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Influence of balancing parameters in achieving magnetic field uniformity in a large cylindrical volume

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Large diameter, air-cored, coaxially configured magnetic coils of rectangular winding cross sections have been envisaged for generating extremely homogenous magnetic fields over a large volume of space. Optimized magnetic field balancing parameters and dimensions for a coil configuration were experimentally developed, verified, and their results are presented. The developed design has made it possible to generate extremely homogenous magnetic fields in a large extended cylindrical region of more than $\frac{1}{2}$ of the coil's diameter, which requires the winding cross-section thickness to have a width ratio of a relatively small integer. Furthermore, field uniformity regions can be extended to more than one and a half times as can be achieved with coils of negligible winding cross section by employing balancing parameters method. © 2008 American Institute of Physics.

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I. INTRODUCTION

Magnetic field induction coils with the greatest possible degree of uniformity are of paramount importance in the domain of experimental physics for adequate electron beam focusing and to attain high degree of parallelism.¹ These are currently being utilized for applications^{2–6} where uniform fields are required over large volumes of space. For example, magnetic field uniformity in terms of magnitude and distribution is incumbent to avoid trailing and to ensure the precision of intense charge particle beams transported in vacuum.⁷ The availability of a uniform external magnetic field over an extended region of space confers several promising features: greater gun to worksite distance is possible, optimum field uniformity, specimens can be immersed in the field, and stable and simpler power supplies as compared to electrostatic ones are possible. Furthermore, high voltage feed throughs are not required and the electrostatic charging problems can be reduced to a significant extent.⁸ It is pertinent to use air-core coils for achieving extended field uniformity, particularly when frequent access to the free experimental space from all directions is required. In practical design, coil winding must occupy an extended volume in space in order to limit the power dissipation per unit volume, or, to keep the current density below the quenching value especially in case of super conducting windings.⁹ Realization of coil geometries with negligible winding cross section to foster uniform fields over extended regions of space was described elsewhere;^{10–12} however, the effects of the finite winding cross section on field uniformity are difficult to characterize and do not support optimization of system pa-

rameters. This review article will focus mainly on how an optimum coil design can be obtained experimentally for generating extremely homogenous magnetic fields over an extended region of space.

Garrett¹³ proposed a theory based on the idealization of negligible winding cross sections for generating uniform axial magnetic fields. Later, by extending Garrett's theory, Franzen¹⁴ analytically discussed the effect of rectangular finite winding cross sections on field uniformity and proposed that the effect of a finite winding area can be eliminated to a certain extent by choosing an appropriate thickness to width ratio. However, Franzen's value for the optimum thickness to width ratio is given by an irrational number, which cannot be approximated to a sufficient accuracy for the ratio of relatively small integers. Also, actual coils are wound with a wire of constant thickness that results in an integer thickness to width ratio which possesses another difficulty in realizing such coils. Keeping in view of such constraints, we have adopted an experimental approach to show that the requirement for obtaining a uniform field over an extended region of space is equivalent to making the field on the axis as uniform as possible. The present study infers that optimum coil designs are always possible for the air-core coils when the ratio of thickness to width of the actual rectangular cross-sectional winding area is an integer ratio. As a consequence, the magnetic field generated by properly designed coils can have better uniformity over much wider regions of space as compared to the field produced by the idealized coils with negligible winding cross section.

The determination of the adequate rectangular cross-sectional area of winding by finding out the most appropriate thickness to width ratio and the estimation of the resultant field at optimized coil separation are of prime significance. The proposed technique makes use of all the practical design

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balancing parameters in a systematic fashion to determine the optimum conditions for the best possible coil arrangement. Real field measurements are presented graphically to illustrate the optimum coil configuration parameters for generating extremely homogenous magnetic fields over a large region of space. This approach can be used to produce highly uniform fields over the extended region of space for greater coil separation than the radius of the coils. The field optimization was achieved by investigating the behavior of several rectangular winding cross sections with varying degrees of thickness to width ratio and by studying the effect of each ratio on other balancing parameters. It was observed that the region of field uniformity is strongly influenced by the thickness to width ratio for distinct coil separations. These characteristics are summarized as: (i) a pair of coils with a finite rectangular cross-sectional winding area at optimized coil separation can generate more uniform magnetic fields as compared to a coil pair with negligible winding cross section. (ii) Optimized coil designs are always applicable to air-core coils provided that the thickness to width ratio of the actual rectangular cross-sectional winding area is an integer number. (iii) There exists a unique optimum value of thickness to width ratio of the winding cross section to maximize the extended uniform magnetic field region. (iv) The design parameters of the remaining coils contribute significantly to validating an extended uniform field region in a large volume of space with extreme homogeneity.

The experimentally determined values of balancing parameters have extended the uniform region to more than 1.5 times as that can be achieved with coils of negligible winding cross section. The developed geometrical configuration with accurate design and functionality features can produce extremely homogenous magnetic fields in a large extended cylindrical region of more than $\frac{1}{2}$ of the coil's diameter as compared to the commercially available designs capable of generating field uniformity in a much lesser volume of space.

II. DESIGN CONSIDERATIONS

In this section, the design considerations of coaxial configuration electromagnetic coils for producing extremely homogenous fields in a large cylindrical volume are summarized. The circular cross-section coils are mainly used for low-current electromagnets whose maximum current is less than 20 A,⁸ while flat cross sections (rectangular wires) are used for magnets with high currents. The magnetic induction B produced through a pair of coils in cgs units is given by $B = \mu H \approx 0.899 \mu NI / R$, where μ is the relative permeability which is unity for unloaded coils and R is the coils' radius in centimeters. In order to obtain maximum uniform magnetic induction at a given excitation NI (where N is the number of turns and I is the coils' current) at an optimized separation, the magnetic coils must be properly dimensioned.¹⁵

The geometrical configurations of conductors designed to generate uniform fields are generally characterized by the axis of symmetry. We are primarily interested in the central uniform field region. By central region, we mean, interior of a cylinder concentrated at the origin and of such dimensions that it does not enclose any conductors. The cylindrical vol-

FIG. 1. Illustration of the configuration and the shape of uniform cylinder region within coils.

ume of the uniform field region, plane of symmetry, axis of symmetry, and other important balancing parameters are schematically described in Fig. 1. The q and p represent the thickness and width of the rectangular cross-sectional area of winding, respectively. We now consider that a coil pair having a configuration of a finite cross section of conductors with uniform current density is arranged to lie on a certain sphere with a radius r . The R_0 is the distance from the origin to the center of the winding cross section and z is the coil displacement on the axis of symmetry, whereas, a and b represent the radius and length of the cylinder, which define the entire inside region of the highly uniform magnetic field. We will restrict our discussion only to the uniform regions of space for which the generated field does not differ from the field at the origin. This fact makes it possible to express the field characteristics everywhere within the central field region in terms of the axial field at the center. The appropriate configuration of conductors can produce homogenous fields in a large specific volume and can eliminate all sorts of non-uniformities from its field edges. The requirement for obtaining a uniform field over an extended region of space is equivalent to making the field on the axis as uniform as possible. By the proper choice of all balancing parameters and by optimum thickness to width ratio, the same current must pass through both coils, which is the condition desirable for practical reasons. The condition can be extended in a straightforward manner to other coil configurations, such as solenoids, solenoids with gaps, or solenoids with over wound ends. However, our interest will be directed primarily toward air-core coils in which the principal advantage is a frequent access to the free experimental space from all directions over a much wider uniform field region.

The optimization was based on experimentally investigating the real field measurement results of different balancing parameters including the effects of thickness to width ratios on field uniformity at different values of coil separations. For any rectangular cross section, the effect of finite winding cross section can be eliminated to a high degree of approximation by an appropriate choice of thickness to width ratio of the winding. As a result of this choice, magnetic

FIG. 2. Dependence of b/a on various thickness to width ratios at different values of p/b .

FIG. 3. Dependence of b/a on various thickness to width ratios at different values of q/a .

FIG. 4. Correction values to the experimental condition needed for a magnetic coil pair of various cross-sectional areas to generate highly homogeneous magnetic field.

FIG. 5. Dependence of the magnetic field, B , on the magnetic coil pair of the finite winding area on the coils' position z/r . The curve $z/r=0.61$ is for the optimized condition.

fields generated by relatively thick coils can have better uniformity over the extended region of space as compared to that achieved with current filaments of negligible cross-sectional area of winding, which is of considerable practical interest.

The dimensions of thickness q and width p , which define the winding cross-sectional area, are necessarily small as compared to the coils' diameter and separation. The experimental technique enabled us to determine quantitatively the extent to which these dimensions can be adequately optimized and to obtain qualitatively maximum regions of uniformity at optimized separation. This leaves us free to choose the optimum ratio in order to successfully achieve large homogeneous volumes of space. Thus, if two coils are wound with finite cross section, copper wires in the same optimum predetermined cross-sectional area of winding at optimized separation, the resultant generated field will be highly uniform over a wider region.

For completeness, a brief description of the experimental procedure and details about its precision follow. The field influencing balancing parameters discussed in Sec. III (see Figs. 2–6) were verified by a final test coil pair and a superior quality field was produced as illustrated in Figs. 7–9. Other than the variations in tested winding cross-sectional areas, the rest of the procedure adopted for the investigation of field characteristics remained the same. The 16 standard wire gauge (SWG) copper wire of current carrying capacity 5.80 A and the fine quality paper insulation of 0.25 mm

FIG. 6. Summing field configuration at separation equal to coils' radius.

FIG. 7. Summing field configuration at separation equal to $\sqrt{1.64 \times}$ coils' radius at optimized parameters.

thickness were used to verify the results. A final test coil pair with a total of 1064 turns (19 layers of 56 turns) was wound for this purpose. The well-calibrated Redcliffe gauss meters were used for all real field measurements. The calibration accuracy of the gauss meters was $\pm 0.1\%$. Field mapping was carried out at all points of distances every 2.5 mm apart with field measuring accuracy of 99.9%. The optimum field uniformity was observed within an entire volume of 70-cm-diameter cylinder of length 50 cm and in this region, the rate of variation of field strength was found to be zero. With such design arrangements, a homogenous field was produced within a large cylindrical region of diameter more than $\frac{1}{2}$ of the coils' diameter. The coils can generate a maximum 80 G of uniform field, which gives rise to a coil temperature of 32°C at a water flow rate of 3 l/min. The coils' current was changed by using a regulated power supply at 100 V and 60 A. Such a design construction worked very well within electrical and thermal operating limits. The power required to generate a certain field increases by increasing the coils' dimension. Much larger coils are not recommended because

FIG. 9. Uniform magnetic field regions achieved at optimized design parameters.

the power output of the system may be too low. Electrical isolation is necessary to avoid the loops of Eddy current, therefore necessitates the use of nonmagnetic materials for the coils' construction. The supporting structure used has enough mechanical strength to avoid any sort of vibration and the axial alignment of coils was better than $\pm 0.1^\circ$.

III. RESULTS AND DISCUSSION

The different thickness to width ratios q/p versus overall achievable uniform field regions b/a was studied as a func-

FIG. 8. Field configuration in uniform regions with coils arranged in opposition at optimized parameters.

tion of p/b and results are illustrated in Fig. 2. It was observed that all curves are symmetrical about the thickness to width ratio of $q/p=0.444$ at different values of p/b . At much lower q/p ratios, the overall uniform region decreases to a great extent, whereas, an overall region of optimum uniformity increases to a considerable extent as the ratio of p/b increases. The scientific value of achieving thickness to width ratios in q/p through real field measurements is for clarity and its distinction from other techniques.^{13,14} Similarly, the behavior of q/p ratios was studied versus b/a as a function of q/a and a family of curves obtained is described in Fig. 3. At higher values of q/a , a considerable increase in the overall uniform region was observed which describes the importance of these design parameters. These effects should be taken into account as they significantly contribute to achieving optimum uniformity over much wider regions of space.

It would be very convenient to discuss different experimentally determined cases as an example. Figure 4 shows how correction values depend on the size of the cross-sectional area, where different ratios of q/p have been described versus various correction values of z/r as a function of p/r . Figure 4 indicates that at $z/r=0.61$, the value is symmetrical about $q/p=0.444$. It is evident from Fig. 4 that the intersections of curves with horizontal line are moving away to the right as the value of p/r increases. The result indicates that the optimum wider uniform region was achieved at a value of $p/r=0.25$. In addition, the optimum value of the thickness to width ratio is quite rational and was found to be realistic for practical applications (unlike Franzen¹⁴ who eliminated the effect of the finite winding cross section). In contrast to the standard cases, we were able to investigate the optimum condition for different sizes of rectangular winding cross sections and obtained an optimum condition at $q/p=0.444$, $p/r=0.25$ for $z/r=0.61$.

For the sake of citing an example, we consider a case in which the amounts of the first adjustment values of z/r were chosen to be $z/r=0.5$ (Helmholtz condition), $p/r=0.16$, and $q/r=0.125$. With this arrangement, the extent of the uniform region was only 26%. However, this situation can be improved markedly by merely taking $p/r=0.19$ and adjusting z ; a change of z/r by 0.06 from the original value of 0.5 leads to the same uniformity, and the extent of the uniform region is 34% in the case of a finite winding cross section (see Fig. 4). These correction values, in most cases, remain fairly small for the different values of p/r . An example shows that the amount of correction applied in p/r and z/r is indeed effective for achieving uniform fields over a wider region. Furthermore, the balancing parameter is useful in achieving better uniformity by coils of finite winding cross section even in cases where the constructed coils were different from the designed shape; the effects of any error produced in winding coils can be compensated by choosing the proper value of z/r which can be exactly determined for actual coil dimensions. The most appropriate extended uniform field region cannot be successfully achieved merely by taking q/p as a balancing parameter. This, in turn, makes it possible in practice to realize other important balancing parameters. For example, for the dimension of $p/R_0 \leq 0.10$, the extent of the

uniform region is widened by 5% at most, but this widening becomes about 17% for $p/R_0=0.15$. It is of practical interest to show that by retaining the value q/p as close as possible to $q/p=0.444$, one can find the proper value of p/R_0 which makes the resulting field homogenous over large regions of space, suppose we approximate the value of $q/p=0.444$, with a realistic value of 0.44, a ratio of two small integers 8 and 18, while retaining $z/r=0.61$. Under this condition, we find that the value of $p/R_0=0.24$ provides an extended region of field uniformity 50% more than the case of the negligible winding cross section, which can be realized by choosing $p=0.24$, R_0 , and $q=0.11 R_0$. We find that it is always possible to approximate closely the widest region of field uniformity with coils of finite winding cross section while retaining $z/r=0.61$. These findings can be applied to cases where we can approximate the value of q/p with any other ratio of integers, which is larger than 0.444.

In the proceeding results, we have concentrated on the determination of a coils' configuration, which fulfills the conditions of balancing parameters. However, q/p is not the only essential condition for realizing a highly uniform field over an extended region. Such a field also requires a slightly positive value of z/R_0 as a balancing parameter to achieve as wide a uniform region of space as possible. It is of practical interest, therefore, to see how this balancing parameter works in coil designs. When z/r is taken as a balancing parameter, we can treat the problem under the assumption of the finite winding cross section. The reason is that all balancing parameters contribute significantly to achieving optimum uniformity over a wider region regardless of whether we treat the optimized case of finite winding cross section or not. Figure 5 shows the behavior of the magnetic field on the z -axis for various values of z/r . The curve $z/r=0.61$ is for the optimized coils' design condition. It is evident from Fig. 5 that with the proper choice of z/r , one can expect to have about 50% wider extent of uniform region as compared to the Helmholtz condition, i.e., at $z/r=0.50$. A coil pair designed to produce such a wide uniform region can only provide such a region if the optimized distance is varied by a very small amount. It was noted that for cases where field uniform areas were smaller as compared to the coils, the distance of the coils' separation at its best position will become an important parameter. Also, for larger uniform areas as compared to the coils there occurs deterioration in the field uniformity on departing from the optimized position.

If we take q/p as a parameter while retaining $z/r=0.61$, we find that the behavior of the magnetic field B for various values of q/p is very similar in shape to curves in Fig. 5. From such a graph we find that the values of q/p by which the extent of the uniform region exceeds that for optimum condition are much smaller than unity for large cross-sectional areas, whereas they are always smaller than unity for small cross-sectional areas. The result ($p > q$) is quite contrary to intuition regarding the coils' shape, i.e., to a flat cross section of usual coils, but also easy to realize in practice. Furthermore, the use of q/p as a balancing parameter represents another important feature: coil pair is designed to extend the uniform region to about 50% more than the standard case in negligible winding cross section. Considering all

the aspects of these two balancing parameters, we can immediately conclude that the design method based on the appropriate use of both balancing parameters q/p and z/r is far more accurate, effective, and practical. In view of their importance in the design, it is desirable to extend the balancing parameters to the finite winding cross-section case in order to achieve the desired field uniformity over an extended region of space.

Figure 6 represents the standard Helmholtz pair with $z/r=0.50$, i.e., separation equal to coil radius and field adding in case of negligible winding cross section. In the central region where B_z is most uniform, B_r is zero. This happens because the maxima of B_r from the two coils have been arranged to coincide and oppose at center. Figure 7 represents the case example of the coils' configurations having all the values of balancing parameters optimized for an optimum value of separation $r=\sqrt{1.64}$. The resultant field was observed within a large cylindrical region of homogenous magnetic field with a diameter of more than half of the coils' diameter. At center, if there is no radial field, the axial field has to be uniform and it achieves optimum uniformity over a 50% wider region than the case of the finite winding cross section. In any design arrangement, the adequate selection of all balancing parameters plays a critical role in the design optimization process to generate highly uniform fields in a specifically large volume of space.

The field symmetry of optimized coils' design was also checked. In the design, the location of the center of symmetry is critical and it must be positioned exactly at the center. It was important to analyze the field symmetry because each thickness to width ratio represents a particular pattern of field configuration. When the coils' current is in the opposite sense, the magnetic field at the center must be zero.¹⁶ The field pattern, in which all field lines are changing least quickly with distance at different coil currents and smoothly distributed within uniform region, represents perfect symmetry. This perfect symmetry was accurately obtained at optimized design parameters only. In this profile, the weaker the radial field, the slower the axial field must change. It is illustrated in Fig. 8. Such a coil design in conjunction produces additive field effects with homogeneity throughout the specifically large volume of space. After precisely measuring the field, data were set out graphically as described in Fig. 9. The field uniformity over a wider region resulted as optimized separation puts the points of inflection from the two coils at the same place. The rate of increase in field from one coil exactly cancels the rate of decrease from the other, just where two rates are changing themselves least quickly with distance. It is obvious from the above discussion that when all field balancing parameters are optimized, the designed pair of coils having a finite rectangular cross-sectional area can generate a field distribution that possesses even better uniformity as compared to the field generated by a coil pair of negligible winding cross section.

In contrast with a solenoid, sheet loop, and field canceling design arrangements,^{17–19} this design method is practical and offers comparatively larger overall extended region of highly uniform magnetic field. The technique based on minimization of root-mean-square deviation of field²⁰ is useful to

some extent. The best focusing cannot be accomplished with mere average uniformity introduced by magnetic coils^{3,21} even at standard separation of coils. In such cases, compromise is necessary over the design of the coils' geometry and the benefits of extreme homogeneity over large region of space are not achieved. Commercially available coils²² at standard separation provide a comparatively smaller region of uniform field and also homogeneity variations of $\pm 10\%$ are reported in the uniform field region. In order to remove these variations and to achieve homogeneous fields in a large volume of space, design configurations based on optimized balancing parameters must yield the best results. The magnetic field produced by such properly designed relatively thick pairs of coils has better uniformity over much wider extended region of space as compared to the field produced by idealized coils with negligible winding cross section.

IV. CONCLUSION

The configuration of each coil has its own particular field form and larger coils yield better field uniformity than smaller ones, but the power required to produce a certain compensation field increases by increasing the dimensions. It was found that whenever Franzen's condition is applied in experimental design, the resulting field from the designed coils had a poor uniformity even after adjustment of q/p , a parameter left free in the Franzen's condition. Considering these preliminary results, it would seem to be the best approach in the design of a coil pair to take the q/p , z/r , z/R_0 , p/b , q/a , and b/a as design balancing parameters and to maximize the extent of uniform magnetic field regions under the condition that the ratio of thickness to width of winding cross section should be a ratio of relatively small integers.

The determination of the appropriate balancing parameters is a vital aspect, which greatly influences the maximum homogenous field region in a large cylindrical volume. We believe that such an approach enables us to obtain coil configurations, which can generate fields of the type shown in Figs. 7 and 9. It was concluded that such coil pairs can generate magnetic fields over a much wider region, which has substantially even better uniformity as compared to the field generated by a coil pair of negligible winding cross section.

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