LETTER TO THE EDITOR

The saturating effect of $N^*(1234)$ on the binding of light nuclei

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Abstract. The binding energy shifts arising from the explicit inclusion of intermediate states involving a $N^*(1234)$ are calculated for the light nuclei ${}^4\text{He}$, ${}^{12}\text{C}$ and ${}^{16}\text{O}$. An oscillator approximation is made for the single-nucleon wavefunctions and the two-body correlations are taken from an LOC calculation. It is observed that the Pauli effect per nucleon is essentially the same in all three nuclei and is similar to that found in the nucleon fluids. The dispersion effect in the light nuclei is smaller than in the nucleon fluids.

It is concluded that these contributions will have a large effect on the saturation of finite nuclei.

Phenomenological N–N interactions designed to fit the two-nucleon data, scattering phase-shifts and deuteron properties presumably include all possible intermediate processes that might occur in the interaction of two nucleons. In particular, they will include the amplitude represented in figure 1 where, in constructing the two-nucleon interaction, the intermediate nucleon state N_3 and resonance state N_4^* are completely summed over and apart from the rest-mass difference between N and N^* the propagator for the intermediate state is determined entirely by the kinetic energies of the nucleon states N_1 , N_2 and N_3 and of the resonance state N_4^* . This amplitude contributes strongly to the intermediate-range attraction of the N–N interaction.

When an amplitude such as that illustrated in figure 1 occurs within a nucleus then it is modified in two ways: the nucleon N_3 has to obey the Pauli exclusion principle and must only be summed over states not occupied by the other nucleons (Pauli effect). Both the intermediate nucleon N_3 and resonance N_4^* will move in the mean field of the other nucleons (dispersion effect). Inclusion of these effects reduces the attractive contribution to the N–N interaction within a nucleus. Thus calculations of nuclei binding with phenomenological N–N interactions which ignore these effects should lead to predictions of overbinding.

The role of these effects in the homogeneous nucleon fluids, neutron matter and nuclear matter have been studied by Green and co-workers (Green and Haapakoski 1974, Green and Niskanen 1975), Day and Coester (1976) and Holinde and Machleidt (1976). These workers conclude that the effects are significant contributing 6–10 MeV/N loss of binding at nuclear matter saturation densities. In these systems

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Figure 1. The contribution to the N-N scattering amplitude from a two-boson exchange graph with an intermediate $N^*(1234)$.

the dispersion effect dominates with the Pauli effect only contributing 1–2 MeV/N. Both effects are extremely density dependent and increase rapidly with increasing density.

Owen *et al* (1976, 1977) have developed a constrained variational approach for calculating the bulk properties of homogeneous quantal fluids. For nuclear matter, for a range of N-N phenomenological interactions, they predicted a significant over-binding with saturation at 20–23 MeV/N at densities corresponding to Fermi momenta $k_F = 1.65$ – 1.75 fm^{-1} . They also noted that if the results of Green and Niskanen or Coester and Day were added to their nuclear matter calculations the saturation point was moved to $\sim 16 \text{ MeV/N}$ at $k_F \sim 1.4 \text{ fm}^{-1}$, in good agreement with the best-fit values of the semi-empirical mass formulae. Subsequently, in a study of nucleon fluids at arbitrary proton to neutron abundance, Howes *et al* (1978) demonstrated that the inclusion of these effects also led to an improved prediction of the symmetry coefficient in the semi-empirical mass formulae.

In the present work we attempt to calculate the Pauli and dispersion effects for some light nuclei. Specifically, we calculate the energy shift ΔE_{N^*} ,

$$\Delta E_{N^*} = \Delta E(P) + \Delta E(D) \quad (1)$$

where the Pauli term is

$$\Delta E(P) = \frac{1}{2} \sum_{\substack{N_1 N_2 N_3 \\ N_4^*}} \frac{|\langle N_1 N_2 | V_{NN^*} | N_3 N_4^* \rangle|^2}{E_4^* + E_3 - E_1 - E_2} \quad (2)$$

and the dispersion term is

$$\Delta E(D) = \frac{1}{2} \sum_{\substack{N_1 N_2 \\ N_3 N_4^*}} |\langle N_1 N_2 | V_{NN^*} | N_3 N_4^* \rangle|^2 [(T_4^* + T_3 - T_1 - T_2)^{-1} - (E_4^* + E_3 - E_1 - E_2)^{-1}]. \quad (3)$$

In calculating the Pauli shift $\Delta E(P)$, the sums on the nucleon states N_1 , N_2 and N_3 are all restricted to states occupied in the nuclear ground-state configuration while the resonance state N_4^* is summed over a complete set of states. In calculating the dispersion shift $\Delta E(D)$, the nucleon states N_1 and N_2 are restricted to occupied states while both intermediate nucleon N_3 and resonance N_4^* are summed over

complete sets of states. The nucleon single-particle energies

$$E_i = T_i + U_i \quad (4)$$

are composed of a kinetic energy T_i and a mean field potential energy U_i , while the resonance energy contains an additional mass difference term

$$E_4^* = T_4 + U_4 + \Delta \quad (5)$$

where $\Delta = m_{N^*} - m_N \simeq 300$ MeV. Similarly, in the dispersion shift we have

$$T_4^* = T_4 + \Delta. \quad (6)$$

We shall assume, in common with most workers in the field, that the N-N* interaction is similar to the N-N interaction and hence the mean field felt by the N* will be similar to that experienced by the nucleon. For the bound occupied single-particle states we shall use an oscillator approximation. We shall assume that there is no potential acting in the unbound, unoccupied intermediate single-particle states.

The two-nucleon wavefunctions are assumed to be antisymmetrised and correlated by a phenomenological N-N interaction, i.e.

$$\begin{aligned} |N_1 N_2\rangle = & \sum (1 + \delta_{12})^{-1/2} [1 - (-1)^{l+s+T}] (l_1 m_1 l_2 m_2 | \lambda \mu) (\frac{1}{2} \sigma_1 \frac{1}{2} \sigma_2 | SM_S) \\ & \times (\frac{1}{2} \tau_1 \frac{1}{2} \tau_2 | TM_T) (lm_l LM_L | \lambda \mu) (SM_S lm_l | jm) \langle n_1 l_1 n_2 l_2 \lambda | nlNL\lambda \rangle \\ & \times f_{jlST}^{(i)}(r) \phi_{STM_T nljm}(r) \phi_{NLM_L}(R) \end{aligned} \quad (7)$$

where we have used the standard Brody-Moshinsky transformation to separate the two-body wavefunction into a product of relative and centre-of-mass oscillator functions in the coordinates $r = (1/\sqrt{2})(r_1 - r_2)$ and $R = (1/\sqrt{2})(r_1 + r_2)$ respectively. The two-body correlation functions $f_{jlST}^{(i)}(r)$ are obtained from a LOCV calculation on the finite nuclei (Bishop *et al* 1978) using the Reid (1968) soft-core interaction.

For the resonance state N_4^* we use a complete set of oscillator functions with the *same oscillator size* as that for the nucleon states. Because of the N-N* mass difference, this implies an oscillator frequency ω^* for the N* related to the frequency ω for the N's by

$$\omega^* = (m_N/m_{N^*})\omega = 0.758 \omega. \quad (8)$$

The intermediate N-N* state is neither antisymmetrised nor correlated:

$$\begin{aligned} |N_3 N_4^*\rangle = & \sum (l_3 m_3 l_4^* m_4^* | \lambda \mu) (\frac{1}{2} \sigma_3 \frac{3}{2} \sigma_4^* | SM_S) (\frac{1}{2} \tau_3 \frac{3}{2} \tau_4^* | TM_T) \\ & \times (l^* m_l^* LM_L | \lambda \mu) (SM_S lm_l | jm) \langle n_3 l_3 n_4^* l_4^* \alpha | n^* l^* NL\lambda \rangle \\ & \times \phi_{STM_T n^* l^* jm}(r) \phi_{NLM_L}(R) \end{aligned} \quad (9)$$

and the relative and centre-of-mass functions are oscillator functions with the same size as those arising from the two-nucleon states in equation (7).

The kinetic energies are now

$$T_i = \frac{1}{2}(2n_i + l_i + \frac{3}{2})\hbar\omega \quad (10)$$

for the nucleons and

$$T_4 = \frac{1}{2}(2n_4^* + l_4^* + \frac{3}{2})\hbar\omega^* \quad (11)$$

for the N^* while the potential energies are

$$\begin{aligned} U_i &= \frac{1}{2}(2n_i + l_i + \frac{3}{2})\hbar\omega - U_0 && \text{occupied states} \\ &= 0 && \text{unoccupied states} \end{aligned} \quad (12)$$

for both N 's and N^* 's. The depth of the potential U_0 is determined for each nucleus and each value of $\hbar\omega$ by requiring that

$$\langle E \rangle \simeq \left\langle \frac{1}{2} \sum_{\text{occupied } i} (T_i + E_i) \right\rangle = \text{BE}(A, Z) \quad (13)$$

where $\text{BE}(A, Z)$ is the empirical binding energy (less Coulomb energy) for the nucleus of mass number A and charge number Z .

We use the transition potential V_{NN^*} given by Green (1976) which assumes π - and ρ -exchange with a short-range cut-off corresponding to the exchange of a particle of mass ~ 1.5 GeV to kill the $1/r^3$ behaviour near the origin:

$$\begin{aligned} V_{NN^*} &= \frac{1}{3}\sqrt{2} m_\pi \frac{f_\pi f_\pi^*}{4\pi} \tau_1 \cdot T_2 \left[S_{12}^{\text{II}} \left\{ V(\mu_\pi r') - \left(\frac{\lambda}{\mu_\pi} \right)^3 V(\lambda r') - \frac{f_\rho^2}{f_\pi^2} \frac{m_\rho}{m_\pi} \right. \right. \\ &\quad \times \left[V(\mu_\rho r') - \left(\frac{\lambda}{\mu_\rho} \right)^3 V(\lambda r') \right] \left. \right\} + \sigma_1 \cdot S_2 \left\{ V_0(\mu_\pi r') - \left(\frac{\lambda}{\mu_\pi} \right)^3 \right. \\ &\quad \times V_0(\lambda r') + 2 \frac{f_\rho^2}{f_\pi^2} \frac{m_\rho}{m_\pi} \left[V_0(\mu_\rho r') - \left(\frac{\lambda}{\mu_\rho} \right)^3 V_0(\lambda r') \right] \left. \right\} \left. \right] \end{aligned} \quad (14)$$

where $r' = r_1 - r_2 = \sqrt{2} r$ and where we have used $m_\pi = 138.1$ MeV, $m_\rho = 775.4$ MeV, $\mu_\pi = 0.7 \text{ fm}^{-1}$, $\mu_\rho = 3.85 \text{ fm}^{-1}$, $f_\pi^2/4\pi = 0.081$, $f_\pi^*/4\pi = 0.35$ and $f_\rho^2/4\pi = 1.726$. The results are insensitive to λ provided it is large enough. We have used $\lambda = 7.5 \text{ fm}^{-1}$, corresponding to a mass ~ 1.5 GeV. The potential form factors are

$$V_0(x) = e^{-x}/x \quad V(x) = (1 + 3/x + 3/x^2)V_0(x). \quad (15)$$

In equation (14) σ_1 is the familiar 2×2 Pauli spin matrix while S_2 is the 2×4 spin matrix which converts a spin $\frac{1}{2}\hbar$ nucleon into a spin $\frac{3}{2}\hbar$ N^* and similarly for the isospin operators τ_1 and T_2 . S_{12}^{II} is the 2×4 generalisation of the familiar tensor operator

$$S_{12}^{\text{II}} = 3\sigma_1 \cdot \hat{r} S_2 \cdot \hat{r} - \sigma_1 \cdot S_2. \quad (16)$$

The matrix elements of $\tau_1 \cdot T_2 S_{12}^{\text{II}}$ and $\tau_1 \cdot T_2 \sigma_1 \cdot S_2$ are given in table 1. In this work we shall restrict ourselves to a calculation involving the $NN \ ^1S_0 \rightarrow NN^* \ ^5D_0$ and the $NN \ ^3P_1 \rightarrow NN^* \ ^5P_1$ channels and noting that, while these give the largest contributions, the other channels are not negligible according to Holinde and Machleidt (1976).

When the matrix elements in equations (2) and (3) are evaluated the only free parameter indicating the sum over intermediate states which remains is the relative quantum number n^* . There are three factors conspiring to produce a rapid convergence in the sums over intermediate states. In order of ascending importance these are (i) the increasing energy denominators, (ii) the Brody-Moshinsky brackets and (iii) the rapid diminution of the integral

$$\int_0^\infty \phi_{nl}(r) f_{jST}^{(i)}(r) V_{NN^*}(\sqrt{2}r) \phi_{n^*l^*}(r) r^2 dr$$

Table 1.

NN	→	NN*	$\langle \tau_1 \cdot T_2 S_{12}^H \rangle$	$\langle \tau_1 \cdot T_2 \sigma_1 \cdot S_2 \rangle$
1S_0		5D_0	4	0
3P_0		3P_0	$\frac{4}{3}$	$\frac{8}{3}$
3P_1		3P_1	$-\frac{2}{3}$	$\frac{8}{3}$
		5P_1	$\frac{161}{60}$	0
		5F_1	$\frac{11}{5}$	0
3P_2		3P_2	$\frac{4}{30}$	$\frac{8}{3}$
		5P_2	$-\frac{6}{5}$	0
		3F_2	$-\frac{49}{30}$	0
		5F_2	$\frac{12}{5}$	0
1D_2		5S_2	$\frac{161}{90}$	0

as $n^* \gg n$. In figure 2 we show this convergence in a plot of $\Delta E(P)$ and $\Delta E(D)$ as a function of $n_{\max}^*$ for ^4He , ^{12}C and ^{16}O assuming these to be simple closed-shell configurations with $\hbar\omega = 41 A^{-1/3}$ MeV yielding 25.82, 17.9 and 16.27 MeV

Figure 2. The calculated energy shifts for the nuclei (a) ^4He , (b) ^{12}C and (c) ^{16}O , with assumed oscillator energies given by $\hbar\omega = 41 A^{-1/3}$ MeV, i.e. 25.82, 17.9 and 16.27 MeV respectively, as a function of the upper limit $n_{\max}^*$ on the sum over the relative principal quantum number n^* . The curves are labelled P and D for Pauli and dispersion contributions with subscripts s and p denoting the $^1S_0 \rightarrow ^5D_0$ and $^3P_1 \rightarrow ^5P_1$ channels respectively. At all $\hbar\omega$ the p-wave effect is much smaller than the s-wave contribution.

Figure 3. The calculated Pauli shifts $\Delta E(P)$ (full curves), dispersion shifts $\Delta E(D)$ (broken curves) and total shifts ΔE_{N^*} (chain curves) for (a) ${}^4\text{He}$, (b) ${}^{12}\text{C}$ and (c) ${}^{16}\text{O}$ as a function of $\hbar\omega$. The depth of the mean field potential is given by equation (13).

respectively. In figure 3 we have plotted our calculated shifts in binding energy as a function of $\hbar\omega$ and we see that the strong density dependence found in the nucleon fluids translates into a strong dependence on the oscillator size parameter.

At oscillator energies appropriate to the size of the nuclei ${}^4\text{He}$, ${}^{12}\text{C}$ and ${}^{16}\text{O}$, we see that our calculated Pauli shift per nucleon is insensitive to the mass number of the nucleus and is in good agreement with the results obtained in nuclear matter at a Fermi momentum $k_F = 1.4 \text{ fm}^{-1}$. On the other hand, our dispersion shift is considerably smaller than that in nuclear matter and this is directly attributable to the smaller potential in finite nuclei compared with nuclear matter (cf 8 MeV/N binding in nuclei against 16 MeV/N in nuclear matter). In figure 4 we illustrate this for the case of ${}^4\text{He}$ by plotting the calculated energy shifts against variation in U_0 (see equation (12)). We see that as the potential well depth increases $\Delta E(P)$ falls slowly while $\Delta E(D)$ increases rapidly. A much smaller source of discrepancy between

Figure 4. The calculated energy shifts $\Delta E(P)$ (full curve), $\Delta E(D)$ (broken curve) and ΔE_{N^*} (chain curve) for ${}^4\text{He}$ as a function of the well depth U_0 . The arrows indicate the value of U_0 for which equation (13) is satisfied with $\hbar\omega = 25.82 \text{ MeV}$.

the present results and the nuclear matter calculations of Green and co-workers and Day and Coester is that we have allowed the intermediate N^* to move in the same mean field as the intermediate N . As in the nuclear matter calculations, we find a much smaller density ($\hbar\omega$) dependence for the dispersion shift compared with the Pauli shift.

We conclude that an explicit treatment of the contribution to the N - N interaction of the amplitude in figure 1 will reduce the calculated binding energy of light nuclei by $\sim 30\%$ at the normal saturation density and, perhaps even more important, the strong density dependence of the effect will help to prevent nuclear collapse and improve the agreement between calculated and observed nuclear radii. In the work of Bishop *et al* (1978) we have combined the results presented here with a LOCV calculation of the saturation of light nuclei using the Reid soft-core interaction.

Recently an improved transition potential has been given by Green *et al* (1978). This is designed to give the box diagram contribution to NN scattering correctly and is much weaker than the NN^* interaction of equation (14). This interaction cannot take account of the crossed pion contributions which will also suffer Pauli and dispersion corrections. The relative effect of these two opposing contributions is not known. Until these uncertainties have been resolved the results presented here and those of the Julich group (Faessler *et al* 1976, Anastasio *et al* 1978) can only be considered indicative of the likely saturating effect of N^* 's in finite nuclei.

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References

- Anastasio M, Faessler A, Mther H, Holinde K and Machleidt R 1978 *Julich preprint*
Bishop R F, Howes C, Irvine J M and Modarres M 1978 to be published
Day B D and Coester F 1976 *Phys. Rev. C* **13** 1720
Faessler A, Mther H, Machleidt R and Schutte D 1976 *Nucl. Phys. A* **262** 389
Green A M 1976 *Rep. Prog. Phys.* **39** 1109
Green A M and Haapakoski P 1974 *Nucl. Phys. A* **221** 429
Green A M and Niskanen J A 1975 *Nucl. Phys. A* **249** 493
Green A M, Niskanen J A and Sainio M 1978 *J. Phys. G: Nucl. Phys.* **4** to be published
Holinde K and Machleidt R 1976 *Nucl. Phys. A* **256** 479
Howes C, Irvine J M and Bishop R 1978 *J. Phys. G: Nucl. Phys.* **4** L81
Owen J C, Bishop R F and Irvine J M 1976 *Nucl. Phys. A* **274** 108
—1977 *Nucl. Phys. A* **277** 45
Reid R V 1968 *Ann. Phys., NY* **50** 411