Homogeneous Turbulence in Superfluid ^4He in the Low-Temperature Limit: Experimental Progress

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Abstract Selected advances in the research on the dynamics of tangles of quantized vortices in superfluid helium with little normal component during the last 50 years are briefly reviewed. The main emphasis is on the experimental techniques of generating and probing homogeneous one-component superfluid turbulence of various energy spectra in superfluid ^4He in the low-temperature limit. The most recent experimental progress, modern theoretical concepts and future outlook are summarized.

Keywords Turbulence · Quantized vortices · Superfluid helium · Turbulence decay

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1 Introduction

The scope of this paper is to outline important advances towards the understanding of homogeneous steady-state turbulence in superfluid helium in the limit of zero temperatures. The coverage is by no means comprehensive. We will mainly cover experimental progress with superfluid ^4He at $T < 1\text{ K}$, briefly mentioning important achievements of the 60 s and 70 s. We then outline the theoretical concepts that were advanced in the last decade and finally focus on recent experimental progress.

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Turbulence generated by vibrating structures, especially in superfluid $^3\text{He-B}$, will be mentioned but not discussed in depth.

A truly homogeneous turbulence is of course impossible to investigate experimentally as liquid helium is always confined in finite-size containers. Thus, the bigger the container the better, as the detectors are further from the walls and a wider range of lengthscales are involved in the turbulent flow. Also, many techniques rely on the observations of the free decay of turbulence, in which the spectrum of turbulence is slowly changing, and hence they should be regarded as quasi-steady at best.

Turbulence, as a general phenomenon in various highly-excited underdamped dynamic systems with many degrees of freedom and wide range of lengthscales, is ubiquitous in nature [1]. However, apart from some general concepts the behaviour of various systems are different. One can single out classical hydrodynamic turbulence in gases and liquids, magnetodynamic turbulence in terrestrial and interstellar plasmas, wave turbulence in oceanic waves and acoustic systems. Change of dimensions, external fields and types of forcing can also produce quite different behaviour within these classes too. The general solution of the problem of turbulence remains elusive and may never be found even though there has been enormous progress in understanding some of the details. The main reason for this is in the strong non-linearity of most of the systems, such as classical fluids described by the Navier-Stokes equation. Comparison of similarities and differences between various types of turbulence helps to generalize the picture of turbulence. This is why it is extremely important to have yet another class of turbulence which is both theoretically tractable and experimentally accessible—especially if in certain domains of parameter space it can fall into some of the previously known classes or sub-classes. Turbulence in superfluid helium in the limit of zero temperature seems to be a good candidate for such a new type.

Since the prediction and discovery of quantized vortices and superfluid turbulence half a century ago [2–8], there were many experimental and theoretical works on turbulence in superfluid ^4He (see reviews [9–14]). These were mainly at relatively high temperatures, 1.1–2.2 K, at which the density of excitations is high. In this case, the excitations behave as a fluid (the normal component) of low viscosity and readily exchange momentum with quantized vortices (which are linear topological defects of the inviscid coherent superfluid component). The system is thus described by two interacting fluids whose general dynamics are enormously complicated. Still, when driven and probed at large length scales it was found to behave as a single classical turbulent liquid [15, 16]. However, one would still expect that the presence of quantized vortices will affect the dynamics at short length scales and the evidence for this has started to appear recently [17]. There are reasonable expectations that at sufficiently low temperatures, when the classical fluid-like normal component vanishes, non-classical features of discrete quantized vorticity in the superfluid component will become even more apparent.

At $T = 0$, there is only the superfluid component of density ρ_s equal to the total density of liquid ($\rho = 145 \text{ kg/m}^3$ for ^4He at SVP). Apart from the Angstrom-thin cores of quantized vortices (the core radius $a_0 = 1.3 \text{ \AA}$ [10]), inside which all vorticity is concentrated, the flow is strictly potential and the velocity circulation around any closed loop is quantized. There is no viscosity. The instantaneous positions of the

filamentary cores of quantized vortices fully define the velocity field, energy spectrum and the subsequent dynamics of the system. The configuration of the tangle of vortex cores is the only remaining phase space; the vortices can both change shape and reconnect. The particular mechanisms of conversion of the kinetic energy of the coherent flow of the superfluid component into incoherent excitations (i.e. heat) is one of the fundamental questions. It is currently believed that bulk turbulence in ^4He loses its energy through the emission of phonons by high-frequency Kelvin waves (helical perturbations of straight vortices) [18]. One challenge is to understand how the energy of large-scale flow reaches these short wavelengths?

At finite temperatures, equilibrium thermal excitations (predominantly, phonons at $T < 0.6$ K and rotons at $T > 0.6$ K) scatter off the vortex lines and exchange momentum. The dominant dissipative component of this force per unit length of vortex line (in reference frame where $\mathbf{v}_n = 0$),

$$f_{mf} = -\alpha\rho_s\kappa\mathbf{v}_L, \quad (1)$$

causes the vortex filament to move with velocity

$$\mathbf{v}_L = \mathbf{v}_s - \alpha\mathbf{s}' \times \mathbf{v}_s, \quad (2)$$

where $\alpha(T) \ll 1$ is the “mutual friction parameter”, $\mathbf{v}_s$ is the local superfluid velocity, $\mathbf{s}'$ is the unit tangential vector in the direction of vortex line’s vorticity and $\mathbf{v}_L$ is the local velocity of the vortex line orthogonal to $\mathbf{s}'$ [10, 19]. Note that at temperatures below some 0.8 K the gas of these excitations (normal component) firstly becomes extremely viscous and eventually ballistic at $T < 0.6$ K. This allows us to disregard any dynamics of the normal component and to set its mean velocity uniformly to zero in the frame of the container. We are thus left with one-component superfluid turbulence. It is expected that this temperature-dependent damping of the vortex motion becomes negligible compared to that of the temperature-independent intrinsic dissipation mechanism (emission of phonons) at $T < 0.4$ K [18, 20, 21]. As the damping of Kelvin waves by mutual friction increases with their frequency, $\alpha(T)$ effectively enforces the dissipative cut-off wavenumber $k_2(T)$ above which they are overdamped [22].

When describing vortex tangles, the main observable mean parameter is the total length of the vortex filaments per unit volume L . The characteristic distance between vortices is thus $\ell \equiv L^{-1/2}$. For typical achievable vortex densities of $L = 10^1 - 10^5 \text{ cm}^{-2}$, ℓ is between 30 μm and 3 mm. The tangle hence always possesses the energy (per unit mass) of quantized flow around each line, $E_q = \frac{\Lambda\kappa^2}{4\pi}L$, where $\Lambda = \ln(\ell/a_0) \approx 13$ for ^4He . On the other hand, correlations in the orientation of vortex lines can add a great deal of excess energy if present. At length scales much greater than ℓ , the coarse grained superfluid velocity field $\mathbf{u}(\mathbf{r})$ will thus add a classical contribution to the energy per unit mass, $E_c = \frac{1}{2}u^2$. Following the classification [23–25], the tangles for which $E_q \gg E_c$ (i.e. predominantly uncorrelated with little or no polarization) will be called “Vinen” or *random* (and the corresponding turbulence *ultraquantum*), while the ones for which $E_q \ll E_c$ (i.e. locally statistically polarized albeit isotropic on average) will be called “Kolmogorov” or *structured* (and the

corresponding turbulence *quasiclassical*). Relevant considerations of the relative importance of the contributions to the total energy and vortex length L of the polarized and unpolarized components of the tangle are discussed in [26–28]. A hydrodynamic account of an advection of a random turbulent tangle by a large-scale flow resulting from the net polarization of the tangle was attempted in [29]. Scaling of vortex polarization as function of length scale for quasiclassical turbulence was considered in [30].

One thus usually deals with the following spectrum of the kinetic energy in the k -space for the given L . At *quantum* lengthscales $k > \ell^{-1}$ there is the $\mathcal{E}_k \propto k^{-1}$ contribution from the potential velocity field around smooth vortex lines. On top of it, one can have a contribution from Kelvin waves of wavenumbers $k > \ell^{-1}$ on individual vortex lines which can be excited to a different extent and can also contribute to an extension of the vortex length L . The range of wavelengths of Kelvin waves will be shortened at elevated temperatures through the decrease with increasing temperature of the dissipative cut-off $k_2(\alpha)$. On the *classical* side of lengthscales, $k < \ell^{-1}$, one either can or cannot have contributions depending on pumping. In containers of size d , the smallest wavenumber of the spectrum is obviously $k_1 \sim d^{-1}$ —provided the turbulence extends throughout the container. Finally, the position of the mesoscale divider $k \sim \ell^{-1}$, as it depends on the total L is higher for more intensive turbulence. Thus, by selecting the container size, wavenumber and intensity of forcing, and temperature, one can control the spectrum of turbulence on all sides, classical and quantum.

In experiments with turbulence in ^4He generated by large-scale flow, at temperatures above 1 K, it was observed that the low-frequency energy spectrum [15] and dynamics of free decay [16] are essentially the same as in developed classical homogeneous isotropic turbulence with a Kolmogorov spectrum. More recent measurements of the spectrum of fluctuations of the vortex line density L in a flow channel have found it to depend on frequency f as $f^{-5/3}$, that is consistent with quasiclassical turbulence at large length scales advecting the underlying small-scale random vorticity as a passive tracer. In the literature, these classical appearances of superfluid turbulence at high temperatures are often explained by the fact that the flow fields of both superfluid and normal component are nearly identical at large length scales, because of strong coupling due to mutual friction, and that the classical nature of the normal component assures the quasiclassical appearance of the combined turbulence (although read [31, 32]). This, for instance, results from the coarse-grained two-fluid equations for smooth (strongly polarized) vortex bundles, known as the Hall-Vinen-Bekarevich-Khalatnikov equations [3, 33–35]. However, even without the normal component and with a substantial non-polarized component of the tangle (i.e. weakly-polarized bundles), there are reasons to expect that the coarse-grained large-scale dynamics of a vortex tangle should reduce to the Euler equation [21] and hence reveal a Kolmogorov spectrum. This actually means that even at high temperatures, provided both normal and superfluid component were stirred together, both have natural tendency to behave as a single Euler liquid at large length scales. The coupling through mutual friction only helps to bind, in space and time, the two classical flows.

In containers of size $k_1^{-1} \sim 1$ cm and $T < 0.4$ K (so that $k_2^{-1} \sim 100$ Å), one thus has a wide range of length scales spanning six orders of magnitude. The theoretical energy spectra for the two extreme limits of turbulence, quasiclassical (pumped at the

Fig. 1 (Color online) Sketches of the energy spectra, for the same vortex density $L = \ell^{-2}$, of quasiclassical (red, spanning the range k_1-k_2) and ultraquantum (blue, spanning the range $\ell^{-1}-k_2$) turbulence, driven at their largest available lengthscale (for instance, during free decay)

scale of container d , $k_1 \sim d^{-1}$) and ultraquantum (pumped at $k \sim \ell^{-1}$) are sketched schematically in Fig. 1 for zero and finite temperature. The turbulence on the quantum side is essentially 1d wave turbulence of non-linear Kelvin waves, while that on the classical side is 3d classical hydrodynamic turbulence. The question about the spectrum of the quasiclassical turbulence at mesoscales, $k \sim \ell^{-1}$, is yet open. This is indicated by the horizontal dashed line connecting two regimes in Fig. 1. At the moment, there are two different theoretical approaches for this region: the microscopic theory of Kozik and Svistunov describing a succession of different reconnection-driven processes [21, 22, 36] and the phenomenological model of L'vov, Nazarenko and Rudenko describing a gradual cross-over from Kolmogorov to purely non-linear Kelvin-wave cascade [28, 30].

2 Experimental Advances

Two liquids can exhibit quantum turbulence: superfluid ^4He (also called HeII) and superfluid $^3\text{He-B}$ (see [37] for their comparison). The general features of the two-fluid hydrodynamics and the dynamics of quantized vortices are basically identical in both. Hence, we might expect very similar general properties of quantum turbulence in both superfluids. On the other hand there are important specific differences in vortex core sizes, pinning forces at container walls, role of remnant vortices in vortex generation, spectra of bulk and core-bound excitations, viscosities of the normal component and values of mutual friction at easily accessible temperatures—these can help to distinguish different dominant mechanisms and models. Perhaps the most surprising difference between the two superfluids is that not many techniques for detecting vortices can be used in both superfluids. While in ^4He one mainly relies on second sound [16, 38] and ion trapping [25, 27], in ^3He they are useless. However, NMR [39] and Andreev scattering of Bogolubov quasiparticles [40] turned out to be powerful techniques for detection of vortices in $^3\text{He-B}$. Generation of turbulence by mechanical techniques, such as rotating containers [27, 39], flow channels [41, 42] and immersed vibrating objects (these can also be used for the detection of turbulence) [40, 43–48], is possible in both liquids.

2.1 Detection of Turbulence in ^4He at $T < 1$ K

Unlike the attenuation of second sound that is sensitive to the presence of vortex lines but can only be used at $T > 1$ K, trapping and deflection of injected ions by vortex lines can be used to detect vortex tangles at almost any temperature. For instance, in the early 60-s, injected ions were already employed by Careri and co-workers to detect turbulence in ^4He [49] and vortex arrays in rotating ^4He [50].

At low temperatures, injected ions have so far turned out to be the most useful technique to study quantized vortices and quantum turbulence. A positive ion is a microscopic sphere of solid helium (“snowball”) of radius ~ 7 Å around a helium molecular ion, while a negative ion is simply an excess electron in liquid helium that self-localizes in a spherical cavity (“electron bubble”) of radius ~ 20 Å devoid of helium [51, 52]. For reviews on the properties and use of injected ions in liquid helium see [10, 53–55]. Ions are attracted to vortex cores and once trapped stay on the cores essentially forever provided that the temperature is sufficiently low ($T < 1.7$ K for negative ions in ^4He at STP). As negative are bound to vortex cores more strongly than positive ions, combined with the fact that it is technically simpler to inject electrons than to ionize helium atoms, negative ions are much more widely used and we will concentrate solely on them.

Singly-charged singly-quantized vortex rings were observed and studied by Reifield and Reif [56]. Apart from being very important elementary test-tubes of vortex dynamics, charged rings can be used to detect vortex arrays [27, 41, 57, 58] at $T < 1$ K. The presence of the trapped ion allows a ring of well-controlled energy and radius to be prepared. It also allows the arrival of such a ring to be detected at the charge collector. On the other hand, the dynamics, velocity and radius of the ring, while being the functions of the ring’s energy, are fully determined by the quantized superfluid flow near the ring. At energies of order 40 eV, the ring’s radius is about $1\text{ }\mu\text{m}$ and the velocity of self-induced motion is some 10 cm/s. As the rings scatter off the other vortices belonging to the turbulent tangle approximately geometrically, such relatively large radius of rings allows quite dilute tangles of density down to $L \sim 10\text{ cm}^{-2}$ ($\ell \sim 3\text{ mm}$) to be detected. The rings are relatively fast on the scale of the velocity of motion of these vortex lines, $\sim \kappa/\ell$, but are slow enough to allow comfortable time-resolved detection of the collector current due to their arrival at characteristic times ~ 1 s.

We should mention some important byproducts of the experiments by McClintock’s group on critical velocities of ions for vortex nucleation in ^4He [59]. In the course of these studies, observations were made of ion-generated turbulence and remnant vortices near the ion injector region and the importance of using isotopically-pure ^4He . As ^3He quasiparticles make another contribution to the density of the normal component that can exchange momentum with vortex lines, it seems important to reduce their concentration down to at least $x_3 < 10^{-6}$ in order to observe the $T = 0$ limit.

There has been extensive use of injected ions to detect or visualize vortices in ^4He and also to study ^3He condensation in their cores [52, 60, 61]. For instance, as ^3He was found to effectively condense in vortex cores at $T < 0.6$ K, it seems that, with even as low ^3He concentration as 10^{-10} per ^4He atom, the cores of vortex lines should

always be full of ^3He . Some unanswered questions remain for $T < 0.7$ K (namely, that the observed mobility of trapped ions along rectilinear vortex lines is higher than that predicted by theory) [61] where our knowledge of the ion-vortex interaction is still rudimentary.

2.2 Creation of Turbulence at $T < 1$ K

At high temperatures, $T > 1$ K, various means of generating turbulence have been successful: forcing through mechanical means (“French washing machine” [15], towed grid [16], flow channel [38]), counterflow [62], ultrasound [63] and ion injection [25]. At lower temperatures, the introduction of moving parts is a complicated engineering task as there are severe limitations on the vacuum insulation and affordable level of heat release due to friction. The latter makes it basically impossible to use, below 1 K, any solid parts rubbing against each other. This is why vibrating structures are the most widely used method for engaging liquid into motion and probing its state, both in superfluid ^4He and ^3He . This simplification comes at a price that the flow direction is alternating by default.

One early example is the experiments by Gamota [43], who used an array of vibrating orifices (a Nuclepore membrane with orifices of diameter $5\text{ }\mu\text{m}$ and resonance frequency of vibration 1.17 Hz) at $0.3\text{--}0.5$ K to generate a beam of monoenergetic vortex rings of diameter $1.7\text{--}3.2\text{ }\mu\text{m}$ and detect them by a transverse beam of charged vortex rings. The emission of individual vortex rings and their tangles was detected and it was found that the tangles decayed within seconds after stopping pumping. Since then, vibrating balls, wires, forks and grids have been used (for the most up-to-date information, see [45, 64–67]). In particular, the experiment by McClintock’s group [68] on generating vortices by a vibrating grid and detecting the turbulence by pulses of bare negative ions were important for attracting interest to the problem of turbulence decay at low temperatures. They quoted 70 mK as the temperature below which the decay rates were independent of temperature but as all other existing experimental and theoretical works point at $T \sim 0.4$ K as the temperature below which the vortex dynamics in ^4He becomes independent of temperature [20, 27, 46], we are inclined to assume that the thermometer readings in [68] were not indicating the actual temperature of the turbulent helium.

Experimental techniques sensitive to the arrival of ballistic vortex rings are of great interest as some theories of tangle dynamics predict emission of small vortex rings [36]. In this connection we refer to the old experiments by Gamota [43] and the much more recent ones by Yano *et al.* [45]. In the former, the arrival of rings emitted at an orifice was detected by their time-resolvable effect on the beam of charged vortex rings at a known distance. In a similar fashion, the latter experiments were capable of detecting the onset of dissipation of a vibrating wire triggered by the arrival of ballistic vortex rings emitted by another vibrating wire. In both experiments, the time delay between the events of emission and detection worked as a sensitive measure of the radius of the ballistic rings (as their time of flight directly depends on their radius).

There were also successes in generating continuous flows of superfluid helium. Guenin and Hess [41, 69] have built fountain pumps capable of producing steady

flow of ^4He at $T = 0.3$ K or lower. By introducing a supercritical jet through an orifice [41], depending on the flow rate, they observed production of either ballistic rings or inhomogeneous vortex tangles. Upon stopping the jet, the vortex tangles were found to decay within seconds. As in [43], the vortex detection was done through the attenuation of a transverse beam of charged vortex rings. We would also like to mention the successful operation of a bellow-driven pump capable of maintaining quasi-steady flow in a channel with superfluid ^3He at as low as $T = 2$ mK by Saundry *et al.* [42]!

To avoid moving parts, one can generate turbulence by a change of the angular velocity of rotation of the whole container (that typically implies spinning up or down of the whole cryostat). This was successfully used in experiments with superfluid ^4He in Manchester [27], described below, and also with generation of a turbulent front in rotating superfluid $^3\text{He-B}$ in Helsinki [39].

Electroconvection (“ion jet”) can also be used to generate turbulence in ^4He at low temperatures. In an applied electric field, the pull of trapped ions on vortex lines is, in some sense, similar to the action of mutual friction force in counterflow turbulence. At low temperatures, random tangles (ultraquantum turbulence) have been successfully generated [25] by this technique.

3 Theoretical Advances and the Need of Further Experiments

3.1 Some Theoretical Ideas about the Dissipation Mechanisms at $T = 0$

We now refer to some milestones in developing the theoretical concepts related to the dissipation of quantum turbulence at $T = 0$. At high T , the bulk dissipation is due to thermal excitations (normal component): either through the momentum transfer between the quantized vortices and normal component (mutual friction) or through the shear viscosity of the normal component itself. At $T = 0$ we need new ingredients.

Feynman [2] outlined a possible cascade of vortex loops upon reconnections down to the atomic size of loops (rotons), although it is unclear what caused dissipation if there is no mutual friction (reconnections themselves?). It is generally believed that reconnections cannot dump substantial energy in ^4He but might be more efficient in ^3He [11]. Kozik and Svistunov [36] commented that these processes are forbidden for isolated rings due to energy-momentum conservation. However, inside dense tangles these restrictions might be lifted.

The fact that compact tangles can relax through the evaporation of small ballistic vortex rings was discussed by Guenin and Hess [41] and observed in numerical simulations [70–72]. Inside inhomogeneous tangles, diffusion of vorticity is expected to occur in parallel with dissipation [71, 73].

When superfluid flows past a rough wall of a container, two possible dissipative mechanisms have been discussed. If there are quantized vortex lines, vortex cores experience effective friction due to the broken translational invariance (repetitive pinning-unpinning) [74] and hence generate Kelvin waves. One can also speculate that the corresponding fluctuations of pressure should emit sound into the container walls. Alternatively, even without vortex lines, time-dependent superflow should result in generating excitations through quantum fluctuations [75].

Svistunov came up with the idea of a quantum cascade of energy to smaller length-scale fueled by vortex reconnections [76]. More theoretical investigations into this problem can be found in papers [77–80] and references therein. For instance, Kozik and Svistunov investigated Kelvin-wave cascade analytically and numerically [81] and studied the role of various reconnection processes in transferring classical energy to Kelvin waves [21, 22]. That the energy is ultimately lost into phonons emitted by very short-wavelength Kelvin waves was suggested by Vinen [18] (see also [82]) and elaborated by Kozik and Svistunov [20].

L'vov, Nazarenko and Rudenko [28, 30] have suggested that, in the absence of any intermediate mechanisms of energy transfer to shorter length scales, the Kelvin wave cascade is inefficient in absorbing the energy flux from classical eddies and will hence cause a pile-up of energy (and hence, vorticity) at mesoscales. However, Kozik and Svistunov [21, 36] point out that various reconnection processes at mesoscales are efficient in transferring this energy to shorter length scales where the pure non-linear Kelvin wave cascade becomes efficient.

It is worth stressing that, although it looks certain that many of the listed decay mechanisms should play a role in quantum turbulence at $T = 0$, there is no direct experimental evidence for any of them so far. Finally, the assumption that the rate of energy decay per unit mass can generally be written as $-\nu(\kappa L)^2$, where the prefactor $\nu \sim \kappa$ (“effective kinematic viscosity”) can depend on the spectrum of turbulence (type of flow) and temperature, was introduced by Stalp et al. [16] in the context of quasiclassical turbulence in ^4He at $T > 1$ K and was later used successfully in relation to the decay of both quasiclassical [27] and ultraquantum [25] turbulence in the $T = 0$ limit.

3.2 Numerical Calculations

Schwarz developed a microscopic description of self-similar vortex tangles and used it for numerical simulations of vortex tangles acted upon by homogeneous counterflow [19, 83, 84]. These have no large-scale flow and thus should belong to the case of *ultraquantum* turbulence (however, see [84]). Moreover, Schwarz used the local-induction approximation which can only deal with uncorrelated tangles at best. It only acts to minimize the local (and thus the total) length of the vortex tangle, completely disregarding the classical energy E_c that can be dominant if the tangle is polarized—unlike the proper Biot-Savart formalism. Also, Schwarz only simulated tangles at temperatures above 1 K, where the mutual friction dominated the dissipation. Later, random tangles at zero temperature were modelled by Tsubota, Araki and Nemirovskii [85] using the local induction approximation and by Tsubota, Araki and Vinen [71] using the full Biot-Savart approach. The range of lengthscales modelled was truncated due to the limitations of computing power. As a result, the energy, cascading down the lengthscales following reconnections was lost not at realistic very short scales but at larger scales limited by the numerical resolution. However, it is believed that the important processes of the cascade and related rates of the energy flux were captured properly. They have observed the free decay of type $L \propto t^{-1}$ from which one can extract $\nu \approx 0.06\kappa$.

Recent analytical calculations by Nemirovskii [73] of 3d diffusion (disregarding bulk dissipation and energy of large-scale flow) in confined inhomogeneous tangles

resulted in free-decay $L(t)$ that, in principle, can account for the numerical simulations [71] and experimental observations of the free decay of a tangle produced by a beam of colliding vortex rings from a vibrating grid in $^3\text{He-B}$ [40]. Note that this model is not applicable to quasiclassical turbulence pumped at large length scales (for instance, flows obtained by injecting energy at the largest scale of the system, such as in the recent spin-down experiments in Manchester [27]).

Simulations of the dynamics of a non-linear Kelvin-wave cascade on a single vortex lines was performed by Kozik and Svistunov [81]. Examples of simulations of rings or small tangles [86, 87] using the non-linear Schrodinger equation and of Kelvin waves on rectilinear vortex arrays [79] should also be mentioned. It should be noted that nobody has yet managed to simulate the dynamics of homogeneous quasiclassical tangles that contain both classical and quantum ranges of lengthscales. Using both the filament [88] and Gross-Pitaevskii [89] model, Tsubota *et al.* managed to demonstrate that a vortex tangle can have the classical Kolmogorov spectrum at large lengthscales $E_k \propto k^{-5/3}$ [91]—when driven by random fluctuations of certain spectrum; although the value of the Kolmogorov constant was found to be $C = 0.55$, unlike the standard value of $C = 1.5$ for classical fluids [90]. We should single out recent simulations by Hanninen [92] (transition from stationary to uniformly rotating superfluid via the propagating turbulent front, and spin-up and spin-down of superfluid helium in a tilted cylindrical container) and Barenghi ([72] crossing of two bundles of quantized vortices) that revealed reconnections of quantized vortices (and of their bundles), emission of small vortex loops and generation of Kelvin waves.

3.3 Questions to be Answered Experimentally

While many theoretical concepts and predictions regarding the behaviour of quantum turbulence at low temperatures look very plausible, experimental evidence is still rather thin, especially at the quantum scales and mesoscales. We single out just a few questions awaiting experimental confirmation:

- Does quantum turbulence behave completely classically at large lengthscales, $\gg \ell$, in the limit of $T = 0$? If yes, can one observe deviations (say, in the energy spectrum) at mesoscales and short length scales?
- Is the dissipation through phonon emission at short length scales, at least in liquid ^4He ? How will large-scale energy reach those short length scales? What type of inertial cascade is relevant at quantum scales: non-linear Kelvin waves, vortex reconnections, or anything else?
- Are the rates of dissipation different for flows with the same L but different spectra (i.e. tangles of different structure/polarization)?
- Can one study the quantum cascade on individual vortex lines (either straight lines or rings), perturbed either by a controlled force or by stochastic interactions with other lines in a tangle?

4 Recent Experimental Results

At the moment, there exist two quantitative studies of free decay of turbulence in superfluid helium in the $T = 0$ limit: Lancaster experiments on vibrating grid turbulence

in $^3\text{He-B}$ and Manchester experiments on turbulence in ^4He generated by spin-down. In this section, we review the latter and provide a brief comparative analysis of the former.

4.1 Quasiclassical Turbulence: Impulsive Spin-Down to Rest of a Cubic Container

To avoid the difficulties of introducing moving parts into the low-temperature environment, Walmsley *et al.* [27] relied on the rotation of the whole cryostat as the means of injecting large-scale flow into superfluid ^4He at $T \sim 0.2$ K. They used a cubic-shaped container of a side $d = 4.5$ cm, rigidly attached to the cryostat that could rotate continuously while maintaining temperature as low as 50 mK. The length of the vortex tangle could be sampled along two orthogonal axes of the container by measuring the loss of the time-resolved short pulse of charged vortex rings at $T < 0.8$ K (or alternatively of bare negative ions at $T > 0.7$ K). The interaction of vortex rings with vortex lines is nearly geometrical, i.e. the effective scattering diameter is of order of the rings' diameter [58]. The sensitivity to the vortex density L was calibrated on arrays of parallel vortex lines of known density at steady rotation [93]. The cell design is described in [94] and the discussion of the experimental technique and its interpretation in [93].

After preparing a nearly solid-body-rotating liquid ^4He (following at least 10 minutes rotation at constant angular velocity Ω), the rotation was suddenly (within about a second) stopped. The measured density of vortex lines along the axial and transverse direction, as well as the observations of the dynamics of the disruption of the transport of injected ions along the rectilinear vortex lines of the initially rotating helium, revealed the following (Fig. 2). During an initial transient of duration $\sim 10/\Omega$, the initial solid-body rotation of the helium was destabilized by the now stationary confining walls of square cross-section. Initially, the density of quantized vortex lines grows near the perimeter walls but then reaches the axial region as well,

Fig. 2 (Color online) $L(t)$ at $T = 0.15$ K for four values of Ω . The average driving fields used for $\Omega = 1.5$ rad/s: 5 V/cm ($\circ$), 10 V/cm (∇), 20 V/cm ($\diamond$), 25 V/cm (Δ). At other values of Ω the electric fields used were either 20 V/cm (0.05 rad/s) or 10 V/cm (0.5 and 0.15 rad/s). The line shows the dependence $t^{-3/2}$. The horizontal bars indicate the initial vortex densities at steady rotation, $L = 2\Omega/\kappa$, at $\Omega = 1.5$ rad/s, 0.5 rad/s, 0.15 rad/s and 0.05 rad/s (from top to bottom)

Fig. 3 (Color online)
 $\Omega^{-3/2}L(t)$ vs. Ωt for
 $T = 0.15$ K (filled symbols) and
 $T = 1.6$ K (open symbols).
 Dashed and solid lines $\propto t^{-3/2}$
 guide the eye through the
 late-time decays at $T = 1.6$ K
 and 0.15 K, respectively

ending up distributed almost uniformly throughout the container. During the subsequent free decay of the resulting turbulence, the vortex line density decays with time t as $L \propto t^{-3/2}$; the prefactor being independent of the initial Ω and direction of measuring L . This technique worked well at any temperatures in the range 0.1 – 1.6 K. The shapes of transients did change slightly between low and high temperatures, but not the transient's duration (the position of maximum at $\sim 10/\Omega$ at all temperatures, Fig. 3). At low temperatures, for the highest initial pumping (angular velocity of initial rotation $\Omega = 1.5$ rad/s), the measured values of the intervortex distance $\ell = L^{-1/2}$ in the decaying turbulence were changing from 0.1 mm to 2 mm—i.e. the range of length scales on the classical side of the spectrum, between $\sim \ell$ and $\sim d = 4.5$ cm, covered almost three decades at the beginning and at least factor of 20 at the end of observations (one hour after the spin-down). From the temperature-dependent prefactor in front of the late-time decay $L \propto t^{-3/2}$, the authors were able to extract the effective kinematic viscosity $\nu(T)$. The interpretation of the observation is that during the transient, the initial kinetic energy of rotation is converted into nearly homogeneous quasiclassical turbulence in which most energy is injected into the smallest wavenumber $k_1 \sim d^{-1}$. The turbulence then decays, at the rate maintained by the rate of the energy release, into the inertial cascade, from the energy-containing eddies of the saturated size $\sim d$. This scenario [16] predicts the energy flux down the cascade to decrease as $\epsilon \propto t^{-3}$, and hence the total squared quasiclassical vorticity as $(\kappa L)^2 \propto t^{-3}$ —assuming the quasi-steady-state regime maintains the self-consistent relation $\epsilon = \nu(\kappa L)^2$.

At high temperatures, a different technique of generating quasiclassical turbulence in a stationary (non-rotating) container was also developed. It turned out that a sustained jet of injected ions through the container produces turbulence that, after switching the current off and removing the injected ions, decays in the same way as the one obtained by spin-down (i.e. with substantial energy of the large-scale flow). Clearly, the drag of the ions on the vortex tangle (either directly through the trapped ions or indirectly through the normal component) was sufficient to pump enough energy into large-scale flow of the superfluid component. So far, this technique was only successful at high temperatures, at which the late-time decay data $L(t)$ were indistinguishable whether after spin-down or ion injection (see Fig. 4) [25, 93]. Moreover, the values of the effective kinematic viscosity ν extracted from these experiments

Fig. 4 (Color online) Free decay of a tangle produced by an impulsive spin-down to rest [27] from 1.5 rad/s and 0.5 rad/s, as well as by a jet of free ions from the bottom injector (●) [25], at $T = 1.60$ K. All tangles were probed by pulses of free ions in the horizontal direction. The line $L \propto t^{-3/2}$ corresponds to (4) with $\nu = 0.2\kappa$

Fig. 5 Free decay of a tangle produced by beams of charged vortex rings of different durations and densities, $T = 0.15$ K. The injection direction and duration, and driving field are indicated. Probing with pulses of charged vortex rings of duration 0.1–0.3 s were done in the same direction as the initial injection, except in one case (*). The line $L \propto t^{-1}$ corresponds to (3) with $\nu = 0.1\kappa$

turned out to be basically identical to those obtained from the analysis of $L \propto t^{-3/2}$ decay of homogeneous quasiclassical turbulence in ^4He at 1.1–1.6 K generated by a towed grid in containers of square cross-section with side $d = 1$ cm [95, 96]. This gives us confidence that the analysis of the spin-down experiments is correct, at least at high temperatures. The values of ν were then extracted from all $L \propto t^{-3/2}$ datasets for spin-downs to rest and are plotted in Fig. 6 along with other relevant values. In this analysis, all changes with temperature in the prefactor of $L \propto t^{-3/2}$ were attributed to $\nu(T)$ alone. Whether some other numerical parameters of order unity in (4) (such as the assumed relation $k_1 d = 2\pi$ and the value of the Kolmogorov constant $C = 1.5$) can effectively change with temperature should be the subject of further experimental and numerical studies [93] (see discussion in Sect. 4.3).

Fig. 6 The effective kinematic viscosity $\nu(T)$ for quasiclassical turbulence in ^4He : $\nu(T)$ inferred using (4) from the free decay $L \propto t^{-3/2}$ of tangles produced by impulsive spin-down (open triangles [27]), ion jet (open circles [25]) and towed grid (open squares [95]). The dashed curve is a fit to our data (open triangles) by theory [22]. For comparison, the values for $\nu(T)$ for the quantum regime of turbulence in ^4He , generated by a beam of colliding CVRs, are shown by crosses (+ and $\times$) [25]. The values of the mutual friction parameter α are indicated on the top horizontal axis

4.2 Ultraquantum Turbulence: Forcing through Ions Trapped on Vortices

At high temperatures, a wind of excitations (counterflow) can be used to pump energy into the vortex tangle at length scales of order ℓ while the large-scale flow (polarization) is kept small [4, 83, 97]. At $T < 1$ K, where there are insufficient thermal quasiparticles, it would be handy to find an alternative way to apply force to vortex lines. Fortunately, injected ions can play this role in ^4He . Namely, it was observed that pulses of injected ions of sufficiently short length (0.1–1 s long), after moving through the cell within some 5–10 s, leave behind a tangle of quantized vortices that are uniform in density and decay at late time as $L \propto t^{-1}$; the prefactor being independent of the density of injected current, propelling electric field and temperatures $0.1 < T < 0.5$ K. So far, this method of generating seemingly random vortex tangles was only successful at these low temperatures. The analysis in terms of the free decay of bulk homogeneous random tangle (ultraquantum, “Vinen” turbulence) has resulted in the value $\nu = 0.1\kappa$ [25] which is in line with the simulations [71, 85] but very different from the values of $\nu = 0.003\kappa$ for quasiclassical “Kolmogorov” turbulence (see the discussion in Sect. 4.3).

4.3 Interpretation of $L(t)$ during Free Decays

Ultraquantum homogeneous turbulence, i.e. the one with negligible amount of mean flow energy at lengthscales greater than ℓ , is expected to decay as

$$L = B\nu^{-1}t^{-1}, \quad (3)$$

where $B \approx \ln(\ell/a_0)/4\pi$ (for superfluid ^4He , $B \approx 1.2$) and ν is the value of the effective kinematic viscosity specific for this type of quantized flow [25]. One should

be careful though as some other types of flows with very different spectra can have decay similar to $L \propto t^{-1}$, namely, the quasiclassical grid turbulence during the transient before its energy-containing length has saturated [16]. As we expected very little large-scale flow in tangles generated by short pulses of charged vortex rings, and as their late-time decays had $L \propto t^{-1}$ with a universal prefactor, we interpret these as following (3) and hence extract $\nu = 0.1\kappa$.

Classical homogeneous isotropic turbulence with the energy-containing length fixed in time by the container size is expected to maintain quasi-steady Kolmogorov energy spectrum with the total energy decaying as $E \propto t^{-2}$. For quasiclassical quantum turbulence, after assuming that the energy decay rate per unit mass is given by $\nu(\kappa L)^2$, and that the classical energy is the main source of this energy, one arrives at

$$L = (3C)^{3/2} \kappa^{-1} k_1^{-1} \nu^{-1/2} t^{-3/2}. \quad (4)$$

One can hence extract the value of ν for homogeneous quasiclassical turbulence from this formula, provided the values of C and k_1 are known. We did this using $C = 1.5$ and $k_1 = 2\pi/d$, and obtained the zero-temperature value of $\nu = 0.003\kappa$ that is much smaller than the corresponding high-temperature value of $\nu \sim 0.2\kappa$.

Let us discuss the potential errors in this technique of determining the absolute value of ν . Firstly, it only applies to homogeneous isotropic turbulence with dominating classical energy concentrated at the energy-containing scale d which is constant in time. While we are certain that in spin-down experiments the energy is mainly injected into the largest length scale (container size d), we still have to prove that the initial rotation has no stabilizing effect on the decay rate of the largest eddies, i.e. that there is no anisotropy in the resulting tangle (there are evidences that the resulting turbulence is reasonably homogeneous and isotropic as the average density of vortex lines is the same during the late-time decay when measured in two orthogonal directions [93]). Secondly, even if the interpretation in terms of (4) is correct, the absolute value of the extracted ν is subject to the values used for C and especially k_1 that can, in principle, change with temperature [93]. For instance, it is still an open question whether the assumption for the smallest wavevector, $k_1 = 2\pi/d$, should hold at all temperatures: both in the limit $T = 0$ when there is no normal component and at $T > 1$ K when there is plenty of normal component with no-slip boundary conditions at the container walls. Also, the value of the constant C might depend on the type of non-linearity possessed by the coarse-grained equation for the microscopic dynamics of the tangle.

To conclude, it was observed in the $T = 0$ limit that quantum turbulence generated mechanically at large length scales decays in agreement with the expectations for quasiclassical turbulence. The experimentally measured value of ν for quasiclassical turbulence is much smaller than that for ultraquantum turbulence at $T = 0$ and quasiclassical turbulence at $T > 1$ K. This means that for the same energy flux towards the dissipative lengthscales, quasiclassical tangles at $T = 0$ build up a larger quasi-steady vortex density L . Two theories [22, 28] support this observation, although they have different explanations.

4.4 Lancaster Experiments on Grid Turbulence in $^3\text{He-B}$ [66, 98]

In the series of elegant experiments, Bradley *et al.* gained an insight into the details of spatial and temporal dynamics of the generation and decay of vortex tangles in superfluid $^3\text{He-B}$ at temperature below $0.2 T_c$, i.e. when $\alpha \sim 10^{-4}$, such that the normal component is deemed negligible. For tangle generation, they used a vibrating grid of width 5 mm and height 3 mm. The detection technique relied on using a vibrating wire to detect the reduction in the damping caused by the Andreev reflection of the excitations by the velocity field due to the quantized vortices forming the tangle. There were two vibrating-wire detectors, positioned 1 mm and 2 mm from the grid. Above a certain amplitude of the velocity of the grid oscillations the grid starts to emit vortex rings of diameter $\leq 5 \mu\text{m}$ [99]. As the density of emitted rings increases at higher drive amplitudes, these rings begin to collide and form a tangle. The vortex line density L was found to fluctuate with the power spectrum $\propto f^{-5/3}$ [66], similar to what was observed in high-temperature experiments with quasiclassical turbulence in ^4He in a flow channel [38]. This observation and the cross-correlation between the two detectors implies that the turbulence is quasiclassical and drifting away from the grid at the mean velocity of order 0.3 mm/s—apparently as a result of accumulating the impulse of the incoming vortex rings. In the steady-state regime with continuous pumping, as the far detector was seeing half the vortex density of the near wire, it is assumed that the effective spread of the tangle is of order $d \sim 1.5$ mm. It seems also reasonable to assume that the transverse spread of the tangle is within 3 mm in the “height” direction and 5 mm across. The classical analog of such flow is an immersed jet, which is known to produce a developed turbulence although rather an inhomogeneous one. After a sudden switching off of the drive and following a quick transient, the tangle’s density was found to decay. This decay was monitored by a stationary detector located 1 mm from the grid. The free-decay measurements of $L(t)$ thus revealed approximately $\propto t^{-1}$ late-time dependence (for waiting time t between 1 and 30 s and vortex densities L from 4×10^3 to $2 \times 10^2 \text{ cm}^{-2}$) at moderate amplitude of pumping, that changes to approximately $\propto t^{-3/2}$ (for waiting time t between 3 and 30 s and vortex densities L from 10^4 to $3 \times 10^2 \text{ cm}^{-2}$) at high drives of the vibrating grid [40]. We understand that the effect of Andreev scattering on the damping of vibrating wire is expected to be linear in vortex density L (as was assumed by the authors) only for random (not quasiclassical) tangles and for small probabilities of Andreev scattering (no multiple scatterings), $adL \ll 1$, where the scattering radius $a \sim 10 \mu\text{m}$. This limits L to $L \ll (ad)^{-1} \sim 10^4 \text{ cm}^{-2}$, which is not met for the reported values of L for the most dense tangles. Hence, the quantitative interpretation of these measurements in terms of L for the highest levels of pumping is probably not certain and should await further theoretical investigations [100].

It is relatively easier to interpret the decay of the low-density tangles (generated at intermediate drive amplitudes) that decay as $L \propto t^{-1}$ and apparently contain less classical energy (of large-scale eddies) E_c than quantum energy E_q . As was pointed in [40], this can imply $\nu \approx 0.3\kappa$; although this might be simply the lower limit on ν if this decay is mixed with a long transient while the energy-containing length has not yet reached the characteristic extent of the tangle [16, 25]. If the interpretation in terms of purely ultraquantum turbulence, (3), is correct, the extracted value of ν is

pretty robust as there are actually no other unknown parameters in the formula. The greatest uncertainty will then come from the absolute values of $L(t)$ as measured; as they are quoted to be accurate within some factor of 2 [40], so should be the values of $\nu \approx 0.3\kappa$ for the ultraquantum turbulence in $^3\text{He-B}$ at low temperatures.

There are more unknowns if one attempts to interpret $L(t)$ for the free decay in the *quasiclassical* regime. As the decaying tangle is still expected to drift through the detector, and its core will cover the distance between the grid and detector in about $1\text{ mm}/0.3\text{ mm/s} = 3\text{ s}$, we might anticipate a decrease in the measured $L(t)$ already after some 3 s—even if there is no bulk decay of turbulence at all. It is hence unclear how to separate the contribution, to the observed late-time dependence $L \propto t^{-3/2}$, from advection of the vortex density from the bulk decay. If we assume that the tangle has suddenly stopped drifting and does not efficiently spread diffusively (say, by emitting vortex rings from its surface), as was done in [40], then (4) would yield $\nu \approx 0.2\kappa$ for the quasiclassical turbulence in $^3\text{He-B}$ at low temperatures. This was obtained using the estimated value of the size of saturated energy-containing quasiclassical eddies $d = 1.5\text{ mm}$. As the value of ν , determined from (4), depends on the choice of d as $\nu \propto d^2$ and on the absolute value of L as $\nu \propto L^{-2}$, the absolute value of ν is probably uncertain to within at least factor of 3 or so. Another complication with the interpretation in terms of quasiclassical turbulence is that the mean inter-vortex distance $\ell = L^{-1/2}$ is not very small when compared to the spatial extent of the tangle, $d \sim 1.5\text{ mm}$: $\ell \sim 0.1\text{ mm}$ in the most dense tangles, and $\ell \sim 1\text{ mm}$ at the end of the observed decay. As coarse-grained quasiclassical behaviour is only expected at length scales greater than ℓ , the classical range, between $\sim d$ and $\sim \ell$ (Fig. 1), is at best one decade in the most dense steady states and hardly present at all at the end of the observed free decay.

5 Ways Forward

It seems that only the first half of the first question from the list in Sect. 3.3 has come close to being answered experimentally: turbulence generated by large-scale flow does behave classically at large length scales—even at $T \rightarrow 0$.

To further our understanding, new experiments with turbulence of well-characterized spectra at $T < 1\text{ K}$ are eagerly desired. This could include towed-grid turbulence, turbulence at steady rotation, Kelvin-wave cascades on individual vortices (say, rings or rectilinear vortex lines in a rotating bucket). Any means of detecting either deviations from the quasiclassical behaviour approaching mesoscales or strictly non-classical hydrodynamic behaviour at mesoscales and quantum scales would be welcome. These will require detectors of pressure or flow velocity sensitive to small lengthscales and short times, as well as detectors of small vortex rings that are expected to be emitted from the boundaries of strongly inhomogeneous tangles. Turbulence produced by either towed grids or steady flow through channels or orifices would be of most interest. Numerical simulations of quasiclassical turbulent flows spanning the range of lengthscales from classical to quantum will be extremely helpful too.

New complementary means of detecting turbulence are required. We can refer to the recent attempts to employ calorimetric and acoustic techniques [101]. It is

worth mentioning that, to achieve the temporal and spatial sensitivity to quantum scales one should not necessarily strive to get kHz and μm resolution. We might expect non-classical behaviour already on the length scale of ℓ and below (in terms of characteristic times of velocity fluctuations this should correspond to ℓ^2/κ). With an experimental cell of characteristic size $d \sim 1\text{ cm}$, the turbulence can be considered developed and homogeneous if $\ell \ll d$, i.e. for $\ell \leq 1\text{ mm}$ and $\ell^2/\kappa \leq 10\text{ s}$. These resolutions seem quite realistic. Making the container's capacity as big as 1 liter (i.e. $d \sim 10\text{ cm}$) will bring these requirements to a really comfortable $\ell \leq 10\text{ mm}$ and $\ell^2/\kappa \leq 10^3\text{ s}$!

Visualization of the vortex tangle and its motion would be especially valuable. Very recently, there was good progress in decorating vortex cores by micron-size hydrogen particles and real-time imaging of them at $T \sim 2\text{ K}$ [17]; although it is unclear whether this technique can be used at $T < 1\text{ K}$. New techniques, potentially applicable at $T < 1\text{ K}$, were recently suggested that rely on trapping by vortex lines of either negative ions [52] or excimer helium molecules [102] whose positions can be repeatedly imaged after application of either acoustic pulse or laser-induced fluorescence. Finally, detecting currents caused by the drift of charge trapped on vortex lines was recently used at to learn about the dynamics of vortex tangles at $T < 1\text{ K}$ [103].

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