High resolution modelling of multi-energy domestic demand profiles

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Highlights

- High resolution multi-energy model (electricity, heat, gas) for domestic buildings is presented
- Domestic energy services considered are space heating, domestic hot water, cooking, and electrical appliances (including lighting)
- Model includes accurate assessment of thermal inertia of different components for heat production, distribution and utilization
- Potential of the model for various network and market applications to current and future electro thermal technologies is demonstrated

Abstract

Increasing interest in low carbon electro-thermal heating technologies, such as electric heat pumps and combined heat and power units, may in the near future significantly change the character of domestic electricity and gas demand profiles. To understand the impact of adoption of such technologies it is necessary to appreciate fully the multi-energy aspects of such technologies, and indeed of any other domestic energy service demands. Such holistic treatment is necessary to fully capture the effects of coincidence in various energy service demands within and across dwellings. To that end this paper introduces and details a high resolution domestic multi-energy model comprised of physically based space heating, domestic hot water (DHW), cooking and electrical appliance models. Given the potential for thermal storage in the heating system and building elements particular attention is given to the space heating model, which includes building, heat emitter, buffer tank and heating unit models, capturing the relevant inter-temporal dependencies, heat transfer rates and system inertia. The power and interest of the presented model is demonstrated through exploration of a selection of various possible applications, including amongst others: effect of input energy/heating technology substitution on consumption profiles of different energy vectors, effect of thermal inertia, and investigation of dwelling diversity. Additionally the importance of high granularity modelling is demonstrated. The presented model can be utilised by many actors involved in energy systems for various purposes. For example electricity distribution network operators for modelling of network peak demand, demand aggregators for estimation of potential demand side flexibility, government agencies for assessing incentive scheme costs, electricity retailers for understanding the impact of electro-thermal technology adoption upon their demand portfolio or potential heat network operators for understanding the demand for heat in an area.

Keywords

Multi-energy systems; high resolution building modelling; electric heat pump; combined heat and power; demand characterization; energy consumption diversity

Abbreviations

ASHP	Air Source Heat Pump
CHP	Combined Heat and Power
COP	Coefficient of Performance
DHW	Domestic Hot Water
DR	Demand Response
EHP	Electric Heat Pump
ETT	Electro-Thermal Technology
GSHP	Ground Source Heat Pump
NEED	National Energy Efficiency Database
WSHP	Water Source Heat Pump

Notation

A	set of electrical appliances
c_p^{water}	specific heat of water (kJ/kg°C)
C	set of cooking appliances
C_c	thermal capacitance of the storage heater core (kJ/°C)
C_{ewo}	thermal capacitance of the outer portion of the external wall (kJ/°C)
C_{ewi}	thermal capacitance of the inner portion of the external wall (kJ/°C)
C_f	thermal capacitance of floor (kJ/°C)
C_{he}	thermal capacitance of heat emitter (kJ/°C)
C_i	thermal capacitance of the inside zone of the building (including furnishings) (kJ/°C)
C_{iw}	thermal capacitance of the internal wall (kJ/°C)
C_p	thermal capacitance of the pipework (kJ/°C)
COP_{T_s, T_w}	EHP coefficient of performance, given source and heating system temperatures
D	set of DHW consuming appliances
E_{CHP}	CHP electricity generation (kW)
E_a	Electrical appliance electricity consumption (kW)
F_b	fuel consumption of gas boiler (kW)
F_{CHP}	fuel consumption of CHP (kW)
F_{EHP}	fuel consumption of EHP (kW)
H_{DHW}	DHW thermal demand (kW)
H_c	cooking thermal demand (kW)
H_b	heat generation, gas boiler (kW)
H_{CHP}	heat generation, CHP (kW)
H_{EHP}	heat generation, EHP (kW)
H_h	heating system net heat transfer (kW)
I_c	cooking heat gain (kW)
I_{he}	energy flux from the heating element (kW)
I_m	metabolic heat gain (kW)
I_{se}	solar gain on external building elements (kW)
I_{si}	solar gain on internal building elements (through glazing) (kW)
I_v	heat exchange through ventilation (with external environment) (kW)

83	$\dot{m}_d$	appliance DHW consumption (kg/s)
84	$\dot{m}$	heating system water mass flow rate (kg/s)
85	$\eta_{FCHP}^{CHP,th}$	thermal efficiency of CHP
86	$\eta_{FCHP}^{CHP,e}$	electrical efficiency of CHP
87	η_{Fb}^b	efficiency of gas boiler
88	p_a	electrical appliance consumption (kW)
89	p_c	cooking appliance consumption (kW)
90	r_c	appliance radiation factor
91	$R_{c\&f}$	combined thermal resistance of ceiling and the floor (°C/kW)
92	R_{c-in}	thermal resistance of the natural heat transfer path between the core and indoor environment (°C/kW)
93		
94	R_{c-if}	thermal resistance of the forced heat transfer path between the core and indoor environment (°C/kW)
95		
96	R'_{ewi}	half of the thermal resistance of the inner portion of the external wall (°C/kW)
97	R'_{ewo}	half of the thermal resistance of the outer portion of the external wall (°C/kW)
98	R_{f-i}	thermal resistance between the floor and indoor environment (°C/kW)
99	$R_{g\&d}$	combined thermal resistance of glazing and doors (°C/kW)
100	R_{he-i}	thermal resistance between heat emitter and indoor environment (°C/kW)
101	R'_{iw}	half of the thermal resistance of the internal wall (°C/kW)
102	R_{p-f}	thermal resistance between the pipe and the floor (°C/kW)
103	R_{se}	external surface resistance (°C/kW)
104	R_{siew}	internal surface thermal resistance-external wall (°C/kW)
105	R_{siiw}	internal surface thermal resistance-internal wall (°C/kW)
106	S_{on-off}^e	external switch
107	S_{on-off}^i	internal switch
108	$T_{delivery}$	delivery temperature of DHW (°C)
109	T_{flow}	heating system flow temperature (°C)
110	T_{flow}^{max}	maximum flow temperature of heating system water (°C)
111	T_e	external temperature evolution (°C)
112	T_{intake}	dwelling water intake temperature (°C)
113	T_{return}	heating system return temperature (°C)
114	T_s	heat source temperature (°C)
115	T_w	heating system water temperature (°C)
116	$u_{a,t}$	operating state of electrical appliance
117	$u_{c,t}$	operating state of cooking appliance
118	$u_{d,t}$	operating state of DHW appliance
119	x_c	temperature of storage heater core node (°C)
120	x_{ewo}	temperature of outer portion of the external wall (°C)
121	x_{ewi}	temperature of inner portion of the external wall (°C)
122	x_f	temperature of floor node (°C)
123	x_{he}	temperature of heat emitter node (°C)
124	x_i	temperature of indoor environment (including furnishings) (°C)
125	x_{iw}	temperature of the internal wall (°C)
126	x_p	temperature of the pipework node (°C)

1 Introduction

Climate change, increasing energy costs and concerns regarding energy security are encouraging interest in new low-carbon energy technologies. In this context, besides the efforts to decarbonise electricity through renewable electricity sources, more and more emphasis is recently being put onto decarbonisation of the heating sector, through the shifting of primary energy usage away from fossil fuel based heating systems or through more efficient use of such fossil resources. The domestic sector, in particular, is seen as one of the main targets for heating sector decarbonisation, particularly in the UK where domestic heating accounts for 57% of heat usage [1]. Various strategies have been proposed for decarbonisation of heating [2], and all of them somehow involve a closer interaction of electricity and heat. Hence, it is expected that household-level electro-thermal technologies (ETT - as any technology which links electricity and heat may be termed) will be increasingly important components of the low carbon energy technology mixes of the future. ETT that are likely to become more and more relevant include electric heat pumps (EHP), including air source, water source and ground source versions (ASHP, WSHP and GSHP, respectively), and combined heat and power (CHP) units. Additionally, electric boilers and electric storage heater based heating systems and, with regards to process heat, electric cooking may, given the expected substantial decarbonisation of the electricity grid, also become more pertinent.

Increased penetration of such ETT may have significant implications for the relevant energy businesses and networks. In particular, significant changes are expected to happen for both the gas (currently most of domestic heat is produced in the UK through gas boilers) and the electricity sectors from multiple perspectives of the businesses, for instance in terms of the volume of gas and electricity to be traded in long term and short term energy markets. In addition, relevant to “fast” electricity balancing markets (with a rolling horizon of 30 minutes or even shorter in some cases), whose importance is rising with the penetration of variable renewable sources in the system, ETT are being more and more investigated in terms of potential to provide Demand Response (DR) [3]–[5], particularly thanks to the intrinsic thermal storage that is often available in buildings (in “physical” stores such as hot water tanks or “virtual” stores such as building fabric). DR modelling and changes in energy market engender a need for modelling energy demand profiles when different ETT are applied in much greater detail than done so far. In addition, this more detailed modelling will also be more and more relevant for local distribution network impact and in particular for the electrical network [6]. In fact, ETT units may be characterized by substantial electrical power rating (whether for generation, as for CHP, or consumption, as for EHP, for instance) and are prone to coincidence across dwellings, which could cause substantial changes in the electrical network loading (increase in the case of EHP and decrease in the case of CHP, potentially). On the other hand, the gas network will also be affected in a somehow complementary way (loading increase in the case of CHP and decrease in the case of EHP).

All the potential issues mentioned above that might arise with different types of ETT need to be quantified with a suitable time resolution and a powerful methodology, being capable to model and quantify not only energy consumption in individual dwellings, but also across a range of them, particularly for network impact purposes besides market trading aspects. Hence, there is a clear scope to develop a new domestic multi-energy model with several key features that, to the authors’ knowledge, are not found together in any previously presented model. More specifically, key features of the proposed model are as follows:

1. Separation of multi-energy services from multi-energy inputs (with respect to the building envelope, which means modelling of inputs from relevant networks and markets) – To model the effect of adoption of various ETT on energy input flows, it is necessary to understand the demand for the energy service that the ETT is to serve. The creation of a clear separation between service and inputs (in a multi-energy context) creates clarity of dwelling energy demands and consumptions and enables the effect of switching energy services between input energy vectors to be studied. Such distinctions have been employed at the city and country level, at annual resolution for the purposes of policy level study [7], [8] but not at a resolution useful to deal with the above challenges, distribution network

and “fast” market analysis in particular, as also mentioned below. In addition, the proposed model automatically captures the correlation between electricity and heat consumption while taking into account the activity profiles, while other models usually focus on electricity or heat consumption only [9]–[11]

2. Accurate representation of system energy storage – Representation of thermal storage associated with ETT is crucial as the inertia created by such storage is likely to be material when considering thermal demand at high resolution and this feature is critical for DR modelling purposes. Whilst thermal storage in the form of hot water tanks [4], [12] and in building fabric [13]–[16] has previously been considered, there is no approach which considers the possibly considerable impact of heat storage in heat emitters (together with heat storage in hot water tanks, building fabric, buffer tanks and the interactions of such components).
3. Representation of diversity – For network impact, new energy market, and DR studies, accurate modelling of diversity of dwellings (including behavioural aspects, which can have significant effect [17]) is necessary, as the diversity of the energy demand of an aggregation of dwellings can have significant impact on network peak demands [18]. Additionally such understanding may be useful for aggregators seeking a diverse aggregation to mitigate the challenge of demand forecasting. Thus the ability to model several dwellings, and correctly capture the effect of diversity and, in case, coincidence, is necessary, particularly when considering a multi-energy view (for instance, electricity and heat may have different diversity and peak characteristics as resulting from different drivers, which need to be opportunely modelled). Capturing at the requisite detail, for multiple energy vectors, both diversity across dwellings and coincidence within dwellings is difficult given amount of information and the size of the modelling task. Previous approaches either focus on creating large scale datasets using distributions of appliance ownership and use, for electricity only [19], [20] or for multiple energy vectors [7], which capture diversity but, due to the problem size, not coincidence of demand within dwellings; or “archetype” dwellings are created, such as in [21] (electricity only) and [22] (multiple energy vectors), which reduce the number of individual dwellings, allowing computational power to be utilised in modelling coincidence of energy demands/appliances rather than in modelling a large number of dwellings. As opposed to the above works this work captures both diversity and coincidence, for multiple energy vectors. In addition, the proposed model has intrinsic flexibility to implement different control strategies and quantify the outcomes in a diversified fashion and at a very high time resolution for different buildings, which is again crucial to estimate collective building behaviour in network impact and demand response studies.
4. High granularity – A model which is capable of operating at one minute time steps may be considered necessary for network impact studies [6] and provision of DR services and “fast” market interaction [23], given the potentially large effect of model resolution on results [24]. Whilst there does exist a comprehensive electricity model at this resolution [20] this does not separate energy service from input energy vectors, and thus does not facilitate a multi-energy perspective, and is thus inadequate, in particular, for the study of ETT penetration and in general to assess the energy system changes due to alternative ETT options.

Whilst some efforts have been made to develop models which address some of these key features [9]–[11], [20], [25]–[29], these models fail to qualify as truly multi-energy as they are either technology specific with respect to the ETT considered, for example considering only heat pumps [10], [25] or only CHP plant [9], [26], [27], do not include all energy demands (e.g. cooking) [9], [10], [25]–[27], or fail to consider all possible energy inputs [11], [20]. As mentioned above, the ability to easily and realistically represent diversity and coincidence of various energy demands across many dwellings, if required, is another key advantage with respect to established building simulation tools, such as for instance [28], [29]. In fact, such tools are able to produce relevant demand profiles for a single building for appliances that affect the thermal balance of a building; however, these tools cannot easily capture diversity/coincidence of energy demands across and within dwellings, nor generate demand and fuel profiles for multiple dwellings (in fact such tasks would be very

laborious in such tools) or generate demand profiles for electrical appliances that do not affect the thermal balance of a building. Conversely the presented model is designed to realistically model diversity and coincidence of various energy demand profiles (including electrical appliances) within and across many dwellings, with ease.

On the above premises, this paper presents a novel high-resolution domestic multi-energy model that can be adopted to deal with the aforementioned challenges in the context of sustainable and “smarter” energy systems which feature high penetration of distributed energy resources. More specifically, in Section 2 we detail a series of interacting domestic energy use models which address the four significant energy uses at domestic level (DHW, space heating, cooking, and electrical appliances, the latter also including lighting). These models produce profiles of end use energy demand (thermal energy in the case of space heating, DHW and cooking; electrical for electrical appliances) which can then be mapped to the appropriate input energy vector dependent on the technology selection. Interactions and commonalities between the models (such as heat gain from cooking, which affect the demand for space heating, and occupant activity profiles, which drive demand for multiple energy services) are opportunely captured to ensure realistic coincidence of energy demands. In Section 3, some of the key features of the model (separation of energy services from input energy vector, representation of storage, representation of diversity, and high granularity) and their impacts are demonstrated. Section 4 contains the concluding remarks of the paper and bridges to future works.

2 Model description

The domestic energy model described in this paper is comprised of DHW, space heating, cooking and electrical appliance models. Dependent on the ETT used for the provision of the DHW, space heating and cooking energy services the energy profiles of the models will map to either gas¹ or electricity as the input energy vectors to the dwelling (see Figure 1).

Figure 1: Domestic energy models to energy profile mapping

The constituent models detailed above are described subsequently, with particular focus on the space heating model which is more complex given the heat storage in building and heating system components. All presented models share a common basis in active occupancy and occupant activity profiles, which describe when occupants are present and awake and when they are employed in various household activities (washing,

¹ It is important to highlight that here natural gas is assumed to be the input energy vectors in terms of fossil “fuel” and relevant network. However, the modelling approach is general enough to be easily adapted to other types of fuels such as, for instance, biofuels.

cooking, watching TV etc.). These profiles are generated following pseudo-random processes as required (allowing the capture of dwelling diversity). Additionally the space heating model and electrical appliance model share a common basis in solar irradiance profiles. These common bases are crucial as they ensure realistic representation of the coincidence in the energy demand profiles which characterise domestic energy use and can have a substantial impact on energy demand profiles at the levels of individual dwellings as well as aggregations.

2.1 DHW model

The fundamental bases of the DHW model, as described by Figure 2, are fourfold. The first basis is a set of DHW using appliances present within the dwelling, including bath and/or shower units. This list enables the mapping of DHW related activity profiles, which constitute the second basis, to DHW usage. These activity profiles describe, for the dwelling in question, all the activities of all occupants which are potentially DHW using (cooking, laundry, washing). Taking into account the list of appliances present these activities can then be mapped to appliance operation profiles, for each appliance. Subsequently the third basis, a set of parameters related to the specific (in terms of time) DHW consumption of each appliance, can be applied to translate appliance operation profiles (per appliance) to DHW volume profiles (per appliance). These are then aggregated to produce the dwelling level DHW volume profile. Finally the fourth basis, the temperature differential between water that is brought into the dwelling and the water that is delivered as DHW, is applied to translate the DHW volume profile to a DHW thermal demand profile.

Figure 2: DHW model

The DHW model can be represented, given the set of DHW consuming appliances (D), appliance operating profiles ($u_{d,t}$), appliance DHW consumption ($\dot{m}_d$) and the temperature of DHW delivery ($T_{delivery}$) and intake (T_{intake}), as shown in equation (1) below:

$$H_{DHW} = \sum_{d=1}^D (T_{delivery} - T_{intake}) c_p^{water} u_{d,t} \dot{m}_d \quad (1)$$

2.1.1 DHW model calibration

Calibration of the DHW model was performed using data from a study conducted by the UK Energy Savings Trust [30]. The study measured DHW consumption by volume at 112 properties. To create a comparable dataset for the calibration 100 DHW profiles were created using activity profiles and DHW appliance presence statistics from the CREST model [20] (with the distribution of occupant number per dwelling set according to [31]), appliance DHW consumption statistics from [32] and intake water and DHW delivery temperatures from

[30]. The DHW model average daily consumption per dwelling was calibrated by varying the bath/shower water volume, whilst the shape of the daily consumption profile was calibrated by varying the split of morning to evening showers/baths. The overall average daily consumption of the calibration data set (123 litres) compares very well with the equivalent measure from [30] (122 litres), as does the consumption distribution profile.

2.2 Space heating model

The space heating model is constituted of a building sub-model and a heating system sub-model, linked through the heating system heat emitter which transfers heat from the heating system to the building. The space heating model is structured and connected as shown in Figure 3.

Figure 3: Space heating model composition

The building model represents the building structure and the heat emitter (radiator, under-floor system, storage heater) as a series of nodes. Those nodes are characterised by their thermal capacity and the paths between them by their thermal resistance. Disturbances to the system, in the form of the temperature evolution of connected nodes (such as the external air node and heating system water node), solar radiation (applied to the building internal wall and building external wall) and internal gains (related to occupant metabolism, cooking and building ventilation) are also represented and applied. The building model can be described using the electrical analogue approach, through extension of this approach, which has been often applied in the field of building modelling [13], [14], [16], [33], [34], to include the heat emitter system [35].

The second sub-model of the space heating model is the heating system sub-model. This sub-model is an energy transfer model which describes the generation, storage and distribution of heat. For hydronic (water based) systems it is linked to the building model through the heat emitter node. The temperature of the heat emitter node is dictated by the energy transfer from the heating unit/buffer. For both hydronic and storage heater systems the heating system model describes the generation, storage and provision of DHW.

Below the building sub-model is described through detailing the component electrical analogues and their state-space equation systems. Subsequently the heating system sub-model is described through detailing the interaction of the system elements, the various possible heating unit models and the heating system controls.

2.2.1 Building model

The exact form of the building model varies dependent on the type of heat emitter (radiator, under-floor system or storage heater) and the state of the heat distribution system (on or off). The form varies by emitter type to accurately reflect the differing composition of the modelled heat emitters and the differing heat transfer paths. The form also varies by the state of the heat distribution system to reflect the dual role of heating system; as a medium for the distribution of heat when the heat distribution is active, and as additional thermal capacitance in the system when the heating system is inactive. Below (Figure 4) the building electrical analogue model is described and subsequently the various heat emitter electrical analogue models are also described. The heat emitter models are accompanied by the relevant sets of state-space equations which

formalise the relationship between the building system nodes and the relevant temperature evolutions and energy fluxes. The state-space equation systems are discretised using the Crank-Nicolson method [36], a numerically stable finite difference method.

The building model employed is made up of four nodes, which are characterised by node thermal capacitances. The paths between these nodes are characterised by their thermal resistances. There are two nodes for the outer wall (outer leaf and inner leaf), one for the indoor environment and one for the internal wall. This form is deemed appropriate as it allows the possibility to accurately represent buildings with cavity wall construction (given the two nodes for the outer wall) which are common in temperate climates, such as in England where 66% of domestic buildings have cavity wall construction [37]. The system is perturbed by the temperature evolution of the external environment (T_e) and by energy fluxes related to solar radiation (I_{se} , I_{si}), cooking (I_c), occupant metabolism (I_m) and ventilation (I_v). The building model is described in Figure 4.

Figure 4: Building model electrical analogue

The thermal resistance of the paths between the nodes are determined by the physical properties of the nodes (building components), as are the various thermal capacitances. The system disturbances, external temperature evolution and energy fluxes, are user defined inputs or, such as for metabolic heat gain, a function of a user defined input (in this case dwelling occupancy).

Given the building electrical analogue model the complete building model can be described fully by appending the appropriate heat emitter electrical analogue. As previously described the form of the heat emitter model varies by heat emitter and system state. Five options are possible. Each is described below with its associated state-space equation system.

2.2.1.1 Radiator systems

Heat transfer between the radiator and the indoor environment is modelled through defining the resistance of the heat transfer path (R_{he-i}). When the heating system is in an active state the temperature evolution of the heating system water (T_w) is dictated by the heating system energy transfer model (see 2.2.2). When the radiator heating system is not operating (i.e. the water of the heating system is not circulating and hence is not acting as the heat distribution medium) the heating system is considered inactive. In the inactive state the thermal capacity of the water of the emitter (C_{he}) becomes relevant (thermal capacity of the structure of the radiator, typically steel, is small and considered negligible). The active and inactive radiator sub-models are shown in Figure 5.

Figure 5: Active (left) and inactive (right) radiator electrical analogues

The building system described by the appending of either radiator sub-models (Figure 5) onto the building sub-model (Figure 4) can be formalised by a state-space equation system. The state-space equation system for the active radiator system are shown in (2), with the equivalent system for the inactive radiator system shown in (3). A , B , C and D are the matrices of the state-space systems. The non-trivial elements of matrices A , B , C and D can be found in Appendix A.

$$\frac{d}{dt} \begin{pmatrix} x_{ewo} \\ x_{ewi} \\ x_i \\ x_{iw} \end{pmatrix} = A \begin{pmatrix} x_{ewo} \\ x_{ewi} \\ x_i \\ x_{iw} \end{pmatrix} + B \begin{pmatrix} T_e \\ I_{se} \\ I_c + I_m - I_v \\ I_{si} \\ T_w \end{pmatrix} \quad (2)$$

$$\frac{d}{dt} \begin{pmatrix} x_{ewo} \\ x_{ewi} \\ x_i \\ x_{iw} \\ x_{he} \end{pmatrix} = C \begin{pmatrix} x_{ewo} \\ x_{ewi} \\ x_i \\ x_{iw} \\ x_{he} \end{pmatrix} + D \begin{pmatrix} T_e \\ I_{se} \\ I_c + I_m - I_v \\ I_{si} \end{pmatrix} \quad (3)$$

In (2) and (3) A and C dictate the relationship between each system node according to the heat transfer paths illustrated in Figure 4 and Figure 5. B and D define the effect of the external system disturbances, that is temperature evolutions and energy fluxes, on the system nodes. Note that, in the inactive state, the radiator heat emitter becomes another system node, interacting with other nodes as defined by C whilst the T_w temperature evolution does not exist.

2.2.1.2 Under-floor heating system

The design of under-floor heating systems necessitates a different model to that of radiator systems. Under-floor heating systems consist of pipework, carrying the heating system water, underneath a floor structure which acts as the heat emitter. The floor structure may have significant thermal capacity. This necessitates the structure shown in Figure 6, which specifies heat transfer paths between the pipework and the floor (R_{p-f}) and between the floor and the indoor environment (R_{f-i}). Also acknowledged is the thermal capacity of the floor node (C_f) and, when in the inactive state, the thermal capacity of the water in the pipework (C_p).

Figure 6: Active (left) and inactive (right) under-floor system electrical analogues

Again the state-space equations of the system are described (4) and (5) for the active and inactive states respectively. In comparison to (2) and (3) the equations have in addition the floor node (x_f). The non-trivial elements of matrices E , F , G and H are given in Appendix A.

$$\frac{d}{dt} \begin{pmatrix} x_{ewo} \\ x_{ewi} \\ x_i \\ x_{iw} \\ x_f \end{pmatrix} = E \begin{pmatrix} x_{ewo} \\ x_{ewi} \\ x_i \\ x_{iw} \\ x_f \end{pmatrix} + F \begin{pmatrix} T_e \\ I_{se} \\ I_c + I_m - I_v \\ I_{si} \\ T_w \end{pmatrix} \quad (4)$$

$$\frac{d}{dt} \begin{pmatrix} x_{ewo} \\ x_{ewi} \\ x_i \\ x_{iw} \\ x_f \\ x_p \end{pmatrix} = G \begin{pmatrix} x_{ewo} \\ x_{ewi} \\ x_i \\ x_{iw} \\ x_f \\ x_p \end{pmatrix} + H \begin{pmatrix} T_e \\ I_{se} \\ I_c + I_m - I_v \\ I_{si} \end{pmatrix} \quad (5)$$

2.2.1.3 Storage heater

The structure of the storage heater sub-model (Figure 7) varies from that of the hydronic systems described above as the system does not feature a heat distribution medium. Heat is generated in situ by an electric resistance heater. When operated (by switch S_{on-off}^i) the heater produces an energy flux which acts on the storage heater core, which has significant heat capacity due to the presence of ceramic bricks (for that purpose). The core stores heat which is transferred to the environment through natural heat loss and forced heat loss. Natural heat loss occurs at all times whilst forced heat loss occurs when enacted (by switch S_{on-off}^e) by forcing air over the core and into the indoor environment. The natural heat transfer is characterised by the thermal resistance of the storage heater envelope (R_{c-in}). The forced heat transfer is characterised an equivalent thermal resistance value (R_{c-if}). The thermal capacity of the store is characterised by C_c ; I_{he} represents the energy flux from the electric resistance heater. The state-space equations for the storage heater system are given in equation (6). Again the non-trivial elements of the coefficient matrices I and J are given in Appendix A.

Figure 7: Storage heater electrical analogue (based on [38])

$$\frac{d}{dt} \begin{pmatrix} x_{ewo} \\ x_{ewi} \\ x_i \\ x_{iw} \\ x_c \end{pmatrix} = I \begin{pmatrix} x_{ewo} \\ x_{ewi} \\ x_i \\ x_{iw} \\ x_c \end{pmatrix} + J \begin{pmatrix} T_e \\ I_{se} \\ I_c + I_m - I_v \\ I_{si} \\ I_{he} \end{pmatrix} \quad (6)$$

2.2.2 Heating system model

The heating system model is linked to the building model through the heat emitter for hydronic systems. In the general case the heating system consists of four components, linked together by flows of hot water and complementary return flows of cooled water. The components are: the heating unit(s) (constituted of a primary heater and possibly an auxiliary heater), the buffer tank, the emitter and the DHW tank (see Figure 8). The basic layout of the system is based on descriptions of systems which incorporate all the detailed elements [39]. For hydronic heating systems the system may reduce through removal of the buffer tank or DHW tank, which are not compulsory features. For storage heater based systems the heating system will reduce to a

maximum of two nodes (heating unit and DHW tank), and in case one node (heating unit) as space heating is modelled through the storage heater electrical analogue (Figure 7).

Figure 8: General heating system layout

The heating system model is based on net heat transfer between nodes, see Figure 8. Each transfer is based on water flow rate ($\dot{m}$), water flow temperature (T_{flow}), water return temperature (T_{return}) and the specific heat of water (c_p^{water}), see (7).

$$H_h = (T_{flow} - T_{return})c_p^{water}\dot{m} \quad (7)$$

2.2.2.1 Heating unit models

The heat transfers from the heating units are dependent on the rated heating power of the heating unit and the remaining heat capacity of the return heating system water. The model for each type of unit is defined below. The exception is the storage heater. Given the storage heater's role as a heat emitter and heat store as well as a heating unit it is modelled as part of the building model, see 2.2.1.3.

2.2.2.1.1 Gas boiler

For the gas boiler, heat generation is set according to the gas boiler gas consumption (F_b , set either to maximum or according to dwelling heating load if variable heat transfer is considered, see 2.2.2.2.5), the boiler efficiency ($\eta_{F_b}^b$, which varies by operating level) and the heat capacity of the heating system water. The heat capacity is determined by the specific heat capacity of the heating system water (c_p^{water}), the maximum flow temperature (T_{flow}^{max}), the return temperature (T_{return}) and the heating system flow rate ($\dot{m}$) (8). This definition ensures that the heating system water is not heated beyond its upper temperature limit.

$$H_b = \min\{F_b\eta_{F_b}^b, (T_{flow}^{max} - T_{return})c_p^{water}\dot{m}\} \quad (8)$$

2.2.2.1.2 Combined heat and power system

Heat generation by the CHP unit is defined similarly to that of the gas boiler (9). Electrical generation of the CHP is also defined in (9) given the defined CHP electrical efficiency ($\eta_{F_{CHP}}^{CHP,e}$, which again varies by unit operating level).

$$\left(\frac{H_{CHP}}{E_{CHP}}\right) = \left(\frac{\min\{F_{CHP}\eta_{F_{CHP}}^{CHP,th}, (T_{flow}^{max} - T_{return})c_p^{water}\dot{m}\}}{F_{CHP}\eta_{F_{CHP}}^{CHP,e}}\right) \quad (9)$$

Additionally there may be periods of unit start-up/shut-down of the CHP during which there is electrical consumption and no heat production.

2.2.2.1.3 Electric heat pump

Heat generation by an EHP (ASHP, GSHP or WSHP) is defined by the unit's relevant coefficient of performance (COP_{T_s, T_w} , which varies according to the source temperature, T_s , and heating system water temperature, T_w , according to a performance map that the user can input which should capture the changes in efficiency with the mentioned temperature characteristics, for instance based on information from EHP manufacturers), the EHP unit's consumption (F_{EHP} , again, set either to maximum or according to dwelling heating load, see 2.2.2.2.5) and the heat capacity of the heating system water, as shown in (10).

$$H_{EHP} = \min\{F_{EHP} COP_{T_s, T_w}, (T_{flow}^{max} - T_{return}) c_p^{water} \dot{m}\} \quad (10)$$

In addition if the source temperature of the EHP is lower than the activation requirement of defrost cycle, then the EHP will run at full power during defrost cycles, delivering no useful heat (as the heat is directed to defrosting the heat exchanger). Duration and frequency of defrost cycles are model inputs.

It should be noted that EHP starting inrush current [40], which can produce power spikes for several seconds, has not been considered here. The effects of such spikes are transient and can be considered an issue of power quality and as such out of scope for the present work.

2.2.2.2 Heating system controls

In the above described heating system models the heat transfers illustrated are subject to controls which dictate the system behaviour. For each energy transfer the relevant control is described below.

2.2.2.2.1 Heating unit(s)-DHW tank/buffer tank/heat emitter

Given the dual purpose of the heating unit(s) to supply heat to the DHW tank and the space heating system (buffer tank or heat emitter) there is necessarily a control to dictate how heat is supplied in the event of coincident demand (i.e. priority for DHW or priority for space heating).

2.2.2.2.2 Heating unit(s)-DHW tank

The charging of the DHW tank is subject to a number of threshold controls designed to ensure that heat is supplied at an acceptable temperature and that operating limits of the storage tank are not breached (see right of Figure 9). These threshold controls dictate that if the DHW tank falls below the 'primary unit on' threshold then the primary heating unit turns on until the 'primary unit off' temperature is reached. If, despite the primary unit being activated, the DHW tank falls below the 'auxiliary unit on' threshold then the auxiliary unit turns on until the 'auxiliary unit off' temperature is reached. This control system favours heat production by the primary unit whilst making additional heat available when the lower temperature limit (which relates to the minimum desirable DHW temperature) is approached.

2.2.2.2.3 DHW tank-DHW demand

Discharging of the DHW tank is subject to DHW demand, with heat delivered dependent on the DHW demand profile produced by the DHW demand model.

2.2.2.2.4 Heating unit(s)-buffer tank

The charging of the buffer tank is, like the DHW tank, subject to a number of threshold controls (again see right of Figure 9). Once again the primary heating unit turns on when the 'primary unit on' threshold is crossed, staying on until the 'primary unit off' threshold is reached. If, despite the primary unit being activated the temperature falls below the 'auxiliary unit on' threshold (dictated by the desired heat emitter temperature) then the auxiliary unit is also activated until the 'auxiliary unit off' threshold is reached.

Figure 9: DHW/buffer tank thresholds for control

2.2.2.2.5 Buffer tank/heating unit(s)-Heat emitter

The buffer tank/heating unit(s)-heat emitter controls are numerous and operate in a hierarchy, with each subsequent control only becoming relevant if the previous control is positive. All the controls which involve set-point controls are accompanied by a hysteresis value. This means that when the relevant system metric crosses the boundary of the hysteresis zone (for example set-point-(hysteresis value/2)) the control becomes positive until the opposing hysteresis zone boundary is crossed (set-point+(hysteresis value/2)). Those controls are:

1. Set-point control on the outdoor temperature². If the outdoor temperature falls below the lower bound the control is positive until the upper bound is surpassed.
2. Set-point control on the indoor temperature. If the indoor temperature falls below the lower bound the control is positive until the upper bound is surpassed.
3. Set-point control on the heat emitter temperature. If the heat emitter temperature falls below the lower bound the control is positive until the upper bound is surpassed.

In addition, if the heating unit is capable of part-load running (to reduce unit cycling) or if a buffer tank is present, a fourth control is relevant:

4. Variable heat transfer control: when the heat emitter temperature is at the set-point of control 3 above, the heating unit or buffer tank output is set to the emitter-indoor environment heat transfer of the last time step. This ensures a more consistent indoor temperature and, if no buffer tank is present, reduces heating unit cycling.

2.2.2.2.6 Storage heater system controls

For storage heater systems the controls which dictate the actuation of the internal and external switches (see Figure 7) must be described. The internal switch, which controls the storage heater charging, actuates according to a user defined profile. Typically storage heaters are charged during electricity system off-peak periods. The external switch is governed by the first two controls detailed in 2.2.2.2.5.

² Although the control may be considered unnecessary, as comfort can be ensured without such a control, evidence as shown the existence of such a set-point (15.5°C in the UK) [55]

2.2.3 Space heating model calibration

Space heating model calibration was conducted using comprehensive survey data on English and Welsh domestic gas usage from National Energy Efficiency Data (NEED) framework [41]. NEED data gives the average annual gas consumption (for properties with gas consumption between 100 and 50,000kWh) for a sample of 2.9 million properties (2011) in England and Wales, disaggregated by dwelling type age. Given the known predominance of gas boiler/radiator systems in the UK, the model (with gas boiler heating unit and radiator heat emitters) may be calibrated against this data by adjusting the most uncertain (due to the uncaptured possibility of insulation improvements) and influential parameters, the building resistance and ventilation parameters. Initial thermal resistance and ventilation, along with thermal capacitance values were taken from [42]. Other physical parameters were derived from technical brochures and practical literature (for heat emitter and storage parameters) [43]–[50]. For the gas boiler heating units the capacity parameters were set to 12/18/24/30kW_{thermal} for flats/terraced/semi-detached/detached dwellings respectively (to reflect a high degree of boiler oversizing which exists in reality [51]) and boiler efficiency was set to 75% (to reflect the average efficiency of the population of currently installed gas boilers [52]). Outdoor temperature and radiation profiles, for south and north England, from [53] were used for the calibration (calibration data was an average of north and south consumption, weighted by the regional split in [41]). The behavioural parameters required were the dwelling set temperatures and the dwelling number of occupants (necessary for DHW consumption and for generating active occupancy profiles). Values for these parameters were derived from [31], [54]. Additionally the assumption was made that heating is operated whenever there is an active occupant, as indicated in [54]. For the control parameters, the hysteresis values for the outdoor temperature, indoor temperature and heat emitter temperature set points were set to 1°C, 1°C and 14°C respectively.

The calibration resulted in a discrepancy between the model and NEED annual consumption figures (using seasonal representative days to estimate model annual consumption) of less than 5% for each dwelling type/age combination.

2.3 Cooking model

The cooking model is applicable to cooking appliances (hob/oven) which may switch between energy vectors (gas/electricity). As with the DHW and electrical appliance model, the cooking model has as a fundamental basis an occupant activity profile, as in [20]. This profile describes when a dwelling occupant is engaged in hob/oven cooking. This profile, together with cooking appliance type (whether an electric/gas hob/oven is present in the dwelling) and consumption parameters, generates a cooking energy profile. Given the material effect incidental heat gains from cooking can have on space heating demand the model also produces a cooking heat gain profile, for integration with the space heating model. This is achieved by applying a cooking radiation factor to the cooking energy profile. The architecture of the cooking model is described in Figure 10.

Figure 10: Cooking model

The two outputs of the cooking model can be represented mathematically by equations (11) and (12) below, which give the cooking thermal energy demand and the cooking incidental heat gain, dependent on the cooking appliance power (p_c), the appliance operating state ($u_{c,t}$) and the appliance radiation factor (r_c).

$$H_c = \sum_{c=1}^c p_c u_{c,t} \quad (11)$$

$$I_c = \sum_{c=1}^c p_c u_{c,t} r_c \quad (12)$$

2.4 Electrical appliance model

The electrical appliance model (defined here as including lighting) is based on that presented by Richardson et al. [20], with several adjustments in order for the electrical appliance model to serve to model only those appliances for which substitution by non-electrical (e.g. gas) powered appliances is not realistic. Hence the architecture of the electrical appliance model is altered from that presented in [20], given consideration of the ‘cooking’ activity profile in the above described cooking model. Additionally all electro-thermal appliances, such as storage heater, water heater, cooking hobs/oven (but with the exception of small cooking appliances such as microwave, kettle, toaster etc.) are excluded from the list of electricity using appliances, as their energy usage is captured by the above described models. The adjusted architecture is presented in Figure 11.

Figure 11: Electrical appliance model, derived from [20]

Mathematical representation of the electrical appliance model is as below, with the electrical appliance electricity demand as a function of the electrical appliance power (p_a) and the appliance operating state ($u_{a,t}$):

$$E_a = \sum_{a=1}^A p_a u_{a,t} \quad (13)$$

3 Demonstration of model features

The presented model has a number of features which offer important contributions to the field of domestic energy modelling which will be especially relevant for application in future low carbon energy systems. To demonstrate those features a number of tests have been undertaken. Firstly the model’s ability to separate the various energy services from the energy inputs which provide those services, as mentioned in the Introduction, is demonstrated. This shows the possibility of mapping one set of energy service demand profiles to numerous input energy vector consumption profiles for exploration of the effect of heating unit substitution when considering different ETT with respect to a “base case” gas boiler. Secondly the impact of the presence

and type of heat emitter on the system thermal inertia is studied, showing how increased heat emitter mass affects thermal behaviour of a dwelling's indoor environment. Thirdly the ability to represent the diversity of an aggregation of dwellings, and the importance of this feature, is demonstrated through a study on the effect of ETT adoption on the aggregation electricity consumption coincidence factor, an important metric for distribution network planning purposes. Lastly the effect of model granularity on model results is studied, showing the impact of model resolution on the characteristics of generated profiles.

3.1 Input energy vector/ETT analysis

Although it may be considered intuitive that the provided energy service is separate from the energy input used to power that service, the distinction between the two is often not clearly defined. This hampers attempts to understand the effect of energy input/ETT substitution. A valuable contribution of the presented model is the ability to map any energy service demand profile to the relevant input energy vector profile, dependent on the ETT modelled. To demonstrate these energy service demand profiles for DHW, electrical appliance and cooking profiles, along with an active occupancy profile (the main determinant of space heating demand) were generated for a three occupant dwelling (see Figure 12). Note that space heating demand cannot be simply represented as a generic profile (applicable to any dwelling type/scenario) as are the other energy service demands. This is because space heating demand is dependent on the temperature of building elements and the heat emitter, which are dynamic and coupled, as well as the outdoor temperature, solar radiation and building ventilation and thermal resistance characteristics, which makes space heating demand dependent on the considered dwelling/environmental scenario.

Figure 12: Energy service demand profiles and dwelling active occupancy

These demand and active occupancy profiles were then applied to the models of two dwelling types, semi-detached house and flat. The semi-detached dwelling has total floor space (over two floors) of 96m² and total window area of 17m², whilst the flat has floor space of 54m² (over 1 floor) and total window area of 12m². The semi-detached dwelling was insulated to the standard of 'modern' buildings (defined here as those built between 1944 and 1984) whilst the flat was insulated to standard of 'new' buildings (built after 1984). Each dwelling was modelled with a set temperature of 21°C during times of active occupancy (there is no set temperature at other times) and a 145 litre DHW tank. Environmental conditions (external temperatures and irradiance) were defined as for a typical winter day, as shown in Figure 13.

Figure 13: Outdoor temperature and irradiance for a typical winter day

Figure 14 shows the gas and electricity input demand if the space heating and DHW is fulfilled by a gas boiler³ (through the associated heating system which incorporates radiator heat emitters) and the cooking is fulfilled by gas. The relationship between electrical appliance demand and electricity input can clearly be seen as can the relationship between cooking demand and gas consumption. The large size of the gas boiler (typical in UK applications) results in frequent cycling, as the operation of the gas boiler is constrained by the heat transfer capability of the heat emitters. Note that the relationship between DHW demand and gas boiler operation cannot be discerned due to the disconnection between heat generation for DHW applications and DHW demand caused by the presence of a DHW tank.

Figure 14: Energy input demands for semi-detached (left) and flat (right) dwellings, for gas boiler/radiator/gas cooking options

If the space heating and DHW is instead fulfilled by a CHP unit (with gas boiler auxiliary heater)⁴ then the electricity and gas consumption will be as shown in Figure 15. The spikes in gas consumption relate to the gas

³ Efficiency of 75%

⁴ CHP thermal efficiency 78%, electrical efficiency 13%. Auxiliary gas boiler efficiency 90%

boiler operation (which is triggered in this case as the CHP unit takes some minutes to start producing heat). Again, inspection of Figure 15 shows additional gas consumption related to cooking demand. During times of CHP operation the electrical generation of the CHP can be seen to offset electrical appliance consumption or produce excess electricity when electrical appliance consumption is low.

Figure 15: Energy input demands for semi-detached (left) and flat (right) dwellings, CHP/radiator/gas cooking options

Switching space heating, DHW and cooking energy services to fulfilment by electricity results in elimination of dwelling gas consumption. **Error! Reference source not found.** and Figure 17 show the electrical consumption of the dwelling given such a fully electrical switch, with space heating and DHW supplied by ASHP⁵ (with auxiliary electric boiler, COP=1), for a radiator based and under-floor heating based system respectively. Electrical consumption can be seen to generally coincide with dwelling active occupancy as the ASHP operates to supply space heating. Exception to this coincidence can be seen in the radiator case as the ASHP cycles more as the set temperature is reached.

Figure 16: Energy input demands for semi-detached (left) and flat (right) dwellings, ASHP/radiator/electric cooking options

⁵ Reference COP values, taken from the a popular model available on the market [49]:

Source\Flow temperatures	35°C	55°C
0°C	3.05	1.93
20°C	4.60	3.43

Figure 17: Energy input demands for semi-detached (left) and flat (right) dwellings, ASHP/under-floor heating/electric cooking options

Figure 18 shows the electrical consumption of the dwelling given fulfilment of space heating through a storage heater, DHW through an electric boiler and cooking through electric cooking. As the storage heater charging window is between midnight and 7am there is significant load until just before 6am for the semi-detached house and just before 5am for the flat (when the storage heater has been charged enough to serve the days heat demand). The additional load just before 5am is due to the electric boiler heating up the DHW tank, as is the load starting at 12pm.

Figure 18: Energy input demands for semi-detached (left) and flat (right) dwellings, storage heater/electric cooking

In all cases the lower space heating demand of the flat, due to its smaller floor space and better insulation, can be inferred from the lower dwelling gas/electricity consumption of the flat example in each case. Further the energy input type, together with the heat generator and emitter type, has differing effects on the thermal behaviour of the dwelling. These effects are depicted in Figure 19. The primary differences can be seen to be between the different types of emitters. The storage heater heats the dwelling fastest. This is due to its size which is large compared to the size required to produce the necessary heat at that time. This is because the unit is sized to be able to maintain the required heat transfer at later times of the day when the core temperature is reduced. Also of note is slower rate of cooling of the storage heater heated dwellings. This is due to the natural uncontrolled emission of heat from the storage heater core. The radiator cases heat the dwellings the next quickest, with the under-floor system heating the slowest due to the lower temperature of that system (under-floor systems are primarily designed for constant or near constant operation, which would render the slow heat rise shown here irrelevant). Also of note is slower rate at which the under-floor heated dwellings cool, which is due to the larger thermal mass of the under-floor emitter (the pipework and the floor

slab), as also elaborated below.

Figure 19: Building and external temperatures, by case, for semi-detached cases (left) and flat cases (right)

3.2 Heat emitter thermal inertia impact

Another key feature of the model to highlight is the incorporation of the thermal mass of the heat emitters, which may be capable of storing significant amounts of heat in the water within the heat emitter and, in the case of under-floor heating, within the floor. The heat stored in the emitter is defined as in (14) for the radiator case and (15) for the under-floor heating case.

$$(x_{he} - x_i)C_{he} \quad (14)$$

$$[(x_p - x_i)C_p + (x_f - x_i)C_f] \quad (15)$$

Figure 20 demonstrates such heat storage, in the radiators and in the under-floor heating system⁶ (pipework and floor), for a modern semi-detached house on a typical winter day with a gas boiler with a target temperature of 21°C (with 1°C hysteresis) from 6am until 11am and from 4pm until 10pm.

Figure 20: Heat stored by heat emitter type, for a modern semi-detached house with gas boiler heating

From Figure 20 it is possible to see the significant amount of heat that can be stored in building heat emitters, particularly in the under-floor heating case. Of particular note is the decay in the amount of heat stored during heating unit cycling and after the heating unit is turned off. Understanding this characteristic is crucial to understand the thermal behaviour of a building. This is particularly salient when considering the impact of DR calls (given heating by an ETT), as the capability of the heat emitter to buffer the effect of the loss (or in the case of CHP, increase) of heating power becomes a key characteristic.

⁶ Defined as the temperature difference between the radiator and the indoor environment

To demonstrate further the effect of heat emitter inertia on indoor temperature, Figure 21 shows the indoor temperature for the above described cases, with the addition of a case in which there is no emitter (heat generated by the heating unit is injected directly in the indoor environment – this could for instance be the case with utilization of fan heater systems). Considering the various cases from around 9/10pm, a drop of approximately 1°C/hour can be observed for the no emitter case, 0.6°C/hour for the radiator case and 0.4°C/hour for the under-floor heating case. More markedly, considering the cases from 6am, the effect of the heat emitter in limiting the heat transfer from heating unit to indoor environment can be observed. For the no emitter case a rise of approximately 4.5°C/hour can be observed, with the equivalent numbers for the radiator and under-floor heating cases 0.9°C/hour and 0.7°C/hour respectively. This effect is due to the limited heat transfer capability of heat emitters (dictated by their size) and further demonstrates the importance of incorporating the heat emitter into space heating modelling.

Figure 21: Indoor temperature, by heat emitter type for a modern semi-detached house with gas boiler heating

3.3 Diversity study

Another key characteristic of the model developed is the ability to represent the energy consumption, in particular electricity consumption, diversity of an aggregation of multiple dwellings, for various applications such as in the field of distribution network impact studies or resource portfolio modelling. One of the primary metrics pertinent to distribution network impact studies is the coincidence factor⁷ (inverse of the diversity factor) of a group of dwellings, as this measure informs the capacity of the network needed to serve the dwellings. Naturally the coincidence factor can vary significantly given the varying penetration of ETT. Given a significant penetration of ETT the heat demand of the dwellings may also become relevant as better insulation reduces heat demand, thus reducing space heat related electricity demand or increasing electricity generation, dependent on technology. Varying amounts of thermal inertia in the system (for example through the presence of thermally massive under-floor heating, rather than a radiator system) may also impact the coincidence factor through their effect on space heating demand profiles.

To illustrate the impact the adoption of ETT may have on dwelling electricity consumption diversity Figure 22 show how the coincidence factor varies for aggregations of semi-detached dwellings, of various ages, heated by a ASHP unit with electric boiler back-up and radiators. Results are shown for an typical winter and extreme winter day (defined by shifting the typical winter outdoor temperature profile down so that the profile hits -5°C at its lowest point, irradiance remains the same). For each dwelling the ASHP unit is sized to be able to provide the thermal load at design conditions (21°C internal temperature, -2.2°C external temperature) and the electric boiler back-up is sized at 3kW. Active occupancy (it is assumed heating is only required when the occupants are present and active), DHW demand profiles (dependent upon the number of occupants, which is

⁷ Defined as the ratio of maximum demand of an aggregation to the sum of each building's maximum demand.

synthesised according to [31]) and base electricity demand are assigned randomly for each dwelling. Each dwelling has a DHW tank of 145 litres capacity and the cooking fuel is electricity. For comparison the equivalent coincidence factor plot is shown for dwellings heated by a 24kW gas boiler, with gas cooking (thus electrical consumption only from electrical appliances and lighting).

Figure 22 shows that when switching heating from a gas boiler to an ASHP the coincidence factor at 500 semi-detached dwellings increases from 0.13 to 0.29/0.28/0.25 for old/modern/new buildings for a typical winter, and to 0.32/0.28/0.25 for old/modern/new buildings for an extreme winter. The general increase in the coincidence factor is due to lengthy running times for the ASHP units which increase the times of high load across the aggregation, serving to increase aggregation peak demand more than individual peak demands. The exact change of electricity demand coincidence varies according to dwelling age. This is due to the effect of insulation level on unit running times. Although heating units are sized according to design heat demand, average unit run times increase as insulation level decreases given that incidental heating loads (which are not considered during unit sizing) have a proportionately larger effect on better insulated dwellings (given their smaller heat demand), reducing unit run times more than in less insulated dwellings. Hence the highest coincidence is for the old dwelling aggregation (and lowest for the new dwelling aggregation) and there is greater coincidence in the extreme winter case.

Figure 22: ASHP (electric boiler auxiliary)/radiator case electrical coincidence factor plot, typical winter day (left) and extreme winter day (right) for semi-detached dwellings of various ages, with a gas boiler case for comparison

As well as the coincidence factor metric, which is primarily of use to network operators, another useful metric may be the number of dwellings at which a certain level of coincidence is achieved. This measure (taken in this work at the 98th percentile of the difference between the maximum and asymptotic minimum coincidence) may be useful for potential demand side aggregators. For the cases shown here this measure is 98 dwellings, for all insulation levels, in the typical winter case and 69 in the extreme winter case. This compares to 55 in the gas boiler case. This reduction in the dwelling number metric, between average and extreme winter cases, is a result of dwelling electricity profiles becoming more similar as the heating load increases, a phenomenon which will occur for electricity producing as well as electricity consuming ETT.

3.4 Granularity study

The high resolution of the model represents another principal contribution of this work. The possibility of operating at one minute resolution is a crucial feature for distribution network impact studies given the extremely peaky nature of electrical consumption related to many electrical appliances (such as kettles, microwaves, electric cookers etc.) [18] and, if present, electric boilers. Additionally the resolution is important for the management of the whole electricity system given the need to balance an electricity system at every moment and in the light of increasing balancing requirements due to variable generation, including local renewable generation such as photovoltaic. To demonstrate the effect of the varying the model resolution Figure 23 shows plots of average electricity demand profiles for an aggregation of 2200 modern semi-detached dwellings heated by an ASHP with radiators on a typical winter day at varying resolutions. The plots demonstrate, in this case, little difference between resolutions of 1-15 minutes. However there are significant

discrepancies between the smaller and larger resolutions (30-60 minutes) of up to 0.5kW which may, depending on the context, be significant. Of particular note is the discrepancy in the measure of peak demand. The 60 minute resolution plot shows the 7am peak at around 3kW whilst the 1 minute resolution plot shows the peak to be about 3.3kW, a difference which results in the 60 minute plot depicting the daily peak as lying in the evening rather than in the morning. Such a discrepancy may be significant for distribution network impact studies, including for voltage regulation issues [6].

Figure 23: Electrical demand profile for a semi-detached dwelling heated by an ASHP with radiators on a typical winter day, at various resolutions

4 Conclusion

This paper describes a modelling framework that is capable of producing profiles for the four main domestic energy uses (DHW, space heating, cooking and electrical appliances) and mapping them to dwelling input energy vector (electricity and gas) profiles. The principal contributions of the framework are: the separation of energy services from input energy vector, which allows: investigation of the effect of input energy and ETT substitution; accurate representation of energy storage (in particular with relation to heat emitters), enabled by the physical basis of the space heating model which allows realistic representation of the effect of thermal inertia on dwelling thermal behaviour; representation of diversity, enabled by the avoidance of onerous complexity; and high granularity, which ensures adequate representation of dwelling power demand, especially relevant for distribution network studies.

Tests have been conducted to demonstrate these contributions. These tests have highlighted some specific results including: mapping of energy service profiles to dwelling energy demand profiles dependent on ETT (EHP, CHP, storage heater) selection, the material effect of heat emitter thermal inertia on building thermal behaviour (with significant variation dependent on heat emitter type), the variability of electrical load coincidence of dwellings with EHP units dependent on dwelling age (although these results are sensitive to the specific control parameters used), and the propensity of lower resolution modelling to introduce discrepancies when compared to high resolution profiles and potentially underestimate and incorrectly locate the peak demand of an aggregation. However it should be noted that these are just a sample of the possible ways the model may be utilised; the parametric nature of the model enables further studies in multiple directions, opening the possibility for the presented model to produce valuable insights into many areas of multi-energy system studies.

Acknowledgements

N. Good would like to thank E.ON New Build & Technology for the sponsorship of his PhD studies.

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857 Appendix A

$$\begin{aligned}
 858 \quad a_{11} &= \frac{-1}{C_{ewo}} \left(\frac{1}{R_{se} + R'_{ewo}} + \frac{1}{R'_{ewo} + R'_{ewi}} \right) \\
 859 \quad a_{12} &= \frac{1}{C_{ewo}} \left(\frac{1}{R'_{ewo} + R'_{ewi}} \right) \\
 860 \quad a_{21} &= \frac{1}{C_{ewi}} \left(\frac{1}{R'_{ewo} + R'_{ewi}} \right) \\
 861 \quad a_{22} &= \frac{-1}{C_{ewi}} \left(\frac{1}{R'_{ewo} + R'_{ewi}} + \frac{1}{R'_{ewi} + R_{siew}} \right) \\
 862 \quad a_{23} &= \frac{1}{C_{ewi}} \left(\frac{1}{R'_{ewi} + R_{siew}} \right) \\
 863 \quad a_{32} &= \frac{1}{C_i} \left(\frac{1}{R_{siew} + R'_{ewi}} \right) \\
 864 \quad a_{33} &= \frac{-1}{C_i} \left(\frac{1}{R_{siew} + R'_{ewi}} + \frac{1}{R_{siiw} + R'_{iw}} + \frac{1}{R_{g\&d}} + \frac{1}{R_{c\&f}} + \frac{1}{R_{he-i}} \right) \\
 865 \quad a_{34} &= \frac{1}{C_i} \left(\frac{1}{R_{siiw} + R'_{iw}} \right) \\
 866 \quad a_{43} &= \frac{1}{C_{iw}} \left(\frac{1}{R_{siiw} + R'_{iw}} \right) \\
 867 \quad a_{44} &= \frac{-1}{C_{iw}} \left(\frac{1}{R'_{iw} + R_{siiw}} \right) \\
 868 \quad b_{11} &= \frac{1}{C_{ewo}} \left(\frac{1}{R_{se} + R'_{ewo}} \right) \\
 869 \quad b_{12} &= \frac{1}{C_{ewo}} \left(\frac{R_{se}}{R_{se} + R'_{ewo}} \right) \\
 870 \quad b_{31} &= \frac{1}{C_i} \left(\frac{1}{R_{g\&d}} + \frac{1}{R_{c\&f}} \right) \\
 871 \quad b_{33} &= \frac{1}{C_i} \\
 872 \quad b_{34} &= \frac{1}{C_i} \left(\frac{R'_{iw}}{R'_{iw} + R_{siiw}} \right) \\
 873 \quad b_{35} &= \frac{1}{C_i} \left(\frac{1}{R_{he-i}} \right) \\
 874 \quad b_{44} &= \frac{1}{C_{iw}} \left(\frac{R_{siiw}}{R'_{iw} + R_{siiw}} \right) \\
 875 \quad c_{11} &= \frac{-1}{C_{ewo}} \left(\frac{1}{R_{se} + R'_{ewo}} + \frac{1}{R'_{ewo} + R'_{ewi}} \right) \\
 876 \quad c_{12} &= \frac{1}{C_{ewo}} \left(\frac{1}{R'_{ewo} + R'_{ewi}} \right) \\
 877 \quad c_{21} &= \frac{1}{C_{ewi}} \left(\frac{1}{R'_{ewo} + R'_{ewi}} \right) \\
 878 \quad c_{22} &= \frac{-1}{C_{ewi}} \left(\frac{1}{R'_{ewo} + R'_{ewi}} + \frac{1}{R'_{ewi} + R_{siew}} \right) \\
 879 \quad c_{23} &= \frac{1}{C_{ewi}} \left(\frac{1}{R'_{ewi} + R_{siew}} \right) \\
 880 \quad c_{32} &= \frac{1}{C_i} \left(\frac{1}{R_{siew} + R'_{ewi}} \right) \\
 881 \quad c_{33} &= \frac{-1}{C_i} \left(\frac{1}{R_{siew} + R'_{ewi}} + \frac{1}{R_{siiw} + R'_{iw}} + \frac{1}{R_{g\&d}} + \frac{1}{R_{c\&f}} + \frac{1}{R_{he-i}} \right) \\
 882 \quad c_{34} &= \frac{1}{C_i} \left(\frac{1}{R_{siiw} + R'_{iw}} \right) \\
 883 \quad c_{35} &= \frac{1}{C_i} \left(\frac{1}{R_{he-i}} \right) \\
 884 \quad c_{43} &= \frac{1}{C_{iw}} \left(\frac{1}{R_{siiw} + R'_{iw}} \right) \\
 885 \quad c_{44} &= \frac{-1}{C_{iw}} \left(\frac{1}{R'_{iw} + R_{siiw}} \right) \\
 886 \quad c_{53} &= \frac{1}{C_{he}} \left(\frac{1}{R_{he-i}} \right) \\
 887 \quad c_{55} &= \frac{-1}{C_{he}} \left(\frac{1}{R_{he-i}} \right)
 \end{aligned}$$

$$\begin{aligned}
888 \quad d_{11} &= \frac{1}{C_{ewo}} \left(\frac{1}{R_{se} + R'_{ewo}} \right) \\
889 \quad d_{12} &= \frac{1}{C_{ewo}} \left(\frac{R_{se}}{R_{se} + R'_{ewo}} \right) \\
890 \quad d_{31} &= \frac{1}{C_i} \left(\frac{1}{R_{g\&d}} + \frac{1}{R_{c\&f}} \right) \\
891 \quad d_{33} &= \frac{1}{C_i} \\
892 \quad d_{34} &= \frac{1}{C_i} \left(\frac{R'_{iw}}{R'_{iw} + R_{siiw}} \right) \\
893 \quad d_{44} &= \frac{1}{C_{iw}} \left(\frac{R_{siiw}}{R'_{iw} + R_{siiw}} \right) \\
894 \quad e_{11} &= \frac{-1}{C_{ewo}} \left(\frac{1}{R_{se} + R'_{ewo}} + \frac{1}{R'_{ewo} + R'_{ewi}} \right) \\
895 \quad e_{12} &= \frac{1}{C_{ewo}} \left(\frac{1}{R'_{ewo} + R'_{ewi}} \right) \\
896 \quad e_{21} &= \frac{1}{C_{ewi}} \left(\frac{1}{R'_{ewo} + R'_{ewi}} \right) \\
897 \quad e_{22} &= \frac{-1}{C_{ewi}} \left(\frac{1}{R'_{ewo} + R'_{ewi}} + \frac{1}{R'_{ewi} + R_{siew}} \right) \\
898 \quad e_{23} &= \frac{1}{C_{ewi}} \left(\frac{1}{R'_{ewi} + R_{siew}} \right) \\
899 \quad e_{32} &= \frac{1}{C_i} \left(\frac{1}{R_{siew} + R'_{ewi}} \right) \\
900 \quad e_{33} &= \frac{-1}{C_i} \left(\frac{1}{R_{siew} + R'_{ewi}} + \frac{1}{R_{siiw} + R'_{iw}} + \frac{1}{R_{g\&d}} + \frac{1}{R_{c\&f}} + \frac{1}{R_{f-i}} \right) \\
901 \quad e_{34} &= \frac{1}{C_i} \left(\frac{1}{R_{siiw} + R'_{iw}} \right) \\
902 \quad e_{35} &= \frac{1}{C_i} \left(\frac{1}{R_{f-i}} \right) \\
903 \quad e_{43} &= \frac{1}{C_{iw}} \left(\frac{1}{R_{siiw} + R'_{iw}} \right) \\
904 \quad e_{44} &= \frac{-1}{C_{iw}} \left(\frac{1}{R'_{iw} + R_{siiw}} \right) \\
905 \quad e_{53} &= \frac{1}{C_f} \left(\frac{1}{R_{f-i}} \right) \\
906 \quad e_{55} &= \frac{-1}{C_f} \left(\frac{1}{R_{f-i}} + \frac{1}{R_{p-f}} \right) \\
907 \quad f_{11} &= \frac{1}{C_{ewo}} \left(\frac{1}{R_{se} + R'_{ewo}} \right) \\
908 \quad f_{12} &= \frac{1}{C_{ewo}} \left(\frac{R_{se}}{R_{se} + R'_{ewo}} \right) \\
909 \quad f_{31} &= \frac{1}{C_i} \left(\frac{1}{R_{g\&d}} + \frac{1}{R_{c\&f}} \right) \\
910 \quad f_{33} &= \frac{1}{C_i} \\
911 \quad f_{34} &= \frac{1}{C_i} \left(\frac{R'_{iw}}{R'_{iw} + R_{siiw}} \right) \\
912 \quad f_{44} &= \frac{1}{C_{iw}} \left(\frac{R_{siiw}}{R'_{iw} + R_{siiw}} \right) \\
913 \quad f_{55} &= \frac{1}{C_f} \left(\frac{1}{R_{p-f}} \right) \\
914 \quad g_{11} &= \frac{-1}{C_{ewo}} \left(\frac{1}{R_{se} + R'_{ewo}} + \frac{1}{R'_{ewo} + R'_{ewi}} \right) \\
915 \quad g_{12} &= \frac{1}{C_{ewo}} \left(\frac{1}{R'_{ewo} + R'_{ewi}} \right) \\
916 \quad g_{21} &= \frac{1}{C_{ewi}} \left(\frac{1}{R'_{ewo} + R'_{ewi}} \right) \\
917 \quad g_{22} &= \frac{-1}{C_{ewi}} \left(\frac{1}{R'_{ewo} + R'_{ewi}} + \frac{1}{R'_{ewi} + R_{siew}} \right) \\
918 \quad g_{23} &= \frac{1}{C_{ewi}} \left(\frac{1}{R'_{ewi} + R_{siew}} \right) \\
919 \quad g_{32} &= \frac{1}{C_i} \left(\frac{1}{R_{siew} + R'_{ewi}} \right)
\end{aligned}$$

$$\begin{aligned}
920 \quad g_{33} &= \frac{-1}{C_i} \left(\frac{1}{R_{siew} + R'_{ewi}} + \frac{1}{R_{siiw} + R'_{iww}} + \frac{1}{R_{g\&d}} + \frac{1}{R_{c\&f}} + \frac{1}{R_{f-i}} \right) \\
921 \quad g_{34} &= \frac{1}{C_i} \left(\frac{1}{R_{siiw} + R'_{iww}} \right) \\
922 \quad g_{35} &= \frac{1}{C_i} \left(\frac{1}{R_{f-i}} \right) \\
923 \quad g_{43} &= \frac{1}{C_{iw}} \left(\frac{1}{R_{siiw} + R'_{iww}} \right) \\
924 \quad g_{44} &= \frac{-1}{C_{iw}} \left(\frac{1}{R'_{iww} + R_{siiw}} \right) \\
925 \quad g_{53} &= \frac{1}{C_f} \left(\frac{1}{R_{f-i}} \right) \\
926 \quad g_{55} &= \frac{-1}{C_f} \left(\frac{1}{R_{f-i}} + \frac{1}{R_{p-f}} \right) \\
927 \quad g_{56} &= \frac{1}{C_f} \left(\frac{1}{R_{p-f}} \right) \\
928 \quad g_{65} &= \frac{1}{C_p} \left(\frac{1}{R_{p-f}} \right) \\
929 \quad g_{66} &= \frac{-1}{C_p} \left(\frac{1}{R_{p-f}} \right) \\
930 \quad h_{11} &= \frac{1}{C_{ewo}} \left(\frac{1}{R_{se} + R'_{ewo}} \right) \\
931 \quad h_{12} &= \frac{1}{C_{ewo}} \left(\frac{R_{se}}{R_{se} + R'_{ewo}} \right) \\
932 \quad h_{31} &= \frac{1}{C_i} \left(\frac{1}{R_{g\&d}} + \frac{1}{R_{c\&f}} \right) \\
933 \quad h_{33} &= \frac{1}{C_i} \\
934 \quad h_{34} &= \frac{1}{C_i} \left(\frac{R'_{iww}}{R'_{iww} + R_{siiw}} \right) \\
935 \quad h_{44} &= \frac{1}{C_{iw}} \left(\frac{R_{siiw}}{R'_{iww} + R_{siiw}} \right) \\
936 \quad i_{11} &= \frac{-1}{C_{ewo}} \left(\frac{1}{R_{se} + R'_{ewo}} + \frac{1}{R'_{ewo} + R'_{ewi}} \right) \\
937 \quad i_{12} &= \frac{1}{C_{ewo}} \left(\frac{1}{R'_{ewo} + R'_{ewi}} \right) \\
938 \quad i_{21} &= \frac{1}{C_{ewi}} \left(\frac{1}{R'_{ewo} + R'_{ewi}} \right) \\
939 \quad i_{22} &= \frac{-1}{C_{ewi}} \left(\frac{1}{R'_{ewo} + R'_{ewi}} + \frac{1}{R'_{ewi} + R_{siew}} \right) \\
940 \quad i_{23} &= \frac{1}{C_{ewi}} \left(\frac{1}{R'_{ewi} + R_{siew}} \right) \\
941 \quad i_{32} &= \frac{1}{C_i} \left(\frac{1}{R_{siew} + R'_{ewi}} \right) \\
942 \quad i_{33} &= \frac{-1}{C_i} \left(\frac{1}{R_{siew} + R'_{ewi}} + \frac{1}{R_{siiw} + R'_{iww}} + \frac{1}{R_{g\&d}} + \frac{1}{R_{c\&f}} + \frac{1}{R_{c-in}} + E_{on-off} \left(\frac{1}{R_{c-if}} \right) \right) \\
943 \quad i_{34} &= \frac{1}{C_i} \left(\frac{1}{R_{siiw} + R'_{iww}} \right) \\
944 \quad i_{35} &= \frac{1}{C_i} \left(\frac{1}{R_{c-in}} + S_{on-off}^e \left(\frac{1}{R_{c-if}} \right) \right) \\
945 \quad i_{43} &= \frac{1}{C_{iw}} \left(\frac{1}{R_{siiw} + R'_{iww}} \right) \\
946 \quad i_{44} &= \frac{-1}{C_{iw}} \left(\frac{1}{R'_{iww} + R_{siiw}} \right) \\
947 \quad i_{53} &= \frac{1}{C_c} \left(\frac{1}{R_{c-in}} + S_{on-off}^e \left(\frac{1}{R_{c-if}} \right) \right) \\
948 \quad i_{55} &= \frac{-1}{C_c} \left(\frac{1}{R_{c-in}} + S_{on-off}^e \left(\frac{1}{R_{c-if}} \right) \right) \\
949 \quad j_{11} &= \frac{1}{C_{ewo}} \left(\frac{1}{R_{se} + R'_{ewo}} \right)
\end{aligned}$$

$$\begin{aligned}
950 \quad j_{12} &= \frac{1}{C_{ewo}} \left(\frac{R_{se}}{R_{se} + R_{tewo}} \right) \\
951 \quad j_{31} &= \frac{1}{C_i} \left(\frac{1}{R_{g\&d}} + \frac{1}{R_{c\&f}} \right) \\
952 \quad j_{33} &= \frac{1}{C_i} \\
953 \quad j_{34} &= \frac{1}{C_i} \left(\frac{R'_{iw}}{R'_{iw} + R_{siiw}} \right) \\
954 \quad j_{44} &= \frac{1}{C_{iw}} \left(\frac{R_{siiw}}{R'_{iw} + R_{siiw}} \right) \\
955 \quad j_{55} &= S_{on-off}^i \left(\frac{1}{C_c} \right)
\end{aligned}$$

DRAFT