown as Figure 3. The four figures in every sub-graph of Figure 3 are the sensitivity distributions between the excitation coil and receiving coil 1 to 4 respectively. Figure 4 shows the sensitivity maps between four coil pairs obtained by the H dot H solution. **The scales used in the figures are the same and there's no scaling here.**

The figures show that the sensitivities are larger in the position near the coils than that in the center of the measuring region. The sensitivity maps are saddle-shaped on the whole for all objects presented, but the different shapes make a difference in the value and the symmetrical characteristic of the sensitivity distributions. For exactly symmetric objects, e.g. sphere, the sensitivity maps are basically symmetric. But for the objects without perfect x-axis symmetry, the sensitivity distributions are not in perfect symmetric in some cases.

Comment [WY3]: 把徐凯的数据要过来，自己画一下。和你的上面图的格式就对应起来了。

Comment [WY4]: ?? 哪些，图3和图4？

Figure 4 is obtained by the H dot H formulation with different parameters, and here it is acted only as a comparison. The relative positions of the coils are the same as in our simulation. The four sub-graphs are four typical sensitivity maps between the excitation coil and the receiving coils 1-4 respectively.

From the pictures shown above, we can see that the sensitivity distributions are in general agreement with what were reported in previous literatures, showing typical saddle shapes. The sensitivity maps are generally the same for five different perturbation shapes, which indicate that objects with different shapes but same volume is likely produce similar sensitivities. In comparison to the pictures obtained by the H dot H formulation, it shows that the sensitivity distributions are in good agreement with the theoretical H dot H solution [25].

5. Experimental procedure

To evaluate the validity of the calculation of the sensitivity maps using BEM, experiments were also conducted using the high frequency MIT system developed by our teams. The hardware comprised several parts- the main FPGA board, the front-end circuit, the host PC, and the sensor array [26].

Figure 5. Schematic diagram of the experiment for two-channel measurements.

Measurements were carried out with one excitation coil and two receiving coils. The three coils are of the same size and their radius is 15mm. The distance from the center of the coils to the center of the measured space is 90mm. Figure 5 shows the positions of the coils. The measured space which was divided into a number of rectangular elements is an 80-mm-diameter circle. A metal object was scanned across the sensing space and the voltages of two receiving coils were recorded. Two kind of metal objects were used in the experiment. One is a 12-mm-diameter, 100-mm-height aluminum bar and the other one is an 8-mm-diameter, 100-mm-height copper bar. To increase the accuracy, the aluminum foil, which acted as a shielding layer, was used to surround the box where the experiment was carried out. In the experiment, the metallic bar is moved in the grids successively shown as Figure 5 and for each measurement turn, we note a voltage with empty space and a voltage of that with the bar. With the help of equation (25), the sensitivity distributions can be deduced between different coil pairs as needed. The system diagram is given as Figure.6.

Sensitivity distributions were calculated using the formula:

$$S_{en} = -\frac{V_{mea} - V_{void}}{V_{void}} \quad (25)$$

where S_{en} is the sensitivity distribution of the measured space. V_{mea} is the measured voltage with the perturbation target present and V_{void} is the voltage with empty space.

Comment [WY5]: 这句话不能有。他问你为什么不能自己按照一样的尺寸做一下？

Figure 6. System diagram of the experiment.

Figure 7. Sensitivity maps for coil pairs 1-2, 1-3 for the copper bar respectively.

Figure 8. Sensitivity maps for coil pairs 1-2, 1-3 for the aluminium bar respectively.

After the measurement, we obtained two groups of sensitivity distributions for both aluminum bar and copper bar. One group was the sensitivity distribution between the excitation coil and the receiving coil 1 and the other one was that between the excitation coil and the receiving coil 2 shown as Figure 7 and Figure 8.

The sensitivity maps are all saddle-shaped. As we known, the conductivity of copper is higher than the conductivity of aluminum, but the copper bar used is thinner than the aluminum bar in the experiment, hence the voltages measured with a copper are smaller than that with an aluminum bar.

From figures 7 and 8, we know that the sensitivity maps are in good agreement with the sensitivity maps obtained by MATLAB and theoretic solution shown as Figure 3 and Figure 4 respectively. It should be noted that quantitative comparison is not possible because non-ideal characteristics exist for both measurement circuits and MIT sensor.

Comment [WY6]: 省去！

6. Conclusions

In this paper, the magnetic fields of high frequency MIT system have been solved numerically and the sensitivity maps of different coil pairs drawn with MATLAB. The paper presented formulas for the high frequency MIT system where the metal object can be treated as a PEC. **The current formulation is implemented in the paper and related information is given to obtain the results.** 3-D Green's function and its gradient are both considered in the computation and the scalar potential is used to calculate the magnetic field in the outside region. In the simulation, the sensitivity maps obtained by the method of perturbation are saddle-shaped, **and the sensitivities in the positions near the coils is larger than that in the center** which fit with the H dot H formulation. Experiments are also conducted with the help of the system developed in our lab. The measured results have confirmed good consistency with the sensitivity maps obtained by BEM although quantitative comparison was not possible due to non-ideal characteristics in measuring circuit and sensor. Nevertheless, from the comparison, we could see that BEM is an effective way to calculate sensitivity maps in the high frequency MIT system.

Comment [WY7]: ??

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