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Location-dependent Distributed Control of Battery Energy Storage Systems for Fast Frequency Response

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Abstract

This paper proposed a distributed strategy to control multiple battery energy storage systems (BESS) delivering fast frequency response in low-inertial power systems with high penetration of renewable energy sources. A novel distributed control algorithm is designed to optimise the operation of an arbitrary number of BESS while incorporating frequency-related constraints to meet the service requirements of fast frequency response. The proposed algorithm uses a consensus-based alternating direction method of multipliers (ADMM) method to solve the formulated optimisation problem. Since the BESS contribution to the system frequency response depends on the BESS locations in the network, a location-dependent term is incorporated into the control strategy to account for the effect of BESS locations on the frequency response. Using a multi-agent system framework the proposed control strategy can be implemented in a distributed fashion to efficiently utilise and coordinate the flexibility of multiple storage systems. Simulation studies demonstrate the effectiveness of incorporating both a location term and frequency-related constraints into the control strategy to provide a cost-effective improved frequency response as well as the suitability of the proposed solution to coordinate a large and arbitrary number of BESS.

Keywords: Distributed optimisation, battery energy storage systems, fast frequency response, low-inertial power system.

1. Introduction

In the future power system renewable energy sources (RES) are expected to supply a large portion of the electricity demand. Unlike traditional syn-

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chronous generators (SG), RES contribute little or nothing to inertia response or primary frequency response [1]. Therefore additional efforts are required to keep frequency deviations within security limits (e.g., 50 ± 0.2 Hz in Great Britain (GB)), in particular when large and sudden active power imbalances occur in a system with high RES penetration levels [2].

To mitigate the challenges caused by the high share of RES, emerging technical solutions are explored by several system operators worldwide [3], particularly focusing on the potential use of fast frequency response ([fast-FR](#)). For instance, the Australian Energy Market Operator (AEMO) is exploring the potential of fast-FR for managing high rate of change of frequency (RoCoF) [3]; Eirgrid in Ireland has introduced an alternative solution called *DS3 Program* [4] to evaluate and assess fast-FR; National Grid (GB transmission operator) has introduced an ancillary service named *Enhanced Frequency Response* (EFR) to provide a fast and reliable frequency regulation service [5]. In addition, National Grid recognised that an improvement is needed in its current frequency management services, which represent a key market for energy storage where a high success rate for bidders is ensured. Therefore, they plan to simplify and overhaul the existing fast-FR and they are developing new faster-acting frequency response services [6].

Specific technical requirements can vary but generally the definition of fast-FR is to provide active power within less than 2s response time after the disturbance occurred, which mitigates high RoCoF and decrease in the frequency nadir [3]. In order to prevent the system frequency to experience higher RoCoF and lower frequency nadir within a few seconds novel frequency control strategies must be designed for the delivery of fast-FR [7], which can efficiently coordinate technologies able to provide the required flexibility. Technologies like battery energy storage systems (BESS) with rapid response time (in the range of milliseconds) have been advocated as an excellent candidate for delivering fast-FR [8]. The technical and economic viability of BESS to provide fast-FR has been analysed and supported by several studies [9, 10, 11]. Because of their promising contribution to support safe and stable system operation, a very large number of BESS is expected to be installed in future power grids. However, the increasing number of BESS would decrease the performance of the frequency support if they are not coordinated properly [12]. Furthermore, BESS are life-constrained due to battery degradation aspects, therefore BESS should be carefully operated to prolong battery life [13]. BESS lifetime and degradation related effects while providing primary frequency regulation have been analysed in the literature, including consideration of market aspects, e.g., [14]. Hence, battery degradation needs to be considered in the control and optimisation of BESS, which are commonly neglected in the existing control frameworks including BESS. Some authors explored the use of BESS for frequency support [15, 16] focusing on extending the Optimal Power Flow (OPF) framework to include primary frequency response and formulating a frequency-constrained economic dispatch. These works adopt simplified frequency dynamic model neglecting the load-damping and do not consider frequency response dynamics, therefore they are not suitable for fast-FR provision. In fact, several stud-

ies [15, 17] suggest that more accurate frequency dynamic models and the integration of frequency-related constraints into the BESS control could improve their frequency response. Besides, existing research focusses on centralised unit commitment frameworks and manage single or a very small number of BESS. These methods are strongly dependent on central controllers, which collect and process global information and distribute the calculated setpoints to all units. As a result, the currently available control frameworks can be sensitive to communication failures and lack of scalability, hence they are not suitable for managing a large number of dispersed storage units [18], particularly for fast-FR, which requires fast response times. Since thousands of BESS are likely to be available in future power systems [19], BESS control frameworks for frequency regulation service have to be scalable so as to optimally coordinate from hundreds to thousands of storage units spread across vast geographical areas [20]. Because of that, distributed approaches are more efficient and suitable solution than centralised methods. Distributed methods, which rely only on limited, local and neighbouring information, can not only reduce computational times, but also be robust against system topology changes and do not require sharing sensitive information with a central entity. Recent studies on distributed control and optimisation in power systems have clearly shown the effectiveness of distributed control, e.g., DC OPF and economic dispatch [21, 22]. In particular, in [23], the authors analyse the application of the conventional distributed alternating direction method of multipliers (ADMM) to frequency regulation using BESS. Authors in [24] propose a hierarchical framework to control a group of BESS for frequency response while reducing the impacts on systems and users. However, most of the existing works are not fully distributed and still need a centralised coordinator to process global information. Furthermore, and most importantly, existing control solutions do not take the BESS location in the network into account and do not address the time-related challenges associated to the fast-FR.

Main contributions of the study. As illustrated in [6], National Grid is overhauling frequency response services, in particular fast-FR. A new high-value fast-FR product - static containment (SC) - is proposed to contain large frequency events, such as generator or demand trips. It is a frequency service activated when a fixed frequency limit is breached (when grid frequency drops or raises by 0.4 Hz), with the characteristics of fast delivery of active power (within 1s) and short duration of delivery (e.g., 20 minutes). The SC product is currently undergoing the standardisation phase [25].

The development of suitable control frameworks that facilitate the coordination of multiple BESS of any size to delivery future static fast-FR is still lacking. In addition, the design of such frameworks has to take into account the effect of BESS locations in order to efficiently assess their contribution to the frequency service provision as previous studies have shown that BESS locations affect the performance of the frequency response [12].

Given the increasing number of diversified assets to control and manage, the increasing uncertainty and the current trend of reducing the time scale of grid operation and services, centralised approaches are not anymore practical

and realisable. On the other hand, it is well known that the fully decentralised approaches can achieve limited and sub-optimal performance, since the decision making is based only on local information: therefore these approaches have limited applicability in the context of optimising and smartening the power grid. Distributed approaches combine the advantages of both centralised and decentralised approaches: they are far more scalable than centralised approaches and able to achieve a far better solution than decentralised approaches. Therefore, this paper proposes a control methodology to optimally coordinates multiple BESS to provide fast-FR services in low-inertial power systems with RES. The focus of this study is on the future static fast-FR. A novel optimisation framework is proposed to meet this objective. It minimises the BESS degradation cost and accounts for the impact of BESS locations in the network on the service provision, while incorporating constraints related to frequency dynamics to guarantee the feasibility of fast-FR service. In order to solve this coordination problem more efficiently and in a fully distributed manner the conventional distributed alternating direction method of multipliers (ADMM) is combined with a consensus-based average estimator. By doing so, the proposed approach can timely calculate optimal BESS power setpoints, which adapt to the time-varying grid conditions and guarantee an adequate delivery of the fast frequency support. The optimisation problem is solved in a distributed fashion using a multi-agent system (MAS) framework, where each battery unit is modelled as an agent that can interact with its neighbours through a local communication network. Taking a stringent static fast-FR service, SC, as an example in this paper, the simulation results show that the proposed methodology is applicable to static fast-FR services. Note that the proposed framework covers the time scales and the control objectives of the traditional primary and secondary control approaches. It not only targets the same control goals as traditional primary and secondary control, but it also achieves those goals in an optimal and real-time manner, minimising the BESS degradation and power losses, as well as maximising the revenue deriving from the frequency service provision. To further clarify this, the proposed methodology is compared with the current practice, a droop-based frequency controller with integral action [26], since the frequency regulation is commonly achieved through PI-based control methods [26].

2. Test network modelling

This section introduces the models of the physical power network and the communication network used in the study. The design of the physical network and the local communication network for distributed BESS could be realistically assumed to be decoupled or weakly coupled. Existing communication technologies could be taken into account [27].

2.1. Power network model

The physical power network is modelled as a directed graph $\mathcal{G}_{\text{power}} = (\mathcal{E}_l, \mathcal{E}_n)$, where the set of edges, denoted as $\mathcal{E}_l$, represents the set of electric lines and the

set of nodes, denoted as $\mathcal{E}_n$, represents the set of buses. The connectivity of the network is captured by the incidence matrix, denoted as $\mathcal{A}$, of the directed graph. The weighted Laplacian of the directed graph is defined by $L_w := \mathcal{A}^T Z^{-1} \mathcal{A}$, where $Z = \text{diag}(|Z|_e, e \in \mathcal{E}_l)$ with $|Z|_e$ being the matrix of magnitudes of the line impedances. It is proved in [28] that there exists a matrix Q_w such that $Q_w L_w = I - \mathbf{1}\mathbf{1}_0^T$ and $Q_w \mathbf{1}_0 = 0$, where $\mathbf{1} = [1, \dots, 1]^T$; if v is a node, $\mathbf{1}_v^T$ is the column vector with 1 in position v , and 0 otherwise.

2.2. Communication network model

Consider a power system with N_B BESS in a MAS framework. A communication network is then modelled as a connected graph $\mathcal{G}_{com} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{\nu_1, \dots, \nu_{N_B}\}$ is the set of the agents, i.e., a battery system, and $\mathcal{E}$ is the edge set. Each edge (ν_i, ν_j) indicates that the i th agent and the j th agent can communicate and transfer information when needed. Let also $\mathcal{N}_i$ matrix be a matrix of the neighbours of i th agent with cardinality $|\mathcal{N}_i|$.

3. Problem Formulation

3.1. Objective function

In this paper the focus is on the discharging BESS modes as the current fast-FR services are mainly designed for supporting the frequency drop in low inertia power systems with RES. Therefore, BESS are to be controlled to discharge active power to compensate for the frequency drop. The objective function is defined as

$$\min \sum_{i \in \mathcal{B}} C_i := \left(\varepsilon_1 \sum_{i \in \mathcal{B}} C_i^{\text{degrad.}} + \varepsilon_2 \sum_{i \in \mathcal{B}} F_i^{\text{dist.}} - \varepsilon_3 \sum_{i \in \mathcal{B}} p P_{B,i} \right), \quad (1)$$

where $\mathcal{B}$ is the set of BESS with N_B being the total number of BESS. $C_i^{\text{degrad.}}$ represents the degradation cost, while $F_i^{\text{dist.}}$ accounts for the BESS locations in the network with regard to the disturbance location. The last, i.e., $p P_{B,i}$, indicates the reward received by the system operator for the frequency service provision, with p being the known reward/contract price and $P_{B,i}$ is a nonnegative optimisation variable representing the power output of i th battery. Since the three functions in (1) have different units, i.e., the price and the electrical distance, and their orders of magnitude can be significantly different, three scaling factors, i.e., ε_1 , ε_2 and ε_3 , are introduced in order to derive an dimensional objective function with comparable terms.

3.1.1. Battery degradation cost

The degradation cost of a battery cell during repeated charging/discharging cycles is key factor for the optimisation of battery operation, as shown in [29]. Different researchers have explored the battery degradation behaviour under different operating conditions. Experiments in [30] show that the marginal degradation cost associated with the increase in the cycle depth is constant

within a given operating range. Because of that, in this work a specific range of depth of discharge (DoD) is considered as introduced in [29]

$$\rho_i(P_{B,i}) := \frac{P_{B,i}}{Nb_i((D_i^{\max} - D_i^{\min})/100\%) \cdot 2C_i^{\text{cap}}}, \quad (2)$$

where Nb_i is the maximum number of cycles between the minimum and the maximum DoD, i.e., $D_i^{\min}$ and $D_i^{\max}$ respectively, for the i th battery, and C_i^{cap} is the capacity of the i th battery. Hence, the degradation cost for each i th battery is expressed as

$$C_i^{\text{degrad.}} := a_i P_{B,i}^2 + C_{\text{ini}} \rho_i(P_{B,i}), \quad (3)$$

where C_{ini} is the initial investment cost of the i th battery unit. The first term of (3) penalises too many charging/discharging cycles which could be harmful to the battery life.

3.1.2. Location impact

BESS can be placed at different voltage levels and locations within the power network, according to their capacities and technical requirements. As shown in [12], the BESS location can play a significant role in frequency support to a power system with increasing RES penetration levels. Furthermore, the BESS location can affect both the electricity and battery costs [31]. This study considers the Z-bus matrix, i.e., the impedance matrix, with the off-diagonal elements being the effective impedances. Thus, the location information is captured by the Z-bus matrix, in particular by the magnitude of the Z-bus matrix ($|Z_{\text{bus}}|$). A location-dependent cost term for each battery unit is introduced in the objective function to weight the response from the i th battery unit according to its location in the network, i.e.,

$$F_i^{\text{dist.}} := \sum_{l \in \mathcal{M}_l} |Z|_{il} P_{B,i}, \quad (4)$$

where $|Z|_{il}$ is the location-dependent weighting factor of the battery unit located at bus i ; $\mathcal{M}_l$ is the set of fault locations. Note that the weight factors can be calculated offline for a given power network and stored in the controller of each battery unit; once the frequency drop occurs and triggers the algorithm execution, only the relevant weight factors will be extracted and used online.

3.2. Frequency-related constraints

The modified form of swing equation (5) is used to describe the evolution of the system-level frequency deviations following a supply-demand mismatch [32]

$$2H\Delta\dot{f}(t) + DP_L\Delta f(t) = P_B(t) - \Delta P_m(t), \quad (5)$$

where H is total system inertia; D is the load damping constant; P_L is system load; $P_B(t)$ is total frequency response from BESS at time t ; ΔP_m is the supply-demand mismatch to be covered by the BESS (which could also represent the contracted BESS power to be delivered). It should be noted that the focus here is on the BESS coordination for fast-FR, therefore $P_B(t)$ includes the decision variables of the optimisation problem to be formulated.

According to the fast-FR technical requirements, the appropriate frequency response must be delivered by a predetermined time t_E . The value of t_E depends on the different service criterion (e.g., $t_E = 1$ for the SC introduced by National Grid in GB). Note that different BESS could have different delivery times, which can be represented by different response slopes during the transient, as expressed in the following BESS modelling

$$P_B(t) = \sum_{i \in \mathcal{B}} P_{B,i}(t); \quad P_{B,i}(t) = \begin{cases} 0, & t < 0, \\ \frac{\alpha_i P_{B,i}}{t_E} t, & 0 \leq t \leq t_E, \\ P_{B,i}, & t > t_E, \end{cases} \quad (6)$$

where α_i is a factor depending on the specific technology of the i th battery unit. Since the control objective is to calculate $P_{B,i}$ per battery so as to guarantee that the required overall BESS power output, $P_B(t)$, is delivered by the required time t_E , according to the specific fast-FR service criteria. The BESS transient response for fast-FR in (6) is included in (5) to obtain the dynamic model used to derive the constraints on the rate of change of frequency (RoCoF) and on the frequency nadir

$$2H\Delta\dot{f}(t) + DP_L\Delta f(t) = \frac{\sum_{i \in \mathcal{B}} \alpha_i P_{B,i}(t)}{t_E} t - \Delta P_m. \quad (7)$$

Constraint on the Rate of Change of Frequency. Given the very short response time required by fast-FR, the frequency deviations are very small during that time interval and the contribution of load damping to it can be neglected. By integrating (7) with initial condition $\Delta f(0) \cong 0$ and making $\Delta f(t = t_{\text{roc}}) \geq \underline{\Delta f}_{\text{roc}}$ (i.e., minimum RoCoF security level) it is obtained

$$\frac{t_{\text{roc}}^2 \sum_{i \in \mathcal{B}} \alpha_i P_{B,i} - 4H\underline{\Delta f}_{\text{roc}} t_E - 2\Delta P_m t_{\text{roc}} t_E}{4H t_E} \geq 0, \quad (8)$$

where t_{roc} is the evaluation time window of RoCoF and $\Delta f(t = t_{\text{roc}})$ is the RoCoF in the given time window. Since $4H t_E > 0$, the constraint on the RoCoF is derived as

$$\sum_{i \in \mathcal{B}} \alpha_i P_{B,i} \geq \frac{4H\underline{\Delta f}_{\text{roc}} t_E + 2\Delta P_m t_{\text{roc}} t_E}{t_{\text{roc}}^2}. \quad (9)$$

Constraint on the frequency nadir. To derive a constraint on the frequency nadir, the following methodology is employed [33]: *i)* integrate the frequency dynamic model with a given initial condition; *ii)* solve $\Delta\dot{f}(t) = 0$ for t_{nadir} ; *iii)* let $\Delta f_{\text{nadir}} = \Delta f(t = t_{\text{nadir}}) \leq \Delta f_{\text{max}}$, where Δf_{max} is the affordable threshold set by the system operator.

Hence, integrating (7) with $\Delta f(0) = 0$ the evolution of frequency deviation is given as

$$\Delta f(t) = \left(\frac{\Delta P_m}{DP_L} + \frac{2HP_B}{t_E(DP_L)^2} \right) (1 - e^{-\frac{DP_L}{2H}t}) - \frac{P_B}{t_E DP_L} t, \quad (10)$$

where $P_B = \sum_{i \in \mathcal{B}} \alpha_i P_{B,i}$. Using (7) and imposing $\Delta\dot{f}(t) = 0$, the time t_{nadir} when the frequency reaches the nadir is calculated as

$$t_{\text{nadir}} = -\frac{2H}{DP_L} \ln \left(\frac{2P_B H}{t_E \Delta P_m DP_L + 2P_B H} \right). \quad (11)$$

The value of the frequency nadir is calculated by replacing $t = t_{\text{nadir}}$ in (10)

$$\Delta f_{\text{nadir}} = \frac{\Delta P_m}{DP_L} + \frac{2P_B H}{t_E(DP_L)^2} \ln \left(\frac{2P_B H}{t_E \Delta P_m DP_L + 2P_B H} \right). \quad (12)$$

To prevent the maximum frequency deviation exceeding the affordable threshold Δf_{max} [6], the following constraint is then included in the optimisation problem to be formulated

$$\frac{\Delta P_m}{DP_L} + \frac{2P_B H}{t_E(DP_L)^2} \ln \left(\frac{2P_B H}{t_E \Delta P_m DP_L + 2P_B H} \right) \leq \Delta f_{\text{max}}. \quad (13)$$

Note that the left side of (12) is a monotonically decreasing function, thus there is a unique solution P_B^* that satisfies $\frac{\Delta P_m}{DP_L} + \frac{2P_B^* H}{t_E(DP_L)^2} \ln \left(\frac{2P_B^* H}{t_E \Delta P_m DP_L + 2P_B^* H} \right) = \Delta f_{\text{max}}$. As a result, the condition $|\Delta f_{\text{nadir}}| \leq \Delta f_{\text{max}}$ is satisfied if and only if

$$\sum_{i \in \mathcal{B}} \alpha_i P_{B,i} \geq P_B^*. \quad (14)$$

Remark 3.1. *A high penetration of power electronics connected generation would change the system inertia. The change of system inertia can be calculated following the method in [34], which would lead to some changes in the constraints on RoCoF (9) and frequency nadir (13). However, the methodology illustrated in this section to derive the constraints and formulate the optimisation problem, and the algorithm to solve the problem in a distributed fashion presented in Section 4, are not tailored to a specific swing equation and are still applicable. Furthermore, the proposed methodology can be equally well applied to high-frequency service provision, by deriving a similar constraint on the frequency zenith for high-frequency services and considering the charging efficiency and a positive sign in the BESS dynamic equation (15c).*

3.3. Optimisation problem for fast-FR

The optimisation problem for discharging BESS to meet the fast-FR requirements is formulated as

$$\min_{P_B} \quad (\varepsilon_1 \sum_{i \in \mathcal{B}} C_i^{\text{degrad.}} + \varepsilon_2 \sum_{i \in \mathcal{B}} F_i^{\text{dist.}} - \varepsilon_3 \sum_{i \in \mathcal{B}} p P_{B,i}) \quad (15a)$$

$$\text{s.t} \quad \sum_{i \in \mathcal{B}} P_{B,i} = \Delta P_m \quad (15b)$$

$$S_{B,i}^{\min} \leq S_{B,i}(t) - \eta_i^{\text{dis}} \frac{P_{B,i} \cdot T_{\text{delivery}}}{C_i^{\text{cap}}} \leq S_{B,i}^{\max}, \quad \forall i \in \mathcal{B} \quad (15c)$$

$$P_{B,i}^{\min} \leq P_{B,i} \leq P_{B,i}^{\max}, \quad \forall i \in \mathcal{B} \quad (15d)$$

$$\sum_{i \in \mathcal{B}} \alpha_i P_{B,i} \geq \max \left\{ P_B^*, \frac{4H \Delta f_{\text{roct}} t_E + 2 \Delta P_m t_{\text{roct}} t_E}{t_{\text{roct}}^2} \right\}, \quad (15e)$$

where $P_{B,i}^{\min}$ and $P_{B,i}^{\max}$ are the minimum/maximum power limits of the i th battery unit. The constraint (15b) maintains the system active power balance. The constraints (15c) prevent individual battery unit from over-discharging during the whole delivery time T_{delivery} by limiting the operating range within the minimum/maximum state of charge (SoC) ($S_{B,i}^{\min}/S_{B,i}^{\max}$ respectively), where $S_{B,i}(t)$ is the (measured or estimated) SoC of the i th battery unit at time t (when the disturbance occurs and the algorithm is triggered) and η_i^{dis} is the discharging efficiency [15]. The constraint (15e) combines the RoCoF constraint (9) and the frequency nadir constraint (14). It should be noted that this study focuses on frequency drops, hence low fast-FR, which is currently mostly required. The proposed service, however, can be easily adopted to provide high fast-FR by replacing the discharging efficiency in (15c) with the charging efficiency and considering a positive instead of a negative sign. This way BESS can contribute to the containment of frequency raise events.

4. Distributed control for efficient management of BESS

In this study, a novel ADMM-based distributed algorithm combined with a consensus average estimator is proposed to solve the problem (15) in order to design a scalable control framework for multiple BESS delivering static fast-FR. To solve the optimisation problem in a distributed manner different distributed techniques can be applied. They can be grouped into two main categories: *i) augmented Lagrangian decomposition*: dual decomposition, analytical target cascading (ATC) and ADMM; *ii) decentralized solution of the Karush-Kuhn-Tucker (KKT) conditions*: optimality condition decomposition (OCD) and consensus + innovation (C+I). These techniques have recently drawn attention within power system applications mainly because of their scalability, which is a necessary feature a control framework has to possess considering the increasing number of controllable units to be managed. The multi-level architecture of ATC would lead to a more complex and expensive implementation than ADMM, while OCD and C+I techniques requires assumption hard to check and may not hold in a large and complex power systems. A detailed comparison among different distributed optimisation techniques is provided in [35]. The online convex optimisation (OCO) framework has also, recently, received attention within the power system field, e.g., [36]. This framework, however, is suitable for dynamic services that have to be continuously provided with a time-varying modulated power output, which is not the static case analysed in this study. In conclusion, an ADMM-based algorithm is developed in this study to take advantage of the decomposability of dual ascent methods and of the robustness and improved convergence properties of augmented Lagrangian [37]. The proposed distributed ADMM with a consensus-based average estimator is illustrated in the next section. The devised control framework aims at generating the optimal active power setpoint for each battery unit to be delivered within t_E seconds after the disturbance has occurred and to be sustained for a given time, depending on the specific fast-FR requirements (e.g., 30 minutes for SC). Each battery unit is required to communicate only with its neighbouring

BESS and computes the optimal active power locally, which results in flexible and scalable operation of BESS providing static fast-FR.

4.1. Main Algorithm

The problem defined in (15) can be rewritten as

$$\min_{\substack{P_{B,i} \in \mathcal{F}_i, \\ m \geq 0, i \in \mathcal{B}}} \sum_{i \in \mathcal{B}} C_i(P_{B,i}) \quad (16a)$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{B}} P_{B,i} = r \quad (16b)$$

$$\sum_{i \in \mathcal{B}} \alpha_i P_{B,i} - m - d = 0, \quad (16c)$$

where $C_i(P_{B,i}) = (w_1 C_i^{\text{degrad.}} + w_2 F_i^{\text{dist.}} - w_3 p P_{B,i})$, $r = \Delta P_m$ and $d = \max\{P_B^*, \frac{4H\Delta f_{\text{roc}} t_E + 2\Delta P_m t_{\text{roc}} t_E}{t_{\text{roc}}^2}\}$; $\mathcal{F}_i$ is the feasible set of i th battery unit consisting of (15d) and (15c), m is an introduced slack variable to deal with (15e). Denote λ as the Lagrange dual variable associated with (16b) and z as the Lagrange dual variable associated with (16c). The problem (16) at t is equivalent to its dual problem

$$\min_{\lambda, z} \sum_{i \in \mathcal{B}} (\phi_i(\lambda, z) + \lambda r^* + z d^*), \quad (17)$$

where $\phi_i(\lambda, z) := \max_{\substack{P_{B,i} \in \mathcal{F}_i, \\ m \geq 0}} \{-C_i(P_{B,i}) - \lambda P_{B,i} - z(\alpha_i P_{B,i} - m^*)\}$, $\forall i \in \mathcal{B}$, with $r^* = r/N_B$, $d^* = d/N_B$ and $m^* = m/N_B$. To solve the problem defined by (17) in a distributed fashion, each battery i is allowed to have a local copy of λ and z , i.e., λ_i and z_i . The problem (17) is therefore reformulated as

$$\min_{\lambda_i, z_i} \sum_{i \in \mathcal{B}} (\phi_i(\lambda_i, z_i) + \lambda_i r^* + z_i d^*) \quad (18a)$$

$$\text{s.t.} \quad \lambda_i = \xi_{ij}, \quad \lambda_j = \xi_{ij} \quad (18b)$$

$$z_i = \theta_{ij}, \quad z_j = \theta_{ij} \quad \forall j \in \mathcal{N}_{b,i}, \quad \forall i \in \mathcal{B}, \quad (18c)$$

where ξ_{ij} and θ_{ij} are auxiliary variables to enforce $\lambda_i = \lambda_j$ and $z_i = z_j$ respectively, $\mathcal{N}_{b,i}$ is the set of neighbouring BESS to i th battery. The augmented Lagrangian function of (18) is

$$\begin{aligned} L := \sum_{i \in \mathcal{B}} \Big\{ & \phi_i(\lambda_i, z_i) + \lambda_i r^* + z_i d^* + \sum_{j \in \mathcal{N}_{b,i}} \Big[(\bar{u}_{ij}(\lambda_i - \xi_{ij}) \\ & + \underline{u}_{ij}(\lambda_j - \xi_{ij})) + (\bar{v}_{ij}(z_i - \theta_{ij}) + \underline{v}_{ij}(z_j - \theta_{ij})) \\ & + \frac{c_1}{2} (\|\lambda_i - \xi_{ij}\|_2^2 + \|\lambda_j - \xi_{ij}\|_2^2) \\ & + \frac{c_2}{2} (\|z_i - \theta_{ij}\|_2^2 + \|z_j - \theta_{ij}\|_2^2) \Big] \Big\}, \end{aligned} \quad (19)$$

where $\bar{u}_{ij}$, $\underline{u}_{ij}$, $\bar{v}_{ij}$ and $\underline{v}_{ij}$ are Lagrangian variables, and $c_1 > 0$ and $c_2 > 0$ are penalty parameters. A distributed algorithm as in Algorithm 1 is proposed for

solving (18), where the details for each step k are shown below

$$\kappa_i^k = \kappa_i^{k-1} + c \sum_{j \in \mathcal{N}_{b,i}} (\lambda_i^k - \lambda_j^k) \quad (20a)$$

$$\psi_i^k = \psi_i^{k-1} + c \sum_{j \in \mathcal{N}_{b,i}} (z_i^k - z_j^k) \quad (20b)$$

$$\gamma_i^k = \sum_{j \in \mathcal{B}} w_{ij} \gamma_j^k; \quad \delta_i^k = \sum_{j \in \mathcal{B}} w_{ij} \delta_j^k \quad (20c)$$

$$\begin{aligned} (\lambda_i^k, z_i^k) = \operatorname{argmin}_{\lambda_i, z_i} & \left\{ \phi_i(\lambda_i, z_i) + \lambda_i \gamma_i^k + z_i \delta_i^k + \lambda_i \kappa_i^k \right. \\ & + z_i \psi_i^k + c_1 \sum_{j \in \mathcal{N}_{b,i}} \left\| \lambda_i - \frac{\lambda_i^{k-1} - \lambda_j^{k-1}}{2} \right\|_2^2 \\ & \left. + c_2 \sum_{j \in \mathcal{N}_{b,i}} \left\| z_i - \frac{z_i^{k-1} - z_j^{k-1}}{2} \right\|_2^2 \right\}, \end{aligned} \quad (20d)$$

where $\kappa_i^k := \sum_{j \in \mathcal{B}} (\bar{u}_{ij}^k + \underline{u}_{ij}^k)$ and $\psi_i^k := \sum_{j \in \mathcal{B}} (\bar{v}_{ij}^k + \underline{v}_{ij}^k)$; $w_{ij} = 2/(|N_{b,i}| + |N_{b,j}| + e)$ if $j \in \mathcal{B}_{b,i}$, and $w_{ij} = 1 - \sum_{j \in \mathcal{N}_{b,i}} (2/(|N_{b,i}| + |N_{b,j}| + e))$ if $i = j$, otherwise, $w_{ij} = 0$. Note that the update in (20c) is a consensus-based average estimator by which a local estimation of the global information, i.e., the power mismatch, is obtained. The mismatch estimation is locally processed based on λ_i and z_i by each battery agent i and then transmitted to neighbouring agents. Because of this consensus-based average estimation, the proposed solution is fully distributed and is robust against single-point failures, since there is no need to send this global information directly to all the battery agents.

Algorithm 1 Consensus-based distributed ADMM algorithm.

Initialize: Each agent initialize locally $P_{B,i}^0 \in \mathbb{R}$, $\lambda_i^0 \in \mathbb{R}$, $z_i^0 \in \mathbb{R}$; $\kappa_i^0 = 0$, $m^0 \in \mathbb{R}^+$, $\psi_i^0 = 0$, $\gamma_i^0 = r/N_B$ and $\delta_i^0 = d/N_B \forall i \in \mathcal{B}$;

Iteration: at the time t , for $k \geq 0$:

1: for $i \in \mathcal{B}$ (in parallel):

- a) Transmit λ_i and z_i to its neighbouring BESS through the communication network
- b) Update κ_i^k and ψ_i^k according to (20a) and (20b), respectively.
- c) Update γ_i^k and δ_i^k according to (20c)
- d) Update $(P_{B,i}^k, m_i^k)$ by solving (21a)
- e) Update λ_i^k and z_i^k using (21b) and (21c), respectively

2: $k = k + 1$ and go to Step 1

3: **return** $P_{B,i}^k$ (until a predefined stopping criterion (e.g., a maximum iteration number) is satisfied).

As shown in (20d), it is difficult to calculate (λ_i^k, z_i^k) directly since $\phi_i(\lambda_i, z_i)$ is a maximum subproblem. By using the minmax theorem [38], and the strong

convexity of (20d), one has

$$\begin{aligned}
(P_{B,i}^k, m_i^k) = & \operatorname{argmin}_{\substack{P_{B,i} \in \mathcal{F}, \\ m_i \geq 0}} \left\{ C_i(P_{B,i}) + \frac{c_1}{4|\mathcal{N}_{b,i}|} \left\| \frac{1}{c_1} (P_{B,i} - \gamma_i^k) - \frac{\kappa_i^k}{c_1} \right. \right. \\
& + \sum_{j \in \mathcal{N}_{b,i}} (\lambda_i^{k-1} + \lambda_j^{k-1}) \Big\|_2^2 + \frac{c_2}{4|\mathcal{N}_{b,i}|} \left\| \sum_{j \in \mathcal{N}_{b,i}} (z_i^{k-1} + z_j^{k-1}) \right. \\
& \left. \left. - \frac{\psi_i^k}{c_2} + \frac{1}{c_2} (\alpha_i P_{B,i} - m/N_B - \delta_i^k) \right\|_2^2 \right\} \quad (21a)
\end{aligned}$$

$$\lambda_i^k = \frac{1}{2|\mathcal{N}_{b,i}|} \left(\sum_{j \in \mathcal{N}_{b,i}} (\lambda_i^{k-1} + \lambda_j^{k-1}) - \frac{\kappa_i^k}{c_1} + \frac{1}{c_1} (P_{B,i}^k - \gamma_i^k) \right) \quad (21b)$$

$$\begin{aligned}
z_i^k = & \frac{1}{2|\mathcal{N}_{b,i}|} \left(\sum_{j \in \mathcal{N}_{b,i}} (z_i^{k-1} + z_j^{k-1}) - \frac{\psi_i^k}{c_2} \right. \\
& \left. + \frac{1}{c_2} (\alpha_i P_{B,i}^k - m_i^k - \delta_i^k) \right). \quad (21c)
\end{aligned}$$

The following theorem, the proof of which is provided in Appendix Appendix A, is stated to prove the convergence property of the proposed algorithm.

Theorem 4.1. *Considering a connected communication network $\mathcal{G}_{com} = (\mathcal{V}, \mathcal{E})$, given an initial condition of BESS as in Algorithm 1, the proposed solution using (20a) - (20c) and (21) converges to the optimal solution of the problem (16).*

Remark 4.1. *The proposed algorithm features a distributed implementation, it does not rely on a control centre and its implementation is based on local communication. Therefore, as long as the communication network is connected or there exists a spanning tree, the proposed algorithm can always provide the optimal solution. However, even if a spanning tree does not exist, the distributed algorithm can adapt since it has a plug-and-play feature. Therefore, if one or more battery agents fail, the other non-failed BESS will not be affected and a new optimal solution can be found based on the changed total number/capacity of BESS.*

5. Case Study

In order to illustrate and assess the proposed methodology, three case studies are considered. Each battery unit is equipped with a local controller consisting of: i) a local estimator for estimating global information i.e., the current system power balance; ii) an optimiser receiving information about disturbance location executing the designed solution to generate a power reference.

The case studies consider a low-inertia power system with a sudden and sustained supply-demand mismatch caused by either an SG outage or a RES power reduction. Without loss of generality, it is assumed that the total BESS capacity

is sufficient to provide the contracted power output for the delivery time. Since PI-based control methods are widely adopted in practice for frequency regulation [39], the proposed distributed approach is compared with a PI-based controller designed based on [26]. Although the control objective of this PI-based controller is the same as the proposed control framework, i.e., to regulate frequency to the nominal value, the proposed approach provides additional benefits: i) it can optimally coordinate an arbitrary number of BESS for provision of static fast-FR; ii) it minimises the degradation and maximises the profit of all BESS, while accounting for BESS location with respect to the fault location(s). This is achieved in real-time, therefore without adding any complexity and additional computational burden with respect to PI-based approaches. All test power systems are modelled within MatPower [40] and the studies are carried out in Matlab/Simulink.

5.1. Simulation Setup

In the following three case studies it is assumed that the disturbance occurs at $t = 0$ s, and the algorithm is then triggered. Constants c_1 and c_2 are set to 0.01 and 0.05, respectively. They can be selected based on the required convergence performance as long as $c_1, c_2 > 0$. The system nominal frequency is $f_0 = 50$ Hz and the inertial constant of each SG and the load damping constant are 10 s and 1 Hz^{-1} , respectively [33]. A total capacity of the installed RES in the system is 120 MW. The static fast-FR with a response time of $t_E = 1$ s [6] is assumed to be the service to be provided by the BESS. The RoCoF limit is -1 Hz/s over $t_{\text{roc}} = 0.5$ s recommended by [41], which implies $\Delta f_{\text{roc}} = -0.5 \text{ Hz}$. The reward price is 9.44 £/MW [6]. The parameters of the cost functions are set as in [29] and scaled up to MW. In the first two case studies the IEEE 14-bus system with 10 BESS is considered, with a rated capacity 2.5 MWh and power limits from -4 MW to 4 MW. In the last case study a modified IEEE 118-bus system with 100 BESS is simulated, where the rated capacity of each battery unit is 2 MWh, and the minimum/maximum power limit is -3 MW/3 MW, respectively. In all cases, the discharging efficiency is 1.1 and the safe SoC range safety is from 0.2 to 0.8[10]. The marginal degradation cost is linearly dependent on the power output [30].

5.2. Case 1: load change at one location

In this case study, a sudden and sustained load increase of 12 MW is assumed to occur at Bus 11 at $t = 0$ s. Given the RES penetration, BESS are deployed to provide static fast-FR and contain the frequency drop.

5.2.1. BESS location impact on the frequency response

BESS at different network locations can have different impact on frequency response while delivering an fast-FR service. In order to explore this three scenarios are considered: i) Scenario 1: all BESS are grouped at the bus, which is electrically the closest to the fault location (Bus 11); ii) Scenario 2: all BESS are grouped at the bus with the largest electrical distance from the fault location; iii) Scenario 3: BESS are equally distributed around the fault location.

The location-dependent weight factors in (4) are given in Table 1. As shown in Table 1, the closest and farthest buses to the disturbance location are Bus 10 (0.772p.u.) and Bus 2 (0.883p.u.) respectively. The results are given in Figs. 1-2. Fig. 1 depicts the heat map of the BESS output power in Scenario 3 and it illustrates that: i) the contribution from BESS to the frequency response is affected by their location in the power network; ii) BESS closer to the drop location contribute more to the frequency regulation. The system frequency response using different scenarios reveals the best performance is achieved in the Scenario1 (BESS located close to the fault location), where the RoCoF is reduced by up to 24% (from -0.14 Hz/s to -0.1 Hz/s) with respect to the other two scenarios. The frequency nadir is also slightly improved in this case, as illustrated in Fig. 2.

Table 1: Magnitudes of Z-bus matrix of each BESS depending on the disturbance location

Bus	2	3	4	5	6
E-Distance	0.883	0.882	0.875	0.877	0.801
Bus	7	9	10	12	14
E-Distance	0.838	0.808	0.772	0.801	0.805

Figure 1: Case 1: Heat map of BESS output power in the Scenario 3

Figure 2: Case 1: Comparison of frequency responses in the three scenarios

5.2.2. Comparison study

The frequency response obtained following the application of the algorithm described in Algorithm 1 is compared with the PI-based frequency control response, as shown in Table 2. The BESS are equally distributed around the fault location (Scenario 3). As shown in Table 2 the BESS coordinated through the proposed approach can restore the frequency more quickly with smaller RoCoF and improved frequency nadir, i.e., the RoCoF is reduced by 38.8% and the frequency nadir is improved by 0.4%. However, if the system inertia is reduced from 10 to 6, the frequency response improvements brought by the proposed approach is more significant, i.e., RoCoF is reduced by 53% and the frequency nadir is improved from 49.18 Hz to 49.58 Hz.

Table 2: Case 1: Performance of the PI-based control and the proposed solution (21)

	PI-based control	Proposed control (21)
frequency nadir	49.53 Hz	49.74 Hz
RoCoF	-0.18 Hz/s	-0.11 Hz/s

The solution resulting from the application of the designed algorithm is compared with the benchmark PI-based control for the three scenarios in terms of optimised battery degradation costs and revenue (as modelled by the objective function (15)). The results presented in Fig. 3 show that the solution obtained by the proposed algorithm outperforms the benchmark PI-based control yielding smaller battery degradation costs and higher revenue. Please, note that the proposed framework is flexible and the control objective can be modified according to the requirements of different applications.

Figure 3: Comparison of economic operation

To further illustrate the advantages of the proposed solution, the designed control algorithm is compared with representative existing work [23], where a conventional ADMM algorithm is applied for frequency control. In particular, the single point failure is investigated in this comparison. It is assumed that there is no power mismatch at the beginning, and that one BESS (BESS 3) is disconnected or fails at 1 s, after both algorithms have converged. The results are shown in Fig. 4. It can be observed that both algorithms can still converge after the failure, but the one proposed in [23] yields significant worse performance in terms of convergence time and frequency response. In fact, a central coordinator is required in the solution developed in [23], to receive and broadcast the latest system information to the rest of BESS, and it needs more iterations to converge, especially when a large number of BESS is controlled, resulting in a worse frequency response. The solution developed in this paper provides an effective and innovative modified ADMM incorporating a distributed estimation algorithm, it can be implemented in a fully distributed manner and it can adapt more efficiently to changes in the network. The results show that the proposed algorithm outperforms the one in [23] yielding a faster convergence rate and a better frequency response following a BESS failure.

Figure 4: Comparison under failure of BESS 3

5.3. Case 2: Analysis of the convergence rate

One of the major factors that affects the convergence rate of distributed frameworks is the communication topology as the convergence rate is quantified by algebraic connectivity of a certain communication topology. The proposed solution is therefore applied to different topologies to further analyse this impact. The same system architecture as in Case 1 is considered, with different communication topologies, namely line, ring and random connections with a spanning tree. The convergence rates for each topology are given in Table 3. As shown in the table, the topology and, in particular, the algebraic connectivity have an impact on the convergence rate. The convergence rates, however, are very fast for all the topologies, with differences in the order of a few milliseconds.

Table 3: Case 2: Convergence rates under different communication topologies

Test model	Line	Ring	Random Connection
Algebraic connectivity	0.1910	0.3820	0.3098
Convergence rate (s)	0.0156	0.0102	0.0119

5.4. Case 3: load change at two locations

In this case, sudden and sustained load changes occur at two locations, at Bus 11 and Bus 5, and the overall load increases by 24 MW at $t = 0$ s. The same three scenarios as defined in the Case 1 are also analysed. The closest and farthest buses to the disturbance locations are Bus 9 and Bus 2 respectively. As illustrated in Fig. 5, the frequency regulation is improved when BESS are placed closer to the fault locations (Scenario 1), i.e., RoCoF is reduced by 34.3% and the frequency nadir is also slightly improved.

The performance achieved when BESS are evenly distributed around the fault locations, i.e., according to their electrical distances (Scenario 3), is very similar to those obtained in Scenario 1. Scenario 3 assumes distributed BESS across the network and it is the scenario most likely to happen. The proposed control framework is compared with the PI-based control [26] as in the previous case study considering the Scenario 3. As shown in Table 4, the proposed method can handle multiple disturbance locations more effectively than a more traditional PI-based control, yielding an improved frequency response (i.e., RoCoF reduced by 36.8% and frequency nadir slightly reduced). It is evident that the proposed control framework not only optimally coordinates an arbitrary number of BESS within a very short time, but also selects the BESS that can provide more effective contribution to the frequency regulation.

Figure 5: Case 3: Frequency response in the three scenarios

Table 4: Case 3: Performance of PI-based control and the proposed solution in (21)

	PI-based control	Proposed control in (21)
frequency nadir	49.945 Hz	49.968 Hz
RoCoF	-0.19 Hz/s	-0.12 Hz/s

5.5. Case 4: scalability

In this case study the scalability of the proposed control framework is analysed. A modified IEEE 118-bus system with 100 BESS randomly distributed around the network is considered. The results are illustrated in Fig 6. It can be seen that the BESS coordinated by the algorithm proposed in Section 4 can successfully regulate the frequency achieving a better performance than the PI-based controller. The computational time of the algorithm is 0.18 s, which clearly meets the fast-FR service criterion of response time ($t_E = 1$ s) and is comparable with PI-based control computational time (0.112s). Hence the proposed control framework is suitable for implementation in large-scale power systems.

Figure 6: Case 4: Frequency responses resulting from the application of the Algorithm described in Section 4 and the PI-based controller [26]

6. Conclusion

A distributed control strategy for multiple BESS coordination is proposed using a consensus-based distributed ADMM algorithm. An optimisation problem

is formulated to schedule the BESS operation for an efficient delivery of fast-FR in low inertial power systems with RES. The optimisation problem is solved in a distributed fashion in order to reduce computational times and communication costs and it yields a more flexible and scalable control framework to coordinate a large number of BESS providing fast-FR. The proposed approach is shown to be more suitable for faster-acting frequency support than centralised methods and commonly applied control. Future studies will focus on extending the proposed framework so that it is robust against several other sources of uncertainty, including the communication delays.

Appendix A. Proof of Theorem 4.1

Note that (20c) is the average consensus estimator with the definition of w_{ij} as $w_{ij} = 2/(|N_{b,i}| + |N_{b,j}| + e)$ if $j \in \mathcal{B}_{b,i}$, and $w_{ij} = 1 - \sum_{j \in \mathcal{N}_{b,i}} (2/(|N_{b,i}| + |N_{b,j}| + e))$ if $i = j$, otherwise, $w_{ij} = 0$. Then, if $\gamma_i^0 = r/N_B$ and $\delta_i^0 = d/N_B$, the following result is obtained after a sufficient long time K , i.e.,

$$\gamma_i^K = \frac{1}{N_B} \sum_{j \in \mathcal{B}} \gamma_j^0 = \frac{r}{N_B}; \quad \delta_i^K = \frac{1}{N_B} \sum_{j \in \mathcal{B}} \delta_j^0 = \frac{d}{N_B}, \quad (\text{A.1})$$

which indicates that each battery unit can access the global information locally. What is to be proved now is that λ_i^k and $P_{B,i}^k$ converge to their corresponding optimal points. Due to the page limit, a simplified proof is provided. By using the results in [42], it can be concluded that $\forall i \in \mathcal{B}, j \in \mathcal{N}_{b,i}$,

$$\lambda_i^k \rightarrow \lambda^*, \quad \lambda_i - \lambda_j = 0; \quad z_i^k \rightarrow z^*, \quad z_i - z_j = 0. \quad (\text{A.2})$$

Then, it must be proved that $P_{B,i}^k \rightarrow P_{B,i}^*$, i.e., the solution of the optimisation problem (15) as $k \rightarrow \infty, \forall i \in \mathcal{B}$,

$$\partial C_i(P_{B,i}) + \lambda^* + \alpha_i z^* \rightarrow 0 \quad (\text{A.3a})$$

$$\sum_{i \in \mathcal{B}} P_{B,i} = r, \quad \sum_{i \in \mathcal{B}} \alpha_i P_{B,i} = m + d. \quad (\text{A.3b})$$

To show that $P_{B,i}^k, \forall i \in \mathcal{B}$ converges to the optimal condition (A.3), the following two steps are provided, the first one focusing on (A.3a) and the second step proving—(A.3b)

1) Step.1. The KKT (Karush-Kuhn-Tucker) conditions of—(21a) are given as

$$\begin{aligned} & \partial C_i(P_{B,i}) + \frac{1}{2|\mathcal{N}_{b,i}|} \left(\frac{1}{c_1} (P_{B,i} - \gamma_i^k) + \sum_{j \in \mathcal{N}_{b,i}} (\lambda_i^{k-1} + \lambda_j^{k-1}) \right. \\ & \left. - \frac{\kappa_i^k}{c_1} \right) + \frac{\alpha_i}{2|\mathcal{N}_{b,i}|} \left(\sum_{j \in \mathcal{N}_{b,i}} (z_i^{k-1} + z_j^{k-1}) - \frac{\psi_i^k}{c_1} \right. \\ & \left. + \frac{1}{c_2} (\alpha_i P_{B,i} - m/N_B - \delta_i^k) \right) = 0, \quad P_{B,i} \in \mathcal{F}. \end{aligned} \quad (\text{A.4})$$

Using the update of (21b) and—(21c) it follows that

$$\partial C_i(P_{B,i}) + \lambda_i^k + \alpha_i z_i^k = 0, \quad P_{B,i} \in \mathcal{F}. \quad (\text{A.5})$$

Since (A.5) holds for all k and (A.2) as $k \rightarrow \infty$, (A.3a) is then obtained.

2) Step.2. The equations (21b) and (21c) can be rewritten as

$$\begin{aligned} & c_1 \sum_{j \in \mathcal{N}_{b,i}} (\lambda_i^k + \lambda_j^k + \lambda_i^{k-1} + \lambda_j^{k-1}) + \kappa_i^k - (P_{B,i}^k - \gamma_i^k) \\ & + 2c_1 \sum_{j \in \mathcal{N}_{b,i}} (\lambda_i^k - \frac{\lambda_i^k + \lambda_j^k}{2}) = 0. \end{aligned} \quad (\text{A.6})$$

Given that $\sum_{i \in \mathcal{B}} \kappa_i^k = \sum_{j \in \mathcal{B}} (\bar{u}_{ij}^k + \underline{u}_{ij}^k) = 0$, one has

$$\sum_{i \in \mathcal{B}} P_{B,i}^k + \sum_{i \in \mathcal{B}} \gamma_i^k = \sum_{i \in \mathcal{B}} c_1 \sum_{j \in \mathcal{N}_{b,i}} (\lambda_i^k + \lambda_j^k + \lambda_i^{k-1} + \lambda_j^{k-1}). \quad (\text{A.7})$$

Since $\sum_{i \in \mathcal{B}} \gamma_i^k = r$ as $k \rightarrow K$ and $\lambda_i^k - \lambda_i^{k-1} \rightarrow 0$ following (A.2) as $k \rightarrow \infty$, (A.3b) is obtained. The proof is completed.

Lemma Appendix A.1. The proposed method converges to the optimal solution at a $\mathcal{O}(1/k)$ convergence rate.

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Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

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Tianqiao Zhao: Methodology, Software, Validation, Formal Analysis, Writing - Original Draft, Visualization, Data Curation

Alessandra Parisio: Methodology, Conceptualization, Supervision, Project administration, Writing - Review & Editing, Data Curation, Funding acquisition

Jovica V. Milanovic: Conceptualization, Funding acquisition, Writing - Review & Editing, Supervision, Project administration