Co-movements between US and UK stock prices: the role of time-varying conditional correlations

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Co-movements between US and UK stock prices: The role of time-varying conditional correlations

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Abstract

We provide evidence on the nature of co-movement in monthly US and UK stock returns by investigating time-varying correlations in returns since 1980. There is a marked increase in correlations between these markets around 2000, which we attribute to globalisation and model with a time-varying smooth transition conditional correlation (STCC) GARCH specification. A double transition correlation model with time and US stock price volatility indicates that correlations not only increase over time but also during periods of high volatility. The STCC models perform well in comparison with constant (CCC) and dynamic (DCC) conditional correlation models in terms of both statistical and economic criteria, but demonstrates that an investor will gain little from portfolio diversification across these two important markets.

1. Introduction

There is a great deal of interest, and a correspondingly large literature, on the relationship between international financial markets. In particular, it is now well established that the correlations of returns across international stock markets are not only strong, but also time-varying and that correlations increase in more volatile bear markets. For important contributions to understanding the nature of this phenomenon see Ang and Bekaert (2002), King, Sentana and Wadhwani (1994), Longin and Solnik (2001), and Ramchand and Susmel (1998), among others. It has also been observed that stock markets have become more integrated over time. For example, Cappiello, Engle and Sheppard (2006) find a structural break in international equity return correlations in January 1999. Although this date corresponds with the introduction of the European single currency, evidence for this break extends beyond these markets; see also Savva, Osborn and Gill (2009). Therefore, broader effects may be operating[1].

The present paper tests for and models changes in the correlations of returns between two of the major stock markets of the world, namely the US and UK. Investors need to understand the nature of changes in co-movements in order to evaluate the potential benefits of international portfolio diversification. For example, if stock markets have become more integrated over time due to the effects of globalisation, then the benefits of international diversification will be reduced, presumably permanently. On the other hand, if movements across markets are closely correlated only during periods of high volatility, then international diversification may be beneficial particularly during periods of relative calm. In this paper we model changing correlations between stock market price movements in the US and UK

[1] We refer to these broader effects as globalisation, which may encompass deregulation of financial markets, increased international trade and cross-border financial flows, improved information technology including the growth of internet usage around the globe, etc.

since 1980 with a smooth transition model in conditional correlations and use time to capture the increased correlation of the markets due to globalisation. We also allow the possibility that US market volatility influences the correlation function.

To preview our results, we find strong statistical evidence for time-varying correlations, which are adequately captured by a smooth transition conditional correlation model that implies a strong increase in correlations around 2000. We also find some evidence that the correlation between markets increases with high volatility phases in the US market. We investigate the economic performance of our models by comparing the certainty equivalent rates of return (CER) for an international investor over a range of time-varying correlation models during the post-sample period from July 2006 to December 2008. Especially for the sub-period when the financial crisis has unfolded and markets fallen, the smooth transition specification that allows correlations to change with both time and volatility performs best for the less risk-averse investor.

The organisation of this paper is as follows. Section 2 describes the time-varying correlation models and discusses the testing procedures we employ. This section also details how the economic performance of the models is compared using CER. Substantive results are then reported and discussed in Section 3. Conclusions in Section 4 complete the paper.

2. Econometric Methodology

We explicitly consider the existence and nature of time-varying conditional correlations using dynamic conditional correlations (Engle, 2002)[2] and the recently developed smooth transition conditional correlation GARCH (STCC-GARCH) model: see Berben and Jansen (2005), Silvennoinen and Teräsvirta (2005) and recently Sivennoinen and Teräsvirta (2007) who extend this to a double STCC-GARCH with two transition variables. The latter are preferred to the Markov-switching approach of Ang and Bekaert (2002) and Pelletier (2006), among others, since it allows the regime to be modelled as a continuous function of one or more so-called transition variables.

After outlining our approach for mean and volatility (Section 2.1), Section 2.2 describes the time-varying conditional correlation models. Specification testing and estimation are then presented in Sections 2.3 and 2.4, while Section 2.5 discusses how we compare the economic implications of our econometric models.

[2] A similar methodology is proposed by Tse and Tsui (2002).

2.1. Mean and Volatility

The two-dimensional vector (y_t) of stock returns for the US and UK markets can be written as

$$y_t = \mu + u_t, \qquad t = 1, 2, \ldots, T \tag{1}$$

where $\mu = (\mu^{SP}, \mu^{FT})'$ is a 2x1 vector of constants. In line with recent literature on international stock market returns, the conditional covariances of the shocks in equation (1) are time-varying, such that

$$u_t | \Im_{t-1} \sim N(0, H_t) \tag{2}$$

where $\Im_{t-1}$ is all available information at t-1. From equation (2), each univariate error process can be written

$$u_{i,t} = h_{ii,t}^{1/2} \varepsilon_{i,t}, \; i = 1, 2 \tag{3}$$

where $h_{ii,t} = E(u_{i,t}^2 / \Im_{t-1})$ and $\varepsilon_{i,t}$ is a sequence of independent random variables with mean zero and variance one. As common in empirical analyses, each conditional variance is assumed to follow the univariate GARCH(1,1) process

$$h_{ii,t} = \alpha_{i0} + \alpha_{i1} u_{i,t-1}^2 + \beta_{i1} h_{ii,t-1} \tag{4}$$

with non-negativity and stationarity restrictions imposed.

Rather than modelling the off-diagonal elements of H_t directly, the definition

$$\rho_t = h_{12,t}(h_{11,t} h_{22,t})^{-1/2} \tag{5}$$

allows the focus to be placed on the conditional correlations ρ_t. The constant conditional correlation (CCC) model assumes that ρ_t is constant over time, while the dynamic conditional correlation (DCC) and (double) smooth transition conditional correlation (STCC) models allow distinct patterns of time-variation in ρ_t.

2.2. Time-Varying Conditional Correlations

Engle (2002) specifies the DCC model through the GARCH(1,1)-type process

$$q_{ij,t} = \bar{\rho}_{ij}(1 - \alpha - \beta) + \alpha \varepsilon_{i,t-1} \varepsilon_{j,t-1} + \beta q_{ij,t-1}, \quad i, j = 1, 2 \tag{6}$$

where $\bar{\rho}_{12}$ is the assumed constant unconditional correlation between the standardized residuals $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$, α is the news coefficient and β is the decay coefficient. The quantity $q_{12,t}$ from equation (6) is normalized using

$$\rho_t = \frac{q_{12,t}}{(q_{11,t}\, q_{22,t})^{1/2}} \tag{7}$$

in order to ensure a conditional correlation between -1 and +1. The model is mean-reverting provided $\alpha + \beta < 1$, while the conditional correlation process in equation (6) is integrated when the sum equals 1. However, the latter case violates the assumption of a constant unconditional correlation $\overline{\rho}_{12}$, which is embedded in equation (6).

Rather than assuming a constant unconditional correlation, the STCC model developed by Berben and Jansen (2005) and Silvennoinen and Teräsvirta (2005)[3] assumes the presence of two extreme states (or regimes) with state-specific correlations. These correlations are, however, allowed to change smoothly between the two regimes as a function of an observable transition variable s_t. More specifically, the conditional correlation ρ_t follows

$$\rho_t = \rho_1 \left(1 - G_t\left(s_t;\gamma,c\right)\right) + \rho_2 G_t\left(s_t;\gamma,c\right) \qquad (8)$$

in which the transition function $0 \le G_t\left(s_t;\gamma,c\right) \le 1$ is a continuous function of s_t, while γ and c are its parameters.

Since equation (8) implies $\rho_t = \rho_1$ when $G_t = 0$ and $\rho_t = \rho_2$ when $G_t = 1$, extreme values of the transition function identify the distinct correlations that apply in these regimes. A weighted mixture of these correlations applies when $0 < G_t < 1$. A plausible and widely used specification for the transition function is the logistic function

$$G_t\left(s_t;\gamma,c\right) = \frac{1}{1+\exp[-\gamma\left(s_t - c\right)]}, \quad \gamma > 0 \qquad (9)$$

where the parameter c locates the midpoint between the two regimes. The slope parameter γ determines the smoothness of the change in G_t as a function of s_t. When $\gamma \to \infty$, $G_t\left(s_t;\gamma,c\right)$ becomes a step function ($G_t\left(s_t;\gamma,c\right) = 0$ if $s_t < c$ and $G_t\left(s_t;\gamma,c\right) = 1$ if $s_t > c$), and the transition between the two extreme correlation states becomes abrupt. In that case, the model approaches a threshold model in correlations.

An important special case of the STCC model uses time as the transition, $s_t = t/T$, which gives rise to the time-varying conditional correlation (TVCC) model employed by Berben and Jansen (2005)[4]. This allows one (smooth) change between correlation regimes, and as $\gamma \to \infty$ captures a structural break in the correlations. This transition variable may be

[3] The model of Berben and Jansen (2005) is bivariate with a time trend as the transition variable, while the framework of Silvennoinen and Teräsvirta (2005) is multivariate and their transition variable can be deterministic or stochastic.

[4] The scaling implied by defining $s_t = t/T$ aids interpretation; see Berben and Jansen (2005).

particularly relevant in order to capture the effects of increasing integration of financial markets over the last thirty years.

Silvennoinen and Teräsvirta (2007) generalise (8) to allow for two transition variables to govern the extreme states, such that

$$\rho_t = (1-G_{2t})(\rho_{11}(1-G_{1t})+\rho_{21}G_{1t}) + G_{2t}(\rho_{12}(1-G_{1t})+\rho_{22}G_{1t}) \qquad (10)$$

where the transition functions $G_{1t}(s_{1t};\ \gamma_1,\ c_1)$ and $G_{2t}(s_{2t};\ \gamma_2,\ c_2)$ are logistic functions as defined in (9). The model in (10) then becomes a double smooth transition conditional correlation (DSTCC) model. By defining one of the transition variables as time; say $s_{2t} = t/T$ and the other (s_{1t}) as a volatility measure, this allows us to investigate the roles of globalisation and the state of the markets on the conditional correlations. At the beginning of the sample period when $G_{2t} = 0$, the correlations vary smoothly between the extremes ρ_{11} ($G_{1t} = 0$) and ρ_{21} ($G_{1t} = 1$) depending on the values of s_{1t}. As time passes, the correlations ρ_{11} and ρ_{21} change smoothly to ρ_{12} and ρ_{22}, respectively. The use of four regimes, each with an associated correlation, allows the transition variable s_{1t} to have distinctive effects in the earlier and later parts of the sample period, reflecting potentially differing impacts of high versus low volatility states as globalisation progresses.

2.3. Specification Tests

Before applying either time varying correlation models, tests are applied to investigate the constancy of the conditional correlations in equation (5). Two residual-based tests suggested by Bollerslev (1990) are particularly suitable for testing against a DCC specification. The first is the Ljung-Box statistic for testing autocorrelation up to m lags in the cross product of the residuals (u_{1t} and u_{2t}) each standardized using the GARCH(1, 1) model of equation (4), which under the null hypothesis is asymptotically distributed as χ^2 with m degrees of freedom (we use $m = 18$). The second is an F test of the significance from a regression of the sample values of $u_{1t}u_{2t}h_{12,t}^{-1} - 1$ on $h_{12,t}^{-1}$, $r_{1,t-1}^2 h_{12,t}^{-1}$, $u_{2,t-1}^2 h_{12,t}^{-1}$ and lags $u_{1,t-k}u_{2,t-k}h_{12,t}^{-1}$ (in which we include $k = 1, \ldots, 12$). In addition, we apply the Lagrange Multiplier (LM) test of Tse (2000), which considers the null hypothesis $\delta = 0$ in the ARCH-type structure

$$\rho_{12,t} = \rho_{12} + \delta\, u_{1,t\text{-}1}u_{2,t\text{-}1} \qquad (11)$$

Under the null hypothesis, the statistic is distributed as χ^2 with 1 degree of freedom[5]. We perform the Tse (2000) test in an estimation of the complete system (including mean equations).

Silvennoinen and Teräsvirta (2005) derive a Lagrange Multiplier test LM_{CCC} for the constancy of the correlations against a STCC with a particular transition variable by applying a first-order Taylor series expansion to the transition function (9) and then testing the significance of the additional terms that arise compared to a CCC specification. We perform this test with time as the transition variable. Moreover, based on previous studies that find co-movements to be stronger in volatile times than in more tranquil periods (Ang and Bekaert, 2002; Ang and Chen, 2002; Baele, 2005; Forbes and Rigobon, 2002; Longin and Solnik, 1995, 2001; Patton, 2004; Ramchand and Susmel, 1998), we also test constancy of correlations against a STCC model with the conditional variance of the US stock returns as the transition variable. After estimation, the STCC model can be tested against the more general double transition STCC, as in (10), using the Lagrange Multiplier test (LM_{STCC}) of Silvennoinen and Teräsvirta (2007).

Finally, the adequacy of the DCC and (D)STCC models are checked using diagnostic tests applied to the standardised residuals from the bivariate system. Following Engle (2002), the required standardised residuals $v_t = H_t^{-1/2} u_t$ are computed through the triangular decomposition of H_t, so that

$$v_{1,t} = u_{1,t} / h_{11,t}^{1/2}$$

$$v_{2,t} = u_{2,t} \frac{1}{(h_{22,t}(1-\rho_{12,t}^2))^{1/2}} - u_{1,t} \frac{\rho_{12,t}}{(h_{11,t}(1-\rho_{12,t}^2))^{1/2}} \qquad (12)$$

in which unknown parameters are replaced by their sample analogues. Since v_{2t} depends on the (estimated) dynamic correlations, tests on this are more revealing than those on v_{1t} (Engle, 2002, p.344). We apply the Ljung-Box test to both the standardized residuals and the squares of these standardized residuals.

2.4. Estimation

We estimate the CCC, DCC and (D)STCC models by quasi-maximum likelihood (QML), where standard errors that are robust to the violation of the assumption of normality in (2)

[5] Tse (2000) notes that it may be more natural to use standardised values of $r_{i,t-1}$ in equation (11), but prefers the unstandardized form for analytical tractability. Nevertheless, this choice may affect the power of the test. Power may also be affected by applying this two-sided test, in a context where δ is positive under the alternative.

are used for the parameter estimates (Bollerslev and Wooldridge, 1992). All equations (that is, for the means, conditional volatilities and conditional correlation) are estimated jointly. Although Engle (2002) and Cappiello *et al*. (2006) use a two step approach for estimation of DCC models, this does not allow for computation of QML standard errors that are robust to the violation of the assumption of normality in equation (2). Furthermore, through joint estimation taking account of (changing) cross-market conditional correlations, we aim for efficiency gains in estimation[6].

Nevertheless, nonlinear estimation of the resulting models is not always easy to achieve and specification of starting values plays a crucial role. The procedure used to obtain starting values is discussed in Appendix 1.

2.5. Economic Implications

To evaluate the potential benefits of portfolio diversification between the US and UK markets, we calculate the realized Certainty Equivalent Rates of Return (CER) for our models using actual post sample data between July 2006 and December 2008. Over this sub-sample the markets in the US and the UK both experienced losses as the credit crunch began to bite in the summer of 2007. Based on coefficients estimated using data to June 2006, we compute optimal portfolio weights for each month of the post-sample period and for each model that we examine.

Following Guidolin and Hyde (2008), we assume an investor spreads their investments over the two markets using mean-variance CER preferences described by:

$$V_t = E\left[W_{t+1}\middle|\Im_t\right] - 0.5\lambda\, Var\left[W_{t+1}\middle|\Im_t\right] \qquad (13)$$

where one-step ahead wealth is:

$$W_{t+1} = \omega_t^{SP}\left(1 + y_{t+1}^{SP}\right) + \left(1 - \omega_t^{SP}\right)\left(1 + y_{t+1}^{FT}\right)$$

in which ω_t^{SP} is the proportion of the portfolio invested in the US market at t and (1 - ω_t^{SP}) is the proportion invested in the UK. Here λ is interpreted as the coefficient of risk aversion that trades-off predicted mean and variance of the one-step ahead wealth; the larger λ the more risk averse is the investor.

At each month t in the post-sample period, the investor empirically estimates $E[W_{t+1}|\Im_t]$ as

[6] In practice we estimate the CCC and DCC models using the GARCH wizard in RATS 6.3. The reported (D)STCC estimates are obtained using GAUSS, where our programs are adapted from code supplied to us by Christos Savva.

$$\hat{W}_{t+1} = \hat{\omega}_t^{SP}\left(1+\hat{\mu}^{SP}\right)+\left(1-\hat{\omega}_t^{SP}\right)\left(1+\hat{\mu}^{FT}\right) \quad (14)$$

where $\hat{\mu}_t^{SP}$, $\hat{\mu}_t^{FT}$ are the estimated constants from the respective mean equations (1). The variance equation in (13) is estimated as:

$$\hat{Var}\left[W_{t+1}\right] = (\omega_t^{SP})^2 h_{t+1}^{SP} + \left(1-\omega_t^{SP}\right)^2 h_{t+1}^{FT} + 2\omega_t^{SP}\left(1-\omega_t^{SP}\right)\hat{\rho}_{t+1}\sqrt{h_{t+1}^{SP}h_{t+1}^{FT}} \quad (15)$$

Substituting from (14) and (15) into (13), we then solve for ω_t^{SP} by maximising this expression by setting $\frac{dV_t}{d\omega_t^{SP}} = 0$. This yields the weight

$$\omega_t^{SP} = \frac{\left(\hat{\mu}^{SP} - \hat{\mu}^{FT} + \lambda h_{t+1}^{FT} - \lambda\hat{\rho}_{t+1}\sqrt{h_{t+1}^{SP}h_{t+1}^{FT}}\right)}{\lambda\left(h_{t+1}^{SP} + h_{t+1}^{FT} - 2\hat{\rho}_{t+1}\sqrt{h_{t+1}^{SP}h_{t+1}^{FT}}\right)} \quad (16)$$

Finally, using (16), we calculate the realized CER for various values of λ as:

$$CER = \sum_{t=1}^{n}\frac{W_{t+1}}{n} - 0.5\lambda\sum_{t=1}^{n}\frac{(W_{t+1}-\overline{W})^2}{n} \quad (17)$$

where n is the number of post sample observations.

Our post-sample period of July 2006 to December 2008 contains two distinct sub-periods, which can be categorized as a bull market, with rising returns and relatively low volatility, until May 2007 and a bear market with falling returns and relatively high volatility after that date. Therefore, we also split the post sample into these two sub-samples, in order to compare the performance of the models in periods of increasing and falling returns.

3. Empirical Results

We simultaneously model monthly changes in the logarithm of the index of US and UK stock prices using data over the sample January 1980 to June 2006. More precisely, the US stock price is the Standard and Poor's composite index (*SP*) and the UK stock price is the Financial Times All Share Index (*FT*), with end-of-month values employed for each. The starting date of 1980 is selected as it is subsequent to the financial liberalisations that occurred during the latter part of the 1970s[7].

[7] Also, the early/mid-1970s were crisis years in the UK, with accelerating inflation, rising unemployment, massive industrial unrest and the first oil price shock (Dow, 1998). In their Markov switching model for UK returns Guidolin and Timmermann (2003) associate one regime, with negative mean returns and a large variance, primarily with this period.

Our sample period includes the stock market crash in October 1987, which affects both UK and US stock prices. The effect of the Long Term Capital Management crisis in 1998 is marked for the US stock price index. To ensure these events do not unduly influence the estimated models, we replace these outliers by the average value of the series over the sample period, computed excluding the outlier observations (see Appendix 2 for details of data, including outliers removed).

Table 1 presents summary statistics for stock returns along with sample cross correlations to provide some descriptive evidence on the changing correlations of the monthly stock market returns series that we model. As can be seen, these markets exhibit a strong positive correlation over our entire sample period, but over (approximately) five year sub-samples, this correlation varies between 0.45 and 0.87. Indeed, the contrast between the correlations for the second half of the 1990s and the high correlation in first part of the new century (including our post-sample period) is particularly marked[8].

Section 3.1 provides a summary comparison of results for different conditional correlation specifications. Then in Section 3.2 we discuss the economic implications of the different models.

3.1. Statistical Comparisons

Initially we run a battery of tests in the context of a CCC model that allows for time varying volatility. Table 2 shows that an assumption of constant cross-market conditional correlations is not adequate. According to the Ljung-Box, Bollerslev and the Tse residual diagnostic tests, this assumption is rejected against a DCC model.

Further, the constancy test of Silvennoinen and Teräsvirta (2005) against the alternative of a STCC model is performed, with time and conditional volatility of the US market as candidate transition variables. In the latter case, conditional volatility is measured through the GARCH (1, 1) model of (4) (the resulting variable denoted as $SPvol_t$), with the US market employed due to its dominant international role. The results, also reported in Table 2, show that the CCC model is particularly strongly rejected against the STCC model with a time transition, and hence we prefer this specification to one based on a volatility transition. Nevertheless, rejection of the CCC model at the 5 percent level using the $SPvol_t$ transition indicates this may also contain some information. Although the Silvennoinen and

[8] Goetzmann, Li and Rouwenhorst (2005) examine the correlation structure of world equity markets for a period of 150 years and find that correlations between stock markets were relatively high during the late nineteenth century, the Great Depression and the late twentieth century.

Teräsvirta (2007) test for the null of the time transition STCC model against a DSTCC with volatility as the additional transition does not reject the former model, we nevertheless include such a DSTCC model in the comparisons, in the light of previous literature (see references in Section 2.3) that finds correlations to increase in periods of high volatility.

The estimation results for the DCC, STCC and DSTCC models are summarised in Tables 3-5. The estimated constant correlation in the CCC model is 0.636, just below the 0.656 cross-correlation sample average reported in Table 1[9]. Tables 3 and 4 present the DCC and STCC model results, respectively, while their resulting time varying conditional correlations are shown in Figure 1 along with the post sample calculation of these series. Note that the DCC correlations are computed each month over the post sample period using the most recent observations, while the STCC ones are constant over this period. In terms of the temporal pattern of the STCC correlations, c in Table 4 defines the middle of the transition period, with this value expressed as a fraction of the sample size, and the corresponding estimated mid-point date of February 2000 is also indicated. It might also be noted that the slope parameter of the transition function of 31.7 results in a relatively fast change over time in the cross-market conditional correlations evident in Figure 1. For both specifications the implied correlations grow fairly dramatically from around 0.5 at the beginning of the sample to around 0.9 in the latter part, which may reflect increasing globalisation and integration of international stock markets. Although Cappiello *et al*. (2006) associate an increase in correlations of stock markets in the recent past with the introduction of the euro currency, the increase in the correlation between the US and UK cannot be attributed to this source and appears to be a consequence of broader international financial market integration; see also Savva *et al*. (2009).

The DSTCC results in Table 5 with the US stock market volatility and time as transition variables indicate four regimes for the correlations. Firstly, in the sample period until around 1999, the correlation of shocks to the markets varies between 0.53 and 0.69, depending on the value of $SPvol_t$ in relation to the threshold value c_1=18. Therefore, stock market volatility appears to play a substantive role for correlations over this period, and the implication is that correlations increase in more turbulent times to 0.69 and fall back in calmer periods to 0.53. Subsequently, from around 2001, these correlations rise to 0.91 (during the high volatility state) and fall back to 0.84 in 2003 (returning to the low volatility

[9] Detailed estimation results are not presented for the CCC model, but they are available from the authors upon request.

state). Although the estimates for this later period have to be treated with considerable caution due to the relatively small number of observations available after the time transition takes effect, the results nevertheless suggest that volatility changes play little role for the correlations at the end of the sample. The estimated DSTCC conditional correlations are shown along with the simultaneously estimated S&P conditional variance in Figure 2.

A statistical comparison of all these models is made in Table 6. Here we see that the STCC model with the time transition has the lowest value of both AIC and SIC, indicating that this model is superior to the DCC specification in capturing temporal correlation effects. Although the DSTCC model results in a relatively marginal increase in the log likelihood compared with the single transition case, this model would be preferred to both the CCC and DCC specifications according to the information criteria. In the next sub-section we compare the models' economic performance.

3.2. Economic Implications

Given the parameter estimates, we calculate optimal portfolio weights to assess the out-of-sample portfolio performance of our models. In particular, we calculate the realized Certainty Equivalent Rates of Return (CER) using actual post sample data during 2006:m7 – 2008:m12 (30 observations). However, two different sub-periods can be isolated within this: during 2006:m7-2007:m5 stock returns are positive and scarcely volatile; on the contrary, over 2007:m6-2008:m12 stock returns become more volatile (especially from around July 2008) and ultimately turn very negative. The latter part of this period, in particular, is characterized by the credit crunch. We evaluate the economic implications of our models for the whole post sample period as well as for the aforementioned post sub-sample periods.

The mean-variance preference function in (13) is solved both without and with restrictions on the admissible range for ω_t^{SP}. That is, the empirical solution given by (16) does not impose the restriction $0 \le \omega_t^{SP} \le 1$, which can be interpreted as preventing the investor from selling securities short. Therefore, we present two sets of results. The first three columns employ weights obtained from (16), which allows short selling, while the last three columns correspond to no short selling. To obtain these latter results, for any month where ω_t^{SP} becomes negative, this is replaced by the boundary value of zero and consequently $(1-\omega_t^{SP})$ becomes one. An analogous method is used when $(1-\omega_t^{SP})$ is negative.

Table 7 provides summary statistics of the portfolio weights (mean values over 2006:m7-2008:m12) induced by the models across investors with different values of the risk aversion parameter λ. In contrast to the DCC model, the CCC and (D)STCC models imply very little demand for US stocks and particularly so for less risk-averse investors (λ=0.2, 0.5)[10]. Interestingly, only in the case of the DCC model does the bulk of the portfolio remain invested in the US market throughout the post-sample period. Removing short selling implies similar results. Further, over this period only the investor with relatively high risk aversion (λ=2) will diversify the portfolio by investing similar amounts in each market.

Table 8 then presents the realized CER results over the full post-estimation sample and then over the two sub-samples within this. For the least risk averse investor, and allowing for short-selling, the DCC performs best for the full post sample and this investor gains utility (when this figure is greater than one) for the first part of the post sample with positive returns. When the market falls in the second part of the post sample, this investor would then minimise their loss by investing on the basis of the DSTCC model with time and S&P volatility transitions. When the investor places more weight on risk with λ=1, then over the full post sample the STCC model gives the highest realized CER, but in the falling market the CCC model performs best. For the most risk averse investor with λ=2 the CCC model performs best over the full sample and in this case the STCC model does better than the DCC in the increasing market up until May 2007.

When no short selling is allowed, the STCC model yields the best results over the whole post sample period, irrespective of λ, with this typically resulting from its relatively good performance in the period of negative returns. As can be seen from Table 7, this model places most weight on the UK market, which experiences a less steep fall than the US market over this period (see the means reported in Table 8). Indeed, when λ = 0.2, 0.5, the DSTCC model implies that all funds are invested in the UK market throughout, and the good performance of this model implies that international diversification does not yield any benefits to the investor.

[10] The investor has greatest disutility of risk when λ = 2 and hence the larger λ the more risk averse the investor.

4. Concluding remarks

This paper provides two contributions to understanding the time varying nature of co-movements between monthly US and UK stock prices. Firstly, we show the usefulness of time-varying conditional correlation models both statistically and judged by their post sample economic performance. Secondly, our results imply an increase in correlations between these markets from around 1999, which is handled well by the smooth transition conditional correlation (STCC) specification of Silvennoinen and Teräsvirta (2005) with time as the transition variable.

The robustness of our STCC results is strengthened by estimating a DSTCC model that includes time and US stock market volatility as transition variables. This model confirms the high degree of co-movement between the US and UK equity markets in recent years and indicates that, after allowing for time effects, correlations increase within phases of high volatility.

From a statistical perspective, the STCC and DSTCC models are preferred to constant and dynamic conditional correlation specifications according to conventional information criteria. To investigate economic performance of these models, we compute certainty equivalent rates of return for investors with a portfolio across the US and UK markets and for different levels of risk aversion. These returns are analyzed over the post sample period to 2008, during which the credit crunch financial crisis took hold. Over our full post sample from July 2006 and with no short selling, the STCC model demonstrates the best economic performance over all levels of risk aversion considered. Furthermore, the DSTCC model minimizes the utility loss for less risk-averse investors when both markets are falling over 2007 and 2008. Therefore, we believe that these smooth transition time-varying correlation models capture the important changes in co-movements between the US and UK stock markets since 1980. In particular, they emphasize the dramatic increase in correlations around 2000, which leaves little scope for portfolio diversification across the two markets.

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Table 1: Summary Statistics and Cross Correlations

Statistic	US S&P 500	UK FT all share
Mean	0.9075	0.9652
Median	1.0268	1.2948
Minimum	-11.656	-12.738
Maximum	12.378	12.976
Standard Deviation	4.0109	4.2537
Skewness	-0.2210	-0.4846
Excess Kurtosis	0.3665	0.5498
Correlations between markets over time		
Sample	**No. of observations**	**Cross Correlation**
1980m1-2006m6	318	0.656
Sub-Sample		
1980m1-1984m12	60	0.509
1985m1-1989m12	60	0.668
1990m1-1994m12	60	0.664
1995m1-1999m12	60	0.449
2000m1-2006m6	78	0.867
Post-Sample		
2006m7-2008m12	30	0.866

Notes: Summary statistics are presented for the outlier corrected nominal stock returns which is 100 times the first difference of the log of the price index.

Table 2: Tests for Constant Conditional Correlations

Test	Statistic	p-value
Constancy tests against DCC		
Ljung Box test	29.56	0.042*
Bollerslev test	2.007	0.017*
Tse test	8.866	0.003**
Constancy tests against STCC		
$SPvol_t$ transition	3.951	0.046*
t/T transition	15.53	8.09e-005**

Notes: *, ** denote significance at the 5% and 1% level, respectively. The Ljung-Box statistic tests autocorrelation up to 18 lags in the cross products of the GARCH standardized residuals, distributed as χ^2 with 18 degrees of freedom. Bollerslev's (1990) residual based diagnostic is the F test from a regression of $u_{i,t}u_{j,t}h_{i,j,t}^{-1}-1$ on $h_{i,j,t}^{-1}$, $u_{i,t-1}^2 h_{i,j,t}^{-1}$, $u_{j,t-1}^2 h_{i,j,t}^{-1}$ and $u_{i,t-1}u_{j,t-1}h_{i,j,t}^{-1},\ldots,u_{i,t-12}u_{j,t-12}h_{i,j,t}^{-1}$. The Tse (2000) test is the Lagrange Multiplier statistic for constant correlations, distributed as χ^2 with 1 degree of freedom. Tests against a single transition STCC model are those of Silvennionen and Teräsvirta (2005), distributed as χ^2 with 1 degree of freedom.

Table 3: DCC-GARCH Model Estimates

	ΔSP_t	ΔFT_t
a. Mean equations		
Constant	1.013 [0.1841]	0.9988 [0.1873]
b. Volatility equations		
Constant	0.7113 [0.2593]	1.1480 [0.5167]
$r_{i,t-1}^2$	0.0796 [0.0285]	0.0979 [0.0362]
$h_{i,t-1}$	0.8732 [0.0265]	0.8417 [0.0396]
c. Correlation equation $E(\varepsilon_{i,t}\varepsilon_{j,t} \mid \Im_{t-1}) = q_{i,j,t}$		
$\varepsilon_{i,t-1}\varepsilon_{j,t-1}$	0.0752 [0.0502]	
$q_{i,j,t-1}$	0.8873 [0.1225]	
Diagnostics		
$LB(v_{i,t},18)$	16.36 (0.567)	9.916 (0.935)
$LB(v_{i,t}^2,18)$	16.96 (0.525)	14.56 (0.691)

Notes: Values in square brackets are robust standard errors (Bollerslev-Wooldridge, 1992). The sample period is January 1980 to June 2006 (318 observations). $LB(.,18)$ is the Ljung-Box statistic for testing autocorrelation up to 18 lags calculated for both the standardized residuals ($v_{i,t}$) (see (12)) and the squared standardized residuals, both distributed as χ^2 with 18 degrees of freedom under the null hypothesis (where 18 is approximately the square root of 318). Figures in parentheses are p-values. The mean equation is given by (1) and the volatility by the GARCH(1,1) model of (4). Finally, the correlation equation is described by the DCC specification of (6) and (7).

Table 4: STCC-GARCH Model Estimates with Time Transition

	ΔSP_t	ΔFT_t
a. Mean equations		
Constant	0.9423 [0.2168]	1.0714 [0.2205]
b. Volatility equations		
Constant	0.6487 [0.3512]	1.6799 [0.8645]
$r_{i,t-1}^2$	0.0679 [0.0230]	0.0813 [0.0367]
$h_{i,t-1}$	0.8887 [0.0235]	0.8236 [0.0554]
c. Correlation equation $\rho_t = \rho_1(1 - G_t(t/T;\gamma,c)) + \rho_2 G_t(t/T;\gamma,c)$		
ρ_1	0.5633 [0.0468]	
ρ_2	0.8813 [0.0210]	
γ	31.739 [22.759]	
c	0.7600 [0.0152] (Date: 2000:m2)	
<u>*Diagnostics*</u>		
$LB(v_{i,t},18)$	16.44 (0.562)	8.989 (0.960)
$LB(v_{i,t}^2,18)$	17.92 (0.461)	16.72 (0.542)

Notes: See Table 3, except that the correlation equation is here given by the STCC specification of (8) and (9).

Table 5: DSTCC-GARCH Model Estimates with Time and S&P Volatility Transitions

	ΔSP_t	ΔFT_t
a. Mean equations		
Constant	0.9613 [0.2175]	1.0670 [0.2166]
b. Volatility equations		
Constant	1.0873 [0.4054]	1.5062 [0.8089]
$r^2_{i,t-1}$	0.1030 [0.0297]	0.1012 [0.0415]
$h_{i,t-1}$	0.8263 [0.0038]	0.8185 [0.0497]
c. Correlation equation	$\rho_t = (1-G_{2t}(t/T))(\rho_{11}(1-G_{1t}(SPvol_t))+\rho_{21}G_{1t}) + G_{2t}(\rho_{12}(1-G_{1t})+\rho_{22}G_{1t})$	
ρ_{11}	0.5316 [0.0553]	
ρ_{21}	0.6861 [0.0536]	
ρ_{12}	0.8400 [0.0408]	
ρ_{22}	0.9104 [0.0217]	
γ_1	500 [.]	
γ_2	26.958 [8.6078]	
c_1	18.049 [1.7406]	
c_2	0.7537 [0.0155] (Date: 1999:m12)	
<u>*Diagnostics*</u>		
$LB(v_{i,t},18)$	16.55 (0.554)	12.26 (0.833)
$LB(v^2_{i,t},18)$	18.57 (0.418)	18.65 (0.413)

Notes: See Table 3, except that the correlation equation is the DSTCC model of (10). No standard error is reported for γ_1 because this estimate is large and imprecisely estimated. To achieve convergence, $\hat{\gamma}_1$ is fixed at 500.

Table 6: Log Likelihood and Information Criteria Values

	Log-Likelihood	AIC	SIC
CCC	-1706.69	10.791	10.897
DCC	-1695.12	10.724	10.842
STCC	-1688.76	10.697	10.716
DSTCC	-1687.01	10.704	10.728

Table 7: Summary Statistics for Mean-Variance Portfolio Weights

	Allowing for Short Selling			No Short Selling		
	Investing in US market	Investing in UK market	Standard Deviation	Investing in US Market	Investing in UK market	Standard Deviation
λ=0.2						
CCC	-2.885	3.885	1.166	0	1	0
DCC	1.044	-0.044	0.193	0.953	0.047	0.094
STCC	-7.513	8.513	5.088	0.018	0.982	0.072
DSTCC	-12.349	13.349	4.039	0	1	0
λ=0.5						
CCC	-0.746	1.746	0.438	0.010	0.990	0.042
DCC	0.878	0.122	0.190	0.848	0.152	0.128
STCC	-2.217	3.217	2.044	0.137	0.863	0.285
DSTCC	-4.436	5.436	1.607	0	1	0
λ=1						
CCC	-0.033	1.033	0.203	0.074	0.926	0.118
DCC	0.823	0.177	0.190	0.800	0.200	0.135
STCC	-0.452	1.452	1.043	0.269	0.731	0.409
DSTCC	-1.799	2.799	0.814	0.008	0.992	0.043
λ=2						
CCC	0.324	0.676	0.102	0.324	0.676	0.102
DCC	0.795	0.205	0.191	0.774	0.226	0.138
STCC	0.431	0.569	0.567	0.490	0.510	0.501
DSTCC	-0.480	1.480	0.449	0.042	0.958	0.166

Notes: The table shows summary statistics for the portfolio weights ω_t^{SP} and (1 - ω_t^{SP}) associated with investing in the US and UK markets, respectively, calculated over the post-sample period 2006m7-2008m12.

Table 8: Realized Certainty Equivalence Rates of Return over Post-Sample Period

	Allowing for Short Selling			No Short Selling		
	Full post-sample	Positive Returns	Negative Returns	Full post-sample	Positive Returns	Negative Returns
	July 06 Dec 08	July 06 May 07	June 07 Dec 08	July 06 Dec 08	July 06 May 07	June 07 Dec 08
Means: SP	-1.1364	1.6954	-2.7759			
FT	-0.9836	1.3395	-2.3285			
λ=0.2						
CCC	0.9826	0.9998	0.9726	0.9899	1.0134	**0.9764**
DCC	**0.9878**	**1.0171**	0.9709	0.9892	**1.0168**	0.9733
STCC	0.9814	1.0076	0.9661	**0.9900**	1.0134	**0.9764**
DSTCC	0.9674	0.9531	**0.9754**	0.9899	1.0134	**0.9764**
λ=0.5						
CCC	0.9863	1.0096	0.9729	0.9890	1.0133	0.9750
DCC	**0.9872**	**1.0164**	0.9705	0.9886	**1.0164**	0.9727
STCC	0.9861	1.0140	0.9700	**0.9905**	1.0158	**0.9759**
DSTCC	0.9800	0.9915	**0.9733**	0.9896	1.0133	**0.9759**
λ=1						
CCC	0.9870	1.0128	**0.9724**	0.9873	1.0133	0.9725
DCC	0.9860	**1.0161**	0.9696	0.9878	**1.0161**	0.9717
STCC	**0.9871**	1.0161	0.9705	**0.9900**	1.0160	**0.9752**
DSTCC	0.9837	1.0043	0.9718	0.9885	1.0133	0.9744
λ=2						
CCC	**0.9865**	1.0143	**0.9710**	0.9865	1.0143	**0.9710**
DCC	0.9850	1.0159	0.9678	0.9864	1.0159	0.9700
STCC	0.9865	**1.0169**	0.9695	**0.9871**	**1.0169**	0.9705
DSTCC	0.9844	1.0106	0.9697	0.9854	1.0132	0.9698

Notes: The table records the values given by the empirical CER of (17) for each model and for various values of λ. Bold font indicates the preferred specification(s).

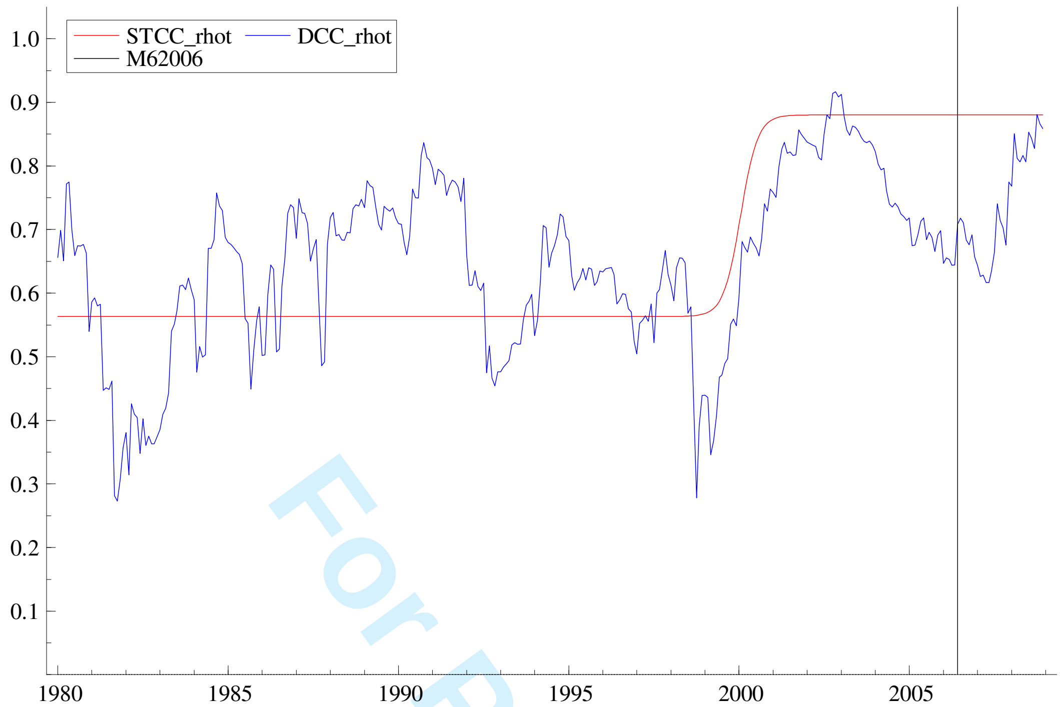

Figure 1: Monthly time-varying conditional correlations from the DCC and STCC models.

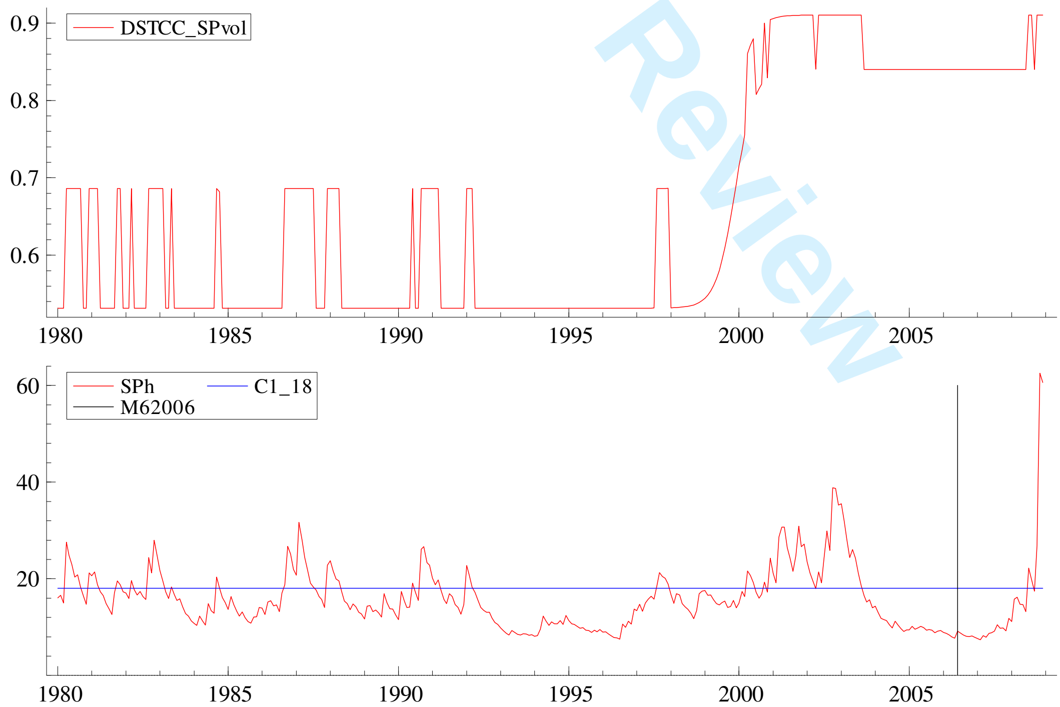

Figure 2: Monthly time-varying conditional correlations from the DSTCC model and S&P estimated conditional variance (SPh).

Appendix 1: Initialisation of the Nonlinear Estimation

An important practical issue in nonlinear modelling is the selection of starting values for the estimation. Starting values for the DCC models are based on linear estimates for the constant mean equations with all parameters in the GARCH part of the equation initialised as 0.05. For the correlation parameters, the news parameter α is initialised at 0.05. While we experimented with different values for the decay parameter, the likelihood maximum was achieved with β initialised at 0.05. The DCC estimates were obtained in RATS 6.3.

As far as the (single transition) STCC model is concerned, we use starting values from the OLS estimation of the constant mean equations (1) and initial univariate estimates of the volatility equation (4) to obtain estimates of the respective parameters and also the associated series $u_{1,t}$, $u_{2,t}$, $h_{11,t}$ and $h_{22,t}$. Using these, we perform a grid search[11] where we select initial values for the remaining parameters as those that minimise the square of the distance between the cross products of the standardised residuals and the implied correlations, namely

$$\min_{\gamma,c,\rho_1,\rho_2}\left\{\left[\frac{\hat{\varepsilon}_{1,t}\hat{\varepsilon}_{2,t}}{(\hat{h}_{11,t}\hat{h}_{22,t})^{1/2}}\right]-\rho_1(1-G_t(s_t;\gamma,c))-\rho_2 G_t(s_t;\gamma,c)\right\}^2 \qquad \text{(A.1)}$$

After obtaining initial estimates, all parameters of the STCC model are estimated jointly by quasi-maximum likelihood in Gauss 6.0. A similar procedure is followed for the estimation of the DSTCC model.

As a check, we also estimated the (D)STCC models conditional on OLS results for the constant mean equations and then apply the iterative procedure of Silvennoinen and Teräsvirta (2005) that separates the parameters of the GARCH, correlation, and the transition function.[12] These estimates are very close to those obtained from the joint estimation of all parameters using our Gauss program.

[11] See Sensier, Osborn and Öcal (2002) for an example of grid search techniques applied to nonlinear estimation.

[12] This procedure was applied using Ox programs supplied by Annastiina Silvennoinen. These programs are written such as that the returns are the residuals from a filtered time series. Also, they do not allow for the computation of QML standard errors.

Appendix 2: Data

Table A.1: Variable Descriptions and Sources

Name	Variable Description	Source	Code
SP	Standard and Poors' composite index (EP), NSA	Datastream	USS&PCOM
FT	Financial Times all share index (EP), NSA	Datastream	UKFTALL

Notes: EP – end of period; NSA – not seasonally adjusted.

Table A.2: Outliers Removed

UK	US
1981m9, 1987m10	1987m10, 1998m8