LETTER TO THE EDITOR

A two-fluid model for backbending

M A Z Habeeb†, R F Bishop‡, J M Irvine‡ and M R Strayer§

† Department of Theoretical Physics, University of Manchester, Manchester M13 9PL, UK

‡ Department of Theoretical Physics, University of Manchester, Manchester M13 9PL, UK
and Daresbury Laboratory, Science Research Council, Warrington WA4 4AD, UK

§ Daresbury Laboratory, Science Research Council, Warrington WA4 4AD, UK

Received 10 March 1977

Abstract. We refine a simple two-fluid model of nuclear rotations which we have previously shown can describe the backbending phenomena observed in the rare-earth nuclei. The model is applied to a study of the nuclei $^{164,168}_{88}\text{Er}$ and $^{166,168,170}_{70}\text{Yb}$, which provide a very sensitive test. The results obtained are extremely encouraging.

The present authors, in a paper to be referred to as I (Habeeb *et al* 1976), have formulated a simple phenomenological two-fluid (namely, normal plus superfluid) model of nuclear rotations, and have applied it to give a description of the ‘backbending’ phenomena associated with the nuclear moments of inertia (e.g. Johnson and Szymanski 1973, Sorensen 1973). The predictive power of the model was, however, found to be severely limited due to the need to introduce a mass-dependent parameter (referred to as P in I) which was primarily associated with the assumption of a uniform density within the nuclear volume, and with the use of the simple Fermi-gas model to relate the superfluid pairing correlations in the otherwise independent neutron and proton fluids. It is the purpose of the present Letter to relax both of these assumptions, thereby eliminating the unknown parameter P . In so doing, we are led to consider the parameters which describe the superfluid components as being different and independent for protons and neutrons, but which are the same for the isotopes and isotones of a given nucleus respectively. This refinement of the model automatically relates the backbending behaviour of neighbouring nuclei, and thus provides us with an extremely stringent test for our model to undergo.

In the notation of I we have that the energy of a nuclear state of angular momentum J is given by

$$E_J = \frac{J(J+1)\hbar^2}{2PI_0} \frac{T^4}{(1+\eta^2 T^6)} \frac{A_n A_p}{(NF_n A_p + ZF_p A_n)A^{-1}} + \frac{9}{128} ka_0^2 T^{-4} (1-T^2)^2$$

$$+ \frac{1}{2} \sum_{\tau=n,p} C_\tau (v_\tau - \Delta_\tau)^2 \quad (1)$$

where $\tau = n, p$ labels the neutron and proton fluids respectively. The nucleus is assumed to be of spheroidal shape with major and minor semi-axes a and b respectively (a_0 and b_0 for the $J = 0$ ground state), and the incompressibility assumption requires

$ab^2 = a_0 b_0^2$. The parameter $T = b/b_0$ describes the stretching, and k is the strength of a phenomenological harmonic restoring force. The functions F_τ and A_τ are given in terms of the pairing gap parameters v_τ ($v_\tau = \Delta_\tau$ at $J = 0$) as

$$F_\tau = [\xi \delta_\tau (1 + \eta^2 T^6) + v_\tau (1 - \eta^2 T^6)]^2 \quad (2)$$

$$A_\tau = (v_\tau^2 + \xi^2 \delta_\tau^2) (1 + \eta^2 T^6)^2 + 2\xi \delta_\tau v_\tau (1 - \eta^4 T^{12}) \quad (3)$$

where $\eta = b_0/a_0$ is the ground-state deformation parameter. The parameter ξ provides a measure of the ratio of normal to superfluid components and, on the basis of a Fermi-gas model,

$$\delta_\tau = \begin{cases} (N/Z)^{1/3} & \tau = n \\ 1 & \tau = p. \end{cases} \quad (4)$$

The pairing spring constants C_τ , given by,

$$C_\tau = \left(2\rho_\tau^l + \frac{2}{\pi} \frac{1}{\Delta_\tau} \right) (1 - \rho_\tau^l G_\tau) \quad (5)$$

where ρ_τ^l is the single-particle level density at the Fermi surface, and G_τ is the pairing force strength, have been tabulated by Ma and Rasmussen (1970). The parameter $I_0 = 4\pi\rho a_0^5 \eta^2/15$ in equation (1) absorbs some of the constants in the ground-state rigid-body moment of inertia for a uniform spheroid of density ρ , and the parameter P was introduced for reasons already cited, as well as to take crude account of possible uncertainties in the values of the ground-state parameters a_0 and η . It is argued in I that P should be approximately unity.

In applications of the model, the ground-state parameters a_0 and η are determined by the experimental quadrupole moments, assuming equivalent mass and charge distributions and an effective nuclear radius $R_0 \simeq 1.2A^{1/3}$ fm. The ground-state gap parameters Δ_τ are obtained from the experimental odd-even mass differences. The energy E_J of equation (1) is then minimized with respect to the parameters T and v_τ , and the remaining parameters k , ξ and P are adjusted to give a good fit to the low-lying members of the ground-state rotational band. It was demonstrated in I that the model was capable of reproducing the experimentally observed backbending behaviour in the rare-earth nuclei.

In the present Letter the refinements that we make to the model are twofold, and are as follows.

(i) We shall assume a non-uniform density distribution, taken for simplicity to be the deformed axially-symmetric version of a trapezoidal radial distribution, with a half-density radius $R_{1/2} = 1.07A^{1/3}$ fm and surface thickness $t = 3.1$ fm (Krutov 1968). We note that the Cusson (1968) formulation of the general linear flow problem, on which our model is based, requires the density distribution to be incompressible, since the transformations in this formulation are volume conserving. For the trapezoidal distribution the incompressibility assumption is only valid to first order in t . We shall assume that, since the amount of matter in the surface is small compared with that in the bulk of the nucleus, higher-order corrections can be ignored. There is little doubt that this is a very good approximation. The explicit form of the first term in equation (1) has now to be recalculated, along the same lines as in I, by calculating the rigid-body moment of inertia for a spheroid with this density distribution.

(ii) We also remove the restrictions of the Fermi-gas approximation, which was used to relate the superfluid to normal fluid ratio in the proton and neutron fluids.

Table 1. The parameters k and ξ_τ for the nuclei $^{164,166}_{68}\text{Er}$ and $^{166,168,170}_{70}\text{Yb}$.

Nucleus	k (keV fm $^{-2} \times 10^4$)	ξ_n (keV)	ξ_p (keV)
$^{164}_{68}\text{Er}$	1.62	278.5	74.9
$^{166}_{68}\text{Er}$	1.64	350.0	74.9
$^{166}_{70}\text{Yb}$	0.99	278.5	10.3
$^{168}_{70}\text{Yb}$	1.19	350.0	10.3
$^{170}_{70}\text{Yb}$	15.20 ^a	318.7	10.3

^a The particularly high value of k obtained for $^{170}_{70}\text{Yb}$ implies that this nucleus is considerably stiffer than the others. We have investigated this point with care and have concluded that a good fit to the first three members of its ground-state rotational band can only be obtained if the stretching is inhibited.

We thus combine the free parameter ξ and the fixed parameter δ_τ in equations (2) and (3) into two new parameters, ξ_τ . We further assume that ξ_n and ξ_p are the same for all isotones and isotopes respectively.

Having relaxed the two most restrictive assumptions made in I, we now set $P \equiv 1$. We now minimize the energy E_J with respect to T and ν_τ at each value of J , and fit the three parameters k and ξ_τ to the *first three members only* of the ground-state rotational band in a nucleus of interest. Furthermore, by working with a group of neighbouring nuclei, the assumption that ξ_n is the same for all isotones and ξ_p is the same for all isotopes of a given nucleus reduces the already small number of fitting parameters even more.

Figure 1. The experimental (full circles) and theoretical (open circles) values of the moment of inertia ($2J/h^2$) against $(\hbar\omega)^2$ for (a) ^{166}Er and (b) ^{168}Er .

It has been suggested (Faessler *et al* 1974) that the five nuclei $^{164,166}_{68}\text{Er}$ and $^{166,168,170}_{70}\text{Yb}$ may provide a good test of any model of nuclear rotation, since they display an interesting alternating pattern in their backbending behaviour. Thus, we fix the parameters k and ξ_r to yield the best χ^2 fit to the first three members of the ground-state rotational band in $^{164}_{68}\text{Er}$. In $^{166}_{68}\text{Er}$ we assume that ξ_p is the same as in $^{164}_{68}\text{Er}$, and we fit k and ξ_n to give the best fit to the first three states in $^{166}_{68}\text{Er}$. In $^{166}_{70}\text{Yb}$ we assume that ξ_n has its $^{164}_{68}\text{Er}$ value, and fit k and ξ_p to the lowest three states in $^{166}_{70}\text{Yb}$. In $^{168}_{70}\text{Yb}$ and $^{170}_{70}\text{Yb}$, ξ_p is assumed to have its $^{166}_{70}\text{Yb}$ value and, in addition, in $^{168}_{70}\text{Yb}$ ξ_n is assumed to have its $^{166}_{68}\text{Er}$ value. This leaves only k to be chosen to give a best fit to the first three states in $^{168}_{70}\text{Yb}$, while in $^{170}_{70}\text{Yb}$ both k and ξ_n are fitted to the first three states of that nucleus. The parameters thus determined are shown in table 1. With these parameters fixed, the computation of the energies of the higher spin states is carried out, and the results of these calculations are presented in figures 1 and 2, where the moments of inertia $\mathcal{J}$ and energy parameters

Figure 2. As figure 1, but for the nuclei (a) $^{170}_{70}\text{Yb}$, (b) $^{168}_{70}\text{Yb}$ and (c) $^{166}_{70}\text{Yb}$.

$\hbar\omega$ are calculated as in equation (22) of I. We see that the present model is easily able to distinguish between the backbending and non-backbending cases, in an at least qualitative fashion. In view of the small number of adjustable parameters, the overall agreement with the experimental results is most impressive. We note that, as in I, the backbending is again associated with a sudden drop in the value of the neutron pairing gap v_n . Coriolis anti-pairing (CAP) of protons and centrifugal stretching effects are found to be of minor importance.

The present model also gives us some insight into the conditions necessary for backbending. The parameter ξ_τ which measures the strength of the pairing is given by (and see I)

$$\frac{L_p^{(\tau)}}{R_0} = \frac{\xi_\tau}{v_\tau} \propto \frac{1}{\rho_\tau^{\frac{1}{2}} v_\tau} \quad (6)$$

where $L_p^{(\tau)}$ is the pair correlation length and R_0 is a measure of the size of the nucleus. As $\xi_\tau \rightarrow \infty$, the superfluid component vanishes and the nucleus behaves as a rigid rotor. Experimentally, the backbending is most extreme in $^{164}_{68}\text{Er}$ and $^{166}_{70}\text{Yb}$, which are found by us to have the smallest values of ξ_n , i.e. the greatest degree of neutron pairing. The next most dramatic case is $^{170}_{70}\text{Yb}$, which is found to have an intermediate value of ξ_n , and finally $^{166}_{68}\text{Er}$ and $^{168}_{70}\text{Yb}$, which are found to have the largest value of ξ_n , show very little evidence of backbending.

We interpret these results as follows. The larger ξ_n is, the weaker is the neutron pairing, and hence quite modest Coriolis forces can break the pairs, leading to a gradual erosion of the neutron pairing gap. As ξ_n decreases and the pairing becomes stronger, a greater Coriolis force is required to destroy the pairing, and the associated backbending effect is postponed to higher spin states, where the collapse becomes more dramatic. If this interpretation is correct, then from the values of ξ_p in table 1 we would expect proton CAP effects at much higher spin states in these nuclei, with a more pronounced effect in the Yb isotopes than in the Er isotopes, but with the effect possibly occurring at lower spins in the Er isotopes than in the Yb isotopes.

References

- Cusson R Y 1968 *Nucl. Phys. A* **114** 289
 Faessler A, Grummer F, Lin L and Urbano J 1974 *Phys. Lett.* **48B** 87
 Habeeb M A Z, Bishop R F, Irvine J M and Strayer M R 1976 *Nucl. Phys. A* **272** 174
 Johnson A and Szymanski Z 1973 *Phys. Rep.* **7** 183
 Krutov V A 1968 *Ann. Phys., Lpz* **21** 263
 Ma C W and Rasmussen J O 1970 *Phys. Rev. C* **2** 798
 Sorensen R A 1973 *Rev. Mod. Phys.* **45** 353