Multi-Objective Vehicle Routing Problem: Case studies in retail distribution

A thesis submitted to The University of Manchester for the degree of Doctor of Philosophy in the Faculty of Humanities

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# Abbreviations

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<tr>
<th>Abbreviation</th>
<th>Full Form</th>
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<tbody>
<tr>
<td>3PL</td>
<td>Third Party Logistic</td>
</tr>
<tr>
<td>ACO</td>
<td>Ant Colony Optimization</td>
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<tr>
<td>AHP</td>
<td>Analytic Hierarchy Process</td>
</tr>
<tr>
<td>ALNS</td>
<td>Adaptive Large neighbourhood Search</td>
</tr>
<tr>
<td>CO2</td>
<td>Carbon Dioxide</td>
</tr>
<tr>
<td>CSR</td>
<td>Corporate Social Responsibility</td>
</tr>
<tr>
<td>CTP</td>
<td>Covering Tour Problem</td>
</tr>
<tr>
<td>DARP</td>
<td>Dial-a-Ride Problem</td>
</tr>
<tr>
<td>DC</td>
<td>Distribution Centre</td>
</tr>
<tr>
<td>DM</td>
<td>Decision Maker</td>
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<tr>
<td>EA</td>
<td>Evolutionary Algorithm</td>
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<td>FC</td>
<td>Fuel Consumed</td>
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<td>FTA</td>
<td>Freight Transportation Association</td>
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<tr>
<td>FTL</td>
<td>Full Truck Load</td>
</tr>
<tr>
<td>GA</td>
<td>Genetic Algorithm</td>
</tr>
<tr>
<td>GB</td>
<td>Great Britain</td>
</tr>
<tr>
<td>GDP</td>
<td>Gross Domestic Products</td>
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<tr>
<td>GRASP</td>
<td>Greedy randomized adaptive search procedure</td>
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<tr>
<td>GUI</td>
<td>Graphical User Interface</td>
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<tr>
<td>H&amp;S</td>
<td>Hub and Spoke</td>
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<tr>
<td>HGV</td>
<td>Heavy Goods Vehicle</td>
</tr>
<tr>
<td>HR</td>
<td>Human Resource</td>
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<tr>
<td>IRP</td>
<td>Inventory Routing Problem</td>
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<td>JIT</td>
<td>Just in Time</td>
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<td>KPI</td>
<td>Key Performance Indicators</td>
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<td>LNS</td>
<td>Large neighbourhood Search</td>
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<td>LRI</td>
<td>Location Routing and Inventory</td>
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<td>LRP</td>
<td>Location Routing Problem</td>
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<tr>
<td>LTL</td>
<td>Less than Truck Load</td>
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<tr>
<td>MDVRP</td>
<td>Multi-depot Vehicle Routing Problem</td>
</tr>
<tr>
<td>MIP</td>
<td>Mixed Integer Programming</td>
</tr>
</tbody>
</table>
MIP  Mixed Integer Programming
MOVRP  Multi-Objective Vehicle Routing Problem
MVRPB  Mixed Vehicle Routing Problem with Backhaul
NN  Nearest Neighbour
OD  Origin-Destination
OR  Operational Research
PRI  Production Routing and Inventory problem
PSO  Particle Swarm Optimization
QoS  Quality of Service
RDC  Regional Distribution Centre
SKU  Stock Keeping Unit
SS  Scatter Search
TSP  Travel Salesman Problem
TTRP  Truck and Trailer Routing Problem
TTVRP  Truck and Trailer Vehicle Routing Problem
UAV  Unmanned Aerial Vehicle
UK  United Kingdom
VED  Vehicle Excise Duty
VNS  Variable Neighbourhood Search
VRP  Vehicle routing Problem
VRPB  Vehicle Routing Problem with Backhaul
VRPPD  Vehicle routing problem with pickup and delivery
VRPSPD  Vehicle Routing Problem with Simultaneous Pickup and Delivery
Abstract

The University of Manchester
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Multi-Objective vehicle routing problem: Case studies in retail distribution

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Vehicle routing and scheduling are fundamental problems encountered in retail distribution, in the context of both primary and secondary distribution. For the purpose of primary distribution, the services of third party logistics companies (3PL) are usually acquired. In contrast, the secondary distribution is generally performed using a fleet of in-house vehicles. The optimization of distribution at both levels is challenging and realistic routing plans may often require the consideration of multiple, conflicting objectives, and the identification of a suitable compromise solution.

In this thesis, two real-world case studies are considered that can be formulated as special cases of the vehicle routing problem (VRP). One of these is about the primary distribution carried out by a 3PL company; while the other relates to secondary distribution in an urban context. The two cases differ significantly in the structure and the details of the problem modelled, but the overall underlying aim is the same, i.e. to transport goods as efficiently as possible and thus improve customer service, which is critical for long-term sustainability.

The first case study focuses on inter-depot trunking carried out by a 3PL company that operates a network of depots in the UK. Salient features of this problem include paired pickup and delivery points, adjustable compartment space, and the swap of trailers between vehicles. As independent plans are currently generated by human planners who are based at different depots, the resulting overall plan is typically suboptimal. VRPs in such a network-based operations context have been given very little attention in the literature. Here, a linear programming model is proposed that takes into account the entire network when generating routing plans. Results show that this centralized approach has the potential to lead to significant savings in terms of the vehicles used, the distance travelled and CO₂ emissions. Furthermore we show that the choice of objectives has a significant impact on the structure of the routes suggested.

The second case study pertains to delivery plans in an urban context and, specifically, routing under time-varying congestion. As travel speed varies and is time-dependent, plans that are generated by considering constant speeds become unreliable in terms of time-estimation and may lead to the violation of time-window constraints. In this work, multiple objectives (distance, time, CO₂, equity among drivers, fleet size and customer satisfaction) are considered simultaneously while historical speed information is used as a proxy for the time-varying congestion in the road network. A hybrid, GA-based interactive optimisation method is developed that takes into account the planner’s aspiration levels and weights for different objectives through several iteration cycles in order to guide the search process and reach a most preferred solution.
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Preface

Parts of the work presented in this thesis have been published in conference proceedings or are currently reviewed for journal publication.

Published Conference proceedings

Chapter 1

Introduction

Logistics is the transfer of goods, information, people and other resources from a point of origin to a point of destination [1]. Different logistics activities such as warehousing, facility location, inventory management, scheduling, and freight routing have always been key areas of interest for the Operational research community. The focus of this thesis is on one specific area of Logistics namely the routing of freight vehicles in the context of retail distribution.

Retailing involves all activities in selling goods and services to people for their personal or household consumption. Retailing and Logistics both are concerned with making the right product available at the right time to the right people. Both of these sectors contribute significantly to the overall UK economy. For example, sales from the retail sector account for almost 20% of the UK GDP [2]. In addition to their huge economic impact, the logistics and retail sectors are also amongst the largest employers in the UK. Almost 8% of the UK workforce is employed in the logistics sector while the retail sector employs nearly 11% of the UK workforce [3].

Since the 1980s, retailing in the UK has undergone significant changes to improve operational efficiencies. A shift to a centralised distribution by adopting region-based distribution structures is one of those changes. That is, instead of making direct deliveries from suppliers to stores or retail outlets, supplies are made from a regional distribution centre (RDC) to retail outlets in its respective region [4]. In the last three decades, the number and variety of products that are offered by retailers have increased many folds due to internationalization, global competition and reduction in trade barriers. While the exact number of different stock keeping
units (SKU) per store may vary according to store format, they are typically found to
be in the thousands.

Retail giants like TESCO, ASDA or Sainsbury source products from thousands
of suppliers that could be dispersed across a wide geographic area. Local deliveries
from an RDC to its associated outlets are usually done using vehicles that are
available in house. In contrast, the transport of goods from suppliers to regional
distribution centres is a full time job. Furthermore, both for retail giants and for
suppliers, it can be economically unviable to transport products themselves and this
job is, therefore, often outsourced to third party logistics companies (3PL). The 3PL
companies collect the goods from suppliers and then either deliver to the retail
company’s RDCs directly (if the goods amount to a full truck load), or bring it back
to their warehouses for consolidation. The consolidated goods from various runs are
then transported to the RDCs. Sometimes the 3PL companies also establish their
own network of depots or establish links with other hauliers/3PL companies to
optimise the flow of goods through the network and reduce the transportation costs
even further. The transportation of goods from suppliers to the RDCs is generally
referred to as primary distribution; whereas the delivery from the RDCs to local
outlets is called secondary distribution. A typical primary and secondary retail
distribution network is depicted in Figure 1-1.

Figure 1-1: A typical Primary and Secondary distribution network in the retail sector of the UK, where S
indicates Suppliers, R indicates the Retailers and 3PL indicates the depots.

From the perspective of retail companies, the overall process for the transport
of goods from suppliers to retail outlets is usually organized on a Day-1 to Day-3
(D1-D3) basis. That is, collection from suppliers is performed on day-1 by a 3PL
company. Inter-depot trunking (if required) of consolidated goods takes place during the night and products are delivered to the RDC on day-2. The vehicles based at each RDC are then used to deliver the products to its associated outlets on day-3. In other words three to four separate plans are made to accomplish the transport operations efficiently.

1. Collection of products from suppliers on day-1.
2. Inter-depot trunking (if required) in the evening of day-1.
3. Deliveries of products to the RDC or to other customers on day-2.
4. Delivery of products from the RDC to the outlets on day-3.

In any retail outlet, whenever the inventory levels of different SKUs fall below certain levels, a request is issued to the local RDC to replenish these items. Vehicles stationed at the RDC are used to transport the demanded goods on the following day. Usually, the accumulated demand of any outlet is less than a full truckload (LTL); therefore a single vehicle could serve the demand of multiple outlets thus requiring the need to determine a delivery sequence for its route. That is, there is a need to construct a routing plan.

The problem of allocating delivery points to multiple vehicles and sequencing the deliveries for each vehicle falls under the umbrella of the Vehicle Routing Problem (VRP) in the academic literature. In the VRP, routes are constructed for different vehicles in order to satisfy the demand of all customers while optimizing a single or a set of objectives and abiding by a set of specified constraints [5]. Numerous variants of the VRP have been modelled by researchers in order to optimise certain economic and/or social objectives. A review of the literature suggests that the five most commonly modelled objectives relate to the distance, cost, fleet size, customer satisfaction, and total time associated with a routing plan [6]–[10]. All of these objectives can be measured in economic terms and contribute either directly or indirectly to the overall profitability of the company and one may thus classify these as representative objectives of the economic domain. Because of changes in business context, legal requirements and human resource management practices, the focus in real-life as well as in the literature has gradually shifted towards the inclusion of social objectives in the modelling process. One of the commonly considered social objectives is the maintenance of equity among drivers. Well trained drivers play an important role in achieving reliable delivery operations
and are considered as a crucial resource for the long-term sustainability of a company. Due to the aging population and other factors, the hiring and retention of drivers has become a potential human resource issue. Among other factors, maintaining equity among drivers by balancing their workload is considered to be an important factor in driver’s retention and has been modelled by several researchers [11], [12]. Other commonly modelled objectives from the social domain include the minimization of risk [13]–[16] and the maximization of coverage [17]–[19].

In addition to the efficient use of physical and human resources, the long-term sustainability of a company is also dependent on a loyal customer base. In addition to direct measures of the quality of the service to customers (e.g. the extent to which delivery time windows are met), the perception of a company as a socially and environmentally responsible organization also plays a crucial role. To improve their Corporate Social Responsibility (CSR) image, companies are paying attention to the implementation of sustainable business practices and to the reduction of their carbon footprint in all aspects of their business including the scheduling of delivery operations. In the past few years, there has been an increasing body of literature concerned with the modelling (and minimization) of emissions [20], noise reduction and transportation of hazardous waste - a strand of research now commonly referred to as the sustainable or green VRP [21]. As emissions from logistics have a wider impact on society and the environment, the minimization of CO₂ emissions falls into the environmental domain.

In the majority of the VRP literature, a single objective is considered; however, due to the above changes in socio-economic context, the focus is now shifting towards the simultaneous consideration of multiple objectives [22]. The consideration of multiple objectives (that could, potentially, be from different domains) poses new challenges for researchers. One of the challenges is how to handle multiple objectives when these are conflicting and incommensurable in nature [23]. In other words, a gain in one objective can often only be achieved at the cost of deterioration in another objective. For example, the minimization of vehicles may result in longer driving time. Similarly, the minimization of distance may result in imbalance in the drivers’ workload and an increase in CO₂ emissions [24]. From a decision making perspective, the ultimate aim of the optimization process is often to find a single recommended solution. When objectives are conflicting and
incommensurable, it may be impossible to find a single solution that optimises all objectives at the same time. In this setting, optimization must then deliver a solution that provides the best compromise between the different conflicting objectives [25]. Here, the definition of “the best” compromise is dependent on the decision maker’s preferences, thus making the elicitation of preferences and the identification of a final solution an important part of multi-objective VRP optimization. In the literature on the multi-objective vehicle routing problem (MOVRP), researchers have usually adopted an a-priori approach or a posteriori approach. In the a-priori approach, the decision maker’s preferences are identified in advance of the optimization, i.e. prior to the generation of any candidate solutions. The most commonly used a-priori technique is the weighted sum method. In the posteriori approach, a set of non-dominated solutions are generated first, and, subsequently, the decision maker is asked to evaluate and pick one solution for final implementation. Both of these approaches have their advantages and limitations, and interactive approaches have been designed to combine some of their strengths.

This thesis is presented as a collection of three research papers. The common theme between these papers is that all three of them focus on the multi-objective VRP in the context of the retail sector. The first paper (i.e. Chapter 2) is a review paper, whereas the second and third papers (Chapter 3 and 4) study particular instances of the VRP problem as commonly encountered in the logistics sector. In particular, Chapter 3 looks at an important aspect of primary distribution i.e. inter-depot trunking in a 3PL context. The problem is considered using formulations with different single objectives, as well as a weighted sum approach. In contrast, Chapter 4 deals with a VRP application related to secondary distribution in an urban context. While solving this problem, an interactive approach is used to incorporate the decision maker’s preference. The focus on retail distribution is particularly pronounced in Chapter 3 and Chapter 4 - to ensure completeness, the review paper (Chapter 2) also considers non-retail applications of the MOVRP. Overlap between the three papers has been minimized as much as possible, but has been unavoidable for a few subsections such as those dealing with interactive approaches in VRP literature and conflict in different objectives (which provide relevant background for both Chapter 2 and 4).
Chapter 2

The primary aim of this chapter is to explore the MOVRP literature in detail. This chapter is divided into different sub-sections. In section 2.2, the components of routing problems are described. Different objectives that are found in the MOVRP are listed and categorised according to their impact on stake-holders and according to routing components. Since researchers have defined and measured objectives in numerous ways, we discuss different facets of the same objective and different ways in which the measurement of the same facet may be undertaken. The focus of Section 2.3 shifts towards the multi-objective VRP and the relationships that exist between different objectives. Section 2.4 explores methodologies that incorporate the decision maker’s (DM) preferences into the optimization process and the application of different methods to solve variants of the MOVRP. In section 2.5, we highlight important issues that may surface during the modelling phase, the validation stages or the presentation of the result to decision makers.

In addition to the primary objective of exploring the academic literature related to the MOVRP, the secondary objective of this chapter was to consider the practitioners’ perspective. As a first step, secondary data from the website of a leading routing software provider was collected. The purpose was to identify the objectives that practitioners genuinely value. The ideas formed at this stage were validated through observations from field visits and direct discussion with transport planning managers and transport planners. An interview with the managing director of a leading VRP software provider was carried out to identify his views related to different aspects of the MOVRP. The aim of collecting these secondary and primary data was to compare and contrast the focus of the academic literature with the realities and need of vehicle routing in practice.

As expected, the results from this analysis suggest that the strongest emphasis is given to the optimization of economic objectives not only in the academic literature but also in practice; however, there appears to be some discrepancies not only in terms of focus but also in terms of the definition and frequency of the measurement of different objectives. In the academic literature, there is an increasing trend of using posteriori approach to solve the MOVRP problem. In the posteriori
approach, a set of non-dominated solutions is created and then it is left at the discretion of the decision maker (who is usually a transport planner) to perform trade-off analysis and to select one solution for further implementation. However, in the academic literature, insufficient attention is paid to assess the desirability of this approach by practitioners. Despite the fact that real-life planners would benefit from the exploration of solutions and the possibility to conduct a trade-off analysis, factors such as the skill set, cognitive span and the limited time available to transport planners appear to constrain the usability of post-eriori approaches in industry. Interactive approaches may present a more viable alternative to address this issue, but the academic literature dealing with interactive MOVRP is currently very limited.

Chapter 3
This chapter aims to investigate the problem of inter-depot trunking which is frequently encountered by third party logistics (3PL) companies that operate network-based operations. In network-based operations, geographic area is usually partitioned into fixed regions and each region is served by a specific depot. Collection and deliveries are performed using a heterogeneous fleet of vehicles that is based at each depot. Operations are organized on day-1 to day-2 (D1-D2) basis. That is, collection from warehouses and manufacturers is performed on day-1, and deliveries to the RDCs and the retail outlets take place on day-2. From each collection point, multiple orders are usually picked up where each order corresponds to a different origin-destination pair. From the collection point of view, one vehicle may often be enough to serve that customer; however, from a delivery point of view multiple vehicles might be used, as the destinations of the orders could be in separate regions. If the origin – destination pairs fall in the same region, then collection on day-1, storage in the warehouse during the night and delivery on day-2 are performed by the same depot. However, when origin and destination fall in different regions, then deliveries for the same destination region are consolidated and the planners from those two regions coordinate with each other to devise a plan for inter-depot trunking. For the purpose of inter-depot trunking, vehicles based at either of the two depots (i.e. the depot associated with the origin or the destination regions) could be used. In practice, when two planners coordinate this process, they only
consider the interests of the two depots concerned, with no consideration of the holistic picture. For this reason, plans that are constructed at this stage are likely to be suboptimal, and the development of an efficient method that can optimize the network-wide routing plan may be warranted.

The inter-depot trunking problem is difficult to solve by practitioners in real-life. The salient features of this problem are 1) the transportation of two categories of products in separate compartments; 2) adjustable payload space; 3) swapping of semi-trailers between two vehicles at some suitable location; and 4) deterministic demand and absence of delivery time windows. Semi-trailer swaps, in particular, are increasingly common in the logistics sector and present a unique challenge to fleet planning that has not been given sufficient attention in the literature.

In its simplest form, this problem could be described as an adaptation of the vehicle routing problem with a fleet of heterogeneous vehicles and paired pickup and delivery points. Different from standard versions of this problem, the vehicle fleet in inter-depot trunking is stationed at several depots rather than a single depot. A new multiple objective mixed integer optimization model is proposed to take into account this feature systematically and rigorously. Furthermore, our formulation of the problem caters for adjustable compartment sizes, as well as truck-meets-truck events (or “semi-trailer swaps”).

Using a small-scale synthetic data set, the impact of different optimization objectives is first analysed and discussed. The proposed model is then applied to a real-world case study for a UK company, and its solutions are compared with the plans generated by a group of human experts with many years of experience in fleet planning. Our optimization approach provides valuable problem insights and planning alternatives, and the results indicate the potential for significant cost savings compared to the distributed planning approaches currently employed by the company.

Chapter 4

In this chapter, a secondary distribution problem in an urban context is considered. While planning deliveries from the RDC to retail outlets, different objectives need to be considered simultaneously. In this research, six different conflicting objectives
from the economic, social and environmental domains are considered. Fleet size, distance, and total time contribute directly to the transportation cost; therefore, these objectives are always considered by practitioners while evaluating any plan. Access restriction e.g. to the outlets located in the city centre, warrant deliveries to be made within specified time-windows. Failing to do so will lead to the postponement of deliveries which could potentially lead to the loss of sales; therefore, to avoid financial consequences, time-window restrictions need to be accommodated in any urban delivery routing plan. In addition to the above-mentioned four objectives that fall into the economic domain, objectives from the social and environmental domains that are considered in this research include the maintenance of equity among drivers by balancing their workload and the minimization of CO$_2$ emissions of the fleet.

Reliability of routing plans can be affected by time-varying congestion that is frequently observed in large cities. The formation of congestion may lead to increases in travel times. If time calculations are made based on fixed average speed, the presence of time-varying congestion may lead to the violation of time-windows, imbalance in workload, and inaccurate estimations of CO$_2$ emissions and total travel times. Consequently, incorrect estimation may hamper the efficiencies of inter-linked operations such as picking of pallets in the warehouse for the following shifts. Therefore, in order to make reliable delivery plans, one needs to take time-varying congestion into account.

As mentioned before, commonly adopted approaches to solve MOVRP have advantages and limitations. For example, one limitation of a-priori approach is that the derivation of preference information for different objectives at a global level is not easy, in practice, and especially in circumstances when some objectives are carefully monitored on a daily basis, while others are monitored on a weekly or monthly basis. Similarly, the biggest limitation of a posteriori approach is the complexity of performing a full trade-off analysis for a high dimensional problem. In particular, it is very difficult and time consuming when the frequency of decision making is quite high and there is a strong need to consider the dynamics of inter-linked processes in conjunction with those of the routing process. Interactive approaches can overcome the limitations of the above-mentioned two approaches, as they provide an opportunity for the decision maker to provide his preferences by
inspecting small subsets of solutions and using this information to guide the search process in the most preferred direction.

In this research, an interactive reference point approach is applied to support the trade-off analysis. Time-varying congestion data that is derived as time-dependent historical average travel speed data from the UK road network is used. A hybrid algorithm embedding the Floyd algorithm within an evolutionary algorithm is developed to generate sets of efficient solutions for this nonconvex multi-objective optimisation problem with mixed integer and continuous variables. The proposed interactive approach is then applied which alternates between solution generation and preference elicitation from a decision maker and guides the search process towards the preferred direction. The effectiveness of the approach is illustrated using a case study that combines synthetic demand data for a company with the actual road and congestion information for the UK. The results obtained using the proposed interactive approach are compared to those obtained for the optimization of individual, single objectives.
Chapter 2

A review of Multi-objective Vehicle Routing Problems (MOVRP)

2.1 Introduction

The Vehicle routing problem (VRP) is one of most highly researched areas in Operations research. Since the introduction of this problem, in 1959 by Dantzig and Ramser [26], thousands of researchers have tried to solve different variants of this real-life problem. In a VRP, a set of routes that are to be traversed by a fleet of vehicles are determined such that some specific objective(s) is optimised while meeting the demands of customers without violating the specified constraints [5].

For any transport planner, the pre-requisite to finding feasible and high-quality solutions to real-life problem instances is to have a thorough understanding of the network structure and components of a problem; awareness about overall routing objectives which usually are reported in the context of key performance indicators (KPIs) in company reports; knowhow about processes that are closely linked with the routing and scheduling process; and knowledge about the company's overall mission and vision. For a researcher who is interested in solving real-life VRPs, it is equally important to understand the full context of the routing problem as it will have significant impact not only on the model formulation but also on the acceptability of the proposed solutions to practitioners and their subsequent implementation.

In real-life, practitioners usually consider multiple objectives simultaneously while making or evaluating routing plans. Similarly, in the organizational context, various processes are inter-linked; therefore, in addition to routing objectives,
planners also consider the requirements of associated processes while evaluating the routing plans. As many objectives conflict with each other, planners usually conduct an implicit trade-off analysis in their mind before finalising any solution. For any researcher working on the multi-objective VRP, it is therefore an important concern how to elicit, understand and incorporate the decision maker’s preference in the solution process.

The primary aim of this chapter is to conduct a comprehensive literature review on the multi-objective VRP. The secondary objective of this chapter is to take practitioners’ point-of-view about different aspects of MOVRP. To do so qualitative interviews and observations were made. Details about this survey are provided in appendix 2.A. To support the reader, this chapter is organized in four main sections. Section 2.2 provides some background knowledge. Specifically, details about network components; effect of components on route structure; and linkages of routing process with other organizational processes are explained. As the same objective can often be measured in different ways, different facets of commonly modelled objective and different types of constraints are also explained. In section 2.3, the multi-objective VRP is discussed. The focus in this section is to highlight the rationale behind considering multiple objectives and to explore links between those objectives. Section 2.4 deals with different approaches used by researchers to incorporate the decision maker’s preferences. Similarly, this section also provides some details of different methods that have been employed to find solutions of MOVRP variants. Section 2.5 briefly discusses practical issues that researchers dealing with the MOVRP may face. A conclusion is given in section 2.6.

2.2 Problem Components

When asked about the importance of understanding the contextual information, the managing director of a leading VRP software provider in UK responded in the following word:

“It is very vital. We always spent quite a lot of time with our customers”

In response to the question about possible consequences of spending less time on this activity, the respondent cited an implementation in which little time was spent on this activity and said:

“I would argue that it was a less successful implementation”.
For any researcher who is interested in finding solution to real-life problems, these quotations indicate the important relationship of contextual knowledge with successful implementation of the proposed solutions. By acquiring contextual knowledge, here we mean to have knowledge about different network components and interlinking of various organizational processes with routing process. This information helps to identify the constraints and objectives to be considered. In the freight routing context, advantages of acquiring this knowledge are numerous. For example, it helps 1) to understand the rationale, behind choice of different objective or constraints, which could ultimately lead to the underlying problem that needs to be addressed; 2) to understand and identify ways to incorporate preferences; and 3) to refine the measurement of overall objective function(s) and constraints.

### 2.2.1 Network Components

To construct efficient and robust solutions, a planner needs to have a clear understanding of network components which includes 1) overall road network; 2) fleet composition and characteristics; 3) customers’ demand and its pattern, and 4) monetary and non-monetary costs [27].

While modelling a VRP, a real-life road network is represented as a graph in which depot(s) and customers are shown as nodes; and roads are represented as links or arcs between nodes. Customers are usually dispersed across a wide geographic area around a single or multiple depot /distribution centres (DC). Single or multiple costs are associated with each link. Cost could be defined in monetary or non-monetary terms and could be proportional to distance, travel time, or some other measure. The network is called symmetrical, if two-way costs between any two nodes in the network are same, otherwise the network is asymmetrical. Cost associated with links could be time-dependent. Change in cost due to change in direction or time may affect the value of objective function(s) and thus may change the routes thus constructed. Since different categories of roads cater for different type of vehicles which could mean that certain roads might be out of bounds for certain vehicle types.

The vehicle fleet(s) could be stationed at single or multiple depots. The fleet could be homogenous or heterogeneous depending on the vehicles characteristics such as payload capacity, cost structure, and emission profiles. Vehicles may have multiple compartments to transport multiple products with different compatibility
requirements. Vehicle may have, instead of fixed, adjustable compartments thus offering opportunities for better capacity utilization. Vehicles can also be categorised into rigid and articulated truck. In the rigid trucks, payload compartment is permanently fixed; whereas the articulated trucks have detachable trailers. Rigid trucks have less capacity than articulated trucks and are mostly used for urban and short-route deliveries. Detaching of trailers in articulated trucks provides an advantage of leaving the trailer at some place for loading/unloading and meanwhile the tractor could be used to pull some other trailer. If a company has a heterogeneous fleet then it increases challenges for transport planners. While making plans, not only do they need to consider the capacity of each truck and/or compartment but also they need to determine which customer node will be served by which vehicle as usage of certain vehicle types might be restricted. Possible reasons that may contribute to this restriction include 1) a dependency between vehicles and drivers, 2) vehicle characteristics that dictate the type of payload it could carry, 3) access restriction on different arcs, or 4) parking or unloading facilities available at different nodes [22].

Customer nodes in a network could have a transportation of goods/people or provision of service demand. The amount to be transported could be deterministic (i.e. known in advance), dynamic (i.e. revealed with passage of time) or stochastic (i.e. known with probability). If demand of a node exceeds more than vehicle capacity or full truck load (FTL), then more than one vehicle is required to fulfil that demand; however, if demand is less than full truck load (LTL) capacity then generally a single vehicle is sent to fulfil that demand. Though in practice, increase in overall gain is sometimes achieved by splitting of demand and allowing more than one vehicle to serve the same node. Demand may pertain to single or multiple products which may have specific transportation and compatibility requirements; and similarly it could pertain to one or more than one time-periods. Sometimes, if demand is stable over a longer horizon, then a customer may specify frequency of visits and the planner in that case needs to determine when each customer will be visited. Due to operational constraints or conditions surrounding the customer node, a customer may demand a service in a specific time window.

Monetary costs associated with route planning usually include vehicles, drivers, road usage, and service penalty costs. Vehicle usage can be divided into fixed cost and variable cost. Fixed cost which is usually measured on per day basis includes depreciation, Vehicle excise duty (VED) and insurance cost, while variable
cost includes fuel, tyre and maintenance [28]. Similarly fixed cost could vary w.r.t. vehicle type; though the variable cost for different vehicles is usually considered the same for accounting purposes. In the case where services are partially outsourced to a 3PL, then a fixed rate (i.e. without variable cost) for a round trip is used to make calculations. Costs associated with drivers’ salary could be a part of fixed or variable cost depending on terms of employment contract (i.e. full-time, part-time or agency driver). In addition to road usages costs such as toll tax, vehicles may have to pay an additional fee such as congestion charging fees which are being imposed by local governments to reduce the traffic influx during daytime in big cities. Similarly formation of a low emission zone may ban entry of different vehicles and needs to be considered while route planning. Violation of service time windows at customer nodes could increase penalty costs and therefore it needs to be avoided. Non-monetary cost could include the emissions, noise level, and risk to surrounding population (in case of waste collection or hazardous material transport).

In the retail distribution context, usually the objective function is an aggregation of different costs; therefore, all good planners need to take into account these costs while making routing plans. Though, in practice planners tend to consider only monetary costs in the routing stage; however increased legislation in future may eventually lead to the inclusion of non-monetary costs in consideration.

2.2.1.a Network Components and its impact on route structure, constraints and objectives

Characteristics of network components act as an input to the modelling of constraints and objective value and consequently change the structure of routes. For example, depending on the service requirements (i.e. collection, delivery or both) and the loading constraint in vehicles, different route structures are possible. If loading could only be performed on one side, then routes are constructed in a way to allow deliveries before doing pickups – it is termed as VRP with backhaul (VRPB). This not only ensures the availability of sufficient space before any pickup but also removes the need to rearrange items during journey. In contrast, if loading can be done from any direction, then deliveries and pickups can be planned simultaneously (Mixed VRP with backhaul -MVRPB) provided that capacity constraints are not violated at any time. In VRPB and MVRPB, each customer either has delivery or pickup demand; however, a customer can have both demands and may need to be
served in a single visit. This is called VRP with simultaneous pickup and delivery (VRPSPD) [27]. Another example in which demand and vehicle characteristics change the route structures is when multiple products are to be transported that have compatibility constraints. Different types of routes are possible e.g. each commodity could be loaded on separate vehicles if vehicles have single compartments. Alternatively, same vehicle could be used to transport multiple products if there are multiple compartments. In the above-mentioned examples, a single depot is assumed where the fleet is stationed and planners need to make decisions related to routing/scheduling only. However, in retail distribution it is quite common that a company is engaged in network-based operations i.e. the company has more than one depot and customers can be provided service from any depot. To make the routing task easy, customers are usually split into regions and served accordingly. However, in addition to usual routes involving a single depot and multiple customers, inter-depot routes are also planned in which vehicles based at one depot are allowed to visit other depots to replenish en-route. This type of problem is commonly termed as multi-depot VRP (MDVRP).

Network structure and its characteristics have a profound effect on the constraints and thus the objective function. For example, when demand can be split and fulfilled by more than one vehicle, then the constraint that each customer is to be served by a single vehicle is relaxed. This type of problem is termed as VRP with divisible deliveries and pickups (VRPDDP). Sometimes demand may comprise point-to-point delivery requests i.e. an item is to be picked from some node and delivered to another node. This requirement warrants that the same vehicle first visits the pickup node and then the delivery node. This type of problem is commonly termed as a pickup and delivery problem (PDP). As it can be understood that for each of these cases (VRPSPD, VRDDDP, and PDP) one needs to ensure that the capacity constraint is not violated on any link [27].

Usually routing plans are made by considering the requirements/limitations of direct stake holders only; however, there are examples in which routes are constructed by giving more weightage to the interests of indirect stake holders that usually have little or no say in the routing process. One such example is about the transportation of hazardous material. Road accidents or other unpleasant incidents to vehicles carrying dangerous material may pose risks to the health and safety of the general public[29]. If routing is done by considering the monetary aspects of the
transport provider only, then shortest distance routes will be adopted which could pose a danger to the public living around that shortest route. On the contrary, consideration of minimization of risk to indirect stakeholders, may lead to routing solutions in which longer routes will be adopted so as to avoid greater risk to the general public.

2.2.1.b Relationship with interlinked processes

In any organizational context, different departments and processes are interlinked with each other. Facility location, vehicle routing, inventory control, order picking in warehouses, scheduling of drivers, and production planning are key elements of any production and distribution logistics system. As these elements are highly dependent on each other; therefore decisions made in one element will affect subsequent decisions in other elements. To achieve the overall organizational objectives, disparate decisions in these different elements need to be integrated to provide value-added services as per customer need [30]. Though, order picking in warehouses and drivers’ scheduling are closely tied with the outcome of the routing process; the majority of the companies, in retail distribution, tackle these problems at the post-optimization stage. However, even when companies make independent routing plans (i.e. without considering objectives of interlinked processes) they ensure, in the words of one respondent, that

“Consequences of plan have no effect on the efficiency of picking operations”

Perhaps the most noticeable interlinked process is the inventory control in which instead of providing a fixed demand for a single period [31], customers provide their continuous and deterministic demand rate. In this case inventory and routing decisions are to be taken together i.e. one also needs to determine the frequency of visit to each customer in order to avoid stock-out conditions while keeping the long-term distribution cost low for a given planning period. This type of problem is commonly referred as inventory routing problems (IRP) [32]–[36].

Facility location is considered a very strategic decision as it has far reaching consequences on the delivery operations. Though in most of cases facility opening decisions are taken very rarely; however, in a few real-life applications the decision to open temporary distribution centres may be taken more frequently. For example, researchers have modelled a problem about opening or closing of retail outlets in response to shifts in population demographics or competitive economic conditions
While choosing the locations, the decision maker not only needs to be aware of population demographics, target customer segments, and information about competitors but also needs to consider how day-to-day routing will be performed to replenish the outlets. These types of problems are termed as Location routing problem (LRP). For more examples, other than retail, about LRP readers are referred to [32], [33], [34]–[36].

 Sometimes the location, routing and inventory decision are taken together. This type of problem is called as Location routing and Inventory problem (LRI) [43].

In manufacturing companies that distribute their products through an in-house logistics department, often the production and distribution decisions are taken independently or sequentially. As distribution cost represents a significant portion of overall product cost, therefore integration of production and distribution decisions offer an opportunity not only to reduce the overall cost in a planning period but also helps to avoid unnecessary delays. This type of problems is called Production and inventory routing problem (PRI) [44], [45].

A good understanding of network components and interlinked processes help in identifying and formulating the constraints and objectives. The most common constraints and objectives that have been modelled by researchers are explained in the next section.

### 2.2.2 Constraints

Constraints can be broadly divided into internally driven, service related or regulatory constraints. Internally-driven constraints are imposed due to intra-company reasons that arise because of technical reasons, resource limitations or human resource related organizational practices. As excellent customer service is essential for long-term profitability, therefore, to meet or exceed customer satisfaction level, routing plans need to abide by service constraints. Service related constraints pertain to meeting specific requirements, and providing deliveries by considering the capacities or limitations at customer nodes. In the above-mentioned two categories of constraints, the needs/limitations of service provider and service receiver are considered only; however, to cater for the broader direct or indirect interests of society, regulatory bodies generally provide guidelines that can be formulated as constraints and are to be followed strictly by the logistics service provider. Figure 2-1 provides some commonly considered constraints. Constraints
that are considered critical from retail distribution practitioners’ perspective are shown in italic.

2.2.2.a Internally driven constraints
From the perspective of the logistics service provider, the most important resources available to any company are vehicles and drivers. Any restriction on these resources or capability of these resources could have impacts not only on individual routes but also on overall routing plans. In the majority of real-life VRP problems, a limited number of vehicles are available to perform operations. Demand of customers could be met by using a fleet of in-house or outsourced vehicles. As costs associated with usage of outsourced vehicles is higher than that of in-house vehicles; therefore, logistics providers always try to use in-house vehicles in a cost-efficient manner so as to avoid subletting the job to external carriers/providers. Commonly considered constraints associated with vehicles include the size and type of fleet available [12], [46]–[49] and capacity constraints for different types of vehicle [7], [8], [10], [38], [46], [50]–[54]. Depending on the requirement, capacity constraint has been implemented by researchers in different ways. For example, when items being transported are assumed to be of the same type then simply an upper bound on
quantity being carried by each vehicle type is added as a constraint in formulation. The complicated implementation of capacity constraints includes the bounds on total weight carried, footprint area or volume restrictions [55].

Another commonly modelled constraint is the bound on the maximum distance travelled [10], [39], [56]–[60] or maximum time required to complete each route [12], [39], [45], [55], [61]–[63]. Underlying reasons to use maximum distance or time as constraints are quite diverse and depend on context. For example, when perishable products are transported and there are no specific measures such as control of temperature to ensure maintenance of product quality during transportation, then quality of product deteriorates with distance/time travelled. In this case, a bound on maximum distance or time ensures that the quality of products does not deteriorate below a certain level. Similarly these constraints can also be used to ensure that a driver’s duty time limit is not violated. Vehicles are an expensive resource for logistic companies. When a logistic company is providing round the clock services then the same vehicle resource could be used in morning and evening shifts. This helps companies to reduce the payback period and to increase efficiency. As the same resource is used in multiple shifts, so longer duration routes in one shift could have knock on effects on the following shift. So to ensure that a vehicle is available before the start of the next shift this bound could be added. Distance travelled and travel times are sometimes used as a surrogate for fuel consumption. A common assumption in VRP is that after performing service all vehicles have to come back to the originating depot. If the depot is operating for limited time period instead of round-the-clock, then to ensure that all vehicles return back before the depot closing time a global bound can be added for all vehicles [64].

The above-mentioned constraints are specific to the routing process within a company. As mentioned in section 2.2.1.b, the routing process is closely linked with other associated processes; therefore, when decisions are taken jointly, then related constraints also need to be formulated in the model. For example, when production and routing decisions are taken simultaneously for a planning horizon, then how much to produce becomes a decision variable. As in a given day, amount produced cannot exceed production capacity; therefore an upper bound on production quantities is added as a constraint [44], [65]. Similarly if amount produced exceeds amount delivered to customers in a given day, then surplus is stored as an inventory. As inventory holding capacity at a plant is a limiting factor, therefore a constraint is
added to decide upon daily production and delivered quantities [44], [65]. Amount delivered from a manufacturing plant cannot exceed production capacity and inventory already available; therefore, it leads to imposition of another constraint [39]. Similarly, customer nodes also have limited inventory capacities and this dictates the delivered quantities in a given day [65]. In ref [33], researchers imposed a bound on the number of trips per route. Practitioners often impose this constraint so as to avoid “heavy routes i.e. routes that serve a large percentage of the total sales volume.”

2.2.2.b Service related constraints

While internal constraints cater for the requirements/limitation of the logistic provider, service constraints pertain to meeting specific requirements of customers. One of the common service-related constraints is a time-window constraint in which a customer has a requirement to be served in a specific period which is generally given in the form of earliest arrival and latest arrival times [6], [10], [50], [52]–[54], [66], [67]. In context of goods or home deliveries, this situation is frequently observed where customers specify a time period in which specific resources such a person, equipment or other resources e.g. loading or unloading bays are available. If demand is strictly to be met in that time window i.e. a customer cannot be served outside the time limits, then the time window is said to be hard. To meet this requirement, two constraints are added in formulation to ensure that the start of service takes place between earliest and latest service start times. There are problem instances in which instead of providing a time period customers only provide a single time after which service is required. Now if a vehicle arrives before that time then the driver has to wait. But if vehicle arrives later than the specified time then customer might become unhappy. However, arrival before the specified time leads to wait time by the driver. Researchers have adopted different strategies to model similar situations. Few researchers have considered the earliest time as hard but made the latest arrival time as soft [68]. To avoid a vehicle coming later, a penalty proportional to tardiness is made a part of the objective function. Another approach used by researchers is to impose a maximum tardiness constraint in order to avoid customers becoming frustrated [68]. Another approach used is due to [8] in which researchers applied a fuzzy membership function to measure the satisfaction level of customers. As per researchers time windows could be violated due to operational or
economic reasons however a limit was imposed on the minimum satisfaction grade for each customer to ensure the minimum service levels [69].

Other commonly considered service-related constraints include coverage, ride-time and precedence constraints; however, these are more applicable to the non-retail sector. Coverage constraint is mostly considered in the covering tour problem (CTP) in which vehicles cannot serve all customer nodes. Instead vehicles visit those nodes that are within a specified distance from customer nodes. An interesting application is due to [70] in which researchers have modelled the problem of providing primary healthcare. As it is not possible for mobile facility to visit all population centres; therefore routes are constructed such that the population centres are within a specified distance from the visited nodes. In this type of problem, while selecting a node to be visited, a constraint is added that population centres should be within a specified distance.

Ride time constraints are prevalent in applications where humans are transported. For example, in school bus routing problems, usually an upper bound on ride time is added to ensure service quality. Another interesting example is a dynamic bus routing problem in an industrial campus [71]. Since in off-hours, the density of customer requests is quite low thus making it unviable to have fixed bus routes. The bus is diverted from its current route, when a new request arrives, so as to meet demand at the same time reducing cost and maintaining service levels. When a bus is diverted, ride time of customers on board increases; therefore, to maintain a balance between cost, efficiency and service, a bound was added on customer ride time. Precedence constraint is a requirement in pickup and delivery problem (e.g. Dial-a-ride problems). One needs to ensure that the same vehicle visits both the pickup and delivery nodes [72] while ensuring that the pickup node is visited prior to visiting the delivery node [73].

2.2.2.c Regulatory constraints
Failing to abide by the regulatory guidelines could result in serious financial or other consequences. One of the most important regulatory requirements is about drivers’ daily work times which includes driving hours [38], [64], lunch time break [55]) and mandatory rest breaks. Drivers working hours rules [74] could be implemented in a variety of ways in real-life; therefore these rules are very difficult to implement from a modelling perspective. Due to fleet size or other limitations, situations may arise in
which duty timings of a few drivers exceed the shift duration and overtime is to be paid. Though there are regulatory restrictions on the number of hours a driver can do overtime in a day or weekly basis. However, the reason to implement this constraint is more social than regulatory. Other drivers, who are not offered to do overtime, often complain when they see a fellow driver earning more than them; therefore to cope with this situation a constraint on maximum overtime a driver can do is added not only to meet legal but social constraints. [57].

Roads in real life are designed to cater for different types of traffic and may have speed limitations for different vehicle types. For example, in UK Heavy goods vehicles (HGV) cannot travel on the C-type roads that are designed for neighbourhood traffic. These constraints need to be included while designing routes and making calculations such as travel times [66], [75].

When a routing consultant was asked to tell which constraints were difficult to model, then his reply was

"Anything that is related with schedule (drivers’ time)"..... "Variable times of day"... "Time windows are quite difficult."

2.2.3 Objectives

By looking at the academic literature, the boundary between constraint and objectives seem to overlap. So when a question was asked how in practice differentiation is made between these two, the answer of a respondent was.

"I think that fundamentally you have got to distinguish between objectives and things we can satisfyingly put in constraints."

It is very interesting to see that the same thing is treated as constraint in one context and as an objective in another. Similarly one can also find some very interesting differences between academic literature and practice. For example, time windows are usually treated more as a constraint than as an objective in academic literature. In retail distribution context, it is usually treated as part of the objective function by turning it into a cost. Perhaps the prime reason is that though this is important for customers, it is usually measured by them on a monthly basis rather on a daily basis; therefore, strictly treating as a constraint on a daily basis might not be a good option.
The following subsections provide details about commonly modelled objectives and different ways to measure the same thing by different researchers.

2.2.3.a Objective classification

The objectives modelled in literature could be classified according to the impact on stakeholders and according to the problem components.

Depending on the impact on the stakeholder, objectives could be divided into economic, social and environmental domains [76]. The objectives that fall in economic domain are related directly or indirectly with growth, efficiency and profitability of the company providing the logistics services. As the stakeholders of a Logistics Company are direct beneficiaries therefore the focus on these objectives is internally driven. One important aspect of these objectives is that these can be quantified in economic terms quite easily. Social objectives are concerned with health, safety, access to service, and equity between people providing or using the services. Social objectives are difficult to quantify in economic terms and could be internally or externally regulated. However, internal decision makers are often not interested as focusing on these objectives increase the economic cost. Objectives from the environmental domain have a wider impact on the society and environment and include optimization of CO₂ emissions, air quality, noise reduction and management of waste. It is quite difficult to convert the value of these objectives into monetary terms. The focus on these objectives is due to improve the CSR image of the company or it is due to some regulatory requirement.

From the operations component’s perspective, objectives could be subdivided into tour, node, resource, and items being transported related objectives [77]. The scope of tour-related objectives ranges from each individual tour/route to the fleet as a whole. Node-related objectives include meeting the specific requirements of each node such as non-violation of time-windows and site dependencies. The resource-related objectives include fair and/or efficient use of physical and human resources. Specific objectives related to goods/people being transported include the maintenance of comfort of passengers or quality of perishable goods being transported. A list of various objectives and the domains in which these objectives fall are given in the Table 2-1.
<table>
<thead>
<tr>
<th>Objective Domain (w.r.t impact)</th>
<th>Economic</th>
<th>Social</th>
<th>Environment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tour</strong></td>
<td>Cost</td>
<td>- Risk (Perceived)</td>
<td>- CO₂</td>
</tr>
<tr>
<td></td>
<td>Distance</td>
<td>- Population Coverage</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Number of violated constraints</td>
<td>- Equity among population</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Reliability</td>
<td>- Equity in regions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Regional Compactness</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Time</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Profit</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Node</strong></td>
<td>Time window</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Missed / split delivery</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Customer-driver relationship</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Resources</strong></td>
<td>Fleet Size</td>
<td></td>
<td>- Equity among Drivers</td>
</tr>
<tr>
<td></td>
<td>Vehicle Utilization</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Driver/Vehicle wait Time</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Capacity utilization</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Risk (to resources)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Efficiency of workforce deployment</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Transported goods/People</strong></td>
<td>Perishability of products</td>
<td>- Make span (Dial-a-ride)</td>
<td></td>
</tr>
</tbody>
</table>

By looking at Table 2-1, it is evident that the majority of the objectives in the classical literature are in the economic domain. This indicates the relative importance of these objectives from decision maker’s perspective and the direct or indirect impact of these objectives on the monetary performance of the logistics company. The increase in consideration of objectives in the social domain is probably due to the enhancing of legal & regulatory requirements. The inclusion of environmental concerns in the corporate agenda, in the past one decade, has also added another dimension to classical VRP. Though literature dealing with the environmental domain is comparatively limited but is growing with the passage of time. Some objectives could overlap in two domains. For example, wait time by driver when converted into monetary terms falls in the economic domain; however, when considered in terms of fatigue/boredom (from driver’s perspective) then it could fall in social domain.
2.2.3.b **Some Important Objectives**

The following are a few of the important objectives that are considered in literature dealing with multi-objective optimization in retail and non-retail contexts. Objectives rationale, facets and definition of objectives are also provided.

2.2.3.b.i **Distance related objectives**

As shown in Figure 2-2, distance related objectives include the minimization of total distance travelled by all vehicles and maximum distance travelled (make span) by any vehicle. Distance travelled is directly related with the total travel time and fuel consumed. As drivers’ wages and fuel consumed are two big expenses for any logistics company, therefore, considering reduction of distance will automatically result in the reduction of overall cost [78]. Perhaps that is the prime reason for making distance as one of the highly modelled objectives considered by researchers. This objective has been formulated in two ways in literature. A simple and most common way is to add up the total distance by all vehicles [6][10], [24], [46], [47], [50], [54], [61], [67], [75], [79][88] and another way is to formulate it as average distance by all vehicles [89], [90]. The problem with the latter formulation is that if vehicle used is a decision variable then it could lead to an increase in the number of vehicles thus increasing the fixed cost associated with vehicle usage. In the case of the distance constrained VRP, an upper limit on the route length is set to fulfil regulatory requirements or other service related reasons. Longer distance journeys on one hand increase stress in drivers that subsequently increase chances of an accident or affect a driver’s satisfaction level; and on other hand are associated with quality of service. For example longer routes may cause tiredness of passengers in the case of a bus routing problem or may lead to perishability of short shelf-life products. Instead of making make-span as a constraint, a few researchers have made it as an objective [90][92]. This is done, perhaps, to explore different possible trade-off solutions which otherwise couldn’t be obtained if make-span is made a constraint.

![Figure 2-2: Distance based objectives](image)
2.2.3.b.ii Cost related objectives

For long-term profitability and sustainability, reduction of monetary cost plays a significant role and this has made measurement and controlling of cost an important part of any logistic managers’ job. Drivers and vehicles are two important resources, so every effort is made to reduce cost associated with these. Vehicle related costs, could be split into fixed and variable costs. According to FTA report [28], fixed cost amounts to 12% of vehicle operating cost and variable cost is around 50% of total vehicle cost. Often long-term averages are taken to calculate cost per unit distance for modelling purpose [33], [93], [94]. An important component of variable cost is fuel consumed which is proportional to distance travelled; therefore, modelling of distance in the objective function indirectly models fuel consumption. Though, fuel consumption also depends on other factors, therefore few researches have included cost of fuel separately in the objective function [49], [66], [95], [96]. Drivers’ related cost is approximately 25% of total vehicle operating cost. Though, costs per unit travel and service time are used as a proxy to model driver’s wages; however, wait time by drivers is often ignored in literature. As waiting incurs cost therefore few researchers have accounted for it in their models [81], [84], [94], [97]–[102]. Additional costs that have been modelled by researchers include remuneration [41], [66], [72], [99], overtime [103], administrative [104], diversion [105], deadheading [59], carrier compensation cost [106], cost of adding new vehicle in case of disruption management [107], and docking and multimodal costs [53], [108].

To outperform competitors, many companies now make combined decisions with other actors – either in forward or backward direction – in their supply chain. Better integration in the supply chain leads to significant cost reduction and improved customer services [44]; therefore, now-a-days managers are often tasked with reducing the cross functional costs. This is particularly common in companies that are vertically integrated. That is they own not only manufacturing plants, but also distribution centres, logistics operations and distribution channels i.e. retail outlets. If a company is inclined to focus on key competencies, then they may choose to outsource transportation and inventory management [36]. Holding inventory in outlets incurs cost and to effectively manage it one needs to look at the longer time-horizon. Focusing on minimization of transportation cost only for individual periods may increase inventory holding cost which is not desirable. Minimization of costs in an Inventory routing problem, by looking at system-wide and multi-period picture,
has been the subject of many researchers. For example [32], [33], [35] have modelled situations in which the outlets have a constant and deterministic demand rate for a single product. The objective was to reduce the average transportation cost of a homogenous fleet and holding costs at outlets without causing stock outs during the whole planning horizon. Researchers in ref [45] have tackled a similar problem but with multiple products and a heterogeneous fleet, while [34] solved the distribution of petrol by using multi compartment vehicles.

![Cost based objectives diagram]

**Figure 2-3: Cost based objectives**

In the case of LRP problems, one needs to make the facility location and routing decision together. So in addition to the routing cost, facility opening cost was also modelled by researchers [38], [39], [42], [44]. In case of Location routing and Inventory (LRI), facility opening cost, inventory holding cost, and routing cost are
considered simultaneously [40]. In IRP, LRI and LRP problems, decisions in forward direction are integrated; however in the case of the production routing problem, the decisions in the backward direction are jointly taken. That is, production and distribution plans are made simultaneously. It is worth noting that this integration gives fruit only when production cost and transport cost are comparable and demand is comparatively stable [65].

2.2.3.b.iii Customer Satisfaction related Objectives

Satisfying customer needs is an important part of any business. This objective has been defined from different angles. Different facets of this objective include quantity delivered to customers, timings of delivery, length of distance travelled by customers to get the service, or customer-driver relationships. Figure 2-4, provides various facets of this objective. The ones that are strongly considered by practitioners in retail distribution are shown in italic.

The most common way of defining customer service is in terms of time. A customer is considered satisfied when service is provided within his specific time-window. Tardiness which is service after the latest arrival as a measure of customer dissatisfaction has been subject of many researchers [68], [98], [109]–[111]. This objective can be measured as a sum of tardiness by all vehicles [112], or by adding a penalty cost for being late [113]. Minimization of maximum tardiness has also been modelled in ref [113]. In case of on-line deliveries or dial-a-ride problems, the lower bounds of time-windows are considered as the most desirable times. Wait time which is the difference between lower bound and actual service time is construed as quality of service. In ref [62], wait time by customers is treated as an objective. Wait time by driver, if a vehicle arrives before earliest arrival time, costs money so in ref [92] it is treated as an objective. In [114], the vehicles were allowed to make deliveries outside the time windows, but sum of difference in arrival before lower time bound and arrival after upper time bound were treated as customer dissatisfaction. The same objective was optimised in [115] in the context of a Dial-a-ride problem.

In the above-mentioned problems, violation of time-windows only led to customer dissatisfaction which usually results in financial consequences; however, in some real-life cases this could lead to serious safety issues. For example, in [116], an interesting problem related to a de-icing vehicle routing problem at a Scandinavian
airport was considered. Before take-off, ice and frost needs to be removed from aircraft as any thin layer of ice may have detrimental effects on the lifting force and control of an aircraft. Specialised vehicles are used to perform de-icing in specified time-windows before flight departure. De-icing long before take-off time will result in formation of fresh ice on an aircraft body; whereas de-icing later than the scheduled time may lead to flight delays. One of the objectives was to reduce the overall delay where delay was calculated as the difference between service finish time and scheduled flight departure time. Another interesting application is described in [117]. In this problem delivery of medical records that are to be picked from storage location to doctors before scheduled appointments with patients is modelled. If the medical record does not reach the doctor on time then it could cause delay or rescheduling of appointments thus resulting in patients’ dissatisfaction.

Figure 2-4: Customer Service / Satisfaction related objectives

Journey time in case of bus routing problems is usually perceived as a proxy for quality of service [93]. Excess journey time is a particular concern in the case of school bus routing, therefore, researchers tried to minimize it by formulating it in different ways. For example, ref [51] formulated it as total travel time spent by all
pupils at all pickup points. In ref [118] average customer time was taken to measure the quality of service. In ref [119], [120] the longest time that any student stays in a bus was used. Whereas, in ref [72], [105] modelled total excess riding time by adding the difference of riding time and time of direct route from pickup point to school. Another interesting application of a bus routing problem in a Chinese industrial campus is due to [105]. Since in off rush hours, the density of customers’ requests is very low, therefore a strategy to follow flexible routes was adopted. Whenever a new customer request came then the bus route was diverted from its existing route and this increased the journey time of existing passengers on the bus. So in their model, they considered different customer services measures such as average wait time by new request, average excess riding by existing customers on board, and new request response times. Another interesting customer service related problem is due to [72] in which researchers addressed the problem of transporting disabled children from home to a specialised school. If children arrive earlier than the school opening time then an extra carer is needed. To cope with the situation, in their research in addition to average ride time and average excess ride time, they also considered wait time at the destination as one of their objectives. Researchers [121] modelled and solved the problem of supplying fodder to farmers in Sweden. When placing an order, customers also specified consecutive feasible days in which service was required. To reduce the overall wait time by customers and to ensure that deliveries were made closer to first feasible days, a penalty cost was associated that increased polynomially with wait time. This was done to yield solutions that favoured shorter waiting times for many customers instead of longer wait times for few customers.

 Provision of efficient healthcare services is usually a part of the manifesto of governments. In rural settings of developing countries, distance between patients’ homes and facilities is a main factor contributing to the utilization of healthcare services. In Ref [122], a situation pertaining to mobile healthcare was presented in which average accessibility was made as one of the objectives. Average accessibility was the average time required by an inhabitant of a population centre to the nearest mobile healthcare facility.

 When a natural disaster hits an area, then provision of relief goods to victims is of utmost priority for governments. To mitigate further damage and loss to human life, essential supplies need to be delivered to affected areas in sufficient quantities
on ASAP basis. While transporting relief goods, it is made sure that high priority goods reach on time. Researchers in ref [64] have modelled a situation in which due to limited numbers of vehicles, supplies were needed to be sent over a period of time. The objective of their model was to minimise the total unsatisfied demand especially for high priority items. Similar objectives have been used by [18], [123].

Good driver-customer relationships can lead to customer retention and increase in sales in the long-run; therefore, some companies encourage their drivers to establish good relationships with their customers. This is often achieved by sending the same driver to the same customer. When demand is low but frequent then it might not be economically viable that the same driver serves the same customer. Ref [48] modelled a problem in which the trade-off between driver-customer relationship and other objectives was considered. In multi-period time horizon, driver-customer objective was measured by taking a weighted sum of the difference between timings of two consecutive deliveries made by the same truck to the same customer. To ensure that at least the best customers are served repeatedly by the same drivers in the same planning horizon, total demand by a customer was used as weight.

2.2.3.b.iv Fleet and other efficiency-based Objectives
Number of vehicles is the most commonly modelled resource. Interestingly, when minimization of the number of vehicles is made as a sole objective then it may lead to multiple solutions having the same objective value; therefore, in most of literature researchers either made it a constraint or used it as an objective along with other objectives. Distance [6], [9], [54], [61], [67], [81], [97], [124]–[127], time [10], [51], [52], [80], [128], [129], customer satisfaction [51], [119], [120], make span [63], [130], [131], equity among drivers [51], [55], [80], [89], [132], coverage [17], regional compactness [55], risk [16], and CO₂ emissions [61] are used as additional objectives along with fleet minimization.

![Figure 2-5: Fleet and other efficiency based measures](image)

One of the issues, faced by practitioners, concerning fleet is how to efficiently utilise the capacity of vehicles. Capacity utilization has been defined in different ways. For example, ref [89], [133] tried to maximise the sum of ratios of weight
loaded on each vehicle to capacity of that vehicle; whereas ref [134], reframed the capacity utilization into non-full loaded factor and tried to minimise it. Non-full loaded factor was defined as sum of (1- ratio of load to capacity) of all vehicles. In ref [35] the researchers converted unutilised capacity into cost and tried to minimise it. When a homogenous fleet is used then maximizing capacity utilization automatically leads to fleet size minimization; however, in case of a heterogeneous fleet, maximization of capacity utilization may lead to solutions in which more vehicles of lesser loading space are used up to their maximum capacity, whereas the same amount of load could be transported by using less vehicles of higher load capacity. A better strategy is to consider both number of vehicles used as well as capacity utilization together. In ref [101], [135], [136], researchers modelled to maximise not only the load focusing degree but also space utility. Load focusing degree is the proportion of total vehicles that are not used and space utility is the average of load to capacity ratios of all vehicles that are used. An advantage of this arrangement is that minimization of vehicles used and adjusting of vehicle types is automatically done.

In mobile healthcare services, patients are generally least concerned with how much time medical staff spends on non-medical tasks such as travelling or camp setup time; rather they are more interested in how much time medical staff spends on attending the patients. From a government perspective, the actual time spent on checking or attending patients is quite important in the provision of basic health service. In ref [122], researchers modelled a health routing problem in which the effectiveness of the workforce was considered in conjunction with other objectives. The effectiveness was measured as the ratio of working time in the provision of healthcare to customers to total routing time.

2.2.3.b.v Time related Objectives

In section 2.2.3.b.iii, time related objectives that measure customer satisfaction are described only; however, in this subsection non-customer service objectives that are related to time are described.

One of such objectives is driving time. Since, it is a common practice by drivers to turn off the vehicle’s engine when waiting or unloading goods; therefore many researchers have used driving time in lieu of total time that includes wait and
service time to make models simple. For example, in ref [61], the focus of the paper is on modelling of fuel consumption so only diving time was considered while calculating total time of all vehicles. Interestingly this simplification might not be correct in case of UAVs as it is not possible to turn off engines while waiting or performing some operations. Perhaps for simplicity in ref [130] just driving time is considered in model.

![Diagram](image)

**Figure 2-6: Time related Objectives**

Another time related objective is the wait time by drivers at customer locations that is considered in different models. One of the reason to consider it as an objective is that as it is a part of driver’s duty so it incurs cost which is to be minimised. It has been formulated as either by taking a sum of wait times by all vehicles [81], [84], [97] or by taking average [69], [137]. In the context of UAV routing, the reason to model it as an objective is different. If a UAV on a mission arrives earlier than the actual time, then it has to hover over the target area and hence chances of getting spotted by an enemy increase. It is therefore treated as an objective in the work of [131].

Since wait time and service time are part of driver’s duty and have cost and time implications, therefore, these have been included while calculating total time by all vehicles by many researchers [7], [29], [51], [52], [80], [88], [114], [129], [138]. When hazardous material is transported then the longer is the total route times, the longer is the risk of exposing hazardous material to the public [10], [15], [139], [140]. So reducing total time may lead to reduction in duration of risk exposure. In
the case of distribution of relief goods, shorter the total time required implies that distribution of relief goods is done on quicker basis [64], [96].

Another time related objective is the make-span of any route. Perhaps the most prominent reason to model route duration is that it is often associated with a driver’s duty time and it could have implications for depot opening times; therefore make-span has been modelled by different researchers [86], [92], [141]. Another reason for considering this objective is reported in application of relief goods distribution in disaster affected areas. Minimizing the route duration of any vehicle ensures some kind of fairness and efficiency. That is, it ensures that relief goods in all areas arrive within a certain time [142].

Objectives that are considered important from a retail distribution perspective are shown in italic in Figure 2-6.

\[\text{2.2.3.b.vi Equity related Objectives}\]

Maintenance of equity among drivers is an important social issue. Not only does it carry significance from an HR perspective but also it is important in the provision of excellent customer services. Figure 2-7 shows the equity related objectives that are used in multi-objective VRP literature. Equity related objectives that planners, in the retail sector, really consider while making routing plans are shown in italic.

Maintaining equity among drivers is the most commonly modelled equity related objective. This can further be sub-divided into workload or remuneration related equities. Workload related equity has been measured in different ways. One of such ways is in terms of distance. For example, in ref [24], [79], [143], it has been calculated as the difference between the longest and shortest routes. While others measured it as the sum of squared deviations from the mean [144] or the difference between the longest route and the mean route [132]. The second way to define workload equity is in terms of the physical load carried by vehicles. Maintaining load equity may lead to robust solutions i.e. any unpredicted demand could be accommodated in balanced plans which otherwise could not be incorporated in unbalanced plans (i.e. few vehicles are fully-loaded vehicles while others are partially loaded) [145]. Furthermore, in a few real-life applications drivers may be responsible not only to unload from vehicles but also to carry it in their hands and deliver it to shops. Balancing of physical loads is usually performed in multi-period
time horizon, and is calculated by taking the standard deviation or the sum of absolute deviations of load transported [48], [64], [121]. The third way of defining workload equity is in the form of travel time. Again it has been measured differently e.g. difference of longest and shortest time routes [38], [51]; sum of deviations of all routes from the shortest route [12]; and standard deviation of route time [146]. Most researchers calculated equity by looking at one aspect, however few researchers considered more than one aspect simultaneously e.g. equity in load and time [38]; and equity in distance and load [82], [147]. Maintaining equity among drivers’ workloads is also significant from a customer services point of view. For example, in a school bus routing problem, it is generally the responsibility of the driver to ensure that all pupils get off or get on the bus safely. So balancing the number of pupils on different buses ensures the equity in customer service time [51].

In a few real-life applications, drivers may also do sales on behalf of companies and are paid commission proportional to their sales. Contrary to the previous case where a driver has to carry loads to shops with his hands and drivers usually prefer to have as little load as possible; in the sales of goods case, drivers wish to take as much load on their vehicles as possible to increase their commission. So in this case another facet of equity among drivers is to balance the number of customers [148] or the amount of goods [149] so as to ensure equity in the drivers’ sales commission.

![Figure 2-7: Equity related objectives](image-url)
Equity in drivers’ workload is usually important in for-profit organizations. In a similar way, maintenance of equity in the provision of service among people served is important for governments or not-for-profit organizations. In the case of disaster relief operations, deliveries of prioritised goods are to be delivered into different population centres as quickly as possible. As resources are scarce therefore it might not be possible to deliver all items in required quantities to all population centres. In ref [64], [150], the difference in satisfaction rate between different population centres is minimised, where the satisfaction rate of a population centre is measured as the ratio of delivered amount to actual demand. Another interesting case pertains to transportation and treatment of hazardous material. If an accident happens during transportation then the population along the road gets affected; whereas if an accident happens in the treatment facility then the lives of the people living around the treatment facility are affected. To reduce the impact of accident risk on the treatment site, the facility should be located as far from the population as possible; while to reduce the impact of accident risk in transportation, vehicles should be routed through safest possible routes. In ref [13], researchers have provided an example in which if total risk is to be minimised then it yields to solutions in which certain roads/population centres will be overloaded with hazmat traffic while less hazmat traffic will be flowing through remaining population centres. So minimisation of spatial inequity of risk in different population centres was used as an objective. It is worth noting that the problems of treatment facility location and hazmat routing are interlinked as choice of location affects the routing plans. So in ref [14], the researcher tried to optimise two objectives of equitable risk distribution and equitable distribution of disutility caused by the operation of treatment facilities. In ref [86] the problem of collecting recyclable waste and delivery to a recycling facility was modelled. One of the objectives considered was to balance the number of collection routes to different facilities. This was done to ensure that each facility gets sufficient amount of waste so as to maintain jobs of deprived people secured who are working on that facility.

While the previous two paragraphs were concerned with maintenance of equity in individuals or in population centres in same region, this paragraph deals with equity between regions. In ref [80], researchers modelled the problem of tobacco distribution in different districts. Due to unbalanced workload, fixed routes proved to be inefficient as well as costly. To solve this issue, average demand of different
customers for a period was taken to model the partition balancing problem and considered balancing the number of routes, distance, time and load for different regions.

2.2.3.b.vii Risk related Objectives
Risk related objectives could be sub-divided into risk to people or risk to resource categories as shown in Figure 2-8. While providing logistics services, not only interests of direct stakeholders (i.e. service providers or service consumers) are to be taken care of but also interests of indirect stakeholders are to be considered. One of such objectives is the minimization of risk posed to the general population when transportation and treatment of dangerous material is performed. Risk posed while travelling has been measured by researchers in different ways. For example, ref [29] calculated risk on a road as a product of probability times the adverse consequence of an event. In ref (191) calculated risk as a product of size of population in the vicinity of road and individual perceived risk, whereas the amount of hazardous material transported through population centres was used as surrogate of individual perceived risk. In ref [15], [16], [139], probability was multiplied with size of population that lived within a specified distance from accident place to calculate risk. In contrast Boffey [13] also combined probability with the risk posed during accident while traversing a link; however, he calculated risk posed to population not only in vicinity but also living in other zones when a link was traversed. In ref [140], total risk was calculated as a product of accident probability, conditional probability of release of hazardous material after accident, conditional probability of incident after release of hazmat material, conditional probability of a consequence after incident, and population in neighbourhood.

![Figure 2-8: Risk related objectives](image-url)
Risk to resources is modelled in context of UAV routing. One of factors that lead to successful mission completion by UAVs is its ability to navigate the enemy terrain without getting caught by the enemy’s air defences. To achieve this objective often UAVs fly at a low fixed altitude in their area of interest. To maintain a fixed altitude requires frequent climbing and descending in rugged areas which is a costly operation. In ref [151], researchers modelled the problem of routing a swarm of heterogeneous UAVs for specific missions and one of the objectives was to reduce the risk of getting caught by enemy’s defences.

2.2.3.b.viii Profit/Sales related Objectives
Profit is the most important criteria to judge the performance of for-profit organizations. In the vehicle routing context, it is very rare that profit is made as an objective perhaps due to the fact it is difficult to consider all direct and indirect costs. There are few examples in which profit is modelled as one of the objectives. For example, in [152], a real-life problem related to extraction of oil by using mobile oil recovery truck is modelled. The trucks start journey from the depot, visit multiple wells and extract oil before returning to the depot at the end of the day. The amount of profit in this case was considered proportional to the amount of oil extracted each day.

In ref [149], maximization of total sales is modelled in context of OVRP. In the modelled problem, sales amount was subject to fierce competition and reaching a customer before a rival was crucial to increase the sales. To calculate the expected sales of each customer, the probability of reaching a customer before a rival was multiplied with the demand. The demand of each customer contained two parts. One part was time-independent i.e. it did not depend whether the customer was visited before or after a rival’s visit; and the second part of demand was time dependent i.e. depended on reaching the customer before a rival’s visit. Probabilities of reaching the customer before a rival required an estimation of rival’s visit time (earliest arrival and latest departure) which was done by using stochastic methods.

2.2.3.b.ix Reliability
One of the assumptions used in formulation of a model is that links in a network are always accessible. This might be true for most of VRP variants; however there are some instances in which access is not deterministic but is rather probabilistic. For
instance, in the case of relief distribution after an earthquake in hilly areas, certain roads might become blocked or inaccessible; however, exact information about this blockage might not be available to transport planners. Sometimes, post-earthquake shocks keep coming for some days which may lead to road blockages due to land sliding. In that scenario, information that certain road segments will get blocked is available with some probability. Distribution of urgent supplies may reduce further damage to human life. Reliability with which supplies can be delivered safely and timely is of utmost importance to disaster response teams and it has been modelled by researchers [42], [142]. Reliability of a link is defined as the probability with which a vehicle can traverse a link in affected areas in post-earthquake scenarios; and reliability of a route is calculated as the product of reliabilities of all individual links in that route [42]. The higher the reliability means that drivers and shipment will reach safely to their destinations. In ref [142], a situation has been modelled in which access to nodes is given in probabilistic form in addition to routes. To calculate overall reliability of transport operations, probability of individual links was determined by looking at historical data and expert judgement from logistics people.

2.2.3.b.x Perishability
Products with shorter shelf lives often require quick delivery from pickup to delivery points. From the retailers’ point of view, perishability or loss of freshness results in reduction in time available to sell a product and hence may lead to loss of profit. Though this objective has been modelled by researchers, direct quantification of perishability is a difficult task for transport planners. Therefore, from a distribution perspective perishability is often linked to remaining shelf lives [153]. In ref [57], [58], perishability was assumed to deteriorate once a certain time lapses after visiting a pickup point. If a product reached its destination before the lapse of that fixed time, it was assumed no deterioration had occurred. In ref [153], researchers assumed that perishability is a linear function of time and products start perishing at a constant rate as soon as they leave their pickup locations. Instead of using the same constant rate of decay for all products, in ref [154], they considered different rates for different products and calculated perishability in a linear way.
2.2.3.b.xi  Regional Compactness

Clustering and combining customers from the same region in one route promotes regionalization. This allows assigning the same driver to the same region and as a consequence of geographical familiarity of the driver, customer service improves. For example, a driver can decipher the illegible or incomplete addresses provided that he is familiar with area [146]. Compactness of a route can be measured by adding the Euclidean distance between different nodes in the same route. Minimizing the total compactness of all routes not only allows clustering of customers automatically but also leads to inter-cluster homogeneity [145]. In ref [146], regionalization in the context of courier company operations was modelled. Courier companies usually divide delivery areas into zones and subzones; and to promote regional familiarity, it is usually preferred that individual couriers make deliveries in their respective subzone. However, due to the dynamic nature of demand and to reduce the costs, a courier may be utilised in more than one sub-zone. To ensure that couriers provide service in their familiar areas, a deviation-based objective that counted the number of zones served by non-designated vehicles was modelled and minimised [146].

2.2.3.b.xii  Deviation-based Objectives

Imposition of strict constraints may, at times, result in infeasibility without providing a clear clue to planners about what actually caused it [128]. To overcome this issue, constraints are sometimes relaxed and deviation from those constraints is measured and used as one of the objective functions. Technically these are not objectives from practitioner’s point of view; rather these provide a way to overcome modelling difficulties. Most common deviation based objectives are related with time windows and capacity violations. For example, in ref [110], [115] [62], hard time-windows were replaced with soft time-windows and deviation from time-windows was made an objective. In ref [155] loading of vehicle more that its capacity is used as objective in the context of a multi-period split delivery problem. Similarly in ref [98], [156], four different violation-based constraints were made as objectives and included 1) Time-window violation measured as the sum of tardiness of all customers, 2) Number of tardy orders, 3) Total amount by which capacity of all vehicles was violated, and 4) number of vehicles that exceeded capacity limits.
Another interesting deviation-based formulation is described in the context of UAV mission planning. Due to changes in battle conditions, modifications in initial plans are inevitable. As excessive changes in the initial plan may undermine or reduce the overall mission effectiveness, therefore, every effort is made that deviation from the initial plan should be minimized. Different changes in the initial plan such as reassignment of resources (UAV), sequence or timings of assigned tasks have been considered by [128].

![Figure 2-9: Deviation based objectives](image)

### 2.2.3.b.xiii Coverage

Coverage related objectives can be categorised into two i.e. coverage of population served and coverage of network/links. However, in both cases the need to model it as an objective arises due to shortage of resources. One needs to make a decision not only about nodes/links to be visited but also about routing.

![Figure 2-10: Coverage related objectives](image)

Most of applications using coverage as objective are in the provision of public services. However, one of the applications in retail distribution is about delivery of perishable goods in an urban context [46]. Due to time-window and drivers’ duty-hour’s constraints it was not possible to serve all customers in a given planning day by using vehicles available in-house. Therefore, a subset of customers was served using an internal fleet and the remaining jobs were outsourced. However, to ensure that the maximum number of customers is served by in-house vehicles, maximization of total gain (proportional to total number of customers served) was used as an objective.

Provision of primary healthcare is one of the applications in which coverage is treated as an objective. For example in ref [17], due to the availability of a limited number of vehicles, maximisation of the number of customers served was used as an objective along with distance. Another healthcare related application in the rural
areas of Ghana is reported in the work of [70]. In their work population associated with each node was taken into account in the maximization of coverage criteria. In contrast to the previous case where the demand of each node was equal to population size, in ref [122], the demand of each node was considered equivalent to the number of inhabitants living within a specified distance. They defined coverage as a percentage of population served; however, in their formulation they assumed demand as a step function i.e. inhabitants within a limit are served and outside are unserved. But, in ref [18], demand of each node was assumed as a decreasing function of the distance of the population centre from the selected facility. At the same time they also took account of the capacity of open facilities which was somehow ignored in previous formulations.

Another application relates to home delivery of prepared meals to elderly and poor [19]. In the case of healthcare applications, information about the demand of each node was available; however, in the case of home-delivery this was to be estimated. Food was prepared in temporary kitchens that had different capacities. Routes were constructed by estimating the demand of different nodes with one of aims to maximize the number of people served. Once a vehicle reached a node, volunteers delivered the food in the surrounding area on foot.

In previous cases, coverage was measured in terms of people; however, coverage has also been measured in terms of time. One of such applications is maximisation of police patrol coverage on highways. Visibility of police on highways or motorways acts as a deterrent for people and reduces traffic violations. In ref [41], the problem of patrol routing on highways was addressed as a dynamic location routing problem. One way to improve patrol efficiency and to keep operational cost to low is to patrol those areas that have a high frequency of crashes. In their model, the overall aim was to minimise total system cost and at the same time to increase the coverage. Coverage benefit was measured in minutes i.e. difference of end of service and start of service of troopers.

2.2.3.b.xiv CO₂ Emissions
Due to increased legislation and awareness, reporting and reducing carbon emissions has become an important concern in logistic companies. Modelling of emissions has
been performed in literature to satisfy different managerial/modelling requirements. Figure 2-11 provides some reasons to model emissions as an objective.

![Figure 2-11: Reasons to include CO2/emissions as an objective](image)

One managerial requirement is to measure the net gains. For example, in ref [157], researchers presented a situation in which there was a cap on emissions released by the fleet; and if the emitted amount exceed that limit then an extra carbon allowance was to be purchased. To calculate net gains, the cost of the additional carbon allowance was made a part of the objective function. The cost of the additional allowance was calculated by multiplying the per unit carbon trading cost with the difference between the fuel requirement and cap limit. The amount of fuel requirement for each vehicle type and for each link was assumed to be known.

To reduce emissions, one needs to explicitly make it a part of the model. To estimate emissions some researchers have used averages. For example, researchers used average emissions per unit distance in [158] and fuel efficiency in [159] to estimate emissions. In ref [73], separate average emission values per unit distance for each vehicle type were used to estimate total emissions for a heterogeneous fleet. In contrast to previous cases where average values per unit distance were used, in ref [160], average emission per unit load was used to calculate emissions; whereas, in ref [49], payload and distance travelled information was used to calculate emissions. Linear regression was applied to find the relationship between load (in %) and average fuel efficiency (litres/distance) which was multiplied with distance travelled to estimate fuel consumed for each vehicle on different roads. Similarly in ref [88], researchers considered distance, payload and speed information.

To estimate emissions, some researchers have used various complicated models which can be sub-divided into 1) average based and 2) instantaneous models. For more details, readers are referred to the work of [21]. Similarly few researchers have also considered real-life network in which average speed of travel on links
keeps on changing because of congestion. For example, in ref [138] a framework was provided to model emissions in time-varying network by using MEET model that linked speed and distance. In ref [161], not only emissions while travelling but also emissions during idling were calculated. Similarly in ref [75], the researcher considered the time-varying speed on different roads.

Some researchers also used speed as a decision variable. For example [162], speed was used as decision variable without considering time-varying congestion. Their work was extended in [66] by initially generating solutions based on average speed but in the next stage optimised the speed to reduce emissions. In ref [163], researchers modelled departure time and speed on each road as decision variables. Qian [164] in her PhD research considered fuel minimization as the primary objective by considering speed optimization on time-varying network. Similarly, in ref [85], a real-life recyclable waste collection problem was modelled and CO₂ emissions were calculated by estimating the energy requirement.

In contrast to the above-mentioned literature where emissions was modelled either to calculate net gains or to actually reduce emissions; one researcher used as an objective but the purpose was to come up with a solution that either minimized the distance or vehicles [165].

The following graph (Figure 2-12) displays the number of times different objectives appear in the multi-objective literature that have been reviewed in this paper. Perhaps it can be seen that first five objectives fall in economic domain as these are directly linked with the profitability of a company. Three out of the next five pertain to social domain, while only CO₂ emission is from environment domain.
2.2.3.c  Practitioners views about different objectives

To find out which objectives practitioners value more and why, the following responses, from a leading consultant having more than 26 years of consultancy, development and implementation of VRP software in UK, were received.

<table>
<thead>
<tr>
<th>Equity among drivers</th>
<th>“I wouldn’t view it an objective as such. They (customers) care only to the extent which got to be sufficiently equalled”…. “Satisfying to a certain level.”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compactness</td>
<td>“Compactness is not an objective. It is a way in which people try to evaluate the quality of solutions. There is nothing intrinsic to the company that it is good or bad.”</td>
</tr>
<tr>
<td>Profit and Cost</td>
<td>“They (planners) can’t look at profit; they will look at cost because it is easier to look at.” …. “And they measure cost”… “includes number of vehicles,</td>
</tr>
</tbody>
</table>
Vehicle utilization

“Vehicle utilization is another way of measuring (the) quality of solutions. It is not intrinsically an objective.”

Customer Driver relationship

“Customers – driver (relationship) is probably another valid secondary objective”

Number of violated constraints

“When we have soft time windows, we put that in cost function. We pre-define it as part of cost. It is no longer a constraint or objective as such.”

CO₂ emissions

“They(emissions) just have to look good. The level of awareness in our customers about CO₂ is virtually negligible.”

Speed optimization

“It is impractical and it is fundamentally wrong concept for road traffic.” ..... “If you drive a truck slowly along the road, you are going to cause problem for other people. You will have a negative impact on overall CO₂. Your truck might be OK but everyone else CO₂ (will be) worse.”

From the interview, in the first look, it appears that there are some gaps between academia and real-life problems. For example, from Figure 2-12, distance appears to be the most important objective in academic literature; whereas in practice, fleet size is considered the most important objective. Retail companies by definition are for-profit companies, so highest emphasis is given to the minimization of cost while making routing plans. Even when cost is used as a primary objective, the cost function is hugely dominated by the number of vehicles followed by distance and time. Few of the measures that are explicitly treated as objectives in academic literature, are actually used in real-life to specify the quality of solutions. Perhaps, if academics are interested to solve real-life problems then they have to use these measures in a similar way as used by the practitioners. Another disconnect that appears to be present is about the awareness level among practitioners about objectives that could become very critical in future if these are not given due attention now. As practitioners are usually busy in finding solutions to day-to-day problems; therefore, they might not be aware of problems that may pop up in future. For example, it appears that practitioners do not care too much about CO₂ emissions.
at present; therefore, it is probably the task of academia to spread awareness about
importance of this objective.

2.3 Multi-Objective VRP

The majority of the classical literature related to VRP deals with a single objective;
however, in the past one decade, there is an increasing trend to accommodate more
than one objective at the same time. The following subsections provide some
information about reasons for considering multiple objectives and relationships
between different objectives.

2.3.1 Why consider multiple objective

Considering multiple objectives in the problem is done either to extend the classical
problem, to generalize a classical problem, or to accommodate the real-life problems
[166]. Figure 2-13 summarises different reasons to consider multiple objectives while
solving VRP problems.

![Figure 2-13: Different reasons to consider multiple objectives simultaneously in VRP problems](image)

In classic VRP literature, usually a single objective is considered. Extension of
classic VRP problems has been done by researchers due to various reasons, a few of
which are as under:
- Generally the objective is to reduce some solution cost such as distance, time, or monetary value. These objectives usually cater for the interest of a single functional unit in an organization and/or often consider monetary aspects only. However, in real life, routing of vehicle has cross-functional implications. For example, routing plans are related with drivers’ duty timings/pattern so they have HR implications. Similarly routing plans may affect customer services so they have marketing implications as well. So to enhance applicability, additional objectives are considered in addition to classic objectives. Often this is done without making significant changes in problem formulations. Readers are referred to the work of ref [22] to see some examples related to extension of classic problems into multi-objective problems.

- Another reason for adding objectives is to differentiate between multiple equivalent solutions by looking at another aspect simultaneously. For example, if vehicle minimization is used as an objective, then it may result in multiple plans with the same number of vehicles but with different values of distance travelled. In ref [165], the reason to add CO₂ minimization as an additional objective along with distance and number of vehicles was to obtain solutions that either improved distance or number of vehicles.

- Another reason could be that few objectives when optimised alone do not lead to intended or desired solutions; therefore these objectives are to be optimised in conjunction with other objectives. An example could be drivers’ equity which is usually defined as the difference between longest and shortest routes. If optimised alone then it may lead to solutions in which equity is high, but all journeys are long. A suitable combination would be to minimize total travel time by all vehicles in addition to maintaining equity.

- Another advantage is that it may reduce solution generation time taken by the solver as it will converge in less number of iterations.

Classic VRP problems have also been generalised by adding objectives which otherwise have been considered as constraints by other researchers [13]. For example,
in ref [115] hard time-windows were replaced with soft time-windows and degree of violation of time-windows (tardiness) as a surrogate to customer service was minimised. A few other deviation based objectives are given in subsection 2.2.3.b.xii. Multiple motives are attributed for replacing the constraint with an objective. A few of these motives are as under:

- For example, tardiness is used as an objective to acknowledge the limitation of a logistics provider to adhere to customer-defined time-windows. This limitation could arise because of shortage of resources or due to demand/network structure and due to these reasons usually companies aim to maintain customer satisfaction above a certain level for a longer period of time [167].

- Another motive is to broaden the search by relaxing the constraint and penalizing violations in the objective function. In this way feasible solutions could be found which otherwise would not have been explored in the presence of hard constraints (e.g. time-windows) [11].

- Another motive is to provide flexibility in modelling i.e. there is less need to design special operators or heuristics to check solution feasibility or to repair infeasible solutions [156]. Making constraints as objectives also provide options to decision maker to make trade-off between resources and service related aspects [167].

- Furthermore, it was very easy to switch from the general problem to the original classic problem by imposing a large penalty for violating time-windows.

Another motive to consider multiple objectives simultaneously in VRP formulation is to solve real-world problems, a few of which are mentioned in subsections 2.2.1.a and 2.2.1.b. These real-world problems, by and large, are inherently multi criteria in nature due to various reasons. A few of such reasons with examples are described as below:

- The foremost reason is that as solutions are to be implemented therefore solutions should satisfy all stake holders which may have differing perspectives and conflicting objectives [29]. Any potential solution should satisfy direct as well as indirect stakeholders’ interests; otherwise chances
of successful implementation will be limited. For example, when transportation of dangerous goods is planned by looking at interests of logistics provider then most economically viable routes will be preferred. However, these routes might be risky from a safety point of view which is a concern of government and general public. Due to sensitivity and potential impact, both cost and risk need to be considered simultaneously [29], [139]. Another example, modelled in [159], is related with production and distribution of dairy products. Since the dairy industry is highly regulated and is subject to environmental legislation; therefore, consideration of emissions in addition to cost is imperative in all aspects of production and routing.

- In today’s business environment, success of a company depends not only on its own performance alone but also on how well supply chains, which that company is part of, compete with rival supply chains. Interconnectedness in inter and intra-company operations increases the long-term competitiveness of a company. In a supply chain various processes such as production planning, inventory management, routing, facility location, and crew scheduling are interlinked [38]. Due to the complexity and hierarchical nature of these processes, usually decisions about these processes are taken independently [39]. However, a wrong choice in one process may affect the success of subsequent processes. Due to interlinking of these processes, decisions need to be taken at the same time.

- Some researchers used decomposition methods to solve the interlinked problems. However, there may be real-world applications in which due to legal or other contractual obligations, decomposition of problem is not possible. In these cases, different aspects/objectives of inter-related problems need to be considered together. For example, in ref [104], provision of rehabilitative services to home-bound patients was considered. The aim was to construct weekly routes for multi-skilled therapists so as to serve patients while optimizing routing, treatment and administrative costs and at same time ensuring that customers service, labour laws and other contractual obligations are fulfilled. Due to the complication of labour law
rules, it wasn’t possible to decompose the problem so weekly plans were constructed by considering all different aspects.

- In ref [53], a multimodal problem was presented. While doing long-haul transportation, freight had to be transported through multiple modes. When multiple modes are considered then due to different schedules, frequency, limitations, and cost structures of different modes, the number of possible solutions becomes quite wide. In that context, to compare these solutions and to present it to end-customers so that they can pick their preferred mode, multiple criteria are considered.

- Choosing a single solution for implementation is quite political in nature and often demands justification. By considering multiple objectives and generating non-dominated solutions thus assists decision makers to cope with problem complexity [122]. Different solutions could be visualised and trade-offs could be evaluated and thus it renders political acceptance to the chosen solution.

The following graph shows the interest level of people in multi-objective VRP over the last three decades.

![Figure 2-14: MOVRP Literature over a period of time](image)
The above two graphs are drawn by using literature that is considered in this research only. Obviously this literature is not exhaustive. As it can be seen from Figure 2-14, interest in MOVRP has increased in the last one decade. From Figure 2-15, one can see that majority of researchers have considered either two or three objectives simultaneously. In an objective function when different criteria that conflicted with each other but are usually measured in monetary terms, were observed these were recorded as a single objective in Figure 2-12. For example, holding cost and routing cost both are recorded in monetary terms but conflict with each other.

In order to find out the objectives that are deemed important by practitioners, secondary data was collected from the website of a leading routing software provider i.e. Paragon Software Systems. The sample data contained information about 55 companies that are operating, in different sectors, mostly within the UK. These companies, after implementation of the off-the-shelf routing software, have posted their success stories on the Paragon’s website. In these success stories not only the changes observed in the value of different objectives/KPIs before and after the implementation of software; but also impacts observed on other related processes are reported. Information about self-reported KPIs/objectives of each company and sector in which that company was operating was then recorded in an excel sheet. The information reported by the companies operating in the same sector was grouped together. Similarly, the self-reported KPIs and objectives were grouped into economic, social and environment domains and are given in Table 2-2.
<table>
<thead>
<tr>
<th>Sector / Industry</th>
<th>Economic</th>
<th>Social</th>
<th>Environment</th>
<th>Fuel Cost or CO₂</th>
<th>Other factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distance</td>
<td>Time</td>
<td>Cost</td>
<td>Vehicle</td>
<td>Driver</td>
</tr>
<tr>
<td>Agriculture / Animal Feed</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Distribution/3PL</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Healthcare</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>Home Delivery</td>
<td>4</td>
<td>6</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manufacturer</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Oil &amp; Gas</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parcels/Post/News</td>
<td>7</td>
<td>8</td>
<td>13</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>Retail</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Service Management</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Waste Management</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Wholesale</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>11</td>
<td>1</td>
<td>24</td>
<td>37</td>
<td>12</td>
</tr>
</tbody>
</table>

The following conclusions could be drawn from Table 2-2.

1. Majority of the self-reported KPIs/objectives fall in the economic category.
2. Importance given to different objectives by practitioners varies from sector to sector. For example, in retail distribution sector, the most frequent self-reported KPI/objective is the fleet utilization followed by the customer service; whereas in the case of healthcare (delivery of medicines to patients’ homes), the highly frequent KPI/objective is the making of deliveries within the customer-specified time-windows. One can argue that as the overall aim of retail distribution companies is to maximize their profits, therefore more attention is given to minimization of number of vehicles used rather making deliveries on time. In contrast, in healthcare sector, the primary aim is maximising the welfare of patients, so more emphasis is given to meeting patients’ demand within specified time-windows.
3. If we look at all sectors, then the most important self-reported KPI appears to be fleet size, followed by meeting customers’ expectations (i.e. delivery within time-windows).

4. The average number of objectives per company is almost 2.43. The frequency distribution is as under:

Table 2-3: Frequency distribution of self-reported KPIs.

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>

While responding to the question of “How many objectives, people actually consider or are concerned about before purchasing VRP software?”, then following response was given by an expert analyst.

“In pre-sales presentation, we typically go as far as vehicles and distance. They know these numbers. It is rare that they will have time (total travel time) information. It is a poorly recorded statistics. Sometimes they talk about driver’s overtime.”

The answer to the question, of “Why they consider these objectives?”, was very interesting.

“Other drivers complain about it (if another driver gets extra overtime). So it is in consideration. They would not ask you for something to optimise for which they don’t have some conceptual measure.”

2.3.2 Relationship between objectives

When multiple objectives are considered then it could lead to three possible scenarios i.e. the objectives are in 1) total concordance, 2) total conflict or 3) partial conflict [168]. When objectives considered in model are in total concordance then the problem could be treated and optimized as a single objective problem. That is, optimising one objective will automatically optimise the other objectives. When
objectives are in total conflict then all feasible solutions also become optimal [168]. However, the majority of the real-life VRP problems fall in the partial conflict category and thus leading to a set of potential solutions.

Usually a combination of important objectives such as number of vehicles used, distance, time, customer services (on-time delivery), drivers’ workload balance, and emissions are considered simultaneously while making retail distribution plans. These objectives conflict with each other in a non-linear way. In any realistic situations, faced by planners that are engaged in making urban delivery plans, time-varying congestion and presence of delivery time-windows are mostly there. This further enhances the conflict between objectives such as CO₂, distance, and travel time. As a consequence, knock on effects can be observed on other objectives. This topic is specifically addressed in the next section. However, in the following paragraphs the relationship between other objectives is described.

Vehicles are usually considered the most important resources; therefore, majority of planners wish to optimise it more than any other objective. It has been shown by researchers that when the number of vehicles is minimised then it could lead to increase in total distance travelled and vice versa [54], [83]. Additionally reducing total distance may result in a situation in which imbalance between routes increases i.e. distance travelled by one vehicle is significantly higher than that of other vehicles thus leading to workload inequity among drivers [24], [79], [82], [143], [147]. Conversely if one tries to reduce workload imbalance (in distance terms), then it can increase the total travel time by all vehicles [169] or total distance travelled by fleet [24], [82], [143], [147] and thus can increase total cost [38].

In the context of hazardous material delivery, distance minimization objective may lead to solution in which shorter paths are adopted that pass through population centres and thus increase the overall risk of an accident. If risk is optimised, then it could lead to longer distance [139] and hence increases total travel time [29], [140]. Similarly while distributing relief goods in disaster affected areas, if emphasis is high on maximization of coverage or maintaining equity between different regions then it could result in increase in total cost [19], total time [57] or distance [46].

Similarly maximisation of customers’ satisfaction requires deliveries to be made within specified delivery time windows. This can be achieved by using more vehicles, thus resulting in an increase in number of vehicles [94] required to provide
the deliveries and consequently resulting in an increase of total distance travelled by all vehicles [81], [84]. Vehicle related costs are the most dominant component of total routing cost; therefore, cost minimization would imply that fewer vehicles will be used to do same deliveries. This will increase the distance travelled and thus will result in loss of quality of perishable product [154]. A similar phenomenon could be observed when routes for school buses are planned. Usage of fewer buses could result in longer journey times for pupils and lowers the customer satisfaction [51].

2.3.3 Interplay of Travel time, Distance and CO\textsubscript{2} emissions under Time-varying Congestion and Time window restrictions

Fluctuation in road traffic conditions and presence of time-windows may lead to situations in which conflict between different objectives becomes pronounced. For example, if speed of travel in a road network is assumed to be the same throughout a day on all links, then the shortest path with respect to distance between any two vertices will also be the shortest path with respect to time. Since CO\textsubscript{2} emissions are highly correlated with distance; therefore, the shortest distance route will also be lowest emitting route. In this case (i.e. presence of no time windows, same speed on all links and static traffic conditions), three objective of CO\textsubscript{2}, distance and time minimization will be in concordance with each other and minimizing one objective will automatically minimise other objectives. However, in reality, the speed of travel is not same on all links. Even speed on the same link is dynamic [170]. This implies that in the presence of dynamic road network, the shortest route between any two nodes could change if the objective is changed. [88]. CO\textsubscript{2} emissions from a vehicle not only depend on distance and speed but also depend on the amount of payload [171][172]. Therefore, if CO\textsubscript{2} minimization is considered as primary objective then it may lead to different solutions as offloading of heavy load first will help reduce emissions [173]. In VRP context, this may result in a change of delivery sequence within the same route or allocation of customers to other vehicle routes. Consequently, these changes may result in an increase in total time and distance travelled.

Many researchers have considered CO\textsubscript{2} minimization as an objective along with other objectives [49], [61], [66], [85]–[87], [138], [157], [161], [169], [174], however literature that models distance, time and CO\textsubscript{2} appears to be very limited.
One of such example is due to Kuo [88]. In his research, he used benchmark instance, five time-zones and three different artificial speeds. Calculation of fuel consumption was performed by considering the laden load and speed of travel. His results highlighted the conflict between three objectives. However, his results would have been more realistic if he had considered customer’s delivery time windows.

Time-varying congestion results in uncertainties in meeting the delivery time windows and consequently has some implications. One of such implications is the reduction in customer satisfaction and logistics operation’s ability to accommodate urgent customer call-out and last-minute changes. Increase in customers’ satisfaction has a direct effect on customers’ retention and the long-term profitability of the company. Another implication could be the stretching of route delivery times, reduction in number of deliveries per run, and inefficient use of vehicles/drivers. Inefficiencies in resource utilization may have a domino effect on the associated processes such as warehouse management and inventory management. Stretching of route time not only causes negative effects on the carbon footprints but also may lead to reduction in shelf lives of products. As a consequence of this customer satisfaction and CSR image is affected. Contingency measures such as giving over-time to drivers to accomplish the job or sub-contracting could be taken to cope with this situation. However, these actions increase cost and thus reduce the profitability. Another implication is the rendering of driver schedules to be unreliable. This could cause inequity among drivers’ schedules and could lead to high job turn over.

2.4 Multi-Objective Optimisation, Methods & Approaches

In order to solve a MOVRP, the solution process passes through three phases which include 1) formulation of the problem, 2) choosing of most suitable method to search solutions, and 3) elicitation of preference from DM so as to select a single solution for further implementation. As mentioned in section 2.1, problem formulation phase requires a thorough understanding of the context, decision variables, constraints and objective function and these have been discussed in subsection 2.2.

The subsection 2.4.2 deals with different approaches used by researchers to incorporate or elicit the preference from DM. And subsection 2.4.4 provides information about most common methods employed by researchers to solve MOVRP problems.
2.4.1 Multi-Objective Optimization

Multi objective optimisation problems can be represented as

\[
\text{minimize } F(x) = \{ f_1(x), f_2(x), \ldots, f_l(x), \ldots, f_k(x) \}
\]

Subject to:
\[
\begin{align*}
& g_i(x) \leq 0 & i = 1, \ldots, m_1 \\
& h_j(x) = 0 & j = 1, \ldots, m_2
\end{align*}
\]

Where \( x \) is a solution vector \( x = [x_1, x_2, x_3, \ldots, x_n]^T \) and \( x_i \) is a decision variable. Objective function \( f_l(x) \) is non-linear and \( g_i(x) \) and \( h_j(x) \) represent non-linear inequality and equality constraints. In multiple objective optimisations there is no single solution that optimises all objectives simultaneously; therefore, the task is to find a set of trade-off or Pareto optimum. A solution \( x \) is said to be Pareto optimum if there does not exist a solution \( x^t \) such that \( F(x^t) \leq F(x) \).

2.4.2 Multi-objective Approaches used in VRP literature

To accommodate multiple objectives, researchers have used a variety of methods. Sometime a single method is used and sometimes hybrid methods are used. At times, more than one method is used to compare the results obtained from different methods. The ultimate objective of any method applied is to come up with a single or multiple solutions from which a decision maker can pick one to implement. So any final solution (if there is any) that is to be implemented requires acceptability of the decision maker. To elicit the preference of a decision maker, some sort of decision maker’s involvement is required while generating solutions. Depending on how decision makers preferences are incorporated in the solution process, methods used by researchers can be grouped into no-preference, a-priori, posteriori, and interactive approaches [175]. Table 2-4 provides a glimpse that the majority of researchers, in the literature considered in this research, have used either an a-priori or posteriori approach to incorporate decision makers’ preferences.

<table>
<thead>
<tr>
<th>Approach</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Preference</td>
<td>3</td>
</tr>
<tr>
<td>A-Priori</td>
<td>77</td>
</tr>
<tr>
<td>Posteriori</td>
<td>70</td>
</tr>
<tr>
<td>Interactive</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 2-4: Frequency of different approaches to incorporate decision maker's preference

The following subsections provide details about these approaches; different methods used in each approach; and shortcoming of these approaches.
2.4.2.a  No-Preference approach

In the no preference approach, it is usually assumed that no decision maker is present. As preference information is not available, the main task is to find some neutral compromise solution. In VRP literature, one can find examples in which the preference from the decision maker is not available, yet one has to find a solution by considering multiple objectives simultaneously.

One of such examples pertains to a school bus routing problem in which researchers considered minimization of the number of buses and maximum route length as objectives [91]; whereas each objective is considered important from point of view of different stakeholders. Reduction in buses is important for the logistic provider from an economic perspective; while minimization of maximum make-span serves the interest of pupils travelling on these buses. Researchers neither aggregated these objectives into a single function nor treated these separately. Instead they proposed a local search heuristic with dynamic self-tuning parameters. Starting from an initial feasible solution $X$, a neighbourhood solution $Y$ was accepted if acceptance conditions were met i.e. $f_1(Y) \leq \alpha_1 f_1(X)$ and $f_2(Y) \leq \alpha_2 f_2(X)$ are true, where $\alpha_1$ and $\alpha_2$ are self-tuning parameters and are based on two criteria 1) the number of local optimum solutions and 2) quality of these solutions. If no neighbouring solution met the criteria, then the values of these parameters were dynamically updated.

Another example is about reducing the production, holding and distribution costs [44]. Economies of scale demand production to be made in large quantities which might increase holding and distribution cost and vice versa. The problem was decomposed into sub-problems which were solved iteratively. Initially a production plan was made by using an integer linear program and was followed by a routing plan by using Tabu search such that it didn’t change the production plan. In next iteration, the production plan was revised by treating the routing plan as a constraint and the cycle continued till both plans became stable.

Another example of the no preference method is due to ref [176] in which the number of vehicles and distance travelled was optimised. To solve the problem, ant colony optimization was used and to avoid the algorithm from getting trapped in local optima, cellular optimization was applied. Separate ant colonies were generated to optimise a different single objective independently. Through the information exchange between different colonies both objectives were optimised.
The advantage of the no-preference method is that one can come up with solutions which do not favour any individual objective. However, a few researchers based on their experience have observed that at times decision makers are reluctant to implement or to utilize these results especially when these pertain to enacting of future policies. For this reason, it is imperative that DM be involved in the solution generation process [122].

2.4.2.b  A-priori approach
In the a-priori approach, decision maker’s preference at global level [177] is sought before solving the actual problem. After elicitation, preference of the decision maker could be incorporated in solution generation process in a variety of ways. A few of these ways and commonly adopted methods in literature are explained in the following subsections.

2.4.2.b.i  Weighted Sum
The most widely used a-priori approach is the weighted sum approach in which multiple objectives are each weighted and then summed to form a single composite objective function [177]. Mathematically the objective function could be written as minimize \( F(x) = \sum w_i f_i(x) \), where \( w_i \geq 0 \) represents the weight of the objective \( f_i \) and \( \sum_i w_i = 1 \). While using the weighted sum method, two issues always arise and these include 1) proper scaling for non-commensurable objectives and 2) choice of weights for each objective. To overcome these issues, different approaches have been used by researchers.

The most common way is to convert all objectives into the same unit. This is usually done by converting all objectives into their monetary equivalents. Usually when objectives are from same domain and only one stakeholder is involved, then
this strategy makes sense as conversion formulas could be determined with ease by looking at the company’s historical data. For example researchers in ref [31], [35], [66], [153], [178] associated different objectives such as distance, fleet size, travel time, unutilised capacities, wait time by driver, and product perishability with monetary values; and formed composite objective functions despite the fact that these individual objectives conflict with each other.

Conversion into monetary terms becomes arguable when there is more than one stakeholder involved. An example could be quoted from the work of ref [34] in which a fuel delivery inventory routing problem by using multi-compartment homogenous fleet was presented with an aim to reduce routing and inventory carrying costs. Reduction in routing cost (proportional to distance travelled) is important from the supplier’s perspective; however it increases inventory holding cost which is critical from the customer’s perspective. Both objectives conflict with each other and serve interests of different stakeholders; however, with the implementation of vendor managed inventories (VMI) in which vendor/supplier is responsible for managing the inventory at a customer’s place, the onus shifts to one stakeholder only. In this case both objectives could be combined. So authors combined these objectives into a composite function of reducing overall cost and solved the problem by using Variable neighbourhood search method. Once the problem is converted into a single objective problem then it could be solved by using different exact and heuristics methods. For example, to solve IRP variants, researchers have employed Tabu search with ad-hoc mixed integer programming [179], linear programming [45], greedy heuristics
followed by local searches [65], and heuristics [180]. To solve LRP, Genetic algorithm hybridized with Tabu search [37] and Lagrangian relaxation-based methods [43] have been deployed by researchers. Please refer to appendix-1 for more details.

In the previous case, conversion into a single objective by means of monetary value was plausible as objectives were easily convertible and eventually catered for the perspective of a single stake holder. However, scalarization by means of some monetary value could become arguable when objectives not only conflict but also fall in different domains or cater for different stakeholders. For example objectives such as customer’s perception of quality/tardiness[30], [178], emissions [61], [66], [161], and distance/travel time not only belong to different domains, but also cater for the interests of different stakeholders, have different units and scales. To resolve the scaling/commensurability issue, researchers normalized the value of different objectives (i.e. converted into unit less form) before taking the weighted sum [169][42][136]. However, to normalize one needs to find out the lowest and highest possible values of each individual objective. The difference in maximum and minimum values provides a range over which an objective value can vary in different solutions. To normalise, the range of maximum and minimum of each objective is put in the denominator; whereas in the numerator, the difference of the solution and the ideal point values of each objective are taken. The following example indicates how maxima and minima were derived to perform scaling of different objectives. In ref [109], a pickup and delivery problem with time window and fixed fleet size was modelled. Objectives considered included the minimization of distance and tardiness. Not only objectives have different units but also represent interests of different stakeholders (i.e. logistics provider and customers respectively). Genetic algorithm was used as a search engine while the weighted sum approach was used to combine the two objectives. To bring these objectives to the same units each objective was divided by the respective optimal solution when considered individually. For example, to calculate lower bound of travel cost, the problem was converted into a TSP and solved by incorporating capacity and precedence constraints. Similarly a lower bound of tardiness was calculated by finding the number of customers served by each vehicle.

To derive weights for different objectives, different ways have been reported in VRP literature. The weights could either be assigned by a decision maker based on their expert judgement/experience [57], [58], [71]. One very interesting example in
which weight is derived from experience is due to [116]. This practical and interesting problem relates to de-icing of aeroplanes. De-icing is supporting activity and is usually outsourced. De-icing of an aeroplane needs to be performed within a certain time-frame before the departure of the plane. Delay in it could cause a delay in flight departure. Two conflicting objectives were considered simultaneously. The first objective was to minimise the delay in departure which is important from the carriers perspective; and the second objective was to minimise the distance travelled by vehicles which is important for the company performing de-icing. To cope with the situation and to accommodate points of view of both stakeholders, a weighted sum approach was adopted and four different heuristics were applied to solve and compare results. The first heuristic represents the intuitive strategy followed in practice. Second and third heuristics were greedy heuristics that considered separate objectives of distance and time, while the fourth heuristic was based on another greedy algorithm (GRASP) to construct the solutions. As units of both objectives were different i.e. time was expressed in minutes and distance was measured in meters. So to cope with the scaling problem, distance travelled was converted into time by assuming an average travel speed. Since each objective is important for different stake hold er, so based on the expertise of the authors, delay was given double the weight of distance travelled.

Sometimes better understanding of context makes derivation or preference of weight very easy. For example, in ref [64], delivery of emergency supplies (medication, food and water) to disaster affected area was modelled. Three different objectives were considered and included minimization of unsatisfied demand (imposed in the form of penalty), minimization of total travel time by all vehicles and minimization of difference of satisfaction rate between different regions. Again, all of these objectives have different units and scales. Now from a practitioner’s perspective, the most important objective is meeting the demand of customers by reaching emergency supplies on time; whereas minimization of travel time is the last objective. Considering the practicality and nature of the problem, highest weight was given to minimization of unmet demand and lowest weight was given to travel time. Interestingly there was no mention of how the scaling issue was tackled.

Weights can also be driven by using the company’s historical data. For example in ref [101], five different objectives tardiness, distance, wait-time, service satisfaction, and delivery efficiency were considered simultaneously. The weighted
sum method was used to form a fitness function within a genetic algorithm. Weights of all objectives were derived by looking at the cost contribution of each objective to the total cost function by looking at the historical data.

Alternatively if the historical record is not available and the decision maker doesn’t have a very clear idea then weights could be derived from decision makers by using pair-wise comparison techniques such as AHP [116]. In a same context problem i.e. distribution of emergency relief, researchers [47] considered minimization of distance and number of vehicles. Weights were derived using AHP pair-wise comparison method and scaling objectives by taking the log of the value of distance objective. However, in this case usage of the log method to scale objectives casts some doubts on the usage of the log method. As reported in the article, value of distance ranged between 800 and 1000, so log of that will be between 2.9 and 3; and the value of vehicles varied from 9 - 22. The weight assigned to the distance objective was 0.354 and weight allocated to vehicle objective was 0.646. It appears that more preference was given to vehicle minimization.

In ref [67], VRPTW problem with two objectives distance and vehicles were minimised. Genetic algorithm was used and two different ways to calculate fitness were used. When weighted sum was used in the fitness function, no scaling was performed and weights were established empirically. For more literature about weighted sum in VRP context, readers are referred to Appendix-2.

The biggest advantage of weighted sum method is that when objectives are converted into a single composite objective, then the problem could be resolved by using any single objective optimization method. Another advantage is that by changing the weights different solutions could be found. Perhaps due to these advantages, weighted sum is highly used a-priori approach in literature, though this approach is not free from shortcomings. One of the foremost shortcomings is that weighted sum cannot find all Pareto optimum solutions when the problem is non-convex. Another shortcoming is that when objectives are interrelated with each other and weights are changed then this approach may yield unexpected results. For example, it is quite possible that changing weights significantly may produce the same solution while varying weights a bit may yield entirely different solutions [181].
2.4.2.b.ii  **Lexicographic ordering**

Another commonly used a-priori approach is lexicographic method. In the lexicographic approach, the decision maker provides absolute preference or hierarchy of objective functions. The most important objective is optimised first by considering original constraints. If optimization process yields more than one solution then the second most important objective is optimised by adding the value of the first objective as a constraint. The cycle continues in this hierarchical fashion. Two issues need to be addressed while applying this approach i.e. 1) Checking the uniqueness of solution at each optimization cycle, and 2) getting of absolute preference information [181].

Derivation of absolute preference information requires a thorough understanding of the context. In literature, the most preferred objective that is optimised first is fleet size, followed by other objectives such as distance, workload balance, time, cost or longest route [9], [50], [52], [182].

Lexicographic ordering approach has been implemented by researchers by using different methods. For example, in ref [8], an exact method (cutting plane algorithm) was used to minimise distance while ensuring that fuzzy time windows are not violated. In the next step, a sub gradient based algorithm was used to improve the service level.

In addition to exact methods, heuristic methods are also used by researchers. For example, in ref [75], a construction and improvement heuristic was developed that first minimised the vehicles followed by the distance related cost. In ref [50], a multi-start local heuristic was proposed to solve VRPTW. After construction of initial routes, the heuristic tries to first reduce the number of routes and in the next phase it tries to reduce the distance travelled. In ref [9], evolution strategy was used to reduce the primary objective of vehicle minimization. The procedure allowed accepting solutions that even worsened the 2nd objective of distance. In later stages, Tabu search and Guided local search procedures were applied to improve the distance. In ref [52], time dependent VRP was modelled to optimise number of vehicles and travel time in a hierarchical way. A multi-ant colony based algorithm was used that first minimises the number of routes followed by travel time. In ref [126], two objectives vehicles and distance were considered. Lagrangian relaxation was used to create a relaxed version of the problem to find the minimum number of vehicles required. In the next step, Tabu search and other local search heuristics were applied to minimise the
distance by treating value of vehicles as a constraint. In ref [46], ALNS was used that first considered the maximization of customer served and then minimized the total distance travelled by all vehicles. In ref [183], two objectives were modelled i.e. minimization of maximum overtime and minimization of distance. Genetic algorithm with local search improvement was used to solve the issue. At the evaluation stage, different solutions were ranked according to value of the objective function in a hierarchical way. Overtime was considered first followed by the distance objective. Readers are referred to Appendix-3 for further related literature.

Please note that exact methods in lexicographic approach do not allow an iota of trade-off between an important and less important objective. In other words, the solution found at the end is Pareto optimal. Though lexicographic approach does not require normalization or scaling; however, elicitation of absolute preference at global level from DM could prove to be difficult.

2.4.2.b.iii  ε-Constraint Method

In the ε- constraint method, only one objective is optimised and all other objectives are converted into constraints. In mathematical form this could be written as:

\[
\text{minimize } f_l(x) \\
\text{Subject to: } \begin{cases} 
  f_m(x) \leq \varepsilon_m & \text{where } m = 1, 2, \ldots, k \text{ and } m \neq l \\
  g_i(x) \leq 0 & i = 1, \ldots, m1 \\
  h_j(x) = 0 & j = 1, \ldots, m2
\end{cases}
\]

While using this approach, one needs to answer the following two questions. 1) Which objective needs to be optimised, and 2) what should be the bounds of objectives that are turned to constraints? Sometime the answer to these questions could be derived from context. For example, in ref [41], a problem related to coverage of highways to reduce the traffic crash frequencies was modelled. Two objectives i.e. maximisation of coverage and minimization of overall cost not only conflicting with each other but also are incommensurable i.e. coverage is measured in minutes while cost is in ($). To solve this problem, ε-constraint method was used for two reasons: 1) maximization of coverage is indirectly linked with reduction of accidents and it is difficult to covert loss of a human life or injury of a person into monetary equivalent; 2) bounds on routing cost were easy to calculate by looking at annual budget which is usually allotted on a fixed term basis. Problem was solved by using LP after converting the routing cost into a constraint. Since in this particular problem, historical data related to crash frequency was known that is demand of each arc/node
travelled was known and budget information was also available beforehand so it was easy to calculate bounds; however in real-life when demand keeps changing on a regular basis, it could be difficult to estimate the value of those bounds $\varepsilon_m$. If too ambitious targets are set then it may result in infeasibility of solution; therefore one needs to use some methods to estimate the value of $\varepsilon_m$.

**2.4.2.b.iv Goal Programming**

In the goal programming method, decision maker expresses aspiration levels $TL_i$ for each objective $f_i$. These objectives and their aspiration levels are made part of goal constraint after adding deviation variable $P_i$. Sum of deviations are then minimised in the achievement function to solve the problem. In real life, some constraints may carry higher importance than other constraints, so weights could be used to indicate relative importance of different objectives. Mathematically the objective function could be written as minimize $F(x) = \sum w_i P_i$ and subject to $f_i(x) - P_i \leq TL_i$, where $w_i \geq 0$ represents the weight of the objective $f_i$; and $\sum_i w_i = 1$.

In ref [14], researchers formulated a problem in which decisions of identification of waste treatment locations and routing of hazardous waste from different population centres to those sites were taken together. Four different objectives of total cost, total perceived risk, individual perceived risk, and individual disutility of treatment site were considered. As these objectives are strongly interlinked with each other, this necessitated the consideration of all objectives simultaneously. Weighted goal programming was used to find a solution. Since cost, risk and disutility have different units; normalization was performed by expressing each objective as a percentage of target value. The deviation variables in that case became the percentage deviation from respective target values. A small hypothetical problem was used to test the model. While using one needs to be very careful while setting the aspiration levels as these may affect the final solution. For setting these targets and their respective weights a thorough understanding of problem context is necessary.

**2.4.2.b.v Hybrid/Heuristic Method**

In the above mentioned a-priori examples, researchers used one approach to solve the problem. However, to take advantage of strengths of different approaches while avoiding shortcomings, some researchers developed hybrid methods that combined more than one of the above-mentioned a-priori approaches. For example, in ref [169],
four different methods namely weighted sum, weighted sum after normalization, $\varepsilon$-constraint, and a hybrid method that combined adaptive weighting with $\varepsilon$-constraint method were used to solve a bi-objective routing problem. Objectives included minimization of emissions and driving time. All four methods were applied and results were compared. Results suggested that weighted sum methods (with and without normalization) were good in terms of generating the extreme solution but could not generate enough Pareto solutions. On contrary, $\varepsilon$-constraint method generated sufficient solutions but generally these were inferior, however, hybrid method outperformed other three methods. In this research, ALNS was used as a search engine and all four a-priori approaches were embedded in it.

In addition to hybrid, researchers have developed heuristics that combined principles of one or more of the above-mentioned a-priori approaches. For example, in ref [62], two conflicting objectives i.e. vehicle travel time and tardiness were modelled. In addition to goal programming formulation, a heuristic was also given that first clusters the customers by looking at insertion cost (weighted sum of two objectives), and in the next stage pre-emptive goal programming was applied on each group to minimise the overall objectives. Another such example can be traced in the work of ref [57], in which three different objectives namely distance, perishability and fulfilment of emergent services were considered. A heuristic was developed that first clustered the customers and in the routing phase iterative goal programming was applied to each customer so as to limit the deviations from distance and perishability target values. However, this approach may yield suboptimum results. Similarly in ref [121], a dynamic multi-period routing problem related to a large distributor in Sweden was solved. Three different objectives, travel time, customer wait time and balance of daily workload over a planning period were optimised. A three phase approach was adopted to solve the issue. Since total travel time was considered the most important objective; so in Phase one, customers were selected for different time periods based on a travel time objective. In phase two, variable neighbourhood algorithm was used and solutions were generated by considering the other two objectives. In the post optimization phase, travel time on any given day were formulated as CVRPs and were solved by using Tabu Search. Another example is due to ref [53] in which three different objectives i.e. distance, cost and mean sharing index were optimised in context of long-haul multi-modal freight transportation. A heuristic was developed that in the first phase used multi-label Martin’s algorithm (a
variant of Dijkstra). This algorithm was used to determine the shortest path by considering two objectives (time and cost) on dominance based principle. In the next stage, a third objective was evaluated. In ref [117], a heuristic was developed to solve a problem containing three objectives of vehicles expenses (fixed plus variable), tardiness and travel time. Customers were clustered according to due time before making sub routes. Concurrent scheduling approach was used that allowed adding the same customer in different routes. Each sub-route was evaluated for feasibility. If multiple feasible routes existed then the best route was selected by looking at savings made. Saving function was an aggregating of different criteria in which weights were determined by using AHP. In the end each route was assigned to a different vehicle.

The overall advantages of all a-priori approaches include 1) the relative easiness with which these methods could be implemented and 2) the generation of a single efficient solution at the end of the optimization process that can be implemented without any further deliberation. However, each of the above-mentioned a-priori approaches has limitations as well. For example, weighted sum method is incapable to deal with nonconvex problems. ε-constraint approach might result in infeasible solutions if ε values are too optimistic. Similarly, choice of aspiration levels in the case of goal-programming may affect the final solutions. Lexicographic method requires absolute preference information of objectives which DMs might be reluctant to provide when the problem is one-off. Similarly, in practice, it may be unrealistic to ask the decision maker to provide his overall utility function or a set of weights as his overall preferences. This is because it is generally difficult to tell what solution one really wants before he knows what solutions are available. Due to these limitations, many researchers have resorted to Posteriori approaches which provide more than one solution at the end of the optimization cycle.

2.4.2.c Posteriori approach
In the posteriori approach, all or a representative set of Pareto optimal solutions are generated. These non-dominated solutions are then presented to the decision maker who selects a single solution after performing trade-off analysis based on problem requirements and his past experience. Different techniques have been used to generate a non-dominated solution set, and a few of these are described in the following subsections.
2.4.2.c.i By changing weights

If different objectives were aggregated into a single composite function, then by changing the weights of different objectives, non-dominated solution sets could be obtained. If there are two objectives, then weight \( \alpha \) of any objective could be varied uniformly by using some formula while allocating the remaining weight (1-\( \alpha \)) to the second objective. Same approach of changing weight is adopted in the work of [93]. Two objectives of total cost and passengers’ perceived quality of service (QoS) were modelled to solve a home-to-work bus service problem. In evaluating and choosing the neighbourhood move by using Tabu search, weight \( \alpha \) was systematically varied in aggregated function of \( \alpha \text{*Cost} + (1-\alpha)\text{*QoS} \). In the first phase, the value of \( \alpha \) was set to be 0.001, however to generate Pareto set, the value of \( \alpha \) was given a uniform increment of \( 0.998 / (\beta-1) \), where \( \beta \) represents the number of phases and each phase lasts for \( \lambda \) local search iterations. The values of \( \beta \) and \( \lambda \) are parameters and were set before running the algorithm.

In above-mentioned example, changing of weights systematically seems to work fine as there were only two objectives. However, when there are more than two objectives, then one needs to resort to some other method. For example, in ref [146], the problem of providing delivery services by a large courier company was modelled. In this problem, three different objectives i.e. Cost, deviation from tactical plan, and workload imbalance were considered. These objectives were aggregated in a single objective function and Tabu search meta-heuristic was used to find solutions. To generate a set of non-dominated solutions, an algorithm was used to generate different combination of weights in an incremental step of 0.1 to guide the direction of the search process.

Though by changing weight one cannot generate all Pareto optimal solutions; nonetheless this method of generating Pareto solutions remains an attractive option for many researchers as it is quite simple to implement.

2.4.2.c.ii By changing value of \( \epsilon \)

Another approach to generate non-dominated solution set is based on \( \epsilon \)-constraint method. In the \( \epsilon \)-constraint method, one of the objectives is optimized while all remaining objectives are turned into constraints by providing upper limits for them. In this way, in each algorithmic run another objective is optimised and the cycle continues. At the end of process, a set of Pareto optimal solutions can be achieved and
presented before the decision maker to choose the most preferred solution from it. While applying this method, one needs to determine 1) which objective is to be optimised and 2) over what range the values of objective turned into constraint will vary in different algorithmic iterations? Following are a few examples in which researchers have addressed these questions.

In ref [154], two conflicting objectives i.e. average freshness of perishable products and total distribution time, were considered in a VRPTW problem. Both of these objectives can take continuous values. Since from the practitioner’s perspective, travel time is more important, therefore product freshness (as a percentage of shelf life) was made as a constraint. To generate Pareto optimum solutions, the bound on freshness value was varied systematically in different algorithmic iterations. Before generating any solution, the lower and upper bound values of freshness were determined. Lower bound was calculated by making assumptions that each vehicle serves a single customer and there is no wait time at the customer location. And for upper bound, it was assumed that vehicles depart at the earliest possible departure time from the depot and serve customers at the finishing time of customers’ time window. Once the bounds were found, then a MOEA based heuristic that embedded $\varepsilon$-constraint approach was applied to small problem instances. In the first iteration, an upper bound of freshness was used as R.H.S. of constraint and distribution time was calculated. This represented an extreme solution that maximized the freshness objective. In the next iteration cycle, value of average freshness was decreased by 0.05 and a new optimum was generated. Cycle continued till the R.H.S of freshness constraint reached its lower bound. Another example in which both objectives take continuous values is due to ref [184]. Total cost (proportional to vehicle usage and distance travelled) and penalties (proportional to tardiness) were modelled. As cost is the main concern from planners’ perspective, so it was optimised while treating penalty as a constraint. Contrary to the previous example, in which bounds were calculated, in this research an upper bound of 500 on penalty was provided by planners which was based on their past experience. In the first iteration, penalty of zero was used thus making soft time widows hard to ensure that each customer was served in the given time-slots. This extreme solution, thus generated, was added to the set of Pareto solutions. Route construction and improvement heuristics were used to solve the problem. The bound was gradually increased in a step size of 20. During the solution improvement stages, if no improvements in solution were found then the
limit was increased again. Cycle continued till penalty reached to its maximum level. Similarly in ref [90], two objectives of average route length and maximum route length were optimised by using Tabu search method. In each iteration cycle, different solutions from existing Pareto set were picked and neighbourhood improvement searches were performed. If a non-dominated solution was found then it was added in the Pareto set. In the Parallel Tabu search method, in the initial run, the value of 2nd objective was set very high which was gradually reduced to generate different solution in next iterations. In this way by using both methods Pareto fronts were generated and compared in the end.

In the above-mentioned three cases, only one objective was treated as constraint. However, it is quite possible that, for any given value of bound, there may exists more than one solution such that value of the optimised objective is the same, but the value of objective that is turned into a constraint might be different. To cope with this situation, in ref [148], a two steps approach was adopted (lexmin) in different iterations. In the first step, one objective was optimised while treating the other as a constraint. In the next step, second objective was optimised while putting the value of the first objective as a constraint found in the previous step. Branch and cut algorithm that embedded $\varepsilon$-Constraint approach was used to generate trade-off solutions of two objectives i.e. travel cost (time/distance) and maximum route length of any vehicle. In the first step of iteration one, upper bound on maximum route duration was set to infinity and travel cost was optimised. In the second step, the value of travel cost obtained in the previous step was made a constraint and route duration was optimised. In the next iteration, the bound on the second objective was reduced, and a two steps process was followed again in similar fashion. A similar approach was adopted in the work of ref [18].

In the previously mentioned case, all objectives could take continuous or a large range of values, however, in real life one might need to solve a problem in which an objective can take limited and discrete values. For example, in ref [119], a problem of school bus routing in an urban context was modelled. Two conflicting objectives fleet size and maximum journey time any pupil spends on a bus were modelled by using a scatter search algorithm. Please note that fleet size can take discrete and limited values while journey time can take continuous values. As each of these objectives is important from the perspective of different stakeholders, therefore trade-off solutions were provided to evaluate different scenarios. Value of maximum journey time was
varied while number of buses was optimised. However, in this research no mechanism to calculate bounds or to change the values was given. Contrary to previous research, researchers in ref [120] used number of buses as a constraint instead of maximum journey time by any pupil. The rationale behind this decision was that since the number of buses used is a discrete number and can take only limited number values, considering number of buses as constraint reduces the number of times an algorithm needs to be run.

One can notice that only two objectives are considered in all of the above-mentioned examples. If however, there are more than two objectives and then this method could prove to be limited. Perhaps that is the reason that majority of the researchers did not adopt this approach to generate a Pareto front.

2.4.2.c.iii Generation of a population of solutions simultaneously
The above-mentioned posteriori approaches appear to be quite limited in terms of the number of objectives that could be handled. Furthermore, the algorithm has to be run many times to approximate the Pareto-set and in each run parameters (weights or bound) are to be adjusted. To overcome these problems, the majority of researchers have used population based heuristics. These heuristics not only have the potential to generate the non-dominated solution set in a single algorithmic run but also are capable to accommodate many objectives at the same time having different scales.

Evolutionary algorithm is a population-based meta-heuristic and is the most commonly used posteriori method. For more details about multi-objective evolutionary algorithm (EA) for VRP, readers are referred to the work of [185]. Different variants of EA such as MOGA [67], [73], [84], [122], [159], [186], NSGA-II [7], [73], [112], [142], [159], VEGA [122], Memetic [19], and PAES [40] have been used by researchers. For hybrid version of EA, readers are referred to the work of [6], [24], [63], [81], [83], [89], [94], [143], [147]. In addition to EA, researchers have also used other meta-heuristics. For example Particle Swarm optimization [149]. For more details, readers are referred to Appendix-9.

2.4.2.c.iv Hybrid
A few researchers combined different approaches to generate Pareto front. For example, in ref [54], a goal programming approach was embedded in a genetic algorithm. Aspiration level of fleet size and total travelling distance were taken from the decision maker. At the evaluation stage, different solutions were ranked by
looking at the deviation from these aspiration levels. Another interesting example that combined multiple approaches to estimate Pareto front is due to ref [86]. In their research, a real-life application of recyclable waste collection in Portugal was considered. In the routing phase, three objectives i.e. distance, CO₂ and drivers maximum working hours were considered. An augmented ε-constraint method was used that combined different approaches. First of all a lexicographic approach was used for each objective function to determine the payoff table. As there was no guarantee that solutions obtained in the previous step are Pareto optimal, so in the next step a goal programming approach was used in the objective function to minimize the value of a penalty variable to come up with efficient solutions. Finally a compromise solution was obtained by minimizing distance between an ideal point and Pareto front.

Another method that combined the ε-constraint and dynamic weight adjustment approaches to generate non-dominated solutions is reported in the work of ref [11]. A problem related to scheduling and routing of long-haul trips was presented by researchers. In long-haul journeys, usually the journey time of the entire trip could be up to eight days; therefore, while constructing routes different regulations related to hours of service need to be strictly followed. Primary objectives considered in that research included 1) reduction of overall cost which was proportional to distance travelled and 2) reduction of inconvenience to drivers. To solve this problem, Tabu search method was used after aggregating the objectives. In addition to primary objectives, the composite function also included violation-based objectives related to capacity, drivers’ on-duty timings, time-windows, and total duration. Penalties were added as co-efficient of violation-based objectives, and weights were added for the primary objectives. Penalty coefficients were updated dynamically throughout the search process. If a specific constraint was feasible for current solution then its penalty coefficient was reduced, and if it was infeasible then the value of the co-efficient was increased. Requirement of minimum fleet size was determined by using a heuristic method. However, to generate a non-dominated solution set Tabu search was applied by making more vehicles available. For each value of fleet size, each solution was evaluated by calculating the Manhattan distance of each solution from an ideal point. Weights of primary objectives were updated dynamically. Three different strategies of changing weight have been proposed and implemented. In the
end non-dominated solutions considering three criteria 1) distance, 2) inconvenience, and 3) fleet size were presented.

The advantage of these posteriori methods is that computation of Pareto set could be done without the presence of the decision maker. But disadvantages of this approach are that 1) approximating an efficient frontier for a high dimensional problem is computationally intensive or even infeasible [73], 2) asking the transport planners to judge a large number of efficient solutions may be problematic, and 3) at times final results may become difficult to justify [58]. Perhaps generation of Pareto front is most suitable when decisions related to policy making are to be evaluated. Posteriori approaches do not appear to be a good choice when a person has to make such a decision on very regular basis. If posteriori methods are not used on a regular basis, even then it may become difficult for a decision maker to do trade-off analysis if the problem is of high dimension. The absence of suitable visualization methods makes trade-off analysis even more difficult.

2.4.2.d Interactive approach
In the interactive approach, the decision maker provides his preference information gradually within the solution generation process. A decision maker not only can use his expertise while exploring the search space but also gains a better understanding of the routing system [58]. Following are few examples of literature that specifically deal with interactive and multi-objective VRP.

Park and Koeling [58] were probably the first who solved a multi-objective VRP problem by using an interactive approach. Optimization of total distance, perishability of products, and fulfilment of emergent services were considered as objectives. They adopted cluster first-route second approach to solve this problem. After getting the weights and upper bounds for each objective, iterative goal programming was used to sequence the customers in each cluster. Once initial solutions were generated, decision maker was invited to inspect the solution by looking at the value of different objectives for each route. Information about new weights and target value for each objective were taken to construct new routes for each cluster, if the decision maker was not satisfied with the earlier solution. Similarly, the decision maker was allowed to shift the customers between different clusters. The process was allowed to continue till the decision maker became satisfied. One of shortcomings of the proposed method is that level of interactivity is
limited to a single route (one at a time) thus resulting in sub-optimal solutions. Similarly, presence of unrealistic assumption, such as lack of delivery time-windows and deterministic travel times, somehow contradicts with real-life scenarios.

An interesting application of an interactive approach pertains to a real-life school bus routing problem in Hong Kong city [51]. In this research, four objectives i.e. minimization of vehicles, maximum journey time of each pupil, total travel time and equity among drivers were considered simultaneously. A lower bound of number of buses required was first determined. A heuristic was used that involved different methods such as Dijkstra, kth route methods and Hungarian algorithm to construct routes. Initial solutions were then improved by using heuristics and an interaction with the decision maker was allowed to allocate the vehicles to different routes. In this research, the mechanism of interaction used is not clearly explained but it appears to be quite limited.

Geiger and Wenger in their research [111] considered two objectives i.e. distance and tardiness. They proposed a framework for interactive multi-objective VRP for an application in which transportation orders were placed in the market and vehicle agents placed their bids for orders. The researcher developed a GUI model in which software acts as an intermediary between the decision maker and vehicle agents. For each order, vehicle agents placed their bids by taking into account potential change when the new order was integrated into current routes. Vehicle agents were allowed to change existing routes by using local search methods. In their research, the utility of the decision maker was calculated in the form of a weighted sum for each solution. In their follow-up research [113], global utility was calculated by aggregating the partial utilities of each objective. Allocation of an order to a vehicle was based on the minimization of the maximum regret principle. At any stage, the decision maker was able to change his preference and based on which the utility was recalculated for each route. As a result, the software could remove an order from an existing route and could place it back into the market for rebidding. The advantage of weighted-sum approach used in this research is the relative simplicity with which the decision maker can express his preferences. As in real-life, often more than two objectives are considered therefore Wenger and Geiger in their next research [187] considered six different criteria including total distance, time, vehicles used, total tardiness, maximum tardiness and number of tardy orders. The decision maker provided a weight for each objective and this information was used to
derive a composite utility function. During the interaction phase, the composite utility function was used to reroute and re-cluster the orders by using the variable neighbourhood metaheuristic. The deficiency in this approach is that if the decision maker wishes to express an aspiration level for different objectives then this approach appears to be somehow limited.

Another example involving an interactive approach is due to ref [188]. Cost and average customer satisfaction objectives were considered in this research; and genetic algorithm embedding local heuristics was used to generate solutions. After each iteration cycle, solutions were presented to the decision maker. Solutions that the decision maker considered acceptable were then added into a satisfactory solution pool. Additional solutions were generated, if required by DM, by breeding of top solutions from current population with solutions from the solution pool. New solutions thus generated were again presented to the decision maker so that he could shortlist more solutions. This cycle continued till the decision maker was satisfied and he selected one final solution for implementation.

Table 2-5 summarises the sub approaches used by researchers in the literature that is reviewed in this chapter. It can be seen that usage of weighted sum to combine different objectives is no doubt the most commonly adopted method by researchers.

<table>
<thead>
<tr>
<th>Sub-approaches</th>
<th>No Preference</th>
<th>A-Priori</th>
<th>Interactive</th>
<th>Posteriori</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goal programming</td>
<td>2</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Hybrid</td>
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<td>2</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Weighted Sum</td>
<td>38</td>
<td>2</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Weighted Sum - (Single objective conversion)</td>
<td>18</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lexicographic</td>
<td>12</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\epsilon$-Constraint</td>
<td>1</td>
<td></td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>Population generation simultaneously</td>
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<td></td>
<td></td>
<td>57</td>
</tr>
<tr>
<td>Heuristics</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Total   | 77 | 6   | 70      |

94
2.4.3 Multi-objective VRP in practice (UK Retail sector)
While responding to the question about level of awareness about multi-objective VRP, the following response was given by a respondent.

“Our customers are not sufficiently astute to MOVRP. They are not much sophisticated.”

However, as customers for VRP software do care about vehicles, distance, time, time-windows, overtime, and customer-driver relationship; therefore software vendors consider all of these objectives while constructing routes. In the words of one respondent

“We use weighted sum cost function to deal with multiple objective.”

As above-mentioned objectives are from different domains and have different scales, so a question was asked to know the aggregation method used by them. The following response was received.

“The customers in an ideal world should create policy which allows them to trade them off against each other in such a way which can be done mechanically by the guys doing planning. Planners are not in any way in a position to make policy. They should be following policy.”

In response to a follow up question of how policy would be formulated, the following response was received.

“We would have to challenge the customers to define what is best for them. We would generate candidate solution by changing parameters and ask them which best represents their policy.”

From the conversation, it appears that enacting of policy meant to derive an overall utility function that would later be converted into a cost function. As per respondent,

“…..linear incorporation of two criteria. If they can do that then we will do that into a cost function. So multi-objective would not come up on daily basis.”

In response to a question about interactivity and the reasons to allow interactivity, a response was received in the following words.
“Planners do it as data or some aspect of problem hasn’t been captured by software and the planner doesn’t really understand why and what. He makes modification.”

“….. they just to change from that to that. May be because, they just prefer it or may be entirely legitimately because then have visually spotted a combination that our optimization process has failed to find”.

In response to a question about kind of interaction that planners do, the following response was received.

“…. (planners) change constraints, generate another solution and compare two solutions. Generate solution independently serially. There are not many constraints that they can change.”

From the conversation, it became apparent that in practice 1) multiple objectives are considered simultaneously, 2) weighted sum method is used to combine these objectives; 3) weights are usually provided by management; 4) planners do interact with the system if they are not happy with the solution; and 5) planners generate very few solutions (typically between two to three) to do some trade-off analysis.

2.4.4 Methods/Methodologies

While choosing an appropriate method, one needs to ensure that the method is capable of finding Pareto front as it is quite possible that solutions generated by a method, after optimization, are local optima. Therefore, at the post optimization stage, these solutions are subject to critical analysis by the decision maker. If a decision maker is not satisfied or solutions generated do not match with DM’s experience due to interdependencies in objectives, then the model or parameters in the algorithm may need to be adjusted in the next iteration till psychological convergence is achieved [175].

Table 2-6 provides a summary of different methods/methodologies that have been used by researchers to guide the search process while solving different variants of multi-objective VRP. Please note that a variety of heuristics/methods have also been used within these search engines/methodologies either to initialize initial solutions, to make local improvements or to compare the results. Some researchers have also hybridized different methods so as to overcome limitations and to get
combined benefits of individual methods. However, in the following table, only those methods/methodologies have been listed that were used as main search engines to explore the solution space.

Table 2-6: Methods / Methodologies commonly adopted to solve MOVRP

<table>
<thead>
<tr>
<th>Methods Type</th>
<th>Method</th>
<th>No-Preference</th>
<th>A-Priori</th>
<th>Posteriori</th>
<th>Interactive</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact</td>
<td>Branch &amp; cut</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
<td>6</td>
</tr>
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<td></td>
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<td>12</td>
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<tr>
<td></td>
<td>Branch &amp; Bound</td>
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<td></td>
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<tr>
<td>Heuristics</td>
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<td>30</td>
<td>13</td>
<td>2</td>
<td>47</td>
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<td>Saving</td>
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<td></td>
<td>Greedy (NN, GRASP)</td>
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<tr>
<td>Meta-heuristics</td>
<td>Evolution based</td>
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</tr>
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<td>GA</td>
<td>14</td>
<td>8</td>
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<td>MOGA</td>
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<td>VEGA</td>
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<td>Memetic</td>
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<td>5</td>
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<td>SPEA/PAEC</td>
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<td>NSGA/II</td>
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<tr>
<td></td>
<td>MO - Quantum EA</td>
<td>1</td>
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<tr>
<td></td>
<td>Ant Colony</td>
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<td>6</td>
<td>3</td>
<td>10</td>
<td>20</td>
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<td></td>
<td>Multiple ACO</td>
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<td>2</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>PSO</td>
<td>3</td>
<td>3</td>
<td></td>
<td></td>
<td>6</td>
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<tr>
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<td>MO PSO</td>
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<td></td>
<td>Simulated Annealing</td>
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<tr>
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<td>5</td>
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<td>2</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>LNS/ALNS</td>
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<td></td>
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<td></td>
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<tr>
<td></td>
<td>Scatter Search</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

As it can be seen in Table 2-6, the majority of researchers have preferred to use meta-heuristic algorithms. There are various reasons for choosing meta-heuristics as preferred method e.g. these have a mechanism to come out of local optimum; can solve a variety of problem instances without making any changes in algorithm; and form a framework within which other methods could be used. In addition to using existing meta-heuristics, a lot of researchers also developed their own heuristics that
were tailored to meet specific needs of problems. Very few people have ventured to use exact methods, and probably this is due to the limitation of exact methods to generate solutions for large-sized problem instances. Most of the researchers, who used exact methods, either used these to either solve small instances for proof of concept or to draw insights that could be in developing heuristics for large test instances.

As it can be seen from Figure 2-18 and Table 2-7 that more than one third of researchers chose evolutionary algorithm (and associated variants) thus making it the most frequently used methodology to model MOVRP. This indirectly highlights the power of evolutionary algorithm to accommodate different variants of VRP. However, when real-life problems are modelled then it somehow necessitates the requirement to develop problem specific operators. Second and third most frequently used methodologies are Tabu search and Ant colony optimization methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact</td>
<td>16</td>
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<tr>
<td>Heuristics</td>
<td>51</td>
</tr>
<tr>
<td>EA</td>
<td>73</td>
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<tr>
<td>ACO</td>
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<tr>
<td>PSO</td>
<td>8</td>
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<tr>
<td>SA</td>
<td>9</td>
</tr>
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<td>TS</td>
<td>19</td>
</tr>
<tr>
<td>VNS</td>
<td>6</td>
</tr>
<tr>
<td>LNS/ALNS</td>
<td>4</td>
</tr>
<tr>
<td>Scatter Search</td>
<td>2</td>
</tr>
</tbody>
</table>

To understand the approach to incorporate the decision maker’s preference and its impact on choice of optimisation method/methodology, Figure 2-19 was drawn.
As it can be seen from Figure 2-19, when a-priori approach is adopted, heuristic methods are used more frequently in comparison to EA. Perhaps this is due to the fact that many of these articles are about solving real-life applications. So, not only decision maker’s information was available but also better knowledge about the problem situation led to developing or tailoring of heuristics rather than using or customising a general purpose method. One cannot forget to notice that the proportion of local-search based heuristics such as ACO, SA, TS, VNS, and LNS/ALNS is significantly higher than population based heuristics such as EA and PSO. Probably the reason behind this is that in almost three quarters of a-priori literature, multiple objectives are aggregated into a single objective function thus making single objective based local search methods more attractive than population based methods that are computationally more expensive.

In contrast to literature dealing with a-priori MOVRP, usage of population based heuristics is more than 60% in the literature dealing with posteriori approach. More than half of posteriori literature uses EA and its variants. As the literature dealing with no-preference and interactive multi-objective VRP is quite limited, so any conclusion drawn will be insignificant.

The following sub-sections provide details about the main methods/methodologies that have been extensively used by researchers in the multi-objective VRP literature. Focus in this section was not only to get basic information about these methods, but also to investigate how solutions were evaluated when different approaches of incorporating decision maker’s preference were used.

2.4.4.a Tabu Search (TS)
Tabu search is a local search heuristic which allows moving to a non-improving solution if a local optimum is reached while exploring neighbourhood solutions. However, in order to stop jumping back to previously visited solutions (taboos) in the next iteration cycle; it keeps a list of previous moves in its short term memory. The reverse of those transformative moves are prohibited for a limited number of iterations (called Tabu tenure). In Tabu search, three different types of memory structures are used while each structure serves a specific purpose. Short-term memories are used to maintain Tabu list, intermediate-term memory is used to guide the search process towards promising areas to intensify the search process, whereas, long-term memory is used to maintain diversity. One of the limitations of TS is that
the search could be restricted to a small area, if the problem under consideration is of high dimensionality or the search area is too vast. To solve this issue, sometimes multiple lists are maintained instead of one. Alternatively instead of storing solutions in short term memory, an attribute of a solution could be stored into the list. However, storing of attributes may lead to banning of more than one solution. As it is quite likely that a few of the banned solutions might be very attractive in terms of quality; therefore, to solve this issue, aspiration values or criteria may be specified to override the Tabu list [189].

Before applying Tabu Search to MOVRP, one needs to make decisions about 1) Operators to explore neighbourhood; 2) evaluation of neighbourhood solution when there is more than one objective under consideration; 3) Aspects of problem components to be included in the construction of the Tabu list; 4) mechanism to get out of local optima; and 5) strategy adopted to intensify and diversify search process. The following few examples provide a glimpse of how different researchers have addressed these issues in the context of MOVRP.

In ref [179], an inventory routing problem for a given period was solved by using Tabu search methodology so as to reduce the value of composite objective function consisting of inventory holding and routing costs. After initialization procedure, Tabu search methodology was invoked that worked by considering two Tabu lists based on delivery days for different customer nodes. One of the lists was used to stop serving the same customer on the same day in neighbourhood solutions; whereas the other list was used to stop changing the service time of the same customer. Neighbourhood solutions were generated by not-visiting or visiting a customer on a specific day, or by shifting the delivery date or by swapping delivery dates of two customers that are scheduled to be served on different dates. To reduce transportation costs, routes in new solutions were improved by solving TSP problem for each route. To improve the results, two mixed integer programming (MIPs) problems were solved in sequence. In the first MIP, routes were rescheduled on some other day without making any changes in the routes themselves. Whereas in the second MIP, removal of customers from one route and insertion into some other route by using cheapest insertion method was performed while keeping the delivery day of that route unchanged. If no improvements were found after a certain number of iterations i.e. a local optimal had been reached, then a jump procedure was applied.
that moved customers to time periods not typically visited in previous solutions so as to explore new search areas.

In ref [126], a multi-depot coordinated pickup and delivery problem was modelled by using Tabu search. A parallel insertion algorithm was used to construct the initial solution. Different inter and intra route operators were applied to explore the neighbourhood. To escape from a local optimum, three actions were performed 1) best solution found so far was used as starting point, 2) an inter / intra route operator was randomly selected and applied, and 3) the Tabu list was cleared.

In ref [11], three different objectives vehicles, cost and drivers’ inconvenience were modelled. For different values of fleet size, Tabu search was applied to find the solution that minimised the weighted Manhattan distance between an ideal point and the current solution. To calculate the Manhattan distance two criteria i.e. cost and drivers inconvenience were used. After a certain number of Tabu iterations or if no feasible solution was found then Tabu list was cleared and the weights were re-adjusted.

In ref [157], initial solutions were generated by first adding the unassigned customers at the end of the route of a vehicle having the largest capacity. If a customer could not be accommodated in this way then it was assigned to the vehicle with next largest capacity. Solutions thus constructed were then improved by 3-opt method. Neighbourhood solutions were explored by using relocating a customer into another route or by swapping two customers in two different routes.

In ref [90], Tabu Search was applied to solve distance constrained VRP with two objectives i.e. minimization of average route length and maximum make-span. As initial routes were generated randomly; therefore, the objective of initial adaptation runs was to bring the route lengths within the distance limits. Afterwards, few non-dominating solutions were randomly selected and improved by running Parallel version of Tabu Search. An external Pareto archive was used to store non-dominated solutions obtained from parallel searches.

In ref [99], used least cost insertion heuristics to generate initial solutions. Five different operators were then applied to generate neighbourhood solutions. A multi-stage operator selection mechanism was used that switched the operators systematically after a specific number of iterations in each stage had lapsed. To keep memory consumption of Tabu list at a manageable level, fingerprints of each solution by using computable checksum procedure were calculated.
2.4.4.b  Ant colony optimization (ACO)

Ants leave a trail of a chemical called pheromone along the path as they travel in search of food. The presence of pheromone increases the probability that other ants will follow the same path. After finding food, the same trail is followed on return and more pheromone (proportional to quality and quantity of food) is laid on it. In nature, pheromone evaporates with the passage of time. The ant that finds a shorter path deposits another layer of pheromone sooner than other ants following the longer path. This makes the shorter path more attractive and thus the probability increases that subsequent ant will follow it. Ant colony optimization (ACO) is a local search meta-heuristics that mimics the collective foraging behaviour of ants. That is, ACO follows the same behaviour of selecting paths according to some probability; evaporating of pheromone at the end of the iteration; and depositing of pheromone according to quality of solution on the return trip. However, ACO algorithm differs from the behaviour of real ants in a sense that it keeps track of past action and has some other information (e.g. distance between locations) available.

ACO starts by assigning same probability of selection for different links. Routes are constructed by looking at the probability of different links that could be accessed from current node. Probabilities are usually calculated on the basis of current pheromone quantity alone. Sometimes, in addition to pheromone quantity, a local heuristic function (also called visibility) that measures the attractiveness of a link is also used to calculate the probability of selection. After route construction, updating of pheromone which consists of two parts takes place. First part is about evaporating of existing pheromone and second part is about depositing of additional pheromone on each link by looking at the goodness of the solution. Sometimes additional quantity of pheromone is also laid on the links that are part of the best found solution so far. The process continues until some stopping criterion is satisfied.

When multiple objectives are considered, then different criteria need to be considered simultaneously while finding goodness of solutions. The following are a few examples in which researchers have used ACO when different approaches to combine objectives were used.

In ref [52], two ant colonies were initialised to solve MOVRP consisting of two objectives in a hierarchical way. Primary objective of first colony was to minimise the fleet size; whereas, objective of second colony was to minimise the total travel time while ensuring that fleet size does not exceed the value of global best. While each ant
colony optimized a different objective, coordination between different colonies took place at the end of iterations when different colonies shared their information with each other to update pheromone at the global level. To calculate probabilities of selection for each link, a product of current pheromone level and a local heuristic function was used. Local heuristic functions that incorporated distance, wait time, and difference of arrival and end of time windows information were used to calculate the attractiveness of the feasible links. Updating of pheromone for each link of two colonies took place at two levels. At local level, depositing of pheromone was performed by considering the number of customers and total distance calculated through nearest neighbour heuristics in each route. At global level, pheromone was incremented according to the global best. To improve the solution further, seven different local search procedures such as customer/branch relocation, exchange and 2-opt, shuffling of tour orders were used.

In ref [176], separate ant colonies were initialised to minimize two objectives i.e. Fleet size and transportation cost. To stop getting stuck in partial optimization, a hybrid of ant colony with cellular automation was used. Each colony was tasked to optimise a separate objective while considering a global optimum solution in sight. To calculate the probabilities of selection for each link, instead of using a heuristic function, travel cost of each link was used together with pheromone levels. Pheromone updating was performed by looking at the goodness of solution. Instead of optimizing objectives in a hierarchical way, each colony optimised a separate objective. At the end of iteration, each colony shared information with each other. If a solution was found that optimized both objectives and was better than the current global best then the global best was updated accordingly and pheromone updating at global level in each colony was performed.

In ref [140], a hazardous material routing problem that considered two objectives (travel time and risk) was modelled. Since a real-life road network was used, therefore not all network nodes had transportation demand; however, with each link a risk cost was associated. Labelling algorithm was used to generate shortest non-dominating paths between different demand nodes. An initial solution was generated by using nearest neighbourhood heuristics to find initial Pareto optimal set. Initial pheromone levels for arcs were set by taking a reciprocal of product of objective function values and fleet size used in the initial solution. Instead of adding customers, feasible arcs were added based on a pseudorandom rule to construct the routes. Since
a single ant colony was used, therefore the probability of each arc was calculated by considering two separate heuristic value functions for risk and travel time in addition to existing pheromone levels. After construction of routes, an insertion based local search method was applied to improve the solutions. While updating Pareto set, not only risk and travel time but also fleet size was considered. While updating pheromone, deposition of new pheromone took into account the fleet size and values of two objective functions. Average values of fleet size, total risk and total time of all solutions in Pareto set were used to update pheromone at global level if exploitation was favoured.

In ref [122], researchers adopted a different strategy to estimate the Pareto Optimal solutions while solving a mobile healthcare CTP problem. In each iteration cycle, weights for each objective were randomly generated and normalised so that all objectives could be aggregated into a single objective function and solved as single objective ACO problem. However, at the end of each iteration cycle, cost for each objective was calculated individually so as to evaluate its non-dominance w.r.t. existing solutions in Pareto set. Weighted transition probabilities were calculated for each link. To update pheromone for each link, instead of using an aggregated function, three different pheromone values (one for each objective) were calculated for each link and pheromone increment was performed in proportion to the weights for each objective. Local improvements were performed before calculating objectives values for three different criteria to check the Pareto dominance.

2.4.4.c Simulated annealing (SA)

This stochastic based meta-heuristics simulates the annealing process in solids. In annealing, crystals lattices are formed when a solid is heated above its melting point and then gradually cooled down. Controlling the temperature is the key to crystal formation as quick cooling could result in retarded formation [190]. Starting from an initial solution, SA generates a new solution by making changes in the previous solution. In case of minimization objective, if the value of the objective function decreases then the new solution replaces the old solution. However, if the objective function increases, then the move is allowed with some certain probability $e^{-\frac{\Delta f}{T}}$ to avoid getting entrapped in local optima, where $\Delta f$ represents the change in the objective function values (i.e. New – Old value) and $T$ is the temperature. The temperature in the initial iterations is set high and as a
consequence of it the probability to accept a worse solution is higher in earlier iterations. The temperature is gradually cooled down thus making the probability of accepting a worst move smaller with the passage of time. Similarly, if the $\Delta f$ is large i.e. the new solution is worse than previous solution then probability will be smaller.

When applying SA to solve MOVRP, one needs to decide how to evaluate new solutions in the context of multiple objective functions; how to calculate the probability to accept worse solution; and how to control the temperature cooling mechanism. Depending on how the decision maker’s preference is incorporated, researchers have addressed above-mentioned concerns in different ways, a few of which are mentioned below.

While solving dial-a-ride problem with three objectives (i.e. travel time, customer dissatisfaction and fleet size), the researcher in [115] incorporated the decision maker’s preferences by adopting a-priori weighted sum approach. Simulated annealing was used to cluster the customers while a nearest neighbourhood heuristic was adapted to solve routing for each cluster. An initial solution was generated randomly and neighbourhood solutions were explored by exchanging customers from different clusters or by moving individual customers from one cluster to another. Routes for each customer were generated by looking at the shortest move which was based on normalised weighted sum of time and tardiness. To improve performance, a Tabu list was also used so as to avoid cycling to previous solutions. Similarly in ref [42], a weighted sum approach to combine objectives was adopted. However, instead of generating an initial solution randomly a heuristic was used; and to explore the neighbourhood a random selection between 1-opt and 2-opt operators was made and applied to the current solution. To generate Pareto front, weights were varied in different iterations. Controlling of temperature was performed by using a geometric function.

In ref [88], a shortest path method was used to generate an initial solution and SA was used to optimise different objectives. Since the value of initial and final temperature and cooling mechanism have a profound impact on overall SA performance; therefore, special attention was paid to address these issues. Temperature cooling function was applied after a single SA iteration. This appears to be slightly different from standard SA version where a certain number of iterations/transition takes place between two successive cooling iterations. Similarly to decide the values of initial and final temperature an algorithm was used.
Instead of adopting weighted sum approach, in ref [40] a posteriori approach to construct Pareto front was used after evaluating each objective function independently. To evaluate each solution and to find $\Delta f$, a different way was adopted. A parallel simulated annealing version was used in which multiple initial solutions were generated. Neighbourhood solutions were generated after applying mutation, assimilation and crossover operators. If the new solutions were better than previous solutions then new solution were made as current solutions. However, if the solutions were non-dominated then these were elected as current with a probability. The value of $\Delta f$ was calculated as sum of percentage change of both objectives. Value of each objective was also calculated separately so as to find non-dominated solutions. The number of function calls was used as the stopping criteria.

In ref [82], instead of generating a single solution, a population of initial solutions with different initial temperatures were generated by using insertion heuristics. To explore the neighbourhood, ten different search operators were applied. If the child solution was indifferent to the parent solution then it replaced it. But when the parent dominated the child solution, then the metropolis criterion was applied to select the child solution by looking at the variation in the value of objective function and temperature. The new solution was added in external Pareto archive if it was not dominated by any existing member.

2.4.4.d Variable neighbourhood search (VNS)

Variable neighbourhood search algorithm works on the concept of modifying the neighbourhood systematically within a local search algorithm. Solution improvement mechanism is done by including multiple neighbourhoods simultaneously. VNS algorithm consists of three main elements which include 1) generation of initial solutions, 2) neighbourhood generation, and 3) local improvement. Once an initial solution is obtained, different neighbourhoods could be generated by applying different neighbourhood structure (operators). From each neighbourhood, a solution is randomly selected and local search is initiated to find the local optima in the surroundings of a randomly selected solution. If the local optimum is better than the initial solution then the current best solution is updated, otherwise another solution from the next neighbourhood is selected and the search process starts again. The process continues till some stopping criterion is satisfied [191].
VNS meta-heuristics have been used in MOVRP in different ways. One way is to use it within the framework of some other metaheuristics. For example, in ref [97], VNS was used within PSO framework to increase solution convergence speed. However, VNS can be used as main search framework and other meta-heuristics could be used within that framework. For example, in ref [121], VNS was used to model dynamic multi-period delivery problem. A Sweep heuristic was used to create initial solution and Tabu Search was used as local search method. In Tabu search, randomly selected customers from one route were removed and reinserted into another route by using insertion heuristics that considered the weighted sum of wait times and workload balance. If no improved solution was found then a shaking procedure based on ruin and recreate was applied to create another neighbourhood solution of the current best solution.

In ref [34], VNS was used to solve an inventory routing problem for the delivery of fuel products. In IRP one needs to determine the quantities as well as customers served on each day in the planning horizon. Initial solutions were generated by solving inventory problem by using three different methods. Once the delivery quantities and customer served for each day became available, a sweep heuristic was used to construct the routes. Since the studied problem was a multi-compartment and multi-product problem; therefore four different operators that were based on context specific information were used to generate neighbourhood solutions. These shaking operators constructed new neighbourhoods by changing the delivery time of all or the same fuel compartments in a given day or station. Local search was implemented at two different levels. In the intra-period search, the neighbourhood was explored by either transferring a customer from one route to another, by swapping two customers in different routes, and by using 2-opt method. The criterion to evaluate a new solution was based on minimisation of total distance travelled for that day. In the inter-period local search method, all possible solutions were evaluated by using the shaking procedures mentioned above. To evaluate different solutions a weighted sum approach that combined routing and inventory holding cost was used.

In above-mentioned examples, weighted sum function was used to evaluate different solutions. However, in the ref [160], VNS was invoked within MOPSO meta-heuristics to explore non-dominated solutions. Pareto front and set of current population was used as an input to VNS meta-heuristics. To start local search, initial solution was picked by using two different strategies. First strategy was based on
selecting an initial solution from the current population set by using Roulette wheel method. Alternate strategy was to choose one member of Pareto set by using crowding distance procedure. As the problem involved optimizing of total cost and environmental impact of operations in context of 2-echelon Location routing problem; therefore, problem specific operators in addition to ordinary operators were used to generate neighbourhood of initial selected solution. From each neighbourhood, a solution was randomly selected and local search procedure was applied. If the new solution $\varepsilon$-dominated the initial solution then it was added in Pareto set. If no one dominated each other, a random solution selection between these two was made to update the Pareto set accordingly.

2.4.4.e Large neighbourhood search (LNS/ALNS)

Large neighbourhood search method is based on the principal of destroy and repair. In the destruction phase, a fraction of customer nodes are eliminated from current solution by using some heuristics. While in the repair phase, the eliminated nodes are inserted back at different locations to reconstruct a neighbourhood solution. If in the destroy phase, only a small part is destroyed then chances of getting stuck in local optimum are quite high. Similarly if a considerable portion is destroyed then the process could take a lot of time and may give poor results. So to avoid this situation, researchers have suggested either to randomly select the degree of destruction in each iteration cycle or to increase it gradually. In the repair process, some heuristics or MIP solvers could be invoked to reconstruct the destroyed solution. If the new solution thus obtained is better than all previous solutions then it replaces the global best and current solutions. However to avoid getting trapped in local optima, inferior solutions could be accepted, as a current solution, by using probability based acceptance criteria that is commonly adopted in simulated annealing method. The iterations continue till some stopping criteria are satisfied. Adapted large neighbourhood search (ALNS) extends LNS by allowing usage of more than one destroy and repair method. With each method a weight is associated that defines probability of using that method in the search process. Based on the performance of the method in a fixed number of previous consecutive iterations, weights are dynamically adjusted during search process [192]. The following are a few examples that provide a glimpse of how researchers have used ALNS in MOVRP context.
In ref [46], a lexicographic a-priori approach was adopted that first maximised the number of customers served followed by minimization of distance. An insertion heuristic based on smallest detour was used to generate initial solutions. In the ruin stage, destroy operators were applied to single customer or route levels. At each level, customers or parts of routes were either eliminated from the existing solution either randomly or by using proximity rule. When destroy operators were implemented at customer level then proximity was determined based on linear aggregation of spatial and temporal characteristics; while at route levels proximity was determined based on smallest distance between any two nodes between different routes. In the reconstruction phase, customers not present in the destroyed solution were selected randomly. These customers were inserted at feasible places by either considering least cost or regret based procedures. At the evaluation stage, a simulated annealing based criterion was used to accept or reject the new solution. At this stage both objectives were scalarized by converting maximization of one objective into its equivalent minimization problem. To scale both objectives, numbers of customers were multiplied with a constant big number.

In the previous case, destroy and repair methods were implemented by using operators that are commonly found in the literature; however, in ref [72] researchers introduced some problem specific and criterion specific operators. Five different objectives were considered in the context of transportation problem of disabled people. Since the destination of many passengers was the same, so this problem specific information helped to design problem specific removal operators. For example, one of the problem specific removal operators was based on destinations in which a destination was randomly picked and all customer requests with the same destination were eliminated from the current solution. Similarly instead of using all five criteria, one criterion was randomly chosen and customers were removed or inserted from/to current solution based on that single criterion. Similar to the work of [72], researchers in ref [66] developed some specific operators to solve the pollution routing problem. A saving heuristics was used to generate initial solutions by looking at capacity and time window constraints only. In addition to using nine existing operators, three different removal operators namely neighbourhood, zone, and node-neighbourhood were proposed. Four existing insertion operators were adapted and one additional insertion operator based on time-windows was proposed. For
acceptance criteria a weighted sum of three objectives was taken within a simulated annealing framework.

2.4.4.f  Particle swarm optimization (PSO)

The particle swarm optimization mimics the cognitive and social behaviour of insects and birds in a swarm. This algorithm starts by initializing a population of solutions randomly. Each individual in the population is called a particle and is analogous to a chromosome in EA algorithm. The particles keep changing their places in cost surface with a velocity. The velocity of each particle and its position is updated according to its personal and global best solutions. Personal best is the position that gives the best objective function of all the positions from which the particle has visited; whereas global best is position that gives best objective function value of all positions that all particles have visited. Once a particle reaches a position where its objective function is better than its own best or swarms best, then both bests are updated accordingly. At each iteration cycle, the velocity of a particle is determined by inertia, local and global best positions. Inertia is a force that keeps moving a particle in the same direction and is product of current velocity and inertia weight. The cognitive and social terms force a particle to move towards its local and global best positions. The cognitive and global terms are the product of random number, acceleration, and difference between current and best positions. The direction of particle is affected by inertia and acceleration weights. High inertia weight means a particle tends to move in the same direction. In the initial iteration, the inertia weight is set to high thus allowing particles to move freely to explore new areas on the cost surface thus enabling to find local and global optimum quickly. However, in later iterations the weight is decreased. As the number of iterations increases, the majority of particles end close to global optima. The biggest advantage of PSO is that it is easy to implement and is able to explore a cost surface which has many local minima [190]. One key step to implement PSO for VRP is route encoding in the form of a particle. Following are few examples in which PSO is applied to solve the MOVRP.

In ref [97], a hybrid version of PSO was used. Objectives considered in the model included fleet size, distance and wait time. Instead of generating the swarm randomly, a variant of greedy algorithm (GRASP) was utilised. Particles were evaluated by taking weighted linear sum of three objectives. To improve solutions, VNS was used. To update velocity not only information about current, local and
global best position but also neighbour information was used. So before updating velocity, nearest neighbour was determined by taking the fitness to distance ratio for each particle.

In ref [149], MOPSO was used to generate Pareto set. Initial population was generated randomly and current, local and global best information were used to update the velocity. Since in Pareto set all solutions are non-dominating, therefore, to find the global best values, a hypercube was generated in 3-dimensional objective space. In ref [160], to select the global best, two different strategies i.e. Grid and crowding distance were used. To improve performance, VNS was used to intensify local search.

2.4.4.g Evolutionary algorithms (EA)

Algorithms that are based on theory of evolution are collectively called evolutionary algorithms. These methods are population based and can be used to solve discrete and continuous optimization problems alike. However, these algorithms do not guarantee an optimum solution. Good solutions could be generated, if certain design principles are followed. The necessary ingredients for an EA to work include encoding solutions into strings, a fitness functions and operators that could be applied to solutions to maintain inheritance and diversity. A fixed population of individual solutions is generated randomly. Individual solutions are evaluated and better solutions are selected to reproduce. The new solutions formed after mating are evaluated and depending on fitness values replace entire or a part of the existing population. The process continues in this way, until the required number of iterations are completed thus yielding the final population of solutions. As mentioned above, EA does not guarantee optimum solutions. The results are stochastic as different runs might yield different results at the end. The reason for EAs to be widely used is their ability to work with multiple non-linear constraints, rugged and discontinuous cost surfaces. However, it is recommended to use EAs if only approximate solutions are required and furthermore there is no alternate method available to model the problem [190].

Different variants of EA have been used. For example when weighted sum approach was used to combine objectives, then evaluation of child solution was very easy so a simple genetic algorithm (GA) was sufficient to solve the problem [37], [42], [64], [101], [109]. Usually fleet is considered the most important asset for any logistics company, therefore, while making plans usually fleet size is minimised first
followed by other objectives i.e. lexicographic approach is adopted. Since in any population, one can find multiple solutions with same number of vehicles but different values of second objective, therefore, it is always good to explore more solutions that minimize second objective while keeping value of first objective as a constraint. GA is often hybridized with local search heuristics to do this task. For example, local search procedures such as Ant colony [182] and Tabu Search [9] have been combined with GA to do this. To generate non-dominated solution set, different variants of EA such as MOGA [54], [84], [159], [186], HMOEA [6], [24], [79], [83], [89], [94], [143], [147], [159], Memetic [19], SPEA/PAEC [40], NSGA/II [7], [19], [40], [112], [142], [148], [149], [154], [159] have been used to solve MOVRP.

2.4.4.h Scatter Search (SS)

Scatter search provides a framework that consists of five different methods. At the first step, some method is used to generate an initial solution set which has not only high quality but also diverse solutions. In the next stage, an improvement method(s) is applied which either tries to improve solution quality or feasibility of different solutions as the initial solution set may contain infeasible solutions. At this stage, usually a local search heuristic is applied that systematically explores the neighbourhood and is capable of escaping local optima. At this stage, a reference set, usually of fixed size, is generated that contains a mix of high quality and diversified non-dominated solutions. Different subsets of the reference set are generated, and elements in each subset are combined to generate new solutions. New solutions are subject to an improvement process. If a new solution is found that dominates any member of the reference set then the dominated solution is removed and the new solution is added. The cycle of subset generation, combination, improvement and reference set updating continues till some stopping criterion is satisfied [193]. The following few examples relate to SS implementation in MOVRP context.

In ref [39], initial solutions were generated by using Clark Wright saving heuristics based on total cost criteria. Pareto archive set was dynamically updated and was used to store non-dominated solutions. A member from this set was deleted if a solution became dominated. Similarly if membership in Pareto archive set reached its upper limit, then a member might need to be eliminated; however, it would result in the destruction of a part of Pareto front. To stop deterioration of non-dominated
solutions, a procedure to update archive set was described. New non-dominated solution was only added if it was dissimilar to the nearest solution in the archive set. Dissimilarity was determined by comparing the Euclidean distance of the new solution from its closest neighbour in the archive set with the “duplication area” of the closest neighbour in the archive. If a new solution was found to be dissimilar then it was added in the archive set thus increasing its size. To generate diversified solutions, freak path algorithm was used that generated auxiliary solutions. To select diversified and high quality solutions, dominance or highest crowding distance based criteria were used. To improve filtered solutions, pair wise exchange and insert procedures were used. Crowding distance criterion was used to select high quality, unique and non-dominated solutions so as to be added in the reference set. Improved solutions on each front were sorted and were selected according to a formula to select diversified solutions. Two-element subsets were generated and a GA-based crossover operator was used to recombine the solutions.

In ref [119], a bus routing problem considering fleet size and maximum travel time by any student was considered. The problem was solved by using scatter search framework. Two different constructive heuristics were proposed which build routes by fixing the number of buses used. Improvement in routes’ travel times were performed either by swapping customers’ location within the same route or between different routes, or by combining different routes into one. Separate solution pools were generated for different values of fleet size. The reference set for each pool was generated and allowed to undergo the improvements stage. Pareto set was generated so decision maker can make trade-off analysis as per his requirements.

2.5   Issues / Concerns in Multi-Objective optimization

In this section, some practical problems and issues are presented that could be faced or need to be thought about by a modeller during different phases of MOVRP i.e. during formulation, implementation, testing, validating, and presentation stages.

2.5.1   Objectives, objective functions and combining approach

A few of the issues/concerns, related to objectives, that one can face when dealing with MOVRP are shown in Figure 2-20.
If there are multiple objectives, then one needs to ask these questions, “Is it really an important objective to model?” and “Can we achieve this objective by some other way?” For example, ‘equity among drivers’ is measured and reported either on a daily basis or a weekly basis in practice. Instead of making it as an objective, one can devise a mitigation strategy. While explaining about this measure, the answer of a respondent was

“Instead of paying overtime on daily basis, make it on weekly basis. If it is financial, people would like to do it on daily basis; if it is non-financial then people tend to calculate it on weekly basis. Try to have some mitigation strategies.”

The definition of objectives at times could be very misleading. For example, one of the commonly reported KPI in companies’ reports is ‘Empty miles run by all vehicles’ which usually is defined as distance travelled by vehicles without any payload. Absence of payload on any journey leg is usually frowned upon by management as only expenses are incurred without generating any revenue on that particular leg. Now if reduction of empty miles is treated as a sole objective then even for the same route with the same travel time and distance, by just reversing the sequence of customers, one can reduce the empty miles run by a vehicle. However, this new solution may lead to an increase in fuel consumption. To cope with this problem one might be tempted to aggregate empty miles and fuel consumption objectives. However by doing so, one introduces a new problem i.e. proper scaling of two objectives. Instead of aggregation, perhaps it would be more sensible to transform the original objective into another form. For example, instead of
formulating empty miles as per above-mentioned definition, it would be better to use tonne-miles as an objective. In other words, at the modelling stage, one needs to consider multiple definitions of the same objective. This could be done by interacting with the decision maker in order to find a surrogate measure that somehow contains the essence of different objectives. Even when different incommensurable objectives are present, one needs to at-least investigate if there is any other way to formulate multiple objectives into a single objective without losing sight of actual objectives and compromising solution quality.

As in real-life, the optimization process may need to be initiated many times a day, therefore any objective function that involves a lot of calculation may consume more time thus slowing down the solution generation process. If possible surrogate objective functions could be used as a proxy to actual objective functions to evaluate different candidate solutions; then actual objective function(s) could be applied when a final solution is picked at the end of the optimization process.

A quick and easy solution, to get answers to the above-mentioned question, would be to talk with practitioners in addition to checking existing literature. However, it appears that there are some gaps between the academic community and practitioners; and these gaps need to be bridged. While responding to the issue of the relationship between academia and industry, one respondent said

“They don’t talk to real people. They talk to other academics.”

Before combining multiple objectives in a weighted sum approach, one needs to ensure that combining of objectives should provide meaningful solutions. For example if minimization of fleet size and minimization of route imbalance (in terms of distance travelled or travel time) are considered as objectives then it may yield unsatisfactory results. As for the same number of vehicles one can generate numerous solutions with varying degrees of route imbalances and distance travelled. Minimization of imbalance may in fact increase overall distance travelled by all vehicles. Instead of aggregating fleet size and route imbalances, perhaps reduction of total distance travelled by all vehicles and distance imbalance between different routes would be a more sensible approach when a weighted sum is used to combine both objectives. If consideration of fleet size and distance imbalances simultaneously is necessary, then a better alternative would be to change the preference incorporation
approach i.e. instead of using a weighted sum approach, generation of a non-dominated solution set would be a better choice.

Similarly, when different objectives are aggregated then scaling and assigning of weights should be performed with extreme care especially when multiple stakeholders are involved.

2.5.2 Choice of method & correct Parameterization

Any proposed method to solve MOVRP should be capable of solving a variety of instances; providing robust solutions in a realistic time; exploiting information that becomes available during the search process; and exploring diverse areas of the search space. As mentioned before, exact methods are limited in solving real-life size instances. Heuristic methods can provide feasible solutions; however, feasible solutions might be quite far from optimal solutions. Also, performance of heuristics vary significantly with the size of problem instances noted by Braysy and Gendreau (cited in ref [50]). In contrast, meta-heuristics provide a general framework within which different heuristics/methods could be fit in and thus are capable of solving very large-scale problem instances. However, meta-heuristics may consume a lot of computational time to converge the solutions and may even fail to always provide a robust solution that is acceptable to decision makers. In order for meta-heuristics to provide a robust solution, it is mandatory to have an awareness of problem specific knowledge which is then translated in designing different heuristics/operators that could be fit into the meta-heuristics framework. Performance of meta-heuristics is not only dependent on the quality of initial solutions and parameters settings, but also on neighbourhood structure. If a simple neighbourhood structure is used then subsequent evaluation of moves might not lead to the right direction through the search space.

Different strategies have been used by researchers to maintain a balance between intensity, diversity, and convergence speed. Random generation of initial solutions may lead to late convergence. Whereas, using heuristics to generate good quality initial solutions may lead to early convergence without exploring all potential areas in the search space. Initial solution generation by using a mix of randomness and greedy heuristics has been found to be an effective strategy by researchers [91]. To maintain balance between intensity and diversity, one needs to fine tune heuristic settings. Performance of many meta-heuristics is sensitive to parameter settings and
could be improved by adjusting parameters; however, it is not always obvious what
the right combination is. Usually a hit-and-trial approach is used to calibrate
parameters. It is quite possible that by upscaling problem size, parameter settings may
require readjustment. In order to solve a variety of instances, a few researchers have
opted to use algorithms with self-tuning parameters. Context specific knowledge
could be very helpful in making parameter adjustment policies [91].

2.5.3 Testing, validating, and performing sensitivity analysis of model
Access to real-life data instances on one hand helps deepen understanding about
problem complexity; and on other hand, it provides an opportunity to compare the
model with solutions from current practice. However, access to real-life data
instances has always been an issue for any researcher. In the absence of real-life data,
one resorts to one of three solutions i.e. to use benchmark data sets, to adapt
benchmark data sets, or to generate new datasets. However, commonly used
benchmark instances such as Solomon datasets were designed to solve single
objective problems and thus are not entirely suitable for MOVRP. Assumptions used
in designing these classic sets are not realistic such as Euclidean distance between
nodes and travel time is proportional to distance. As a consequence, when multiple
objectives are evaluated by using benchmarks then one might find the correlation
between different objectives to be very weak thus indicating that the benchmark
instances are partially inadequate to test MOVRP [92]. Therefore, many researchers
adapt the classic benchmark instances. As there is no common framework to compare
the performance of proposed algorithms by using adapted data sets, researchers often
apply methods developed by other researchers to compare the results. Comparisons
performed, in this way, can only be meaningful, if the proposed algorithm and the
algorithm used for comparison are subject to the same parameters or testing
conditions.

One problem that a researcher might face, who used an adapted data set, is that
it would be difficult to convince practitioners to adopt a model which is not based on
real data. Therefore, an alternate strategy is to generate new test instances while
ensuring that the generated instances are as close to real-life as possible. To generate
such instances, researchers have used different methods. For example, in ref [92],
researchers generated test instances based on knowledge obtained through observing
operations of a distribution company. Similarly researchers, in ref [165], generated
test instances by using open street maps. A data generator that is capable of producing different instances when provided with different designing parameters such as service policies and vehicle characteristics is also developed by researchers [194]. Similarly researchers in ref [195] have also investigated the characteristics of existing robust test instances; have proposed three frameworks to generate test instances; and shown that challenging instances could be generated by adjusting different parameters.

Checking of model validity and sensitivity analysis has been done by researchers in different ways. For example, checking of solution quality and sensitivity analysis was performed by changing weights in an aggregated objective function; varying instance size by either increasing customers; changing demand patterns; modifying network structure; varying different parameters of heuristics/algorithms; comparing results with real-life or with that of other researchers; and comparing with lower bounds [33], [50], [51], [64], [91], [143]. Sometimes comparisons are performed by solving the same problem with different variants of the same meta-heuristic [38]; by using approaches to incorporate decision makers’ preferences e.g. a-priori and Pareto set (89); by applying different algorithms in separate runs [6], [7], [34], [39], [79], [94], [97], [123], [147], [149], and by creating and testing multiple scenarios [159].

2.5.4 Presentation of results to decision makers

As it can be seen, in Table 2-5, a significant number of researchers have adopted posteriori approach to incorporate the decision maker’s preference. When a posteriori approach is used then presenting the solutions to the decision maker in a format that he can understand is a serious issue. Usually, when two objectives are present, results are reported in a two-dimensional graph. Though for more than two objectives, one can resort to 3D, parallel coordinate graphs or inverse radar graphs; however, the main problem is how to ensure that the decision maker is able to comprehend the solution and to do trade-off analysis. The gravity of the problem compounds when a person looks at the profile of decision makers. In the VRP context, usually the decision makers who evaluate the solution are the transport planners; and the majority of these planners started their careers as drivers. It would be very interesting to investigate if these transport planners are capable of doing trade-off analysis especially in circumstances when the frequency of making decision is very regular and the time available is quite limited. At present, it appears that usage of posteriori
approach of handling a MOVRP problem is limited by factors such as skill set, cognitive span and the limited time available to transport planners.

When asked about these issues, the following response was received from a respondent about the number of solutions and the way comparisons are performed by planners in retail distribution.

“….. change constraints, generate another solution and compare two solutions. Generate solution independently serially”

“I don’t think they are going to look more than two solutions. What the average cognitive span of a typical user could be?”

2.6 Conclusion
Multi objective problems frequently arise in real-life situations. In the context of the VRP, various factors such as inter-process dependencies, focus on cross-function activities, and the presence of different stakeholders has intensified the attention of practitioners and academics towards the consideration and solution of multi-objective VRPs. Since objectives from the economic, social and environmental domains conflict with each other, the identification of a single solution that optimises all objectives is not usually possible. Therefore, to find a compromise solution, researchers often resort to a-priori or posteriori approaches and use meta-heuristics to solve different variants of the MOVRP.

This chapter has provided a review of academic work related to the MOVRP, with a focus on the retail sector. To investigate the practical relevance of this work, additional secondary and primary data was collected from practitioners. The results from this analysis suggest that there is a strong alignment between academia and practitioners in certain aspects; however, there are still some gaps that require bridging. One such gap relates to the priority and definition of different objectives. In the academic literature, distance travelled is the most commonly modelled objective, while practitioners consider the number of vehicles as the most important objective. Similarly there are differences in the definitions of objectives e.g. for objectives related to workload balance and customer services.

Another difference relates to the approaches adopted in incorporating multiple objectives. In the retail distribution context, the weighted sum method is the most
commonly used approach to deal with instances of the MOVRP. While the weighted sum approach is widely used in the academic literature, the use of posteriori approaches to deal with the MOVRP is increasing. The most common assumption behind a posteriori approach is that the decision maker can do a trade-off analysis before picking a particular solution. However, in high dimensional, complex problems that give rise to a large number of trade-off solutions, it may be very difficult to conduct a full trade-off analysis, as time may be an important bottleneck. Other factors that may inhibit the adoption of posteriori approaches in the retail sector include the limited skill set of transport planners (DM) and their limited attention span. These remaining challenges provide an opportunity to researchers to find innovative ways of allowing the decision maker to guide the search process in the preferred direction, and facilitating trade-off analysis.

The review further highlights that a common issue faced by researchers in this field is the absence of benchmark or real-life data sets, an issue that needs to be addressed through a closer cooperation between academics and practitioners.
2.A Survey Information about practitioners’ view

To get the views of the practitioners, qualitative interviews were conducted with the relevant staff members working at different management levels in different Logistic companies. Semi-structured formal interviews were conducted with the top and middle-management level managers; while informal interviews were conducted with the managers/planners that were engaged in operational levels activities. Observations were also made in few of the companies to gain further understanding about the overall routing processes that were in place in those companies. While making observations, emphasis was given to understand how planners actually made plans and overall objectives considered and constraints considered while making the routing plans. An interview with the managing director of a routing software providing company was also carried out. The following table provides some details about the profile of people that were interviewed and some relevant information about companies’ overall network and services offered.

<table>
<thead>
<tr>
<th>Visit Dates / Month</th>
<th>Designation and number of years of experience in Logistics sector</th>
<th>Data collection method</th>
<th>Company</th>
<th>Network Details / Services offered</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 2012</td>
<td>Director operations (30+ years)</td>
<td>Formal Interview</td>
<td>AKW Group PLC</td>
<td>1 DC and 100+ trucks Distribution, Warehousing, Contract Packing &amp; International Freight Services</td>
</tr>
<tr>
<td>July 2013 – Nov 2014</td>
<td>Depot Manager (35+ years)</td>
<td>Formal Interview &amp; informal chats on multiple occasions</td>
<td>Nagel-Langdons</td>
<td>8 DC, 240+ vehicles Storage, Order picking, Haulage, Pickup &amp; Delivery</td>
</tr>
<tr>
<td>July 2013 – Nov 2014</td>
<td>Transport Manager</td>
<td>Informal Interview /</td>
<td>Nagel-Langdons</td>
<td>--- Same as above ---</td>
</tr>
<tr>
<td>Time Period</td>
<td>Position</td>
<td>Method</td>
<td>Company</td>
<td>Observations</td>
</tr>
<tr>
<td>-------------------</td>
<td>-----------------------------------------------</td>
<td>---------------------</td>
<td>----------------------------------</td>
<td>--------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Sep 2014 – Feb 2015</td>
<td>Transport Support Manager (15+ years)</td>
<td>Interview / Observation</td>
<td>Yearsley Group</td>
<td>13 Sites, 300+ vehicles, Storage, distribution and freight forwarding or Frozen food.</td>
</tr>
<tr>
<td>Sep 2014 – Feb 2015</td>
<td>Transport Data Analyst (2 years)</td>
<td>Interview / Observation</td>
<td>Yearsley Logistics</td>
<td>--- Same as above ---</td>
</tr>
<tr>
<td>Aug 2014 – Feb 2015</td>
<td>Network Planning Manager (20+ years)</td>
<td>Formal Interview</td>
<td>NFT</td>
<td>7 Sites, 450+ vehicles, Distribution, Cross-docking, Order picking, Rework; Co-packing; Relabelling; RFID tag writing, Warehousing</td>
</tr>
<tr>
<td>Aug 2014 – May 2015</td>
<td>Senior Planner (25+ years)</td>
<td>Informal Interviews / Observation</td>
<td>NFT</td>
<td>--- Same as above ---</td>
</tr>
<tr>
<td>7th–8th Sep 2015</td>
<td>Managing Director (20+ years)</td>
<td>Formal Interview</td>
<td>Optrak</td>
<td>Routing consultants and software provider.</td>
</tr>
</tbody>
</table>
Chapter 3

Optimization of inter-depot trunking with semi-trailer swap

3.1 Introduction

Distribution logistics play an important role in meeting the demand of customers in the retail sector. In the past few decades many changes have been noted in distribution networks. This includes increases in the distances between manufacturers and retailers, as well as increases in the number of products and the demand for just-in-time (JIT) deliveries [196]. For small companies, it becomes unviable to run their own logistic operations, if customers are geographically dispersed. Similarly, for large companies, running their own logistics operations may be inefficient in terms of capacity utilization, as backhaul trips will frequently amount to empty runs. In an endeavour to increase efficiencies and to focus on core competencies, many companies are now outsourcing transport operations to third party logistic (3PL) companies.

From the perspective of 3PL companies, source-to-destination (manufacturer-to-retailer) direct deliveries may be worth considering, if orders from a customer are large or are associated with strict lead-time restrictions [197]. However, if orders are small, it becomes difficult to justify separate point-to-point shipments from a cost-perspective. For this reason, 3PL companies generally operate in a hub-and-spoke (H&S) fashion. In other words, they have depots located at different parts of the
country and customers are served by their local depots. This allows companies to consolidate orders from different customers before transporting products across long distances [196], and may even extend to the consolidation of payload from several depots.

In line with this, large 3PL companies that operate in the UK’s food retail sector typically organize their operations in an H&S fashion. While some products with short shelf lives may warrant an immediate delivery to their end destinations, this leads to an increase in overall transportation cost. To maintain a balance between delivery cost and service quality, many 3PL companies run their operations on D1-D2 basis i.e. pickup operations take place on the first day (D1), followed by delivery operations on the second day (D2). This particular sector has further specific requirements, most prominently the maintenance of an appropriate temperature for certain goods during transportation. To meet variations in order size and differing temperature requirements between products, 3PL companies may use a heterogeneous fleet, or use special vehicles that can split payload space into multiple compartments, such that separate temperature requirements can be met in each compartment.

The arrangement of logistics networks in an H&S fashion with D1-D2 operations allows for the decomposition of transport planning into a number of sub-problems, as illustrated in Figure 3-1. If supply and customer nodes for a particular product are associated with the same depot (hub), the product would usually be considered as a part of two separate planning problems. The first of these relates to the pick-up of products and their subsequent transport to the closest depot (associated hub) on day one. The second relates to the delivery of products from the depot to the customer nodes on day two. If the collection and delivery nodes of the product are associated with different depots, however, then an intermediate planning problem needs to be addressed. This third problem addresses the efficient transport of consolidated orders between regional depots (network hubs).
This manuscript is focused on investigating the last problem, which is referred to as the problem of inter-depot trunking (line haul plan). The problem (as encountered in the UK food retail sector) has a number of unique features, which have not previously been addressed in the literature. The foremost feature is the possibility of truck-meets-truck events with semi-trailer swaps between depots that are crucial in ensuring that the maximum travel times for individual vehicles are not exceeded. Other interesting features include the consideration of a heterogeneous fleet, adjustable compartment sizes and multiple objectives.

If we neglect the aspects of semi-trailer swaps and consider the problem from the perspective of each individual depot, the delivery of orders to the associated depots can be formulated as a standard (single-depot) vehicle routing problem (VRP), with all other depots taking the role of customers. However, such a distributed formulation is unlikely to lead to optimal solutions, as it does not account for the possibility of co-operation between depots, such as cross-depot consolidation of orders and the implementation of back-hauling. While it is generally clear that a systems approach should lead to a better solution, there has been no previous investigation into the inefficiencies of a distributed approach in this particular problem scenario.

At a systems level, if we continue to neglect the aspect of semi-trailer swaps, inter-depot trunking can be said to have some similarities with vehicle routing problems that involve paired pick-ups and deliveries from customers (VRPPD), as each order has an associated source and destination node. The problem we consider
is novel though in the sense that there are defining differences to the existing literature on the VRPPD, which prevent the application or adjustment of existing models. Most prominently, these differences include the possibility of truck meets-truck events, but other key differences lie in the subsidence of customers and depots, the distribution of the fleet across all depots, and the possibility of adjusting compartment space at each individual leg of a journey.

In this manuscript, we first describe and (practically) motivate a finite set of local route choices for inter-depot trunking that constrain the space of feasible routing plans, thus reducing the computational complexity of the problem. We then proceed to describe a mathematical programming formulation of inter-depot trunking that optimizes the routes subject to the set of local route choices defined. In this context, we consider a set of alternative objective functions, and discuss and analyse the implications of this choice. Finally, the usefulness of the model is demonstrated using real-world data from a UK-based 3PL company.

The remainder of this paper is organized as follows. Section 3.2 surveys the relevant literature and highlights the defining features of inter-depot trunking. A mixed integer formulation is presented in Section 3.3, and we describe its practical implementation. In Section 3.4, we use a small, synthetic data set to analyse the impact of changes to the optimization objective, considering minimization of distance travelled, number of vehicles, a weighted sum of these objectives, or carbon emissions. In Section 3.5, we introduce details of a real-life case study, and the results of our model are compared to the plans designed by human experts. Section 3.6 provides a final discussion of our results and concludes the paper.

### 3.2 Background

#### 3.2.1 Problem characteristics

We focus on a special form of the problem of inter-depot trunking (or hub-to-hub routing) in a hub-and-spoke style network. More specifically, the problem is characterized as follows:

1) Demand is given in the form of origin-destination pairs, where the origin and destination nodes correspond to different depots. Demand may exceed the capacity of an individual truck, thus requiring splitting and transporting using
multiple vehicles. It may further be categorised in terms of temperature requirements.

2) A heterogeneous vehicle fleet (heterogeneous w.r.t. capacity) is distributed across the set of depot nodes (hubs). The payload space of all trucks may be split into two compartments to accommodate demand with different temperature requirements. This compartmentalization may be flexibly adjusted for each leg of the journey, with the single constraint that an even number of pallets will be accommodated for by each sub-compartment. The fleet includes vehicles capable of transporting and exchanging semi-trailers. Further details about vehicle types, product categories, pallet types and multiple compartments are given in appendix-3.A.

3) All vehicle routes for inter-depot trunking are closed. In other words, they start and end at the same depot.

4) The set of nodes considered in this problem comprise all depots of the networks, as well as a set of swapping points, which are designated locations for truck-meets-truck events. Vehicles that meet at these swapping points or a depot, may exchange the semi-trailers that they currently carry.

5) A plan for inter-depot trunking is a routing plan for the available vehicle fleet that ensures that demand at all depots is fulfilled. Feasible plans may be subject to additional constraints such as the number of vehicles available at a particular depot, vehicle capacities, driving time restrictions, etc. An optimal plan may be characterized by the minimization of the number of vehicles utilized, total distance travelled or other measures of economic interest.

With regard to the routing of individual vehicles, we limit our attention to three key types of route choices, as illustrated in Figure 3-2. More complicated routes are possible in principle, but introduce the need for complex synchronization constraints and have been observed to be unrealistic in practice.

1) **Direct source-destination routes**: If the total journey time (round trip) between two depots does not exceed constraints on maximum journey times, then a direct delivery/pickup is possible. Products destined from one depot to another depot could be transported by a vehicle based at either of these two depots. This type of journey does not require a trailer swap and therefore makes no assumption on vehicle type. Note that this type of journey is the most efficient when the load to be delivered/picked up (line-haul and back-
haul) between two depots is close to the full capacity of the available vehicle (full truck-load).

2) **Trailer Swap routes**: If the round trip between two depots exceeds the maximum journey time, then a direct delivery/pickup is not possible. In this case, both depots need to send a vehicle and arrange an en-route trailer-swap. The swapping could take place at another depot or at a designated swapping location. Following the swapping event, both vehicles will return to their home depots. This type of journey requires trucks that can transport (and exchange) semi-trailers. We do not consider situations in which an en-route trailer swap is infeasible (in terms of constraints on journey time), as this should have been accounted for during network design.

3) **Round-Robin routes**: If the load to be transported between two depots is less than the vehicle capacity (i.e. less than truck load), then the vehicles may be sent via other depots. Specifically, any vehicle based at a source-depot can travel to its destination-depot via an intermediate-depot and pickup or deliver further orders in passing. We refer to this type of journey as a round-robin trip and we limit our attention to round-robin trips involving at most three depots. This type of journey does not include a trailer swap and therefore makes no assumption on vehicle type.
3.2.2 Related literature

As the problem shares some similarities with multi-depot VRP and pickup and delivery problems a review of the relevant literature is presented in the following subsections. To avoid confusion, we also review previous literature on “trailer swaps”, which has been recently considered in a context quite different to that of inter-depot trunking.

3.2.2.1 Vehicle Routing with Pickup and Delivery

Pickup and delivery vehicle routing problems (VRPPD), involve the collection of a commodity (goods or people) from origins and their delivery to a finite set of destinations. Researchers have studied different variants of VRPPD and have classified these into different categories. When the number of origins and destinations of each commodity is used as basis of classification then the VRPPD can be sub-categorised into a) many-to-many (M-M), b) one-to-many-to-one (1-M-1) and c) one-to-one (1-1) problems. When there are many possible origins or destinations for the commodity being transported, then the problem is grouped under many-to-many (M-M) pickup and delivery problem. If the commodity is delivered from a single depot to many potential customers and vice versa then the problem
falls into the one-to-many-to-one (1-M-1) category. And if each object has a single, designated source and destination then it falls in the one-to-one (1-1) pickup and delivery problem category.

An alternative view is to distinguish based on the point of time at which full information about the problem becomes available. In this case, we can differentiate between a) the static VRPPD where all information is known a-priori and b) the dynamic VRPPD where a part of the information (e.g. the route network) is known in advance and the remaining information (e.g. demand) becomes available in real time.

Finally, we may differentiate between VRPPDs dependent on the sequence in which pickup and delivery are fulfilled. In this case we can identify a) simultaneous VRPPDs, where pickup and delivery demand of each customer is fulfilled simultaneously; b) backhaul VRPPDs, where pickup takes place once all delivery demand has been met; and c) mixed pickup VRPPDs. where a combination of the previous two options is used. For an up-to-date literature review and mathematical formulations of the various VRPPD problems, the reader is referred to [198], [199]. For a specific literature review of static and dynamic problems, we refer to [200], [201].

To solve VRPPD, different exact and heuristic methods have been used. These methods include branch-and-cut, branch-and-cut-and-price, column generation, branch-and-price, Tabu search, large neighbourhood search approaches. For a specific literature review about methods and applications of the VRPPD, we refer the readers to the work of [202] and [203].

### 3.2.2.2 Hub-and-Spoke models

The hub-and-spoke structures utilized in 3PL networks are similar in nature to the approaches used in the airline industry, shipping and the rail freight sectors. For example, in the context of air transport, each passenger/baggage has a specific origination and destination point. Demand (in the form of passengers/baggage) is typically consolidated by using small feeder aeroplanes to different hubs and then large aeroplanes are used to transport people between those hubs. Planners will try to improve utilization by assigning aeroplanes of appropriate capacity to each route [204].
The design of such networks has been considered e.g. by Aykin [205] who described the capacitated hub-and-spoke network design problem in the airline sector. Mathematical formulations for the combined hub location and routing problem in this sector have been presented e.g. in ref [206], which addresses the sub-problems in an iterative manner. Other work has considered the routing problems in isolation. Related work in the context of rail freight transport and cargo shipping includes [207] and [208]. For related work in the context of road freight, readers are referred to [197], [209], [210].

3.2.2.3 Multi-Depot Vehicle Routing

Contrary to the classical VRP, where there is only one depot, multi-depot vehicle routing problems (MDVRPs) allow for the presence of multiple depots. The existing literature related to MDVRP considers both strategic (i.e. long-term) and operational (short-term) decisions. Typical strategic decisions include the determination of the optimal number and location of depots and the definition of geographical boundaries between depots [211]–[213]. In the location-routing problem the routing of vehicles and the choice of depots are addressed together, but the problem is typically sub-divided and then solved sequentially. Furthermore, some research has considered the solution of the problem under additional constraints such as capacity and cost restrictions or in the presence of a heterogeneous fleet and fleet size constraints [212], [213].

Operational decisions in this context include the assignment of customers to depots and vehicles; as well as the sequencing of customers in order to minimize single/multiple objectives under a given set of assumptions and constraints. In addition to usual cost (total distance or time) minimization objectives, the literature has also considered additional objectives such as route balancing, customer satisfaction and environmental cost [214]–[217]. Constraints in the problem may relate to limits on fleet composition and fleet size, limits on the maximum distance travelled by an individual vehicle or its travel time (e.g. to obey driving hour regulations), and compliance with the delivery time-windows requested by customers [218]–[220].

Different exact and meta-heuristic techniques have been described and compared by researchers [217], [221]–[225]. Three different approaches have been used to solve the MDVRP. The most common approach is based on a decomposition
of the problem and a separate analysis of the assignment and routing sub-problems. Customers are first assigned to their nearest depots, through the use of clustering techniques, or based on distance, ratio or urgency [219], [226]. This reduces the problem for each depot to a standard VRP problem and routes associated with each depot can then be constructed in isolation using dedicated heuristics [214], [215], [222], [227], [228]. Alternatively, the assignment and routing decisions can be considered simultaneously [219], [220]. The final approach is conversion of MDVRP into a standard VRP through the introduction of a virtual central depot [229].

3.2.2.4 Trailer Swap VRP

The literature that explicitly considers trailer-specific decisions covers three different types of problems. The first of these relates to the problem of truck and trailer routing (TTRP) in which an additional full trailer is attached to rigid/artic vehicle to form a road train. During the vehicle’s journey, the trailer may be left at a suitable parking place, so that customers with certain access restrictions can be served. After these customers have been served, the vehicle returns and re-attaches the trailer for the remainder of the journey. Contrary to the standard VRP, the TTRP includes additional decision variables related to the best parking location of the trailer [230]–[232]. Literature in this area has considered a number of real-world problems including the consideration of a variety of constraints such as the number of times a trailer can be de-coupled, driving hour limits and unit demand of customers [233]–[235]. A variety of exact and heuristic methods have been developed and compared in recent years [236]–[243].

Trailers also feature in situations where customer demand corresponds to pickup and delivery of requests of semi-trailers. These types of problems are known as truck and trailer vehicle routing problems (TTVRPs). Suitable trailers are left at customers’ locations; once they have been filled by customers, they may then be picked up by another tractor and transported to their destination. In this case, the delivery and pickup of trailers are treated as separate requests with precedence restrictions, and the planning horizon for the problem may extend across multiple days. The primary objective in this problem typically relates to the determination of a schedule that minimizes the overall distance travelled and the number of vehicles.
used, while considering e.g. constraints related to delivery time-windows and trailer availabilities [6].

3.2.2.5 Originality

Analysing inter-depot trunking in the view of the above literature, we can conclude that the problem may be classified as a multi-vehicle, static, one-to-one, simultaneous pickup and delivery VRP. However, our work differs from previous models in the VRPPD literature in a number of aspects.

An important feature in inter-depot trunking is the presence of multi-depots. While multiple depots are considered in the MDVRP literature, a unique feature here is that all depots act as each other’s customers, i.e. both the pickup-and-delivery points correspond to depots, and the demands for all pick-up-and-delivery requests are paired. In other words, each delivery request needs to be served by a particular depot, whereas, in the traditional MDVRP, the demand of a customer can be met by any depot.

Furthermore, while the problem has some similarities with the general hub-and-spoke models (e.g. when compared with the airline industry), a defining difference lies in the consideration of trailer swaps between depots. Specifically, such trailer swaps require the synchronization of two vehicles dispatched from different depots and are different from the trailer-swaps previously considered in the literature.

In addition to the trailer swaps, the problem has a number of other interesting features. Specifically, the demand from a single depot may be split i.e. the same node may be visited by many vehicles. Finally, we consider a heterogeneous fleet (where only some vehicles are suitable for trailer swaps) and the trucks contain multiple compartments whose space is adjustable to adjust for the demand associated with specific temperature requirement.

3.3 Problem Formulation

A full problem formulation of inter-depot trunking as a mixed integer linear programming problem is provided in this section.
3.3.1 Mathematical programming formulation

Let $G = (N, A)$ represent a graph, where $N$ is the vertex set and $A$ is the arc set. The distance of each arc $(l, m) \in A$ is symmetric and is represented by $c_{(l,m)}$ which is a positive value. The set $N$ includes both the depot nodes $D = \{1, 2 \ldots |D|\}$ and the non-depot nodes $S = \{1, 2\ldots |S|\}$ i.e. $N = D \cup S$. Depot nodes are associated with potential shipment demand and have a fleet of vehicles stationed. Non-depot nodes have neither shipment demand nor any vehicles situated with them but are used to swap trailers en-route. Let $P = \{\text{Frozen, Chill}\}$ be a set of product types and let $q_{(p,i,j)}$ be the number of pallets of product type $p \in P$ that are to be transported from depot $i$ to depot $j$ ($i \neq j \in D$). Let $V$ be an index set of vehicles and $T = \{\text{Artic, Rigid}\}$ be a set of vehicle types which could have different fuel consumption profiles and maximum capacities which are represented by $Q_t$ ($t \in T$). The cost associated with the usage of vehicle of type $t$ is represented as $VC_t$, while the cost associated with CO$_2$ emission per mile is represented by $w_e$. Each depot has a heterogeneous fleet of variable size. Let $MV_{(i,t)}$ represent the maximum number of vehicles of type $t \in T$, based at depot $i \in D$. The maximum distance a vehicle can travel in a day is represented by $C_{\text{max}}$.

The CO$_2$ emissions from a vehicle are directly proportional to the amount of fuel consumed by the vehicle. Fuel consumed between any two nodes $l$ & $m$ is represented using a linear function of payload to capacity ratio.

\[
\text{Fuel Consumed} \ (l, m) = \left( F_e^t + F_p^t \cdot \frac{\text{Payload}}{Q_t} \right) \cdot c_{(l,m)} \tag{3.1}
\]

Where $F_e^t$ represents the fuel consumed (in g/km) when the vehicle is running empty and $F_p^t$ represents the fuel consumed (g/km) when there is a payload. Further details about how the values of $F_e^t$ and $F_p^t$ are calculated can be found in appendix-3.B. Let $FC^t_{i,v}$ represent the total amount of fuel consumed by vehicle $v$ of type $t$ based at $i$ on its entire trip.

Two types of decision variables need to be used. A set of binary variables are used to represent the usage of vehicle; whereas a set of integer variables are used to determine the number of pallets to be loaded during each leg of the journey. The decision variables are given as follows:
<table>
<thead>
<tr>
<th>Decision Variables</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>VD_{i,j}^{v,t}</td>
<td>Binary</td>
<td>It equals 1, if vehicle ( v ) of type ( t ) based at ( i ) goes directly to ( j ) (( i \neq j )) and returns back; otherwise it is set to 0, ( i \neq j ).</td>
</tr>
<tr>
<td>VV_{i,j,k}^{v,t}</td>
<td>Binary</td>
<td>It equals 1, if vehicle ( v ) of type ( t ) based at ( i ) goes to ( j ) via ( k ) (( i \neq j \neq k )) and returns back; otherwise it is set to 0. Route is ( i \rightarrow k \rightarrow j \rightarrow i ).</td>
</tr>
<tr>
<td>VT_{i,j,l}^{v,w}</td>
<td>Binary</td>
<td>It equals 1, if trailers are swapped between vehicle ( v ) based at ( i ) and vehicle ( w ) based at ( j ) at swap point ( l ) (( i \neq j \neq l )); otherwise it is set to 0. The value of ( t='Artic' ).</td>
</tr>
<tr>
<td>PDD_{i,j}^{v,t,p}</td>
<td>integer</td>
<td>The number of pallets, of product type ( p ), delivered from depot ( i ) to ( j ) (( i \neq j )) on a vehicle ( v ) of type ( t ) that is based at ( i ).</td>
</tr>
<tr>
<td>PDP_{i,j}^{v,t,p}</td>
<td>integer</td>
<td>The number of pallets, of product type ( p ), picked up from depot ( j ) by a vehicle ( v ) of type ( t ) that is based at ( i ) (( i \neq j )).</td>
</tr>
<tr>
<td>PVD_{i,j,k}^{v,t,p}</td>
<td>integer</td>
<td>The number of pallets of category ( p ) delivered to ( j ) by a vehicle ( v ) of type ( t ) based at ( i ) via depot ( k ) (( i \neq j \neq k )). The route is ( (i \rightarrow k \rightarrow j \rightarrow i) ).</td>
</tr>
<tr>
<td>PVP_{i,j,k}^{v,t,p}</td>
<td>integer</td>
<td>The number of pallets of category ( p ) picked up by vehicle ( v ) of type ( t ) based at ( i ) from ( j ). The route is via ( k ) (( i \neq j \neq k )) and is ( (i \rightarrow k \rightarrow j \rightarrow i) ).</td>
</tr>
<tr>
<td>PVED_{i,j,k}^{v,t,p}</td>
<td>integer</td>
<td>The number of pallets of category ( p ) delivered en-route to node ( k ), by a vehicle ( v ) of type ( t ) based at ( i ) and going to ( j ) (( i \neq j \neq k )). The route is ( (i \rightarrow k \rightarrow j \rightarrow i) ).</td>
</tr>
<tr>
<td>PVEP_{i,j,k}^{v,t,p}</td>
<td>integer</td>
<td>The number of pallets of category ( p ) picked-up en-route from ( k ) (for ( i )) by a vehicle ( v ) of type ( t ) based at ( i ) and going to ( j ) (( i \neq j \neq k )). The route is ( i \rightarrow k \rightarrow j \rightarrow i ).</td>
</tr>
<tr>
<td>PTD_{i,j,l}^{v,t,p,w}</td>
<td>integer</td>
<td>The number of pallets of category ( p ) sent from ( i ) to ( j ) on trailer carried by vehicle ( v ) of type ( t='Artic' ) based at depot ( i ) and does a trailer swap at ( l ) (( i \neq j \neq l )).</td>
</tr>
<tr>
<td>PTED_{i,j,k}^{v,t,p,w}</td>
<td>integer</td>
<td>The number of pallets of category ( p ) delivered en-route.</td>
</tr>
</tbody>
</table>
Minimization of Total Vehicles Used (Objective -1)
\[
\sum_{t} \sum_{v} \sum_{(i \neq j)} (VD_{i,j}^{v,t} + \sum_{k \neq i \neq j} VV_{i,j,k}^{v,t} + \sum_{l} \sum_{w} VT_{i,j,l}^{t,v,w})
\]  (3.2)

Minimization of Total Distance (Objective -2)
\[
\sum_{t} \sum_{v} \sum_{(i \neq j)} ((VD_{i,j}^{v,t} * c_{(i,j)} * 2) + \sum_{k \neq i \neq j} VV_{i,j,k}^{v,t} * (c_{(i,k)} + c_{(k,j)} + c_{(j,i)})) + \sum_{l} \sum_{w} (VT_{i,j,l}^{t,v,w} * c_{(i,l)} * 2))
\]  (3.3)

Minimization of weighted sum of objective 1 & 2
\[
\sum_{t} \left( \sum_{v} \sum_{(i \neq j)} (VD_{i,j}^{v,t} + \sum_{k \neq i \neq j} VV_{i,j,k}^{v,t} + \sum_{l} \sum_{w} VT_{i,j,l}^{t,v,w}) \right) * VC_t
\]  (3.4)

Minimization of emissions (Objective -3)
\[
\sum_{t} \sum_{v} \sum_{(i,j)} \left( VD_{(i,j)}^{v,t} \ast FC_{i}^{vt} \right) + \sum_{k} \left( VV_{(i,j,k)}^{v,t} \ast FC_{i}^{vt} \right) + \sum_{l} \sum_{w} \left( VT_{(i,j,l)}^{v,t,w} \ast FC_{i}^{vt} \right)
\]

Subject to the following constraints

(3.6)
\[
\sum_{j} \left( VD_{i,j}^{v,t} + \sum_{k} VV_{i,j,k}^{v,t} + \sum_{l} \sum_{w} VT_{i,j,l}^{v,t,w} \right) \leq 1 \quad \forall (v, t, i)
\]

(3.7)
\[
\sum_{j} \sum_{v} \left( VD_{i,j}^{v,t} + \sum_{k} VV_{i,j,k}^{v,t} + \sum_{l} \sum_{w} VT_{i,j,l}^{v,t,w} \right) \leq MV_{(i,t)} \quad \forall (t, i)
\]

(3.8)
\[
\sum_{p} \left[ \frac{PDD_{i,j}^{v,t,p}}{2} \right] = \frac{Q_{t} \ast VD_{i,j}^{v,t}}{2} \quad \forall (v, t, i, j)
\]

(3.9)
\[
\sum_{p} \left[ \frac{PDP_{i,j}^{v,t,p}}{2} \right] = \frac{Q_{t} \ast VD_{i,j}^{v,t}}{2} \quad \forall (v, t, i, j)
\]

(3.10)
\[
\sum_{p} \left[ \frac{(PVD_{i,j,k}^{v,t,p} + PVED_{i,j,k}^{v,t,p})}{2} \right] \leq \frac{Q_{t} \ast VV_{i,j,k}^{v,t}}{2} \quad \forall (v, t, i, j, k)
\]

(3.11)
\[
\sum_{p} \left[ \frac{(PVD_{i,j,k}^{v,t,p} + PVEP_{i,j,k}^{v,t,p})}{2} \right] \leq \frac{Q_{t} \ast VV_{i,j,k}^{v,t}}{2} \quad \forall (v, t, i, j, k)
\]

(3.12)
\[
\sum_{p} \left[ \frac{(PVEP_{i,j,k}^{v,t,p} + PV_{i,j,k}^{v,t,p})}{2} \right] \leq \frac{Q_{t} \ast VV_{i,j,k}^{v,t}}{2} \quad \forall (v, t, i, j, k)
\]
\[
\sum_{p} \left[ \frac{(PTD_{i,j,l}^{v,t,p,w} + (PTP_{i,j,l}^{v,t,p,w}))}{2} \right] \leq \frac{Q_t \cdot VT_{i,j,l}^{t,v,w}}{2} \quad \forall (t, v, i, w, j, l) \tag{3.13}
\]

\[
\sum_{p} \left[ \frac{(PTD_{i,j,l}^{v,t,p,w} + (PTED_{i,j,l}^{v,t,p,w}))}{2} \right] \leq \frac{Q_t \cdot VT_{i,j,l}^{t,v,w}}{2} \quad \forall (t, v, i, w, j, l) \tag{3.14}
\]

\[VT_{i,j,l}^{t,v,w} = VT_{j,i,l}^{t,w,v} \quad \forall (t, v, i, w, j, l) \tag{3.15}\]

where \( t = 'Artic' \)

\[
\sum_{i,j} \sum_{w} VT_{i,j,l}^{t,v,w} \leq 1 \quad \forall (t, v, i) \tag{3.16}
\]

\[
\sum_{t} \sum_{v} (PDD_{i,j}^{v,t,p} + PDP_{j,i}^{v,t,p}) \quad \forall (p, i, j) \tag{3.17}
\]

\[
+ \sum_{k} (PVD_{i,k}^{v,t,p} + PVP_{j,k}^{v,t,p} + PVED_{i,k,j}^{v,t,p} + PVEP_{j,k,i}^{v,t,p})
+ \sum_{w} (PTED_{i,j}^{v,t,p,w} + (PTP_{k,j,i}^{v,t,p,w}))
+ \sum_{i} \sum_{w} (PTD_{i,j,l}^{v,t,p,w})) = q(p,i,j)
\]

\[
(c_{i,j} + c_{j,i}) \cdot VD_{i,j}^{v,t} \leq C_{max} \quad \forall (v, t, i, j) \tag{3.18}
\]

\[
(c_{i,k} + c_{k,j} + c_{j,i}) \cdot VV_{i,k,j}^{v,t} \leq C_{max} \quad \forall (v, t, i, j, k) \tag{3.19}
\]

\[
(c_{i,l} + c_{l,i}) \cdot VT_{i,j,l}^{t,v,w} \leq C_{max} \quad \forall (t, v, i, w, j, l) \tag{3.20}
\]

Equation (3.2), (3.3) and (3.5) represent the vehicle, distance and emission minimization objectives. Equation (3.4) is the weighted sum objective which is obtained by converting the objectives (3.2) and (3.3) into respective monetary values. Constraints that each vehicle is used only once are represented by equation...
(3.6). Equation (3.7) ensures that the number of vehicles of any type departing from a depot cannot exceed the number of vehicles of that specific type available at the depot. Equations (3.8)-(3.14) ensure that the payload on each vehicle does not exceed the vehicle capacity on different legs of journey. Constraints (3.15-3.16) ensure that one trailer can only be swapped with one and only one other trailer. Similarly, swapping can only be performed when both sides send an artic truck to the swap point, i.e. swapping between an artic and rigid truck is not possible. Constraint (3.17) ensures that demand is met and constraints (3.18) – (3.20) ensure that the maximum distance requirement is not violated. Similarly these constraints also ensure that if distance exceeds the maximum distance limit, then swapping will be performed.

The payload on each leg of the journey can be different. For this reason, the fuel consumed on these legs can be different even if their distance remains same. Assuming that a vehicle travels from depot $i$ to depot $j$ and back, i.e. the journey consists of two legs, the total fuel consumed can be calculated as:

$$ FC_{i}^{vt} = \left( F_e^t + F_p^t * \frac{\sum_p PDD_{ij}^{v,t,p}}{Q_t} \right) * C_{(i,j)} + \left( F_e^t + F_p^t * \frac{\sum_p PDP_{ij}^{v,t,p}}{Q_t} \right) * C_{(j,i)} \quad (3.21) $$

If the vehicle takes a round robin trip or is involved in a trailer swap, the fuel consumption needs to be calculated in a similar manner, with the number of journey legs and the payload varying accordingly.

The variable $FC_{i}^{vt}$ requires pallet information which itself is a decision variable; therefore, the objective function i.e. eq. (3.5) becomes non-linear. To linearize this, we can use the following method. Note that this method requires additional variables and constraints for each leg of the journey to estimate fuel consumption at the cost of increased computation time. The fuel consumed on any leg can be divided into two parts as shown below:

$$ Fuel\ Consumed = \begin{cases} F_e^t * c_{i,j} & \text{Related to curb weight} \\ F_p^t * \frac{\text{Payload}}{Q_t} * c_{i,j} & \text{Related to Payload} \end{cases} \quad (3.22a) $$
Let $ED_{ij}^{vt}$ represent additional fuel consumed because of payload when a vehicle goes from depot $i$ to depot $j$. To help in estimating the value of $ED_{ij}^{vt}$, we add the following constraints.

$$ED_{ij}^{vt} \leq F_p^t * c_{ij} * VD_{ij}^{vt}$$  \hspace{1cm} (3.23)

$$ED_{ij}^{vt} \leq F_p^t * \frac{\text{Payload}}{Q_t} * c_{ij}$$  \hspace{1cm} (3.24)

$$ED_{ij}^{vt} \geq F_p^t * \frac{\text{Payload}}{Q_t} * c_{ij} - F_e^t * c_{ij} * (1 - VD_{ij}^{vt})$$  \hspace{1cm} (3.25)

$$ED_{ij}^{vt} \geq 0$$  \hspace{1cm} (3.26)

Equations (3.23) and (3.26) set the limits on the value of this variable; whereas equations (3.24) and (3.25) estimate the amount of emissions within that range. Note that in equation (3.25) the value of fuel consumption without payload ($F_e^t$) is always greater than payload-specific consumption ($F_p^t$).

### 3.3.2 Pre-Optimization

The problem size grows exponentially with an increase in the number of depots, swap points, trucks and product types. The problem size can be reduced through the introduction of a pre-processing step that directly assigns some vehicles to those sub-routes that require full-truck loads.

Specifically, we consider pair-wise demand between all depots and identify all those depots for which the required load amounts to a full truck load or more in both directions (line haul and backhaul). If the return-trip between two depots does not exceed the constraints on the maximum travel distance, then a direct trip can be made. If the return between two depots exceeds the permitted travel distance, then vehicles carrying a full truck load are sent from both depots, and an en-route trailer swap is arranged. The most suitable swap point is selected as the one that minimises the overall distance travelled by the two vehicles.

Demand and vehicles that have been accounted for through these direct routing decisions are eliminated from the overall problem formulation. This ensures a reduced, smaller optimization problem is obtained that focuses on the planning of the less-than-truck-load trips only.
3.3.3 Implementation and post optimization

The model was implemented in AIMMS (ver. 4.1) which invokes CPLEX 12.6 solver to optimize the MIP. A maximum of 2 million iterations or a gap of 3% from the best LP bound was used as the stopping criterion. After the optimization, the total travel time and distance for each route, the associated CO\textsubscript{2} emission, and the total cost are calculated. More details are given in appendix-3.B.2 & 3.C3.

3.4 Small-scale analysis

3.4.1 Data set

For an initial analysis of the model, a small case study consisting of four depots and a single swap point was considered. Pair-wise distances between depots, availability of vehicles and demand (in pallets) from each depot are assumed as shown in Figure 3-3 below. We assumed a heterogeneous fleet with two different vehicle types (Artic and Rigid Trucks, where Artic Trucks are required for all routes involving a trailer swap).

![Demand Data (in pallets) & Distances (in miles) & MaxVehicles & Parameters]

<table>
<thead>
<tr>
<th>Demand Data (in pallets)</th>
<th>Distances (in miles)</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand (i,j)</td>
<td>From \ To</td>
<td>Arctic Rigid</td>
</tr>
<tr>
<td>D</td>
<td>D</td>
<td>290 115 150 162</td>
</tr>
<tr>
<td>L</td>
<td>L</td>
<td>290 176 175 134</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>115 176 59 50</td>
</tr>
<tr>
<td>P</td>
<td>P</td>
<td>150 175 59 90</td>
</tr>
<tr>
<td>W</td>
<td>W</td>
<td>162 134 50 90</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Demand Data (in pallets)</th>
<th>Distances (in miles)</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>MaxVehicles</td>
<td>Arctic Rigid</td>
</tr>
<tr>
<td>Depot</td>
<td></td>
<td>6 1</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>6 1</td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>6 1</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>6 1</td>
</tr>
<tr>
<td></td>
<td>P</td>
<td>6 1</td>
</tr>
</tbody>
</table>

N.B: D,T,L,P are depots
W is a swap point

Figure 3-3: Initial data for small case study.

3.4.2 Parameters and Assumptions

The following (simplifying) assumptions were made: the distance between all nodes is considered symmetric. Loading and unloading time is considered a constant (independent of load and vehicle type), and vehicles of the same type are assumed to have the same emission characteristics (Euro-4 standard). To calculate the value of weighted sum objective, values of distance and vehicles minimization objectives were converted into monetary cost (£) by using suitable weights. The costs associated with vehicles’ usage (VC\textsubscript{i}) were obtained from the company. The company assumes a fixed vehicle usage cost whenever a vehicle is used and it
includes drivers’ cost, fuel cost, insurance and etc. The distance travelled by all vehicles was converted into emission cost. To calculate emission cost, value of \( w_e \) (in pence / mile), was taken from Freight Transport Association [78]. The following table provides the values of different parameters used in calculations.

<table>
<thead>
<tr>
<th>Table 3-1: Parameters used in the model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Distance</strong></td>
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<tr>
<td></td>
</tr>
<tr>
<td><strong>Cost</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Capacity</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Fuel Consumed (g/km)</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

### 3.4.3 Results

In the analysis of this problem, we focus on the impact of the objective function used during the optimization process. Comparisons are made between distance minimization, minimization of the number of vehicles and a weighted sum of the two. Finally, we also consider the optimization of an emission-objective that depends on distance travelled as well as the composition of the fleet due to differences in the emission characteristics of truck types.
Comparing the results of distance and vehicle minimization, it is clear that the two objectives are in conflict. When minimizing distance alone, savings of 338 miles in distance, 8+ hours in total time and 129 litres in fuel consumption are obtained compared to a solution focused on vehicle minimization. In contrast though, the minimization of the number of vehicles allows for a reduction of fleet size by one vehicle. It is unsurprising that a reduction of fleet size would be accompanied by an increase in total distance and average journey time of the vehicles. Specifically, in the context of our model, it is indicative of an increased use of round robin trips.

If a homogenous fleet is used, then the results of emission minimization and distance minimization are exactly the same i.e. there is no conflict between these objectives. However, if a heterogeneous fleet is used then a decrease in 5 litres of fuel consumed is obtained at an expense of an additional 60 miles when compared with the results of distance minimization objective. When the results of emission minimization for a heterogeneous fleet are compared to those of vehicle minimization, then it is evident that a saving of 135 litres and a reduction of 278 miles are obtained at an expense of using two additional vehicles.

The most interesting results are those obtained under the weighted sum objective where one vehicle is saved at an expense of an additional 60 miles and 26 litres of fuel, compared to the results obtained for the distance minimization. In
terms of total cost, since the cost associated with vehicle usage has a high weight, the total cost generally increases with the number of vehicles. But, when comparing the weighted sum with vehicle minimization, a cost saving of £231 is obtained for two reasons. 1) a rigid truck is used instead of an artic truck and 2) a difference in total distance travelled due to choice of routes.

![Parallel coordinate plot of Objectives vs criteria](image)

Figure 3-4: Comparison of different solutions

The results above indicate that savings in one objective can be achieved at the expense of other objectives. More interestingly though, we identify situations in which savings in a second objective are possible without compromising the primary objective. Specifically, Figure 3-4 provides a comparison of all four solutions using parallel co-ordinate plots (note that objective values have been normalized). It can be seen that the solution from the weighted sum objective is superior to (dominate) the plan obtained through vehicle minimization. In contrast to this, the solutions obtained through emission minimization method, distance minimization and weighted sum present optimal trade-off solutions that are non-dominated w.r.t. each other. To understand these results in more detail, we provide simple examples that highlight the reasons behind some of the differences observed.

The domination of the result for vehicle minimization is possibly less surprising and illustrates the weakness of this measure as a sole objective. In case-1 of Figure 3-5, the round trip distance between depots is less than the maximum distance constraint, and the swap point Z lies on the shortest route between the depots X & Y (i.e. \( c(x,Z) + c(Z,Y) = c(x,Y) \)). Two possible routes for this sub-problem are shown in case-1a and case-1b. If the objective is to minimize distance, then the total distance travelled in both routes is the same. For the first route, however, two vehicles are used, while the second route utilizes a single vehicle. A similar situation
is present in case 2. Here, the number of vehicles utilized for two possible routing plans (case-2c and case-2d) are the same, but the plans differ significantly in terms of the combined distance travelled. This difference in quality is captured by the weighted sum objective, resulting in a better plan than achieved from vehicle minimization alone.

The results obtained by using emission minimization (heterogeneous fleet) are interesting from the perspective of the impact this objective has on the values of other objectives, and the types of routes obtained. For example, Table 3-2 shows that the total emissions are reduced at the cost of an increase in the number of vehicles used. The following simple case is used to illustrate why this happens.
Consider Case-3 (Figure 3-6), which presents a case with three depots, where depot B is located on the route between A to C. Three possible solutions to this scenario are highlighted as case-3e, 3f and 3g. Under the distance objective, all of these solutions yield the same total value and are hence indifferent to each other. If considering the minimization of vehicles, solution 3e will be preferred over solution 3f and 3g, as a single artic vehicle is used. Under an emissions objective, however, 3e is the least preferable solution as pallets from depot C to B and B to A have to travel an additional distance ‘B-A-B’ and ‘B-C-B’ respectively. Solution 3g would be preferable in this setting, as a rigid truck (with a more favourable emissions profile) is employed to take care of transportation between B and A, instead of an artic truck.

We also note that the emissions from vehicles may also vary when solely the order in which depots are served is changed. For example, in case-4 (Figure 3-7), a single artic vehicle from depot A is sufficient to execute a round robin delivery. When distance or vehicle minimization is used as an objective, then the sequence in which nodes are visited (i.e. ‘A-B-C-A’ or ‘A-C-B-A’) has no impact on the objective function value. However, when emission minimization is used as the main objective, the direction of travel starts to impact on the objective function value - in case-4, the route ‘A-B-C-A’ becomes the optimal solution.

**3.5 Case Study**

The real-world case study relates to a 3PL company that deals with pickup and delivery of temperature controlled products throughout Great Britain (GB). There are eight depots located in different parts of GB. Each depot is responsible for providing
delivery and collection services in its own specified area (regions). A heterogeneous fleet of variable size, containing artic (tractor with semi-trailer attached) and rigid trucks, is located at each depot. The customers are located throughout GB and can be divided into two categories i.e. the originating customers (warehouses, wholesaler, and RDC - from which pallets are collected) and delivery customers (shop keepers, retailers and etc. - where pallets are to be delivered). The minimum order size is one pallet. If the order is too large then delivery/pickup could be done by many vehicles.

Figure 3-8 depicts the overall network of this company. The area served by each depot is shown in a different colour. A supply customer $S_1$ located in a region operated by depot $D_1$ can send pallets destined for delivery customers $C_1$ & $C_2$ (in the same region as that of $S_1$) and to customer $C_5$ (located in a region serviced by depot $D_4$). In these types of cases, the customer’s request is treated as three separate orders as there are three different source destination pairs i.e. $S_1$-$C_1$, $S_1$-$C_2$ and $S_1$-$C_5$. The collection of such pickup orders can be done by a single vehicle, but for delivery more than one vehicle may need to be used as the destinations are different and may lie in different regions.

The company aims to achieve next-day (Day-1 to Day-2) delivery to anywhere in the country. Deliveries and collections specific to each depot are scheduled by the local depot planner for each region. If the collection and delivery nodes fall in the same region then the respective regional depot manages the entire process i.e. collection on day one, storage in warehouse overnight, and delivery on the next day. But if the delivery and source nodes are in different regions, then pallets are transferred overnight from the originating region to the delivery region. The resulting problem of inter-depot trunking is highly time-constrained. It is executed using the same vehicle fleet that is used for regional collection and delivery, and needs to be finished before the start of the morning shift on the next day.
Figure 3-8: Delivery Network in GB showing eight depots and regions (in different colour) served by each depot.
3.5.1 Data set

Detailed data describing the company’s network is given in Table 3-3 and Table 3-4. This includes the distance matrix between different nodes (depots and swap points), total number of vehicles stationed at each depot and demand between different origin destination pairs. A homogeneous fleet (consisting of artic trucks only) is assumed here, as the company avoids the use of rigid trucks for inter-depot trunking. As highlighted in the results small case study, when a homogenous fleet is used then results of emission minimization are similar to that of distance minimization. Therefore, in the large case study, the model was not run for emission minimization as an objective.

Table 3-3: Distance (in miles) between different nodes (left) and number of artic vehicles available at each depot (right). The nodes starting with D indicate a depot node, while nodes starting with the S suffix indicate the non-depot swap points. N.B: The swapping can also take place at depots.

<table>
<thead>
<tr>
<th>From \ To</th>
<th>\ D1</th>
<th>\ D2</th>
<th>\ D3</th>
<th>\ D4</th>
<th>\ D5</th>
<th>\ D6</th>
<th>\ D7</th>
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<td>183</td>
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<td>53</td>
<td>195</td>
<td>331</td>
<td>160</td>
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</tbody>
</table>

Table 3-4: Delivery demand (in pallets) data between different OD pairs.

<table>
<thead>
<tr>
<th>From \ To</th>
<th>D₁</th>
<th>D₂</th>
<th>D₃</th>
<th>D₄</th>
<th>D₅</th>
<th>D₆</th>
<th>D₇</th>
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<td>57</td>
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<tr>
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<td>70</td>
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<td>101</td>
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<tr>
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149
3.5.2 Results

The pre-processing step described in section 3.2, resulted in the identification of two-way full truck loads. This load was directly accounted for through a total of 34 direct and 4 trailer swap trips. Table 3-5 gives the demand that remains after the elimination of these two-way full truck loads.

Table 3-5: Data (remaining pallets) after elimination of full truck loads

<table>
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<tr>
<th>Demand (i,j)</th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
<th>D5</th>
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We compared the results of our model to the plans currently in place in the 3PL company (i.e. made by a group of human planners). Our model results included the plans for the minimization of the number of vehicles, minimization of the distance travelled and the weighted sum of these two objectives. The emission objective was not employed, as a single vehicle type was considered in the data used in large case study, and due to a strong correlation between total emissions and distance travelled. The values of different criteria when different objectives are optimized, the gaps from best LP bounds, and computational times are shown in Table 3-6.
In Table 3-6 above, four different results are compared. The % Δ column compares the results obtained from the model to the results of real planners using the heuristics mentioned in appendix-3.D, in which delivery and collection between pairs of depots is considered in isolation. However, in the plans made by the model a holistic (network wide) picture is considered in which direct, trailer swap and round robin deliveries are planned. Core conclusions from these results are presented as follows:

It can be seen that regardless of the objective optimized in the model, fewer vehicles are required to carry out the same job without violating constraints regarding the distance and the driving hours. A 14-19% reduction in vehicle usage is achievable if plans are made by considering the holistic picture rather than considering the interests of two depots. This observation has significant implications for the company. On one hand the fixed cost associated with vehicle usage (such as insurance) could be reduced, while on the other hand, fewer drivers would be required during the night shift. A saving of 14-19% in the number of vehicles per day implies the potential for significant (close to seven digit (£)) savings over the course of a year.

Following routing plans from our model, fewer vehicles would be used to complete the same operations and this has a number of implications including an
increase in the average number of pallets carried by each vehicle by 16-24%. Another implication is a 15-27% increase in the average distance travelled (miles/vehicle) which indicates a better utilization of vehicles. If average distance increases, it also increases the average driving time (that excludes rest, loading and swap times) by 15-27% thus indicating a better utilization of drivers.

Another interesting observation from Table 3-6 is the difference in the % increase in driving and that of total trip length. There is difference of 6% in these two figures which is attributed to the fact that the use of less number of vehicles reduces the loading, unloading and swap activities.

If we compare the results obtained when objective-1 and objective-2 are optimized, it is consistent with the findings of the small case study. In other words, a reduction in the value of one objective can only be obtained at the expense of other objectives. For example, the number of vehicles used is reduced at the cost of increase in the total distance travelled.

We also consider the effect of different objectives on the frequency of different type of trips as shown in the Figure 3-9.

![Breakup of vehicle usage vs objective](image)

Figure 3-9: Breakup of vehicle usage vs objective.

As shown in Figure 3-9, almost 60% of trips planned by human planners are direct trips, whereas the remaining 40% are trailer-swap trips. However, our network-based optimization models produce plans where direct trips correspond to between 52-53% of tours, and the remaining tours either correspond to round robin or trailer swap routes.
In the network-based solutions, regardless of the objective function, the direct trips remain more or less the same (~ 53%) perhaps due to the fact that the demand between different OD pairs was more than Full truck load (FTL) on both ways so a direct route was the best choice. When the minimization of vehicles is considered as a primary objective, more ‘Round Robin’ trips are planned. These ‘Round Robin’ trips actually increase the distance travelled and therefore the average trip length. On the contrary, when distance is chosen as the main objective, then trailer-swap trips are more frequent while only a few ‘Round Robin’ trips are utilized. With the weighted sum objective a compromise solution is achieved i.e. fewer round robin trips and more trailer swaps are used compared to vehicle minimization, and similarly more ‘Round Robin’ trips and fewer trailer swaps are used compared to distance minimization.

3.6 Discussion & Conclusion

This paper investigates a new type of multi-depot, split-delivery and heterogeneous vehicle routing problem in which full truck load (FTL) and less-than truck load (LTL) are planned together. This type of problem is routinely encountered by practitioners in real world vehicle routing and is difficult to solve satisfactorily and efficiently. However, it has not been adequately addressed in the existing literature. Several features make this problem distinctive from problems found in literature, such as the fact that inter-depot demand between two depots can be fulfilled by vehicles based at different depots, and the possibility of performing trailer swaps (at pre-defined locations), if the distance between depots exceeds a pre-specified threshold. In this paper, a multiple objective mixed integer linear programming model was proposed to formulate this new type of vehicle routing problem for systematic analysis. Three objectives were considered including the minimization of the number of vehicles used, total distance travelled, and total CO₂ emissions from all vehicles used.

Two different case studies were presented in this paper. In the small case-study, the results of vehicle minimization, distance minimization and emission minimization (heterogeneous fleet) are compared with each other. These objectives are found to be conflicting with each other and gains in one objective value are typically obtained at the cost of the other objectives. A weighted sum method was used to find a suitable compromise solution.
In the large case-study, the results generated by using distance and vehicle minimization objectives are compared with those obtained by the existing planning method used by a company in the UK. The main findings from the case studies include that in comparison with the current planning practice, the employment of the proposed model could lead to a saving between 14-19% in vehicles used, which is equivalent to 6-7 digit savings in the annual cost of the UK company. The average trip length of vehicles and the number of pallets/trip carried by each vehicle also increase. In the large case study, the impact of the optimization objective on the type of route choices were also analysed and some interesting patterns were observed. For instance, when the objective is to minimize the number of vehicles, more round robin trips are planned; whereas, if the objective is to minimize the distance, then more direct or trailer swaps at a third depot are planned.

It needs to be noted that while the network-based proposed model offer many benefits, there are some real-life operational and human-related issues that are closely linked with vehicle scheduling. In particular, the model currently does not consider system-wide and long-term constraints. For example, the company works round the clock and uses the same resources (i.e. vehicles) in all shifts, making changes in one shift could have knock on effects on the second shift. Most customer interactions take place during the morning shift and timely deliveries during the morning shift are crucial in order to sustain long-term profitability. Longer routes during the night shift, as proposed by our model, may increase the risk of a late return of vehicles and therefore violation of delivery windows during the morning shift. Furthermore, it is crucial to maintain a delicate balance between efficiency and flexibility, and too ambitious routing plans could potentially affect the company’s ability to accommodate last minute changes in customers’ orders (and hence affect the quality of customer service). Finally, implications of the changes on driver-shift pattern should be another important consideration. The drivers of the company are permanent staff members and their shift patterns are decided once per month, while the number of deliveries between depots changes on a daily basis. To account for these long-term implications of the routing problem, it is likely to be necessary that optimized solutions continue to be further adjusted by human planners.

There are a number of avenues to further extend the research presented in this manuscript. First of all, a switch to heuristic (rather than exact) optimization methods would be valuable to ensure a more rapid generation of routing plans. The model
currently contains a number of simplifications, and these present points where further complexity may be added to the model. For example in Round Robin trips, pickup and deliveries between non-vehicle-originating depots could also be considered. This could potentially reduce vehicle usage and, therefore, the distance travelled. Also, vehicles of the same type (e.g. Artic trucks) may of different standards e.g. Euro-4, Euro-5 or Euro-6, and multiple emission functions may therefore be considered in future research. Similarly, the model could be extended to cater for the presence of pallets of different standards. Finally, further realistic constraints could be integrated such as the regulation that drivers are only allowed to do 10 hours of driving twice per week.
3.A Supplementary Material

The following section provides some background information about types of truck generally in use by 3PL companies in GB, the pallets types and how payload space is compartmented.

3.A.1 Truck and Trailer Types

To cope with the varying demands of customers in an efficient manner, often heterogeneous fleets of vehicles are stationed at different depots. The vehicles that are used in practice can broadly be divided into two categories i.e. Rigid and Artic. A rigid truck has a trailer fixed to the tractor which can’t be detached; whereas, an artic truck has a tractor with an attachable/ detachable trailer. The coupling and uncoupling of trailers provides flexibility in a way that trailers could be left at some customer location for loading/unloading, while a tractor could be used to pull another trailer during that time.

The trailers that are used, in practice, can also be broadly divided into two categories i.e. Full trailer and semi-trailer. A full trailer has its own front and rear axles and is pulled by a truck; whereas the semi-trailer has rear axle only and the weight is supported by detachable tractor. Sometimes a full trailer is also attached with a rigid or arctic truck to form a road train. The rational of using a road-train is to bring further operational efficiency and reduction in cost. Figure 3-10 shows different combinations of truck, tractor and trailer in which road haulage is carried out in practice.

Figure 3-10: Different truck and trailer combinations used in practice. Source: [235]
3.A.2 Pallets type and temperature requirement

Two types of pallets are commonly used in GB. Both types of pallets could have a weight of up to 1 ton; however, the dimensions of both types are different. The UK standard pallet has more foot-print area (surface space occupied) and has a height less than the EU standard pallet. As the EU standard has lower foot print area, three EU pallets could be adjusted in the same surface area, in place of 2 UK pallets. From a collection perspective, as prior information about type of pallets being transported is not available, therefore, UK and EU pallets are treated in the same way. However, for inter-depot trunking, a combination of EU and UK pallets provides an opportunity to a 3PL to transport more pallets in the same vehicle provided that the overall weight of all pallets doesn’t violate the recommended regulatory guideline.

With respect to the temperature requirement, the orders (in pallets) that are collected and delivered could be categorised into two types i.e. Chill (temperature requirement 0° to 5°C) and Frozen (-24° to -16°C). Sometimes the pallets that fall in the ambient category (i.e. temperature requirement ≥ 6°C) are also collected but are considered along with the chill and are treated accordingly.

3.A.3 Payload Capacities

As mentioned above, rigid and artic trucks are commonly used to carry out the transport plans. Both types of trucks have different payload capacities in terms of the number of pallets. A rigid truck can accommodate up to 16 Pallets (UK standard) or 20 Pallets (EU standard), whereas an artic can accommodate up to 26 Pallets (UK Standard) or 33 Pallets (EU Standard). As semi-trailers could be uncoupled from the tractor pulling it and swapped with any other semi-trailer located at depots or attached with any other tractor, generally large companies have more semi-trailers than the number of tractors. This helps them to refill extra trailers while tractors are carrying haulage on the remaining trailers. Rigid trucks have tail lifts for loading and unloading pallets and are used mostly in urban areas, whereas artic trucks require specific bays to load and unload pallets and thus are mostly used to fulfil the demand of large warehouses, regional distribution centres (RDCs), and for inter-depot trunking. In rigid trucks, sometimes a pump truck is also placed which is used to lift pallets. However, a pump truck reduces the capacity of rigid trucks by one pallet.
3.4 Multiple Compartments and temperature requirements

As the customer’s order could have any mix of products which could have different transportation requirements, e.g. temperature and/or compatibility issues with other products, multi-compartment vehicles are used to meet these requirements. Often these compartments have fixed capacities. In real-life, it is quite likely that, due to the varying demands of different product types, one compartment is filled to capacity and the other compartment is empty due to no demand for the second product. To cope with this situation, sometimes trailers with adjustable capacities are used. These adjustable capacity trailers provide flexibility to vary the capacity of one compartment so that trailer capacity is utilized in a better way. This feature is particularly helpful when a vehicle is making the en-route pickup and deliveries of multiple types of products.

To accommodate the temperature requirement associated with each product category, the payload space of each vehicle could be partitioned into two compartments by means of a splitter (also called bulk head). Each vehicle can accommodate 2 UK pallets or 3 EU pallets in a column as shown in figure-9. A splitter can be removed if required, but if it is used then each compartment space can accommodate pallets in multiples of 2 (UK Standard). In this research only UK standard pallets are considered. One can use any combination. For example in an artic truck the combination could be 2-24, 4-22, 6-20 and etc. There are two refrigeration units, one at the front and the other at the back. These units help maintain different temperatures in each compartment. Compartment space can be adjusted en-route if required. Figure 3-11 shows the position of ACs, splitter and the layout in an artic vehicle.

![Splitter, refrigeration units and compartments in an arctic truck.](image-url)

Figure 3-11: Splitter, refrigeration units and compartments in an arctic truck.
Since separate temperatures for each compartment could be maintained, each compartment could be reserved for any product type. In other words if the majority of frozen pallets are to be delivered in the last leg of a journey after delivering chill items, the frozen pallets will be stacked at the front (i.e. close to the driver side) of a trailer and chill pallets will be loaded at the back of the trailer (i.e. close to the trailer door). It is not necessary that pallets are stacked in LIFO order. For instance, in the case where a combination of Chill and Frozen pallets are to be delivered to a specific customer, a driver may have to shuffle pallets at the back, by using pump truck (in case of rigid truck), in order to get access to the front compartment. Drivers are trained to use pump-truck and know how to shuffle pallets (en-route) to take a pallet out from the front compartment.

3.B Emissions calculations

To calculate CO$_2$ emissions from vehicles, first the amount of fuel consumed (FC) is calculated. After FC calculation, the conversion of FC to CO$_2$ is done by using molecular mass formulas. The fuel consumption formulas are taken from TRL Ltd. UK [244]. The following table gives the amount of FC (g/km) for two different vehicles at differed payload to capacity ratios. Please note that 0% payload/capacity ratio means that no pallets are loaded on vehicle (running empty); whereas, 100% payload/capacity means that vehicle is filled up to the maximum capacity.

**Table 3-7: Fuel consumed (g/km) for a 40-50 tonnes – Euro 4 standard Artic Vehicle**

<table>
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<tr>
<th>Expression</th>
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<tbody>
<tr>
<td>$171.76 + 541.21 \times e^{-0.0552 \times Speed} + 2947.22 \times e^{-0.5005 \times Speed}$</td>
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</tr>
<tr>
<td>$208.9517 + 549.5422 \times e^{-0.03606 \times Speed} + 13534.946 \times e^{-0.77604 \times Speed}$</td>
<td>50%</td>
</tr>
<tr>
<td>$1$</td>
<td>100%</td>
</tr>
<tr>
<td>$0.00106 + 2.97306 \times 10^{-5} \times Speed$</td>
<td></td>
</tr>
</tbody>
</table>

**Table 3-8: Fuel consumed (g/km) for a 20-26 tonnes – Euro 4 standard rigid vehicle**

<table>
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<tr>
<th>Expression</th>
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</tr>
</thead>
<tbody>
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<td>$155.32 + 517.62 \times e^{-0.0638 \times Speed} + 2785.13 \times e^{-0.5248 \times Speed}$</td>
<td></td>
</tr>
<tr>
<td>$178.21 + 492.96 \times e^{-0.051326 \times Speed} + 5083.974 \times e^{-0.62857 \times Speed}$</td>
<td>50%</td>
</tr>
<tr>
<td>$192.69 + 489.69 \times e^{-0.040712 \times Speed} + 11992.63 \times e^{-0.775104 \times Speed}$</td>
<td>100%</td>
</tr>
</tbody>
</table>

**N.B:** Speed in the above mentioned formula is expressed in km/hours.

The formulas given in Table 3-7 and Table 3-8 are plotted in Figure 3-12 and Figure 3-13 which show the amount of FC (g/km) for two different vehicles at
different payload/capacity ratios and at different speeds. As it can be seen from both graphs, more fuel (g/km) is consumed at lower speeds and at higher speeds the amount of fuel consumed (g/km) decreases. Similarly, the amount of FC also varies with the amount of payload loaded on a truck. The higher the payload, the more fuel is consumed. As it can be seen, if we compare the two graphs, fuel consumption also increases when gross vehicle weight (gvw) increases.

**Figure 3-12: Fuel Consumption profile of Artic (40-50 tonnes - Euro 4 standard)**

**Figure 3-13: Fuel Consumption profile of rigid truck (20-26 tonnes - Euro 4 standard)**

### 3.B.1 Fuel Estimation Formulas

For inter-depot trunking, a major part of a journey consists of travelling on motorways. In the UK the maximum allowed speed for HGV (Heavy good vehicles) on motorways is 56 mph (90 km) and in this particular case study, drivers are generally asked to maintain this speed. Therefore, in this model a speed of 56mph
(90 km/h) is used to have an approximation of CO₂ emissions. The following table gives fuel consumed at various payload/capacity ratios at 56 mph for two different vehicles (Euro-4 standard).

Table 3-9: Fuel consumed (g/km) at different loads (expressed as % of total capacity)

<table>
<thead>
<tr>
<th>Load (%)</th>
<th>Artic (40-50 t)</th>
<th>Rigid (20-26 t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>175.56</td>
<td>157.00</td>
</tr>
<tr>
<td>0.5</td>
<td>230.52</td>
<td>183.11</td>
</tr>
<tr>
<td>1</td>
<td>267.94</td>
<td>205.31</td>
</tr>
</tbody>
</table>

The data from Table 3-9 is plotted in Figure 3-14 and Figure 3-15. In the two graphs, load ratio (i.e. pallets loaded to the actual capacity) is plotted against fuel consumed (g/km).

Figure 3-14: Fuel Consumed at different payload/capacity ratios (Artic vehicle)
The following regression models were obtained to interpolate FC at various payload/capacity ratios.

\[
\text{Fuel Consumed (g/km) = 92.387} \frac{\text{Payload}}{\text{Capacity}} + 178.48 \quad \text{(Artic)} \quad R^2 = 0.9881 \quad (3.27)
\]

\[
\text{Fuel Consumed (g/km) = 48.316} \frac{\text{Payload}}{\text{Capacity}} + 157.65 \quad \text{(Rigid)} \quad R^2 = 0.9978 \quad (3.28)
\]

The regression models given in equation 3.27 and 3.28 suggest that when vehicles are running empty (i.e. without payload), fuel consumed is 178.48 g/km or 157.65 g/km for an artic or rigid vehicles respectively; and when these vehicles are running at full capacity 92.387 or 48.316 g/km are additionally consumed for an artic or rigid vehicles respectively. If we compare the payload related fuel consumption part of both vehicles types, then for an artic vehicle loading one pallet consumes 3.553 g/km/pallet (92.387 * \frac{1}{26}), whereas, for a rigid vehicle the consumption is 3.0197 g/km/pallet (48.316 * \frac{1}{16}) which is 15% less than that for an artic truck.

3.B.2 Conversion of Fuel consumed into \( \text{CO}_2 \) emissions

As \( \text{CO}_2 \) emissions are directly proportional to the amount of fuel consumed, FC (g/km) could easily be converted into \( \text{CO}_2 \) emissions by using molecular masses. The calculations are shown as follows:

\[
\text{Fuel (assume CH1.85) = 13.85 g/mol} \\
\text{CO}_2 = 44 \text{ g/mol}
\]
So \( \text{CO}_2 = FC \times \frac{44}{13.85} \)

Fuel consumed can also be converted into litres by dividing it with the density of the type of fuel used. For example, the density of diesel used in UK is 850 g/litre approximately.

\[
\text{Number of litres of diesel consumed} = \frac{\text{Fuel consumed}}{850} \tag{3.29}
\]

### 3.C Calculation of Time

If all drivers are permanent employees and they are paid for a complete shift regardless of the number of driving hours driven by them, as is the case in the real-life case study reported in this paper, time is not considered as an objective by planners. However, to abide by driving time regulations [245], there is a need to calculate the overall time. Four different types of times are calculated in this model and are described as follows:

#### 3.C.1 Driving Time

It is calculated by dividing distance (miles) by maximum allowed speed. Inter-depot trunking is performed during night and the majority of distance travelled is on motorways. Drivers are instructed to maintain the maximum allowed speed of 56 mph (90 km/hour) on motorways. Therefore, this speed limit is used to calculate driving time as well as \( \text{CO}_2 \) emissions (as explained in appendix-3.B).

#### 3.C.2 Rest Time

As per VOSA driving hour regulations, after every 4.5 hours of duty (i.e. travelling or other official duty during journey), a driver has to take a mandatory rest of 45 minutes. This mandatory rest time will be added if total journey exceeds 4.5 hours.

#### 3.C.3 Swap Time

In order to swap a trailer with another trailer, a driver has to park his vehicle, decouple the existing trailer and couple the trailer that has arrived with the swapping vehicle. A maximum of 30 minutes is required to finish this task. This time is considered as a part of a driver’s duty time.
3.4 Load/Unload Time

The loading/unloading time includes time to park, to do paper work and to shuffle pallets. Up to 40 minutes time is required to finish this task. This task could be performed by driver or any other staff member from a depot where the vehicle has stopped/arrived. Be clear that loading/unloading time is calculated to calculate the total journey time, i.e. time between vehicle departures from and return time at originating depot. It doesn’t include time to load/unload at the originating depot as that is not considered as a part of total journey time.

3.D Heuristics followed by the human planners in the case study

The following heuristic is used while making the plans. Please note that this heuristic was obtained after multiple discussions with and observing (for more than ten days) the experienced route planners in the company where the case study reported in this paper was conducted. Another important aspect to reiterate here is that, while making deliveries between two depots only respective planners co-ordinate with each other. In other words, the solution that they obtain is sub-optimal as a network-wide optimization objective is not considered.

1. For any pair of depots, if the round journey distance is less than the maximum limit and load is FTL on both sides then go for a direct delivery and pickup. However, the provision of vehicles from each depot will be as follows:
   a. If two vehicles are required then both sides will provide one vehicle each.
   b. If only one vehicle is required and pallets transported in both directions are equal then any side can provide the vehicle to complete the operation.
   c. If only one vehicle is required and pallets to be transported on both sides are unequal, then side with more pallets to ship will send the vehicle.

2. If the distance on both sides is more than the maximum limit and load is
   a. FTL (both directions), then choose a swap point that minimizes the total distance travelled by both vehicles.
b. FTL (One direction) and LTL in other direction, then choose either option ‘c’ or ‘d’ as mentioned below:

c. LTL and if there is no other depot where delivery/pickup is required then chose any swap point that minimizes the distance and do trailer swap.

d. LTL and if there is an intermediate depot where any or both coordinating depots as mentioned above require delivery and pickup, then chose that third depot as a swap point provided that it saves the journey of one additional vehicle to make that delivery to intermediate depot.

e. LTL and if there is no common intermediate depot where both coordinating depots require delivery and pickup. Instead both coordinating depots need to make deliveries to two different depots, then chose that non-coordinating depot as a swap point that saves the total distance the most.

While making routes, the constraints that are considered are summed up as follows:

1. The capacity of any vehicle on any segment of trip can’t exceed the maximum capacity of 26 in case of arctic vehicle and 16 in case of rigid truck.

2. The sum of \(\frac{\sum_{ij} \text{Chill Pallets}_{ij}}{2}\) + \(\frac{\sum_{ij} \text{Frozen Pallets}_{ij}}{2}\) should be less than 13 (for arctic vehicle) or 8 (for rigid truck).

3. The total driving time for entire journey for each vehicle can’t exceed 9 hours; whereas the length of any journey leg can’t exceed 4.5 hours.

4. All departing vehicles must return back to their originating depots.

5. The number of vehicles of any type departing from any depot can’t exceed its total number of vehicles of that type based at that depot.

6. If the journey time of any leg of a journey is more than 4.5 hours then trailers should be swapped at some suitable point.

7. All Pallets should reach their destinations before morning.

8. For practical reasons, a vehicle originating from any depot can’t visit more than two other depots even if its total journey time is less than 9 hours.
Chapter 4

Interactive multi-objective vehicle routing via GA-based hybrid algorithm

4.1 Introduction

Radial deliveries in an urban context are an important part of distribution logistics. Goods from a regional distribution centre are supplied to different outlets or drop points surrounding the distribution centre. Generally, the demand of outlets is less than full truck load (FTL); therefore in the literature urban delivery problems are modelled as vehicle routing problems (VRP). In a VRP, a set of routes for a fleet of vehicles need to be constructed so that some specific objective is optimized while the demands of customers are met within some specified operational, legal and other constraints [5]. The choice of constraints and objectives used are usually dependent on the problem context.

To accommodate needs arising due to changes in the socio-economic business context, transport planners consider multiple objectives while making urban delivery routing plans. Similarly, a growing number of academic studies focus on the consideration of multiple objectives from different domains [77]. In this research, six conflicting objectives that include criteria related to the economic, social and environmental domains are considered. The objectives from the economic domain amount to the minimization of total distance, total time, tardiness, and the number of vehicles used to serve all customers. In this research, delivery time-windows are considered semi-hard in the sense that the earliest arrival time at each customer is a
hard constraint but the latest arrival time is treated as a soft constraint. In other words, if a vehicle arrives earlier than the earliest arrival time at a customer location, then the driver needs to wait outside the customer premises and is allowed to start service at the earliest allowed arrival time. However, if a vehicle arrives later than the latest allowed arrival time at a customer location, then driver is still allowed to serve the customer. In the latter case, the associated tardiness is recorded as a representation of customer dissatisfaction and a penalty proportional to the length of the delay (the number of minutes a vehicle is late) is charged. The imbalance between route durations is interpreted as a measure of driver dis-satisfaction that is to be minimised and is included as a representative of social domain objectives. CO$_2$ minimization is the only objective considered from the environmental domain. These six objectives are deemed important, as identified in Chapter 2, by practitioners engaged in retail distribution planning.

When modelling multiple objectives (especially where these are representatives of different domain), one of the problems faced by researchers and practitioners alike is the combination of objectives that are conflicting and incommensurable in nature [23]. Since there may not be a single solution that optimises all objectives, the aim in such a setting is to find a compromise solution that optimises the decision maker’s overall utility function [25]. As the definition of compromise is dependent on the decision maker’s preferences, eliciting preferences from a decision maker and incorporating these in solution process becomes an important part of the optimization process in the multi-objective VRP.

To provide solutions to different real-life and theoretical vehicle scheduling problems, researchers have used various exact, heuristic and meta-heuristic methods. To accommodate multiple objectives and to take into account the decision maker’s preferences, previous work commonly adopts either an a-priori or posteriori approach [22]. In an a-priori approach, the decision maker’s preferences for each objective is sought before running the model [246]. In contrast, in a posteriori approach, a set of efficient (Pareto optimal or non-dominated) solutions to approximate the efficient frontier is first generated, and, subsequently, the decision maker’s preferences are taken into account to choose one of the efficient solutions as the most preferred solution. Both of these approaches have their own advantages and limitations, as discussed previously.
In an organizational context various organizational processes are interlinked (also indicated in section 2.2.1.b). For example, the vehicle routing process may affect drivers’ duty roster, vehicle maintenance schedules, inventory management or picking (of pallets) operations in a warehouse, or other location related decisions (and vice versa). For this reason, a practical challenge for researchers is the provision of solutions that are compatible with other inter-linked processes, which may not have been modelled directly. The involvement of expert human planners is essential in such a setting and many companies rely on a combination of human expertise and specialized software packages to make scheduling plans. Interactive approaches could potentially help reduce some of the limitations of a-priori and posteriori approaches as indicated in section 2.5.4; at the same time, interactive approaches can help find solutions that are compatible with other inter-linked processes. However, there is only a very limited amount of existing literature that deals with interactive approaches to the multi-objective VRPs. In this manuscript, an interactive reference point approach is applied to further help bridge this research gap. The aim of the model proposed here is to explore and identify a compromise solution which, in each iteration cycle, takes into account the decision maker’s preferences regarding six different objectives that are of practical relevance in the retail distribution sector. Several realistic features such as time-varying congestion and the more complex structure of a real-life road network are also integrated in the proposed model.

The rest of the paper is organized as follows. In Section 4.2, a literature review is presented. The problem is explained in Section 4.3, and the overall methodology, calculation method and algorithm details are described in Section 4.4. Section 4.5 provides details about the real-life road network and how demand data was generated. A discussion of the results and conclusions are provided in section 4.6 and 4.7 respectively.

4.2 Literature Review

Due to its practical applicability and continuous changes in business requirements, VRP has always gained the attention of researchers thus making it one of the highly researched areas in operations research (OR). A variety of objectives have been modelled by researchers. The choice of objective modelled depends on the network structure, availability of resources, requirement of customers being served, and the special requirements imposed by the context in which an organization is operating.
Depending on impact on stakeholders, objectives could be divided into
economic, social and environmental domains [76]. The objectives that fall in the
economic domain are related directly or indirectly to the growth and efficiency of a
company providing logistics services. Social objectives are concerned with health,
safety, access to service, and equity between people providing or using the services.
Objectives from environmental domain have wider impact on the society and
environment, and include optimization of CO$_2$ emissions, air quality, noise reduction
and management of waste.

4.2.1 Multi-objective VRP
The majority of the VRP literature deals with a single objective; however, in the past
one decade, the trend is increasing to accommodate more than one objective at the
same time. Considering of multiple objectives is done either to extend a classical
problem, to generalize a classical problem, or to accommodate real-life requirements
[166]. When multiple objectives are considered simultaneously, it could lead to three
possible scenarios, with the objectives being in 1) total concordance, 2) total conflict
and 3) partial conflict [168]. When objectives considered in a model are in total
concordance, the model could be treated and optimized as a single objective
problem. When objectives are in total conflict, all feasible solutions also become
optimal [168]. However, the majority of the real-life VRP problems fall in the partial
conflict category and therefore require the decision maker to choose one from the set
of trade-off solutions.

As mentioned before, six different objectives have been considered in this
research, including minimization of total distance travelled by all vehicles, total
travel time by all vehicles, fleet size (Number of Vehicles used), CO$_2$ emissions; and
maximization of customer satisfaction and equity among drivers. Maximization of
customer satisfaction is equivalent to minimization of customer dissatisfaction
which, in this research, is measured as total number of minutes by which the upper
time window limited is violated for all customers. Maximization of equity among
drivers is equivalent to minimization of difference (in time units) between longest
and shortest routes.

Different researchers have modelled a combination of these objectives
depending on the problem requirements. For example, distance has been modelled
with fleet size [6], [9], [47], [54], [61], [67], [75], [80], [81], [83], [84], [89], [97],
[112], [126], [182], [186], with travel time [7], [10], [80], [88], [139], with customer satisfaction [8], [81], [109], [112], [116], [136], [247], with drivers’ equity [24], [79], [80], [82], [89], [143], [147] and with CO$_2$ emissions [61], [85]–[88]. Similarly fleet size has been modelled along with travel time [10], [51], [52], [80], [128], [129], with customer’s satisfaction [81], [112], with drivers’ equity [51], [80], [89] and with CO$_2$ emissions [61]. Travel time has been modelled with customers’ satisfaction [114], [117], equity [51], [80] and CO$_2$ [88], [138], [169], and customer satisfaction with equity and CO$_2$ in [121] and [73] respectively. In all of the above-mentioned research, the majority of researchers considered only two objectives at the same time, whereas the remaining few researchers considered three or four objectives at the same time. Since these six objectives are of practical relevance to practitioners in retail distribution; therefore, all of these need to be considered simultaneously while making routing plans in the retail distribution context. Figure 4-1 indicates a graphical representation on how frequently these six objectives appear with each other in the literature.

Figure 4-1: Six objectives and how frequently these have been modelled with each other

In Figure 4-1, the presence of an arc between any two nodes is an indication that researchers have modelled that pair in their respective research. The width of each arc is a representation of how frequently that pair of objectives appears in the literature considered in this paper. The border width of the objective node indicates
the relative frequency of that objective with regards to other objectives. Whereas the colour of objective nodes is an indication of the domain (Pink ~ Economic; Blue ~ Social; Green ~ Environmental) that the objective belongs to.

As it can be seen, distance and fleet size are considered the most by researchers, whereas it appears that no one has modelled CO₂ emissions and equity among drivers in the same research. Distance is the objective that appears as the most common objective in the literature, whereas CO₂ and drivers equity appears to be comparatively less common objectives in the body of literature.

It has been shown by many researchers that conflict exists between these different objectives. For example, distance could be minimized but at cost of increase in fleet size [54], [83]. Reducing the imbalance in distance travelled by vehicles/drivers may lead to increase in total time [169] and total distance travelled by all vehicles [24], [82], [143], [147]. Similarly maximisation of customers’ satisfaction requires deliveries to be made within specified delivery time windows. This can be achieved by using more vehicles, thus resulting in an increase in number of vehicles [94] required to provide the deliveries and consequently resulting in increase of total distance travelled by all vehicles [81], [84].

Furthermore, it has also been shown in the above mentioned research that these objectives show conflicting behaviour; however, few researchers have reported that some objectives show both conflicting and non-conflicting behaviours depending on the data instances used and inclusion of other objectives. For example, Ghannadpour et al in their research observed that vehicles and total distance objectives are in conflict when vehicles increase from 13 to 15. However, the two objectives are in concordance when vehicles increase beyond 15. It should be noted that, in their research, they also considered customer satisfaction as an objective. A possible explanation of the complex relationship observed by them between distance and vehicles could be that when more vehicles are made available to improve customer satisfaction then it consequently will increase the distance. Though, these observations might be very problem specific but nonetheless can occur in real-life and highlight the complex relationship when more than two objectives are considered. More details about such behaviours can be found in [81], [84]. Similarly, the relationship between distance, time and CO₂ appears to be quite interesting in literature. Sometime these objectives appear to be in concordance and
sometimes in conflict. Perhaps the main reason behind this is the absence or presence of time-varying congestion; non-imposition or imposition of delivery time-windows by customers; and consideration of different factors while measuring CO$_2$ emissions.

4.2.2 Travel time, Distance and CO$_2$ emissions under Time-varying Congestion and Time window restrictions

As observed in practice, due to time-dependant access restrictions or for other practical reasons, a customer may demand deliveries in some specified delivery time-windows. Similarly changes in road usage at different points of the day, by other road users, may lead to fluctuation in traffic conditions and may cause time-varying congestion. The time-window constraint and dynamic aspects in the road network may cause or compound the conflict in these objectives which otherwise appear to be in concordance with each other. For example, if the travel speed on all roads is assumed to be the same throughout a day, then the shortest route with respect to (w.r.t.) distance between any two nodes also corresponds to the shortest route w.r.t. time. As the amount of CO$_2$ emissions released is positively correlated with the distance travelled, the more distance is travelled, the more CO$_2$ is released by a vehicle. So if the speed of travel on all roads is assumed to be the same throughout a day, then the shortest route w.r.t. distance between any two nodes also corresponds to the shortest route w.r.t. CO$_2$. Now if the assumptions of constant speed on all roads, absence of delivery time-windows, and distance as the only factor in CO$_2$ calculation are applied to network-wide routing decision, then the three objectives appear to be in concordance i.e. the minimization of any objective will lead to the minimization of other objectives.

However, in reality, the speed of travel could be different on different roads. Similarly, the speed of travel on the same road could be different at different points of day [170]. Once we consider the presence of time-varying congestion as is found in real-life road networks, then the route that is the shortest w.r.t. distance may no longer correspond to the shortest route w.r.t. time and CO$_2$ emissions. In other words, the minimization of distance could result in an increase in CO$_2$ emissions and vice versa. Similarly the minimization of total travel time may only be achievable at the cost of travelling a longer distance or incurring an increase in CO$_2$ emissions [88]. As the amount of CO$_2$ emissions released depends on payload in addition to speed of travel, distance and other factors [171][172], if CO$_2$ minimization is
considered as an objective then it may lead to different solutions as the offloading of a heavy load first will help reduce emissions [173]. This may lead to a change in the sequencing of customers in the same route or the shifting of customers to other routes. As a consequence of this shifting or sequencing, when coupled with time-window restrictions, it could result in an increase in total time and distance travelled.

In brief, when many objectives which exhibit complex relationships are considered simultaneously; then it may become difficult to quantify the effect of changes in one criterion on other criteria especially when there are time-varying congestion and time-window constraints. Since practitioners not only consider multiple objectives while making routing plans for urban deliveries under time-varying congestion and delivery time-window conditions, but also wish to explore different compromise solutions; therefore, there is a requirement to facilitate a trade-off analysis to reach the best compromised solution. However, the literature review shows that there has been no study that is dedicated to addressing this important problem of practical significance in real life vehicle routing.

4.2.3 Multi-objective Approaches in VRP
To accommodate multiple objectives, different methods have been adopted by researchers, which could be grouped into a-priori, posteriori and interactive approaches. For each of these approaches, commonly used methods by researchers considering multi-objective VRP are briefly discussed in the following sub-sections.

4.2.3.a A-priori approaches
As mentioned in Section 4.1, in the a-priori approach, the decision maker’s preference is sought before solving the actual problem. Different methods have been adopted by researchers to accommodate preference information a-priori. Out of these methods, the most widely used method is probably the weighted sum approach, in which multiple objectives are each weighted and then summed to form a single objective [177]. For example, when all objectives considered in the model are from the economic domain, a composite objective function can be constructed by associating monetary value with each objective [45][37][59]. However, scalarization by means of a monetary value becomes arguable when objectives are conflicting and fall in different domains. To resolve this issue, the value of different objectives can be normalized before taking the weighted sum [169][42][136]. The weights could
either be assigned by the decision maker based on his expert judgement/experience [57][58][105] or could be derived by using historical data [101]. Alternatively, pairwise comparisons can be used to derive weights such as in AHP [117][47].

The second commonly used a-priori method is the lexicographic method in which objectives are optimised in sequence. The choice of objective to be optimised with the highest priority is generally determined from the context. In the vehicle routing literature dealing with the lexicographic approach, the most preferred objective that is optimised first is fleet size, followed by other objectives such as distance [50][9], workload balance [51], time [52], cost [182] or longest route [148]. The preference given to minimization of the number of vehicles used before considering other objectives is an indication of relative importance or monetary impact of the number of vehicles as a resource over other objectives.

Another commonly used a-priori method is Goal programming in which the desired levels of each objective (i.e. goals) are expressed as constraints with variation variables and the deviations from the goals are minimised. For example, in [62] the vehicle travel time and customer wait time are modelled by using Linear goal programming.

The advantage of the a-priori approach is the relative easiness with which these methods could be implemented and the generation of a single efficient solution at the end of the process. Although one can argue that it is not appropriate to apply these a-priori methods to deal with nonconvex problems or discrete problems. Furthermore, a-priori methods also require the decision maker to provide his overall preferences a-priori. In practice, it may not be realistic to ask the decision maker to provide his overall utility function or a set of weights as his overall preferences. This is because it is generally difficult to tell what solution one really wants before he knows what solutions are actually available or possible.

4.2.3.b Posteriori approaches

In the posteriori approach, all or a representative set of Pareto optimal solutions are generated either by running the algorithm many times after changing different parameters or by running algorithm only once by using population-based metaheuristics.
Mathematical programming based methods could be used to generate a single solution in each algorithmic run. A change of parameters in the next run could yield another solution. In this way, after running an algorithm many times, a set of non-dominated solutions could be generated. Weighted sum is among the commonly used methods in the posteriori approach [146]. By changing weights of different objectives, either at the start of each run or by adjusting dynamically by using some algorithm [7], different solutions could be generated.

Another posteriori method used by researchers is the $\varepsilon$-constraint method [18], [154]. In the $\varepsilon$-constraint method, one of the objectives is optimized while all remaining objectives are turned into constraints by providing upper limits for them. In this way, in each algorithmic run another objective is optimised and the cycle continues. At the end of process, a set of Pareto optimal solutions can be achieved and presented before the decision maker to choose the most preferred solution.

In the above mentioned two approaches, the algorithm has to run many times to approximate the Pareto-set and in each run parameters are to be adjusted. But in the case of an evolutionary algorithm, a non-dominated set could be obtained in a single algorithmic run. An evolutionary algorithm is a population-based meta-heuristics and is the most commonly used posteriori method [6], [24], [67], [81], [83], [89],[143]. More details about multi-objective evolutionary algorithms for VRP can be found in [185].

The advantage of these posteriori methods is that computation of the Pareto set could be done without the presence of the decision maker. But the disadvantages of this approach are that 1) approximating an efficient frontier for a high dimensional problem is computationally intensive or even infeasible [73], 2) asking the transport planners to judge a large number of efficient solutions may be problematic, and 3) at times final results may become difficult to justify[58].

4.2.3.c Interactive VRP
In the interactive approach, the decision maker articulates his preferences progressively. As the decision maker is involved in the solution generation process, he can use his knowledge / expertise while exploring the search space and at the same time gains a better understanding of the system [58]. However, the literature dealing with interactive and multi-objective VRP is very limited and is given below.
The early research that deals with interactive multi-objective VRP is due to Park and Koeling [58]. In their research, three different objectives including total distance, perishability of products and fulfilment of emergent services were considered. A two-step approach was adopted to tackle the problem. In the first step, the customer nodes were clustered by using ‘Flexible clustering Method’. In step 2, the sequencing of customers for each cluster was performed by using an iterative goal programming method after getting the weights and upper bounds for each objective. After generation of initial solutions, the decision maker was asked to inspect the solution by looking at the value of objectives for each route. If the decision maker was not satisfied with the current solution then his preferences about weights and target value of each objective function were taken and new routes for each cluster were generated. Similarly, the decision maker was allowed to shift the customers between different clusters. The shortcoming of their proposed method is that the level of interaction is limited to a single route (one at a time) thus resulting in generation of local optimum solutions. Similarly, the lack of delivery time-windows and the assumption that travel times are deterministic somehow contradicts with real-life situations.

The later research that includes the decision makers interaction with the system pertains to a real-life bus routing problem to provide pickup and drop services to kindergarten students in Hong Kong city [51]. Four different objectives were considered in that research, including minimization of vehicles, maximum journey time of each pupil, total travel time and equity among drivers. A two-step approach was adopted. In the first phase, the minimum number of vehicles was determined, thus giving an upper bound required to carry on the operations. And in step 2, the route for each vehicle was constructed by using different methods such as Dijkstra, kth route methods and Hungarian algorithm. The initial solutions were improved by using a heuristic and interaction with the decision maker was allowed to allocate vehicles to different routes. Though the interaction process is not clearly explained in the research, it appears that the interaction is quite limited to the allocation of vehicles to routes only.

For all VRP problems, two types of decision (i.e. clustering and sequencing) need to be made. As both decisions are interdependent, the outcome of one decision influences the other, whether both decisions are made in sequence or in parallel.
Geiger and Wenger in their research [111] proposed a framework for interactive multi-objective VRP in which both decisions are made in parallel. They considered two objectives (i.e. distance and tardiness) and developed a GUI model in which software acts as an intermediary between the decision maker and vehicle agents. Transportation orders were placed in the market; and for each order, vehicle agents placed their bids by taking into account the potential change when a new order was integrated into current routes. Vehicle agents were able to modify the routes by using local search methods and the decision maker could provide his preferences in the form of weights. The utility of the decision maker was calculated in the form of a weighted sum for each solution. Similarly in their follow-up research [113], global utility was calculated by aggregating the partial utilities of each objective. The software (decider) allocated the order to a vehicle in order to minimize the maximum regret. At any stage, the decision maker was able to change his preferences and based on which the utility could be recalculated for each route. As a result, the software could remove an order from an existing route and could place it back into the market for rebidding. The advantage of the weighted-sum approach used in this research is the relative simplicity with which the decision maker can express his preferences. As more than two objectives are often considered in real-life Wenger and Geiger in their next research [187] considered six different criteria including total distance, time, vehicles used, total tardiness, maximum tardiness and number of tardy orders. The decision maker’s preferences in the forms of weights were used to derive a composite aggregated utility function. During the interaction phase, the composite utility function was used to reroute and re-cluster the orders by using variable neighbourhood metaheuristics. The deficiency in this approach is that if the decision maker wishes to express an aspiration level for different objectives then this approach appears to be somehow limited.

In [188], researchers modelled cost and average customer satisfaction as objectives and used GA and local heuristics to provide solutions. After each iteration cycle, solutions were presented to the decision maker and were allowed to choose the solutions acceptable to him. These solutions were then added into a satisfactory solution pool. If the decision maker wished to produce more solutions then the top solutions from the current population and satisfactory solution pool were allowed to breed and generate the next population. The decision maker was again asked to
shortlist the solutions that were acceptable to him and added in the satisfactory solution pool. The cycle continued till the decision maker was satisfied and he selected one final solution for implementation. The problem in this approach is that, at the end, in the final solution pool there could be a large number of solutions and doing trade-off analysis before picking one final solution could be a very tedious job. As indicated before in section 2.6, several factors limit the planners to perform trade-off analysis when there is a large number of a solution available. Thus there is a strong need to implement some method which not only allows interactivity to guide the search process and the same time does not burden the decision maker to perform trade-off analysis for a large set of solutions.

4.3 Problem formulation and Decomposition

In subsection 4.3.1, mathematical notations are provided which are then used in the subsequent sections. To solve a VRP in a real-life setting, one needs to consider a road network that not only has depot and customer nodes but also includes intermediate nodes (i.e. non-depot or non-customer vertices). Therefore, to find a solution, one needs to solve not only a routing/scheduling problem but also a shortest path problem between different nodes. To cope with problem complexity, the problem was decomposed into two sub-problems details of which are given in subsection 4.3.2.

4.3.1 Mathematical Notations

Let $G = (N, A)$ be a connected graph, where $N$ is the vertex set and $A$ is the arc set. Vertex set $N$ contains customer, depot and intermediate nodes. There are $n$ customers which require service from a single depot. Let $V \subseteq N$ represent the customer and depot nodes i.e. $V = \{0, 1, \ldots, n, n+1\}$ where the single depot is represented by 0 and $n+1$ nodes while all remaining $n$ nodes in $V$ are customer nodes. All other nodes in $N$ but not in $V$ are intermediate nodes. Each customer node has a demand of $q_i > 0$ which can be served by a single vehicle. A time window $[e_i, l_i]$ is associated with each node $i \in C$ during which the vehicle is expected to provide the service. The earliest arrival time $e_i$ is considered hard. This implies that if a vehicle arrives at this node earlier than this time, then it has to wait; however, if a vehicle arrives after the latest arrival time $l_i$ then the vehicle is allowed to serve but penalty $p_c$ (proportional to time) is charged. Once the service on any node starts then it takes $g_i$ units of time.
to complete the service. A fleet of \( K \) homogenous vehicles is stationed at the single depot. The objective is to find \( K \) simple routes, starting from node 0 and ending at node \( n+1 \), while considering multiple objectives. While making plans, the following assumptions are made.

1. Vehicles are used to deliver demand (in pallets) from the depot to customers only, so no pickup en-route or backhaul is allowed.
2. Once a vehicle serves a customer, it departs to the next customer immediately without any delay.
3. The engine is turned off while waiting or serving at a customer location.
4. The emission profiles of all vehicles are assumed to be identical.

Following notations are used to describe the objectives and constraints.

**Graph**

\[ G = (N, A) \]

Where \( N \) and \( A \) are sets of vertices and arcs.

\[ V = \{0, 1, 2, 3... n, n+1\} \]

Where 0 and \( n+1 \) represent same depot

\[ C = \{1...n\} \]

Customer nodes \( C = V \setminus \{0, n+1\} \)

\[ A = \{(i, j): i \neq j \land i, j \in N\} \]

Represents a set of arcs.

**Vehicles**

\[ M = \{1, 2, 3 \ldots K\} \]

A set to represent vehicles of homogenous capacity.

\( Q_{\text{max}} \)

Maximum Capacity of homogenous Vehicles.

**Customers**

\( q_i \)

Demand of any customer \( i \in C \)

\( q_0 \text{ or } q_{n+1} = 0 \)

Demand of Depot.

\([e_i, l_i]\)

Service time window of customer \( i \in C \)

\( e_i \)

Lower bound of service start time-window at customer node \( i \in C \)

\( l_i \)

Upper level of service start time-window at customer node \( i \in C \)

\( g_i \)

Time to complete serving customer \( i \in C \), once the service starts.

\([e_0, l_0]\)

Depot working timings.
\( g_0 = 0 \)  

**Road**  

- \( d_{mn} \): Distance of an arc \((m,n) \in A\)  
- \( D_{ij} \): Shortest distance between node \(i\) and \(j \in V\)  
- \( y^i_k + g_i \): Departure time from customer \(i\). Vehicle should depart immediately after providing service.  
- \( t_{ij}(y^i_k + g_i) \): Time taken to go from node \(i\) to \(j\), when vehicle \(k\) departs from \(i\) \((i,j \in V\)\).  
- \( y^i_k \): Service start time of vehicle \(k\) at customer \(i \in C\) when served by a vehicle \(k\) coming from node \(j\);  
- \( E_{ij}(y^i_k + g_i) \): Emissions released while going from \(i\) to \(j\) and departing at time \(y^i_k + g_i\).  
- \( X^k_{ij} = \{0, 1\} \): if vehicle \(k\) travels from vertex \(i \to j\) \((i,j \in V)\), then value=1, otherwise value=0.

The overall aim is to find \(K\) simple routes, starting from node 0 and ending at node \(n+1\) such that multiple objectives are optimised.

\[
\begin{align*}
    f_1(x) &= \sum_{k \in M} \sum_{i,j \in V} X^k_{ij} D_{ij} \\
    f_2(x) &= \sum_{k \in M} \sum_{j \in C} X^k_{0j} (y^k_{n+1} - y^k_0) \\
    f_3(x) &= \sum_{k \in M} \sum_{i,j \in V} X^k_{ij} E_{ij}(y^i_k + g_i) \\
    f_4(x) &= \sum_{k \in M} \sum_{j \in V \setminus \{0, n+1\}} X^k_{0j} \\
    f_5(x) &= \max \left\{ (y^k_{n+1} - y^k_0) \sum_{j \in C} X^k_{0j} : k \in M \right\} \\
    f_6(x) &= \sum_{k \in M} \sum_{i \in V \setminus \{n+1\}} \sum_{j \in C} X^k_{ij} \max\{0, (y^k_j - l_j)\} p_c 
\end{align*}
\]

Subject to the following constraints:
\[
\sum_{i \in C} q_i \sum_{j \in V} X_{ij}^k \leq Q_{\text{max}} \quad \forall k \in M \quad (4.7)
\]

\[
\sum_{k \in M} \sum_{j \in V \setminus \{i\}} X_{ij}^k = 1 \quad \forall i \in C \quad (4.8)
\]

\[
\sum_{i \in V \setminus \{i,n+1\}} X_{li}^k - \sum_{j \in V \setminus \{i,0\}} X_{ij}^k = 0 \quad \forall l \in C, \forall k \in M \quad (4.9)
\]

\[
\sum_{j \in V \setminus \{0\}} X_{0j}^k = 1 \quad \forall k \in M \quad (4.10)
\]

\[
\sum_{j \in V \setminus \{n+1\}} X_{j,n+1}^k = 1 \text{ where } n + 1 \text{ is depot, } \forall k \in M \quad (4.11)
\]

\[
y_i^k = \max\{e_i, y_j^k + g_j + t_{ji} (y_j^k + g_j)\} \quad (4.12)
\]

\[
\sum_{k \in M} \sum_{i \in V} X_{i0}^k = 0 \quad (4.13)
\]

\[
\sum_{k \in M} \sum_{i \in V} X_{n+1,i}^k = 0 \quad (4.14)
\]

\[
X_{ij}^k = \{0,1\} \quad \forall i, j \in V, k \in M \quad (4.15)
\]

Equations (4.1)-(4.6) represent six different objectives that need to be optimized and include total distance travelled, total time taken to serve all customers, total emissions by all vehicles, total vehicles used, Route imbalance, and total penalty (customer service) respectively.

Equation (4.7) states that the maximum load carried by any vehicle cannot exceed the maximum vehicle capacity. Equation (4.8) and (4.9) state that demand of all customers should be served. Any customer should be visited by one vehicle and the vehicle that arrives should leave it. Equation (4.10) and (4.11) are about that all vehicles leave and return back to the depot. Equation (4.12) is used to calculate the service start time of a vehicle at a customer node. This equation also serves as sub tour elimination constraint. Equations (4.13) and (4.14) ensure that no vehicle can go back to depot 0 and no vehicle can come out from depot n+1.
4.3.2 Problem decomposition

The problem was decomposed into sub-problems of 1) allocation and sequencing of customers and 2) determination of the shortest route between two nodes. The decomposition of the problem may lead to sub-optimality; however, the ultimate aim is to come up with a single compromise solution when the decision maker can provide weights and aspiration levels for each objective.

A hybrid method was developed and implemented in Visual C++ to solve this problem. Allocation and sequencing of customers was done by using a genetic algorithm and the shortest route between any two customer nodes was identified using the Dynamic programming based Floyd algorithm. In the decomposed problem, it was assumed that all vehicles start their journeys at the same time i.e. depot start time. The capacity constraint was checked at the assignment of customers to vehicle stage. In the context of time-varying congestion, the constraint on travel speed was implemented by looking at the departure time of a vehicle from a node. Further details are provided in sub-section 4.4.1.

4.3.2.a Allocation and Sequencing of Customers

For a real life vehicle scheduling problem, the size and complexity of the above mixed nonlinear and non-smooth model can make it impractical to use an enumeration or exhaustive search method to examine all possible allocation and sequencing plans. As such, a genetic algorithm (GA) was implemented in Visual C++ to solve the allocation and sequencing sub-problem. A genetic algorithm is a population based meta-heuristic which borrows concepts from evolution theory by natural selection. In GA, a population of solutions is generated; solutions are selected and recombined to produce offspring. Fittest solutions go through a mutation process with the hope that fitness of offspring will be better than the parent solutions. The process is repeated till further improvement in fitness function stops or some specific criteria (e.g. number of generations) have been reached. GA was chosen because of its ability to handle multiple objectives at the same time and that it can produce a number of solutions in a single run [246].

The initial population was generated randomly. Each individual chromosome represents a solution (a permutation of n customers). In each solution, the i-th gene contains the label of a customer. To avoid the duplication of customers at the
generation stage, the inversion method was applied as proposed in ref [249]. Further
details about the inversion method are given in appendix 4.A.

![Figure 4-2: Allocation of customers to Vehicles](image)

Customers were allocated to vehicles/routes while ensuring that the capacity
constraint is not violated. For example, in Figure 4-2, the first three customers are
assigned to route 1. As the inclusion of the 4th customer in route 1 will violate the
capacity constraint, the 4th customer is added to route 2 and the allocation process
continues in the same way. The ranking of solutions was performed based on the
ranking method as explained in Section 4.4.2. Parents are selected from top ranking
chromosomes and crossover is performed. The crossover operator was randomly
picked up and applied to selected parents to produce offspring. Available crossover
operators included single point or double point crossover based on the inversion or
PMX methods adopted. Further details about these operators are given in appendix
4.B. Mutation was performed by randomly picking an operator. Three mutation
operators were used and included 2-swap, 2-opt and shift operators. For additional
explanation about these operators, please see appendix 4.C. The ranking of solutions
is performed based on the reference point approach. In the reference point approach
the weights for each objective and the aspiration level for each objective are taken
from the decision maker. Further details about the ranking method are given in
section 4.4.2.

4.3.2.b Shortest route between two customer/depot nodes

In real-life road network, while travelling from one node to another node, a vehicle
may have to travel through intermediate nodes. Similarly there may exist multiple
routes between two points and the distance of each route could be different. As
mentioned in section 4.2.2, in the absence of any speed restriction a vehicle may
travel at constant speed and thus the shortest distance route is also the shortest in
terms of travel time and CO₂ emissions. Due to congestion, average travel speed on
different links keeps changing thus making it difficult to travel at a constant speed.
The average travel speed reduces in the morning and evening peak hours when there
is a lot of traffic travelling to and from the work place. On the contrary, in the off-
peak times, a vehicle can travel at much higher speed (up to the maximum allowed speed limits). As a consequence of these fluctuations in travel speed, the travel time and thus CO\textsubscript{2} emissions keep changing throughout the day. An implication of this is that a route that is the shortest in terms of distance may or may not be the shortest in terms of time or CO\textsubscript{2} at different points of time.

By looking at the historical data, one can divide the day-long planning horizon into different time slots such that the average speed on each road remains the same in that time slot. Eglese et al. [170] in their research suggested a way to include the time-dependent fluctuations to construct a road time table which could then be used to provide estimated times and travel paths between different locations for journeys starting at different points of day. A road timetable was constructed by using the Floyd algorithm which is a DP based algorithm. This algorithm can find the shortest path among all nodes in one algorithmic run. This algorithm was adapted to find the shortest route for each time slot. Secondly, while identifying the shortest routes among all pairs of nodes, lexicographic ordering was implemented so that if there is more than one shortest route then the one with the shorter travel time is selected. Please note that time required to traverse a route and CO\textsubscript{2} emitted on that route depend on the departure time of the vehicle. The calculation of total time and CO\textsubscript{2} emissions is explained in the following Section 4.4.1.

4.4 Methodology & Calculations

In addition to time and CO\textsubscript{2} calculation, the following subsections provide details about how solutions in the current population are ranked after taking information about the decision maker’s aspiration levels and weights for different objectives. The overall process is summarised in subsection 4.4.3.

4.4.1 Calculation of Time and CO\textsubscript{2} emissions

The time taken from node \( m \) to \( n \) \((m,n \in N)\) and the amount of CO\textsubscript{2} emitted from a vehicle are a function of the distance and speed at which vehicles traverses that link. So when a vehicle moves at low speed, then it will take more time to traverse the same distance. Conversely at higher speed, less time will be required to traverse the same route. In addition to speed and distance, CO\textsubscript{2} emissions also depend on the weight loaded and other factors. As other factors such as road grade, wind speed, engine condition, driving behaviour are difficult to model so these factors are
ignored in this research. As the amount of CO$_2$ released is proportional to the amount of fuel consumed, fuel consumed (FC) can be used as a proxy for CO$_2$ emissions. For calculating fuel consumption, formulas given in Table 4-1 (taken from Transport Research Lab of UK) are used [244]. Values of parameters (a-e) are given in Table 4-2. These formulas link the amount of fuel consumed (in g/km) with speed $Sp$ (km/hour) at various load levels.

Table 4-1: Fuel consumed (in g/km) when an arctic truck (40-50 tons, Euro 4 standard) moves with speed $Sp$ with different load levels

<table>
<thead>
<tr>
<th>Load</th>
<th>Formula</th>
<th>Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>($e+a\exp(-1<em>b</em>Sp)+c\exp(-1<em>d</em>Sp)$)</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>($e+a\exp(-1<em>b</em>Sp)+c\exp(-1<em>d</em>Sp)$)</td>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
<td>$(1/(a+(b*Sp)))$</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Load</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>541.2069</td>
<td>0.055235</td>
<td>2947.224</td>
<td>0.500523</td>
<td>171.7586</td>
</tr>
<tr>
<td>0.5</td>
<td>549.5423</td>
<td>0.036062</td>
<td>13534.95</td>
<td>0.77605</td>
<td>208.9517</td>
</tr>
<tr>
<td>1</td>
<td>0.001063</td>
<td>2.97E-05</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 4-3 shows link between fuel consumption, speed and load level of an artic truck (Euro 4 standard). Load of 0% indicates that there is no payload, whereas load of 100% indicates that vehicle is filled up to the capacity.

Figure 4-3: Fuel consumption (g/km) at different speed and load levels
As it can be seen from Figure 4-3, at lower speeds the fuel consumption is high and when speed increases between the range 6km/h to 86 km/h, the fuel consumption (g/km) decreases. Due to the speed restriction on the various roads in UK as imposed by the highway agency, a goods vehicle can travel at a maximum speed of 56mph (~90km/h) on motorways; whereas speed limits in urban and build-up area are much lower. Therefore we can use the formula as mentioned in Table 4-1 to estimate the fuel consumption. In practice, drivers are recommended to travel at a fixed speed (as close to maximum allowed limit as possible). In the case of congestion or speed restriction a vehicle may have to reduce speed to conform to the speed limits.

By looking at the historical data which is now commonly available from respective agencies/departments under the freedom of information act, one can divide the day long planning horizon into different time slots and for each time slot each road may have a different average travel speed. Let \( T = \{t^1, t^2, t^3 \ldots t^{\text{max}}\} \) represent a set of unequal timeslots. Associated with any timeslot \( t \), there are upper and lower limits represented by \( t^- \) and \( t^+ \). The lower limit of first time slot is 0, and the upper limit of the last time slot will be \( +\infty \). Let \( F_{mn}^t \) represent the maximum speed (imposed by law or due to traffic conditions based on historic data) at which a vehicle can travel in time interval \( t \) from node \( m \) to \( i \).

While going from node \( i \) to \( j \) (\( i, j \in V \)), a vehicle adopts the shortest distance route and may have to pass through intermediate nodes. Let \([0,1,2,\ldots,p,p+1]\) represent a sequence of nodes that form the shortest path between any two nodes \( i \) and \( j \) \( (i,j \in V) \) where 0 represents the node \( i \) and \( p+1 \) represents the node \( j \). Let \( b_i \) be the departure time of that vehicle from node \( i \). So the shortest distance between nodes \( i \) and \( j \) will be given as follows:

\[
D_{ij} = \sum_{m=0}^{p+1} d_{m,m+1}
\tag{4.19}
\]

Similarly the values of \( t_{ij}(b_i) \) and \( E_{ij}(b_i) \) will be calculated accordingly. Note that arrival time at any intermediate node is also the departure time from the same node.

\[
E_{ij}(b_i) = \sum_{m=0}^{p+1} E_{m,m+1}(b_m)
\tag{4.20}
\]
\[ t_{ij}(b_i) = \sum_{m=0}^{p+1} t_{m,m+1}(b_m) \]  

(4.21)

The time taken and emissions released on each link will be calculated as follows. To make formulation simple, it is assumed that one can reach any adjacent node in the same time interval, should the vehicle depart at the start of time interval i.e. \( t^- \). However, if the vehicle departs at some later point \( \left( t^+ - \frac{d_{mn}}{F_{mn}^t} \right) < b_m \leq t^+ \) in the same time window such that it cannot complete journey in the same time-slot, then a part of journey will be travelled in the current time slot and the remaining in the next time slot. Assuming that a vehicle departs from \( m \) to \( n \) in time interval \( t \) (i.e. \( t^- \leq b_m \leq t^+ \)), then in that case the \( t_{mn}(b_m) \) will be calculated as follows:

\[
t_{mn}(b_m) = \begin{cases} 
\frac{d_{mn}}{F_{mn}^t} & \text{if Condition (A)} \\
(t^+ - b_m) + \left( \frac{d_{mn} - (t^+ - b_m) \cdot F_{mn}^t}{F_{mn}^{t+1}} \right) & \text{if Condition(B)}
\end{cases}
\]  

(4.22)

Similarly, emissions \( E_{mn}(b_m) \) will be calculated by using formulas mentioned in Table 4-1. In the case of condition A, the speed and distance will be \( F_{mn}^t \) and \( d_{mn} \) respectively. Whereas, in case of Condition B a part of distance of that arc i.e. \( (t^+ - b_m) \cdot F_{mn}^t \) will be travelled at speed \( F_{mn}^t \) and the remaining distance of that arc i.e. \( (d_{mn} - (t^+ - b_m) \cdot S_{mn}^t) \) will be travelled at speed \( F_{mn}^{t+1} \).

The following are conditions and their explanations.

**Condition (A)** \( \left( t^- \leq b_m \leq \left( t^+ - \frac{d_{mn}}{F_{mn}^t} \right) \right) \)  

(4.23)

Vehicle can complete journey from \( m \) to \( n \) in time slot \( t \).

**Condition (B)** \( \left( t^+ - \frac{d_{mn}}{F_{mn}^t} < b_m \leq t^+ \right) \)  

(4.24)

Vehicle cannot complete journey from \( m \) to \( n \) in time slot \( t \).
4.4.2 Ranking of Solutions

For ranking chromosomes in each population, the following two different methods can be used, depending upon how the decision maker’s preferences are acquired.

- Min-Max weighted sum Method
- Reference Point Method

In the Min-Max weighted sum method, for each population \( p \), first of all the minimum \( (M_{O}^{P-}) \) and maximum \( (M_{O}^{P+}) \) of each objective \( o \) were calculated. The ranking of chromosomes were done based on the value of \( N_{i}^{P} \) which is the weighted sum of the normalized values of each objective value of each chromosome \( i \) in population \( p \) and can be represented in a mathematical equation as follows:

\[
N_{i}^{P} = \sum_{o=1}^{n} W_{O} \times \frac{V_{i}^{O_{p}} - M_{O}^{P-}}{M_{O}^{P+} - M_{O}^{P-}}
\] (4.25)

where \( V_{i}^{O_{p}} \) represents the corresponding value of the \( O_{th} \) objective for chromosome \( i \) in the \( p_{th} \) population and \( W_{O} \) represents the relative weight of the objective \( o \). To avoid a denominator equal to zero, a small positive number was added in the denominator.

The weighted sum method can be used if the decision maker is very clear about the weights. Similarly, the problem could be converted to a single objective problem when full weight is assigned to that objective. This method could be used to find the global minimum \( Z_{O} \) of each objective function.

In the reference point method, the decision maker provides the aspiration level \( A_{O}^{r} \) and the weight \( W_{O}^{r} \) of each objective \( o \) in iteration \( r \) of GA. In each population \( p \), the rank \( RV_{i}^{P} \) of chromosome is calculated by using the following formula:

\[
RV_{i}^{P} = \sum_{o=1}^{n} W_{O}^{r} \times \left[ \frac{V_{i}^{O_{p}} - Z_{O}}{A_{O}^{r} - Z_{O}} \right]^{y}
\] (4.26)

In each iteration cycle \( r \), the decision maker looks at the current populations minimum \( (M_{O}^{P-}) \) and maximum \( (M_{O}^{P+}) \) and provides his aspiration level and weight of each objective. In this way, the search process is directed in that preferred direction.
4.4.3 Overall Process

The overall process is explained below:

1. At the pre-processing stage, the shortest route between all pairs of customer nodes is calculated using an approach based on lexicographic ordering. The aim here is to create a look-up table to make things efficient at run-time. The shortest route is first identified based on distance alone; where several routes corresponding to the same minimal distance are identified, the route associated with the shortest travel time was used. Due to congestion the travel speed (and hence travel time) may vary across time slots, and this analysis is therefore repeated for all time slots.

2. In terms of the evolutionary algorithm, a set of parameters (e.g. population size, number of generations) need to be defined, and suitable operators should be identified for mating, crossover, mutation, and ranking. Alternatively, the strategy to allow the algorithm to pick any operator randomly can be chosen. The initial population is generated randomly after reading instance and vehicle-specific information. To use the reference based ranking method, we need to first identify the optimum of each objective function. This was done in separate runs of the evolutionary algorithm using the min-max ranking approach by assigning a weight of 100% to the considered objective and zero weight to other objectives.

3. Once the minimum of each objective function has been found, a new population is initialized and all individuals are ranked according to a set of initial weights provided by the decision maker. At the beginning of iteration $I$, the objective values of the current population are shown to the decision maker along with the minimum and maximum of each objective value across the entire population.

4. Based on those values the decision maker provides his preferences in the form of reference values, and this information is used to generate the ranking of solutions [175]. The algorithm runs for a number of generations. The top-ranked individuals are selected for mating and after crossover and mutation the objective values of each child are calculated. After each iteration cycle, the results (the minimum and maximum of each objective
value across the current population) are shown to the decision maker who then provides his new preferences. The cycle continues till the decision maker finds a solution that maximises his implicit utility function.

5. The details of the final solution are shown to the decision maker and the algorithm terminates. In Figure 4-4, the flowchart of the overall process is displayed.
Figure 4.4: Overall process to find the most preferred solution
4.5 Data Description

Multiple sources of real-world data are used to parameterize our model. At the pre-processing stage, the road network information, the congestion and details about time-slot are used. The instance-specific information includes customer & demand information, as well as vehicles and emissions profile. Figure 4-5 depicts the input and output from the model.

In this research, a real-life road network will be used in which each node has direct connections to a limited number of other nodes. These nodes can be divided into two groups: 1) Pickup/delivery nodes to represent depot and customer location, and 2) intermediate nodes. In a real-life road network, in order to go from one pickup/delivery node to another, there may exist more than one route and a vehicle may pass through more than one intermediate node. As mentioned in Section 4.2.2, due to the presence of time-varying congestion, the average travel speed on any road may change during the course of a day [250]. For example, in the morning and evening peak hours, vehicles typically travel at low speeds; while during off peak hours, vehicles can move in correspondence with the speed limit. Clearly, travel speed has an impact on the choice of routes between any two customer nodes during peak and off peak hours. Therefore, at the pre-processing stage of our method, the road-network information together with the congestion and time-slot information is used to generate the shortest routes between any two customer nodes during the different available time-slots. The pre-processing is done to gain efficiency during the later stages of the interactive process.

For this research, data pertaining to the county Surrey in the UK was used. The road network file contained nodes, distances, and average travel speed information for 1986 roads comprising 1525 different pickup/delivery and intermediate nodes. Frequency distribution of road lengths is given in Table 4-3. Almost 70% of the
roads have a length of less than a km, whereas only 1% roads have length more than 5 km.

<table>
<thead>
<tr>
<th>Meters</th>
<th>Count</th>
<th>% Freq.</th>
<th>% C. Freq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-300</td>
<td>404</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>301-500</td>
<td>352</td>
<td>18</td>
<td>38</td>
</tr>
<tr>
<td>501-700</td>
<td>280</td>
<td>14</td>
<td>52</td>
</tr>
<tr>
<td>701-1000</td>
<td>338</td>
<td>17</td>
<td>69</td>
</tr>
<tr>
<td>10001-2000</td>
<td>441</td>
<td>22</td>
<td>91</td>
</tr>
<tr>
<td>2001-3000</td>
<td>111</td>
<td>6</td>
<td>97</td>
</tr>
<tr>
<td>3001-4000</td>
<td>36</td>
<td>2</td>
<td>99</td>
</tr>
<tr>
<td>4001-5000</td>
<td>14</td>
<td>1</td>
<td>99</td>
</tr>
<tr>
<td>5000+</td>
<td>10</td>
<td>1</td>
<td>100</td>
</tr>
</tbody>
</table>

As the length of the majority of roads is less than 5 kms and the length of all time-windows is equal to or more than three hours, it is a quite reasonable assumption that when a vehicle that travels from customer node \( i \) to \( j \), it traverses most of the intermediate nodes in the same time slot.

Roads could be categorised into 6 different types i.e. Motorways, Trunk, city Primary, A-category, B-Category, C-Category [251]. Road category has an effect on the maximum allowed speed imposed by the law. The travel speed information for each road was available for five time-slots including AM & PM rush hours and off peak hours. Table 4-4 shows the boundaries for different time bands over which average speed information was available.

<table>
<thead>
<tr>
<th>Time Slot #</th>
<th>Start Time</th>
<th>End Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>00:00</td>
<td>07:00</td>
</tr>
<tr>
<td>1</td>
<td>07:00</td>
<td>10:00</td>
</tr>
<tr>
<td>2</td>
<td>10:00</td>
<td>16:00</td>
</tr>
<tr>
<td>3</td>
<td>16:00</td>
<td>19:00</td>
</tr>
<tr>
<td>4</td>
<td>19:00</td>
<td>23:59</td>
</tr>
</tbody>
</table>

To generate the instance specific information, a major UK retailer was selected and information about its all outlets (27 outlets) in the Surrey County and distribution centre was extracted from the web and mapped onto the digitised road network file. Information about store opening time, store format and vicinity of store
was also recorded. The chosen retailer operates two different types of store formats: local convenience store and supermarket. Local convenience stores cater for small neighbourhoods, whereas superstores serve a wider area and hence attract a large number of customers. Store format and locality in which a store is situated have an effect not only on store opening hours and delivery time windows but also on quantities demanded (measured in number of pallets) by these stores from the distribution centre. As local convenience stores have less floor area and cater for a smaller number of people, the delivery demand of these stores is less than that of superstores. If a store is located in a city centre then due to access restriction deliveries are to be made either before the start of morning peak hours or after evening peak hours. But if a store is located in an area that has no access restrictions then delivery can be made any time after store opening time but one hour before store closing time. The duration of service time is dependent on the quantities to be delivered. The more the pallets, the more is the service time. According to the communication with the transport planners of the company, while planning routes on average, one minute is reserved per pallet in addition to the necessary documentation time of 10-15 minutes.

By considering the above mentioned factors, synthetic data pertaining to demand, earliest arrival, latest departure and service time was randomly generated for each of these outlets. A homogenous fleet of articulated vehicles of Euro standard IV with a capacity of 26 pallets each was assumed. The emission profile information from the Transport Research Lab of UK was used to estimate the CO₂ emissions [252].

The output of our model provides 1) a routing plan for each vehicle in the fleet; and 2) performance values for each route and the total fleet, with respect to each objective.

4.6 Results

In order to understand more about the conflict and complex relationships between different objectives, first of all each objective was optimized independently by giving 100% weight to that single objective. In addition to six objectives that are considered in this research, four other objectives were also considered at this stage. These additional objectives were 1) Total wait time by drivers in case if vehicle
arrives before earliest arrival time, 2) the number of unsatisfied customers if service starts after latest arrival time, 3) the number of vehicles returned late after depot closing time, and 4) the number of vehicles violating driving time limit. Each objective was optimised in 35 separate runs; and the averages and standard deviations of the best solution from each of 35 runs were calculated for each criterion. The results are presented in Table 4-5. The objectives to be optimised are given as row heading, whereas the criteria are shown as column heading. The minimum and maximum of each criterion were calculated and were used to calculate the normalised values as shown in Table 4-6.
Table 4-5: Averages and Std. Deviation of different criteria when different single objectives are optimized in 35 runs

<table>
<thead>
<tr>
<th>Objective</th>
<th>Total distance travelled</th>
<th>Total time by all vehicles</th>
<th>Total CO$_2$</th>
<th>Customer Satisfaction (measured as Penalty)</th>
<th>Total wait time by drivers</th>
<th># of Vehicles Used</th>
<th># of Unsatisfied Customers</th>
<th># of Vehicles Returned Late</th>
<th>Drivers' Equity (Longest-Shortest)</th>
<th># of Max. Driving Time Violations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance</td>
<td>618</td>
<td>41</td>
<td>174685</td>
<td>906</td>
<td>11.20</td>
<td>5.40</td>
<td>5.00</td>
<td>4.37</td>
<td>4.11</td>
<td>0.82</td>
</tr>
<tr>
<td>Time</td>
<td>742</td>
<td>19</td>
<td>219</td>
<td>1740</td>
<td>11.20</td>
<td>5.40</td>
<td>5.00</td>
<td>4.37</td>
<td>4.11</td>
<td>0.82</td>
</tr>
<tr>
<td>CO$_2$</td>
<td>623</td>
<td>19</td>
<td>4816</td>
<td>3965</td>
<td>11.20</td>
<td>5.40</td>
<td>5.00</td>
<td>4.37</td>
<td>4.11</td>
<td>0.82</td>
</tr>
<tr>
<td>Penalty</td>
<td>733</td>
<td>35</td>
<td>4664</td>
<td>8763</td>
<td>11.06</td>
<td>5.40</td>
<td>5.00</td>
<td>4.37</td>
<td>4.11</td>
<td>0.82</td>
</tr>
<tr>
<td>Wait Time</td>
<td>777</td>
<td>36</td>
<td>4176</td>
<td>93</td>
<td>11.40</td>
<td>4.73</td>
<td>4.11</td>
<td>3.79</td>
<td>3.56</td>
<td>0.75</td>
</tr>
<tr>
<td>Vehicles used</td>
<td>748</td>
<td>47</td>
<td>4795</td>
<td>152</td>
<td>11.00</td>
<td>5.23</td>
<td>4.94</td>
<td>4.73</td>
<td>4.52</td>
<td>0.75</td>
</tr>
<tr>
<td>Un Satisfied Customers</td>
<td>738</td>
<td>38</td>
<td>4668</td>
<td>211</td>
<td>11.09</td>
<td>5.23</td>
<td>4.94</td>
<td>4.73</td>
<td>4.52</td>
<td>0.75</td>
</tr>
<tr>
<td>Vehicles Returned Late</td>
<td>747</td>
<td>36</td>
<td>4823</td>
<td>179</td>
<td>11.29</td>
<td>5.23</td>
<td>4.94</td>
<td>4.73</td>
<td>4.52</td>
<td>0.75</td>
</tr>
<tr>
<td>Drivers' Equit (Longest-Shortest)</td>
<td>744</td>
<td>34</td>
<td>5003</td>
<td>167</td>
<td>11.00</td>
<td>5.23</td>
<td>4.94</td>
<td>4.73</td>
<td>4.52</td>
<td>0.75</td>
</tr>
<tr>
<td>Max. Driving time Violation</td>
<td>746</td>
<td>37</td>
<td>4673</td>
<td>167</td>
<td>11.00</td>
<td>5.23</td>
<td>4.94</td>
<td>4.73</td>
<td>4.52</td>
<td>0.75</td>
</tr>
</tbody>
</table>

| Minimum                                | 618                      | 4147                        | 174685       | 1910                                       | 11.00                     | 5.40              | 5.00                        | 4.37                            | 4.11                            | 0.82                             |
| Maximum                                | 777                      | 5003                        | 214862       | 1740                                       | 11.46                     | 5.40              | 5.00                        | 4.37                            | 4.11                            | 0.82                             |
### Table 4-6: Normalized values of each criterion when single objective are optimized

<table>
<thead>
<tr>
<th>Objective</th>
<th>Total Distance</th>
<th>Total Time</th>
<th>Total CO₂</th>
<th>Cust Satisfaction (Penalty)</th>
<th>Wait Time</th>
<th># of Vehicles Used</th>
<th># of Unsatisfied Customers</th>
<th># of Veh. Returned Late</th>
<th>Drivers' Equity</th>
<th># of Max Driv Time Viol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance</td>
<td>0.00</td>
<td>0.66</td>
<td>0.03</td>
<td>0.97</td>
<td>0.83</td>
<td>0.63</td>
<td>0.97</td>
<td>0.17</td>
<td>0.89</td>
<td>0.80</td>
</tr>
<tr>
<td>Time</td>
<td>0.78</td>
<td>0.00</td>
<td>0.78</td>
<td>0.65</td>
<td>0.02</td>
<td>1.00</td>
<td>0.85</td>
<td>0.67</td>
<td>1.00</td>
<td>0.73</td>
</tr>
<tr>
<td>CO₂</td>
<td>0.03</td>
<td>0.78</td>
<td>0.00</td>
<td>1.00</td>
<td>0.95</td>
<td>0.44</td>
<td>1.00</td>
<td>0.33</td>
<td>0.81</td>
<td>0.92</td>
</tr>
<tr>
<td>Penalty</td>
<td>0.72</td>
<td>0.60</td>
<td>0.75</td>
<td>0.00</td>
<td>0.62</td>
<td>0.13</td>
<td>0.06</td>
<td>0.00</td>
<td>0.42</td>
<td>0.25</td>
</tr>
<tr>
<td>Wait Time</td>
<td>1.00</td>
<td>0.03</td>
<td>1.00</td>
<td>0.61</td>
<td>1.00</td>
<td>0.88</td>
<td>0.81</td>
<td>0.67</td>
<td>0.97</td>
<td>0.63</td>
</tr>
<tr>
<td>Vehicles used</td>
<td>0.82</td>
<td>0.76</td>
<td>0.83</td>
<td>0.98</td>
<td>0.76</td>
<td>0.00</td>
<td>0.97</td>
<td>1.00</td>
<td>0.84</td>
<td>1.00</td>
</tr>
<tr>
<td>Un Satisfied Customers</td>
<td>0.75</td>
<td>0.61</td>
<td>0.77</td>
<td>0.00</td>
<td>0.62</td>
<td>0.19</td>
<td>0.00</td>
<td>0.00</td>
<td>0.36</td>
<td>0.27</td>
</tr>
<tr>
<td>Vehicles Returned Late</td>
<td>0.81</td>
<td>0.79</td>
<td>0.83</td>
<td>0.85</td>
<td>0.78</td>
<td>0.63</td>
<td>0.91</td>
<td>0.00</td>
<td>0.83</td>
<td>0.98</td>
</tr>
<tr>
<td>Drivers' Equit (Longest-Shortest)</td>
<td>0.79</td>
<td>1.00</td>
<td>0.76</td>
<td>0.20</td>
<td>1.00</td>
<td>0.00</td>
<td>0.36</td>
<td>0.00</td>
<td>0.00</td>
<td>0.14</td>
</tr>
<tr>
<td>Max. Driving time Violation</td>
<td>0.80</td>
<td>0.61</td>
<td>0.80</td>
<td>0.35</td>
<td>0.62</td>
<td>0.00</td>
<td>0.59</td>
<td>0.00</td>
<td>0.44</td>
<td>0.00</td>
</tr>
</tbody>
</table>
As expected, solutions identified are non-dominated. When distance is minimized only, for example, the resulting solution performs badly in terms of time, vehicles and customer satisfaction criterion. Similarly the minimization of time, as the sole objective, yields an increase in total distance, emissions, vehicle usage, and etc. From the above-mentioned examples one can conclude that the objectives considered in this problem appear to be in conflict with each other, as the minimization of one objective can only be achieved at the expense of other objectives. One can easily attribute the conflict in minimization of distance, time and CO$_2$ to time-varying congestion and delivery time-windows. As the CO$_2$ emissions from a vehicle depend on distance along with other factors including speed of travel and weight loaded, minimizing the distance also minimizes the emissions in most of the cases. In fact, as shown in Table 4-5, it can be seen that the values of distance and CO$_2$ criteria are very close to each other when distance or CO$_2$ is minimised as sole objective respectively. The difference in the values could be attributed to weight loaded and speed of travel due to congestion. Due to these the sequence of customers changes in order to optimise the overall CO$_2$ emissions.

By looking at Table 4-6, one can derive some very interesting results. For example, when total time by all vehicles is minimised then it is achieved by minimizing the wait time by all drivers and at the same time violating the latest arrival time thus resulting in increase in customers’ dissatisfaction level (measured as Penalty). More vehicles are used and at the same time the inequity among drivers trip time is quite high. Conversely the inequity among drivers is minimised by increase in total time and wait time by drivers. This implies that total time and wait time are in concordance with each other but are in conflict with Drivers’ equity. Similarly if the number of vehicles is optimised then it is achieved at a cost of increase in travel time and wait time, which means that fleet size is in conflict with total time and wait time. If the results of fleet size vs drivers’ equity objectives are compared then one can observe that when fleet size is optimised then drivers’ inequity increases. But conversely, if drivers’ equity is optimised then it appears to be in concordance with fleet size objective. Perhaps the main reason for this anomaly is the inability of GA to differentiate between solutions when a single criterion is used. In this case, as the fleet size can only take limited values, in a population many different solutions can have the same value for this criterion, leading to the same
rank. Perhaps the best strategy would be to consider more than one criterion at the same time while ranking solutions in a population. The same anomaly could be observed when we compare the number of vehicles used vs the number of unsatisfied customer objectives. Reduction in drivers’ wait time is also in conflict with total distance, indicating that on average trips are of longer length which consequently causes an increase in CO₂ emissions. If the number of vehicles is to be optimised then it causes an increase in total distance travelled. Since fewer vehicles are used to serve the same number of customers, so journey time of a few vehicles becomes longer. This not only results in an increase in driving hours limit violation but also causes an increase in the number of vehicles returning late. The normalised values as shown in Table 4-6 are drawn in the parallel coordinate plot (Figure 4-6) to highlight the conflict in different objectives. As mentioned earlier, distance and CO₂ emissions appear to be more in concordance than in conflict, while other objectives appear to be in partial conflict.

![Parallel Coordinate plot for different single objectives](image)

**Figure 4-6**: Parallel coordinate plot of normalized value of different criteria
Table 4-7: Comparison of normalized values of each criterion obtained in two iterations of preference-based method vs the single objective minimization method

<table>
<thead>
<tr>
<th>Objective</th>
<th>Distance</th>
<th>Time</th>
<th>CO2</th>
<th>Cust. Distat</th>
<th>Vehicles</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance</td>
<td>0.00</td>
<td>0.66</td>
<td>0.03</td>
<td>0.97</td>
<td>0.63</td>
<td>0.89</td>
</tr>
<tr>
<td>Time</td>
<td>0.95</td>
<td>0.00</td>
<td>0.95</td>
<td>0.65</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>CO2</td>
<td>0.04</td>
<td>0.78</td>
<td>0.00</td>
<td>1.00</td>
<td>0.44</td>
<td>0.81</td>
</tr>
<tr>
<td>C. Dissatisfaction (Penalty)</td>
<td>0.88</td>
<td>0.60</td>
<td>0.90</td>
<td>0.01</td>
<td>0.13</td>
<td>0.42</td>
</tr>
<tr>
<td>Vehicles used</td>
<td>1.00</td>
<td>0.75</td>
<td>1.00</td>
<td>0.98</td>
<td>0.00</td>
<td>0.84</td>
</tr>
<tr>
<td>Equity</td>
<td>0.97</td>
<td>1.00</td>
<td>0.92</td>
<td>0.21</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Iteration 1</td>
<td>0.51</td>
<td>0.34</td>
<td>0.55</td>
<td>0.01</td>
<td>0.00</td>
<td>0.38</td>
</tr>
<tr>
<td>Iteration 2</td>
<td>0.52</td>
<td>0.31</td>
<td>0.44</td>
<td>0.00</td>
<td>0.00</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Figure 4-7: Parallel coordinate plot of normalized value of different criteria

As objectives are in conflict, focusing on subsets of objectives or changing of preferences may lead to different solutions. As mentioned in section 4.2.3.b, in a high dimension problem, the number of non-dominating solutions could be too large; therefore, it would be problematic or impossible to evaluate all non-dominating solutions. In practice, the decision maker is usually interested in one solution that matches their criteria and to explore the surrounding solutions. Therefore, to meet this requirement, the reference point method is applied by taking the aspiration levels and weights of each objective from the decision maker. In this experiment, equal weights were assigned to each of the six objectives. During the interaction phase, the aspiration levels of each criterion were adjusted by looking at the minimum and maximum values of each criterion in the current population. It has been observed that the solution converges after two or three iterations. This appears to be good as in practice the decision maker might not be interested to go through a lot of iterations. The normalized values of different criteria for the six single objectives and two
iterations of the reference point method are shown in Table 4-7 and are drawn in a parallel coordinate plot in Figure 4-7. As it can be seen, the solutions obtained in both iterations are non-dominating. From Figure 4-7, one can see that gain in equity and CO₂ emission is obtained at a cost of increase in total time.

![Comparison plots](image)

*Figure 4-8: Comparison of results of two iterations with other single objective results*

Figure 4-8 provides comparisons of solutions obtained from two iterations with the solution of different single objectives. It can be seen that solutions from two iterations dominate the single objective solutions of fleet size and customer dissatisfaction; however, all other solutions are non-dominated. Though one can argue that these results are highly subjective as these are based on the reference values and weights used in each iteration cycle, nonetheless these provide a means to reach a compromise solution that is acceptable to the decision maker.

### 4.7 Conclusions

In this research, the problem of routing of vehicles in an urban context is considered, using an approach that takes into account time-varying congestion information. To
find a compromise solution in the presence of multiple and conflicting objectives, an interactive reference point approach is used. A hybrid algorithm based on an evolutionary algorithm and the Floyd algorithm is employed to search for promising compromise solutions. The decision maker’s preferences in the form of reference values are taken into account in each iteration cycle to inform ranking and to guide the creation of the subsequent generations in the evolutionary search. The search space is explored along a preferred direction until the decision maker finds his most preferred solution. The results of this method are compared with the results obtained from single-objective optimization. While the results obtained are subjective in the sense that they depend on the aspiration level and weights assigned, the analysis indicates that the preferred solutions tend to lie in the middle of the Pareto front rather than at the extremes of individual objectives.

In addition to reducing the cognitive effort for the decision maker compared to an a posteriori approach, an interactive method has the advantage of allowing the implicit consideration of the problem-specific context. Furthermore, as there are strong linkages between routing and other organizational processes, the interaction with an expert during the optimization may have the additional benefit of ensuring the identification of a compromise solution that complies with the requirements of associated processes which are not modelled explicitly as constraints or objectives.
4.A Inversion Method

In the inversion method, a permutation of customers (labelled from 1 to \( n \)) is converted into its inversion sequence. Let \( p_1, p_2, p_3, \ldots, p_n \) be a permutation and let \( b^1, b^2, b^3, \ldots, b^n \) be the corresponding inversion sequence, where \( b^j \) represents the number of customers that precede the customer \( j \) in that permutation and have a label number higher than \( j \). To understand the inversion method, let us take a very simple example consisting of six customers (i.e. customers are labelled from 1 to 6).

![Permutation to conversion](image)

In the Figure 4-9, a permutation is shown on the left and its corresponding inversion sequence is shown at the right bottom. In the permutation sequence, one can see that there are two customers to the left of customer labelled 1 and values of label number of these customers are higher than the customer # 1. This is reflected, in the inversion sequence, by setting the value of \( b^1 \) (i.e. 1\(^{st} \) element in the inversion sequence) to be 2. Similarly, the value of \( b^2 \) (2\(^{nd} \) element in the inversion sequence) is 1 and it indicates that there is only one customer in the permutation sequence that precedes customer # 2 and has label number higher than 2. By following the same process, one can show that the inversion of permutation 5-2-1-3-6-4 is 2-1-1-2-0-0.

At the time of initial population generation, instead of generating the permutation, the inversion sequence is generated. To explain how this is done, consider a simple example consisting of six customers. The maximum number of customers that can precede customer #1 and have label value higher than 1 is 5; therefore, a random number between 0 and 5 is generated to get the value of \( b^1 \). Similarly the value of \( b^2 \) is generated randomly between 0 and 4. One can use a simple formula of (n- label#) to generate the inversion sequence for all customers.

To demonstrate how permutation could be generated from inversion sequence, a simple example is given in the Figure 4-10 and explained in the subsequent paragraph.
In the inversion sequence 2-1-1-2-0-0 as shown in Figure 4-10, the value of $b^1$ is 2 indicating that two customers with higher label values precede customer #1. So the first two cells in the permutation sequence are left blank and customer #1 is placed in the 3rd cell. The value of $b^2$ is 1 so it can be placed in cell # 2. The value of $b^3$ is 1, so starting from the left side, one unoccupied cell is left empty and customer # 3 is placed in the next available cell i.e. cell # 4. The value of $b^4$ is 2, so the left two unoccupied cells are left empty and customer # 4 is placed into 6th position in the permutation sequence. When the same process is repeated for the remaining two customers, then it leads to the final permutation of 5-2-1-3-6-4.

The method of generation permutation via inversion method helps to avoid duplication/missing of a customer in a permutation sequence.

### 4.B Crossover method

After ranking of solutions, parents are selected from top ranking chromosomes to perform crossover in order to produce offspring. The crossover operator was randomly picked up and applied to the selected parents. Available crossover operators included single point or double point crossover based on the inversion or PMX methods. The following subsection explains the single point crossovers with inversion or PMX methods.

#### 4.B.1 Single point PMX crossover

In the single point PMX crossover, a suitable point between two adjacent customers in a permutation is randomly selected. Then the part of permutation sequence to the
left of that point in both parent chromosomes is swapped to generate two offspring. However, this swapping may result in the duplication or missing of customers; therefore, in the next step this missing or duplication is rectified. To elaborate the PMX operator, the following simple example is presented.

![Figure 4-11: Single point PMX Crossover](image)

In the Figure 4-11, the crossover point between two parent solutions represented by permutations 5-2-1-3-6-4 and 3-1-4-5-2-6 is picked between second and third element (gene). To create off springs, partial permutations to the left of the crossover point of both parents are swapped. However, as mentioned above, this might lead to the duplication/missing of one or more customers. For example, when 1st element in both permutations are swapped (i.e. when customer 3 is swapped with customer 5), then it may lead to duplication of customer 3 in parent 1. To rectify this error (duplication), customer # 3 (4th element in parent 1) is replaced with the swapped customer (i.e. 5). After swapping of 1st element in the permutation of parent 1 in step 1, we get intermediate permutation of 3-2-1-5-6-4. When the 2nd element of permutation is swapped (i.e. when 2 in parent 1 is replaced by 1 in parent 2), then it leads to another duplication which after rectification in step 2 leads to the final permutation (offspring 1) of 3-1-2-5-6-4. The same process is repeated to generate the second offspring.

4.B.2 Two point PMX crossover
The example shown in Figure 4-12 will be used to explain the two-point crossover. Consider two parent solutions represented by permutations 4-1-8-2-7-5-6-3 and 5-1-7-3-6-8-2-4. Two crossover points are selected randomly as shown by the dotted line. The values of genes between these two points are swapped between the parent solutions. However, this leads to duplication of genes (customers). For example, in step 1 the customers 3, 6 and 8 are repeated in parent-1. To rectify this error, non-shaded duplicates are swapped with their corresponding values in the second parent. This leads to the permutation of 4-1-8-3-6-8-2-4 as offspring-1. When the same process is repeated then one can get the permutation of second offspring as 4-1-8-2-7-5-6-4.

4.B.3 Single and Two-point crossover with inversion method

![Figure 4-12: Two-point crossover](image)

![Figure 4-13: Single and two-point crossover with inversion method](image)
inversion sequences are swapped. However, in contrast to PMX method, no duplication or missing of customers occurs. Examples of these operators are shown in Figure 4-13.

4C Mutation Operators

Mutation was performed by randomly picking an operator. Three mutation operators were used and included 2-swap, 2-opt and shift operators. Figure 4-14 depicts how the sequence of customers changes when these different operators are applied. When 2-swap mutation operator is applied, it randomly swaps the position of two customers in a permutation. For example, the customers labelled 1 and 6 can be randomly chosen and swapped. In the 2-opt method, two points are chosen randomly in the permutation and then the sequence of customers is swapped between those points. For example, when contiguous chunk 8-2-7-5 is reversed then it changes the permutation 4-1-8-2-7-5-6-3 into 4-1-5-7-2-8-6-3. In the shift mutation operator, a partial sequence (contiguous chunk) is picked randomly in the permutation and is then moved by few places in the forward or backward direction. For example, in the following example, partial sequence 1-8-2 is moved two spaces in the forward direction.

<table>
<thead>
<tr>
<th>Parent</th>
<th>4 1 8 2 7 5 6 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-Swap mutation</td>
<td>4 1 8 2 7 5 6 3</td>
</tr>
<tr>
<td>2-opt mutation</td>
<td>4 1 8 2 7 5 6 3</td>
</tr>
<tr>
<td>shift mutation</td>
<td>4 1 8 2 7 5 6 3</td>
</tr>
</tbody>
</table>

Figure 4-14: Mutation operators
Chapter 5

Conclusions

Motivated by the applicability of multi-objective optimization, this thesis focuses on the vehicle routing problem in the retail distribution context. Two case studies have been considered in this research. These case studies are based on the observations made during visits to different retail distribution companies in UK. The first case study, addressed in Chapter 3, relates to primary distribution i.e. the transportation of goods from manufacturers to Regional distribution centres (RDC). Specifically, this case study deals with the problem of inter-depot trunking that is usually performed by 3PL companies in the evenings. In contrast, the second case study, addressed in Chapter 4, relates to primary distribution i.e. the deliveries that are undertaken from a RDC to its associated retail outlets. These deliveries are usually performed during day time, using the vehicles available in-house. To complement chapter 3 and 4 that deal with modelling of two specific problems, Chapter 2 presents an up-to-date literature review that also incorporates the practitioners’ point of view concerning different aspects of the MOVPRP. For this purpose, interviews with different transport planners, transport managers, and VRP software developer were carried out. The information that was thus obtained was considered in finding practical solutions for the above-mentioned case-study problems.

Chapter 2

The primary aim of this chapter is to undertake a comprehensive literature review on MOVPRP problems. However, different from conventional VRP literature review papers, this chapter incorporates key points from qualitative interviews that were conducted with practitioners. As the majority of real-life VRP applications are multi-
objective in nature, it was necessary to understand how the individuals directly involved in making routing plans view and handle these different objectives.

This chapter starts with defining different routing problem components. In an organizational context, routing plans are not made in isolation. Instead, planners always consider other interlinked processes that either provide input to or take output from routing processes. Even if a plan is made using software, it is always inspected by transport planners before it is implemented. At this stage, planners usually ensure that any plan does not affect the other associated processes. Better understanding of the overall context not only leads to the generation of good solutions but also improves the acceptability of the generated plans by other practitioners. Various factors such as inter-process dependencies; focus on cross-function activities; and the presence of different stakeholders have led practitioners as well as academics to consider multiple objectives simultaneously. These objectives can be categorised into economic, social and environmental objectives. Since these objectives often conflict, one needs to find a compromise solution. To find a solution that provides the best compromise with respect to the objectives considered, researchers often adopt either a-priori or posteriori approaches, and various meta-heuristics may be employed to support this process.

Objectives from the economic domain are not only commonly considered in the literature but are also valued highly by practitioners; yet, the priority and the definition of these objectives may differ between these two groups. For example, the minimization of distance is the most commonly used objective in the academic literature, whereas transport planners give more preference to fleet size minimization. There are differences in the definition of objectives as well, e.g. workload equity. Similarly, differences also exist whether or not a criterion is an objective.

The usage of posteriori approaches to incorporate a decision maker’s preferences is increasing in the academic literature. However, there continues to be very little acceptance of this approach by practitioners working in retail distribution. The assumption that a decision maker will conduct a trade-off analysis from a set of solutions has been met with some reservations from practitioners. There are multiple reasons for this such as the frequency of solution generation, the skill set of decision makers, the need to identify solutions in a limited time frame, and the cognitive span of the decision makers limit the adoption of this approach. However, there is a strong requirement to undertake trade-offs within the above-mentioned constraints. By
applying interactive approaches to MOVRP, search processes could be directed in the preferred direction and trade-off analysis process could be facilitated.

It appears from the literature, that one of the most common problems faced by researchers is the absence of real-life instances on which they can test their algorithms. Closer communication and collaboration between academia and industry will be necessary to bridge these gaps.

**Future Research:**

In this research, interviews were conducted with practitioners working in retail distribution; in future work, it may be useful to expand this focus by conducting interviews with representatives from other industries. Qualitative interviews can help in identifying the problems faced by the industry and consequently can provide future research opportunities.

**Chapter 3**

In this chapter, the problem of inter-depot trunking that is often faced by network-based 3PL companies is modelled. Several characteristics make this problem different from the ones found in the VRP literature. These features include fulfilling of demand by using vehicles based at different depots, as well as the swapping of trailers at some suitable location, if the distance of a trip exceeds a certain limit. Other features of this problem include the splitting of demand, the presence of a heterogeneous fleet, and the planning of FTL and LTL simultaneously. This problem is difficult to solve and is regularly faced by practitioners but, nevertheless, has not been addressed in the existing literature.

A mixed integer linear programming model with multiple objectives is proposed to solve this problem. Minimization of fleet size, distance, CO$_2$ emissions, and a weighted sum of distance and fleet size are considered as possible objectives. In the small case-study, the results of the first three objectives are compared with each other. The results illustrate that these objectives are in conflict, and the weighted sum method was therefore used to find a compromise solution.

In the large case-study, results from current practice are compared with the results generated by the model when distance, fleet, or weighted sum is used as the objective. The results suggest that a reduction of 14-19% in fleet size is possible.
which, when extrapolated, could result in 6-7 digit saving in transportation costs. Since fewer vehicles are used to meet the same demand, an increase in average trip length and the fill rate of vehicles is also observed. The minimization of distance or the weighted sum objective also offers the potential to reduce CO₂ emissions by 1-3%. The results of the large case study also reveal interesting patterns of the impact of objectives on route structure. For example, when fleet size is minimised then more round robin trips are planned; when distance is minimized, more direct and trailer swap trips are included.

Since this model does not take into account system-wide effects, there could be some implications for other associated processes. For example, the same vehicle resources are used in the morning shifts, and longer journey times in the evening shift may thus affect the picking operation performed by warehouse staff for the morning deliveries. As a majority of customer interaction takes place in the morning shift, any delay in the morning schedule may affect the long-term profitability of the company. While usage of fewer vehicles may lead to better capacity utilization, it may also reduce the flexibility to accommodate last minute changes in the customers’ orders. Therefore, it is crucial to maintain a delicate balance between efficiency and flexibility of operations. Another potential implication of reduction in fleet size is the effect on the drivers’ schedule. To account for these system-wide and long-term implications, it remains necessary that optimized solutions are further adjusted by human planners.

**Future Research:**

This research can be extended in the following directions. Instead of using an exact method, the application of a heuristic method may aid in the quick generation of solutions. Further complications related to pallet types, route structures, emission profile, and driving hour’s regulations could be added to the model. For example, in this model only UK type pallets are assumed; in practice, both UK and EU type pallets are transported in the same vehicle. This will have implications not only in terms of the number of pallets that can be loaded, but also in terms of the arrangements of the pallets in the compartments. Similarly, in this research, only three types of route structures are considered. Additional type of route structures could potentially lead to further reduction in the fleet size and the distance travelled; therefore, these should be investigated in future research. While calculating CO₂
emissions, it was assumed that vehicles of the same type exhibit the same emission profiles. However, emission profiles for the same vehicle type can change with respect to engine standards e.g. an articulated vehicle of a Euro-5 standard emits less emissions in comparison to that of a Euro-4 standard vehicle. Incorporation of different emission profiles can help accurate estimation and reduction of the emissions from fleet. VOSA guidelines about drivers’ driving hours provide some flexibility to deviate from daily driving limit. The incorporation of these guidelines may, therefore, have implications for trailer swap routes, and this could be considered to extend this research.

Chapter 4

In this second case study, six different objectives are considered simultaneously while making routing plans for a secondary distribution network. Objectives that are considered in this research conflict with each other and include the minimization of fleet size, distance, time, and CO₂ emissions, and the maximization of customer-service and drivers’ equity. In the urban context, the reliability of routing plans is an important concern when there is a time-varying congestion in the road network and there are delivery time-windows. As a consequence of inaccurate time estimation, the operations of the inter-linked processes could be affected.

In the presence of multiple objectives, a single optimal solution may no longer exist. Therefore, this situation necessitates input from a decision maker in determining a desirable compromise solution. Preferences can be elicited a-priori before generating a solution. However, defining preferences at a global level for different objectives is not easy especially when priorities can change regularly. Alternatively, a set of non-dominating solutions could be generated and the decision maker could then be asked to pick one after performing some trade-off analysis. In realistic problems, the number of Pareto optimal solutions may be very large, and a full trade-off analysis may be difficult.

In this research, an interactive reference point approach is used that overcomes the limitation of the above-mentioned approaches. Historical average travel speed data is used to estimate the travel time and CO₂ emissions. A hybrid algorithm that embeds the Floyd algorithm within an evolutionary algorithm is used to generate a set of efficient solutions. The proposed interactive approach switches between
solution generation and preference elicitation stages. After each iteration cycle, in addition to the current best solution, the maximum and minimum values of each objective in the current population are presented to decision maker. By looking at these statistics and considering the requirements of other associated processes, the decision maker can provide reference values and weights for each objective which can then be used to inform and to guide the search process in the preferred direction.

Since the results are sensitive to the reference values and weights, the results of this approach are very subjective. However, this study demonstrates that this approach can better balance the decision maker’s preference which is very desirable in real life MOVRP problems.

**Future Research:**

In this research, information obtained through observations was used to mimic the behaviour of decision maker (i.e. transport planners); however, in future research, one may implement a reference point approach with the real planners. Furthermore, one could create multiple scenarios and repeat this experiment in order to find out how weights change with each scenario. While devising routing plans in this model, a few simple assumptions were used. One of these assumptions is that all vehicles depart from the depot at the same time, an assumption that may be violated in many real-life instances. Therefore, future research may consider a post-optimization process to determine the departure time from the depot. In addition, other simple assumptions were made about the presence of a single type of pallet and single compartment vehicles. However, in certain retail applications, this assumption might not be accurate especially when there are different categories of products. Hence, future research may consider the presence of different products and multiple compartments in vehicles.
## Appendices

### Appendix-1: Weighted Sum (Conversion to Cost)

<table>
<thead>
<tr>
<th>Ref</th>
<th>Objectives</th>
<th>Problem Type/ Context</th>
<th>Treatment of Multiple Objectives</th>
<th>Actual Method Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>[33]</td>
<td>Min: long-run average transportation and holding cost (system-wide)</td>
<td>IRP</td>
<td>Weighted sum of all costs</td>
<td>Regional Partitioning Procedure</td>
</tr>
<tr>
<td>[34]</td>
<td>Min : Cost (Vehicle routing distance for all period + inventory cost based on average daily inventory level)</td>
<td>IRP</td>
<td>Weighted Sum as Everything is cost</td>
<td>Variable Neighbourhood search heuristics (VNS)</td>
</tr>
<tr>
<td>[61]</td>
<td>Min: Travel Distance Min: Fleet Size Min: Fuel consumption Min: Total Cost</td>
<td>PDVRP</td>
<td>Converted to single objective (took cost of each objective)</td>
<td>ILP</td>
</tr>
<tr>
<td>[178]</td>
<td>Total Cost (Transportation Cost, Early Arrival and delay penalty)</td>
<td>VRP</td>
<td>single objective converted to cost</td>
<td>Simulation</td>
</tr>
<tr>
<td>[179]</td>
<td>Min Cost (inventory proportional to inventory level + transportation proportional to distance)</td>
<td>IRP</td>
<td>Weighted Sum Improvement done by MIP</td>
<td>Heuristics Tabu search scheme with ad hoc mixed integer programming</td>
</tr>
<tr>
<td>[35]</td>
<td>Min: Transportation Cost Min: Capacity not</td>
<td>IRP</td>
<td>Weighted sum (Cost only)</td>
<td>MILP</td>
</tr>
<tr>
<td>Reference</td>
<td>Description</td>
<td>Model</td>
<td>Methodology</td>
<td>Notes</td>
</tr>
<tr>
<td>-----------</td>
<td>-------------</td>
<td>-------</td>
<td>-------------</td>
<td>-------</td>
</tr>
<tr>
<td>[161]</td>
<td>Min: Transportation cost Min: Emission cost</td>
<td>VRP</td>
<td>Converted to cost and single objective function</td>
<td>Continuous approximation model</td>
</tr>
<tr>
<td>[45]</td>
<td>Min Cost (Distribution &amp; inventory holding cost)</td>
<td>IRP</td>
<td>Added (Expressed in cost)</td>
<td>Linear program Heuristics proposed</td>
</tr>
<tr>
<td>[43]</td>
<td>Min Total Cost (Facility location, inventory and transportation)</td>
<td>LRI</td>
<td>Single composite function</td>
<td>Nested Lagrangian relaxation-based solution embedded in subgradient optimization Problem decomposition into distribution network design with risk pooling effect + Lagrangian relaxation</td>
</tr>
<tr>
<td>[153]</td>
<td>Min Total Cost (Distance + Time + Penalty + perishability) Perishability = Load * Time to traverse a link</td>
<td>VRP</td>
<td>Weights - All Cost</td>
<td>Heuristics based on Tabu Search</td>
</tr>
</tbody>
</table>
## Appendix-2: Weighted Sum (Scaling and Determination of Weights)

<table>
<thead>
<tr>
<th>Ref</th>
<th>Objectives</th>
<th>Problem Type/Context</th>
<th>Treatment of Multiple Objectives</th>
<th>Actual Method Used</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Pareto Front: By dynamically adjusting the value of alpha</td>
<td></td>
</tr>
<tr>
<td>[67]</td>
<td>Min: Number of Vehicles Min Cost (Total Distance)</td>
<td>VRPTW</td>
<td>Applied</td>
<td>Genetic Algorithm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1) Weighted sum</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2) Equal importance - Pareto method</td>
<td></td>
</tr>
<tr>
<td>[47]</td>
<td>Min: Travel distance Min: Vehicles</td>
<td>Disaster Relief</td>
<td>Weighted sum of objective function after scaling</td>
<td>ACO</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>For scaling (took a log of distance)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Min Z = 0.354*\lg(\text{Total Distance})+0.646*\text{Total vehicles}</td>
<td></td>
</tr>
<tr>
<td>[116]</td>
<td>Min: Distance Min: Delay</td>
<td>VRP</td>
<td>Weighted sum</td>
<td>Fast greedy construction</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>GRASP</td>
<td></td>
</tr>
<tr>
<td>[64]</td>
<td>Min: Penalty function (total unsatisfied demand)</td>
<td>Disaster Relief</td>
<td>Weighted sum</td>
<td>Two heuristics</td>
</tr>
<tr>
<td></td>
<td>Min: Total travel time for all tours</td>
<td>Operations</td>
<td></td>
<td>First based on</td>
</tr>
<tr>
<td></td>
<td>Min: Difference between satisfaction rate between nodes</td>
<td></td>
<td></td>
<td>genetic algorithm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Second based on</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>decomposition method</td>
</tr>
<tr>
<td>[129]</td>
<td>Min: travel time Min: Number of Vehicles</td>
<td>Disaster Relief</td>
<td>Scalar (heavy weight to vehicles used)</td>
<td>Tabu Search</td>
</tr>
<tr>
<td>[109]</td>
<td>Min: Total Travel Cost (Distance) Min: Total tardiness time</td>
<td>m-PDPTW</td>
<td>Scalarized (weights and scaling co-efficient)</td>
<td>Genetic Algorithm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>For Scaling he has used lower bound method by relaxing constraints</td>
<td></td>
</tr>
<tr>
<td>[42]</td>
<td>Min: Cost (facility setup + transportation) Max: Probability of delivery to customers</td>
<td>LRP</td>
<td>For VRP Normalised weighted sum</td>
<td>Two stages</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Facility location problem (Stochastic set-covering problem)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>MO-MDVRP (Simulated Annealing hybridized by genetic algorithm)</td>
</tr>
<tr>
<td>[128]</td>
<td>Max: Overall mission effectiveness Min: Changes in initial mission plan Min: Total travel time for all resources Min: Total UAVs</td>
<td>UAV</td>
<td>Combined into one using scaling parameters</td>
<td>Integer Linear programming</td>
</tr>
<tr>
<td>Reference</td>
<td>Objectives</td>
<td>Problem</td>
<td>Solution Method</td>
<td></td>
</tr>
<tr>
<td>-----------</td>
<td>------------</td>
<td>---------</td>
<td>----------------</td>
<td></td>
</tr>
</tbody>
</table>
| 97        | Min: Number of vehicles  
Min: total travel distance  
Min: Total wait time | PDPTW (pickup and delivery time-window) | MILP hybrid particle swarm optimization and uses variable neighbour search |
| 136       | Max: Delivery efficiency (Load focusing degree, Space utility)  
Max: Service Satisfaction  
Min: Waiting Time of vehicles  
Min: Transportation Distance | VRPFTW | Genetic Algorithm |
| 115       | Min: Cost (Travel Time)  
Min: Customer disutility/dissatisfaction (degree of violation of time windows)  
Min: Number of trips i.e. number of vehicles used | Dial-a-ride | Simulated Annealing |
### Appendix-3: Lexicographic Ordering

<table>
<thead>
<tr>
<th>Ref</th>
<th>Objectives</th>
<th>Problem Type OR Context</th>
<th>Treatment of Multiple Objectives</th>
<th>Actual Method Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>[75]</td>
<td>Min: Fleet&lt;br&gt;Min: Cost (proportional to Distance)</td>
<td>VRP</td>
<td>Lexicographic ordering&lt;br&gt;First fleet and then distance related cost</td>
<td>Heuristics Route construction and improvement</td>
</tr>
<tr>
<td>[50]</td>
<td>Min: Total Routes i.e. Vehicles&lt;br&gt;Min: Distance of each route</td>
<td>VRP</td>
<td>Lexicographic (First vehicles then distance)</td>
<td>Three step heuristics&lt;br&gt;1) Step 1: Construct routes with min vehicles&lt;br&gt;2) Step 2: reduce distance: CROSS-exchanges&lt;br&gt;3) Step 3: Post optimization: Reduce the distance further by using TA (Threshold Accepting) algorithm which is a simulated annealing</td>
</tr>
<tr>
<td>[52]</td>
<td>Min: Number of Tours&lt;br&gt;Min: total Travel Time</td>
<td>VRPTW</td>
<td>Hierarchical way first number of tours then time</td>
<td>Ant Colony Optimization with local search procedures&lt;br&gt;&lt;strong&gt;Local Search procedures&lt;/strong&gt;&lt;br&gt;1. Customer relocation&lt;br&gt;2. Customer exchange&lt;br&gt;3. 2-K Opt&lt;br&gt;4. Branch relocation&lt;br&gt;5. Branch exchange&lt;br&gt;6. Post insertion&lt;br&gt;7. Shuffle Tour Order</td>
</tr>
<tr>
<td>[9]</td>
<td>Min: Fleet Size&lt;br&gt;Min: Distance</td>
<td>Open vehicle routing problem</td>
<td>Hierarchical order (first fleet and then distance)</td>
<td>Hybrid evolution strategy (ES)&lt;br&gt;Improve offspring: Memory based trajectory local search (Tabu search + guided local search)</td>
</tr>
<tr>
<td>[8]</td>
<td>Min: Travel Distance&lt;br&gt;Max: Service level of supplier to customers (deviation of service time)</td>
<td>VRP</td>
<td>Sequential&lt;br&gt;First stage: Distance minimize&lt;br&gt;2nd stage: Maximize service level</td>
<td>Two stage algorithm&lt;br&gt;Stage 1: Traditional VRPTW-α&lt;br&gt;Stage 2: Service improvement problem&lt;br&gt;Fuzzy linear: Cutting plane algorithm&lt;br&gt;Fuzzy concave: Sub gradient based algorithm</td>
</tr>
<tr>
<td>[51]</td>
<td>Min: Number of Buses&lt;br&gt;Min: Total travel time spent by pupil at all pick-up points&lt;br&gt;Min: Total bus travel time&lt;br&gt;Balance loads&lt;br&gt;Balance travel times between buses</td>
<td>School Bus Routing</td>
<td>Lexicographic ordering</td>
<td>Heuristics &amp; Optimisation methods&lt;br&gt;Hungarian Algorithm, Lawler's kth shortest route algorithm, Dijkstra</td>
</tr>
<tr>
<td>[182]</td>
<td>Min: Number of tours&lt;br&gt;Min: Total travel cost (total travel)</td>
<td>TD-VRPTW</td>
<td>Hierarchical: First vehicles then cost For cost</td>
<td>Hybrid (Genetic algorithm + multi-ant colony system)</td>
</tr>
<tr>
<td>Reference</td>
<td>Objective</td>
<td>Constraint</td>
<td>Methodologies</td>
<td></td>
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<tr>
<td>-----------</td>
<td>-----------</td>
<td>------------</td>
<td>---------------</td>
<td></td>
</tr>
<tr>
<td>[126]</td>
<td>Min: Vehicles utilized Min: travel distance</td>
<td>VRPSPD (Simultaneous pickup and delivery)</td>
<td>Lexicographic Lagrangian relaxation technique with Tabu Search</td>
<td></td>
</tr>
<tr>
<td>[125]</td>
<td>Min: Fleet Size Min: Distance</td>
<td>Open VRP</td>
<td>Lexicographic : Minimize Fleet followed by distance Evolutionary Algorithm Initial population by using greedy algorithm like GRASP Self-adaption parameters offspring improved - tabu search survivor selection - deterministic scheme</td>
<td></td>
</tr>
<tr>
<td>[253]</td>
<td>Min: Number of Vehicles Min: Total Travel Time</td>
<td>VRP + Queuing schedule</td>
<td>Lexicographic ordering of objectives Importance of vehicles &gt; that of travel time Multi-stage local search</td>
<td></td>
</tr>
<tr>
<td>[46]</td>
<td>Max: Maximize the number of customers Min: Total distance travelled by all vehicles</td>
<td>VRP</td>
<td>Hierarchical ALNS</td>
<td></td>
</tr>
</tbody>
</table>
### Appendix-4: ε-constraint

<table>
<thead>
<tr>
<th>Ref</th>
<th>Objectives</th>
<th>Problem Type/Context</th>
<th>Treatment of Multiple Objectives</th>
<th>Actual Method Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>[86]</td>
<td>Min: Total distance travelled &lt;br&gt; Min: CO2 emissions &lt;br&gt; Min: Maximum working hours among all drivers in planning horizon</td>
<td>MDVRP</td>
<td>A-priori &lt;br&gt; At Step 1: Only vehicles are considered &lt;br&gt; At Step 2: All three objectives are considered &lt;br&gt; Hybrid: Lexicographic to find ranges of objectives goal programming ε-Constraint</td>
<td>Heuristics: Step 1: Set partitioning problem - Generate set of feasible routes &lt;br&gt; Step 2: augmented ε-constraint &lt;br&gt; Compromise solution</td>
</tr>
<tr>
<td>[154]</td>
<td>Min: Distribution Cost &lt;br&gt; Max: Freshness of perishable food</td>
<td>VRP</td>
<td>ε-Constraint Freshness &gt; a level and minimize Cost</td>
<td>ε- Constraint (short test instance) Multi-objective evolutionary algorithm (NSGA-II)</td>
</tr>
<tr>
<td>[148]</td>
<td>Min: total Travel Cost &lt;br&gt; Min: Length of longest route</td>
<td>VRP</td>
<td>Pareto Solution. By changing value of c) ε in each iteration a non-dominated solution is generated.</td>
<td>Based on adaptive c) ε-Constraint method Branch-and-cut used to solve resulting single objective sub problem Incumbent solution generated by single objective GA and NSGA-II</td>
</tr>
<tr>
<td>[18]</td>
<td>It is a two stage process: &lt;br&gt; In stage one there are two objectives Stage 1 &lt;br&gt; Min: Cost (facility opening cost + routing cost) &lt;br&gt; Expected uncovered demand &lt;br&gt; Stage 2 &lt;br&gt; Min: Uncovered demand (Total Demand - Total Supply)</td>
<td>LRP Humanitarian logistics</td>
<td>ε-Constraints</td>
<td>Two stage stochastic program with recourse Branch and cut within ε-Constraint &lt;br&gt; By using iteratively Pareto front is generated</td>
</tr>
<tr>
<td>[41]</td>
<td>Min : Cost (facility + salary + routing) &lt;br&gt; Max: Coverage (benefit)</td>
<td>LRP</td>
<td>ε-Constraint</td>
<td>Problem decomposition</td>
</tr>
<tr>
<td>[184]</td>
<td>Min Cost (Distance and vehicles fixed cost) &lt;br&gt; Min: Penalty (delayed delivery)</td>
<td>VRPTW</td>
<td>ε-Constraint (converted 2nd objective into constraint)</td>
<td>Heuristics</td>
</tr>
</tbody>
</table>
### Appendix-5: Goal Programming

<table>
<thead>
<tr>
<th>Ref</th>
<th>Objectives</th>
<th>Problem Type / Context</th>
<th>Treatment of Multiple Objectives</th>
<th>Actual Method Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>[254]</td>
<td>Goals are fuzzy Total transportation cost Min: Independence value of customers</td>
<td>VRP</td>
<td>Fuzzy lexicographic ordering</td>
<td>Pre-emptive Fuzzy Goal Programming Assignment: Transportation model</td>
</tr>
<tr>
<td>[14]</td>
<td>Min: Total Operating Cost (transportation cost + cost of opening a treatment facility) Min: Total perceived Risk (Quantity of waste * total individual in population)</td>
<td>LRP - Hazardous waste</td>
<td>Normalized and given weights before using goal programming</td>
<td>Weighted Goal Programming</td>
</tr>
</tbody>
</table>

### Appendix-6: A-priori Hybrid/Heuristics

<table>
<thead>
<tr>
<th>Ref</th>
<th>Objectives</th>
<th>Problem Type OR Context</th>
<th>Treatment of Multiple Objectives</th>
<th>Actual Method Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>[57]</td>
<td>Min: Travel distance of vehicles Min: Total deterioration of goods Max: Total Fulfilment of emergent services</td>
<td>VRP</td>
<td>Used in Sequencing (after clustering) Pre-emptive priorities or weights. Upper bound for constraints (first priority goal)</td>
<td>Two steps: Clustering - Flexible cluster method Sequences: Iterative goal programming heuristics applied to each cluster Sequential Linear Goal programming together with mixed-integer programming algorithm</td>
</tr>
<tr>
<td>[117]</td>
<td>Minimize Vehicle Expenses Tardiness Travel Time</td>
<td>Health Maintenance (DARP)</td>
<td>No Preference in terms of objective at customer insertion stage: weighted sum is used</td>
<td>Heuristics method developed AHP (Eigen value) for weights</td>
</tr>
<tr>
<td>[53]</td>
<td>Min: Travel Time Min: Route Cost Max: Mean sharing Index</td>
<td>Multi-Modal freight transport</td>
<td>Two phases Path Search Phase Martins shortest path algorithm : Considers two objectives (time and Cost) on dominance based principle second phase All non-dominated</td>
<td>Heuristics Martins algorithm (Dijkstra based) Greedy routine</td>
</tr>
<tr>
<td>Reference</td>
<td>Objective(s)</td>
<td>Method</td>
<td>Details</td>
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<td>-------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>[121]</td>
<td>Min: Travel Cost</td>
<td>VRP</td>
<td>Weighted sum method</td>
<td>Three phase rolling horizon heuristics</td>
</tr>
<tr>
<td></td>
<td>Min: Customer Wait Time</td>
<td></td>
<td>In Phase one, only time is considered as it is considered most important.</td>
<td>Phase 1: Customer Selection</td>
</tr>
<tr>
<td></td>
<td>Balance daily workload over a planning period</td>
<td></td>
<td>Phase 2: Weighted sum of wait time and workload balance</td>
<td>Phase 2: Variable Neighbourhood search</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Phase 2a: Initialise: Sweep heuristics</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Phase 2b: Tabu Search to minimise time on any given day</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Phase 2c: Shaking</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Phase 3: Tabu Search</td>
</tr>
<tr>
<td>[62]</td>
<td>Min: Total vehicle travel Time</td>
<td>VRP</td>
<td>Clustering was done on cost (weighted sum of two objectives)</td>
<td>1. Linear Goal programming</td>
</tr>
<tr>
<td></td>
<td>Min: Customer wait time</td>
<td></td>
<td>For each cluster: Pre-emptive Goal programming</td>
<td>2. Heuristics (Cluster and then routing)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2a. Clustering: Parallel insertion method</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2b. Routing: Sequential goal programming procedure</td>
</tr>
<tr>
<td>[169]</td>
<td>Min: Fuel Consumption</td>
<td>PRP</td>
<td>A-priori (four methods)</td>
<td>ALNS</td>
</tr>
<tr>
<td></td>
<td>Min: Driving Time</td>
<td></td>
<td>1. Weighting method</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2. Weighting method with normalization</td>
<td></td>
</tr>
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<td></td>
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<td></td>
<td>3. ε-constraint</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4. A new hybrid method (Adaptive weighting with epsilon-constraint)</td>
<td></td>
</tr>
</tbody>
</table>
### Appendix-7: Posteriori – By changing weights

<table>
<thead>
<tr>
<th>Ref</th>
<th>Objectives</th>
<th>Problem Type OR Context</th>
<th>Treatment of Multiple Objectives</th>
<th>Actual Method Used</th>
</tr>
</thead>
</table>
| [93] | **Min**: Total Cost  
**Max**: Passenger perceived quality of service | Bus routing              | A-priori: Weighted sum of objective function after scaling  
$\alpha$*Cost + (1-$\alpha$)*QoS  
Pareto Front: By dynamically adjusting the value of $\alpha$.  
In initial run $\alpha = 0.001$ and in last iteration it reaches 0.999 | Tabu Search            |
| [146]| **Min**: Total transportation Cost  
**Min**: deviation from tactical plan (counting regions assigned to different region)  
**Min**: Workload imbalance (Square root of sum of squared deviations from average route time) | VRP                      | To evaluate solutions, weighted exponential sum of normalised values was taken. By changing weights, different areas are explored. | Multi-neighbourhood search |

### Appendix-8: Posteriori – By changing value of $\varepsilon$

<table>
<thead>
<tr>
<th>Ref</th>
<th>Objectives</th>
<th>Problem Type OR Context</th>
<th>Treatment of Multiple Objectives</th>
<th>Actual Method Used</th>
</tr>
</thead>
</table>
| [148]| **Min**: total Travel Cost  
**Min**: Length of longest route | VRP                      | Pareto Solution. By changing value of $\varepsilon$ in each iteration cycle a non-dominated solution is generated. | Based on adaptive E-Constraint method Branch-and-cut used to solve resulting single objective sub problem Incumbent solution generated by single objective GA and NSGA-II |
| [18] | It is a two stage process: In stage one there are two objectives  
**Stage 1**  
Min: Cost (facility opening cost + routing cost)  
Expected uncovered demand (1) unfulfilled demand if customers are not within a curtain distance from DC, (2) unfulfilled demand because of distribution centre capacity limit, (3) demand unfulfilled | LRP Humanitarian logistics | E-Constraints  
First objective is optimised, by adding 2nd as constraint, then optimise 2nd by putting 1st as constraint. Do next iteration to generate Pareto | Two stage stochastic program with recourse Branch and cut within E-Constraint By using iteratively Pareto front is generated |
because of vehicle capacity constraints

**Stage 2**
Min: Uncovered demand (Total Demand - Total Supply)

| Min: Distribution Cost | VRP | E-Constraint
Freshness > a level and minimise Cost | E- Constraint (short test instance)
For large instance: Multi-objective evolutionary algorithm (NSGA-II) |
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>[154]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Min: Number of buses used</th>
<th>School Bus Routing</th>
<th>At construction stage: generate solutions by changing target value/bound of objective 2.</th>
<th>Route construction: Heuristics based on clustering and Sweep heuristics Scatter search (based on evolutionary approach)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min: Transportation Time (maximum time a student spends en-route)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[119]</td>
<td></td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Min: Operational Cost (Number of buses)</th>
<th>Bus Routing Problem</th>
<th>Lexicographic &amp; non-dominated For each value of objective 1 i.e. # of routes (it is discrete), they try to minimize the distance For each value of routes &amp; corresponding minimum distance, the Non-dominated solutions presented to decision make (school planners)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min: Longest time a student stay in bus</td>
<td></td>
<td>Initial route construction (Fisher &amp; Jaikumar &amp; Insertion mechanism) Route improvement - Tabu search intensification: Path relinking methodology</td>
</tr>
<tr>
<td>[120]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Min: Average route length</th>
<th>VRP</th>
<th>Pareto efficient solutions</th>
<th>Tabu Search &amp; Parallel Tabu Search (Parallel Pareto archived Tabu search)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min: Maximum route of a single vehicle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[90]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Min Cost (Distance and vehicles fixed cost)</th>
<th>VRPTW</th>
<th>E-Constraint (converted 2nd objective into constraint) Change limit in a step of 20, maximum penalty 500</th>
<th>Heuristics Solomon I1 insertion heuristics improvement: ejection chain, Or-opt and 2-opt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min: Penalty (delayed delivery)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[184]</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

**Appendix-9: Posteriori – Population based & other meta-heuristics**

<table>
<thead>
<tr>
<th>Ref</th>
<th>Objectives</th>
<th>Problem Type OR Context</th>
<th>Treatment of Multiple Objectives</th>
<th>Actual Method Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>[54]</td>
<td>Min: Fleet Size Min: travelling Distance</td>
<td>VRPTW</td>
<td>Pareto Optimality Min of Weighted Deviations</td>
<td>Genetic algorithm with goal programming Local exploitation (Pop initialize: randomly, push forward insertion)</td>
</tr>
<tr>
<td>Reference</td>
<td>Objective</td>
<td>Method</td>
<td>Details</td>
<td></td>
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<tr>
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</tr>
<tr>
<td>[67]</td>
<td>Min: Number of Vehicles Min Cost (Total Distance)</td>
<td>VRPTW</td>
<td>Applied heuristics and λ-interchange mechanism</td>
<td></td>
</tr>
<tr>
<td>[112]</td>
<td>Min: Distance Min: Total Vehicles Min: Service punctuality (for being early or late)</td>
<td>VRP</td>
<td>Pareto Crowding Distance</td>
<td></td>
</tr>
<tr>
<td>[79]</td>
<td>Min: Total length Min: Longest - shortest route (Balance of route lengths)</td>
<td>VRP</td>
<td>Pareto Target Aiming Pareto Search Compared with (Hybrid MOGA with local searches)</td>
<td></td>
</tr>
<tr>
<td>[143]</td>
<td>Min: Total Distance Min: Longest Route - Shortest Route</td>
<td>VRP</td>
<td>Pareto Evolutionary with explicit collective memory method</td>
<td></td>
</tr>
<tr>
<td>[7]</td>
<td>Min: Total distance Min : Travelling Time</td>
<td>VRP</td>
<td>Pareto Fuzzy Logic Guided NSGA-II Fuzzy logic is to dynamically adjust crossover rate and mutation rate after 10 consecutive generations</td>
<td></td>
</tr>
<tr>
<td>[39]</td>
<td>Max: Total Demand served Min: Cost (Facility start up, fixed and variable + delivery cost)</td>
<td>LRP</td>
<td>Pareto Multi-objective scatter search Two phases done iteratively</td>
<td></td>
</tr>
<tr>
<td>[89]</td>
<td>Min: travel distance Min: Routes Max: Load Rate of vehicles Min: Average travel distance</td>
<td>VRP</td>
<td>Pareto (Distance + routes) Other objectives are used to measure routes in route pool Hybrid multi-objective evolutionary algorithm. Heuristics for local improvement</td>
<td></td>
</tr>
<tr>
<td>[149]</td>
<td>Min: travel Cost of routes Max: Obtained Sales Balance goods distributed [Sum of Absolute of (Expected sales of each vehicle - Mean expected sales)]</td>
<td>Open VRP</td>
<td>Pareto (Hyper cubes) For NSGA - Crowding distance Multi objective Particle Swarm optimization Compared with NSGA-II</td>
<td></td>
</tr>
<tr>
<td>[94]</td>
<td>Max: average Customer Satisfaction = min average customer dissatisfaction Min: Travel Cost (travel + fixed + waiting cost)</td>
<td>VRP</td>
<td>Pareto Challenge cup rule is used for Pareto set Multi-objective quantum evolutionary algorithm</td>
<td></td>
</tr>
<tr>
<td>Page</td>
<td>Objective</td>
<td>Sub-problem</td>
<td>Algorithm</td>
<td>Comparison</td>
</tr>
<tr>
<td>------</td>
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<td>-----------</td>
<td>------------</td>
</tr>
</tbody>
</table>
| 123  | Min: Route Cost  
Min: Uncollected demand | VRPSPD | Pareto selection procedure - Crowding distance | Multi-objective iterated local search (MOILS) Compared with NSGA and IBMOLS |
| 147  | Min: Distance  
Min: workload balance (distance & Pallets/load) | VRPTW | Pareto (based on Metropolis criterion)  
Multi temperature annealing | Hybrid (MOEA by using Simulated Annealing as acceptance criteria) Multi-start simulating annealing strategy |
| 82   | Min: Distance  
Min: Route imbalance (distance + load) | VRPTW | Pareto - Parallel island version | Multi temperature-Pareto Simulated annealing  
Sequential and island based parallel version of MT-PSA SPEA2 |
| 186  | Min: Total distance  
Min: total vehicles used | VRP | Pareto | MOGA initial population  
nearest neighbour search |
| 83   | Min: number of Vehicles  
Min: Total Distance | VRPTW | Pareto | MO Evolutionary Algorithm |
| 84   | Min: Fleet Size  
Min: Travelling Distance  
Min: Waiting time imposed on vehicles  
Max: Customers preference for service | Dynamic VRP | Pareto | GA |
| 160  | Min: Total fixed and variable cost of supply chain  
Min: Environmental impact of supply chain | LRI | Pareto | Hybrid approach (MOPSA and adapted multi-objective variable neighbourhood search) |
| 140  | Min: total scheduled travel time  
Min: Total Risk | Hazardous material | Pareto - based on dominance rule | ACO with local search |
| 122  | Min: Ineffectiveness  
[(travel time + facility setup time) / Total Time] i.e. weighted average of stop and tour length  
Min: Average distance to nearest tour stops  
Max: Coverage (Min % of population > pre-defined maxim distance limit) | LRP | P-ACO  
(weighted sum) - Weights are selected randomly  
VEGA/MOGA (sub-problems - lexicographic method)  
MOGA | Simultaneously decide (location and routing)  
P-ACO (weighted sum)  
Sub-Problems  
VEGA  
MOGA |
| 73   | Min: Cost  
Max: Quality of Service  
Min: Impact on environment | DARP | Pareto | NSGA -II  
Multi-objective genetic algorithm based on optimal timing algorithm |
| 19   | Min: Total Cost  
Max: Customer served = Min (Customer not served) | LRP | Pareto | GA with local improvement  
(Memetic Algorithm) |
| 40   | Min: Total Cost (facility opening cost + transportation + inventory (holding + ordering + safety stock))  
Min: Maximum mean time for delivering to customers | LRI | Pareto | - Multi Objective imperialist competitive algorithm (MOICA)  
- MO - parallel simulated annealing (MOPSA)  
NSGA-II and PAES |
<table>
<thead>
<tr>
<th>Reference</th>
<th>Problem Description</th>
<th>Algorithm</th>
<th>Optimization Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>[142]</td>
<td>Min: Maximum vehicle route travelling time Min: Distribution Cost (fixed + variable) Max: minimum route reliability</td>
<td>Open VRP</td>
<td>NSGA and non-dominated sorting differential evolution algorithm</td>
</tr>
<tr>
<td>[108]</td>
<td>Min: Cost (travel, docking, hiring cost, penalty for violating time window) Max: Service level (Proportion of voyages made without violating time-windows)</td>
<td>Oil tanker scheduling</td>
<td>Multi-objective ACO Multi pheromone structure involves non-dominated sorting</td>
</tr>
<tr>
<td>[63]</td>
<td>Min: UAV cruise distance Min: Number of UAVs</td>
<td>UAV</td>
<td>Evolutionary Algorithm</td>
</tr>
<tr>
<td>[81]</td>
<td>Min: Number of Vehicles Min: Total distance Max: Customer Satisfaction Min: waiting time (if arrive before earliest arrival time)</td>
<td>VRPTW</td>
<td>Evolutionary search with various heuristics and local search</td>
</tr>
<tr>
<td>[159]</td>
<td>Min: Total Cost Min: CO₂</td>
<td>Pareto</td>
<td>NSGA-II, MOGA-II and hybrid</td>
</tr>
</tbody>
</table>
### Appendix-10: Posteriori – Hybrid approaches

<table>
<thead>
<tr>
<th>Ref</th>
<th>Objectives</th>
<th>Problem Type OR Context</th>
<th>Treatment of Multiple Objectives</th>
<th>Actual Method Used</th>
</tr>
</thead>
</table>
**Min**: Driver inconvenience                                                                 | VRP                      | Weighted sum weight dynamically updated                           | Multi-criteria Tabu search                                                                            |
| [38] | **Min**: Total Cost (facility opening, routing -fixed, variable) 
**Min**: workload imbalance (load, working time)                                                                 | LRP                      | Efficient frontier                                                | Three phases (used iteratively)  
1. Facility location (Greedy selection) 
2. Routing (meta-heuristics - Tabu search or simulated annealing) 
2a. Initial routes: Clarke wright, nearest neighbour 
2b. Performance non-dominated solutions 
2c. Improvement - nearest neighbour - two swap - Tabu list 
3. Assignment of routes to vehicles (multiple routes) - Modelled as bin packing problem (route time is item size and bin capacity is working hours of vehicle)  
Another approach is location decided first and then routing and assignment done simultaneously (Simulated Annealing) |                                                                                       |
| [86] | **Min**: Total distance travelled 
**Min**: CO2 emissions 
**Min**: Maximum working hours among all drivers in planning horizon                                                                 | MDVRP                    | A-priori 
At Step 1: Only vehicles are considered 
At Step 2: All three objectives are considered 
Hybrid: Lexicographic to find ranges of objectives goal programming E-Constraint | Heuristics:  
Step 1: Set partitioning problem - Generate set of feasible routes 
step 2: augmented E-constraint Compromise solution |


## Appendix-11: Interactive Methods

<table>
<thead>
<tr>
<th>Ref</th>
<th>Objectives</th>
<th>Problem Type OR Context</th>
<th>Treatment of Multiple Objectives</th>
<th>Actual Method Used</th>
</tr>
</thead>
</table>
| [51] | Min: Number of Buses  
Min: Total travel time spent by pupil at all pick-up points  
Min: Total bus travel time  
Balance loads  
Balance travel times between buses | School Bus Routing | Lexicographic ordering | Heuristics & Optimisation methods  
Hungarian Algorithm, Lawler's kth shortest route algorithm, Dijkstra |
| [111] | Min: Distance  
Min: Tardiness (Arrival after latest arrival time) | VRP | Interactive Method | Local search based on variable neighbourhood search |
| [113] | Min: Distance  
Min: Tardiness (Arrival after latest arrival time) | Rich VRP | weighted sum of utility function | Local search neighbourhood |
| [188] | Min: Transportation Cost  
(Fixed cost + travel cost)  
Max: Average Customer Satisfaction (Represented as concave function) | VRP | Randomly selecting one objective as fitness function. | Hybrid (GA + local search) |
| [187] | Min: Time (Travel Time + Service Time + Waiting Time)  
Min: Total Distance  
Min: Number of Vehicles  
Min: Total Tardiness due to violating time windows  
Min: Maximum tardiness  
Min: Number of tardy orders | VRP | Weighted sum based on partial utilities  
Utility is the normalised value based on pre-calculated lower and upper bounds  
Weights are provided by decision makers | variable Neighbourhood Search |
| [58] | Min: Travel distance of vehicles  
Min: Total deterioration of goods  
Max: Total Fulfilment of emergent services | VRP | Used in Sequencing (after clustering)  
Pre-emptive priorities or weights.  
Upper bound for constraints (first priority goal)  
deviation from target value | Two steps:  
Clustering - Flexible cluster method  
Sequences: Iterative goal programming heuristics applied to each cluster  
Sequential Linear Goal programming together with mixed-integer programming algorithm |
References


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W. Xuefeng and X. Wang, “An integrated multi-depot location-inventory-routing problem for logistics distribution system planning of a chain enterprise,” in


