A TWO FLUID MODEL FOR NUCLEAR ROTATIONS

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Abstract: A simple macroscopic model based on two-fluid liquid drop ideas is proposed for studying rotational band structure in heavy deformed doubly even nuclei. The pairing and centrifugal degrees of freedom are taken into account in a harmonic approximation. The model is then applied to study the interplay of Coriolis, centrifugal and pairing forces in some well-known backbenders and non-backbenders and it is concluded that the observed anomalies at high spins could be the result of such an interplay. Calculations for the actinides ^{232}Th and ^{240}Pu are also presented.

1. Introduction

With the increasing interest in the experimental study of high-spin states in deformed nuclei, strong deviations from simple rigid rotor behaviour have been revealed ¹). One such deviation is the anomaly of the moment of inertia at high spins called “backbending”.

Many theoretical models have been suggested for the explanation of these phenomena, some of the most common of which are:

(i) The Coriolis-antipairing (CAP) model based on the original prediction of Mottelson and Valatin ²) that the coherent action of the Coriolis force causes a coherent destruction of pairing. This basic idea has been extended in recent years by the incorporation of various microscopic models ^{3–8}).

(ii) The Stephens-Simon model ⁹), where the Coriolis force causes the rotation-aligned (RAL) coupling scheme to be more favoured energetically at high spins, thus leading to band-crossing and possibly backbending.

(iii) Models that suggest shape phase transitions, asymmetries, etc., as causes of backbending. These have been of limited success ^{10–12}).

The first two effects represent the most widely accepted mechanisms for the explanation of backbending and it is still an open question which is more responsible in the different regions of the periodic table. Excellent reviews of the subject have been given by Johnson and Szymanski ¹³⁾, Sorensen ¹⁴⁾, Stephens ¹⁵⁾ and Faessler ¹⁶⁾.

Our aim in the present work is the construction of a phenomenological macroscopic model which incorporates the most important ingredients for studying the interplay of pairing, Coriolis and centrifugal forces, and which thus contains the possibility of a phenomenological description of CAP effects and backbending. However, no attempt will be made to include the RAL degree of freedom. It is hoped that such phenomenological models, whose parameters have some direct physical interpretation, might help to achieve a better understanding of the backbending phenomena, and also serve as a guide for more sophisticated microscopic investigations.

The motivation for the present work stems from the success of macroscopic hydrodynamic and phenomenological models at low spins ^{17, 18)} and the recent application of two-fluid hydrodynamics to high-spin states ¹⁹⁾. The idea of an intermediate nuclear fluid flow ²⁰⁾ is adopted to replace the irrotational flow assumption of previous hydrodynamic models of nuclei. This is consistent with the experimental observation that ground-state moments of inertia are intermediate between what is expected for a superfluid (irrotational) and a normal (rigid) flow. The model thus defined involves two free parameters, with well-defined physical meaning. However, it is found that the results can be further improved by introducing a third parameter whose freedom to vary accounts for some of the major approximations made in the model.

In sect. 2 the model is formulated and in sect. 3 the results of its application to some deformed doubly even nuclei are presented and discussed. Sect. 4 includes a summary and the main conclusions.

2. Formulation of the model

The nucleus is viewed as an axially symmetric liquid drop with semiaxes a and b , rotating with angular velocity ω about an axis perpendicular to its axis of symmetry, as shown in fig. 1. We assume that neutrons and protons move as independent non-interacting fluids.

Since the experimental situation indicates intermediate ground-state moments of inertia we seek a nuclear fluid flow that is also intermediate between the superfluid (irrotational) and normal (rigid) flows. Since it is known that the moment of inertia depends on the strength of the pairing interaction ¹⁴⁾ it is also necessary that the nuclear fluid flow depends on the pairing interaction.

It was shown by Cusson ²⁰⁾ that such an intermediate flow exists, which in turn is a special case of a more general incompressible linear flow, which is a mixture of

Fig. 1. The rotating nucleus viewed as a spheroid with axial symmetry in equilibrium (solid curve) and stretched (dashed curve) states.

superfluid and normal flows. Cusson²⁰) also proved that if the ratio of the two velocity-field components, γ , is fixed before quantization, then the resultant quantized Hamiltonian represents a rotational spectrum with a moment of inertia intermediate between the superfluid and rigid values, depending on the ratio γ :

$$H_{\text{rot}}^{(\tau)} = J(J+1)\hbar^2/2\mathcal{J}_{\text{rig}}^{(\tau)}f_{\tau} \quad (1)$$

$$f_{\tau} = [\gamma_{\tau} + |B|]^2/(1 + 2\gamma_{\tau}|B| + \gamma_{\tau}^2), \quad (2)$$

where τ labels neutrons ($\tau = n$) and protons ($\tau = p$), and $|B|$ is a geometric factor defined by

$$\mathcal{J}_{\text{sup}}^{(\tau)} = B^2\mathcal{J}_{\text{rig}}^{(\tau)} \quad (3)$$

where $\mathcal{J}_{\text{sup}}^{(\tau)}$ and $\mathcal{J}_{\text{rig}}^{(\tau)}$ are the pure superfluid and rigid flow moments of inertia respectively and where J is the angular momentum. It can easily be seen that the effective moment of inertia $\mathcal{J}_{\text{rig}}^{(\tau)}f_{\tau}$ in eq. (1) approaches the limits $\mathcal{J}_{\text{sup}}^{(\tau)}$ for $\gamma_{\tau} \rightarrow 0$ and $\mathcal{J}_{\text{rig}}^{(\tau)}$ for $\gamma_{\tau} \rightarrow \infty$.

To produce a pairing-dependent flow, and hence effective moment of inertia, it is necessary to relate the ratio γ_{τ} to the microscopic pairing structure²⁰). There is no unique way of doing this, but the simplest ansatz that also reproduces the two limits of the moment of inertia $\mathcal{J}_{\text{rig}}^{(\tau)}$ for zero pairing and $\mathcal{J}_{\text{sup}}^{(\tau)}$ for infinite pairing is²⁰)

$$\gamma_{\tau} \propto L_p^{(\tau)}/R_0^{(\tau)}, \quad (4)$$

where $L_p^{(\tau)}$ is the pair correlation length defined by

$$L_p^{(\tau)} = \frac{2\varepsilon_F^{(\tau)}}{k_F^{(\tau)}v_\tau} = \text{const} \sqrt{\varepsilon_F^{(\tau)}} \frac{1}{v_\tau}, \quad (5)$$

where $\varepsilon_F^{(\tau)}$ and $k_F^{(\tau)}$ are the Fermi energy and momentum respectively and v_τ is the state-dependent pairing gap parameter ²¹). The quantity $R_0^{(\tau)}$ in eq. (4) is a measure of the size of the system, hence, the ratio γ_τ as given in eq. (4) represents a qualitative measure of the degree of pairing ^{20, 22}). From eqs. (4) and (5) we obtain

$$\gamma_\tau = \xi \sqrt{\varepsilon_F^{(\tau)}} \frac{1}{v_\tau}, \quad (6)$$

where ξ is a mass-dependent parameter of the model and it is assumed that neutrons and protons occupy the same region of space, so that we may drop the τ on R_0 and ξ . To simplify the calculations we assume the density of the fluid ρ_τ to be uniform. It follows from the geometry of fig. 1 and the definitions of rigid and irrotational velocity fields that

$$\mathcal{J}_{\text{rig}}^{(\tau)} = \frac{4}{15} \pi \rho_\tau a b^2 (a^2 + b^2), \quad (7)$$

$$\mathcal{J}_{\text{sup}}^{(\tau)} = \frac{4}{15} \pi \rho_\tau a b^2 (a^2 - b^2)^2 / (a^2 + b^2), \quad (8)$$

and hence from eq. (3)

$$B^2 = \left(\frac{a^2 - b^2}{a^2 + b^2} \right)^2. \quad (9)$$

For a system of neutrons and protons the total rotational Hamiltonian is then

$$H_{\text{rot}}^{(\text{tot})} = J(J+1) \hbar^2 / 2 (\mathcal{J}_{\text{rig}}^{(n)} f_n + \mathcal{J}_{\text{rig}}^{(p)} f_p). \quad (10)$$

We make the assumption that centrifugal stretching (CS) is controlled by harmonic restoring forces and thus contributes to the total energy a term of the form ¹⁸)

$$V_{\text{cent}} = \frac{1}{2} k (X - X_0)^2, \quad (11)$$

where k is a mass-dependent stiffness parameter and $X - X_0$ is the displacement from equilibrium of the mass centres of the two halves of the spheroidal nucleus shown in fig. 1.

The important pairing degree of freedom ^{21, 23}) is included in the total energy as a harmonic approximation to the pairing energy. This approximation clearly holds rigorously only for low-spin states ²¹). For high-spin states one expects large changes in v_τ and therefore it is expected that this approximation may break down in the regime in which we are interested. This is clearly one inherent weakness in our model. However, our philosophy is that it is of interest to understand the behaviour of these high-spin states with this simple harmonic form before embarking on more complicated forms that might introduce new adjustable parameters and tend to obscure the basic physics. Also, this harmonic approximation has been used for moderate

spins by Ma and Tsang ²⁴⁾ and more recently for high spins by Gupta ²⁵⁾. In partial defence of this basic approximation, we note that no appreciable change in our results was found on substituting for the harmonic approximation, the complete form for the BCS pairing energy as used by Ma and Rasmussen ²¹⁾ in their study of backbending. We conjecture that the fact that the harmonic approximation for pairing is much better in this high-spin regime than one expects beforehand, is in large measure due to the fact that our results indicate at most only a 50 % reduction in v_τ at the backbending rather than a complete collapse of pairing, as indicated later. The harmonic approximation thus achieves a certain degree of internal consistency. For these reasons, we feel justified in using this harmonic approximation as a first step towards an understanding of the model, and to avoid the complications of additional parameters associated with more complicated forms for the pairing energy.

The harmonic form can be derived from number-projected BCS theory in the continuous model (21) as

$$V_{\text{PBCS}}(v_\tau) \approx \text{const} + \frac{1}{2} C_{v_\tau} (v_\tau - \Delta_\tau)^2, \quad (12)$$

where C_{v_τ} is a pairing-spring constant related to the single particle level density ρ_τ^l at the Fermi surface, and the ground-state pairing gap Δ_τ and pairing force strength G_τ are as given in ref. ²¹⁾ where these spring constants are tabulated.

Within our model the total energy $E_J(v_n, v_p, X)$ is thus the sum of the above contributions for both neutrons and protons

$$E_J(v_n, v_p, X) = \frac{J(J+1)\hbar^2}{2(\mathcal{I}_{\text{ns}}^{(n)} f_n + \mathcal{I}_{\text{ns}}^{(p)} f_p)} + \frac{1}{2} k (X - X_0)^2 + \frac{1}{2} \sum_{\tau=n,p} C_{v_\tau} (v_\tau - \Delta_\tau)^2, \quad (13)$$

where the constant in $V_{\text{PBCS}}(v_\tau)$ is dropped since we are interested only in calculating excitation energies. To proceed further, we now estimate the ratio $\sqrt{\epsilon_F^{(n)}/\epsilon_F^{(p)}}$, from the Fermi gas model as (26)

$$\sqrt{\frac{\epsilon_F^{(n)}}{\epsilon_F^{(p)}}} = \left(\frac{N}{Z} \right)^{\frac{1}{3}}, \quad (14)$$

and utilize the uniform density assumption to write

$$\begin{aligned} \rho_\tau &= \frac{N_\tau}{A} \rho, \\ N_\tau &= \begin{cases} N & \text{for } \tau = n \\ Z & \text{for } \tau = p, \end{cases} \end{aligned} \quad (15)$$

where ρ is the total density; and A , N and Z are the mass, neutron and atomic numbers respectively. Eq. (13) may now be written in terms of the stretch variable ¹⁸⁾ $T = b/b_0$, with the aid of eqs. (7), (9), (14) and (15) and the incompressibility assump-

tion, as

$$E_f(v_n, v_p, T) = \frac{J(J+1)\hbar^2}{2\mathcal{J}_0} \frac{T^4}{1 + \eta^2 T^6 N F_n A_p + Z F_p A_n} \frac{A_n A_p}{1 + \eta^2 T^6 N F_n A_p + Z F_p A_n} + \frac{9}{128} k a_0^2 \frac{[1 - T^2]^2}{T^4} + \frac{1}{2} \sum_{\tau=n,p} C_{v_\tau} (v_\tau - A_\tau)^2, \quad (16)$$

where

$$\eta = b_0/a_0, \quad (17)$$

$$F_\tau = [\xi \delta_\tau (1 + \eta^2 T^6) + (1 - \eta^2 T^6) v_\tau]^2, \quad (18)$$

$$A_\tau = [(1 + \eta^2 T^6)^2 v_\tau^2 + 2\xi \delta_\tau v_\tau (1 - \eta^4 T^{12}) + \xi^2 \delta_\tau^2 (1 + \eta^2 T^6)^2], \quad (19)$$

$$\delta_\tau = \begin{cases} (N/Z)^{\frac{1}{2}} & \text{for } \tau = n \\ 1 & \text{for } \tau = p, \end{cases} \quad (20)$$

$$\mathcal{J}_0 = \frac{4\pi}{15A} \rho a_0^5 \eta^2, \quad (21)$$

and where some constants have been absorbed in ξ .

In the application of the model a_0 and b_0 are determined from the experimental quadrupole moments Q_0 [ref. ²⁷], assuming a nuclear radius $R_0 = r_0 A^{\frac{1}{3}} = 1.2 A^{\frac{1}{3}}$ fm and equivalent mass and charge distributions. The gaps A_n and A_p and the pairing-spring constants C_{v_n} and C_{v_p} for some of the nuclei studied are taken from Ma and Rasmussen ²¹). The total density ρ is assumed to be $A/\frac{4}{3}\pi R_0^3 = 0.14$ nucleons/fm³.

It is found that the results are sensitive to some theoretical and experimental uncertainties as follows:

(i) Small uncertainties in Q_0 lead to larger uncertainties in $\mathcal{J}_0$ because a_0 appears in $\mathcal{J}_0$ as a_0^5 . Similar uncertainties arise from the assumption of equivalent mass and charge distributions. The experimental uncertainty in r_0 and the possibility that it is different for neutrons and protons also amount to similar uncertainties in $\mathcal{J}_0$ since a fixed value of $r_0 = 1.2$ fm is used in our calculations.

(ii) The actual mass distributions in nuclei are known not to be uniform. Therefore, ρ should be regarded as an effective density which could well be different from 0.14 nucleons/fm³.

(iii) The non-uniqueness of the ansatz for relating γ_τ to v_τ suggests that other possible forms would lead to different predictions. However, some of the alterations could be partly accounted for by altering $\mathcal{J}_0$.

It is found that the calculational results could be improved if a factor P is introduced to multiply $\mathcal{J}_0$ in eq. (16). This factor is introduced to accomodate the uncertainties mentioned earlier and allows us to study the behaviour of the results for a range of values of P , which could be tolerated within these uncertainties. The final expression for $E_f(v_n, v_p, T)$ is then that of eq. (16), but with $\mathcal{J}_0$, multiplied by P . The energies for different angular momenta are determined from this expression

by minimisation for a given J with respect to T , v_n and v_p , and at the same time adjusting the parameters ξ and k to get the best fit to the experimental spectrum for a given P . The interplay of $H_{\text{rot}}^{(\text{tot})}$ and V_{cent} in the minimisation with respect to T constitutes centrifugal stretching while its interplay with V_{PBCS} in the minimisation with respect to v_n and v_p constitutes the CAP effect in our model. We note here that eq. (16) and the calculational scheme mentioned above resembles an extended VMI model²⁸⁾ where the single generalized stretch variable is now extended to three variational parameters T , v_n and v_p , thus allowing for some differentiation between neutrons and protons, as well as accounting for the CAP effect explicitly.

3. Discussion of the results

The results are displayed in figs. 2–8 in the customary way by plotting $(2\mathcal{J}/\hbar^2)$ versus $(\hbar\omega)^2$ where these quantities are derived from the spectra using the formulae that give a straight line for the VMI model¹⁴⁾:

$$\frac{2\mathcal{J}}{\hbar^2} = \frac{4J-2}{\Delta E_{J,(J-2)}},$$

$$(\hbar\omega)^2 = \frac{[\Delta E_{J,(J-2)}]^2(2J-1)}{\binom{2}{4}^3 \{ [J(J+1)]^{\frac{1}{3}} - [(J-1)(J-2)]^{\frac{1}{3}} \}^3}. \quad (22)$$

For the typical nucleus $^{164}\text{Er}_{68}$ we also display plots of the gap v_r and the deformation parameter β_J versus J (see fig. 2). The later plots show the CAP and CS effects, as well as the increase in v_p and the Coriolis-anti stretching (CAS) effects at the critical spin, similar to what was found in microscopic models^{3,5–8)}. The experimental spectra for all nuclei are taken from ref. ²⁹⁾ and references therein.

The main conclusions from these results are summarised in table 1, where the three parameters ξ , k and P as well as an estimation of the quality of the empirical fit are given. We achieve a good fit to all nuclei studied, with the exception of ^{170}Yb , ^{154}Gd , and ^{186}Os . However, the behaviour of the high-spin states in these nuclei cannot be attributed solely to a pairing phase transition¹⁵⁾ and since our model does not allow for the possibility of RAL, this effect cannot be excluded, at least in these nuclei. The results for the actinides ^{232}Th and ^{240}Pu are not sensitive to P and show no sign of CAP backbending even at very high spins. This leads us to believe that if backbending occurs in these nuclei at high spins, it must be due to band-crossing or some other phenomena not contained in our model³⁰⁾.

It should also be mentioned that all backbending found at high spins in these calculations is due to neutron CAP. However, there are also cases which show backbending due to proton CAP at spins above the neutron CAP (see fig. 3a).

Fig. 2. (a) Experimental (●) and theoretical (○) plots of $(2J/h^2)$ versus $(\hbar\omega)^2$ for $^{164}\text{Er}_{68}$. The $P = 1.3$ curve is the result of adjusting ξ only since this resulted in the best fit. (b) Neutron (●) and proton (○) pairing gaps v_n and v_p respectively as functions of J . (c) The deformation parameter $\beta_J = \sqrt{\frac{1}{3}\pi}(a-b)/R_0$ as a function of J .

Fig. 3. As in fig. 2a, but for the nuclei (a) $^{160}\text{Dy}_{66}$, (b) $^{168}\text{Yb}_{70}$ and (c) $^{170}\text{Yb}_{70}$. The $P = 1.2$ and 1.5 curves for $^{170}\text{Yb}_{70}$ are the results of adjusting ξ only for the same reason as in fig. 2a.

Fig. 4. As in fig. 2a, but for $^{154}\text{Gd}_{64}$. Both experimental forking levels are shown here. Only experimental levels below the forking levels are included in the fit due to the uncertain nature of higher levels.

Fig. 5. As in fig. 4, but for $^{186}\text{Os}_{76}$.

Fig. 6. As in fig. 2a, but for $^{188}\text{Os}_{76}$. Fig. 7. As in fig. 2a, but for the nuclei (a) $^{162}\text{Dy}_{66}$, (b) $^{168}\text{Er}_{68}$, (c) $^{172}\text{Yb}_{70}$ and (d) $^{178}\text{Hf}_{72}$. All theoretical curves shown, except the $P = 1.3$ for $^{172}\text{Yb}_{70}$, are the results of adjusting ζ only for the same reason as in fig. 2a.

Fig. 8. As in fig. 2a, but for the nuclei (a) $^{232}\text{Th}_{90}$ and (b) $^{240}\text{Pu}_{94}$. Parameters Δ_n and Δ_p for these nuclei were calculated from the mass table of ref. ³²⁾, then C_{v_n} and C_{v_p} were calculated as in ref. ²¹⁾.

TABLE I

Table of parameters k and ξ for values of P which give the best χ^2 fit to the experimental energies

Nucleus	P	ξ (keV)	k (keV · fm ⁻² × 10 ⁴)	Qualitative fit up to highest exp. level	Comments
¹⁶⁴ Er	1.6	249.8	2.299	good (see fig. 2a)	Backbending is reproduced and attributed to CAP. No upper band exp. observed
¹⁶⁰ Dy	1.5	323.3	1.739	good (see fig. 3a)	Model shows backbending, whereas exp. curve upbends. No upper band exp. observed
¹⁶⁸ Yb	1.3	325.4	0.449	good (see fig. 3b)	Model exhibits upbending at spins higher than presently measured. No upper band exp. observed
¹⁷⁰ Yb	0.9	439.5	0.314	bad (see fig. 3c)	Slight backbending is exp. observed. Model either over-estimates or under-estimates effect. Other CAP calculations have similar difficulties *). No upper band exp. observed
¹⁵⁴ Gd	0.9	534.9	0.137	bad (see fig. 4)	Band crossing is experimentally confirmed. Our CAP fits not sensitive to parameters and show no backbending. No other CAP calculations available
¹⁸⁶ Os	0.8	384.5	large (no stretching)	bad (see fig. 5)	Forking is exp. observed and hence CAP model alone is not applicable. Model shows no backbending
¹⁸⁸ Os	0.7	393.4	1.105	good (see fig. 6)	Fit shows no backbending. No other CAP calculations available. No exp. observed upper band
¹⁶² Dy	1.3	432.6	large (no stretching)	good (see fig. 7a)	Fit does not show CAP backbending. Confirms calculations ^b). No exp. observed upper band
¹⁶⁸ Er	0.8	694.6	large (no stretching)	good (see fig. 7b)	Fit does not show CAP backbending. Confirms calculations ^b). No upper band exp. observed
¹⁷² Yb	0.8	592.5	large (no stretching)	good (see fig. 7c)	Same as ¹⁶² Dy

TABLE 1 (continued)

Nucleus	P	ξ (keV)	k (keV · fm ⁻² × 10 ⁴)	Qualitative fit up to highest exp. level	Comments
¹⁷⁸ Hf	0.8	471.4	large (no stretching)	good (see fig. 7d)	Same as ¹⁶² Dy
²³² Th	1.0	460.2	1.650	good (see fig. 8a)	No CAP backbending, even at very high spins. Results independent of parameter P
²⁴⁰ Pu	1.0	480.7	1.896	good (see fig. 8b)	Same as ²³² Th

a) Ref. 7).

b) Ref. 21).

4. Summary and conclusions

We have presented a macroscopic hydrodynamical model of CAP backbending which has the following features:

(i) The nucleus is represented as a rotating deformed liquid drop comprised of two fluids which are to be thought of as a normal fluid and a superfluid. The moment of inertia as a function of the deformation and the pairing gap is then obtained from geometry and the Cusson ansatz ²⁰).

(ii) There are three free parameters k , ξ and P in the current study. The first two have well defined physical significance. The parameter P is introduced to improve the results and to account for some inherent uncertainties. A more sophisticated model for the nuclear ground state may allow for an elimination of some components of P .

(iii) The results demonstrate that a two-fluid model of the kind described in this work shows backbending associated with CAP effects. The model is predictive to the extent that a good fit to the low-spin states prescribes a very narrow range of values for P . Within these values of P our predicted high-spin spectrum qualitatively agrees with the experimental results over a wide range of nuclei.

(iv) The model does not allow for the RAL effect, since this is a pure single-particle effect. However, it allows for the CAP effect and we note that at the critical spin J_c there is no complete collapse of pairing, but rather a decrease in v_r of order 50 %.

(v) Improvements could be obtained from a more detailed investigation of the ansatz for γ_r , a better treatment of the pairing energy, inclusion of shell effects, etc. Work along these lines is now proceeding ³¹).

Due to the inherent simplicity of the model, we retain the capability of including additional terms in the Hamiltonian, which will thereby extend the range of phenomena that we may describe. For example, within the spirit of the original

liquid drop model, extended now to two-fluid hydrodynamics, the only truly macroscopic term that we have omitted is a surface-pairing term. There is also the possibility of extending the model to include vibrations. Clearly, a two-fluid system has an incomparably richer vibrational spectrum than that of a single fluid. For example, in analogy with second sound in liquid ^4He , a two-fluid system may show a vibrational mode in which the normal and superfluid phases vibrate antiphase with the c.m. remaining at rest. It will clearly be of interest to attempt to extend the present model along these lines.

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